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兩部門異質性廠商下的最適碳稅與資源重分配

Optimal Carbon Tax and Resource Reallocation under Two-Sector Model with Heterogeneous Firms

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摘要

本論文建立一套具備異質性廠商的多部門一般均衡模型，用以評估不同碳稅政策對經濟與環境的影響。該模型以歐盟 27 國 2019 年的資料進行校準，並用來分析四種稅制方案：統一碳稅、差異化稅率、部門豁免，以及小型排放源豁免。結果顯示，碳稅能有效減少排放，並影響生產要素在部門間的重新配置。與未課徵碳稅的情況相比，統一碳稅可將碳排放量減少 73%，並使消費增加 4.13%。若依部門特性調整稅率，整體福祉可進一步提升。相對地，以豁免為基礎的政策往往會降低減碳效率並導致資源錯置，進而造成福祉損失。然而，若豁免碳排放密集度低且生產力高的部門，則可透過引導資源配置提高整體福祉。至於依廠商規模設計的豁免政策，則會導致資源在部門內部及部門間重新分配。

關鍵字：碳稅、租稅赦免、資源重分配、產業結構、氣候變遷

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Abstract

This thesis develops a multi-sector general equilibrium model with heterogeneous firms to evaluate the impact of different carbon tax policies on economic and environmental performance. The model is calibrated using 2019 data from 27 EU countries and used to analyze four tax schemes: uniform carbon tax, differentiated tax rates, sectoral exemptions, and small emitter exemptions. The results show that carbon taxes effectively reduce emissions and affect inter-sectoral resource reallocation. A uniform carbon tax reduces emissions by 73% and increases consumption by 4.13% compared with no carbon tax. If tax rates are adjusted based on sectoral characteristics, welfare improves further. Sector-based exemption often reduces carbon mitigation efficiency and causes resource misallocation, leading to welfare loss. However, exempting the low-carbon-intensive and high-productivity sector could result in higher welfare through guiding resource allocation. Size-based exemption creates resource allocation within and across sectors.

Keywords: Carbon tax, Tax exemption, Resource reallocation, Industrial structure, climate change

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Chapter 1 Introduction

Climate change poses an urgent global challenge, with rising carbon dioxide emissions intensifying the greenhouse effect and increasing the frequency and severity of extreme weather events. In response, carbon taxation has emerged as a leading market-based instrument to reduce greenhouse gas emissions by internalizing the environmental cost of carbon.

Despite its theoretical appeal, the real-world implementation of carbon taxes frequently involves trade-offs between environmental effectiveness and political or economic feasibility. To mitigate compliance costs and gain political support, many governments introduce exemptions or reduced rates for specific sectors and small emitters. For instance, Sweden’ s 1991 carbon tax allowed industrial users to pay only half the standard rate, with sectors such as mining and horticulture fully exempted (International Energy Agency (2008)). Similarly, the European Union Emissions Trading System (EU ETS) permits installations emitting fewer than 25,000 tons of CO_2-equivalent annually to opt out of the scheme. The rationale for the regulation lies in the observation that smaller emitters incurred relatively higher transaction costs under the initial phases of the EU ETS compared to larger firms (Jaraitėucd et al. (2010), Heindl (2015), and Kurz (2024)).

However, these exemptions may dilute the effectiveness of carbon taxes, introduce

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distortions in resource allocation, and reduce the overall efficiency of climate policy. As a result, understanding how the design of carbon tax schemes—particularly with respect to tax base coverage and rate differentiation—shapes macroeconomic and environmental outcomes is critical for informed and effective policy-making. Timilsina (2022) highlights the need for more comprehensive studies to assess whether granting exemptions to specific sectors or small emitters is economically justified.

While a growing body of literature has explored the optimal design of carbon taxes, most existing studies rely on representative agent models or highly stylized general equilibrium frameworks. These approaches, though analytically tractable, often abstract away from the firm-level heterogeneity and sector-specific asymmetries that characterize real-world economies. Consequently, the quantitative consequences of exemptions and heterogeneous tax treatments remain insufficiently understood. This lack of detailed empirical grounding limits our ability to assess the true economic costs, mitigation trade-offs, and structural implications of exemption-based policies.

To address this gap, this paper develops a multi-sector general equilibrium model with heterogeneous firms to evaluate the macroeconomic and environmental impacts of alternative carbon tax designs. The model is calibrated to reflect the structural characteristics of the 27-member European Union(EU) in 2019, drawing on detailed data from Eurostat and the EU KLEMS database, including firm size distributions, input shares, and emission factors. We consider four carbon pricing regimes: (i) a uniform tax applied across all sectors; (ii) a differentiated tax across sectors; (iii) sectoral exemptions, in which selected industries are excluded from taxation; and (iv) exemptions for small emitters, defined as firms emitting less than 25,000 tons of CO_2 annually.

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These four policy regimes vary in the extent of the emission reduction, economic performance, and the level of resource allocation. In regimes without exemptions—namely, the uniform and differentiated tax scenarios—carbon pricing operates as expected: emission-intensive sectors contract in response to higher effective energy costs, while low-emission sectors, particularly commercial sectors, expand. This reallocation of capital, labor, and especially energy reflects relative price signals and market share changes, contributing simultaneously to welfare gains and emission reductions.

By contrast, sectoral exemption regimes signify the asymmetry between sectors. Exempted dirty sectors are shielded from the carbon tax cost and thus absorb disproportionate resources, crowding out taxed but cleaner sectors. This distortion weakens the carbon price signal and leads to inter-sectoral resource misallocation. However, exempting clean but productive sectors, such as the commercial sector, maintains most of the emissions in the carbon tax base and leads the resource to a more productive sector, resulting in a higher welfare level than a uniform tax scheme.

In the case of size-based exemption policy schemes, first, the resulting distortions give rise to inefficiencies within sectors. When exemptions based on firm size or emissions thresholds are implemented, More resources are reallocated to the low-productivity small producers, lowering the sector's productivity. Second, as shown in Kurz (2024), exempting small emitters can lead to within-sector carbon leakage, allowing inefficient firms to remain active while burdening more productive firms with higher costs. As a result, this size-based exemption policy will undermine the effectiveness of the emissions price as a mechanism for reducing aggregate emissions. This within-sector carbon leakage is observed in our research. Lastly, we further emphasize the heterogeneous impacts on sectors. When implementing a 25,000 tons CO_2 threshold, different portions of firms

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in each sector are exempted, leading to notable inter-sectoral resource reallocation.

To assess these effects, we compare the macroeconomic and environmental performance of four carbon tax regimes. Under the uniform tax, a rate of 1220 EU per ton of carbon emission reduces emissions by 73% and increases steady-state consumption by 4.13% relative to the no-tax baseline. A differentiated tax aligned with sector-specific emission factors performs slightly better, increasing welfare by 4.22%. Interestingly, exempting the commercial sector yields a 4.18% welfare gain, driven by the final good production expansion due to more resources allocated to the productive commercial sector. However, other exemption scenarios lead to lower welfare levels due to higher climate damage rooted in a limited carbon tax base. These results suggest that carefully targeted exemptions, particularly in low-emission, high-productivity sectors, may offer second-best efficiency outcomes.

Also, we examine the performance of different scenarios for a given carbon emission reduction target, which is set to be the carbon emission level under an optimal uniform scenario. Our quantitative results highlight that the policy performance diverges markedly. First, the regime with the widest tax base, which is the uniform tax scenario, exhibits the highest efficiency in achieving the emission goal. Meeting the highest consumption level(C=3.2784) with the lowest tax rate($\tau = 0.0122$) among all the scenarios. However, exemption-based regimes require significantly higher tax rates on non-exempt sectors to reach the same goal. For example, under the small emitters exemption scenario, the higher tax rate($\tau = 0.0235$) on taxed firms curtails the sectoral output and eventually results in lower consumption (C = 3.2625). The industrial sector exemption even fails to meet the target under maximum taxation. These findings show that partial exemptions compromise the mitigation efficiency of emissions and shift burdens inequitably to non-exempted sec-

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tors, causing more welfare loss to reach the same carbon emission target.

Since the carbon tax also caused the resource reallocation, our paper is also related to Restuccia and Rogerson (2009), Hsieh and Klenow (2009), and especially Ferro et al. (2024). Ferro et al. (2024) presents quantitative evidence suggesting that the carbon tax could reshape industrial structure by reallocating resources from energy-intensive to less-affected sectors. Our study also highlights the inter-sectoral resource reallocation impacts of carbon taxation, notably shifting labor, capital, and energy toward clean industries, such as the commercial sector. This redistribution enhances the clean industry's competitiveness, changing the country's industrial structure.

Our main finding in achieving certain carbon emission goals is consistent with the consensus that a carbon tax would be most efficient to reduce GHG emissions if it has a wider base (Oates (1995), Marron and Toder (2014)). Also, Böhringer and Rutherford (1997), the pioneering research on examining the exemption of the carbon tax challenges the idea of viewing the sectoral exemption as an instrument to reduce the employment impact of a carbon tax, suggesting that a uniform tax, combined with targeted wage subsidies, offers a more effective strategy to achieve the same level of carbon emissions and employment at a fraction of the cost of sectoral exemption, indicating the sectoral exemption treatment is inefficient. Our research offers an alternative perspective by considering the feedback mechanism from cumulated CO_2 emissions to economic activity. Quantitative results also point out the inefficiency of exempting sectors in obtaining specific emission targets.

The subsequent sections are structured as follows: Chapter 2 details the theoretical model setup, specifying the key assumptions and equations governing sectoral production

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and carbon taxation. Chapter 3 discusses the data sources and calibration methods used to estimate model parameters. Chapter 4 presents qualitative and quantitative results, focusing on the economic impact of carbon taxation and resource reallocation. Finally, Chapter 5 concludes with policy recommendations and potential avenues for future research.

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Chapter 2 The model

This chapter introduces a multi-sector economic model that explicitly accounts for heterogeneity across sectors, which can be considered as an extension beyond the framework developed by Restuccia and Rogerson (2009). Unlike prior models that often assume homogeneous sectors or a single final good sector, this study incorporates multiple sectors characterized by distinct firm productivity distributions, input intensity, and emission factors.

The model consists of two fundamental layers: final good producers and sectoral producers. Final good producers aggregate outputs from heterogeneous sectors to produce composite final goods for consumption. Sectoral producers manufacture intermediate goods by employing labor, capital, and energy inputs. This multi-sector heterogeneity framework allows the analysis to capture differential responses to various carbon taxation policies in heterogeneous sectors, thereby providing a more nuanced understanding of resource reallocation and economic adjustment across and within sectors. Also, by following Golosov et al. (2014), we specify the economic damages associated with CO_2 emissions. These damages are modeled through a climate damage function that directly affects final good production, reflecting the adverse effects of atmospheric carbon concentration on economic output.

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2.1 The Consumer's Problem

The economy is populated by a representative household that maximizes the expected lifetime utility derived from consumption. Formally, the household's intertemporal optimization problem can be expressed as:

$$\sum_{t=0}^{\infty} \beta^t u(C_t) \tag{2.1}$$

where C_t denotes consumption at time t, $\beta \in (0, 1)$ is the subjective discount factor, and $u(\cdot)$ is a strictly concave utility function capturing diminishing marginal utility of consumption.

The household faces the following budget constraint at each period t:

$$C_t + K_{t+1} = w_t N_t + r_t K_t + \Pi_t + (1 - \delta) K_t + T_t, \ \forall t \tag{2.2}$$

where N_t and K_t represent labor and capital supply, respectively; w_t and r_t denote the wage rate and rental rate of capital; Π_t is the aggregate profits distributed from firms within and across sectors; T_t corresponds to government transfers; and $\delta \in (0, 1)$ is the capital depreciation rate.

The optimal consumption-saving decision of the household is characterized by the Euler equation given by the first-order condition:

$$u'(C_t) = \beta u'(C_{t+1})[r_{t+1} + (1 - \delta)] \tag{2.3}$$

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where $u'(\cdot)$ denotes the marginal utility of consumption.

2.2 Final Good Producers

The representative final good producer operates in a perfectly competitive market and chooses sectoral inputs to maximize profits. Formally, the optimization problem is:

$$\begin{aligned} &\max_{Y_{l,t}} Y_t - \sum_{l=1}^{L} P_{l,t} Y_{l,t} \\ &s.t.\ Y_t = F(Y_{1,t}, ..., Y_{L,t}, S_t) = (1 - D(S_t))\tilde{Y}_t(Y_{1,t}, ..., Y_{L,t}) \end{aligned} \tag{2.4}$$

Here, Y_t is the actual final output after accounting for the climate damage loss at time t, and the price of the final good is normalized to one. $P_{l,t}$ denotes the relative price of the sector l good, and $Y_{l,t}$ is the quantity of intermediate good from sector l. The undamaged output $\tilde{Y}_t$ aggregates sectoral outputs via a constant elasticity of substitution (CES) function:

$$\tilde{Y}_t = \left[\sum_{l=1}^{L} a_l^{\frac{1}{\sigma}} Y_{l,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{2.5}$$

where a_l are share parameters and $\sigma > 0$ is the elasticity of substitution across sectoral goods. A higher σ implies greater substitutability between different sectoral goods.

To incorporate environmental externalities, the model introduces a climate damage function $D(S_t)$ that reduces final output based on atmospheric carbon concentration S_t. Specifically, we adopt the exponential damage function:

$$1 - D(S_t) = \exp(-\gamma(S_t - \overline{S})) \tag{2.6}$$

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where γ is the climate damage sensitivity parameter and $\overline{S}$ represents the pre-industrial CO_2 baseline concentration.

This damage function simplifies the complex climate dynamics modeled in Nordhaus's Dynamic Integrated Climate-Economy (DICE) framework. While the DICE model employs a detailed multi-reservoir carbon cycle and a quadratic damage function to capture temperature impacts and economic losses, this exponential specification, following Golosov et al. (2014), approximates the climate-economy interaction by modeling damages as a proportional output loss linked directly to carbon concentration.

Despite its reduced complexity, this approach produces quantitatively comparable optimal carbon tax recommendations and preserves clear analytical tractability, making it well-suited for integration into multi-sector general equilibrium models.

The atmospheric carbon stock S_t evolves as a function of past global emissions according to:

$$S_t - \overline{S} = \sum_{s=0}^{t+T} (1 - d_s) G_{t-s}^{w} \tag{2.7}$$

where G_{t-s}^{w} is global CO_2 emissions at time $t-s$ and d_S is the fraction of emissions removed s periods after release. Assuming the European Union emits a fixed fraction θ of global emissions over time, we have $G_{t-s}^{w} = G_{t-s}/\theta$ with G_{t-s} denoting carbon emissions within the EU at time $t - s$.

The carbon retention fraction $1 - d_s$ follows:

$$1 - d_s = \phi_L + (1 - \phi_L)\phi_0(1 - \phi)^s \tag{2.8}$$

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where ϕ_L is the long-term atmospheric retention share, and a portion $1-\phi_0$ of the remaining carbon emissions is absorbed by the biosphere and the surface oceans immediately, and the rest share decays at a geometric rate ϕ.

Solving the producer's profit maximization yields sectoral demand:

$$Y_{l,t}^d = a_l \tilde{Y}_t \left(\frac{P_{l,t}}{1-D(S_t)} \right)^{-\sigma} \tag{2.9}$$

2.3 Sectoral Good Producers

Each sector l comprises a continuum of heterogeneous firms characterized by firm-specific productivity levels s_l. A firm with productivity s_l chooses labor $n_{l,t}$, capital $k_{l,t}$, and energy $e_{l,t}$ inputs to maximize its profit:

$$\max_{k_{l,t},n_{l,t},e_{l,t}} \pi_{l,t} = \left\{ P_{l,t} y_{l,t} - w_t n_{l,t} - r_t k_{l,t} - \nu_t^E \left(1 + \frac{\tau_t^{CE} \varphi_l}{\nu_t^E} \right) e_{l,t} \right\} \tag{2.10}$$

subject to the Cobb-Douglas production function:

$$y_{l,t} = s_l n_{l,t}^{\alpha_l} k_{l,t}^{\beta_l} e_{l,t}^{\gamma_l} \tag{2.11}$$

where $p_{l,t}$ is the sectoral output price; w_t, r_t, and ν_t^E denote the wage, capital rental, and energy prices, respectively; τ_t^{CE} is the carbon emission tax; and φ_l is the sector-specific emission factor, representing carbon emissions emitted per unit of energy consumed. Therefore, the effective energy price faced by firms in sector l is thus $\nu_t^E + \tau_t^{CE} \varphi_l$. The exponents α_l, β_l, γ_l are input elasticities with respect to labor, capital, and energy and

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satisfy $0 < \alpha_l + \beta_l + \gamma_l < 1$, ensuring decreasing returns to scale at the firm level.[1]

The firm's profit maximization leads to the following first-order conditions equating the marginal products of inputs to their prices:

$$P_{l,t}\frac{\partial y_{l,t}}{\partial n_{l,t}} = \frac{\alpha_l P_{l,t} y_{l,t}}{n_{l,t}} = w_t \tag{2.12}$$

$$P_{l,t}\frac{\partial y_{l,t}}{\partial k_{l,t}} = \frac{\beta_l P_{l,t} y_{l,t}}{k_{l,t}} = r_t \tag{2.13}$$

$$P_{l,t}\frac{\partial y_{1,t}}{\partial e_{l,t}} = \frac{\gamma_l P_{l,t} y_{l,t}}{e_{l,t}} = \nu_t^E + \tau_t^{CE}\varphi_l \tag{2.14}$$

Solving these conditions yields the optimal input demands:

$$\overline{n_{l,t}}(s_l) = (P_{l,t}s_l)^{\frac{1}{1-\alpha_l-\beta_l-\gamma_l}} \left(\frac{\alpha_l}{w_t}\right)^{\frac{1-\beta_l-\gamma_l}{1-\alpha_l-\beta_l-\gamma_l}} \left(\frac{\beta_l}{r_t}\right)^{\frac{\beta_l}{1-\alpha_l-\beta_l-\gamma_l}} \left(\frac{\gamma_l}{\nu_t^E + \tau_t^{CE}\varphi_l}\right)^{\frac{\gamma_l}{1-\alpha_l-\beta_l-\gamma_l}} \tag{2.15}$$

$$\overline{k_{l,t}}(s_l) = \frac{\beta_l w_t}{\alpha_l r_t}\overline{n_{l,t}} \tag{2.16}$$

$$\overline{e_{l,t}}(s_l) = \frac{\gamma_l w_t}{\alpha_l\left(\nu_t^E + \tau_t^{CE}\varphi_l\right)}\overline{n_{l,t}} \tag{2.17}$$

The empirical literature provides strong support that firm productivity and size distributions are well approximated by lognormal distributions across sectors in the EU. Following this evidence (Artige and Bignandi (2022); Musa et al. (2024);Cabral and Mata (2003)), we assume the firm productivity distribution $\mu_l(s)$, which indicates the number of firms with productivity s in the sector l, follows a lognormal distribution:

$$\mu_l(s) = M_l\hat{\mu}_l(s) \tag{2.18}$$

where M_l is the total number of firms in the sector l and $\hat{\mu}_l(s)$ is the lognormal density

[1] These decreasing returns to scale at the establishment level are crucial for ensuring non-degenerate firm size or productivity distributions within different sectors, as indicated in Restuccia and Rogerson (2009).

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function, which integrates to one in its support.

2.4 Market Clearing Conditions

The following equations represent the market clearing conditions for the labor, capital, and energy markets, ensuring that the total demand for each factor equals its total supply:

$$\sum_{l=1}^{L}\int_{s}\overline{n_{l,t}}(s)\mu_l(s)ds=\sum_{l=1}^{L}N_{l,t}=N_t \tag{2.19}$$

$$\sum_{l=1}^{L}\int_{s}\overline{k_{l,t}}(s)\mu_l(s)ds=\sum_{l=1}^{L}K_{l,t}=K_t \tag{2.20}$$

$$\sum_{l=1}^{L}\int_{s}\overline{e_{l,t}}(s)\mu_l(s)ds=\sum_{l=1}^{L}E_{l,t}=E_t \tag{2.21}$$

Here, $N_{l,t}$,$K_{l,t}$, and $E_{l,t}$ denote the aggregate demand for labor, capital, and energy, respectively, originating from sector l. Conversely, N_t,K_t, and E_t represent the economy-wide total supply of labor, capital, and energy.

The government levies a carbon tax, τ_t^{CE}, on emissions and is assumed to return all collected revenue to the representative households in a lump-sum fashion, ensuring a balanced budget. The government budget constraint is thus given by:

$$\sum_{l=1}^{L}\int_{s}\tau_l^{CE}\varphi_l\overline{e_{l,t}}(s)\mu_l(s)ds=T_t \tag{2.22}$$

Here, T_t represents the total carbon tax revenue transferred.

The sectoral output market clearing conditions satisfy:

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$$a_l\tilde{Y}_t\left(\frac{P_{l,t}}{1-D(S_t)}\right)^{-\sigma} = \int_s \overline{y_l}(s)\mu_l(s)ds,\ \forall l \tag{2.23}$$

In this equation, the left-hand side signifies the total demand for the sectoral good l from the final good producer, while the right-hand side represents the aggregate supply of good l from all firms within that sector.

Finally, the aggregate resource constraint for the economy is given by:

$$C_t + K_{t+1} = Y_t + (1-\delta)K_t \tag{2.24}$$

where C_t is aggregate consumption, K_{t+1} is the next period's aggregate capital stock, and δ is the depreciation rate of capital. This equation ensures that total output is allocated between consumption and investment, accounting for capital depreciation.

2.5 Competitive Equilibrium

The economy reaches a competitive equilibrium when all factors and goods markets clear simultaneously, and agents optimize given prices. Therefore, a competitive equilibrium at time t is defined as a set of prices $\{w_t, r_t, \nu_t, P_{l,t}\}_{=1}^{L}$, the carbon tax rate τ_t^{CE} , household allocations C_t, K_{t+1}, N_t, government transfers T_t, and firm input and output decisions $\{n_{l,t}(s), k_{l,t}(s), e_{l,t}(s), y_{l,t}(s)\}_{l=1}^{L}$, such that:

- Households maximize lifetime utility subject to budget constraints;
- Firms maximize profits given input prices and carbon tax;
- Factor markets clear according to the Equation 2.19, 2.20, and 2.21;

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- Goods markets clear according to the Equation 2.23, and 2.24;
- The government budget constrains the Equation 2.22.

Next, we aim to determine the optimal carbon tax policy. To achieve this, we first solve the social planner's problem of determining the welfare-maximizing allocation of resources. By comparing this socially optimal allocation with the decentralized market outcome, we can infer the appropriate level of carbon taxation required to internalize the climate externality.

2.6 Social Planner's Problem

The social planner seeks to maximize the lifetime utility of the representative agent, subject to the economy's technological and resource constraints. The problem is formulated as follows:

$$\max_{\{\{N_{l,t}\}_{l=1}^{L},\{K_{l,t}\}_{l=1L},\{E_{l,t}\}_{l=1}^{L},\{Y_{l,t}\}_{l=1}^{L},C_t,K_{t+1},S_t,G_t,Y_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{2.25}$$

$$\text{s.t. } Y_t = F(S_t, Y_{1,t}, ..., Y_{L,t}),\ \forall t \tag{2.26}$$

$$Y_{l,t} = F_l(K_{l,t}, N_{l,t}, E_{l,t}),\ \forall l, t \tag{2.27}$$

$$\sum_{l=1}^{L} K_{l,t} = K_t,\ \forall t \tag{2.28}$$

$$\sum_{l=1}^{L} N_{l,t} = N_t,\ \forall t \tag{2.29}$$

$$\sum_{l=1}^{L} \varphi_l E_{l,t} = G_t,\ \forall t \tag{2.30}$$

$$S_t = \tilde{S}_t(G_{-T}, G_{-T+1}, ...G_t),\ \forall t \tag{2.31}$$

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$$C_t + K_{t+1} = Y_t + (1-\delta)K_t,\ \forall t \tag{2.32}$$

We assign the Lagrangian multiplier λ_t, $\lambda_{l,t}$, θ_t^K, θ_t^N, θ_t^E θ_t^G, and Λ_t to Equations 2.26, 2.27, 2.28, 2.29, 2.30, 2.31 and 2.32 respectively and solve this social planner problem. Notably, the first-order condition with respect to G_t and $E_{l,t}$ can then be written as:

$$\frac{1}{\varphi_l}\frac{\partial Y_t}{\partial Y_{l,t}}\frac{\partial Y_{l,t}}{\partial E_{l,t}} = \Lambda_t \tag{2.33}$$

where Λ_t is defined as:

$$\Lambda_t = \sum_{j=0}^{\infty}\beta^j \frac{u'(C_{t+j})}{u'(C_t)}\frac{\partial Y_{t+j}}{\partial S_{t+j}}\frac{\partial S_{t+j}}{\partial G_t} \tag{2.34}$$

Equation 2.33 summarizes the trade-off between the marginal benefits and marginal costs of emitting carbon dioxide. The left-hand side represents the marginal benefit of emissions, derived from the marginal product of energy in final goods production scaled by the inverse of the emission factor. The right-hand side, λ_t, represents the marginal social cost of carbon (SCC), which is the discounted sum of future economic damages caused by an additional unit of CO_2 emitted at time t.

2.7 Optimal Carbon Tax

To determine the optimal carbon tax, we compare the first-order condition from the decentralized market outcome (Equation 2.14) with the corresponding social planner's optimality condition (Equation 2.33). This comparison allows us to derive the Pigouvian tax rate required to internalize the carbon externality. Specifically, the optimal carbon tax

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for sector l at time t should be:

$$\tau_{l,t}^{CE} = \Lambda_t - \frac{\nu_t^E}{\varphi_l} \tag{2.35}$$

This equation provides a clear economic intuition for the optimal carbon tax. The term Λ_t represents the marginal social cost of carbon (SCC). The term $\frac{\nu_t^E}{\varphi_l}$ denotes the marginal private cost of energy usage, expressed in terms of its carbon emission equivalent. In a setting where energy consumption directly leads to emissions, a portion of the total cost of emissions is already borne by firms through the energy price. Therefore, the optimal carbon tax, $\tau_{l,t}^{CE}$, is effectively the difference between the total marginal social cost of carbon and the market price of energy. If the private cost of energy usage does not fully account for the social marginal externality of CO_2 emissions, then a positive carbon tax is necessary to bridge this gap and fully internalize the environmental damage.

2.8 Steady-State Equilibrium

Our analysis primarily focuses on the model's steady-state competitive equilibrium. In a steady state, all aggregate variables remain constant over time. Given the first-order condition of the representative household in Equation 2.3, a solution with constant consumption (C) and rental rate (r) for capital must satisfy the following condition:

$$r = \frac{1}{\beta} - (1 - \delta) \tag{2.36}$$

Here, r represents the constant steady-state real interest rate (or rental rate of capital). This formulation precisely establishes the steady-state interest rate based on the exogenous parameters δ, the depreciation rate of capital, and β, the discount factor.

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To ensure consistency within the steady-state framework, the damage function must also be specified for the long run. Under steady-state conditions, the atmospheric carbon concentration S will converge to a constant level, which can be expressed as a function of steady-state global emissions G^w. Recalling the carbon accumulation Equation 2.31 and the assumption that emissions from the EU constitute a fixed proportion θ of global carbon emissions. We can derive the steady-state damage function as:

$$D(G) = 1 - e^{-\gamma\left(\frac{\phi_0}{\phi}\frac{G}{\theta}\right)} \tag{2.37}$$

The detailed derivation for this steady-state damage function is provided in Appendix A. This formulation highlights that, given the assumption of the EU's fixed share of global emissions, the long-run climate damage is solely determined by the level of carbon emissions within the EU.

Therefore, a steady-state competitive equilibrium for this economy is defined as a set of constant prices $(w, r, \nu^E, \{P_l\}_{l=1}^L)$, a lump-sum transfer T, L sectoral firm productivity distributions $\{\mu_l(s)\}_{l=1}^L$, firm-level policy functions for labor, capital, energy, and output$(\{\overline{n_l}(s)\}_{l=1}^L, \{\overline{k_l}(s)\}_{l=1}^L, \{\overline{e_l}(s))\}_{l=1}^L, \{\overline{y_l}(s))\}_{l=1}^L)$, and aggregate steady-state quantities for consumption C, labor N, capital K, energy E, emission G, and final good output $\tilde{Y}$, such that the following conditions are satisfied:

1. **Consumer optimization problem:**

$$r = \frac{1}{\beta} - (1 - \delta) \tag{2.38}$$

2. **Sectoral producer's profit Maximization:**

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Given (w, r, ν), the policy functions $\overline{x_l}(s)$, $\overline{k_l}(s)$, $\overline{n_l}(s)$, $\overline{e_l}(s)$ solve the sectoral producer's problem.

3. **Market clearing:**

$$N = \sum_{l=1}^{L} \int_s \overline{n_l}(s)\mu_l(s)ds \tag{2.39}$$

$$K = \sum_{l=1}^{L} \int_s \overline{k_l}(s)\mu_l(s)ds \tag{2.40}$$

$$E = \sum_{l=1}^{L} \int_s \overline{e_l}(s)\mu_l(s)ds \tag{2.41}$$

$$a_l\tilde{Y}\left(\frac{P_l}{1-D(G)}\right)^{-\sigma} = \int_s \overline{y_l}(s)\mu_l(s)ds,\ \forall l \in \{1, ..., L\} \tag{2.42}$$

$$C + \delta K = (1 - D(G))\tilde{Y} \tag{2.43}$$

4. **Government budget balance:**

$$\sum_{l=1}^{2} \tau_l^{CE}\varphi_l \int_s \overline{e_l}(s)\mu_l(s)ds = T \tag{2.44}$$

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Chapter 3 Calibration and Parameterization

We calibrate the model parameters to match key features of the European Union in the baseline year 2019. The calibration process is guided by both empirical data availability and established macroeconomic theory. Parameters are categorized according to their functional roles in the model: preference and intertemporal parameters, production technology parameters, climate system and energy price parameters, and sectoral heterogeneity and distributional parameters. For each category, we either adopt values directly from the literature, estimate them using statistical data, or calibrate them internally to replicate observed economic moments.

The year 2019 is chosen as the calibration baseline due to its broad data coverage among all the datasets and its status as a stable pre-pandemic reference year. Also, to ensure consistency across datasets, we map NACE Rev.2 industrial classifications into four representative sectors: industrial, commercial, transportation, and utility. Details are provided in Appendix C.

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3.1 Preference and Intertemporal Parameters

The preference and intertemporal parameters are taken directly from the literature. The discount factor β is set to 0.96, which implies an annual real interest rate of approximately 4.17%, consistent with Restuccia and Rogerson (2009). The depreciation rate δ is calibrated at 0.08 per annum, which is a standard value in macroeconomic models. In addition, we normalize the total labor supply to unity since the individual does not value leisure.

3.2 Climate System and Energy Price Parameters

This group includes exogenous parameters that govern the climate damage process and energy market structure. Following the formulation introduced by Golosov et al. (2014), the decay rate of atmospheric CO_2, ϕ is set to 0.0228, and the immediate absorption share $1 - \phi_0$ is calibrated at 0.607, implying $\phi_0 = 0.393$. The damage sensitivity parameter γ is set to 2.3792×10^{-5}, based on a weighted average of moderate and catastrophic climate scenarios [1]. These parameters govern how atmospheric CO_2 levels translate into output losses via the climate damage function. The EU carbon emission accounts for 9.86% of global emissions, pinning down the model parameter θ to 0.0986. The energy price parameter ν is calibrated at 0.0138, expressed in units of 100,000 euros per ton of oil equivalent (toe). This value is computed as a weighted average of electricity, natural gas, and petroleum prices [2], with weights reflecting their respective shares in final energy

[1]This is calculated from $\gamma = p\gamma_H + (1-p)\gamma_L$, Here, $\gamma_L = 1.060 \times 10^{-5}$ is the damage sensitivity parameter for moderate damages, implying a 0.48% loss at 3 degrees of heating, and $\gamma_H = 2.046 \times 10^{-4}$ is the damage sensitivity parameter for catastrophic damages, implying a 30% loss at 6 degrees of heating. The parameter $p = 0.068$ represents the ex-ante probability of the high value of the damage parameter.

[2]We calibrate the exogenous energy price using "The dashboard for energy prices in the EU and main trading partners in 2024" from Eurostat. This dashboard reports producer-side purchasing prices for elec-

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consumption[3] within the European Union. Although this calibration excludes certain minor energy sources, these three categories jointly account for approximately 83% of total final energy use in the EU, providing a sufficiently representative basis for the benchmark energy price.

3.3 Production Technology Parameters

Under the assumption of a Cobb-Douglas production function, each sector's value added is generated by a combination of labor, capital, and energy inputs. The input elasticities can be inferred from the respective factor shares in gross value added.

First, we estimate sectoral energy expenditures by multiplying the energy consumption [4] with the calibrated average energy price. By dividing energy expenditures by gross value added [5], we recover the energy input share, which corresponds to the energy elasticity γ_l. Second, labor compensation data [6] together with the previous gross value-added data allow us to compute the labor share, and therefore labor elasticity α_l. The remaining share is attributed to capital, implying $\beta_l = 1 - \alpha_l - \gamma_l$. To ensure positive operating margins and the coexistence of heterogeneous firms within each sector, we follow the approach of Restuccia and Rogerson (2009) by assuming decreasing returns to scale. Specifically, we proportionally scale down all elasticities so that $\alpha_l + \beta_l + \gamma_l = 0.85$, thereby preserving the relative input shares while introducing concavity into the produc-

tricity, natural gas, and petroleum-based fuels—three primary energy.

[3]Here we utilize the IEA dataset "Evolution of total final energy consumption in Europe since 2000". This dataset contains annual data on the composition of energy use across all major fuel categories within the EU, which enables us to calculate the energy consumption shares of these three primary energy types.

[4]Here, we use the 'Complete energy balance' dataset from Eurostat (dataset code: nrg-bal-c), which reports energy consumption by end-use sector, including industry, commercials, transportation, and utilities

[5]This gross value added is derived from variable VA-CP in the EU KLEMS national account dataset, representing value added in the current prices

[6]we use the variable COMP in the EU KLEMS national account dataset, representing employee compensation including wages and social contributions)

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tion function. Table 3.1 summarizes the calibrated values for these input intensities.

	Labor intensity	**Capital intensity**	**Energy intensity**
Industry	0.4297	0.4031	0.0172
Commercial	0.4471	0.3905	0.0123
Transportation	0.4483	0.3094	0.0923
Utility	0.2733	0.5099	0.0668

Table 3.1: Input intensities by sector

Two parameters must be calibrated for the final good producer: a_l and σ. For σ, we follow the assumption in Acemoglu et al. (2012), whose model setup is similar to our Multi-sector model with the final good produced using "dirty" and "clean" sectoral input. Therefore, we assume the elasticity of substitution σ to be 3. For the share parameter a_l, we use the calibration method proposed in Sancho (2009). It is common to select units of measurement in which one unit of good is worth one unit of value during the calibration process. Therefore, we rescale the units for Y_l and Y to make the P_l in each sector match the unit value in the base year. Therefore, the formula can be simplified to:

$$a_l = (\frac{Y_0}{Y_{l,0}})^{\frac{\sigma}{1-\sigma}} \tag{3.1}$$

where $\frac{Y_{l,0}}{Y_0}$ represents the output share of different sectors in the base year. [7]

Therefore, we can solve for a_l based on the assumed value of σ. The calibrated sectoral share parameters are 0.3449 for the commercial sector, 0.2822 for industry, 0.0141 for transportation, and 0.0027 for utilities. These weights reflect each sector's relative contribution to aggregate output, with larger values assigned to sectors that produce a greater share of GDP. As expected, the commercial and industrial sectors dominate the

[7] We also calibrate sectoral share parameters by using the 'GO-CP' variable from the EU KLEMS dataset, which represents gross output in the current price. By computing the share of each sector's gross output relative to total GDP across the EU27, we determine the output share weights that are used in our model to reflect each sector's contribution to final good production.

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final good composition, while transportation and utilities play a smaller role.

In the model, the total effective price of energy faced by firms in sector l is given by $\nu + \tau_l \varphi_l$, where ν denotes the exogenous energy price and $\tau_l \varphi_l$ captures the carbon tax burden per unit of energy input. To ensure dimensional consistency in the model' s equilibrium conditions and comparative statics, we define the unit of the carbon tax τ_l as 100,000 euros per ton of carbon dioxide emissions. This scaling guarantees that the tax component and the energy price are expressed in comparable monetary units per unit of energy.

3.4 Sectoral Heterogeneity and Distributional Parameters

Sectors differ in their emission factor, which is defined as the ratio of carbon emissions to energy consumption. By dividing total sectoral carbon emissions [8] by corresponding energy consumption data, we compute the emission factor φ_l for each sector, which captures how much tons of carbon emission is created per tons of oil equivalent(toe) is used. The resulting values are 3.8521 for the industrial sector, 0.951 for the commercial sector, 1.7068 for the transportation sector, and 11.8745 for the utility sector—the highest among all, reflecting its status as the most "dirty" sector.

To incorporate firm-level heterogeneity in productivity within each sector, we follow the methodology of Restuccia and Rogerson (2009), which proxies firm productivity using employment size. In our model, the relative labor demand between two firms in the same

[8] We calibrate sectoral carbon emissions using the 'Air emissions accounts for greenhouse gases by NACE Rev. 2 activity' dataset from Eurostat (dataset code: env-ac-aigg-q). This dataset provides disaggregated data on CO_2-equivalent greenhouse gas emissions by production activity across the EU27, allowing us to map carbon emissions to each sector consistent with our model classification.

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sector, i and j, is given by:

$$\frac{\overline{n_l^i}}{\overline{n_l^j}} = \left(\frac{S_i}{S_j}\right)^{\frac{1}{1-\alpha_l-\beta_l-\gamma_l}} \tag{3.2}$$

To implement this, we utilize two datasets from Eurostat. First of all, the Business Demography dataset [9] Although helpful, it lacks detail for larger firms. We, therefore, complement this with the Structural Business Statistics dataset [10] By combining these two sources, we obtain a more complete picture of the firm size distribution across the EU27.

With detailed data on firm size distribution, we also calibrate the number of establishments in each sector(M_l). The commercial sector has the largest number of firms, which is 22.3565 million firms, followed by the industrial sector (6.0075), the transportation sector (1.4249), and the utility sector (0.1717). The distribution of firm productivity across all sectors is derived from the distribution of firm size, proxied by the number of employees ranging from 1 to 500. By normalizing the productivity level of the smallest establishments to 1, this mapping yields a firm-level productivity range from 1 to 2.5401.

Since we assume a lognormal distribution for firm-level productivity within each sector in our model, we approximate the cumulative distribution function (CDF) of firm size using a lognormal distribution fitted to the firm count data obtained from Eurostat. By aligning the fitted CDF with the empirical distribution of firm size classes based on employment brackets, we derive sector-specific firm share curves. As illustrated in Figure 3.1, the fitted curves closely track the observed data across all sectors, confirming the suitability of the lognormal assumption.

[9]The Business Demography dataset (dataset code: bd-size) provides firm counts by employment size class in narrow ranges—specifically 0, 1-4, 5-9, and greater than 10 employees.

[10]The Structural Business Statistics dataset (dataset code: sbs-ovw-smc) reports firm counts in higher employment brackets: 10-19, 20-49, 50-249, and over 250 employees. For calibration purposes, we cap the largest category (>250) at an upper bound of 500 employees.

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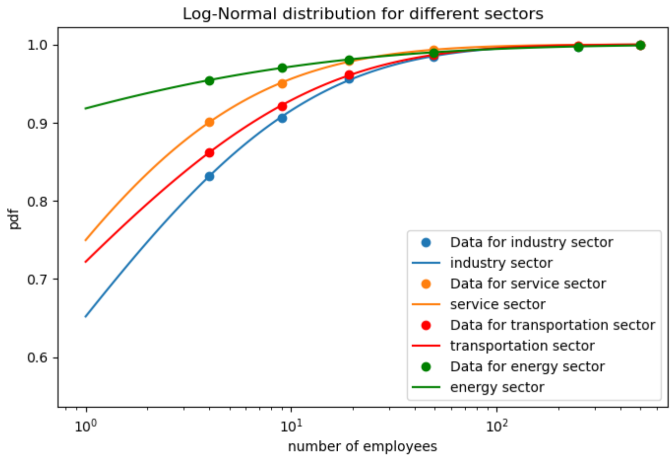

Figure 3.1: Fitting a log-normal distribution to the firm size distribution data of different sectors.

Following Eurostat's classification scheme, we categorize firms as Micro (0–9 employees), Small (10–49), Medium (50–249), and Large (>250), and summarize the shares of establishments, carbon emissions, and output across sectors in each category by sector accordingly in Table 3.2.

The firm size and output distribution across sectors display several notable patterns. First of all, the industrial and transportation sectors share a similar structure. The Small firm categories in the industrial sector(output share= 38.09%) and transportation sector(output share= 37.65%) contribute the largest share of output. By contrast, although 96.91% firms belong to the micro category, the utility sector' s output is predominantly concentrated in the Medium (output share=41.29%) and large (output share=26.25%) categories, which is consistent with the capital-intensive nature and larger operational scale of utility infrastructure.

Interestingly, the commercial sector displays a unique feature: the Micro category not only accounts for the vast majority of firms (94.15%) but also contributes a significantly higher share of output (31.97%) than in other sectors. This highlights the distinct structure of the commercial economy, reflecting the high fragmentation and prevalence of small-

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scale producers in these sectors. As for the emission share, it will follow the pattern of output if there is no within-sector distortion.

Table 3.2: Firm size distribution, output share, and emission share across sectors

(a) Industrial sector

	share of firms	**share of output**	**share of emissions**
Micro (0–9)	0.898167	0.223884	0.223884
Small (10–49)	0.085867	0.381850	0.381850
Medium (50–249)	0.015015	0.319752	0.319752
Large (>250)	0.000950	0.074514	0.074514

(b) Commercial sector

	share of firms	**share of output**	**share of emissions**
Micro (0–9)	0.945440	0.319688	0.319688
Small (10–49)	0.047357	0.361815	0.361815
Medium (50–249)	0.006758	0.255695	0.255695
Large (>250)	0.000445	0.062802	0.062802

(c) Transportation sector

	share of firms	**share of output**	**share of emissions**
Micro (0–9)	0.914610	0.207978	0.207978
Small (10–49)	0.071142	0.376496	0.376496
Medium (50–249)	0.013396	0.337169	0.337169
Large (>250)	0.000852	0.078357	0.078357

(d) Utility sector

	share of firms	**share of output**	**share of emissions**
Micro (0–9)	0.969101	0.101769	0.101769
Small (10–49)	0.021586	0.222972	0.222972
Medium (50–249)	0.007836	0.412906	0.412906
Large (>250)	0.001476	0.262533	0.262533

To sum up, Table 3.3 provides a comprehensive summary of all calibrated parameters used in the model, along with their corresponding data sources and references. Each parameter is grounded in either empirical data from Eurostat, EU KLEMS, IEA, or in established literature such as Acemoglu et al. (2012), Golosov et al. (2014), and Restuccia and Rogerson (2009). This calibration framework ensures that the model reflects both the

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structural features of the EU economy and widely accepted empirical regularities, providing a credible foundation for the policy simulations that follow.

Description	Parameter	Value [I, C, T, U]	Source
Time Preference	β	0.96	Restuccia and Rogerson (2009)
Depreciation Rate	δ	0.08	Restuccia and Rogerson (2009)
Elasticity of substitution	σ	3	Acemoglu et al. (2012)
decay rate of CO_2	ϕ	0.0228	Golosov et al. (2014)
ratio of CO_2 absorbed instantly	$1-\phi_0$	0.607	Golosov et al. (2014)
damage parameter	γ	2.3792×10^{-5}	Golosov et al. (2014)
US carbon emission share	θ	0.0986	Estimated from IEA
energy price	ν	0.0138	Estimated from IEA and Eurostat
share parameter	a_l	[0.2822, 0.3449, 0.0141, 0.0027]	Estimated from EU KLEMS
emission factor	φ_l	[3.8521, 0.951, 1.7068, 11.8745]	Estimated from Eurostat
mean of ln(s) distribution	μ_l	[0.8862, 0.1676, 1.518, 0.6241]	Estimated from Eurostat
std of ln(s) distribution	σ_l	[0.1509, 0.2500, 0.0936, 0.2736]	Estimated from Eurostat
mass of the sector	M_l	[6.0075, 22.3565, 1.4249, 0.1717]	Estimated from Eurostat
labor intensity	α_l	[0.4297, 0.4471, 0.4483, 0.2733]	Estimated from EU KLEMS and Eurostat
capital intensity	β_l	[0.4031, 0.3905, 0.3094, 0.5099]	Estimated from EU KLEMS and Eurostat
energy intensity	γ_l	[0.0172, 0.0123, 0.0923, 0.0668]	Estimated from EU KLEMS and Eurostat

Table 3.3: Calibration Results

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Chapter 4 Quantitative Analysis

In this section, we conduct a quantitative evaluation of the long-term effects of carbon taxation, using parameter values calibrated in Chapter 3. Our analysis considers four distinct policy scenarios: uniform taxation, heterogeneous taxation, sectoral exemption, and small emitters' exemption. The uniform taxation scenario applies a consistent carbon tax rate across all sectors. The heterogeneous taxation scenario allows for different tax rates in heterogeneous sectors. Under the sectoral exemption scenario, we sequentially exempt different sectors to evaluate the economic impact of such policies. In the exempting small emitters scenario, the firms with carbon emissions below the given levels will be exempted from the carbon tax.

First, we describe how carbon taxes affect key macroeconomic variables under each policy scenario. While the transmission mechanisms, such as input substitution, market share reallocation, and resource reallocation, are broadly similar across scenarios, the differences in macroeconomic outcomes arise due to the heterogeneous impact of each policy scenario on specific sectors.

Subsequently, we turn to a quantitative comparison across scenarios, evaluating the optimal carbon tax rate and its associated effects on aggregate variables such as undamaged GDP [1], consumption, and sectoral variables like market share and energy share. This

[1] The undamaged GDP here indicates the final good production before accounting for the climate dam-

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comparative exercise enables us to identify the most effective carbon tax design from a welfare-maximizing perspective.

Lastly, we conduct an efficiency-equivalent comparison. For each policy scenario, we compute the level of carbon taxation required to achieve the same reduction in emissions under a uniform tax scenario and assess the resulting welfare loss. This allows us to determine which policy design delivers a given environmental outcome at the lowest possible cost to social welfare.

4.1 Uniform tax

In the uniform carbon tax policy scenario, the same level of carbon tax is imposed across all sectors. Given that our utility function is a monotonically increasing function solely dependent on consumption, maximizing social welfare is equivalent to maximizing aggregate consumption. Figure 4.1 illustrates steady-state consumption levels under various uniform carbon tax rates. The resulting curve exhibits an inverted U-shape, indicating that consumption initially rises with the tax rate and then declines after a certain point. We observe that consumption reaches its maximum when the carbon tax rate is set at 0.0122, corresponding to the optimal tax rate under a uniform taxation scheme.

The response of aggregate consumption to changes in the carbon tax rate can be analytically decomposed into two opposing effects:

$$\frac{\Delta C}{\Delta \tau} = \frac{\Delta(\tilde{Y} - \delta K)}{\Delta \tau} - \frac{\Delta(D(G)\tilde{Y})}{\Delta \tau} \tag{4.1}$$

age and the capital depreciation, which is $\tilde{Y}$

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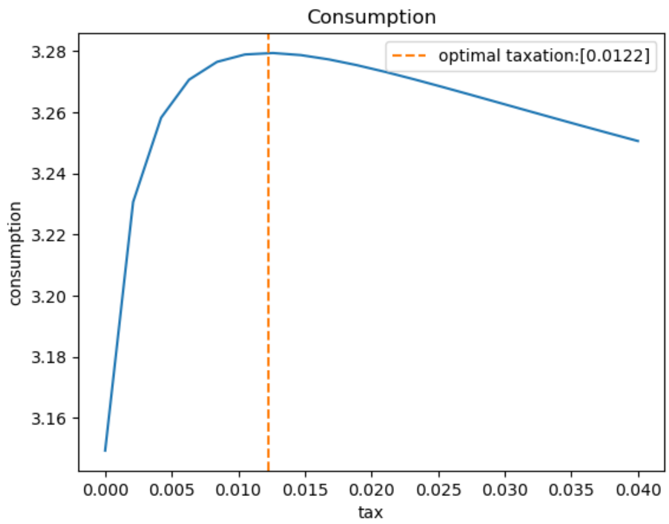

Figure 4.1: Aggregate consumption to carbon tax under uniform tax scenario

The first term captures the decline in the undamaged net domestic product (NDP)[2] that arises from increased production costs under carbon taxation. As illustrated in Figure 4.2, higher carbon taxes raise energy prices, compressing sectoral outputs, particularly in energy-intensive industries. This contraction in undamaged NDP translates into a lower consumption level.

The second term reflects the climate damage mitigation effect. By curbing fossil fuel use, carbon taxation reduces greenhouse gas emissions, thereby lowering the economy's exposure to future climate-related damages. This effect enhances effective output, $(1 - D(G))\tilde{Y}$, and thus increases long-run consumption.

Carbon taxation reduces fossil fuel consumption and, consequently, carbon emissions, thereby mitigating future climate-related economic damages. At low tax rates, this mitigation effect dominates the contraction in undamaged net domestic product (NDP), leading to a net increase in aggregate consumption. However, as the tax rate rises, the marginal benefit from further emission reductions declines, while the distortionary im-

[2] Undamaged NDP, defined as $\tilde{Y} - \delta K$, refers to final good production net of capital depreciation, before accounting for climate damages.

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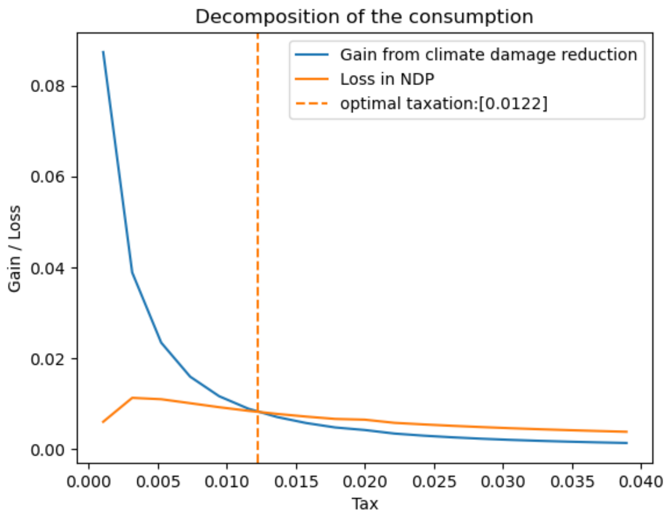

Figure 4.2: Decomposition of the change of consumption under a uniform tax scenario

pact on production intensifies. These opposing forces converge at an interior maximum, where aggregate consumption peaks—corresponding to the welfare-maximizing carbon tax rate. Beyond this point, the output loss outweighs the environmental gains, causing consumption to decline and giving rise to the inverted-U pattern depicted in Figure 4.1.

4.1.1 Mechanism

In this section, we will focus on discussing the mechanism behind how the carbon tax changes the main economic indicator.

First of all, total factor productivity (TFP) at the sectoral level is closely related to the number and distribution of firms within each sector under our decreasing return-to-scale setting. This relationship is derived analytically in Appendix B. In the steady-state equilibrium, we estimate TFP using a Solow residual approach:

$$TFP_l = \frac{Y_l}{N_l^{\alpha_l} K_l^{\beta_l} E_l^{\gamma_l}} \tag{4.2}$$

Among all sectors, the commercial sector, characterized by the largest number of

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firms, exhibits the highest estimated productivity ($TFP_C = 1.8161$). It is followed by the industrial sector ($TFP_I = 1.6270$), which has the second-largest number of firms. The smaller transportation sector ($TFP_T = 1.2797$) and utility sectors ($TFP_U = 0.8506$) have lower productivity estimates.

The upper half of the Figures 4.3 illustrate that increasing carbon tax rates are associated with reductions in energy consumption, carbon emissions, and the climate damage ratio. Notably, the marginal rate of decline for these indicators decreases as the carbon tax intensifies.

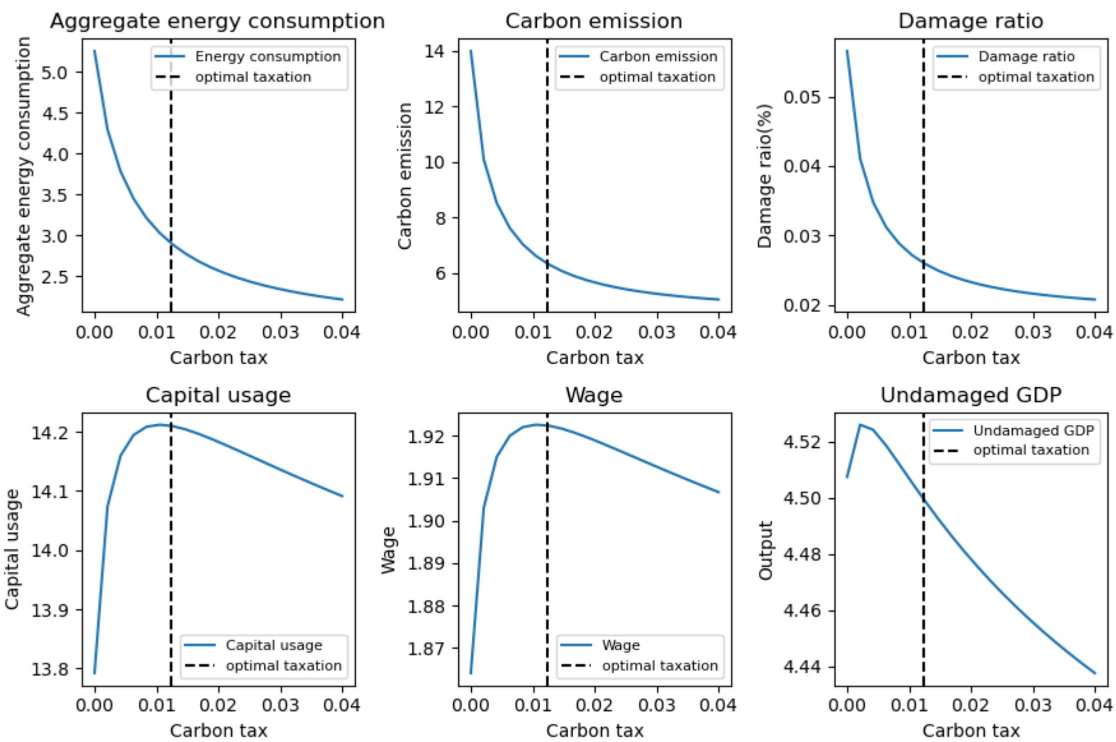

Figure 4.3: The change of capital usage, wage, and undamaged GDP to carbon tax under the uniform tax scenario

As shown in the lower half of Figure 4.3, several macroeconomic aggregates also exhibit systematic responses to changes in the carbon tax, including capital usage, wage, undamaged GDP. As energy becomes more expensive, firms increasingly substitute capital and labor for energy input. Since labor supply is exogenously fixed in the model, the

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heightened demand for labor exerts upward pressure on wages. Capital prices, determined by the representative household's intertemporal optimization in a steady state, remain stable, prompting firms to increase capital deployment.

At modest tax levels, undamaged GDP experiences a slight increase as resources shift from low-productivity, energy-intensive sectors, such as the utility sector, toward higher-productivity sectors, such as the commercial sector. However, as taxation becomes more severe, the rise in production costs surpasses the gains from reallocation, leading to a pronounced decline in undamaged GDP.

	0.00	**0.01**	**0.02**	**0.03**	**0.04**
Industrial sector	0.774	+4.612%	+5.494%	+5.900%	+6.137%
Commercial sector	0.751	+3.352%	+3.799%	+3.957%	+4.026%
Transportation sector	0.675	+8.894%	+12.472%	+14.828%	+16.598%
Utility sector	0.525	+14.887%	+18.871%	+21.207%	+22.857%

Table 4.1: Sectoral price responses under carbon tax rates
(Note: We report the percentage changes compared to the price in the first column)

	0.00	**0.01**	**0.02**	**0.03**	**0.04**
Industrial sector	2.304	-1.450%	-2.687%	-3.555%	-4.220%
Commercial sector	3.082	+2.199%	+2.158%	+1.953%	+1.731%
Transportation sector	0.174	-12.623%	-19.698%	-24.348%	-27.755%
Utility sector	0.071	-25.596%	-31.982%	-35.674%	-38.244%

Table 4.2: Sectoral output change under varying carbon tax rates
(Note: We report the percentage changes compared to the output in the first column)

Table 4.1 displays the evolution of sectoral output prices as the carbon tax increases. The prices in all sectors rise with the tax rate, indicating that firms pass on their increased production costs, induced by the carbon levy, to downstream producers and ultimately consumers. Importantly, the magnitude of price increases correlates with the emission factor and energy intensity of each sector. When the carbon tax is 0.01, the utility sector(+14.887%), with the highest emission factor, exhibits the greatest cost pass-through as expected, followed by the transportation sector (+8.894%), the industrial sector

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(+4.612%), and finally the commercial sector (+3.352%). These results demonstrate how emission-intensive sectors bear higher implicit carbon costs and are more likely to transmit those costs to the final stage of production.

Table 4.2 presents the change in sectoral output levels in response to the carbon tax. We observe that the industrial, transportation, and utility sectors, all characterized by relatively high emissions factors and energy intensities, experience a steady decline in output as the tax increases. When the carbon tax is 0.01, the utility sector(-25.596%) exhibits the greatest output downturn, followed by the transportation sector (-12.623%), and the industrial sector (-1.450%). This decline becomes more pronounced with higher tax rates, as rising production costs increasingly suppress activity in these sectors. On the other hand, the commercial sector displays a non-monotonic pattern: its output initially rises with low levels of taxation but eventually falls once the tax surpasses a certain threshold. When the carbon tax is 0.01, which is still modest, the commercial sector experiences a relatively high increase in output(+2.199%). However, when the carbon tax goes up to 0.02, the increase in output is less profound(+2.158%). This behavior reflects the role of resource reallocation. In the early phase, the commercial sector benefits from the contraction of carbon-intensive sectors by absorbing displaced capital and labor, thereby boosting its output. However, as the tax becomes more stringent, the adverse effects of increased energy costs outweigh the benefits from input reallocation, leading to a decline in commercial sector output. We explore the resource reallocation in greater detail in the following sections.

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4.1.2 Resource reallocation

In this section, we focus on how carbon taxation affects the inter-sectoral allocation of resources. From the first-order conditions of firm optimization (Equations 2.12, 2.13, and 2.14), we derive the following expressions for the relative use of labor, capital, and energy across sectors:

$$\frac{N_1}{N_2} = \frac{\alpha_1}{\alpha_2} \frac{P_1 Y_1}{P_2 Y_2} \tag{4.3}$$

$$\frac{K_1}{K_2} = \frac{\beta_1}{\beta_2} \frac{P_1 Y_1}{P_2 Y_2} \tag{4.4}$$

$$\frac{E_1}{E_2} = \frac{\gamma_1}{\gamma_2} \frac{P_1 Y_1}{P_2 Y_2} \frac{(\nu + \tau^{CE} \varphi_2)}{(\nu + \tau^{CE} \varphi_1)} \tag{4.5}$$

These equations reveal two key mechanisms through which carbon taxes alter sectoral resource use. First, the relative revenue share term captures the composition effect, whereby sectors that are less adversely affected by the tax expand in the market, attracting more factor inputs. This mechanism corresponds to the composition effect described in Antweiler et al. (2001), where structural changes in the economy, induced by environmental policy, redirect resources from pollution-intensive sectors toward cleaner, more productive sectors, thereby mitigating environmental degradation despite ongoing economic growth. However, it operates indirectly through changes in market shares across sectors. Its influence is generally more modest. This is in contrast to the more immediate and quantitatively significant relative price effect. The relative price effect arises from differences in effective energy costs, as sectors with higher pollution multipliers face steeper increases in marginal energy prices under taxation. This price differential shifts energy use toward sectors less penalized by the tax, further reinforcing inter-sectoral resource reallocation.

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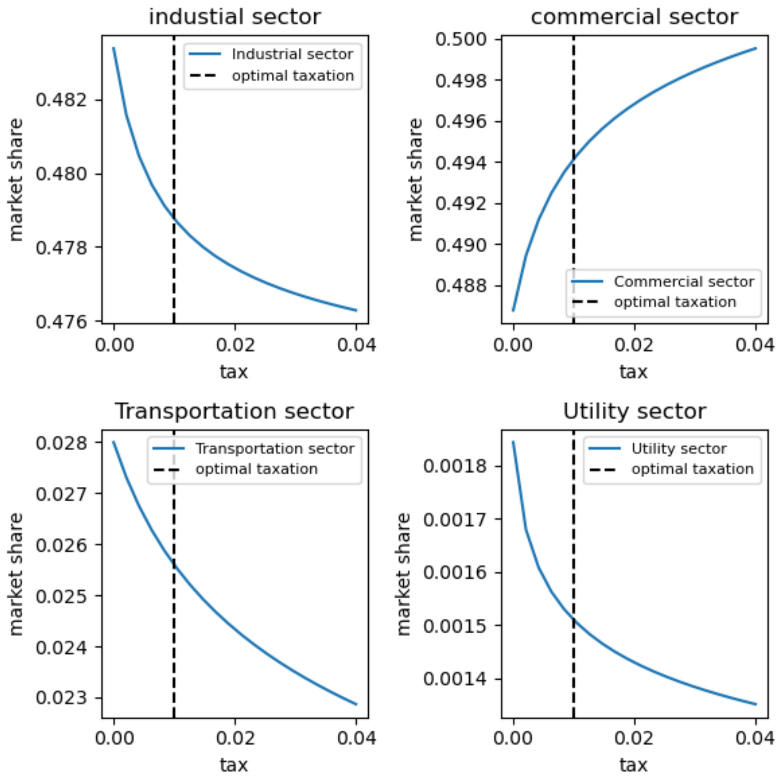

Figure 4.4: The sectoral market share change over carbon tax under uniform tax

As shown in the Figure 4.4, the market shares [3] of each sector respond differently to the carbon tax. The industry, transportation, and utility sectors, all relatively energy-intensive, experience a contraction in market share, and the extent of this contraction increases with the severity of the tax. In contrast, the commercial sector, which is less exposed to carbon costs, expands its share in the economy. This pattern reflects the reallocation of economic activity away from carbon-intensive sectors and highlights how carbon taxation can also reshape the structural composition of the economy.

Moreover, in Figure D.1 and Figure D.2 [4], we observe that sectoral shares of labor and capital inputs evolve similarly with the tax rate, closely tracking the changes in market shares. This is consistent with the predictions derived from eq 4.3, eq 4.4, and eq 4.5 above, where input demand is proportional to sectoral output levels.

[3]The market share is defined as $\frac{P_i Y_i}{\sum P_i Y_i}$

[4]Since the similarity, we locate the Figure in appendix D to avoid repetition.

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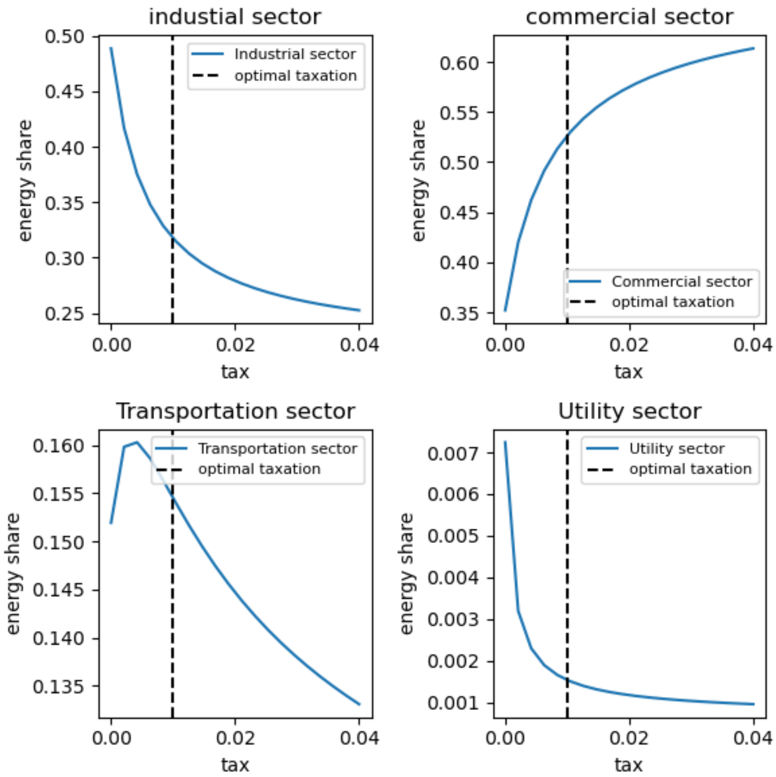

Figure 4.5: The sectoral energy share change over carbon tax under the uniform tax

In contrast, the patterns for energy input shares exhibit greater heterogeneity due to the influence of relative price effects. First, the magnitude of change in energy shares is considerably larger than that of labor and capital, reflecting the central role the direct price effect plays in shaping energy allocation across sectors. In Figure 4.5, we can observe that the transportation sector, the second least carbon-intensive sector, initially increases its energy share under low carbon tax rates. This occurs because more emission-intensive sectors contract more rapidly, temporarily boosting transportation's relative position. However, as the tax rate continues to rise, the transportation sector eventually begins to lose its energy share as well. Meanwhile, the commercial sector, least exposed to the tax, contracts the usage of energy at a relatively slower pace. Therefore, it experiences a continuous increase in energy share. The resulting reallocation further underscores the dynamic role of relative prices in determining energy usage patterns.

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4.2 Heterogeneous tax

As derived in Chapter 2, the optimal sectoral carbon tax (τ_l^{opt}) can be expressed as [5]:

$$\Lambda^s = \tau_l^{opt} + \frac{\nu}{\varphi_l} \tag{4.6}$$

The left-hand side represents the external cost of climate damage per unit of carbon emissions, which are identical across sectors. The right-hand side reflects the effective cost per unit of carbon emissions borne by producers in sector l, which consists of the explicit tax burden(τ^{opt}) and the implied energy cost per unit of emissions $\frac{\nu}{\varphi_l}$.

Sectors with higher emission factors, such as the utility sector($\varphi_U = 11.87$), or the industrial sector($\varphi_I = 3.85$), face lower implied energy costs and thus require higher carbon taxes to internalize their externalities appropriately. Conversely, in sectors with lower emission factors, like the commercial sector ($\varphi_C = 0.95$) or transportation ($\varphi_T = 1.71$), the externality can be addressed with relatively lower tax rates. This ensures that the combined marginal cost of carbon across sectors is equated to the external damage.

Notably, the order of optimal tax rates derived from our numerical simulations precisely mirrors the ranking of emission factors across sectors: the utility sector($\tau_U = 0.0136$), with the highest emission factor, faces the highest optimal tax rate, followed by industry($\tau_I = 0.0121$), transportation($\tau_T = 0.0086$), and finally the commercial sector($\tau_C = 0.0013$). This consistent ordering reinforces the theoretical principle that carbon taxes should scale with emission factors to achieve efficiency.

[5] We deduce the optimal carbon tax in equation 2.35, This formula is a rearranged version under the steady-state condition.

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In Table 4.3, the commercial gets more factor share under a heterogeneous tax scenario. This resource reallocation to the most productive sector boosts the final good production. As shown in Table 4.4, the heterogeneous tax scenario leads to a higher undamaged GDP than the uniform tax scenario. Despite the slightly higher carbon emission, overall the increase in undamaged GDP is enough to offset the downturn on higher climate damage, resulting in a higher consumption level. The quantitative result shows that tax differentiation leads to a 4.22% increase in aggregate consumption compared with the no carbon tax scenario, which outperforms the uniform tax scenario (+4.13%). This aligns with theoretical expectations, showing that the heterogeneous tax can lead to the global maximum in aggregate consumption.

	Market share	**Labor share**	**Capital share**	**Energy share**
No carbon tax	0.5444	0.5553	0.5387	0.3929
Uniform tax	0.5532	0.5638	0.5475	0.5894
Heterogeneous tax	0.5558	0.5663	0.5502	0.6931

Table 4.3: Factor and market share of the commercial sector under different scenarios

	Carbon emissions	**Damage ratio**	**Undamaged GDP**	**Consumption**
No carbon tax	13.9953	0.0565	4.5075	3.1493
Uniform tax	3.7782 (-73.00%)	0.0156	4.4983	3.2794 (+4.13%)
Heterogeneous tax	3.9997 (-71.42%)	0.0165	4.5061	3.2823 (+4.22%)

Table 4.4: Comparison of macroeconomic outcomes under different scenarios
(Note: We report the percentage changes compared to the first row)

4.3 Sectoral exemption

Carbon tax exemptions for specific sectors are a common policy feature in practical implementations. To evaluate their implications, we conduct a series of counterfactual simulations in which each sector is exempted individually from the carbon tax.

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4.3.1 Resource reallocation

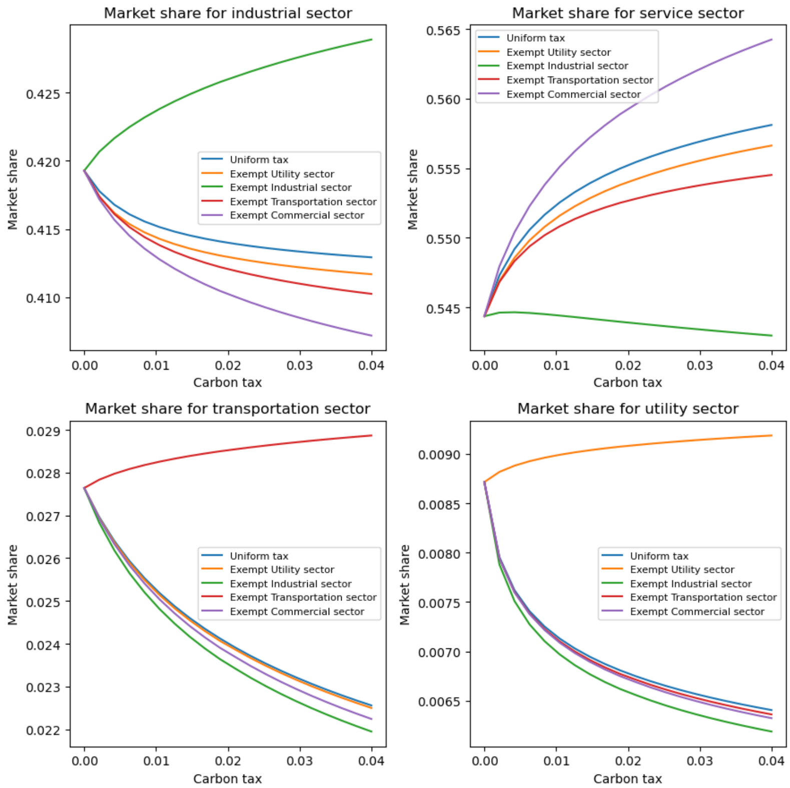

Figure 4.6: The sectoral market share change over carbon tax under the sectoral exemption scenarios

Figure 4.6 illustrates how sectoral market shares evolve under these exemption scenarios. Two key patterns emerge. First, the exempted sector enjoys a relative advantage over taxed sectors, expanding its market share. For instance, when the industrial, transportation, or utility sector is exempted, its market share reverses from a downward trend under uniform taxation to an upward trajectory. Similarly, the commercial sector, although already benefiting under a uniform tax, experiences an even greater increase in market share when exempted.

Second, as the exempted sector expands, it imposes downward pressure on the market shares of other sectors. For example, under the scenario where the transportation sector is exempted, this crowding-out effect is particularly visible in the commercial sector. While

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it normally sees a rising market share under uniform taxation, its growth is curtailed when transportation sectors are exempted. Also, the industrial sector’s market share falls more steeply than under the uniform tax, as the transportation sector absorbs a disproportionate share of market resources. Similar effects can be observed across the other exemption cases.

These results highlight how partial exemptions, while potentially easing the burden on specific industries, can distort sectoral competition and shift the burden toward unexempted sectors, ultimately affecting the overall structure of the economy.

Correspondingly, the reallocation of resources change follows the two key principles observed earlier in the movements in market shares: (1) the exempted sector gains a comparative advantage and expands its market share, and (2) the expansion of the exempted sector suppresses others via the reallocation of inputs.

As the reallocation of resources under sectoral exemptions broadly follows key principles we discover and the mechanism we deliver in section 4.1.1, we present the full set of visualization results for labor share(Figure D.3), capital share (Figure D.4), and energy shares(Figure D.5) under different exemption schemes in Appendix D.

4.3.2 Macroeconomic outcomes

The upper half of Figure 4.7 presents the changes in energy consumption and climate damage ratio across different sectoral exemption scenarios. Among the cases, exempting the industrial sector results in the weakest reduction in energy use, followed by exemptions for the commercial, transportation, and utility sectors. This pattern reflects the underlying composition of energy use across sectors. Table 4.5 shows, in the absence of a carbon tax,

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the steady-state shares of emissions are 61.17% for the industrial sector, 14.02% for the commercial sector, 9.59% for transportation, and 15.22% for utilities. Hence, exempting sectors with higher energy shares, such as the industrial sector, undermines the tax’ s effectiveness in reducing emissions and amplifies residual climate damages. As a result, climate-induced economic harm remains elevated under these exemption schemes.

	Industrial sector	**Commercial sector**	**Transportation sector**	**Utility sector**
Energy share	0.423198	0.392922	0.149719	0.034161
Emission share	0.611694	0.140210	0.095885	0.152211

Table 4.5: Sectoral energy and emission shares under no carbon tax scenario

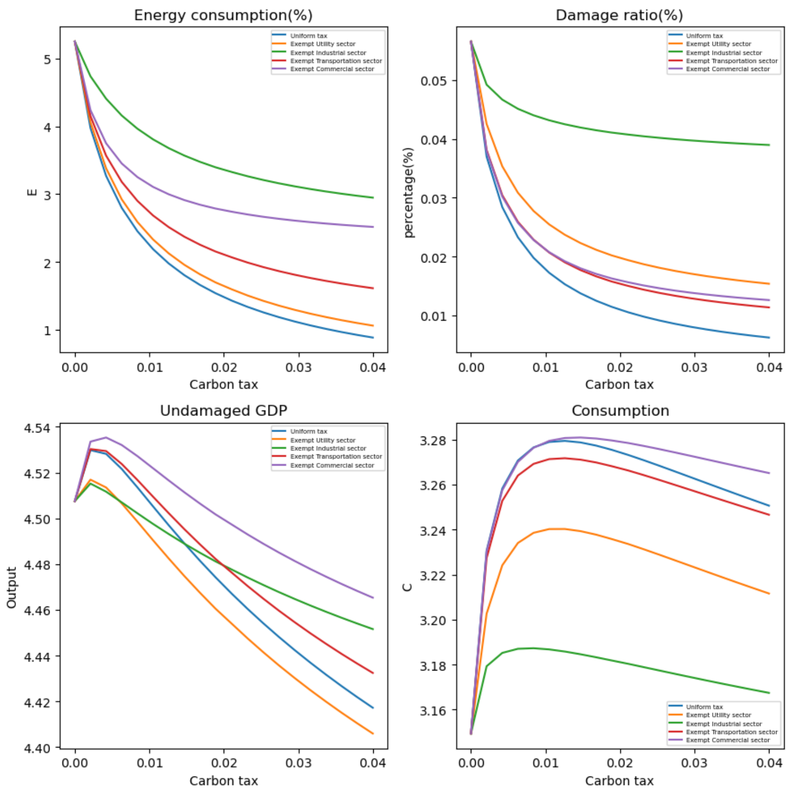

Figure 4.7: The change of aggregate variables of interest to carbon tax under the sectoral exemption scenarios

The lower half of Figure 4.7 illustrates how undamaged GDP and consumption respond to different exemption schemes as the carbon tax varies. The exempted sectors experience additional input expansion since they are shielded from carbon cost burdens. Notably, under the commercial sector exemption, more resources allocated to the com-

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mercial sector help boost undamaged GDP to levels higher than in the uniform tax case. Despite incurring slightly higher climate damages, the exemption still yields higher consumption at certain tax rates. However, the other exemption scenarios do not raise the undamaged GDP enough to offset the higher climate damages, resulting in a lower undamaged GDP and consumption compared to the uniform tax scenario. In conclusion, exempting low-productivity sectors from carbon taxation can lead to a misallocation of resources, thereby reducing overall production efficiency. Furthermore, if the exempted sector also contributes significantly to total carbon emissions, such exemptions not only weaken the effectiveness of emissions reduction but also amplify climate-related damages.

4.4 Exempting small emitters

In this section, we turn to another commonly adopted carbon tax policy. In practice, governments often exempt small emitters, typically small and medium-sized enterprises (SMEs), from carbon tax obligations to reduce their vulnerability to external shocks and adjustment costs. For example, as documented in the Kurz (2024) and International Energy Agency (2008), the European Union Emissions Trading System (EU ETS) exempts small producers whose annual emissions fall below 25,000 tons of CO_2 equivalent.

However, such exemptions can have heterogeneous effects across sectors due to differences in sectoral characteristics. Using our multi-sector heterogeneous firm model, we simulate the macroeconomic implications of this kind of policy by computing firm-level emissions based on productivity levels. Since productivity distributions, emission factors, and energy intensities differ across sectors, the proportion of firms emitting below the 25,000-ton CO_2-equivalent exemption threshold varies. To be more specific, 76.59%

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firms are exempted in industry, 97.36% in the commercial sector, 74.40% in transportation, and 77.69% in utilities. As a result, only the most productive firms in the commercial sector are subject to carbon taxation, whereas a much larger share of industrial, utility, and transportation firms bears the tax burden. This policy design thus implicitly redistributes tax liabilities across sectors based on firm-level emission profiles and sectoral structural characteristics.

Overall, approximately 92% of firms across all sectors benefit from the exemption. For comparative purposes, we also consider a counterfactual scenario where an equivalent 92% share of firms in each sector is exempted, which is the average ratio of exemption under the 25,000-ton CO_2-equivalent exemption scenario. This allows us to isolate the additional differential impacts arising specifically from the 25,000-ton exemption threshold in a heterogeneous multi-sector context.

4.4.1 Intra-sector resource reallocation

Figure 4.8 presents three distinct scenarios: uniform tax, equal portion exemption scenario, whose exemptions are granted to 92% of firms across all sectors, and 25,000-ton exemption scenario, whose exemptions target firms emitting less than 25,000 tons of CO_2-equivalent. For each scenario, we plot the distribution of market shares across firms with varying productivity levels at the optimal carbon tax rate, illustrating intra-sectoral resource reallocation. First, we can see that all sectors exhibit notable intra-sectoral resource redistribution under the equal exemption portion scenario.

Compared to the no-tax scenario, exempted small producers occupy a larger share of the market. This shift in market structure affects sectoral productivity, as estimated using

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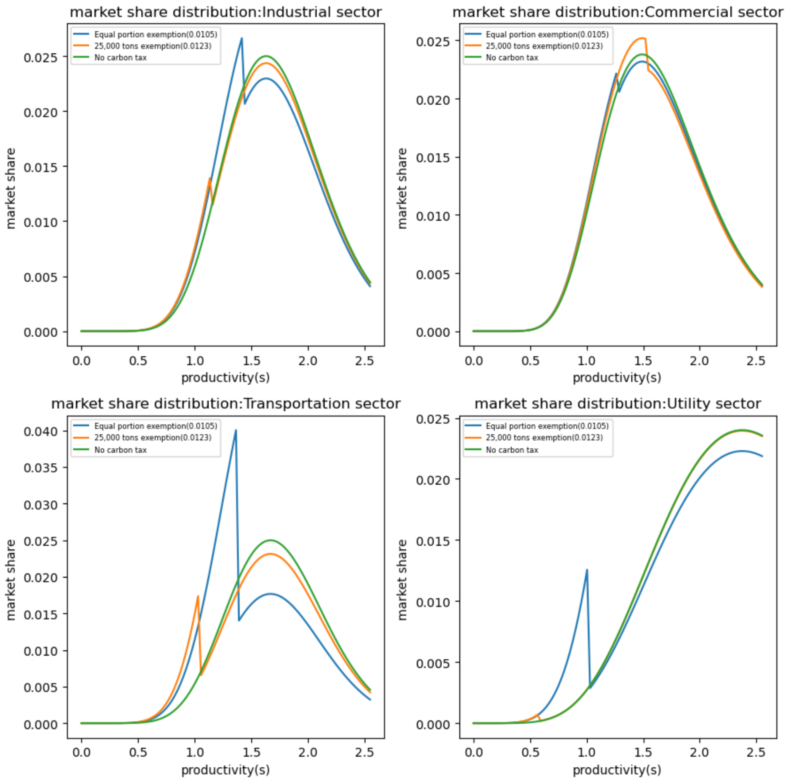

Figure 4.8: Intra-sectoral market share redistribution under exempting small emitters scenario

Equation 4.2. The results, reported in Table 4.6, show a decline in total factor productivity (TFP) across all sectors. The increased market share of smaller, less productive firms lowers the representative productivity of each sector. Notably, the commercial sector still retains the highest productivity level, it falls slightly to 1.8154 (−0.04%). Under the scenario in which emitters below 25,000 tons of CO_2 are exempted, the commercial sector —receiving a greater share of exemptions—experiences a further productivity decline to 1.8149 (−0.06%), while other sectors see marginal improvements. These observations underscore how carbon tax exemptions for small emitters can alter intra-sectoral market share distributions and resource allocations, ultimately influencing the representative productivity of each sector.

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	No carbon tax	Equal exemption portion	25,000-ton exemption
Industrial sector	1.6270	1.6203(-0.41%)	1.6229(-0.26%)
Commercial sector	1.8161	1.8154(-0.04%)	1.8149(-0.06%)
Transportation sector	1.2797	1.2655(-1.11%)	1.2737(-0.47%)
Utility sector	0.8506	0.8330(-2.07%)	0.8498(-0.09%)

Table 4.6: Sectoral productivity(TFP) across different small emitters exemption scenarios

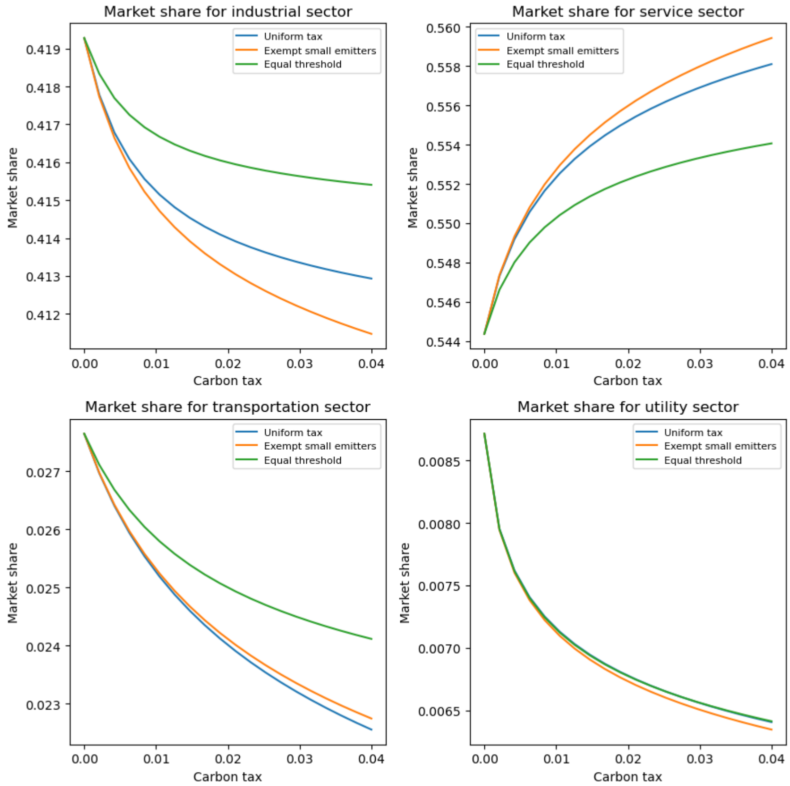

Figure 4.9: The sectoral market share change over carbon tax under the small emitters exemption scenario

4.4.2 inter-sector resource reallocation

Figure 4.9 illustrates the evolution of market shares across sectors under varying carbon tax policy scenarios. Focusing first on the comparison between the uniform carbon tax and the equal exemption portion scenario, the impact of the carbon tax on each sector attenuates since a portion of firms are exempted. As a result, market share shifts are less pronounced relative to the uniform tax case.

However, under the 25,000 tons exemption scenario, more firms in the commercial

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sector are exempted and fewer firms in other sectors are exempted compared with the equal exemption portion scenario. This asymmetry impact leads to a profound market share increase in the commercial sector that surpasses even the uniform tax scenario. Meanwhile, other high-emission sectors exhibit consistent declines in market shares since more firms are taxed, with the industrial sector suffering a more severe reduction than under the uniform tax scenario. Resource reallocation follows the patterns of sectoral market share adjustment driven by these mechanisms. Detailed results on cross-sectoral reallocation under these policy scenarios are provided in Appendix D.

Overall, exempting small emitters equally will lead to a less profound reallocation across sectors since only a portion of firms is under the scope of the carbon tax. However, If the exempting small emitters scenario includes heterogeneous impacts on sectors, It might end up leading to a more notable market structure change. Just like the 25,000 tons exemption scenario, distributing the tax burden unevenly.

4.4.3 Macroeconomic outcomes

The upper half of Figure 4.10 illustrates how energy consumption, and climate damage, change with carbon tax. The equal exemption portion scenario exhibits the least effective reduction in energy use. In contrast, the 25,000-ton exemption scenario achieves more substantial decreases in energy consumption. This difference arises because the 25,000-ton exemption scenario spares a smaller share of high-emission firms and a larger share of low-emission firms. Nonetheless, compared to the uniform carbon tax scenario, energy use, and climate damages remain elevated under both exemption policies.

The lower half of Figure 4.10 shows how undamaged GDP and consumption change

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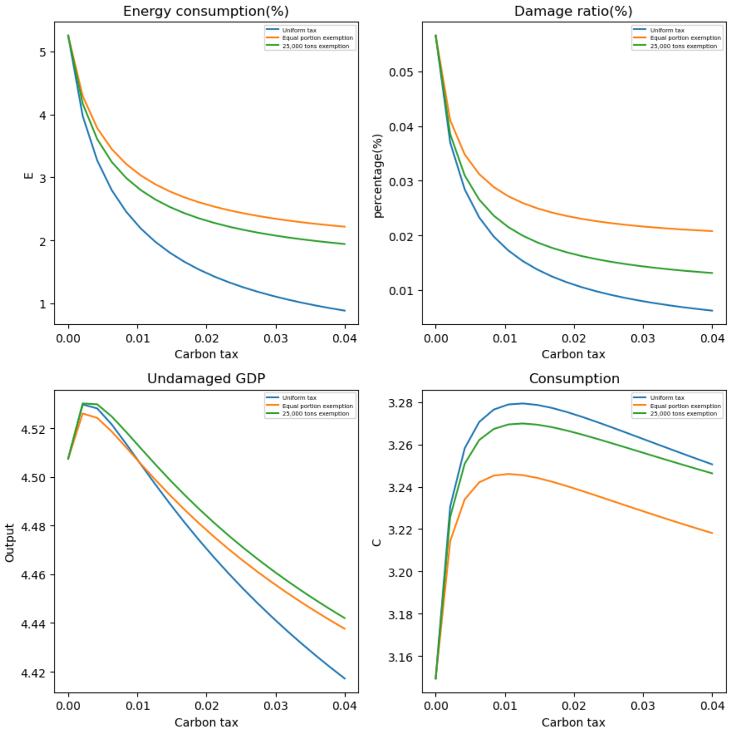

Figure 4.10: Aggregate variables of interest change to carbon tax under the small emitters exemption scenario

with carbon tax. Under the 25,000 tons exemption scenario, although the intra-sectoral reallocation lowers the sectoral productivity, the undamaged GDP still outperforms the uniform tax scenario. This is due to the notable resource reallocation to the commercial sector. The equal portion exemption scenario, on the other hand, does not have this profound reallocation effect, leading to smaller undamaged GDP at a low carbon tax rate. However, when accounting for increased climate damages and capital depreciation, aggregate consumption under these exemptions falls short of the uniform tax scenario.

4.5 Comparison

We have observed inverted U-shaped consumption curves across the various policy regimes introduced in the preceding sections, which allow us to define an optimal carbon tax rate that maximizes overall social welfare. In Table 4.7, we summarize the optimal

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	Industrial	Commercial	Transportation	Utility
No carbon tax	0.0	0.0	0.0	0.0
Uniform tax	0.0122	0.0122	0.0122	0.0122
Exempt Industrial sector	0.0	0.0047	0.0047	0.0047
Exempt Commercial sector	0.0116	0.0	0.0116	0.0116
Exempt Transportation sector	0.0101	0.0101	0.0	0.0101
Exempt Utility sector	0.0099	0.0099	0.0099	0.0
Exempt small emitters	0.0102	0.0102	0.0102	0.0102
Heterogeneous tax	0.0121	0.0013	0.0086	0.0136

Table 4.7: Sectoral tax rates under different scenarios

rates under different taxation regimes.

4.5.1 Comparison of the optimal level in different scenarios

	Carbon emissions	Damage ratio	Undamaged GDP	Consumption
Uniform tax	3.7782(-73.00%)	0.0156	4.4983	3.2794(+4.13%)
Heterogeneous tax	3.9997(-71.42%)	0.0165	4.5061	3.2823(+4.22%)
Exempt Commercial sector	4.4340(-68.32%)	0.0183	4.5126	3.2809(+4.18%)
Exempt Transportation sector	4.6884(-66.50%)	0.0193	4.5036	3.2717(+3.89%)
Exempt small emitters	4.8889(-65.07%)	0.0201	4.5060	3.2699(+3.83%)
Exempt Utility sector	5.9694(-57.35%)	0.0245	4.4863	3.2404(+2.89%)
Exempt Industrial sector	10.8882(-22.20%)	0.0443	4.5034	3.1873(+1.21%)
No carbon tax	13.9953	0.0565	4.5075	3.1493

Table 4.8: Comparison of optimal condition under the carbon tax scenarios

Table 4.8 summarizes the macroeconomic outcomes under each policy scenario. We report not only absolute values for consumption and emissions but also percentage changes in consumption and emissions relative to the no-tax scenario. Notably, the uniform carbon tax achieves the most effective emissions reduction and brings climate damage to the lowest level. However, it also imposes the greatest drag on undamaged GDP. In contrast, the heterogeneous tax regime delivers a relatively higher level of undamaged GDP. Although this regime results in higher emissions and thus greater climate damages, it yields higher aggregate consumption than the uniform tax.

A similar pattern is observed under the commercial sector exemption. Despite higher

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emissions, the elevated undamaged GDP offsets part of the climate damage, leading to higher consumption relative to the uniform tax case, though still below the level under the heterogeneous tax. Other exemption policies might or might not lead to higher levels of undamaged GDP. However, the resulting increase in climate damages offsets these gains, ultimately preventing any improvement in aggregate consumption greater than the uniform tax scenario, but does bring different levels of improvement compared to the no carbon tax scenario.

Other

In conclusion, these results underscore the value of implementing carbon pricing: every scenario analyzed in this study contributes to emissions mitigation and welfare improvement relative to inaction. Also, exempting or taxing less on less pollutive and more productive sectors can reallocate the resources to the sectors. This boosts the output while maintaining the main control of carbon emissions, leading to higher welfare levels. This highlights the urgency of carefully designing and implementing a carbon tax to achieve more meaningful environmental and economic outcomes.

4.5.2 Comparison of achieving certain emission goal

	Transportation	**Industrial**	**Utility**	**Commercial**
uniform tax	0.0122	0.0122	0.0122	0.0122
Exempt Transportation sector	0.0	0.0194	0.0194	0.0194
Exempt Industrial sector	1.0	0.0	1.0	1.0
Exempt Utility sector	0.0384	0.0384	0.0	0.0384
Exempt Commercial sector	0.0212	0.0212	0.0212	0.0
Exempting small emitters	0.0235	0.0235	0.0235	0.0235

Table 4.9: sectoral tax burden to achieve emission goal($G = 3.7781$)

This study also investigates which carbon tax policy can achieve the emissions re-

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duction target at the lowest welfare cost. We constrain the maximum allowable tax rate at $\tau = 1$ and set the emissions reduction target as the carbon emission level ($G = 3.7781$) attained under the optimal uniform tax scenario. Tables 4.9 report the effective tax burdens across sectors under this target-based comparison.

	Carbon emissions	**Damage ratio**	**final good**	**Consumption**
uniform tax	3.7782	0.0156	4.4983	3.2794
Exempt Commercial sector	3.7782	0.0156	4.4969	3.2784
Exempt Transportation sector	3.7782	0.0156	4.4809	3.2678
Exempting small emitters	3.7782	0.0156	4.4752	3.2625
Exempt Utility sector	3.7782	0.0156	4.4093	3.2134
Exempt Industrial sector	8.8468	0.0361	4.2710	3.0462

Table 4.10: Comparison of economic performance to achieve emissions goal($G = 3.7783$)

Tables 4.10 show the associated macroeconomic outcomes under this target-based comparison. A key finding is that in the scenario where the industrial sector is exempted, the emissions level remains above the target despite imposing the tax ceiling of $\tau = 1$. This implies that too large a share of emissions escapes the scope of the carbon tax, making the policy ineffective in reaching the desired target.

Additionally, in all exemption scenarios, including those targeting specific sectors or small emitters, achieving the same emissions target requires a significantly higher tax rate on unexempted sectors or firms compared to the uniform baseline. These elevated tax levels strongly depress undamaged GDP across all alternative scenarios. Given that climate damages are fixed at the same level by construction, the resulting higher tax burden and lower undamaged GDP translate into lower consumption compared to the uniform tax baseline.

In conclusion, These findings underscore the efficiency of a broader tax base in achieving carbon emissions goals and highlight the welfare costs associated with narrowing the tax base.

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Chapter 5 Conclusion

This thesis quantitatively evaluates the macroeconomic and environmental effects of carbon taxation under multiple policy scenarios in a four-sector general equilibrium framework with heterogeneous firms. A key contribution of the study lies in its systematic comparison of four different carbon tax designs: uniform taxation, sectoral exemption, small emitters exemption, and heterogeneous taxation—and in identifying the policy that best balances emission reductions, efficiency, and welfare outcomes.

Our analysis finds that a uniform carbon tax at a rate of 0.0122 achieves a 73% reduction in carbon emissions, lowers the climate damage ratio from 5.65% to 1.56% and raises steady-state consumption by 4.13% compared to the no-tax baseline. This configuration delivers the most effective emissions mitigation among all scenarios, though it imposes the largest contraction in undamaged GDP due to broad-based production cost increases.

In contrast, heterogeneous taxation delivers the highest welfare gain among all policy designs. Under this regime, the utility sector, being the most carbon-intensive, faces a tax of 0.0136, whereas the low-emission commercial sector faces only 0.0013. This tailored approach aligns the marginal private cost of emissions with the marginal social damage across sectors, resulting in a slightly lower emissions reduction (71.4%), but a marginally higher consumption improvement of 4.22% due to a higher undamaged GDP compared

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with the uniform tax scenario.

Partial exemption policies, such as exempting a single sector or small emitters, substantially weaken mitigation outcomes and introduce stronger inter-sectoral distortions. For example: Exempting the industrial sector results in only a 22.2% emission reduction and the smallest consumption gain (+1.21%) among all cases. Exempting the commercial sector, by contrast, retains much of the tax base, delivering a 68.3% emission reduction and a relatively strong consumption gain (+4.18%) that even outperforms the uniform tax scenario. The small emitters exemption, though politically expedient, leads to uneven burden sharing: over 97% of firms in the commercial sector are exempt, while only 76% are exempt in the industry sector. This structural asymmetry causes hidden redistributions of tax burdens and lowers the net consumption gain (+3.83%), with emissions falling only by 65.1%.

A further efficiency-equivalent comparison reinforces these findings. When holding the emissions target constant (G = 3.7782), all exemption-based regimes require higher tax rates in non-exempt sectors to achieve the same reduction, which results in lower undamaged GDP and consumption. For instance, in the small emitter exemption scenario, the required tax rate rises from 0.0201 to 0.1032 across taxed firms, yet consumption falls to 3.2625, below the uniform tax outcome of 3.2794.

These results yield several clear policy implications: First, the commercial sector consistently benefits and takes greater market share across most scenarios due to its low emission factor, which emphasizes the carbon tax potential of changing the industrial structure. Second, full tax coverage is essential to maximize emissions reduction. Third, heterogeneous carbon taxation is not only theoretically optimal but also practically supe-

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rior in welfare terms. Lastly, the size-based exemptions create heterogeneous impacts on different sectors. Fourth, while sectoral or size-based exemptions may relieve localized burdens, they weaken environmental effectiveness and reduce the aggregate efficiency of carbon pricing. Furthermore, partial exemption might lead to potentially costly reallocation distortions.

In conclusion, this thesis provides robust quantitative evidence of different common carbon tax practices and delivers the pros and cons of each scenario. It highlights the urgency of carefully designing and implementing a carbon tax to achieve more meaningful environmental and economic outcomes. Future extensions could explore transitional dynamics, endogenous technology adoption, or political economy constraints to better inform feasible and effective climate policy in complex economies.

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Appendix A — Steady-state damage function

Notably, the term ϕ_L does not appear in the expression of the steady-state damage function. This omission is intentional to ensure the existence of a steady state. If ϕ_L were not zero, the CO_2 concentration would continuously increase as long as the energy is still being used, preventing the existence of the steady state.

At the steady state, S_t and G_t must maintain the same constant level, denoted as S and G. Under these steady-state conditions, Equation 2.7 can be revised as:

$$\begin{aligned} S - \overline{S} &= \sum_{s=0}^{\infty}(1-d_s)G^w \\ &= \sum_{s=0}^{\infty}\phi_0(1-\phi)^s\frac{G}{\theta} \\ &= \frac{1}{(1-(1-\phi))}\frac{\phi_0 G}{\theta} = \frac{\phi_0}{\phi}\frac{G}{\theta} \end{aligned} \tag{A.1}$$

Therefore, the steady-state damage function is:

$$\begin{aligned} D(G) &= 1 - e^{-\gamma(S_t-\overline{S})} \\ &= 1 - e^{-\gamma\frac{\phi_0}{\phi}\frac{G}{\theta}} \end{aligned} \tag{A.2}$$

We can see that it only relates to the steady-state level of CO_2 emission within the EU.

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Appendix B — The aggregation

To analyze the macroeconomic implications of sectoral production, it is necessary to aggregate firm-level outcomes within each sector. Since firms exhibit heterogeneity in productivity, we derive aggregate sectoral output and resource demand by integrating across the firm distribution.

Based on the log-normal productivity distribution assumption. Sector l have a productivity distribution described by $\mu_l(s)$, which follows the following formula:

$$\mu_l(s) = M_l \hat{\mu}_l(s) \tag{B.3}$$

where M_l represents the total number of firms producing in the market. $\hat{\mu}(s)$ represents the distribution with the total mass of firms in the market as 1, following the log-normal distribution:

$$\hat{\mu}_l(s) = \frac{1}{s\sigma_l\sqrt{2\pi}} e^{-\frac{(\ln s - \mu_l)^2}{2\sigma_l^2}} \tag{B.4}$$

Therefore, the aggregate sectoral labor demand is determined by summing the labor demand of all firms in sector l:

$$N_{l,t} = \int_0^\infty \overline{n_{l,t}}(s)\mu_l(s)ds \tag{B.5}$$

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We then can express aggregate sectoral output as:

$$\begin{aligned}
N_{l,t}^{d} &= \int_{0}^{\infty} \overline{n_{l,t}} \mu_l(s)\, ds \\
&= \int_{0}^{\infty} s^{\frac{1}{1-\alpha_l-\beta_l-\gamma_l}} C_l^n M_l \hat{\mu}_l(s)\, ds \\
&= M_l C_l^n \left\{ \int_{0}^{\infty} \frac{1}{s\sigma_l\sqrt{2\pi}} s^{\frac{1}{1-\alpha_l-\beta_l-\gamma_l}} e^{-\frac{(\ln s-\mu_l)^2}{2\sigma_l^2}}\, ds \right\} \\
&= M_l e^{\frac{(\mu_l')^2-\mu_l^2}{2\sigma_l^2}} C_l^n \\
&= \tilde{s}_l^{\frac{1}{1-\alpha_l-\beta_l-\gamma_l}} C_l^n
\end{aligned} \tag{B.6}$$

Here, μ_l and σ_l denote the mean and the standard deviation of $\ln(s)$. μ_l' is an abbreviation for $\frac{(1-\alpha_l-\beta_l-\gamma_l)\mu_l+\sigma_l^2}{1-\alpha_l-\beta_l-\gamma_l}$. $N_{l,t}^d$ represents the total labor demand for sector l in period t. Lastly, $\tilde{s}_l$ denotes the aggregate productivity, which follows the functional form:

$$\tilde{s}_l = \left(M_l e^{\frac{(\mu_l')^2-\mu_l^2}{2\sigma_l^2}} \right)^{1-\alpha_l-\beta_l-\gamma_l} \tag{B.7}$$

Therefore, we can view the sectoral output and resource demand as being provided or demanded by a representative firm with aggregate productivity $\tilde{s}_l$, which is proportional to the mass of the sector M_l and related to the parameter controlling the productivity distribution in the sector.

To attain $K_{l,t}^d$, $E_{l,t}^d$, and $Y_{l,t}$, we can also implement the same trick on the following equation.

$$Y_{l,t} = \int_{0}^{\infty} s_l \overline{n_{l,t}}(s)^{\alpha_l} \overline{k_{l,t}}(s)^{\beta_l} \overline{e_{l,t}}(s)^{\gamma_l} \mu_l(s) ds \tag{B.8}$$

$$K_{l,t}^d = \int_{0}^{\infty} \overline{k_{l,t}}(s) \mu_l(s) ds \tag{B.9}$$

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$$E_{l,t}^{d} = \int_{0}^{\infty} \overline{e_{l,t}}(s)\mu_l(s)ds \tag{B.10}$$

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Appendix C — The sector mapping

To better match the various datasets, especially energy-related datasets, used during calibration. The mapping from the NACE Rev. 2 to the energy end-use sectors is necessary. According to Eurostat, the Industrial sector corresponds to sections B, C, and F in NACE Rev. 2. Sections H and D in NACE Rev. 2 correspond to the transportation-related industry and the utility-related industry, so it is straightforward to assign them to the transportation and utility sectors in the end-use sector definition in the energy-related dataset, respectively. The remaining industries are assigned to the Commercial sectors. In Table C.4, we can observe that all the industries are service-related.

Description	**NACE sections**
Mining and quarrying	B
Manufacturing	C
Construction	F

Table C.1: Industrial Sector Descriptions and correspondence in NACE Rev. 2

Description	**NACE sections**
Transportation	H

Table C.2: Transportation Sector Descriptions and correspondence in NACE Rev. 2

Description	**NACE sections**
Utilities	D

Table C.3: Utility Sector Descriptions and correspondence in NACE Rev. 2

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Description	**NACE sections**
Water supply; sewerage, waste management, and remediation activities	E
Wholesale and retail trade; repair of motor vehicles and motorcycles	G
Accommodation and food service activities	I
Information and communication	J
Financial and insurance activities	K
Real estate activities	L
Professional, scientific, and technical activities	M
Administrative and support service activities	N
Public administration and defense; compulsory social security	O
Education	P
Human health and social work activities	Q
Arts, entertainment and recreation	R
Other service activities	S

Table C.4: Commercial Sector Descriptions and correspondence in NACE Rev. 2

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Appendix D — The graph of resource reallocation

Since the changes in the input share show a similar trend as the market share change, we put a detailed plot of how the labor, capital, and energy share change under different policy scenarios in this section to avoid repetition in the content.

D.1 The uniform tax

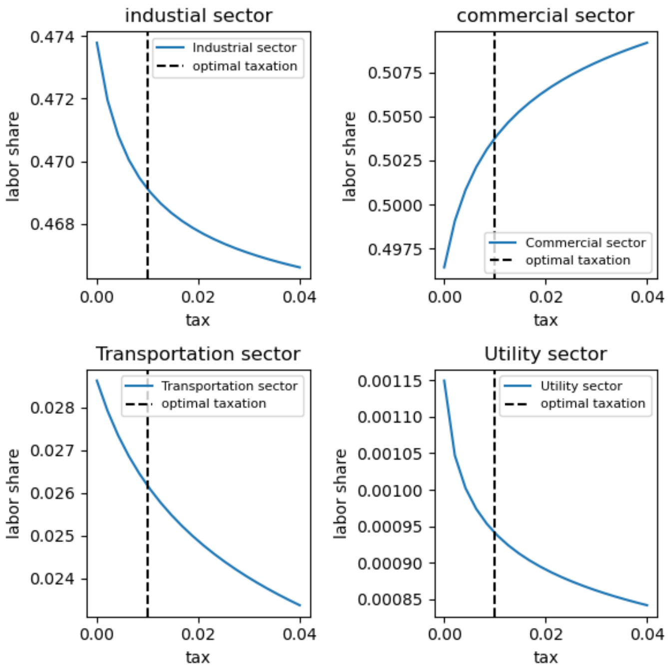

Figure D.1: The sectoral labor share change over the carbon tax under a uniform tax

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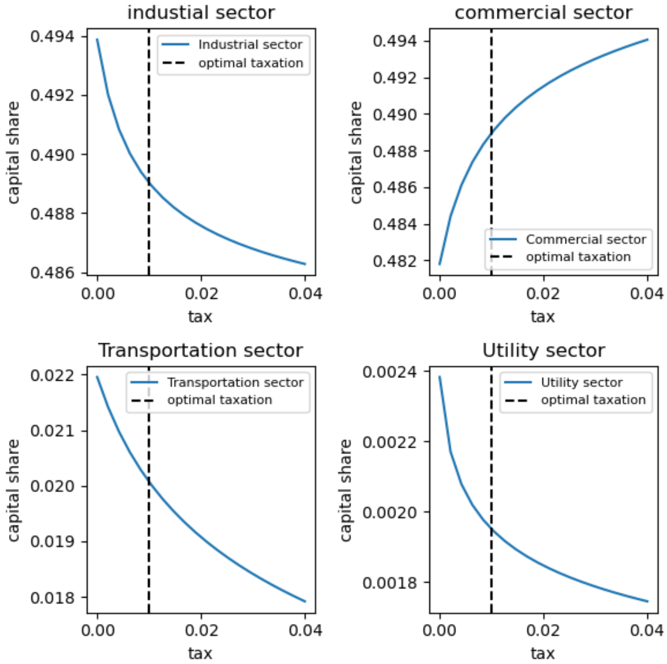

Figure D.2: The sectoral capital share change over the carbon tax under a uniform tax

D.2 The sectoral exemption scenarios

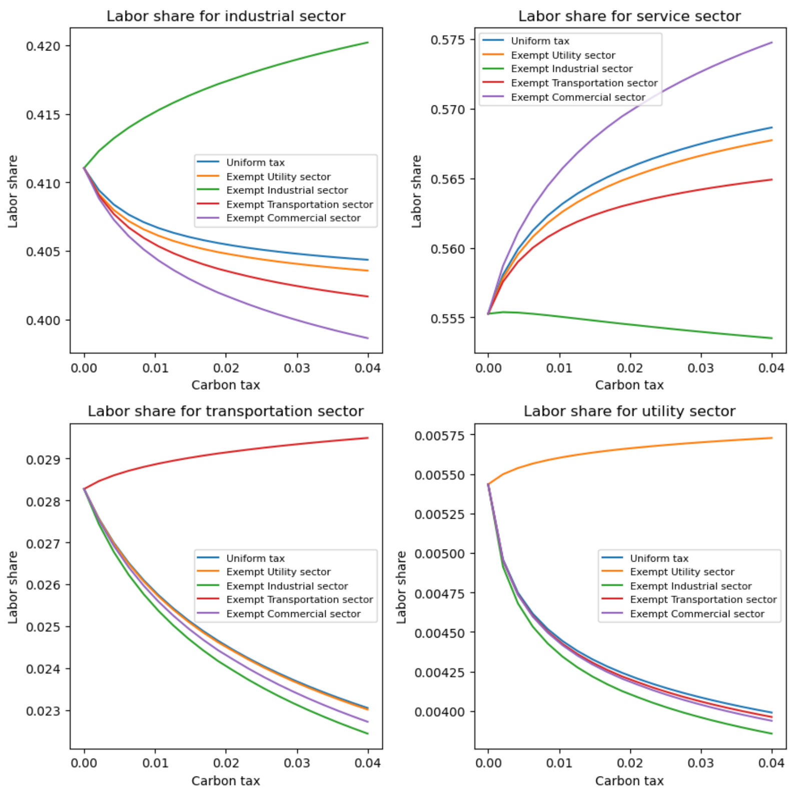

Figure D.3: The sectoral labor share change over the carbon tax under sectoral exemption

doi:10.6342/NTU202500782

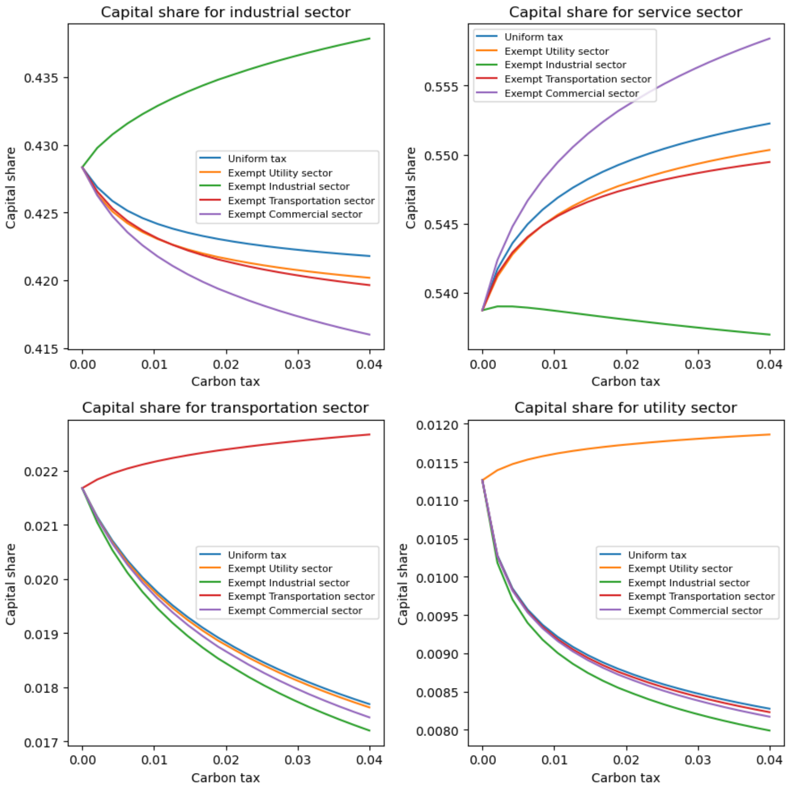

Figure D.4: The sectoral capital share change over the carbon tax under the sectoral exemption

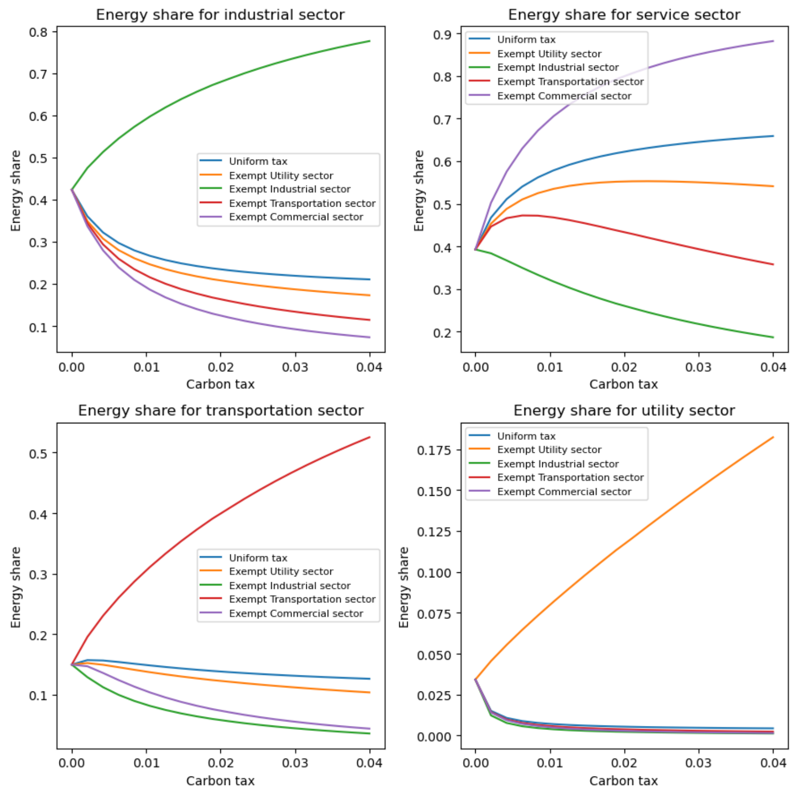

Figure D.5: The sectoral energy share change over the carbon tax under the sectoral exemption

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D.3 The small emitters exemption scenario

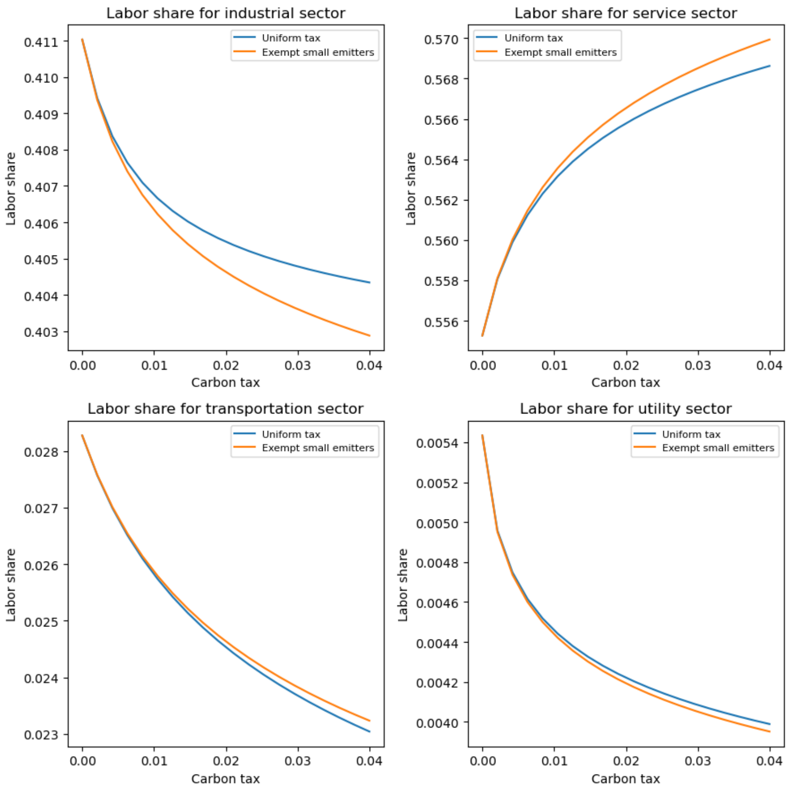

Figure D.6: The sectoral labor share change over the carbon tax under the small emitters exemption

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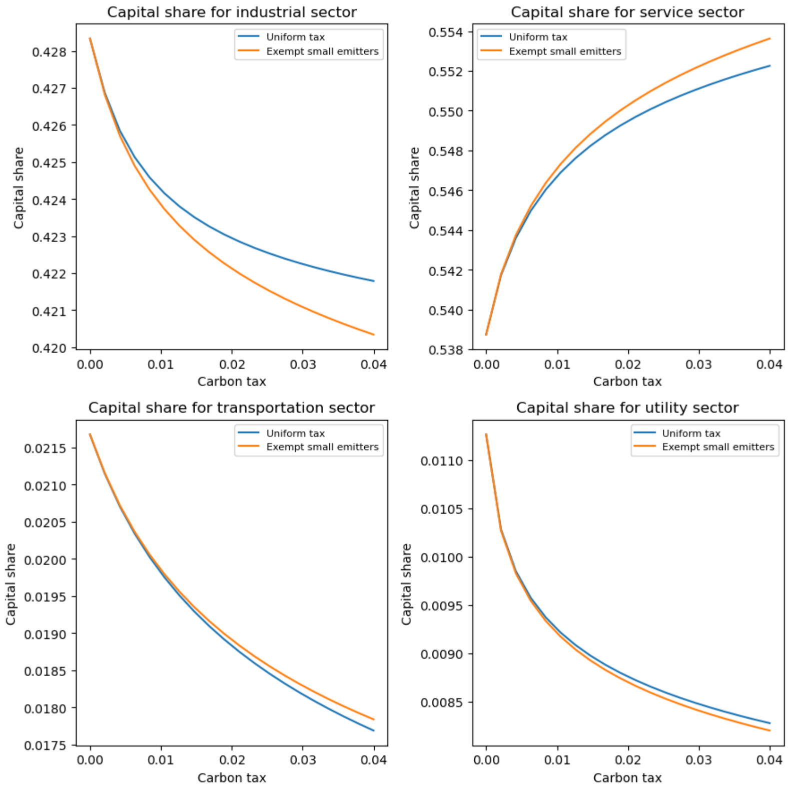

Figure D.7: The sectoral capital share change over the carbon tax under the small emitters exemption

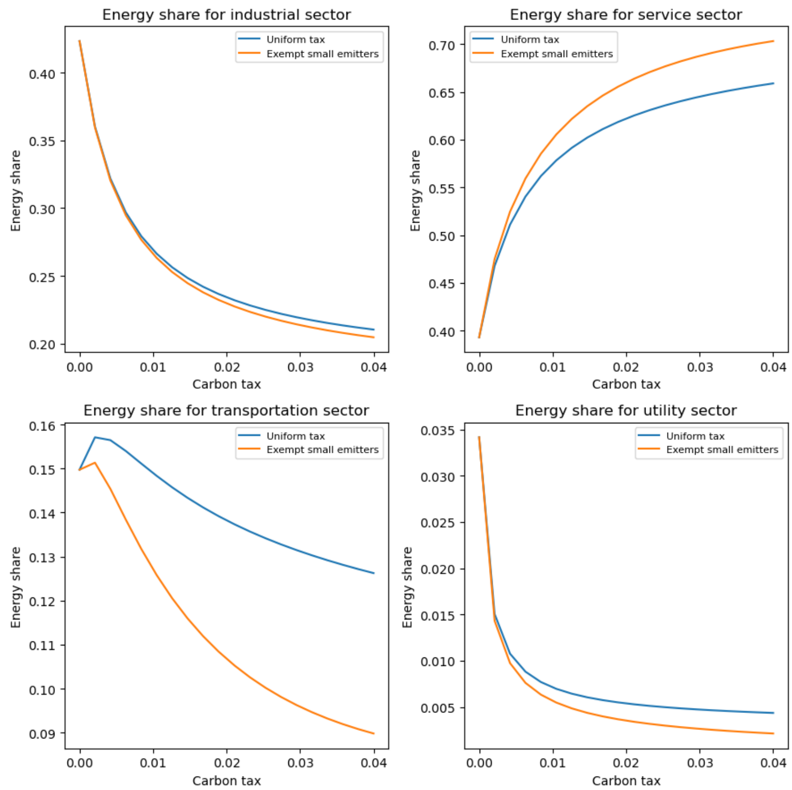

Figure D.8: The sectoral energy share change over the carbon tax under the small emitters exemption

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