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紅超巨星的物理起源及其質量散失

The Physical Origin of Red Supergiants and Their Mass
Loss

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致謝

多少回，在深夜或凌晨離開研究室，看見高掛在天上的紅超巨星參宿四，頓時驚覺，即使走出門外也未能脫離論文主題。或苦惱於未解的謎團，或欣喜於稍早突來的頓悟，總要讚嘆宇宙造化扣人心弦的壯闊景致，以及自然生生滅滅的循環演化。我們正好位於太陽系的適居帶上，加上諸多幸運的機緣巧合，於是能夠以理性來探究創生我們的宇宙。研究宇宙的過程雖然艱辛，卻是如此難能可貴。

完成這本博士論文，是極其艱難而彌足珍貴的歷程。論文題目挑戰數十年未解的天文物理難題，期間遭遇的阻礙遠超過預期，甚至多次將自己的理論推翻。所幸，在師長、同僚與親友的支持下，終於逐一突破難關，今夏將博士論文收尾，邁向人生的下一階段。

非常感謝我的碩士與博士指導教授朱有花老師，不辭辛勞地栽培學生。老師平時極其忙碌，卻願意為學生細心指導研究，一字一句幫忙修訂論文與觀測計畫，並以幽默風趣的語言，將繁複的概念說到學生能夠理解。遇到如此有耐心栽培學生的師長，實在幸運。師徒之間，我不僅習得老師的學識，更是努力效法老師嚴謹治學的態度。此外，一路上老師不吝於傳授經驗與提供研究資源，讓我受益無窮。老師常分享學術界的小故事，不僅是茶餘飯後的閒談，更是為後輩指引明燈。

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容。每有新的研究想法，老師總是願意即時展開討論。每回討論皆是高度「燒腦」的對話，但是老師總是很有耐心地一起設法釐清難題。也要感謝老師給予寬闊自由的創造空間，讓我得以將研究範疇向知識邊界開拓，挑戰全新的領域。老師也時時為學生著想，盡可能提供最好的研究資源，且不論遭遇瓶頸或成功破關時，皆不斷給予鼓勵。由衷感激老師一路上的支持，才能突破種種挑戰。

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摘要

紅超巨星是演化晚期的大質量恆星，膨脹至約 1000 倍太陽半徑，具有顯著的質量散失，且為核心坍縮超新星的常見前身星，對於周遭星際介質影響巨大。雖然紅超巨星階段在恆星演化模型中總是能夠再現，其物理起源長期存在爭論，至今仍不清楚。透過恆星模型的建造與分析，本論文全面探討恆星天文物理中長期存在的一個問題：「為何恆星會演化成為紅巨星或紅超巨星？」

首先，本研究使用 MESA 程式建構約 2,000 個恆星演化模型，探討不同質量與不同金屬量之大質量恆星演化，並從這組模型中發現了一個臨界金屬量——唯有金屬量 $Z \gtrsim 0.001$ 的恆星可演化為紅超巨星，並歷經龐大的質量散失。這些模型亦用來計算恆星回饋，並與宇宙學模擬中常用的經驗式比較。

本文進一步分析 MESA 模型，探討何種機制決定恆星能否演化為紅超巨星。結果顯示一個修正版的「鏡像原理」：恆星外層的移動方向始終與其內部邊界的移動方向相反。此內部邊界由氫燃燒殼層的外緣所定義。根據此鏡像原理，本文建立出一套通用的判準及演化路線圖，判斷恆星能否演化為紅巨星或紅超巨星。此外，當恆星演化至接近紅巨星或紅超巨星時，可找到一個劇烈的結構轉變階段。

為深入考察紅巨星及紅超巨星的物理，本文以數值方法解出穩態恆星結構方程組。結果證實恆星外層的穩態解符合修正版之鏡像原理，並顯示紅巨星及紅超巨星不但對應於恆星外層膨脹的極限半徑，亦為結構上獨特的階段，與較緻密恆

星的結構截然不同。這些穩態解也合理解釋，巨星及超巨星分為紅色及藍色兩個分支。



接著，為進一步探討臨界金屬量的物理起源，本研究利用更大量的模型組，測試多個物理參數的效應。結果顯示，對於給定的恆星質量，在主序終點時存在一個半徑閾值——唯有半徑超過此閾值的恆星可演化為紅超巨星。金屬量之所以影響演化結果，是藉由影響不透明度及核反應，因而改變恆星半徑，於是決定恆星能否在主序終點達到紅超巨星形成的半徑閾值。

本文藉由數值方法解出恆星結構與演化，建立一套完整的物理框架，解釋恆星外層朝向紅巨星及紅超巨星階段的演化。此框架不僅在理論上提供後主序星演化機制的理解，亦能說明觀測上巨星及超巨星在赫羅圖的分布。此外，臨界金屬量的確認及恆星回饋的計算，則對早期宇宙的貧金屬星演化有深遠意義。

關鍵字：紅超巨星、紅巨星、恆星演化、恆星物理、質量散失



Abstract

Red supergiants (RSGs) are evolved massive stars that expand to $\sim 1,000$ solar radii, exhibit substantial mass loss, and are common progenitors of core-collapse supernovae, significantly shaping their surrounding interstellar environments. Although the RSG phase is consistently reproduced in stellar evolution models, its physical origin has been long debated and remains unclear. Through the construction and analysis of stellar models, this thesis provides a comprehensive investigation into a longstanding question in stellar astrophysics: why do stars evolve into red giants (RGs) or RSGs?

As a first step, a grid of approximately 2,000 stellar evolution models is computed using the MESA code to explore the evolution of massive stars across a wide range of masses and metallicities. From this grid, a critical metallicity is identified: only stars with $Z \gtrsim 0.001$ evolve into RSGs and undergo substantial mass loss. These models are also used to calculate stellar feedback, which is compared with prescriptions commonly adopted in cosmological simulations.

To explore the mechanism that determines whether a star evolves into the RSG phase, further analysis of MESA models suggests a refined version of the “mirror principle”: the stellar envelope consistently moves in the opposite direction to its inner boundary, defined by the outer edge of the hydrogen-burning shell. This principle leads to general criteria and an evolutionary roadmap for determining whether a star will evolve into an RG or RSG. Moreover, a dramatic structural transition is identified as stars approach the RG/RSG phase.

To further investigate the underlying physics of the RG/RSG phase, steady-state stellar structure equations are solved numerically. The resulting envelope solutions validate the refined mirror principle and show that the RG/RSG phase corresponds to both a limiting radius for envelope expansion and a distinct structural phase, markedly different from that of more compact stars. These solutions also naturally explain the bifurcation between red and blue branches of giants and supergiants.

The physical origin of the critical metallicity is further investigated using extended grids of models designed to test the effects of various physical parameters. The results reveal a threshold radius at the terminal-age main sequence (TAMS) for a given stellar mass; only stars exceeding this radius evolve into RSGs. Metallicity governs the evolution outcome by influencing stellar radius through its effects on opacity and nuclear burning,

thereby determining whether the threshold TAMS radius for RSG formation is reached.

By numerically solving stellar structure and evolution, this thesis establishes a comprehensive physical framework for explaining envelope expansion toward the RG/RSG phase. This framework not only offers theoretical insight into the evolutionary mechanisms of post-main-sequence stars, but also accounts for the observed distribution of giant and supergiant stars on the Hertzsprung–Russell diagram. The identification of a critical metallicity and the computation of stellar feedback further provide implications for the evolution of metal-poor stars in the early universe.

Keywords: Red supergiant stars, Red giant stars, Stellar evolution, Stellar physics, Mass loss



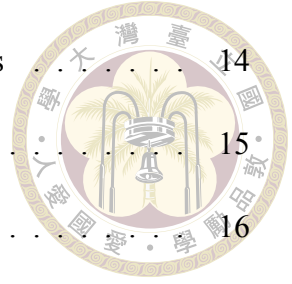




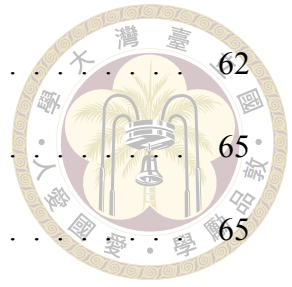
Contents

	Page
致謝	iii
摘要	v
Abstract	vii
Contents	xi
List of Figures	xv
List of Tables	xix
Chapter 1 Critical Metallicity of Cool Supergiant Formation and Mass Loss	1
1.1 Introduction	2
1.2 Methodology	5
1.2.1 MESA Code	5
1.2.2 Physical Processes	5
1.2.3 Mass-Loss Prescriptions	6
1.2.4 Stellar Evolution Simulations	8
1.3 Stellar Evolution and Mass Loss Rate	9
1.3.1 Stellar Evolution	9
1.3.2 Evolution of Mass Loss Rate	11
1.4 Total Mass Loss and Metallicity	13

1.4.1	Initial Mass and Metallicity Dependence of Mass Loss	14
1.4.2	Critical Metallicity	15
1.4.3	Mass Loss from Different Wind Schemes	16
1.4.4	Cool Supergiants and the Critical Metallicity	18
1.5	Feedback from Stellar Winds	22
1.5.1	Kinetic Energy Released by Winds	22
1.5.2	Wind Feedback from a Star Cluster	24
1.6	Discussions	27
1.6.1	Two Effects of Metallicity on RSG Evolution	27
1.6.2	Possible Uncertainties	29
1.6.2.1	Uncertainties from stellar models	29
1.6.2.2	Uncertainties from mass-loss prescriptions	32
1.6.3	Implications of the Critical Metallicity	34
1.7	Conclusions	36
	Appendix A: Wind Prescriptions	37
Chapter 2	Expansion Mechanism Toward Red Giants and Superigants	49
2.1	Introduction	50
2.2	Methods	52
2.2.1	Grid of different masses and metallicities	52
2.2.2	Experiments with $25 M_{\odot}$ stars	53
2.3	Physical Conditions for Transition to Red Supergiants	54
2.3.1	Energy Budget	55
2.3.2	The Mirror Principle	59



2.3.3	Role of Envelope Base Luminosity	62
2.4	General Criteria for Stars to Turn Red	65
2.4.1	Evolution Across Different Masses and Metallicities	65
2.4.1.1	Examples of 5, 15, and $25 M_{\odot}$	66
2.4.1.2	Relationship Across the Full Grid of Models	67
2.4.2	Roadmap Toward Red Giants or Supergiants	69
2.5	Physics of the Mirror Principle	70
2.6	Effects of Envelope Properties	72
2.6.1	The Role of Opacity	72
2.6.2	Structural Transition into the RG/RSG Phase	74
2.7	Discussions and Conclusions	76
Chapter 3	Steady-State Solutions of Supergiant Stellar Structure	91
3.1	Introduction	92
3.2	Methods	94
3.2.1	Polytropic models	97
3.2.2	Realistic models	98
3.3	Results from Polytropic Models	101
3.4	Results from Realistic models	105
3.4.1	Structural Transition Toward the Red Giant/Supergiant Phase	106
3.4.2	Physical Origin of the Red Giant/Supergiant Solution	109
3.4.3	Red and Blue Solutions	114
3.5	Discussions	115
3.6	Conclusions	116



Chapter 4 Physical Mechanism of the Critical Metallicity



4.1	Introduction	125
4.2	Models	126
4.3	Numerical Experiments on Opacity and Nuclear Reaction Rates . . .	132
4.3.1	Grid (b): Metallicity and Opacity	132
4.3.2	Grid (c): Hydrogen Burning Rates and Opacity	133
4.3.3	Grid (d): Shell Hydrogen Burning Rate and Opacity	135
4.3.4	Grid (e): Helium Burning Rate and Opacity	137
4.4	Direct Indicator for RSG Formation	137
4.4.1	Threshold for Evolution into the RSG Phase at a Constant Mass . .	138
4.4.2	Threshold TAMS Radii for Different Masses	140
4.5	Metallicity Effect on Stellar Radius	142
4.5.1	Metallicity Dependence of Stellar Evolutionary Tracks	142
4.5.2	Homology Relation between Metallicity and Radius	144
4.6	Classification of Supergiant Evolution Pathways	148
4.7	Envelope Expansion and Core Condition	152
4.8	Discussions and Conclusions	156
References		171

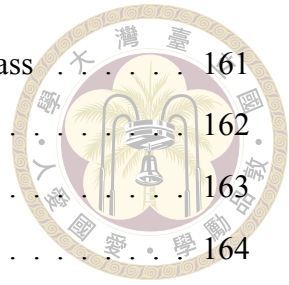


List of Figures

1.1	HR diagram of massive stars and their mass-loss histories	13
1.2	Total mass loss as a function of the initial mass	15
1.3	Total mass loss as a function of initial metallicity	16
1.4	Fraction of total mass loss in each of 1900 models	17
1.5	Contributions of the hot, cool, and WR winds to the total mass loss	18
1.6	Fractions of mass loss to the initial masses caused by the hot, cool, and WR winds	41
1.7	Evolutionary tracks and mass-loss rates of 25 and 50 $M_{\odot}$ stars	42
1.8	Maximum stellar radius of each model in the $M - Z$ grid	43
1.9	Evolution of wind terminal velocity and mechanical luminosity	44
1.10	Total kinetic energy released by stellar winds throughout the stellar lifetimes	45
1.11	Integrated mass-loss rate and energy-injection rate from stellar winds in a star cluster	46
1.12	Mass and kinetic energy released by stellar winds in a star cluster	47
2.1	Schematic diagram of the post-main-sequence stellar structure	55
2.2	Evolution and energy budget of 25 $M_{\odot}$ stars with metallicity $Z = 0.001$ but different He-burning rates	60
2.3	Evolution of the stellar radius alongside He core radius and H-burning shell radius	61
2.4	Evolution of radius and luminosity at interface between the H-burning shell and the envelope	64
2.5	Evolution of energy generation rates	80
2.6	Evolution of 5, 15, and 25 $M_{\odot}$ stars with different metallicities	81

2.7	Evolution of central pressure	82
2.8	Scatter plots showing characteristic radii at key time point during core-He burning	82
2.9	Flow chart illustrating the evolutionary pathways toward the RG/RSG phase	83
2.10	Temperature and pressure at the shell-envelope boundary and the center of the star	84
2.11	Pressure and density profiles during the envelope expansion phase	85
2.12	Models with different metallicities and opacity parameters	86
2.13	Evolution of surface luminosities (L_s) and nuclear energy generation rates (L_{nuc})	86
2.14	Kippenhahn diagrams for stellar models with 5, 15, and 25 $M_{\odot}$	87
2.15	Structural profiles during the phase transition into RGs/RSGs	88
2.16	Opacity from the gs98 opacity table in MESA	89
3.1	Schematic diagram illustrating the problem setup of steady-state models .	95
3.2	Polytropic models of different γ	102
3.3	Pressure and density profiles for polytropic models with $\gamma = 1.3$	105
3.4	Steady-state solutions for 25 $M_{\odot}$ stars	119
3.5	Steady-state solutions for 5 $M_{\odot}$ stars	120
3.6	Structural profiles for red supergiants	121
3.7	Experiments on convection, surface pressure, and opacity	122
3.8	Structural profiles for models with a minimum opacity floor of $\kappa \geq 0.1 \text{ cm}^2 \text{ g}^{-1}$	123
3.9	MESA "OP_gs98" Opacity	124
4.1	Maximum stellar radius during the core He-burning stage for Grid (b) . .	133
4.2	Maximum stellar radius during the core He-burning stage for Grid (c) . .	134
4.3	Maximum stellar radius during the core He-burning stage for Grid (d) . .	135
4.4	Maximum stellar radius during the core He-burning stage for Grid (e) . .	136
4.5	ZAMS and TAMS temperatures and pressure	159
4.6	ZAMS and TAMS radii	160
4.7	Distribution of TAMS radii and He core radii	160

4.8	Relationship between the TAMS radius and initial stellar mass	161
4.9	HR diagram for different opacity parameters	162
4.10	HR diagram for different CNO-cycle reaction rates	163
4.11	HR diagram for different metallicities	164
4.12	Coefficient of radius from homology relations	165
4.13	Classification of evolutionary pathways	166
4.14	Evolution of central pressure	167
4.15	Evolution of nuclear energy generation rates	168
4.16	Evolution of central pressure and He-burning luminosity	169
4.17	Flow chart illustrating the evolutionary stages of the stellar core and envelope	170







List of Tables

4.1	Summary of model grids and varied parameters	131
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Chapter 1 Critical Metallicity of Cool Supergiant Formation and Mass Loss

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Abstract for Chapter 1

This chapter systematically studies the relation between metallicity and mass loss of massive stars. We perform one-dimensional stellar evolution simulations and build a grid of ~ 2000 models with initial masses ranging between 11 and 60 $M_{\odot}$ and absolute metallicities Z between 0.00001 and 0.02. Steady-state winds, comprising hot main-sequence winds and cool supergiant winds, are the main drivers of the mass loss of massive stars in our models. We calculate the total mass loss over the stellar lifetime for each model. Our results reveal the existence of a critical metallicity Z_c at $Z \sim 10^{-3}$, where the mass loss exhibits a dramatic jump. If $Z > Z_c$, massive stars tend to evolve into cool supergiants, and a robust cool wind is operational. In contrast, if $Z < Z_c$, massive stars usually remain as blue supergiants, wherein the cool wind is not activated and the mass loss is

generally weak. Moreover, we calculate the wind feedback in a $10^5 M_{\odot}$ star cluster with the Salpeter initial mass function. The kinetic energy released by winds does not exhibit any significant transition at Z_c because the wind velocity of a cool supergiant wind is low and contributes little to the kinetic energy. The effects of critical metallicity provide implications for the fates of metal-poor stars in the early universe.

1.1 Introduction

Massive stars lose a substantial amount of mass via stellar winds, which inject energies and enriched material into the interstellar medium (ISM). Main-sequence OB stars can produce *hot winds* that are driven by the radiation pressure of the line emission (Puls et al. 2008; Vink 2022, for review), and red supergiants (RSGs) produce even stronger *cool winds* that are driven by the dust continuum (Decin 2021; Willson 2000, for review). The mass loss from these winds influences the evolutionary tracks and the fates of massive stars (Langer 2012; Smith 2014; Vink et al. 2015) and reshapes the ambient environment (e.g., Chu 2008).

The mass from the stripped-off envelopes of massive stars form circumstellar media (CSM). When a massive star explodes as a supernova (SN), it may interact with the CSM produced by its progenitor star. This interaction plays an important role in the SN observables (e.g., Chen et al. 2020; Maeda et al. 2015; Moriya et al. 2017; Smith 2014). If the SN explosion was preceded by a blue supergiant (BSG) or Wolf-Rayet (WR) phase, its fast wind would have swept up the CSM to form a wind-blown bubble. The SN ejecta and shock wave will encounter the circumstellar bubble, and strongly affect the dynamical evolution of SNR (e.g., Dwarkadas 2005; Ou et al. 2018; Patnaude et al. 2017). Therefore,

stellar winds have a profound impact on SNe and their resulting SNRs, which drive the ISM ecosystem.



On a larger scale, stellar winds serve as a critical ingredient of stellar feedback in galaxies ([Abbott 1982](#); [Freyer et al. 2003](#); [Leitherer et al. 1992](#)). Although the contribution of stellar winds is thought to be less powerful than that of SNe (e.g., [Dale 2015](#); [Walch, & Naab 2015](#)), some ISM simulations indicate that stellar winds are more efficient than SNe in disrupting giant molecular clouds ([Fierlinger et al. 2016](#); [Rey-Raposo et al. 2017](#); [Rogers, & Pittard 2013](#)) and retaining energies in superbubbles ([Krause et al. 2013](#)). For cosmological simulations, stellar feedback is essential for reproducing the observed star-formation rates in galaxies, and appropriate expressions of stellar mass loss are often required (e.g., [Agertz, & Kravtsov 2015](#); [Emerick et al. 2019](#); [Hopkins et al. 2022, 2018, 2014](#); [Marinacci et al. 2019](#); [Pillepich et al. 2018](#); [Schaye et al. 2015](#); [Springel et al. 2018](#); [Stinson et al. 2006](#); [Wang et al. 2015](#)).

To evaluate the energy feedback of massive stars, we need to consider the total mass loss and total kinetic energy injected by stellar winds. The mass loss rate ($\dot{M}$) of a stellar wind depends on metallicity (Z), and the dependence is typically expressed as $\dot{M} \sim Z^m$, where m is a constant (e.g., [Mauron & Josselin 2011](#); [Mokiem et al. 2007](#); [Vink et al. 2001](#)). In the aforementioned cosmological simulations, a single-value Z appropriate for a particular epoch is used for the $\dot{M}$ and energy feedback estimates. It is well known that Z can strongly impact the evolutionary tracks of stars ([El Eid et al. 2009](#); [Langer 2012](#); [Maeder 2009](#)), and Z varies along the stellar evolution tracks. It is conceivable that the dependence of $\dot{M}$ on Z varies along the stellar evolution tracks. To estimate the total mass loss (ΔM) of a massive star, it is necessary to integrate the $\dot{M}$ for its contemporary Z over the entire evolutionary track. Likewise, the total kinetic energy injected by a massive star's

stellar winds has to be determined by integrating the wind mechanical luminosity over the evolutionary track using $\dot{M}$ and wind velocity appropriate for their contemporary Z .

Previous calculations of ΔM of massive stars have been carried out for a wide range of initial masses (e.g., [Beasor & Davies 2018](#); [Limongi 2017](#); [Renzo et al. 2017](#)). It is found that different wind prescriptions cause $\sim 50\%$ uncertainties in the resulting ΔM ([Renzo et al. 2017](#)). Recently, [Fichtner et al. \(2022\)](#) calculated the total mass and energy released by stellar winds within a star cluster and showed that they generally increase with metallicity; they also found that the rate of increase is quite different for single and binary stars. The above studies either considered an entire cluster or included only a small number of discrete Z values; thus they do not allow a detailed examination of metallicity effects.

We have systematically studied the metallicity effects on the total mass loss and kinetic energy release of massive stars by employing the MESA code to perform a series of stellar evolution simulations. We establish models of single non-rotating stars with wind physics. The numerical methods and stellar wind prescriptions are described in Section 1.2. Based on this model grid, we describe the general features of stellar evolution in Section 1.3. Then in Section 1.4, we present the main results of the total mass loss as a function of the initial mass and metallicity of massive stars. We report a *critical metallicity* (Z_c) of mass loss and show that it is associated with the cool supergiant formation. In Section 1.5 we calculate the energy feedback of winds using our grid of models. Finally, we discuss the implications of our results in Section 1.6 and present the conclusions in Section 1.7. In the Chapter 4, we will further investigate the physical mechanism of Z_c and explain it according to the fundamental understanding of cool supergiant formation.

1.2 Methodology



In this section, we introduce the stellar evolution code for building our models, describe the prescriptions of stellar mass loss, and summarize the parameter space of our models.

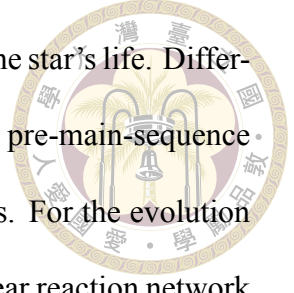
1.2.1 MESA Code

We perform massive-star evolution simulations using the Modules for Experiments in Stellar Astrophysics (MESA; [Paxton et al. 2019, 2018, 2015, 2013, 2011](#)) version No. 10108. We primarily use MESArstar, a 1D stellar evolution code that can model stellar evolution from the pre-main-sequence stage to the zero-age main sequence (ZAMS) stage, and from the ZAMS to the iron core-collapse stage. It hydrodynamically evolves stars using microphysics such as nuclear reaction networks, equations of state, and opacities, which are the key physics of stellar evolution and are available in MESA.

MESA provides several stellar wind functions that can be used to calculate the mass-loss rates using stellar parameters at each timestep during the stellar evolution. When a mass-loss routine is turned on, the code adjusts the stellar structure by rescaling the mass coordinates of the mass cells, according to the mass-loss rate given by the wind functions ([Paxton et al. 2011](#)).

1.2.2 Physical Processes

Our simulations include two steps. First we evolve a pre-main-sequence star to reach the ZAMS stage, and then, we use the ZAMS model as the initial condition in the stellar



evolution and evolve the model from the main sequence to the end of the star's life. Different nuclear reaction networks are employed in these two steps. For the pre-main-sequence evolution, we simply adopt the “basic” network with eight isotopes. For the evolution from the ZAMS stage onward, we employ the more complicated nuclear reaction network “approx21 (Cr60),” which includes 21 isotopes ¹.

In the stellar evolution simulations, we mainly use the default parameters for massive stars in the “inlist_massive_defaults” file in the MESAstar module. Additionally, the Ledoux criterion and Henyey's mixing-length theory (Henyey et al. 1965) are applied, and the mixing-length scaling parameter α (i.e., the ratio of mixing length to pressure scale height) is set at 1.5. The Type 2 OPAL opacity tables (Iglesias & Rogers 1996) are applied during and after the helium burning. In addition to the default parameters, we limit the change of helium abundance per timestep to $\leq 10\%$ to prevent abrupt changes.

1.2.3 Mass-Loss Prescriptions

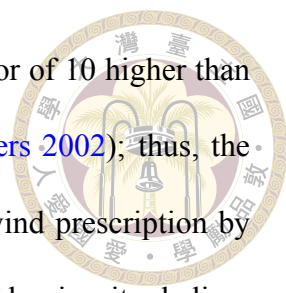
Although the mass-loss rates can be calculated using theoretical approaches (e.g., Vink et al. 2001, 2000), such calculations are time-consuming and only available for hot winds. Instead, simple formulae of the mass-loss rates derived from observations or theoretical calculations are usually adopted in stellar evolution. Most of these widely used mass-loss prescriptions have been built in as modules in MESA. In our stellar evolution simulations, we apply the hot wind prescription of Vink et al. (2001, 2000, hereinafter, V01), cool wind prescription of de Jager et al. (1988, hereinafter, dJ88), and WR wind prescription of Nugis & Lamers (2000, hereinafter, NL00). These prescriptions are de-

¹¹H, ³He, ⁴He, ¹²C, ¹⁴N, ¹⁶O, ²⁰Ne, ²⁴Mg, ²⁸Si, ³²S, ³⁶Ar, ⁴⁰Ca, ⁴⁴Ti, ⁴⁸Cr, ⁵²Fe, ⁵⁴Fe, ⁵⁶Fe, ⁵⁶Ni, ⁶⁰Cr, proton, neutron

scribed below.

For hot OB stars, metal ions in the stellar atmospheres absorb photons in the ultra-violet (UV) resonance lines and are radiatively accelerated, yielding steady-state winds (Castor et al. 1975; Lucy & Solomon 1970; Puls et al. 2008). Using multi-line radiative transfers calculated by the Monte Carlo methods (Abbott & Lucy 1985), Vink et al. (2001, 2000) presented a grid of wind models, fitted the models with a linear regression, and established a mass-loss prescription for hot stars with effective temperatures $T_{\text{eff}} > 10^4$ K.

When a massive star evolves into an RSG, its cool wind is driven by the radiation pressure on dust grains, similar to the winds from the asymptotic-giant-branch stars. To date, although empirical laws have been established, no well-established theoretical predictions for cool winds exist, because the origin and structure of RSG winds are very complex and the cool wind mechanism is poorly understood (Smith 2014). Among the mass-loss rates suggested by the various empirical laws, up to a factor of 10 scatter has been seen (dJ88; Beasor et al. 2020; Beasor & Davies 2018; Goldman et al. 2017; Humphreys et al. 2020; Maun & Josselin 2011; Nieuwenhuijzen & de Jager 1990; Reimers 1975; van Loon et al. 2005). In our simulations, we adopt the most commonly used cool-wind prescription by dJ88, which is an empirical function based on the fitting of the observed mass-loss rates of a set of O–M stars. Although $\dot{M} \sim Z^m$ has been included in some recent cool wind functions (e.g., Groh et al. 2019; Maun & Josselin 2011), the exponent m is uncertain. Recent observations have shown that the mass-loss rates of RSGs are nearly independent of metallicity (Beasor et al. 2020; Goldman et al. 2017). Therefore, we adopt the dJ88 prescription without metallicity dependence available in MESA, similar to other works such as Choi et al. (2016).



WR stars have $T_{\text{eff}} > 10^4$ K, but their mass-loss rates are a factor of 10 higher than those of hot O stars exhibiting the same luminosity (Nugis & Lamers 2002); thus, the hot-wind prescription is not suitable for them. We adopt the WR wind prescription by NL00, which is an empirical law of mass-loss rate as a function of luminosity, helium abundance, and metallicity. It was derived from the observations of 64 Galactic WR stars with the correction of clumping in the wind. This prescription is supported by an optically thick radiation-driven wind theory (Nugis & Lamers 2002).

The hot, cool, and WR wind schemes operate under different stellar evolution stages. The switches between different wind schemes depend on T_{eff} and the surface hydrogen abundance (X_H) of the simulated stars. The wind prescriptions are explained in detail in Appendix A.

In the “Dutch” hot/WR wind scheme in MESA, a “low-T” regime is present where the dJ88 prescription is adopted rather than the V01 prescription. For simplicity, we use the term “hot wind” to denote only the mass loss derived from the V01 prescription, and use the term “cool wind” to denote all the mass loss derived from the dJ88 prescription.

1.2.4 Stellar Evolution Simulations

We establish a set of models to investigate the influence the initial mass and metallicity on the stellar mass loss. In our grid, the initial mass values are sampled successively in the range of 11–60 $M_{\odot}$ with 1- $M_{\odot}$ steps. Moreover, 38 different values of initial absolute metallicities Z are chosen: $Z = (1, 2, \dots, 9) \times 10^{-5}$, $(1, 2, \dots, 9) \times 10^{-4}$, $(1, 2, \dots, 9) \times 10^{-3}$, and $(1.0, 1.1, 1.2, \dots, 2.0) \times 10^{-2}$. For reference, the solar metallicity is $Z_{\odot} = 1.34 \times 10^{-2}$ (Asplund et al. 2009). Overall, the total number of models in our grid is 1900.

Our stellar models are typically evolved until the iron core-collapse stage. However, some models with initial masses of $11\text{--}17 M_{\odot}$ stall at the carbon or oxygen burning stages, usually because of abrupt changes in abundance that cannot be easily treated numerically. As these stars have already passed the RSG phase, we adopt the masses at this point as the final masses.

We perform our simulations using the TIARA-TC cluster ² maintained by the Institute of Astronomy and Astrophysics, Academia Sinica (ASIAA). This cluster has 50 nodes, 600 CPU cores, and 2.3 TB memory. Our grid simulations were completed in about 100,000 core-hours.

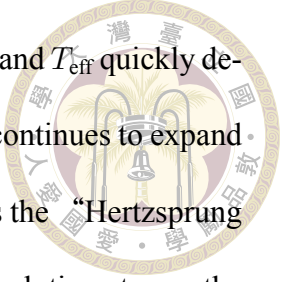
1.3 Stellar Evolution and Mass Loss Rate

Stellar mass loss depends on the physical parameters of stars, which vary along the stellar evolution. In this section, we first describe the stellar evolution and then their associated mass loss.

1.3.1 Stellar Evolution

We perform simulations of stellar evolution with different initial masses and metallicities. Their evolutionary tracks in the theoretical Hertzsprung–Russell (HR) diagram, i.e., luminosity (L) versus T_{eff} diagram, are shown in the left panels of Figure 1. During the main-sequence phase of $20 - 60 M_{\odot}$ stars, T_{eff} decreases by 0.1–0.25 dex and L increases by 0.3–0.15 dex over the main-sequence lifetime of $\sim 8\text{--}4$ Myr. A star reaches the terminal-age main sequence (TAMS) when the hydrogen in its stellar core is exhausted.

²<https://www.tiara.sinica.edu.tw/facility.php>



At this time, a hydrogen-burning shell forms, the star starts expanding, and T_{eff} quickly decreases. Then, the core-helium burning starts, and the outer envelope continues to expand and becomes much cooler. In the HR diagram, the star quickly crosses the “Hertzsprung gap” and moves to the right side. After these post-main-sequence evolution stages, the star becomes a supergiant. Supergiants with $\log(T_{\text{eff}}/\text{K}) < 3.6$ (or $T_{\text{eff}} \lesssim 4,000 \text{ K}$) are often called RSGs, $3.6 < \log(T_{\text{eff}}/\text{K}) < 3.8$ (or $4,000 \text{ K} \lesssim T_{\text{eff}} \lesssim 6,300 \text{ K}$) are called yellow supergiant (YSGs), and $\log(T_{\text{eff}}/\text{K}) > 3.8$ (or $T_{\text{eff}} \gtrsim 6,300 \text{ K}$) are called BSGs (e.g., [Georgy 2012](#)).

A star’s lowest T_{eff} in its evolutionary track depends on its initial mass and metallicity. As shown in Figure 1.1, for stars with $Z = 0.02$, those with initial masses $\gtrsim 50 M_{\odot}$ can only become YSGs instead of RSGs. Furthermore, comparison of the models of $Z = 0.02$ and $Z = 0.0005$ shows that the evolutionary tracks of lower- Z stars usually end at higher T_{eff} . This feature is critical in the relation between mass loss and metallicity, and will be elaborated in later sections. For convenience, we use the term “cool supergiant” as long as the dJ88 wind prescription is fully active, i.e. $T_{\text{eff}} < 10,000 \text{ K}$, or $\log(T_{\text{eff}}/\text{K}) < 4.0$.

During the advanced burning stages, stars with initial masses $\gtrsim 50 M_{\odot}$ can reach luminosities $\gtrsim 10^6 L_{\odot}$. These stars can be regarded as luminous blue variables (LBVs) ([Humphreys, & Davidson 1994](#)), but our simulations do not include eruptive mass loss, which is a common feature of LBVs. Furthermore, some LBVs lose their envelopes and evolve back to a hotter region in the HR diagram to become WR stars. We use the criterion of surface hydrogen abundance (X_S) below 0.4 to identify WR stars in our work.

1.3.2 Evolution of Mass Loss Rate



As a star evolves through different burning phases, its T_{eff} and radius vary, resulting in different types of stellar winds and mass-loss rates. As described in Section 1.2.3, the mass-loss rates are calculated using a combination of mass-loss prescriptions chosen according to T_{eff} and X_S . In the HR diagrams in Figure 1.1, most high- Z and some low- Z stars shift from the hot wind region to the cool wind region in the post-main-sequence stage. The massive stars of 50 and 60 $M_{\odot}$ with $Z = 0.02$ further evolve back to the hot region and enter the WR phase. The criterion $X_S < 0.4$ is used to activate the WR wind scheme.

The right panels of Figure 1.1 present the evolution of the mass-loss rates in our stellar evolution simulations. During the main-sequence phase, $\log T_{\text{eff}}$ is in the range of $\sim 4.45 - 4.75$, and thus, hot winds dominate. For models of the same Z , higher-mass stars have higher luminosities and higher mass-loss rates; furthermore, a main-sequence star's mass-loss rate is usually low and stable because the T_{eff} and luminosity changes are not large. For models of different Z but the same initial mass, a higher Z leads to a higher mass-loss rate, but the $\dot{M}$ dependence on Z is not a simple Z^m because $\dot{M}$ also depends on L and T_{eff} , which are also dependent on Z . For $Z = 0.0005$, $\dot{M}$ for 20–60 $M_{\odot}$ main-sequence stars is in the range $\sim 10^{-8} - 10^{-6} M_{\odot} \text{ yr}^{-1}$, and for $Z = 0.02$, $\sim 10^{-7.5} - 10^{-5.5} M_{\odot} \text{ yr}^{-1}$. When a star evolves into the post-main-sequence stage, T_{eff} suddenly decreases. The change in T_{eff} causes the enhancement of mass-loss rate by about 1–3 orders of magnitude, because the T_{eff} passes through two jumps in the mass-loss rate included in the wind prescriptions. These two jumps correspond to two physical transitions in the supergiant winds:

(1) *The “bi-stability jump” in early-type supergiant winds.* When the T_{eff} is less than $\sim 25,000$ K, the mass-loss rate suddenly increases and the wind velocity decreases, which is called the “bi-stability jump” (Pauldrach & Puls 1990). This is caused by the recombination of Fe IV. When the temperature decreases below $\sim 25,000$ K, the ionization fraction of Fe III below the sonic point significantly increases, and Fe III lines are more efficient drivers of winds than Fe IV lines (Vink et al. 1999). The mass-loss prescription of V01 illustrates the bi-stability jump by setting different mass-loss functions in the two regimes with T_{eff} values higher and lower than the jump temperature, which ranges between 22,500 and 27,500 K.

(2) *The transition from hot winds to cool supergiant winds.* When the T_{eff} decreases below 10,000–12,000 K, the star evolves into a cool supergiant. The cool, dust-driven wind becomes the dominating means of mass loss, and the mass-loss rate significantly increases.

These two jumps in the mass-loss rate occur at the post-main-sequence stage, marked as the bi-stability jump and the hot-to-cool wind transition region in Figure 1.1. The stellar evolution timescale between these two jumps are usually so short, $< 10^4$ yr, that these two jumps are unresolvable in the evolution of mass loss rate plots (the right panels of Figure 1.1).

Nevertheless, for some low- Z stars with $Z = 0.0005$, their T_{eff} only decreases to the range between 10,000 and 25,000 K at the onset of the post-main-sequence stage, and they remain as early-type supergiants. Some of the other low- Z stars do cool down to $T_{\text{eff}} < 10,000$ K, but this happens in the very late evolutionary stage after the core-carbon burning commences; thus, the separation between the two jumps is about several hundred

thousand years. For example, for the star with $30 M_{\odot}$ and $Z=0.0005$, the second jump in the mass-loss rate occurs at the very end of its lifetime.

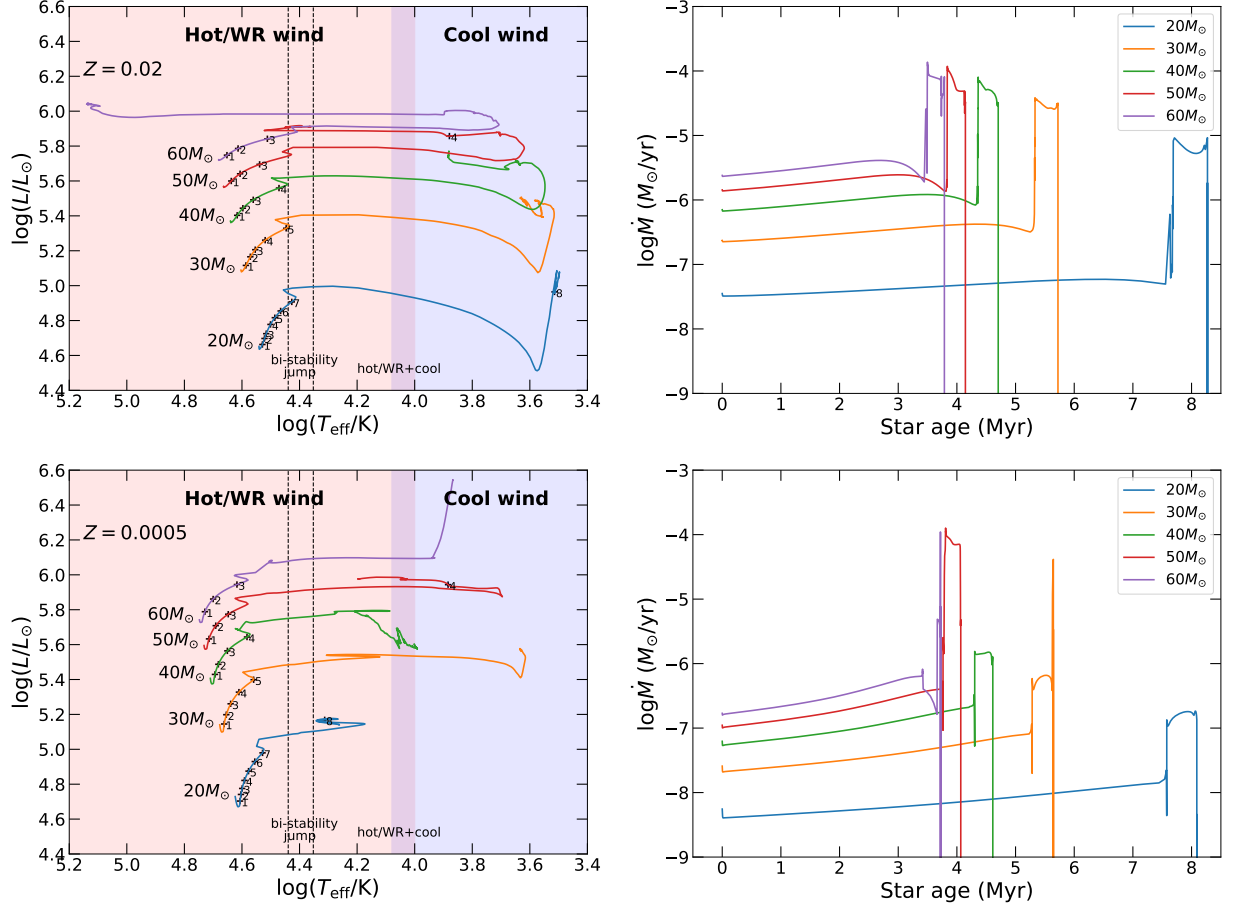


Figure 1.1: The HR diagram of massive stars and their mass-loss histories. *Left panels:* the stellar evolutionary tracks of 20 – 60 $M_{\odot}$ stars with $Z = 0.02$ and 0.0005 . The region between dashed lines denotes the temperature range of the bi-stability jump. The red and blue regions represent hot and cool winds, respectively, instead of the colors of stars. When $10,000 \text{ K} < T_{\text{eff}} < 12,000 \text{ K}$, a combination of hot and cool winds is adopted. The "+" marks and the numbers beside them record the stellar ages in the unit of Myr. *Right panels:* The evolution of the mass-loss rates of stars with absolute metallicities $Z = 2 \times 10^{-2}$ (*top panel*) and $Z = 5 \times 10^{-4}$ (*bottom panel*). The mass loss of massive stars depends on their initial masses and metallicities, and their mass-loss rates in the post-main sequence can be $10^3 - 10^4$ higher than those in the main sequence.

1.4 Total Mass Loss and Metallicity

Since stellar mass loss is very sensitive to the evolutionary phases, the total mass loss in the lifetime of a star depends on its initial mass and metallicity. This section presents

our results of the total mass loss integrated through stellar lifetimes.



1.4.1 Initial Mass and Metallicity Dependence of Mass Loss

Figure 1.2 displays representative relations between a star's initial mass (M_i) and the total mass loss (ΔM), which is the integrated mass loss over the stellar lifetime. The total mass loss generally increases with the initial stellar mass, but the detailed relation depends on the metallicity. At a first glance, three different trends of mass loss are observed at different metallicities:

(1) For $Z \gtrsim 5 \times 10^{-3}$, the high- Z cases, the total mass loss of a star generally increases with its initial mass. For example, for $Z = 0.02$ ($> 1 Z_{\odot} = 0.0134$), a $15 M_{\odot}$ star loses $\sim 2 M_{\odot}$, and a $60 M_{\odot}$ star loses $> 30 M_{\odot}$ during its lifetime.

(2) For $Z \lesssim 3 \times 10^{-4}$, the low- Z cases, the total mass loss of a star also increases with its initial mass, but it is considerably lower than that of the $Z \gtrsim 10^{-2}$ cases. For example, for $Z = 10^{-4}$, a $15 M_{\odot}$ star loses $\sim 0.01 M_{\odot}$, and a $60 M_{\odot}$ star loses $\sim 0.5 M_{\odot}$. Some low- Z high-mass stars have high mass losses, but their total mass loss is still $< 20\%$ of the initial mass.

(3) For stars with $Z \sim 10^{-3}$, the total mass loss is not a smooth function of the initial mass; instead, it oscillates between the high values for the high- Z cases and the low values for the low- Z cases.

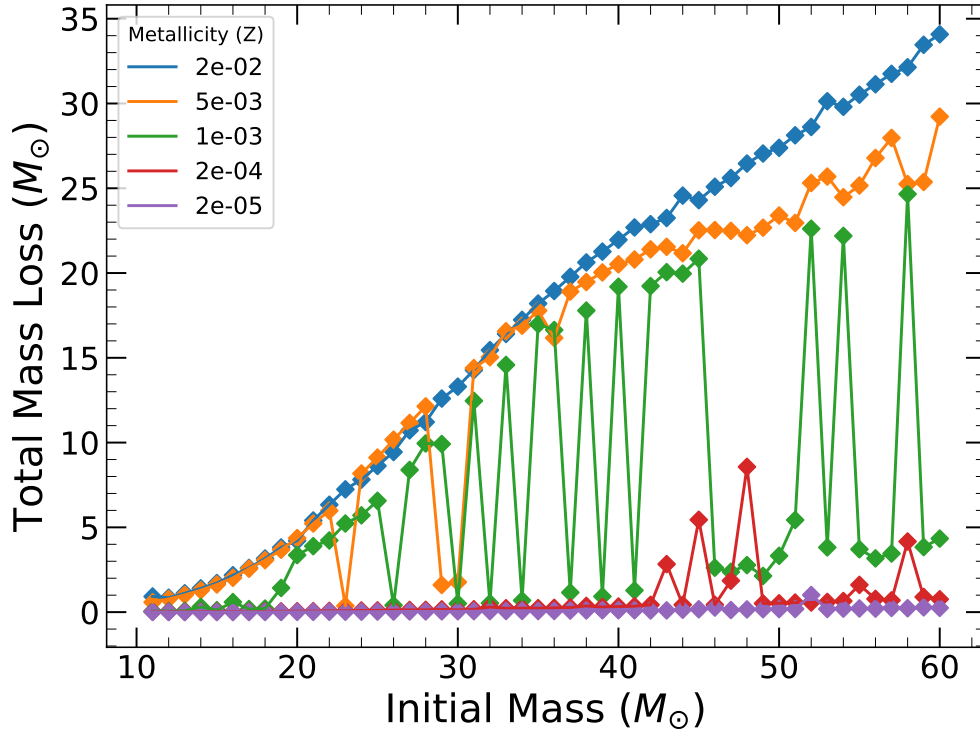


Figure 1.2: Total mass loss as a function of the initial mass for stars with different initial metallicities.

1.4.2 Critical Metallicity

We have shown that the total mass loss during a star's lifetime largely depends on its metallicity. To further illustrate the metallicity effects on mass loss, we plot the total mass loss as a function of metallicity in Figure 1.3.

Figure 1.3 displays a significant jump in the total mass loss at $Z \sim 10^{-3}$. When $Z \lesssim 3 \times 10^{-4}$, the mass loss is generally low. When Z approaches $\sim 10^{-3}$, the mass loss jumps to a significantly higher level.

Herein, we call the $Z \sim 10^{-3}$ as the critical metallicity (Z_c) for the mass loss of massive stars. Generally, significant mass loss occurs only when $Z > Z_c$. Nonetheless, Z_c is not a sharp boundary as shown in the stellar initial mass versus metallicity ($M_i - Z$)

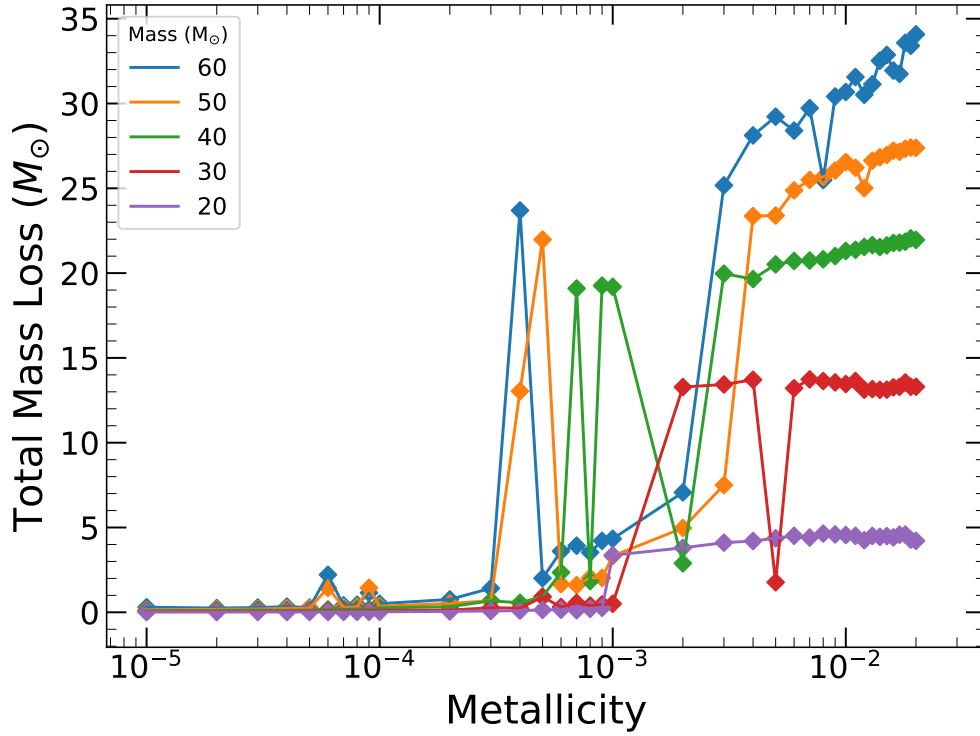


Figure 1.3: Total mass loss as a function of initial metallicity for stars with different initial masses.

diagram of mass loss fraction ($\Delta M/M_i$) in Figure 1.4, where all the 1900 models are included. Two distinct regions are present in this diagram: the high- Z region with mass loss fraction $\gtrsim 40\%$ and the low- Z region with mass loss fraction $\lesssim 10\%$. The separation of the high- Z and low- Z regions is clear, and the boundary at $Z \sim 10^{-3}$ is what we call Z_c . For initial masses $\lesssim 30 M_\odot$, the boundary between these two regions is sharp; for initial masses $\gtrsim 30 M_\odot$, the boundary is fuzzy.

1.4.3 Mass Loss from Different Wind Schemes

To understand the origin of Z_c , we compare the individual contributions of mass loss by hot, cool, and WR winds in order to identify a specific stellar evolutionary stage that is most accountable for Z_c . Figure 1.5 displays the contributions of different winds for initial

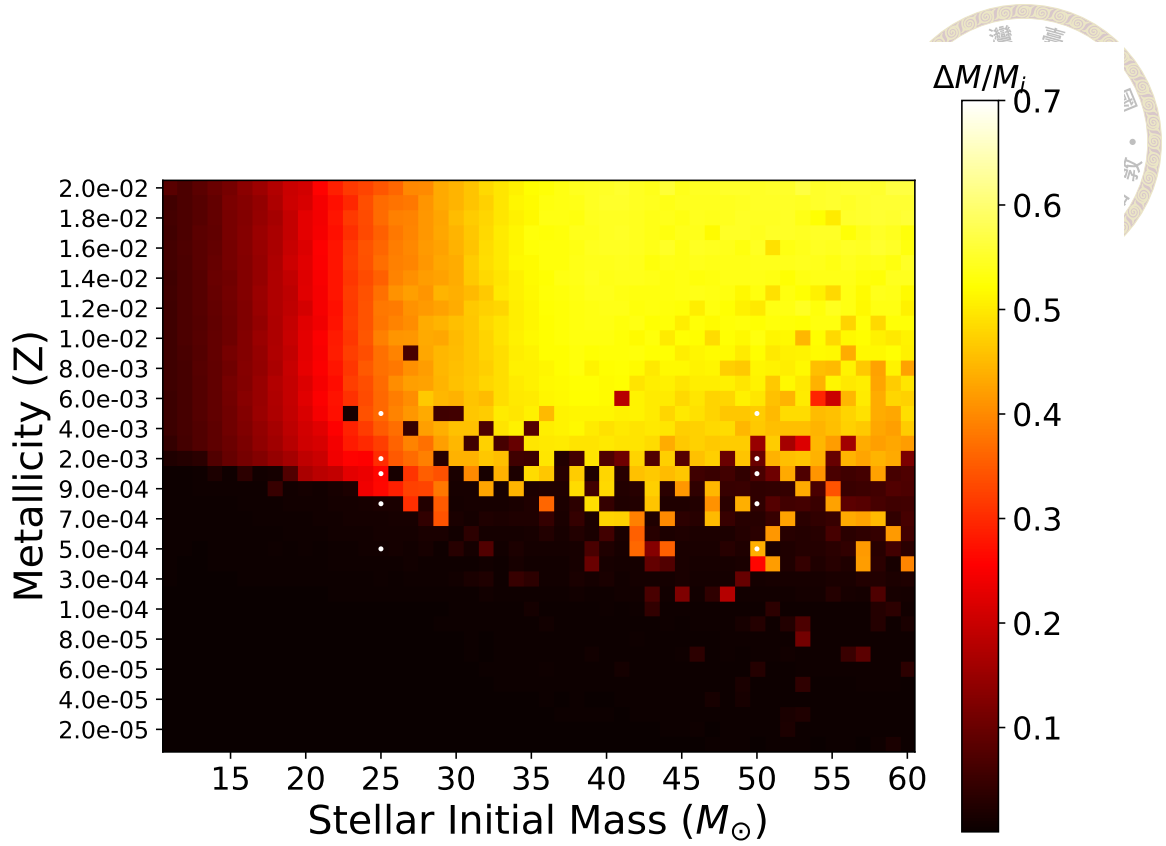


Figure 1.4: Fraction of total mass loss in each of our 1900 models is shown in the initial mass–metallicity phase diagram. The color scale represents the fraction of initial mass that is lost before a star dies. Each pixel in this map represents a stellar model. Note that the scale of the y-axis is nonuniform. A significant transition of the fraction value exists around $Z \sim 10^{-3}$, which separates the bright (high- Z) and dark (low- Z) sides of the map. This transition is called the *critical metallicity*. The white dots mark the models chosen in Section 1.4.4 to investigate the origin of the critical metallicity.

stellar masses of 25 and 50 $M_{\odot}$. In both cases, a distinct jump in the mass loss occurs only in the cool wind near Z_c , indicating that the cool wind is responsible for Z_c .

The $M_i - Z$ diagrams of mass loss fraction for hot, cool, and WR winds are presented in Figure 1.6. For cool winds, going from high Z to low Z , the high mass loss makes a sharp transition into a low mass loss near a threshold Z of $\sim 1 \times 10^{-3}$. The threshold Z is slightly lower for stars with $M_i = 25 - 30 M_{\odot}$ and higher for the more massive stars. For hot winds, the transition of high mass loss to low mass also occurs near a threshold Z of 1×10^{-3} , but the transition is gradual. For WR winds, the threshold Z is higher, near 3×10^{-3} . As the total contribution of hot and WR winds to the mass loss is small,

$\Delta M/M_i \lesssim 20\%$, the $M_i - Z$ diagram of cool wind is very similar to that of the total mass loss (all winds combined) in Figure 1.4; furthermore, the Z_c of the total mass loss is similar to the threshold Z of the cool winds. It should be noted that at low Z , the hot winds may lose more mass than the cool winds up to the threshold Z of the cool winds; however, the mass lost via hot winds is so much smaller than the mass lost via cool winds that the cool winds' threshold Z dominates.

Therefore, Z_c of mass loss mainly stems from cool winds. This result is reasonable because the mass-loss rates of cool winds are usually considerably higher than those of hot winds, as included in the wind prescriptions (Section 1.3.2).

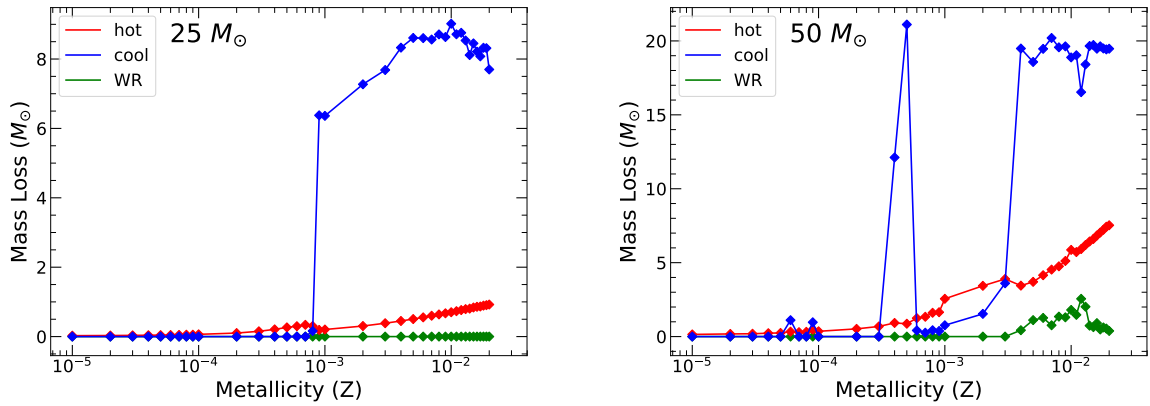


Figure 1.5: Contributions of the hot, cool, and WR winds to the total mass loss of 25 and 50 $M_\odot$ stars. The significant jump in mass loss at $Z \sim 10^{-3}$ mainly arises from cool winds.

1.4.4 Cool Supergiants and the Critical Metallicity

We have shown that the mass-loss fraction ($\Delta M/M_i$) of a massive star is high for high Z and low for low Z , making a transition at Z_c that is caused mainly by the cool winds. To gain more insights into the dichotomy of mass-loss fraction and its scatter around Z_c , we use 25 $M_\odot$ and 50 $M_\odot$ stars with different metallicities that are near Z_c as examples, and look into their post-main-sequence evolution. For each stellar mass, five models with $Z =$

0.0005, 0.0008, 0.001, 0.002, and 0.005 are examined. These models are marked by white dots in Figure 1.4. It can be seen that the $25 M_{\odot}$ stars show monotonic increase of mass loss fractions with increasing Z , while the $50 M_{\odot}$ stars show non-monotonic variations of mass loss fraction across Z_c . We deliberately choose these two masses with contrasting mass loss variations across Z_c , and expect the comparison between them may shed light on the physical origin of the transition and the scatter of mass loss fraction near Z_c .

To investigate the crucial differences among these models, we plot their stellar evolutionary tracks and mass-loss rates in Figure 1.7. For the $25 M_{\odot}$ stars, the three models yielding high ΔM ($Z = 0.001, 0.002$, and 0.005) are those that evolve to radii $R > 1,000 R_{\odot}$ and $\log(T_{\text{eff}}/\text{K}) < 3.6$ (i.e., $T_{\text{eff}} < 4,000 \text{ K}$). In other words, these stars end their lives as RSGs. Due to stellar expansion, their mass-loss rates increase by 2–3 orders of magnitude in the supergiant stages. In contrast, the two models yielding in low ΔM ($Z = 0.0005$ and 0.0008) only expand to $\sim 100 R_{\odot}$. After moderate expansions, they bounce back to the left (i.e., higher T_{eff}) in the HR diagram, and their T_{eff} remains higher than $\sim 10,000 \text{ K}$. Such a high T_{eff} prevents the cool wind from operating, and thus, their mass-loss rates only slightly increase during the post-main-sequence stages. In summary, the two different behaviors of stellar expansion result in the dichotomy of the total mass loss.

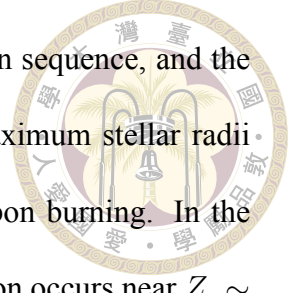
For the $50 M_{\odot}$ stars, the evolutionary tracks are more complicated. At the age of 4 Myr (marked by “+” in the HR diagram in Figure 1.7), the two models yielding high ΔM ($Z = 0.0005$ and 0.005) have already evolved into the cool-wind domain, and eventually expand to $R > 1,000 R_{\odot}$. On the other hand, the three models yielding low ΔM ($Z = 0.0008, 0.001$, and 0.002) are still in the hot-wind domain at 4 Myr. These three models eventually enter the cool-wind domain at the very end of their lifetimes when the core-

carbon burning commences, and approach a high mass-loss rate of $\sim 10^{-4} M_{\odot} \text{ yr}^{-1}$. However, their cool winds only last a few $\times 10^4$ yr. Thus, the integrated ΔM is still small, yielding $\Delta M/M_i < 10\%$.



The above examples demonstrate that the crucial process that determines the total mass loss is the expansion toward the formation of a supergiant. If a star expands to a cool supergiant with $R > 1,000 R_{\odot}$ at the beginning of the post-main-sequence stage, significant mass loss will occur. Otherwise, if a star never expands to the extent of a cool supergiant or only expands to that extent at a very late stage (i.e., after the core-carbon burning begins), only a minimal amount of mass loss will occur.

The crucial role played by the supergiant expansion in the dichotomy of mass loss is further illustrated in Figure 1.8, which presents maps of maximum radii in the $M_i - Z$ grid. We have considered maximum radii over two time spans: the entire lifetime (left panel of Figure 1.8) and the time span from the ZAMS to the beginning of core carbon burning (right panel of Figure 1.8). The left panel shows a complex map with a visible dichotomy only for stellar masses of $14\text{--}27 M_{\odot}$. The complexity stems from the maximum radii arising from different evolutionary stages and paths for stars of different masses. For high metallicities ($Z > Z_c$), stars reach maximum radii during the core helium burning; while for low metallicities ($Z < Z_c$), $14\text{--}27 M_{\odot}$ stars reach the maximum radii during the core helium burning, but the higher mass stars reach maximum radii after the core carbon burning. The core carbon burning stage is pretty late in a massive star's lifetime, and the duration for high mass loss rate is short; thus, massive stars reach maximum radii during the core carbon burning stage have small mass loss fractions. The complications at the core carbon burning stage makes the maximum radius over the entire lifetimes an ineffective indicator of total mass loss.



As the core helium burning stage lasts the longest after the main sequence, and the bulk of mass loss occur during this stage, we next consider the maximum stellar radii during the time span from the ZAMS to the beginning of core carbon burning. In the right panel of Figure 1.8, a cleaner dichotomy is seen and the transition occurs near $Z_c \sim 0.001$. In fact, this map of maximum stellar radii resembles closely the map of mass loss fraction (Figure 1.4): both show dichotomy across the $Z_c \sim 0.001$ and larger scatter around Z_c at high stellar masses. Examined closely, even the scatter exhibits one-to-one correspondences between these two maps. These similarities indicate that the maximum stellar radius before core carbon burning is an effective indicator of total mass loss.

We further find that models with high mass loss are almost exactly those that have expanded to cool supergiants with $R > 1,000 R_\odot$ at the core-helium burning stage, and these models are usually of $Z \gtrsim 10^{-3}$. Therefore, we conclude that the Z_c of mass loss is essentially the *critical metallicity of cool supergiant formation*.

Generally, cool supergiant formation and the corresponding mass loss only occur when Z is higher than the critical metallicity ($Z_c \sim 10^{-3}$). For stars with $Z > Z_c$, they usually expand to $R > 1,000 R_\odot$ and become cool supergiants at the beginning of the post-main-sequence stage. These stars can lose large amounts of mass due to cool winds. In contrast, stars with $Z < Z_c$ either do not reach $R > 1,000 R_\odot$ throughout their lifetimes or only expand to this extent at the very final stage of stellar evolution. For these low- Z stars, the cool wind scheme is not activated in most of their stellar lifetime; thus, their total mass loss is very low. For $Z \sim Z_c$, a stars either successes or fails to become a cool supergiant, and thus show erratic variations of the total mass loss as a function of metallicity (Figure 1.2 and 1.4). The physical mechanism that determines whether a star can become a cool supergiant will be investigated in Chapter 4.



1.5 Feedback from Stellar Winds

Our grid of stellar evolution models can be used to determine the feedback from stellar winds. We first compute the kinetic energy released by winds from individual stars, then consider the wind energy feedback for a star cluster. For star clusters we investigate the metallicity dependence of the total mass and kinetic energy injected by stellar winds into the ISM, and the evolution of mass and energy injection rates.

1.5.1 Kinetic Energy Released by Winds

The flux of kinetic energy carried by a stellar wind is called mechanical luminosity of the wind, defined as $L_w \equiv \frac{1}{2}\dot{M}v_\infty^2$, where v_∞ is the wind terminal velocity. The determination of $\dot{M}$ has already been described in Section 1.2. Our stellar evolution models do not provide v_∞ directly, but provide escape velocity (v_{esc}) that can be scaled to determine v_∞ . For hot winds, we follow V01 and directly use the v_∞/v_{esc} scaling relations given in Eq. 1.1 and 1.2 in Appendix A. For cool winds, we follow [Leitherer et al. \(1992\)](#) and set the RSG wind terminal velocity at a constant 30 km s^{-1} . More details of v_∞/v_{esc} can be found in Appendix A. A star's $\dot{M}$ and v_∞ are used to compute L_w , then L_w is integrated over the star's lifetime to determine the total kinetic energy injected by its winds into the ISM.

Figure 1.9 illustrates v_∞ and L_w of $25 M_\odot$ stars with different Z values around $Z_\odot$. Main-sequence O stars produce fast winds of $\sim 1500\text{--}3000 \text{ km s}^{-1}$. When a star evolves into an RSG, its v_∞ suddenly drops by a factor of 100. At the same time, its $\dot{M}$ goes up by 2–3 orders of magnitude (Figure 1.7). As a result, L_w decreases by 1–2 orders of

magnitude when the star evolves from a main-sequence O star to an RSG. Although cool RSG winds release large amounts of mass, they only carry relatively small amounts of kinetic energy because of their slow wind velocities. For stars with higher initial masses and metallicities, the domination of hot winds in the energy output is even more significant.

Figure 1.10 presents the total kinetic energy output over the stellar lifetime of each model. This figure also shows the contributions of the hot and cool winds separately. It is evident that the hot winds dominate the energy output, while the contributions from cool winds are almost negligible. The dominance of hot winds is attributed to their velocities being much faster than the cool winds' velocities, and their duration (including the main-sequence stage) spanning $\sim 90\%$ of the stellar lifetime. It can also be seen from the top panel of Figure 1.10 that the most massive stars with near-solar metallicities each inject $10^{50} - 10^{51}$ erg of kinetic energy into the surrounding ISM, which is comparable to the explosion energy of a SN. In contrast, the cool winds of such a massive star contribute only up to $\sim 10^{47}$ erg to the total kinetic energy.

It is interesting to note that the Z_c for the dichotomy of total mass loss show some effects only the cool winds' total kinetic energy. This is understandable because Z_c is closely associated with the onset of cool winds, as discussed in Section 1.4.3. The stellar winds' total kinetic energy injected into the ISM is dominated by contributions from hot winds. As cool winds play a negligible role in the total wind energy, its closely associated Z_c does not affect the total wind energy or total hot wind energy. In other words, whether a star becomes an RSG has no bearing on the total wind kinetic energy injected to the ISM.

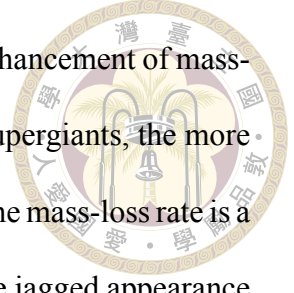
1.5.2 Wind Feedback from a Star Cluster



The mass loss and kinetic energy output from stellar winds of individual stars can be used to evaluate the feedback from a star cluster. We consider a star cluster from a single burst of star formation with an initial mass function (IMF) $\xi(M) = \xi_0 M^{-\alpha}$, where M is the initial stellar mass, ξ_0 is a scaling factor, and α is a constant. Here, ξ_0 is scaled by the number of stars in the system N so that $dN = \xi_0 M^{-\alpha} dM$. In our calculations, we apply $\alpha = 2.35$ (the Salpeter IMF; [Salpeter 1955](#)). For comparison, the widely used IMF in [Kroupa \(2001\)](#) gives $\alpha = 2.3$ for stars with masses $> 1 M_{\odot}$, which is similar to the Salpeter IMF.

We have calculated the feedback for a cluster with a total mass of $10^5 M_{\odot}$. Although this mass is associated with a super star cluster, the feedback scales linearly with the cluster mass and can be used to determine feedback of clusters of any other masses. For the cluster members, we adopt the canonical initial stellar mass range of $1 - 150 M_{\odot}$. We note that the upper mass limit is not important because the population of very massive stars is much smaller than that of lower-mass stars. With the above assumptions, we integrate the $\dot{M}$ and L_w of individual stars to calculate the wind feedback from the entire cluster. The feedback we present only include the contributions from massive stars of $M_i = 11 - 60 M_{\odot}$, which is the mass range of our simulations; thus, our results show the lower limit of wind feedback.

In Figure 1.11, we choose a high- Z ($Z = 0.02$) case and a low- Z ($Z = 0.0002$) case and plot their integrated mass-loss rates and energy-injection rates of a $10^5 M_{\odot}$ cluster before the age of 10 Myr. At about 3–4 Myr, the most massive stars evolve into the post-main-sequence stage. In the high- Z cluster, these massive stars successively become cool supergiants and significantly enhance the overall mass-loss rate. However, the lifetimes



of cool supergiants are short, so each star can only contribute to the enhancement of mass-loss rate for a short time. When stars of lower masses become cool supergiants, the more massive stars have already ended their lifetimes. The overall trend of the mass-loss rate is a gradual decrease after the sudden enhancement at about 3–4 Myr. The jagged appearance of the curve of mass-loss rate is caused by the limit of mass resolution ($1 M_{\odot}$) in our simulations. For the low- Z cluster, the stars do not evolve into cool supergiants at the helium-core burning stage, so the mass-loss rate after 4 Myr can only increase to $10^{-5} M_{\odot}/\text{yr}$ due to the bi-stability jumps. Some sudden spikes of mass-loss rate occur at about 4 Myr because some low- Z stars reach cool supergiants in the very late stage of their lifetimes, and thus they have enhanced the mass-loss rate for very short times.

While the mass-loss rate of the high- Z cluster is enhanced by cool winds at the ages of 3–4 Myr, cool winds do not contribute much to the energy injection rate. Thus, for both high- and low- Z clusters, their energy injection rates do not rise drastically but continue to decline after 3–4 Myr, when the most massive stars end their lifetimes.

We compare our mass-loss rates and energy injection rates with those adopted by the FIRE-3 cosmological simulations ([Hopkins et al. 2022](#)), which are shown by the blue dash lines in Figure 1.11. The mass-loss rate in FIRE-3 is expressed by a function of time and Z with four segments separated by 1.7, 4, and 20 Myr. For the high- Z model, the post-main-sequence mass-loss rate from our simulations is in reasonable agreement with the FIRE-3 curve, while for the low- Z model, our mass-loss rate after ~ 4 Myr is about one order of magnitude lower than the FIRE-3 rate. The essential difference is because the low- Z stars in our models do not evolve into cool supergiants when they enter the post-main-sequence stage, so the mass-loss rate is low. For the times before 4 Myr, the comparison of mass-loss rates is out of our scope as we do not consider the very massive

stars with $> 60 M_{\odot}$. We also compare our energy injection rate the FIRE-3 function. For both high- and low- Z cases, our energy injection rate is lower than the FIRE-3 rate by about a order of magnitude after ~ 4 Myr. FIRE-3 uses the entire mass-loss rate and the average velocity, which is $\sim 1,000 \text{ km s}^{-1}$ at ~ 4 Myr, to estimate the wind feedback. In contrast, we separately consider hot and cool winds, with hot winds having velocities of $> 1,000 \text{ km s}^{-1}$ but low mass-loss rates and cool winds having high mass-loss rates but a velocity of 30 km s^{-1} . Thus, our calculations lead to a lower energy injection rate than the FIRE-3 function.

We have calculated the feedback from stellar winds in a cluster using the Salpeter IMF. The critical metallicity of cool supergiant formation significantly affects the total mass loss within a cluster, and we suggest that this effect be considered in order to refine the treatment of stellar feedback in cosmological simulations. In contrast, the kinetic energy feedback is dominated by hot winds, which are irrelevant to the critical metallicity of cool supergiants.

We further integrate the mass and energy injected by a $10^5 M_{\odot}$ cluster over time, and the results are shown in Figure 1.12. The contribution of wind feedback in our calculations is from star with $M_i = 11 - 60 M_{\odot}$. For comparison, we also plot the feedback that only includes the contribution of $11-30 M_{\odot}$ stars. From these plots, hot winds dominate the kinetic energy and momentum feedback for all of the metallicities, while the cool winds surpasses hot winds in the contribution to mass loss when $Z > Z_c$. The effect of critical metallicity occurs in the output of mass, but not in that of kinetic energy. The WR wind is never the dominating component, although its contribution to kinetic energy is larger than cool wind in an environment of solar metallicity. We note that these integrated quantities are the lower limit, as we do not consider the very massive stars with $> 60 M_{\odot}$.

1.6 Discussions



This section discusses some implications of and remaining issues with our results.

In Section 1.6.1, we discuss the effects of metallicity on cool supergiant evolution. In Section 1.6.2, we point out the possible uncertainties in our models. Finally, in Section 1.6.3, we discuss the implications of critical metallicity on the fates and environments of massive stars.

1.6.1 Two Effects of Metallicity on RSG Evolution

Our study finds that the critical metallicity of cool supergiant formation causes a significant increase in mass loss. This effect arises from stellar evolution rather than the treatment of mass-loss prescription. We discern two different effects of metallicity on RSG evolution and review the previous theoretical works that have shown a similar result of critical metallicity.

From the evolutionary tracks shown in Figure 1.7, there are two apparent effects of metallicity on the evolution of cool supergiants. First, at a lower metallicity, the overall evolutionary tracks, from the ZAMS to the supergiant stage, shift to higher T_{eff} or bluer locations. Second, the evolutionary tracks of stars with $Z < Z_c$ end at hotter T_{eff} , whereas stars with $Z > Z_c$ can successfully evolve into cooler regimes. This is a sharp transition rather than a continuous change. In the following paragraphs we discuss these two different effects.

(1) *The overall shift of the evolutionary tracks with metallicity.* This effect has been discussed with regard to the two ends of stellar evolution: the shift of ZAMS and the



color of supergiants. First, the displacement of ZAMS toward the bluer locations with decreasing Z has been reported by stellar models (e.g., [Groh et al. 2019](#); [Mowlavi et al. 1998](#); [Schaller et al. 1992](#)). Among several physical parameters that directly change with metallicity (opacity, energy generation rate, molecular weight, etc.), the ZAMS shift has been attributed to opacity κ (see e.g., [Maeder 2009](#)). In general, decreasing Z leads to lower κ , transferring more radiation outward and leading to higher luminosities, and the stars also need to contract further to maintain the energetic equilibrium. With a larger L and a smaller R , a higher $T_{\text{eff}} (\propto L/R^2)$ is required for lower- Z stars.

At the end of stellar evolution, supergiants with lower Z also shift toward bluer locations in the HR diagram. This feature has been observed in stars in the local group ([Elias et al. 1985](#); [Humphreys 1979a,b](#)), and has been reproduced in stellar models (e.g., [Groh et al. 2019](#); [Meynet et al. 1994](#); [Schaller et al. 1992](#)). Surveys of massive stars have confirmed that the RSGs in galaxies with lower metallicities have earlier spectral types on average: M2 I in the Milky Way, M1-M1.5 I in the LMC, and K3–K7 I in the Small Magellanic Cloud (SMC) ([Levesque et al. 2006](#); [Massey & Olsen 2003](#)). This can be explained as the shift of the Hayashi line, which represents the coolest extent of giant stars, to higher T_{eff} at lower Z ([Elias et al. 1985](#); [Levesque et al. 2006](#)). These stars shift to bluer positions at lower Z due to opacity ([Hayashi & Hoshi 1961](#); [Kippenhahn et al. 2012](#)), which is a similar physical effect to the ZAMS shift.

(2) *The critical metallicity of cool supergiant formation.* Instead of causing continuous shifts in the HR diagram, the critical metallicity acts as a boundary between two different types of evolution tracks in the supergiant stage. The stars with $Z > Z_c$ successfully become RSG with $R \gtrsim 1,000 R_{\odot}$, and those with $Z < Z_c$ remain as smaller BSGs. This effect causes the significant increase in mass loss.



Some previous models have shown that low- Z stars can never evolve into RSGs ([Arnett 1991](#); [Baraffe & El Eid 1991](#); [Brocato & Castellani 1993](#); [El Eid et al. 2009](#); [Groh et al. 2019](#); [Hirschi 2007](#); [Limongi 2017](#)). The metallicity value has been identified as $Z \sim 10^{-3}$ ([Baraffe & El Eid 1991](#); [El Eid et al. 2009](#)), which is consistent with our results. We start from the investigation of mass loss and identify the same phenomenon in cool supergiant formation.

However, the physical origin of the critical metallicity of cool supergiant formation is still poorly understood. A profound question remains: Why do massive stars of $Z > Z_c$ successfully expand to RSGs, but those of $Z < Z_c$ fail to become RSGs? In Chapter 4, we will present some experiments that study the physical origin of critical metallicity. The explanation of its mechanism will help us understand the fundamental physics of RSG formation.

1.6.2 Possible Uncertainties

In this subsection, we discuss the uncertainties in our models from both stellar models and mass-loss prescriptions. We then evaluate the impacts of these factors on our main results.

1.6.2.1 Uncertainties from stellar models

For simplicity, this work only considers single and non-rotating stars. In fact, many recent studies of stellar evolution consider the effects of rotation and binarity (e.g., [Langer 2012](#), for review).

Stellar rotation can change the evolution tracks through processes such as rotational

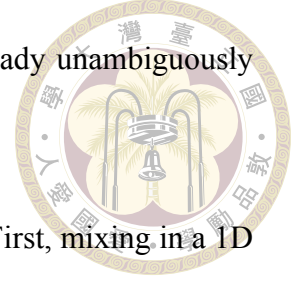
mixing ([Maeder 2009](#)). For RSGs, rotation is suggested to prevent a large intermediate convective zone, thus preventing the star from evolving backward to the blue side ([Maeder 2009](#)). It has also been suggested that at low metallicities, evolving into RSGs is easier for rotating stars than for non-rotating stars ([Hirschi 2007](#)).

Binarity is another important factor in recent stellar evolution studies, as most massive stars are found in binaries ([Duchêne, & Kraus 2013](#); [Sana et al. 2012](#)). Mass transfer is one of the essential processes in interacting binary stars that dramatically changes the post-main-sequence evolution (e.g., [Iben 1991](#); [Langer 2012](#); [Smith 2014](#); [Taam & Sandquist 2000](#); [Yoon et al. 2010](#)). For example, an RSG can strip off its envelope due to mass transfer and move back to bluer locations in the HR diagram, becoming a YSG or WR star ([Levesque 2017](#)).

Therefore, for rotating and binary stars, we cannot presume that the feature of critical metallicity is the same as our results of non-rotating and single stars. Furthermore, internal mixing processes such as semiconvection and overshooting can affect the evolution of supergiants ([Schootemeijer et al. 2019](#)), and thus they may impact the critical metallicity. The effects of rotation, binarity, and mixing processes on cool supergiant formation and the corresponding mass loss are thus worth studying in future works.

Another minor issue in our model is that some models of $11\text{--}17 M_{\odot}$ do not successfully run until the iron core collapses. Instead, they stall at the carbon or oxygen burning stage. This situation does not affect our results, as the criterion of high or low mass loss refers to whether a star becomes a cool supergiant at the beginning of the core-helium burning stage. The final evolutionary stages only last for a very short time and thus do not contribute any significant mass loss via steady-state winds. Moreover, the models of

$>17 M_{\odot}$, which smoothly evolve until the iron core collapses, already unambiguously demonstrate the feature of critical metallicity.



We further point out the limits of 1D stellar evolution models. First, mixing in a 1D stellar model is done through the mixing length theory, which assumes all the chemical elements homogeneously mix together within the local pressure scale height if fluid instabilities occur. However, the mixing length theory likely becomes invalid inside the shell-burning region where violent burning, dynamics, and nucleosynthesis coevolve. Precise mixing then becomes challenging. Since a full 3D stellar evolution model is still unavailable, recent efforts ([Jørgensen et al. 2018](#); [Mosumgaard et al. 2018](#); [Trampedach et al. 2014b,a, 2013](#)) have started to use local or global 3D hydro simulations of stellar convective zones to physically calibrate the parameters of mixing length theory used in 1D stellar evolution models.

Another critical issue is the opacities of the stellar atmosphere. One-dimensional stellar evolution models of MESA primarily focus on interior nuclear burning and structure evolution, and their stellar envelopes contain numerical artifacts due to their poor resolution in the Lagrangian code. In addition, the gas in the stellar atmosphere is at non-local thermal equilibrium, and its ionization states are difficult to calculate correctly. These issues can alter the surface opacities and significantly affect the physical properties of wind. Solving this problem requires multi-D radiation-hydro simulations, including sophisticated atomic physics, to evolve the wind using the first principles.

1.6.2.2 Uncertainties from mass-loss prescriptions



The currently available mass-loss prescriptions are based on the fitting of data obtained from simulations or observations, as described in Section 1.2. The best-effort prescriptions remain uncertain, and various prescriptions have been proposed. A comprehensive review of these prescriptions was given by [Mauron & Josselin \(2011\)](#). More recently, some new prescriptions have been proposed for hot winds (e.g., [Björklund et al. 2022](#)) and cool winds (e.g., [Beasor et al. 2020](#); [Goldman et al. 2017](#)). A new study of RSG mass loss by [Beasor et al. \(2020\)](#) suggested that dJ88 overestimated the mass-loss rate by a factor of nine.

To explore the impact of uncertain mass-loss rates on stellar evolution, [Renzo et al. \(2017\)](#) performed a grid of stellar evolution simulations with MESA using different wind prescriptions. They also re-scaled the wind prescriptions by multiplying the mass-loss rates by a wind efficiency parameter η and carried out simulations using η of 0.1, 0.33, and 1. They found $\sim 50\%$ variations in the final stellar mass within all the combinations of η and wind prescriptions and $\sim 15\text{--}30\%$ changes in the final stellar mass for a fixed $\eta = 1$.

Another limit in our study is that we only consider steady-state winds. In addition to steady-state winds, the LBVs have eruptive mass loss driven by super-Eddington winds ([Humphreys, & Davidson 1994](#); [Owocki et al. 2019, 2017](#); [Shaviv 2000](#); [Smith & Owocki 2006](#); [Vink 2018](#)), such as η Carinae. These eruptive LBV winds may be a promising source of metal enrichment for Pop III stars in the early universe ([Smith & Owocki 2006](#)).

The physical properties of a star cluster can also affect the mass loss of its resident stars. For example, in a cluster with multiple stellar populations, as a new star starts to form in the dense stellar environment shaped by the old stars, the accretion disk of its

proto-star is disrupted by the gravity of old stars, leading to the formation of a rapidly rotating star that evolves longer during its red giant phase, and produce more mass loss (Tailo et al. 2021, 2020, 2015). Although this environmental effect is based on the low-mass stars, we speculate that it may apply to forming massive stars and altering their mass loss.

The extension of the mass-loss prescriptions to low metallicities is another issue. For hot winds, Vink et al. (2001) compared their mass-loss prescription with observations of stars in the SMC, where $Z \sim 1/5 Z_{\odot}$, and found reasonable agreement. It has been suggested that at $Z \sim 1/7 Z_{\odot}$ mass-loss rates derived from non-LTE stellar atmospheric model fittings of the optical spectra cannot be described by the V01's prescription (Tramper 2014; Tramper et al. 2011). Nonetheless, subsequent analyses of far-UV spectra of the same targets indicate that Tramper et al. (2011)'s low- Z stars actually have metallicities $\sim 1/5 Z_{\odot}$, making the mass-loss rates consistent with V01's prescription (Bouret et al. 2015). In regards to the cool winds, dJ88's prescription has also been tested in the SMC (Mauron & Josselin 2011). Generally speaking, the mass-loss prescriptions have been tested in low- Z stars with $Z \sim 1/5 Z_{\odot}$. For even lower metallicities, it is not known whether any breakdown of these scaling relations occurs. Few extremely metal-poor stars have been observed, and most of them are low-mass stars (Beers, & Christlieb 2005; Da Costa et al. 2019; Yong et al. 2013), thus yielding no information on the mass-loss rates of massive stars. For now, adopting the widely-used mass-loss prescriptions in low- Z regimes is the only feasible method.

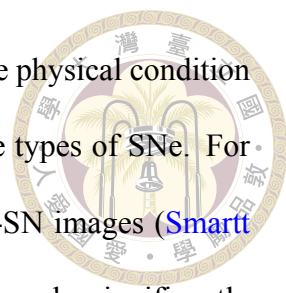
It is also unclear what kind of Z -dependent function should be included in mass-loss rate, as uncertainty exists, especially for cool winds. Earlier prescriptions of cool winds do not include the Z -dependence (de Jager et al. 1988; Nieuwenhuijzen & de Jager 1990;

van Loon et al. 2005), but some later works adopt $\dot{M} \sim Z^m$ in their simulations (e.g., Groh et al. 2019; Mauron & Josselin 2011). Nevertheless, some recent observations show that mass-loss rates of RSGs are nearly independent of Z (Beasor et al. 2020; Goldman et al. 2017). We use the conventional dJ88 prescription without Z -dependence, but are aware of uncertainties in this function.

Fortunately, most of these uncertain factors in mass-loss prescriptions only slightly affect our main results. The major difference in mass loss between high- and low- Z stars depends on whether a star evolves into the RSG phase rather than the mass-loss prescriptions. In other words, the physical origin of the critical metallicity lies in stellar evolution, regardless of the adopted recipe of mass loss, although these two processes can couple together to some extent. We carried out a simple test and found that even if the mass loss is fully turned off, the critical metallicity of cool supergiant formation still exists. As long as the mass-loss rate in the RSG phase is much higher than that in the main-sequence phase, the critical metallicity of cool supergiant formation leads to the bimodal distribution of mass loss. Therefore, whether the mass-loss rate is a function of $\dot{M} \sim Z^m$ is only a minor issue.

1.6.3 Implications of the Critical Metallicity

Our study reveals a critical metallicity of cool supergiant formation that leads to a significant difference in mass loss between high- and low- Z environments. The low- Z stars do not evolve into cool supergiants, and thus only have minimal mass loss through steady-state winds over their lifetimes. In the following paragraphs, we discuss the consequences of the critical metallicity.

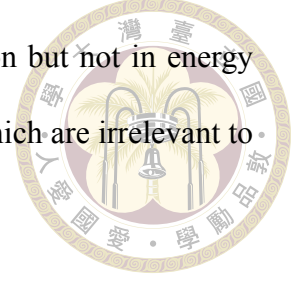


It is widely known that pre-SN evolution and mass loss set up the physical condition for SN explosions. The types of progenitor stars often determine the types of SNe. For example, RSGs are the progenitors of SNe II-P, as identified in pre-SN images ([Smartt 2015, 2009](#), for review). It has also been shown that the types of SNe can be significantly affected by the interaction with CSM produced by RSGs (e.g., [Beasor et al. 2021, 2020](#); [Ekström et al. 2012](#); [Moriya 2021](#); [Smartt 2015](#); [Smith 2014](#); [van Loon 2010](#)). For example, the early-phase light curves of SNe II-P can be better explained if dense CSM is considered ([Moriya et al. 2018, 2017, 2011](#)). Moreover, it is suggested that CSM not only affects SN types but also makes SN explosions more likely to happen ([Morozova et al. 2018](#)).

Based on these understandings and the critical metallicity we found, we can expect the low- Z stars to behave differently from high- Z stars in SN explosions. First, if the low- Z stars indeed end up as BSGs instead of RSGs, whether they will still explode as SNe is uncertain. Even if they successfully explode, the resulting SN type may differ from SNe II-P. Low- Z stars never go through the RSG phase, or only become RSGs in the very final stage, so they only lose a negligible amount of mass and may not end as typical SNe II-P. Their circumstellar material can be too dilute for interacting SNe, although we have not considered eruptive mass loss that can generate dense CSM.

On a larger scale, the critical metallicity also impacts the stellar feedback in a star cluster. From our results, the total mass returned to the ISM by stellar winds shows a significant increase at Z_c . In the high- Z ($Z > Z_c$) domain, these results are consistent with the current treatment of feedback in cosmological simulations such as FIRE-3. However, if the low- Z ($Z < Z_c$) stars do not evolve into cool supergiants, as our simulations predict, the current treatment of feedback may overestimate the mass injected to the ISM in these

low- Z environments. This discrepancy exists only in mass injection but not in energy output, as the kinetic energy feedback is dominated by hot winds, which are irrelevant to the critical metallicity.



As the critical metallicity $Z_c \sim 0.001$ is lower than $0.1 Z_\odot$, it is still difficult to test it using current observational instruments. Nevertheless, it will be promising to test the stellar evolution below this metallicity with the next-generation telescopes. For example, observations of massive stars in the low- Z galaxies are listed as one of the scientific goals of the LUVOIR observatory ([Garcia et al. 2021](#)).

In future works, we plan to apply these 1D mass loss models to 2D and 3D radiation hydro simulations to further understand how the mass loss shapes the CSM and ISM around massive stars. With the advancement of numerical models and observational data, we may reveal the mystery of the mass loss of massive stars and their feedback.

1.7 Conclusions

We studied the total mass loss of massive stars and their dependence on metallicity by performing 1D stellar evolution simulations with MESA. In the initial mass range $11\text{--}60 M_\odot$, the total mass loss increases dramatically if their metallicity becomes higher than $Z \sim 10^{-3}$. We call this *critical metallicity*, though the exact metallicity values for this mass-loss jump are not very well defined. Cool winds that operate in the cool supergiant phase are responsible for this mass-loss jump. Massive stars with $Z < Z_c$ usually stay blue during the post-main sequence; thus, the cool winds are not operating, and the mass loss is low. In contrast, massive stars with $Z > Z_c$ successfully become cool supergiants, leading to significantly higher mass loss. The critical metallicity of cool supergiant formation

gives rise to the bimodal distribution of mass loss.

We also calculated the feedback of stellar winds in a star cluster. In low- Z environments, we may overestimate the integrated mass loss in a cluster if we do not consider the effect of critical metallicity. While the mass loss of a cool supergiant is much higher than that of a hot main-sequence star, its wind velocity is much lower, so its kinetic energy output is lower. Consequently, the critical metallicity affects mass loss but not kinetic energy feedback.

In summary, we identified the critical metallicity of cool supergiant formation in our stellar models. A striking consequence of the critical metallicity is the significant jump in mass loss when we adopt the widely used wind prescriptions. This evolutionary effect may have various consequences on the fates of metal-poor stars in the early universe.

Appendix A: Wind Prescriptions

This appendix presents the details of the mass-loss rate and wind velocity functions that we adopt in our simulations.

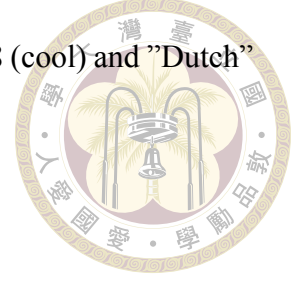
Mass-loss rate

The settings of mass-loss rate in our simulations follow the wind schemes established in MESA. We apply the "Dutch" scheme for hot and WR winds and the dJ88 prescription for cool winds. The criteria to activate these winds are as follows:

- (1) If $T_{\text{eff}} > 12,000$ K, the "Dutch" scheme (*hot/WR wind*) is fully operating.
- (2) if $T_{\text{eff}} \leq 8,000$ K, the *cool wind* is fully operating, and the prescription of dJ88 is used.



(3) Between 80,000 K and 12,000 K, a linear interpolation of the dJ88 (cool) and "Dutch" (mainly hot/ WR) winds is applied.



The "Dutch" scheme is not a single function, but a combination of V01's hot wind prescription, NL00's WR wind prescription, and the extension of dJ88's cool wind prescription. In detail, the wind configuration of the "Dutch" scheme is as follows:

- (1) If $T_{\text{eff}} \geq 11,000$ K and $X_H \geq 0.4$, the V01's hot wind prescription is adopted.
- (2) If $T_{\text{eff}} \geq 11,000$ K and $X_H < 0.4$, the NL00's WR wind prescription is adopted.
- (3) If $T_{\text{eff}} \leq 10,000$ K (low-T "Dutch" scheme), the dJ88 prescription, which is also used to express the cool wind, is adopted.
- (4) If $10,000 \leq T_{\text{eff}} \leq 11,000$ K, a linear interpolation of dJ88 and V01/NL00 prescriptions is adopted.

There are criteria to turn off the mass loss at the very end of the stellar lifetime. If the central temperature is higher than 2×10^9 K, mass loss will be fully turned off. If the central temperature is between 1×10^9 and 2×10^9 K, the mass-loss rate weakens linearly with the central temperature. These settings simply follow the default in MESA.

Wind velocity

As our MESA simulations do not include wind models, the wind terminal velocity (v_∞) is estimated using empirical functions of v_{esc} and other stellar parameters.

- (1) *Hot wind*. If the V01's prescription of mass loss is turned on, we follow the wind

velocity formulae adopted by [Vink et al. \(2001\)](#). These expressions originate from [Leitherer et al. \(1992\)](#) and [Lamers et al. \(1995\)](#).



a. If $T_{\text{eff}} > 27,500$ K, which is higher than the bi-stability jump, then

$$v_{\infty}/v_{\text{esc}} = 2.6(Z/0.019)^{0.13}. \quad (1.1)$$

b. If $T_{\text{eff}} < 22,500$ K, which is lower than the bi-stability jump, then

$$v_{\infty}/v_{\text{esc}} = 1.3(Z/0.019)^{0.13}. \quad (1.2)$$

c. If $22,500 < T_{\text{eff}} < 27,500$, the mass-loss rate is the linear combination of the high- T mass-loss rate ($\dot{M}_h$) and the low- T mass-loss rate ($\dot{M}_l$):

$$\dot{M} = \alpha \dot{M}_h + (1 - \alpha) \dot{M}_l, \quad (1.3)$$

where $\alpha \equiv (T_{\text{eff}} - 22,500)/500$. The wind velocity we adopt is the weighted average of the high- T wind velocity ($v_{\infty,h}$) and the low- T wind velocity ($v_{\infty,l}$), and weighting coefficients are based on ratio of $\dot{M}$ from high- and low- T components. The weighted average velocity $\bar{v}_{\infty}$ can be expressed as

$$\bar{v}_{\infty} = \frac{\alpha \dot{M}_h v_{\infty,h} + (1 - \alpha) \dot{M}_l v_{\infty,l}}{\alpha \dot{M}_h + (1 - \alpha) \dot{M}_l}. \quad (1.4)$$

(2) *Cool wind*. We follow [Leitherer et al. \(1992\)](#) to set the cool wind velocity a constant 30 km s^{-1} . As long as the dJ88 prescription is used, even if it is called from the "Dutch"

low- T scheme, we regard this part of mass loss as the cool wind.



(3) *WR wind*. We use the wind velocities given by [Nugis & Lamers \(2000\)](#) for WR stars.

a. WN stars:

$$\log(v_{\infty}/v_{\text{esc}}) = 0.61 - 0.13 \log L + 0.3 \log Y. \quad (1.5)$$

b. WC stars:

$$\log(v_{\infty}/v_{\text{esc}}) = -2.37 - 0.43 \log L - 0.07 \log Z. \quad (1.6)$$

In some transition temperatures, the mass-loss rate is composed of more than one wind component. In such cases, we use the ratios of mass-loss rates of these components as the weighting coefficients to calculate the average wind velocity. The expression is similar to Equation (A4), but with a linear combination of the hot/WR and cool wind velocities.

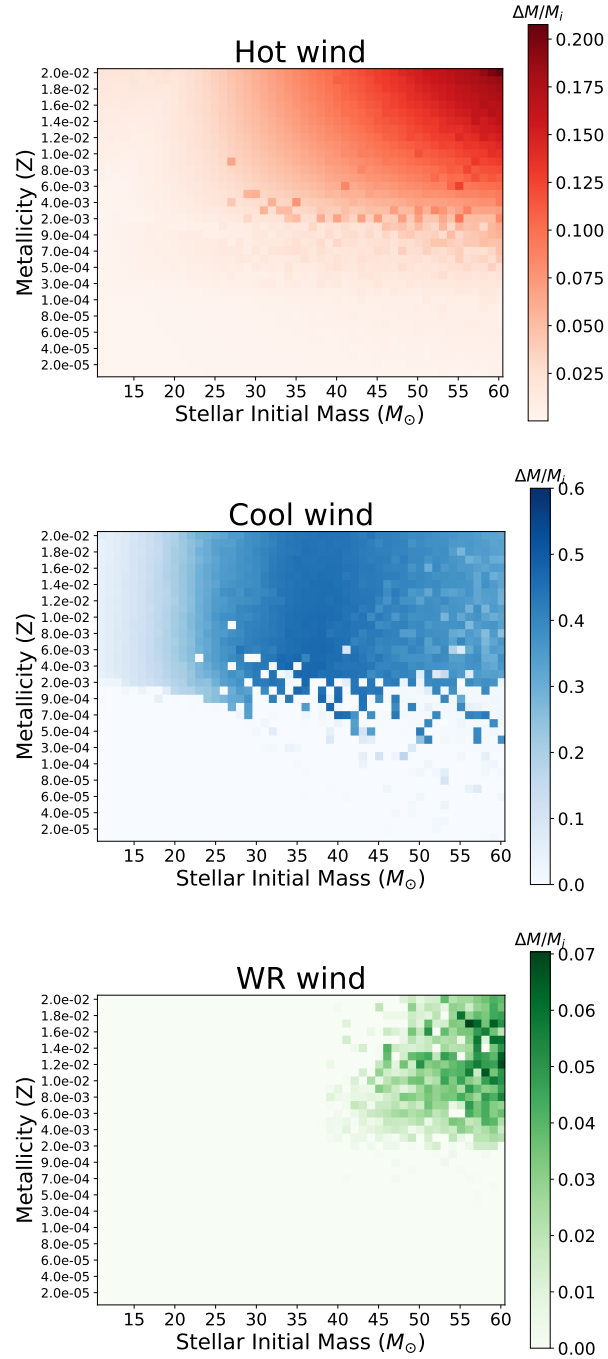


Figure 1.6: Fractions of mass loss to the initial masses caused by the hot, cool, and WR winds. These diagrams are similar to Figure 1.4, but the three wind components are separately plotted.

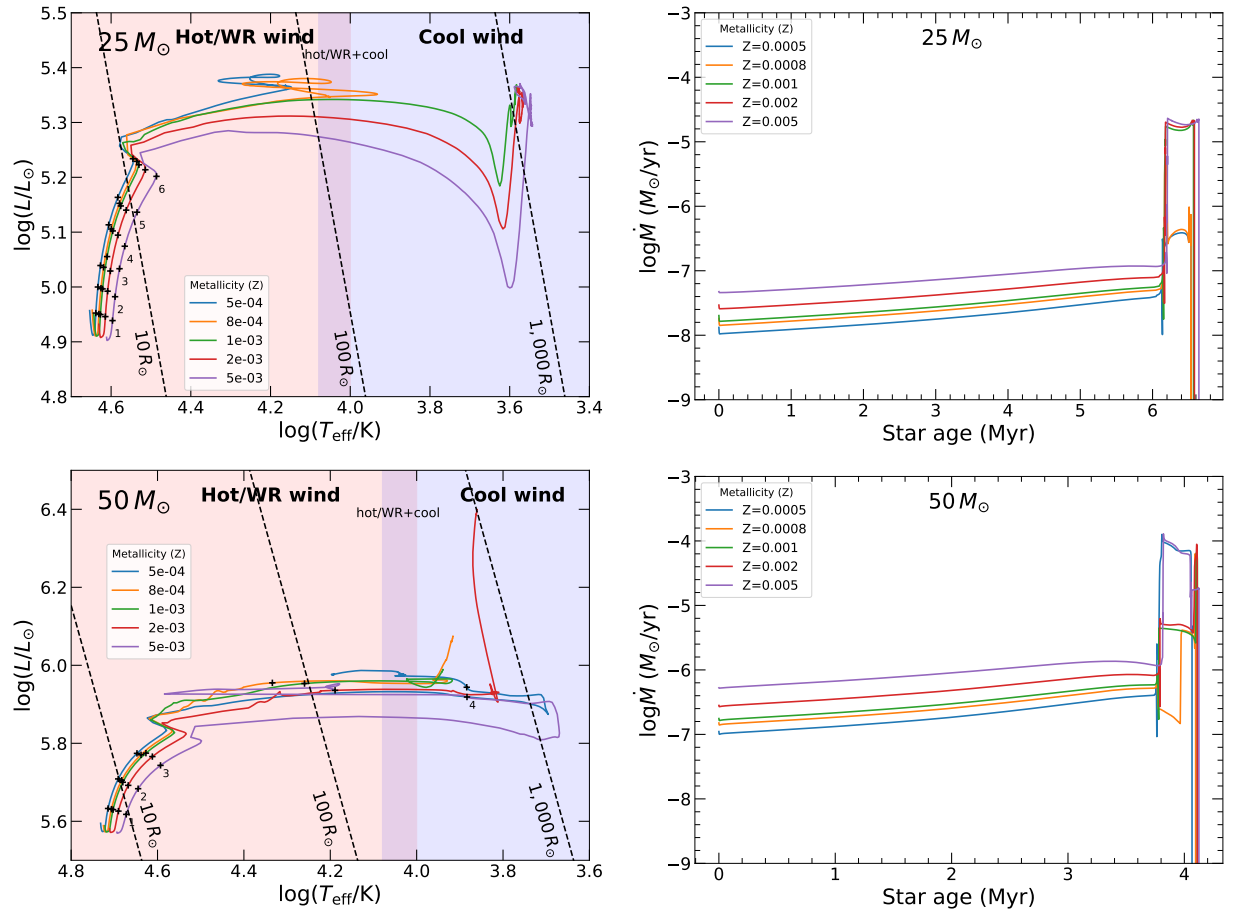


Figure 1.7: Evolutionary tracks (*left panel*) and mass-loss rates (*right panel*) of 25 and 50 $M_{\odot}$ stars with different Z that are all near the critical metallicity. The bimodal behavior of the mass loss corresponds to two different trends of evolutionary tracks; the high-mass-loss models are those that successfully evolve into cool supergiants at the beginning of the post-main-sequence stage.

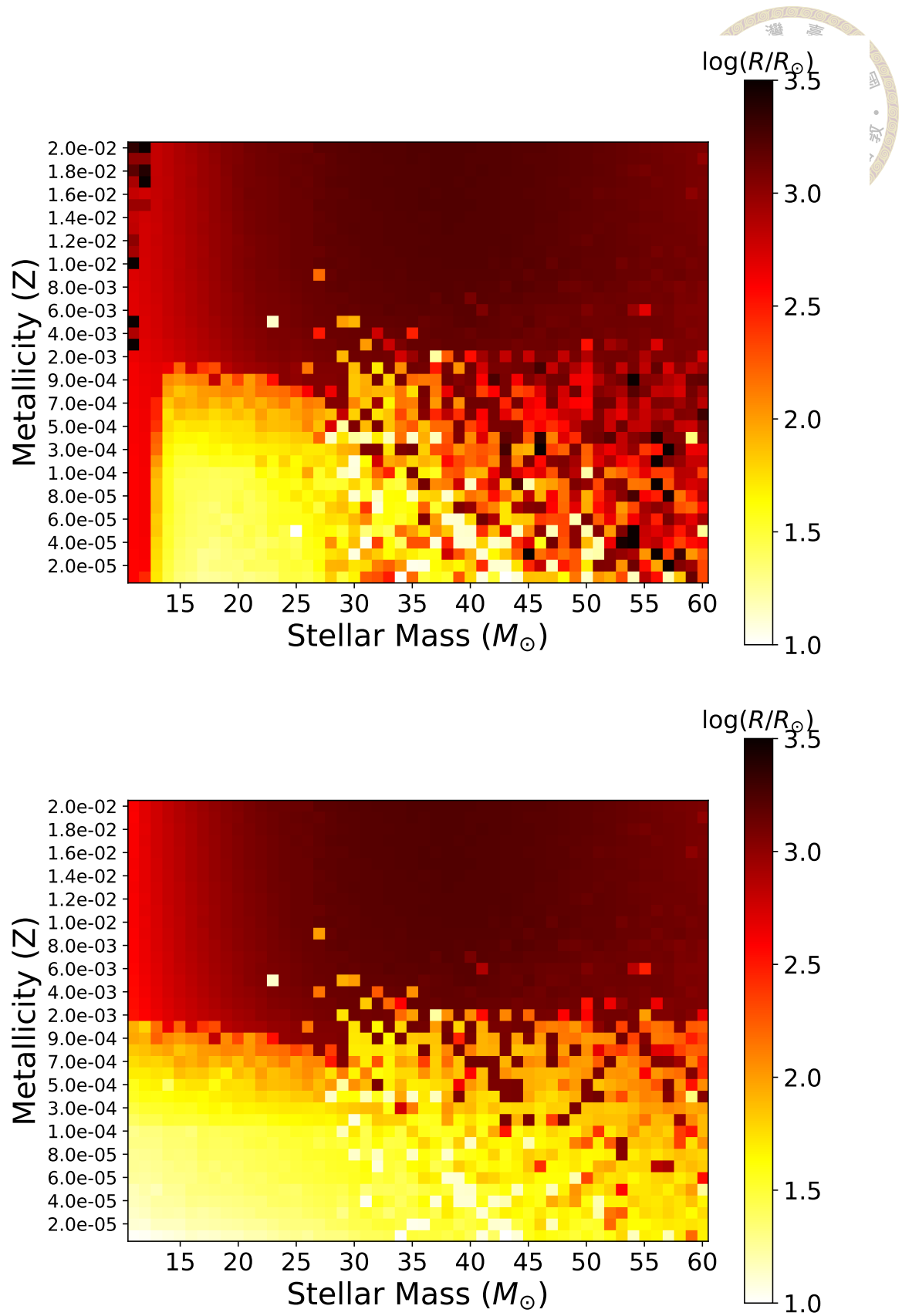


Figure 1.8: Maximum stellar radius of each model in the entire stellar lifetime (*upper panel*) and before the core-carbon burning commences (*lower panel*). The distribution of values in the right panel is almost identical to that in Figure 1.4, indicating that the low- Z models with negligible mass loss are those that fail to expand to cool supergiants before the core-carbon burning commences.

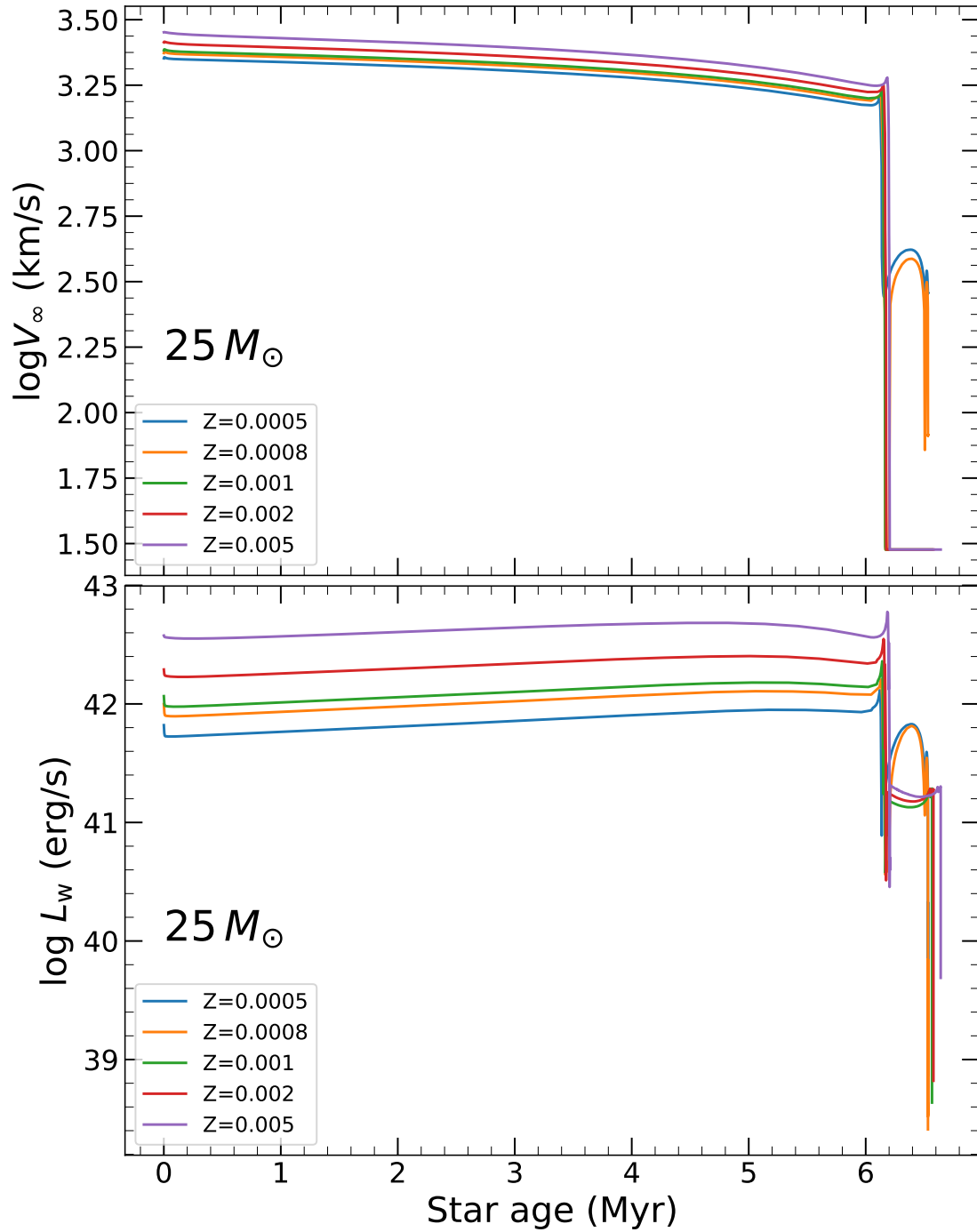
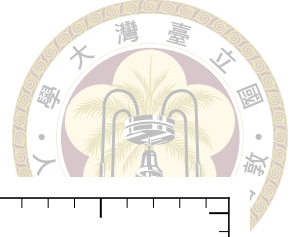


Figure 1.9: The evolution of wind terminal velocity v_{∞} (*top panel*) and mechanical luminosity L_w (*bottom panel*) of stars with initial mass $25 M_{\odot}$ and different metallicities near $Z_{\odot}$. Sudden transitions of v_{∞} and L_w occur when the stars enter the post-main-sequence stage after the age of 6 Myr.

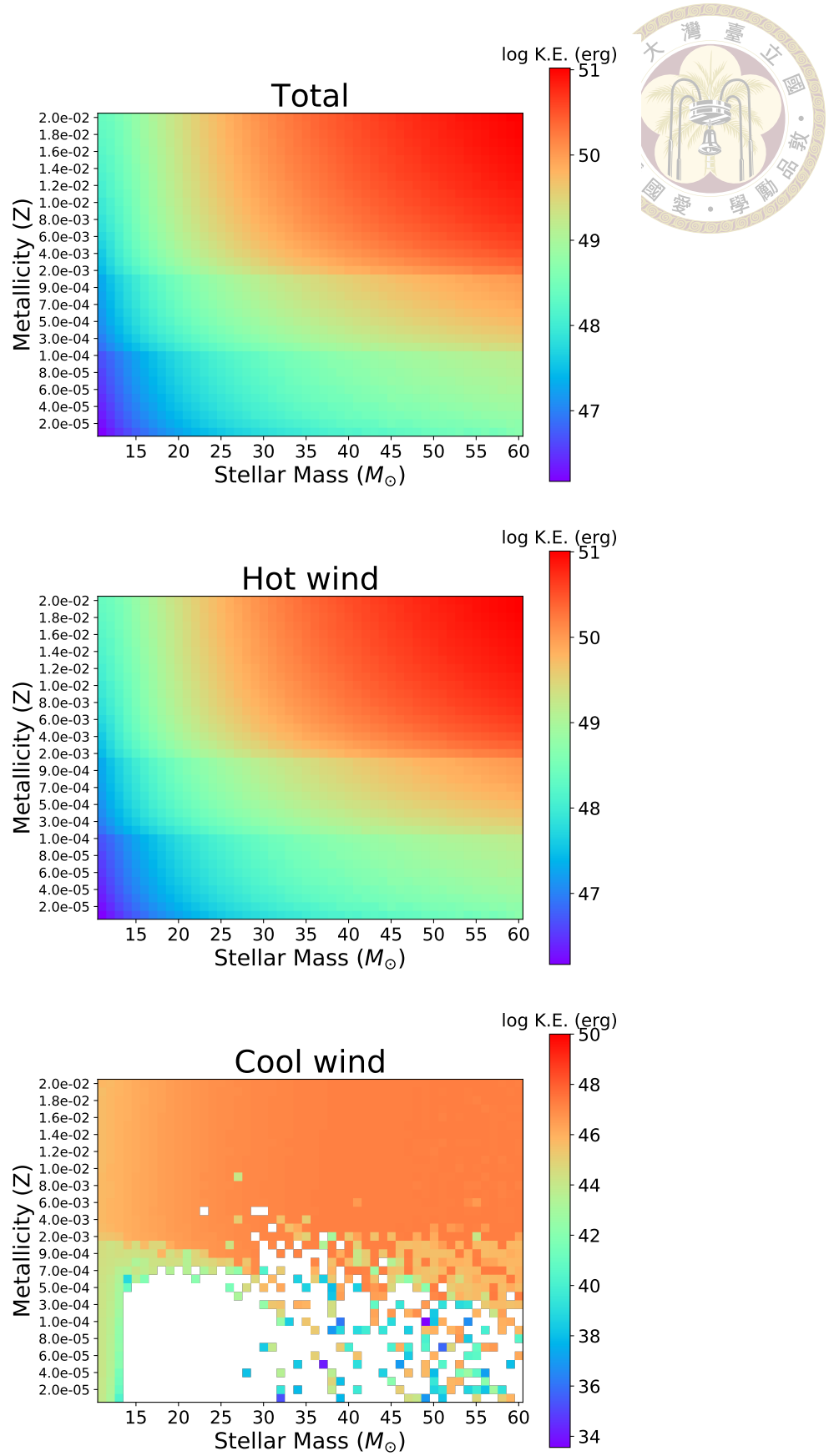


Figure 1.10: The total kinetic energy released by stellar winds throughout the stellar lifetimes. Each pixel in this map represents a stellar model. Note that the y-axis scale is non-uniform. In addition to the total energy, we separately plot the contributions from hot and cool winds, and hot winds dominate the kinetic energy released by stellar winds.

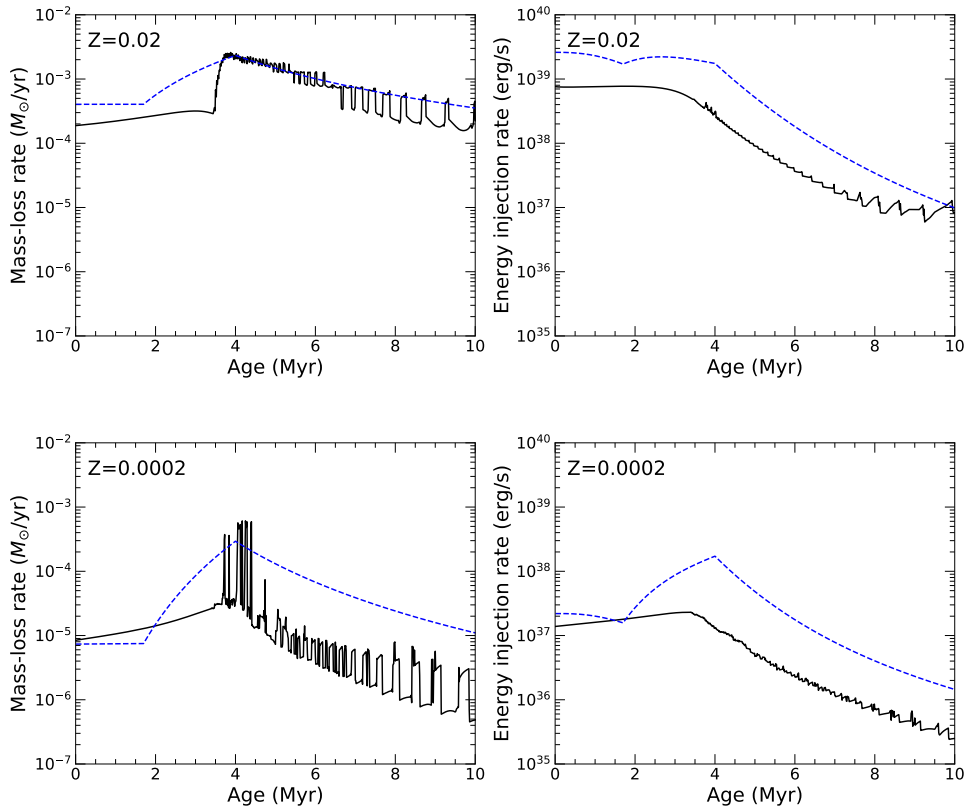


Figure 1.11: The integrated mass-loss rate and energy-injection rate from stellar winds in a star cluster of $10^5 M_{\odot}$, in which the Salpeter IMF is assumed. The upper and lower panels show the results of a $Z = 0.02$ cluster and a $Z = 0.0002$ cluster, respectively. The blue dash lines show the functions adopted in the feedback treatment of the FIRE-3 cosmological simulation (Hopkins et al. 2022).

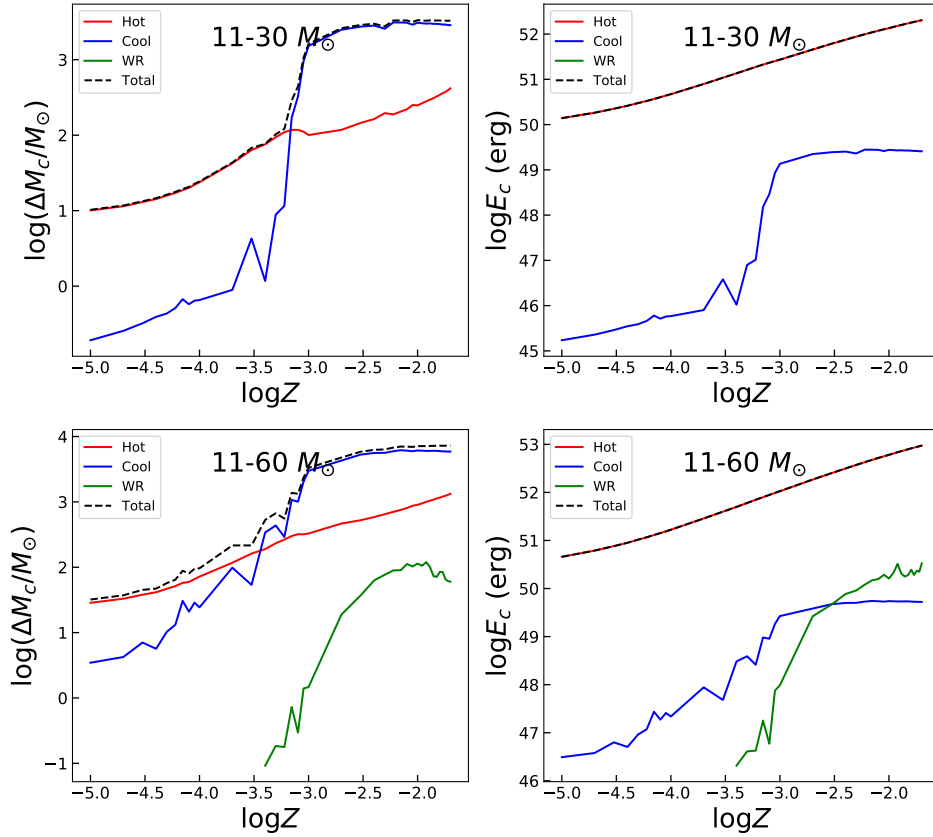


Figure 1.12: The mass (left panels) and kinetic energy (right panels) released by the stellar winds in a star cluster of $10^5 M_{\odot}$, in which the Salpeter IMF is assumed. The upper panels only include the contribution from stars of 11–30 $M_{\odot}$, and the lower panels include the effect from all of our models within 11–60 $M_{\odot}$. The curves of mass loss show a significant increase around the critical metallicity of ($Z \sim 10^{-3}$), where the cool winds exceed the hot winds. The corresponding kinetic energy curves are dominated by hot winds and thus do not show any sudden increase around the critical metallicity.



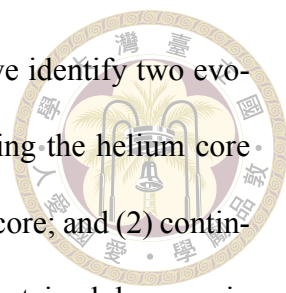


Chapter 2 Expansion Mechanism Toward Red Giants and Supergiants

* This chapter has been submitted to the AAS journals.

Abstract for Chapter 2

We revisit the long-standing question: Why do post-main-sequence stars evolve into red giants (RGs) or red supergiants (RSGs)? Using the 1D stellar evolution code MESA, we perform numerical experiments to compare stellar models that successfully transition into RGs/RSGs with those that remain blue. We identify the key physical criteria and mechanisms that govern envelope expansion when stars evolve from the core hydrogen-burning to helium-burning stage. We propose that the expansion is governed by a refined version of the "mirror principle": the stellar envelope consistently moves in the opposite direction to the outer edge of the hydrogen-burning shell (i.e., the shell-envelope boundary), rather than to the helium core. Instead of the simple energy absorption in the envelope proposed in previous studies to drive the expansion, this modified mirror principle naturally arises from hydrostatic equilibrium, as the burning shell sets a constant-pressure



boundary that moves inward or outward. Following this principle, we identify two evolutionary pathways to the RG/RSG phase: (1) direct expansion during the helium core contraction, as the burning shell keeps moving inward along with the core; and (2) continued envelope expansion after core contraction halts, regulated by a sustained decrease in nuclear energy generation rate, accompanied by the inward motion of the shell-envelope boundary. Additionally, we identify a structural transition that occurs in the envelope during the final approach to the RG/RSG phase, characterized by mass redistribution and changes in gas opacity in the envelope. These results provide a unified physical framework for understanding the post-main-sequence envelope expansion and the formation of RGs/RSGs.

2.1 Introduction

When a star exhausts its core hydrogen (H) fuel, it evolves into the post-main-sequence stage. The helium (He) core begins to contract due to the Schönberg–Chandrasekhar limit (e.g., [Kippenhahn et al. 2012](#)), while the stellar envelope expands. This expansion leads to the formation of red giants (RGs; for initial masses $\lesssim 8 M_{\odot}$) or red supergiants (RSGs; for initial masses $\gtrsim 8 M_{\odot}$). The inverse relationship between the core contraction and envelope expansion is often described as the "mirror effect" or "mirror principle" ([Kippenhahn et al. 2012](#); [Maeder 2009](#)). However, the physical mechanism that drives stellar envelopes to expand to significantly large radii with a red color remains unclear.

Various theories have been proposed to explain why a star becomes an RG/RSG, attributing the key role to factors such as degree of core contraction ([Eggleton & Cannon 1991](#); [Faulkner 2005](#)), thermal imbalance governed by core luminosity and envelope

opacity (Renzini 2023; Renzini & Ritossa 1994; Renzini et al. 1992; Renzini 1984; Ritossa 1996), gradients of the opacity and mean molecular weight (Sugimoto & Fujimoto 2000), and the pressure and density contrast between the shell and the core (Miller Bertolami 2022). Some other authors suggested that no single factor plays a major role (Iben 1993; Whitworth 1989). Despite these varied theories, there has been no consensus on what precisely drives stars to turn red.

A clue to understanding the physics of stellar evolution into RG/RSG is the concept of "critical metallicity" required for RSG formation, as suggested by models of massive stars in Chapter 1. According to these models, not all stars successfully evolve into RSGs during their core-He burning phase. Only stars with metallicities above a specific threshold can transition into RSGs; those below this threshold remain as BSGs with smaller radii.

Due to the complex physics underlying stellar expansion and the transition between blue and red stars, we divide this study into two parts. In this chapter, we investigate the mechanisms that govern the evolution of stars into RGs/RSGs, focusing on identifying the criterion that determines whether a star turns red or remains blue. Chapter 3 will follow, where we construct steady-state envelope structures to elucidate the physical principles behind the behaviors observed in our MESA models.

The structure of this chapter is as follows. Section 2.2 describes the construction of our stellar models. In Section 2.3 we use a set of $25 M_{\odot}$ models to identify the criterion for a star to evolve into the RSG stage. Section 2.4 extends this analysis to stars with varying masses and metallicities. Section 2.5 discusses the physics of the mirror principle during envelope expansion. Section 2.6 explores how envelope properties, including opacity and the structural transitions, play a role during expansion. Section 2.7 concludes the chapter

and highlights open questions for future study.



2.2 Methods

To investigate the physical origin of RGs and RSGs, we construct stellar models using the Modules for Experiments in Stellar Astrophysics (MESA; [Jermyn et al. 2023](#); [Paxton et al. 2019, 2018, 2015, 2013, 2011](#)) with the version No.10108. MESA is a one-dimensional stellar evolution code that solves the stellar structure and composition equations, incorporating physical modules such as nuclear reaction networks, equations of state, opacities, and the mixing length theory (MLT).

2.2.1 Grid of different masses and metallicities

We develop a grid of non-rotating stellar models with initial masses from 1 to $30 M_{\odot}$ in $1 M_{\odot}$ intervals and 38 different metallicities: $Z = (1, 2, \dots, 9) \times 10^{-5}$, $(1, 2, \dots, 9) \times 10^{-4}$, $(1, 2, \dots, 9) \times 10^{-3}$, and $(1.0, 1.1, \dots, 2.0) \times 10^{-2}$.

For stars with masses $\geq 11 M_{\odot}$, we adopt the models from Chapter 1. We first calculate pre-main-sequence evolution to create zero-age-main-sequence (ZAMS) models, and then continue to evolve these models from the ZAMS to iron core collapse. After the ZAMS, we primarily adopt the default settings for massive stars from the “in-list_massive_defaults” file within the MESASTAR module. In this setup for massive stars, convection is treated using the MLT following [Heney et al. \(1965\)](#), with a mixing-length parameter $\alpha_{\text{MLT}} = 1.5$. The Ledoux criterion is adopted to allow for semi-convection, with an efficiency parameter $\alpha_{\text{sc}} = 0.01$. Thermohaline mixing is included using the [Kippenhahn et al. \(1980\)](#) prescription, with an efficiency parameter $\alpha_{\text{th}} = 2$. We employ the

”approx21 (Cr60)” nuclear reaction network, which includes 21 isotopes. Opacities are set using the ”gs98” option, and Type 2 opacities are used to account for carbon (C) and oxygen (O) enhancement in the post-main-sequence stage. Mass loss is treated using the hot wind prescription from [Vink et al. \(2001\)](#), the cool wind prescription from [de Jager et al. \(1988\)](#), and the WR wind prescription from [Nugis & Lamers \(2000\)](#). For more details of these models, see Chapter 1.

For stars with masses of $\leq 10 M_{\odot}$, we employ the settings from the test suite ”1M_pre_ms_to_wd” to evolve the stars from pre-main-sequence stage to the white dwarf phase. The nuclear reaction network ”o18_and_ne22” includes 10 isotopes: ^1H , ^3He , ^4He , ^{12}C , ^{14}N , ^{16}O , ^{18}O , ^{20}Ne , ^{22}Ne , and ^{24}Mg . We use the ”gs98” option for high-temperature opacities and the ”lowT_fa05_gs98” option for low-temperature opacities. The mixing length parameter α_{MLT} is set to be 2 as default. For mass loss, the wind prescription from [Reimers \(1975\)](#) is used during the red giant branch phase, and the wind prescription from [Bloeker \(1995\)](#) is used during the asymptotic giant branch phase.

2.2.2 Experiments with $25 M_{\odot}$ stars

We also perform a series of experiments to examine how variations in nuclear reaction rates and opacity influence stellar evolution toward RGs/RSGs. In the experiment described in Section 2.3, we focus on a $25 M_{\odot}$ model with metallicity $Z = 0.001$, which lies near the critical metallicity separating BSGs and RSGs (Chapter 1). At this transitional metallicity, the evolutionary outcome is particularly sensitive to parameter changes, such that even a small perturbation can cause the star to shift between BSG and RSG paths. In this experiment, we vary the triple-alpha reaction rate by applying a linear scaling factor, $\eta_{3\alpha}$, to investigate how changes in the He-burning rate within the core affect the star’s

evolutionary fate.

We also perform experiments to investigate the effect of opacity on stellar expansion by introducing the parameter ζ_κ , which specifies the base metallicity (Z_{base} in MESA) used to retrieve opacity values from the opacity tables. For $25 M_\odot$ stars, we explore various combinations of Z and ζ_κ , where Z represents the absolute metallicity of the star, and ζ_κ serves as a separate parameter controlling the opacity.



2.3 Physical Conditions for Transition to Red Supergiants

It has been shown that while some stars successfully evolve into RSGs in the core-He burning phase, others remain as BSGs without further expansion into the RSG regime (Chapter 1). Comparing these two types of stellar evolution can provide valuable insight into the criteria and physical mechanisms that determine RSG formation. In this section, we examine MESA models of $25 M_\odot$ stars with metallicity $Z = 0.001$, varying only $\eta_{3\alpha}$, to investigate the conditions under which a star can evolve into an RSG.

For our analysis, we define three distinct layers within a star at the beginning of the post-main-sequence stage, as illustrated in the schematic diagram in Figure 2.1: the He core, the H-burning shell, and the envelope. The extent of the He core, with a radius denoted as R_{core} , is defined as the location where the H mass fraction (X_{H}) falls below 0.1, following the default definition adopted in MESA. The boundary between the H-burning shell and the envelope, with radius denoted as R_{shell} , is defined as the location where the local energy generation rate from shell burning falls below $10^3 \text{ erg s}^{-1} \text{ g}^{-1}$.

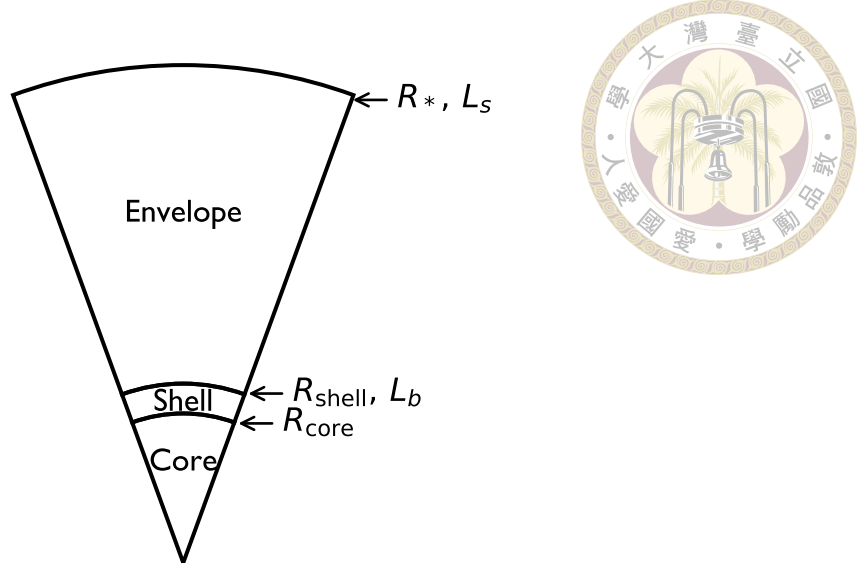


Figure 2.1: Schematic diagram of the post-main-sequence stellar structure discussed in this chapter, consisting of a He core, a H-burning shell, and an envelope. The radii and luminosities at the layer boundaries are labeled as follows: R_* and L_s at the stellar surface, R_{shell} and L_b at the interface between the envelope and the H-burning shell, and R_{core} at the boundary between the shell and the He core.

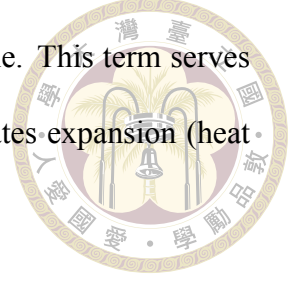
2.3.1 Energy Budget

An intuitive explanation for RSG formation, from an energy perspective, is that the envelope expands by absorbing gravitational energy released from the core contraction. To clarify this concept and explore the physical mechanism driving the transition to the RSG phase, we analyze the energy budgets associated with envelope expansion during the BSG and RSG evolutionary stages.

We begin by defining the key quantities used to evaluate the energy budget in a star. In MESA outputs, the local energy generation rate consists of three terms: the nuclear energy generation rate ϵ_{nuc} , the neutrino loss rate ϵ_{ν} , and the gravothermal term ϵ_{grav} , which describes the rate of gravothermal heat absorption or liberation. The gravothermal term is defined as

$$\epsilon_{\text{grav}} \equiv -T \frac{\partial s}{\partial t}, \quad (2.1)$$

where T is the temperature, s is the specific entropy, and t is the time. This term serves as a diagnostic for stellar expansion or contraction: $\epsilon_{\text{grav}} < 0$ indicates expansion (heat absorption), while $\epsilon_{\text{grav}} > 0$ indicates contraction (heat release).



The energy conservation equation in stellar interiors is written as

$$\frac{dl}{dm} = \epsilon_{\text{nuc}} + \epsilon_{\text{grav}} - \epsilon_{\nu}, \quad (2.2)$$

where l is the local luminosity and m is the mass coordinate. By integrating this equation over a given mass range, we obtain the luminosity difference (ΔL) between the base (L_b) and the surface (L_s) of the stellar envelope:

$$\Delta L = L_s - L_b = \dot{E}_{\text{nuc}} - \dot{Q} - L_{\nu}, \quad (2.3)$$

where $\dot{E}_{\text{nuc}}$, $\dot{Q}$, and L_{ν} are the integrals of ϵ_{nuc} , ϵ_{grav} , and ϵ_{ν} , respectively, along the stellar mass coordinates. Specifically, we define the integral of the gravothermal term as

$$\dot{Q} \equiv - \int_{M_1}^{M_2} \epsilon_{\text{grav}} dm, \quad (2.4)$$

with M_1 and M_2 denoting the lower and upper mass boundaries of the integration region. A positive $\dot{Q}$ implies net heat absorption, while a negative value indicates heat release.

In our stellar models, L_{ν} are typically negligible, so Equation (3) simplifies to $\dot{Q} \simeq \dot{E}_{\text{nuc}} + (L_b - L_s)$. Within the stellar envelope, there is nearly no nuclear reaction, so $\dot{E}_{\text{nuc}} \simeq 0$. Therefore, the envelope's gravothermal term can be approximated by the thermal imbalance between the base and the surface:

$$\dot{Q}_{\text{env}} \simeq L_b - L_s \quad (2.5)$$

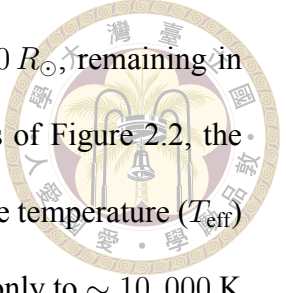
This imbalance serves as a robust diagnostic of envelope expansion or contraction (e.g., [Renzini & Ritossa 1994](#); [Renzini et al. 1992](#); [Ritossa 1996](#)). During expansion, $L_b > L_s$ as radiative energy is partially trapped and used to perform P - V work. Conversely, during contraction, $L_b < L_s$ as gravothermal energy is released and radiated away.

In Figure 2.2, we compare the energy budget of two MESA models, both with an initial mass of $25 M_\odot$ and a metallicity of $Z = 0.001$, differing only in the value of $\eta_{3\alpha}$. Model A adopts $\eta_{3\alpha} = 1$, corresponding to the standard triple-alpha reaction rate, and successfully evolves into an RSG in the post-main-sequence phase. In contrast, Model B uses $\eta_{3\alpha} = 2$ and remains a BSG throughout the core-He burning stage, without transitioning into an RSG.

For each model, we plot the evolution of various physical quantities in the top panels of Figure 2.2 to analyze the energy budget: stellar radius (R_*), surface luminosity (L_s), the envelope's base luminosity (L_b), energy generation rate by H burning (L_H), energy generation rate by He burning (L_{He}), and gravothermal heat absorption rate of the core ($\dot{Q}_{\text{core}}$) and the envelope ($\dot{Q}_{\text{env}}$).

At the beginning of the post-main-sequence phase, before the onset of core-He burning, Models A and B evolve similarly. At ~ 6.13 Myr, core contraction releases gravothermal energy, indicated by negative $\dot{Q}_{\text{core}}$. Soon after the onset of core contraction, the shell-H burning is ignited. At this time, the envelope begins to expand, resulting in an increasing stellar radius R_* . *The energy source of this expansion includes the gravothermal energy released by the contracting core and energy produced by the H-burning shell.*

The evolution paths of the two models begin to bifurcate after core-He burning commences. Model A successfully evolves into an RSG, reaching a radius of $\sim 1,000 R_\odot$.



In contrast, Model B stops expanding when its radius reaches $\sim 150 R_{\odot}$, remaining in the BSG phase. As shown in the HR diagrams in the bottom panels of Figure 2.2, the surface temperature of Model A continues to cool down to an effective temperature (T_{eff}) of $\sim 4,000$ K, becoming an RSG. Model B, on the other hand, cools only to $\sim 10,000$ K before reheating, never reaching the RSG regime.

If the extent of envelope expansion were solely determined by amount of energy supplied by the core contraction, then we should see that the core of Model A release more energy than that of Model B, since it expands to a larger radius. However, Figure 2.2 shows that this is not the case. Instead, the evolution of $\dot{Q}_{\text{core}}$ is nearly identical for both models: a negative peak at ~ 6.13 – 6.16 Myr, followed by a return to ~ 0 . After ~ 6.16 Myr, while the envelope of Model A continues to expand, its core nearly no longer releases gravothermal energy. There is no additional energy released from the core in Model A compared to Model B. Thus, the release of gravothermal energy from the core alone cannot account for the formation of the RSG.

After ~ 6.17 Myr, the positive $\dot{Q}_{\text{env}}$ of Model A is primarily contributed by the decrease of L_s , rather than enhanced L_b . In fact, L_b in Model A decreases even more rapidly than in Model B. While in Model B L_s gradually increases to approach the level of L_b , in Model A L_s instead declines. Notably, during 6.17 – 6.18 Myr, Model A exhibits a prominent bump in $\dot{Q}_{\text{env}}$, indicating enhanced energy absorption by the envelope. This increase is not accompanied by any corresponding rise in $\dot{Q}_{\text{core}}$, indicating that it is not due to additional energy input from the core. Instead, it results from a larger fraction of the existing nuclear luminosity being trapped within the envelope, as reflected by the drop in L_s .

Therefore, according to these models, the evolution toward an RSG can be divided

in two distinct stages, based on the energy budget of stellar envelope:

(1) **From TAMS to BSG.** Both models undergo this stage, expanding to $\sim 100 R_{\odot}$ as BSGs. At this stage, core contraction releases gravothermal energy, resulting in negative $\dot{Q}_{\text{core}}$. In response, the envelope absorbs energy from core contraction and shell H burning, showing a positive peak in $\dot{Q}_{\text{env}}$.

(2) **From BSG to RSG.** At the BSG phase, the evolution paths of Model A and B bifurcate. While Model B halts its expansion and remains a BSG, Model A continues expanding into the RSG regime as its envelope increasingly traps outgoing radiation. This second stage of expansion is not powered by gravothermal energy from the core, and therefore is not a necessary consequence of core contraction.

2.3.2 The Mirror Principle

To further investigate the how the envelope responds to core contraction, we plot the evolution of R_* alongside R_{core} for Models A and B, as shown in Figure 2.3. At the beginning of the post-main-sequence stage, both models evolves similarly: as the He core contracts, the envelope expands. However, when $R_* \sim 100 R_{\odot}$, their evolutionary paths bifurcate. In Model B, the core radius reaches a minimum of $R_{\text{core}} \sim 0.48 R_{\odot}$ and subsequently begins to increase, while the envelope stops expanding at $\sim 150 R_{\odot}$ and then gradually contracts to $\sim 40 R_{\odot}$. In contrast, Model A successfully evolves into an RSG with $R_* \sim 1,000 R_{\odot}$ before the core radius rises again.

Generally, the stellar envelope tends to move in the opposite direction to the core: when the core contracts, the envelope tends to expand; when the core radius grows again at the BSG stage, the envelope ceases expanding and begins to contract. However, this

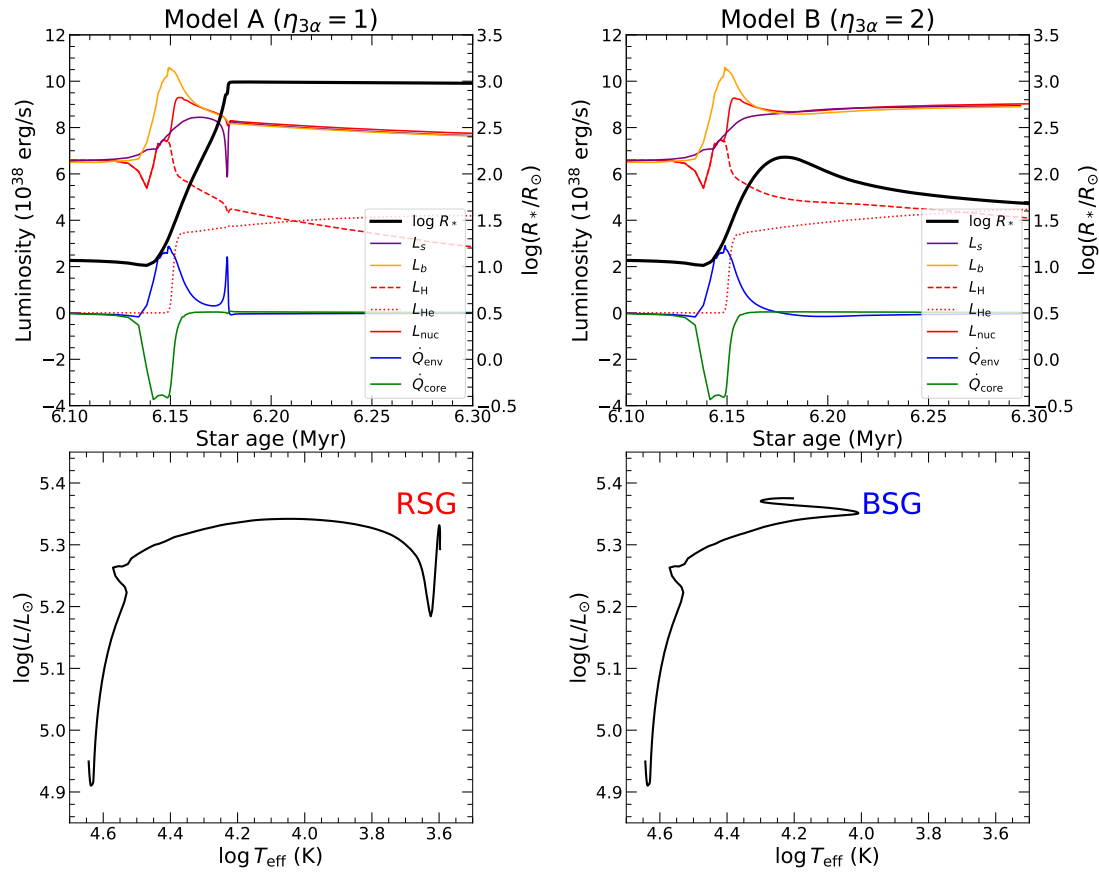


Figure 2.2: Evolution of $25 M_{\odot}$ stars with metallicity $Z = 0.001$ but different He-burning rates, controlled by a linear scaling factor $\eta_{3\alpha}$. Model A (*left panels*) adopts the standard rate ($\eta_{3\alpha} = 1$) and evolves into an RSG, while Model B (*right panels*) has an enhanced rate ($\eta_{3\alpha} = 2$) and remains a BSG. The top panels show the time evolution of the stellar radius (R_*), surface luminosity (L_s), luminosity at the base of the envelope (L_b), energy generation rate by H burning (L_H), by He burning (L_{He}), the total nuclear luminosity (L_{nuc}), and the gravothermal heat absorption rates for the core ($\dot{Q}_{core}$) and the envelope ($\dot{Q}_{env}$). The bottom panels display the evolutionary tracks of the two models on the Hertzsprung–Russell diagram.

inverse relationship does not always hold. As shown in the left panel of Figure 2.3, Model A exhibits significant envelope expansion by a factor of ~ 10 in radius, from $R_* \sim 100 R_{\odot}$ to $R_* \sim 1,000 R_{\odot}$, even though the core remains stable with $R_{core} \sim 0.47 R_{\odot}$ rather than continuing to contract. This indicates that core contraction is not responsible for the continued expansion. This is consistent with the energy budget shown in Figure 2.2, which shows that the gravothermal energy input from the core (represented by $\dot{Q}_{core}$) is negligible during this transition from BSG to RSG. Thus, the continued expansion in Model A should

be driven by another mechanism instead of core contraction.

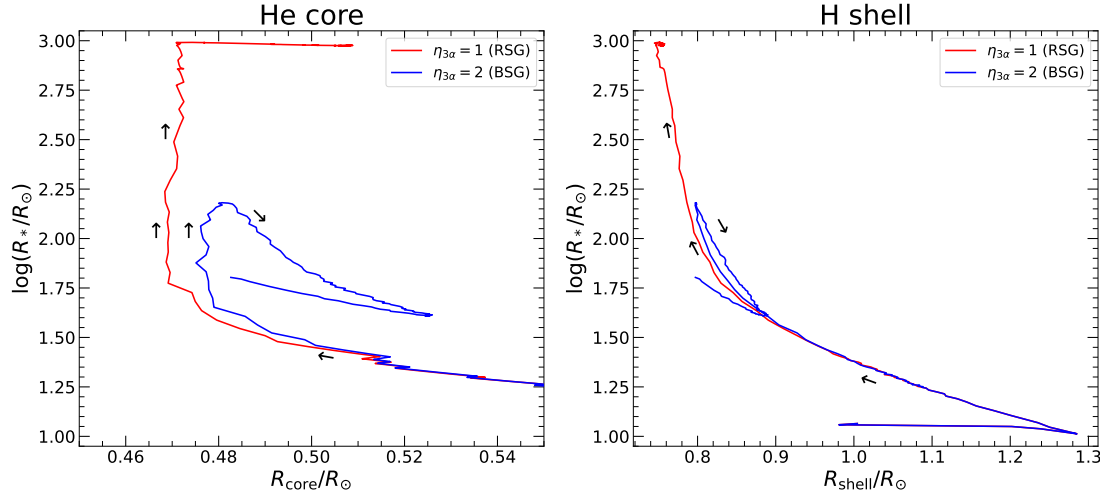
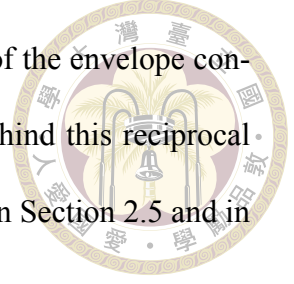


Figure 2.3: Evolution of the stellar radius (R_*) alongside the He core radius (R_{core}) and the radius at the outer edge of the H-burning shell (R_{shell}). Arrows indicate the direction of time progression during stellar evolution. Notably, R_* consistently evolves in the opposite direction of R_{shell} .

A more robust correlation appears when R_{core} is replaced by the radius at the outer edge of the H-burning shell, R_{shell} , as shown in the right panel of Figure 2.3. For both models, the envelope’s outer boundary (traced by R_*) and shell-envelope boundary with the shell (traced by R_{shell}) consistently move in the opposite directions. Initially, for both models, the burning shell moves inward while the envelope expands. After the two models diverge in their evolution when $R_* \gtrsim 100 R_\odot$, this inverse relationship between R_{shell} and R_* still persists: in Model A, R_{shell} continues to move inward while R_* increases to $\sim 1,000 R_\odot$; in Model B, at $R_* \sim 150 R_\odot$, R_{shell} reverses its motion and begins to move outward, and the envelope begins to contract at the exact same time. At all times, changes in the location of the shell-envelope boundary lead to a nearly instantaneous response of the entire envelope, either expanding or contracting accordingly.

This result refines the traditional description of the “mirror principle” for stars with a burning shell: rather than an inverse relationship between the envelope and the core,

the more accurate description is that the outer and inner boundaries of the envelope consistently evolve in opposite directions. The physical mechanism behind this reciprocal behavior, as revealed by our stellar models, will be further explored in Section 2.5 and in Chapter 3.



2.3.3 Role of Envelope Base Luminosity

A key turning point in supergiant evolution occurs when the motion of the shell-envelope boundary (at radius R_{shell}) reverses, thereby halting further expansion toward the RSG regime. In this subsection, we examine the physical conditions that trigger this reversal, focusing on the role of the local luminosity at the shell-envelope boundary (L_b).

Figure 2.4 shows the relationship between R_{shell} and L_b . Notably, in Model B, the reversal of R_{shell} from inward to outward motion occurs exactly when L_b changes from decreasing to increasing. In other words, the turning points of L_b , R_{shell} , and also R_* all occur at the same time. In contrast, in Model A, both L_b and R_{shell} continue to decrease until R_{shell} reaches $\sim 0.75 R_{\odot}$. These results suggest that once L_b reaches its maximum, its subsequent evolution closely correlates with that of R_{shell} . The turning point of envelope expansion is likely regulated by this relationship.

At this turning point of L_b , the core contraction has already ended, as shown in Figure 2.3. Thus, the luminosity supplied from the base of the envelope is primarily provided by nuclear burning, including the shell-H burning and the core-He burning. As a result, L_b closely approximates the total nuclear energy generation rate, L_{nuc} . During this phase, given the established correlation between L_b and R_{shell} , and the anti-correlation between R_{shell} and R_* , an anti-correlation between L_{nuc} and R_* is also expected. To test this re-

lationship, we computed additional models with $25 M_{\odot}$ and $Z = 0.001$, varying the $\eta_{3\alpha}$ parameter, with particular focus values between those of Models A and B.

Figure 2.5 shows the relationships between R_* and L_{nuc} , L_{H} and L_{He} for the set of models with various $\eta_{3\alpha}$. Models with larger $\eta_{3\alpha}$, shown in blue, evolve into BSGs, while those with smaller $\eta_{3\alpha}$, shown in red, evolve into RSGs. In all cases, after L_{nuc} reaches its peak, it decreases as R_* continues to increase. For models with larger $\eta_{3\alpha}$, L_{nuc} subsequently begins to rise again, coinciding with the onset of envelope contraction, i.e., a decrease in R_* . The turning points of L_{nuc} and R_* occur simultaneously in all the models. In contrast, for models with smaller $\eta_{3\alpha}$, L_{nuc} continues to decline as R_* increases, until the envelope approaches the RSG regime at $\sim 1,000 R_{\odot}$. These results further reinforce the correlation between the luminosity supplied at the base of the envelope and the global expansion or contraction of the envelope.

We further examine the evolution of L_{H} and L_{He} , which are the components contributing to changes in L_{nuc} . In models with higher $\eta_{3\alpha}$, L_{H} begins to decline earlier, at smaller values of R_* . Shortly afterward, their L_{He} rises more sharply compared to models with lower $\eta_{3\alpha}$. This rapid increase in L_{He} compensates for the decline in L_{H} , causing the total L_{nuc} to reverse its downward trend and begin rising. Once L_{nuc} starts to increase, the burning shell moves outward, and the stellar envelope begins to contract. In contrast, in models with lower $\eta_{3\alpha}$, L_{He} increases more slowly and fails to offset the faster decline of L_{H} , resulting in a continued decrease in L_{nuc} . Consequently, these stars maintain envelope expansion and evolve toward the RSG phase.

Based on the results above, we propose a criterion to determine whether a BSG that

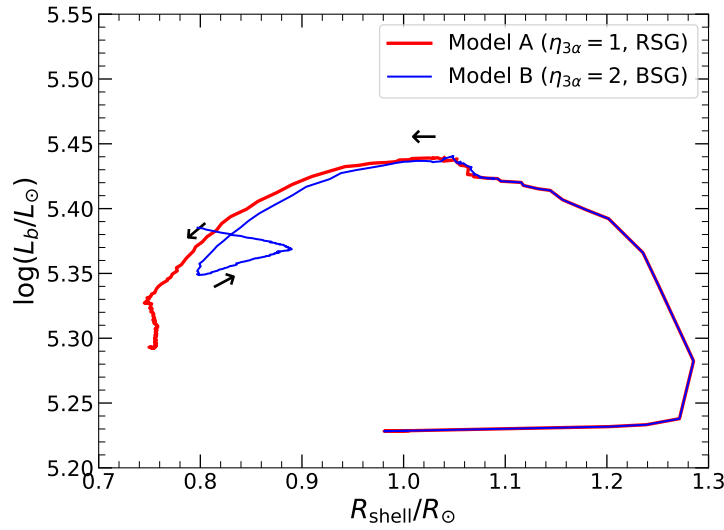


Figure 2.4: Evolution of the radius and luminosity at interface between the H-burning shell and the envelope for Models A and B. The radius and luminosity at this location are denoted as R_{shell} and L_b , respectively. The arrows indicate the directions of evolution in time.

has completed core contraction will continue to expand at a given time

$$\frac{dL_{\text{nuc}}}{dt} = \frac{d}{dt}(L_{\text{H}} + L_{\text{He}}) < 0. \quad (2.6)$$

As long as this condition is satisfied, the envelope continues to expand. Once this quantity changes sign, i.e., $dL_{\text{nuc}}/dt > 0$, the stellar envelope of the BSG begins to contract immediately.

By varying $\eta_{3\alpha}$, we effectively control the nuclear burning rates and find a clear correlation between nuclear energy generation and envelope motion. This suggests that the envelope's behavior is directly regulated by nuclear burning.

Importantly, the mechanism by which the central region of the star governs the envelope expansion toward the RSG phase is not simply a matter of energy supply. From the TAMS to the BSG stage, while the core is still contracting, an enhanced energy supply to the base of the envelope does correspond to envelope expansion. However, once core

contraction halts, continued expansion requires a decline—rather than an increase—in L_b (or equivalently, L_{nuc}). If L_{He} increases too rapidly, it can lead to a rise in L_{nuc} and trigger contraction instead. Therefore, the energy supply alone cannot fully explain RSG formation, contradicting earlier claims that increasing core luminosity makes stars more likely to expand (e.g., [Renzini & Ritossa 1994](#)).

Instead of driving expansion by simply supplying energy, *the evolution of stellar burning region governs the supergiant expansion as follows: changes in L_{nuc} determine the motion of R_{shell} , and the stellar radius R_* consistently moves in the opposite direction.* The envelope's expansion or contraction does not depend directly on the amount of energy it receives, but rather on the motion of the burning shell. Even as the energy supply from the stellar interior decreases, expansion toward the RSG phase can be feasible by trapping a larger fraction of outgoing radiation, resulting in a drop in surface luminosity L_s .

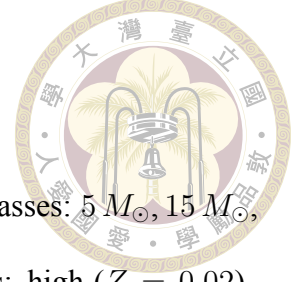
In summary, the $25 M_{\odot}$ models with varying values of $\eta_{3\alpha}$ allow us to refine the description of the mirror principle and identify a clear criterion (Equation 6) for continued envelope expansion after core contraction. In the following section, we extend this criterion to stars across a broad range of masses and metallicities.

2.4 General Criteria for Stars to Turn Red

2.4.1 Evolution Across Different Masses and Metallicities

To generalize the criteria for RSG formation, we examine the grid of stellar models with initial masses ranging from 1 to $30 M_{\odot}$ and metallicities from $Z = 10^{-5}$ to 2×10^{-2} , as described in Section 2.2.1.

2.4.1.1 Examples of 5, 15, and 25 $M_{\odot}$



As illustrative examples, we first analyze the evolution of three masses: 5 $M_{\odot}$, 15 $M_{\odot}$, and 25 $M_{\odot}$. For each mass, we present models at three metallicities: high ($Z = 0.02$), intermediate ($Z = 0.002$), and low ($Z = 0.0002$). Figure 2.6 presents the post-main-sequence evolution of R_{shell} , R_{core} , the central pressure (P_c), and L_{nuc} before the formation of carbon core for each case.

In all cases shown in Figure 2.6, the stellar radius R_* generally evolves in the opposite direction of R_{shell} throughout the post-main-sequence evolution. This behavior is consistent with the refined mirror principle suggested in Section 2.3.

Next, we assess the applicability of the criterion for continued expansion at the BSG stage given by Equation (6). As discussed in Section 2.3, a prerequisite for applying this criterion is that core contraction has already ceased, so that the motion of the burning shell is no longer simply following the core. We therefore begin by determining when core contraction ceases.

In some cases, it is difficult to use the defined R_{core} (the boundary of $X_{\text{H}} = 0.1$) to determine whether the core is still contracting, so we use the central pressure (P_c) as a more robust diagnostic. Figure 2.7 shows a zoom-in of the evolution of P_c as a function of R_* for the 25 $M_{\odot}$ models. For $Z = 0.002$, P_c reaches a maximum at $\log(R_*/R_{\odot}) \sim 1.75$ and then begins to decline. In contrast, for $Z = 0.02$, it is notable that P_c continues to rise even as the star expands into an RSG with $R_* > 1,000 R_{\odot}$. Returning to Figure 2.6, among the nine models shown, five exhibit ongoing core contraction while the star is already at RG/RSG radii—specifically, all three high-metallicity models ($Z = 0.02$) and the two 5 $M_{\odot}$ models at lower metallicities.

These results reveal that in some models, P_c continues to rise throughout the envelope expansion toward the RG/RSG phase. In such cases, since R_{shell} continues to move inward together with the core, the envelope growth into the RSG regime occurs independently of the behavior of L_{nuc} .

For the remaining models, core contraction completes before the star turns red. In these cases, Equation (6) provides the criterion for determining whether the envelope can continue expanding. For the intermediate-metallicity ($Z = 0.002$) models with $15 M_{\odot}$ and $25 M_{\odot}$, L_{nuc} steadily declines after core contraction ends, allowing the envelope to expand continuously into the RG/RSG regime. In contrast, in the low-metallicity ($Z = 0.0002$) models with the same masses, L_{nuc} initially decreases after core contraction ends, but then begins to rise again during the BG/BSG stage. At the turning point of L_{nuc} , expansion halts and contraction begins. This clear correlation between L_{nuc} and R_* aligns with the behavior observed in the models discussed in Section 2.3.

2.4.1.2 Relationship Across the Full Grid of Models

After presenting several representative examples, we test the criterion for evolution toward the RG/RSG phase across the full grid of models spanning a broad range of initial masses and metallicities. Among the 760 massive star models ($11\text{--}30 M_{\odot}$), we exclude 35 low-metallicity cases (all between $24\text{--}30 M_{\odot}$) in which the C core forms at the BSG radii while the He core has not yet ceased contraction. These cases fall outside the scope of this study. For lower-mass stars ($1\text{--}10 M_{\odot}$), we include all 380 models. In total, our analysis covers 1,105 stellar models.

For each stellar model, we focus on the evolutionary phase in which the He core has

already formed but the C core has not yet developed. Within this time duration, we define the maximum stellar radius as $R_{\max}$ and identify the corresponding timestep. We then retrieve the central pressure P_c at each timestep and identify its local maximum value at the core-He burning stage, denoting the corresponding stellar radius as $R_{P_{\max}}$. The top panels of Figure 2.8 shows scattering plots of $R_{P_{\max}}$ versus $R_{\max}$. We classify models as red or blue based on the effective temperature at $R_{\max}$ (denoted $T_{R_{\max}}$), using $\log T_{R_{\max}} < 3.7$ (i.e., $T_{R_{\max}} \lesssim 5,000$ K) as the threshold for RG/RSG classification.

In many cases where the star evolves into an RG or RSG, $R_{P_{\max}}$ coincides with $R_{\max}$, indicating that the envelope reaches its maximum radius while the core is still contracting. In such cases, the expansion to $R_{\max}$ is driven directly by core contraction and does not depend on the criterion discussed in Section 2.3.

For the remaining cases, $R_{\max} > R_{P_{\max}}$, indicating that the envelope continues to expand after core contraction has completed. For these models, we identify the minimum nuclear energy generation rate, L_{nuc} after the peak of P_c and define the corresponding stellar radius as $R_{L_{\min}}$. The bottom panels of Figure 2.8 display $R_{L_{\min}}$ versus $R_{\max}$. Remarkably, in all BG/BSG cases, $R_{L_{\min}}$ and $R_{\max}$ are nearly identical, indicating that the envelope halts its expansion and begins to contract exactly when L_{nuc} stops decreasing and begins to rise. In RG/RSG cases, $R_{L_{\min}}$ and $R_{\max}$ are also closely aligned, indicating that L_{nuc} continues to decline throughout the expansion until the star reaches its final RG/RSG size. These results confirm that Equation (6) serves as a robust and general criterion for continued expansion following core contraction, applicable to stars across a wide range of stellar masses and metallicities.

2.4.2 Roadmap Toward Red Giants or Supergiants



We summarize the physical mechanism and criteria for a star to evolve into an RG/RSG using the flow chart shown in Figure 4.17. At the TAMS, the core H burning ends and the He core begins to contract. During this contraction phase, an H-burning shell forms just outside the core and gradually moves inward along with it. Meanwhile, the envelope begins to expand in the opposite direction of the inward-moving boundary of the burning shell. The extent of this envelope expansion by the end of core contraction depends on stellar mass and metallicity. In some cases, the star already reaches RG/RSG dimensions while the He core is still contracting and the central pressure continues to rise. This represents the first evolutionary channel toward becoming an RG or RSG.

If the stellar core completes its contraction and stabilizes at the blue giant (BG) or BSG stage, further envelope expansion depends on the rate of nuclear energy generation—specifically, core-He burning and shell-H burning. At the base of the envelope, there exists a strong correlation between the local luminosity and radius ($L_b \simeq L_{\text{nuc}}$ and R_{shell}). Initially, as L_{nuc} declines, R_{shell} moves inward, and the envelope continues to expand in the opposite direction. However, if L_{nuc} subsequently begins to rise during the BSG stage, such as due to a rapid increase in L_{He} , R_{shell} instantaneously reverses and begins to move outward. At the same time, the envelope begins to contract, and the star remains in the BG/BSG phase. In contrast, if L_{nuc} continues to decline, R_{shell} persists in moving inward, allowing the envelope to expand further until reaching the RG/RSG regime. Generally, after core contraction ceases, the net energy generation rate from core He-burning and shell H-burning determines which evolutionary path the star follows.

These evolutionary paths generally follow the modified “mirror principle,” in which

the outer and inner boundaries of the envelope evolve in opposite directions. While this currently remains an empirical relation observed in our models, the next section presents a deeper physical explanation for this principle.



2.5 Physics of the Mirror Principle

Instead of the traditional description, we propose a refined formulation of the “mirror principle” for stars with an H-burning shell: the stellar envelope expands or contracts in the opposite direction to its inner boundary, defined by the outer edge of the H-burning shell, instead of the He core. In this section, we explore the physical mechanism underlying this principle.

An important feature of the shell-envelope boundary is its nearly constant temperature, corresponding to the threshold required to ignite the CNO cycle, typically a few times 10^7 K. As shown in the top panels of Figure 2.10, while R_{shell} moves inward or outward and the central temperature continues to rise, the temperature at the shell-envelope boundary remain nearly constant at $\sim (3 - 4) \times 10^7$ K, i.e., $\log(T/\text{K}) \sim 7.5 - 7.6$.

Meanwhile, the P - T stratification in the central region of the star generally follows the adiabatic temperature gradient, which remains nearly constant (~ 0.25 in these $25 M_{\odot}$ stars). As a result, the pressure at the outer edge of the burning shell also stays nearly constant. As shown in the bottom panels of Figure 2.10, while the central pressure continues to change through the evolution of R_{shell} , the pressure at the shell-envelope boundary remains the same.

To explore the physical process during the envelope expansion, Figure 2.11 demonstrates the pressure and density profiles at two timesteps of a $25 M_{\odot}$, $Z = 0.02$ star, plot-

ted in both radius and mass coordinates. Between these two timesteps, the shell-envelope boundary moves inward in both radius and mass coordinates, while the pressure at that boundary remains nearly constant. This behavior enables us to explain the physics behind the reciprocal motion between the outer and inner boundary of the envelope using the equation of hydrostatic equilibrium:

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}, \quad (2.7)$$

where P is the local pressure, m is the enclosed mass, r is the radial coordinate, and G is the gravitational constant.

At the shell-envelope boundary, we denote the radius as R_{shell} and define the enclosed mass as M_{shell} . From Figure 2.11, when the shell-envelope boundary moves inward between the two timesteps, R_{shell}^4 decreases more rapidly than M_{shell} . Specifically, M_{shell} decreases by $\sim 20\%$, from $9.8 M_{\odot}$ to $7.8 M_{\odot}$, while R_{shell} decreases by $\sim 80\%$, as R_{shell} shrinks from $1.38 R_{\odot}$ to $0.9 R_{\odot}$. Therefore, according to Equation (7), the inward motion of the shell-envelope boundary demands a steeper pressure gradient $|dP/dm|$ to satisfy hydrostatic equilibrium. Since the pressure at the inner boundary of the envelope is fixed by the burning shell, the only way to steep pressure gradient is to reduce the pressure adjacent to the shell in the overlying mass layers of the envelope. Indeed, as shown in Figure 2.11, $|dP/dm|$ significantly steepens near the shell-envelope boundary, and the pressure across most of the envelope drops by more than two orders of magnitude between the two timesteps. According to the equation of state, this pressure drop leads to a corresponding decrease in density. Due to mass conservation, this lower density must be accompanied by an increase in volume, resulting in an outward expansion of the enve-

loped. Therefore, the inward motion of the shell-envelope boundary naturally causes the envelope to expand. This provides the physical foundation of the refined mirror principle proposed in this study.



In previous studies, the mirror principle has often been derived from energy conservation and the virial theorem. However, such interpretations do not fully capture the physical processes revealed by stellar models. Instead, our results show that the refined mirror principle arises directly from hydrostatic equilibrium and mass conservation under the condition of a fixed pressure at the shell-envelope boundary. Beyond this conceptual explanation drawn from MESA models, we will further validate this mechanism in Chapter 3 by solving the steady-state hydrostatic structure of the envelope.

2.6 Effects of Envelope Properties

We have demonstrated that the properties of the core and burning shell can determine whether the envelope expands into the RSG phase. Nevertheless, since many previous studies have attributed RSG formation to the intrinsic properties of the envelope, we now examine the roles of envelope characteristics, specifically opacity and internal structure.

2.6.1 The Role of Opacity

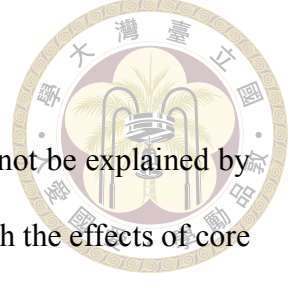
Earlier studies based on the concept of thermal imbalance proposed that envelope expansion is driven by heat absorption, with higher core luminosities and greater envelope opacity favoring the transition toward RGs/RSGs (Renzini 2023; Renzini & Ritossa 1994; Renzini et al. 1992; Renzini 1984; Ritossa 1996). This view has been challenged by Iben (1993), who argued that heat absorption in the envelope is not the cause of expansion, but

rather a consequence of the development of convective envelope.

In Section 2.3, we showed that the expansion toward RSGs cannot be explained by increased energy supply. We now revisit this issue by considering both the effects of core luminosity and envelope opacity. To this end, we construct a set of models combining different actual stellar metallicities ($Z = 0.001$ and 0.0001) with different values of opacity parameter ζ_κ (0.01, 0.001, and 0.0001; see Section 2.2 for the definition of ζ_κ). The model with $Z = 0.001$ and $\zeta_\kappa = 0.001$ corresponds exactly to Model A discussed in Section 2.3. Figure 4.1 shows the evolution of L_{nuc} and P_c as functions of $\log R_*$ for these models (A, C, D, E, and F).

We begin by comparing Models A and F. Although Model F has higher opacity parameter than Model A, it does not expand into an RSG, whereas Model A does. This indicates that higher opacity alone does not necessarily favor RSG formation. One might still argue, under the heat absorption scenario, that Model F fails to expand because its L_{nuc} is lower than that of Model A, implying insufficient energy input despite its high opacity. However, a comparison between Models F and C undermines this explanation. Models C and F share the same ζ_κ (and thus similar opacity at the same values of temperature and density) but differ in Z , leading to different L_{nuc} . If heat absorption were the dominant mechanism driving expansion, Model C, which has a lower L_{nuc} and thus weaker energy supply, should be even less likely to expand than Model F. Yet, it successfully expands into an RSG. Therefore, the heat absorption scenario fails to provide a consistent explanation for RSG formation.

While the heat absorption scenario does not hold for the envelope expansion, opacity does influence stellar evolution during this phase, with higher opacity generally favoring



expansion. In Figure 4.1, we also present evolution of P_c for each model. We find that higher opacity tends to shift the peak of P_c corresponding to larger R_* . For instance, with the same Z but different ζ_κ values, Model A (with lower ζ_κ) reaches its pressure peak at $R_* \sim 30\text{--}40 R_\odot$, while Model C (with higher ζ_κ) does so at $R_* > 100 R_\odot$. In other words, at a comparable stage of core evolution, a model with higher opacity retains a larger envelope radius. Moreover, opacity also influences the nuclear burning in the stellar interior. Since L_{nuc} directly affects whether a star expands or contracts in the BSG phase, opacity-driven changes in L_{nuc} may significantly alter the evolution.

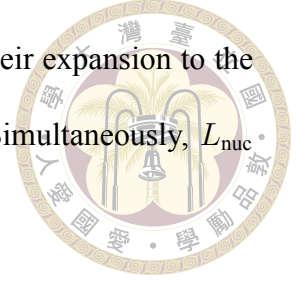
Thus, although higher opacity does not directly drive envelope expansion through energy trapping, it can still influence the evolutionary pathway toward the RSG phase by modifying stellar structure. These changes may enable the star to meet the physical conditions for sustained expansion. The precise influence of opacity on post-main-sequence evolution remains a topic for future investigation.

2.6.2 Structural Transition into the RG/RSG Phase

While the core and shell properties primarily determine whether a star evolves into an RSG or BSG, the envelope properties become increasingly important during the final approach to the RSG phase. In this subsection, we identify a structural transition of the envelope that occurs in the final stage of expansion.

As shown in Figure 2.5, the evolution of L_{nuc} exhibits a sharp decline at $R_* \gtrsim 700\text{--}1,000 R_\odot$ ($\log(R_*/R_\odot) \sim 2.85\text{--}3.0$), just before the star enters the RSG phase. This drop is primarily due to a decrease in L_{H} , with a smaller contribution from L_{He} . Figure 2.13 further shows the evolution of L_s and L_{nuc} toward the RG/RSG phase for $5 M_\odot$ and $25 M_\odot$.

models at different metallicities. In all models, before completing their expansion to the RG/RSG phase, L_s exhibits a sharp dip followed by a rapid rise. Simultaneously, L_{nuc} shows a similar dip.



To understand the gap in L_s , we present Kippenhahn diagrams in Figure 2.14 in both mass and radius coordinates, showing the internal structures of the stars with $Z = 0.02$, highlighting the convective zones inside the stars. We provide a zoomed-in view focused on the evolutionary phase during which the stars transition into the RG/RSG stage. As the stellar radius approaches $\log(R_*/R_\odot) \sim 1.2, 2.2$, and 2.6 for the $5, 15$, and $25 M_\odot$ models, respectively, expansion temporarily halts. This brief pause lasts for $\sim 50,000$ years for the $5 M_\odot$ star, $\sim 1,000$ years for the $15 M_\odot$ star, and only ~ 500 years for the $25 M_\odot$ star. During this pause, a surface convection zone begins to develop and extends inward. Once the star resumes its expansion, the convective zone rapidly deepens, extending to the base of the envelope just above the H-burning shell.

To further investigate the origin of this transition, we plot the profiles of radius, density, temperature, opacity, and radiative temperature gradient for the $5, 15$, and $25 M_\odot$ models, all with $Z = 0.02$ in Figure 2.15. These profiles, taken during the transitional phase, reveal significant structural changes. Prior to this transition, only a small fraction of the envelope mass near the surface extends to large radii, while the bulk of the envelope remains compact. During the transition, however, most of the mass above the H-burning shell redistributes outward, forming a low-density, extended envelope. In other words, the apparent pause in surface expansion shown in Figure 2.14 refers to a major structural transition within the stellar envelope, with a substantial mass redistributing outward.

This mass redistribution also alters the temperature profile within the envelope. Be-

fore the transition, only the outermost layers have temperatures $\lesssim 10^5\text{--}10^6\text{ K}$. After the structural adjustment, however, nearly the entire envelope cools to below a few times 10^5 K . This shift in the temperature profile has important consequences. As shown in Figure 2.16, when stellar material cools to a few times 10^5 K , the opacity rises steeply with decreasing temperature, in following Kramers' law. Since most of the envelope mass reaches these lower temperatures during the transition, the average opacity throughout the envelope becomes significantly higher. The higher opacity contributes to the sudden drop in surface luminosity observed at this stage. Moreover, the enhanced opacity raises the radiative temperature gradient, which in turn triggers the development of an extended convective zone that penetrates deep into the base of the envelope. Once convection becomes efficient throughout most of the envelope, energy transport is restored and the luminosity rapidly recovers.

Thus, the evolution toward the RG/RSG phase involves not only an increase in stellar radius but also a structural phase transition. Once this phase transition completes, the star stabilizes at the RG/RSG dimension with a fully developed convective envelope.

We also note that this abrupt structural change in the envelope not only causes a drop in surface luminosity, but also influences the energy generation rates from both H-shell and He-core burning, as reflected in the evolution of L_{nuc} shown in Figures 2.5 and 2.13.

2.7 Discussions and Conclusions

In this study, we revisit the long-standing question: Why do stars become RG/RSGs? We identify the central physical mechanism that governs envelope expansion in post-main-sequence stars as a refined form of the “mirror principle.” Contrary to the traditional view

that the burning shell simply mirrors the contraction of the core with the expansion of the envelope, our results reveal a more accurate description: the expansion and contraction of the envelope consistently follow the opposite direction of its inner boundary, defined by the outer edge of the H-burning shell. This manifests as a strong anti-correlation between R_* and R_{shell} .

We further elucidate the physical foundation of this refined mirror principle. Unlike the previous interpretations based on energy absorption of the envelope, we demonstrate that this mirror-like behavior is a direct consequence of hydrostatic equilibrium and mass conservation. The key point lies in the fact that the inner boundary of the envelope is defined by the H-burning shell, which maintains a nearly constant temperature and pressure. As the shell-envelope boundary moves inward to smaller radii, gravitational acceleration increases. To maintain hydrostatic equilibrium, a steeper pressure gradient is then required. Since the pressure at the base of the envelope remains fixed by the shell, the only way to achieve this steeper gradient is by reducing the pressure, and consequently the density, throughout the envelope. This drop in density leads naturally to an expansion in radius. In Chapter 3, we will further validate this mechanism by solving steady-state stellar structure equations.

Applying this refined mirror principle, we propose a general set of criteria for post-main-sequence evolution toward the RG/RSG phase, as summarized in the flow chart in Figure 4.17. The process includes the following stages:

(1) Core contraction. After the H fuel in the core is exhausted, the He core begins to contract, and an H-burning shell is established just above it. As this burning shell moves inward along with the contracting core, the envelope expands outward. In some cases,

this is sufficient for the star to reach RG/RSG dimensions during core contraction.

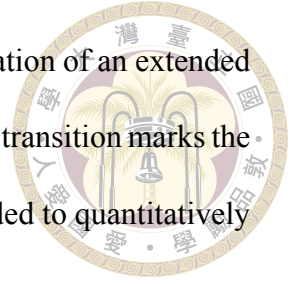
(2) Regulation by nuclear burning. After core contraction halts, the shell-H burning becomes the primary energy source, and He core burning begins to develop. After the core stabilizes, the burning shell continues to move inward as L_{nuc} decreases. If L_{nuc} later rises again during the BSG stage, the shell-envelope boundary will immediately reverse direction and move outward, causing the envelope to contract. Only if L_{nuc} continues to decline can the envelope maintain its outward expansion and eventually reach the RG/RSG phase.

Therefore, there exist two pathways for evolution toward the RG/RSG phase: (1) direct expansion during core contraction, and (2) continued expansion from blue to red after core contraction has ceased, regulated by a sustained decline in nuclear energy generation rate. In both cases, expansion is driven by the refined mirror principle, which arises naturally from hydrostatic equilibrium.

We emphasize that the thermal imbalance arising from energy absorption, often invoked to explain the evolution toward RG/RSG, is not a cause of expansion, but rather a consequence of it. The extent of a supergiant's expansion is not determined by the amount of energy supplied, but by the new configuration required to sustain hydrostatic equilibrium. Energy is redistributed according to the structure established by this balance, rather than driving the expansion itself.

We also identify a phase transition that occurs as the stellar radius approaches the RG/RSG regime. During this stage, much of the envelope mass redistributes toward the outer layers and cools significantly. This cooling causes a large fraction of mass to have significantly increased opacity, leading to a sharp drop in surface luminosity. At the same

time, the radiative temperature gradient steepens, triggering the formation of an extended convective zone that penetrates deep into the envelope. This structural transition marks the final step toward becoming an RG/RSG. Further investigation is needed to quantitatively determine the conditions that trigger this envelope restructuring.



This study does not address envelope expansion occurring after core-He exhaustion. However, expansion during the final evolutionary stages may have important implications in binary systems, particularly in triggering Case C mass transfer ([Lauterborn 1970](#); [Podsiadlowski et al. 1992](#)). This aspect warrants further investigation in future work.

In this chapter, we have established a unified physical framework explaining why post-main-sequence stars evolve into the RG/RSG phase. In the next chapter, we will further explore the underlying physical mechanisms through steady-state models of stellar envelopes.

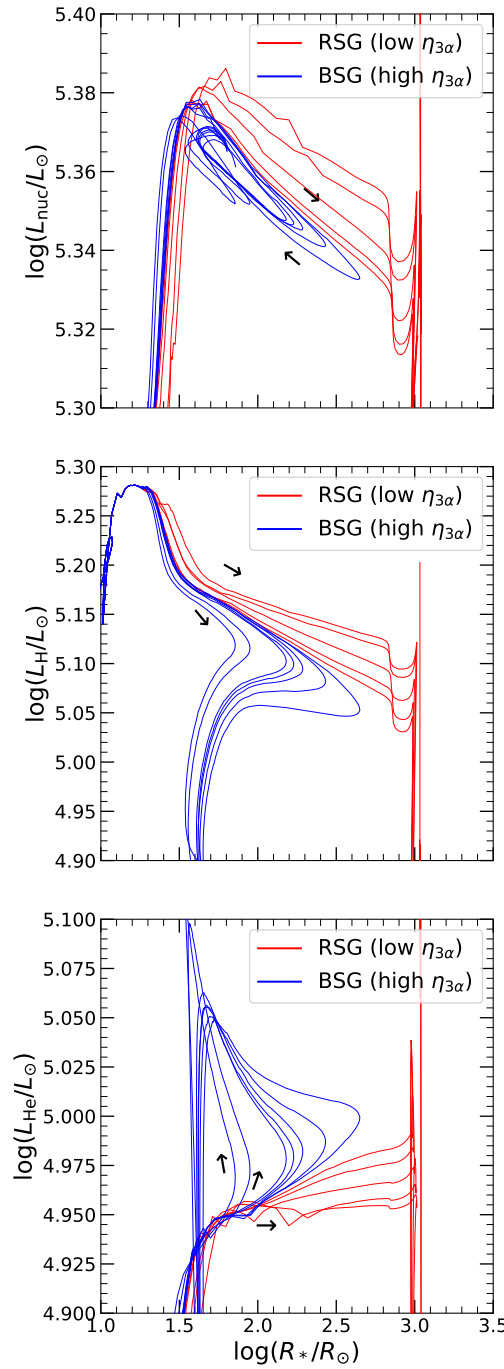


Figure 2.5: Evolution of the energy generation rates as a function of stellar radius (R_*). The three panels show the total nuclear energy generation rate (L_{nuc}), the contribution from hydrogen burning (L_{H}), and from helium burning (L_{He}). Red curves represent models with lower $\eta_{3\alpha}$ (0.1, 0.2, 0.5, 1.0, and 1.2) that eventually evolve into RSGs, while blue curves represent models with higher $\eta_{3\alpha}$ (1.33, 1.4, 1.6, 1.8, 2.0, 5.0, 10) that contract after some expansion and remain as BSGs. In general, stellar evolution proceeds from smaller to larger R_* .

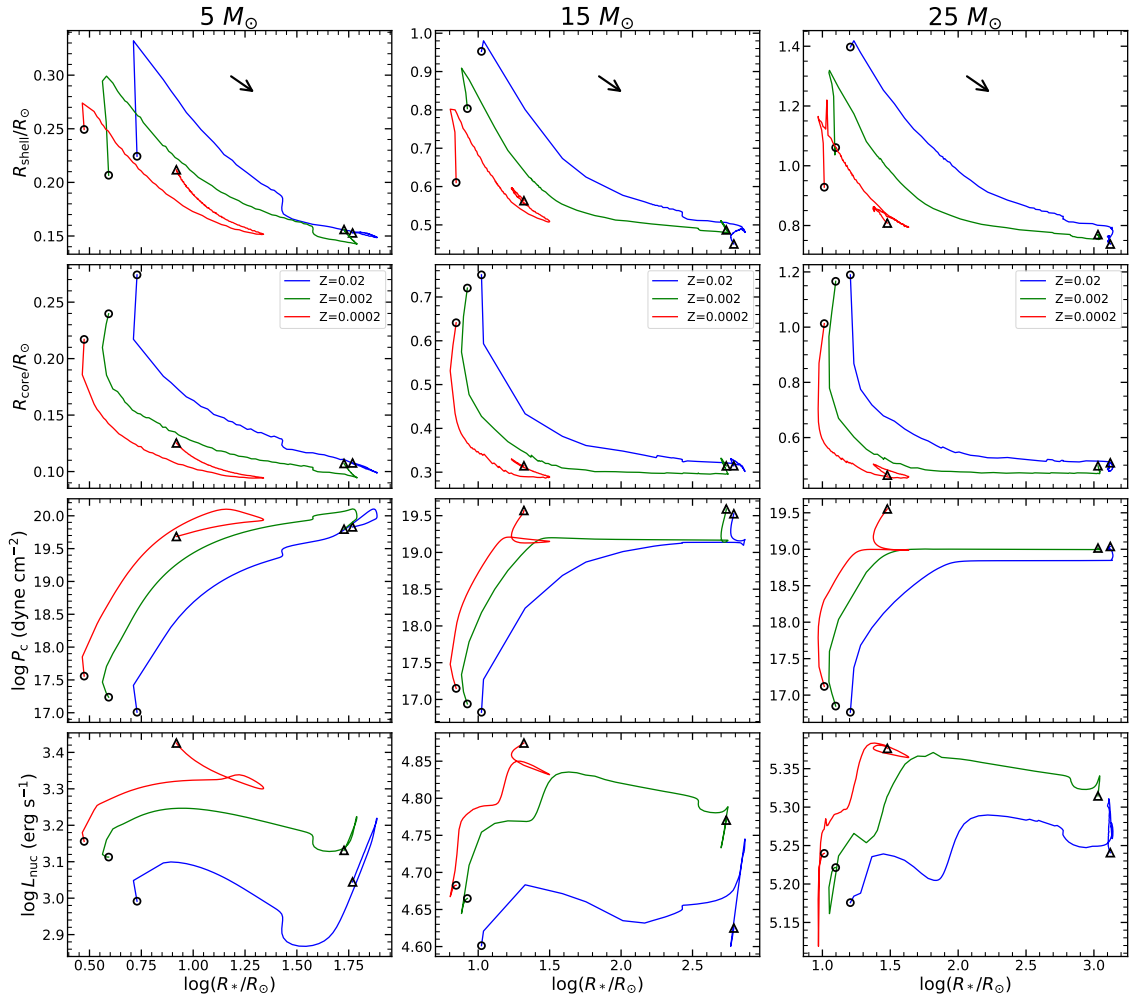


Figure 2.6: Evolution of shell radius (R_{shell}), core radius (R_{core}), central pressure (P_c), and nuclear luminosity ($\log L_{\text{nuc}}$) as functions of stellar radius (R_*) during the post-main-sequence stage, prior to the formation of C core, for 5, 15, and $25 M_{\odot}$ stars with different metallicities. Circles and triangles indicate the formation of the He core and C core, which define the start and end of the time span shown, respectively. Arrows in the top row indicates the general direction of evolution in time, with R_{shell} decreasing and R_* increasing.

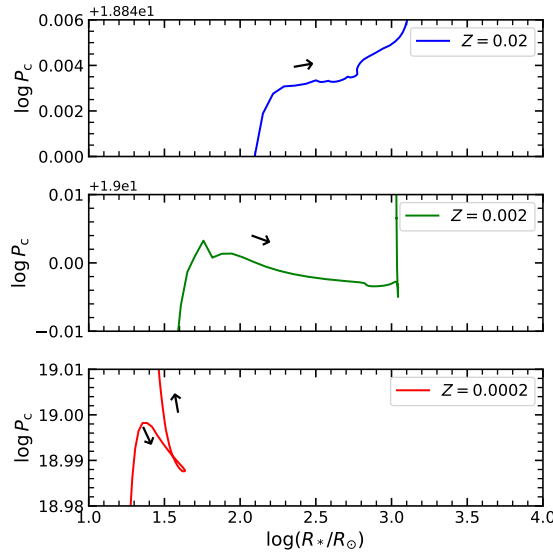


Figure 2.7: Evolution of central pressure (P_c) in the cgs unit for stars with $25 M_\odot$ at different metallicities. The arrows indicate the directions of evolution in time.

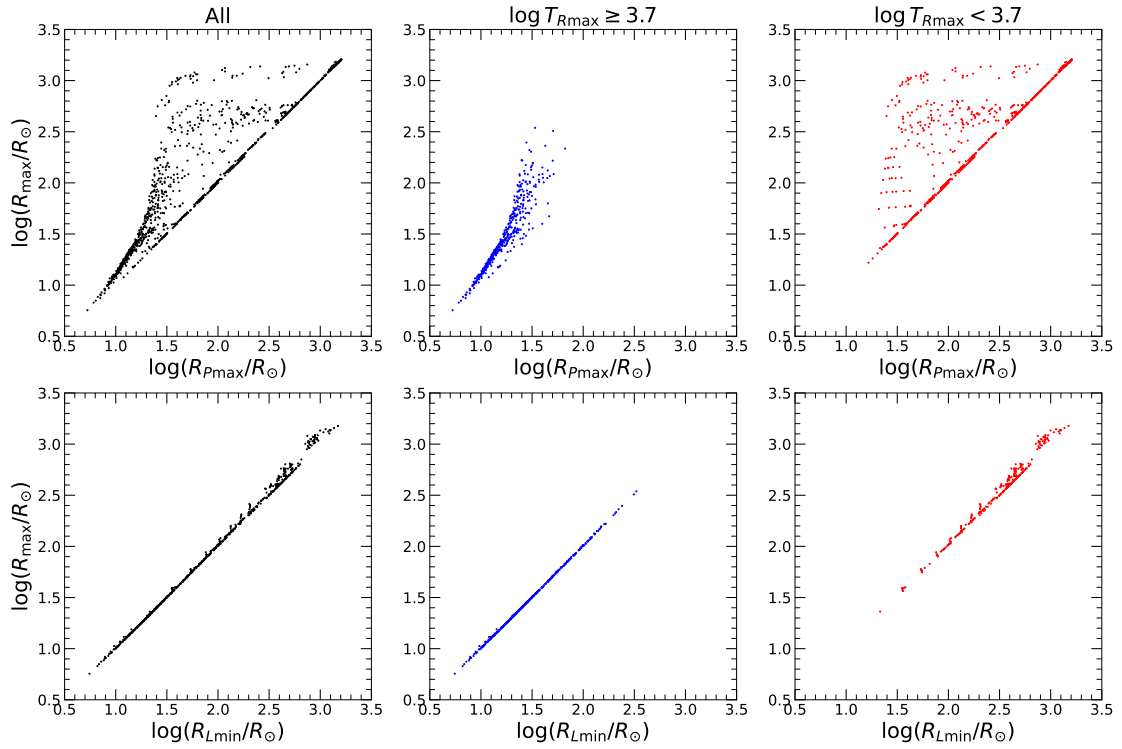


Figure 2.8: Scatter plots showing characteristic radii at key time point during core-He burning for the full grid of 1,105 stellar models with initial masses between $1\text{--}30 M_\odot$ and metallicities from $Z = 1.0 \times 10^{-5}$ to 2.0×10^{-2} . The top panels display the relationship between the maximum stellar radius during the core-He burning phase ($R_{\max}$) and the radius at which the central pressure reaches a local maximum ($R_{P\max}$), marking the end of core contraction. The bottom panels show the relationship between $R_{\max}$ and the radius at which the nuclear energy generation rate (L_{nuc}) reaches its minimum after the cessation of core contraction ($R_{L\min}$). Left panels include all models; middle panels show only BG/BSG models with effective temperature at $R_{\max}$ (denoted $T_{R\max}$) satisfying $\log T_{R\max} \geq 3.7$; right panels present the RG/RSG cases with $\log T_{R\max} < 3.7$.

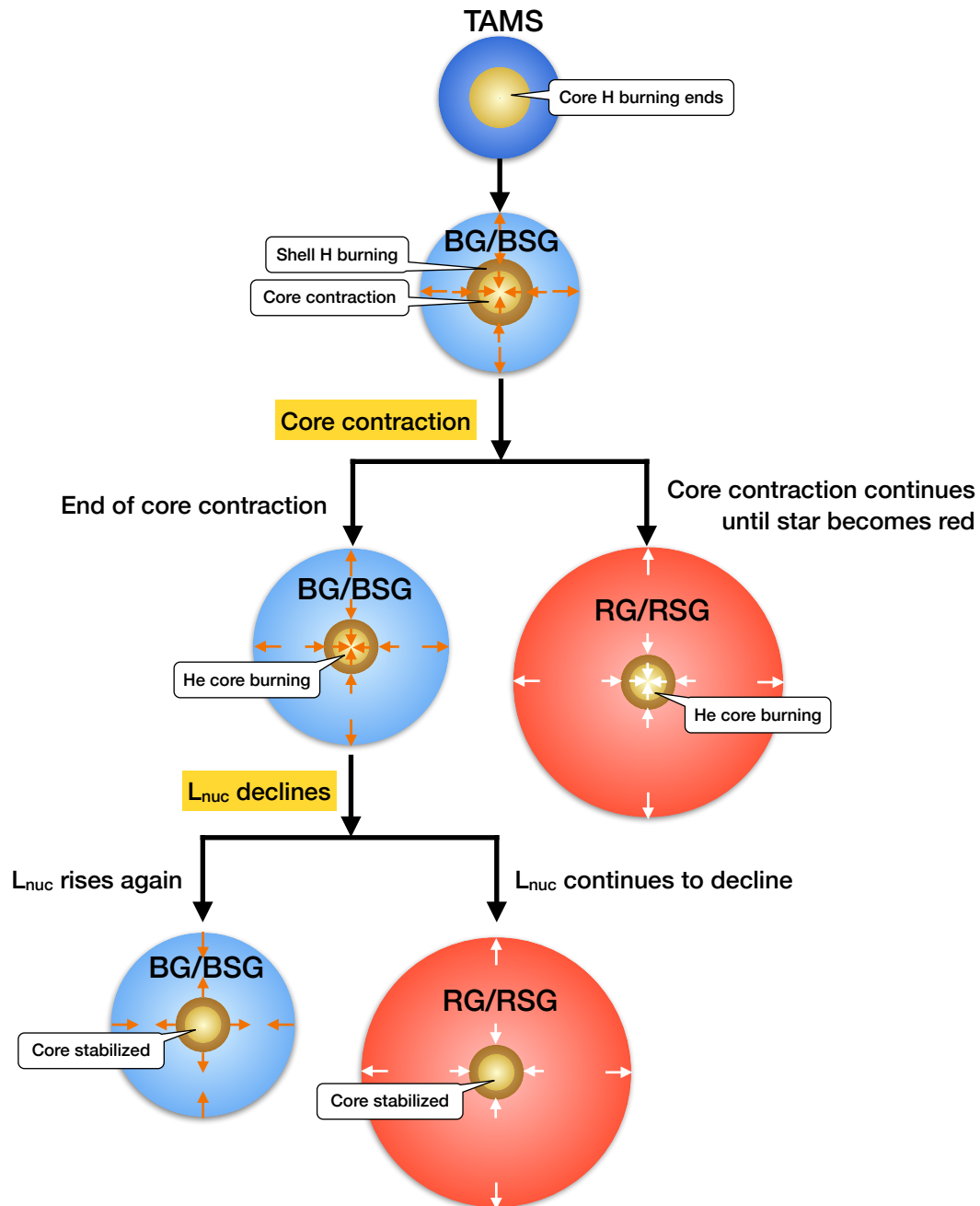


Figure 2.9: Flow chart illustrating the evolutionary pathways toward the RG/RSG phase.

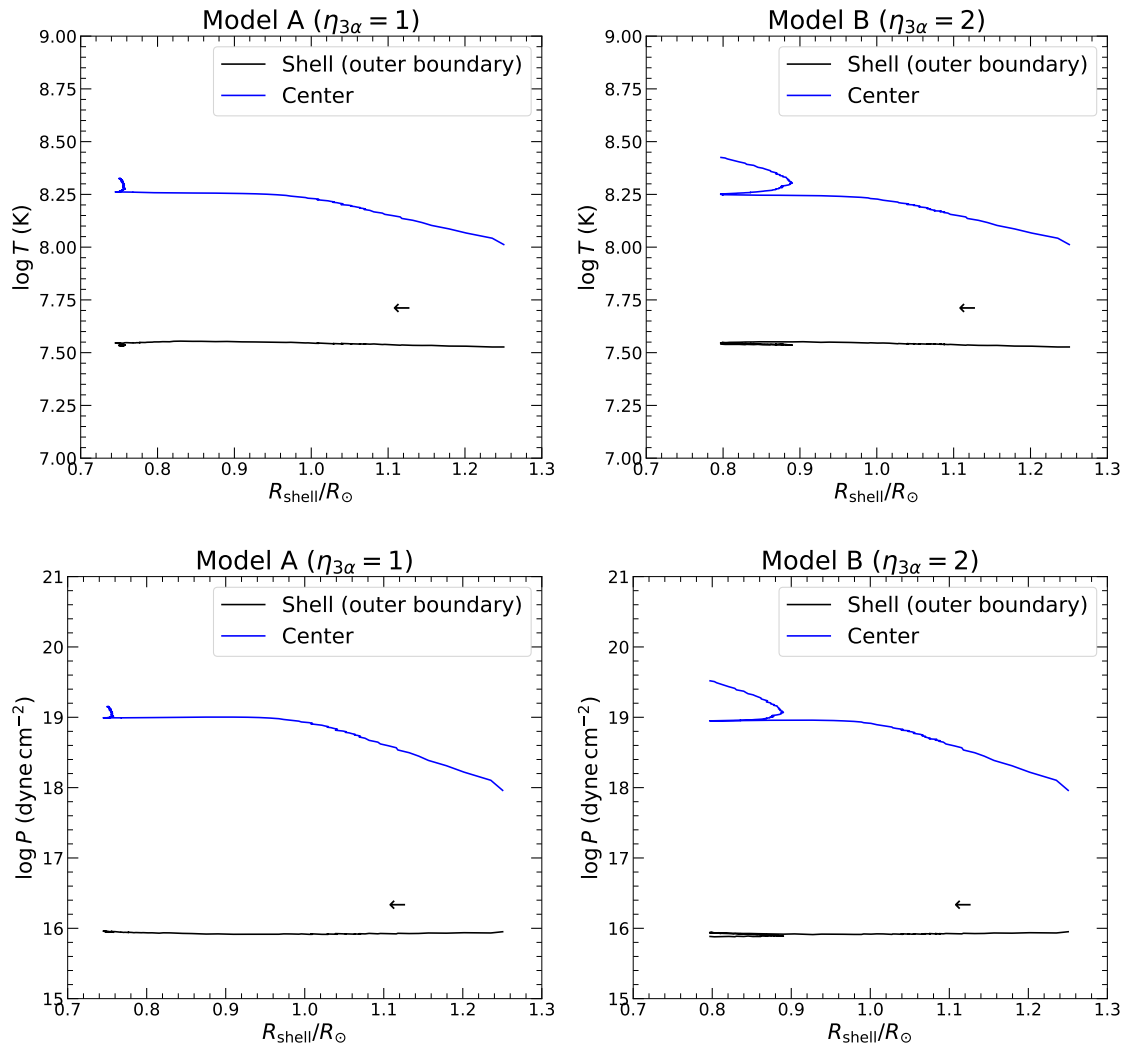
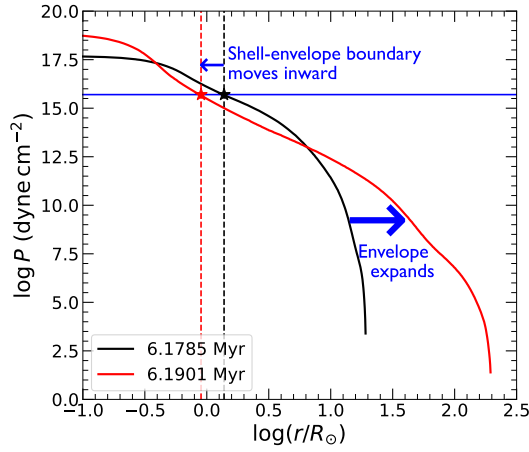
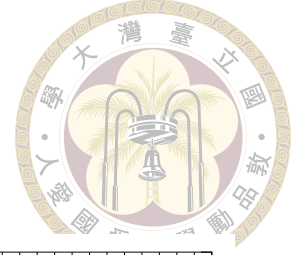
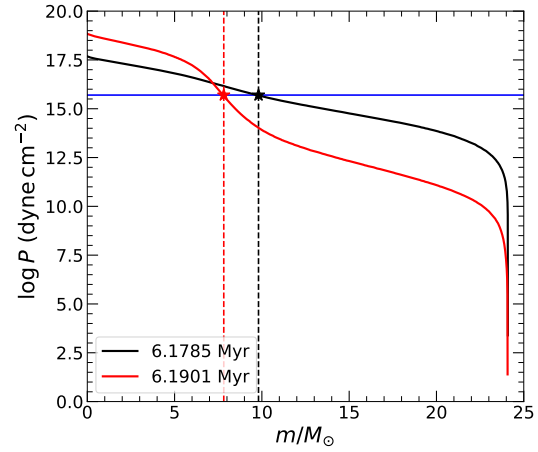


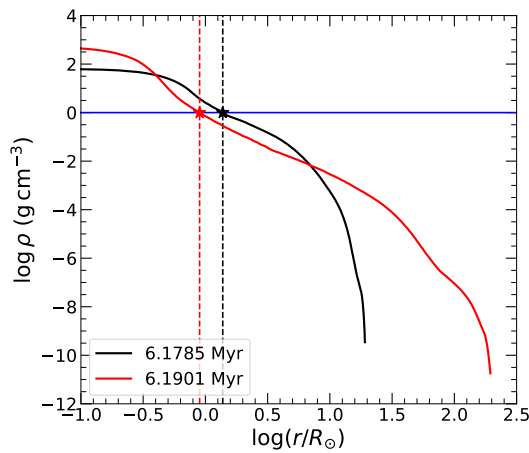
Figure 2.10: Temperature and pressure at the shell-envelope boundary and the center of the star from Models A and B. In both models, the temperature and pressure at the shell-envelope boundary remain nearly constant throughout the evolution. The arrows indicate the general evolution direction.



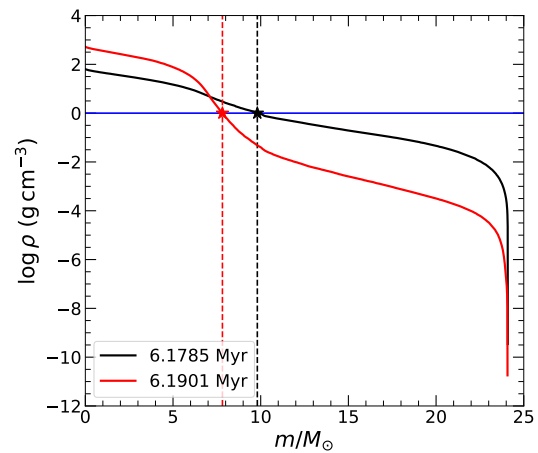
(a) Pressure vs Radius



(b) Pressure vs Mass



(c) Density vs Radius



(d) Density vs Mass

Figure 2.11: Pressure (*top panels*) and density (*bottom panels*) profiles at two different timesteps for a stellar model with $25 M_{\odot}$ and $Z = 0.02$ during the envelope expansion phase in both radius (*left panels*) and mass (*right panels*) coordinates. Dashed lines mark the outer edge of the H-burning shell, which coincides with the inner boundary of the envelope. Star-shaped symbols indicate the pressure and density at this boundary for each snapshot, which remain nearly constant. Blue horizontal lines represent these constant values. As the shell-envelope boundary moves inward, the pressure gradient (dP/dm) in the inner envelope steepens to maintain hydrostatic equilibrium. This steepening leads to a reduction in both pressure and density throughout the envelope, thereby driving its expansion.

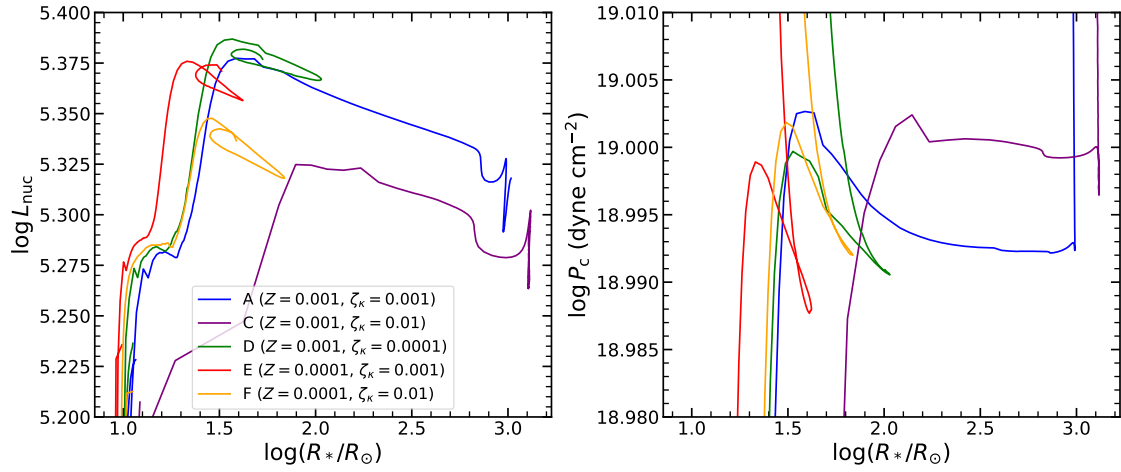
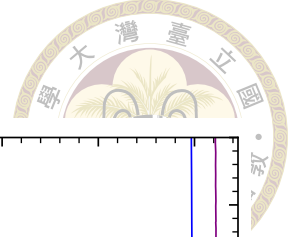


Figure 2.12: Evolution of the energy generation rate from nuclear burning (L_{nuc}) and central pressure (P_c) as functions of the stellar radius (R_*) for models with different metallicities (Z) and opacity parameters (ζ_κ).

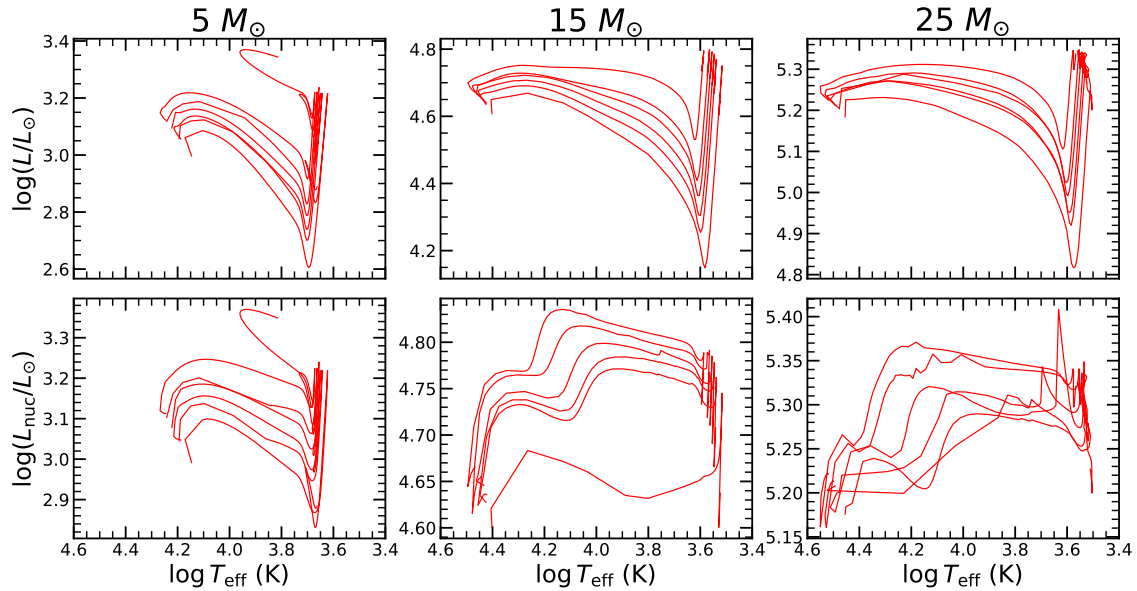


Figure 2.13: Evolution of surface luminosities (L_s) and nuclear energy generation rates (L_{nuc}) as a function of effective temperature for 5, 15, and 25 $M_\odot$ stars with various metallicities ($Z = 0.002, 0.004, 0.006, 0.008, 0.01, \text{ and } 0.02$). The lower-metallicity models generally show higher surface luminosities and higher nuclear energy generation rates. A notable feature is the sharp drop in both L_s and L_{nuc} as the star cools to $\sim 5,000\text{--}4,000$ K, marking the transition into the RG/RSG phase. The direction of evolution is generally from the higher T_{eff} to lower T_{eff} .

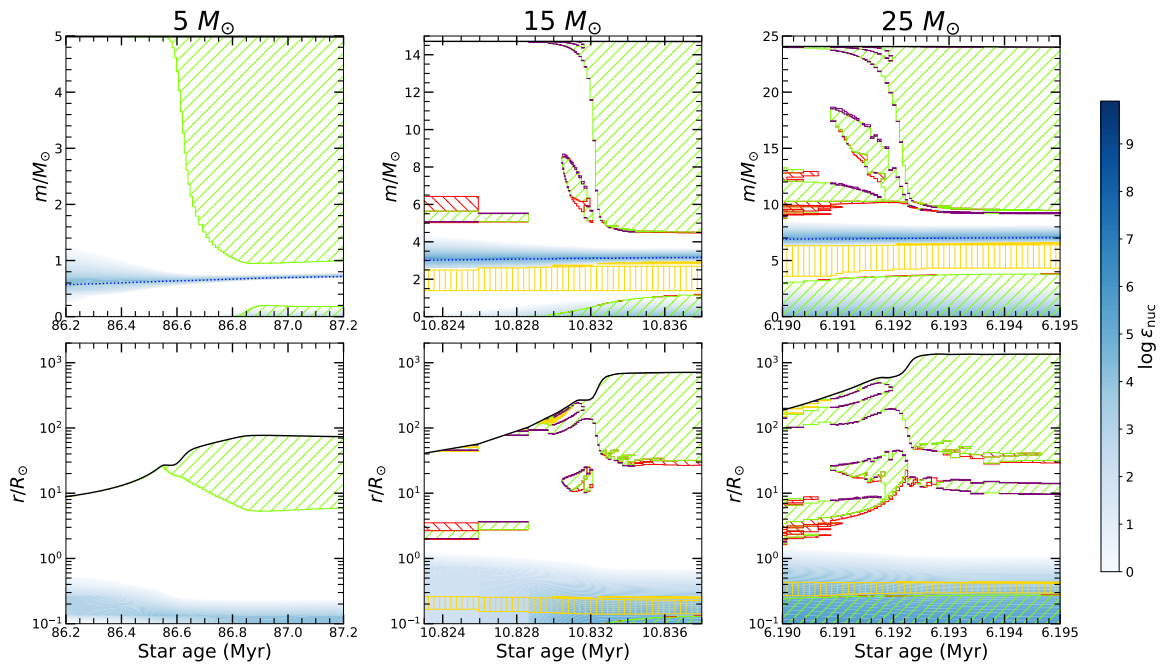


Figure 2.14: Kippenhahn diagrams for stellar models with 5, 15, and $25 M_{\odot}$, all at $Z = 0.02$. The top panels use mass coordinates, and the bottom panels use radius coordinates. Regions with striped lines indicate mixing: green for convection, red for semi-convection, yellow for thermohaline mixing, and purple for overshooting. The blue background indicates $\log \epsilon_{\text{nuc}}$ in the unit of $\text{erg s}^{-1} \text{g}^{-1}$. Each diagram is zoomed in on the evolutionary phase during which the star transitions into the RG/RSG stage and develops an extended convective envelope. Notably, a brief pause in radial expansion occurs just before the envelope becomes convective, at stellar ages of approximately 86.55, 10.831, and 6.1917 Myr for the 5, 15, and $25 M_{\odot}$ models, respectively.

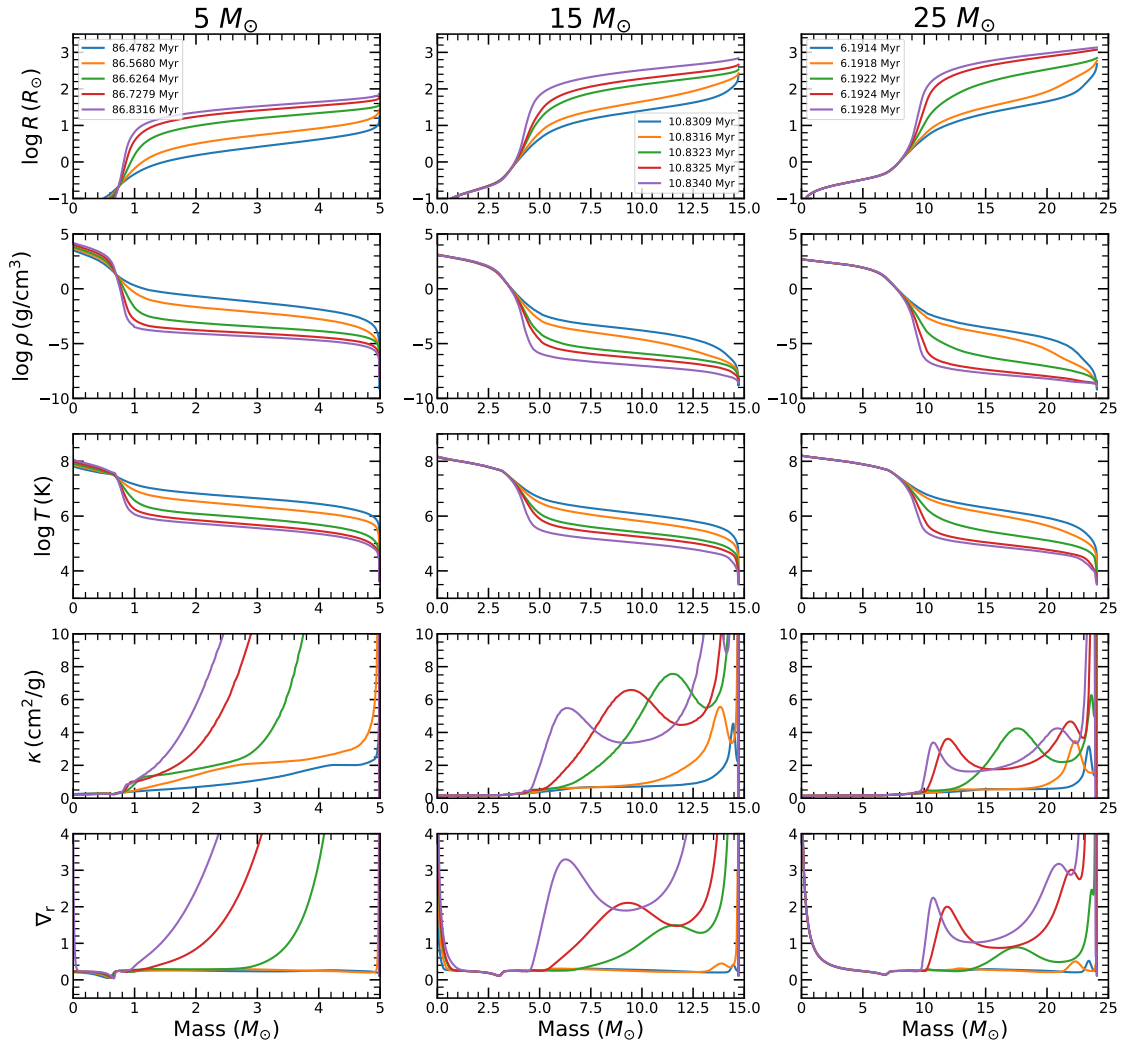


Figure 2.15: Profiles of radius, density, temperature, opacity, and radiative temperature gradient (∇_r) plotted in mass coordinates during the phase transition into RGs/RSGs for stars with 5, 15, and 25 $M_{\odot}$, all with $Z = 0.02$. During this transition, the redistribution of mass in the envelope causes a significant cooling of most of the stellar mass, which in turn increases the opacity and steepens the radiative temperature gradient, leading to the formation of an extended convective zone.

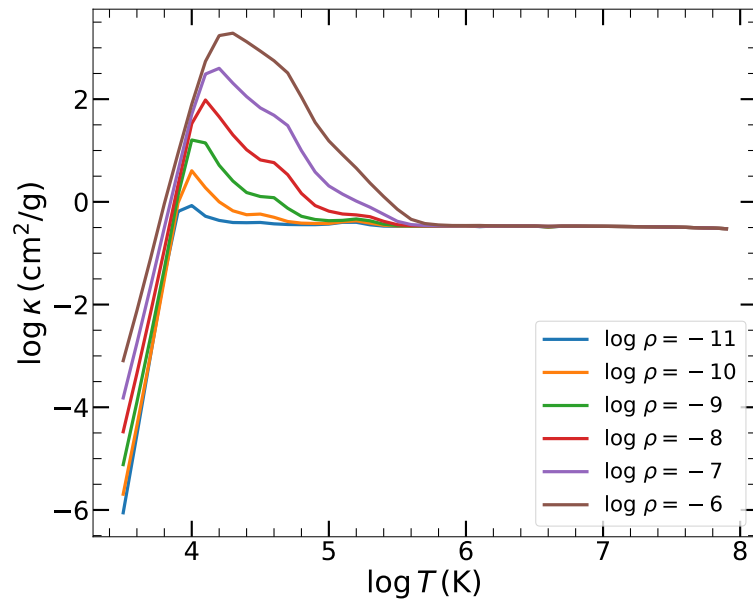


Figure 2.16: Opacity as a function of temperature and density from the "gs98" opacity table in MESA. As stellar material cools from high temperatures to a few times 10^5 K, the opacity rises steeply with decreasing temperature, following Kramers' law.





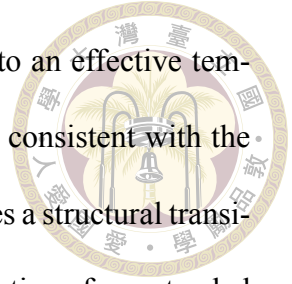
Chapter 3 Steady-State Solutions of Supergiant Stellar Structure

* This chapter has been submitted to the AAS journals.

Abstract for Chapter 3

To investigate the physical origin of red giants (RGs) and red supergiants (RSGs), we construct steady-state models of stellar envelopes by explicitly solving the stellar structure equations, with boundary conditions set at the stellar surface and the hydrogen burning shell. For comparison, we consider both polytropic and realistic models. Polytropic models adopt a polytropic equation of state (EOS) and neglect energy transport, while realistic models incorporate radiative and convective energy transport, along with tabulated EOS and opacities. Our steady-state solutions reproduce three key features relevant to the evolution toward the RG/RSG phase. First, the refined mirror principle of Chapter 2 is reproduced: the stellar radius varies inversely with the radius of the envelope's inner boundary, defined by the burning shell's surface. This feature arises purely from hydrostatic equilibrium, as it appears in both polytropic and realistic models. Second, realistic

models reveal an upper limit to envelope expansion, corresponding to an effective temperature of $\sim 4,000$ K, which is characteristic of RG/RSG stars and consistent with the classical Hayashi limit. Approaching this limit, the envelope undergoes a structural transition marked by a significant change in the density profile and the formation of an extended convective zone. The location of this limit is governed by the sharp drop in H^- opacity as the envelope cools below $\sim 10,000$ K. Finally, our solutions show that even a small shift in the envelope's inner boundary can induce substantial envelope expansion throughout the yellow regime, naturally explaining the bifurcation of giants and supergiants into red and blue branches.



3.1 Introduction

The physical origin of red giants (RGs) and red supergiants (RSGs) remains one of the fundamental questions in stellar astrophysics. In Chapter 2, we unveiled the mechanisms that drive stars to evolve into the RG/RSG using models with the stellar evolution code Modules for Experiments in Stellar Astrophysics (MESA; [Jermyn et al. 2023](#); [Paxton et al. 2019, 2018, 2015, 2013, 2011](#)), version 10108. We showed that previous explanations based on energy absorption by the envelope are insufficient to account for the transition to the RG/RSG phase. Moreover, the conventional "mirror principle," which posits that the envelope expands in response to core contraction, does not always hold.

Instead, we identified a refined form of the mirror principle that persists throughout the envelope's evolution: the envelope expands or contracts in the opposite direction to the motion of the boundary between the envelope and the hydrogen-burning shell. Specifically, there exists an inverse relationship between the stellar radius (R_*) and the inner

radius of the envelope (R_{in} , denoted as R_{shell} in Chapter 2). R_{in} is initially controlled by helium (He) core contraction and later by total nuclear energy generation from both the core and the hydrogen-burning shell, but consistently moves in the opposite direction to R_* .

By analyzing MESA models, we elucidated the physical origin of this refined mirror principle: as the shell-envelope boundary moves inward, its local gravitational acceleration increases. To maintain hydrostatic equilibrium, the pressure gradient must therefore steepen. Nonetheless, the pressure at the shell-envelope boundary is effectively fixed by nuclear burning conditions. The only way to increase the gradient under this constraint is to reduce the pressure, and hence the density, within the envelope. This reduced average density naturally leads to an expansion of the envelope.

However, it is difficult to validate this refined mirror principle using stellar evolution models alone, as they trace physical variables along specific evolutionary paths and do not allow exploration of all the configurations permitted by the stellar parameter space. To examine the relationship between R_* and R_{in} more rigorously, we turn to solving steady-state stellar structures, which enable systematic examinations of the envelope structure for given stellar parameters.

In addition to the refined mirror principle, Chapter 2 identified a notable structural transition that occurs as the star approaches the RG/RSG phase. Just before a star enters this phase, the outward expansion of the stellar surface temporarily stalls, whereas a large fraction of the interior mass is redistributed toward the outer envelope. This transition leads to increased opacity in most regions of the envelope, resulting in an extended convective zone that penetrates deeper toward the envelope's base. In the present chapter, we

demonstrate that this structural transition naturally emerges from steady-state envelope solutions and investigate its physical origin in detail.



Furthermore, we aim to address a fundamental question: why do giant and supergiant stars converge to two distinct branches, red and blue? In our previous studies (Chapter 1), we demonstrated a bimodal distribution of red and blue supergiants across stellar models with varying masses and metallicities, with very few models residing in the intermediate yellow regime. In the present study, by analyzing steady-state stellar envelope configurations, we gain new insights into the physical origin of this bimodal distribution.

This chapter solves steady-state stellar envelope structures to investigate the refined mirror principle and uncover the physical significance of the RG/RSG phase. Section 3.2 defines the physical problem addressed in this work and describes the method we use to solve this problem. Section 3.3 begins with simplified polytropic envelope models. Section 3.4 presents realistic models that incorporate energy transport, using tabulated opacities and equations of state, together with convective mixing. Section 3.5 discusses the advantages, limitations, and implications of our approach. Finally, Section 3.6 summarizes our findings and concludes this study.

3.2 Methods

This study aims to solve a key physical question raised by the refined mirror principle introduced in Chapter 2: the relationship between the stellar radius and that of the shell-envelope boundary. As illustrated schematically in Figure 3.1, we approach this question by solving the envelope structure rather than modeling the evolution of the entire star. The envelope is defined as the stellar interior that does not contain active nuclear burning.

Thus, the inner boundary in our calculation is not the center of the star but the shell-envelope boundary.

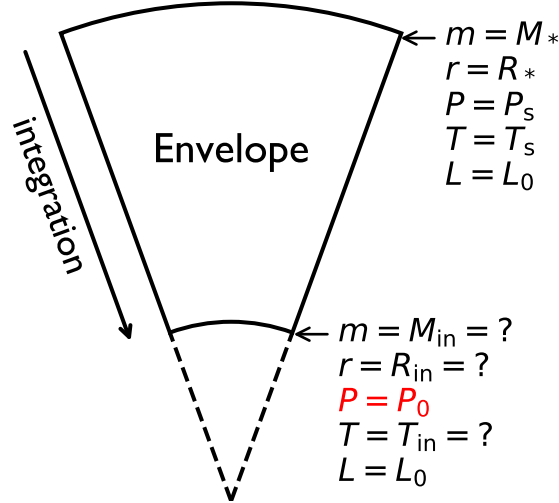


Figure 3.1: Schematic diagram illustrating the problem setup in this study. We integrate the stellar structure equations inward from the surface down to the inner boundary of the envelope, which is defined by a termination pressure P_0 .

Most importantly, as demonstrated by the MESA models in Chapter 2, the inner boundary of the envelope is not set by a fixed enclosed mass but rather by a fixed pressure that reflects the physical conditions necessary to ignite shell hydrogen (H) burning. In our setup, we impose a constant pressure $P = P_0$ as the termination condition for the inward integration. The outer boundary of the envelope is defined at the stellar surface, where the radius is R_* and the enclosed mass is M_* . By integrating inward from the stellar surface, we aim to determine the radius and enclosed mass at the envelope's inner boundary, denoted by R_{in} and M_{in} , respectively. We note that R_{in} and M_{in} are not prescribed parameters, but are instead outputs of the integration.

To compute the envelope structure, we solve the standard stellar structure equations

along the mass coordinate m . The first equation expresses mass conservation:

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}, \quad (3.1)$$

where r is the radius, and ρ is the density. The second equation enforces hydrostatic equilibrium:

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}, \quad (3.2)$$

where P is the pressure, and G is the gravitational constant. The third equation governs energy transport by computing temperature T :

$$\frac{dT}{dm} = -\frac{GmT}{4\pi r^4 P} \nabla, \quad (3.3)$$

The value of temperature gradient ∇ depends on the mode of energy transport, either radiative or convective. We do not solve the fourth differential equation that governs energy conservation:

$$\frac{dl}{dm} = \epsilon_{\text{nuc}} + \epsilon_{\text{grav}} - \epsilon_{\nu}. \quad (3.4)$$

Here, l is the local luminosity, ϵ_{nuc} is the nuclear energy generation rate, ϵ_{ν} is the neutrino loss rate, and ϵ_{grav} is the gravothermal term. Within the envelope, ϵ_{nuc} is negligible due to the absence of nuclear burning, and ϵ_{ν} is typically small compared to the radiative flux, allowing both terms to be ignored. The remaining term ϵ_{grav} depends on structural changes such as expansion or contraction. Thus, this term contains an evolution effect that requires information from the preceding time step. By assuming a steady-state envelope, we simply set $\epsilon_{\text{grav}} = 0$. Therefore, the luminosity is taken as a constant parameter L_0 and is uniform throughout the envelope. This simplification eliminates any internal luminosity gradient, effectively removing time-dependent factors and allowing us to focus on the steady-state



structure.

With constant luminosity in the envelope, we solve the steady-state stellar structure within the envelope using the direct-shooting method. For a given stellar radius R_* , we specify the surface boundary conditions at $r = R_*$, which include the surface temperature T_s and the surface pressure P_s . Starting from these surface conditions, we solve the stellar structure equations by integrating physical quantities inward. The integration is terminated when the pressure reaches the target value, $P = P_0$, which corresponds to the condition of the shell-envelope interface. Here, we adopt the fiducial value of $P_0 = 1.0 \times 10^{16} \text{ dyne cm}^{-2}$, estimated from Figure 2.10 in Chapter 2.

3.2.1 Polytropic models

Before solving the full set of equations, we first consider a simplified polytropic model, in which the gas obeys a power-law equation of state (EOS) relating the pressure P to the density ρ :

$$P = K\rho^\gamma, \quad (3.5)$$

where K is the polytropic constant and γ is the polytropic exponent. In this polytropic model, the density depends solely on pressure and is independent of temperature. Therefore, the equation of hydrostatic equilibrium is decoupled from energy transport (e.g., [Prialnik 2009](#)). This simplification allows us to solve only the two fundamental structure equations, Equations (1) and (2), without calculating the temperature.

To integrate these equations, we impose a surface boundary condition on the pressure:

$$P_s = \frac{\tau_s g}{\kappa_s}, \quad (3.6)$$

where we adopt a surface optical depth of $\tau_s = 2/3$. Since temperature is not included in this model, we cannot use opacity tables that depend on temperature, density, and metallicity (e.g., [Iglesias & Rogers 1996](#)). Instead, we assume a constant opacity $\kappa_s = 0.34 \text{ cm}^2 \text{ g}^{-1}$, representative of electron scattering in fully ionized hydrogen. The surface gravity is given by $g = GM_*/R_*^2$, and hence the surface pressure scales as $P_s \propto R_*^{-2}$.

3.2.2 Realistic models

To construct realistic models, we require detailed gas physics such as the EOS and opacity. These data are retrieved from MESA (version 10108), with linear interpolation applied over the tabulated grids.

For the EOS, we adopt the MESA “eosPT” tables ([Paxton et al. 2011](#)), which take pressure and temperature as input parameters. Specifically, we average two tables, both with metallicity $Z = 0.02$, but with hydrogen mass fractions of $X = 0.6$ and $X = 0.8$. The resulting table is treated as representative of $X = 0.7$ and $Z = 0.02$. From this table, we extract several thermodynamic quantities as functions of P and T : the density $\rho(P, T)$, adiabatic temperature gradient $\nabla_{\text{ad}}(P, T)$, specific heat at constant pressure $C_P(P, T)$, $\chi_T(P, T) \equiv (\partial \ln P / \partial \ln T)_\rho$, and $\chi_\rho(P, T) \equiv (\partial \ln P / \partial \ln \rho)_T$.

The opacity $\kappa(\rho, T)$ is taken from the “OP_gs98” table in MESA, which combines the OPAL ([Iglesias & Rogers 1996, 1993](#)) and OP ([Seaton 2005](#)) tables. For simplicity, we do not use Type 2 opacities that consider the effect of carbon and oxygen enhancement.

We now define the two surface boundary conditions. The surface temperature (T_s) is

taken to be the effective temperature (T_{eff}) of the star:

$$T_s = T_{\text{eff}} = \left(\frac{L_0}{4\pi R_*^2 \sigma} \right)^{1/4}. \quad (3.7)$$



where L_0 is the stellar luminosity and σ is the Stefan-Boltzmann constant. L_0 is treated as a constant throughout the envelope and is specified as an input parameter in our calculations.

The surface pressure is set using the “simple photosphere” prescription in MESA (Paxton et al. 2011):

$$P_s = \frac{\tau_s g}{\kappa_s} \left[1 + 1.6 \times 10^{-4} \kappa_s \left(\frac{L_0/L_\odot}{M_*/M_\odot} \right) \right], \quad (3.8)$$

where we adopt $\tau_s = 2/3$ and express the surface gravity as $g = GM_*/R_*^2$. Since κ_s is a function of ρ and T , and ρ is itself a function of P via the EOS, the right-hand side of Equation (8) implicitly depends on P_s . Therefore, P_s cannot be expressed explicitly, and we solve Equation (8) numerically to determine P_s for a given R_* .

Unlike in the polytropic models, the realistic models solve for the temperature profile via the energy transport equation. Whether energy is transported by radiation or convection is determined by comparing the radiative temperature gradient ∇_{rad} with the adiabatic gradient ∇_{ad} . The radiative gradient is computed by

$$\nabla_{\text{rad}} = \frac{3}{16\pi acG} \frac{\kappa l P}{m T^4}, \quad (3.9)$$

where a is the radiation density constant and c is the speed of light. The adiabatic gradient $\nabla_{\text{ad}}(P, T)$ is obtained from the EOS table. If the Schwarzschild criterion for convective stability ($\nabla_{\text{rad}} < \nabla_{\text{ad}}$) is satisfied, the energy is transported predominately by radiation, and we set the actual temperature gradient ∇ equal to ∇_{rad} .

If instead $\nabla_{\text{rad}} \geq \nabla_{\text{ad}}$, then the transport of convective energy must be taken into

account using mixing-length theory (MLT). Following the formulation of [Kippenhahn et al. \(2012\)](#), the MLT framework solves for five quantities: the radiative flux F_{rad} , the convective flux F_{con} , the element temperature gradient ∇_e , the actual temperature gradient ∇ , and the convective velocity v . The total flux is the sum of the radiative and convective components:

$$F_{\text{rad}} + F_{\text{con}} = \frac{4acG}{3} \frac{T^4 m}{\kappa P r^2} \nabla_{\text{rad}}. \quad (3.10)$$

The radiative flux is given by

$$F_{\text{rad}} = \frac{4acG}{3} \frac{T^4 m}{\kappa P r^2} \nabla. \quad (3.11)$$

The convective velocity is calculated using

$$v^2 = g\delta(\nabla - \nabla_e) \frac{l_m^2}{8H_P}, \quad (3.12)$$

where $\delta \equiv -(\partial \ln \rho / \partial \ln T)_P = \chi_T / \chi_\rho$, and $H_P \equiv -dr / d \ln P$ is the pressure scale height. The mixing length l_m is taken as $l_m = \alpha H_P$, with a mixing-length parameter $\alpha = 1.5$ used in this work. The convective flux is then expressed as

$$F_{\text{con}} = \rho C_P T \sqrt{g\delta} \frac{l_m^2}{4\sqrt{2}} H_P^{-3/2} (\nabla - \nabla_e)^{3/2}. \quad (3.13)$$

Finally, the relationship between the four temperature gradients is given by

$$\frac{\nabla_e - \nabla_{\text{ad}}}{\nabla - \nabla_e} = \frac{6acT^3}{\kappa \rho^2 C_P l_m v}. \quad (3.14)$$

Using this MLT framework, we can compute the actual temperature gradient ∇ in Equation (3), accounting for the transport of radiative and convective energy.

However, directly solving the full set of five MLT equations can be a numerical challenge. To simplify the calculation, we adopt the approach described by [Kippenhahn et al. \(2012\)](#), which introduces two dimensionless quantities: $U \equiv (3acT^3/C_P\rho^2\kappa_m^2)\sqrt{(8H_P/g\delta)}$ and $W \equiv \nabla_{\text{rad}} - \nabla_{\text{ad}}$. With these definitions, the actual temperature gradient ∇ can be obtained by solving a single cubic equation:

$$(\xi - U)^3 + \frac{8U}{9}(\xi^2 - U^2 - W) = 0, \quad (3.15)$$

where ξ is defined by this relation

$$\nabla = \xi^2 + \nabla_{\text{ad}} - U^2. \quad (3.16)$$

Once the root ξ in Equation (15) is found, the actual temperature gradient ∇ can be obtained. This value of ∇ can then be substituted back into the energy transport equation in Equation (3) to solve the stellar structure.

3.3 Results from Polytropic Models

To explore the relationship between R_{in} and R_* solely under the effect of hydrostatic equilibrium, we construct polytropic models for $25 M_{\odot}$ stars. We assume a fixed $K = 5.0 \times 10^{15}$, such that the pressure and density scale as $P = (5.0 \times 10^{15})\rho^\gamma$. Figure 3.2 presents the resulting values of R_{in} and M_{in} as functions of R_* for various choices of γ . The value of γ is adjusted to reproduce the typical values of R_{in} and M_{in} found in MESA stellar evolution models (as shown in Chapter 2) for $25 M_{\odot}$ stars during the core He-burning phase: $R_{\text{in}} \sim 1.0 R_{\odot}$ and $M_{\text{in}} \sim 10 M_{\odot}$. The results show that when $\gamma \sim 1.3$ and $R_* \gtrsim 10 R_{\odot}$, the above values R_{in} and M_{in} from the stellar evolution models can be reproduced.

Remarkably, in these parameter regimes, the models consistently exhibit negative slopes between R_{in} and R_* , indicating that the mirror principle holds. Conversely, when the mirror principle is not satisfied, such as when γ is far from ~ 1.3 , or when $R_* \lesssim 10 R_\odot$, the resulting values of R_{in} and M_{in} deviate substantially from those expected from realistic stellar evolution. It is worth noting that post-main-sequence stars of $25 M_\odot$ typically have radii greater than $\sim 10 R_\odot$, as shown in Chapter 2; therefore, they are in the parameter regime where the mirror principle is satisfied. To sum up, in these polytropic models, the mirror principle generally holds for envelope configurations that are physically reasonable for post-main-sequence stars.

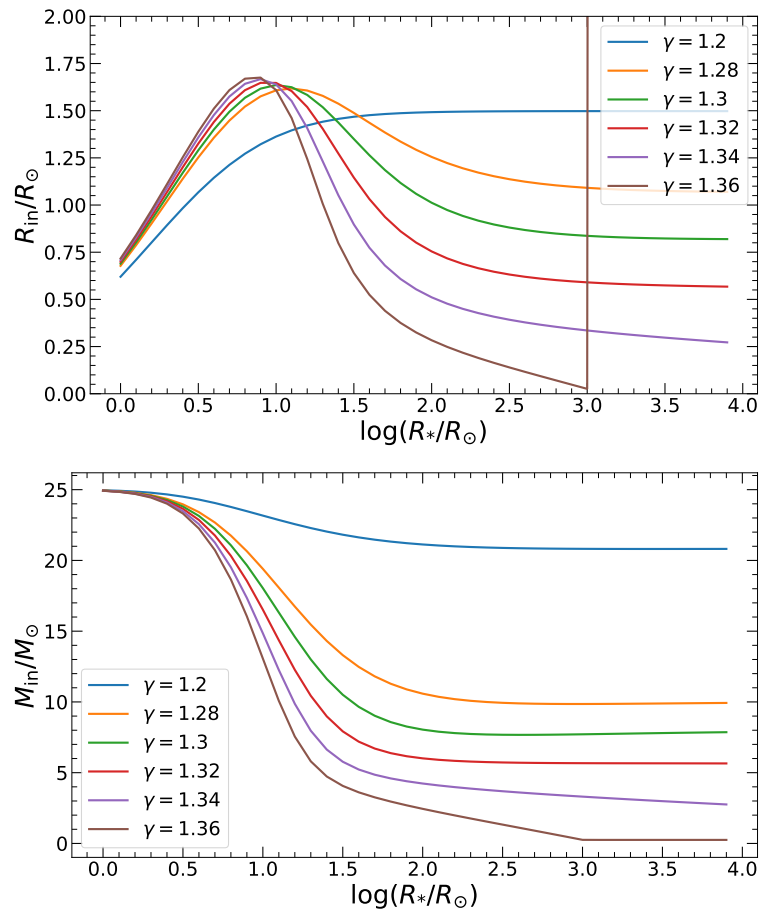
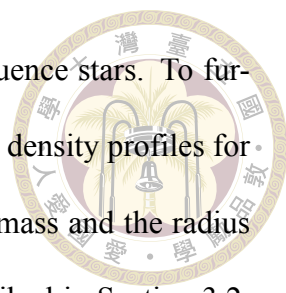


Figure 3.2: Radius (R_{in} , *top panel*) and enclosed mass (M_{in} , *bottom panel*) at the envelope's inner boundary for polytropic models of $K = 5.0 \times 10^{15}$ but different γ .

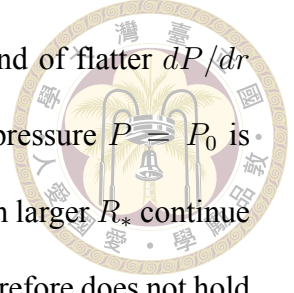
These results demonstrate that hydrostatic equilibrium alone, without involving en-



energy transport, can reproduce the mirror principle for post-main-sequence stars. To further investigate its physical origin, Figure 3.3 shows the pressure and density profiles for models with $\gamma = 1.3$ but different values of R_* , plotted in both the mass and the radius coordinates. For clarity, we briefly recall the integration setup described in Section 3.2. Each model is integrated inward from $m = 25 M_\odot$ and $r = R_*$ until the pressure reaches the termination value $P = P_0$, at which point the values of M_{in} and R_{in} are determined.

We first examine the $\log P$ versus m profiles. At the stellar surface, all models share the same enclosed mass $m = 25 M_\odot$, so the pressure gradient follows $dP/dm \propto r^{-4}$. Thus, a larger R_* results in a flatter dP/dm in the outer envelope. After integrating inward to a given P , models with a larger R_* and therefore flatter dP/dm traverse a larger mass, leading to a smaller enclosed mass m . This smaller m reduces the gravitational acceleration g , which in turn affects the $\log P$ versus $\log r$ profiles. At a given P , the density is fixed due to the polytropic relation. According to the hydrostatic equilibrium equation in radius coordinates, $dP/dr = -\rho g$, a fixed ρ implies that dP/dr is set directly by g . Therefore, at a given P , the models with larger R_* exhibits flatter dP/dr due to weaker g , as shown in the $\log P$ versus $\log r$ profiles.

The difference in dP/dr within the envelope for models with different R_* further determines the resulting R_{in} . We compare the three models with $R_* \gtrsim 10 R_\odot$ in Figure 3.3, corresponding to $\log(R_*/R_\odot) = 1.5, 2.0$, and 2.5 . Since models with larger R_* exhibit flatter dP/dr , their $\log P$ versus $\log r$ profiles intersect and reverse order near $\log(r/R_\odot) \sim 1.0$. Beyond the intersection point, the shallower dP/dr in models with larger R_* requires integration toward smaller radii to reach the termination pressure $P = P_0$. As a result, models with larger R_* end up with smaller R_{in} , illustrating the mirror principle.



For models with $R_* \lesssim 10 R_\odot$ (i.e., $\log(R_*/R_\odot) \lesssim 1$), the trend of flatter dP/dr for larger R_* still holds. However, in this regime, the termination pressure $P = P_0$ is reached before the pressure profiles intersect. As a result, models with larger R_* continue to produce larger R_{in} , and no reversal occurs. The mirror principle therefore does not hold in this parameter range. Nevertheless, as previously noted, these models yield M_{in} values that are significantly higher than those expected in the post-main-sequence evolution.

The above discussion explains the mirror principle from the perspective of our integration scheme, which proceeds inward from the stellar surface. We now consider the same physical system from the opposite viewpoint, starting from the inner boundary, to understand how the envelope “responds” to the core. In this perspective, for models with $R_* \gtrsim 10 R_\odot$, the pressure gradient dP/dm near the inner boundary becomes steeper as R_{in} moves inward. For example, comparing models with $\log(R_*/R_\odot) = 1.5$ and 2.5, the latter exhibits a smaller R_{in} by about 30% and a reduced M_{in} by approximately 20%. Since the pressure gradient near the inner boundary scales as $|dP/dm|_{\text{in}} \sim M_{\text{in}} R_{\text{in}}^{-4}$, the model with $\log(R_*/R_\odot) = 2.5$, which has the smaller R_{in} and M_{in} , exhibits a much steeper gradient. This steep pressure gradient causes the pressure to drop rapidly outwards, which in turn leads to a steep decline in density. The resulting low-density envelope extends to a much larger radius, making the overall envelope more extended. This physical mechanism is consistent with the result from the MESA models shown in Chapter 2.

Thus, the “response” of the envelope to its inner boundary can be understood as follows: With a smaller R_{in} , the stronger gravity at the shell-envelope interface requires a steeper outward pressure gradient to sustain hydrostatic equilibrium, which is achieved through envelope expansion that reduces the pressure and density in the outer layers. As long as M_{in} is not excessively large (i.e., the core does not dominate the total mass), the

inverse relationship between R_{in} and R_* is generally valid.

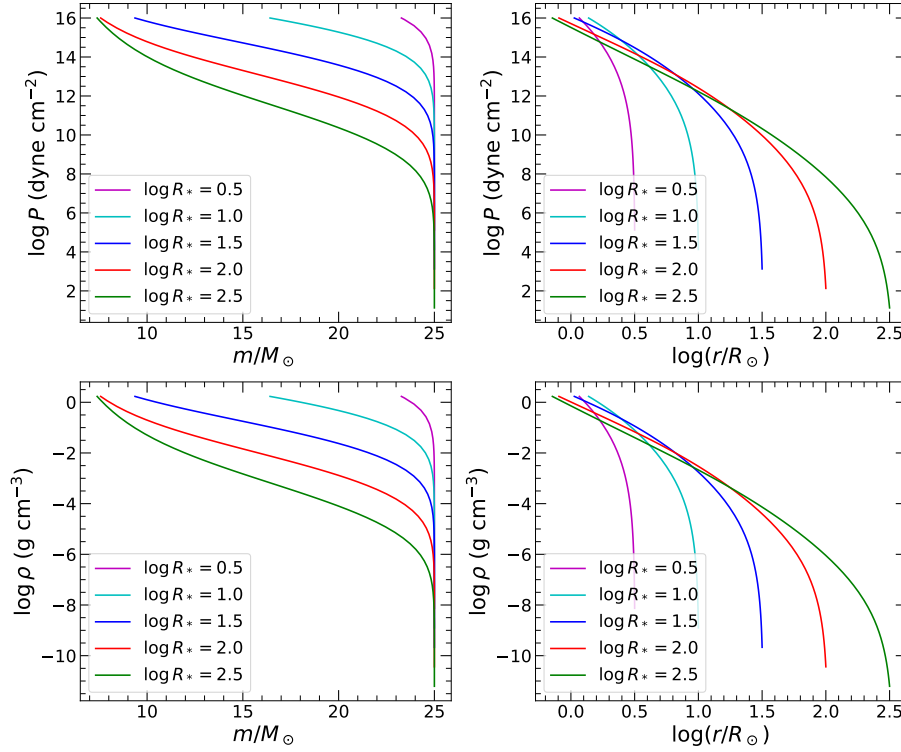


Figure 3.3: Pressure and density profiles for polytropic models with $\gamma = 1.3$ and varying stellar radius R_* . Profiles are shown as functions of both mass and radius coordinates. Stellar radii R_* are in units of $R_\odot$.

3.4 Results from Realistic models

After analyzing the polytropic models, we now construct realistic stellar envelope models that incorporate temperature calculations, EOS and opacity tables, and the MLT treatment for convection. As representative cases, we present models with initial masses of 5 and $25 M_\odot$. For each model, we compute the structure of the steady-state envelope adopting a fixed internal luminosity L_0 , which is assumed to be constant throughout the envelope. Based on the MESA models presented in Chapter 2, we set $L_0 = (6-10) \times 10^{38} \text{ erg s}^{-1}$ for the $25 M_\odot$ models, and $L_0 = (4-12) \times 10^{36} \text{ erg s}^{-1}$ for the $5 M_\odot$ models.

Figure 3.4 presents the results for the $25 M_\odot$ models with different values of L_0 ,

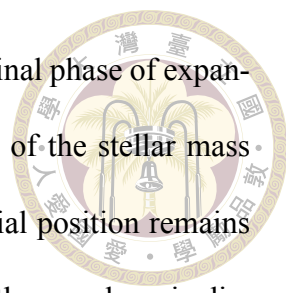


showing the relationships among R_{in} , M_{in} , R_* , and T_{eff} . The general trend of R_{in} versus R_* resembles that seen in polytropic models: the mirror principle appears to hold when M_{in} lies in the physically reasonable range ($\sim 10 M_{\odot}$). However, a notable difference from the polytropic models emerges near $R_* \sim 1,000 R_{\odot}$, the characteristic radius of RSGs. In this region, R_{in} initially increases slightly with R_* before undergoing a sharp decline. As shown in the bottom panel of Figure 3.4, during this sharp drop, T_{eff} for all models converge at the surface temperatures of ~ 4000 K (i.e., $\log(T_{\text{eff}}/\text{K}) \sim 3.6$), consistent with the typical effective temperatures of RSGs. The $5 M_{\odot}$ models, presented in Figure 3.5, exhibit a similar pattern. The mirror principle begins to apply when $R_* \gtrsim$ a few $R_{\odot}$, and an abrupt drop in R_{in} occurs at $R_* \sim 100 R_{\odot}$ and $T_{\text{eff}} \sim 4,000\text{--}4,500$ K, marking the RG phase for lower-mass stars.

Importantly, this sharp drop in R_{in} near the RG/RSG radii defines an upper limit to stellar expansion. Even as R_{in} contracts to small values, the envelope cannot expand beyond this threshold. Hence, the RG/RSG phase emerges as a limiting configuration for envelope expansion in steady-state solutions. Furthermore, in the RG / RSG solution, the curve R_{in} becomes nearly vertical in the $R_{\text{in}} - R_*$ plane, indicating that variations in R_{in} have little impact on R_* . As a result, once a star enters this regime, the RG/RSG structure behaves as a stable state, remaining largely unaffected by any moderate change in R_{in} .

3.4.1 Structural Transition Toward the Red Giant/Supergiant Phase

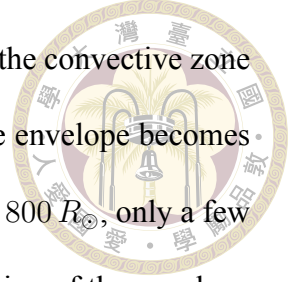
To understand the physical significance of the RG/RSG solution, Figure 3.6 presents the structural profiles of $25 M_{\odot}$ stellar models at three stages as they expand toward the RSG regime, with radii of approximately $800 R_{\odot}$, $1,000 R_{\odot}$, and $1,050 R_{\odot}$ (corresponding to $\log(R_*/R_{\odot}) = 2.9, 3.0$, and 3.02 , respectively).



These profiles reveal pronounced structural changes during the final phase of expansion toward the RSG radii. At $R_* \sim 800 R_\odot$, only a small portion of the stellar mass extends to large radii. At a mass coordinate of $m \sim 23 M_\odot$, the radial position remains relatively compact at $r \sim 30 R_\odot$. Only the outermost $\sim 2 M_\odot$ of the envelope is distributed between $\sim 30 R_\odot$ to $\sim 800 R_\odot$. As R_* increases, the mass distribution changes dramatically. At $R_* \sim 1,050 R_\odot$, the position of $m \sim 12 M_\odot$, which is only $\sim 2 M_\odot$ above the inner boundary of the envelope, is already located at $r \sim 150 R_\odot$. As the star expands from 800 to 1,050 $R_\odot$, a significant portion of the envelope mass is redistributed outward, markedly altering the radius profile. This redistribution of mass is also evident in the density profiles. At $R_* \sim 800 R_\odot$, most of the envelope remains relatively compact, with densities $\rho > 10^{-4} \text{ g cm}^{-3}$, and only the outermost layers reach lower densities. In contrast, at $R_* \sim 1,050 R_\odot$, the density drops sharply near the base of the envelope, from $\sim 1 \text{ g cm}^{-3}$ at $m = 10 M_\odot$ to $< 10^{-6} \text{ g cm}^{-3}$ at $m = 11 M_\odot$.

The temperature profiles also exhibit dramatic differences as R_* increases. At $R_* \sim 800 R_\odot$, the region near $m = 23 M_\odot$ remains hot, with temperatures near 10^6 K . Only the outermost layers cool below $\sim 10^6 \text{ K}$. By the time the star reaches $R_* \sim 1,050 R_\odot$, nearly the entire envelope has cooled to $\lesssim 10^6 \text{ K}$. This temperature drop has a profound effect on the opacity. In the temperature range 10^4 – 10^7 K , the opacity generally follows the Kramers' law, $\kappa \propto \rho T^{-3.5}$, and thus increases as the temperature decreases. As the envelope cools by 1.5–2 orders of magnitude between $R_* \sim 800$ and $1,050 R_\odot$, the opacity increases significantly throughout most of the envelope mass.

This enhancement in opacity leads to a critical structural transition: the development of an extended convective envelope. The elevated opacity increases the radiative temperature gradient ∇_{rad} , well above the adiabatic gradient ∇_{ad} , thus triggering convection. As



the star expands and deeper layers cool into the high-opacity regime, the convective zone penetrates into the base of the envelope. Eventually, nearly the entire envelope becomes convective. As shown in the bottom-left panel of Figure 3.6, at $R_* \sim 800 R_\odot$, only a few localized convective zones are present, each confined to a small fraction of the envelope mass. By $R_* \sim 1,000 R_\odot$, the surface convective zone has been extended to reach a mass coordinate of $m \sim 16.5 M_\odot$. When the star reaches $R_* \sim 1,050 R_\odot$, the convective region spans almost the entire envelope.

These processes of mass redistribution and the formation of an extended convective zone are highly consistent with the MESA stellar evolution models, as presented in Section 2.6 of Chapter 2. Our steady-state solutions now reproduce this structural transition, demonstrating that the RG/RSG configuration is not merely an outcome of stellar evolution but a physically required state for stars achieving such large radii. Specifically, for a $25 M_\odot$ star, once the stellar radius exceeds $\gtrsim 1,000 R_\odot$, the envelope necessarily adopts an RG/RSG structure with an extended convection zone.

This result highlights the physical significance of the phase transition. The nearly vertical drop in R_{in} with respect to R_* in $R_* \sim 1,000 R_\odot$ coincides with the completion of the phase transition to RG / RSG. This indicates that, beyond this point, the stellar radius cannot increase further. The structural profiles reveal that after the transition, the pressure and temperature gradients across the envelope become extremely shallow. If the radius were to increase further, these gradients would flatten even more, making it impossible for the integration to reach the termination pressure P_0 within the remaining stellar mass. Therefore, for a star with an envelope above an H-burning shell, no steady-state solution exists beyond this limiting radius.

3.4.2 Physical Origin of the Red Giant/Supergiant Solution

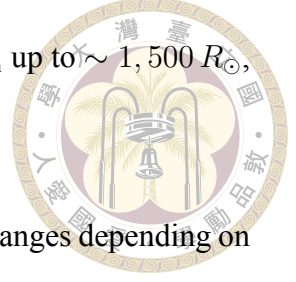


To explore the physical origin of the RG/RSG solution, we conduct a series of controlled experiments by disabling individual physical treatments: convection, surface boundary condition from atmospheric models, and opacity interpolation from tables. All models assume a stellar mass of $25 M_{\odot}$ and a fixed luminosity of $L_0 = 8.0 \times 10^{38} \text{ erg s}^{-1}$. We consider the following model setups:

- (1) Standard model: Includes MLT for convection, interpolated opacity from tabulated data, and a simple atmospheric model to determine the surface pressure boundary condition.
- (2) No convection: Convection is disabled by turning off the MLT. The energy transport throughout the envelope is purely radiative, i.e. $\nabla = \nabla_{\text{rad}}$ everywhere.
- (3) Constant surface pressure: The outer boundary condition is replaced by a fixed surface pressure. Three values are tested: $P_s = 1.0 \times 10^4 \text{ dyne cm}^{-2}$, $1.0 \times 10^3 \text{ dyne cm}^{-2}$, and 1.0 dyne cm^{-2} .
- (4) Opacity floor: Imposes a minimum opacity using four values: $\kappa \geq 0.01, 0.1, 0.34$, and $1.0 \text{ cm}^2 \text{ g}^{-1}$.

Figure 3.7 presents the $R_{\text{in}}-R_*$ relationships for these models. We begin by comparing the standard model with the case in which convection is disabled. In the absence of convection, the $R_{\text{in}}-R_*$ trend remains similar to the standard case for $R_* < 1,000 R_{\odot}$. However, an apparent difference emerges beyond this radius. In the convective model, R_{in} drops sharply once $R_* \gtrsim 1,000 R_{\odot}$, indicating the transition to the RSG phase. In

contrast, the non-convective model shows a continued increase in R_{in} up to $\sim 1,500 R_{\odot}$, followed by a sudden drop in larger radii.



For models with constant surface pressure, the RSG transition changes depending on the value P_s . A value of $P_s = 1.0 \times 10^3 \text{ dyne cm}^{-2}$ produces results similar to the standard model. Increasing P_s to $1.0 \times 10^4 \text{ dyne cm}^{-2}$ moves the RSG transition to a smaller radius ($R_* \sim 700 R_{\odot}$). On the other hand, reducing the surface pressure to $P_s = 1.0 \text{ dyne cm}^{-2}$ produces a result very similar to the standard case, indicating that even a significantly reduced P_s has minimal effect on the RSG solution.

Opacity modifications produce the most dramatic changes in the $R_{\text{in}}-R_*$ relation. For $\kappa \geq 0.01 \text{ cm}^2 \text{ g}^{-1}$, the RSG solution occurs at $R_* \sim 1,500 R_{\odot}$. A stricter floor of $\kappa \geq 0.1$ pushes the solution to an even larger radius of $R_* \sim 5,000-6,000 R_{\odot}$. For $\kappa \geq 0.34$, the RSG solution extends to $R_* \sim 10,000 R_{\odot}$. When the opacity floor is increased to $\kappa \geq 1.0$, the RSG limit effectively disappears.

To examine the effects of minimum opacity on stellar structure, Figure 3.8 shows the internal structure profiles for models with $\kappa \geq 0.1 \text{ cm}^2 \text{ g}^{-1}$ at $\log(R_*/R_{\odot}) = 3.2, 3.6$, and 3.75 . These models show mass redistribution and the formation of extended convective zones similar to the standard case (Figure 3.6), but the transition occurs at much larger radii.

The effect of enforcing $\kappa \geq 0.1 \text{ cm}^2 \text{ g}^{-1}$ mainly influences stellar material that has cooled to temperatures $\lesssim 10,000 \text{ K}$. In this regime, H^- opacity dominates, and the opacity drops sharply as the temperature decreases. As shown in the profiles of the standard models (Figure 3.6), values of $\kappa < 0.1$ occur only in the outermost layers where $T < 10^4 \text{ K}$. By imposing a minimum opacity, we effectively remove this sharp decline associated with



H^- opacity in the surface layers.

Opacity affects stellar structure in two main ways. The first modifies the surface boundary condition. From Equation (8), the surface pressure P_s depends on the surface opacity κ_s , with higher opacities leading to lower P_s . As seen in Figure 3.8, the model with $\log(R_*/R_\odot) = 3.75$ shows an extremely low surface pressure, $P < 1.0 \text{ dyne cm}^{-2}$, due to the imposed opacity floor. However, according to the experiments shown in Figure 3.7, reducing P_s to below $10^3 \text{ dyne cm}^{-2}$ has a minimal effect on the location of the RSG limit. Therefore, the change in surface pressure alone does not explain the significant shift of the RSG solution to such large radii caused by modifying the opacity.

The second effect of opacity is its influence on radiative energy transport. We now simply consider the energy transport equation via radiation:

$$\frac{dT}{dm} = -\frac{3}{64\pi^2 ac} \frac{\kappa l}{r^4 T^3}. \quad (3.17)$$

In our models, the luminosity l is fixed to L_0 throughout the envelope, and the surface condition gives $L_0 \sim R_*^2 T_{\text{eff}}^4$. Therefore, at the surface the energy transport equation can be approximated as:

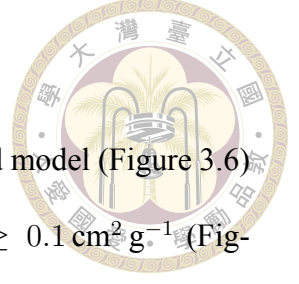
$$\frac{dT}{dm} \sim \kappa_s T_{\text{eff}}^5. \quad (3.18)$$

Here, κ_s is highly sensitive to temperature. In the H^- opacity regime, the opacity decreases rapidly with decreasing temperature. Figure 3.9 presents the opacity as a function of temperature and density, based on data from MESA's "OP_gs98" opacity table. At a density of $10^{-8} \text{ g cm}^{-3}$, the opacity scales approximately as $\kappa \sim T^{11}$ in the temperature range of 3,000–10,000 K, leading to $dT/dm \sim T_{\text{eff}}^{16}$. Therefore, as the star expands and T_{eff} drops, the magnitude of $|dT/dm|$ becomes increasingly shallow, resulting in flatter

temperature, density, and pressure profiles.

We compare the opacity and temperature profiles of the standard model (Figure 3.6) with those of the model imposing a minimum opacity floor of $\kappa \geq 0.1 \text{ cm}^2 \text{ g}^{-1}$ (Figure 3.8). In the standard case, when $\log(R_*/R_\odot) = 3.02$ (i.e., $R_* \sim 1,050 R_\odot$), the opacity drops below $10^{-3} \text{ cm}^2 \text{ g}^{-1}$ near the stellar surface, leading to a flattened $|dT/dm|$, compared to the steeper $|dT/dm|$ in models with smaller R_* . As integration proceeds inward, the opacity increases substantially, which causes ∇_{rad} to exceed ∇_{ad} , triggering convection. In these convective regions, despite the large opacity, $|dT/dm|$ remains shallow, maintaining a flat temperature profile. In contrast, the experimental model with $\kappa \geq 0.1 \text{ cm}^2 \text{ g}^{-1}$ in Figure 3.8 exhibits a different behavior. At $\log(R_*/R_\odot) = 3.2$, which is already larger than the limiting radius in the standard case, still presents a steep $|dT/dm|$ because the opacity is prevented from being extremely low. However, as the star expands further and a larger portion of the mass adopts $\kappa = 0.1 \text{ cm}^2 \text{ g}^{-1}$, this opacity is sufficiently low to flatten the temperature profile. At $\log(R_*/R_\odot) = 3.75$, the outer envelope cools below 2,000 K, and the region where H^- opacity dominates extends inward to $m \sim 20 M_\odot$. As a result, the outer envelope ($m \gtrsim 22 M_\odot$) exhibits significantly reduced opacity compared to models with smaller R_* . This leads to a structural transition to the RSG state, although it occurs at a much larger radius and a lower effective temperature than in the standard model.

In summary, the stellar temperature profile is highly sensitive to surface opacity, particularly in the low-temperature regime where H^- dominates. The flattening of dT/dm , driven by changes in opacity, is the key mechanism behind the structural transition to the distinct RG/RSG phase. In this context, the RSG solution emerges as a limiting radius established by the low-opacity conditions near the surface. By contrast, in polytropic



models where we only consider the electron scattering opacity, this limiting radius does not appear, and the distinct RSG phase does not present.



This limit of envelope expansion corresponds to the classical Hayashi limit or Hayashi line ([Hayashi & Hoshi 1961](#)), which is a nearly vertical line in the HR diagram that corresponds to the solutions of fully convective stars in hydrostatic equilibrium. The region to the right of this line is a forbidden zone in the HR diagram, where no hydrostatic solution exists with the MLT treatment. For our steady-state solutions, each model has a given stellar luminosity, so the effective temperature and radius at the Hayashi line are determined. Once the stellar envelope becomes fully convective after the structural transition, it reaches the configuration corresponding to the Hayashi limit. Therefore, the star cannot expand beyond this point. Across models with different L_0 , their limiting effective temperatures converge at $\sim 4,000$ K (Figure 3.4 and 3.5), consistent with the Hayashi line being nearly vertical in the HR diagram, i.e., stars with a wide range of luminosities attain similar effective temperatures.

The Hayashi limit can be derived from the intersection between radiative atmospheric models and the adiabatic P – T stratification of a fully convective stellar interior, and it is highly sensitive to the opacity profile in the envelope ([Hayashi & Hoshi 1961](#); [Kippenhahn et al. 2012](#)). In our steady-state models, artificial modifications to the opacity significantly affect the limiting radius of the convective envelope, which can be interpreted as a shift of the Hayashi line. This dependence of the limiting radius on opacity aligns with the classical interpretation of the Hayashi limit.

3.4.3 Red and Blue Solutions



Our envelope solutions provide insight into the distribution of giant and supergiant stars on the HR diagram. Based on the $R_{\text{in}}-R_*$ and $R_{\text{in}}-T_{\text{eff}}$ relations shown in Figures 3.4 and 3.5, the solutions can be broadly divided into three distinct regimes:

(1) *Red giant/supergiant phase*: Near $T_{\text{eff}} \sim 4,000$ K, the RG/RSG solution defines a limiting stellar radius and an effective temperature for envelope expansion. The RG/RSG configuration represents a distinct physical state, with an internal structure that differs fundamentally from that of stars with smaller radii. As a star approaches this regime, it undergoes a dramatic structural transition, characterized by substantial mass redistribution and the development of an extended convective envelope.

(2) *Blue giant/supergiant regime*: In the temperature range $T_{\text{eff}} \sim 10,000-30,000$ K, a wide range of steady-state envelope solutions exist, determined primarily by R_{in} , which is set by the core and H-burning shell properties. Generally, for a given luminosity, a smaller R_{in} corresponds to a larger R_* within this blue regime.

(3) *Yellow gap*: Between $T_{\text{eff}} \sim 4,000-10,000$ K, the R_{in} curves are nearly horizontal with respect to R_* or T_{eff} , indicating that only a small inward shift in R_{in} can cause the stellar radius to evolve significantly across this regime. This behavior is consistent with the observed Hertzsprung gap, where stars pass through quickly and few are observed in this temperature range. As a result, most giants and supergiants tend to converge either to the red or blue branches, rather than remaining in the yellow regime.

3.5 Discussions



Compared to full stellar evolution models, the advantage of our steady-state approach is that it allows us to explore all plausible combinations of physical quantities in stellar structures, rather than being restricted to specific evolutionary tracks. This flexibility enables us to isolate the problem from evolutionary effects and investigate the underlying physics that governs stellar envelope structures. Moreover, in stellar evolution simulations, modifying the opacity, surface boundary condition, or convection treatment often causes the computation to stall or fail. In contrast, steady-state models are not affected by prior evolutionary history and thus provide a more feasible framework to test a wide range of physical assumptions. Additionally, we solve only the envelope structure, defining the inner boundary using a constant pressure. This treatment allows us to focus specifically on the behavior of the envelope itself, and in particular, to examine the validity of the mirror principle within the realistic range of M_{in} . By excluding the nuclear burning zones in the central region of the star, we also avoid the complexity arising from nuclear reactions.

Nevertheless, several limitations must be acknowledged. In our models, we assume a fixed luminosity throughout the envelope. This simplification allows us to avoid the gravothermal energy term, ϵ_{grav} , which depends on time derivatives and thus reflects the evolutionary history. However, as shown in Section 3.4, the input luminosity significantly influences the structure of the envelope. Because our steady-state models do not incorporate time evolution, they cannot determine which luminosity values correspond to actual stellar conditions. In fact, luminosity evolves substantially during the post-main-sequence stage. As shown in Chapter 2, the local luminosity at the base of the envelope correlates strongly with the radius at that point. Consequently, R_{in} and L are coupled. In stellar

evolution, a star does not follow a single $R_{\text{in}} - R_*$ curve at fixed luminosity, as shown in Figures 3.4 and 3.5, but instead may traverse multiple such curves corresponding to various luminosities. Therefore, the relationship between R_{in} and R_* derived from our constant luminosity models should be interpreted as general characteristics of the stellar structure, not as an evolutionary track. Furthermore, the full stellar structure requires solving both the core and the envelope self-consistently. Since our calculations omit the core, they do not include the structural constraints imposed by the core properties. To partially compensate for this, we use MESA stellar evolution models to guide the choice of inner boundary conditions and assess the validity of our envelope solutions. In summary, our approach has two main limitations. First, by adopting steady-state models to explore a wide range of structural configurations, we lose information about which configurations are realized in actual stellar evolution. Second, by isolating the envelope and excluding the core, we also lose the information about the physical constraints that arise from the core.

Despite these limitations, our method provides a powerful tool for understanding the fundamental physics of stellar envelopes. Beyond the application to RGs and RSGs presented in this chapter, this approach can be extended to the study of other evolutionary phases, including advanced stages following core carbon burning.

3.6 Conclusions

To understand the physical origin of the mirror principle in envelope expansion and the distinct phase of RG/RSG in post-main-sequence evolution, we construct models of stellar envelopes to solve for steady-state solutions of their structures. These models allow

us to explain three key features:

(1) *The mirror principle for envelope expansion.* Both realistic models and simplified polytropic models without energy treatment can generally reproduce the mirror principle in the regime of physically reasonable core masses: the inner and outer boundaries of the envelope move in the opposite directions. Therefore, this relationship is a natural consequence of the hydrostatic equilibrium regardless of energy transport. As the inner boundary of the envelope moves inward, its gravitational acceleration increases, requiring a steeper pressure gradient to maintain hydrostatic equilibrium. Since the pressure at this boundary is fixed by nuclear burning condition of the burning shell, the only way to achieve a steeper pressure gradient is to reduce the pressure and, therefore, the density in the outer envelope. As a result, the envelope expands in response to the contraction of the inner boundary of the envelope.

(2) *The red giant/supergiant phase.* The RG/RSG phase represents a distinct state in the stellar structure, characterized by significantly different temperature, density, and pressure profiles compared to stars with smaller radii. To reach this phase, a star undergoes a structural transition involving mass redistribution and the formation of an extended convective zone. Completion of this phase transition sets a limiting stellar radius and effective temperature ($\sim 4,000$ K). Once the star reaches this status, further inward motion of the envelope's inner boundary no longer results in additional envelope expansion. The limiting extent of the stellar envelope is governed by a sharp decline in opacity as the temperature drops in the regime where H^- opacity dominates. The resulting reduction in opacity near the stellar surface flattens the temperature gradient, thereby altering the internal structure of the envelope. This physical limit on envelope expansion is consistent with the classical Hayashi limit.

(3) *The Hertzsprung gap between blue and red regimes.* Steady-state solutions show that a small change in the envelope's inner boundary can drive the envelope to expand significantly across the yellow regime, corresponding to the temperature range of $\sim 4,000$ - $10,000$ K. Therefore, stars tend to settle as either red or blue giants/supergiants, instead of remaining yellow. This behavior explains the bifurcation into red and blue supergiants revealed in the grid of stellar models Chapter 1, and accounts for the observed Hertzsprung gap in the HR diagram.

By combining these structural features revealed by steady-state envelope models with the evolutionary mechanisms presented in Chapter 2, we provide a comprehensive explanation for the long-standing question: why stars become red giants or supergiants.

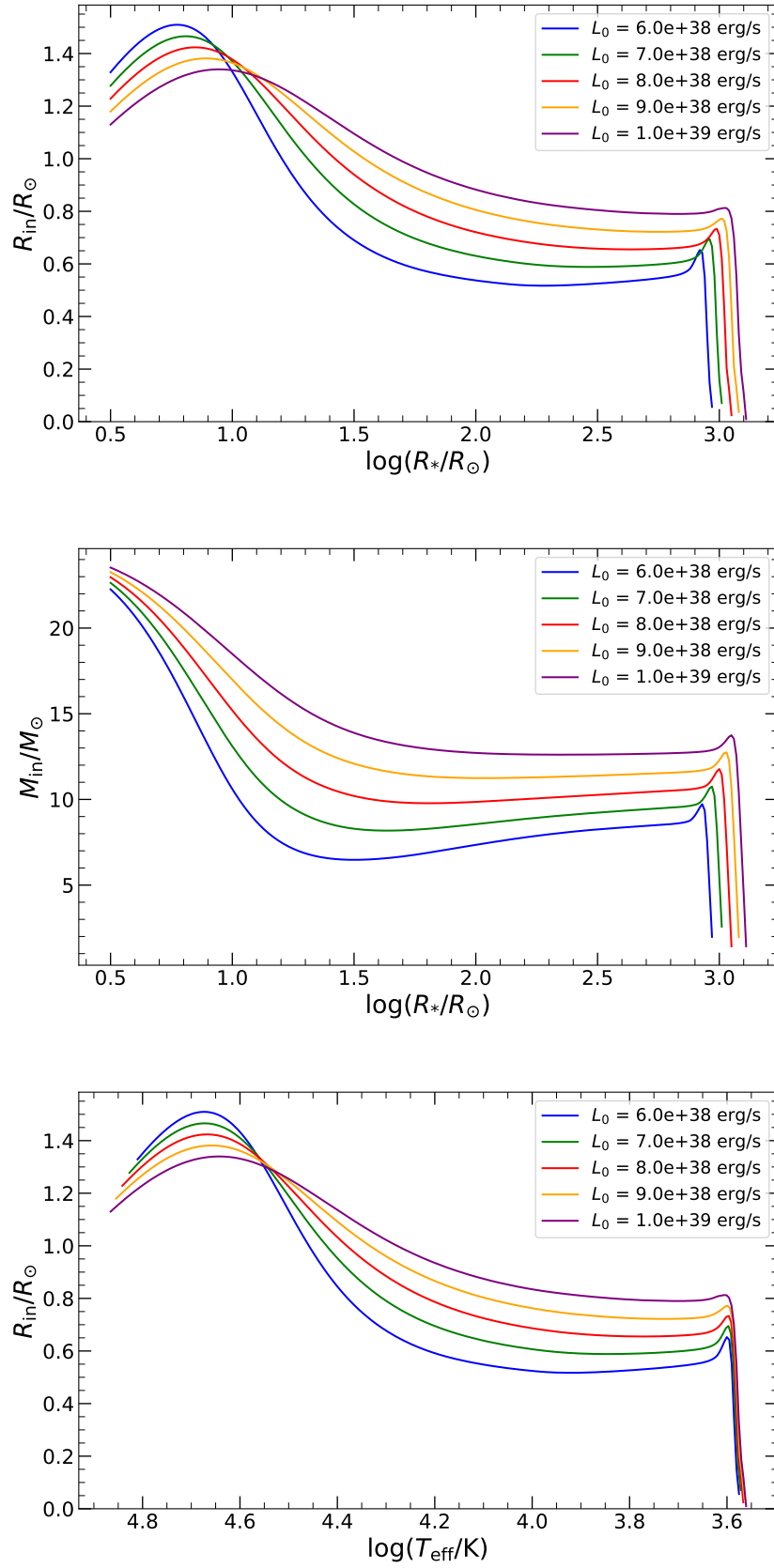
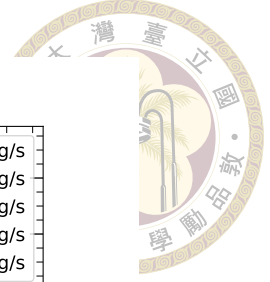


Figure 3.4: R_{in} versus R_* (top panel), M_{in} versus R_* (middle panel), and R_{in} versus T_{eff} (bottom panel) of $25 M_{\odot}$ stars with different luminosities L_0 .

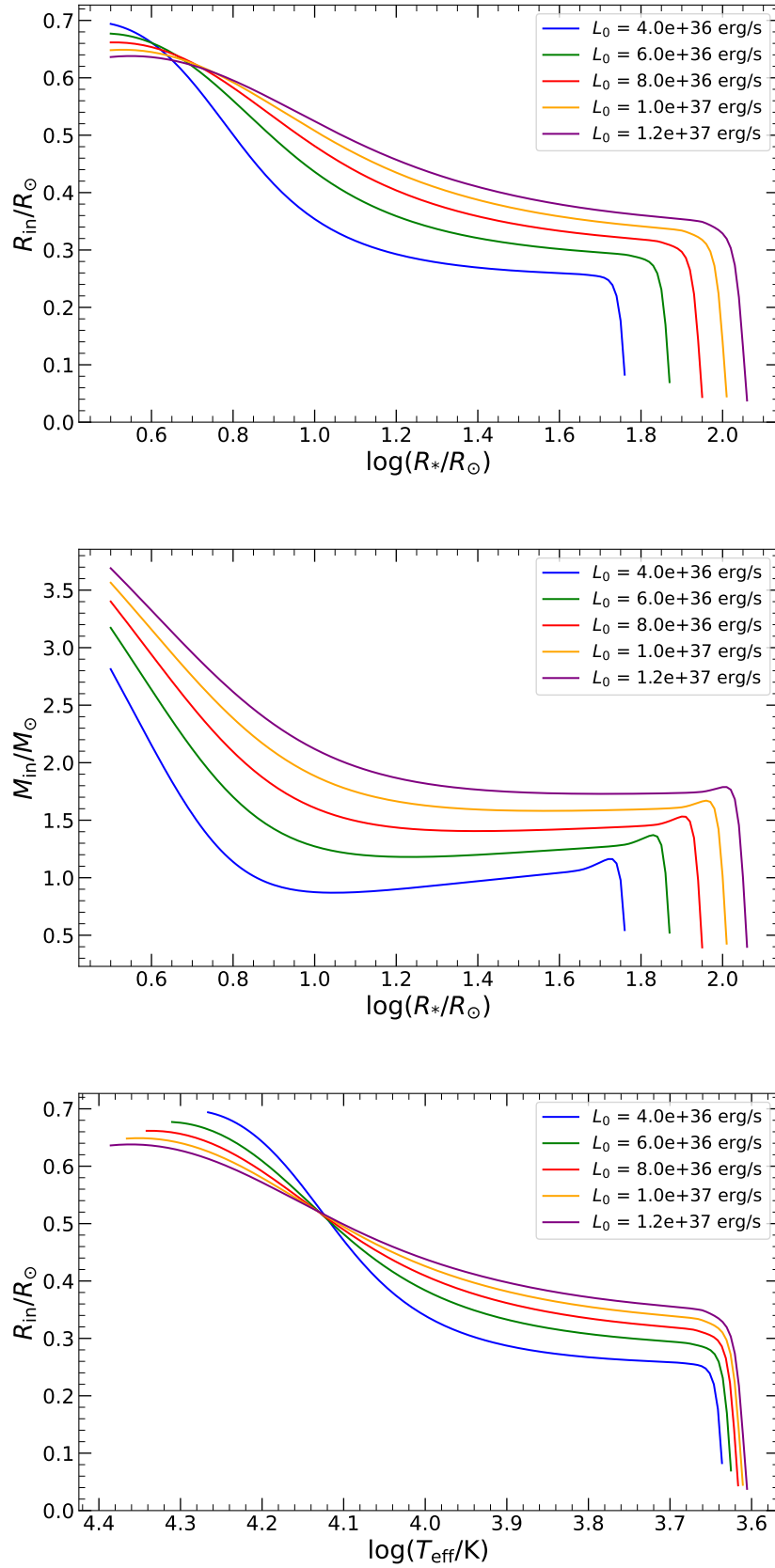
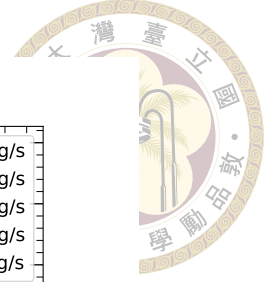


Figure 3.5: R_{in} versus R_* (top panel), M_{in} versus R_* (middle panel), and R_{in} versus T_{eff} (bottom panel) of $5 M_\odot$ stars with different luminosities L_0 .

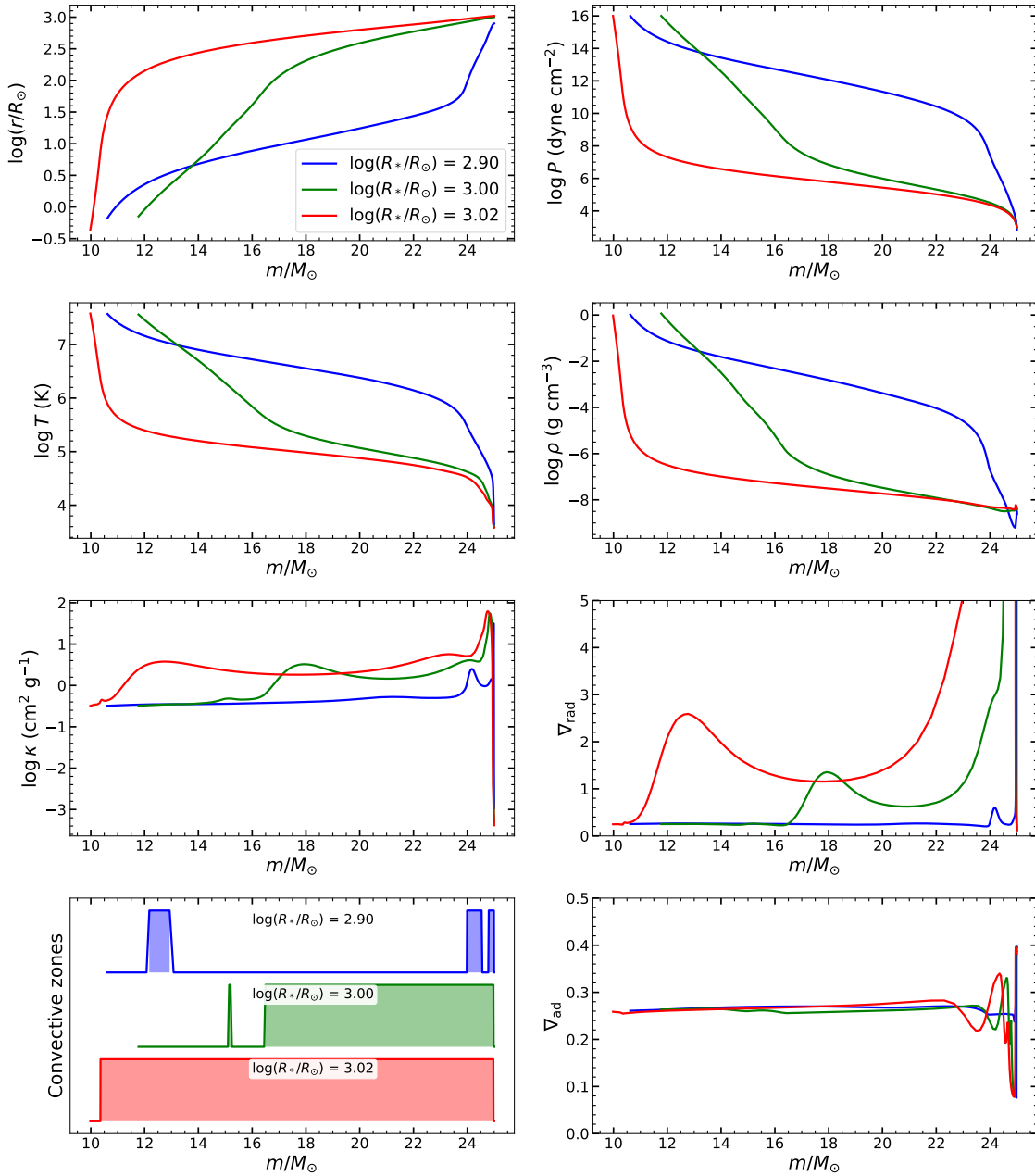
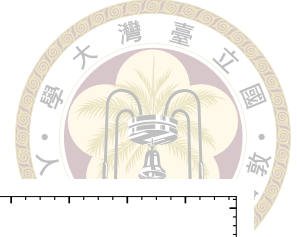


Figure 3.6: Profiles of radius (r), pressure (P), temperature (T), density (ρ), opacity (κ), radiative temperature gradient (∇_{rad}), convective zones, and adiabatic temperature gradient (∇_{ad}) for $25 M_{\odot}$ stars with different stellar radii. The blue, green, and red curves correspond to stellar radii of $\log(R_*/R_{\odot}) = 2.90$, 3.00 , and 3.02 ($R_* \sim 800$, $1,000$, and $1,050 R_{\odot}$), respectively, illustrating steady-state envelope solutions that roughly represent several time points as the star approaches the RSG phase. In the bottom-left panel, the color blocks represent the location of convective zones for different R_* .

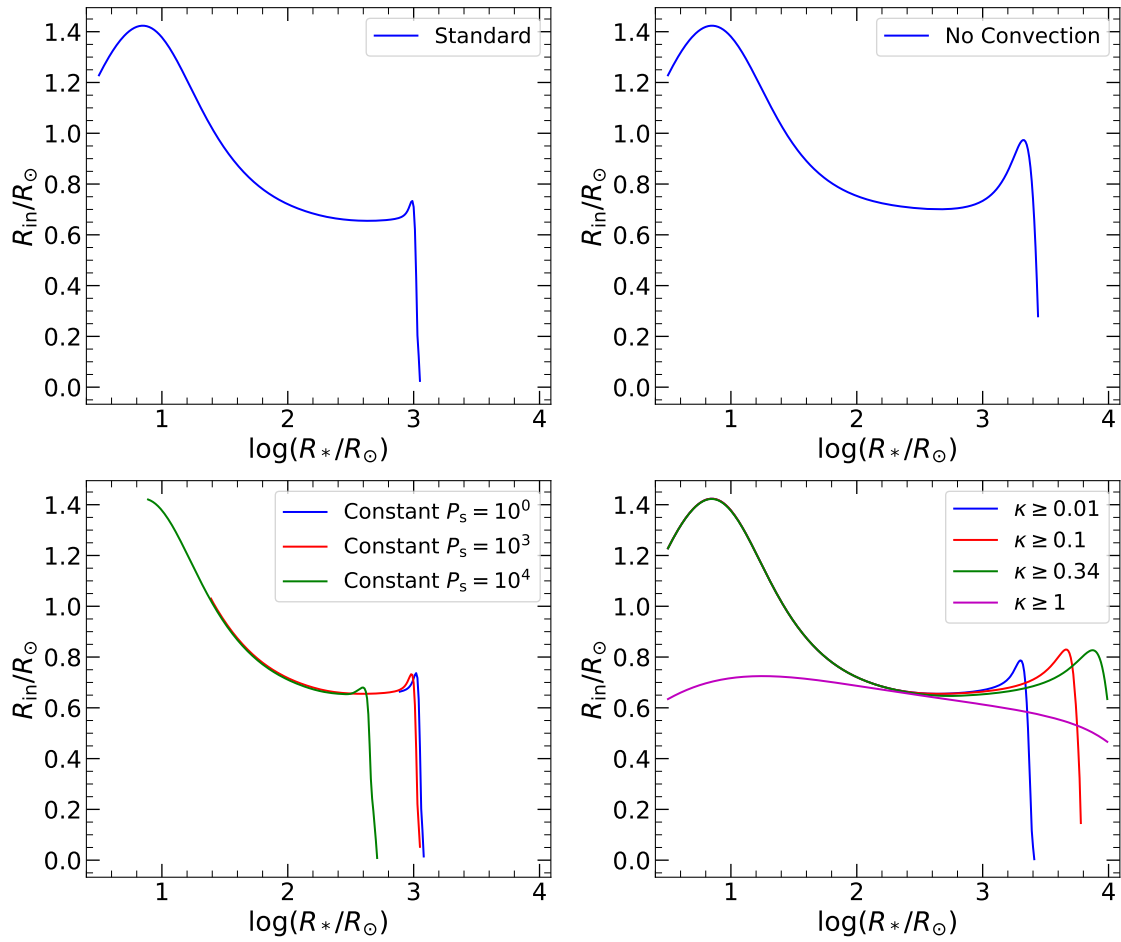


Figure 3.7: Relationship between R_{in} and R_* for experiments modifying envelope properties on a $25 M_{\odot}$ star. The upper-left panel shows results from the standard setup, while the upper-right panel presents the model without convection. The lower-left panel displays models with constant surface pressures of 10^0 , 10^3 , and 10^4 dyne cm^{-2} . The lower-right panel shows models with a minimum opacity floor of $\kappa = 0.01, 0.1, 0.34$, and $1.0 \text{ cm}^2 \text{ g}^{-1}$.

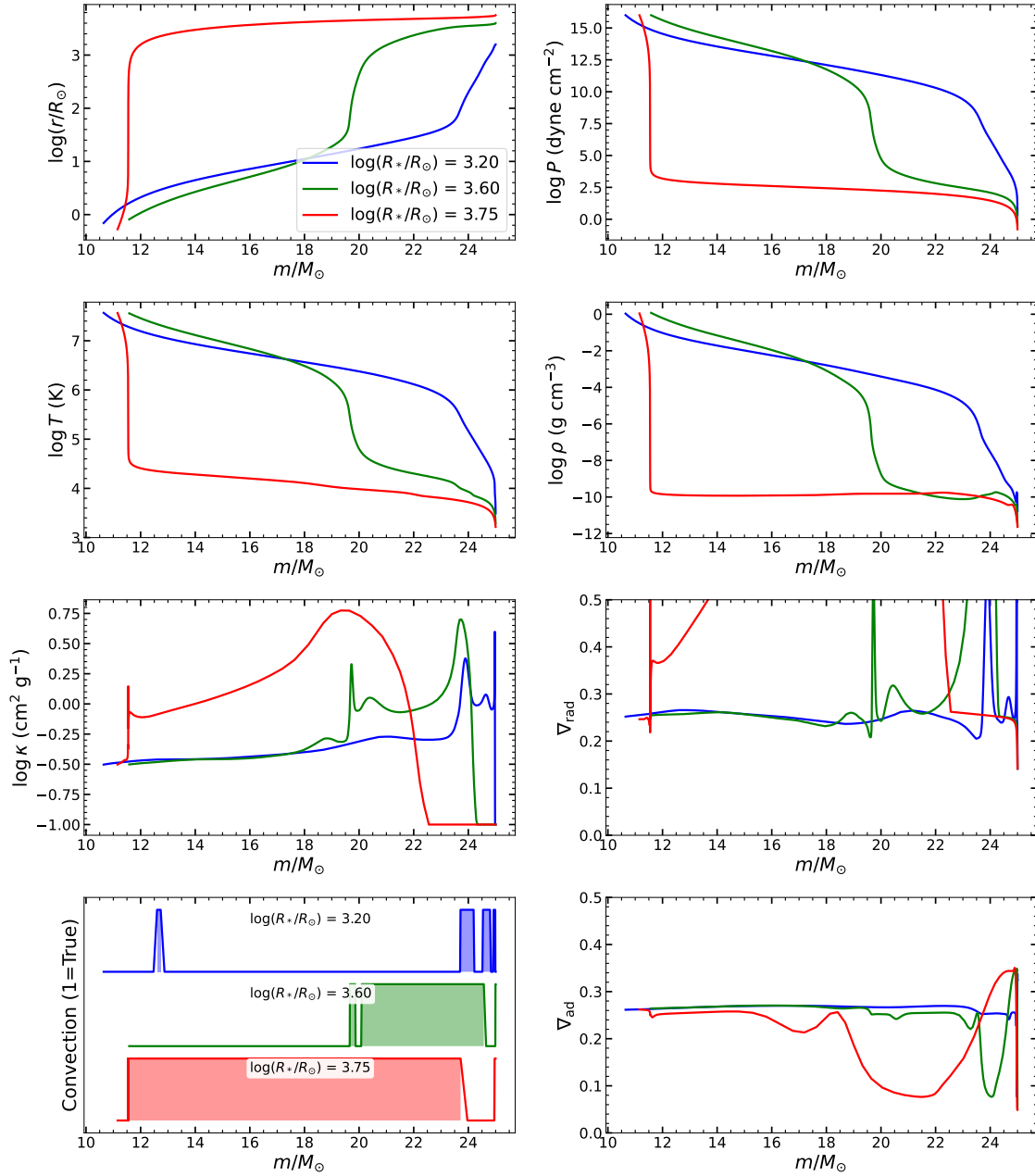
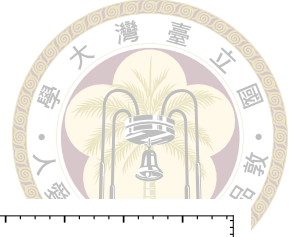


Figure 3.8: Profiles for $25 M_{\odot}$ stars, similar to those in Figure 3.6, but with a minimum opacity floor of $\kappa \geq 0.1 \text{ cm}^2 \text{ g}^{-1}$. The blue, green, and red curves correspond to stellar radii of $\log(R_*/R_{\odot}) = 3.20, 3.60$, and 3.75 , respectively, illustrating steady-state envelope solutions that roughly represent several time points as the star approaches the RSG phase. In the bottom-left panel, the color blocks represent the location of convective zones for different R_* .

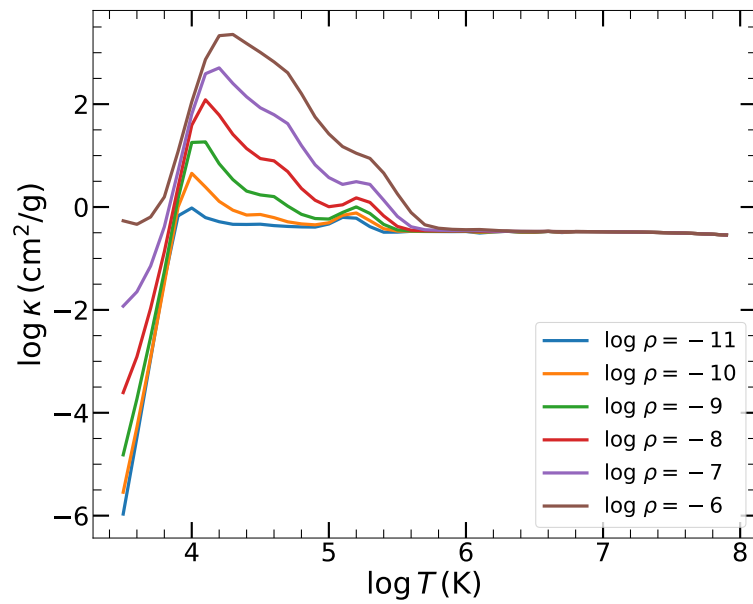


Figure 3.9: Opacity as a function of temperature for various densities retrieved from the MESA "OP_gs98" opacity tables at $X = 0.7$ and $Z = 0.02$.



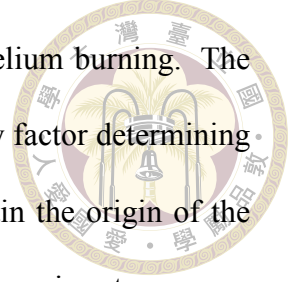
Chapter 4 Physical Mechanism of the Critical Metallicity

* This chapter has been submitted to the AAS journals.

Abstract for Chapter 4

This study investigates the physical origin of the critical metallicity ($Z_c \sim 0.001$) required for the formation of cool supergiants, as revealed by stellar evolution models. Using model grids that vary in mass, metallicity, opacity, and nuclear reaction rates, we identify a threshold radius (R_T) at the terminal-age main sequence (TAMS) that determines whether a star of a given mass can evolve into the red supergiant (RSG) phase. Through stellar models and homology relations, we show that metallicity affects the TAMS radius via its influence on opacity and nuclear burning energy generation. By classifying the evolutionary pathways of supergiants, we demonstrate how TAMS radius, shaped by metallicity, plays a decisive role in determining the evolution outcome: Stars with metallicities $Z < Z_c$ exhibit TAMS radii smaller than R_T and reach advanced advanced core helium or carbon burning while their envelopes remains compact, preventing preventing further expansion into the RSG regime. In contrast, stars with $Z > Z_c$ have TAMS radii

exceeding R_T and expand into the stable RSG phase during core helium burning. The corresponding envelope radius at each core evolution stage is the key factor determining whether a star becomes a red or blue supergiant. Our results explain the origin of the critical metallicity and offer insight into the evolution of metal-poor massive stars.

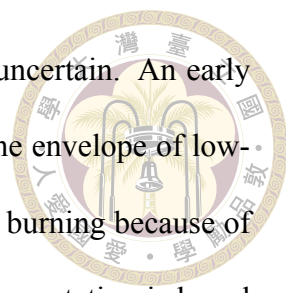


4.1 Introduction

In the Chapter 1, we presented a grid of ~ 2000 stellar models and identified a critical metallicity for the formation of cool supergiants (i.e., red or yellow), $Z_c \sim 0.001 \approx 0.1 Z_\odot$ (For reference, $Z_\odot = 0.0134$; [Asplund et al. 2009](#)). We found that massive stars with metallicities below this threshold ($Z < Z_c$) typically do not evolve into cool supergiants during the core-helium (He) burning phase. This bifurcation of evolutionary paths has a significant impact on stellar mass loss, as cool supergiants exhibit considerably higher mass-loss rates than main-sequence stars. In this chapter, we further explore the physical origin of this critical metallicity.

Our finding of a critical metallicity is consistent with various earlier stellar models, which likewise show that low- Z stars tend to remain as blue supergiants (BSGs) rather than evolving into red supergiants (RSGs) ([Arnett 1991](#); [Baraffe & El Eid 1991](#); [Brocato & Castellani 1993](#); [Brunish & Truran 1982](#); [El Eid et al. 2009](#); [Hirschi 2007](#); [Limongi 2017](#)). Similarly, in the Hertzsprung–Russell (HR) diagram of the recent Geneva stellar evolution models, stars with $Z = 0.0004$ also remain blue during the core-He burning phase ([Groh et al. 2019](#)). These results suggest that the existence of a critical metallicity is a general outcome across different stellar evolution models.

Although the metallicity dependence of RSG formation is consistent between mod-



els, the underlying physics behind the effect of metallicity remains uncertain. An early attempt to explain it was made by [Ritossa \(1996\)](#), who argued that the envelope of low-metallicity stars cannot effectively trap radiation energy from central burning because of reduced opacity. Therefore, they cannot evolve into RSGs. This interpretation is based on a framework in which thermal imbalance within the envelope, governed by the core luminosity and the envelope opacity, drives the inflation of the stellar envelope towards red-giant or supergiant dimensions ([Renzini 2023](#); [Renzini & Ritossa 1994](#); [Renzini et al. 1992](#); [Renzini 1984](#)). However, this explanation for RSG formation based on heat absorption has been questioned by subsequent studies ([Faulkner 2005](#); [Iben 1993](#); [Miller Bertolami 2022](#); [Sugimoto & Fujimoto 2000](#)).

Understanding the metallicity effect on RSG formation is closely tied to the long-standing question: Why do stars become red supergiants? In Chapter 2, we revisited this question and identified the physical criteria that determine whether a star can evolve into the RSG phase. Building on the results from that work, we now aim to investigate how metallicity influences RSG¹ formation and identify the physical origin of the critical metallicity using stellar models.

In Section 4.2, we briefly describe the models employed in this study. In Section 4.3, we present a series of numerical experiments to investigate the metallicity effect on supergiant evolution. Section 4.4 identifies a threshold radius at the end of main sequence for RSG formation and explores its dependence on mass. To explore the connection between

¹We briefly clarify our use of the terms "red supergiant" and "cool supergiant." In Chapter 1, we adopted the term cool supergiant to describe massive stars that expand to $\gtrsim 1,000 R_{\odot}$. We avoided the conventional term red supergiant because very massive stars (e.g., $\gtrsim 50 M_{\odot}$) often maintain effective temperatures above 4,000 K, placing them in the yellow rather than strictly defined red regime. Nevertheless, from a physical standpoint in terms of stellar radius, such stars do not differ intrinsically from classical red supergiants. Moreover, in this chapter, our focus is mainly on stars with $\lesssim 30 M_{\odot}$, which cool below 4,000 K and fulfill the criteria of red supergiants. Therefore, we adopt the more common term red supergiant throughout this work.

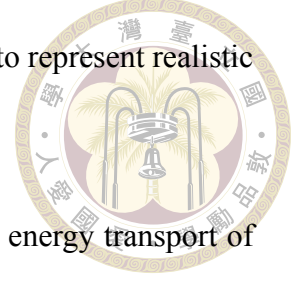
metallicity and this threshold radius, Section 4.5 study how metallicity influences stellar radii. Section 4.6 classifies the evolutionary pathways for stellar models with different masses and metallicities, using the RSG formation criteria proposed by Chapter 2. In Section 4.7, we offer a physical interpretation of the threshold radius and critical metallicity based on the core properties. Finally, Section 4.8 discusses the broader implications of our findings and concludes the chapter.

4.2 Models

We begin with the grid of massive star models presented in Chapter 1, which includes models with initial masses in the range $11 - 60 M_{\odot}$ and metallicities spanning $Z = 1 \times 10^{-5} - 2 \times 10^{-2}$. The models were computed using the Modules for Experiments in Stellar Astrophysics (MESA; [Jermyn et al. 2023](#); [Paxton et al. 2019, 2018, 2015, 2013, 2011](#)) version No. 10108. Each model evolves from the pre-main-sequence stage using the “basic” nuclear reaction network with eight isotopes, and then proceeds from the zero-age main sequence (ZAMS) using the “approx21 (Cr60)” network, which includes 21 isotopes. The models incorporate Henyey’s mixing-length theory, semi-convection, convective overshooting, Type 2 opacity, and mass loss. Details of the model setup are provided in Section 1.2 of Chapter 1.

Following the analysis in Chapter 1, we adopt the $25 M_{\odot}$ models as representative cases, as they exhibit typical evolutionary behaviors across different metallicities. Using these $25 M_{\odot}$ models from the grid described above, we carry out a series of experiments on stellar models by modifying the opacity and nuclear reaction rates to examine their effects on supergiant evolution. These extended models are designed as numerical experiments

to examine the influence of specific physical parameters, rather than to represent realistic stellar evolution scenarios.

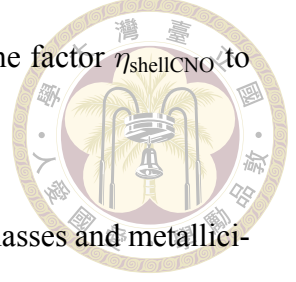


Opacity, as a metallicity-dependent factor, strongly impacts the energy transport of the entire star. Adjusting opacity in stellar models requires careful consideration as opacity is coupled with the evolution of gas density, temperature, and metallicity. A robust method to modify opacity is to adjust the value of Z_{base} in MESA, which specifies the base metallicity used to retrieve opacities from the opacity tables. Importantly, Z_{base} governs the opacity alone and does not represent the metallicity of the star. For example, a model with $Z = 0.02$ and $Z_{\text{base}} = 0.001$ has a metallicity of 0.02 but evolves with an opacity comparable to that of a star with $Z = 0.001$. This approach enables us to isolate the effects of opacity from other metallicity-dependent effects in stellar evolution. To avoid confusion in terminology, we denote the MESA input parameter Z_{base} as ζ_{opacity} throughout the discussion in this chapter.

Nuclear reaction rates are modified using the `reaction_for_special_factor` option in the `star_job` section of the MESA inlist. For the CNO cycle, we adopt the parameter, η_{CNO} to linearly scale the rates of five key reactions included in the approx21 nuclear reaction network: (1) $^{12}\text{C} + ^1\text{H} \rightarrow ^{13}\text{N} + \gamma$, (2) $^{14}\text{N} + ^1\text{H} \rightarrow ^{15}\text{O} + \gamma$, (3) $^{16}\text{O} + ^1\text{H} \rightarrow ^{17}\text{F} + \gamma$, (4) $^{15}\text{N} + ^1\text{H} \rightarrow ^{16}\text{O} + \gamma$, and (5) $^{15}\text{O} + ^1\text{H} \rightarrow ^{12}\text{C} + ^4\text{He}$. In addition, we use another factor, $\eta_{3\alpha}$ to linearly scale the reaction rate of the triple-alpha process.

We also designed an experiment in which the CNO reaction rates are modified only during the shell H-burning phase in the post-main-sequence stage. Specifically, the adjustment is applied starting from the time when the He core is formed and only in regions outside the core. By default in MESA, the He core is defined as the region where the

H mass fraction satisfies $X_{\text{H}} \leq 0.1$. For this experiment, we used the factor η_{shellCNO} to linearly scale the CNO reaction rates in the H burning shell.



In addition to the extensive grid of stellar models with varying masses and metallicities presented in Chapter 1, we have constructed four new distinct grids of MESA models spanning different parameter spaces. Together, these five grids comprise a total of 6,896 models. A summary of the model configurations is provided below and in Table 4.1.

- (a) Initial stellar mass (M_i) and metallicity (Z), from Chapter 1.
- (b) Metallicity (Z) and opacity (set by ζ_{opacity}).
- (c) Hydrogen-burning rate (scaled by η_{CNO}) and opacity (set by ζ_{opacity}).
- (d) Shell Hydrogen-burning rate (scaled by η_{shellCNO} for the burning shell) and opacity (set by ζ_{opacity}).
- (e) Helium-burning rate (scaled by $\eta_{3\alpha}$) and opacity (set by ζ_{opacity}).

Grid (a), as presented in Chapter 1, consists of realistic stellar models used to demonstrate the existence of a critical metallicity. On the other hand, the models in Grids (b)–(e) are not intended to represent realistic stars; instead, they act as experimental models to investigate the effects of specific stellar parameters. For comparison, we vary only two parameters in each grid of models, while keeping the remaining parameters identical to those in Grid (a). Grids (b)–(e) are based on $25 M_{\odot}$ stars, with Grids (c)–(e) further restricted to $Z = 0.001$.

Table 4.1: Summary of model grids and varied parameters

Grid	$M_i (M_\odot)$	Z	ζ_{opacity}	η_{CNO}	η_{shellCNO}	$\eta_{3\alpha}$
(a)	11, 12, 13, ..., 59, 60	$(1, 2, \dots, 9) \times 10^{-5}$, $(1, 2, \dots, 9) \times 10^{-4}$, $(1, 2, \dots, 9) \times 10^{-3}$, $(1.0, 1.1, \dots, 2.0) \times 10^{-2}$	1.0	1.0	1.0	1.0
(b)	25	$(1, 2, \dots, 9) \times 10^{-5}$, $(1, 2, \dots, 9) \times 10^{-4}$, $(1, 2, \dots, 9) \times 10^{-3}$, $(1.0, 1.1, \dots, 2.0) \times 10^{-2}$	$(1, 2, \dots, 9) \times 10^{-5}$, $(1, 2, \dots, 9) \times 10^{-4}$, $(1, 2, \dots, 9) \times 10^{-3}$, $(1.0, 1.1, \dots, 2.0) \times 10^{-2}$	1.0	1.0	1.0
(c)	25	0.001	$(1, 2, \dots, 9) \times 10^{-5}$, $(1, 2, \dots, 9) \times 10^{-4}$, $(1, 2, \dots, 9) \times 10^{-3}$, $(1, 2, 3, 4, 5) \times 10^{-2}$	$(1, 2, \dots, 9) \times 10^{-3}$, $(1, 2, \dots, 9) \times 10^{-2}$, $(1, 2, \dots, 9) \times 10^{-1}$, $(1, 2, \dots, 10)$	1.0	1.0
(d)	25	0.001	$(1, 2, \dots, 9) \times 10^{-5}$, $(1, 2, \dots, 9) \times 10^{-4}$, $(1, 2, \dots, 9) \times 10^{-3}$, $(1, 2, 3, 4, 5) \times 10^{-2}$	1.0	$(1, 2, \dots, 9) \times 10^{-3}$, $(1, 2, \dots, 9) \times 10^{-2}$, $(1, 2, \dots, 9) \times 10^{-1}$, $(1, 2, \dots, 10)$	1.0
(e)	25	0.001	$(1, 2, \dots, 9) \times 10^{-5}$, $(1, 2, \dots, 9) \times 10^{-4}$, $(1, 2, \dots, 9) \times 10^{-3}$, $(1, 2, 3, 4, 5) \times 10^{-2}$	1.0	1.0	$(1, 2, \dots, 9) \times 10^{-3}$, $(1, 2, \dots, 9) \times 10^{-2}$, $(1, 2, \dots, 9) \times 10^{-1}$, $(1, 2, \dots, 10)$

Notes. M_i : Initial stellar mass. Z : Stellar metallicity. ζ_{opacity} : Equivalent to the base metallicity (Z_{base}) adopted for interpolating opacities from the opacity tables, independent of the stellar metallicity Z . η_{CNO} : Scaling factor for the CNO cycle reaction rate. η_{shellCNO} : Scaling factor for the CNO cycle reaction rate, applied only to the H-burning shell.

4.3 Numerical Experiments on Opacity and Nuclear Reaction Rates



Metallicity affects stellar evolution through its impact on several key physical quantities, including opacity, nuclear burning rates, and mean molecular weight (e.g., [El Eid et al. 2009](#)). In this section, to disentangle the primary factors through which metallicity impacts evolution toward the supergiant phase, we conduct a series of experiments for $25 M_{\odot}$ stars using Grid (b)–(e), as described in Section 4.2. In particular, we focus on the roles of opacity and nuclear reaction rates.

For each model in these grids, we assess whether it evolves into an RSG by calculating the maximum radius it reaches during the core He-burning phase, prior to the onset of C-core burning. This maximum radius is denoted as $R_{\max}$.

4.3.1 Grid (b): Metallicity and Opacity

We begin by examining the effect of opacity on envelope expansion by constructing a grid of stellar models, Grid (b), with varying metallicities and opacities.

Figure 4.1 shows $R_{\max}$ for each model in Grid (b). The models that reach RSG dimensions ($\log(R/R_{\odot}) \gtrsim 3$) are clearly separated from those that remain within BSG dimensions ($\log(R/R_{\odot}) \lesssim 2$). For $Z \lesssim 10^{-3}$, the boundary dividing these two areas is vertical, indicating that opacity, controlled by ζ_{opacity} , has little influence on the supergiant outcome in this low- Z range. Only when $Z \gtrsim 10^{-3}$ does opacity begin to significantly affect whether the star becomes an RSG or remains a BSG. Notably, for $Z \gtrsim 0.003$, stars can still expand to RSG regime even when ζ_{opacity} is extremely low (a few times 10^{-5}).

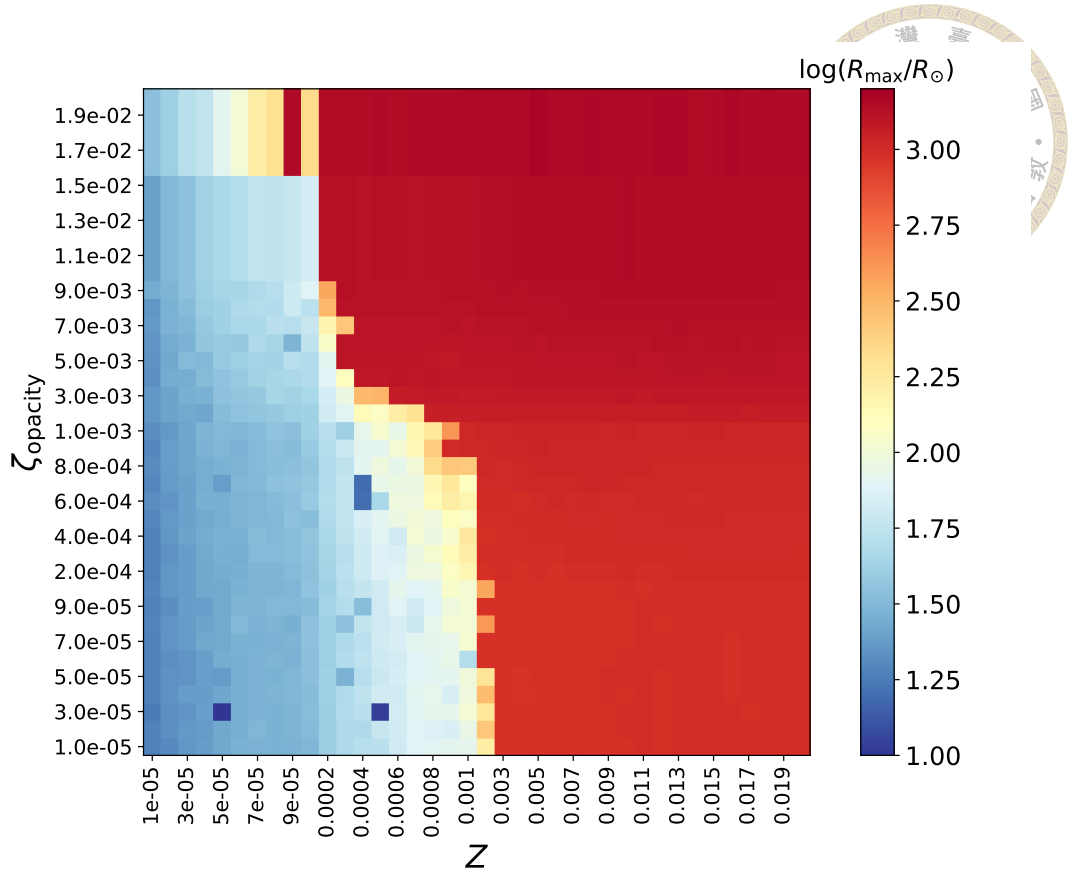


Figure 4.1: Maximum stellar radius during the core He-burning stage ($R_{\max}$) for Grid (b), which includes models with an initial mass of $25 M_{\odot}$, varying in metallicity (Z) and opacity parameter (ζ_{opacity}). The boundary between red and blue supergiants depends on ζ_{opacity} in the metallicity range $Z \gtrsim 10^{-3}$, but becomes nearly independent of ζ_{opacity} when $Z \lesssim 10^{-3}$.

This suggests that additional metallicity-dependent factors, beyond opacity alone, play a role in determining the supergiant's fate.

4.3.2 Grid (c): Hydrogen Burning Rates and Opacity

In addition to opacity, metallicity also significantly influences nuclear burning within a star. In our second experiment, Grid (c), we examine the combined effects of these two factors by varying both the opacity parameter ζ_{opacity} and the linear scaling factor for the reaction rates of the CNO cycle, η_{CNO} . In this experiment, η_{CNO} is applied consistently throughout the entire stellar lifetime, scaling the reaction rates of both core H burning

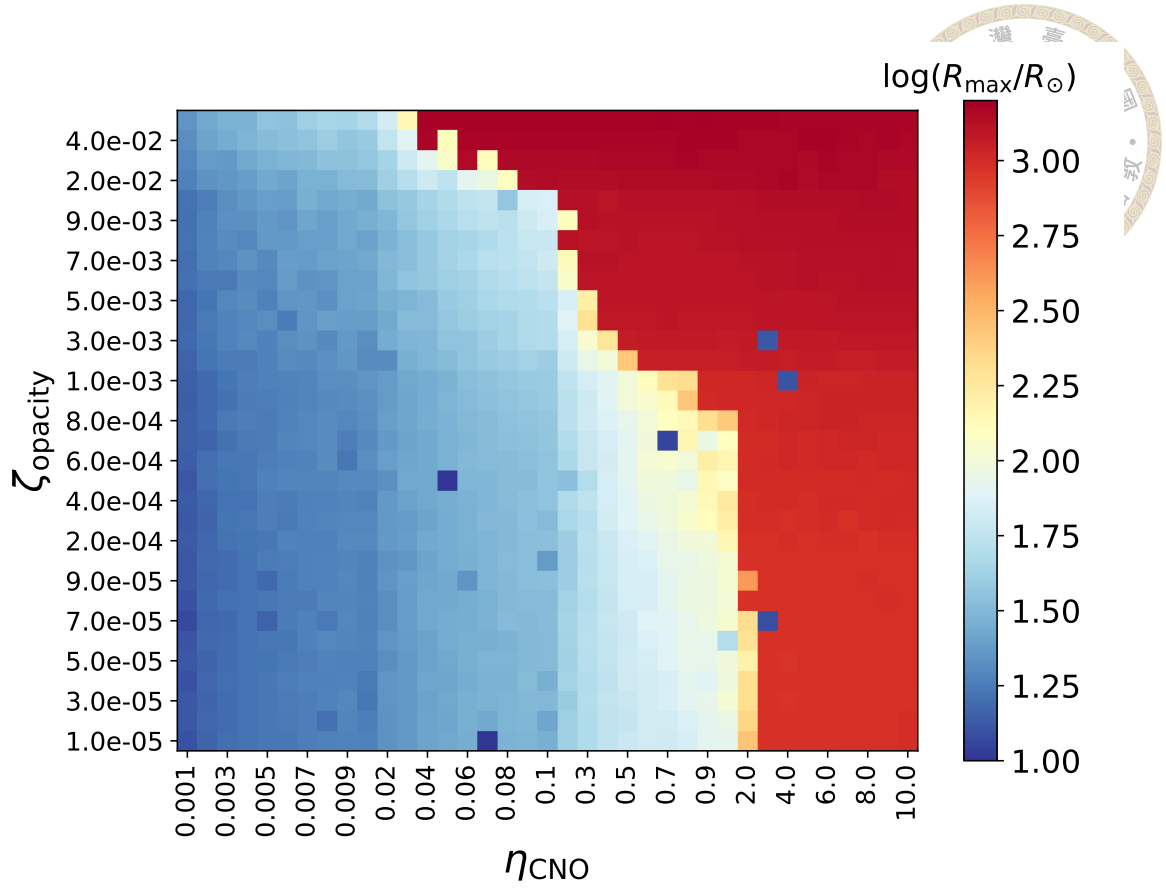


Figure 4.2: Maximum stellar radius during the core He-burning stage ($R_{\max}$) for Grid (c): models with an initial mass of $25 M_{\odot}$ and metallicity $Z = 0.001$, computed for varying values of the CNO-cycle reaction rate (scaled by η_{CNO}) and opacity parameter (ζ_{opacity}). Both η_{CNO} and ζ_{opacity} influence the evolutionary outcome, determining whether the star becomes a red or blue supergiant.

during the main-sequence phase and shell H burning during the post-main-sequence evolution.

Figure 4.2 shows $R_{\max}$ for each model in Grid (c). A clear boundary separates models that evolve into RSGs from those that remain as BSGs. In general, both higher opacity and higher CNO reaction rates favor the transition to RSGs. When the opacity is low, a larger CNO rate is required for the star to expand to RSG radii. *In particular, even models with low opacity with ζ_{opacity} on the order of a few times 10^{-5} can still evolve into RSGs if the CNO cycle is sufficiently strong (i.e., $\eta_{\text{CNO}} \geq 3$).*

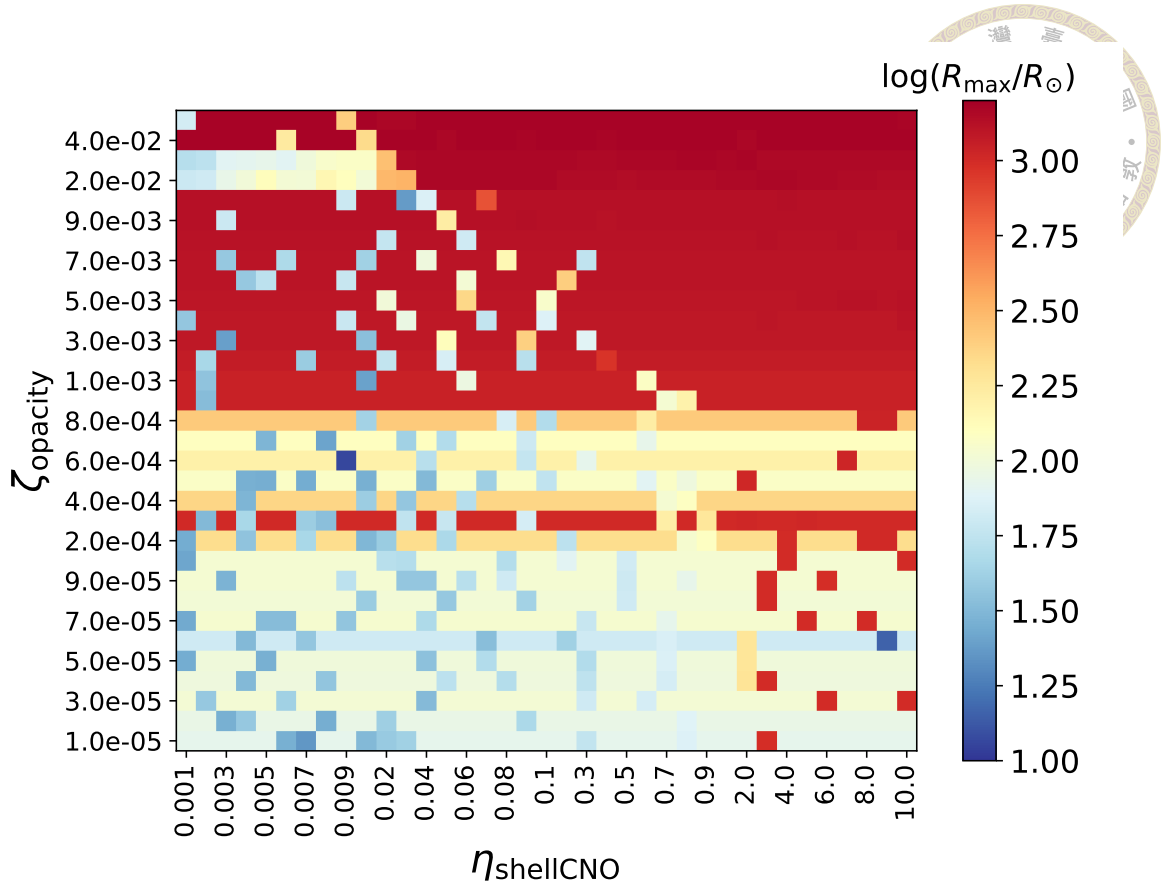


Figure 4.3: Maximum stellar radius during the core He-burning stage ($R_{\max}$) for Grid (d): models with an initial mass of $25 M_{\odot}$ and metallicity $Z = 0.001$, computed for varying values of the CNO-cycle reaction rate during the shell H-burning stage (scaled by η_{shellCNO}) and opacity parameter (ζ_{opacity}). For comparison, the parameter η_{CNO} used in Figure 4.2 is applied throughout all evolutionary stage, whereas η_{shellCNO} in this figure is applied only during the shell H-burning stage. While $R_{\max}$ exhibits some scatter, the overall trend indicates that η_{shellCNO} has little impact on the supergiant outcome.

4.3.3 Grid (d): Shell Hydrogen Burning Rate and Opacity

The reaction rates of H burning can significantly influence whether a star evolves into an RSG; however, it remains unclear whether this effect originates primarily from core H burning during the main-sequence phase or from shell H burning in the post-main-sequence phase. To address this question, we constructed another set of models, Grid (d), in which the CNO cycle reaction rates are scaled by η_{shellCNO} only during the post-main-sequence stage, while remaining unchanged during the main sequence.

Figure 4.3 shows the resulting $R_{\max}$ values for models with varying η_{shellCNO} . Despite

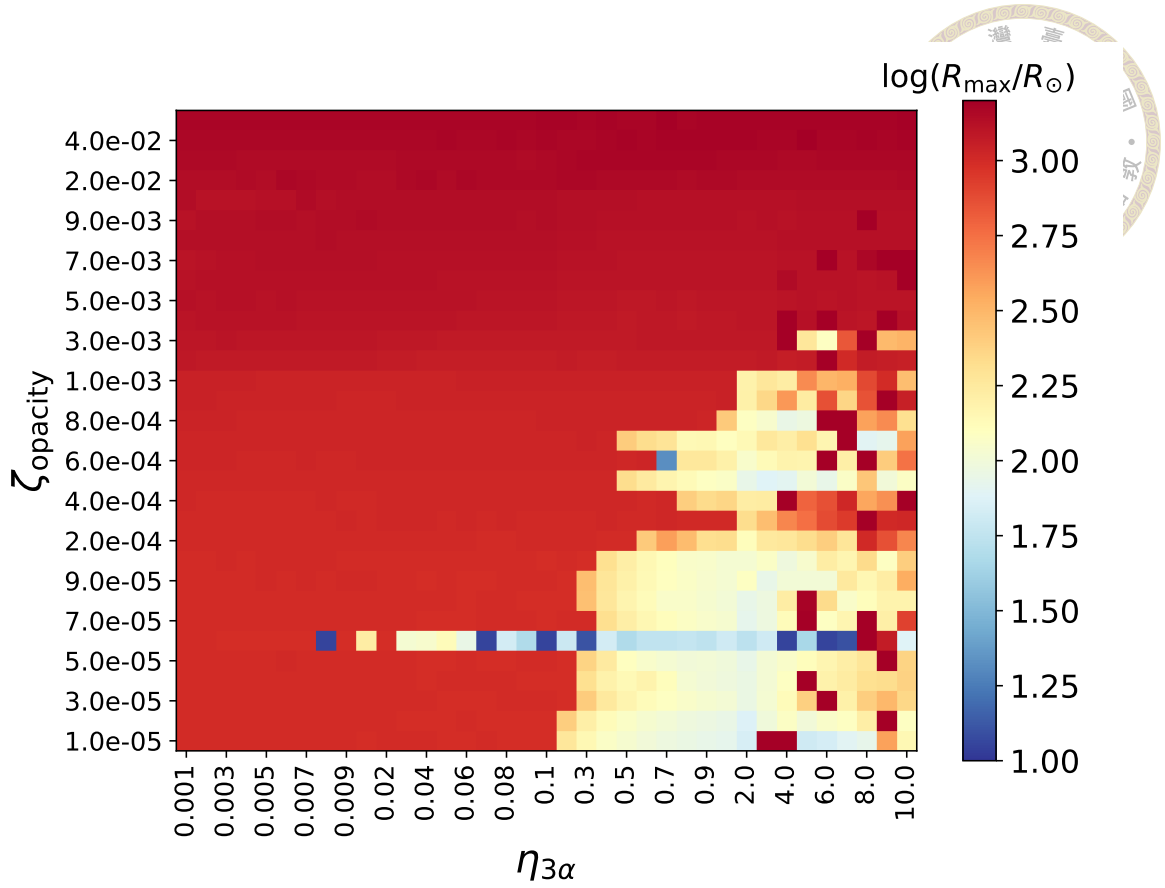


Figure 4.4: Maximum stellar radius during the core He-burning stage ($R_{\max}$) for Grid (e): models with an initial mass of $25 M_{\odot}$ and metallicity $Z = 0.001$, computed for varying values of the triple-alpha reaction rate (scaled by $\eta_{3\alpha}$) and opacity parameter (ζ_{opacity}). Increasing $\eta_{3\alpha}$ does not promote evolution toward the RSG phase and may even suppress it in the low-opacity regime ($\zeta_{\text{opacity}} \lesssim 10^{-3}$).

some scatter in the data, it is clear that η_{shellCNO} has little impact on the final radii of supergiants. In general, even when η_{shellCNO} is extremely low, stars with sufficiently high opacity can still expand to RSG dimensions.

This suggests that the strong dependence of $R_{\max}$ on η_{CNO} observed in Grid (c) primarily originates from core H burning during the main sequence, rather than from shell H burning in the post-main-sequence phase. In other words, our results reveal that nuclear burning during the main sequence can significantly affect the degree of envelope expansion during the post-main-sequence stage.

4.3.4 Grid (e): Helium Burning Rate and Opacity



In Chapter 2, we showed that enhanced He burning can actually suppress the expansion toward the RSG phase. To further demonstrate this effect, we construct a dedicated set of models, Grid (e), in which the triple-alpha reaction rate is scaled by a linear coefficient $\eta_{3\alpha}$.

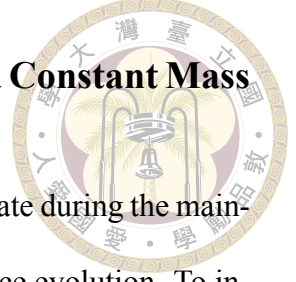
Figure 4.4 displays the Grid (e) with varying $\eta_{3\alpha}$ and ζ_{opacity} . In the lower-right region of the diagram, some models with low ζ_{opacity} and high $\eta_{3\alpha}$ fail to evolve into RSGs. In other words, an enhanced $\eta_{3\alpha}$ does not promote RSG formation; rather, it can prevent stars from reaching RSG dimensions.

In summary, we conducted a series of experiments to investigate the effects of opacity and nuclear reaction rates on the formation of RSG. Our results show that both metallicity-dependent factors, opacity and the H-burning rate during the main-sequence phase, play important roles in promoting the evolution into an RSG. In contrast, an enhanced core He-burning rate tends to suppress RSG formation.

4.4 Direct Indicator for RSG Formation

The results in Section 4.3 indicate that both opacity and core H-burning rates contribute to promoting stellar evolution into the RSG phase. In this section, we aim to develop a unified picture that explains how these two factors jointly determine the final radii of supergiants.

4.4.1 Threshold for Evolution into the RSG Phase at a Constant Mass



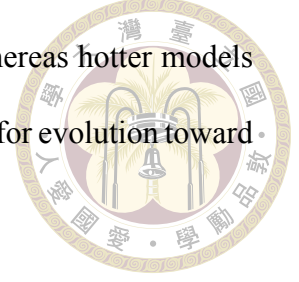
One key finding from the previous section is that the H-burning rate during the main-sequence phase significantly influences the star's post-main-sequence evolution. To investigate this delayed effect, we analyze how the stellar parameters in the main sequence correlate with the maximum radius ($R_{\max}$) reached during the core He-burning stage. In particular, we examine two key moments in the history of stellar evolution: the ZAMS and the terminal-age main sequence (TAMS).

To avoid the complication of varying stellar mass, we begin with Grid (c), in which the initial mass is fixed at $25 M_{\odot}$. Figure 4.5 shows the effective temperature (T_{eff}), central temperature (T_c), surface pressure (P_{surf}), and central pressure (P_c) at both the ZAMS and TAMS for each model. The data points are plotted as functions of η_{CNO} , and colored by the resulting $R_{\max}$. Our main goal is to identify the boundary separating models that evolve into RSGs (red points) from those that remain as BSGs (blue points), and to explore how this transition correlates with the physical parameters at ZAMS and TAMS. If a threshold value in one of these parameters can clearly distinguish the red and blue points, it may provide an important clue to the underlying evolutionary mechanism.

At ZAMS, while RSG and BSG models show a clear visual separation, the dividing line does not correspond to any constant value of T_{eff} , T_c , P_{surf} , or P_c . Thus, these ZAMS parameters alone do not serve as direct predictors of whether a star will become an RSG.

At the TAMS, neither T_c nor P_c shows a clear separation between the RSG and BSG models, implying that physical conditions of the stellar core at this stage are also not robust indicators of the final supergiant outcome. Among the physical parameters examined, the most reliable discriminator is the effective temperature at the TAMS. Models with

$T_{\text{eff}} \lesssim 30,000\text{--}35,000\text{ K}$ at the TAMS tend to evolve into RSGs, whereas hotter models do not. This implies that T_{eff} at the TAMS acts as a practical threshold for evolution toward the RSG phase.



T_{eff} is tightly related to the stellar radius as the envelope cools during expansion. Thus, we further examine how the radii at the ZAMS and TAMS correlate with the maximum radius, R_{max} . In Figure 4.6, we present the ZAMS and TAMS radii as functions of η_{CNO} and ζ_{opacity} , with colors of data points indicating the corresponding values of R_{max} .

At the ZAMS, when core H burning has just begun, the stellar radius (R_{ZAMS}) shows minimal dependence on η_{CNO} , but displays moderate variation with ζ_{opacity} . At this stage, the boundary separating RSG and BSG data points appears as a tilted line rather than aligning with any constant radius. This suggests that the ZAMS radius alone is not a robust predictor of whether a star will eventually evolve into an RSG.

As shown in the middle and right panels of Figure 4.6, by the TAMS, after the main-sequence evolution is complete, the stellar radius (R_{TAMS}) exhibits a much stronger dependence on η_{CNO} , while opacity also contributes to increasing the radius to some extent, particularly for $\zeta_{\text{opacity}} \gtrsim 10^{-3}$. RSG and BSG models are now clearly divided by a horizontal boundary. Specifically, if the TAMS radius exceeds a threshold of $\sim 12 R_{\odot}$, a $25 M_{\odot}$ star eventually evolves into an RSG; otherwise, it remains a BSG. *These results indicate that the stellar radius attained by the end of the main-sequence phase is a decisive factor for whether a star will ultimately evolve into an RSG.*

To assess whether this threshold radius correlates with the He core radius, we plot the distribution of TAMS stellar radius versus the He core radius at its formation, with point colors indicating R_{max} . We find that, for any given TAMS radius, the models span a broad

range of core radii. The boundary separating RSGs (red points) from BSGs (blue points) at $R_{\text{TAMS}} \sim 12 R_{\odot}$ does not correspond to any fixed He core radius, but encompasses various He core sizes. This confirms that whether a star evolves into an RSG is not determined by He core radius, but rather by the radius of the entire star at the onset of the post-main-sequence phase.

These results lead to a unified principle of the metallicity effect on supergiant evolution. At a fixed initial mass, a star can eventually evolve into an RSG if its radius at the onset of post-main-sequence stage exceeds a certain threshold. Conversely, if the star fails to reach this threshold, it will remain a BSG. Both opacity and the core H-burning rate are influenced by metallicity and affect the stellar structure during the main-sequence phase. Together, they shape the stellar radius at the TAMS, which ultimately determines whether the star will evolve into an RSG.

We note that altering the core He-burning rates during the post-main-sequence phase, as explored in Grid (e), can still influence the final evolutionary outcome. However, this is not the primary mechanism by which metallicity affects supergiant evolution, as it does not account for the systematic trend in which higher metallicity favors RSG formation over BSGs. For this reason, we exclude this effect from our main analysis.

4.4.2 Threshold TAMS Radii for Different Masses

To investigate how stellar mass influences the threshold radius required for RSG formation, we conduct a similar analysis using Grid (a). We examine the boundary between red and blue data points, which marks the transition between stars that eventually become RSGs and those that remain BSGs. As shown in the left panel of Figure 4.8, this boundary

is no longer located at a constant TAMS radius but increases systematically with stellar mass.



To obtain a coarse fit for the threshold TAMS radius required for RSG formation as a function of mass, we multiply the TAMS radius (R_{TAMS}) of each model by the stellar mass (M_{TAMS}) raised to a trial exponent. By manually adjusting the exponent, we identify a scaling relation that yields an approximately horizontal boundary separating RSG and BSG cases. As shown in the right panel of Figure 4.8, this leads to the following approximate criterion for RSG formation:

$$\left(\frac{R_{\text{TAMS}}}{R_{\odot}} \right) \gtrsim \left(\frac{M_{\text{TAMS}}}{M_{\odot}} \right)^{3/4}, \quad (4.1)$$

This relation shows the approximate mass dependence of the threshold radius for RSG formation. We note that M_{TAMS} differs only slightly from the initial mass, as mass loss during the main sequence is typically modest.

We propose the TAMS radius as a practical criterion for determining whether a star evolves into the RSG phase. However, the underlying physical mechanism responsible for this threshold, and its connection to metallicity, requires further investigation. In the following sections, we address two key questions:

- (1) How does metallicity affect the stellar radius?
- (2) How does the TAMS radius influence post-main-sequence expansion and define a threshold for RSG formation?

We begin by examining the effect of metallicity on stellar structure and evolution in Section 4.5, followed by an overview of the evolutionary pathways of supergiants in Section 4.6. Based on these pathways, we then analyze how the TAMS radius regulates su-

pergiant expansion in Section 4.7.



4.5 Metallicity Effect on Stellar Radius

To evaluate how metallicity influences stellar radius, we begin by analyzing evolutionary tracks from Grids (a) and (c), and then apply homology relations to perform a dimensional analysis.

4.5.1 Metallicity Dependence of Stellar Evolutionary Tracks

In previous sections, we demonstrated that both η_{CNO} and ζ_{opacity} can influence whether a star evolves into an RSG by modifying its TAMS radius. To investigate how these parameters affect stellar evolution in detail, we examine evolutionary tracks on the HR diagram. We focus on two subsets of models from Grid (c): one in which η_{CNO} is varied while ζ_{opacity} is held fixed, and the other in which ζ_{opacity} is varied while η_{CNO} is fixed.

Figure 4.9 presents the HR diagram for models with an initial mass of $25 M_{\odot}$ and metallicity $Z = 0.001$, where ζ_{opacity} is varied, and the reaction rate of the CNO cycle is fixed at its standard value ($\eta_{\text{CNO}} = 1$). Changes in ζ_{opacity} lead to noticeable differences in luminosity starting from the ZAMS, along with moderate variations in T_{eff} . In general, a higher opacity (larger ζ_{opacity}) results in a lower luminosity, which would suggest a smaller radius. However, higher opacity also reduces T_{eff} , partially offsetting by the luminosity effect on radius. As a result, the ZAMS positions of models with varying opacity lie nearly parallel to iso-radius lines on the HR diagram. As the stars evolve, the relationship between opacity and radius shifts. By the time they reach the TAMS, just before the expansion into the supergiant phase, models with higher opacity display systematically

larger radii.

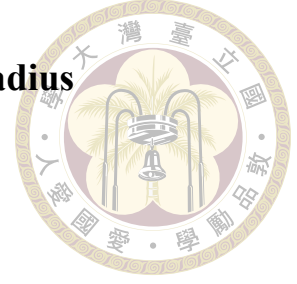
Figure 4.10 displays the HR diagram for another subset of $25 M_{\odot}$ models at $Z = 0.001$, this time with fixed opacity ($\zeta_{\text{opacity}} = 0.001$, the standard value for this metallicity) and varying η_{CNO} . In this case, the ZAMS positions are not significantly affected by changes in η_{CNO} , but the subsequent evolutionary tracks exhibit a systematic shift in T_{eff} across different values of η_{CNO} . Throughout the main-sequence phase and up to the TAMS, higher values of η_{CNO} result in lower T_{eff} , while the luminosities remain similar. Consequently, stars with enhanced CNO-cycle rates attain larger radii by the TAMS.

These results show that during the main-sequence phase, both increased opacity and a higher CNO-cycle reaction rate shift the stellar evolutionary tracks toward cooler effective temperatures and larger radii on the HR diagram. At higher metallicities, where both the opacity and the CNO-cycle rate are elevated, stars are thus expected to develop systematically larger TAMS radii.

To examine the cumulative effect of metallicity on TAMS radius, Figure 4.11 presents evolutionary tracks of $25 M_{\odot}$ models with varying metallicities from Grid (a). While the ZAMS positions show a relatively irregular distribution across different Z , the subsequent evolutionary tracks exhibit a more systematic trend toward the TAMS. In general, higher-metallicity models tend to reach lower luminosities but also lower T_{eff} at the TAMS, resulting in systematically larger stellar radii. This trend reflects the combined influence of metallicity on stellar evolution through its effects on both opacity and nuclear burning.



4.5.2 Homology Relation between Metallicity and Radius



The effects of opacity and nuclear energy generation rates on stellar radius can be understood through homology relations (see, e.g., [Kippenhahn et al. 2012](#); [Maeder 2009](#)). The central idea of the homology method is to compare two stars with similar internal structures. At a given relative mass coordinate, defined as $\xi = m/M$, where M is the total stellar mass and m is the local mass coordinate, the fundamental physical quantities in the two stars can be transformed with simple scaling constants. In the analysis below, we simply use the global stellar parameters such as M or the average values within the entire star to express the homology relations to avoid complexity.

The four differential equations governing the stellar structure can be transformed into four homology relations. The mass conservation equation yields

$$M \sim \rho R^3, \quad (4.2)$$

where M is the total stellar mass, ρ is the mean density, and R is the stellar radius. The hydrostatic equilibrium equation gives

$$P \sim M^2 R^{-4}, \quad (4.3)$$

where P is the central pressure. The energy conservation leads to

$$L \sim \epsilon M, \quad (4.4)$$

where L is the luminosity and ϵ is the average energy generation rate per unit mass. The

radiative energy transport equation is:

$$\frac{dT}{dm} = -\frac{3}{64\pi^2 a c} \frac{\kappa l}{r^4 T^3}, \quad (4.5)$$



where T is the temperature, κ is the opacity, l is the local luminosity, a is the radiation density constant, and c is the speed of light. This equation results in the fourth homology relation

$$R \sim (\kappa L M)^{1/4} T^{-1}. \quad (4.6)$$

Combining Equations (4) and (6), the two relations associated with energy, we obtain

$$RT \sim (\kappa \epsilon)^{1/4} M^{1/2}. \quad (4.7)$$

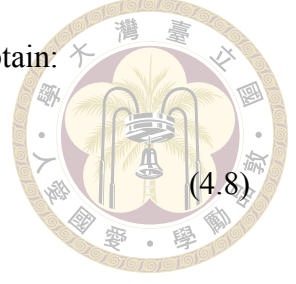
Thus, at fixed stellar mass, increasing either ϵ or κ leads to a larger product RT . Since both ϵ and κ depend on the metallicity Z , variations in Z can significantly affect the stellar structure.

To derive an explicit expression of R as a function of Z , we adopt additional assumptions for the gas properties in a star. Following [Maeder \(2009\)](#), we assume:

- An ideal gas equation of state: $\rho = \frac{\mu m_H}{k} P T^{-1}$, where μ is the mean molecular weight, m_H is the mass of a H atom, and k is the Boltzmann constant.
- Kramers' opacity law: $\kappa = \kappa_0 \rho T^{-3.5}$, where κ_0 is a coefficient that depends on its composition;
- A power-law form for nuclear energy generation rate: $\epsilon = \epsilon_0 \rho T^\nu$, where ϵ_0 is a coefficient that depends on its composition, and the exponent ν depends on the dominant nuclear reaction. For the CNO cycle, $\nu = 17$.

Substituting the expressions for κ and ϵ into Equation (7), we obtain:

$$R^{5/2} \sim (\kappa_0 \epsilon_0)^{1/4} T^{\frac{\nu-7.5}{4}} M. \quad (4.8)$$



To eliminate T , we use a scaling relation derived by combining mass conservation, hydrostatic equilibrium, and the ideal gas law:

$$T \sim \frac{\mu M}{R}. \quad (4.9)$$

Substituting this expression for T into Equation (7) yields:

$$R \sim (\kappa_0 \epsilon_0)^{\frac{2}{2\nu+5}} \mu^{\frac{2\nu-15}{2\nu+5}} M^{\frac{2\nu-7}{2\nu+5}} \quad (4.10)$$

For the CNO cycle with $\nu = 17$, this relation becomes:

$$R \sim (\kappa_0 \epsilon_0)^{0.051} \mu^{0.49} M^{0.69}. \quad (4.11)$$

The three quantities κ_0 , ϵ_0 , and μ all depend on Z . We now express the Z -dependence of each term. For Kramers opacity, which includes both free-free and bound-free absorption, the coefficient can be approximated as

$$\kappa_0 \sim (1 + 10^3 Z). \quad (4.12)$$

When $Z \gg 10^{-3}$, bound-free absorption dominates and thus $\kappa_0 \sim Z$; when $Z \ll 10^{-3}$, free-free absorption dominates, and thus the opacity becomes largely independent of Z . Near the critical metallicity $Z \sim 0.001$ for RSG formation, both contributions are comparable. The coefficient ϵ_0 for the burning of the CNO cycle is proportional to the mass fraction of carbon, nitrogen, and oxygen, and we approximate it as $\epsilon_0 \approx Z$. For a fully

ionized ideal gas, the mean molecular weight is approximated by:

$$\mu \simeq \frac{4}{3 + 5X - Z} \quad (4.13)$$



Assuming a fixed He mass fraction $Y = 0.278$ in our models, the H mass fraction becomes $X = 0.722 - Z$, and thus:

$$\mu^{-1} \simeq \frac{6.61 - 6Z}{4} \quad (4.14)$$

. Substituting these Z dependencies into Equation (11), we obtain:

$$R \sim [Z(1 + 10^3 Z)]^{0.051} (1.1 - Z)^{-0.49} M^{0.69}. \quad (4.15)$$

We define the coefficient dependent on Z as $C_Z \equiv [Z(1 + 10^3 Z)]^{0.051} (1.1 - Z)^{-0.49}$, with contributions from nuclear burning $C_\epsilon \equiv Z^{0.051}$, opacity $C_\kappa \equiv (1 + 10^3 Z)^{0.051}$, and mean molecular weight $C_\mu = (1.1 - Z)^{-0.49}$. The values of these coefficients as functions of Z are plotted in Figure 4.12. At $Z = 2 \times 10^{-2}$, the value of C_Z is ~ 1.7 times greater than at $Z = 10^{-5}$. Although this difference is somewhat smaller than that seen in the TAMS radii between these two metallicities in Figure 4.8 ($\sim 2 - 3$ times difference between $Z = 2 \times 10^{-2}$ and $Z = 10^{-5}$), the expression nonetheless captures the general trend of increasing the radius with metallicity.

Among the three components, the contribution from mean molecular weight is relatively mild, as $(1.1 - Z)^{-0.49}$ remains close to unity and varies only slightly across the metallicity range considered. For $Z \lesssim 10^{-3}$, the C_κ also stays near unity, making C_ϵ the dominant contributor to C_Z . At higher metallicities ($Z \gtrsim 10^{-3}$), the contribution from opacity becomes more significant, while C_ϵ continues to play a role. This trend is consistent with the results from Grid (b) in Figure 4.1 and Grid (c) in Figure 4.2, which show that

stellar radii become sensitive to opacity only when $\zeta_{\text{opacity}} \gtrsim 10^{-3}$. Here, ζ_{opacity} serves as a pseudo-metallicity parameter used for interpolating opacities in the opacity tables, effectively mimicking the metallicity dependence of opacity. Overall, the increase in stellar radius with metallicity during the main sequence is primarily due to enhanced opacity and more efficient energy generation through the CNO cycle.

We now examine the mass dependence of the stellar radius at fixed metallicity. From the homology relation, we obtain $R \sim M^{0.69}$ for the CNO cycle. This can be compared to the threshold radius for RSG formation across different stellar masses, as shown in Figure 4.8, which follows a rough power-law scaling of $R \sim M^{0.75}$. Although this exponent 0.75 is only a coarse fit, it matches closely to the exponent 0.69 of the constant-metallicity relation. This similarity indicates that the mass dependence of the threshold radius for RSG formation is closely aligned with mass dependence of stellar radius at constant metallicity. Consequently, the existence of a nearly constant critical metallicity at $Z \sim 0.001$ across a range of masses can be well justified.

4.6 Classification of Supergiant Evolution Pathways

To understand the origin of the threshold radius at TAMS for the formation of RSG, we draw on the evolutionary pathways proposed in Chapter 2. In that study, we identified two pathways through which stars evolve into the RG/RSG phase, both governed by a refined form of the “mirror principle,” wherein the stellar envelope expands or contracts in the opposite direction of the shell-envelope boundary. Initially, as the shell-envelope boundary recedes inward during core contraction, the envelope expands accordingly. In some models, the envelope expands into the RG/RSG regime during this phase—this

represents the first evolutionary pathway to RG/RSG.

After core contraction ends, the nuclear energy generation rate (L_{nuc}), which includes contributions from shell H burning and core He burning, dominates the energy production of the stellar interior. Although L_{nuc} dominates the energy source during this phase, it is already in decline. As L_{nuc} decreases, the shell-envelope boundary continues to move inward, causing the envelope to expand further. If L_{nuc} subsequently increases after reaching a minimum, the envelope reverses and contracts, and the star remains a BSG. If, on the other hand, L_{nuc} continues to decline monotonically, the envelope continues to expand into the RSG regime, this represents the second pathway to the evolution of RG / RSG.

Based on these evolutionary pathways, we classify each massive star model in Grid (a). In addition to the above scenarios, we identify an additional evolutionary type that occurs in stars with initial masses $\gtrsim 30 M_{\odot}$ and metallicities $Z \lesssim 0.001$. In these cases, the C core begins to form while the He core is still contracting. The envelope may or may not expand further after the onset of core C burning, and this complex evolution in the final stages lies beyond the scope of this study. As our classification focuses on the core He-burning stage, we categorize these stars as BSGs on the basis of their status before the formation of the C core.

Combining all cases, we classify the core He-burning evolution of massive stars into four distinct types:

- (1) **RSG-C**: Core contraction directly leads to expansion into the RSG regime.
- (2) **RSG-L**: The luminosity of nuclear burning (L_{nuc}) undergoes a sustained decline after the core contraction, driving a continued expansion of the envelope in the RSG regime.

- (3) **BSG-L**: The luminosity of nuclear burning (L_{nuc}) increases shortly after core contraction, causing the envelope to contract, and the star remains a BSG.
- (4) **BSG-E**: Early C core formation occurs during ongoing He core contraction; the star remains in the BSG phase at this time.



To perform the classification, we examine the evolutionary tracks of each model using the approach described in Section 2.4 of Chapter 2. We track the evolution of the stellar radius (R_*), central pressure (P_c), and the nuclear energy generation rate (L_{nuc}) from the formation of the He core to the formation of the C core. Within this time interval, we determine the maximum stellar radius (R_{max}) and the maximum central pressure (P_{max}), along with the corresponding stellar radius at the time of P_{max} , denoted $R_{P_{\text{max}}}$.

If $R_{P_{\text{max}}}$ and R_{max} differ by less than 1%, this implies that the maximum radius is reached while the central pressure is still increasing, indicating that the core is contracting at that time. For stars that satisfy this condition and have an effective temperature $\log(T_{\text{eff}}/\text{K}) < 3.8$ at R_{max} , we classify them as **RSG-C**.

If $R_{P_{\text{max}}}$ occurs significantly earlier than R_{max} , it indicates that the envelope continues to expand after the core contraction ends. In such cases, we identify the minimum L_{nuc} following $R_{P_{\text{max}}}$ and define the corresponding stellar radius at this time as $R_{L_{\text{min}}}$. If $R_{L_{\text{min}}}$ and R_{max} differ by less than 5%, we classify the model as **RSG-L** if $\log(T_{\text{eff}}/\text{K}) < 3.8$ at R_{max} , and as **BSG-L** if $\log(T_{\text{eff}}/\text{K}) \geq 3.8$ at R_{max} .

For low-metallicity models, we identify 60 cases where the star remains a BSG throughout core He burning and exhibits $R_{P_{\text{max}}} \sim R_{\text{max}}$. These cases cannot be reliably classified using automated criteria. We therefore assign classifications manually based on the evolution of R_* and L_{nuc} . Models that exhibit envelope contraction when L_{nuc} reaches

the minimum and subsequently rises are classified as **BSG-L**, while those that continue expanding at the time of C core formation are classified as **BSG-E**.

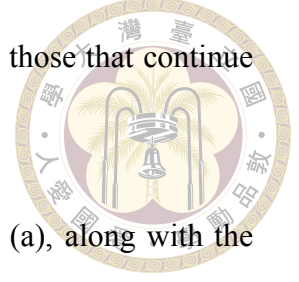


Figure 4.13 illustrates the classification of each model in Grid (a), along with the corresponding $R_{\max}$ values. For high-metallicity stars with $Z \gtrsim 0.01$, nearly all stars with initial masses $\geq 25 M_{\odot}$ directly expand into the RSG regime during core contraction, via the **RSG-C** pathway. For metallicities in the range $Z \sim 0.001 - 0.01$, near the critical metallicity ($Z_c \sim 0.001$), as well as for stars with $< 25 M_{\odot}$ and higher metallicity, many models evolve into RSGs via the **RSG-L** pathway, while others still follow the **RSG-C** pathway. At low metallicities ($Z \lesssim Z_c \sim 0.001$), stars generally remain as BSGs but separate into two categories: stars with $< 30 M_{\odot}$ are mostly **BSG-L**, while those with $M \gtrsim 30 M_{\odot}$ tend to be **BSG-E**.

The critical metallicity for cool supergiant formation presents at $Z \sim 0.001$ across the full mass range of $11 - 60 M_{\odot}$. However, the boundary between the RSG and BSG regimes is not defined by a single transition between two evolutionary subtypes; rather, it is composed of multiple subtype boundaries that vary with stellar mass. For stars with initial masses below $30 M_{\odot}$, the critical metallicity corresponds to the transition between the **RSG-L** and **BSG-L** cases, which is well defined at $Z \sim 0.001$. The physical mechanisms governing this bifurcation have been thoroughly studied in Chapter 2. This mass range ($11 - 30 M_{\odot}$) corresponds to the conventional mass range of red supergiants (Beasor et al. 2025; Meynet & Maeder 2000), many of which are expected to end their lives as core-collapse supernovae (Smartt 2009).

In contrast, for stars with $M \gtrsim 30 M_{\odot}$, the critical metallicity corresponds to the transition between **RSG-C** and **BSG-E** cases. This boundary is less sharply defined, and the

evolutionary behavior in this mass regime becomes more complex. We return to discuss the nature of these **BSG-E** models at the end of Section 4.7.



4.7 Envelope Expansion and Core Condition

To investigate the physical origin of the threshold radius for RSG formation, we examine a set of models with varying values of η_{CNO} and ζ_{opacity} , focusing on the evolution of their core properties. Figure 4.14 presents the time evolution of central pressure (P_c) along with the stellar radius (R_*), illustrating the relationship between the evolution of the core and the expansion of the envelope. The models are sorted by R_{TAMS} in the legend, rather than by the individual parameter values of η_{CNO} or ζ_{opacity} .

All the models in Figure 4.14 exhibit a common evolutionary trend: P_c increases significantly and then temporarily stabilizes before continuing to rise. This behavior corresponds to the contraction and stabilization of the He core. During this stages, the envelope expands to BSG or RSG radii, leading to an increase in R_* . While the overall trend is consistent across models, the tracks are systematically offset in R_* between models. Crucially, this shift aligns with the ordering of R_{TAMS} rather than the input parameters η_{CNO} or ζ_{opacity} . In general, models with smaller R_{TAMS} reach the end of the core contraction and reach a peak P_c at smaller stellar radii.

As shown earlier, the TAMS radius is a strong indicator of whether a star will evolve into the RSG phase. We now investigate the physical connection between the TAMS stellar structure and the subsequent evolution into a supergiant. The results in Figure 4.14 suggest that the core evolution from the TAMS to the supergiant stage may bridge this gap. Stars with smaller TAMS radii undergo core contraction and stabilization at smaller stellar

radii and tend to remain BSGs. In contrast, stars with larger TAMS radii experience core contraction at larger radii and subsequently succeed in expanding to RSG dimensions.

To understand how the shift in envelope size corresponding to a given stage of core evolution influences the fate of supergiants, we refer to the criterion for RSG formation. As discussed in Section 4.6, for stars with initial masses below $30 M_{\odot}$, the critical metallicity marks the boundary between the RSG-L and BSG-L regimes. In these cases, whether a star eventually becomes an RSG is primarily governed by the behavior of L_{nuc} .

To examine the evolution of L_{nuc} in stars with different TAMS radii, Figure 4.15 presents this evolution for the same set of models shown in Figure 4.14. These $25 M_{\odot}$ models follow the typical evolutionary paths of the RSG-L and BSG-L regimes. The bifurcation occurs during the BSG phase: if L_{nuc} begins to rise again, envelope expansion halts, and the star remains a BSG. Conversely, if L_{nuc} continues to decline, the envelope expands further, and the star evolves into the RSG phase.

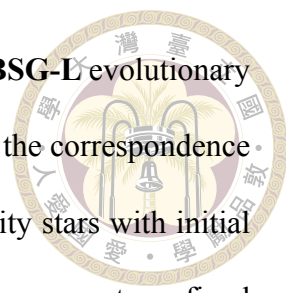
In the same figure, we also plot the individual contributions from the luminosity of H (L_{H}) and the luminosity of the He (L_{He}). Comparing the RSG and BSG models near the peak of L_{nuc} , we find no significant differences in the rate of decline of L_{H} . However, the evolution of L_{He} reveals a marked contrast. In models that remain BSGs, L_{He} exhibits a sharp increase during the BSG phase, and this increase is responsible for the subsequent upturn in L_{nuc} . This rise in L_{nuc} causes the shell-envelope boundary to move outward. This motion, in turn, drives the envelope contraction and prevents the star from expanding into the RSG phase.

In fact, the further increase in L_{He} occurs not only in the BSG models but also in the RSG models. In Figure 4.16, we plot the evolution of P_{c} and L_{He} as functions of time

for representative RSG and BSG models. During the core He-burning phase, both models exhibit similar evolutionary trends. Initially, both P_c and L_{He} rise rapidly due to core contraction. As the core contraction concludes, P_c temporarily decreases before gradually increasing again, while L_{He} continues to rise. After reaching its peak, L_{He} steadily declines. These features are common to both models, indicating that the core evolution follows a similar trajectory, regardless of whether the star ultimately becomes a RSG or a BSG.

The key difference between these models lies in the timing of the increase in L_{He} relative to the expansion of the envelope. In RSG models, the further rise in L_{He} occurs after the star has already expanded into the RSG phase, and thus has little impact on the envelope structure. In contrast, if this increase occurs while the star is still at smaller radii, it can halt the envelope expansion and cause the star to remain in the BSG phase.

Broadly speaking, the primary difference between the models that leads to the bifurcation into RSG and BSG outcomes lies in the correspondence between the evolutionary stages of the core and the envelope. As summarized in the flow chart in Figure 4.17, the post-main-sequence evolution of the core and the envelope is not synchronized; each follows its own sequence of structural transitions. For the envelope, it initially enters the BSG phase and may subsequently expand into the RSG regime. For the core, evolution begins with contraction starting from the TAMS, followed by a decrease in L_{nuc} once the core contraction ends. Afterward, L_{He} continues to rise, which can cause an upturn in L_{nuc} . If the envelope is still in the BSG phase at this stage, the increase in L_{nuc} can lead to envelope contraction, preventing further expansion into the RSG regime. Thus, the timing of these core changes relative to the expansion state of the envelope ultimately determines whether the star evolves into a BSG or an RSG.



So far, our discussion has primarily focused on the **RSG-L** and **BSG-L** evolutionary types. Nevertheless, the **BSG-E** type can also be understood through the correspondence between the evolution of the core and the envelope. In low-metallicity stars with initial masses $\gtrsim 30 M_{\odot}$, which fall into the BSG-E type, the envelope remains compact, confined within the BSG regime, while the core not only undergoes He burning, but has already advanced to the core C-burning stage. Once a star enters the core C burning, its evolution becomes significantly more complex compared to the earlier stages. As shown in Figure 1.8 of Chapter 1, the evolutionary outcomes of low-metallicity models beyond the onset of core C burning become highly irregular. The fundamental reason these stars evolve into the **BSG-E** type is that their small envelope radii correspond to an advanced core evolutionary stage.

As summarized in Figure 4.17, the four evolutionary pathways originates from different correspondence between core condition and envelope radius. The **RSG-C** pathway is followed by stars with large TAMS radii, such that even modest expansion during core contraction is sufficient for the star to enter the RSG phase. If the TAMS radius is smaller than that of **RSG-C** cases but still exceeds the threshold radius for RSG formation, the star follows the **RSG-L** pathway. In this case, the envelope expands to the BSG regime when core contraction ends and continues expanding toward the RSG phase as the H-burning shell moves inward, driven by the continued decline of L_{nuc} . We note that once a star reaches the RSG phase, it stabilizes there and is no longer affected by subsequent changes in the core during advanced He burning.

For stars $\lesssim 30 M_{\odot}$, if the TAMS radius is below the threshold for RSG formation, the star follows the **BSG-L** pathway. For this type, the envelope expands to the BSG radius when the core enters the advanced He-burning stage with further rise of L_{He} , and thus

the upturn of L_{nuc} . The increase of L_{nuc} shifts the H-burning shell outward and causes the envelope to contract, resulting in the star remaining as a BSG. Finally, the **BSG-E** pathway applies to more massive stars ($\gtrsim 30 M_{\odot}$) with TAMS radii below the threshold of RSG formation. They remain compact and enter core C burning while the He core is still contracting.

4.8 Discussions and Conclusions

Stellar models indicate a critical metallicity of $Z_{\text{c}} \sim 0.001$ that determines whether a star successfully evolves into the RSG phase. In this study, we provided a comprehensive physical explanation for the origin of this critical metallicity. Through experiments with $25 M_{\odot}$ models in which we vary opacity and nuclear reaction rates, we identify a threshold TAMS radius that predicts whether a star will expand into the RSG regime. Furthermore, we demonstrate that the mass dependence of this threshold radius is similar to the mass dependence of stellar radius at fixed metallicity, which naturally explains why the critical metallicity for RSG formation remains approximately constant at $Z \sim 0.001$ across a wide range of stellar masses.

Metallicity significantly influences stellar evolution through its effects on opacity, nuclear reaction rates, and the mean molecular weight. Based on evolutionary tracks from stellar models, along with dimensional analysis using homology relations, we have shown that stars with higher metallicities tend to develop larger radii upon entering the post-main-sequence stage. This behavior is primarily driven by enhanced opacity and increased CNO-cycle energy generation at higher Z . As metallicity affects the TAMS radius, and the TAMS radius set a threshold for RSG formation, metallicity ultimately determines

whether a star expands into the RSG regime during post-main-sequence evolution.

We further identify the physical mechanism linking the TAMS radius to supergiant outcomes. Generally, the envelope radius corresponding to a given core evolution stage is the key that leads to bifurcation in supergiant evolution. For stars with initial masses in the range $11\text{--}30 M_{\odot}$, the distinction between the RSG and BSG regimes is primarily governed by the post-main-sequence behavior of L_{nuc} . In this mass range, the critical metallicity marks the boundary between the **RSG-L** and **BSG-L** types. If $Z < Z_c$, the star has a TAMS radius smaller than the threshold, and the envelope remains compact when the continued rise of L_{He} causes an upturn in L_{nuc} . This upturn drives the shell-envelope boundary outward, triggering envelope contraction and preventing expansion into the RSG regime. In contrast, if a star has $Z > Z_c$, its TAMS radius is larger than the threshold, so L_{nuc} continues to decline after the core contraction ends, allowing the envelope to continue expanding into the RSG regime. In these cases, the subsequent rise in L_{He} occurs after the star has already reached the RSG phase, and therefore has little impact on the envelope structure.

For more massive stars ($\gtrsim 30 M_{\odot}$), the threshold metallicity for RSG formation is less sharply defined but still lies near $Z \sim 0.001$, set by the boundary between the **RSG-C** and **BSG-E** regimes. In these low-metallicity stars, the envelope remains compact during the BSG phase, while the core evolves beyond core He burning into the core C-burning stage. After this point, stellar evolution becomes highly irregular, and whether the envelope further expands varies significantly from case to case.

In addition to metallicity, other factors, such as rotation, can also influence stellar evolutionary tracks on the HR diagram starting from the main-sequence stage (e.g.,

[Maeder 2009](#)). These effects may alter the envelope radius corresponding to a given stage of core evolution. In particular, the TAMS radius can be affected and potentially shifted above or below the threshold required for evolution into the RSG phase. In this context, one may expect not only a critical metallicity but also a critical rotation velocity, or threshold values in other physical parameters that determine whether a star evolves into an RSG.

From stellar models, we have identified the physical mechanism that causes low-metallicity massive stars to remain blue rather than evolving into the RSG phase. These distinct evolutionary pathways at low metallicity have important implications for the nature of stellar populations and feedback in the early universe. For example, since low-metallicity stars are less likely to evolve into RSGs, a typical progenitor class for core-collapse supernovae, the resulting supernova types in metal-poor galaxies may differ significantly from those observed in the local universe. Future observations targeting galaxies with metallicities as low as $\sim 0.1 Z_{\odot}$ will be crucial for testing these predictions.

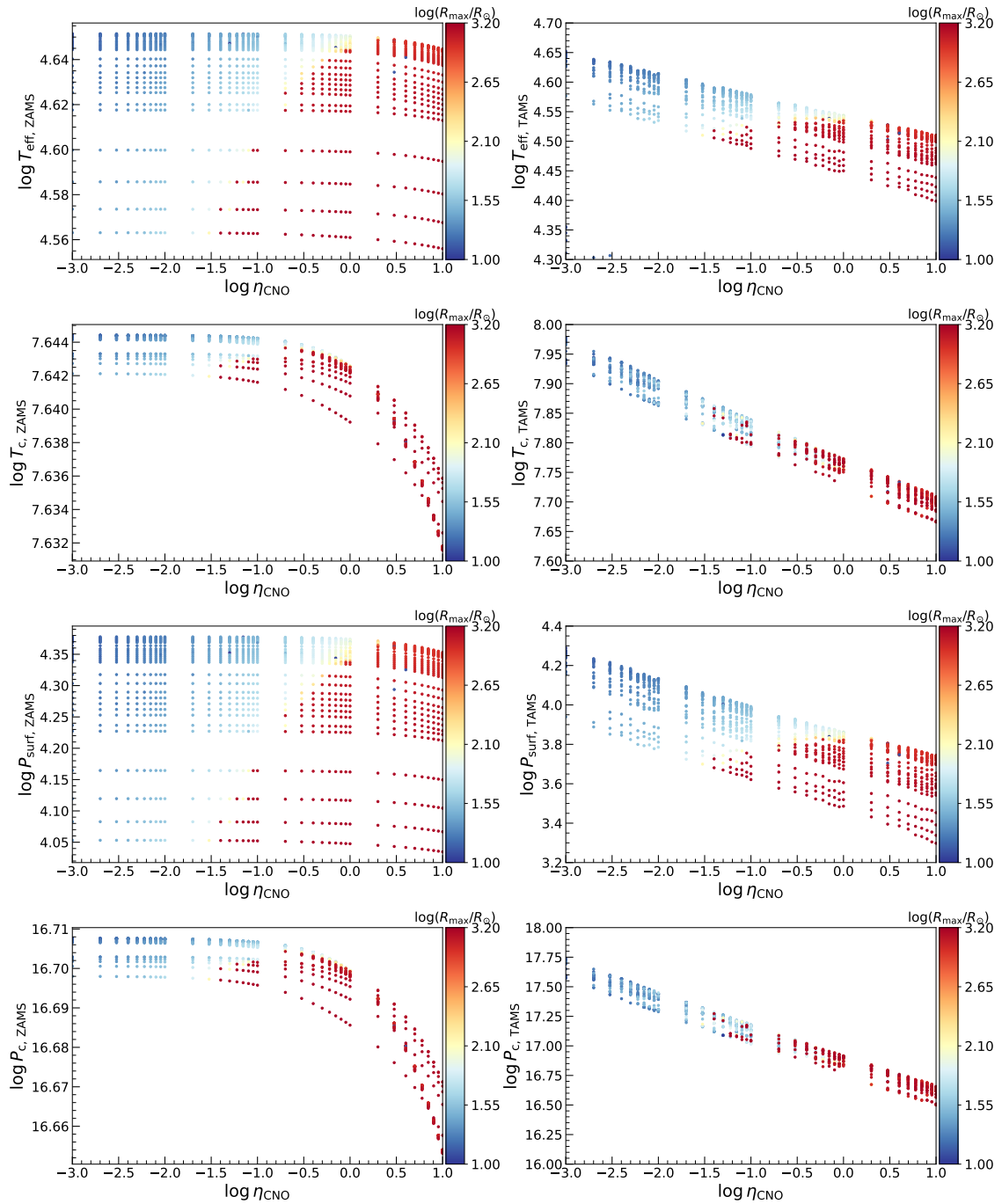
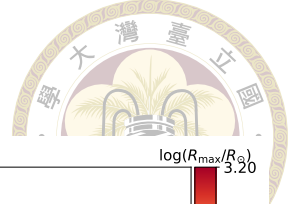


Figure 4.5: Effective temperature (T_{eff}), central temperature (T_c), surface pressure (P_{surf}), and central pressure (P_c) at both the ZAMS and TAMS for $25 M_{\odot}$ models with $Z = 0.001$ from Grid (c), where the CNO cycle energy generation rate is scaled by η_{CNO} and the opacity parameter by ζ_{opacity} . In each panel, models are plotted along the η_{CNO} axis, and the color of each point represents the maximum stellar radius (R_{max}) reached during the core He-burning phase, with red indicating RSG-scale radii and blue indicating BSG-scale radii.

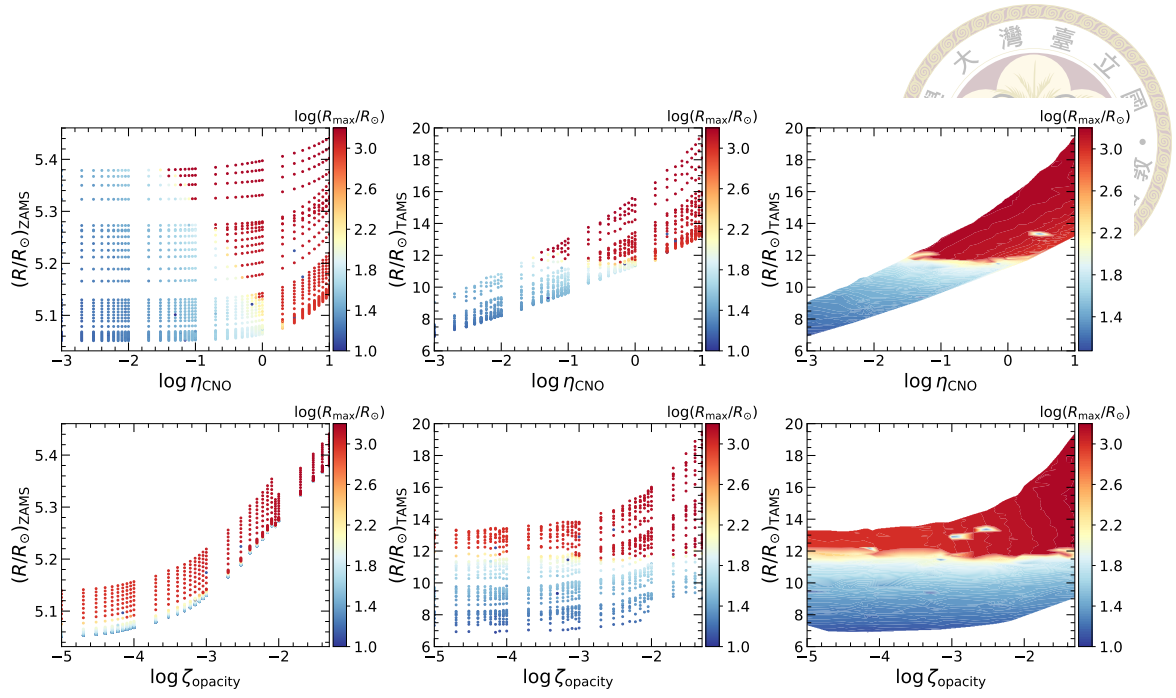


Figure 4.6: ZAMS radii (*left panels*) and TAMS radii (*middle and right panels*) for $25 M_{\odot}$ models with $Z = 0.001$ from Grid (c), systematically varying the CNO cycle energy generation rate (scaled by η_{CNO}) and the opacity parameter (ζ_{opacity}). The top row shows models arranged along η_{CNO} , and the bottom row along ζ_{opacity} . In each panel, colors indicate the maximum radius reached during the core He-burning phase (R_{max}), with red corresponding to RSG sizes and blue to BSG sizes. The middle panels display individual models as scattered points at TAMS, while the right panels present smoothed contour plots using the same dataset. Notably, models with TAMS radii $\gtrsim 12 R_{\odot}$ predominantly evolve into RSGs, whereas those below this threshold remain BSGs.

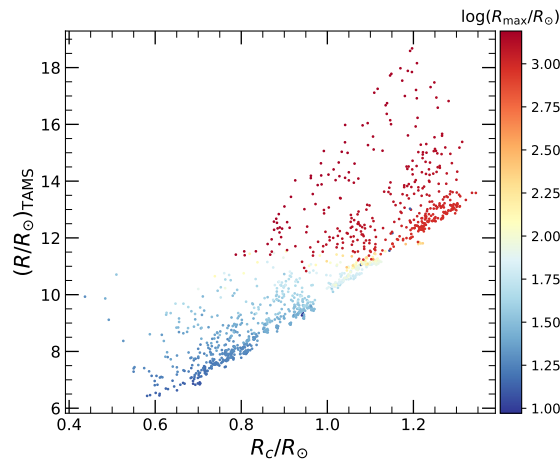


Figure 4.7: Distribution of TAMS radii and He core radii at its formation (R_c), with colors indicating the maximum radius during core He burning (R_{max}). The boundary between RSG and BSG models aligns closely with a horizontal line of constant TAMS radius, rather than with the He core radius.

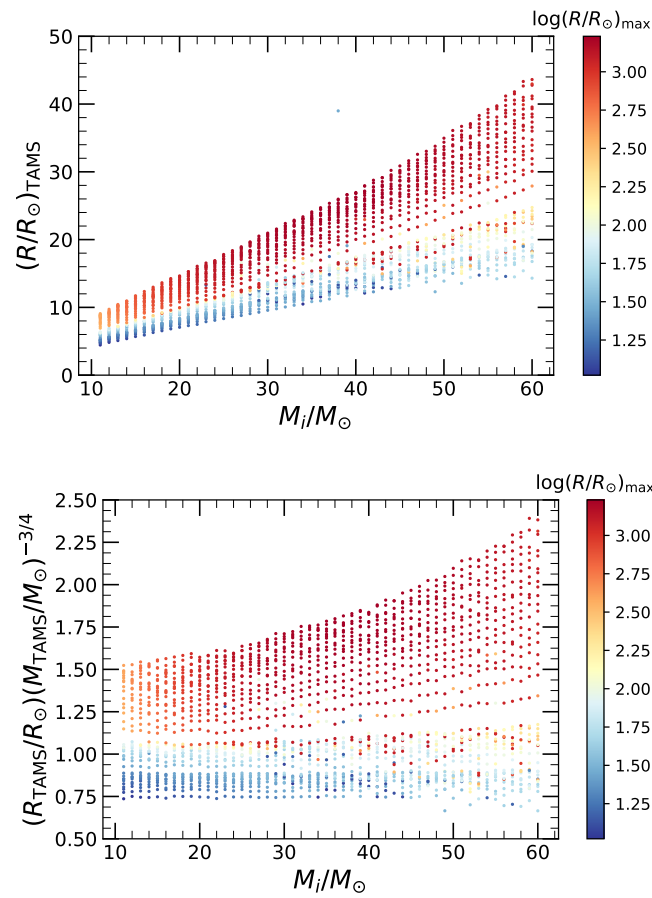


Figure 4.8: The relationship between the TAMS radius and initial stellar mass (M_i), with the colors of the data points representing R_{max} .

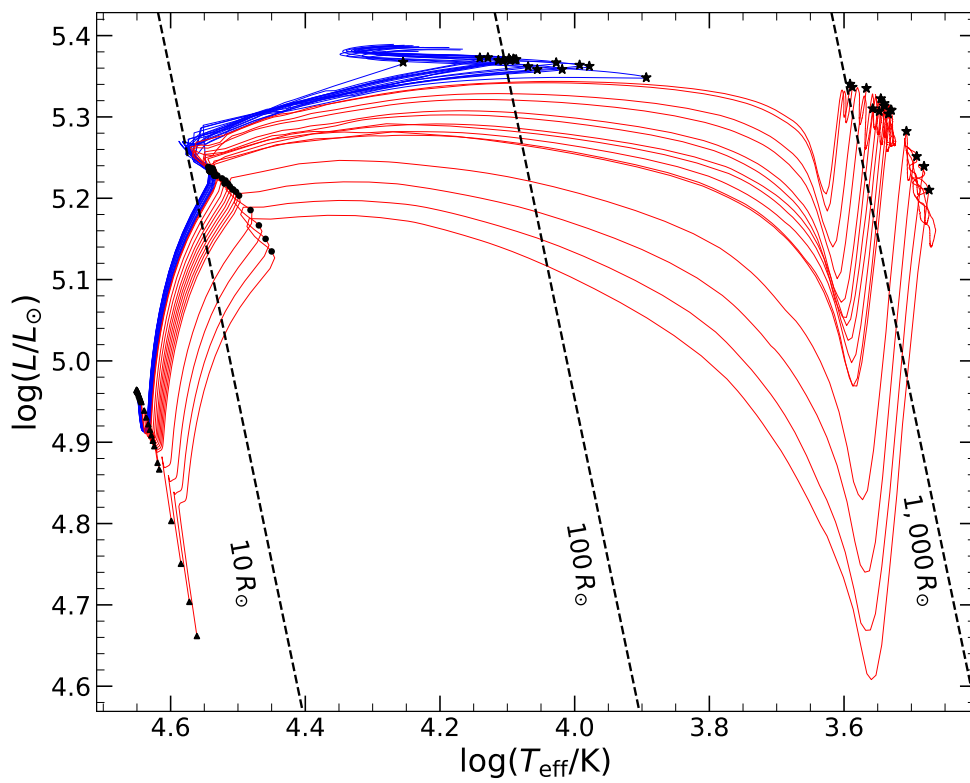


Figure 4.9: Evolutionary tracks of $25 M_{\odot}$, $Z = 0.001$ stars with different ζ_{opacity} on the HR diagram. Dashed lines indicate constant-radius contours at 10 , 100 , and $1000 R_{\odot}$. Black triangles mark the ZAMS, black circles indicate the TAMS, and black stars denote the maximum stellar radius reached during the core He-burning phase. Tracks that evolve into RSGs are shown in red, while those that remain BSGs are shown in blue.

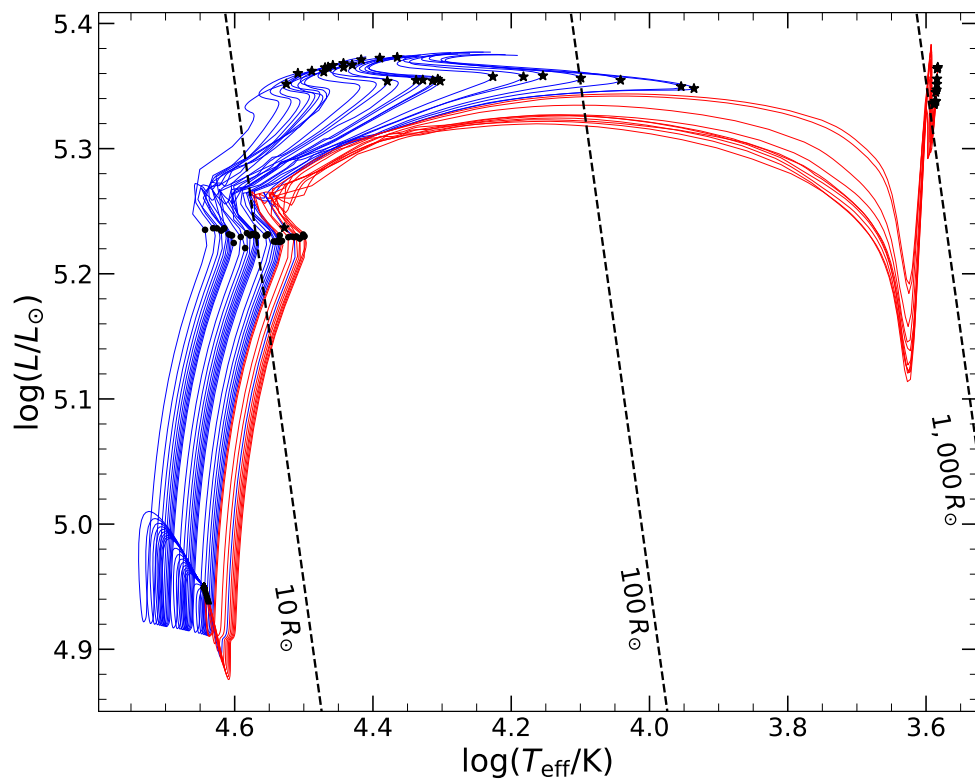


Figure 4.10: Evolutionary tracks of $25 M_{\odot}$, $Z = 0.001$ stars with different η_{CNO} on the HR diagram. Dashed lines indicate constant-radius contours at 10 , 100 , and $1,000 R_{\odot}$. Black triangles mark the ZAMS, black circles indicate the TAMS, and black stars denote the maximum stellar radius reached during the core He-burning phase. Tracks that evolve into RSGs are shown in red, while those that remain BSGs are shown in blue.

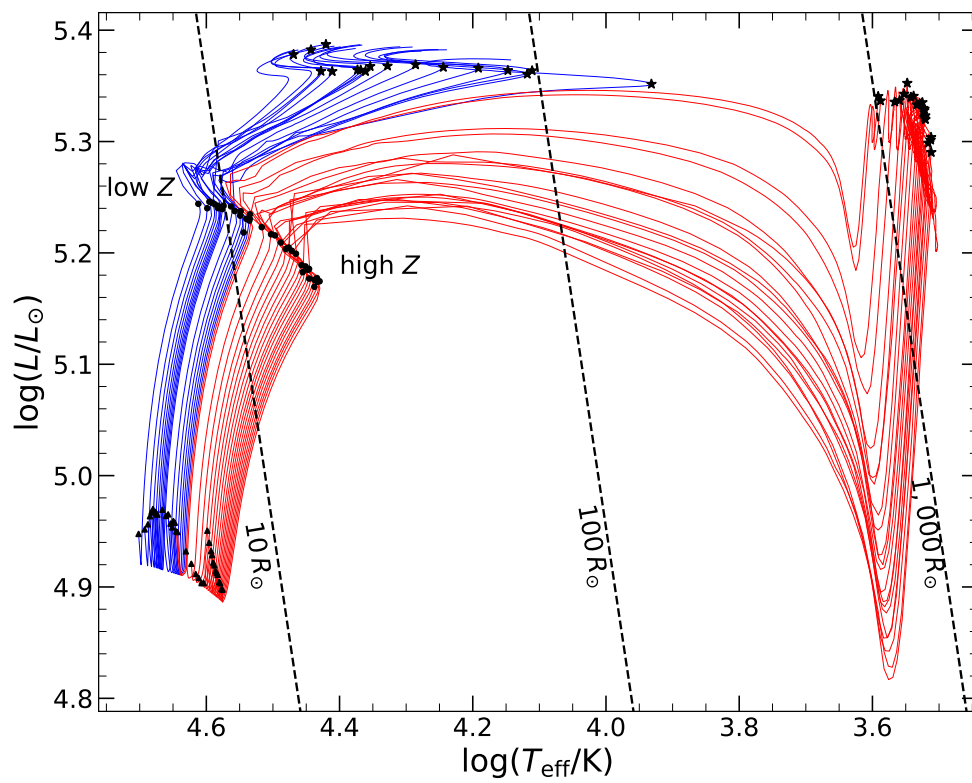


Figure 4.11: Evolutionary tracks of $25 M_{\odot}$ stars with different metallicities on the HR diagram. Dashed lines indicate constant-radius contours at 10, 100, and $1000 R_{\odot}$. Black triangles mark the ZAMS, black circles indicate the TAMS, and black stars denote the maximum stellar radius reached during the core He-burning phase. Tracks that evolve into RSGs are shown in red, while those that remain BSGs are shown in blue.

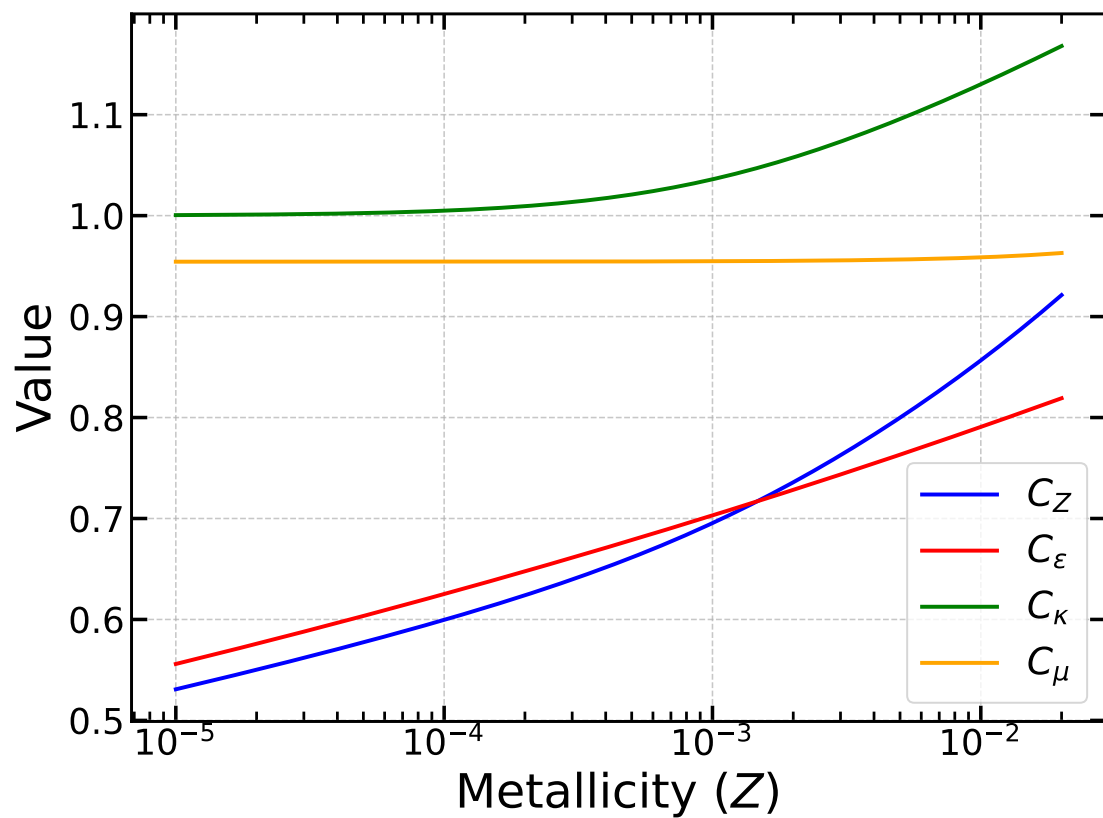


Figure 4.12: The coefficient of radius C_Z as a function of metallicity, along with the individual contributions from nuclear energy generation (C_ϵ), opacity (C_κ), and mean molecular weight (C_μ), as derived from homology relations.

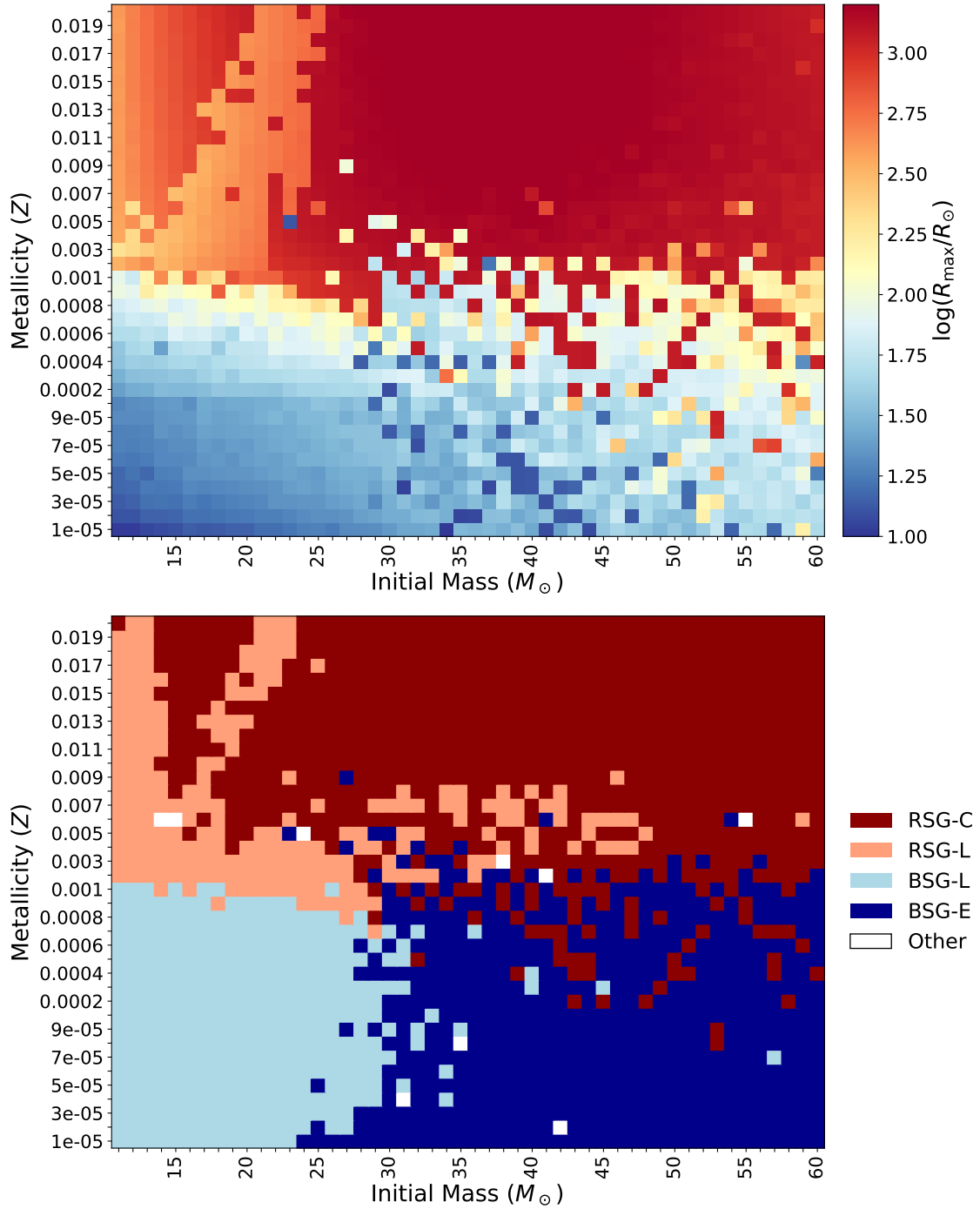
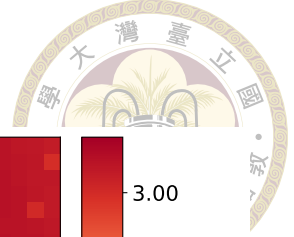


Figure 4.13: *Top panel:* Maximum stellar radius during the core He-burning stage ($R_{\max}$) for each model in Grid (a), spanning initial masses of 11 – 60 $M_{\odot}$ and metallicities of $Z = 10^{-5}$ – 2×10^{-2} . *Bottom panel:* Classification of each model’s evolutionary pathway into these types: **RSG-C**, **RSG-L**, **BSG-L**, **BSG-E**, and **Other**. In general, stars with initial masses $\lesssim 30 M_{\odot}$ and metallicities $Z \lesssim Z_c \sim 0.001$ follow the **BSG-L** pathway, while more massive stars ($\gtrsim 30 M_{\odot}$) with at similarly low metallicities are classified as **BSG-E**. Stars with $Z \gtrsim Z_c$ are classified as either **RSG-C** or **RSG-L**.

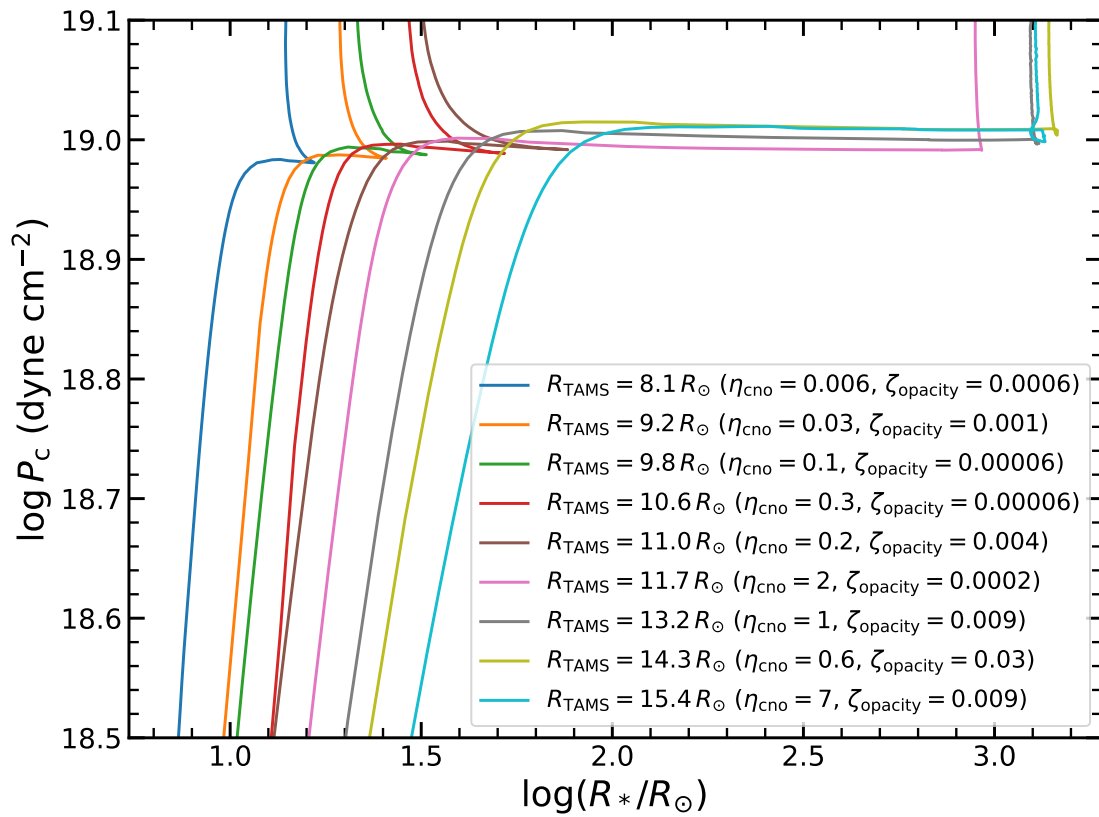


Figure 4.14: Evolution of central pressure (P_c) as a function of stellar radius (R_*) during the core He-burning stage for $25 M_{\odot}$, $Z = 0.001$ models with varying η_{CNO} and ζ_{opacity} . The legend is ordered by the TAMS radius (R_{TAMS}) of each model. Notably, the horizontal shifts in R_* among the evolutionary tracks align with R_{TAMS} , rather than with the individual input parameters η_{CNO} or ζ_{opacity} .

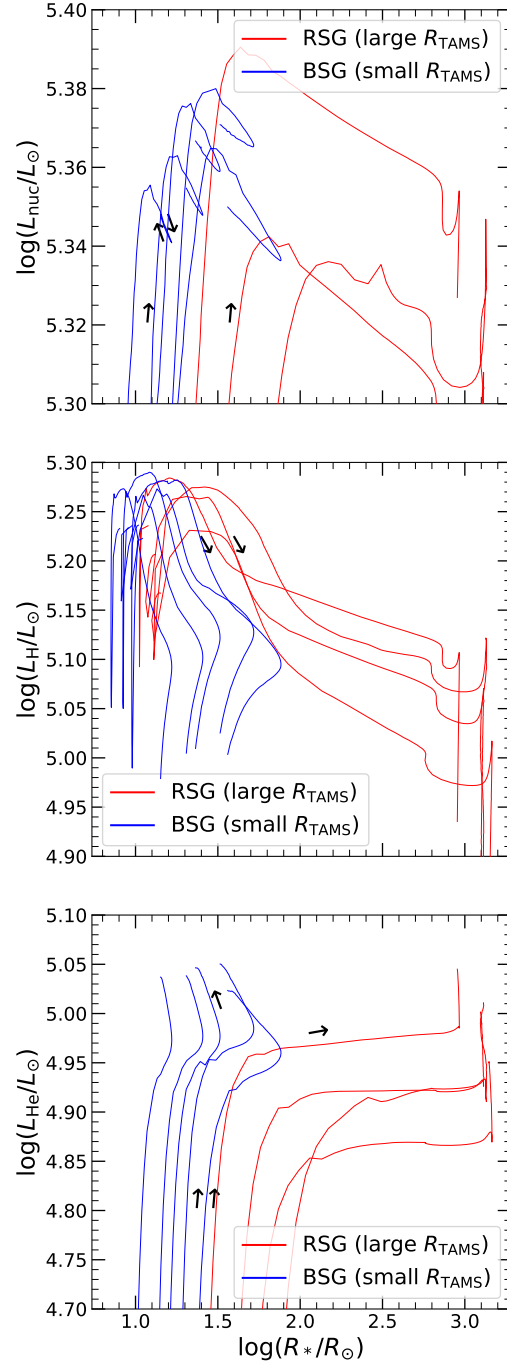


Figure 4.15: Evolution of L_{nuc} , L_{H} , and L_{He} as functions of stellar radius (R_*) for the same models shown in Figure 4.14. Red curves represent models that evolve into the RSG phase, while blue curves correspond to those that remain in the BSG phase. To avoid clutter, the curves are truncated at the point where L_{He} reaches its maximum value.

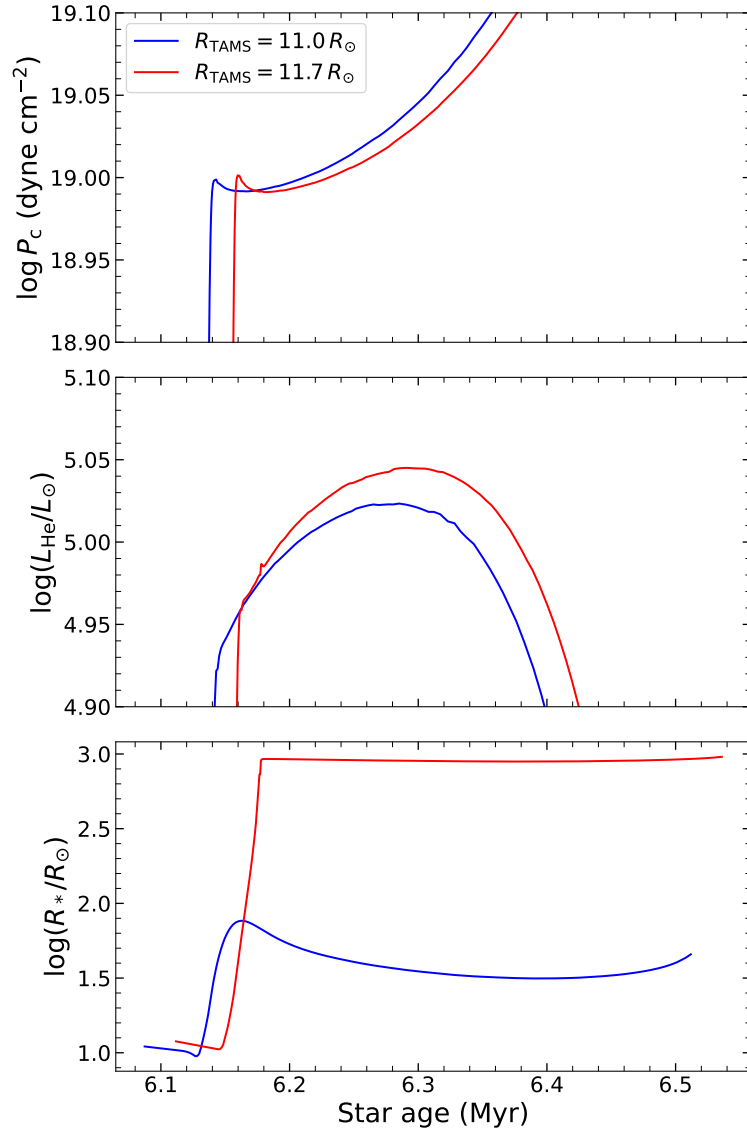


Figure 4.16: Evolution of central pressure (P_c), He-burning luminosity (L_{He}), and stellar radius (R_*) during the core He-burning stage. The red curves correspond to the model with $R_{\text{TAMS}} = 11.7 R_{\odot}$, which evolves into the RSG phase, while the blue curves represent the model with $R_{\text{TAMS}} = 11.0 R_{\odot}$, which remains in the BSG phase.

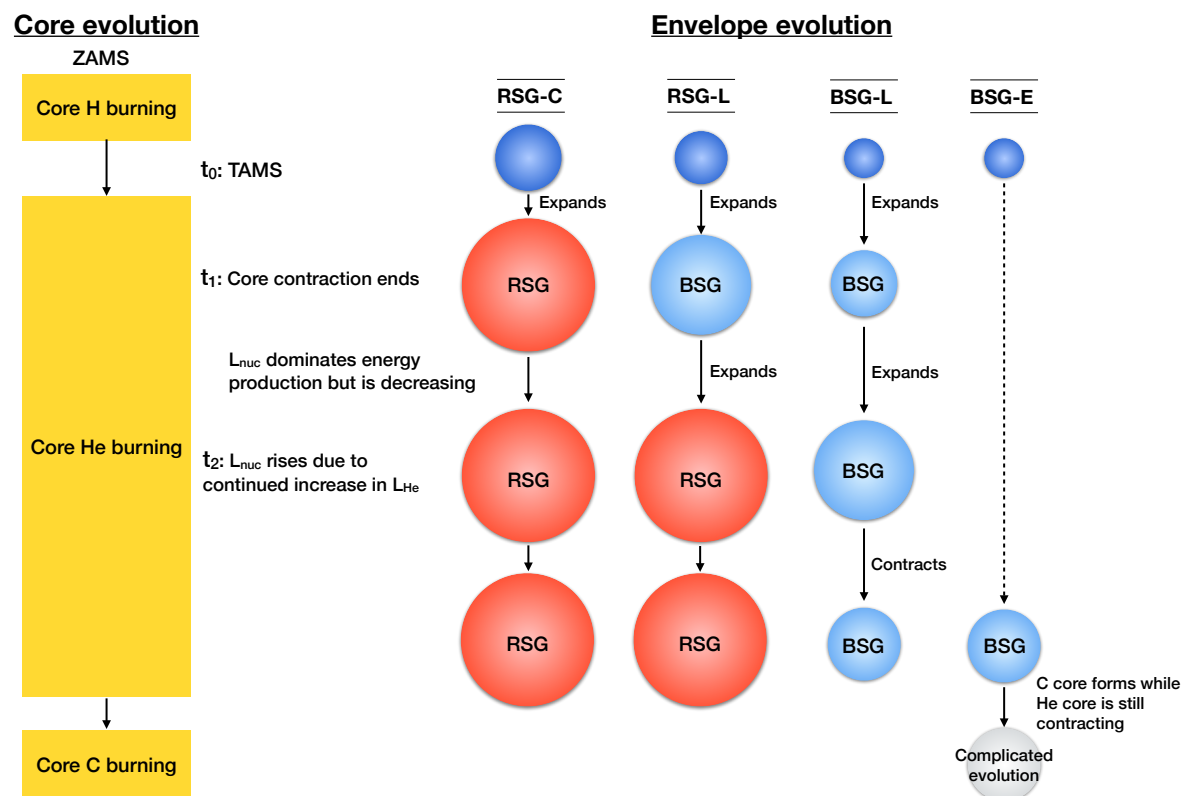


Figure 4.17: Flow chart illustrating the evolutionary stages of the stellar core and envelope. The envelope radius at the TAMS determines the evolutionary pathway and the resulting supergiant type.



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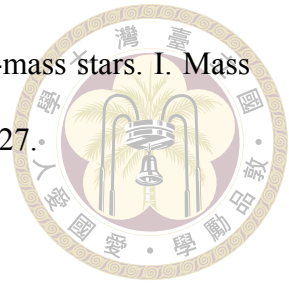
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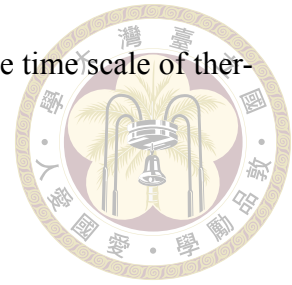
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