國立臺灣大學電機資訊學院電信工程學研究所

碩士論文

Graduate Institute of Communication Engineering
College of Electrical Engineering and Computer Science
National Taiwan University

Master's Thesis

基於資料包絡分析的夏普利值模型在警務人員配置中的應用

A DEA-based Shapley Value Model for Police Personnel Allocations

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中華民國 114 年 5 月
May, 2025

doi: 10.6342/NTU202500911

國立臺灣大學碩士學位論文
口試委員會審定書
MASTER'S THESIS ACCEPTANCE CERTIFICATE
NATIONAL TAIWAN UNIVERSITY

A DEA-based Shapley Value Model for Police Personnel Allocations
基於資料包絡分析的夏普利值模型在警務人員配置中的應用

本論文係宋蘊玟君 (P11942A10) 在國立臺灣大學電信工程學研究所完成之碩士學位論文，於民國 114 年 5 月 29 日承下列考試委員審查通過及口試及格，特此證明。

The undersigned, appointed by the Graduate Institute of Communication Engineering, National Taiwan University on 29 May 2025 have examined a Master's Thesis entitled above presented by YUNG-WEN, SUNG (P11942A10) candidate and hereby certify that it is worthy of acceptance.

口試委員 Oral examination committee:

（指導教授 Advisor）

所　長 Director:

doi: 10.6342/NTU202500911

誌謝

學界與職場，是組織目標及文化截然不同的兩個生態體系。在這三年的學術旅程中，魏宏宇教授持續鼓勵與訓練我們培養獨立思考與批判性思維的能力，這正是學界與職場間最顯著的差異之一。學校與職場兩頭轉的我，就像時常在不同溫度的泉池中切換，兩者的對比在心中激起漣漪。而也正是因為這樣的落差，更深刻體認到「獨立思辨能力」的價值與其對一個人的影響力。期許自己未來無論身處何時何地，都能牢記，讓它成為伴我一生的硬實力。

李侑融是我目前在警界遇過最清醒的朋友，在官場浮沉之中，卻能始終保持警覺、明辨是非，堅守社會價值與自我信念。若非如此，稍有閃神，便容易在這茫茫官海中隨波逐流，失去自我。也可能因為他是個水瓶座的外星人，才不易受地球的磁場滋擾，相較容易保有本心。

特別感謝貴人分局長李憲蒼──我心目中警界的明日之星、未來的警政署長，給了我很大的空間與包容，讓我能任性地多花點時間投入學業。有實力、領導能力，同時又能體恤下屬的好長官，在警界是稀世珍寶，可遇不可求，我真的非常幸運。

謝謝好同學葉晨星和劉育廷，學伴就像同我一起在海中共游的海豚，彼此鼓勵、相互扶持，即使你們先上了岸，還是不停在岸邊為我揮旗吶喊。

謝謝總是帶著濾鏡看我、無腦給予大量支持、給我滿滿情緒價值的陳梅，美貌、知性、性感與智慧兼具，幫我算紫微斗數還要當聽我告解的神父，說來本人可能不信，但善解人意梅一直是我學習的標竿，若我變得更聰明一點或是更自信一點，那一定要謝謝阿梅。

最後，謝謝我最親愛、最寶貝的家人們，謝謝你們一直都在，讓我成為地球上的快樂小精靈。

doi: 10.6342/NTU202500911

摘要

臺灣警力不足問題存在已久，而現代社會對治安需求的日益提升，進一步加重了基層警力的負擔。派出所基層警員除了維持日常勤務的正常運作以外，如遇有節慶活動或是民眾集會遊行活動等此類需要大量警力來維護公共秩序及保護社會安全的情況，警力問題更顯捉襟見肘，現有警力調度的挑戰愈加急迫。

目前警力的分配與調度大多依賴單位主管或執行業務的承辦人員的經驗判斷或主觀決策，然而此種方法是非理性的，缺乏系統化與科學性，可能導致資源分配不均或效率低下。如何引入科學化方法以妥善分配有限警力資源，降低基層警員的負擔與疲勞度，已成為台灣警界不容忽視及亟待解決的關鍵議題。

本文提出一種基於 DEA (Data Envelopment Analysis) 與 Shapley value 的模型，目的是在有限的警力資源下，對於臨時性或突發性的警力支援需求情形，進行最適化分配，將對日常勤務的影響降至最低。以單一警察局為例，警察局轄下有數個派出所，當需要警力支援的陳情抗議事件發生時，該如何科學性且合理地調度各派出所的人力？我們在此 DEA-based Shapley value model 中，首先將各派出所視為決策單元 (Decision-Making Units, DMUs)，利用 DEA 模型計算其效率分數，再透過效用函數 (Utility Function) 將效率分數轉換為代表各單位貢獻能力的效用值 (Utility Value)。接著，將各派出所組成大聯盟 (Grand Coalition) 進行合作賽局，計算 Shapley value，以衡量各派出所在團體合作中的邊際貢獻，藉此進一步推導出各派出所應派遣支援陳抗活動的人力數量。

本研究透過 DEA-based Shapley value model，實現科學化、合理化的人力資源分配。我們將結合台灣警界的實務案例，展示此模型在實務操作上的可行性、實用價值及效益。

關鍵字： 成本分配、資源分配、資料包絡分析、夏普利值、合作賽局

doi: 10.6342/NTU202500911

Abstract

The issue of insufficient police manpower in Taiwan has persisted for a long time, and the growing demand for public security in modern society has further increased the burden on police officers. In addition to maintaining the routine operation of daily duties, officers are also required to be deployed significant manpower for events such as festive activities or public protests, which are critical for maintaining public order and safety, and promoting social stability. The shortage of police resources has become even more pressing, and the challenge of managing police manpower allocation has become increasingly urgent.

The current allocation and scheduling of police manpower largely rely on the judgment or subjective decisions of unit leaders or officers in charge. However, this approach is irrational, lacks systematic and scientific foundations, and may lead to uneven resource distribution or inefficiency. How to introduce scientific methods to properly allocate limited police resources and reduce the burden on police officers has become a pressing concern within Taiwan's police system that requires attention.

This paper presents a model based on DEA (Data Envelopment Analysis) and Shapley value, with the aim of optimizing the allocation of police manpower resources for temporary or emergency support needs under limited police resources, while minimizing the impact on routine duties. Taking a police precinct as an example, with several police stations under its jurisdiction, how should manpower be allocated scientifically and reasonably when

doi: 10.6342/NTU202500911

an event, such as a protest or demonstration requiring police support, occurs?

In this DEA-based Shapley value model, each police station is treated as a decision-making unit (DMU), whose efficiency score is derived from the traditional DEA model. Then, the efficiency score is converted into a utility value, representing each unit's independent contribution capacity, through a utility function. Next, the police stations participate in a cooperative game to form a grand coalition, and the Shapley value is calculated to measure how each station contributes marginally to the overall outcome. This allows us to further derive the amount of manpower each police station should dispatch to support the festive activities or public protests.

By applying the DEA-based Shapley value model, this study aims to achieve a scientific and rational distribution of manpower resources. A real-world case from the Taiwanese police force is incorporated to demonstrate the proposed model ' s practicality and benefits".

Keywords: cost allocation, resource allocation, data envelopment analysis, Shapley value, Cooperative game

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Chapter 1. Introduction

With the growing awareness of human rights, rapid advancements in the internet, and the impact of globalization, the demand for public security has significantly increased to maintain social stability. As a result, the police, responsible for maintaining social order and public safety, are at the forefront of these challenges.

In Taiwan, the issue of insufficient police manpower has persisted for a long time, leading to various subsequent problems. In addition to perform daily routine duties at police stations—such as patrols, household visits, and criminal investigations—police officers are often required to mobilize large numbers of personnel to support temporary or emergency events, such as student movements, public protests or large-scale gatherings. This further highlights the strain on police officers caused by insufficient resources. It is not uncommon to hear of some police units attempting to address this issue by overburdening existing officers, requiring them to work excessive hours and handle overwhelming workloads, leaving them physically and mentally exhausted. Therefore, in a situation where police manpower cannot be replenished in a timely manner, finding an effective and comprehensive strategy to allocate limited police resources has become an urgent and unavoidable challenge.

Beyond routine duty operations, when temporary or emergency police manpower support is required, the most common practice in Taiwan's police system is to rely on the judgment or subjective decisions of unit leaders or officers in charge. However, this approach lacks rationality, objectivity, and systematic scientific methods. As a result, decisions often vary between individuals, leading to uneven distribution of human resources and inefficiencies. This, in turn, gives rise to security lapses, turning public safety into an

doi: 10.6342/NTU202500911

unpredictable ticking time bomb.

Current academic research includes various approaches to resource or cost allocation, such as determining the most efficient police department in each city, or using the DEA model to analyze the operational efficiency of multiple companies and how external costs should be allocated among them. Additionally, some studies focus on how to distribute resources to players in a cooperative game using the Shapley value. However, these approaches do not align with our actual needs and may even deviate from the core topic.

Therefore, this paper proposes a DEA-based Shapley value model, integrating the DEA (Data Envelopment Analysis) model and Shapley value in cooperative games, to address the issue of police manpower allocation. The goal is to optimize the distribution of police resources for temporary or emergency support needs under limited manpower, while minimizing the impact of these additional manpower demands on daily operations.

In this DEA-based Shapley value model, we take a single police precinct as an example, with several police stations under its jurisdiction. When an event such as a protest or demonstration occurs, how can manpower from each police station be allocated reasonably? First, each police station is considered a decision-making unit (DMU), and the classic DEA model is employed to evaluate its efficiency score. Then, the efficiency score is converted into a utility value, representing the independent contribution capacity of each unit, through a utility function. Next, the police stations participate in a cooperative game to form a grand coalition, and the Shapley value is calculated to determine how each station contributes marginally to the overall outcome within the coalition. Based on this, we can further derive the number of officers each police station should dispatch to support a protest or demonstration.

In the following chapters, we will incorporate practical cases from Taiwan's police system to validate the real-world applicability and usefulness of the proposed model. Compared to traditional methods, our approach offers the following key advantages. The main strengths of our work are presented as follows:

- Scientific and Rational Allocation: Unlike conventional police personnel allocation

doi: 10.6342/NTU202500911

methods, which often rely on subjective judgment, our model provides a more systematic and logical approach by incorporating efficiency analysis and cooperative contributions.

- System-wide Impact Consideration: Instead of focusing solely on the allocation of resources, our method also accounts for the impact on the remaining system, balancing efficiency and fairness.
- More Equitable Distribution: Traditional allocation methods often prioritize efficiency scores or total personnel count, potentially leading to operational imbalances. In contrast, our approach considers both each police station's efficiency and its contribution to the grand coalition, leading to a more equitable allocation.

Through simulations based on real-world data, our model demonstrates its effectiveness in improving fairness and rationality in personnel allocation, as detailed in Chapter 5.

The structure of the paper is outlined below. Chapter 2 provides an overview of relevant literature. Chapter 3 provides a introduction to the practical background, as well as an overview of Data Envelopment Analysis and the Shapley value in a cooperative game. Chapter 4 presents our proposed method. Chapter 5 discusses the simulation results of our proposed model. Finally, Chapter 6 concludes the paper with a summary of our findings.

doi: 10.6342/NTU202500911

doi: 10.6342/NTU202500911

Chapter 2. Relate work

The duties, work conditions, and nature of police work share many similarities with nursing. Just as the demand for nursing staff can surge suddenly during a flu outbreak or other public health emergencies, policing also faces unpredictable spikes in personnel needs, such as during unexpected public demonstrations or protests. These parallels make studies on nursing resource allocation valuable references for police staffing research. Therefore, when discussing studies on police work, we also refer to studies on nursing as a reference or for comparison.

In this section, while analyzing the related work on police resource allocation, we also include several studies on nursing for comparison.

2.1 Allocation Studies in Police and Nursing Scenarios

Wilson and Weiss [1] review various police staffing allocation methods, including population size, minimum staffing levels, authorized level, and task-driven methods. Through a literature review and interviews with 20 U.S. police agencies, they evaluate the strengths and weaknesses of these methods in optimizing personnel distribution.

Many studies have also applied various mathematical methods to resource and cost allocation. For instance, He et al. [2] propose a heuristic algorithm to optimize police resource allocation in evacuation scenarios and demonstrates its effectiveness in enhancing resource utilization and evacuation efficiency. Similarly, Rico, Salari, and Centeno [3] discusses a similar scenario and utilizes heuristic optimization for efficient nurse resource allocation.

Additionally, some studies employ a descriptive analytical approach to examine nurs-

doi: 10.6342/NTU202500911

ing resource allocation. An example is 'Human Resources Allocation in the Hospital Emergency Department During the COVID-19 Pandemic' [4], which utilizes this method to optimize staffing during the pandemic.

Regardless of whether in a police or nursing context, integer linear programming (ILP) is widely applied. Adler et al. [5] utilizes ILP to develop a location-allocation model aimed at optimizing traffic police patrol vehicle assignments on an interurban road network. Likewise, Maenhout et al. [6] applies ILP to develop an integrative approach for nurse staffing and shift scheduling.

Other optimization methods have also been explored. Agerström et al. [7] propose a method for optimizing nursing resource allocation using simple linear regression analysis.

To optimize staff assignments, Yinusa and Faezipour [8] develop a MILP-based model which employs mixed-integer linear programming, patient allocations, resource distribution, and overtime management, with the goal of reducing healthcare costs and improving patient care.

Resource allocation for healthcare organizations [9] proposes an auction-based resource allocation model to optimize surgical resources in hospitals, focusing on efficient scheduling and allocation of operating rooms, nursing staff, and other critical resources.

Dunnett, Leigh, and Jackson [10] focuse on cost and resource efficiency in responding to an incident. The proposed problem is similar to ours, where a police commander must decide who should be assigned to react to the incident promptly. It proposes a framework that merges mapping and routing algorithms into a decision-making system to guide optimal selection. However, this paper does not provide sufficient detail on how to assess the efficiency of a response unit or how to improve it to respond quickly and efficiently. Finally, Fair Algorithms for Learning in Allocation Problems [11] develops a learning algorithm for fair resource allocation, addressing allocation bias in settings such as lending and policing.

doi: 10.6342/NTU202500911

2.2 Allocation Studies Based on DEA or the Shapley Value

In this paper, we integrate the DEA model with the Shapley value to explore solutions for manpower cost allocation. The subsequent discussion reviews related work from these two perspectives.

'Factors Related to Police Staffing [12]', 'Measuring the relative efficiency of police precincts [13]', and 'Factors that Impact Police Patrol Allocation [14]' provide valuable references for selecting exogenous and endogenous variables as input and output factors in our DEA-based model.

Cook and Kress [15], as well as Cook and Zhu [16],introduce DEA-based methods for proportionally allocating costs based on each DMU's efficiency, ensuring equitable distribution among various DMUs. Sun [13] applies DEA to evaluate the efficiency of 14 police precincts in Taipei, Taiwan. The study focuses on improving their efficiency while considering the operating environment factors and non-discretionary input variables. By applying DEA, it aims to adjust specific inputs and outputs to explore potential improvements in technical efficiency.

However, beyond evaluating the efficiency of DMUs, additional considerations such as group contributions and member satisfaction could be incorporated to achieve more favorable outcomes in cost or resource allocation. Moreover, while DEA assesses whether the resulting allocation is fair, it does not inherently determine cost allocation among DMUs, highlighting a potential limitation in its application.

Fixed costs are often introduced as additional inputs in the development of DEA-based models for cost allocation. For example, this approach has been adopted in studies such as Beasley (2003) [17], as well as Li et al. (2013) [18], and Li et al. (2018) [19].

Moreover, Lan and Chuang [20] further demonstrate the application of DEA in measuring the efficiency of nonprofit organizations to prevent efficiency dilution in resource allocation. By providing a quantitative framework for decision-makers, this approach aligns with our study ’s objective of optimizing resource allocation through DEA-based models.

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Furthermore, some studies employ multi-stage DEA models. For example, Wu et al. [21] construct a three-stage model to measure all police precincts' efficiency. Similarly, Gorman and Ruggiero [22] utilize a multiple-stage DEA framework to evaluate state police performance in the United States.

There are also studies that apply DEA to assess unit efficiency and optimize cost or resource allocation. Vafaee Najar et al. [23] employ DEA to evaluate nurse performance in a hospital setting, categorizing them into different efficiency groups. Similarly, Al-Refaie et al. [24] utilize DEA to optimize nurse allocation in emergency departments, aiming to shorten the time patients spend waiting, enhance nurse utilization, and expand the number of patients treated.

In addition to DEA, cooperative game theory, with a focus on the Shapley value, offers another approach to resource allocation. A representative example of applying cooperative game theory to resource allocation is Zhang et al. [25]. Similarly, Liao et al. [26] compare Shapley value-based allocation with other methods to assess its effectiveness. Furthermore, Wu and Hu [27] propose a Shapley value-based allocation approach for distributing multi-agent resources and demonstrate its feasibility.

Some studies propose frameworks that integrate two game-theoretic models to allocate costs or resources. For example, Wu et al. [28] allocate police officers to optimal patrol shifts. Likewise, to enhance the efficiency of medical resource deployment, Wu et al. [29] apply two game-theoretic models. Yu et al. [30] optimize supplier allocation. Similar to [29] and [30], Wu, Cheng-Kuang [31] develops a two-level framework to ensure the prompt deployment and allocation of security forces. This approach is designed to enhance response strategies for terrorist events, such as the 2015 Paris attacks.

2.3 Allocation Studies Integrating DEA and Game Theory

Xie et al. propose a framework [32] for a fixed-cost allocation framework that integrates DEA with Nash equilibrium-based non-cooperative game theory for distributing

doi: 10.6342/NTU202500911

water pollutant discharge permits among 31 Chinese provinces. The proposed scheme incorporates environmental efficiency and enables DMUs to engage in negotiations to arrive at a mutually agreed allocation. This paper provides a valuable reference for resource allocation methods.

To tackle the problem of allocating carbon emission abatement quotas, Li et al. (2020) [33] develop an integrated DEA-based cooperative game model. It first evaluates relative efficiency and then formulates a cooperative game with a nucleolus-based allocation plan, offering a unique perspective by incorporating the nucleolus into cooperative game approaches. Similarly, Li et al. (2019) [34] employ an integrated DEA-based and nucleolus-based cooperative game approach to allocate costs.

doi: 10.6342/NTU202500911

doi: 10.6342/NTU202500911

Chapter 3. Background

To address the issue of police resource allocation, this study utilizes Data Envelopment Analysis (DEA) and Shapley value as core tools. DEA quantifies the efficiency of each police station, providing an objective benchmark, while Shapley value calculates the marginal contributions of each unit within a cooperative game framework, enabling fairness in resource distribution. The combination of these methods offers a scientific and practical solution to the problem of police resource allocation. The following sections elaborate on the theoretical foundations and application methods of DEA and the Shapley value.

3.1 Data Envelopment Analysis (DEA)

We use the output-oriented BCC model to conduct and present our analysis in this paper. The DEA model forms a critical foundation for the development of our proposed personnel allocation method.

Data Envelopment Analysis (DEA) is a widely adopted technique to evaluate the relative efficiency of comparable Decision Making Units (DMUs) based on data regarding input consumption and output production. It evaluates a set of DMUs, each of which utilizes certain inputs and generates particular outputs. DEA computes each DMU's efficiency and assigns a normalized efficiency score to them. Efficient units are assigned a score of 1, while inefficient units receive a score between 0 and 1, indicating their degree of inefficiency.

A fundamental assumption in DEA is that each DMU is evaluated in relation to other DMUs for assessing relative efficiency. We use the output-oriented BCC model to assess

doi: 10.6342/NTU202500911

DMUs' relative efficiency. Under the DEA model, a DMU's efficiency is represented by the ratio between the weighted sum of outputs and the weighted sum of inputs.

The DEA model, initially introduced by Charnes et al. (1978) [35], is commonly referred to as the CCR model and is used to evaluate the efficiency of DMUs.

Assume a set of n independent DMUs, where each utilizes m inputs to produce s outputs. The i-th input and r-th output of DMUj $(j = 1, 2, ..., n)$ are denoted as $xij(i = 1, ..., m)$ and $y_{rj}(r = 1, ..., s)$, respectively. The following CCR model is used to compute the efficiency of a given DMU$_d$. Each DMU$_j$ consumes m inputs $x_{ij}(i = 1, 2, ..., m)$ to produce s outputs $y_{rj}(r = 1, 2, ..., s)$. θ_j^* represents the optimal efficiency for DMU$_j$. And u_r, v_i are the optimal weights for the inputs and the outputs. DMU$_j$'s relative efficiency is determined through solving the maximization problem below:

$$\begin{aligned} \theta_j^* &= \max \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}, \\ \text{s.t.} \quad & \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \leq 1, \quad \forall j, \\ & u_r, v_i \geq 0 \end{aligned} \tag{3.1}$$

It can be converted into an equivalent linear programming model through the Charnes–Cooper transformation (Charnes and Cooper 1962), as shown below:

$$\begin{aligned} \theta_j^* &= \max \sum_{r=1}^{s} u_r y_{rj}, \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{ik} - \sum_{r=1}^{s} u_r y_{rk} \geq 0, \quad \forall j, \\ & \sum_{i=1}^{m} v_i x_{ij} = 1, \\ & u_r, v_i \geq 0, \quad \forall r, i. \end{aligned} \tag{3.2}$$

θ_j^* denotes the optimal efficiency of DMU$_j$. If $\theta_j^* = 1$, DMU$_j$ is considered efficient and if $\theta_j^* \leq 1$, it is considered inefficient. The weights u_r,v_i vary according to the specific circumstances of each DMU to achieve maximum efficiency. And when calculating a DMU's efficiency score, the model determines the optimal weights to maximize its efficiency, subject to the constraint that all other DMUs' efficiencies remain below or equal

doi: 10.6342/NTU202500911

to one.

In DEA models, the most common ones are the CCR model, as proposed by Charnes, Cooper, and Rhodes (1978) [35], which assumes that both inputs and outputs exhibit constant returns to scale(CRS). In contrast to the CCR model, Banker, Charnes, and Cooper (1984) [36] introduce the BCC model, which considers variable returns to scale. The key difference is that the BCC model includes an additional constraint: $\sum \lambda_j \equiv 1$, as shown in (3.3). This constraint ensures that the combination of the n DMUs is convex and restricts how the DMUs can be combined linearly.

$$\begin{aligned} \max \quad & \theta_k \\ \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} - x_{ik} \leq 0, \quad \forall i, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq \theta_k y_{rk}, \quad \forall r, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \geq 0, \quad \forall j. \end{aligned} \tag{3.3}$$

In this paper, our DEA-based model calculates the efficiencies of DMUs using the BCC model. Since a police station may not achieve greater efficiency at a larger scale, the BCC model is applied to our methods to take variable returns to scale into consideration.

3.2 the Shapley value

In this work, the DEA efficiency score and the Shapley value are integrated to propose a model for personnel allocation. Our aim is to allocate costs based on each DMU's contribution to the group. The Shapley value can appropriately represent the importance of the DMUs and fulfill this requirement.

Shapley originally proposed the concept of the Shapley value in 1953 [37]. The Shapley value is a method in cooperative game theory used to allocate payoffs according to marginal contributions. It presents the relative contributions of players and can be easily

doi: 10.6342/NTU202500911

calculated. Therefore, it is widely applied in both political science and economics. Below, we outline the formal definition of the Shapley value in the context of a cooperative game characterized by a given characteristic function.

Definition 1. *A cooperative game characterized by a pair (N, v) consists of:*

1. *A finite set N, representing all players involved in the game.*

2. *A characteristic function v, which assigns to every subset $S \subseteq N$ (called a coalition) a real number $v(S)$, indicating the total benefit that coalition can achieve collectively.*

   The function v is typically assumed to satisfy the super-additivity property, expressed as:

$$v(S \cup T) \geq v(S) + v(T)$$

This condition holds for any two coalitions S and T that are disjoint (i.e., $S \cap T = \emptyset$), meaning the value of their union is at least as large as the sum of their separate values. This property ensures that collaboration between disjoint groups leads to no loss in total value and is known as super-additivity. A cooperative game in characteristic function form is defined as a pair (N, v), where N is a finite set representing the players, and v is a characteristic function $v : 2^N \to \mathbb{R}$, with $v(\emptyset) = 0$. Any subset $S \subseteq N$ is called a coalition, and $v(S)$ represents its worth in the game.

Definition 2. *The Shapley value is calculated using the following formula based on the characteristic function v.*

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N| - |S| - 1)!}{n!} [v(S \cup \{i\}) - v(S)]$$

n denotes the number of players in the grand coalition N and s represents the size of any subset $S \subseteq N$. The Shapley value, $\phi_i(v)$, is the only solution concept that fulfills the following four axioms: efficiency, symmetry, null player property and additivity. The value obtained from the formula indicates the payoff or contribution assigned to player

doi: 10.6342/NTU202500911

i in a cooperative game (N, v). Hence, the Shapley value can be interpreted as a payoff distribution across all players in a cooperative game involving n participants. As outlined in definition 2, the initial step in computing the Shapley value involves defining the characteristic function v. When it comes to resource distribution, this function should appropriately reflect the collective value of different DMU groupings. Therefore, we construct the characteristic function to capture the worth of both individual DMUs and their coalitions, based on their relative efficiency within the organization.

Therefore, in our proposed model, we first formulate the characteristic function v for every coalition based on the individual DMUs' relative efficiency. This function represents the supporting personnel capability a police station can demonstrate independently, regardless of other DMUs. We will elaborate on this in chapter 4. Moreover, the Shapley value derived from this function is used to capture each DMU's contribution to the grand coalition within the cooperative game. Based on these values, we determine the appropriate share of personnel for each unit.

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doi: 10.6342/NTU202500911

Chapter 4. Proposed Method

This chapter presents our proposed model, including the problem definition, the reasons for proposing this allocation approach, and the method we use to allocate resources or costs.

4.1 Problem Definition

The issue of temporary police manpower allocation arises when police authorities need to deploy personnel to support large-scale events, protests, or emergency situations. Given the limited number of officers available at each police station, an efficient and fair allocation mechanism is essential to ensure that temporary assignments do not excessively disrupt daily operations.

To formally define the problem, we consider a police precinct comprising multiple police stations, each acting as a decision-making unit (DMU). Each DMU has an available workforce a_j and an efficiency score, derived from a Data Envelopment Analysis (DEA) model. Our objective is to allocate a total of W officers required for temporary duty in a way that balances both efficiency and fairness.

The allocation process must satisfy the following constraints: The allocation process must satisfy the following constraints:

1. Total Personnel Constraint:The total allocated personnel across all DMUs must equal the required manpower:

$$\sum_{j \in J} p_j = W$$

Where p_j represents the number of officers dispatched from DMU_j.

doi: 10.6342/NTU202500911

2. Capacity Constraint: Each DMU cannot dispatch more officers than its available workforce:

$$p_j \leq a_j, \quad \forall j \in J$$

3. Fairness Consideration: The allocation should be based on the contribution capacity of each DMU, determined by its efficiency score and available personnel. To achieve this, we use the Shapley value to ensure that each DMU contributes proportionally to its marginal impact on the coalition.

In summary, this problem requires integrating DEA efficiency scores with cooperative game theory to derive an optimal personnel allocation strategy that is both efficient and fair. The following sections detail our proposed model and its implementation.

4.2 Integration of DEA and Shapley Value for Resource Allocation

In this section, we introduce a model that integrates DEA and the Shapley value to address the cost problem across multiple DMUs, particularly the police personnel cost problem, leveraging the strengths of both approaches.

We will outline the steps of how this model operates, as illustrated in the flowchart below. First, each DMU's efficiency score is calculated using DEA. Based on these efficiency scores, the Shapley value for each DMU is derived. Finally, using the Shapley value, the personnel cost for each DMU is determined.

4.2.1 Efficiency of DMUs in DEA

In our DEA-based model, we apply an output-oriented BCC model. The proposed model uses police sectors as an example and is implemented in the context of police organizations, where each police station is regarded as a DMU. The objective is to evaluate each police station's efficiency score. DMU_j is an independent decision making unit, which is considered to be a police station ($j \in J = \{1, 2, \ldots, n\}$). In our DEA model, we

doi: 10.6342/NTU202500911

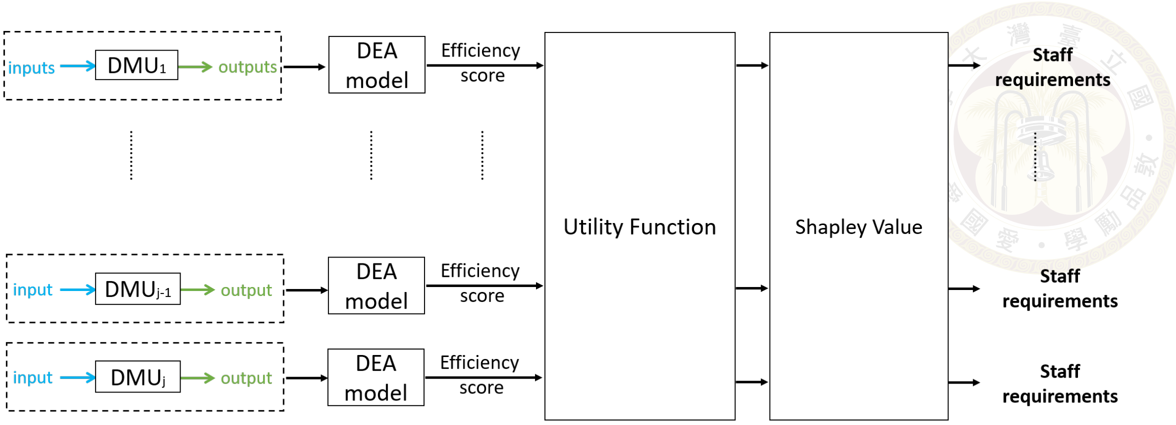

Figure 4.1: the DEA-based Shapley value model

define the input variable as the total number of staff in a police station. The output variables include the number of cases reported by the public to a police station and the inverse of the number of flaws identified by supervisors. This setup allows us to evaluate the efficiency of utilizing available resources to handle public reports and minimize operational deficiencies.

Additionally, due to the many uncertainties in the police environment, there are numerous non-discretionary input variables for a police unit, such as the number of criminal cases reported by the public. On the other hand, output variables can be more directly controlled by police officers than input variables. Therefore, to obtain a reliable efficiency score, it is more appropriate to use an output-oriented DEA model in our proposed method.

At the same time, the efficiency of a police station does not necessarily increase with its scale, such as having more police officers. Therefore, the CCR model is not suitable for our study because it presumes constant returns to scale. On the other hand, the BCC model, which factors in variable returns to scale, is more suitable for our study.

Thus, the model includes one input variable and one output variable. Therefore, the DEA formula can be shown as below:

doi: 10.6342/NTU202500911

$$
\begin{aligned}
\max \quad & \theta_k \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{1j} - x_{1k} \leq 0, \\
& \sum_{j=1}^{n} \lambda_j y_{1j} \geq \theta_k y_{1k}, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0, \quad \forall j.
\end{aligned}
\tag{4.1}
$$

To ensure clarity of notation, the symbols and variables used throughout the model are summarized in Table 4.1.

Table 4.1: Notations

Notation	**Description**
DMU_j	Decision Making Unit j, representing a police station
x_{ij}	Input i used by DMU j
y_{rj}	Output r produced by DMU j
m	Number of inputs (in this study, $m = 1$)
s	Number of outputs (in this study, $s = 2$)
u_r	Weight for output r in the DEA model
v_i	Weight for input i in the DEA model
θ_j^*	Output-oriented DEA efficiency score of DMU k
u_j	Utility function value of DMU j (e.g., $u_j = \theta_j^* \cdot a_j$)
a_j	Available police personnel (resources) in DMU j
$\phi_j(u_j)$	Shapley value of DMUj
N	Set of all DMUs (players) in the grand coalition
S	Subset of DMUs excluding j
R_a, R_b	Resource values allocated to units a and b
$\bar{R}$	Mean of the resource distribution
g	Gini coefficient, measuring inequality in allocation

doi: 10.6342/NTU202500911

With each DMU ’ s efficiency score derived from the DEA model, we proceed to the next step to calculate the Shapley value.

4.2.2 Shapley Value Calculation

4.2.2.1 Characteristic Function

We use the characteristic function as a tool to convert the efficiency score into a specific measure representing the capability of a police station.

We consider each DMU as a player and let all DMUs within an organization participate form a cooperative game. We then use the Shapley value within this framework to represent each DMU's marginal contribution. Before calculating the Shapley value, we first need to define the characteristic function for the cooperative game. We define our characteristic function as below:

$$u_j = \theta_j^* \times a_j \tag{4.2}$$

Here, u_j represents the characteristic function value of DMU$_j$, θ_j^* denotes the optimal efficiency score of DMU$_j$ derived from the previous step, and a_j represent available personnel of DMU$_j$.

The characteristic function value of each DMU represents the number of personnel that each police station can contribute to support temporary manpower demands. Specifically, this value is obtained by multiplying the DMU's efficiency score with its available personnel, reflecting the equivalent number of 'fully efficient' individuals that the police station can deploy. In other words, the same number of personnel, a DMU with lower efficiency has less ability to contribute. This value solely reflects the contribution a police station can make when considered in isolation, without taking into account other factors such as the conditions of other DMUs or its contribution in a group.

In the DEA calculation of each DMU's efficiency score in the previous section, we defined the input parameter as the total personnel number in a police station. However, when defining the variable for the characteristic function, we focus on a different measure: the available and deployable manpower in a single day at the police station. This distinc-

doi: 10.6342/NTU202500911

tion arises because, when evaluating a DMU ’ s efficiency, we aim to consider the overall capability of the entire staff, not just the efficiency of those on duty on a particular day. On the other hand, when discussing the characteristic function value, it refers to situations where there is a temporary need for supporting manpower, which may be required either for several days or even just a single day, rather than for long-term staffing needs.

4.2.2.2 Shapley Value

The Shapley value, unlike the characteristic function value, shows a DMU's marginal contribution within a group. Alternatively, the Shapley value reflects a police station's capability within a precinct to share personnel costs in a cooperative framework. By calculating each unit's Shapley value, we can determine the allocation of personnel costs, ensuring that the allocated cost corresponds to each DMU's marginal contribution.

In the police work environment, no individual or unit can operate entirely on their own or exist in isolation. Everyone and every unit are interdependent, making policing a profession that requires collective action. This collaboration ensures a higher and sufficient level of safety for all, while allowing each individual to maximize their abilities and value, ultimately achieving greater overall effectiveness compared to working independently.

For this reason, police stations within the same precinct are well-suited to the application of cooperative game theory. The police stations within the precinct can collectively share the associated costs, reflecting the collaborative nature of the profession when there is a need for temporary personnel. Every task in policing is fundamentally a team effort requiring unified participation.

In this cooperative game, based on the characteristic function values derived from the previous step, all DMUs form a grand coalition, enabling the computation of individual Shapley values. The Shapley value quantifies each police station's marginal contribution to the collective effort, ensuring a fair and efficient allocation of personnel costs across the coalition.

doi: 10.6342/NTU202500911

4.2.2.3 Personnel Allocation Based on Shapley Value

The Shapley value $\varphi_i(v)$ obtained in the previous step is unique for a DMU and reflects its relative contribution to the organization. However, it does not directly represent the final allocation result. Taking these factors into account, the proportion of personnel cost assigned to DMU_k can be determined as follows:

$$\frac{\varphi_i(v)}{\sum_{i=1}^{n} \varphi_i(v)}, \quad i = 1, \ldots, n$$

4.2.3 Fairness Consideration in the Proposed Model

Although the proposed model primarily aims to optimize personnel allocation in response to temporary demands, fairness remains a key design consideration. In this study, fairness is addressed through the application of the Shapley value within the cooperative game framework. Although the proposed model does not explicitly include a fairness objective function (such as minimizing the Gini coefficient or directly balancing personnel burdens) in the initial optimization formulation, fairness is implicitly incorporated through the mechanism of marginal contribution-based allocation. Specifically, each police station ' s efficiency is first measured using the DEA model, which evaluates the relative operational capacity of each unit by considering its personnel and performance outcomes. Based on these efficiency scores and the available manpower of each station, the characteristic function value is constructed to represent each unit ' s standalone contribution capacity. The Shapley value is then employed to allocate the temporary personnel cost by averaging each DMU ' s marginal contribution across all possible coalition formations. This ensures that the allocation not only reflects each unit ' s standalone ability but also its potential contribution within different collaborative scenarios. In other words, police stations with higher efficiency and greater available manpower—thus having stronger capacity to share the burden—are assigned a larger share of the temporary personnel support responsibilities, while less capable stations are assigned proportionally less. This allocation mechanism aligns with the practical nature of policing, where collective action and mutual support are essential, and helps achieve a fair workload distribution that respects

doi: 10.6342/NTU202500911

both individual capacity and group collaboration. Therefore, although fairness is not directly optimized in the mathematical model, the use of Shapley value naturally embeds a fairness principle consistent with the cooperative nature of police work, balancing workload according to each station ’ s capacity to contribute.

4.2.4 Implementation Tools

The DEA model and Shapley value computations were implemented using Python 3.11 on Google Colab. DEA efficiency scores of DMUs were calculated using self-developed Python scripts, and Shapley value calculations were performed through enumeration of all possible coalitions. All simulation experiments and result analyses presented in Chapter 5 were also conducted using Python.

doi: 10.6342/NTU202500911

Chapter 5. Simulation

In this section, we illustrate the calculation procedure with a simple example involving police sectors and simulate it within the context of police organizations.

We apply our allocation method to a case study involving five police stations from the Zhongzheng Second Police Precinct within the Taipei City Police Department in Taiwan to illustrate its feasibility. Specifically, we examine police duty on the 228 Peace Memorial Day, a national public holiday, during which the president will deliver a speech within the jurisdiction of the Zhongzheng Second Police Precinct.

Next, we conduct additional simulations to test the robustness of the proposed method under different DMU configurations and parameter variations. These include differences in the ability of police personnel across units, variations in personnel requirements, fluctuations in available personnel within a police unit, and changes in the number of DMUs within a police precinct.

5.1 Case Study: Zhongzheng Second Police Precinct

5.1.1 Scenario Description and Problem Statement

Zhongzheng Second Police Precinct is responsible for maintaining security in one of Taiwan ’ s most important administrative areas. A key duty of the precinct is frequently managing temporary public demonstrations. To manage these events, it is necessary to deploy additional personnel for temporary duties, which differ from officers’ routine daily tasks.

These demonstrations pose a recurring challenge for police stations under the precinct,

doi: 10.6342/NTU202500911

as they must continuously address manpower allocation issues. Under such special circumstances, finding an effective solution to this allocation problem remains an ongoing concern.

In current police practice, whenever a demonstration or protest occurs, personnel allocation is typically determined based on the subjective judgment of the responsible officer or superior. For example, the number of personnel required for demonstration duty may be allocated proportionally according to the total staffing in each police station, without considering the actual conditions of each station, such as its efficiency. Making decisions without a rational, logical, and scientific basis can lead to inefficiency in the department. Moreover, police services emphasize solving cases reported by the public as quickly as possible.

We take the special police duty on February 28 as an example, which is conducted within the jurisdiction of the Zhongzheng Second Police Precinct. This special duty is arranged because the president is scheduled to deliver a speech within the precinct's jurisdiction. Therefore, as the police department is responsible for ensuring security, maintaining public safety, and managing traffic to prevent any disruptions, it is crucial for the director of the Zhongzheng Second Police Precinct to effectively command police officers in carrying out this duty. First of all, we need an adequate number of police personnel to perform this task. The personnel assigned to this task will be selected from each police station within the Zhongzheng Second Police Precinct. But how should we determine which police station to assign personnel from, and how can we ensure that the selection process is appropriate? The goal is to minimize the additional workload on the remaining officers as much as possible, such as handling too many public reports per officer, while ensuring that each police station retains sufficient personnel to maintain normal operations. Additionally, the allocation decision should be as fair as possible among all police units.

doi: 10.6342/NTU202500911

5.1.2 Implementation of the DEA-Based Allocation Model

The five police stations in Zhongzheng Second Police Precinct are: Quanzhou St. Police Station, Nanhai Rd. Police Station, Nanchang Rd. Police Station, Xiamen St. Police Station and Siyuan St. Police Station. We consider these five police stations as five decision-making units (DMUs), meaning that each police station is treated as a DMU.

Initially, we compute the standard efficiency score for each police station using an output-oriented BCC DEA model. In our DEA-based model, we define the input as the total number of staff in a police station and the output as the proportion of cases reported by the public compared to the number of flaws identified by supervisors. Using these data, we derive the normal work efficiency score of each DMU. Based on these efficiency scores, we then calculate each DMU ' s utility function value and determine the Shapley value step by step.

Table 5.1 below presents the data of the five police stations in the Zhongzheng Second Police Precinct, derived from our DEA-based Shapley value model. The data include the available personnel at each police station, as well as the efficiency score, utility function value, and Shapley value. Based on these values, we determine the personnel cost that each police station should allocate and cover. Finally, Table 5.2 presents the specific personnel cost assigned to each police unit.

We define the utility function value as the efficiency score of each police unit multiplied by its available personnel. As shown in Table 5.1, both the efficiency score and available personnel are proportional to the utility function value, which in turn influences the Shapley value.

A higher Shapley value indicates a greater contribution to the police precinct. Consequently, in Table 5.2, police stations with higher Shapley values are assigned a greater personnel requirement. Moreover, we observe that a higher efficiency score or greater available personnel does not necessarily correspond to a higher personnel cost. Instead, the personnel cost is determined by the Shapley value.

doi: 10.6342/NTU202500911

Table 5.1: Data of the five police stations derived from our DEA-based Shapley value model.

Police Stations	Efficiency Score	Available Personnel	Utility Function Value	The Shapley Value
Siyuan St. Police Station	0.72	8	5.760	6.800
Xiamen St. Police Station	0.737	6	4.422	5.139
Nanchang Rd. Police Station	0.779	7	5.453	6.192
Nanhai Rd. Police Station	1	8	8.0	11.103
Quanzhou St. Police Station	0.807	5	4.035	4.766

Table 5.2: Personnel costs allocated to each police unit.

Police Stations	The Shapley Value	Total Personnel Needed			
		25	20	15	10
Siyuan St. Police Station	6.800	5	4	3	2
Xiamen St. Police Station	5.139	4	3	2	2
Nanchang Rd. Police Station	6.192	5	4	3	2
Nanhai Rd. Police Station	11.103	8	6	5	3
Quanzhou St. Police Station	4.766	4	3	2	1

doi: 10.6342/NTU202500911

5.1.3 Comparison of Allocation Results and Equity Evaluation

To test whether our allocation method is better than other methods, we use the Gini coefficient to assess fairness among the remaining police personnel across different police stations. Our goal is to minimize the negative impact after manpower cost allocation, ensuring that the workload for the remaining staff is as balanced as possible. Given the same total personnel requirement within a police precinct, an unfair allocation may result in some police units bearing a disproportionately heavy burden while others are assigned significantly less. Such an imbalance could negatively impact public service quality and case resolution efficiency.

5.1.3.1 Equity Evaluation Using the Gini Coefficient

We use the Gini coefficient to evaluate the fairness of the remaining police personnel distribution across the five police stations. By comparing the Gini coefficients from different allocation methods, we assess the effectiveness of each allocation approach.

The Gini coefficient, proposed by the statistician Corrado Gini [38], is widely used in management science to assess the degree of fairness of a resource allocation plan. The Gini coefficient varies between 0 and 1, with 0 denoting absolute equality and 1 indicating maximum inequality. Lower values imply a more equitable allocation of resources, whereas higher values indicate greater disparity. Mathematically, The formula for calculating the Gini coefficient (g) is as follows.:

$$g = \frac{\sum_a \sum_b |R_a - R_b|}{2n^2 \bar{R}} \tag{5.1}$$

n represents the number of DMUs ($a \in \mathbb{N} = \{1, \ldots, n\}$). In this study, we consider police stations as DMUs. R_a and R_b represent the allocated resources of units a and b. n is the total number of units. $\bar{R}$ is the mean value of the resource distribution, defined as:

$$\bar{R} = \frac{1}{n} \sum_{a=1}^{n} R_a \tag{5.2}$$

To make the Gini coefficient more applicable to our specific context and allocation

doi: 10.6342/NTU202500911

problem, and better suited for evaluating the fairness of the allocation plan, we have made slight modifications to the original Gini coefficient. The modified version is as follows:

$$G = \frac{\sum_a \sum_b \left| \frac{\text{cases}}{q_a s_a} - \frac{\text{cases}}{q_b s_b} \right|}{2n^2 \left(\frac{\bar{\text{cases}}}{q_a s_a} \right)} \tag{5.3}$$

In the modified Gini coefficient formula, we use the ratio of caseload to effective contribution to represent the workload of each remaining police officer in a station. These remaining officers are responsible for maintaining normal operations, such as carrying out daily duties, solving criminal cases, and providing public services. The denominator, effective contribution, is defined as a DMU ’ s efficiency score multiplied by the number of remaining officers. This ratio reflects the workload distribution among the remaining staff. We assess the fairness of an allocation method by comparing the workload equality among officers across different stations. Our goal is to ensure that after assigning a certain number of officers to support a temporary protest event, the remaining personnel in each police station still have the capacity to maintain law and order and assist the public effectively. Additionally, we aim for a balanced workload distribution across stations, preventing any single station from being disproportionately burdened.

5.1.3.2 Comparison of Allocation Methods

In this section, we comparing the Gini coefficient resulting from our method with those obtained from two other allocation methods: one that assigns police officers according to the staffing level within a police station and another that allocates them based on the station's efficiency score. The one that assigns police officers according to the staffing level within a police station is closer to current practice, as it aligns with the subjective judgment of the responsible officer or superior.

The Gini coefficient of each allocation method is influenced by the personnel cost covered by a police unit and the remaining personnel in the unit. Using the modified Gini coefficient formula mentioned in the previous section, we obtain the comparison results shown in Figure 5.1.

doi: 10.6342/NTU202500911

Figure 5.1 below presents a comparison of the three allocation methods. As shown, our method consistently results in the lowest Gini coefficient, ensuring the fairest distribution of remaining police staff across police stations, regardless of the total personnel required.

Furthermore, we analyze how the Gini coefficient of each method changes as the total personnel requirement within the Zhongzheng Second Police Precinct increases. Figure 5.1 illustrates these trends, confirming that our method is the fairest among the three. Additionally, our method proves to be superior to the one currently used in police practice.

Moreover, we observe that as the total personnel requirement increases, the Gini coefficient of all three methods slightly decreases. We hypothesize that this is because, with a higher total requirement, more cost is allocated to each unit, and the differences between units become more pronounced.

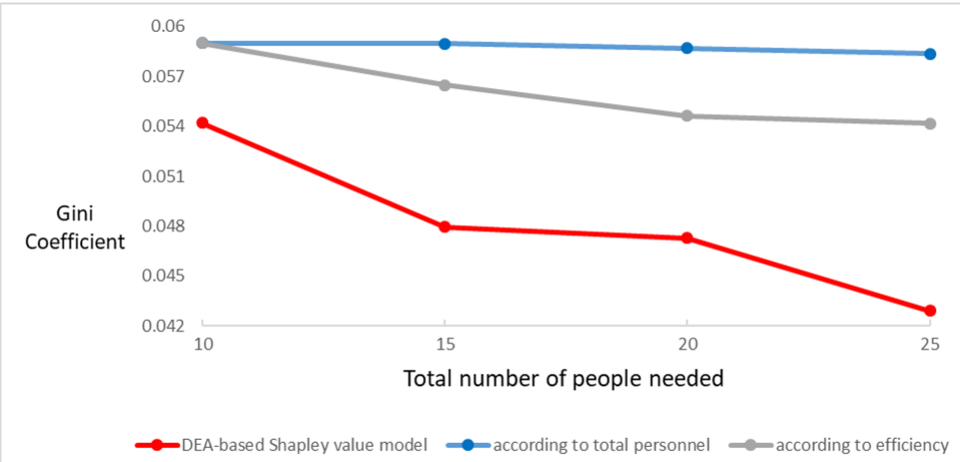

Figure 5.1: The comparison of the three allocation methods as the total personnel requirement varies.

5.2 Additional simulations

To verify that our method is not only effective in the case study of the Zhongzheng Second Police Precinct but also applicable to other practical scenarios, we conduct additional simulations, considering different DMU configurations and parameter variations.

doi: 10.6342/NTU202500911

5.2.1 Variation in Efficiency and Available Personnel at Police Stations

We first simulate five police stations, maintaining the same number of police units as in the Zhongzheng Second Police Precinct. We design four different simulation scenarios by varying efficiency scores and available personnel across the five DMUs:

1. A large efficiency gap between DMUs, with the same available personnel.
2. A large efficiency gap between DMUs, with a large difference in available personnel.
3. A small efficiency gap between DMUs, with the same available personnel.
4. A small efficiency gap between DMUs, with a large difference in available personnel.

The efficiency gap reflects the varying abilities of police stations to solve cases reported by the public and provide public services. A higher efficiency score indicates a greater ability and efficiency in addressing public issues and carrying out daily duties. A station ' s efficiency may vary due to factors such as changes in personnel or leadership.

The variation in available personnel is due to the different tasks each police station has in its daily duties. Moreover, these duties may vary, requiring different personnel to carry them out. As a result, the police stations will not have the same additional staff available to support the protest event.

Under these four scenarios, we simulate and present the calculated efficiency scores, utility function values, and Shapley values, which are presented in Table 5.3 - Table 5.6 below.

When facing different protest events, the required number of police personnel required varies. Since different protests demands a different level of manpower. We assume that the personnel requirement may be 25, 20, 15, or 10. Using the Shapley value, we determine the exact number of officers assigned from each police station for these scenarios. The assignment for each police station under the four previously mentioned scenarios are presented in Table 5.7 - Table 5.10.

doi: 10.6342/NTU202500911

Table 5.3: Data for the scenario with a large efficiency gap between DMUs, with the same available personnel.

Police Stations	Efficiency Score	Available Personnel	Utility Function Value	The Shapley Value
Siyuan St. Police Station	0.64	8	5.120	5.97
Xiamen St. Police Station	0.655	8	5.24	6.1
Nanchang Rd. Police Station	0.932	8	7.456	10.015
Nanhai Rd. Police Station	1	8	8.0	11.391
Quanzhou St. Police Station	0.718	8	5.744	6.523

Table 5.4: Data for the scenario with a large efficiency gap between DMUs, with a large difference in available personnel.

Police Stations	Efficiency Score	Available Personnel	Utility Function Value	The Shapley Value
Siyuan St. Police Station	0.64	6	3.840	4.436
Xiamen St. Police Station	0.655	6	3.93	4.538
Nanchang Rd. Police Station	0.932	8	7.456	9.355
Nanhai Rd. Police Station	1	8	8.0	10.458
Quanzhou St. Police Station	0.718	4	2.872	3.214

Table 5.5: Data for the scenario with a small efficiency gap between DMUs, with the same available personnel.

Police Stations	Efficiency Score	Available Personnel	Utility Function Value	The Shapley Value
Siyuan St. Police Station	0.908	8	7.264	7.498
Xiamen St. Police Station	0.913	8	7.304	7.537
Nanchang Rd. Police Station	0.933	8	7.464	7.668
Nanhai Rd. Police Station	0.984	8	7.872	8.422
Quanzhou St. Police Station	1	8	8.0	8.875

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Table 5.6: Data for the scenario with a small efficiency gap between DMUs, with a large difference in available personnel.

Police Stations	**Efficiency Score**	**Available Personnel**	**Utility Function Value**	**The Shapley Value**
Siyuan St. Police Station	0.908	6	5.448	5.616
Xiamen St. Police Station	0.913	6	5.478	5.646
Nanchang Rd. Police Station	0.933	8	7.464	7.671
Nanhai Rd. Police Station	0.984	8	7.872	8.27
Quanzhou St. Police Station	1	4	4.0	4.797

Table 5.7: Assignment for the scenario where DMUs have a large efficiency gap and the same available personnel.

Police Stations	**The Shapley Value**	**Total Personnel Needed**			
		25	20	15	10
Siyuan St. Police Station	5.970	4	3	2	1
Xiamen St. Police Station	6.100	4	3	2	2
Nanchang Rd. Police Station	10.015	6	5	4	2
Nanhai Rd. Police Station	11.391	7	6	4	3
Quanzhou St. Police Station	6.523	4	3	3	2

Table 5.8: Assignment for the scenario where DMUs have a large efficiency gap and a large difference in available personnel.

Police Stations	**The Shapley Value**	**Total Personnel Needed**			
		25	20	15	10
Siyuan St. Police Station	4.436	3	3	2	1
Xiamen St. Police Station	4.538	4	3	2	2
Nanchang Rd. Police Station	9.355	7	6	4	3
Nanhai Rd. Police Station	10.458	8	6	5	3
Quanzhou St. Police Station	3.214	3	2	2	1

doi: 10.6342/NTU202500911

Table 5.9: Assignment for the scenario where DMUs have a small efficiency gap and the same available personnel.

Police Stations	The Shapley Value	Total Personnel Needed			
		25	20	15	10
Siyuan St. Police Station	7.498	5	4	3	2
Xiamen St. Police Station	7.537	5	4	3	2
Nanchang Rd. Police Station	7.668	5	4	3	2
Nanhai Rd. Police Station	8.422	5	4	3	2
Quanzhou St. Police Station	8.875	5	4	3	2

Table 5.10: Assignment for the scenario where DMUs have a small efficiency gap and large differences in available personnel.

Police Stations	The Shapley Value	Total Personnel Needed			
		25	20	15	10
Siyuan St. Police Station	5.616	4	3	3	2
Xiamen St. Police Station	5.646	4	4	3	2
Nanchang Rd. Police Station	7.671	6	5	3	2
Nanhai Rd. Police Station	8.270	7	5	4	3
Quanzhou St. Police Station	4.797	4	3	2	1

doi: 10.6342/NTU202500911

To visually illustrate the fairness differences among the three previously mentioned allocation methods, we present Gini coefficient charts showing how the Gini coefficient varies with different total personnel requirements. This comparison is presented in Figure 5.2 - Figure 5.5 below, corresponding to the four conditions mentioned above.

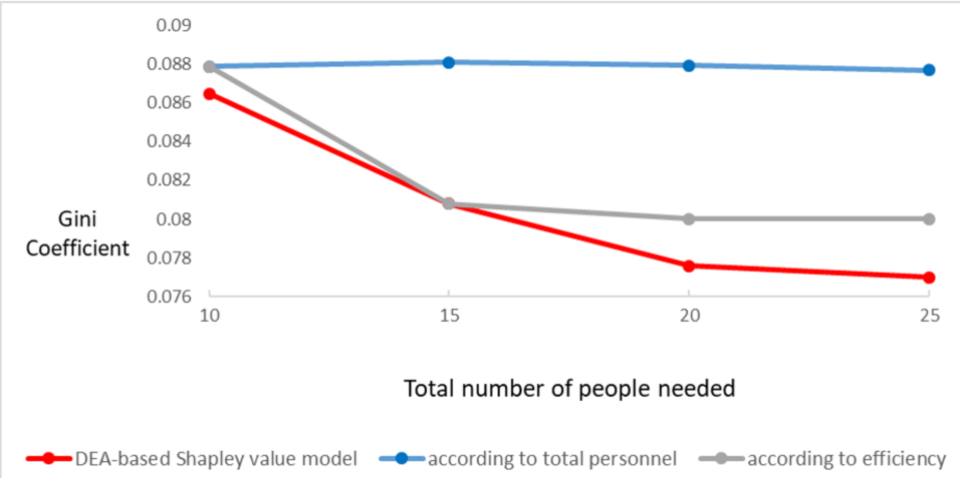

Figure 5.2: Allocation results for five DMUs when the efficiency gap is large and the difference in available personnel is the same under three different methods.

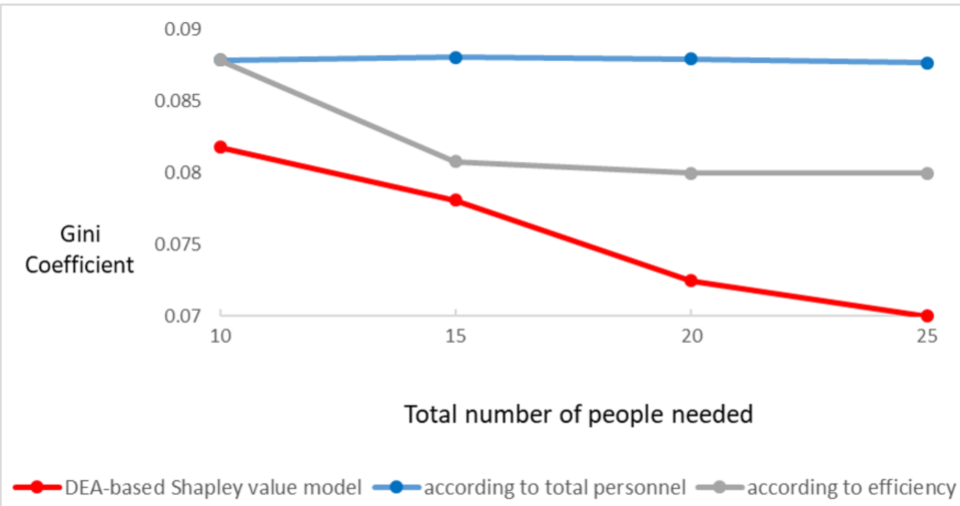

Figure 5.3: Allocation results for five DMUs when the efficiency gap is large and the difference in available personnel is large under three different methods.

We observe that, under all four conditions, the Gini coefficients of our method consistently remain the smallest among the three allocation methods, regardless of the total personnel required. In some cases, the Gini coefficients of our method are very close to those of the efficiency-based method, but our method still yields the smallest Gini coefficient.

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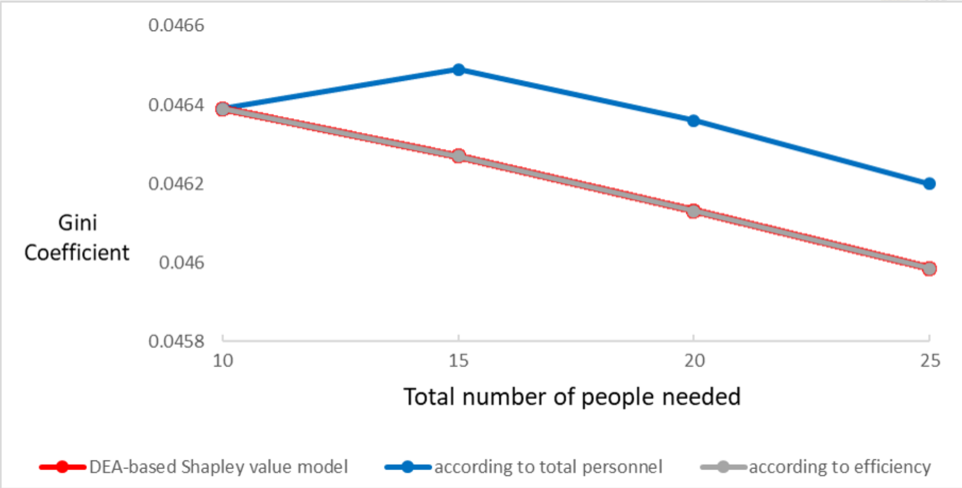

Figure 5.4: Allocation results for five DMUs when the efficiency gap is small and the difference in available personnel is the same under three different methods.

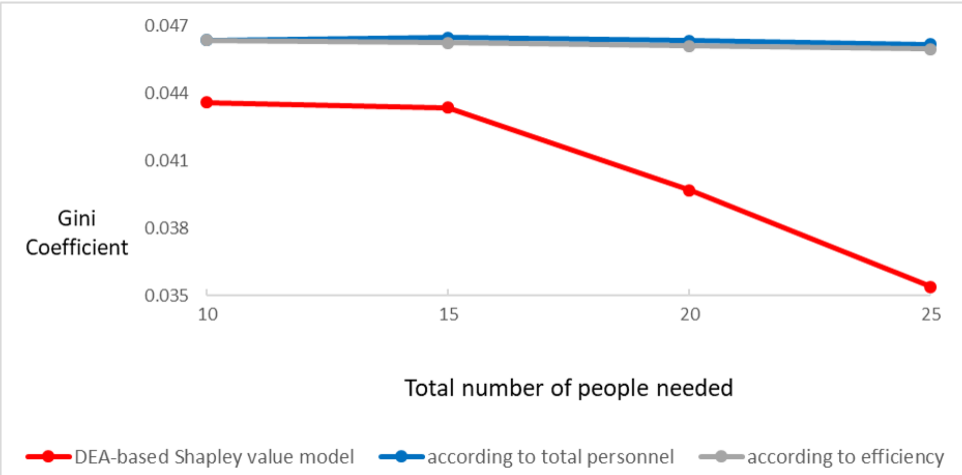

Figure 5.5: Allocation results for five DMUs when the efficiency gap is small and the difference in available personnel is large under three different methods.

This indicates that, despite variations in efficiency scores and available personnel across DMUs, our method consistently produces fairer results. Therefore, the DEA-based Shapley value model ensures a more equitable allocation of resources among police officers.

Furthermore, the overall trend suggests that the efficiency score-based method performs better than the method based solely on the total number of police personnel in each station.

In addition, we observe that as the total number of required personnel increases, the Gini coefficient tends to remain stable or gradually decrease, indicating a more balanced distribution of remaining officers, regardless of the method used. This suggests that higher

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personnel requirements contribute to a more equitable distribution of police resources, making all three allocation methods more effective in achieving fairness.

5.2.2 Different Numbers of DMUs: Varying the Number of Police Stations in a Precinct

Different police precincts have jurisdiction over different numbers of police stations. For example, the Daan Police Precinct of the Taipei City Police Department oversees seven police stations, while the Wenshan Second Police Precinct of the Taipei City Police Department oversees only three police stations.

In the previous section, we conducted simulations with five DMUs. In this section, we extend our analysis by conducting simulations with three and seven DMUs to further test the reliability of our method.

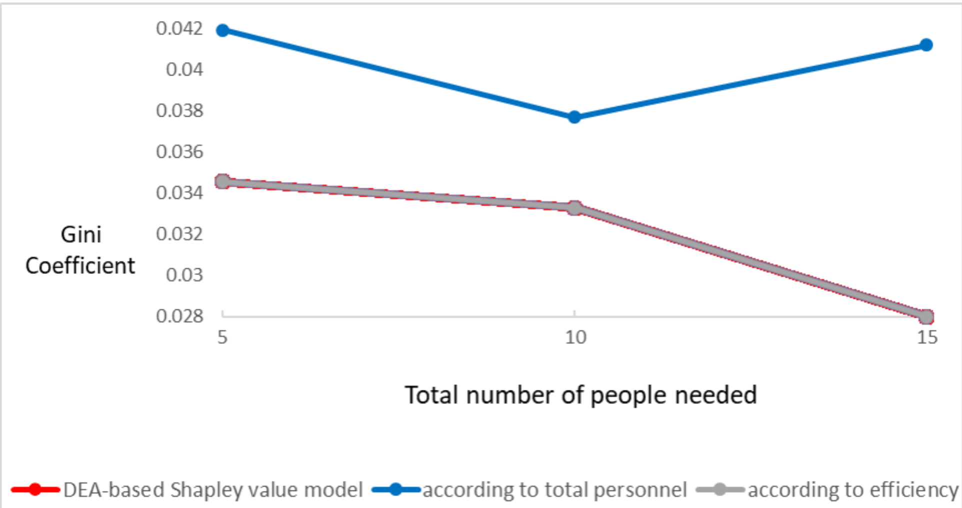

Figure 5.6: Allocation results for three DMUs when the efficiency gap is large and the difference in available personnel is the same under three different methods.

Figure 5.6 - Figure 5.9 present the simulation results with three DMUs under the four conditions outlined in Section 5.2.1, illustrating the Gini coefficients for different allocation methods based on the personnel cost assigned by each police station. Additionally, due to the limited total available personnel in the three-DMU simulation, we assume that the required personnel may be 15, 10, or 5.

Figure 5.10 - Figure 5.13 present the simulation results with seven DMUs under the

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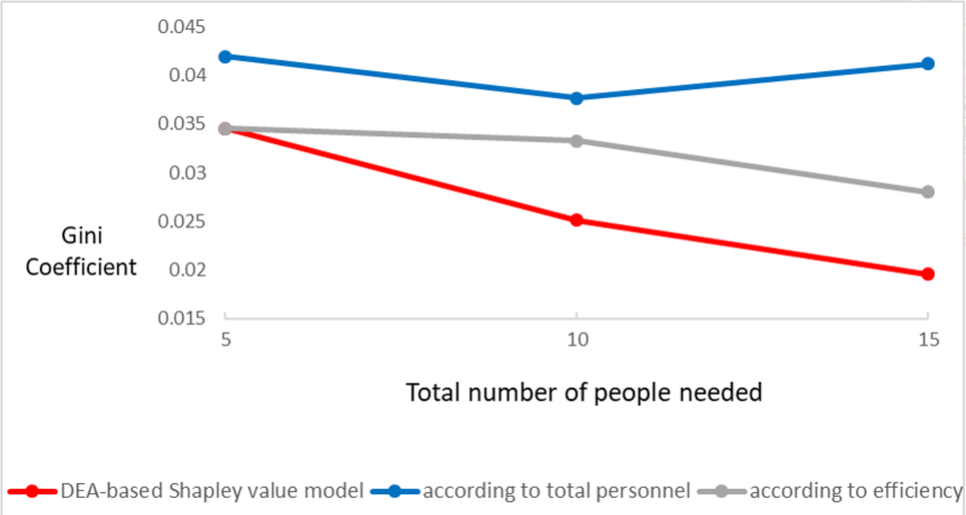

Figure 5.7: Allocation results for three DMUs when the efficiency gap is large and the difference in available personnel is large under three different methods.

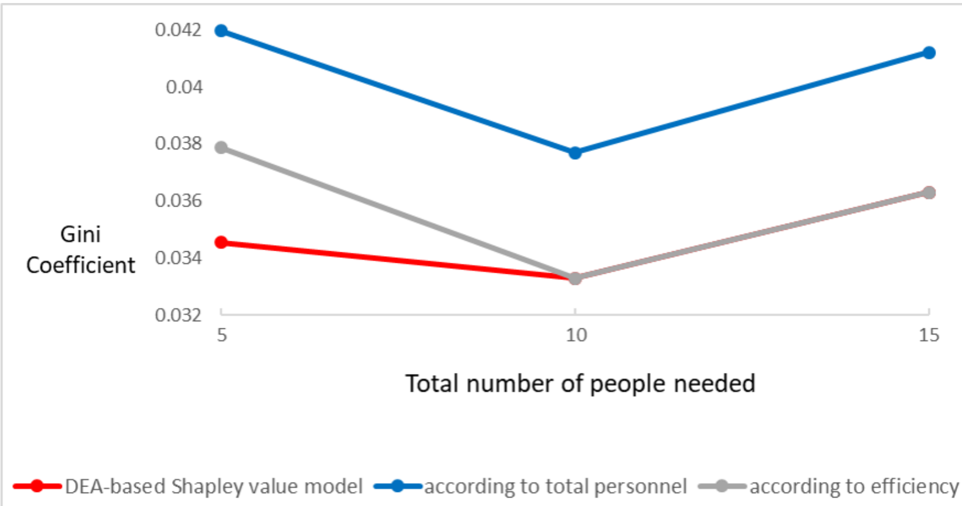

Figure 5.8: Allocation results for three DMUs when the efficiency gap is small and the difference in available personnel is the same under three different methods.

same four conditions, illustrating the Gini coefficients for different allocation methods based on the personnel cost assigned by each police station. Additionally, with the increase in the number of DMUs and the corresponding rise in total available personnel in the seven-DMU simulation, we assume that the required personnel may be 25, 20, 15, or 10.

In the simulation with three DMUs, the Gini coefficients from our method remain the smallest among the three allocation methods. Additionally, the Gini coefficients of the efficiency-based method are lower than those of the total-personnel-based method, which is consistent with the observations from the five-DMU simulation in 5.2.2.

In the seven-DMU simulation, our method consistently produces the fairest alloca-

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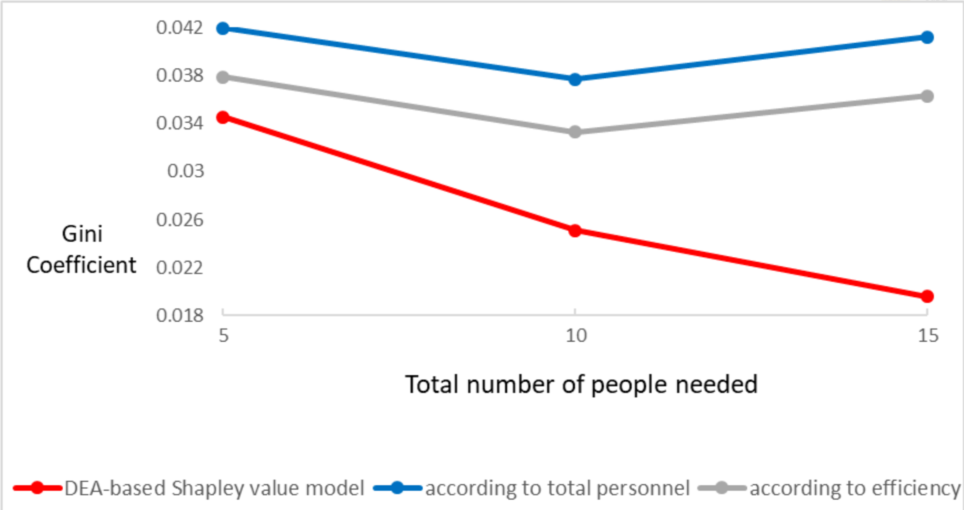

Figure 5.9: Allocation results for three DMUs when the efficiency gap is small and the difference in available personnel is large under three different methods.

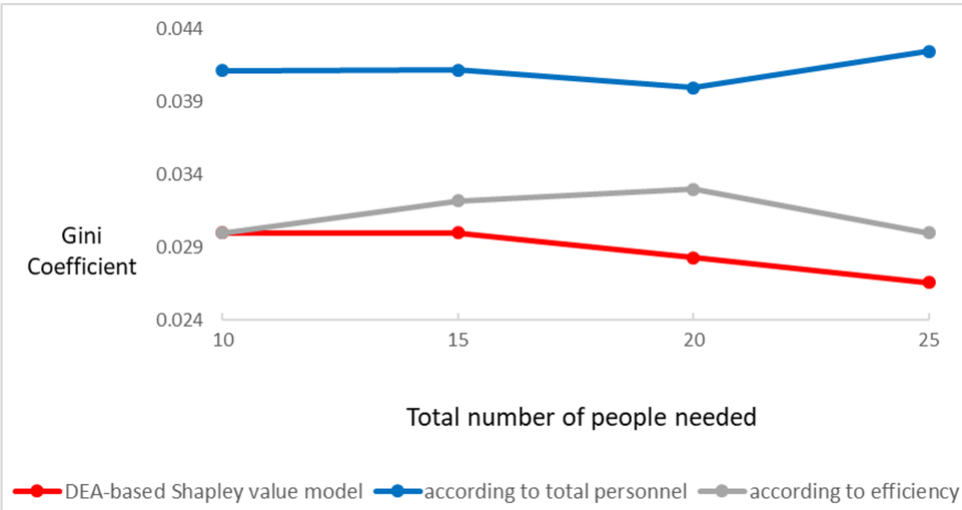

Figure 5.10: Allocation results for seven DMUs when the efficiency gap is large and the difference in available personnel is the same under three different methods.

tion, as evidenced by the lowest Gini coefficients across all tested conditions. Compared to the efficiency-based and total-personnel-based methods, our method demonstrates a more balanced distribution of personnel among the police stations.

Moreover, the overall ranking of fairness among the three methods remains consistent: our method yields the most equitable results, followed by the efficiency-based method, while the total-personnel-based method results in the highest disparity in resource distribution.

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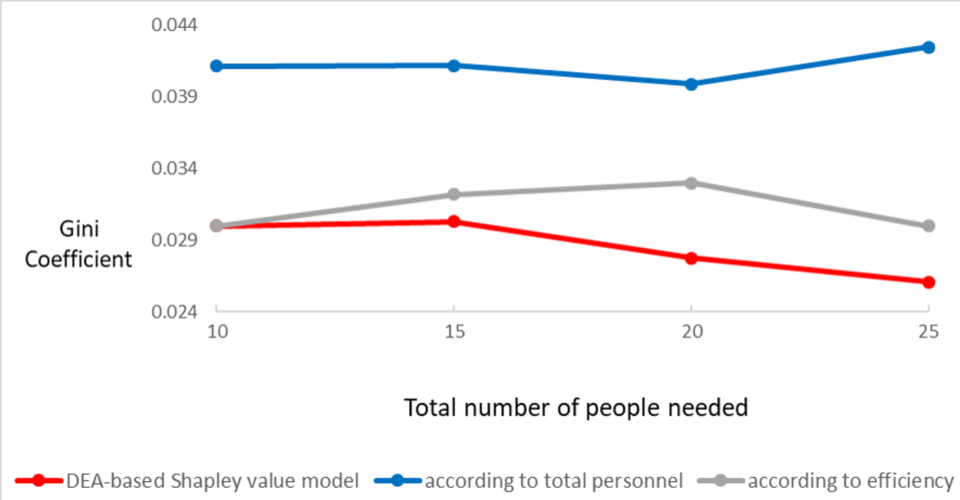

Figure 5.11: Allocation results for seven DMUs when the efficiency gap is large and the difference in available personnel is large under three different methods.

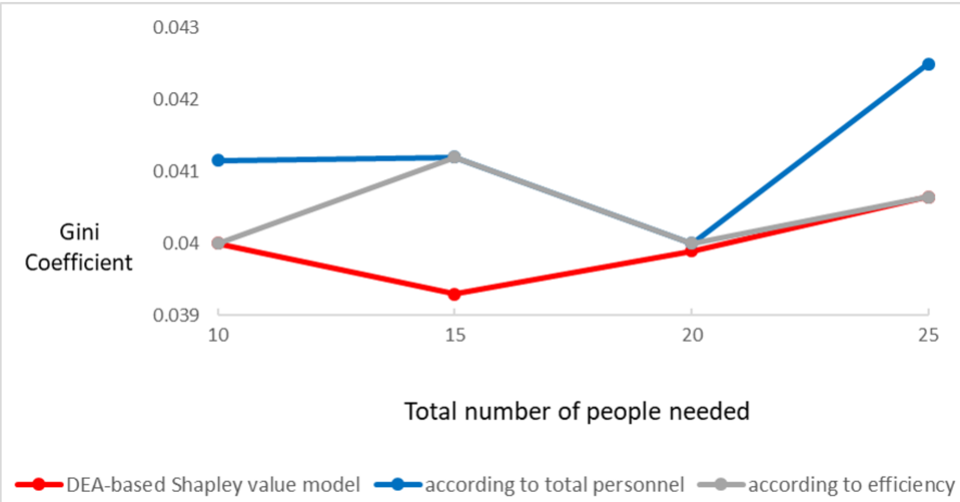

Figure 5.12: Allocation results for seven DMUs when the efficiency gap is small and the difference in available personnel is the same under three different methods.

5.2.3 Allocation Results When a Single Unit's Efficiency Varies

In police practice, operational conditions are constantly changing. Therefore, we aim for the allocation method to adapt to dynamic environments, such as personnel adjustments, changes in police station leadership, or shifts in police policies. Given these unpredictable circumstances, we further evaluate the applicability of our method when a single unit's efficiency varies.

To analyze the effects of different allocation methods under these varying conditions, we extend our simulation to scenarios where a single unit's efficiency changes. In this section, we maintain the same number of police units as in the Zhongzheng Second Police

doi: 10.6342/NTU202500911

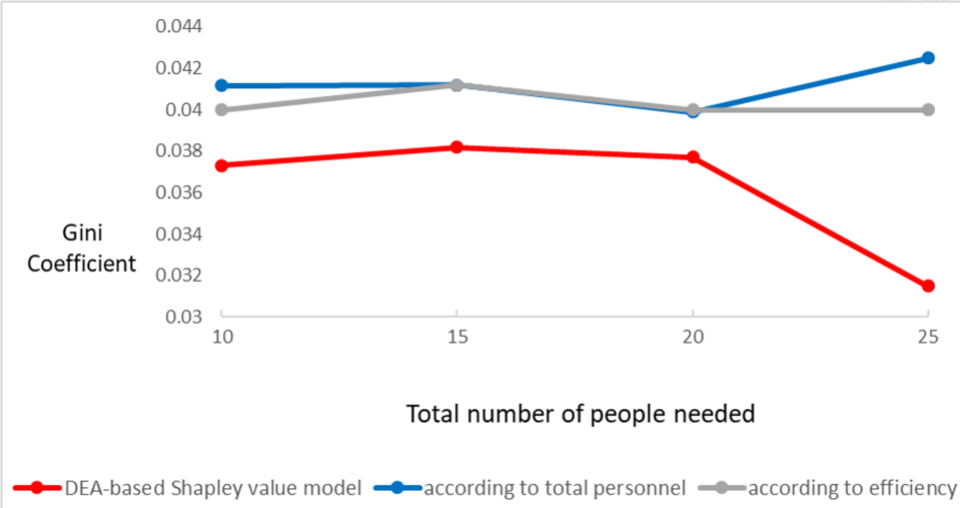

Figure 5.13: Allocation results for seven DMUs when the efficiency gap is small and the difference in available personnel is large under three different methods.

Precinct, which consists of five police stations, and simulate how the allocation results respond to variations in a single police unit ’ s efficiency.

Changes in leadership or personnel structure within a police station can significantly impact its efficiency. To examine this effect, we conduct a simulation to assess how variations in a single police unit ’ s efficiency influence the allocation outcomes under three different conditions:

1. When the efficiency gap among DMUs is large, a single unit experiences a sudden increase in efficiency (e.g., from 0.64 to 0.96).
2. When the efficiency gap among DMUs is large, a single unit undergoes a sharp decline in efficiency (e.g., from 0.98 to 0.68).
3. When the efficiency gap among DMUs is small, a single unit undergoes a sharp decline in efficiency (e.g., from 0.93 to 0.64).

We observe that regardless of whether a single unit undergoes a sharp increase or decrease in efficiency, our method consistently yields the lowest Gini coefficient among the three allocation methods, followed by the efficiency-based method. Therefore, we conclude that our method achieves the fairest allocation, even when a police station's efficiency is affected by unforeseen circumstances.

doi: 10.6342/NTU202500911

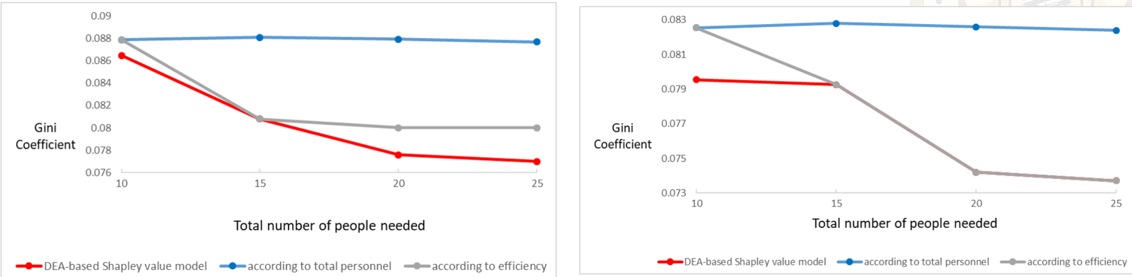

Figure 5.14: When the efficiency gap among DMUs is large, a single unit experiences a sudden increase in efficiency.

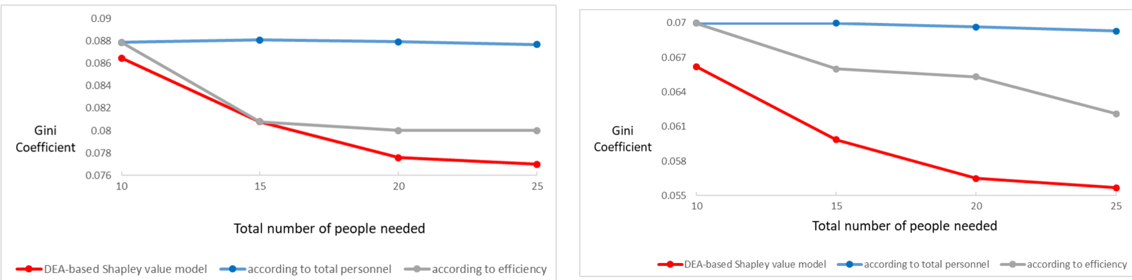

Figure 5.15: When the efficiency gap among DMUs is large, a single unit undergoes a sharp decline in efficiency.

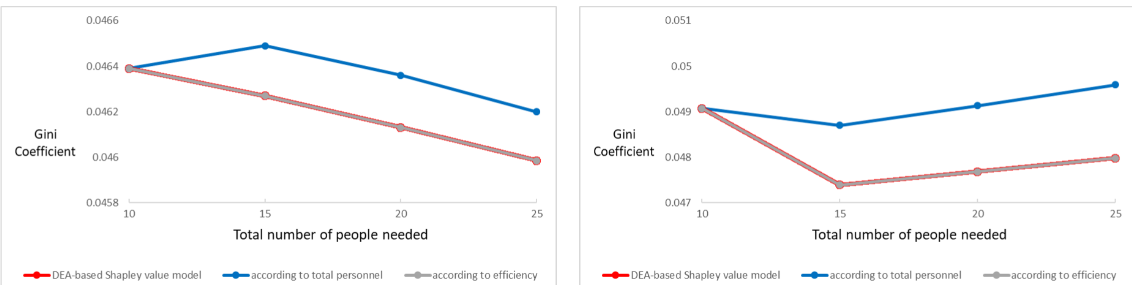

Figure 5.16: When the efficiency gap among DMUs is small, a single unit undergoes a sharp decline in efficiency.

doi: 10.6342/NTU202500911

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Chapter 6. Conclusion

In this study, we integrated the DEA model with the Shapley value as defined in cooperative game theory to propose an allocation method for addressing resource and cost allocation problems. We can apply this DEA-based Shapley value model to meet the police practical requirements. We take the five police stations in the Zhongzheng Second Police Precinct of the Taipei City Police Department in Taiwan as an example to demonstrate the effectiveness of our method. Based on the simulation results, we present the following observations and strengths of the proposed allocation method:

1. Scientific Basis of the Model:Unlike current police practice, our method is more scientific, rational, and logically structured. It takes into account each police station ’ s operational efficiency and actual working personnel, eliminating reliance on subjective human judgment, which can lead to inconsistent outcomes and potential security risks.

2. System-wide Consideration: Unlike general allocation problems, our approach is not solely focused on distributing known quantities of resources or costs. Instead, we also consider the impact on the remaining system after allocation. We take into account the trade-off between efficiency and fairness.

3. More equitable: Traditional methods that allocate personnel solely based on efficiency scores or the total number of officers follow a formal equality approach, which may even be detrimental to the actual operations of police stations. In contrast to these traditional methods, our approach considers both each police station ’ s efficiency and its contribution to the grand coalition. We apply the BCC DEA

doi: 10.6342/NTU202500911

model to determine each station ' s efficiency and use the Shapley value in a cooperative game to assess its contribution within the group. Unlike conventional allocation methods that rely only on efficiency scores or the total number of officers, our method provides a more comprehensive and balanced distribution. The example in Chapter 5 demonstrates that our proposed allocation method is the more equitable one.

doi: 10.6342/NTU202500911

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