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個體經濟學三篇論文

Three Essays on Microeconomics

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個體經濟學三篇論文
Three Essays on Microeconomics

本論文係劉藍一君（學號 D04323003）在國立臺灣大學經濟學系完成之博士學位論文，於民國 113 年 06 月 24 日承下列考試委員審查通過及口試及格，特此證明

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摘要

本論文涵蓋三項個體經濟學議題：不完全資訊中國餐館賽局、獨裁者的競租資源分配決策，以及公平排隊問題中瓦拉斯分配規則的公設刻畫。第一篇文章題目為「附加排隊規則的中國餐館賽局中的阻塞現象和資訊品質探討」，本篇提出一個不完全資訊非合作賽局，並探討玩家的貝氏學習如何影響其決定。本篇文章探討中國餐館排隊問題中，資訊壅塞與訊號品質的抵換關係。餐館排隊的玩家擁有不同的事前資訊，且玩家可以選擇立刻進餐廳或等待並觀察其他玩家的訊號後才進餐廳。當多數玩家同時選擇進餐廳或者多數選擇延遲都會導致壅塞。理想上，我們預期玩家們會釋出餐廳的資訊以增進社會決策效率，但我們證明：事前擁有高品質資訊的玩家沒有動機釋出其資訊給事前擁有低品質資訊的玩家，而社會效率的排隊結果則不是均衡結果。

第二篇文章題目為「生產、衝突與獨裁者的最適偏袒行為」，本篇提出一個包括一名政策制定者及一群玩家的競賽架構，探討政策制定者的偏袒如何影響玩家投入的努力水準。本篇文章建立一個兩階段、多項獎勵競賽，其中玩家於第二階段選擇其生產性及浪費性資源投入數量，在第一階段，領導人透過調整相對權重以扭曲玩家的浪費性投入贏得獎勵的機率。我們發現當獎勵數量較多時，領導人有較強誘因鼓勵玩家投入浪費性資源，若玩家數量很多，領導人也會鼓勵玩家投入浪費性資源。我們發現在多數情況，領導人都會鼓勵玩家多投入浪費性資源，這解釋了常見的「錫鍋式」專制統治下的貪腐行為，以及公司治理中的馬屁文化。

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第三篇文章題目為「具備初始稟賦之排隊問題的瓦拉斯分配規則」，本篇以合作賽局途徑定義一個解規則，並進行公設刻畫。本篇文章探討在具有初始稟賦設定的公平排隊問題下之瓦拉斯均衡概念。在本文模型中，每位決策者均持有初始入場排隊位置，並可透過金錢補償，與其他決策者交換位置到排序較前的位置。本文定義了一種新的分配規則，其結果與瓦拉斯均衡對應的分配結果相同，且此分配規則可以被無嫉妒公理唯一刻畫。

關鍵字：排隊問題，壅塞，資訊品質，負外部性，策略性延遲；多獎品競賽，生產性努力，獨裁制，貪腐，偏袒；公平排隊問題，瓦拉斯均衡，無嫉妒性質，初始稟賦

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Abstract

This dissertation consists of three microeconomics topics: the Chinese restaurant game with incomplete information, the dictator's rent allocation decision, and an axiomatic characterization of Walras allocation rules in the fair queuing problems.

Chapter 2 is titled "Congestion and Information Quality in Chinese Restaurant Games with Priority Rules." This chapter proposes a non-cooperative game with incomplete information and explores how players' Bayesian learning affects their decisions and final social distribution. We explore the trade-off between congestion and signal quality in a Chinese restaurant queuing situation. Players are heterogeneous on the information of the true distribution; players choose entry immediately or wait and see how information goes. Too many players choosing to delay decisions will cause congestion. Ideally, we expect players holding critical information to reveal it to the public to improve social efficiency. We show that players with high-quality signals have no incentive to disclose their information to players with low-quality signals, and the efficient queuing outcome is not

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achieved.

Chapter 3 is titled “Production, Conflict, and Dictator’s Optimal Favoritism.” This chapter proposes a competition structure including a policy maker and a group of players to explore how the policy maker’s favoritism affects the level of effort invested by the players. We consider a two-stage multi-prize contest in which players choose a productive-wasteful effort allocation in Stage 2; in Stage 1, a ruler chooses a weight to enlarge the players’ winning probability of investing unproductive effort. We find that the productive effort investment level is almost zero whenever the number of prizes is small. When the number of prizes is high, the ruler has a stronger incentive to encourage the players to invest in wasteful efforts. The ruler also encourages the players to invest in wasteful efforts as the player group size increases. We find that in most situations the ruler encourages the contestants to invest in wasteful efforts, which explains the tinpot-like autocrats’ bribery behavior and company managers’ favoritism.

Chapter 4 is “The Walrasian Rule in Fair Queuing Problems With an Initial Order.” This chapter defines a new allocation rule and characterizes the allocation rule by an axiomatic approach. We focus on analyzing the Walrasian equilibrium in the context of fair queuing problems with initial order. Each agent, occupying a particular queuing position, seeks to obtain a former position and is permitted to exchange positions with other agents

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by paying a monetary compensation. We define an allocation rule in which the outcome coincides with the Walrasian equilibrium and show that the rule is characterized by the envy-free axiom.

Keywords: Queuing problems, congestion, information quality, negative externality, strategic delay; multiple-prize contests, productive effort, dictatorship, corruption, favoritism; fair queuing problems, Walrasian equilibrium, envy-free, initial endowment

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Chapter 1 Overview

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This thesis consists of three microeconomics essays: the Chinese restaurant game with incomplete information, the dictator's rent allocation decision, and an axiomatic characterization of Walrasian allocation rule in the fair queuing problems.

The first essay is titled "Congestion and Information Quality in Chinese Restaurant Games with Priority Rules", listed in Chapter 2, proposes a non-cooperative game with incomplete information and explores how players' Bayesian learning affects their decisions and final social distribution.

In this chapter, a Chinese restaurant refers to a queuing problem with multiple servers that provide different resources or service capacities. We explore the trade-off between congestion and signal quality in a Chinese restaurant queuing situation. The players are heterogeneous on the information of the true distribution, and the players choose entry immediately or wait and see how information goes. The players were notified of the priority queuing rules suggested by the system or social planner, and the players must choose whether to enter the restaurant immediately or choose a delayed option under this suggestion. However, too many players choosing to delay decisions will cause congestion. Ideally, we expect players holding critical information to reveal it to the public to improve social efficiency.

In the queuing problem with incomplete information, efficiency, or information efficiency, has a different meaning from what is generally called economic efficiency. In this chapter, efficiency means that the social planner hopes that the information learned by high-quality signal type players (strong type) and low-quality signal type players (weak type) will be similar in quality at the end of the game. In other words, efficiency means the

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expected payoff gap between strong type and weak type is minimized. Under the structure of cooperative games, the information efficiency mentioned in this chapter is closer to the concept of fairness.

We show that strong type players have no incentive to disclose their information to weak type players, and the efficient queuing outcome is not achieved. Our simulation results with different numbers of people also support this result.

The second essay is titled "Production, Conflict, and Dictator's Optimal Favoritism," listed in Chapter 3. In this chapter, we propose a competition structure including a policy maker (ruler) and a group of contestants (population) to explore how the policy maker's favoritism affects the level of effort invested by the contestants.

We consider a two-stage multi-prize contest in which players choose a productive-wasteful effort allocation in Stage 2; in Stage 1, a ruler chooses a weight to enlarge the players' winning probability of investing unproductive effort. We find that the productive effort investment level is almost zero whenever the number of prizes is small, and when the number of prizes is high, the ruler has a stronger incentive to encourage the players to invest in wasteful efforts. The ruler also encourages the players to invest in wasteful efforts as the player group size increases. Unfortunately, we find that in most situations the ruler encourages the contestants to invest in wasteful efforts, which explains the tinpot-like autocrats' bribery behavior and company managers' favoritism.

The title of the third essay is "The Walrasian Rule in Fair Queuing Problems With an Initial Order", listed in Chapter 4, constructs a new allocation rule and characterizes the allocation rule by an axiomatic approach.

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We focus on analyzing the Walrasian equilibrium in the context of fair queuing problems with initial order. Each agent, occupying a particular queuing position, seeks to obtain a former position and is permitted to exchange positions with other agents by paying a monetary compensation. We define an allocation rule in which the outcome coincides with the Walrasian equilibrium and show that the rule is characterized by the envy-free axiom.

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Chapter 2 Congestion and Information Quality in Sequential Chinese Restaurant Queuing Games with Priority Rules

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2.1 Introduction

We propose a model to exhibit the trade-off between signal quality and first-mover advantage in queuing problems with various system priority queuing rules. We also investigate how a strategic delay in sequential decisions is shaped by priority queuing rules, congestion, and incomplete information.

Strategic delay detracts from efficiency. Such a kind of delay can be found in the vaccination during pandemics, job hunting, and in many decisions in real life. Waiting in line and choosing when to claim a resource share is a common interaction in daily life, but most queuing orders are not efficient. The reason is that most queuing systems apply a first-come, first-served mechanism instead of letting the good information person gets the service first. Inefficiencies arise because players do not have enough information about target resource distribution and are affected by congestion externalities. The congestion is defined as a population externality in the player's utility. When the number of people lining up for service, the externality becomes greater.

To solve the population externality, the social planner should rearrange the initial queuing order to an efficient allocation. Queuing order refers to the relative waiting positions in a queue. In the incomplete information situation, the efficiency queue refers to letting people with more information choose first. People in the latter position can observe the choices of the former and promote the quality of the latter's choice. In a decentralized situation, a coercive social planner may not exist, we thus explore the case of queuing rules as a non-mandatory recommendation.

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Our model captures delay behavior as a strategic choice for players. To explain the cause of strategic delay and its influence on efficiency, we introduce a Chinese restaurant game to be our analytic framework, which is proposed by Wang et al.[77] of social coordination problems under asymmetric information.

To simplify, we consider a two-stage queuing game containing two independent servers of different sizes. In the first stage, each player simultaneously chooses to join a server immediately or to join later. In the second stage, according to the queue determined by the previous stage, players successively receive services, reveal their private information, and settle payoffs based on the total number of participants on the server.

The true server size distribution is unknown; however, this game contains two types of players: the player who owns more information about the server sizes is called a strong signal type; otherwise, it is called a weak type. The difference between strong type and weak type is the probability of state distribution consisting in their private signals. Players make a server decision sequentially; after one player makes the decision, the choice and player's type are observed by the followers. Therefore, if the strong type player makes a choice first, the follower can update the information about the true distribution to improve the quality of the decision.

Rearranging the order of players can improve the queuing efficiency. In other words, a social planner can propose a queuing rule; moreover, each rule recommends which type of player can serve first through their types. We call this rule the priority rule. A priority rule is a function assigning queuing positions to different types of players. We study the reaction of different types of players to various priority rules. A rule prioritizing strong-

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type players to choose servers immediately and then letting weak-type players delay server selection decisions is called an efficient rule. This rule specifies the strong type to be served first and the weak type to be served later.

Our model can be applied to queuing decisions with incomplete information in daily life, especially when there are policy goals that need to be implemented. For example, delay in vaccination caused an outbreak during the SARS-CoV-2 (COVID-19) epidemic in several countries. Although the vaccine is the ultimate solution to infectious disease, the vaccination speed is significantly slower than expected in many developed countries. Schmidt[56, 57] indicate that the slow speed of vaccination is the inefficiency of prioritization. We provide a further explanation for this problem; that is, citizens face the priority orders for vaccination recommended by the government, and they strategically postpone vaccination. Vaccination is a voluntary activity in most countries. Some countries classify medical staff as a priority group for a vaccine does, but some people choose to postpone vaccination, which is a strategic action performed by people who know more about vaccine information when the domestic epidemic is not severe. This could be explained by the fear of possible side effects brought by the vaccine. On the other hand, after the domestic epidemic broke out, some countries encountered vaccine shortages. The shortage comes from the congestion because most people were in queueing for vaccination regardless of whether they were in or not in the priority group. This indicates that when the epidemic becomes serious, no matter whether people are fully aware of the vaccine's side effects, having the vaccine becomes the first choice for all people. We model a trade-off between congestion and signal quality, which can explain the strategic factors of the delay for the vaccine does and why some governments' vaccination policies fail. Nevertheless, when

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the safety of the vaccine is unclear, distributing a safe SARS-CoV-2 vaccine efficiently has garnered immense citizens' interest. People will delay vaccination decision-making, which leads to the congestion effect. Ideally, we expect or request people holding critical information, such as the knowledgeable estimation on the safety of the vaccine from the medical perspective, to reveal it to the public to improve social welfare, although those people may strategically postpone the disclosure of such information. Our results also explain how the proposed model explains the low vaccination rates of many countries when priority-based vaccination policy is applied without enforcement.

In our model, agents choose whether to have a vaccine immediately and then make their vaccine station selection sequentially, concerning the order determined by the agents. The distribution of vaccine storage among two vaccine stations is unknown for agents. Still, the agents can observe the former agents' choices and their signals to update the beliefs of true state distribution. Selecting the vaccine station earlier leads to a higher probability of receiving the preferred vaccine brand, and postponing the selection increases the probability of choosing a station with the rich vaccine. Our model can be applied to problems such as queuing for location selection of new industrial zones for firms, and provides some intuition about the trade-off between information quality and first-mover advantage.

In literature, the structure we applied is called Chinese restaurant game (CRG, Wang et al.[76]). The CRG describes the strategic interaction structure with the setting of players choosing the set of resources simultaneously. The further extend the strategic-form Chinese restaurant game into the sequential setting, which is called the sequential Chinese restaurant game (SCRG, Wang et al.[77]). Jiang et al.[39] further studied the situation of how a user learns the uncertain network state in a dynamic social learning setting. Kung

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and Wang[44] explores the decision order problem in a two-person SCRG with one specific player who is permitted to deviate from the default decision order. The authors also find that the equilibrium strategy depends on signal quality and the relative resource size distribution. Nevertheless, in the structure of resource selection with congestion, none of the above works studied the strategic behavior when choosing a queuing position is permitted.

The related literature consists of issues of social coordination with externality, rational queuing, and strategic delay. Firstly, our model is related to social coordination games with network externality. A negative externality comes from the congestion effect whenever many people choose the same common facility. Some of the articles addressing this problem include Bala and Goyal[11], Fagiolo[30], and Jackson and Watts[38]. Bala and Goyal [11] has proposed a network game studying the relationship between the structure of neighborhoods and the process of social learning whenever the payoffs of different actions are unknown. Fagiolo[30] assumes that network externality decrease to negative values whenever the neighborhood sizes increase. The authors also found that larger populations attain higher coordination. Jackson and Watts[38] considered the coordination problem with endogenous partners, that is, players can decide how many partners to cooperate, and the authors showed that the social coordination equilibrium might fail on a fixed network. The literature and our result establish that the congestion externality becomes more prominent if the population size increases. We also consider the player's learning behavior, which is different from the literature.

Secondly, the literature on rational queuing games with congestion is related to our model. For example, Johari and Kumar[40] proposed a queue system with identical cus-

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tomers and one server, which jointly considered negative congestion effects and positive network effects. Veeraraghavan and Debo[75] has designed a queuing system that concentrates on a negative waiting cost externality and a positive informational externality. When the player chooses a queuing group, the player acts strategically, and the strategy set is about whether to follow the signal, which is not the same as our model's setting. Veeraraghavan and Debo[75] shows the positive informational externality leads to congestion; moreover, herding is a possible rational strategy, and the consumer would ignore the signal under some conditions.

Our model reached a similar result: strong signal type players would give up the first-mover's advantage (FMA) with a specific priority rule under some parameters. Players have the incentive to lure the others to choose a crowding table. However, we focus on how players choose their queuing positions without considering the waiting costs or the predetermined distribution of players, different from the literature. Other notable studies in this literature include Arnosti et al.[9], Debo et al.[27], Veeraraghavan et al.[73], Kremer et al.[43], Guo et al.[34], Feldman et al.[31], Johari et al.[41], and Debo et al.[28].

Thirdly, this topic is related to rational queuing with incomplete information. Veeraraghavan and Debo[74] discuss incomplete information on server quality. Players in their model have their signals and use them to choose a queue, whereas the quality of servers is uncertain. Their setting is close to our model, but our players only have two-stage choices: in the first stage, choose a former or later waiting group; in the second stage, choose a sharing table. In Large and Norman[46], there are only two types of players: those informed of the queue's length and those uninformed about it. Besides, players decide whether to join the queue or not by their arrival order. If players choose to join

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the queue, their payoff must exceed some threshold depending on their information. In our model, on the other hand, players have no opt-out choice. Xu and Hajek[79] added supermarket games with incomplete information to server quality games. In their model, players have waiting costs. Their goals are to choose a queue and optimal inspection to minimize the waiting costs. In contrast, players do not consider the waiting costs when making their decisions in our model. In sum, the target and methods in the literature are generally different from this chapter even though the models cited above consider strategic delays, network externalities, or incomplete information.

We observed strategic delay behavior in our simulation result related to the literature on delay in rational queuing. For example, players have different types and face entry prices, and the server maximizes profit by setting the entry price. For instance, Zhang et al.[80] has shown the number of priority positions affects the profit and incentive to delay, and Mandjes[47] found if the customer is permitted to choose a priority position, then the price is not incentive compatible. Afeche[3] indicated that if the required service is heterogeneous for different types of customers, the strategic delay may be optimal for customers under fair conditions. Other studies include Afeche et al.[5], Guo et al.[35] and Afeche and Mendelson[4].

Chamley and Zhou[17] discussed that in a social coordination game with incomplete information, players will choose to delay to taking action to wait for more information, or to act preemptively. Different from our model setting, time discounting setting was added for the delayed action, and the authors found that choosing to delay may lead to more delays.

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Recent related literature also covers experimental and empirical results. For example, Avoyan and Ramos[10] proposed an experiment involving information delays. The authors found that with a gradual increase during the information exchange process, that is, after communication, players' utility increases and reputation effects emerge.

Lagzi et al.[45]'s empirical study explores the strategic delay behavior of radiologists when faced with tasks of different priorities. The authors found that radiologists perform low-priority, high-reward tasks first, resulting in longer delays for other low-priority tasks. That is, when the system presets priority rules, the externalities caused by congestion will be stronger then without a priority rule. Although the results are different from ours, Lagzi et al.[45] also found that setting the priority rule will produce negative results.

Anunrojwong et al.[7] adds the setting of player heterogeneity to the incomplete information queuing model. Players are divided into two types: low demand and high demand according to their demand for services, and players will decide whether to join a first-come-first-serve queue or leave, based on information design. The authors proved that if the player homogeneity is high, Pareto efficiency cannot be achieved even by providing complete information. Although the entry point is different from our model setting, this article also proposes negative results of queuing under incomplete information.

In this chapter we show that rational players have no profitable incentive to follow the efficient queuing rule in a simple complete information model. The rule of "following the crowd", which is irrelevant to the player's signal quality, dominates the other priority rules. In the incomplete information case, we show that the preferred equilibrium in which encouraging transparency is never a dominant strategy for both types of players.

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2.2 The Model

There are two possible states drawing by equal probabilities, θ_1 and θ_2, and the set of possible states is denoted by $\Theta = \{\theta_1, \theta_2\}$. Assume that there are two restaurant tables of different sizes, table 1 and table 2, as known as the potential resource. Players believe that size of tables are 1 and r, where the parameter $0 \leq r \leq 1$. If the true state is θ_1, then the revealed table size distribution over table 1 and 2 is 1 and r, respectively; if the revealed status is θ_2, then the true distribution of table 1 and table 2 is r and 1, respectively. The value of r is common knowledge for all players.

There are a set of players $\mathcal{N} = \{1, 2, ..., N\}$ where N is a finite and positive integer. Each player $i \in \mathcal{N}$ chooses a table $x_i \in \{1, 2\}$ without knowing the true state, and the player i's utility is

$$U_i(x) = g_i(\theta)\frac{R_{x_i}(\theta)}{n_{x_i}}, \tag{2.1}$$

where $x = (x_j)_{j \in \mathcal{N}} \in X$ denotes the the vector of all players' choices, $X \equiv \{1, 2\}^{\mathcal{N}}$; $\theta \in \Theta$ denotes the possible state of table distribution; g_i is the player i's belief function of the states; $R_{x_i}(\theta) : \Theta \rightarrow \{1, r\}$ is a mapping determining the player i's table size, and $n_{x_i} : X \rightarrow \mathcal{N}$ denotes the total number of players who choosing the same table with player i.

Each player receives a private signal s_i, which denotes a prior probability to the true state distribution. To simplify the analysis, we assume that there are four types of signals. Each signal s_i is generated independently conditional on the true state as follows: $Pr(s_i = 2|\theta_1) = Pr(s_i = -2|\theta_2) = p_1, Pr(s_i = 1|\theta_1) = Pr(s_i = -1|\theta_2) = p_2, Pr(s_i =$

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$-1|\theta_1) = Pr(s_i = 1|\theta_2) = p_3, Pr(s_i = -2|\theta_1) = Pr(s_i = 2|\theta_2) = p_4$. Let $\Delta(p)$ be the set containing all feasible beliefs of true states and $p = (p_1, p_2, p_3, p_4)$ such that $\sum_{s=1}^{4} p_s = 1$ and $p_1 \geq p_2 \geq p_3 \geq p_4$. The values of p_1, p_2, p_3, p_4 are common knowledge to all players in $\mathcal{N}$, though the signals are the private information of the players.

We interpret that the players who receive signals $s_i = 2$ or $s_i = -2$ as the strong type players; players who receive signals $s_i = 1$ or $s_i = -1$ as the weak type players. Let S be the set containing all the strong type signals and W be the set containing all the weak type signals.

Game procedure: The game contains two stages. A queue is determined at stage 1 according to the priority rule and players' simultaneous announcement of queuing choice. Each player's queuing position is only based on his/her own signal because at stage 1 there is no further information can be obtained. More explicitly, in stage 1, player $i \in \mathcal{N}$ announces $\sigma^{\mathcal{R}}(i) \in \{F, L\}$ which represents player i's the queuing group choice; option $F(L)$ represents the former (later) queuing group. We interpret choosing F as choosing a table at a former group and L as making a table choice at a later group. The notation $\mathcal{R} \in \{FF, FL, LF, LL\}$ denotes the priority rule in this game.

The priority rule recommended by the social planner will affect the player's queuing distribution outcome. All players know the priority rule, which represents one of the four possible allocation policies as follows: The FL rule: players within strong type signals are suggested to choose a table at a former group, and players within weak type signals are suggested to choose a table at a later group. The LF rule: players within strong type signals are suggested to choose a table at a later group, and players within weak type signals

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are suggested to choose a table at a former group. The FF rule: both the strong and weak type players are suggested to choose a table at a former group. Regardless of whether the player has a strong type signal or a weak type signal, they will be randomly assigned a queue order for the table choices. The LL rule: both the strong and weak type players are suggested to choose a table at a later group. That is, regardless of whether the player has a strong type signal or a weak type signal, they will be randomly assigned a queue order for the table choices. For example, suppose that there are five players $\{1, 2, 3, 4, 5\}$ within the signals $s_1 = 2, s_2 = 1, s_3 = -2, s_4 = 2$, and $s_5 = 1$. The FF rule produces orders $(1, 2, 3, 4, 5)$,$(5, 4, 3, 2, 1)$ or any permutation, because the FF rule randomly assigns positions to both types of players. A couple of orders using the LF rule may be $(2, 5, 1, 3, 4)$ or $(5, 2, 4, 1, 3)$, but $(1, 5, 2, 3, 4)$ is not possible using the LF rule.

The priority rule influences the players' expectancy of the other's actions. Specifically, player i with position $\sigma^{\mathcal{R}}(i)$ expects that she can observe the signals and choices of players $j \in \mathcal{N}$ with position $\sigma^{\mathcal{R}}(j)$, where $1 \leq \sigma^{\mathcal{R}}(j) < \sigma^{\mathcal{R}}(i)$. If the priority rule follows the FL rule, then player i expects that the signal quality is better than the signal quality generated by the FF and LL rules, and the LF rule is expected to provide the worst signal quality. The FL rule is called the efficient queue. If this rule is followed, then the strong type players will reveal their signals to the public before the weak type players. If all players do not have an incentive to violate the FL rule, namely, this priority rule encourages information transparency as the high-quality signals will be propagated in the system earlier than in other policies. Therefore, priority rule following the FL rule potentially attains the social optimum.

In stage 2, each player $i \in \mathcal{N}$ selects a table $x_i \in \{1, 2\}$ in the queuing order de-

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termined by stage 1. Each player will be able to observe the choices of the players in the front and their signals, for the selection of the table is public and sequential. That is, each player i with position $\sigma^{\mathcal{R}}(i)$ will receive the player j's signal, for all $j \in \mathcal{N}$ such that $\sigma^{\mathcal{R}}(j) \leq \sigma^{\mathcal{R}}(i)$, $\mathcal{R} \in \{FF, FL, LF, LL\}$. Each player's utility is realized at the end of stage 2.

Accordingly, we solve the two-person case analytic solution as follows. Recall that the utility function is defined as $U_i(x_i = x, x_j, j \neq i, i, j \in \mathcal{N}) = g_i(\theta)\frac{R_{x_i}(\theta)}{n_{x_i}}$. Namely, the player's payoff depends on the size of the table selected by the player, and the total number of players who choose the table. Let $|\mathcal{N}| = 2$ and suppose player i receives the signal $s_i = 2$ under a predetermined FF rule. Then player i's expected payoff of choosing $a_i = F$ is

$$EU_i(a_i = F, s_i = 2|\sigma^{FF}, \theta, r, p)$$

$$= Pr(k_i = 1|\sigma^{FF}, s_i)EU_1(x_i, x_j, s_i = 2, s_j \in \mathcal{I}|\sigma^{FF}, \mathcal{H}_1)$$

$$+Pr(k_i = 2|\sigma^{FF}, s_i)EU_2(x_i, x_j, s_i = 2, s_j \in \mathcal{I}|\sigma^{FF}, \mathcal{H}_2)$$

$$= \frac{1}{2}(\frac{p_1}{p_1 + p_4} + \frac{p_4}{p_1 + p_4}r) + \frac{1}{2}[\sum_{s \in \mathcal{I}} Pr(s_j = s|s_i = 2, FF)EU_2(x, s|FF, \mathcal{H}_2)],$$

where

$$\sum_{s \in \mathcal{I}} Pr(s_j = s|s_i = 2, \sigma^{FF})EU_2(x, s|\sigma^{FF}, \mathcal{H}_2))$$

$$= Pr(s_j = 2|s_i = 2, \sigma^{FF})EU_2(x = (1, 2), s = (2, 2)$$

$$+Pr(s_j = -2|s_i = 2, \sigma^{FF})EU_2(x = (2, 1), s = (-2, 2))$$

$$+Pr(s_j = 1|s_i = 2, \sigma^{FF})EU_2(x = (1, 2), s = (1, 2))$$

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$$+Pr(s_j=-1|s_i=2,\sigma^{FF})EU_2(x=(2,1),s=(-1,2))$$

$$=\frac{p_1^2+p_4^2}{p_1+p_4}(\frac{p_1^2}{p_1^2+p_4^2}\times r+\frac{p_4^2}{p_1^2+p_4^2}\times 1)$$

$$+\frac{2p_1p_4}{p_1+p_4}(\frac{p_1p_4}{2p_1+p_4}\times 1+\frac{p_4p_1}{2p_1p_4}\times r)$$

$$+\frac{p_1p_2+p_3p_4}{p_1+p_4}(\frac{p_1p_2}{p_1p_2+p_3p_4}\times r+\frac{p_3p_4}{p_1p_2+p_3p_4}\times 1)$$

$$+\frac{p_1p_3+p_2p_4}{p_1+p_4}(\frac{p_1p_3}{p_1p_3+p_2p_4}\times 1+\frac{p_2p_4}{p_1p_3+p_2p_4}\times r).$$

Similarly, player i's expected payoff of choosing $a_i=L$ is as follows:

$$EU_i(a_i=L,s_i=2|\sigma^{FF},\theta,r,p)$$

$$=Pr(k_i=1|\sigma^{FF},s_i)EU_1(x_i,x_j,s_i=2,s_j\in\mathcal{I}|\sigma^{FF},\mathcal{H}_1)$$

$$+Pr(k_i=2|\sigma^{FF},s_i)EU_2(x_i,x_j,s_i=2,s_j\in\mathcal{I}|\sigma^{FF},\mathcal{H}_2)$$

$$=\sum_{s\in\mathcal{I}}Pr(s_j=s|s_i=2,\sigma^{FF})\times$$

$$EU_2(x_i,x_j,s_i=2,s_j\in\mathcal{I}|\sigma^{FF},\mathcal{H}_2).$$

The equilibrium strategy is

$$a_i^*=\text{arg}\max\{EU_i(F,s_i|\sigma^{\mathcal{R}},\theta,r,p),EU_i(L,s_i|\sigma^{\mathcal{R}},\theta,r,p)\}.$$

Therefore, given an arbitrary set of signals distribution $p\in\Delta(p)$ and the table size distribution $r\in[0,1]$, the equilibrium strategy can be exactly derived from the above formula.

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2.3 Equilibrium Analysis

2.3.1 Existence of the Optimal Player Distribution

In this subsection, we consider the complete information game as a benchmark. Let $(\mathcal{N}, X, (U_i)_{i\in\mathcal{N}})$ be a two-stage Chinese restaurant game, where $\mathcal{N}$ denotes the set of players containing N players, $X = \{1, 2\}$ denotes the set of tables, and utility U_i is defined as Eqn.(2.1). If the true state $\theta \in \Theta$ is known, then given the total number of players and table size parameter $0 \leq r \leq 1$, there must exist an optimal players distribution $(k^*, N - k^*)$ among the two tables, and

$$k^* = \begin{cases} \arg\min_{k:1\leq k\leq N}(|\frac{1+r}{N} - \frac{1}{k}|) & \text{for } r < \frac{1}{N}; \\ N & \text{otherwise,} \end{cases}$$

where $n^*(\theta_1) = (k^*, N - k^*)$ and $n^*(\theta_2) = (N - k^*, k^*)$ respectively denote the optimal numbers of players for table 1 and table 2.

It is easy to see that (i) the priority rule plays no role under the complete information games, and (ii) following the FF rule or the LL rule yields the pure strategy equilibria. Players choose a table with a larger personal share instead of simply choosing a larger size one. We interpret k^* as the first k^* players who choose the table first can attain the better personal share because the optimal number k^* is the upper bound of players who have the opportunity to choose the better table. Therefore, we call k^* the maximum number of players to achieve the first-mover advantage (FMA). The amount of FMA is $|\frac{1}{k^*} - \frac{r}{N-k^*}| \geq 0$. Following the FF rule is a dominant strategy whenever $r \geq \frac{1}{N}$, because

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the $|\frac{1}{k^*} - \frac{r}{N-k^*}| > 0$ in this case; if $r < \frac{1}{N}$, then $|\frac{1}{k^*} - \frac{r}{N-k^*}| = 0$, following the LL rule is a dominant strategy.

2.3.2 Learning from the Former Players

In an incomplete information case, the true state θ is the critical information all players must learn from the signals to perform accurate estimations of the expected utility and then choose a larger share table. Following Bayes' rule, each player may create a belief $g_i(\theta|\mathcal{H}_i)$ that θ is the true state conditional on history that includes his/her signal and the signals and table choices announced by his/her former players, with the belief function $g_i : \{\mathcal{H}_i, \Theta\} \to [0, 1]$.

Remark 2.1. *An example of player's belief based on signals: assume that there are four players, i.e., $N = 4$. Suppose that player i receives a strong type signal $s_i = 2$ and chooses the former group under the FL priority rule in Stage 1, and suppose the system assigns $k_i = 2$. Then player i's belief about the true state is*

$$g_2(\theta_1|s_i) = \frac{g_1(\theta_1|s_1) \times p_1}{g_1(\theta_1|s_1) \times p_1 + g_1(\theta_2|s_1) \times p_4},$$

where function g_1 denotes the first player's belief and s_1 denotes the first player's signal. Under the FL rule, the first player is expected to have a strong signal if the second player is also of strong type. The probability of being assigned to the second position under the FL rule depends on the signal distribution: $Pr(k_i = 2|\sigma^{FL}, s_i = 2, a_i = F) = Pr(k_i = 2, s_1 = 2, s_2 = 2|\sigma^{FL}, a_i = F) + Pr(k_i = 2, s_1 = -2, s_2 = 2|\sigma^{FL}, a_i = F) = \sum_{\theta\in\Theta}[g_1(\theta)s_2(\theta)\sum_{m=1}^{n-1}\binom{n-1}{m}f_2(p|\sigma^{FL}, s_2 = 2, a_i = F)]$, where the function f represents the probability of a specific combination of signal types occurring. Similarly,

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we have $Pr(k_i = 2, s_1 = 2, s_2 = 2|FL, a_i = F) = (\frac{p_1}{p_1+p_4} \times p_1 + \frac{p_4}{p_1+p_4} \times p_4)\times$ $[\binom{3}{3}\frac{1}{4}(p_1^2 + p_4^2) + \binom{3}{2}\frac{1}{3}(p_1p_2 + p_2p_4 + p_3p_4) + \binom{3}{1}\frac{1}{2}(p_2^2 + p_2p_3 + p_3p_2 + p_3^2)]$.

We interpret the deviation incentive as follows: deviating from the suggested priority rule may move the decision order of player i to a relatively former or latter position. Player i can be the first person in the queue whenever choosing strategy $a_i = F$ under the LL rule. Conversely, player i can be the last person in the queue whenever choosing strategy $a_i = L$ under the FF rule. The players who make decisions later tend to have more information; therefore, they have better estimations of the true state. Nevertheless, later having a position in the decision-making order sacrifices the FMA and leads to fewer opportunities to choose a larger share table in the equilibrium.

We study the equilibrium in several restricted but fairly common cases. Let $(\mathcal{N}, \Theta, A, (g_{k_i})_{i\in\mathcal{N}}, (U_i)_{i\in\mathcal{N}})$ be a two-stage game. For each $i \in \mathcal{N}$, the strategy set $A = \{F, L\}$ contains each player i's possible queuing choices, for all $i \in \mathcal{N}$. Given a priority rule, when the player does not comply with the associating priority rule, we call the player deviates from the rule. If the priority rule is FL, given some player$i$ with a strong signal, then player i deviates from the priority rule if $a_i = L$. Denote that $d(-d) \in A$ be the deviation (no deviation) choice. The expected utility function takes a recursive form, which is defined as follows:

$$EU_i(a_i|s_i, \sigma^{\mathcal{R}}) = \sum_{\theta'\in\Theta} g_k(\theta'|s_i)$$

$$\sum_{k=1}^{n}[Pr(\sigma^{\mathcal{R}}(i) = k|s_i, \sigma^{\mathcal{R}}) \sum_{s'\in\mathcal{I}} Pr(s' = s|s_i, \sigma^{\mathcal{R}})\frac{R_{x_i}(\theta')}{n_{x_i}}|_{x_i=BR_i(s|s_i,\sigma^{\mathcal{R}})}], \tag{2.2}$$

where strategy $a_i \in A$, signal $s_i \in \mathcal{I}$, and player i's permutation outcome $\sigma_i^{\mathcal{R}}$ with a given

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priority rule $\mathcal{R} \in \{FF, FL, LF, LL\}$ for all $i \in \mathcal{N}$. Then the profitable deviation criteria of player i with signal type $M \in \{S, W\}$ under a priority rule $\mathcal{R}$ is denoted by

$$X_i(s_i \in M; \sigma^{\mathcal{R}}, \theta, r, p) \equiv$$

$$EU_i(d, s_i \in M|\sigma^{\mathcal{R}}, \theta, r, p) - EU_i(-d, s_i \in M|\sigma^{\mathcal{R}}, \theta, r, p), \text{ for all } i \in \mathcal{N}. \qquad (2.3)$$

Given the stage 2 best responses, players compute the expected payoff. We study the stage 1 equilibrium behavior as follows. [1]

Definition 2.1. *Let $\langle \mathcal{N}, A, (U_i)_{i\in\mathcal{N}} \rangle$ be the stage 1 of the game defined in Section Section 2.2. Denote $a_{-i} = (a_j)_{j\in\mathcal{N}\setminus\{i\}}$, $\forall i \in \mathcal{N}$. A Nash equilibrium is an action profile $(a_i^*)_{i\in\mathcal{N}}$ such that $EU_i(a_i^*, a_{-i}^*) \geq EU_i(a_i, a_{-i}^*)$, for all $a_i, a_{-i} \in A$.*

The existence of equilibrium is shown by finding a pair of parameters (p, r), that does not violate the deviation criteria under the priority rule.

Proposition 2.1. *Let $(\mathcal{N}, \Theta, A, (g_i)_{i\in\mathcal{N}}, (U_i)_{i\in\mathcal{N}})$ be a two-stage game defined in Section 2.2, where the utility is defined by Eqn.(2.1). Then for each $0 \leq r \leq 1$ and $p \in \Delta(p)$, there exists a pure strategy Nash equilibrium.*

Proof: see Appendix A.

The recursive-form solution in Eqn.(2.3) can be simplified by

$$X_i(s_i \in M; \sigma^{\mathcal{R}}, \theta, r, p)$$

[1] For example, set the parameters $r = 0.5, \beta = 1$, and $p = (p_1, p_2, p_3, p_4) = (0.75, 0.15, 0.07, 0.03)$. Then $EU_i(a_i = F, s_i = 2|FF, \theta, r, \beta, p) \approx 0.94$ and $EU_i(a_i = L, s_i = 2|FF, \theta, r, \beta, p) \approx 0.91$. Thus, $EU_i(a_i = F, s_i = 2|FF, \cdot) - EU_i(a_i = L, s_i = 2|FF, \cdot) > 0$, so there is no profitable deviation for a strong type player in moving from group F to group L. The weak type expected payoff under the same rule is an analog.

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$$= \sum_{\theta \in \Theta} \sum_{k=1}^{n} g_k(\theta; \mathcal{H}_{k_i})$$

$$(Pr(\sigma^{\mathcal{R}}(i) = k, a_i = F | \sigma^{\mathcal{R}}, r, p) - Pr(\sigma^{\mathcal{R}}(i) = k, a_i = L\sigma^{\mathcal{R}}, r, p)) \times$$

$$EU_k(t_i^*, t_j^*, (s_1, ..., s_n) | \sigma^{\mathcal{R}}, r, p, \theta, \mathcal{H}_{k_i})$$

$$= \sum_{\theta \in \Theta} \sum_{k=1}^{n} g_i(\theta | \mathcal{H}_i, \sigma^{\mathcal{R}}(i) = k)(b_k - b'_k)[EU_i(x, (s_1, ..., s_n) | \sigma^{\mathcal{R}}(i) = k, r, p, \theta, \mathcal{H}_i)],$$

where b_k and b'_k denote the probability of player i being assigned to position k when player i respectively chooses queuing group F or queuing group L, and $x = (x_i)_{i \in \mathcal{N}}$ denotes the profile of table selection in stage 2, for all $k = 1, 2, ..., N$.

To derive the expected payoff for being assigned to a queuing position k, $k = 1, 2, ..., N$, we first consider the probability to be assigned to that position. Note that the probability of each player i being assigned to position $\sigma^{\mathcal{R}}(i) = k$ depends on the action a_i that the player chose. Then the expected payoff for being assigned to position k takes the following form:

$$EU_i(s_i, x | \sigma^{\mathcal{R}}, \mathcal{H}_i, r, p, \theta)$$

$$= \sum_{s \in \mathcal{I}^{|\mathcal{N}|}} Pr(\sigma^{\mathcal{R}}(i) = k, (s_1, s_2, ..., s_n) | r, p, \theta) \frac{R_{x_i}(\theta)}{n_{x_i}(\theta)}, \text{ for each } k = 1, 2, ..., N.$$

The following theorem is the main result, providing a sufficient condition so that the FL rule is impossible to be a pure strategy equilibrium.

Theorem 2.1. *Let $(\mathcal{N}, \Theta, A, (g_{k_i})_{i \in \mathcal{N}}, (U_i)_{i \in \mathcal{N}})$ be a two-stage game defined in Section 2.2, where the utility is defined by Eqn.(2.1). Then for both types of players, it is impossible that following the FL rule is an equilibrium strategy whenever $0 \leq r < 1/N$.*

Proof: see Appendix A.

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Theorem 2.1 indicates that, under fair conditions, following the efficient priority rule is not an equilibrium, because the table size constant $r \in [0, 1]$ is players' common knowledge, which means each player can derive the benefits gap from selecting a larger share table relative to choosing a smaller one. Under the FL priority rule, players of weak signals are assigned after the players with strong signals, which means the weak signal players are expected to observe the signals of former players and the weak signal players' expectations of the FMA coincide with the former players.

The following proposition shows that the FF rule is an equilibrium strategy for both types around a large range of parameter r. Indeed, this result points out that following the crowd to fight for the former queuing position is dominant whenever parameter r is larger than one-half, which is irrelevant to population size.

Proposition 2.2. *Let* $(\mathcal{N}, \Theta, A, (g_{k_i})_{i\in\mathcal{N}}, (U_i)_{i\in\mathcal{N}})$ *be a two-stage game defined in Section 2.2, where the utility is defined by Eqn.(2.1). The for all* $p \in \Delta(p)$ *and* $1/2 \leq r \leq 1$*, the* FF *rule is a dominant strategy equilibrium.*

Proof: see Appendix A.

The following shows that the equilibrium priority rule is unique whenever the table size constant $r \geq 1/2$. Thus, the FF rule is dominant to the other priority rules and is unique if $r \geq 1/2$. Using this along with Proposition 2.2, we can see that if the size looks similar between the two tables, then the players have a profitable incentive to join the former queuing group and maximize the possibility of achieving the value of FMA.

Proposition 2.3. *Let* $(\mathcal{N}, \Theta, A, (g_{k_i})_{i\in\mathcal{N}}, (U_i)_{i\in\mathcal{N}})$ *be a two-stage game defined in Section 2.2, where the utility is defined by Eqn.(2.1). Then for all* $p \in \Delta(p)$*, and* $1/2 \leq r \leq 1$*,*

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the FF rule is the unique Nash equilibrium.

Proof: see Appendix A.

2.3.3 Simulations

We conduct simulations to verify the priority policies and recursive best response algorithms. The simulation illustrates the equilibrium strategy distribution, as Figure 2.1 to Figure 2.7. The algorithm to identify the pure strategy Nash equilibrium (PSNE) is as follows. Given the players' private signals, probability distribution, and table size constant r, players first calculate their expected utility according to each table choice they made potentially. Then they calculate their deviated expected utility, that is, the expected utility when other players follow the priority rule but the player does not. Finally, they compare the two expected utilities and decide whether following the priority rule benefits them. When the expected utility is less than the deviated expected utility, a player prefers not to follow the given rule. Then we mark that following the given priority rule is not a PSNE.

We present four scenarios, which contain a different number of players. In the following figures, S denotes that the player is a strong type who receives a signal $s = 2$ or $s = -2$, and W denotes that the player is a strong type who receives a signal $s = 1$ or $s = -1$. We assign the players' initial signals by various signal distribution and table size constant (p, r), and give four priority rule two-stage games in each figure. For each priority rule, the players calculate their expected utility and deviate expected utility in different sequences by predicting other players' signals and best responses when they follow the

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rule.

The figures exhibit the PSNE distribution: Figure 2.1 indicates the three-person case, Figure 2.2 indicates the four-person case, Figure 2.4 indicates the six-person case, and Figure 2.7 indicates the nine-person case. In each figure, N indicates the number of players, $N = 3, 4, 6, 9$; "quotient" means that the player's utility is determined by Eqn.(2.1). The horizontal axis marks the table size constant r, ranging from 0 to 1, indicating the size distribution of two tables. The vertical axis indicates the distribution of the information parameter p, from the strong signal and the weak signal between the small to the obvious information difference. The figures show four types of PSNEs, which involve following the FF priority rule, the FL rule, the LF rule, or the LL rule. Few parameter combinations will lead to no pure strategy Nash equilibrium, which is caused by the calculation exception caused by the specific combination of p and r, which is represented by a black triangle on the figures.

NE(FF), the red dot on the figures, means that regardless of the strong type and weak type, the difference between the utility of choosing a former queuing group and the utility of choosing a later group is positive, that is, the utility of choosing a former queuing group is relatively higher, the equilibrium strategy is both types of players choose to wait in a former group. NE(LL), the blue square on the figures, means that regardless of the strong type and weak type, the difference between the utility of choosing a later group and the utility of choosing a former group, is positive, that is, the utility of waiting in a later group is relatively higher, the equilibrium strategy is both types of players choose to wait in a later group. Double-color asterisks and double-color triangles respectively indicate that there are more than two pure strategy Nash equilibria. The diamond indicates that the strong

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type player choosing a later group is more profitable, and the weak type player choosing a former group is more profitable. It means that in certain parameter combinations marked as a diamond, the strong type player’ s equilibrium strategy is to wait; the weak type player’ s equilibrium strategy is to select a table as quickly as possible.

We use the numerical example to explain the impact of different priority rules on player behavior. Given the FF rule, a player knows that she will randomly be assigned a queuing position if she follows the rule because all other players are randomly assigned to the former group. On the other hand, if she deviates from the rule, she will be the final player to make the table selection but know others’ signals and choices. In Figure 2.1, given $r = 0.9$ and $[p1, p2, p3, p4] = [0.85, 0.07, 0.05, 0.03]$, all players follow the FF rule. The LL rule is similar to the FF rule when a player follows the rule. The player will also randomly be assigned a queuing position because other players choose a table last. However, unlike the FF rule, if she chooses to deviate from the rule, she will be the first one to choose a table but cannot observe others’ signals and choices. In Figure 2.1, given $r = 0$ and $[p1, p2, p3, p4] = [0.85, 0.07, 0.05, 0.03]$, all players follow the LL rule because of parameter combination. Moreover, Figure 2.1 also shows given $r = 0.9$ and $[p1, p2, p3, p4] = [0.85, 0.07, 0.05, 0.03]$, all players follow the LL rule because of the dominant effect of the parameter r.

Under the LF rule, when players receive strong signals, they will choose to postpone the table selection. When a player receives weak signals, they will choose the group to join a former queuing group. Furthermore, when a player deviates from the suggested rule, she will be assigned to the other group. If the player receives a signal $s = 2$ and chooses to deviate from the rule, then she will be randomly assigned a position with the

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players in the former queuing group. In Figure 2.1, given $r = 1$ and $[p1, p2, p3, p4] = [0.45, 0.4, 0.1, 0.05]$, all players follow the LF rule. In Figure 2.1, following the FL rule is not observed to be an equilibrium for any set of parameters r and p.

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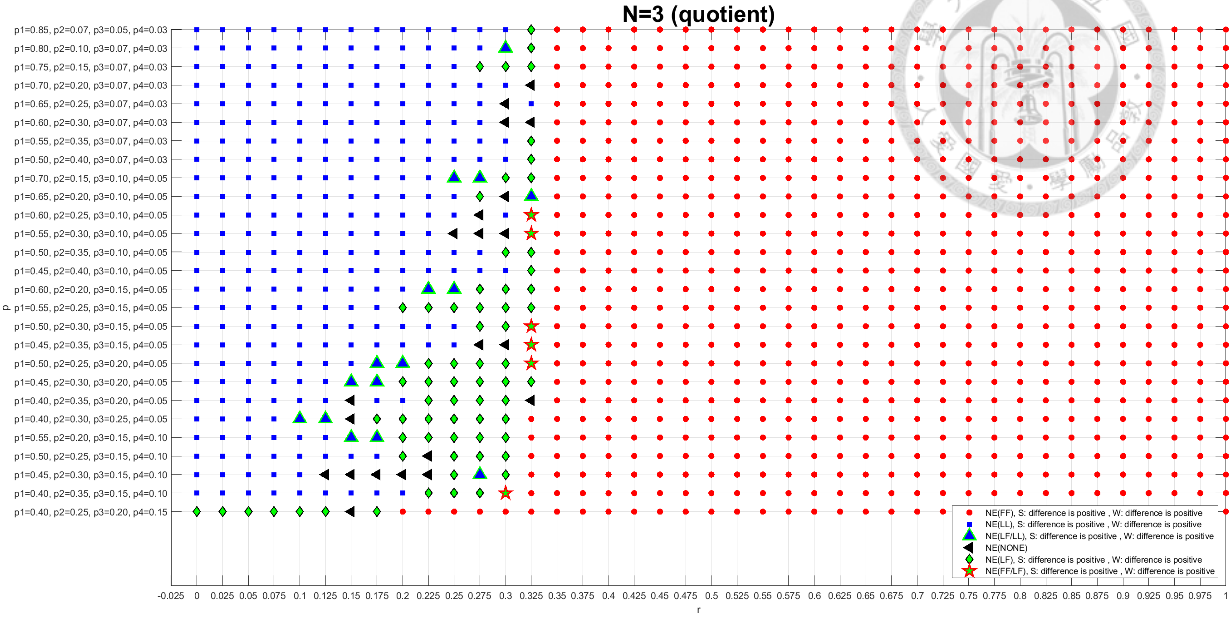

Figure 2.1: Three-person case.

The red dots on the figure represent that "following the FF rule" is a pure strategy Nash equilibrium. The blue squares represent that "following the LL rule" is a pure strategy Nash equilibrium. The green diamonds indicate that "following the LF rule" is a pure strategy Nash equilibrium. The green nested blue triangles indicate that "following both the LF and LL rules" are pure strategy Nash equilibria. The green nested red stars indicate that "following both the FF and LF rules" are pure strategy Nash equilibria. The black triangles indicate that there is no pure strategy equilibrium under a specific combination of (p, r) parameters. Unfortunately, "following the FL rule" never be a pure strategy equilibrium under those parameter combinations. In the case of $r > 1/3$, no matter how the value of p varies, "following the FF rule" is a pure strategy equilibrium.

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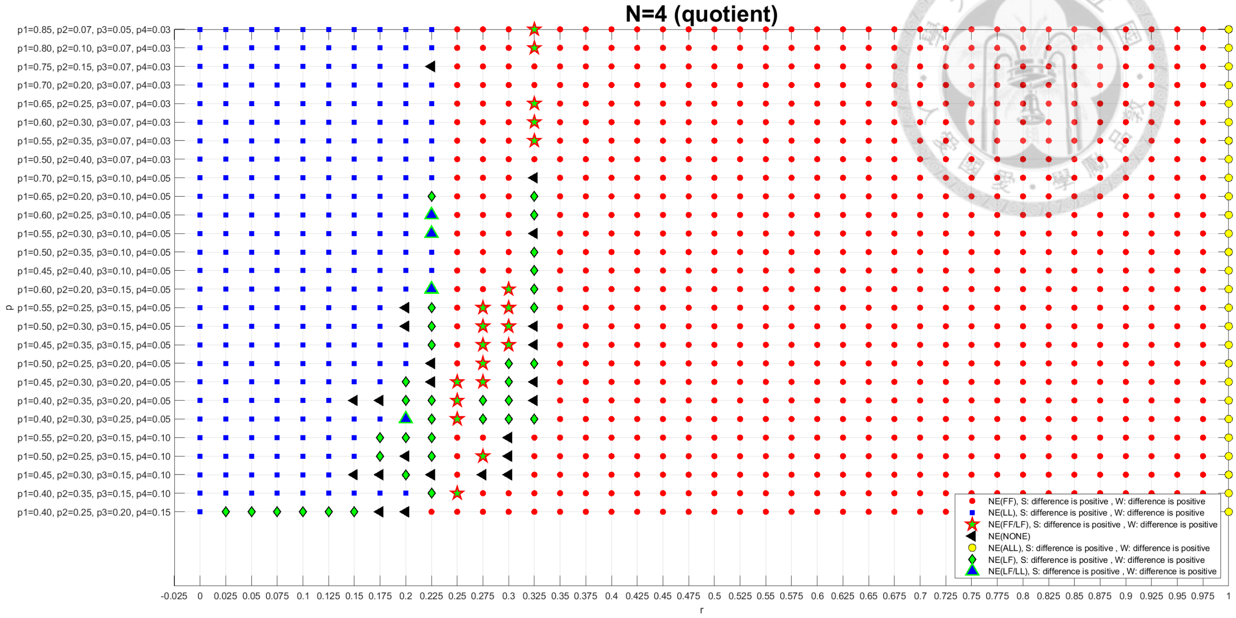

Figure 2.2: Four-person case

The red dots on the figure represent that "following the FF rule" is a pure strategy Nash equilibrium. The blue squares represent that "following the LL rule" is a pure strategy Nash equilibrium. The green diamonds indicate that "following the LF rule" is a pure strategy Nash equilibrium. The green nested blue triangles indicate that "following both the LF and LL rules" are pure strategy Nash equilibria. The green nested red stars indicate that "following both the FF and LF rules" are pure strategy Nash equilibria. The black triangles indicate that there is no pure strategy equilibrium under a specific combination of (p, r) parameters. Unfortunately, "following the FL rule" never be a pure strategy equilibrium under those parameter combinations.

In the case of $r \geq 0.25$, no matter how the value of p varies, "following the FF rule" is a pure strategy equilibrium. The yellow dots indicate that under a specific parameter combination, the four priority rules are all equilibrium strategies.

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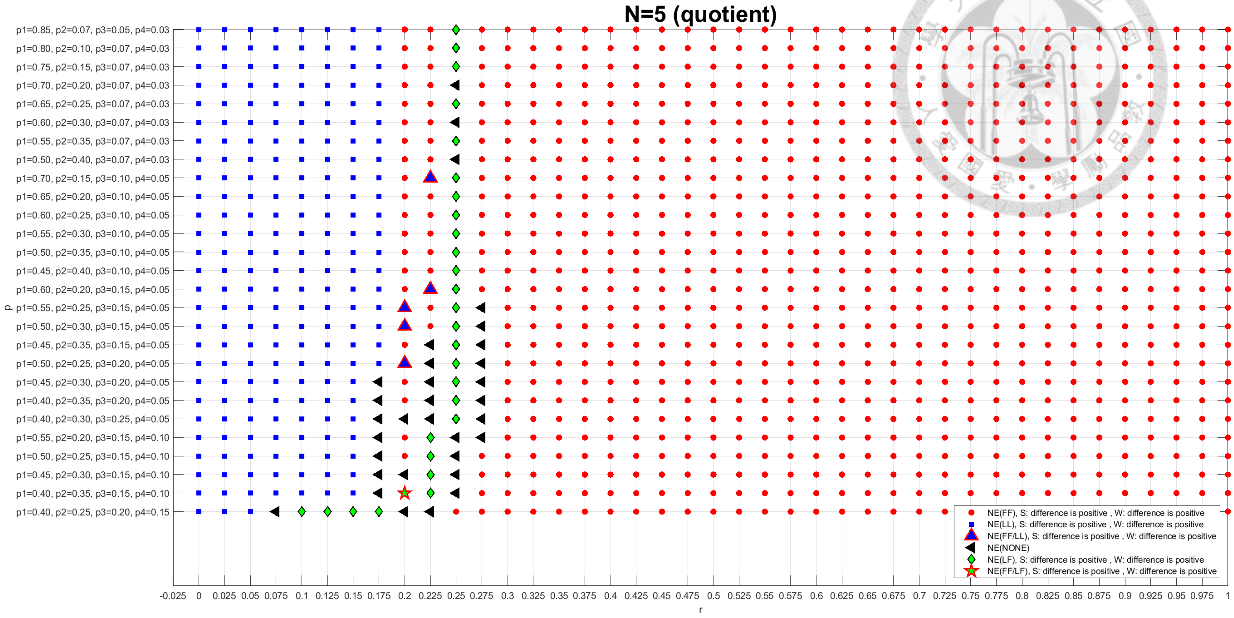

Figure 2.3: Five-person case

The red dots on the figure represent that "following the FF rule" is a pure strategy Nash equilibrium. The blue squares represent that "following the LL rule" is a pure strategy Nash equilibrium. The green diamonds indicate that "following the LF rule" is a pure strategy Nash equilibrium. The green nested blue triangles indicate that "following both the LF and LL rules" are pure strategy Nash equilibria. The green nested red stars indicate that "following both the FF and LF rules" are pure strategy Nash equilibria. The black triangles indicate that there is no pure strategy equilibrium under a specific combination of (p, r) parameters. Unfortunately, "following the FL rule" never be a pure strategy equilibrium under those parameter combinations.

In the case of $r > 1/5$, no matter how the value of p varies, "following the FF rule" is a pure strategy equilibrium.

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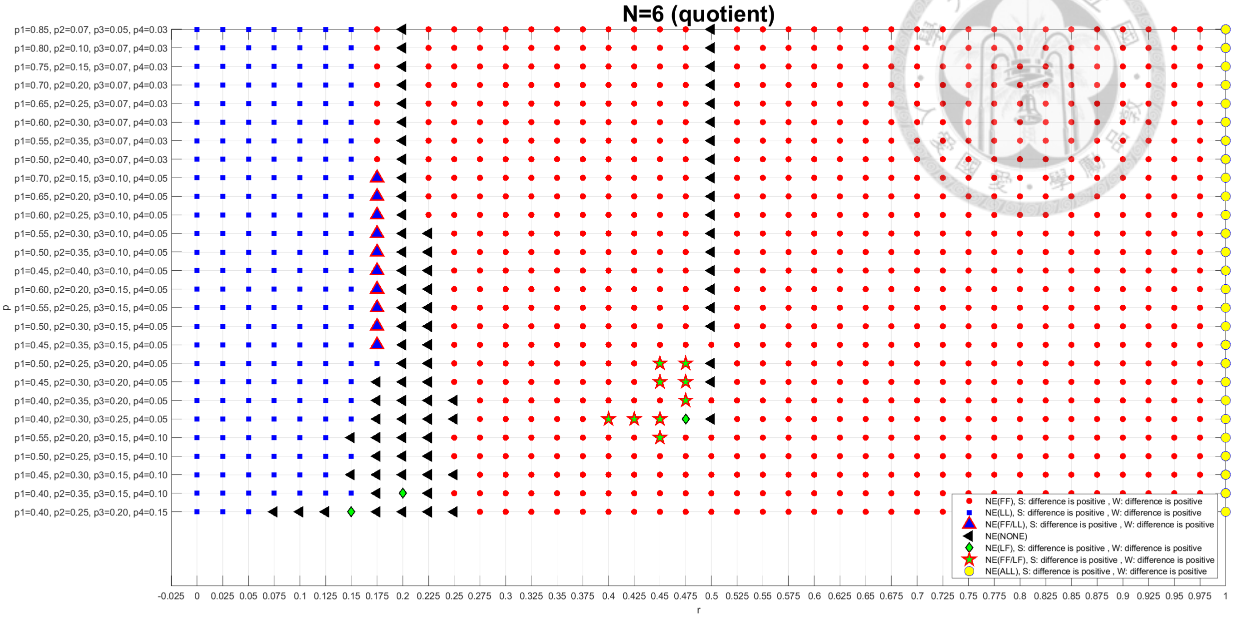

Figure 2.4: Six-person case

The red dots on the figure represent that "following the FF rule" is a pure strategy Nash equilibrium. The blue squares represent that "following the LL rule" is a pure strategy Nash equilibrium. The green diamonds indicate that "following the LF rule" is a pure strategy Nash equilibrium. The green nested blue triangles indicate that "following both the LF and LL rules" are pure strategy Nash equilibria. The green nested red stars indicate that "following both the FF and LF rules" are pure strategy Nash equilibria. The black triangles indicate that there is no pure strategy equilibrium under a specific combination of (p, r) parameters. The yellow dots indicate that under a specific parameter combination, the four priority rules are all equilibrium strategies. Unfortunately, "following the FL rule" never be a pure strategy equilibrium under those parameter combinations.

In the case of $r \geq 1/6$, no matter how the value of p varies, "following the FF rule" is a pure strategy equilibrium at most parameter combinations.

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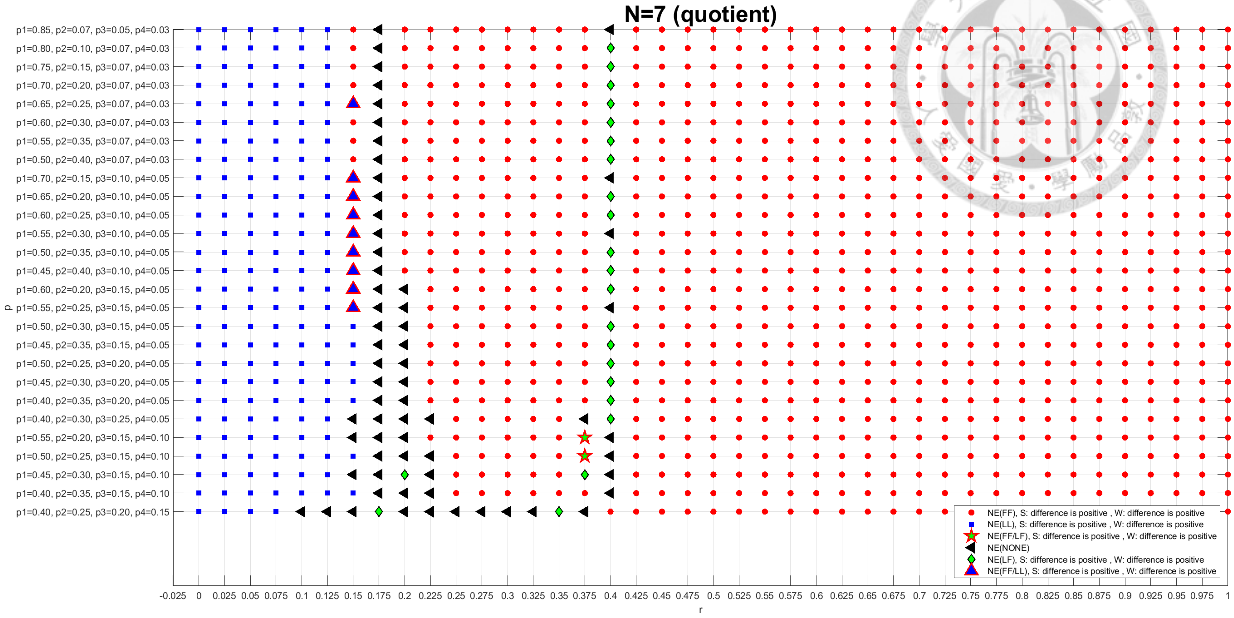

Figure 2.5: Seven-person case

The red dots on the figure represent that "following the FF rule" is a pure strategy Nash equilibrium. The blue squares represent that "following the LL rule" is a pure strategy Nash equilibrium. The green diamonds indicate that "following the LF rule" is a pure strategy Nash equilibrium. The green nested blue triangles indicate that "following both the LF and LL rules" are pure strategy Nash equilibria. The green nested red stars indicate that "following both the FF and LF rules" are pure strategy Nash equilibria. The black triangles indicate that there is no pure strategy equilibrium under a specific combination of (p, r) parameters. Unfortunately, "following the FL rule" never be a pure strategy equilibrium under those parameter combinations.

In the case of $r > 1/7$, no matter how the value of p varies, "following the FF rule" is a pure strategy equilibrium.

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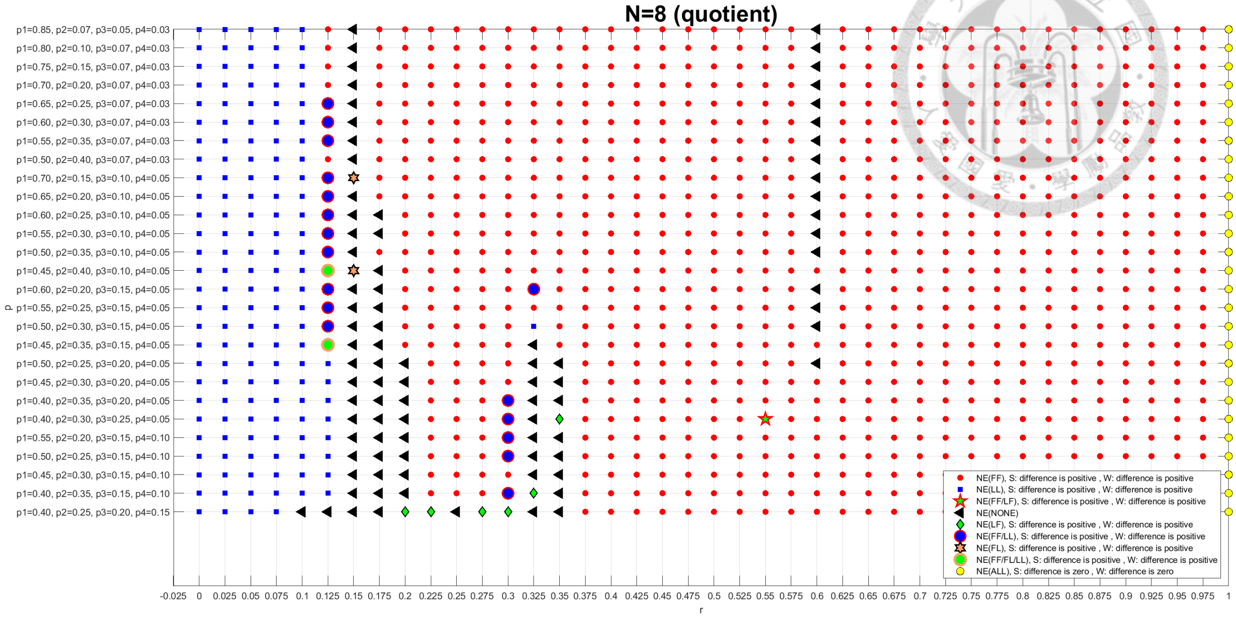

Figure 2.6: Eight-person case

The red dots on the figure represent that “following the FF rule” is a pure strategy Nash equilibrium. The blue squares represent that “following the LL rule” is a pure strategy Nash equilibrium. The green diamonds indicate that “following the LF rule” is a pure strategy Nash equilibrium. The green nested blue triangles indicate that “following both the LF and LL rules” are pure strategy Nash equilibria. The green nested red stars indicate that “following both the FF and LF rules” are pure strategy Nash equilibria. The red nested blue dots indicate that “following both the FF and LL rules” are pure strategy Nash equilibria. The black nested yellow six-pointed stars indicate that ”following the FL rule” is a pure strategy Nash equilibrium. Only two points appear among many parameter combinations, which is caused by a specific parameter combination. The yellow nested green dots indicate that “following the FF, FL, and LL rules” are pure strategy Nash equilibria. The black triangles indicate that there is no pure strategy equilibrium under a specific combination of (p, r) parameters. The yellow dots indicate that under a specific parameter

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combination, the four priority rules are all equilibrium strategies. Unfortunately, "following the FL rule" rarely be a pure strategy equilibrium under those parameter combinations. In the case of $r \geq 1/8$, no matter how the value of p varies, "following the FF rule" is a pure strategy equilibrium at most parameter combinations.

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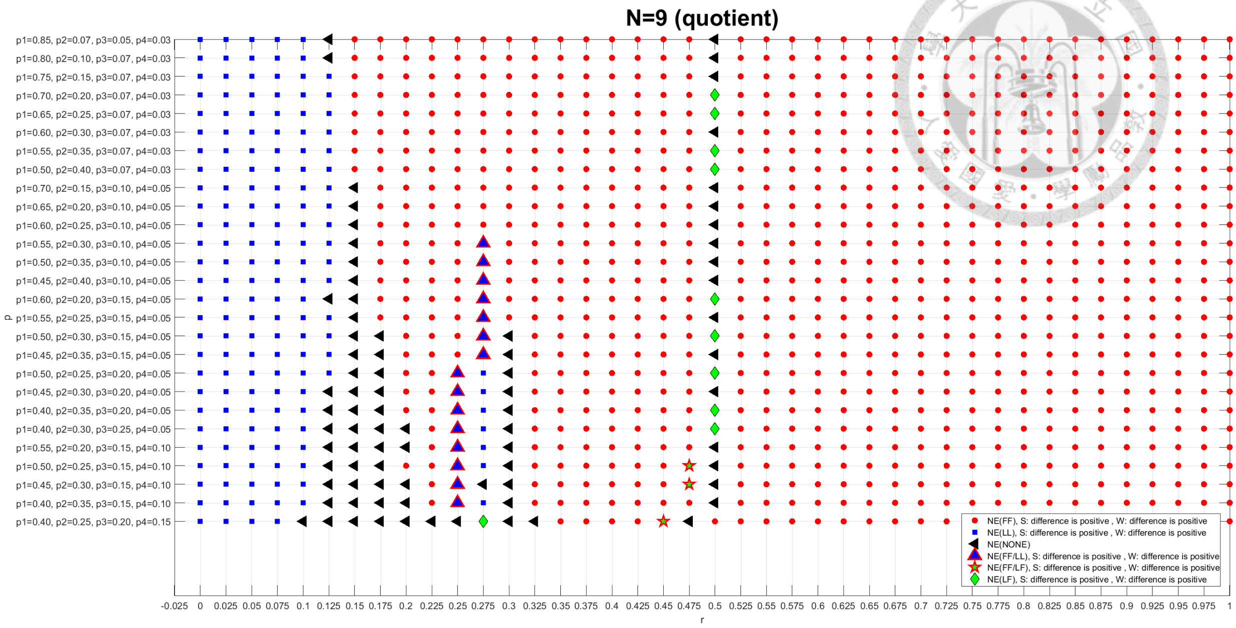

Figure 2.7: Nine-person case

The red dots on the figure represent that "following the FF rule" is a pure strategy Nash equilibrium. The blue squares represent that "following the LL rule" is a pure strategy Nash equilibrium. The green diamonds indicate that "following the LF rule" is a pure strategy Nash equilibrium. The green nested blue triangles indicate that "following both the LF and LL rules" are pure strategy Nash equilibria. The green nested red stars indicate that "following both the FF and LF rules" are pure strategy Nash equilibria. The black triangles indicate that there is no pure strategy equilibrium under a specific combination of (p, r) parameters. Unfortunately, "following the FL rule" never be a pure strategy equilibrium under those parameter combinations.

In the case of $r \geq 1/9$, no matter how the value of p varies, "following the FF rule" is a pure strategy equilibrium at most parameter combinations.

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The influence of the number of players: We can observe the trend of equilibrium strategy from Figure 2.8. The convergent trend of the "FF" rule speeds up whenever the population grows. As the parameter r increases from zero to one, more and more players switch to following the FF rule. We observe that the PSNE distribution has a critical threshold: $r = 1/N$, the threshold is marked with a black dotted line in Figure 2.8.

The figures indicate that following the FF priority rule is a PSNE when $r > 1/n$, where n is the number of the set of players; the figures also exhibit the other three types of PSNE occur only when $r \leq 1/n$. This threshold represents that a smaller table would be selected by at least one player whenever the value of FMA is positive. The figures also indicate that following the LL rule as an equilibrium strategy occurs as r goes to zero, which coincided with the Proposition. The numerical results also observe that following the FL rule is unusual to be an equilibrium strategy, as in the theoretical prediction.

2.4 Concluding Remarks

In this chapter, we study that how players may strategically determine their decision orders when information asymmetry exists and an externality. We propose a model to explore the players' reactions to different priority rules in Chinese restaurant games with priority. We derived the criteria for deviating from the system suggested priority rule and explored the conditions of the existence of pure strategy Nash equilibria. For a finite number of players, there is a closed-form solution. Each player's dominant strategy depends on the expectation of other players' queuing choices under the suggested priority rule—both signal quality and table size matter. Furthermore, we show that following the ef-

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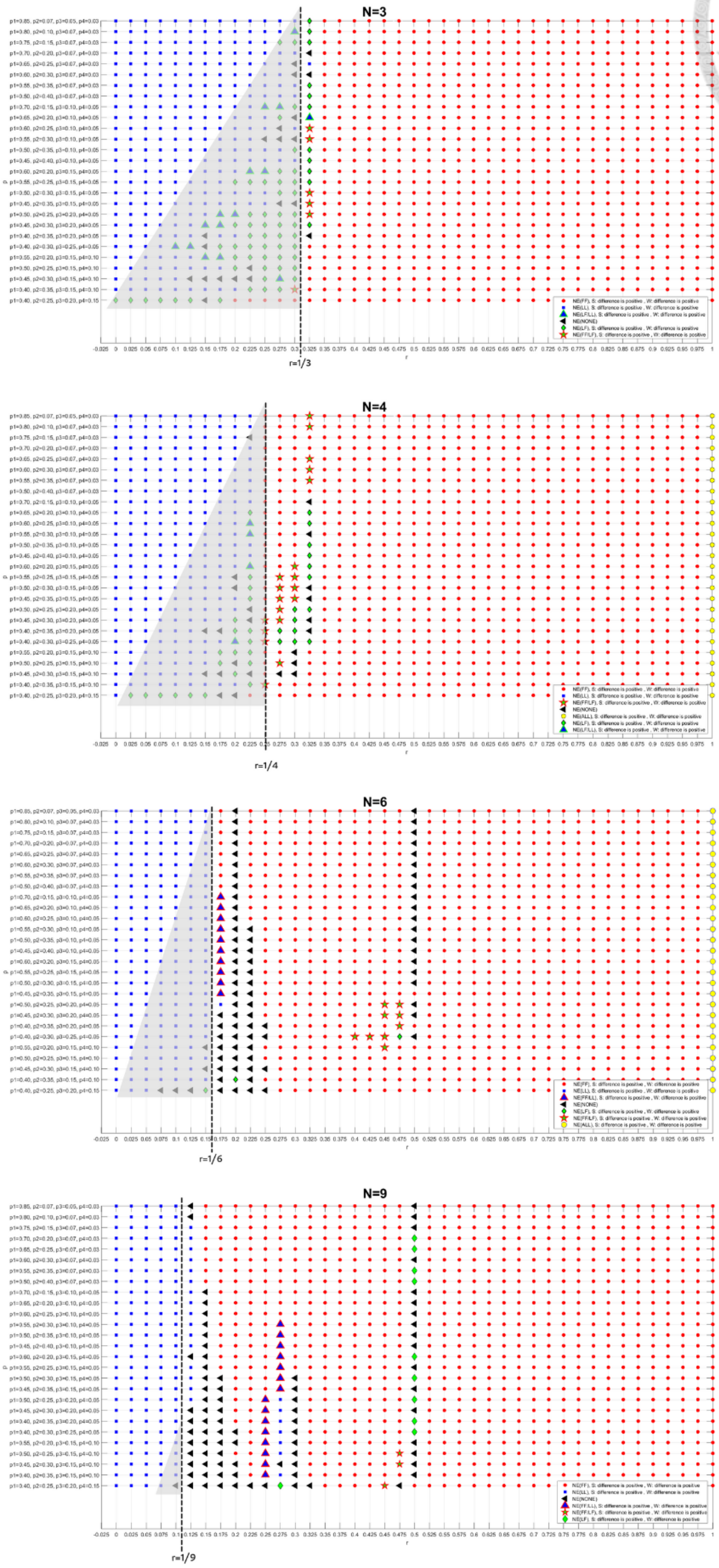

Figure 2.8: Multiple-person case comparison

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ficient priority rule is not an equilibrium strategy for both types of players. In addition, following the FF rule is the players' dominant strategy when the population number is large enough.

There are several future directions for this work. Firstly, the congestion effect is restricted to a quotient form in the player's payoff; it can be extended to a more general function. Secondly, the queuing with priority rules structure can be applied to real-world problems like allocating addresses to potential firms or job search situations for labor markets, which may explain the players' entrance considerations.

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Chapter 3 Production, Conflict, and Dictator's Optimal Favoritism

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3.1 Introduction

We propose a two-stage extensive form game in which the ruler can intervene in the contestants' winning probability by adjusting the weight of a specific effort of contestants. The contestants choose the levels of two types of efforts which are productive and unproductive, and their productive effort decisions depend on the trade-off between rent-seeking efficiency and additional gain from productive activity.

In Tullock's definition, contests specifically refer to situations when the distribution of prizes is in the hands of the government (Tullock[71], p.231). A contest, especially a rent-seeking contest, is a non-cooperative game encompassing the possibility of using lobby, corruption, and other non-productive activities to get the prize. The use of non-productive activities burns the money and causes more welfare losses in society.

In real life, we have observed that dictators in some countries will intensify their efforts to absorb social resources as private property when the economy performs poorly, Wintrobe[78]'s results also confirm this. In the interaction between a state ruler and his/her subordinates, resources and the rules for obtaining them are determined by the ruler, while subordinates need to exert more than one type of effort to gain control of the resources, that is, the prizes. Common prizes include the right to undertake construction projects, the right to develop a certain piece of land, or the position of county magistrate or other local official. In fact, according to the Global State of Democracy 2021 Report, 70% of the world's population lives in countries with democratic regression or authoritarian and hybrid regimes of democracy and dictatorship. It indicates that in the real world, the power to allocate resources(prizes) belongs to the ruler, so those who want resources(prizes) must

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compete according to the rules set by the ruler, that is, the contest designed by the ruler. The imperial examination system in feudal China, government project review laws, and even the company's internal promotion assessment mechanism are common examples of contests.

As another example, company leaders control the promotion opportunities of their subordinates. By adjusting promotion rules, leaders can favor people with specific skills and increase the chances of such people being promoted. The rule-makers of sports or other competitions influence the behavior of participants by adjusting the rules and prize distribution. This kind of interference with competition rules due to favoritism will also have a negative impact on the rule-makers because it involves social unfairness.

In our model, to obtain prizes, contestants can compete through two types of means. The first is to improve their abilities to complete higher-quality work, and the second is to flatter superior bureaucrats or use other methods to bribe them to win over superior bureaucrats' good impression. We call the former a means to invest productive effort and the latter an unproductive effort. Both require different costs.

Our results show that the productive effort investment level is almost zero whenever the number of prizes is small. When the number of prizes is high, the ruler has a stronger incentive to encourage the players to invest in wasteful efforts. The ruler also encourages the players to invest in wasteful efforts as the population size increases. Unfortunately, we find that in most situations the ruler encourages the contestants to invest in wasteful efforts, which explains the tinpot-like autocrats' bribery behavior and company managers' favoritism.

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Contest theory provides a general form dealing with the interactions of players who make wasteful efforts to win some valuable targets. Konrad[42] distinguished three main types of contests: the first-price all-pay auction, standard Tullock model with contest success function(CSF), and evolutionary game, which have different powers of explanation to several kinds of contests. We focus on this topic: Tullock contests, or rent-seeking contests, with multiple prizes and more than one activities.

Nitzan[51] surveyed the early findings of rent-seeking contests. The strategy behavior is the main part of contest (Dixit[29]), then the non-cooperative game are feasible. Corchon[25] collated existing results from the contest into a unified framework and listed the basic axiomatic properties of the contest model. Sisak[61] compiled the existing results of multi-prize contests and pointed out that when the number of prizes increases, the effort invested by players does not increase linearly.

Multiple prize contests have a more complex structure with the interactions among contestants and it changes the results from the single prize structure.

In the one-armed contest with a single prize, Baye et al.[14] showed that there is no pure strategy Nash equilibrium if the discriminatory power parameter $\alpha > 2$ in the CSF. Clark and Riis[23] showed that in a symmetric, simultaneous contest with a Tullock CSF aggregate effort is maximized when only one prize is awarded.

Gradstein and Nitzan[33] studied a contest with multiple advantageous prizes. In their article, pure strategy Nash equilibrium does exist but the symmetric equilibrium does not. Clark and Riis[22] extended a complete information all-pay auction with multiple homogeneous prizes and found that even if the number of prizes changes leads to entirely

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different result from single prize all-pay auction like Baye et al.[13].

Seigel[60] studies an all-pay auction-like contest model, where the heterogeneous players assign the identical prizes to different scores, and the author finds that in a separable contest of the multiple prizes, the total expenditure is higher than the total value of the prizes, i.e., the rent dissipation occurs.

Sisak[61] introduced a question from a mechanism design perspective of contest theory: why the sports contests, or others, provide several prizes like gold, silver, and bronze medals? Why not just provide one winner? What is the optimal prize allocation? In the above consideration, Barut and Kovenock[12] first characterized the equilibrium existence of a symmetric multiple-prize all-pay auction with complete information, for which the number of prizes is less than the number of players. The key setting of this model is the costs of effort and utility of prizes are linear functions, and the prizes by a valued of weakly decreasing orders. Then the authors found that if the value of the lowest-valued k prizes are equal, then the Nash equilibrium arises.

Cohen and Sela[24] combined the results of Clark and Riis[22] and Barut and Kovenock[12] by assuming that the players have full information of all prizes and face linear costs of effort. That is, the players' values on the prizes are common knowledge. The result follows an intuition that the aggregate effort level of two-prize contests is larger than the one-prize contests, and the contest attains efficiency in single prize contest since the highest valuation received the prize.

Some literature recognizes the cost and effort structure of the contests. Arbatskaya and Mialon[8] provided an axiomatic approach to Tullock contests with multiple arms

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and derived a Cobb-Douglass form of effort production. The authors found that if the contestants are almost identical, then an increase in the number of arms leads to more rent dissipation, which coincides with all-pay auction results. Accordingly, this chapter shows that the rent dissipation increases if the number of arms changes from one to more.

Multiple prizes contest structure is more applicable to real-life situations. For example, Terwiesch and Xu[69] demonstrated an inefficiency result in the innovation contests that if the award system changes from a fixed-price award to a performance-contingent award, then the problem solvers will reduce the investment of innovation. The results seem to be more complicated if the number of players or prizes approaches large sizes.

Olszewski and Siegel[52] studied the equilibrium outcomes of an asymmetric all-pay auction with a large size of players and multi-prize by assuming that each player's type follows a Dirac measure. The authors showed that the best response can be implemented by a tariff mechanism with a continuous inverse tariff. Under the type independence setting, the mechanism converges that the prize location in prize distribution is equal to the type location in the type distribution to which the prize is allocated. The authors also found that the equilibrium remains almost the same as the number of players increases.

In applications of multi-prize contests, Bodoh-Creed and Hickman[15] applied the large contest structure to develop a model of college assignment with heterogeneous students in ability. In the calibration, the authors found that each student wipes out about $91,000 net present value of lifetime on average among the competition of admission in both quota-based and admission-based preference systems.

The direction in which our model applies is related to bribery behavior or corruption

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in autocratic regimes and sabotage in rent-seeking contests. Even in democracies, politicians may run for office to gain political office rather than pursue their policy goals, and political candidates are more willing to sacrifice policy for the benefit of office (Callander[16]), that is, to put their favoritism over policy interests.

The sabotage in contests is not the same concept with the wasteful effort in our model. Sabotage, for example, Hirsch and Kastellec[37] studied the sabotage in the policy process that a political party can choose to block the policy implementation of the opponent party. The authors found that, in a democratic system, sabotage can easily lead to resentment among voters. Therefore, whether a party chooses to adopt sabotage against the policies of the opponent depends on how voters feel about the opponent. Chen[18] studied sabotage in promotion situations in organizations. The author found that the sabotage effectively weakens opponents in promotion contests, and the level of sabotage goes down as the number of players decreases, which suggests that the contest is less intense if the number of contestants are low.

In our model, investing wasteful effort can increase a player's probability of winning in a specific contest, but it will not damage the other player's effort invested, whether it is productive or wasteful, while the setting of sabotage directly damages the effort invested by other players, thus to achieve the effect of reducing the opponent's probability of winning. Although both sabotage and wasteful effort reduces the welfare in the social point of view. In addition, in our model setting, investing wasteful efforts will not cause reputation loss, which is different from the setting of Hirsch and Kastellec[37].

The standard definition of corruption is the misuse of public positions to seek pri-

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vate gain (Treisman[70] and Svensson[63]). In most countries, corruption is criminal, but many corrupt behaviors are not easily observed immediately, and corrupt behaviors can also cause huge welfare losses. Types of corruption include politicians directly misappropriating or stealing government public funds, but such behavior is too obvious and easy to detect. Therefore, the more common corruption behavior is accepting personal bribes and interfering with the content of policies or bills through political power. Corruption occurs more frequently in countries where the risk of detection and punishment is lower, i.e., less efficient, making corruption more prevalent in authoritarian regimes. Treisman[70]'s well-known empirical research summarizes the common causes of corruption, and the empirical evidence supports that countries with democratic and open trade institution, or with better economic performance may have lower corruption.

Another well-known conjecture is whether corruption is positively related to a country's abundant natural resources as in the case of oil? Ades and DiTella[2] proposed a model of corruption and natural resources and verified it using empirical data during 1980 to 1990. The authors found that the impact of rents on corruption through restrictions on competition is ambiguous, because by restricting competition, bureaucrats or rulers can obtain rents from firms, but this also means that the public has an incentive to monitor bureaucrats or rulers to avoid corruption because the value of rents is high. However, the authors' empirical analysis showed that the amount of rents is positively related to a country's corruption. Our results confirm the above results from the individual level. We found that when the number of prizes is large, contestants are motivated to increase their investment in wasteful efforts. At the same time, the ruler is also motivated to encourage contestants to invest more wasteful efforts, that is, to encourage them to invest in

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non-productive behaviors, such as sucking up or bribing a superior.

Christensen and Gibilisco[19] explored whether the amount of a government's budget affects autocrats' incentives to share or centralize power. The authors found that because it is costly for autocrats to suppress potential rivals, less government budgets also compress autocrats' expected prospects for power. On the contrary, a rich government budget may lead to a dictator's expectation of being in power for a longer period of time, thereby creating an incentive to share power, which may result in more inclusive institution. Although our model does not include the ruler's sharing or centralizing of power, the impact of the weakness of the social structure on the ruler's decision-making can be observed from the optimal favoritism level of the ruler under different cases. Our results also find that when the country's rent is abundant and the ruler can only obtain a low proportion of benefits from wasteful effort, the ruler induces contestants to invest more wasteful effort. This implies that our findings corroborate the aforementioned results in different direction, that is, under the premise of sufficient budget, rulers are more daring to show their favoritism.

3.2 The Baseline Model

3.2.1 Setup

Our model consists of a two-stage game: Stage 1, where the policymaker determines the contest rule, and the second stage, where the contestants compete. First we describe the Stage 2 settings. A contest is defined by a finite set of players, at least one prize, a set of possible actions taken by the players before the prize is allocated, a contest suc-

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cess function (CSF) which relates to the actions taken by the players to the probability of winning the prize, and a cost function which yields the cost of possible actions.

Formally, let N be a finite set of players, also called contestants, that $N = \{1, 2, ..., n\}$ who join the contest. Each contestant $i \in N$ chooses a pair of productive and wasteful efforts $e_i = (e_i^P, e_i^C) \in \mathbb{R}_+^2$ and the pair of efforts taken by the CSF as the following form:

$$\varphi_i(e_i) = e_i^P + \alpha e_i^C, \forall i \in N, \tag{3.1}$$

where $\alpha > 0$ represents the weight of wasteful effort which is adjusted by a policymaker D. The policymaker, also called the ruler, is the unique player in Stage 1 of the game.

The prizes: in a single-prize case, let the common value of the prize be $V > 0$; in a multiple-prize case, let the common values of the prizes are denoted by $V^1 > V^2 > ... > V^K > 0$, where $K \geq 2$ is a positive integer. The value of each prize does not change no matter what the player chooses to do, that is, the evaluations of the prize are assumed to be independent of efforts and each value of the prize is identical to all the contestants.

Let $p_i = p_i(\varphi_1(e_1), ..., \varphi_n(e_n))$ be the probability that contestant i obtains the prize V when the actions are $(e_i)_{i \in N}$, which is defined by

$$p_i(e) = \begin{cases} \frac{\varphi_i(e_i)}{\varphi_i(e_i) + \sum_{j \neq i, j \in N} \varphi_j(e_j)} & \text{if} \quad \sum_{s \in N} \varphi_s(e_s) > 0, \\ \frac{1}{n} & \text{otherwise}, \end{cases} \tag{3.2}$$

where $e_i = (e_i^P, e_i^C) \in \mathbb{R}_+^2, \forall i \in N$, and $|N| = n$.

Assuming that the contestants are risk-neutral with payoffs linear on the expected

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prize and costs, let $U_i()$ denote the contestant i's payoff function, and

$$U_i(\varphi(e_i), \varphi(e_{-i})) \equiv p_i(\varphi_1(e_1), ..., \varphi_n(e_n))V + \beta e_i^P - C_i(e_i), \tag{3.3}$$

where all $i \in N, e_{-i} \equiv (e_j)_{j \in N \setminus \{i\}}$, the parameter $\beta > 0$ denotes the contestant's additional gain from investing productive effort e_i^P. The cost function $C_i()$ denotes the cost function which represents the cost of taking each pair of effort $e_i = (e_i^P, e_i^C)$, assuming that $C(0) = 0, C' > 0, C'' \geq 0$.

The ruler D's payoff is defined as follows:

$$U_D = \sum_{i \in N}(e_i^P + \gamma e_i^C) - \frac{\omega}{2}(\alpha - 1)^2, \tag{3.4}$$

where the negative part of Eqn.(3.4) represents the disutility encountered by the ruler that $\omega > 0$ denotes the weakness of social structure, the parameter γ represents the importance the ruler attaches to production, and both ω, γ are exogenous.

One of the reasons for setting the parameter of favoritism α to compare with 1 instead of 0 is that for two different types of efforts, the ruler chooses α to be 1 revealing his/her fair tendency to the two types of efforts. When α is specified as 1, two different types of efforts have the same weight in the contest success functions, so contestants cannot change their winning probability by adjusting the relative size of the two types of efforts. Therefore, we determine whether the ruler shows favoritism by observing whether the optimal α level deviates from 1.

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3.2.2 Game Procedure

This is a two-stage game. In the first stage, the ruler chooses a favorable level α to adjust the relative weight between productive effort and wasteful effort. If the ruler has no preference for wasteful input, then the ruler chooses $\alpha = 1$, thus creating zero disutility. In the second stage, the contestants choose the effort level simultaneously to maximize their expected payoffs under a normalized budget constraint. Formally, we have

Stage 1: Ruler D chooses $\alpha \in \mathbb{R}$ to solve the ruler's problem:

$$\max_{\alpha \geq 0} U_D = \sum_{i \in N} (e_i^P + \gamma e_i^C) - \frac{\omega}{2}(\alpha - 1)^2. \tag{3.5}$$

Stage 2: Contestant i chooses the pair of effort levels $e_i = (e_i^P, e_i^C) \in \mathbb{R}_+^2$ to solve the contestant's problem:

$$\begin{aligned} &\max_{(e_i^P, e_i^C) \geq 0} \frac{e_i^P + \alpha e_i^C}{\sum_{j \in N}(e_j^P + \alpha e_j^C)} V + \beta e_i^P - C_i(e_i) \text{ s.t.} \\ &\qquad\qquad e_i^P + e_i^C = 1, \text{ for all } i \in N. \end{aligned} \tag{3.6}$$

3.2.3 The Single-Prize Benchmark

In the very beginning, we deal with a standard ruler-contest case, that is, without the productive effort. Formally, we solve the following game:

Stage 1: Ruler D chooses $\alpha \in \mathbb{R}$ to solve the ruler's problem:

$$\max_{\alpha > 0} U_D = \sum_{i \in N} \gamma e_i^C - \frac{\omega}{2}(\alpha - 1)^2. \tag{3.7}$$

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Stage 2: Contestant i chooses the pair of effort levels $e_i^C \in \mathbb{R}_+$ to solve the contestant's problem:

$$\max_{e_i^C \geq 0} \frac{\alpha e_i^C}{\sum_{j\in N}(\alpha e_j^C)} V - C_i(e_i), \text{ for all } i \in N. \tag{3.8}$$

Optimal wasteful efforts:

Applying the Nash equilibrium concept, the unique effort level in Stage 2 is:

$$e^{C^*} = \frac{(n-1)V}{n^2 C_C}, \tag{3.9}$$

where $C_C \equiv \frac{\partial C(e^C)}{\partial e^C} > 0$ by assumption. It is easy to see that e^{C^*} is irrelevant to the parameter α.

The optimal wasteful effort $e^{C^*} > 0$ for all $n > 1$. We observe that, for all $n > 1$,

$$\begin{aligned} \frac{\partial e^{C^*}}{\partial V} &= \frac{n-1}{n^2 C_C} > 0; \\ \frac{\partial e^{C^*}}{\partial C_C} &= \frac{-(n-1)V}{n^2 C_C^2} < 0; \\ \frac{\partial e^{C^*}}{\partial n} &= \frac{(2-n)nV}{n^2 C_C} < 0, \forall n > 2. \end{aligned} \tag{3.10}$$

The comparative statics Eqn.(3.10) indicates that each contestant pays more on the wasteful effort whenever the value of the prize goes up; when the marginal cost of effort goes up, then the contestant decreases the effort input, and these results coincide with the results of standard contest model. We also observe that there is an opposite direction movement on the optimal wasteful efforts when the population size goes up. The economic intuition behind this is that when the number of contestants increases, it is more beneficial for the contestants to engage in productive activities rather than compete with

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more contestants. This suggests that in places with larger populations, there is no benefit to the contestants from competing with multiple people for a single prize.

Optimal level of favoritism:

Now consider Stage 1, the ruler's payoff is:

$$\begin{aligned} U_D &= \sum_{i\in N}(\gamma e^{C*}) - \frac{\omega}{2}(\alpha-1)^2 \\ &= \gamma n e^{C*} - \frac{\omega}{2}(\alpha-1)^2 \\ &= \frac{\gamma(n-1)V}{nC_C} - \frac{\omega}{2}(\alpha-1)^2 \end{aligned} \tag{3.11}$$

It is easy to see that $\alpha^* = 1$ is the unique solution to maximize the ruler's problem Eqn. (3.11).

From the above results, we know that in an environment without productive effort, all contestants will invest positive wasteful effort, while the ruler has no incentive to favoritism.

3.2.4 The Multiple-Prize Benchmark

Now we turn to the multiple-prize case without productive efforts. We solve for a subgame perfect Nash equilibrium by backward induction. Therefore, we start by considering the contestants' best response as follows:

$$U_i(e^{C_1},...,e^{C_K}) = \sum_{k=1}^{K}\frac{\alpha e_i^{C_k}}{\sum_{j\in N}\alpha e_j^{C_k}}V_k - C_i(e_i^{C_1},...,e_i^{C_K}),\ \text{for all } i\in N, \tag{3.12}$$

where K is the number of prizes, and $e_i^{C_k}$ denotes the contestant i's effort input on the $k-$th specific prize. Let $C_k \equiv \frac{\partial C_i(e^{C_1},...,e^{C_K})}{\partial e_i^{C_k}}$ be the marginal cost of investing the $k-$th

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specific prize. Then we have the optimal wasteful effort

$$e^{C_k^*} = \frac{(n-1)V_k}{n^2 C_k}, k = 1, 2, ..., K, \forall n > 1. \tag{3.13}$$

It is easy to see that the optimal wasteful effort is irrelevant to the parameter α.

Turning to the Stage 1 ruler's problem, we have

$$U_D = \sum_{i \in N} \sum_{k=1}^{K} \gamma e_i^{C_k} - \frac{\omega}{2}(\alpha - 1)^2 = \frac{(n-1)\gamma Z}{n} - \frac{\omega}{2}(\alpha - 1)^2, \tag{3.14}$$

where $Z \equiv \sum_{k=1}^{K}(V_k/C_k) > 0$ denotes the total benefit of prizes.

Similar to the single-prize benchmark, it is easy to see that $\alpha^* = 1$ is the unique solution to maximize the above ruler's problem. Therefore, we find that in an environment without productive effort, all contestants will invest positive wasteful effort. At the same time, the ruler has no incentive to favoritism, even if the country consists of fruitful resources.

3.3 Extensions

3.3.1 Policymaker Who Cares About Social Productivity

Different from the baseline single-prize model, now we consider a specific ruler that only cares about the total productive effort in the payoff function, and like the baseline setting, the ruler will also be damaged by favoritism. That is, we set $\gamma = 0$ in Eqn.(3.4).

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Define the Stage 1 ruler's payoff function as follows:

$$U_{\hat{D}} = \sum_{i \in N} e_i^P - \frac{\omega}{2}(\alpha - 1)^2 = \frac{n\alpha}{\alpha - 1} - \frac{(n-1)V}{n(\beta - C_P)} - \frac{\omega}{2}(\alpha - 1)^2 \quad (3.15)$$

with the CSF $\varphi_i(e_i) = e_i^P + \alpha e_i^C$, for all $i \in N$. The Kuhn-Tucker conditions for the ruler's problem are:

$$\frac{\partial U_{\hat{D}}}{\partial \alpha} = 0, \alpha \geq 0. \quad (3.16)$$

Then we have

$$\frac{-n}{\omega} = (\alpha - 1)^3 \quad (3.17)$$

$$\Rightarrow \alpha = \left[-\frac{n}{\omega} \right]^{\frac{1}{3}} + 1. \quad (3.18)$$

Unfortunately, Eqn.(3.17) has no positive real solution for all the combinations (n, ω) such that $(n/w)^{1/3} > 1$. This indicates that whenever the population size grows, there does not exist any ruler who favors a wasteful effort. More precisely, a ruler does not reveal his/her favoritism for wasteful effort whenever he or she cares about social productivity, which is reasonable and trivial.

In the real world, however, there is no reason to assume that a country' s leader values only the productive efforts of his/her people. The optimal choice that $\alpha^* < 1$ implies that the ruler will impose some punishment on the rent-seeking behavior on specific prizes of the contestants, further inducing the contestants to invest only productive efforts as much as possible, and the economic implications of this situation point to such ruler is a benevolent social planner.

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3.3.2 Multiple-Prize Contests

Now we turn to the multiple-prize case. Assume that there are K infinitely divisible prizes with $K > 1$ a positive integer. Let V_k denote the value of the $k-$th prize and C_k denote the marginal cost of joining the contest for the $k-$th prize, for all $k = 1, 2, ..., K$. Each contestant i chooses a specific wasteful effort $e_i^{C_k}$ on each prize k with contestant i's CSF $\varphi_i^k(e_i) \equiv e_i^P + \alpha e_i^{C_k}$, where the effort is a length $K+1$ vector that $e_i = (e_i^P, e_i^{C_1}, ..., e_i^{C_K})$, $k = 1, 2, ..., K$, all $i \in N$. The payoff of each contestant $i \in N$ is defined by

$$U_i((\varphi_i^k(e_i), \varphi_{-i}^k(e_{-i}))_{k=1}^K) = \sum_{k=1}^K \frac{\varphi_i^k(e_i)V_k}{\sum_{j\in N} \varphi_j^k(e_j)} + \beta e_i^P - C(e_i), \tag{3.19}$$

where $e_i = (e_i^P, e_i^{C_1}, ..., e_i^{C_K}) \in \mathbb{R}^{K+1}$.

Assume that each contestant faces a budget constraint

$$e_i^P + e_i^{C_1} + ... + e_i^{C_K} = 1, \forall i \in N. \tag{3.20}$$

Solving the contestant's problem we have a Nash equilibrium that

$$\begin{aligned} \sum_{k=1}^K \frac{(n-1)V_k}{n^2(e^P + \alpha e^{C_k})} &= C_P - \beta \\ \frac{\alpha(n-1)V_k}{n^2(e^P + \alpha e^{C_k})} &= C_k, k = 1, 2, ..., K, \end{aligned} \tag{3.21}$$

where $C_k = \frac{\partial C(e^P, e^{C_1}, ..., e^{C_K})}{\partial e^{C_k}} > 0$ for $k = 1, 2, ..., K$.

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The optimal effort levels are derived by the above $K+1$ equations:

$$e^P(e^{C_1},...,e^{C_K}) = \frac{\alpha(n-1)}{(K-\alpha)n^2}\sum_{k=1}^{K}\frac{V_k}{C_k} - \frac{\alpha}{K-\alpha}.$$
$$e^{C_k}(e^P) = \frac{(n-1)V_k}{n^2C_k} - \frac{1}{\alpha}e^P, k=1,2,...,K. \tag{3.22}$$

Let $Z \equiv \sum_{k=1}^{K}\frac{V_k}{C_k}$. Then we have

$$e^P(e^{C_1},...,e^{C_K}) = \frac{\alpha}{K-\alpha}\left[\frac{(n-1)Z-n^2}{n^2}\right].$$
$$e^{C_k}(e^P,e^{C_1},...,e^{C_K}) = \frac{(n-1)V_k}{n^2C_k} - \frac{1}{K-\alpha}\left[\frac{(n-1)Z-n^2}{n^2}\right], k=1,...,K. \tag{3.23}$$

In Stage 1, the ruler solves the ruler's problem that

$$\max_{\alpha\geq 0} U_{\hat{D}} = \sum_{i\in N} e_i^P - \frac{\omega}{2}(\alpha-1)^2$$
$$= \frac{n\alpha}{K-\alpha}\left[\frac{(n-1)Z}{n^2}-1\right] - \frac{\omega}{2}(\alpha-1)^2 \tag{3.24}$$

The Kuhn-Tucker conditions for the ruler's problem are:

$$\frac{\partial U_{\hat{D}}}{\partial\alpha} = 0, \alpha\geq 0, \tag{3.25}$$

which implies that

$$(\alpha-1)(K-\alpha)^2 = \frac{K[(n-1)Z-n^2]}{n\omega}, \tag{3.26}$$

where $K>1$ is a positive integer, and Eqn.(3.26) is a cubic equation which usually has one real solution and two imaginary solutions. Consider the existence of positive real solution, we require the condition that $\frac{K[(n-1)Z-n^2]}{n\omega} > 0$, that is, $(n-1)Z \geq n^2$, which approximate to $Z>n$.

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The condition $Z > n$ means that the sum of the prize values almost exceeds the total population. This is a strict condition, especially in large countries with a large population satisfying the above condition, this implies that the country is rich in natural resources.

Therefore, the ruler's optimal favoritism choice α is

$$\alpha = \frac{2K+1}{3} + \frac{\delta^{1/3}}{3\sqrt[3]{2}} + \sqrt[3]{2}\left[(2K+1)^2 + 3K(K+2)\right](3\delta)^{-1/3},$$

where

$$\delta = 9K(2K+1)(K-2) - \frac{27K}{n\omega}[Kn\omega + (n-1)Z - n^2] + 2(2K+1)^3 + 3\sqrt{3}\sigma^{\frac{1}{2}},$$
$$\sigma = \frac{27K^2}{n^2\omega^2}[Kn\omega + (n-1)Z - n^2]^2 - 4K^2(K-2)^2 - K^2(2K+1)^2(K-2)^2$$
$$-[\frac{18}{n\omega}K^2(2K+1)(K-2) + \frac{4K(2K+1)^3}{n\omega}][Kn\omega + (n-1)Z - n^2],$$

given $Z > n$.

(3.27)

Proposition 3.1. *Let $\gamma = 0$. Suppose that $\sum_{k=1}^{K} V_k > n$. Then there is a unique subgame perfect Nash equilibrium with*

$$\alpha^* = \frac{2K+1}{3} + \frac{1}{9\sqrt[3]{2}} + \sqrt[3]{2}(K^2-1)^2\delta^{-1/3} > 1.088,$$
$$e^{P^*} = \frac{3\sqrt[3]{2}(2K+1) + 9(2^{2/3})(K^2-1)^2)\delta^{-1/3} + 1}{3\sqrt[3]{2}(K-1) - 9(2^{2/3})(K^2-1)^2)\delta^{-1/3} - 1}\left[\frac{(n-1)Z - n^2}{n^2}\right];$$
$$e^{C_k^*} = \frac{(n-1)V_k}{n^2C_k} - \frac{1}{9\sqrt[3]{2}}\left[3\sqrt[3]{2}(K-1) - 9(2^{2/3})(K^2-1)^2)\delta^{-1/3} - 1\right]\left[\frac{(n-1)Z - n^2}{n^2}\right];$$

(3.28)

for $k = 1, 2, ..., K$, $\delta = \sqrt[3]{3}[27\sigma^2 - 4\sigma K^3 + 12\sigma K(K-1) + 4\sigma]^{1/2} + 27\sigma - 2K^3 + 6K(K-1) + 2$, and $\sigma = \frac{K[(n-1)Z-n^2]}{n\omega} > 0$.

Therefore, assume that only productive investment provides utility to the ruler, when the total value of prizes is larger than the population size, then the ruler reveals favoritism toward wasteful effort more than productive effort.

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Proof. Let $\sum_{k=1}^{K} V_k > n$. Then we have $\sum_{k=1}^{K} > \sum_{k=1}^{K} \frac{V_k}{C_k} \equiv Z > n$, which implies that $\frac{K[(n-1)Z-n^2]}{n\omega} > 0$. Accordingly, the cubic function $(\alpha-1)(K-\alpha)^2 = \frac{K[(n-1)Z-n^2]}{n\omega}$ has a unique positive real solution, that is,

$$\alpha^* = \frac{2K+1}{3} + \frac{1}{9\sqrt[3]{2}} + \sqrt[3]{2}(K^2-1)^2\delta^{-1/3},$$
$$\delta = \sqrt[3]{3}[27\sigma^2 - 4\sigma K^3 + 12\sigma K(K-1) + 4\sigma]^{1/2} + 27\sigma - 2K^3 + 6K(K-1) + 2,$$
$$\sigma = \frac{K[(n-1)Z-n^2]}{n\omega} > 0. \tag{3.29}$$

Recall that the number of prize $K > 1$ is a positive integer, then we have

$$\begin{aligned}\alpha^* &= \frac{2K+1}{3} + \frac{1}{9\sqrt[3]{2}} + \sqrt[3]{2}(K^2-1)^2\delta^{-1/3} \\ &> \frac{2K+1}{3} + \frac{1}{9\sqrt[3]{2}} \\ &> \frac{2+1}{3} + \frac{1}{9\sqrt[3]{2}} \approx 1 + 0.0881 > 1.0881.\end{aligned} \tag{3.30}$$

The optimal efforts e^{P^*}, $e^{C_k^*}$ follow from Eqn.(3.23), for $k = 1, ..., K$. □

Note that

$$\frac{\partial e^{P^*}}{\partial \alpha} = \left[\frac{(n-1)Z-n^2}{n^2}\right]\frac{K}{(K-\alpha)^2}, \text{ and} \tag{3.31}$$

$$\frac{\partial e^{C_k^*}}{\partial \alpha} = \left[\frac{(n-1)Z-n^2}{n^2}\right]\frac{-1}{(K-\alpha)^2}, \text{ for } k = 1, ..., K. \tag{3.32}$$

The key to judging whether the above Eqn.(3.31) and (3.32) are positive or negative is the relative amounts of Z and n, which represent the total net value of prizes and the population size, respectively.

From Eqn.(3.31) and Eqn.(3.32), we observe that the signs of the two types of efforts

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are opposite, that is, if $(n-1)Z > n^2$, or approximately, $Z > n$, then as α goes up, the productive effort rises, and at this time the wasteful effort changes in the opposite direction. Otherwise, if $(n-1)Z < n^2$, or approximately, $Z < n$, then as α goes up, the wasteful effort rises, and at this time the productive effort changes in the opposite direction.

A policy implication to the situation of dictators and their subordinates, the α value is known as the degree to which a dictator encourages contestants to invest in wasteful spending, which means that the dictator prefers contestants to invest resources in flattering superiors, giving gifts and providing bribes, rather than investing in production. Then the α value represents the degree of corruption of the dictators. The higher the α value, the higher the degree of corruption of the dictators.

The economic intuition contained in Eqn.(3.31) and (3.32) is that, when the value of the total reward is higher than the population size, which means in a country with richer resources relative to the population, the higher the degree of corruption ($\alpha > 1$), the more contestants invest in production and less wasteful efforts. On the contrary, when the value of the total reward is lower than the population size, which means in a country with poorer resources relative to the population, the higher the degree of corruption ($\alpha > 1$), the more contestants invest in wasteful effort and less production.

In the rich literature related to the resource curse, which refers to the phenomenon of countries with an abundance of natural resources, there has been considerable discussion on whether resource-rich countries will experience both low-quality governance and low economic growth (Sachs and Warner[53], de Mesquita and Smith[26], Smith and Waldner[62]). Newer literature points out that institutions are more critical to economic growth

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instead of resource reasons (Acemoglu and Robinson[1]). Our results provide an individual decision-making perspective against the resource curse. Even when dictators become more corrupt, in resource-rich countries, people still have incentives to invest in production rather than all wasteful activities.

3.3.3 General Ruler Preference with Multi-Prize

Consider a ruler who holds multiple prizes and the ruler's preference is defined as follows:

$$U_D = \sum_{i \in N} \left[e_i^P + \gamma \sum_{k=1}^{K} e_i^{C_k} \right] - \frac{\omega}{2}(\alpha - 1)^2 \text{ s.t. } \gamma \geq 0. \tag{3.33}$$

As a weighted element in the ruler's payoff, the parameter γ represents that the benefits to the ruler of people's wasteful efforts. It seems counter-intuitive that wasteful effort investment benefits the ruler. For rulers, especially dictators in non-democratic systems, it is unrealistic to expect the subordinates to continuously strive to enrich their strength and become powerful. Strong subordinates often threaten the ruler's position. On the other hand, a moderate level of corruption among subordinates stabilizes a ruler's position. Similarly, in democratic systems, when potential political competitors are not strong, the incumbent ruler has less pressure to run for re-election.

Also, this setting in Eqn.(3.33) is broader and richer than the previous two subsections. We will discuss it in the following four scenarios: (i) $\gamma = 0$, (ii) $\gamma = 1$, (iii) $0 < \gamma < 1$, and (iv) $\gamma > 1$.

Scenario (i) Multi-prize altruistic ruler $\gamma = 0$.

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In this scenario, Stage 1 ruler's payoff coincides with Section 3.3.2, and the ruler's optimal favoritism α is strictly greater than one, given the total prize value is higher than the population size.

Compared to the situation in Section 3.3.1 where the number of prizes approaches one, we find that the ruler's optimal favoritism α is less than one. This means that as the value of the prize increases, the ruler's favoritism becomes apparent, even if showing stronger favoritism compromises his/her payoff.

In other words, even when the wasteful resource investment in society does not bring benefits to the ruler, that is, the ruler is of a benevolent type when the rent held by the ruler has large quantities and high-value characteristics, the ruler still shows his/her favoritism for contestants' wasteful spending.

Scenario (ii) Multi-prize fair ruler $\gamma = 1$.

In this scenario, the utility term in the ruler's payoff function consists of an equal share of each productive effort and unproductive efforts, that is,

$$U_D = \sum_{i \in N} \left[e_i^P + \sum_{k=1}^{K} e_i^{C_k} \right] - \frac{\omega}{2}(\alpha - 1)^2. \tag{3.34}$$

According to the contestant's budget constraint assumption $e_i^P + \sum_{k=1}^{K} e_i^{C_k} = 1$, the ruler's payoff is equivalent to $U_D = n - \frac{\omega}{2}(\alpha - 1)^2$. It is easy to see that $\alpha^* = 1$ is the only maximizer of the ruler's payoff and the maximal ruler's payoff is n. In this case, the

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contestants' optimal efforts are

$$e^{P^*} = \frac{(n-1)Z - n^2}{(K-1)n^2},$$
$$e^{C_k^*} = \frac{(n-1)V_k}{n^2 C_k} - \frac{(n-1)Z - n^2}{(K-1)n^2}, k = 1, ..., K. \tag{3.35}$$

For each $K > 1$, according to Eqn. (3.35), we observe that each contestant chooses zero productive effort whenever the total value-cost ratio approaches the population size, that is, $Z = n^2/(n-1) \to n$. The contestants chooses positive amount of productive effort whenever $Z > n$.

An economic intuition is that, when the policymaker does not discriminate between general(productive) and specific(wasteful) inputs, and if the policymaker has the power to allocate rents, then the contestants have the incentive to invest more productive efforts to obtain rents and additional personal gains as the value of prizes is higher than the population size.

It is also intuitive that the amount of each specific wasteful effort depends on the amount of the specific value-cost ratio V_k/C_k, for each $k = 1, 2, ..., K$. By the assumption that $V_1 > V_2 > ... > V_k$, we have the conclusion that $e^{C*_1} > e^{C*_2} > ... > e^{C_k^*}$, namely, contestants rank on the amount of wasteful effort invested in each specific prize based on the value of each prize.

The result of Scenario (ii) is the same as our benchmark result, that is, when the contestants' productive efforts and unproductive efforts have the same impact on the ruler's utility, the ruler will not show a favor for the contestants' unproductive efforts.

Scenario (iii) Multi-prize caring ruler $0 < \gamma < 1$. In this scenario, the utility term

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in the ruler's payoff function consists of an unequal allocation between each productive effort and unproductive effort, and the weight on the wasteful efforts is less than the weight on the productive effort. The ruler's payoff is as follows:

$$\begin{aligned} U_D &= \sum_{i \in N} \left[e_i^P + \gamma \sum_{k=1}^{K} e_i^{C_k} \right] - \frac{\omega}{2}(\alpha - 1)^2 \\ &= \left[\frac{n^2 - (n-1)Z}{n} \right] \left[\frac{\gamma K - \alpha}{K - \alpha} \right] + \frac{\gamma(n-1)Z}{n} - \frac{\omega}{2}(\alpha - 1)^2 \end{aligned} \tag{3.36}$$

Solve the ruler's problem under the constraint that $\alpha \geq 0$ and $0 < \gamma < 1$, we have

$$(\alpha - 1)(K - \alpha)^2 = \frac{(1-\gamma)K[(n-1)Z - n^2]}{n\omega}, K > 1, \omega > 0. \tag{3.37}$$

The condition that supports the existence of a unique positive real solution for α in Eqn. (3.37) is $\frac{(1-\gamma)K[(n-1)Z-n^2]}{n\omega} > 0$, which is equivalent to $(n-1)Z > n^2$.

An analogue of Proposition 3.1 is as follows:

Proposition 3.2. *Let* $0 < \gamma < 1$. *Let* $\sum_{k=1}^{K} V_k > n$. *Then there exists a unique subgame perfect Nash equilibrium with*

$$\begin{aligned} \alpha^* &= \frac{2K+1}{3} + \frac{1}{9\sqrt[3]{2}} + \frac{(K^2 - 2K + 1)}{9\hat{\delta}^{1/3}}; \\ e^{P^*} &= \frac{3\sqrt[3]{2}(2K+1) + 3\sqrt[3]{2}(K-1)^2\hat{\delta}^{-1/3} + 1}{9\sqrt[3]{2}K - 3\sqrt[3]{2}(2K+1) - 3\sqrt[3]{2}(K-1)^2\hat{\delta}^{-1/3} - 1} \left[\frac{(n-1)Z - n^2}{n^2} \right]; \\ e^{C_k^*} &= \frac{(n-1)V_k}{n^2 C_k} - \frac{9\sqrt[3]{2}[(n-1)Z - n^2]}{n^2[9\sqrt[3]{2}K - 3\sqrt[3]{2}(2K+1) - 3\sqrt[3]{2}(K-1)^2\hat{\delta}^{-1/3} - 1]}, \end{aligned} \tag{3.38}$$

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where $k = 1, ..., K$, and

$$\hat{\delta} = 3\sqrt{3}[27\hat{\sigma}^2 - 4\hat{\sigma}K^3 + 12\hat{\sigma}K(K-1) + 4\hat{\sigma}]^{1/2} + 27\hat{\sigma} - 2K^3 + 6K(K-1) + 2;$$

$$\hat{\sigma} = \frac{(1-\gamma)K[(n-1)Z - n^2]}{n\omega} > 0. \tag{3.39}$$

That is, assume that wasteful inputs instead of productive investment provide more utility to the ruler. Then when the total value of prizes is lower than the population size, the ruler favors the wasteful efforts more than the productive effort.

Proof. An analogue to Proposition 3.1 proof. □

The economic intuition indicates that when the ruler's payoff structure consists of relatively less wasteful input but more production input, and the society is full of rent, then the ruler favors the contestants' wasteful effort.

The literature points out that countries rich in natural resources are more likely to develop non-democratic regimes, which is commonly known as the resource curse. Our results confirm that when there are more rents in the economy, whereas the ruler is less likely to benefit from wasteful inputs, he/she will be more inclined to show favoritism for wasteful inputs.

Scenario (iv) Multi-prize tin-pots ruler $\gamma > 1$.

In this scenario, the utility term in the ruler's payoff function consists of an unequal allocation between productive and wasteful efforts, and the weight on the wasteful efforts

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is more than the weight on the productive effort. The ruler's payoff is as follows:

$$\begin{aligned} U_D &= \sum_{i \in N} \left[e_i^P + \gamma \sum_{k=1}^{K} e_i^{C_k} \right] - \frac{\omega}{2}(\alpha - 1)^2 \\ &= \left[\frac{n^2 - (n-1)Z}{n} \right] \left[\frac{\gamma K - \alpha}{K - \alpha} \right] + \frac{\gamma (n-1) Z}{n} - \frac{\omega}{2}(\alpha - 1)^2 \end{aligned} \tag{3.40}$$

Solve the ruler's problem under the constraint that $\alpha \geq 0$ and $\gamma > 1$, we have

$$(\alpha - 1)(K - \alpha)^2 = \frac{(\gamma - 1)K[n^2 - (n-1)Z]}{n\omega}, K > 1, \omega > 0. \tag{3.41}$$

The condition that supports the existence of a unique positive real solution for α in Eqn. (3.41) is $n^2 - (n-1)Z > 0$, which approximates to $n > Z$.

Proposition 3.3. *Let $\gamma > 1$. Let $\sum_{k=1}^{K} V_k < n$. Then there exists a unique subgame perfect Nash equilibrium with*

$$\begin{aligned} \alpha^* &= \frac{2K+1}{3} + \frac{1}{9\sqrt[3]{2}} + \frac{(K^2 - 2K + 1)}{9\hat{\delta}^{1/3}}; \\ e^{P*} &= \frac{[3\sqrt[3]{2}(2K+1) + 3\sqrt[3]{2}(K-1)^2\hat{\delta}^{-1/3} + 1][n^2 - (n-1)Z]}{n^2[3\sqrt[3]{2}(2K+1) + 3\sqrt[3]{2}(K-1)^2\hat{\delta}^{-1/3} + 1 - 9\sqrt[3]{2}K]}; \\ e^{C_k^*} &= \frac{(n-1)V_k}{n^2 C_k} - \frac{9\sqrt[3]{2}[n^2 - (n-1)Z]}{n^2[3\sqrt[3]{2}(2K+1) + 3\sqrt[3]{2}(K-1)^2\hat{\delta}^{-1/3} + 1 - 9\sqrt[3]{2}K]}, \end{aligned} \tag{3.42}$$

where $k = 1, ..., K$, and

$$\begin{aligned} \hat{\delta} &= 3\sqrt{3}[27\hat{\sigma}^2 - 4\hat{\sigma}K^3 + 12\hat{\sigma}K(K-1) + 4\hat{\sigma}]^{1/2} + 27\hat{\sigma} - 2K^3 + 6K(K-1) + 2; \\ \hat{\sigma} &= \frac{(\gamma - 1)K[n^2 - (n-1)Z]}{n\omega} > 0. \end{aligned} \tag{3.43}$$

That is, assume that productive investment instead of wasteful input provides more utility to the ruler. Then when the total value of prizes is larger than the population size, the ruler reveals favoritism toward wasteful effort more than productive effort.

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Proof. Let $\sum_{k=1}^{K} V_k < n$. Then we have $Z \equiv \sum_{k=1}^{K} \frac{V_k}{C_k} < \sum_{k=1}^{K} V_k < n$, this implies that $0 < (n-1)Z < (n-1)n < n^2$.

Accordingly, the cubic function $(\alpha-1)(K-\alpha)^2 = \frac{(\gamma-1)K[n^2-(n-1)Z]}{n\omega}$ has a unique positive real solution, that is,

$$\alpha^* = \frac{2K+1}{3} + \frac{1}{9\sqrt[3]{2}} + \frac{(K^2-2K+1)}{9\hat{\delta}^{1/3}};$$
$$\hat{\delta} = 3\sqrt{3}[27\hat{\sigma}^2 - 4\hat{\sigma}K^3 + 12\hat{\sigma}K(K-1) + 4\hat{\sigma}]^{1/2} + 27\hat{\sigma} - 2K^3 + 6K(K-1) + 2;$$
$$\hat{\sigma} = \frac{(\gamma-1)K[n^2-(n-1)Z]}{n\omega} > 0. \tag{3.44}$$

Recall that the number of prize $K > 1$ is a positive integer, then we have

$$\begin{aligned} \alpha^* &= \frac{2K+1}{3} + \frac{1}{9\sqrt[3]{2}} + \frac{(K^2-2K+1)}{9\hat{\delta}^{1/3}} \\ &> \frac{2K+1}{3} + \frac{1}{9\sqrt[3]{2}} \\ &> \frac{2+1}{3} + \frac{1}{9\sqrt[3]{2}} \approx 1 + 0.0881 > 1.0881. \end{aligned} \tag{3.45}$$

Then the optimal efforts follows. □

We have the estimation on the optimal efforts that

$$e^{P^*} = \frac{-\alpha^*}{K-\alpha^*}\left[\frac{n^2-(n-1)Z}{n^2}\right] \leq 0 \text{ if } K \geq \alpha^*. \tag{3.46}$$

That is, the optimal productive effort approaches zero when the number of prizes is larger. It follows that if $K \geq \alpha^*$, then

$$e^{C_k^*} = \frac{(n-1)V_k}{n^2 C_k} + \frac{1}{K-\alpha^*}\left[\frac{n^2-(n-1)Z}{n^2}\right] > 0. \tag{3.47}$$

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Therefore, when the number of prizes is larger, each contestant prefers more wasteful input in the prizes rather than productive investment.

Interestingly, the optimal favoritism level α^* of the ruler in Scenario (i) and Scenario (iii) is greater than 1—the benchmark optimal favoritism level—regardless of whether the ruler's payoff will be amplified or reduced by contestants' wasteful efforts input.

This result is unusual, it is even a negative result because it means that as long as a society allows people to make productive investments and obtain additional benefits, that is, when γ is neither zero nor one, the ruler has an incentive to show his/her favoritism and encourage the contestants to make wasteful efforts.

3.3.4 Rent Dissipation

The rent dissipation D^* is defined by the ratio of total cost and the value of prizes.

$$D^* \equiv n \sum_{k=1}^{K} \frac{C_k}{V_k} - n\beta e^P. \tag{3.48}$$

Then in the multi-prize case, we have

$$\begin{aligned} D^* &= n \left[\sum_{k=1}^{K} \frac{C_k}{V_k} - \frac{\alpha^*\beta((n-1)Z - n^2)}{(K-\alpha^*)n^2} \right] \\ &\geq n \left[Z - \frac{\alpha^*\beta n(Z-n)}{(K-\alpha^*)n^2} \right] \\ &\geq n \left[Z - \frac{\alpha^*\beta(Z-n)}{(K-\alpha^*)n} \right], \end{aligned} \tag{3.49}$$

where $\alpha^* > 0$ and $\beta > 0$. The amount of dissipation is positive if $Z > \frac{\alpha^*\beta(Z-n)}{(K-\alpha^*)n} > 0$, or $Z < n$ as the number of prizes K is large.

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The economic implication is that rent dissipation is inevitable if the country's total resources are small compared to its population.

3.4 Concluding Remarks

In this chapter, we consider a two-stage multi-prize contest in which a ruler chooses a weight to enlarge the players' winning probability of investing unproductive effort in the first stage, and the contestants choose a productive-wasteful effort allocation in the second stage. we show that the optimal productive effort level is almost zero whenever the number of prizes is small.

When the number of prizes goes up, the ruler has a stronger incentive to encourage the contestants to invest in wasteful efforts. The ruler also encourages the contestants to invest in wasteful efforts as the population size increases. Unfortunately, we find that in most situations the ruler encourages the contestants to invest in wasteful efforts, which explains the tinpot-like autocrats' bribery behavior and company managers' favoritism.

Our model assumes that all contestants are homogeneous, and the weight of different wasteful efforts must be treated consistently by the ruler, which is a limitation of our research. Further research can relax the contestant homogeneity assumption, or give different weights to different specific efforts to vary the contestants' winning probabilities, and there may be more different results.

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Chapter 4 The Walrasian rule in Queuing Problems with an Initial Order

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4.1 Introduction

In this chapter, we apply the concept of Walrasian equilibrium in fair queuing problems with an initial order, which is an assignment problem of a finite number of indivisible objects and a unique infinitely divisible good, and each agent holds a heterogeneous endowment. For example, situations that include returning a rental house and renting a new one, replacing a roommate, reselling concert tickets, and waiting for public service, can be modeled as a queuing problem with an initial order.

We study one of the fairness axioms in reordering problems and find the set of allocation rules satisfying Foley's envy-free concept (Foley[32]). To this end, we develop a Walrasian allocation rule which selects the envy-free outcomes in this problem.

Suppose that there is a group of agents with a heterogeneous endowment as their current objects, and each agent demands a new object. Waiting in a queue is costly for the agents, and the agents are different in the unit waiting cost. Agents have quasi-linear utility preferences and their waiting costs are linear in waiting time. A social planner intends to rearrange the initial order to improve the queue-efficiency while maintaining the fairness of each agent's position in the queue. We are interested in natural allocation rules which satisfy queue-efficiency, envy-freeness, and individual rationality.

Relative literature about this chapter is the results of Walrasian equilibrium in exchange economy and the exploration of envy-free property (Mas-Colell[48], Hildenbrand[36], Aliprantis and Burkinshaw[6], Mas-Colell, Whinston, and Green,[49]). Tadenuma[67] explores the Walrasian equilibrium in economy with indivisible goods and shows that

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there may not exist an allocation which is envy-free and efficient whenever each agent can consume more than one objects. In Chun[20]'s exploration of envy-free allocation rules in queuing problems, there is no rule satisfying the queue-efficiency, envy-freeness, and one cost monotonic property. Based on the previous literature, we study the solutions in queuing problems with an initial order that are efficient and envy-free properties, and we show the existence of allocation rules satisfying the above two axioms and an additional axiom.

The model setting in this chapter is closely related to Chun et al.[21]'s setting of reordering an existing queue. Chun et al.[21] show that four axioms, i.e., efficiency, budget balanced, strategy-proofness, and a fairness axiom, are compatible in queuing problems but are not compatible in any queuing problems with initial order. Because the two problems cannot apply to the same results, there is still a lot of room for exploration in the field of reordering problems.

The literature is also related to the Walrasian equilibrium of indivisible goods allocation with an infinitely divisible good (i.e., monetary transfer).

There is a known result that an envy-free and Pareto efficient allocation exists in an exchange economy (Schmeidler and Vind[55], Varian[72]), and Svensson[64] shows that there exists an envy-free Walrasian equilibrium for an indivisible good allocation problem with an equal endowment.

Svensson[64]'s concern is closely related to our model, which is a general setting of exchange economy with n agent, n indivisible good, and one divisible good. Compared with our result, Svensson[64]'s preference setting is more theoretical general, and the au-

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thor proposed a result that an envy-free property implies the Pareto-efficient equilibrium. In this chapter, our characterization theorem provides an if-and-only-if result.

Tadenuma and Thomson[68] study the economies of a finite number of indivisible objects and an infinitely divisible good (i.e., money) without endowments. The authors show that there is a set of envy-free solutions and identify a small set of sub-solutions since the set of envy-free solutions is quite large, and they also show that envy-freeness implies efficiency. Svensson[65] shows that in a model of a market game with indivisible goods, a Walrasian equilibrium is a solution that is Nash implementable; Svensson[66] proposes a truth-telling mechanism for queuing problem without an initial queue. Sakai[54] also shows that the no-envy solution is the only solution satisfying the Maskin-monotonicity, equal treatment of equals, and restricted continuity property in queuing problems without an initial queue.

In this chapter, we define the Walrasian rule as a solution for fair queuing problem with an initial order, and it collects the initial queue and rearranges a new queue satisfying the queue efficiency and envy-free trade property. This rule distributes the final queue and money transfer to each agent, and each transfer that the agent received is the Walrasian equilibrium price gap between the selling price of the initial position and the buying price of the final position. The Walrasian rule satisfies the participation condition, and the outcome recommended by the Walrasian rule also belongs to the core. And the main result is the characterization for the Walrasian allocation rule.

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4.2 The Model

4.2.1 Setup

Let $\mathbb{N}$ denote the set of natural numbers, and let $\mathcal{N}$ be the family of all finite nonempty subsets of $\mathbb{N}$. Let $N \in \mathcal{N}$ denote a finite set of agents who form an initial queue $\sigma^0 = (\sigma_i^0)_{i \in N} \in \Pi^N$, where Π^N denotes the set of all possible permutation over N, and each σ_i^0 denotes the agent i's endowment position. The agents in N are in need of a facility that serves only one agent at a time. The server serves only one agent at a time by the queue order, and the agents' unit waiting costs is a list of real numbers $\theta = (\theta_i)_{i \in N} \in \mathbb{R}_+^N$. To wrap-up, each agent $i \in \mathbb{N}$ is characterized by two kinds of characteristics: (θ_i, σ_i^0), where $\theta_i \in \mathbb{R}_+$ is his unit waiting cost, and σ_i^0 is his initial position.

A reordering problem for N is defined as a list (θ, σ^0), where $\theta = (\theta_i)_{i \in N} \in \mathbb{R}_+^N$ is the vector of unit waiting costs and $\sigma^0 \in \Pi^N$ is the initial queue for N. Let $\mathcal{Q}^N$ be the family of all reordering problems for N, and let $\mathcal{Q} \equiv \cup_{N \in \mathcal{N}} \mathcal{Q}^N$ be the family of all reordering problems.

Given $N \in \mathcal{N}$, an allocation for a problem $(\theta, \sigma^0) \in \mathcal{Q}^N$ is a pair $(\sigma, t) \in \Pi^N \times \mathbb{R}^N$, where $\sigma : \mathcal{Q}^N \to \Pi^N$ denotes a queuing rule, $t : \mathcal{Q}^N \to \mathbb{R}^N$ is a transfer rule, for all $i \in N$. Let $\sigma_i(\theta, \sigma^0)$ denote agent i's position in the final queue, and $t_i(\theta, \sigma^0)$ is agent i's the monetary transfer.

For each $\sigma \in \Pi^N$, let $P_i(\sigma) = \{j \in N | \sigma_j < \sigma_i\}$ denote the set of agents containing all agent i's predecessors in σ, and let $F_i(\sigma) = \{j \in N | \sigma_j > \sigma_i\}$ denote the set of agents containing all agent i's followers in σ.

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The agent's utility is defined as moving distance units multiplied by unit waiting costs and minus money transfers. Formally, let $N \in \mathcal{N}$ be a set of agents, for each problem $(\theta, \sigma^0) \in \mathcal{Q}^N$ and each $i \in N$, assume that each agent $i \in N$ has a quasi-linear utility function from consuming the bundle (σ, t), which is given by

$$U_i(\sigma, t; \theta, \sigma^0) = (-\sigma_i\theta_i + t_i) - (-\sigma_i^0\theta_i + 0) = (\sigma_i^0 - \sigma_i)\theta_i + t_i.$$

In this chapter we require that each allocation does not contain any outside monetary transfer to the agents.

Definition 4.1. *An allocation is **feasible** if the sum of all the transfers is less than zero. Thus, the set of feasible allocations $\mathcal{E}(\theta, \sigma^0)$ consists of all pairs $(\sigma, t) \in \Pi^N \times \mathbb{R}^N$ such that*

$$\sum_{i \in N} t_i \leq 0.$$

An allocation rule, associates to each reordering problem, indicates the allocation outcome of the agents. The agents have no drop out option, which means that each reordering problem is fixed population.

Definition 4.2. *An **allocation rule** is a correspondence $\varphi : \mathcal{Q} \twoheadrightarrow \bigcup_{N \in \mathcal{N}} \bigcup_{(\theta, \sigma^0) \in \mathcal{Q}^N} \mathcal{E}(\theta, \sigma^0)$, which associates with each problem $(\theta, \sigma^0) \in \mathcal{Q}^N$ a non-empty subset $\varphi(\theta, \sigma^0)$ of feasible allocations $\mathcal{E}(\theta, \sigma^0)$.*

Given an allocation $(\sigma, t) \in \varphi(\theta, \sigma^0)$, the pair (σ_i, t_i) represents the position σ_i of agent i in the final queue and his transfer t_i in problem (θ, σ^0). If the monetary transfer of an agent is positive, then this agent receives a compensation from other agents. If it

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is negative, then he has to pay that amount as compensation to other agents. My generic notation for rules is φ. For each $N' \subset N$, each $(\theta, \sigma^0) \in \mathcal{Q}^N$, and each pair $i, j \in N$ with $i \neq j$, I denote $(\theta_k)_{k \in N'}$ and $(\theta_k)_{k \in N \setminus \{i\}}$ by $\theta_{N'}$ and θ_{-i}, respectively.

A queue σ is called **efficient** for a reordering problem $(\theta, \sigma^0) \in \mathcal{Q}^N$ if and only if σ maximizes the coalition value of the problem, that is, $\sigma = \arg\max \sum_{i \in N}(\sigma_i^0 - \sigma_i)\theta_i$. Let $E(\theta)$ be the set of all efficient queue. The efficient queue is not unique if there are two agents with equal unit waiting costs in the reordering problems.

4.2.2 The Walrasian Rule

For each finite set of agents $N \in \mathcal{N}$ and $|N|$ goods, given a price vector $p \in \mathbb{R}^N$ and the agents' utilities $(U_i)_{i \in N}$, a certain bundle of goods $\sigma_i \in N$ is in the demand set of agent $i \in N$ if $U_i(\sigma_i, p; \cdot) \geq U_i(\sigma_i^{'}, p; \cdot)$ for the other bundles $\sigma_i^{'}$. Given a reordering problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, for each $i \in N$, let $D_i(p)$ be the demand set that

$$D_i(p) = \{\sigma_j \in N : U_i(\sigma_i, p; \theta, \sigma^0) \geq U_i(\sigma_i^{'}, p; \theta, \sigma^0).$$

Definition 4.3. *A Walrasian equilibrium (WE) is a price vector p and a feasible allocation σ such that $\sigma_i \in D_i(p)$, for all $i \in N$.*

In reordering problems, the number of goods to be allocated is equal to the number of agents, and each agent consumes one and only one position. The literature often restricts every good which has a positive price is fully allocated and other unallocated items are priced at 0. But due to the specificity of our problem, it is reasonable to set the price of the last position, which is least desired by everyone, to 0. In addition, it is well known that

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a Walrasian equilibrium allocation is efficient, and the set of price vectors is non-empty, which forms a complete lattice(Shapley and Shubik[59], Mishra and Talman[50]).

The following is an example of two-person case Walrasian equilibrium in a reordering problem.

Example 4.1.

Consider a set of agents $N = \{1, 2\}$. Assume that $\theta = (4, 2)$ be a queuing problem with an initial order $\sigma^0 = (2, 1)$. Then we find the $WE(\theta, \sigma^0)$ as follows:

Let $p = (p_1, p_2)$ be a price vector. Agent 1's demand set is

$$D_1(p) = \{\sigma_1 \in \{1, 2\} : U_1(\sigma_1, p; \theta, \sigma^0) = (2 - \sigma_1)\theta_1 + p_2 - p_1 \geq 0\},$$

and agent 2's demend set is

$$D_2(p) = \{\sigma_2 \in \{1, 2\} : U_2(\sigma_2, p; \theta, \sigma^0) = (1 - \sigma_2)\theta_2 + p_1 - p_2 \geq 0\},$$

because there are only two positions in this queue and each agent holds the initial position receives zero net utility, i.e., $(\sigma_i^0 - \sigma_i^0)\theta_i + p_{\sigma_i^0} - p_{\sigma_i^0} = 0$. Thus, given the price vector, to choose a position to maximize each agent's net utility, we solve both equations and find that

$$\theta_1 + p_2 \geq p_1 \geq p_2 + \theta_2,$$

$$p_1 - \theta_2 \geq p_2 \geq p_1 - \theta_1.$$

The prices are non-negative. Consider $p_A = (\theta_1, 0)$, then in this case, we have $U_1(\sigma_1 = 1, p_A; \theta, \sigma^0) = 0$ and $U_2(\sigma_2 = 2, p_A; \theta, \sigma^0) = \theta_1 - \theta_2$. Consider $p_B = (\theta_2, 0)$, then in this case, we have $U_1(\sigma_1 = 1, p_B; \theta, \sigma^0) = \theta_1 - \theta_2$ and $U_2(\sigma_2 = 2, p_B; \theta, \sigma^0) = 0$. The set of

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$WE(\theta, \sigma^0)$ is depicted as Figure 4.1.

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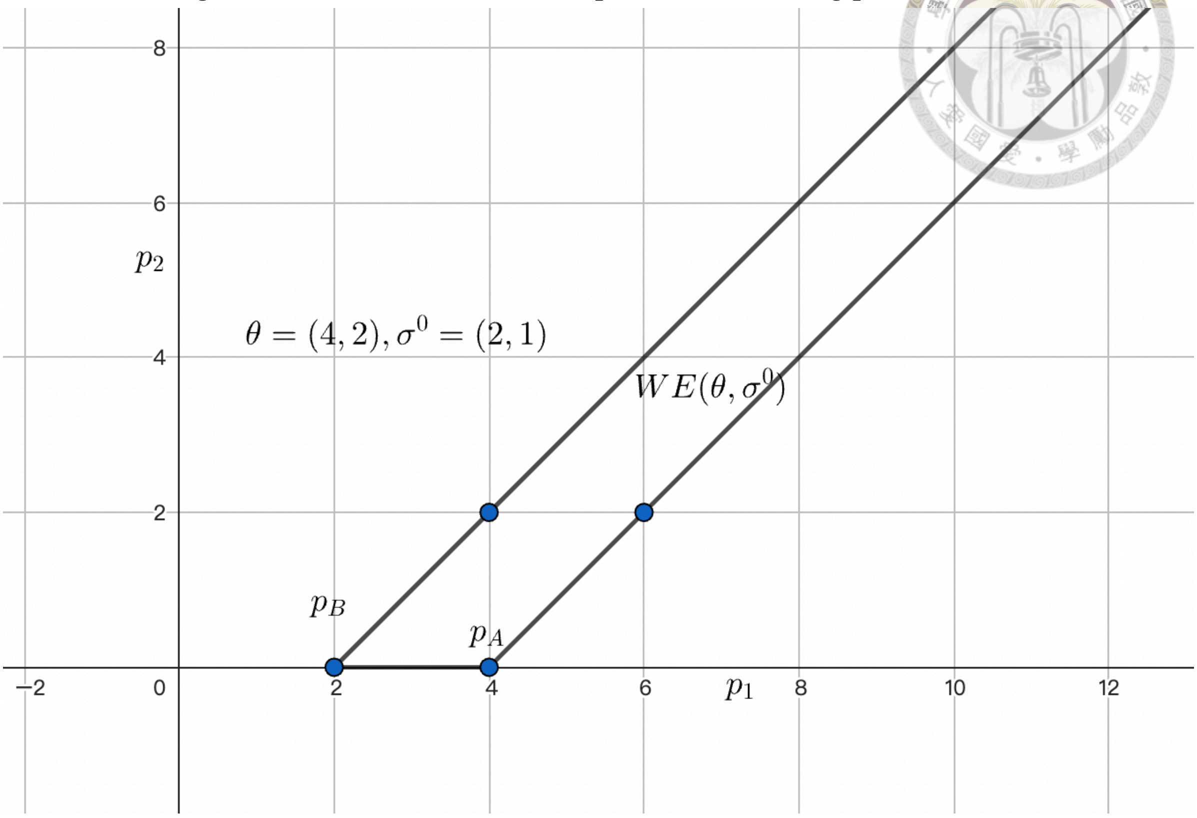
Figure 4.1: Lattice nature of WE prices in reordering problems



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Now we construct the **Walrasian rule**, denoted by φ^W, which is associated with the Walrasian equilibrium, as a solution concept for reordering problems. In reordering problems, a Walrasian equilibrium is the set of queue-price pairs (σ, p) such that each agent weakly prefers the current allocation to any other allocation bundle. Formally, for any problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, let $WE(\theta, \sigma^0)$ denote the corresponding set of Walrasian equilibrium, that is,

$$WE(\theta, \sigma^0) = \left\{ (\sigma, p) \in \Sigma(\theta) \times \mathbb{R}^N_+ | \sigma_i = \arg\max_{j \in N} [(\sigma^0_i - \sigma^0_j)\theta_i + (p_{\sigma^0_i} - p_{\sigma^0_j})], \text{ for all } i \in N \right\}.$$

The vector $p = (p_1, ..., p_n) \in \mathbb{R}^N_+$ denotes the price vector of each queuing position among the queue σ. Each element p_k of the price vector denotes the trading cost of the $k-$th position among the queue σ. If agent i chooses to sell or buy the $k-$th position, then agent i receives or pays the corresponding price p_k, for $k = 1, 2, ..., |N|$. Thus, the total money transfer that agent i received is $p_{\sigma^0_i} - p_{\sigma^0_j}$, where σ^0_i is i's initial position and σ_i is i's final position. Recall that the net utility of agent i is defined by

$$U_i(\sigma_i, p; \theta, \sigma^0) = (\sigma^0_i - \sigma_i)\theta_i + t_i, \text{ where } t_i = p_{\sigma^0_i} - p_{\sigma_i}, \forall i \in N \tag{4.1}$$

Then we say that an allocation rule φ is a **Walrasian rule** whenever φ is defined by

$$\varphi^W(\theta, \sigma^0) = \left\{ (\sigma, p) \in \mathcal{E}(\theta, \sigma^0) | (\sigma, p) \in WE(\theta, \sigma^0) \right\}.$$

In reordering problems, rules satisfying the basic axioms like queue efficiency and individual rationality are our concerned. The first property requires a queue minimizing

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the total waiting costs over the set of agents; the second property requires the net utility is non-negative. Queue efficiency ensures the reordered queue is by the order of highest waiting cost to the lowest.

Given any reordering problem, we seek an allocation which maximizes the possible value of the whole set of agents. Thus, queue efficiency ensures the reordered queue is by the order of highest waiting cost to the lowest. This property requires a queue minimizing the total waiting costs over the set of agents.

Definition 4.4. *(Queue Efficiency) A rule φ is efficient if and only if for each $N \in \mathcal{N}$, each $(\theta, \sigma^0) \in \mathcal{Q}^N$, and each $(\sigma, p) \in \varphi(\theta, \sigma^0)$, $\sigma \in E(\theta)$.*

Lemma 4.1. *Let $(\theta, \sigma^0) \in \mathcal{Q}^N$ be a reordering problem. Then*

$$(\sigma, p) \in \varphi^W(\theta, \sigma^0) \Rightarrow \sigma \in E(\theta)$$

Proof of Lemma 4.1: see Appendix A..

We show that the set of Walrasian equilibria can be rewritten as an adjoint pair of prices and unit waiting costs.

Lemma 4.2. *For each problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, after the position transaction between agent i and agent j, the price difference is between θ_i and θ_j, for any pair of adjacent agents $i, j \in N$. That is,*

$$\varphi^W(\theta, \sigma^0) = \left\{(\sigma, p) \in E(\theta) \times \mathbb{R}^N_+ | \theta_i \le p_{\sigma_j} - p_{\sigma_i} \le \theta_j, \text{ where } \sigma_i = \sigma_j + 1\right\}.$$

Proof of Lemma 4.2: see Appendix A.

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According to Lemma 4.2, the final queue recommended by the Walrasian rule is selected from the set of efficient queues, so we rewrite the Walrasian rule as follows:

Lemma 4.3. *For each problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, the utility of the agent holding the efficient position is at least as high as holding other positions. That is, for any pair of agents $i, j \in N$, we have*

$$\varphi^W(\theta, \sigma^0) = \left\{(\sigma, p) \in E(\theta) \times \mathbb{R}_+^N | -\sigma_i\theta_i - p_{\sigma_i} \geq -\sigma_j\theta_i - p_{\sigma_j}, \textit{ for all } \sigma_j < \sigma_i\right\}.$$

Proof of Lemma 4.3: see Appendix A.

We also show that the outcome recommended by the Walrasian rule is in the core. For a cooperative game with a transferable utility, the core is a set of payoff vectors that cannot be improved upon by any subset of coalition from the population(Shapley and Scarf[58]).

Given a reordering game $(\theta, \sigma^0) \in \mathcal{Q}^N$ with a transferable utility, a utility vector $(U_i)_{i \in N}$ is in the core if it satisfies the following conditions: (1) $\sum_{i \in N} U_i = \sum_{i \in N}(\sigma_i^0 - \sigma_i)\theta_i$; (2) for any subsets $S \subset N$: $\sum_{i \in S}(\sigma_S^0(s) - \sigma_S(i))\theta_S(i) \leq \sum_{i \in S} U_i$.

Lemma 4.4. *Let $q = (\theta, \sigma^0) \in \mathcal{Q}^N$ be a reordering problem. Then each outcome recommended by the Walrasian rule is in the core.*

Proof of Lemma 4.4: see Appendix A.

4.2.3 The Axioms

Due to the setting of queuing problems with an initial queue, any allocation rule that does not satisfy the participation constraint will lead to the result of maintaining the

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status quo, that is, rules that conform to individual rationality are necessary. The rules satisfying the individual rationality, which requires the net utility is non-negative. In the reordering problem, different from the queuing situations without an initial order, agents have a reserved level of utility and choose not to join the reordering process whenever the net utility is negative.

Definition 4.5. *(Individual Rationality, IR) A rule φ satisfies individual rationality if and only if for each $N \in \mathcal{N}$, each $(\theta, \sigma^0) \in \mathcal{Q}^N$, each $(\sigma, p) \in \varphi(\theta, \sigma^0)$, and each $i \in N$, $U_i(\sigma, p; \theta, \sigma^0) \geq 0$.*

The individual rationality property requires that each agent' s utility in the new queue is at least as large as the utility she would receive if the jobs were processed according to the initial queue and no transfers are given. If a new queue does not satisfy this property, then agents may not agree to join the reordered system.

The notion of fairness we highlighted is the **envy-free property**. We define the envy-free property as an envy-free trade during the trading process that agents have no incentive to exchange with the other agent's position-monetary transfer bundle. Namely, the envy-free trade property requires that an allocation is stable for all agents.

Definition 4.6. *(Envy-Free Trade, EFT) For each $N \in \mathcal{N}$, each problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, φ such that $\varphi(\theta, \sigma^0) = (\sigma, p)$. A rule φ is envy-free if and only if for each pair of agents $i, j \in N$, $U_i(\sigma_i, p_{\sigma_i}; \theta, \sigma^0) \geq U_i(\sigma_j, p_{\sigma_j}; \theta, \sigma^0)$, or equivalently, $-\sigma_i\theta_i - p_{\sigma_i} \geq -\sigma_j\theta_i - p_{\sigma_j}$.*

According to Lemma (4.3), the allocation outcomes recommended by the Walrasian rule satisfy the envy-free trade property.

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In queuing problem with an initial priority, the envy-free trade property involves the efficiency and the individually rational property under the quasi-linear utility setting.

Lemma 4.5. *For each problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, if an allocation rule φ satisfying the envy-free trade property, then φ is efficient.*

Proof of Lemma 4.5: see Appendix A.

Then we show that we can exclude the individual rationality axiom whenever providing a characterization.

Lemma 4.6. *For each problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, if an allocation rule φ satisfying the envy-free property, then φ is individually rational.*

Proof of Lemma 4.6: see Appendix A.

Meanwhile, we also require the budget balanced property to ensure there is no welfare to be wasted.

Definition 4.7. *(Budget Balanced) An allocation rule φ satisfies budget balanced if and only if for all for each $N \in \mathcal{N}$, each $(\theta, \sigma^0) \in \mathcal{Q}^N$, $(\sigma, p) \in \varphi(\theta, \sigma^0)$ such that $\sum_{i \in N} t_i = 0$. In particular, $\sum_{i \in N} p_{\sigma_i^0} = \sum_{i \in N} p_{\sigma_i}$.*

Since the monetary transfer we define is the difference of the selling price and the buying price, the nature of budget balanced is naturally satisfied.

We also show that we can exclude the budget balance axiom whenever providing a characterization.

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Lemma 4.7. *For each problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, if an allocation rule φ satisfying the envy-free trade property, then φ is budget balanced.*

Proof of Lemma 4.7: see Appendix A.

4.3 The Characterization

In this section, we provide a characterization of the allocation rule associated with the Walrasian equilibrium.

Theorem 4.1. *Let $N \in \mathcal{N}$ be a set of agents, for each reordering problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, let φ be an allocation rule. Then the following two statements are equivalent:*

(i) The allocation $\varphi(\theta, \sigma^0)$ selects all combinations of queues σ and vectors of prices $p \geq 0$ such that $(\sigma, p) \in WE(\theta, \sigma^0)$. That is, $\varphi = \varphi^W$.

(ii) φ satisfies the envy-free trade (EFT) property.

Proof. (i) $\Rightarrow$ (ii): For a set of agents $N \in \mathcal{N}$, each reordering problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, let φ^W be the Walrasian rule such that $(\sigma, p) \in \varphi^W(\theta, \sigma^0)$.

Then by definition of $WE(\theta, \sigma^0)$ and Lemma 4.3, we have

$$\sigma_i = \arg\max_{j \in N}[(\sigma_i^0 - \sigma_j^0)\theta_i + p_{\sigma_i^0} - p_{\sigma_j^0}], \forall i \in N.$$

$$\Rightarrow (\sigma_i^0 - \sigma_i)\theta_i + p_{\sigma_i^0} - p_{\sigma_i} \geq (\sigma_i^0 - \sigma_j)\theta_i + p_{\sigma_i^0} - p_{\sigma_j} \forall j \neq i, j \in N$$

$$-\sigma_i\theta_i - p_{\sigma_i} \geq -\sigma_j\theta_i - p_{\sigma_j} \forall j \neq i, j \in N,$$

which is equivalent to the EFT.

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(ii) $\Rightarrow$ (i): For a set of agents $N \in \mathcal{N}$, each reordering problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, let $\varphi(\theta, \sigma^0) = (\sigma, t)$ satisfying the envy-free trade (EFT) property. By the EFT, we have

$$-\sigma_i \theta_i - p_{\sigma_i} \geq -\sigma_j \theta_i - p_{\sigma_j} \forall j \neq i, j \in N,$$

and this implies that $(\sigma, p) \in \varphi^W(\theta, \sigma^0)$ by definition and Lemma 4.3. Therefore, we concludes that $\varphi = \varphi^W$. □

Theorem 4.1 states that the Walrasian rule can be uniquely characterized by the envy-free trade property. This result shows that among solutions that satisfy the envy-free property, the Walrasian rule we define is economically intuitive and simple, and echoes the existing literature.

4.4 Concluding Remarks

In this chapter, we investigate the envy-freeness axiom in the context of queuing problems with an initial priority. Based on the concept of Walrasian equilibrium, we developed a new solution to characterize this property, which is also restricted as a single-value solution by adding some selection criteria. The Walrasian allocation rule is the only solution that is equivalent to the envy-free trade solution.

The solution concept mentioned in this chapter can also be further expanded on other queuing models. For example, considering the Walrasian rule for multiple-server queuing problems with initial orders, or relaxing the unit waiting cost assumption in queuing problems may lead to new results.

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In addition, fair queuing problems with an initial queue do not need to consider the situation when the number of agents exceeds the number of queue positions. However, if agents who originally had queue positions are allowed to leave, the welfare changes are also worthy of further exploration. The case that each agent requires different units of services is also a direction that can be extended and explored.

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Appendix A — Proofs

Proof of Proposition 2.1.

Proof. The existence of a Nash equilibrium can be shown by construction. Let $r = 1$, $p = [1, 0, 0, 0]$, and $|\mathcal{N}| > 2$. Denote that $N = |\mathcal{N}|$. Then no matter what the true state is, the optimal player distribution among the two tables is $(k^*, N - k^*)$, where k^* is the maximal number of players to achieve the FMA.

Claim: IF the priority rule is σ^{FF}, then $a_i = F$ is an equilibrium strategy for both signal type of player $i \in \mathcal{N}$.

For a strong type player $i \in \mathcal{N}$, without loss of generality we assume that the signal received is $s_i = 2$. Then player i follows the FF priority rule if

$$EU_i(F|s_i = 2, \sigma^{FF}) - EU_i(L|s_i = 2, \sigma^{FF}) \geq 0.$$

$$EU_i(F|s_i = 2, \sigma^{FF}) - EU_i(L|s_i = 2, \sigma^{FF})$$

$$= \sum_{\theta \in \Theta} \sum_{k=1}^{N} g_i(\theta) Pr(\sigma^{FF}(i) = k | a_i = F, s_i = 2) \frac{R_{x_i}(\theta)}{n_{x_i}} - \sum_{\theta \in \Theta} g_N(\theta) \frac{R_{x_i}(\theta)}{n_{x_i}}$$

$$= \sum_{\theta \in \Theta} \sum_{k=1}^{N} g_{k_i}(\theta) Pr(\sigma^{FF}(i) = k | a_i = F, s_i = 2) \frac{R_{x_i}(\theta)}{n_{x_i}} - \frac{2}{N}$$

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$$= \frac{1}{N/2-1} - \frac{2}{N} = \frac{2}{N-2} \geq 0 \text{ if } N \text{ is odd;}$$

the expected utility is zero if N is an even integer. The weak type player case is analog.

Thus, players have no profitable incentive to deviate from the priority rule σ^{FF}, which shows the equilibrium exists.

$\square$

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Proof of Theorem 2.1.

Proof. Let parameter r such that $0 \leq r < 1/N$. Assume, for the sake of contradiction, that there is a player chooses to follow the FL rule as a dominant strategy. Then given an arbitrary position k, there exists a player $i \in \mathcal{N}$ with $|\frac{1}{k^*} - \frac{r}{N-k^*}| > 0$ whenever $s_i \in S = \{2, -2\}$, and $|\frac{1}{k^*} - \frac{r}{N-k^*}| = 0$ with $s_i \in W = \{1, -1\}$ under the FL rule. That is,

$$g_k(\theta_1|s_i \in S, \sigma^{FL}) \times 1 - g_k(\theta_2|s_i \in S, \sigma^{FL}) \times r = 0,$$

$$g_k(\theta_1|s_i \in W, \sigma^{FL}) \times 1 - g_k(\theta_2|s_i \in W, \sigma^{FL}) \times r > 0.$$

Thus,

$$[g_k(\theta_1|s_i \in W, \sigma^{FL}) - g_k(\theta_1|s_i \in S, \sigma^{FL}))$$

$$+((g_k(\theta_2|s_i \in S, \sigma^{FL}) - g_k(\theta_1 2|s_i \in W, \sigma^{FL}))r] > 0,$$

which is impossible because for both types of players, the belief terms are negative under the parameter range r. $\square$

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Proof of Proposition 2.2.

Proof. Firstly, we consider the case $N = 2$.

Claim: For all $p \in \Delta(p)$ and $r \geq 1/2$, each second player's best response in stage 2 is to choose table $x_2 \neq x_1$.

Proof of the Claim: By backward induction, in stage 2, the expected payoff difference of choosing table (i) $x_2 = x_1$ and (ii) $x_2 \neq x_1$ is

$$EU_2(x_2 = x_1|\mathcal{H}_2, \sigma^{FF}, r) - EU_2(x_2 \neq x_1|\mathcal{H}_2, \sigma^{FF}, r)$$

$$= [g_2(\theta_1)\frac{R_{t_1}(\theta_1)}{2} + g_2(\theta_2)\frac{R_{t_1}(\theta_2)}{2}] - [g_2(\theta_1)r + g_2(\theta_2)]$$

$$= \frac{1+2r}{4}g_2(\theta_2) - \frac{2r-1}{2} \leq 0$$

as $g_2(\theta_2) \in (0, 1/2)$ and $r \in [1/2, 1]$. This completes the proof.

Thus, the best response of player 2 in table selection is $x_2 \neq x_1$, as $r \in [1/2, 1]$. Choosing a former waiting group at stage 1 to improve the probability to be the first to select a larger table x_i and get the non-negative value of FMA is a dominant strategy for each player $i \in \mathcal{N}$, which is irrelevant to the type of player.

We consider cases $|\mathcal{N}| = N > 2$ inductively.

Claim': Assume that $p \in \Delta(p)$ and $r \geq 1/2$, each latter player $i \in \mathcal{N}$'s best response in stage 2 is choosing the table $x_i \neq x_j$ satisfying the optimal number of players k^*, for all $i \neq j, i, j \in \mathcal{N}$.

Proof of Claim' is similar to the Proof of Claim. Then let us consider an induction argument on an $N-$person case.

Let player i be the last player in stage 1. Given that the other $N-1$ players choose a

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former queuing group (group F), player i must be assigned to the last position if player i chooses a later group (group L) under the FF rule. Thus, because of $r \in [1/2, 1]$ the best response of table selection is $x_i \neq x_j$, for all $j \in \mathcal{N} \setminus \{i\}$. Choosing group F at stage 1 improves the probability of being the first to select a larger table t_i to achieve the optimal player numberk^*. Following the FF rule leads to a higher expected payoff, which is irrelevant to the type of player. $\square$

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Proof of Proposition 2.3.

Proof. Suppose not. Then there is at least one player $i \in \mathcal{N}$ with a signal $s_i \in \{S, W\}$ who has a profitable incentive to choose $a_i = L$ under the FF priority rule.

Denote k be the maximal number of players who receives a positive FMA. For the case where $|\mathcal{N}| = 2$, choosing $a_i = L$ under the FF rule leads to $k_i = 2$ for sure, and this implies that player i must be expected to select table $x_i \neq x_j$ $(j \neq i, j \in \mathcal{N})$ to achieve his/her highest expected payoff as $r > 1/2$. Player i lost the FMA in this case. In the same situation, choosing $a_i = F$ under the FF rule increases the probability of being assigned to $k_i = 1$, which increases the expected payoff, then $a_i = L$ is dominated by $a_i = F$.

For $|\mathcal{N}| > 2$, assume, without loss of generality, that player i's belief about $\theta_1 = g_h$, $h = 1, 2, ..., N$, denotes player i's assigned position. Under the FF rule, player i does not observe any other player's signal and expects that the distribution of signals is uniform across the signal set. Then the average gap between g_h and g_h' is small for $h - h' < N$.

Let q_k be the probability of receiving the value of FMA when the player is in position k, $k = 1, 2, ..., N$. Then player i's expected payoff from choosing the L queuing group under the FF rule is

$$EU_i(a_i = L|\sigma^{FF}, r > 1/2, p, \theta) = q_N(g_h\frac{1}{n_1} + (1 - g_h)\frac{r}{n_2}) + (1 - q_N)(\min\{\frac{1}{n_1}, \frac{r}{n_2}\})$$

$$\leq q_l(g_l\frac{1}{n_1} + (1 - g_l)\frac{r}{n_2}) + (1 - q_l)(\min\{\frac{1}{n_1}, \frac{r}{n_2}\}) = EU_i(a_i = F|\sigma^{FF}, r > 1/2, p, \theta),$$

where $q_l > q_N \geq 0$. The no deviation result holds for the worst signal quality case. $\square$

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Proof of Lemma 4.1.

Proof. Suppose not. Then there exists a problem $(\theta, \sigma^0) \in \mathcal{Q}^N$ with a Walrasian allocation $(\sigma, p) \in \varphi^W(\theta, \sigma^0)$ such that $\sigma \notin E(\theta)$. Thus, there is a pair of agents $i, j \in N$ such that $\theta_i > \theta_j$ and $\sigma_i > \sigma_j$.

The definition of the set of Walrasian equilibrium requires that a queue maximizes each agent's utility, that is,

$$(\sigma_i^0 - \sigma_i)\theta_i + (p_{\sigma_i^0} - p_{\sigma_i}) = \max_{k \in N}\{(\sigma_i^0 - \sigma_k)\theta_i + (p_{\sigma_i^0} - p_{\sigma_k})\}.$$

Then we have

$$-\sigma_i\theta_i - p_{\sigma_i} \geq -\sigma_j\theta_i - p_{\sigma_j}$$

and

$$-\sigma_j\theta_j - p_{\sigma_j} \geq -\sigma_i\theta_j - p_{\sigma_i}.$$

Combine the above two inequalities we have

$$\theta_i \leq \frac{p_{\sigma_j} - p_{\sigma_i}}{\sigma_i - \sigma_j} \leq \theta_j,$$

which contradicts with the assumption $\theta_i \geq \theta_j$. Therefore, for all $(\sigma, p) \in \varphi^W(\theta, \sigma^0)$, we have

$$WE(\theta, \sigma^0) = \left\{(\sigma, p) \in E(\theta) \times \mathbb{R}_+^N | - \sigma_i\theta_i - p_{\sigma_i} = \max_{j \in N}(-\sigma_j\theta_i - p_{\sigma_j})\right\}.$$

□

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Proof of Lemma 4.2.

Proof. Let $(\sigma, p) \in \varphi^W(\theta, \sigma^0)$ for any problem $(\theta, \sigma^0) \in \mathcal{Q}^N$ and let $i, i' \in N$ s.t. $\sigma_i = \sigma_{i'} + 1$. Because $\sigma \in E(\theta)$, we have

$$\sigma_i \theta_i + p_{\sigma_i} \leq \sigma_{i'} \theta_i + p_{\sigma_{i'}}$$

and

$$\sigma_{i'} \theta_{i'} + p_{\sigma_{i'}} \leq \sigma_i \theta_{i'} + p_{\sigma_i}.$$

These imply that

$$\theta_i \leq p_{\sigma_{i'}} - p_{\sigma_i} \leq \theta_{i'},$$

because of $\sigma_i - \sigma_{i'} = 1$. □

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Proof of Lemma 4.3.

Proof. Let $\sigma_i = \sigma_j + k$ for some $k > 0$. Lemma 4.2 shows that

$$-\sigma_i\theta_i - p_{\sigma_i} = -(\sigma_i - 1)\theta_i - \theta_i - p_{\sigma_i}$$

$$\geq -\sigma_{i'}\theta_i - (p_{\sigma_{i'}} - p_{\sigma_i}) - p_{\sigma_i}, \text{ where } \sigma_{i'} = \sigma_i - 1,$$

$$= -\sigma_{i'}\theta_i - p_{\sigma_{i'}}$$

$$\geq -\sigma_{i''}\theta_i - p_{\sigma_{i''}}, \text{ where } \sigma_{i''} = \sigma_{i'} - 1,$$

$$\geq ...$$

This completes the proof. □

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Proof of Lemma 4.4.

Proof. For any problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, $N \in \mathcal{N}$, let $\varphi^W(\theta, \sigma^0) = (\sigma, p)$ such that $(\sigma, p) \in WE(\theta, \sigma^0)$. Recall that each agent's utility is $U_i(\sigma_i, t_i; \theta, \sigma^0) == (\sigma_i^0 - \sigma_i)\theta_i + t_i = (\sigma_i^0 - \sigma_i)\theta_i + p_{\sigma_i^0} - p_{\sigma_i}$, for all $i \in N$. Then we have

$$\sum_{i \in N} U_i(\sigma, p; \theta, \sigma^0) = \sum_{i \in N} (\sigma_i^0 - \sigma_i)\theta_i + \sum_{i \in N} p_{\sigma_i^0} - \sum_{i \in N} p_{\sigma_i}$$

$$= \sum_{i \in N} (\sigma_i^0 - \sigma_i)\theta_i.$$

Let S be a subset of N. If $|S| = 1$, then the agent in coalition S has zero net utility because there is no rearrangement in a one person queue. For $|S| \geq 2$, denote the reduced reordering problem as $q_S = ((\theta_S(i))_{i \in S}, (\sigma_S^0(i))_{i \in S}) \in \mathcal{Q}^S$ such that the profile of waiting costs is a segment from the $|N|-$person profile of waiting costs.

Then we have

$$\sum_{i \in S} U_i(\sigma_S(i), t_S(i); \theta_S, \sigma_S^0) = \sum_{i \in S} (\sigma_S^0(i) - \sigma_S(i))\theta_S(i) + \sum_{i \in S} p_{\sigma_S^0(i)} - \sum_{i \in S} p_{\sigma_S(i)}$$

$$= \sum_{i \in S} (\sigma_S^0(i) - \sigma_S(i))\theta_S(i) \leq \sum_{i \in N} (\sigma_i^0 - \sigma_i)\theta_i,$$

where $p_{\sigma_S^0(i)}$ and $p_{\sigma_S(i)}$ denote the prices of positions among the initial queue σ_S^0 and the final queue σ_S, respectively, for all $i \in S$. □

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Proof of Lemma 4.5.

Proof. Let φ be an allocation rule satisfying the envy-free trade property. For each set of agents $N \in \mathcal{N}$, any problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, let $\varphi(\theta, \sigma^0) = (\sigma, p)$. For any pair of agents $i, j \in N$, φ satisfies the envy-free trade means that

$$U_i(\sigma_i, p_{\sigma_i}; \theta, \sigma^0) \geq U_i(\sigma_j, p_{\sigma_j}; \theta, \sigma^0)$$

$$\Rightarrow (\sigma_i^0 - \sigma_i)\theta_i + p_{\sigma_i^0} - p_{\sigma_i} \geq (\sigma_i^0 - \sigma_j)\theta_i + p_{\sigma_i^0} - p_{\sigma_j}$$

$$\Rightarrow -(p_{\sigma_i} - p_{\sigma_j}) \geq (\sigma_i - \sigma_j)\theta_i.$$

Similarly, we have

$$U_j(\sigma_j, p_{\sigma_j}; \theta, \sigma^0) \geq U_j(\sigma_i, p_{\sigma_i}; \theta, \sigma^0)$$

$$\Rightarrow -(p_{\sigma_i} - p_{\sigma_j}) \leq (\sigma_i - \sigma_j)\theta_j.$$

Combining the above two equations we have

$$\theta_i \leq \frac{-(p_{\sigma_i} - p_{\sigma_j})}{\sigma_i - \sigma_j} \leq \theta_j,$$

because $\theta_j \geq \theta_i \geq 0$, either $p_{\sigma_i} \leq p_{\sigma_j}$ and $\sigma_i > \sigma_j$ holds, or $p_{\sigma_i} \geq p_{\sigma_j}$ and $\sigma_i < \sigma_j$ holds.

If the former holds and $\theta_j \geq \theta_i \geq 0$, then we conclude that $\sigma \in E(\theta)$; otherwise, if $p_{\sigma_i} \geq p_{\sigma_j}$ and $\sigma_i < \sigma_j$ and $\theta_j \geq \theta_i \geq 0$ holds, then it contradicts the EFT assumption, which is impossible. Therefore, we have $\theta_j \geq \theta_i$ and $\sigma_i \geq \sigma_j$, for all $i, j \in N$. That is, $\sigma \in E(\theta)$ and φ is an efficient rule.

$\square$

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Proof of Lemma 4.6.

Proof. For each problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, let φ be an allocation rule which satisfies the envy-free trade (EFT), and let $\varphi(\theta, \sigma^0) = (\sigma, p)$. Then for each pair of agents $i, j \in N$ with $\sigma_j = \sigma_i^0$, EFT implies that

$$-\sigma_i\theta_i - p_{\sigma_i} \geq -\sigma_j\theta_i - p_{\sigma_j}$$

$$\Leftrightarrow -\sigma_i\theta_i - p_{\sigma_i} \geq -\sigma_j^0\theta_i - p_{\sigma_i^0}$$

$$\Leftrightarrow (\sigma_i^0 - \sigma_i)\theta_i + p_{\sigma_i^0} - p_{\sigma_i} = U_i(\sigma_i, p_{\sigma_i}; \theta, \sigma^0) \geq 0.$$

Thus, φ satisfies the individual rationality, which completes this proof. $\square$

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Proof of Lemma 4.7.

Proof. For each problem $(\theta, \sigma^0) \in \mathcal{Q}^N$, let φ be an allocation rule which satisfies the envy-free trade (EFT), and let $\varphi(\theta, \sigma^0) = (\sigma, p)$.

We prove this lemma by contradiction. Assume that φ is not budget balanced. Then we have $\sum_{i \in N} p_{\sigma^0} \neq \sum_{i \in N} p_\sigma$. According to Feasibility we have $\sum_{i \in N} t_i = \sum_{i \in N} p_{\sigma^0} - p_\sigma < 0$, then there exists at least a pair of agents $i, j \in N$, $i \neq j$ such that $p_{\sigma_i} = p_{\sigma_j^0} < p_{\sigma_j}$ and $\sigma_j^0 = \sigma_j$. It is easy to see that $p_{sigma_j} > p_{\sigma_i}$ implies that $\sigma_j < \sigma_i$ and then $\theta_j > \theta_i$ because σ is an efficient queue. Accordingly,

$$\begin{aligned} -\sigma_j\theta_i - p_{\sigma_j} &> -\sigma_j\theta_i - p_{\sigma_j^0} \text{ by } p_{\sigma_j} > p_{\sigma_j^0} \\ &> -\sigma_i\theta_i - p_{\sigma_j^0} \text{ by } \sigma_i > \sigma_j \\ &= -\sigma_i\theta_i - p_{\sigma_i} \text{ because } p_{\sigma_i} = p_{\sigma_j^0}, \end{aligned} \tag{A.2}$$

which violates the EFT property and leads to a contradiction. □

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