



國立臺灣大學理學院應用物理研究所

碩士論文

Graduate Institute of Applied Physics

College of Science

National Taiwan University

Master's Thesis

Nb/Sb₂Te₃ 多層薄膜中磁通釘扎行為與渦旋動力學之研究

Investigation of Flux Pinning Behavior and Vortex Dynamics in

Nb/Sb₂Te₃ Multilayer Thin Films

徐辰硯

Chen-Yen Hsu

指導教授：王立民 博士

Advisor: Li-Ming Wang, Ph.D.

中華民國 115 年 8 月

August, 2026

PREFACE



浜までは

海女も蓑着る

時雨かな

滝野瓢水

摘要

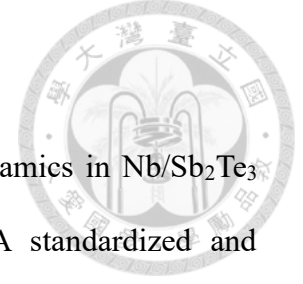


本論文研究不同堆疊配置之 Nb/Sb₂Te₃ 多層薄膜的磁通釘扎行為與渦旋動力學，並建立一套規範化且具再現性的分析架構，由溫度與磁場相依的磁化量測中取得超導轉變溫度、特徵磁場、臨界電流密度 J_c 與釘扎力密度 F_p 。磁性響應呈現多重超導轉變特徵，顯示此多層薄膜無法以單一均勻的超導態描述。透過分析 $J_c(H)$ 、 $F_p(H)$ 、歸一化釘扎力與對數 $J_c(H)$ ，可辨識出不同的釘扎區域與類第二峰值行為，顯示渦旋響應會隨磁場及溫度演變。

最值得注意的是，當磁場平行於 **ab** 平面時，系統於約 3 K 呈現異常的低場釘扎增強。排除受額外磁性訊號影響的區域後，面外方向並未重現相同的增強現象。此方向差異顯示該異常增強具有異向性，且對多層膜介面敏感，而非僅源自傳統的塊材 Nb 釘扎。儘管僅憑磁性量測尚無法唯一判定其微觀來源，本研究結果仍為進一步探討 Nb/Sb₂Te₃ 多層薄膜中受介面調控的渦旋動力學提供了實驗基礎。

關鍵字： 多層薄膜、拓撲超導體、磁通釘扎、渦旋動力學、介面效應

ABSTRACT



This thesis investigates flux pinning behavior and vortex dynamics in Nb/Sb₂Te₃ multilayer thin films with different multilayer configurations. A standardized and reproducible framework was established to extract superconducting transition temperatures, characteristic magnetic fields, critical current density J_c , and pinning-force density F_p from temperature- and field-dependent magnetization measurements. The magnetic responses exhibit multiple superconducting-transition features, indicating that the multilayer films cannot be described by a single homogeneous superconducting state. Analyses of $J_c(H)$, $F_p(H)$, normalized pinning force, and logarithmic $J_c(H)$ reveal distinct pinning regimes and second-peak-like behavior, demonstrating the evolution of the vortex response with magnetic field and temperature.

Most notably, an anomalous low-field pinning enhancement appears near 3 K when the magnetic field is applied parallel to the **ab** plane. No comparable enhancement is reproduced in the out-of-plane configuration after excluding regions affected by additional magnetic signals. This directional contrast indicates that the anomalous enhancement is anisotropic and sensitive to the multilayer interfaces, rather than arising solely from conventional bulk Nb pinning. Although its microscopic origin cannot be uniquely determined from magnetic measurements alone, these results provide an experimental basis for investigating interface-modified vortex dynamics in Nb/Sb₂Te₃ multilayer thin films.

Keywords: Multilayer thin films, topological superconductor, flux pinning, vortex dynamics, interface effects

CONTENTS



PREFACE.....	I
摘要	II
ABSTRACT	III
CONTENTS	IV
LIST OF FIGURES.....	IX
LIST OF TABLES	XIII
CHAPTER 1 INTRODUCTION	1
1.1 Superconductivity	1
1.1.1 Superconductivity	1
1.1.2 Niobium.....	3
1.2 Topological Insulator	4
1.2.1 Discovery of Topological Insulators.....	4
1.2.2 Sb_2Te_3	5
1.2.3 Topological Superconductor	6
1.3 Research Motivation and Objectives	6

CHAPTER 2 BACKGROUND KNOWLEDGE FOR MAGNETIC ANALYSIS 8

2.1	Magnetic Response of Superconductors	8
2.1.1	Superconducting Transition and Zero Resistance	8
2.1.2	Meissner Effect.....	9
2.1.3	Critical Magnetic Field.....	11
2.1.4	Critical Current.....	12
2.2	Critical Fields in Type-II Superconductors.....	14
2.2.1	London penetration depth.....	14
2.2.2	Pippard coherence length.....	17
2.2.3	Ginzburg–Landau Theory and Type-I/Type-II Classification	21
2.3	Vortex Matter and Irreversible Magnetization.....	27
2.4	Interpretation of anomalous magnetization-loop morphologies.....	34
2.4.1	Second Peak Effect.....	34
2.4.2	Abrupt low-field vortex processes.....	41

CHAPTER 3 EXPERIMENTAL METHODS AND DATA PROCESSING

PROCEDURES 46

3.1	Chapter Overview	46
3.2	Sb ₂ Te ₃ Bulk Target Preparation	47
3.2.1	Stoichiometric Calculation and Sintering Process	47
3.2.2	EDS and XRD Characterization of Sb ₂ Te ₃ Bulk Crystal	50
3.2.3	Grinding and Target Compression.....	53

3.3	Thin Film Deposition	55
3.3.1	Sputtering System Configuration	55
3.3.2	Substrate Cleaning and Chamber Pre-conditioning.....	58
3.3.3	Thin Film Deposition Parameters.....	62
3.3.4	Deposition Rate Calibration	65
3.3.5	Multilayer Structure Design	69
3.4	Structural Characterization	71
3.4.1	X-ray Diffraction Measurement	71
3.4.2	Crystallographic Analysis.....	72
3.5	Definitions and Extraction Criteria of Magnetic Quantities.....	73
3.5.1	SQUID Output and Magnetization Conversion.....	73
3.5.2	Superconducting Transition Temperature Determination.....	74
3.5.3	Background Subtraction Procedure	76
3.5.4	Lower Critical Field Determination	77
3.5.5	Upper Critical Field Determination.....	79
3.5.6	Irreversibility Field Determination.....	80
3.6	Automated Data Analysis Workflow	81
3.6.1	Overview of the Analysis Programs	81
3.6.2	Derivative and Local-Slope Processing Module	83
3.6.3	H_{c2} Extraction Module.....	85
3.6.4	J_c , F_p , and H_{irr} Processing Module	87
3.6.5	Dew-Hughes Fitting Module	93
3.6.6	Output Consistency and Workflow Limitations	96

3.7	Summary of Experimental Workflow	97
-----	--	----

CHAPTER 4 MAGNETIC RESPONSE AND VORTEX PINNING BEHAVIOR IN NB/SB₂TE₃ MULTILAYERS 100

4.1	Analysis strategy and evidence chain	100
-----	--	-----

4.2	Temperature-dependent magnetization: superconducting transition and anomalous magnetic response	102
-----	--	-----

4.2.1	Superconducting quality verification of single-layer Nb film	102
-------	--	-----

4.2.2	$M(T)$ curves of Nb/Sb ₂ Te ₃ multilayer samples	105
-------	--	-----

4.3	Field-dependent magnetization from $m(H)$ -curves: characteristic superconducting fields	108
-----	---	-----

4.3.1	Initial $m(H)$ -curves and field-induced suppression of diamagnetism	108
-------	--	-----

4.3.2	Determination of H_{c1} and H_{c2}	110
-------	--	-----

4.3.3	Effective coherence length, penetration depth, and GL parameter	113
-------	---	-----

4.3.4	Definition of H_g as an empirical diamagnetic maximum field	123
-------	---	-----

4.3.5	Temperature dependence of H_{c1} , H_g , and H_{c2}	126
-------	---	-----

4.4	Magnetic hysteresis and irreversible vortex response	129
-----	--	-----

4.4.1	Temperature evolution of $m(H)$ -loops	129
-------	--	-----

4.4.2	Temperature evolution of $J_c(H)$	132
-------	---	-----

4.4.3	Temperature dependence of the zero-field critical current density and Griessen-type pinning analysis	134
-------	---	-----

4.4.4	Irreversibility field $H_{irr}(T)$ and extended phase diagram	140
-------	---	-----

4.5	Pinning force $F_p(H)$ and Field-dependent Pinning Regimes	142
-----	--	-----

4.5.1	Field dependence of $F_p(H)$ and Normalized pinning force $f_p(h)$	142
4.5.2	Dew-Hughes-type analysis of low-field and high-field regimes.....	145
4.6	Evidence for competing vortex-pinning regimes	150
4.6.1	Logarithmic analysis of $J_c(H)$	150
4.6.2	Temperature evolution of H_{SPE} , $H_{collapse}$, $h_{max,L}$ and $h_{max,H}$	153
4.7	Field-orientation dependence of vortex pinning in the 3ML sample	162
CHAPTER 5 CONCLUSION AND FUTURE WORK		171
5.1	Conclusion.....	171
5.2	Future works	174
REFERENCES		176

LIST OF FIGURES

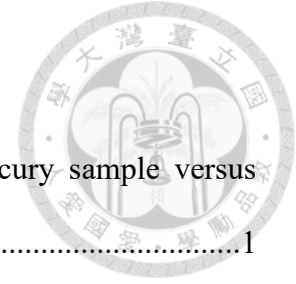


Figure 1.1: Onnes's graph plotting electrical resistance of a mercury sample versus temperature in kelvins [2].	1
Figure 1.2: Historical overview of the discovery dates of superconducting materials and their respective critical temperatures, separated	2
Figure 1.3: Cubic crystalline structure of metallic Nb [9].	3
Figure 1.4: Schematic band diagram of insulator, metal with 2D (and 3D) topological insulators with insulating bulk comprising metallic edges states [14].	4
Figure 1.5: Schematic of Sb_2Te_3 crystal structure [15].	5
Figure 2.1: Fundamental properties of a superconductor [21].	10
Figure 2.2: Exponential decrease of the magnetic flux $\mathbf{B}$ with distance x into a bulk superconductor [29].	17
Figure 2.3: Magnetization loops of II-type superconductor [39].	38
Figure 2.4: Magnetic field H dependence of magnetization in 112-type superconductor	39
Figure 2.5: Low-field magnetization hysteresis and schematic vortex-flipping process in a Ni/Nb/Ni/CoO [47].	44
Figure 3.1: Raw materials of Sb and Te.	48
Figure 3.2: Sintering process of Sb_2Te_3 .	49
Figure 3.3: Crystal of Sb_2Te_3 after sintering process.	50
Figure 3.4: Energy-dispersive X-ray spectroscopy of Sb_2Te_3 bulk.	51
Figure 3.5: X-ray diffraction of Sb_2Te_3 Bulk.	52
Figure 3.6: Sputter system.	56
Figure 3.7: Schematic of the sputtering chamber geometry and the measured target–	

substrate distances.	57
Figure 3.8: Schematic diagram of the auxiliary Ar inlet path.	60
Figure 3.9: AFM landscape for calibration sample.	68
Figure 3.10: Schematic of structural parameters of the Nb/Sb ₂ Te ₃ multilayer samples..	70
Figure 3.11: XRD spectrum of sample (a)2ML and (b)3ML.	72
Figure 3.12: Example of determination of the superconducting transition temperature T_c from the ZFC $m(T)$ curve.....	75
Figure 3.13: UI of local-slope analysis tool.	84
Figure 3.14: UI of H_{c2} extraction module.....	87
Figure 3.15: UI of pinning force analyzer.	91
Figure 3.16: Preview plots of $F_p(H)$	92
Figure 3.17: UI of Dew-Hughes fitting module.	96
Figure 3.18: Magnetic-data processing workflow used in this study.	98
Figure 4.1: $m(T)$ of S _{Nb}	103
Figure 4.2: dm/dT of S _{Nb}	103
Figure 4.3: Magnetic Susceptibility of S _{Nb}	105
Figure 4.4: $m(T)$ curves of the 2ML and 3ML samples. (a)2ML (b)3ML.....	106
Figure 4.5: Corresponding dm/dT curves of the (a)2ML and (b)3ML samples.	107
Figure 4.6: The magnetic susceptibility $4\pi\chi T$ of the (a)2ML (b)3ML samples.	107
Figure 4.7: The raw $m(H)$ -curves of the (a)2ML and (b)3ML samples.....	109
Figure 4.8: The Background-subtracted $m(H)$ -curves for (a)2ML and (b)3ML samples.	110
Figure 4.9: Schematic illustration of the determination criteria for H_{c1} from $m(H)$ - curves [48].	111
Figure 4.10: The temperature dependence of H_{c1} and H_{c2}	113

Figure 4.11: Temperature dependence of the coherence length ξT for (a)2ML and (b)3ML.....	115
Figure 4.12: Temperature dependence of the GL parameter κT for (a)2ML and (b)3ML.	117
Figure 4.13: Temperature dependence of the effective penetration depth $\lambda_{eff} T$ for (a)2ML and (b)3ML over the full measured temperature range.	119
Figure 4.14: Fitting of ξT for 2ML at (a)Low-temperature and (b)High-temperature.	120
Figure 4.15: Fitting of $\lambda_{eff} T$ for 2ML at (a)Low-temperature and (b) High-temperature.	122
Figure 4.16: Definition of H_g in this work.	124
Figure 4.17: Temperature dependence of H_g for the 2ML and 3ML samples.....	125
Figure 4.18: Schematic vortex-matter phase diagram for a type-II superconductor [68].	127
Figure 4.19: The phase diagram of (a)2ML and (b)3ML samples.	128
Figure 4.20: The raw $M(H)$ -loops of the (a)2ML and (b)3ML samples.....	130
Figure 4.21: The background-subtracted $M(H)$ -loops of the (a)2ML and (b)3ML samples.	131
Figure 4.22: the field dependence of $J_c H$ for the (a)2ML and (b)3ML samples.....	133
Figure 4.23: Zero-applied-field critical current density $J_c H_{appl} = 0, T$ for (a)2ML and (b)3ML samples.	136
Figure 4.24: Simplified power-law analysis of $J_c H_{appl} = 0, T$ for (a)2ML and (b)3ML samples.	139
Figure 4.25: The $H_{irr}(T)$ for the 2ML and 3ML samples.....	141
Figure 4.26: The extended phase diagrams of (a)2ML and (b)3ML.	142

Figure 4.27: The field dependence of FpH for the (a)2ML and (b)3ML samples.	143
Figure 4.28: The normalized pinning force fph for the (a)2ML and (b)3ML samples.	145
Figure 4.29: Dew-Hughes-type fitting at $T = 2.5 K$ for the (a)2ML and (b)3ML samples.	147
Figure 4.30: Dew–Hughes-type analysis for the 3ML sample at $T=2 K$	149
Figure 4.31: The logarithmic $J_c(H)$ plots of the (a)2ML and (b)3ML samples.	151
Figure 4.32: Definition of $HSPE$ and $H_{collapse}$ from logarithmic JcH	154
Figure 4.33: Temperature dependence of H_{SPE} for 2ML and 3ML.....	155
Figure 4.34: Temperature dependence of $H_{collapse}$ for 2ML and 3ML.	156
Figure 4.35: Temperature dependence of $h_{max,L}$ and $h_{max,H}$ for 2ML.....	158
Figure 4.36: Temperature dependence of $h_{max,L}$ and $h_{max,H}$ for 3ML.....	159
Figure 4.37: Raw OOP mH -curves of the 3ML sample measured at different temperatures.....	163
Figure 4.38: The upper critical field H_{c2} for 3ML sample at OOP-orientation.....	164
Figure 4.39: Raw OOP mH -loops of the 3ML sample.....	165
Figure 4.40: The apparent critical current density J_c for 3ML sample.	166
Figure 4.41: (a)Apparent pinning-force density Fp and (b)Normalized pinning-force curves of the 3ML sample.	168
Figure 5.1: Optical image of the patterned Au sacrificial mask developed for future electrical-device fabrication.....	175

LIST OF TABLES

Table 3.1: Components of Sb_2Te_3 crystal via EDS data	52
Table 3.2: Standard sputtering parameters for Nb and Sb_2Te_3 thin-film deposition	64
Table 3.3: Sputter power and relative deposition rate R	68
Table 3.4 : Structural design parameters of the Nb/ Sb_2Te_3 multilayer samples	70

Chapter 1 Introduction



1.1 Superconductivity

1.1.1 Superconductivity

Superconductivity refers to a macroscopic quantum state in which a material exhibits zero electrical resistance and magnetic-field expulsion below a critical temperature T_c . The phenomenon was first discovered by H. Kamerlingh Onnes in 1911 when the resistance of mercury suddenly vanished at liquid-helium temperatures [1]. This discovery established superconductivity as a distinct low-temperature phase rather than a simple extrapolation of normal metallic conduction.

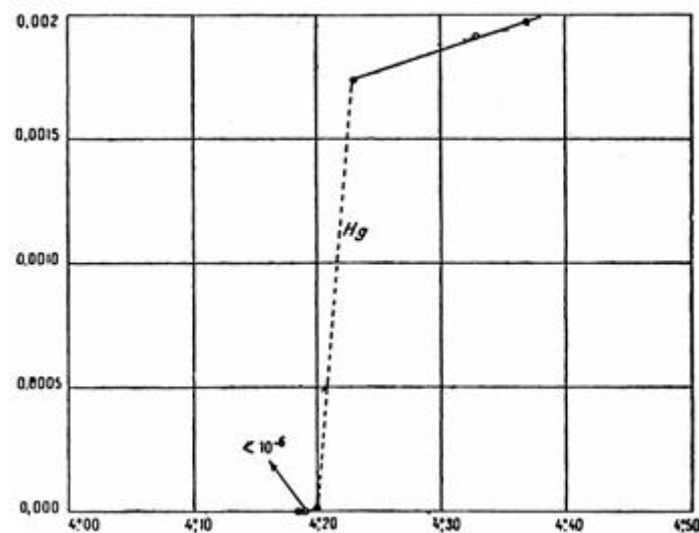


Figure 1.1: Onnes's graph plotting electrical resistance of a mercury sample versus temperature in kelvins [2].

A second essential feature of superconductivity was reported by Meissner and Ochsenfeld in 1933: when a material enters the superconducting state, the magnetic flux is expelled from its interior. This Meissner effect distinguishes a superconductor from a merely perfect conductor [3]. Later theoretical developments, including the Ginzburg–Landau theory and the Bardeen–Cooper–Schrieffer theory, provided phenomenological and microscopic descriptions of superconductivity, respectively [4], [5]. For type-II superconductors, Abrikosov further showed that magnetic flux can penetrate the superconductor in the form of quantized vortices between the lower and upper critical fields, H_{c1} and H_{c2} [6].

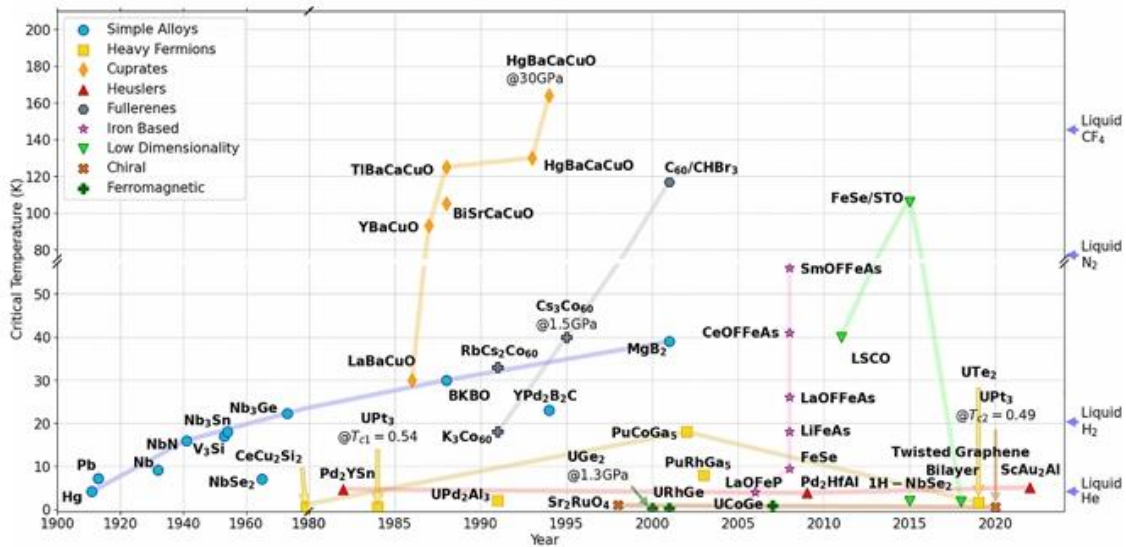
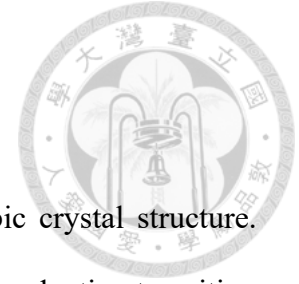


Figure 1.2: Historical overview of the discovery dates of superconducting materials and their respective critical temperatures, separated by “families [7].

1.1.2 Niobium



Niobium (Nb) is a transition metal with a body-centered cubic crystal structure. Among elemental superconductors, Nb has one of the highest superconducting transition temperatures, with T_c close to 9.2–9.3 K [8]. Because of its relatively high T_c , metallic character, and robust superconducting properties, Nb has been widely used as a conventional *s*-wave superconducting material in superconducting devices and hybrid structures.

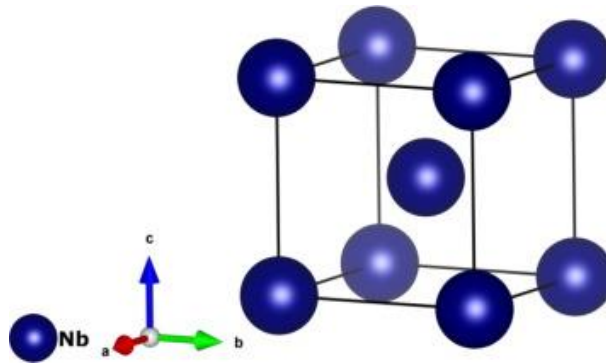


Figure 1.3: Cubic crystalline structure of metallic Nb [9].

In this thesis, Nb is used as the superconducting constituent of the multilayer system. The role of Nb in Chapter 1 is introduced only from the viewpoint of its basic metallic and superconducting properties. The effects of thin-film geometry, multilayer interfaces, vortex pinning, and magnetic hysteresis will be discussed in later chapters based on experimental measurements.



1.2 Topological Insulator

1.2.1 Discovery of Topological Insulators

Topological insulators are quantum materials characterized by an insulating bulk and conducting boundary states protected by time-reversal symmetry. The concept was first developed from the quantum spin Hall effect, in which spin-polarized edge states can exist without an external magnetic field [10]. The experimental observation of the quantum spin Hall state in HgTe quantum wells provided an important foundation for the later development of three-dimensional topological insulators [11].

In three-dimensional systems, angle-resolved photoemission spectroscopy (ARPES) played a central role in identifying topological surface states. $\text{Bi}_{1-x}\text{Sb}_x$ was among the first experimentally observed three-dimensional topological insulators [12]. Later, the Bi_2Se_3 , Bi, and Sb_2Te_3 family was proposed as a simpler class of strong topological insulators with a single Dirac cone on the surface [13]. This material family became especially important because its surface electronic structure is more direct and experimentally accessible.

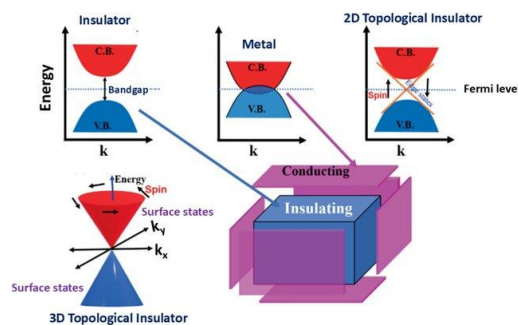


Figure 1.4: Schematic band diagram of insulator, metal with 2D (and 3D) topological insulators with insulating bulk comprising metallic edges states [14].



1.2.2 Sb_2Te_3

Sb_2Te_3 is a layered chalcogenide compound and a member of the Bi_2Se_3 -type topological-insulator family. Its crystal structure consists of quintuple layers stacked along the c -axis, with weak van der Waals bonding between neighboring quintuple layers. First-principles calculations predicted Sb_2Te_3 to be a strong topological insulator with a single Dirac cone surface state [13]. Therefore, Sb_2Te_3 provides a suitable platform for studying surface-state transport and interface-related phenomena.

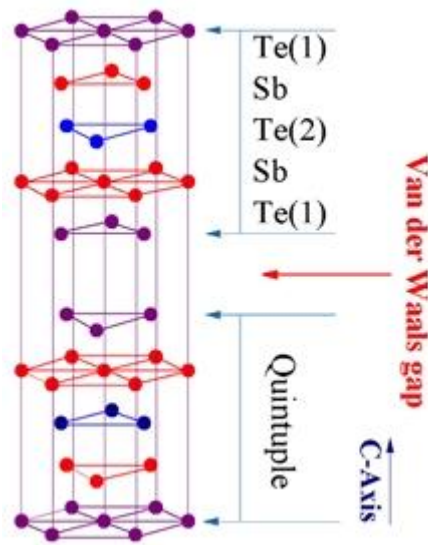


Figure 1.5: Schematic of Sb_2Te_3 crystal structure [15].

In practical samples, the transport properties of Sb_2Te_3 may include contributions from both surface and bulk channels. For this reason, experimental probes such as magnetotransport and weak anti-localization analysis are commonly used to distinguish possible conduction channels. In particular, the Hikami–Larkin–Nagaoka (HLN) fitting

parameter α is often used as an indicator of the effective number of coherent two-dimensional conducting channels [16], [17].



1.2.3 Topological Superconductor

A topological superconductor is a superconducting phase whose quasiparticle excitation spectrum has a nontrivial topological structure. Such systems are of great interest because they may host Majorana bound states at defects, vortices, or boundaries. One important theoretical proposal is to induce superconductivity on the surface of a strong topological insulator through the proximity effect with a conventional *s*-wave superconductor [18]. In this case, the spin-momentum-locked surface states of the topological insulator can combine with superconducting pairing to produce an effective topological superconducting state.

In this thesis, the concept of topological superconductivity is introduced as a broader motivation for studying hybrid systems composed of superconductors and topological insulators. The present work does not assume the direct realization of topological superconductivity. Instead, it focuses on how the superconducting response and vortex behavior are modified in Nb/Sb₂Te₃ multilayer thin films.

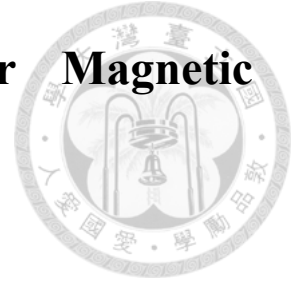
1.3 Research Motivation and Objectives

Topological insulators are important quantum materials because their conducting surface states provide a platform for studying spin-momentum locking, weak anti-localization, and possible interface-induced phenomena. When a topological insulator is

combined with a conventional superconductor, the interface becomes a possible region where superconducting pairing, surface-state transport, and vortex physics may interact. Therefore, Nb/Sb₂Te₃ multilayer thin films provide a suitable system for investigating the competition and coupling between superconductivity and topological-insulator-related electronic channels.

The transport analysis based on the HLN formula suggests that the fitting coefficient α is related to the number of effective coherent conducting channels [16], [17]. This provides a practical motivation for designing and comparing multilayer samples with different possible channel numbers. Instead of directly claiming the formation of topological superconductivity, this thesis investigates how changes in the multilayer structure and possible conduction channels are reflected in superconducting magnetic responses.

Chapter 2 Background Knowledge for Magnetic Analysis



2.1 Magnetic Response of Superconductors

Superconductivity is a macroscopic quantum state characterized by zero electrical resistance, magnetic flux expulsion, and the existence of limiting thermodynamic quantities such as the critical temperature, critical magnetic field, and critical current density. Historically, the disappearance of electrical resistance was first observed by Kamerlingh Onnes in mercury at liquid-helium temperature, while the magnetic flux expulsion from the superconducting state was later reported by Meissner and Ochsenfeld [19]. These two observations established that superconductivity is not merely a state of perfect conductivity, but a distinct thermodynamic phase with a characteristic magnetic response. The microscopic interpretation was later provided by Bardeen, Cooper, and Schrieffer, who described superconductivity as a condensate of Cooper pairs formed by an effective attractive interaction between electrons [4].

2.1.1 Superconducting Transition and Zero Resistance

The most direct transport signature of superconductivity is the disappearance of electrical resistance below the superconducting transition temperature T_c . In the normal metallic state, electrical resistivity originates from scattering processes such as electron–phonon scattering, impurity scattering, and defect scattering. When the system enters the superconducting state, a coherent condensate forms, and the DC resistivity ideally

vanishes:

$$\rho(T < T_c) = 0 \quad (2.1)$$



However, zero resistance alone is not sufficient to define superconductivity. A perfect conductor with zero resistivity would simply preserve the magnetic flux distribution that existed before cooling, whereas a superconductor actively expels magnetic flux from its interior. Therefore, superconductivity should be identified by both zero resistance and Meissner diamagnetism [19], [20]. This distinction is essential for magnetic measurements, because the superconducting transition can be identified not only through transport data but also through the onset of diamagnetic shielding in $m(T)$.

2.1.2 Meissner Effect

The Meissner effect is the expulsion of magnetic flux from the interior of a superconductor when the material is cooled below T_c . This phenomenon demonstrates that the superconducting state is a thermodynamic equilibrium state rather than simply a zero-resistance state [19]. The London brothers later introduced a phenomenological electrodynamic description that successfully explained the exponential decay of magnetic field inside a superconductor [20]. The characteristic length scale of this field penetration is the London penetration depth, which will be discussed in Section 2.1.3.

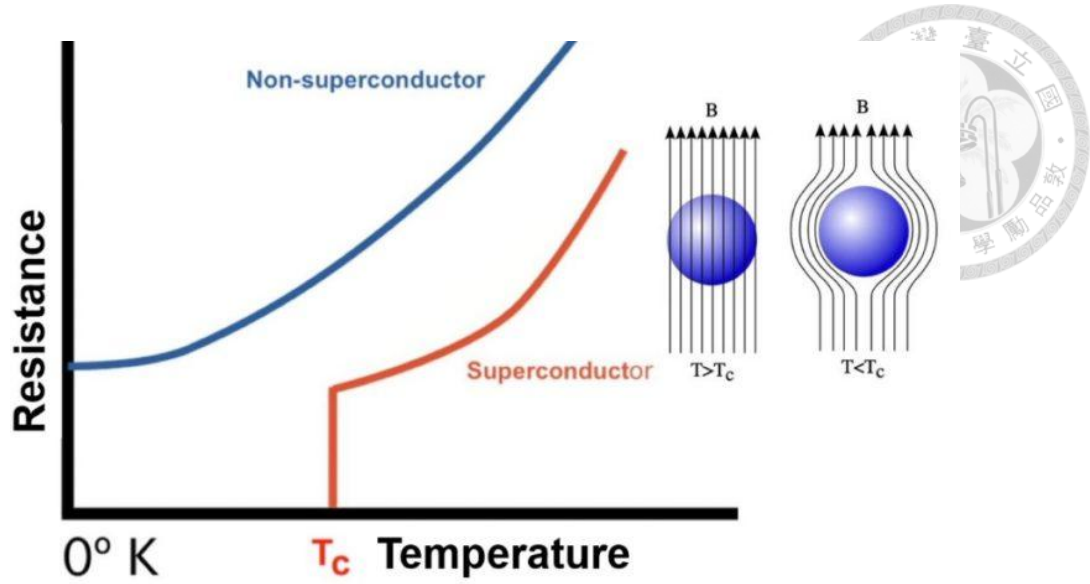


Figure 2.1: Fundamental properties of a superconductor [21].

In the ideal Meissner state, the magnetic induction inside a bulk superconductor is zero:

$$B = 0 \quad (2.2)$$

Using the SI relation between magnetic induction, applied field, and magnetization,

$$B = \mu_0(H + M) \quad (2.3)$$

the ideal full-shielding condition becomes

$$M = -H \quad (2.4)$$

Therefore, the ideal volume susceptibility is

$$\chi_v = \frac{M}{H} = -1 \quad (2.5)$$

In real samples, however, the measured magnetic response is modified by geometry and demagnetization effects. The internal magnetic field can be approximated as

$$H_{\text{int}} = H_{\text{app}} - NM \quad (2.6)$$

where H_{app} is the applied field, H_{int} is the internal field, and N is the demagnetization factor. This correction is particularly important for thin films, because the apparent magnetic susceptibility can change significantly with the direction of the applied field. Therefore, the measured $m(T)$ signal should be interpreted as a geometry-dependent magnetic response rather than a direct measure of ideal shielding.

2.1.3 Critical Magnetic Field

The superconducting state is stable only within a finite region of temperature, magnetic field, and current density. When the applied magnetic field exceeds a characteristic value, superconductivity is suppressed and the system returns to the normal state. For type-I superconductors, the boundary between the superconducting and normal states is described by a single thermodynamic critical field H_c . A common phenomenological expression for its temperature dependence is

$$H_c(T) = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right] \quad (2.7)$$

where $H_c(0)$ is the thermodynamic critical field at zero temperature.

For type-II superconductors, the magnetic response is governed by two critical magnetic fields: the lower critical field H_{c1} and the upper critical field H_{c2} . The theoretical foundation for distinguishing type-I and type-II superconductivity was established through the Ginzburg–Landau theory [22], while Abrikosov showed that superconducting alloys and high- κ superconductors can host quantized flux vortices

between H_{c1} and H_{c2} [23]. The three characteristic field regions are

$$H < H_{c1} : \text{Meissner state}$$

$$H_{c1} < H < H_{c2} : \text{mixed state}$$

$$H > H_{c2} : \text{normal state}$$



The lower critical field H_{c1} marks the field at which magnetic flux first enters the superconductor in the form of vortices, while H_{c2} represents the field at which superconductivity is completely suppressed. In magnetic measurements, these fields are not merely theoretical boundaries; they provide reference scales for interpreting vortex penetration, magnetic irreversibility, and the evolution of field-dependent magnetization curves. Therefore, H_{c1} , H_{c2} , and related field scales play a central role in the magnetic analysis of type-II superconductors.

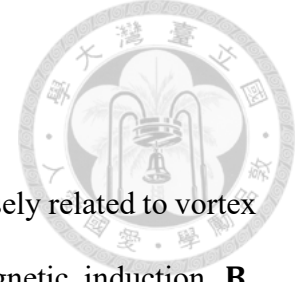
2.1.4 Critical Current

A superconductor can carry electrical current without dissipation only below a limiting current density. This limiting value is called the critical current density, J_c . In a simple geometrical definition,

$$J_c = \frac{I_c}{A} \quad (2.8)$$

where I_c is the critical current and A is the effective current-carrying cross-sectional area. In transport measurements, J_c is usually determined by the appearance of a finite voltage. In magnetic measurements, J_c can be inferred from the irreversible magnetization

associated with flux pinning and persistent screening currents.



For type-II superconductors, the magnetic critical current is closely related to vortex motion. When a current density $\mathbf{J}$ flows in the presence of magnetic induction $\mathbf{B}$, vortices experience a Lorentz force density:

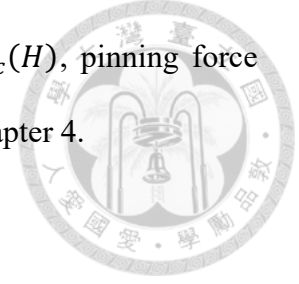
$$\mathbf{F}_L = \mathbf{J} \times \mathbf{B} \quad (2.9)$$

If vortices are not pinned, this force drives vortex motion, leading to dissipation. Therefore, a finite critical current in type-II superconductors is sustained by pinning centers that prevent vortex motion. The balance between Lorentz force and pinning force is the physical origin of the critical-state behavior [24], [25].

The critical-state model proposed by Bean provides a phenomenological framework for connecting magnetization hysteresis to critical current density [25]. In this model, the width of a magnetic hysteresis loop reflects the magnitude of persistent current circulating inside the superconductor. A larger loop width corresponds to a larger irreversible magnetization and therefore to a larger J_c . This relation forms the basis for estimating $J_c(H)$ from $m(H)$ -loop measurements in type-II superconductors.

The general connection between irreversible magnetization, vortex pinning, and critical current has been extensively discussed in later studies of hard superconductors, flux pinning, and vortex matter [24], [26], [27]. In the context of this thesis, J_c is treated as a bridge between measured magnetic hysteresis and vortex pinning behavior. The detailed definition and extraction procedure of J_c from $m(H)$ -loop data will be

introduced in Chapter 3, while the field-dependent behavior of $J_c(H)$, pinning force $F_p(H)$, and normalized pinning force $f_p(h)$ will be analyzed in Chapter 4.



2.2 Critical Fields in Type-II Superconductors

2.2.1 London penetration depth

The Meissner effect demonstrates that a superconductor is not merely a perfect conductor with zero resistivity, but a thermodynamic phase that actively expels magnetic flux from its interior. In order to describe this electromagnetic response phenomenologically, F. London and H. London proposed the London equations, which provide one of the earliest theoretical descriptions of superconducting electrodynamics [20]. Within this framework, the magnetic field is not abruptly excluded at the surface; instead, it penetrates a finite distance into the superconductor and decays exponentially. This characteristic length scale is known as the London penetration depth, λ_L .

The London theory assumes that a superconducting current density $\mathbf{J}_s$ is generated by the coherent motion of superconducting carriers. One of the London equations relates the curl of the superconducting current density to the magnetic induction $\mathbf{B}$:

$$\nabla \times \mathbf{J}_s = -\frac{n_s q^2}{m^*} \mathbf{B} \quad (2.10)$$

where n_s is the superconducting carrier density, q is the carrier charge, and m^* is the effective mass. Combining this equation with Maxwell's equation,

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}_s \quad (2.11)$$

gives

$$\mu_0 \nabla \times \mathbf{J}_s \quad (2.12)$$

Using the vector identity

$$\nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} \quad (2.13)$$

and the condition

$$\nabla \cdot \mathbf{B} = 0 \quad (2.14)$$

one obtains

$$-\mu_0 \frac{n_s q^2}{m^*} \mathbf{B} \quad (2.15)$$

or equivalently,

$$\frac{1}{\lambda_L^2} \mathbf{B} \quad (2.16)$$

where the London penetration depth is defined as

$$\lambda_L = \sqrt{\frac{m^*}{\mu_0 n_s q^2}} \quad (2.17)$$

This equation shows that λ_L is determined by the density and effective mass of superconducting carriers. A larger superconducting carrier density leads to a shorter penetration depth, meaning that the magnetic field is screened more efficiently. Conversely, a smaller n_s results in a larger λ_L , allowing the magnetic field to penetrate deeper into the superconductor.

For a semi-infinite superconductor occupying the region ($x > 0$), with an external magnetic field applied parallel to the surface, the solution of the London equation is

$$B(x) = B(0) \exp\left(-\frac{x}{\lambda_L}\right) \quad (2.18)$$

where $B(0)$ is the magnetic field at the surface. This expression directly demonstrates the physical meaning of λ_L : it is the characteristic distance over which the magnetic field decays to $(1/e)$ of its surface value [28].

The London penetration depth is also connected to the superconducting condensate density. Since the superfluid density is proportional to n_s/m^* , the inverse square of the penetration depth is proportional to the superfluid density:

$$\frac{1}{\lambda_L^2} \propto \frac{n_s}{m^*} \quad (2.19)$$

Therefore, the temperature dependence of $\lambda_L(T)$ provides information about the evolution of the superconducting condensate. As the temperature approaches the superconducting transition temperature T_c , the superconducting carrier density decreases, causing $\lambda_L(T)$ to increase. In the limit near T_c , the weakening of the condensate makes magnetic screening less effective.

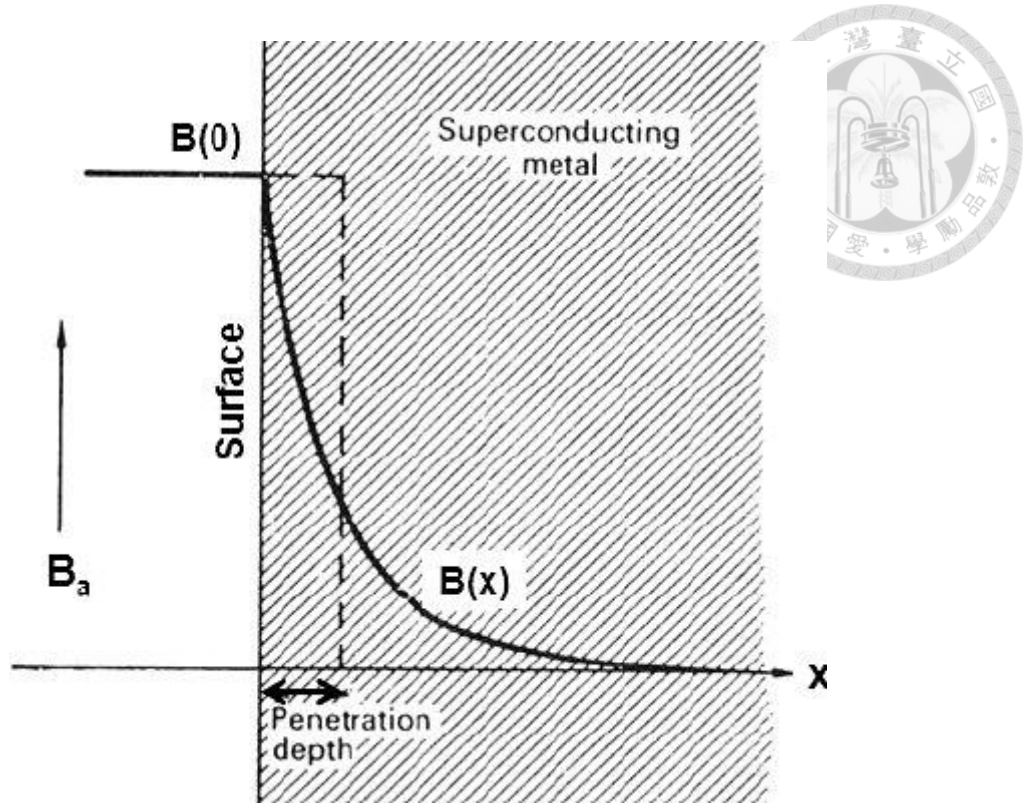


Figure 2.2: Exponential decrease of the magnetic flux $\mathbf{B}$ with distance x into a bulk superconductor [29].

2.2.2 Pippard coherence length

Although the London theory successfully explains the Meissner effect and introduces the magnetic-field screening length λ_L , it is based on a local electrodynamic relation. In the local London description, the superconducting current density at a position $\mathbf{r}$ is determined only by the vector potential at the same position. Under the London gauge, this relation can be written as

$$\mathbf{J}_s(\mathbf{r}) = -\frac{1}{\mu_0 \lambda_L^2} \mathbf{A}(\mathbf{r}) \quad (2.20)$$

where $\mathbf{J}_s$ is the superconducting current density, λ_L is the London penetration depth, and $\mathbf{A}$ is the magnetic vector potential. This local approximation is valid when the

electromagnetic field varies slowly over the characteristic length scale of the superconducting condensate. However, Pippard pointed out that such a strictly local relation is incomplete because the superconducting carriers are correlated over a finite spatial range [30].

Pippard introduced a nonlocal electrodynamic relation by arguing that the current density at a point should be determined not only by the vector potential at that point, but also by the vector potential within a surrounding volume. This spatial range is characterized by the Pippard coherence length, ξ_0 . In this picture, ξ_0 represents the distance over which the superconducting wave function remains spatially coherent, or equivalently the characteristic size over which the motion of superconducting electrons is correlated [30], [31].

A simplified form of Pippard's nonlocal relation can be expressed as

$$\mathbf{J}_s(\mathbf{r}) \propto - \int K(\mathbf{r} - \mathbf{r}') \mathbf{A}(\mathbf{r}') d^3r' \quad (2.21)$$

where $K(\mathbf{r} - \mathbf{r}')$ is a nonlocal kernel describing how the vector potential at $\mathbf{r}'$ contributes to the current density at $\mathbf{r}$. The kernel decays over a characteristic distance of order ξ_0 . Therefore, the London theory can be regarded as the limiting case in which the kernel becomes sharply localized:

$$K(\mathbf{r} - \mathbf{r}') \rightarrow \delta(\mathbf{r} - \mathbf{r}') \quad (2.22)$$

In this limit, the current density depends only on the local vector potential, and the London relation is recovered.

The physical origin of Pippard coherence length becomes clearer from the microscopic BCS theory. In the weak-coupling BCS limit, the zero-temperature coherence length is commonly written as

$$\xi_0 = \frac{\hbar v_F}{\pi \Delta(0)} \quad (2.23)$$

where v_F is the Fermi velocity and $\Delta(0)$ is the superconducting energy gap at zero temperature [32]. Since weak-coupling BCS theory gives

$$\Delta(0) = 1.764 k_B T_c \quad (2.24)$$

the coherence length can also be expressed as

$$\xi_0 \approx 0.18 \frac{\hbar v_F}{k_B T_c} \quad (2.25)$$

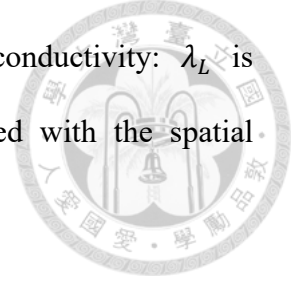
This relation shows that ξ_0 is inversely proportional to the superconducting energy gap and to the transition temperature. A superconductor with a large energy gap or high T_c tends to have a shorter coherence length, while a conventional low- T_c metal with a small gap and large Fermi velocity usually has a long coherence length.

The Pippard coherence length is conceptually different from the London penetration depth. The London penetration depth λ_L describes how far a magnetic field penetrates into a superconductor before being screened, whereas the Pippard coherence length ξ_0 describes the spatial range over which the superconducting condensate remains coherent. In other words,

λ_L : magnetic field screening length

ξ_0 : superconducting coherence length

These two length scales describe different aspects of superconductivity: λ_L is associated with electromagnetic screening, while ξ_0 is associated with the spatial rigidity of the superconducting condensate.



Pippard also emphasized that impurity scattering modifies the effective range of nonlocal electrodynamics. When the electron mean free path ℓ becomes shorter than the intrinsic coherence length, the spatial range over which superconducting electrons remain correlated is reduced. A commonly used phenomenological relation is

$$\frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{\ell} \quad (2.26)$$

where ξ is the effective coherence length and ℓ is the electronic mean free path [28], [31].

In the clean limit,

$$\ell \gg \xi_0 \quad (2.27)$$

the effective coherence length approaches the intrinsic Pippard coherence length:

$$\xi \approx \xi_0 \quad (2.28)$$

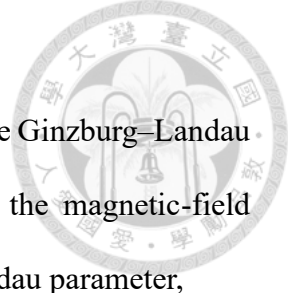
In the dirty limit,

$$\ell \ll \xi_0 \quad (2.29)$$

the effective coherence length is strongly reduced by impurity scattering:

$$\xi \approx \ell \quad (2.30)$$

This result indicates that structural disorder, impurity scattering, and defects can significantly modify the electrodynamic response of a superconductor.



The distinction between λ_L and ξ_0 later became essential in the Ginzburg–Landau description of superconductivity. In GL theory, the ratio between the magnetic-field screening length and the coherence length defines the Ginzburg–Landau parameter,

$$\kappa = \frac{\lambda}{\xi} \quad (2.31)$$

which determines whether a superconductor belongs to type-I or type-II superconductivity. Although the GL coherence length ξ is not identical in definition to the Pippard coherence length ξ_0 , both describe the characteristic spatial scale of superconducting coherence. Therefore, Pippard’s introduction of coherence length provides an important conceptual bridge between London electrodynamics, microscopic BCS theory, and the later GL classification of superconductors [32].

In the context of magnetic analysis, the coherence length is important because it is related to the spatial scale over which the superconducting order parameter can vary. In type-II superconductors, this length scale is also associated with the vortex-core size. Since each vortex contains a normal-like core where superconductivity is suppressed, the coherence length provides an estimate of the characteristic radius of the vortex core. This connection makes ξ essential for understanding vortex penetration, upper critical field, vortex pinning, and the magnetic hysteresis behavior discussed in later sections.

2.2.3 Ginzburg–Landau Theory and Type-I/Type-II Classification

Ginzburg–Landau theory provides a phenomenological framework for describing superconductivity in terms of a complex order parameter. Unlike London theory, which

focuses mainly on electromagnetic screening, and Pippard theory, which introduces the nonlocal coherence length, GL theory describes both the spatial variation of the superconducting condensate and its coupling to the electromagnetic field. Although it was originally formulated phenomenologically by Ginzburg and Landau in 1950 [22], it was later shown by Gor'kov that GL theory can be derived from the microscopic BCS theory near the superconducting transition temperature T_c . Therefore, GL theory serves as an important bridge between macroscopic superconducting electrodynamics and microscopic superconductivity.

The central quantity in GL theory is the complex superconducting order parameter,

$$\psi(\mathbf{r}) = |\psi(\mathbf{r})|e^{i\theta(\mathbf{r})} \quad (2.32)$$

where $|\psi(\mathbf{r})|$ represents the amplitude of the superconducting condensate and $\theta(\mathbf{r})$ is its phase. The squared amplitude is proportional to the local density of superconducting carriers:

$$|\psi(\mathbf{r})|^2 \propto n_s(\mathbf{r}) \quad (2.33)$$

Thus, a spatially uniform superconducting state corresponds to a nearly constant $|\psi|$, while suppression of superconductivity, such as near a boundary, defect, or vortex core, corresponds to a reduction of $|\psi|$.

In the GL framework, the free energy density of a superconductor in a magnetic field can be written as

$$f_s = f_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}|(-i\hbar\nabla - q^*\mathbf{A})\psi|^2 + \frac{B^2}{2\mu_0} \quad (2.34)$$

where f_s and f_n are the superconducting and normal-state free energy densities,

respectively. The parameters α and β are phenomenological GL coefficients, m^* and q^* are the effective mass and charge of a Cooper pair, $\mathbf{A}$ is the magnetic vector potential, and $\mathbf{B} = \nabla \times \mathbf{A}$ is the magnetic induction. The coefficient α changes sign at T_c , and is usually written as

$$\alpha = \alpha_0(T - T_c) \quad (2.35)$$

with $\beta > 0$ to ensure the stability of the free energy.

Each term in the GL free energy has a clear physical meaning. The term $\alpha|\psi|^2$ controls the onset of superconductivity, while $\frac{\beta}{2}|\psi|^4$ stabilizes the magnitude of the order parameter. The gradient term,

$$\frac{1}{2m^*} |(-i\hbar\nabla - q^*\mathbf{A})\psi|^2 \quad (2.36)$$

represents the energy cost associated with spatial variations of the order parameter and its coupling to the electromagnetic field. The final term,

$$\frac{B^2}{2\mu_0} \quad (2.37)$$

is the magnetic-field energy density.

In the absence of magnetic field and spatial variation, the equilibrium value of the order parameter can be obtained by minimizing

$$f_s - f_n = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 \quad (2.38)$$

with respect to $|\psi|$. This gives

$$\frac{\partial}{\partial |\psi|} \left(\alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 \right) = 0 \quad (2.39)$$

and therefore

$$2\alpha |\psi| + 2\beta |\psi|^3 = 0 \quad (2.40)$$

For the superconducting state with $|\psi| \neq 0$, one obtains

$$|\psi_0|^2 = -\frac{\alpha}{\beta} \quad (2.41)$$

Since $\beta > 0$, superconductivity requires $\alpha < 0$, corresponding to

$$T < T_c \quad (2.42)$$

This result shows that the superconducting order parameter grows continuously below T_c , which is consistent with a second-order phase transition.

By minimizing the GL free energy with respect to (ψ^*) , the first GL equation is obtained:

$$\alpha \psi + \beta |\psi|^2 \psi + \frac{1}{2m^*} (-i\hbar \nabla - q^* \mathbf{A})^2 \psi = 0 \quad (2.43)$$

This equation describes how the superconducting order parameter varies in space under the influence of magnetic field and boundary conditions. Minimization with respect to the vector potential $\mathbf{A}$ gives the superconducting current density:

$$\mathbf{J}_s = \frac{q^* \hbar}{2m^* i} (\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{q^{*2}}{m^*} |\psi|^2 \mathbf{A} \quad (2.44)$$

This expression is the second GL equation. It shows that the supercurrent is determined by both the spatial variation of the phase of the order parameter and the electromagnetic vector potential.



Two fundamental length scales naturally emerge from GL theory. The first is the GL coherence length, ξ , which describes the characteristic distance over which the order parameter recovers when it is locally suppressed:

$$\xi = \sqrt{\frac{\hbar^2}{2m^*|\alpha|}} \quad (2.45)$$

The second is the GL penetration depth, λ , which describes the length scale over which magnetic field is screened:

$$\lambda = \sqrt{\frac{m^*}{\mu_0 q^{*2} |\psi_0|^2}} \quad (2.46)$$

These two length scales have different physical meanings. The coherence length ξ measures the spatial rigidity of the superconducting condensate, while the penetration depth λ measures the efficiency of magnetic-field screening. Their ratio defines the Ginzburg–Landau parameter,

$$\kappa = \frac{\lambda}{\xi} \quad (2.47)$$

The value of κ determines whether a superconductor belongs to type-I or type-II superconductivity. The critical boundary is

$$\kappa_c = \frac{1}{\sqrt{2}} \quad (2.48)$$

For

$$\kappa < \frac{1}{\sqrt{2}}$$

the superconductor is classified as type-I. In this case, the normal-superconducting

interface has positive surface energy, and the system tends to minimize the interface area between superconducting and normal regions. As a result, type-I superconductors usually exhibit a complete Meissner state below the thermodynamic critical field H_c , and then transform into the normal state when the applied field exceeds H_c .

For

$$\kappa > \frac{1}{\sqrt{2}}$$

the superconductor is classified as type-II. In this case, the normal-superconducting interface has negative surface energy, making it energetically favorable for magnetic flux to penetrate the superconductor in the form of quantized vortices. Abrikosov showed that, within GL theory, type-II superconductors can support a mixed state between the lower critical field H_{c1} and the upper critical field H_{c2} . This mixed state contains quantized vortex lines, each carrying one flux quantum,

$$\Phi_0 = \frac{h}{2e} \quad (2.49)$$

The basic classification can therefore be summarized as

$$\kappa < \frac{1}{\sqrt{2}} : \text{type-I superconductor}$$

$$\kappa > \frac{1}{\sqrt{2}} : \text{type-II superconductor}$$

In GL theory, the upper critical field is related to the coherence length by

$$\mu_0 H_{c2} = \frac{\Phi_0}{2\pi\xi^2} \quad (2.50)$$

This relation shows that a shorter coherence length corresponds to a larger upper

critical field. The lower critical field is associated with the field at which vortex penetration becomes energetically favorable. For a large- κ type-II superconductor, it is often approximated as

$$\mu_0 H_{c1} \approx \frac{\Phi_0}{4\pi\lambda^2} \ln \kappa \quad (2.51)$$

These equations show that ξ and λ are not only abstract theoretical length scales, but also directly connected to experimentally measurable critical fields.

In the context of this thesis, GL theory provides the theoretical foundation for discussing type-II superconductivity, vortex penetration, and magnetic hysteresis. The London penetration depth λ describes magnetic-field screening, while the coherence length ξ is associated with the spatial variation of the superconducting order parameter and the vortex-core size. Their ratio, κ , determines whether the system can support vortex states. Therefore, GL theory naturally connects the phenomenology of superconductivity to the vortex matter and magnetic analysis discussed in the following sections.

2.3 Vortex Matter and Irreversible Magnetization

For a type-II superconductor with a Ginzburg–Landau parameter satisfying $\kappa > 1/\sqrt{2}$, magnetic flux is no longer completely expelled when the applied field lies between the lower critical field H_{c1} and the upper critical field H_{c2} . Instead, the magnetic flux penetrates the superconductor in the form of quantized vortices. Based on Ginzburg–

Landau theory, Abrikosov demonstrated that these vortices can form a periodic vortex lattice in equilibrium, with the triangular lattice generally possessing the lowest free energy in an isotropic system [33]. The mixed state is therefore not simply a coexistence of macroscopic superconducting and normal regions. It is a collective system composed of a large number of interacting quantized vortices and is commonly referred to as vortex matter [26].

Each Abrikosov vortex carries one magnetic flux quantum:

$$\Phi_0 = \frac{h}{2e} \quad (2.52)$$

where h is Planck's constant and e is the elementary charge. At the center of each vortex is a core region in which the superconducting order parameter is suppressed. The characteristic radius of this core is of the order of the coherence length ξ . The circulating screening current surrounding the vortex core extends over a characteristic length scale determined by the penetration depth λ . Therefore, ξ characterizes the vortex-core dimension, whereas λ determines the spatial distribution of the magnetic field and supercurrent surrounding the vortex.

For a mixed state with an average magnetic induction B , the areal vortex density is

$$n_v = \frac{B}{\Phi_0} \quad (2.53)$$

If the vortices form an ideal triangular lattice, the nearest-neighbor vortex spacing a_0 is

$$a_0 = \left(\frac{2\Phi_0}{\sqrt{3}B} \right)^{1/2} \quad (2.54)$$



The vortex density therefore increases and the intervortex spacing decreases with increasing magnetic field. Overlap between the magnetic fields and circulating currents of neighboring vortices produces a repulsive intervortex interaction, which favors an ordered arrangement in a defect-free equilibrium system. The formation of an individual vortex also requires core and magnetic-field energies. In the London limit of $\kappa \gg 1$, the energy per unit length of an isolated vortex can be approximated as

$$\varepsilon_1 \approx \frac{\Phi_0^2}{4\pi\mu_0\lambda^2} \ln \kappa \quad (2.55)$$

This energy scale gives the vortex line an effective line tension that resists bending and local deformation [26].

Real superconducting materials generally contain crystallographic defects, impurities, voids, grain boundaries, surface roughness, and material interfaces. These spatial inhomogeneities modify either the vortex-core energy or the surrounding supercurrent distribution, causing vortices to preferentially occupy particular positions. The elastic interaction between vortices tends to preserve an ordered vortex arrangement, whereas quenched disorder tends to displace vortices from their equilibrium lattice positions. Larkin and Ovchinnikov demonstrated that even when individual defects are weak, a large number of randomly distributed defects can collectively produce a finite pinning force and distort the vortex lattice beyond a characteristic correlation length [34]. The state of vortex matter is therefore determined by competition among intervortex

interactions, vortex–defect interactions, thermal fluctuations, and external driving forces.

Depending on magnetic field, temperature, disorder strength, and effective dimensionality, the vortex system may exhibit an approximately ordered Abrikosov lattice, an elastically distorted vortex solid, a strongly disordered and metastable vortex-glass-like state, or a mobile vortex-liquid or flowing regime [26]. These terms describe distinct correlation and dynamical behaviors of vortex matter. However, an experimentally observed crossover does not necessarily constitute a rigorous thermodynamic phase transition. Changes in magnetic hysteresis, characteristic fields, or critical current generally identify effective vortex regimes or crossovers. Establishing a true vortex-glass transition or vortex-lattice melting boundary requires additional dynamical evidence, such as magnetic relaxation, current–voltage scaling, or other time-dependent measurements.

When a current density $\mathbf{J}$ flows through a type-II superconductor with magnetic induction $\mathbf{B}$, the vortex system experiences the Lorentz force density

$$\mathbf{F}_L = \mathbf{J} \times \mathbf{B} \quad (2.56)$$

For an individual vortex, the Lorentz force per unit vortex length can be written as

$$\mathbf{f}_L = \mathbf{J} \times \Phi_0 \hat{\mathbf{t}} \quad (2.57)$$

where $\hat{\mathbf{t}}$ denotes the local direction of the vortex line. Neglecting inertia and adopting an overdamped approximation, the motion of the i -th vortex may be represented conceptually by

$$\eta \mathbf{v}_i = \mathbf{f}_L + \sum_{j \neq i} \mathbf{f}_{vv,ij} + \mathbf{f}_{p,i} + \mathbf{f}_{T,i} \quad (2.58)$$

where η is the viscous-drag coefficient, $\mathbf{f}_{vv,ij}$ is the intervortex force, $\mathbf{f}_{p,i}$ is the pinning force produced by material disorder, and $\mathbf{f}_{T,i}$ represents thermal fluctuations. This expression emphasizes that vortex motion is governed by the combined competition among multiple interactions rather than by a single force [26].

Vortex motion with an average velocity $\mathbf{v}$ generates an electric field

$$\mathbf{E} = \mathbf{B} \times \mathbf{v} \quad (2.59)$$

leading to a finite voltage and energy dissipation. If the pinning force is sufficient to oppose the Lorentz force, the vortices remain approximately immobilized and the material can sustain a persistent current. Even when the applied driving force is insufficient to depin the vortices directly, thermal fluctuations may enable vortices to overcome a finite pinning barrier. The corresponding thermally activated rate can be expressed schematically as

$$\Gamma = \Gamma_0 \exp \left[-\frac{U(J, H, T)}{k_B T} \right] \quad (2.60)$$

where $U(J, H, T)$ is an effective activation barrier dependent on current, magnetic field, and temperature. The Anderson–Kim model describes this thermally activated vortex motion as flux creep, explaining why persistent currents and irreversible magnetization relax gradually with time rather than forming an absolutely static state [35].

In an ideal type-II superconductor without defects, surface barriers, or geometrical barriers, a sufficiently slow field sweep would allow the vortex configuration to remain

in thermodynamic equilibrium. The increasing- and decreasing-field magnetization branches would then coincide, producing a reversible magnetic response. In contrast, vortex pinning introduces barriers to flux entry, redistribution, and exit. A nonuniform magnetic-induction profile and persistent screening currents consequently remain inside the sample. The vortex system is trapped in field-history-dependent metastable states, causing the increasing- and decreasing-field branches to separate and producing magnetic hysteresis [36],[35].

For a properly background-subtracted and approximately symmetric $M(H)$ -loop, the two field-sweep branches may be decomposed into approximate equilibrium and irreversible components:

$$M_{\text{eq}}(H) \approx \frac{M_{\text{inc}}(H) + M_{\text{dec}}(H)}{2} \quad (2.61)$$

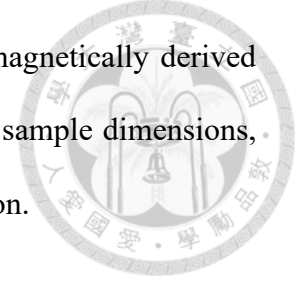
$$M_{\text{irr}}(H) \approx \frac{M_{\text{dec}}(H) - M_{\text{inc}}(H)}{2} \quad (2.62)$$

where M_{inc} and M_{dec} denote the increasing- and decreasing-field branches, respectively. The hysteresis width is therefore

$$\Delta M(H) = |M_{\text{dec}}(H) - M_{\text{inc}}(H)| = 2|M_{\text{irr}}(H)| \quad (2.63)$$

When bulk pinning dominates and the critical-state approximation is applicable, ΔM reflects the persistent current density sustained inside the sample. The Bean critical-state model consequently establishes a quantitative relation between the magnetic hysteresis width and the critical current density J_c [36]. A larger ΔM generally

corresponds to a stronger irreversible magnetization and a larger magnetically derived critical current density. Its quantitative value, however, depends on sample dimensions, field orientation, and the geometrical model adopted in the calculation.



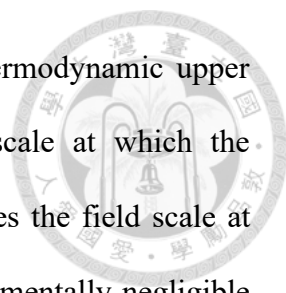
Irreversible magnetization is not produced exclusively by bulk vortex pinning. A microscopic surface barrier may impede vortex entry or exit across the sample boundary, while the nonuniform demagnetizing field of a nonellipsoidal specimen may generate a geometrical barrier. Studies by Zeldov et al. and Brandt demonstrated that thin plates and strips can exhibit magnetic hysteresis and irreversible magnetization even in the absence of substantial bulk pinning [37], [38]. The width, symmetry, and orientation dependence of an $m(H)$ -loop must therefore be considered together, particularly for thin films and multilayer structures. An approximately symmetric hysteresis width is commonly associated predominantly with bulk pinning and persistent currents, whereas pronounced branch asymmetry may contain contributions from surface barriers, geometrical barriers, flux trapping, or other nonequilibrium processes.

Irreversible magnetization generally decreases as temperature or magnetic field increases. When thermal activation, vortex interactions, and external driving reduce the hysteresis width below the experimentally resolvable level, an irreversibility field H_{irr} may be defined. In an idealized form,

$$\Delta M(H_{\text{irr}}, T) \rightarrow 0 \quad (2.64)$$

or equivalently,

$$J_c(H_{\text{irr}}, T) \rightarrow 0 \quad (2.65)$$



The irreversibility field is not, however, equivalent to the thermodynamic upper critical field H_{c2} . The upper critical field specifies the field scale at which the superconducting order parameter disappears, whereas H_{irr} identifies the field scale at which the macroscopic irreversible vortex response becomes experimentally negligible under a particular measurement timescale and criterion. Consequently, H_{irr} may depend on temperature, field-sweep rate, observation time, instrumental sensitivity, and the selected threshold criterion.

In summary, vortex matter is a collective system formed by quantized magnetic-flux lines and their mutual interactions. Irreversible magnetization is the macroscopic manifestation of a nonequilibrium flux distribution and persistent currents produced when vortex motion is restricted by pinning or flux-penetration barriers. The hysteresis width, critical current density, irreversibility field, and magnetic relaxation therefore constitute important experimental quantities for investigating vortex matter. The following section introduces flux pinning and the critical-state model in greater detail, providing the theoretical basis for deriving the critical current density and pinning strength from magnetic hysteresis measurements.

2.4 Interpretation of anomalous magnetization-loop morphologies

2.4.1 Second Peak Effect

In a type-II superconductor with appreciable flux pinning, the width of the magnetic hysteresis loop generally decreases as the applied magnetic field increases. This behavior

reflects the progressive suppression of superconductivity and the reduction of the irreversible critical current density that can be sustained at high fields. In some superconductors, however, the hysteresis width increases again after its initial decrease, producing a secondary maximum away from the central zero-field peak. This nonmonotonic field response is commonly referred to as the Second Peak Effect (SPE), the second magnetization peak (SMP), or the fishtail effect [39], [40].

For a background-subtracted $M(H)$ -loop containing increasing- and decreasing-field branches, the hysteresis width is expressed as

$$\Delta M(H) = |M_{\text{dec}}(H) - M_{\text{inc}}(H)| \quad (2.66)$$

where $M_{\text{inc}}(H)$ and $M_{\text{dec}}(H)$ denote the magnetization on the increasing- and decreasing-field branches, respectively. Within the Bean critical-state framework, the hysteresis width is directly related to the critical current density:

$$J_c(H) \propto \Delta M(H) \quad (2.67)$$

A monotonically decreasing $J_c(H)$ therefore produces a hysteresis loop that progressively closes with increasing magnetic field. If $J_c(H)$ increases again over a finite field interval, the hysteresis width expands correspondingly and produces a secondary peak or shoulder in the $m(H)$ -loop. From this perspective, the direct physical meaning of the SPE is a nonmonotonic enhancement of the irreversible current or effective pinning capability, rather than an additional equilibrium magnetization signal defined solely by the visual appearance of the loop [39], [40].

Within the critical-state model, the spatial distribution of magnetic induction inside

the sample is governed by

$$\frac{dB(r)}{dr} = \pm \mu_0 J_c(B) \quad (2.68)$$

where $B(r)$ is the local magnetic induction at position r . Since the experimentally measured magnetization depends on the magnetic-induction distribution throughout the sample, a local enhancement of $J_c(B)$ modifies the internal flux gradient and consequently changes both the hysteresis width and the $m(H)$ -loop morphology. Johansen et al. introduced an additional peak function into $J_c(B)$ and demonstrated that a nonmonotonic critical-current dependence naturally produces a fishtail-shaped magnetization loop within the critical-state model [40].

Gokhfeld extended this concept to asymmetric magnetic hysteresis loops using an extended critical-state model. In this model, the critical current density is represented by a monotonically decreasing background contribution $J'_c(B)$ multiplied by a localized enhancement term [39]:

$$J_c(B) = J'_c(B)[1 + Af_{\text{peak}}(B)] \quad (2.69)$$

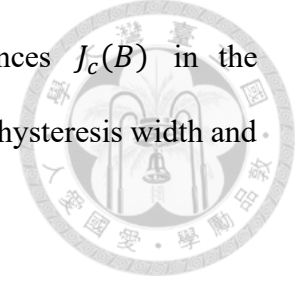
where A represents the enhancement amplitude relative to the background critical current density, and $f_{\text{peak}}(B)$ is a normalized peak function satisfying

$$f_{\text{peak}}(B_{\text{peak}}) = 1 \quad (2.70)$$

The parameter B_{peak} is the magnetic induction at the center of the critical-current enhancement, whereas the width of the peak function describes the field interval over which the enhancement is effective. When $A = 0$, the model reduces to a monotonically



decreasing $J'_c(B)$ without a second peak. Increasing A enhances $J_c(B)$ in the intermediate- or high-field region and correspondingly increases the hysteresis width and the height of the secondary peak.



The internal induction B_{peak} is not necessarily identical to the applied field at which the secondary peak is directly observed. Because the sample contains an internal flux gradient, self-field contributions, and demagnetizing fields, the applied field H is not locally identical to $B(r)$ throughout the specimen. A characteristic field identified from an experimental $m(H)$ -loop should therefore be interpreted as an effective field scale associated with the overall magnetic-flux distribution rather than as a direct measurement of a single local B_{peak} .

In addition to $J_c(B)$, the Gokhfeld model considers a surface region in which equilibrium magnetization is established. The depth of this region is denoted by $L_s(B)$, and its relative thickness with respect to a characteristic sample radius R is

$$P_a(B) = \frac{L_s(B)}{R} \quad (2.71)$$

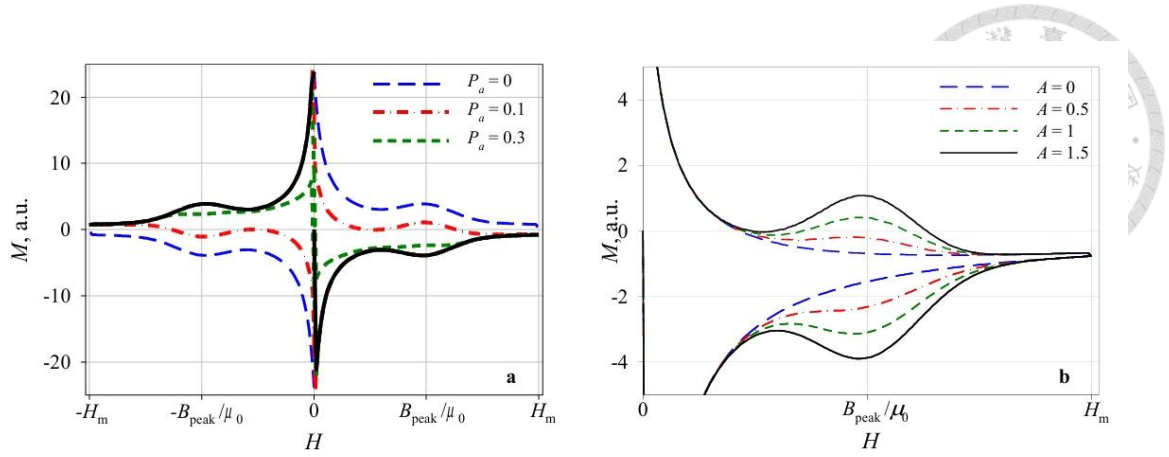


Figure 2.3: Magnetization loops of II-type superconductor [39].

A larger L_s indicates that a greater fraction of the sample is close to an equilibrium magnetic state, reducing the separation between the two field-sweep branches. As $L_s/R \rightarrow 1$, the magnetic response approaches reversibility. Conversely, stronger pinning and screening reduce the depth of the equilibrium surface region. Gokhfeld described the local reduction of $L_s(B)$ using the same peak function responsible for the enhancement of $J_c(B)$ [39]:

$$L_s(B) = \frac{L'_s(B)}{1 + Af_{\text{peak}}(B)} \quad (2.72)$$

where $L'_s(B)$ denotes the background field dependence in the absence of the second-peak enhancement. This relation indicates that the increase of $J_c(B)$ in the SPE regime is accompanied by stronger magnetic screening and a reduced equilibrium surface layer.

This modification is particularly relevant to asymmetric $m(H)$ -loops. Gokhfeld's calculations showed that the secondary peak on one field-sweep branch is governed mainly by $J_c(B)$, whereas the peak height on the other branch is affected by both $J_c(B)$ and $L_s(B)$. Consequently, the two sweep branches may exhibit different secondary-peak

amplitudes even when the underlying $J_c(B)$ enhancement is approximately symmetric with respect to the field direction. Branch asymmetry therefore does not necessarily imply different critical-current mechanisms under positive and negative fields. It may instead result from different combinations of irreversible bulk magnetization and equilibrium surface contributions.

Experimentally, the SPE may be described using several characteristic fields. The onset field H_{on} marks the field at which the original decrease of the hysteresis width or $J_c(H)$ is interrupted and an additional enhancement begins. The field H_{sp} or H_{peak} denotes the local maximum of the second peak. At higher fields, $J_c(H)$ decreases again and eventually approaches the experimental criterion used to define the irreversibility field H_{irr} :

$$H_{on} < H_{sp} < H_{irr} \quad (2.73)$$

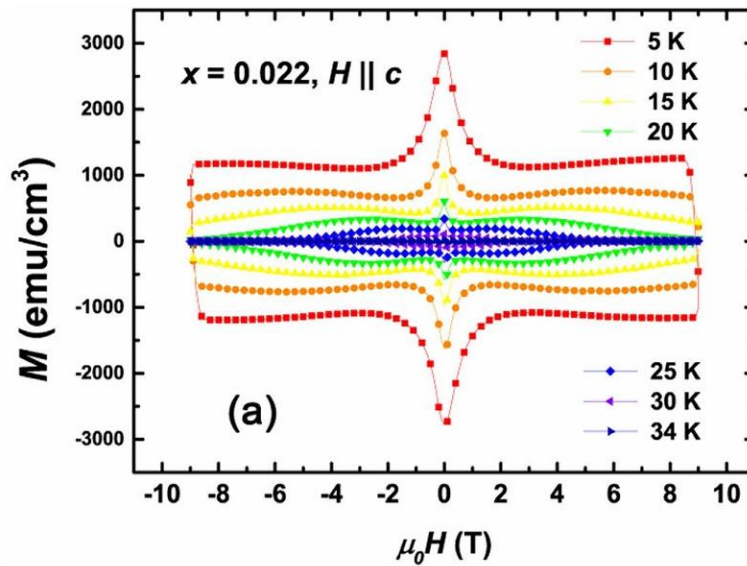
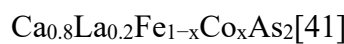
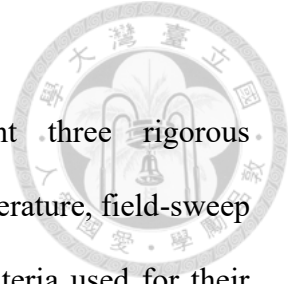


Figure 2.4: Magnetic field H dependence of magnetization in 112-type superconductor



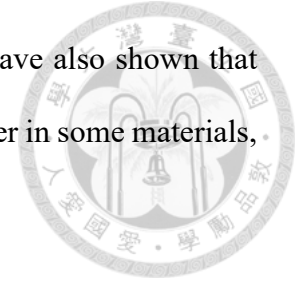


These characteristic fields do not necessarily represent three rigorous thermodynamic phase boundaries. Their values may depend on temperature, field-sweep rate, magnetic relaxation, sample geometry, and the operational criteria used for their extraction. The precise determination criteria must therefore be defined separately in the methodology chapter.

The microscopic origin of the SPE is not unique. Mikitik and Brandt associated the peak effect with a transition from a relatively ordered Bragg-glass-like state to a more disordered vortex-glass-like state. The reduced elastic correlation of the disordered vortex configuration allows the vortices to adapt more effectively to the random pinning landscape, thereby increasing J_c [42]. Other studies have shown that structural transformations of the vortex lattice may also enhance the critical current, including a square-to-rhombic vortex-lattice transition in tetragonal superconductors [43]. History dependence and hysteretic behavior observed by local magnetic measurements further indicate that the second peak may involve transformations and metastable coexistence between ordered and disordered vortex configurations in some materials [44], [45].

The SPE may also be associated with a crossover from collective elastic creep to plastic vortex creep, competition between different pinning mechanisms, modifications of the defect distribution, or spatial inhomogeneity between superconducting and nonsuperconducting regions. Gokhfeld specifically noted that both a vortex-lattice transition and phase separation can produce a nonmonotonic $J_c(B)$. The peak function in the extended critical-state model should therefore be regarded as a phenomenological representation of an effective pinning enhancement rather than as the identification of a

unique microscopic mechanism [39]. Magnetic-relaxation studies have also shown that the second peak can coincide with an elastic-to-plastic creep crossover in some materials, although this correspondence is not universal [46].



In summary, the SPE can be rigorously described as a nonmonotonic enhancement of $J_c(H)$ or the magnetic hysteresis width over a finite magnetic-field interval. The critical-state model establishes the direct relation between an enhancement of $J_c(B)$ and the secondary-peak morphology, whereas the extended critical-state model further demonstrates how an equilibrium surface layer can produce branch asymmetry. The SPE therefore provides an important phenomenological indication of a change in the vortex-pinning state. Nevertheless, the morphology of a secondary peak in an $m(H)$ -loop alone is insufficient to uniquely determine its microscopic origin. The following sections discuss abrupt low-field vortex processes and the limitations of morphology-based physical identification.

2.4.2 Abrupt low-field vortex processes

When a type-II superconductor is swept from a large positive magnetic field toward zero, a fraction of the vortices oriented along the positive-field direction may remain trapped by flux pinning. These remanent vortices do not disappear immediately when the applied field reaches zero or reverses direction. The magnetic response is therefore strongly dependent on the previous field history.

In a conventional type-II superconductor, reversal of the applied field does not directly reverse the orientation of the trapped vortices. When a negative field is applied

while positive vortices remain inside the sample, vortices with the opposite orientation first enter from the sample boundary and annihilate the pre-existing positive vortices. As the negative field is increased further, additional negative vortices are created and progressively replace the original magnetic flux. Conventional flux reversal can therefore be represented schematically as

vortex pinning $\rightarrow$ vortex creation-annihilation $\rightarrow$ vortex creation.

Patiño et al. observed a different low-field process in a Ni/Nb/Ni/CoO superconducting–ferromagnetic spin-valve structure [47]. In their sample, the Nb-layer thickness was smaller than the estimated vortex-core diameter. The vortex circulating currents could therefore extend into the neighboring Ni layers through the proximity effect. Polarized electrons in the ferromagnets were consequently subjected to the vortex magnetic field, while an opposite reaction torque acted on the vortex currents.

When penetration of the vortex currents into the ferromagnet is limited by the ferromagnetic coherence length ξ_F , the characteristic vortex field acting on the adjacent ferromagnetic layers may be estimated as

$$B_v \sim \frac{4\Phi_0}{\pi(d_S + 2\xi_F)^2} \quad (2.74)$$

where d_S is the superconducting-layer thickness. The coupling energy between a ferromagnetic electron moment $\boldsymbol{\mu}$ and the vortex field $\mathbf{B}_v$ can be written as

$$U = -\boldsymbol{\mu} \cdot \mathbf{B}_v \quad (2.75)$$

This energy is reduced when the vortex magnetic moment is aligned with the average magnetization of the ferromagnetic layer. Reversal of the ferromagnetic magnetization within its low-field coercive region can therefore directly drive a reversal of the vortex orientation. Patiño et al. referred to this process as vortex flipping [47].

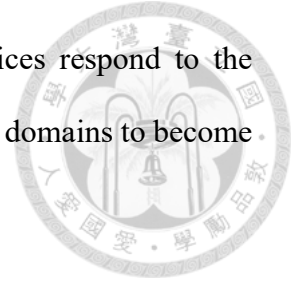
The corresponding physical picture consists of several stages. Starting from a large positive field, positively oriented vortices remain trapped while the field is reduced. When the unpinned Ni layer reverses first, a fraction of the vortices flips with its magnetization. If N_+ and N_- denote the numbers of positively and negatively oriented vortices, respectively, the net vortex magnetization may be represented schematically as

$$M_v \propto N_+ - N_- \quad (2.76)$$

When partial vortex flipping produces $N_+ \approx N_-$, the magnetic contributions from the two vortex orientations approximately cancel. The measured superconducting magnetization may consequently collapse abruptly toward zero. Such a collapse does not imply that all vortices have exited the sample, nor does it necessarily indicate that pinning has vanished. It may instead reflect cancellation between oppositely oriented vortex populations.

As the negative applied field is increased further, the Ni layer pinned by the antiferromagnetic layer gradually rotates. Although the ferromagnetic reversal is gradual, Patiño et al. observed an abrupt reversal of the remaining vortices. Once the average magnetization of the pinned layer becomes negative, the remaining positive vortices rapidly flip into the negative direction, producing a sudden sign reversal of the

superconducting magnetization. This result indicates that the vortices respond to the average ferromagnetic magnetization and do not require all magnetic domains to become fully aligned before flipping occurs.



After vortex flipping is completed, further increase of the negative field creates additional negative vortices. The dominant vortex sequence in the Patiño structure is therefore

vortex pinning → vortex flipping → vortex creation

This mechanism produces abrupt drops, plateaus, or sign changes in the $m(H)$ -loop near zero field or the ferromagnetic coercive field, rather than the smoother flux-annihilation response expected in a conventional superconductor.

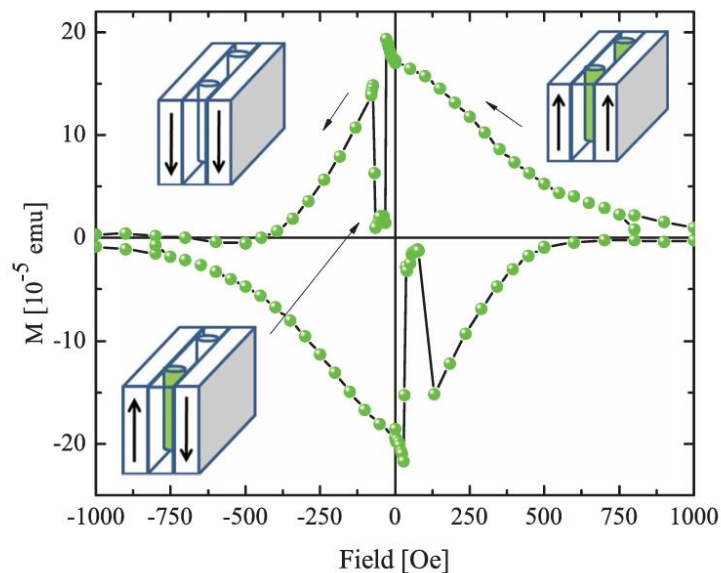
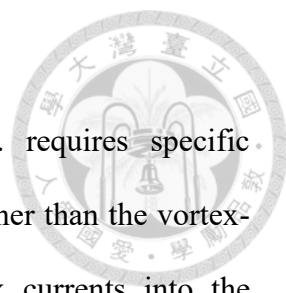


Figure 2.5: Low-field magnetization hysteresis and schematic vortex-flipping process in a Ni/Nb/Ni/CoO [47].



The vortex-flipping mechanism proposed by Patiño et al. requires specific conditions, including a superconducting layer comparable to or thinner than the vortex-core diameter, sufficient proximity-induced penetration of vortex currents into the ferromagnet, and an appropriate ferromagnetic coherence length and exchange interaction. It should therefore not be directly generalized to superconducting systems without ferromagnetic layers.

Nevertheless, this work establishes an important morphological principle: an abrupt low-field collapse or sign reversal of magnetization does not necessarily originate from the Second Peak Effect and does not by itself demonstrate that J_c or the pinning strength suddenly vanishes. It may instead arise from reorientation of trapped flux, cancellation between oppositely oriented vortices, or nonequilibrium vortex reconstruction triggered by the magnetic state of an adjacent material. Abrupt low-field anomalies and intermediate- or high-field SPE features should therefore be treated as potentially distinct vortex processes.

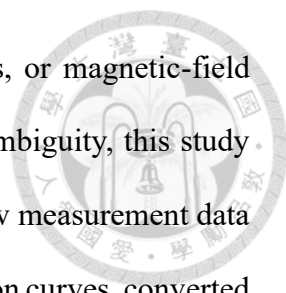
Chapter 3 Experimental Methods and Data Processing Procedures



3.1 Chapter Overview

This chapter describes the sample preparation, structural characterization, magnetic measurements, and standardized data-processing procedures employed in this study. The fabrication and magnetic characterization of the Nb/Sb₂Te₃ multilayer thin films in this work were partly developed based on previous studies of Nb/Sb₂Te₃ heterostructures conducted in our laboratory. That work established the basic experimental procedures for Sb₂Te₃ target preparation, magnetron sputtering deposition, X-ray diffraction (XRD) structural characterization, and $M-T$ and $M-H$ magnetic measurements. It also reported that the Nb/Sb₂Te₃ system may exhibit the Paramagnetic Meissner Effect (PME), the Second Peak Effect (SPE), and complex vortex behavior [48]. Based on this foundation, the present study further extends the investigation to Nb/Sb₂Te₃ multilayer thin films with different numbers of layers and stacking configurations, with particular emphasis on the evolution of magnetic hysteresis, critical current density, and vortex pinning behavior as functions of temperature and magnetic field.

Many characteristic quantities used in superconducting magnetic analysis are not obtained directly from the raw experimental data. Instead, they must be defined through specific criteria and numerical procedures. Examples include the lower critical field (H_{c1}), irreversibility field (H_{irr}), upper critical field (H_{c2}), and SPE characteristic field. Therefore, if the analysis relies solely on manual point selection or visual interpretation of individual



curves, comparisons among different samples, temperature regimes, or magnetic-field ranges may be affected by subjective uncertainty. To reduce such ambiguity, this study establishes a standardized data-processing workflow in which the raw measurement data are classified, background-subtracted, reconstructed into magnetization curves, converted into physical quantities, locally fitted, and analyzed for characteristic field extraction according to fixed procedures. This standardized workflow ensures that the extracted parameters from different samples are comparable and reproducible. It also provides the methodological basis for the discussion of vortex pinning, SPE, and multilayer-structure effects in Chapter 4.

3.2 Sb₂Te₃ Bulk Target Preparation

3.2.1 Stoichiometric Calculation and Sintering Process

To obtain stable and reproducible Sb₂Te₃ thin-film quality, it is essential to first prepare a compositionally uniform Sb₂Te₃ bulk precursor with a stoichiometry close to the ideal compound. The preparation procedure used in this study was adapted from the high-temperature melting and slow-cooling method reported by Sultana et al. for bulk Sb₂Te₃ crystal growth [49], and was further modified based on the Sb₂Te₃ target-preparation procedure previously established in our laboratory for Sb₂Te₃/Nb heterostructure studies [48]. The process was standardized according to the experimental requirements of the present work.

High-purity Sb powder with a purity of 99.5% and Te powder with a purity of 99.8% were used as the starting materials, as shown in Figure 3.1. The preparation procedure for

the Sb_2Te_3 bulk crystal was as follows:

1. Sb powder with a purity of 99.5% and Te powder with a purity of 99.8% were mixed according to a mass ratio of 2:3.
2. The mixed powders were thoroughly ground to improve the homogeneity of the precursor mixture.
3. The ground powder mixture was loaded into a quartz tube.
4. The quartz tube was evacuated to approximately 10^{-2} Torr and sealed to reduce oxidation and contamination during the subsequent heat treatment.
5. The sealed quartz tube was placed in a furnace and heated to 800 °C at a rate of 2 °C/min.
6. The sample was held at 800 °C for 18 hours to allow sufficient melting and reaction between Sb and Te.
7. The furnace was slowly cooled to 650 °C at a rate of 2 °C/h, allowing crystal growth to proceed during the cooling process.
8. Finally, the furnace was turned off and the sample was naturally cooled to room temperature.

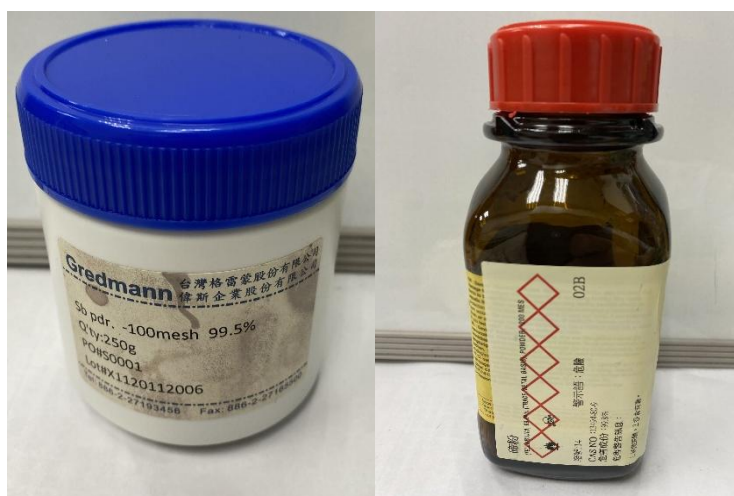


Figure 3.1: Raw materials of Sb and Te.

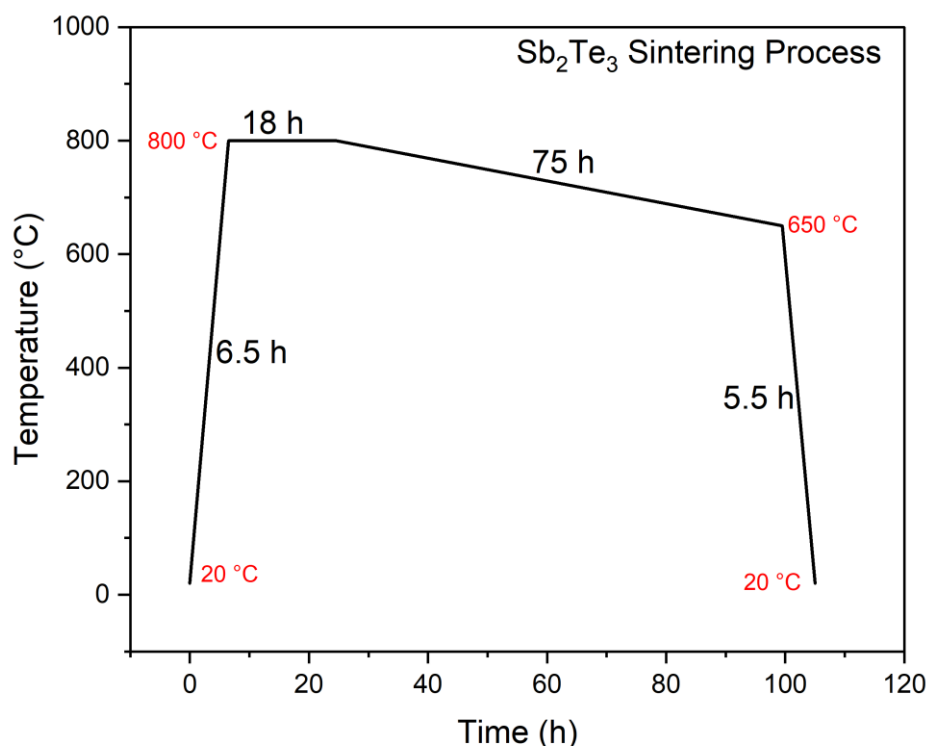


Figure 3.2: Sintering process of Sb₂Te₃.

In Step 1, since the atomic masses of Sb and Te are relatively close, the starting composition can be approximately prepared using a mass ratio of 2:3, corresponding to the stoichiometric composition of Sb₂Te₃. During the preparation process, the size of the quartz tube and the amount of precursor powder loaded into each tube must be carefully controlled. In this study, relatively thick quartz tubes with an outer diameter of approximately 9 mm were used, and the total amount of powder loaded into each tube was limited to less than 5 g. This precaution is necessary because the melting point of Te is approximately 449.51 °C, which is far below the maximum sintering temperature of 800 °C used in this work. In addition, Te exhibits a considerable equilibrium vapor pressure in this temperature range. Reported vapor-pressure data indicate that the vapor

pressure of Te near 800 °C can reach the order of several tens to 10^2 Torr [50], which is much higher than the initial vacuum level of approximately 10^{-2} Torr before sealing. Therefore, molten Te is expected to undergo significant evaporation, or even boiling-like behavior, during heating, leading to a rapid increase in the internal pressure of the sealed quartz tube. If the quartz tube is too narrow, the amount of loaded powder is excessive, or the sealing quality is poor, the risk of quartz-tube rupture will increase. Therefore, controlling the powder amount per tube and using a sufficiently thick quartz tube are essential safety parameters in the standardized Sb_2Te_3 bulk-preparation procedure adopted in this study.

3.2.2 EDS and XRD Characterization of Sb_2Te_3 Bulk Crystal

After sintering, a small amount of white crystalline residue may form on the outer surface of the main Sb_2Te_3 ingot or on the inner wall of the quartz tube. This residue may originate from unreacted elements, locally condensed volatile species, or secondary reaction products. It was therefore removed before subsequent grinding and target pressing to avoid affecting the compositional uniformity of the sputtering target and the quality of the deposited thin films. Product appearance shown as



Figure 3.3: Crystal of Sb_2Te_3 after sintering process.

To confirm whether the sintered Sb_2Te_3 bulk crystal was suitable as the precursor material for sputtering-target fabrication, Energy-Dispersive X-ray Spectroscopy (EDS) and X-ray Diffraction (XRD) measurements were performed before the subsequent grinding and target-pressing processes. EDS was used to examine the Sb/Te elemental ratio in the main ingot region, while XRD was used to verify the crystal structure and phase composition. Sb_2Te_3 is a tetradymite-type layered compound that is generally assigned to the rhombohedral $R\bar{3}m$ space group [51].

Figure 3.4 and Table 3.1 show the EDS result obtained from the main region of the sintered Sb_2Te_3 bulk crystal. The measured weight percentages of Sb and Te were 39.09 wt.% and 60.91 wt.%, respectively, while the corresponding atomic percentages were 40.21 at.% and 59.79 at.%. For stoichiometric Sb_2Te_3 , the ideal atomic ratio of Sb to Te is 2:3, corresponding to 40 at.% Sb and 60 at.% Te. Therefore, the EDS result obtained in this work is very close to the ideal stoichiometric composition, indicating that the main region of the sintered ingot possesses an appropriate elemental ratio.

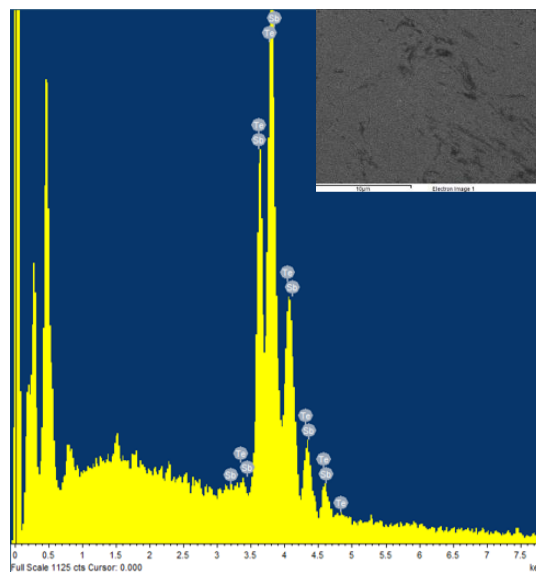


Figure 3.4: Energy-dispersive X-ray spectroscopy of Sb_2Te_3 bulk.

Elements	Weight %	Atomic %
Sb	39.09	40.21
Te	60.91	59.79
Totals	100	

Table 3.1: Components of Sb_2Te_3 crystal via EDS data

Figure 3.5 shows the XRD pattern of the same batch of Sb_2Te_3 bulk crystal. The main diffraction peaks can be assigned to the rhombohedral structure of Sb_2Te_3 , and no obvious diffraction peaks from elemental Sb, elemental Te, or other secondary phases are observed. This result indicates that the weighing, sealing, melting, and slow-cooling procedures adopted in this study effectively produced a bulk material dominated by the Sb_2Te_3 phase. Combining the EDS and XRD results, the sintered main ingot region was confirmed to be compositionally and structurally suitable for further grinding and pressing into an Sb_2Te_3 sputtering target.

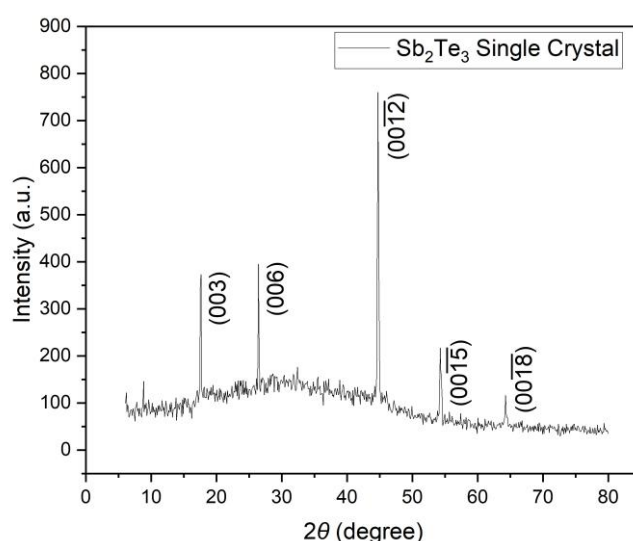


Figure 3.5: X-ray diffraction of Sb_2Te_3 Bulk.



3.2.3 Grinding and Target Compression

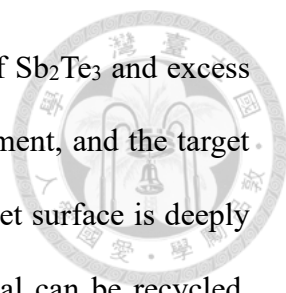
After confirming the composition and structure of the Sb_2Te_3 bulk crystal, the main ingot was further ground and pressed into a sputtering target for magnetron sputtering deposition. Since Sb_2Te_3 thin films may suffer from Te loss during sputtering, deviations from the stoichiometric composition can introduce defects and affect the crystallographic orientation and structural quality of the deposited films. Therefore, controlling the target composition is an important processing parameter in this study. Tan *et al.* reported that highly (00 l) -preferentially oriented layered Sb_2Te_3 films can be obtained by co-sputtering Sb_2Te_3 and Te targets [52]. In the present work, due to the limited target positions in the sputtering chamber, dual-target co-sputtering was not adopted. Instead, following the approach established in previous work, additional Te was introduced directly into the target material to compensate for possible Te loss during sputtering [48].

The preparation procedure for the Sb_2Te_3 mixed target was as follows:

1. The main Sb_2Te_3 ingot region confirmed by EDS and XRD was selected.
2. White crystalline residues or possible secondary products on the outer surface were removed by sandpaper.
3. The Sb_2Te_3 ingot was thoroughly ground into fine powder.
4. Te powder with a molar amount equal to the Te content in the original Sb_2Te_3 was added, giving an overall Sb:Te ratio of 2:6.
5. The mixed Sb_2Te_3 and Te powders were ground again to improve homogeneity.
6. The mixed powder was loaded into a 2-inch mold and pressed into a circular target using a hydraulic press.
7. The pressed target was sintered in a nitrogen atmosphere at 200 °C for at least 12 h.

In this procedure, Step 4 is the key step in the target-composition design. Since 1 mol of Sb_2Te_3 contains 2 mol of Sb and 3 mol of Te, adding another 3 mol of Te results in an overall Sb:Te molar ratio of 2:6 in the mixed powder. This Te-rich mixed target is not a stoichiometric Sb_2Te_3 target, but is intentionally designed to compensate for Te loss during the subsequent sputtering process.

Through this method, the Te content can be increased from the target-composition side without using Sb_2Te_3 /Te dual-target co-sputtering. Previous work in our laboratory also showed that adding extra Te into the Sb_2Te_3 target enables the growth of c-axis-oriented Sb_2Te_3 thin films [48]. Therefore, this mixed-target preparation method was adopted as one of the standardized processes for the subsequent deposition of Nb/ Sb_2Te_3 multilayer thin films.



It should be noted that this target is a mixed target consisting of Sb_2Te_3 and excess Te. As a result, it is consumed more rapidly under plasma bombardment, and the target surface may undergo nonuniform erosion. In practice, when the target surface is deeply eroded or penetrated during sputtering, the remaining target material can be recycled, reground, and pressed again. To maintain the target thickness and sputtering stability, the total amount of precursor powder before repressing was kept at approximately 60 g. If the recovered powder was insufficient, additional $\text{Sb}_2\text{Te}_3/\text{Te}$ powder was supplied according to the same procedure to maintain the designed target composition.

3.3 Thin Film Deposition

3.3.1 Sputtering System Configuration

The Nb/ Sb_2Te_3 multilayer thin films investigated in this study were deposited using a magnetron sputtering system. The basic configuration of the sputtering chamber and the selection of sputtering modes followed the experimental platform previously established in our laboratory for Nb/ Sb_2Te_3 heterostructure fabrication [48]. In this system, high-purity Ar gas is introduced into the vacuum chamber to generate the sputtering plasma. Under an applied electric field, Ar atoms are ionized, and the resulting Ar^+ ions are accelerated toward the target surface. The target atoms are then ejected by ion bombardment and subsequently deposited onto the substrate. Permanent magnets installed behind the sputtering gun confine electrons near the target surface, thereby enhancing the local plasma density and improving the sputtering efficiency [53].

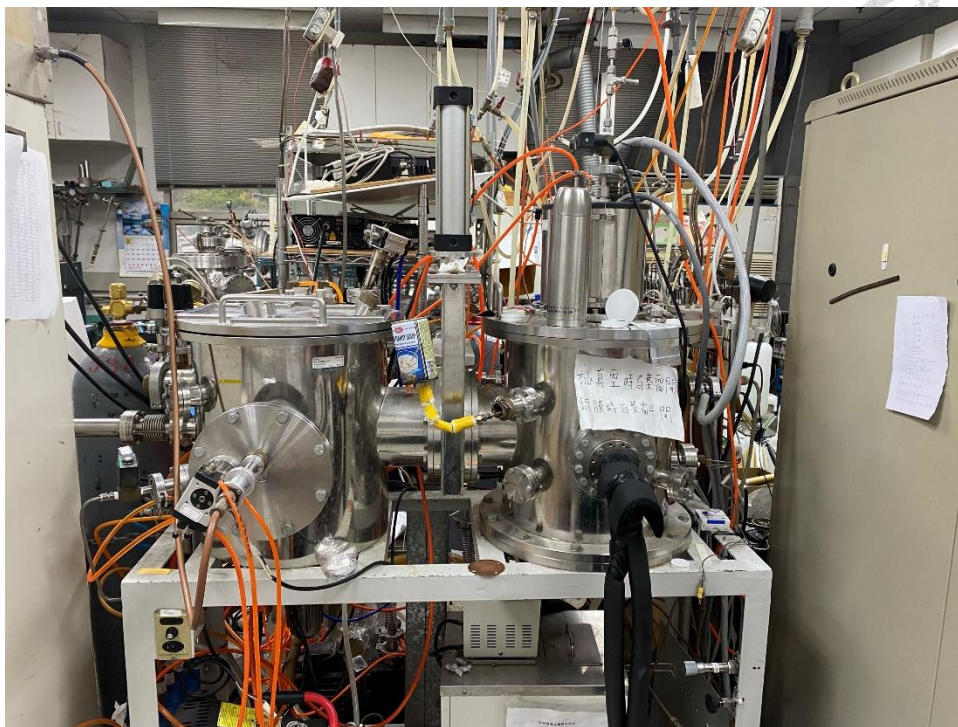


Figure 3.6: Sputter system.

In the present work, Sb_2Te_3 and Nb were deposited using different sputtering modes according to their electrical properties. The Sb_2Te_3 target was operated using Radio-Frequency magnetron sputtering (RF sputtering), whereas the Nb target was operated using Direct-Current magnetron sputtering (DC sputtering). DC sputtering is generally suitable for electrically conductive metallic targets, such as Nb, because charge can be continuously removed from the target surface during ion bombardment. In contrast, RF sputtering is more appropriate for targets with relatively poor electrical conductivity, such as Sb_2Te_3 -based targets, because the alternating electric field helps reduce charge accumulation on the target surface and suppresses arcing during deposition [48], [53].

In addition to following the chamber configuration used in previous work, the target–substrate geometry was explicitly measured in this study to standardize the deposition

conditions. As illustrated in Figure 3.7: , the sputtering system adopts an off-axis geometry, in which the Nb and Te-rich Sb_2Te_3 targets are mounted on opposite sides of the chamber and face the substrate region laterally. Therefore, the deposition geometry was characterized by two parameters: the horizontal offset from each target surface to the substrate rotation axis and the vertical height difference between the target axis and the substrate surface.

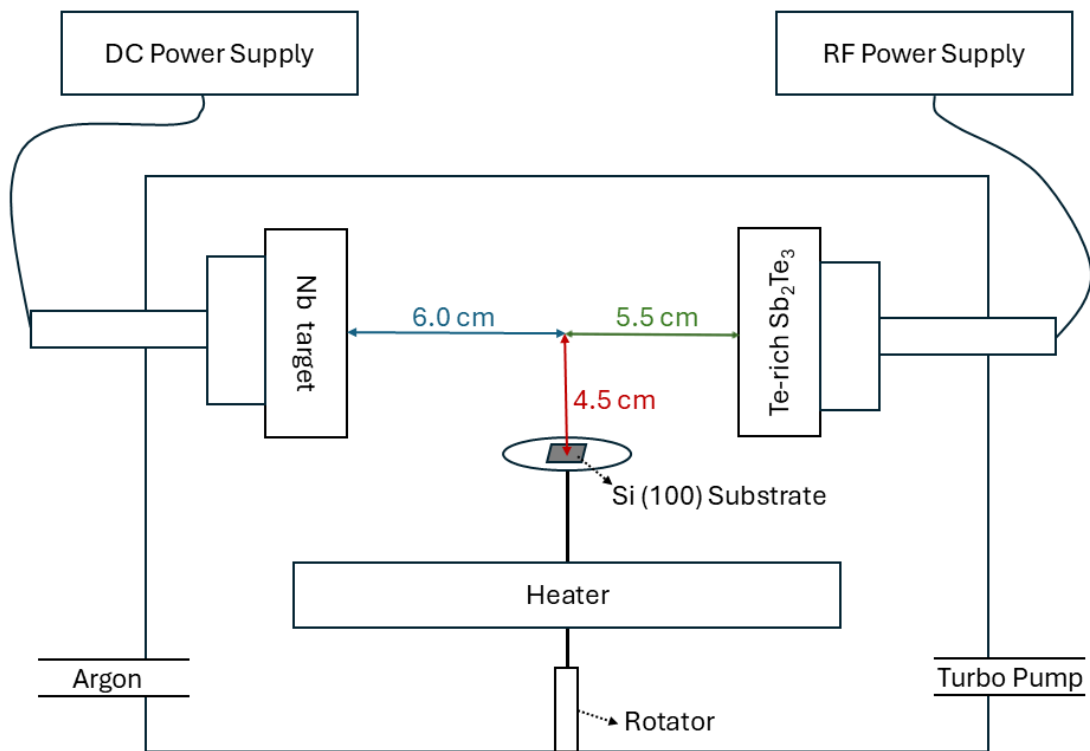


Figure 3.7: Schematic of the sputtering chamber geometry and the measured target–substrate distances.

In this configuration, the horizontal distance from the Nb target surface to the substrate rotation axis was 6.0 cm, while that from the Te-rich Sb_2Te_3 target surface to the substrate rotation axis was 5.5 cm. The vertical distance between the target axis and the

substrate surface was 4.5 cm. Based on these values, the effective line-of-sight distance from the Nb target to the substrate was approximately 7.5 cm, while that from the Te-rich Sb_2Te_3 target to the substrate was approximately 7.1 cm. These geometrical parameters were recorded as part of the standardized sputtering conditions.

The measurement of the target-to-substrate distance is an important part of the system standardization in this work. In magnetron sputtering, the deposition rate and thickness uniformity are affected not only by power, working pressure, and gas flow, but also by the target–substrate geometry. Previous studies have shown that the target–substrate distance can influence the deposition rate and spatial thickness distribution of sputtered films [54], [55]. Therefore, by explicitly recording the target-to-substrate distances for both Sb_2Te_3 and Nb deposition, the deposition conditions used in this study can be described more quantitatively and reproduced more reliably in future experiments.

3.3.2 Substrate Cleaning and Chamber Pre-conditioning

Before thin-film deposition, both the substrate surface and the chamber environment were carefully prepared to improve the reproducibility of the sputtering process. In previous work, oxidized Si substrates with a relatively thick artificial oxide layer were used, and the oxide layer had to be removed through chemical etching before deposition. Such a process required hazardous chemicals such as hydrofluoric acid (HF). In the present work, commercially available Si(100) substrates with a clean silver-gray surface were used. Since the surface of these substrates contains only a naturally formed native oxide layer, no additional HF etching was performed. Even if the native oxide is removed, it can rapidly reform under ambient conditions. Therefore, the substrate treatment in this

work focused on removing organic residues, particles, and surface contamination rather than removing the native oxide layer.

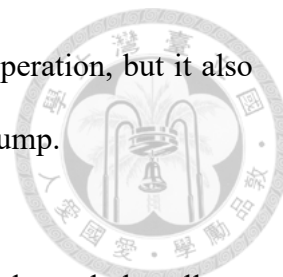


The substrate cleaning procedure was as follows:

1. The required Si(100) substrate size was determined before cutting. In this work, the diced substrate size was typically kept within $5 \times 5 \text{ mm}^2$.
2. A shallow scratch was made on the backside of the Si substrate using a diamond cutter.
3. A small mechanical force was applied to cleave the substrate along the scratch line.
4. The diced substrate was ultrasonically cleaned in acetone for at least 10 min.
5. The substrate was ultrasonically cleaned in isopropyl alcohol for 10 min.
6. The substrate was ultrasonically cleaned in deionized water for 10 min.
7. The substrate was dried using a nitrogen gun.
8. The cleaned substrate was immediately loaded into the sputtering chamber, and the chamber was evacuated for the next processing step.

In addition to substrate cleaning, chamber pre-conditioning was also performed before deposition. This step is particularly important for Nb deposition because Nb is chemically active and can be easily oxidized. A cleaner chamber environment is therefore required to reduce the possible influence of residual oxygen and moisture during sputtering. The sputtering system used in this work is equipped with an automatic protection mechanism to prevent the turbo pump from operating under overload conditions. In addition, the Ar inlet is equipped with a pressure interlock, which automatically closes the Ar valve when the chamber pressure is higher than approximately

10⁻¹ Torr. This design protects the pumping system during normal operation, but it also prevents direct Ar purging of the chamber before starting the turbo pump.



To address this limitation, the system includes an auxiliary gas inlet path that allows Ar gas to enter the chamber through the gas line originally connected to the nitrogen purge valve, as illustrated in Figure 3.8. This configuration allows the chamber to be purged with Ar before turbo-pump operation, while maintaining the intended safety sequence of the pumping system.

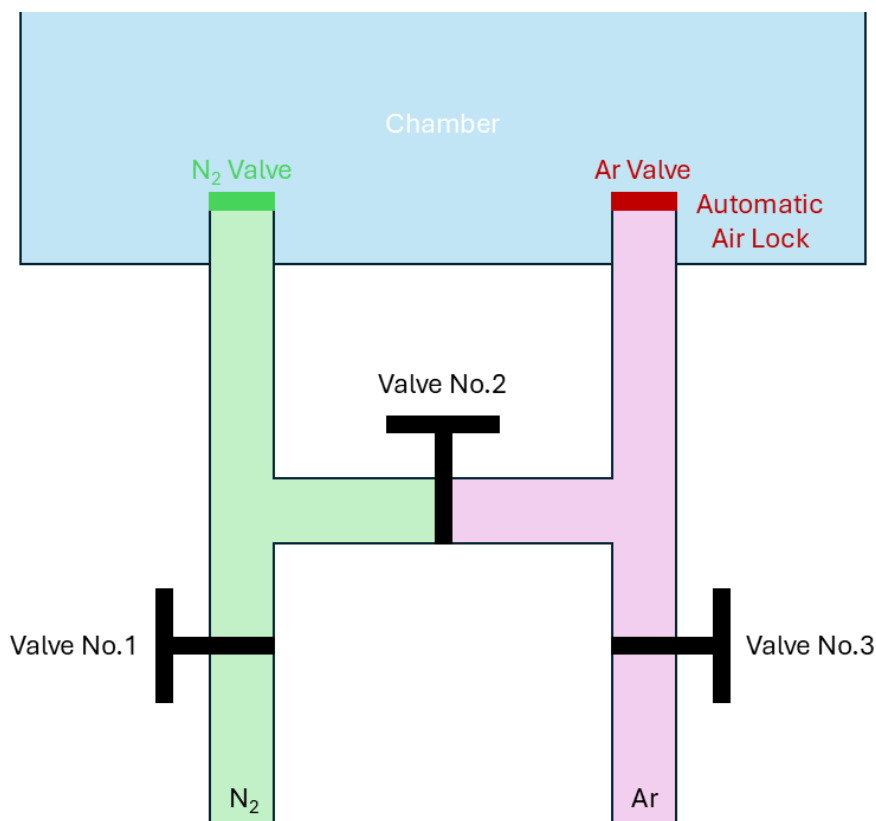


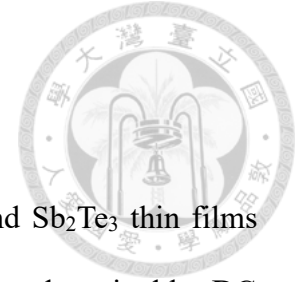
Figure 3.8: Schematic diagram of the auxiliary Ar inlet path.

The chamber pre-conditioning procedure was as follows:

1. The chamber was first evacuated by the mechanical pump to approximately 2×10^{-2} Torr.
2. The valve between the chamber and the mechanical pump was closed.
3. Valve 1 was confirmed to be closed.
4. The N₂ valve was opened.
5. Valve 3 was opened.
6. Valve 2 was opened, allowing Ar gas to enter the chamber through the auxiliary inlet path connected to the N₂ valve.
7. The chamber was filled with Ar until the pressure reached approximately 60 Torr.
8. Valve 2 was closed.
9. The N₂ valve was closed.
10. Valve 3 was kept open for the subsequent pumping procedure.

Through this pre-conditioning process, the chamber can be filled with relatively clean Ar gas before the turbo pump is activated. This procedure helps reduce the amount of residual oxygen and moisture inside the chamber, lowers the initial load on the pumping system, and shortens the waiting time required to reach the base pressure for sputtering. During this Ar purging process, the pressure in the red-region pipeline shown in Figure 3.8 should be maintained higher than that in the green-region pipeline. This pressure gradient prevents nitrogen from flowing backward into the Ar gas path and contaminating the Ar line. Therefore, substrate cleaning, chamber pre-conditioning, and gas-line pressure control were treated as standardized preparation steps before every Nb/Sb₂Te₃ multilayer deposition.

3.3.3 Thin Film Deposition Parameters



After substrate cleaning and chamber pre-conditioning, Nb and Sb₂Te₃ thin films were deposited by magnetron sputtering. In this study, Nb films were deposited by DC magnetron sputtering, whereas Sb₂Te₃ films were deposited by RF magnetron sputtering. Since Nb is a metallic target, DC sputtering is suitable for its deposition. In contrast, Sb₂Te₃ is a compound target, and the target used in this work was a Te-rich mixed target; therefore, RF sputtering was adopted to improve plasma stability and reduce the possibility of target charging or abnormal discharge [48], [53].

Before deposition, the chamber was evacuated and thermally stabilized. The standard procedure was as follows:

1. The chamber was evacuated to a base pressure of approximately 2×10^{-5} Torr using the pumping system.
2. The power supply of the heater was turned on, and the operate power (OP) was set to 18.
3. The chamber and substrate holder were allowed to reach a thermally stable condition.
4. The OP was reduced to 12.
5. The chamber and substrate holder were again allowed to reach a thermally stable condition.
6. The Ar valve was opened, and Ar gas was slowly introduced into the chamber until the target working pressure was reached. During this process, the turbo pump shutter was closed to reduce the gas flow into the turbo pump.
7. The working pressure, sputtering mode, sputtering power, and pre-sputtering

time were set according to the material to be deposited.

8. After pre-sputtering, the target shutter was opened and the deposition time was recorded.



Steps 2–5 were used as a simple bake process. During heating, water vapor and residual gases adsorbed on the chamber wall, substrate holder, and internal components can be desorbed and removed by the pumping system. This procedure helps reduce the amount of residual moisture and oxygen in the chamber and allows the system to reach a lower and more stable base pressure before deposition. In this system, OP = 18 corresponds to a heater current of approximately 9 A, while OP = 12 corresponds to approximately 6 A. The chamber was first heated at the higher OP and then stabilized at the lower OP to promote desorption without excessive heating.

When Ar gas was introduced to reach the working pressure, the turbo pump shutter was adjusted simultaneously. When the shutter is fully open, the turbo pump operates with its maximum pumping conductance. When the shutter is partially closed, the gas flow into the turbo pump is reduced. Therefore, during the transition from high vacuum to a working pressure on the order of 10^{-3} Torr, partially closing the shutter reduces the gas load on the turbo pump and improves the stability of pressure control.



The standard sputtering parameters for Nb and Sb₂Te₃ are summarized in Table 3.2.

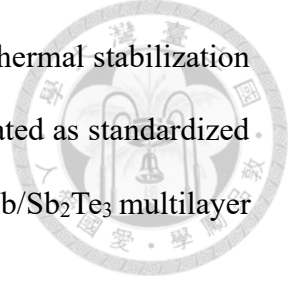
Material	Working pressure	Sputtering mode	Power	Pre-sputtering time
Nb	4×10 ⁻³ Torr	DC	80 W	10 min
Sb ₂ Te ₃	9×10 ⁻³ Torr	RF	20 W	Not applied

Table 3.2: Standard sputtering parameters for Nb and Sb₂Te₃ thin-film deposition

Different working pressures and sputtering powers were used for Nb and Sb₂Te₃ to account for their different electrical properties, plasma stability, and deposition rates. For the first Nb deposition, a longer pre-sputtering time of 10 min was used to remove possible oxide or contamination layers formed on the target surface after exposure to air [56]. For the second or third Nb layer within the same deposition sequence, the Nb target was not significantly exposed to air after the previous sputtering step; therefore, the pre-sputtering time could be reduced to 1–3 min. In contrast, stable plasma ignition could not be achieved for the Sb₂Te₃ target when the target shutter was closed. Therefore, pre-sputtering was not applied to Sb₂Te₃ deposition in this work.

During multilayer deposition, an interval of at least 10 min was introduced when switching between different materials. This waiting time reduces the influence of residual plasma, sputtered particles, or local chamber atmosphere from the previous deposition step, thereby minimizing cross-contamination between Nb and Sb₂Te₃ layers. The

working pressure, sputtering power, pre-sputtering time, heater OP, thermal stabilization procedure, and waiting time between material switching were all treated as standardized deposition parameters in this study to improve the reproducibility of Nb/Sb₂Te₃ multilayer fabrication.



3.3.4 Deposition Rate Calibration

To precisely control the thicknesses of the Nb and Sb₂Te₃ layers, the deposition rates of both materials were calibrated before the fabrication of multilayer samples. Since the deposition process used in this study involves substrate heating, a conventional AZ-1500 photoresist pattern cannot reliably serve as a mask or lift-off layer throughout the entire deposition process. Therefore, Au was used as a sacrificial lift-off layer. An Au pattern was first formed on a Si substrate, followed by deposition of the target film. The Au-covered region was then removed using an iodine/potassium iodide solution, leaving a step height between the deposited film and the exposed substrate. The deposition rate (R) was calculated from the measured film thickness (d) and deposition time (t):

$$R = d/t(nm/min) \quad (3.1)$$

The Au-pattern fabrication procedure was as follows:

1. The Si(100) substrate was thoroughly cleaned according to the procedure described in Section 3.3.2.
2. The substrate was fixed on a spin coater.
3. A small amount of AZ-1500 photoresist was dropped onto the substrate using a dropper.

- 
4. The substrate was spin-coated at high speed for 10 s to form a uniform photoresist layer.
 5. The sample was baked at 90 °C for 10 min to dry the photoresist.
 6. The sample was cooled in a dark environment.
 7. The photoresist was exposed for 75 s using a photomask and exposure system.
 8. The exposed sample was developed for 20 s.
 9. The sample was immersed in DI water to stop the development process.
 10. The sample was dried using a nitrogen gun.
 11. The photoresist pattern was inspected using an Optical Microscope (OM). If the pattern quality was insufficient, the photoresist was removed with acetone and the above steps were repeated.

After the photoresist pattern was formed, a 50 nm Au film was deposited in a separate sputtering chamber. The Au deposition was performed under an Ar working pressure of 5×10^{-3} Torr using a DC power of 20 W, without intentional heating. After Au deposition, the sample was immersed in acetone to remove the photoresist, leaving the Au pattern on the Si substrate as a sacrificial layer for the subsequent deposition-rate calibration.

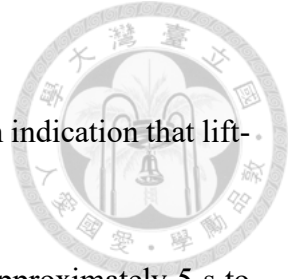
The film-thickness measurement procedure was as follows:

1. The Si substrate with the Au pattern was loaded into the Nb/Sb₂Te₃ sputtering chamber, and the target film was deposited under the standard conditions described in Section 3.3.3.
2. An iodine/potassium iodide etching solution was prepared with a ratio of KI : I₂ : H₂O = 4 g : 1 g : 40 mL.
3. The deposited sample was immersed in the etching solution for approximately 10

min to selectively remove the Au sacrificial layer.

4. Wrinkling or partial peeling of the surface film was used as an indication that lift-off had begun in the Au-covered region.
5. The sample was rinsed in DI water with mild agitation for approximately 5 s to remove the lifted-off film region.
6. The sample was dried using a nitrogen gun.
7. The step height between the deposited film region and the exposed substrate region was measured using AFM.
8. The deposition rate R was calculated from the AFM-measured film thickness d and the deposition time t , using $R = d/t$.

The purpose of this procedure is to create a well-defined and reproducible step edge between the deposited film and the substrate. If a photoresist pattern is used directly as the lift-off layer, the resist may deform, decompose, or lose its pattern integrity during heating or sputtering, making reliable thickness measurement difficult. In this work, Au was therefore used as a thermally stable sacrificial layer. The Au layer can be removed afterward by the iodine/potassium iodide solution, causing the film deposited on top of Au to be lifted off and leaving a measurable step height for AFM analysis.



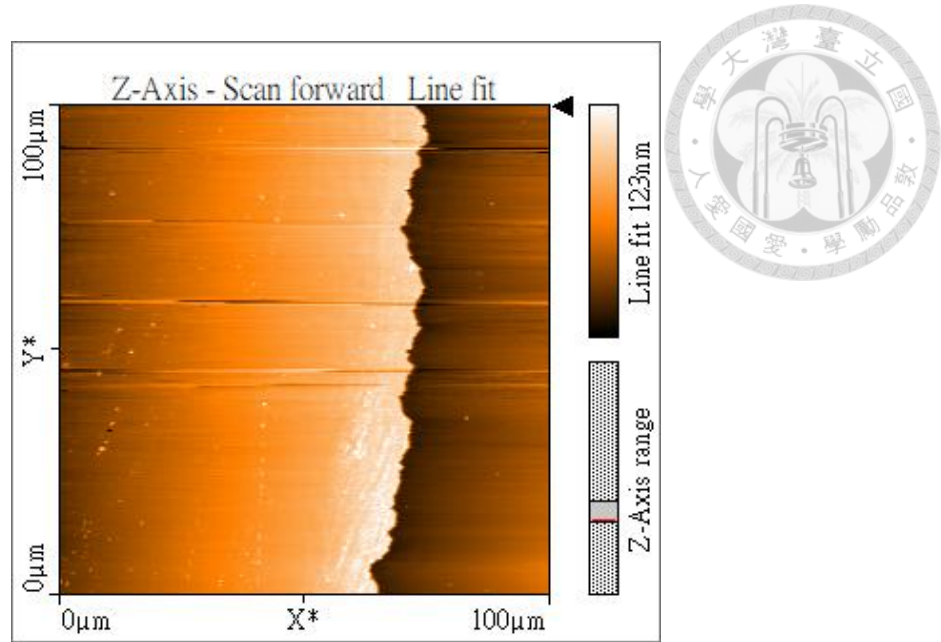


Figure 3.9: AFM landscape for calibration sample.

Figure 3.9 shows the AFM step-height profile obtained from the calibration sample. Based on this calibration, the deposition rates of Nb and Sb_2Te_3 were determined to be 13.33 nm/min and 15.00 nm/min, respectively. The deposition times for the subsequent multilayer samples were designed based on these calibrated rates to ensure reproducible layer thicknesses for both Nb and Sb_2Te_3 .

Material	Sputter mode	Power	deposition rate R
Nb	DC	80 W	13.33 nm/min
Sb_2Te_3	RF	20 W	15.00 nm/min

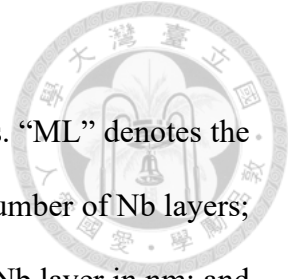
Table 3.3: Sputter power and relative deposition rate R .

3.3.5 Multilayer Structure Design



The design of the Nb/Sb₂Te₃ multilayer structures in this study was based on two main considerations. First, in previous studies of Nb/Sb₂Te₃ heterostructures, the coherence length ξ estimated from magnetic measurements was found to be approximately within the range of 10–14 nm [48]. Although the coherence length estimated from Nb-related samples does not directly represent the superconducting proximity length inside the Sb₂Te₃ spacer layer, it provides an experimentally relevant length scale for designing the separation between adjacent Nb layers. Based on this consideration, the thickness of the Sb₂Te₃ spacer layer between Nb layers was set to 20 nm. This thickness is on the same order as, but slightly larger than, the superconducting coherence length estimated in previous work. The purpose of this design is to allow possible but not fully overlapping interlayer coupling between different Nb layers, thereby enabling the investigation of how the multilayer structure affects magnetic hysteresis, critical current density, and vortex pinning behavior.

Second, to avoid changes in the total amount of superconducting Nb from becoming the dominant factor in the magnetic response, the total Nb thickness was kept at approximately 1000 nm. The primary structural variable was therefore not the total Nb thickness, but the way in which the Nb layer was divided by Sb₂Te₃ spacer layers. In other words, this study compares multilayer structures with approximately the same total Nb thickness but different numbers of Nb metal layers and Nb/Sb₂Te₃ interfaces. This design reduces the influence of the total superconducting volume and allows the subsequent magnetic measurements to more directly reflect the effects of stacking configuration and interface-related behavior.



The sample-naming rule used in this study is defined as follows. “ML” denotes the number of Nb metal layers; the number before “ML” indicates the number of Nb layers; the first number after “ML” indicates the nominal thickness of each Nb layer in nm; and the final number indicates the nominal thickness of the Sb_2Te_3 spacer layer between Nb layers in nm. For example, 2ML50020 denotes a sample consisting of two Nb metal layers, each with a nominal thickness of 500 nm, separated by a 20 nm Sb_2Te_3 spacer layer.

The multilayer structures prepared in this study are summarized in Table 3.4.

Sample	Layers of Nb	Thickness of Nb	Sb_2Te_3 spacer thickness	Abbreviation
2ML50020	2	500 nm	20 nm	2ML
3ML33320	3	333 nm	20 nm	3ML

Table 3.4 : Structural design parameters of the Nb/ Sb_2Te_3 multilayer samples

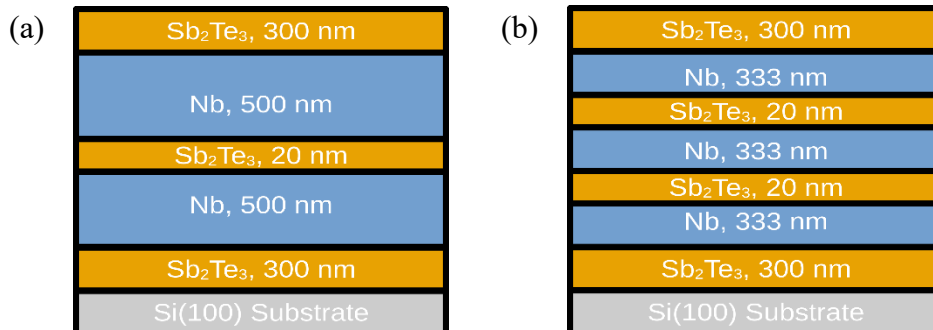
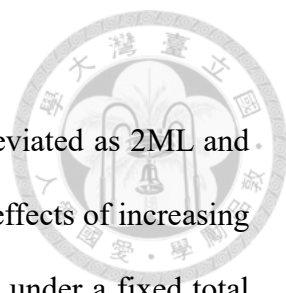


Figure 3.10: Schematic of structural parameters of the Nb/ Sb_2Te_3 multilayer samples.

(a)Sample of 2ML50020 (b)Sample of 3ML33320



In the following sections, 2ML50020 and 3ML33320 are abbreviated as 2ML and 3ML, respectively, unless otherwise specified. With this design, the effects of increasing the number of Nb layers and Nb/Sb₂Te₃ interfaces can be examined under a fixed total Nb thickness and fixed Sb₂Te₃ spacer thickness. This provides a controlled basis for comparing the magnetic response, SPE features, $J_c(H)$ behavior, and pinning-force analysis among different multilayer configurations.

3.4 Structural Characterization

3.4.1 X-ray Diffraction Measurement

X-ray diffraction measurements were carried out on the prepared Nb/Sb₂Te₃ multilayer thin films using a standard scan configuration. The XRD patterns were collected to identify the crystalline phases present in the deposited films and to verify whether the Nb/Sb₂Te₃ multilayer deposition produced the expected structural components. Since the samples contain a total Nb thickness of approximately 1000 nm together with relatively thin Sb₂Te₃ spacer layers, both the strong Nb diffraction peak and the weaker Sb₂Te₃-related diffraction peaks were considered in the phase assignment.

The measured XRD patterns were plotted as intensity versus 2θ . The major diffraction peaks were assigned by comparison with the reported crystal structures of rhombohedral Sb₂Te₃ and body-centered-cubic Nb. The peak positions were used for phase identification rather than for full structural refinement. Therefore, the XRD analysis presented here focuses on the identification of major diffraction features and the

comparison of the structural characteristics of the multilayer samples.



3.4.2 Crystallographic Analysis

Figure 3.11 shows the XRD pattern of the Nb/Sb₂Te₃ multilayer thin film. Several diffraction peaks can be assigned to Sb₂Te₃, including the (015), (1010), (0015), and (0021) reflections. In addition, a strong peak near the Nb (110) position is observed, confirming the presence of crystalline Nb in the multilayer structure [57]. The coexistence of Nb and Sb₂Te₃ diffraction features indicates that both constituent materials contribute to the measured crystalline structure of the deposited multilayer film.

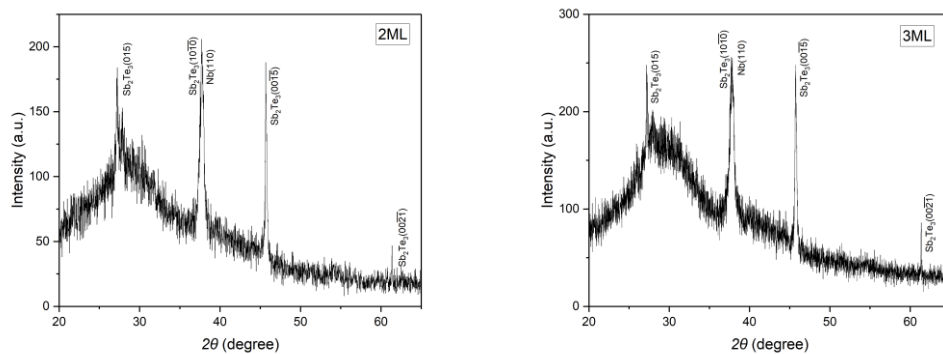
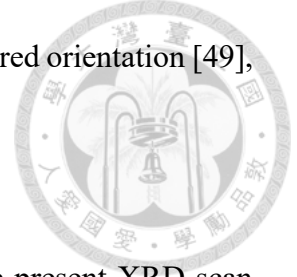


Figure 3.11: XRD spectrum of sample (a)2ML and (b)3ML.

The observation of multiple Sb₂Te₃ diffraction peaks suggests that the Sb₂Te₃ layers retain detectable crystallinity after multilayer deposition. Although the presence of (00 l)-type reflections indicates an out-of-plane layered component associated with the tetradymite structure of Sb₂Te₃, the simultaneous appearance of non-(00 l) reflections indicates that the film should not be described as a purely c -axis-oriented film. Therefore, the XRD result is interpreted as evidence for crystalline Sb₂Te₃ contribution with mixed

or partially oriented texture, rather than as evidence for a single preferred orientation [49], [52].



No dominant unexpected impurity peak is observed within the present XRD scan range. This result suggests that the multilayer deposition process produced films primarily composed of Nb and Sb_2Te_3 crystalline phases. However, XRD alone cannot directly resolve the interface sharpness, local interdiffusion, or the exact thickness of individual 20 nm Sb_2Te_3 spacer layers. Therefore, the structural characterization presented here provides phase and crystallographic-feature confirmation, while the influence of multilayer stacking on superconducting and vortex behavior is further evaluated through magnetic measurements in Chapter 4.

3.5 Definitions and Extraction Criteria of Magnetic Quantities

3.5.1 SQUID Output and Magnetization Conversion

The SQUID magnetometer directly measures the magnetic moment m of the sample as a function of temperature and applied magnetic field. In this study, three types of magnetic datasets were used: temperature-dependent magnetization $m(T)$, field-dependent magnetization $m(H)$ -curve, and magnetic hysteresis loops $m(H)$ -loop. These datasets were used for different analysis purposes and are therefore distinguished explicitly in the data-processing workflow.

The $m(T)$ data were used to identify superconducting transition temperatures and possible

multi-step transition features. The $m(H)$ -curve data were obtained without a full increasing-field and decreasing-field hysteresis cycle. These field-dependent magnetization curves were used to determine the lower critical field H_{c1} , the vortex glass characteristic field H_g , and the upper critical field H_{c2} . In contrast, the $m(H)$ -loop data contain both increasing-field and decreasing-field branches and were used to evaluate the magnetic hysteresis width ΔM , critical current density $J_c(H)$, pinning force $F_p(H)$, second-peak-related features, and irreversibility field H_{irr} .

3.5.2 Superconducting Transition Temperature Determination

The superconducting transition temperature T_c was determined from the temperature-dependent magnetization $m(T)$ measurement. In this study, the zero-field-cooled (ZFC) curve was used as the primary reference for defining T_c , because the onset of superconducting diamagnetic shielding is more clearly reflected in the ZFC response.

To avoid defining T_c only by visual inspection of the raw $m(T)$ curve, the transition temperature was defined from the derivative curve dm_{ZFC}/dT . In the derivative plot, the superconducting transition appears as a rapid change in the magnetic response. The steepest transition region was represented by a linear tangent line, while the non-transition region before the superconducting transition was represented by a nearly horizontal baseline. The intersection of these two lines was defined as the superconducting transition temperature.

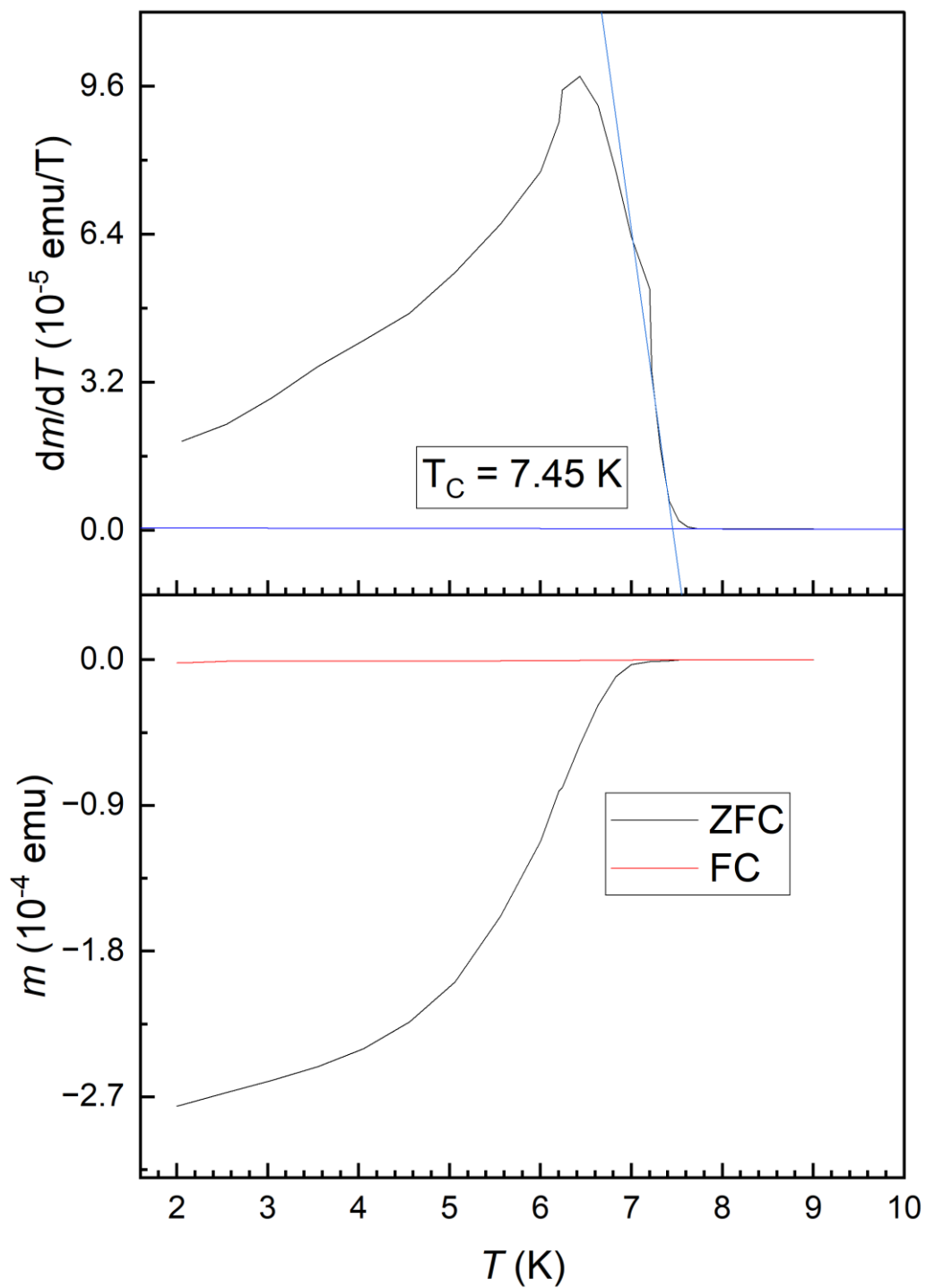
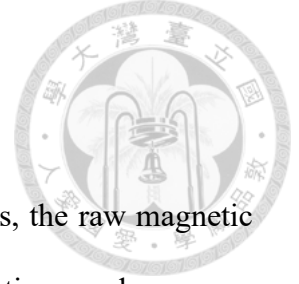


Figure 3.12: Example of determination of the superconducting transition temperature T_c

from the ZFC $m(T)$ curve

3.5.3 Background Subtraction Procedure



Before determining characteristic fields from the $m(H)$ -curves, the raw magnetic moment must be separated into a superconducting contribution and a non-superconducting magnetic background. In this study, the measured magnetic moment is expressed as

$$m_{\text{raw}}(H, T) = m_{\text{SC}}(H, T) + m_{\text{bg}}(H, T) \quad (3.2)$$

where $m_{\text{raw}}(H, T)$ is the measured magnetic moment, $m_{\text{SC}}(H, T)$ is the superconducting magnetic contribution, and $m_{\text{bg}}(H, T)$ is the background contribution.

At sufficiently high magnetic field, the superconducting diamagnetic response is strongly suppressed and the remaining signal is assumed to be dominated by a weak, approximately linear magnetic background. Therefore, the background is described as

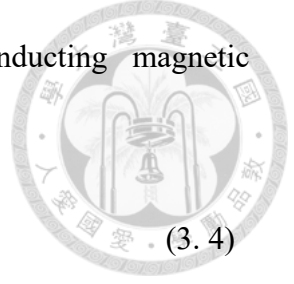
$$m_{\text{bg}}(H, T) = aH + b(T) \quad (3.3)$$

where a represents the field-linear background slope and $b(T)$ represents the temperature-dependent offset.

To define the background consistently for all temperatures, the high-field tail of each $m(H)$ -curve was used as the background reference region. In this work, the terminal 20% of the measured magnetic-field range was selected as the high-field background region. This fixed-field-range criterion was adopted so that the background determination does not depend on the position or magnitude of the superconducting diamagnetic minimum in each curve.

After the linear background is determined, the superconducting magnetic contribution is obtained by

$$m_{\text{SC}}(H, T) = m_{\text{raw}}(H, T) - m_{\text{bg}}(H, T) \quad (3.4)$$



The background-subtracted curve $m_{\text{SC}}(H, T)$ is then used as the basis for determining the characteristic fields in the following sections. Under this definition, the high-field region of $m_{\text{SC}}(H, T)$ approaches zero, while the negative part of the curve represents the superconducting diamagnetic response. Therefore, all characteristic fields extracted from $m(H)$ -curves in this study are defined after the same linear high-field background subtraction procedure.

3.5.4 Lower Critical Field Determination

The lower critical field H_{c1} marks the onset of magnetic-flux penetration into a type-II superconductor. Below H_{c1} , the sample is in the Meissner state, and the magnetic response is expected to follow an approximately linear diamagnetic relation in the low-field region. Once the applied field exceeds H_{c1} , vortices begin to penetrate the sample, causing the measured $m(H)$ -curve to deviate from the initial Meissner linear behavior.

In this study, H_{c1} was determined from the low-field region of the background-subtracted $m(H)$ -curve. Following the criterion used in previous work, the initial Meissner response was first represented by a linear fit to the low-field data:

$$m_{\text{lin}}(H, T) = \alpha(T)H + \beta(T) \quad (3.5)$$

where $m_{\text{lin}}(H, T)$ is the extrapolated Meissner linear response at temperature T , and

$\alpha(T)$ and $\beta(T)$ are the fitting parameters obtained from the low-field region.

The deviation between the measured curve and the low-field linear fit was then evaluated as

$$\Delta m(H, T) = |m_{SC}(H, T) - m_{lin}(H, T)| \quad (3.6)$$

To define a consistent criterion for flux penetration, H_{c1} was taken as the magnetic field where the measured $m(H)$ -curve first deviates from the low-field linear fit by 1% in the magnetic-moment axis. This criterion can be expressed as

$$\frac{|m_{SC}(H_{c1}, T) - m_{lin}(H_{c1}, T)|}{|m_{lin}(H_{c1}, T)|} = 0.01 \quad (3.7)$$

Therefore, H_{c1} in this work is not defined by the maximum diamagnetic response or by the derivative of the $m(H)$ -curve. Instead, it is operationally defined as the first field at which the low-field Meissner linearity breaks down according to the 1% deviation criterion.

This method is suitable for the present $m(H)$ -curve analysis because the low-field linear response directly represents the Meissner shielding regime. Any subsequent oscillation or anomalous magnetic feature at higher fields is treated as behavior occurring after the initial flux penetration. Thus, the extracted H_{c1} corresponds to the boundary between the Meissner state and the mixed state under the adopted 1% deviation criterion.

3.5.5 Upper Critical Field Determination



The upper critical field H_{c2} represents the magnetic field at which the superconducting state is suppressed and the superconducting magnetic contribution disappears. In this study, H_{c2} was determined from the background-subtracted field-dependent magnetization $m(H)$ -curves. Since the raw magnetic moment contains a weak magnetic background, H_{c2} was not defined from the raw $m_{\text{raw}}(H, T)$ curve directly. Instead, it was determined from the superconducting magnetic contribution $m_{\text{SC}}(H, T)$ after the high-field background subtraction described in Section 3.5.3.

After background subtraction, the upper critical field is defined by the condition

$$m_{\text{SC}}(H_{c2}, T) = 0 \quad (3.8)$$

This condition indicates that the superconducting diamagnetic response has disappeared. However, because the measured $m_{\text{SC}}(H, T)$ curve approaches zero gradually in the high-field region, the experimental H_{c2} in this work is defined by an extrapolated zero-crossing criterion. Specifically, the high-field recovery of $m_{\text{SC}}(H, T)$ is represented by a local linear relation,

$$m_{\text{SC}}(H, T) = s(T)H + c(T) \quad (3.9)$$

and H_{c2} is defined as the field at which this linear representation reaches zero:

$$H_{c2}(T) = -\frac{c(T)}{s(T)} \quad (3.10)$$

To keep the definition consistent among different temperatures and samples, the high-field recovery line is taken from the region where the remaining diamagnetic signal is approximately (20%) of the maximum diamagnetic amplitude (A_{dia}). If

$$A_{\text{dia}}(T) = |\min[m_{\text{SC}}(H, T)]| \quad (3.11)$$

then the reference region for defining the high-field recovery is given by

$$|m_{\text{SC}}(H, T)| \approx 0.20A_{\text{dia}}(T) \quad (3.12)$$

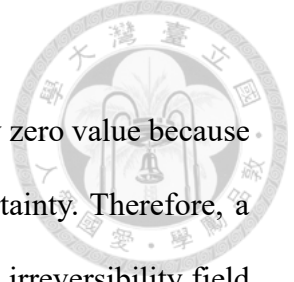
Therefore, the H_{c2} used in this thesis is a magnetization-defined upper critical field. It should be understood as the field where the superconducting magnetic contribution, after background subtraction, disappears according to the adopted high-field extrapolation criterion. This definition provides a consistent basis for comparing $H_{c2}(T)$ among different Nb/Sb₂Te₃ multilayer samples.

3.5.6 Irreversibility Field Determination

The irreversibility field H_{irr} represents the magnetic field above which the irreversible magnetic response associated with vortex pinning becomes negligible. In the magnetic hysteresis analysis, this corresponds to the high-field region where the hysteresis width of the $m(H)$ -loop becomes very small and the magnetically derived critical current density $J_c(H)$ approaches zero.

In an ideal case, H_{irr} may be defined as the field where

$$J_c(H_{\text{irr}}, T) = 0 \quad (3.13)$$



However, in experimental data, $J_c(H)$ rarely reaches an exactly zero value because of instrumental noise, finite field resolution, and background uncertainty. Therefore, a finite critical-current-density criterion was adopted in this study. The irreversibility field was defined as the field at which the high-field branch of $J_c(H)$ decreases to

$$J_{c,\text{crit}} = 100 \text{ A/cm}^2 \quad (3.14)$$

Thus, the operational definition of H_{irr} used in this work is

$$J_c(H_{irr}, T) = J_{c,\text{crit}} \quad (3.15)$$

or explicitly,

$$J_c(H_{irr}, T) = 100 \text{ A/cm}^2 \quad (3.16)$$

Under this criterion, H_{irr} marks the field scale at which the vortex-pinning-induced irreversible current density becomes smaller than the selected experimental threshold.

In this work, H_{irr} was operationally defined as the magnetic field at which the high-field $J_c(H)$ curve decreases to $J_{c,\text{crit}}=100 \text{ A/cm}^2$ [58], [59], [60]. Similar $J_c=100 \text{ A/cm}^2$ criteria have been used in previous superconducting pinning studies to determine H_{irr} from $J_c(H)$ or $M(H)$ -loop-derived critical current density.

3.6 Automated Data Analysis Workflow

3.6.1 Overview of the Analysis Programs

Programs used in this work correspond to different stages of the magnetic-data analysis. The first program calculates local derivatives or local slopes from selected x - y datasets. The second program extracts $H_{c2}(T)$ from background-subtracted $m(H)$ -curve data using a tail-extrapolation method. The third program converts raw $m(H)$ -loop data into volume-corrected magnetization, critical current density, pinning force, normalized pinning force, SPE-related characteristic fields, and H_{irr} . The fourth program performs Dew-Hughes-type fitting of the normalized pinning-force curves and exports fitting parameters for different temperature-dependent regimes.

These programs were designed with a similar graphical-user-interface structure. In each program, the user selects the input data file, assigns the corresponding data columns, specifies numerical parameters, previews the analysis result, and exports the processed data. This design reduces repeated manual processing and ensures that different datasets are treated under the same analysis logic.

3.6.2 Derivative and Local-Slope Processing Module



The derivative and local-slope processing module was used to generate numerical derivative curves from selected x - y datasets. In this study, this module was mainly applied to the temperature-dependent magnetization $m(T)$ data to obtain dm/dT , which was then used for the determination of the superconducting transition temperature described in Section 3.5.2.

The program reads the input data file and allows the user to select the x and y columns. For T_c analysis, the temperature column is selected as $x = T$, and the ZFC magnetic moment is selected as $y = m_{\text{ZFC}}(T)$. The user also specifies a local fitting window size m , which determines the number of neighboring data points used on each side of the target point.

For each data point x_n , the program selects a local window from x_{n-m} to x_{n+m} . Within this local window, the derivative can be estimated using either a first-order or a second-order polynomial fit. For the first-order fitting option, the local data are fitted by

$$y(x) = A_n x + B_n \quad (3.17)$$

and the local slope at x_n is obtained as

$$\left. \frac{dy}{dx} \right|_{x=x_n} = A_n \quad (3.18)$$

For the second-order fitting option, the local data are fitted by

$$y(x) = A_n x^2 + B_n x + C_n \quad (3.19)$$

and the local slope at x_n is calculated as

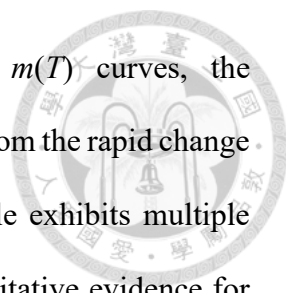
$$\left. \frac{dy}{dx} \right|_{x=x_n} = 2A_n x_n + B_n \quad (3.20)$$

By repeating this local fitting procedure for every data point, the program generates a derivative curve corresponding to the selected input dataset. For $m(T)$ analysis, the output derivative is therefore

$$D(T) = \frac{dm_{ZFC}}{dT} \quad (3.21)$$

Figure 3.13: UI of local-slope analysis tool.

This module is particularly useful for analyzing temperature-dependent



magnetization data. By applying local derivative analysis to $m(T)$ -curves, the superconducting transition temperature can be defined more clearly from the rapid change in magnetization rather than by visual inspection alone. If a sample exhibits multiple transition-like features, the derivative curves can also provide quantitative evidence for distinguishing different transition regions. Therefore, the local-slope module was used as an auxiliary tool to support the identification of superconducting transition temperatures and possible multi-step transition behavior.

3.6.3 H_{c2} Extraction Module

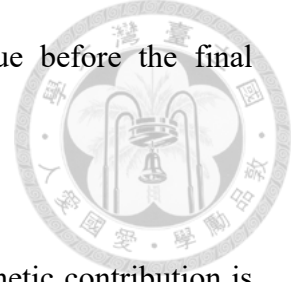
The H_{c2} extraction module was used to determine the upper critical field from field-dependent magnetization $m(H)$ -curves. The program reads the magnetic field H , raw magnetic moment m_{raw} , and temperature T from user-selected columns in the input data file. The data are then grouped according to temperature, and the H_{c2} extraction procedure is applied independently to each $m(H)$ -curve

The extraction procedure is based on the intersection between the magnetic background and the high-field extrapolation of the superconducting signal. First, a linear background is estimated from the high-field region of the raw $m(H)$ -curve. For each temperature, the high-field fitting range is selected according to a user-defined field fraction, and the background is described as.

$$m_{\text{bg}}(H, T) = aH + b(T) \quad (3.22)$$

where a is the background slope and $b(T)$ is the temperature-dependent intercept. In the program, the background slope is estimated from the high-field regions of all valid

temperature groups, and the user may manually adjust this value before the final extraction.



After the background is determined, the superconducting magnetic contribution is obtained by

$$m_{SC}(H, T) = m_{raw}(H, T) - m_{bg}(H, T) \quad (3.23)$$

The program then selects the high-field recovery region of $m_{SC}(H)$, where the superconducting diamagnetic signal gradually approaches zero. This selected region is fitted linearly as

$$m_{SC}(H) = sH + c \quad (3.24)$$

The upper critical field is defined as the field where the extrapolated superconducting contribution disappears:

$$m_{SC}(H_{c2}) = 0 \quad (3.25)$$

Therefore,

$$H_{c2} = -\frac{c}{s} \quad (3.26)$$

Equivalently, this procedure corresponds to finding the field where the extrapolated high-field superconducting signal intersects the fitted magnetic background. The extracted $H_{c2}(T)$ values are exported as a text file, and a preview plot of $H_{c2}(T)$ is generated for inspection. In this work, H_{c2} is therefore defined as an operational upper critical field obtained from the high-field disappearance of the background-subtracted

superconducting magnetic signal in the $m(H)$ -curve.

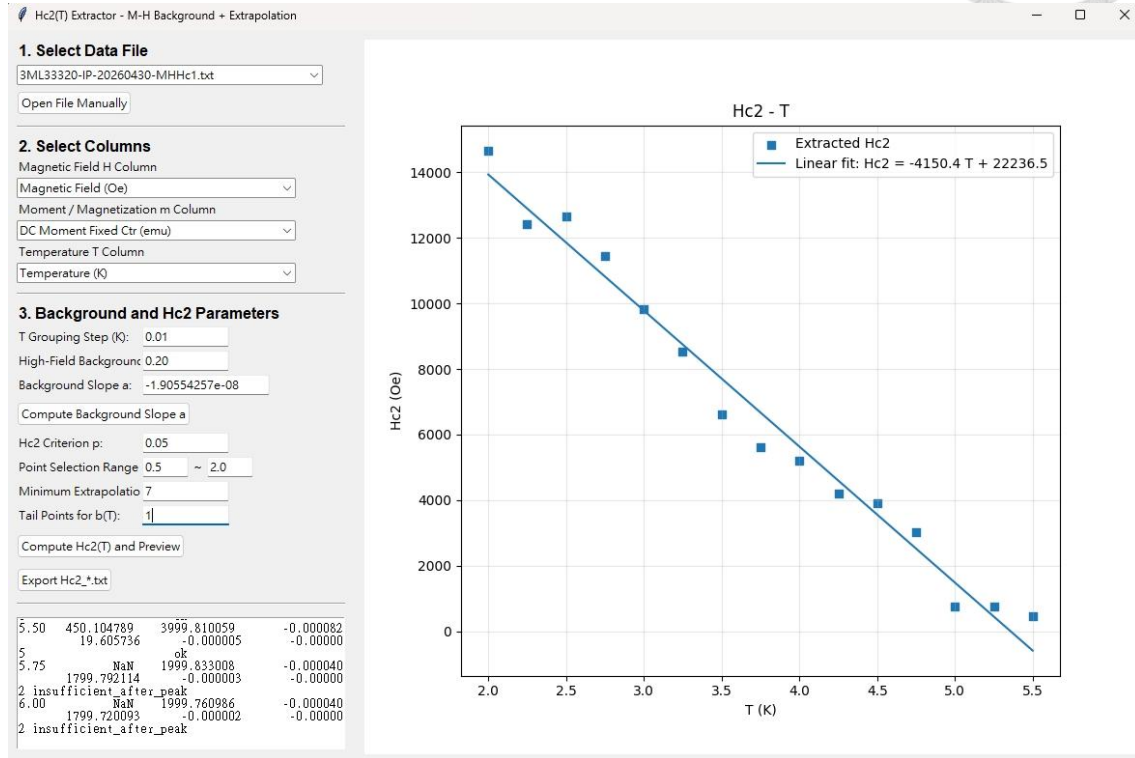


Figure 3.14: UI of H_{c2} extraction module.

3.6.4 J_c , F_p , and H_{irr} Processing Module

The J_c , F_p , and H_{irr} processing module was used to process magnetic hysteresis data obtained from $m(H)$ -loops. The program first reads the magnetic field H , raw magnetic moment m_{raw} , and temperature T from user-selected columns in the input file. After invalid numerical entries are removed, the data are grouped according to temperature using a user-defined temperature resolution. Each temperature group is then processed independently.

The program first converts the raw magnetic moment into volume-corrected magnetization. The sample volume is calculated from the input sample dimensions,

$$V = Lwt \quad (3.27)$$

where L , w , and t are the sample length, width, and film thickness, respectively. The magnetization is then calculated as

$$M(H) = \frac{m_{\text{raw}}(H)}{V} \quad (3.28)$$

The resulting $M(H)$ data are exported as an intermediate output file so that the volume-corrected $m(H)$ -loop can be checked before further analysis.

For each temperature-dependent $m(H)$ -loop, the program separates the data into two field-sweep branches according to the sign of the field increment,

$$\Delta H_i = H_{i+1} - H_i \quad (3.29)$$

Data points with $\Delta H_i > 0$ and $\Delta H_i < 0$ are assigned to two different branches. The two branches are then matched at equivalent magnetic-field values. In the standard setting, the program uses $|H|$ and rounds the field values according to a user-defined precision,

$$H_{\text{key}} = \text{round}(H, N_H) \quad (3.30)$$

where N_H is the selected number of decimal places used for field matching. The two branches are then paired using the same H_{key} . This procedure reduces small mismatches caused by finite field resolution while preserving the original signed magnetic-field

values in the loop.



After the branch matching procedure, the program calculates the branch difference at each matched field,

$$\Delta m(H) = |m_1(H) - m_2(H)| \quad (3.31)$$

And

$$\Delta M(H) = |M_1(H) - M_2(H)| \quad (3.32)$$

The matched branch data, $\Delta m(H)$, and $\Delta M(H)$ are retained in the output table and used in the subsequent calculation steps.

The effective sample dimensions used for the geometrical correction are selected according to the field orientation. The program calculates the geometrical factor as

$$G = a \left(1 - \frac{a}{3b} \right) \quad (3.33)$$

where a and b are the effective dimensions determined from the sample geometry. The critical current density output is then calculated internally as

$$J_c(H) = \frac{20\Delta M(H)}{G} \quad (3.34)$$

The program also converts J_c from A/cm² to A/m², approximates the magnetic flux density by

$$B \simeq |H| \times 10^{-4} \quad (3.35)$$

and calculates the pinning force density as

$$F_p(H) = J_c(H)B \quad (3.36)$$

The normalized pinning force is then generated by

$$f_p(H) = \frac{F_p(H)}{F_{p,\max}} \quad (3.37)$$

These quantities are exported as $J_c(H)$ and $F_p(H)$ output files.

The irreversibility field is extracted from the high-field decay region of $J_c(H)$. The program first defines a J_c criterion, either as a fixed value or as a fraction of $J_{c,\max}$. The high-field search region is selected according to

$$H \geq \eta H_{\max} \quad (3.38)$$

where η is a user-defined high-field fraction. Near the criterion-crossing region, the program selects a local fitting window and fits the data using either a linear J_c - H relation,

$$J_c(H) = sH + c \quad (3.39)$$

The irreversibility field is then obtained from the fitted curve as the field where J_c reaches the selected criterion. This local-fitting procedure prevents H_{irr} from being determined solely by a single high-field data point.

Finally, if a valid H_{irr} is obtained, the program calculates the reduced field



$$h = \frac{H}{H_{irr}} \quad (3.40)$$

which is used in the subsequent normalized pinning-force analysis. The main output files generated by this module include the volume-corrected $M(H)$ data, the $J_c(H)$ table, the $F_p(H)$ and $f_p(H)$ table, and a summary file containing, $F_{p,max}$, and H_{irr} . Preview plots of $J_c(H)$, $F_p(H)$, $f_p(H)$, and $H_{irr}(T)$ are also generated for checking the analysis results.

Figure 3.15: UI of pinning force analyzer.

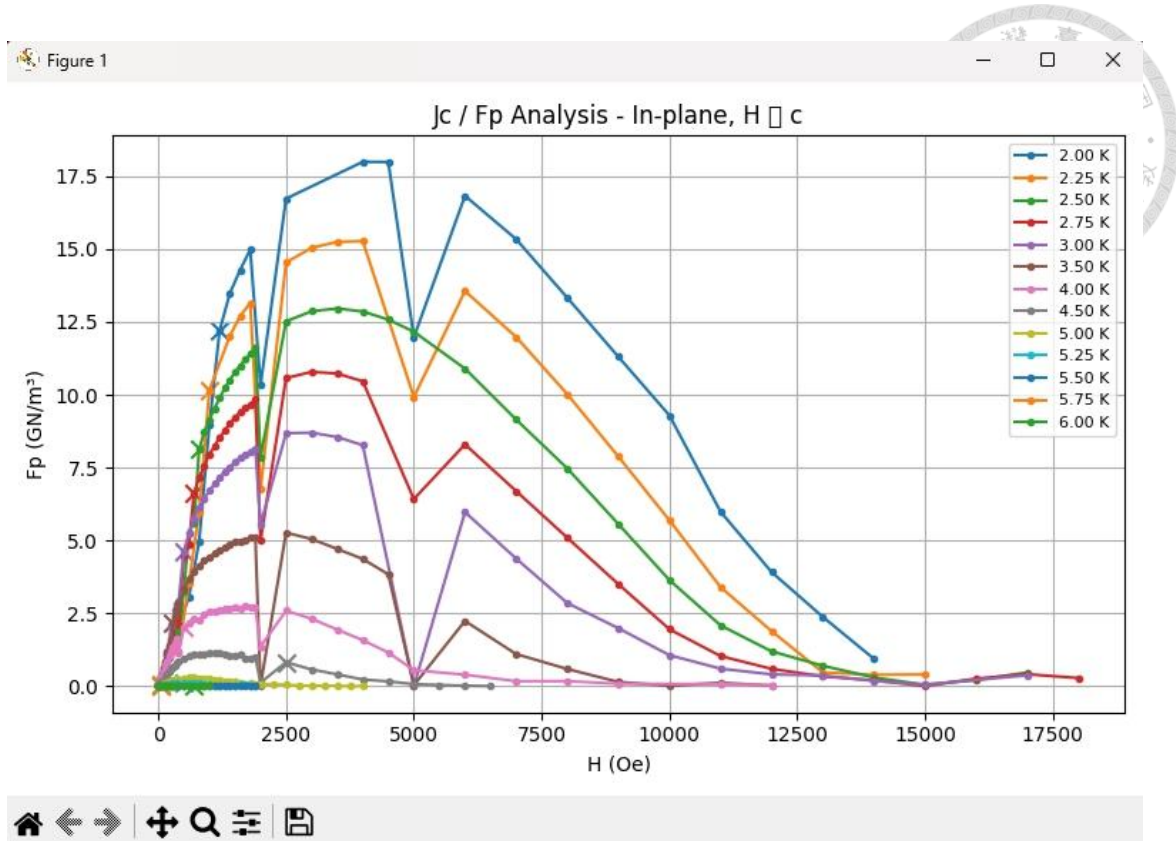


Figure 3.16: Preview plots of $F_p(H)$.

It should be noted that the branch-separation procedure depends on the field-sweep sequence recorded in the raw $m(H)$ -loop file. In some measurements, preliminary field-setting segments may be included before the regular loop sequence starts. For example, the magnetic field may first be increased to selected intermediate fields, such as 2000 Oe and 5000 Oe, before reaching the maximum positive field. The regular loop measurement then begins from the high-field region. In this situation, duplicate data points with the same signed field and the same sweep direction may appear in the raw dataset.

Because the program separates the loop branches according to the sign of the field increment and then matches the two branches by magnetic-field value, such preliminary segments can generate artificial discontinuities in the calculated $\Delta M(H)$, $J_c(H)$, and $F_p(H)$.

curves. These discontinuities are not associated with intrinsic vortex-pinning behavior, but originate from the measurement sequence contained in the raw data file. Therefore, the processed curves were inspected using the preview plots, and isolated jump points caused by preliminary field-setting paths were excluded from the physical interpretation of $J_c(H)$ and $F_p(H)$.

3.6.5 Dew-Hughes Fitting Module

The Dew-Hughes fitting module was used to analyze the normalized pinning-force curves obtained from the $F_p(H)$ output file. The program reads the temperature, magnetic field, pinning force, normalized pinning force, and reduced field from user-selected columns. The reduced field can be taken directly from the $F_p(H)$ file as $h = H/H_{irr}$. Alternatively, when an $H_{c2}(T)$ file is loaded, the program performs a linear regression of the imported $H_{c2}(T)$ data,

$$H_{c2}(T) = \alpha T + \beta \quad (3.41)$$

and generates a new reduced-field column according to

$$h = \frac{H}{H_{c2}(T)} \quad (3.42)$$

This allows the same fitting workflow to be applied using either H_{irr} -normalized or H_{c2} -normalized pinning-force data.

For each temperature, the program displays both the raw $F_p(H)$ curve and the normalized $f_p(h)$ curve. The fitting procedure is performed separately for two user-defined

regimes: the front regime, corresponding to the low-field side of the normalized pinning-force curve, and the back regime, corresponding to the higher-field side. Each regime has its own fitting range, initial parameters, excluded data points, and fitting result. This design allows the low-field and high-field pinning behaviors to be treated independently when the normalized pinning-force curve cannot be described by a single Dew-Hughes function over the entire field range.

The fitting model used in each regime is

$$f_p(h) = Ah^p(1 - h)^q \quad (3.43)$$

where A , p , and q are fitting parameters. For a selected temperature and regime, the program first selects the data points satisfying

$$h_{\min} \leq h_i \leq h_{\max} \quad (3.44)$$

with $0 < h_i < 1$. Data points manually excluded by the user are removed from the fitting set. The fitting is then performed by minimizing the residual between the experimental normalized pinning force $f_{p,i}$ and the model value:

$$\chi^2 = \sum_i [f_{p,i} - Ah_i^p(1 - h_i)^q]^2 \quad (3.45)$$

Depending on the selected fitting option, the program can fit A , p , and q simultaneously, fit only p and q with fixed A , or fit only A with fixed p and q . Manual parameters can also be saved directly as the fitting result. After fitting, the program calculates the fitted curve, the coefficient of determination R^2 , and the root-mean-square error (RMSE) for evaluating the fitting quality.



The peak position of the fitted Dew-Hughes function is calculated from the fitted p and q values as

$$h_{\max} = \frac{p}{p + q} \quad (3.46)$$

This value is recorded for each fitted regime and is used to compare the characteristic pinning-force peak position between the low-field and high-field regimes. In the publication-figure output, the corresponding $h_{\max}$ positions can also be marked directly on the normalized pinning-force plot.

To improve reproducibility during repeated fitting, the program uses a session-based workflow. For each temperature and each fitting regime, the selected fitting range, initial fitting parameters, fixed-parameter settings, excluded points, normalization mode, and fitting results can be stored in a session file. The session is saved in JSON format, allowing the same fitting configuration to be reloaded later without repeating the full manual setup. This is particularly useful when several temperatures require separate low-field and high-field fitting ranges.

The module exports several types of output. The fit table contains the fitting parameters A , p , q , $h_{\max}$, R^2 , RMSE, the number of fitted points, the selected fitting range, and the regime label for each temperature. The fitted-curve output contains the calculated $f_p(h)$ curves for the fitted regimes. The program can also export publication-style figures for individual temperatures, including the experimental normalized pinning-force data, fitted curves for different regimes, optional fitting-range shading, optional extrapolated

dashed curves, and $h_{\max}$ markers. These outputs are used for the final comparison of pinning behavior and reduced-field scaling in Chapter 4.

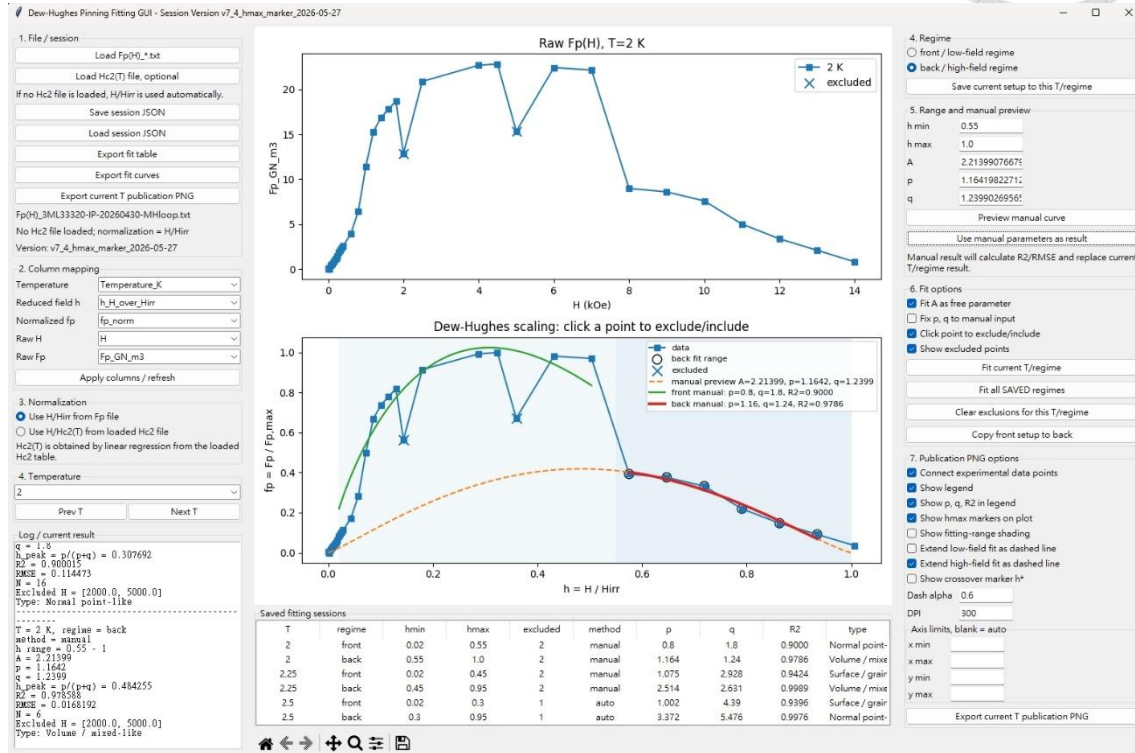


Figure 3.17: UI of Dew-Hughes fitting module.

3.6.6 Output Consistency and Workflow Limitations

The automated workflow provides several advantages for the present study. First, it ensures that all datasets are processed using the same column selection, sample geometry, temperature grouping, field matching, and fitting criteria. Second, it generates intermediate output files, allowing each analysis step to be checked independently. Third, it produces preview figures, which help identify inappropriate fitting ranges, abnormal data points, or unreliable extrapolations before final interpretation.

However, the workflow should be regarded as a standardized numerical procedure rather than a fully automatic physical interpretation. The selection of fitting ranges, thresholds, and normalization fields still requires experimental judgment. Therefore, all extracted characteristic fields and fitting parameters were inspected before being used in the physical discussion. The purpose of the workflow is to reduce subjective point selection and improve reproducibility, while preserving the necessary physical judgment required for superconducting magnetic analysis.

3.7 Summary of Experimental Workflow

This chapter established the experimental and data-processing workflow used in this study. The workflow includes Sb_2Te_3 target preparation, $\text{Nb}/\text{Sb}_2\text{Te}_3$ multilayer deposition, structural characterization, magnetic measurement classification, physical-quantity definition, and automated data processing. The purpose of this chapter is to ensure that the sample fabrication conditions and magnetic-analysis procedures are reproducible and that the characteristic quantities discussed in Chapter 4 are extracted using consistent criteria.

Figure 3.18 summarizes the magnetic-data analysis workflow. The SQUID measurements used in this study were classified into three main data types: $m(T)$, $m(H)$ -curve, and $m(H)$ -loop. The $m(T)$ data were used to determine the superconducting transition temperature T_c and possible transition-like features. The $m(T)$ -curves were used to extract the characteristic fields H_{c1} , H_g , and H_{c2} after the background subtraction procedure. In contrast, the $m(H)$ -loops were used to calculate the hysteresis-derived quantities, including $J_c(H)$, $F_p(H)$, normalized pinning force $f_p(h)$, and H_{irr} .

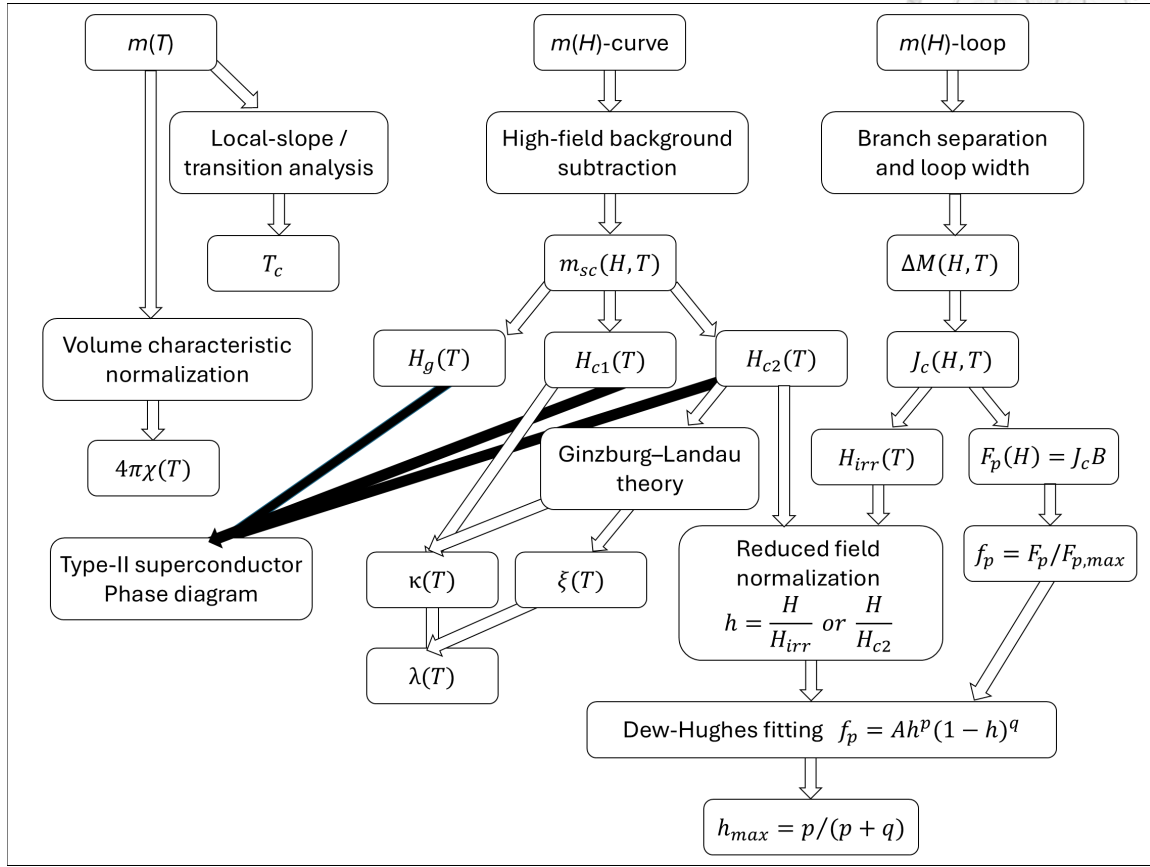


Figure 3.18: Magnetic-data processing workflow used in this study.

The extracted $J_c(H)$, $F_p(H)$, H_{irr} , and H_{c2} values provide the basis for the subsequent vortex-pinning analysis. In particular, the normalized pinning-force curves can be expressed using either $h = H/H_{irr}$ or $h = H/H_{c2}$, and are further analyzed using the Dew-Hughes fitting model. Therefore, the workflow shown in Figure 3.18 connects the raw magnetic measurements to the physical discussion of vortex pinning, irreversibility, and multilayer-structure effects in Chapter 4.

Overall, Chapter 3 defines the experimental foundation and the standardized analysis route of this thesis. By clearly separating $m(T)$, $m(H)$ -curve, and $m(H)$ -loop data, the

extracted magnetic quantities can be assigned to their proper measurement origins and used consistently in the following results and discussion.



Chapter 4 Magnetic Response and Vortex Pinning

Behavior in Nb/Sb₂Te₃ Multilayers



4.1 Analysis strategy and evidence chain

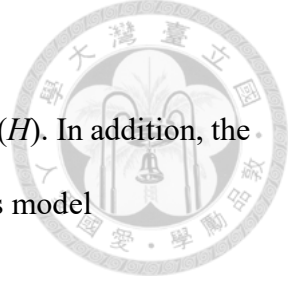
To investigate the vortex-pinning behavior in Nb/Sb₂Te₃ multilayer films, a hierarchical analysis strategy was adopted in this work. Rather than relying on a single physical quantity, multiple magnetic analyses were combined to establish an evidence chain linking experimental observations to the underlying vortex dynamics and pinning mechanisms [26], [61].

The magnetic hysteresis loop (M – H loop) serves as the primary experimental dataset throughout this chapter. According to the Bean critical-state model, the critical current density J_c can be extracted from the width of the hysteresis loop [25]. The corresponding pinning force is then obtained through

$$F_p = \mu_0 H J_c \quad (4.1)$$

which provides a quantitative measure of the interaction between vortices and pinning centers [27].

The analysis presented in this chapter is organized into three levels. The first level focuses on the temperature evolution of the M – H loops, including the hysteresis width, loop morphology, and the appearance of the Second Peak Effect (SPE). Since the SPE is frequently associated with changes in vortex dynamics and pinning behavior, it provides the first indication of vortex-state evolution within the multilayer system [61], [62].



The second level examines the field dependence of $J_c(H)$ and $F_p(H)$. In addition, the normalized pinning-force curves are analyzed using the Dew-Hughes model

$$f_p = Ah^p(1 - h)^q \quad (4.2)$$

to identify characteristic pinning behaviors and their temperature dependence [27].

The third level focuses on the relationship between J_c and magnetic field in logarithmic coordinates. Variations in the slope of the $\log(J_c) - \log(H)$ curves may indicate changes in vortex interactions or vortex configurations [26], [63]. Characteristic fields associated with the onset and evolution of the SPE are therefore identified and tracked as a function of temperature.

By correlating these analyses, a complete evidence chain is established from the raw magnetic measurements to the interpretation of vortex behavior. This framework provides the basis for discussing the competition between bulk superconductivity and interface-related effects in Nb/Sb₂Te₃ multilayer structures.

It should be noted that most of the magnetic measurements discussed in this chapter were performed under the in-plane (IP) field configuration. Therefore, unless otherwise specified, all magnetic data and extracted quantities presented in the following sections correspond to the IP geometry.

In addition to the systematic IP measurements, out-of-plane (OOP) $m(H)$ -loops were acquired for the 3ML sample as an independent orientation control. The IP configuration

aligns the applied field and vortex lines predominantly parallel to the multilayer interfaces, whereas the OOP configuration forces vortices to cross the interfaces. The comparison between these configurations is therefore used in the final stage of the analysis to evaluate whether the anomalous low-field pinning response is associated with the multilayer-interface geometry.

4.2 Temperature-dependent magnetization: superconducting transition and anomalous magnetic response

4.2.1 Superconducting quality verification of single-layer Nb film

Before fabricating the Nb/Sb₂Te₃ multilayer samples, a single-layer Nb reference film was prepared to verify whether the sputtering condition used in this work could produce a superconducting Nb layer suitable for subsequent multilayer growth. The sample, denoted as S_{Nb}, consists of a 1000 nm Nb film deposited using the same Nb sputtering condition described in section 3.3.3. This sample is discussed only in the present subsection as a process-validation reference, rather than as a main object for the vortex-pinning analysis in the following sections.

Figure 4.1 shows the temperature-dependent magnetization $m(T)$ of the S_{Nb} film. A clear diamagnetic transition is observed upon cooling, confirming the emergence of superconductivity in the single-layer Nb film. To determine the superconducting transition temperature more objectively, the derivative dm/dT was calculated according to the definition described in section 3.5.2. As shown in Figure 4.2, the

transition temperature of the S_{Nb} film is estimated to be approximately $T_c \approx 6.35$ K.

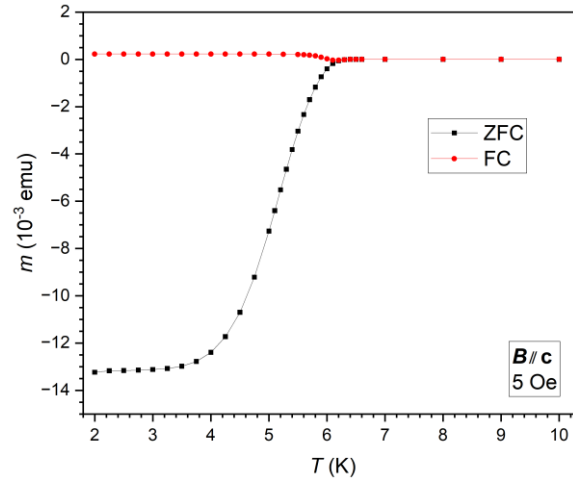


Figure 4.1: $m(T)$ of S_{Nb}

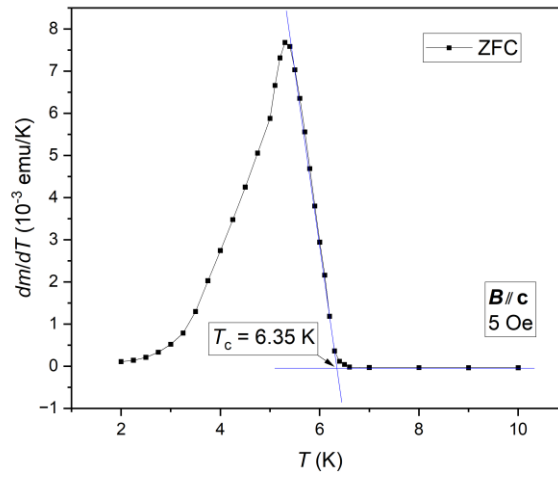


Figure 4.2: dm/dT of S_{Nb}

To further evaluate the magnetic shielding response of the single-layer Nb film, the measured magnetic moment was converted into the volume magnetic susceptibility. The measured magnetic moment m was first normalized by the sample volume V to obtain the

magnetization,

$$M = \frac{m}{V} \quad (4.3)$$



The measured susceptibility is then given by

$$\chi_{\text{meas}} = \frac{M}{H} = \frac{m}{VH} \quad (4.4)$$

where H is the applied magnetic field. Because the magnetic response of a thin film strongly depends on the field orientation, the demagnetization correction must be considered when discussing the absolute value of χ . The corrected susceptibility is expressed as

$$\chi = \frac{\chi_{\text{meas}}}{1 - N\chi_{\text{meas}}} \quad (4.5)$$

where N is the demagnetization factor. For a thin-film geometry, N can be approximated as $N \approx 0$ when the field is applied in the in-plane (IP) direction, whereas $N \approx 1$ when the field is applied in the out-of-plane (OOP) direction. Since the S_{Nb} film was measured under the OOP configuration, the demagnetization correction is particularly important for estimating its superconducting shielding response. In contrast, most of the multilayer data discussed in this chapter were measured under the IP configuration, for which the demagnetization correction is much smaller.

Figure 4.3 shows the temperature dependence of the demagnetization-corrected susceptibility of the S_{Nb} film. After entering the superconducting state, χ approaches the ideal diamagnetic shielding value, indicating that the deposited Nb layer exhibits a strong superconducting shielding response.

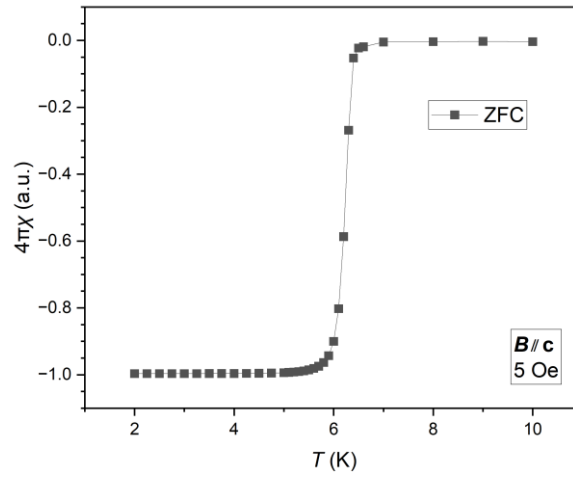


Figure 4.3: Magnetic Susceptibility of S_{Nb}

Therefore, the S_{Nb} measurement serves as a preliminary verification of the Nb deposition quality. Since defects and interfacial disorder can accumulate during multilayer deposition, confirming the superconducting quality of the single-layer Nb film is essential before discussing the more complex magnetic response and vortex-pinning behavior of the Nb/Sb₂Te₃ multilayers.

4.2.2 $M(T)$ curves of Nb/Sb₂Te₃ multilayer samples

After confirming the superconducting quality of the single-layer Nb reference film, the temperature-dependent magnetization of the Nb/Sb₂Te₃ multilayer samples was examined. The multilayer samples were fabricated according to the structures described in Table 3.4. In this section, two representative samples, denoted as 2ML and 3ML, are discussed to compare the superconducting magnetic response of different multilayer configurations.

Figure 4.4(a) and Figure 4.4(b) show the $m(T)$ curves of the 2ML and 3ML samples, respectively. Both samples exhibit a clear diamagnetic transition upon cooling, indicating the emergence of superconductivity in the multilayer films. Compared with the single-layer Nb reference film, the multilayer samples show a more complex temperature-dependent magnetic response. In particular, abnormal hump-like features can be observed in similar temperature ranges in both samples. At the present stage, these features cannot be completely excluded as possible experimental artifacts or isolated jump points. Nevertheless, their repeated appearance in different multilayer samples suggests that they should not be ignored and motivates further field-dependent magnetic measurements.

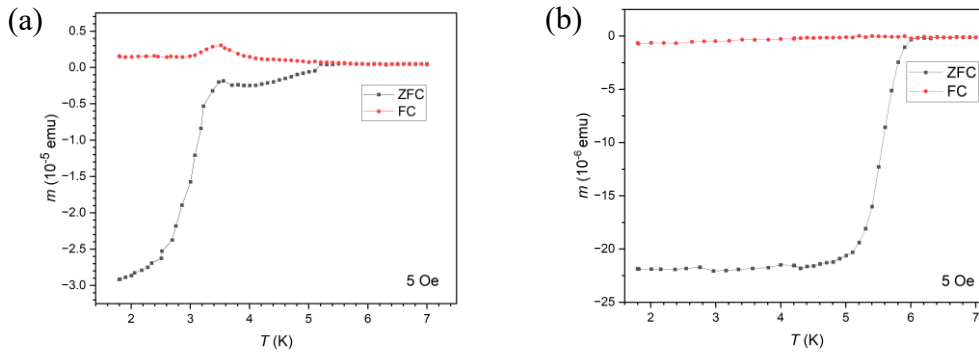


Figure 4.4: $m(T)$ curves of the 2ML and 3ML samples. (a)2ML (b)3ML.

To determine the superconducting transition temperature more objectively, the $m(T)$ curves were differentiated according to the criterion described in Sec. 3.5.2. Figure 4.5(a) and Figure 4.5(b) show the corresponding dm/dT curves of the 2ML and 3ML samples. From the derivative analysis, the superconducting transition temperatures are estimated to be $T_c = 5.73$ K for 2ML and $T_c = 6.08$ K for 3ML. These values confirm that superconductivity is retained after introducing the Sb_2Te_3 layers into the Nb-based multilayer structure.

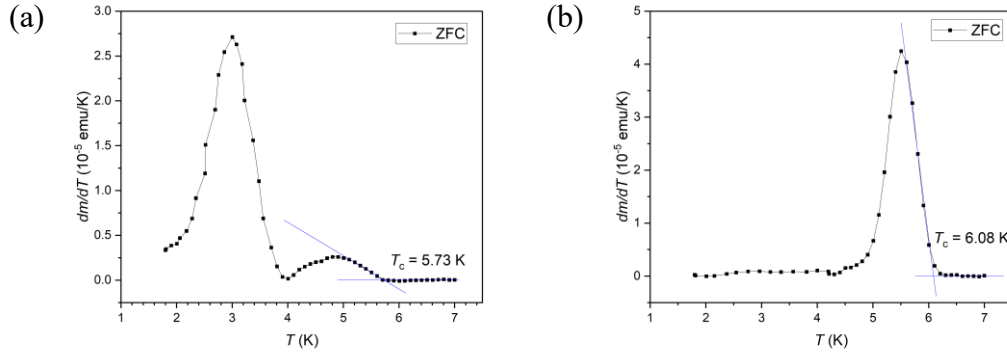
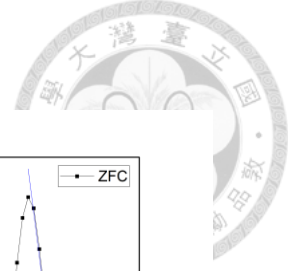


Figure 4.5: Corresponding dm/dT curves of the (a)2ML and (b)3ML samples.

The magnetic susceptibility $\chi(T)$ of the two multilayer samples is shown in Figs. Figure 4.6(a) and Figure 4.6(b). In both samples, the susceptibility exhibits a diamagnetic response after the superconducting transition, confirming the superconducting shielding behavior. However, in addition to the diamagnetic background, anomalous paramagnetic-like features are also observed near the transition region. Such behavior resembles the paramagnetic Meissner effect (PME), also known as the Wohleben effect, in which a positive magnetization or paramagnetic contribution may appear in a superconducting state under low-field cooling conditions [64], [65].

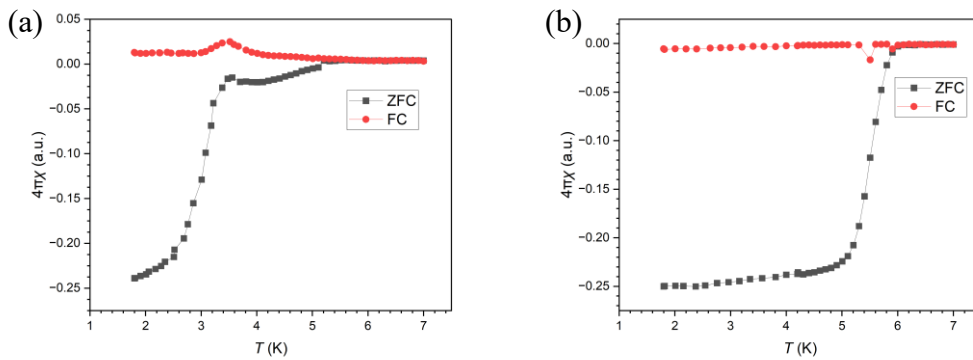
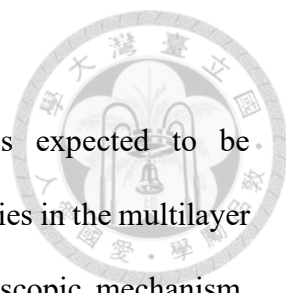


Figure 4.6: The magnetic susceptibility $4\pi\chi(T)$ of the (a)2ML (b)3ML samples.



In conventional superconductors, the Meissner response is expected to be diamagnetic. Therefore, the observation of paramagnetic-like anomalies in the multilayer samples is not treated here as direct evidence of a specific microscopic mechanism. Instead, it is regarded as an experimental indication that the magnetic response of the Nb/Sb₂Te₃ multilayers is more complex than that of a simple single-layer Nb film. Previous studies have shown that PME-like signals may arise from several mechanisms, including flux trapping, flux compression, sample geometry, and inhomogeneous superconducting nucleation [65], [66]. Therefore, the anomalous features observed in $m(T)$ provide a motivation for the subsequent analysis of field-dependent magnetization, magnetic hysteresis loops, critical current density, and pinning force.

It should be emphasized that the $m(T)$ results alone are insufficient to determine the origin of the paramagnetic-like anomalies. The role of this section is therefore to establish three experimental facts: first, both Nb/Sb₂Te₃ multilayer samples exhibit superconducting transitions; second, the transition temperatures can be quantitatively identified from the derivative analysis; and third, both samples show anomalous magnetic features near the superconducting transition. These observations form the starting point for the field-dependent vortex-pinning analysis presented in the following sections.

4.3 Field-dependent magnetization from $m(H)$ -curves: characteristic superconducting fields

4.3.1 Initial $m(H)$ -curves and field-induced suppression of diamagnetism

To further examine the field-induced suppression of the superconducting response, field-dependent magnetization measurements were performed on the Nb/Sb₂Te₃ multilayer samples.

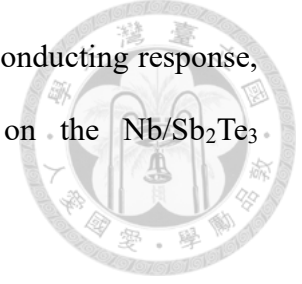


Figure 4.7 and show the raw $m(H)$ -curves of the 2ML and 3ML samples, respectively. In both samples, a strong diamagnetic signal can be observed at low magnetic fields, corresponding to the superconducting shielding response. As the applied field increases, the diamagnetic response is gradually suppressed and the magnetization approaches the high-field background. This behavior reflects the progressive weakening of the superconducting magnetic response under an external magnetic field.

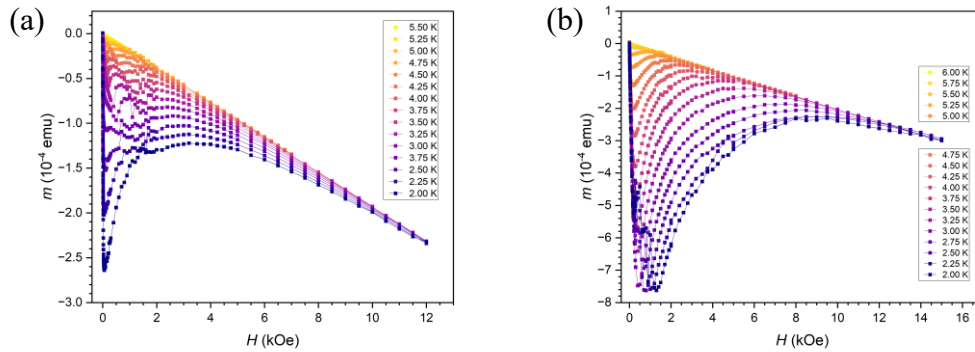


Figure 4.7: The raw $m(H)$ -curves of the (a)2ML and (b)3ML samples.

However, the raw $m(H)$ -curves contain not only the superconducting contribution but also a field-dependent background signal. This background may originate from the normal-state magnetic response of the film, substrate contribution, or small instrumental offsets. Therefore, direct extraction of characteristic fields from the raw curves may lead to ambiguity, especially near the high-field region where the superconducting signal becomes weak.

To isolate the superconducting contribution, the background signal was removed following the definition and procedure described in Sec. 3.5.3 and Sec. 3.6.3. The resulting background-subtracted $m(H)$ -curves are shown in Figure 4.8(a) and Figure 4.8(b) for the 2ML and 3ML samples, respectively. After background subtraction, the suppression of the diamagnetic signal becomes clearer, allowing the characteristic field scales to be identified more consistently.

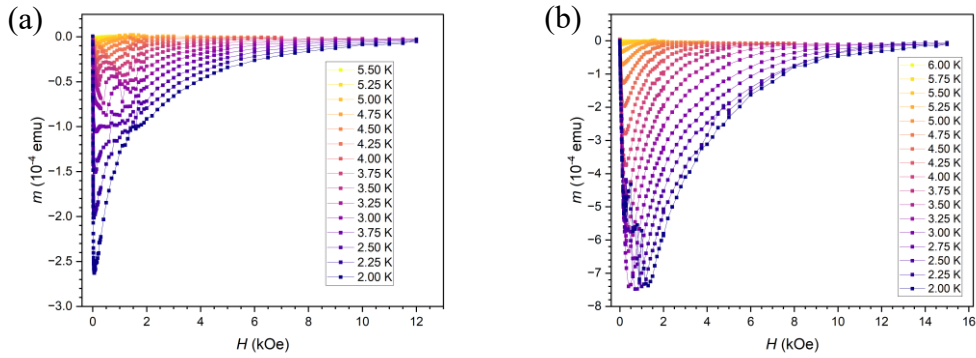


Figure 4.8: The Background-subtracted $m(H)$ -curves for (a)2ML and (b)3ML samples.

The background-subtracted curves provide the basis for the subsequent determination of H_{c1} , and H_{c2} . Here, H_{c1} corresponds to the field scale at which the low-field Meissner-like response starts to deviate from its initial behavior and H_{c2} corresponds to the field scale where the superconducting diamagnetic contribution is suppressed toward the background level. These characteristic fields will be analyzed as a function of temperature in the following subsection.

4.3.2 Determination of H_{c1} and H_{c2}

After the background subtraction of the $m(H)$ -curves, the lower critical field H_{c1} and upper critical field H_{c2} were extracted from the field-dependent magnetization data. The determination criteria used in this section follow the definitions introduced in Sec. 3.5.3, while the numerical procedure for H_{c2} extraction follows the automated workflow described in Sec. 3.6.3.

For H_{c1} , the initial low-field region of the $m(H)$ -curve is treated as the Meissner-like response. A linear fit is applied to this initial region, and H_{c1} is identified as the field at which the measured curve deviates from the fitted linear behavior by 1%. This criterion provides an empirical estimate of the field scale where flux penetration begins to modify the initial diamagnetic response.

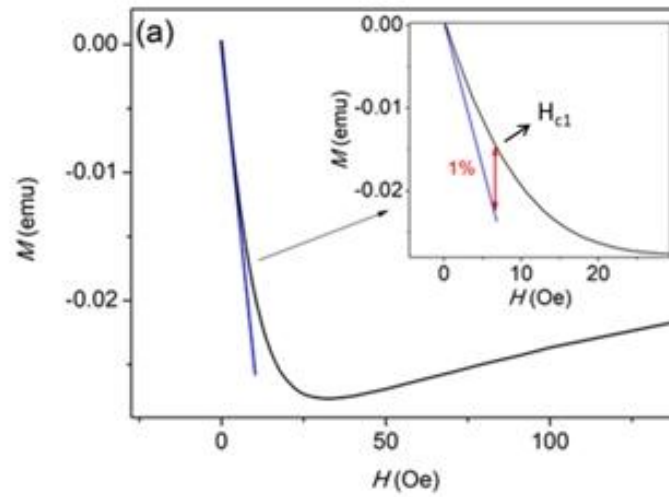


Figure 4.9: Schematic illustration of the determination criteria for H_{c1} from $m(H)$ -curves [48].

For H_{c2} , the high-field part of the background-subtracted $m(H)$ -curve is used. Following the procedure described in Sec. 3.6.3, the terminal region of the

superconducting diamagnetic signal is fitted by a straight line, and the intersection between this fitted line and the zero-magnetization baseline is taken as H_{c2} . In this work, H_{c2} is therefore treated as the field scale at which the superconducting diamagnetic contribution is suppressed toward the normal-state background.

Figure 4.10(a) and Figure 4.10(b) show the temperature dependence of H_{c1} and H_{c2} , respectively, for the 2ML and 3ML samples. As shown in Figure 4.10(a), H_{c1} decreases with increasing temperature in both samples, consistent with the weakening of the Meissner screening response as the system approaches the superconducting transition. The 3ML sample exhibits a larger H_{c1} than the 2ML sample over most of the measured temperature range, indicating that a higher field is required to disturb the initial diamagnetic response in the 3ML structure.

The temperature dependence of H_{c2} is shown in Figure 4.10(b). Both samples exhibit a decreasing H_{c2} with increasing temperature, as expected for the suppression of superconductivity by an external magnetic field. At low temperatures, the 3ML sample shows a higher H_{c2} than the 2ML sample, suggesting a more robust superconducting response under applied magnetic field. The difference between the two samples becomes smaller at higher temperatures, where both H_{c2} values approach the low-field region near the superconducting transition.

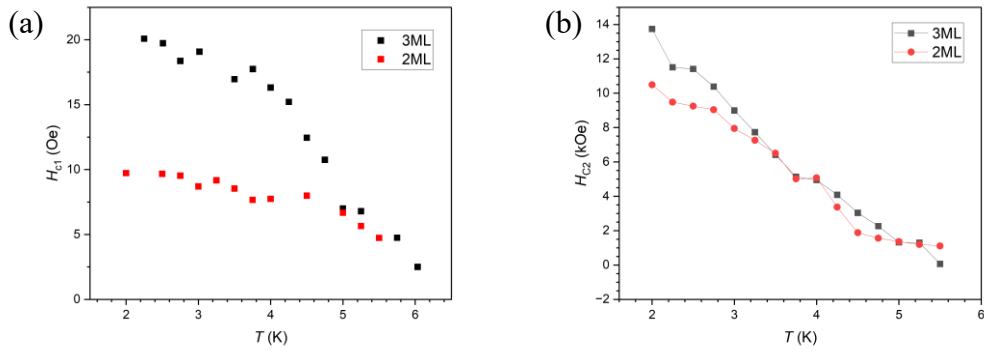


Figure 4.10: The temperature dependence of H_{c1} and H_{c2}

The combined comparison of $H_{c1}(T)$ and $H_{c2}(T)$ indicates that the 3ML sample generally exhibits stronger characteristic superconducting fields than the 2ML sample, especially at lower temperatures. This result suggests that increasing the number of Nb/Sb₂Te₃ interfaces modifies the magnetic response of the multilayer system. However, at this stage, H_{c1} and H_{c2} alone only provide characteristic field scales of the superconducting response. A more complete discussion of vortex pinning and magnetic irreversibility requires the analysis of $m(H)$ -loops, $J_c(H)$, H_{irr} , and $F_p(H)$, which will be presented in the following sections.

4.3.3 Effective coherence length, penetration depth, and GL parameter

In the previous sections, the lower critical field $H_{c1}(T)$ and upper critical field $H_{c2}(T)$ were determined from the field-dependent magnetization data. According to the Ginzburg–Landau (GL) framework, the upper critical field can be used to estimate the superconducting coherence length $\xi(T)$:



$$B_{c2}(T) = \frac{\Phi_0}{2\pi\xi^2(T)} \quad (4.6)$$

Therefore,

$$\xi(T) = \left[\frac{\Phi_0}{2\pi B_{c2}(T)} \right]^{1/2} \quad (4.7)$$

where $\Phi_0 = h/2e$ is the magnetic flux quantum. In the present calculation, H_{c2} was converted from Oe to Tesla before being substituted into the above expression. Since the main purpose here is to compare 2ML and 3ML under the same measurement geometry, the obtained $\xi(T)$ should be regarded as an effective coherence length corresponding to the present field configuration and multilayer structure.

To describe the temperature dependence of $\xi(T)$, two fitting forms were used. The first one is the GL model,

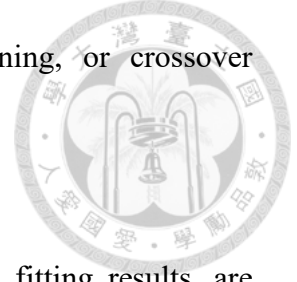
$$\xi(T) = \xi(0) \left(1 - \frac{T}{T_c} \right)^{-1/2} \quad (4.8)$$

The second one is an extended phenomenological model,

$$\xi(T) = \xi(0) \left[\frac{1 + T/T_c}{1 - T/T_c} \right]^p \quad (4.9)$$

where p is an empirical exponent describing the deviation from the simple GL temperature dependence. In this work, the extended model is treated as a phenomenological fitting function rather than a replacement of GL theory. Its purpose is to describe the effective temperature dependence possibly modified by interfacial

coupling, inhomogeneous superconducting response, vortex pinning, or crossover between different vortex states in the multilayer system.



The $\xi(T)$ values converted from $H_{c2}(T)$, together with the fitting results, are shown in Figure 4.11. For 2ML over the full measured temperature range, the GL models give

$$\xi_{\text{GL}}(0) = 13.85 \text{ nm}, \quad R^2 = 0.780,$$

whereas the extended model give

$$\xi_{\text{ext}}(0) = 15.02 \text{ nm}, \quad p = 0.363, \quad R^2 = 0.866$$

For 3ML, the GL and extended model gives

$$\xi_{\text{GL}}(0) = 16.81 \text{ nm}, \quad R^2 = 0.845,$$

and

$$\xi_{\text{ext}}(0) = 9.58 \text{ nm}, \quad p = 0.658, \quad R^2 = 0.971,$$

respectively. Therefore, the extended model improves the fitting quality for both samples, but the improvement and the overall R^2 are much more significant for 3ML than for 2ML.

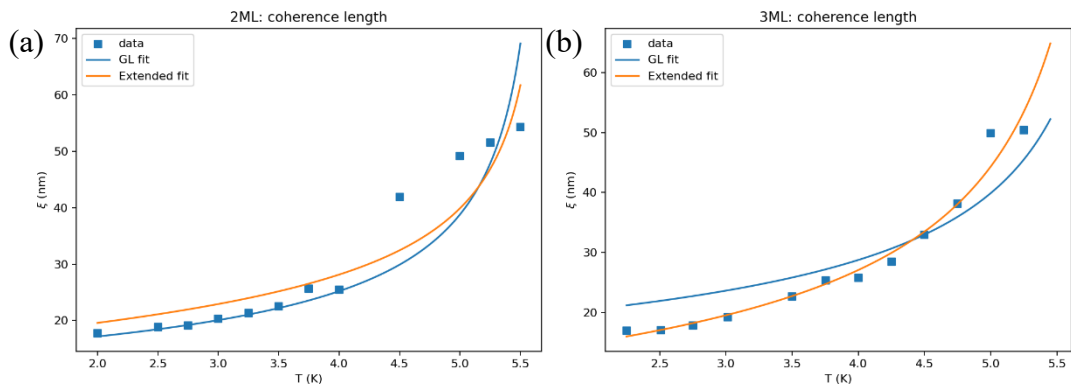


Figure 4.11: Temperature dependence of the coherence length $\xi(T)$ for (a) 2ML and

(b)3ML.



In Figure 4.11, $\xi(T)$ of 3ML increases smoothly and continuously with increasing temperature and can be well described by the extended model. In contrast, although $\xi(T)$ of 2ML also increases with temperature, the full-range fitting quality is poorer. This indicates that the temperature dependence of 2ML is not fully described by a single T_c and a single homogeneous superconducting branch. This behavior suggests that 2ML may exhibit different effective superconducting responses in different temperature regions, which will be further examined together with $\kappa(T)$, $\lambda_{\text{eff}}(T)$, and the $m(T)$ and $\chi(T)$ results discussed in Sec. 4.2.2.

Next, the GL parameter $\kappa(T)$ was obtained from $H_{c1}(T)$ and $H_{c2}(T)$ using the generalized Abrikosov–Ginzburg–Landau relation [5], [33], [67]:

$$\frac{H_{c2}(T)}{H_{c1}(T)} = \frac{2\kappa^2(T)}{\ln \kappa(T) + 0.5} \quad (4.10)$$

Figure 4.12 shows the temperature dependence of $\kappa(T)$. For both samples, the obtained κ values are much larger than $1/\sqrt{2}$, confirming that both 2ML and 3ML are well within the type-II superconducting regime. For 3ML, $\kappa(T)$ decreases relatively smoothly from approximately $\kappa \sim 31$ at low temperature to $\kappa \sim 16$ at higher temperature. In contrast, 2ML exhibits a pronounced drop around $4.0 - 4.5$ K, where κ rapidly decreases from approximately $\kappa \sim 34$ to $\kappa \sim 19$. This temperature region is therefore used as the main boundary for the segmented fitting analysis below.

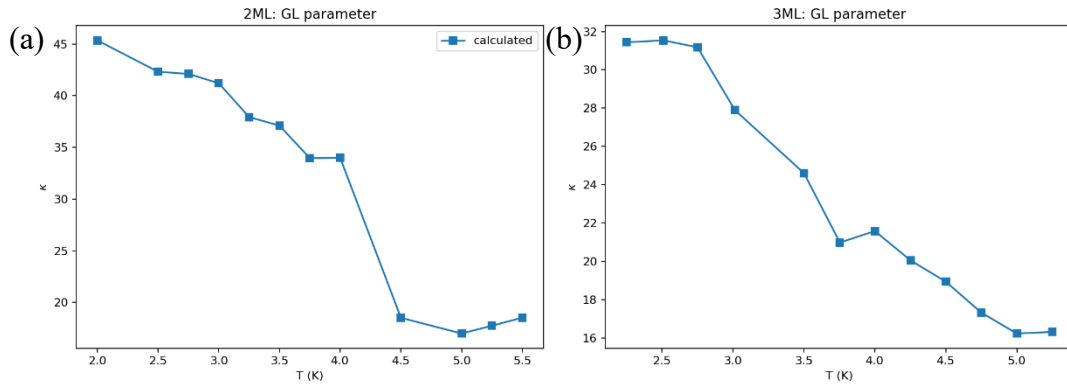


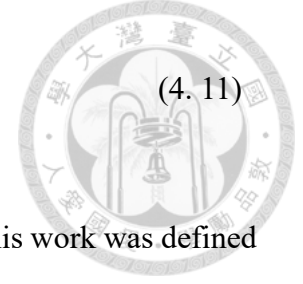
Figure 4.12: Temperature dependence of the GL parameter $\kappa(T)$ for (a) 2ML and (b) 3ML.

In Figure 4.12, although both 2ML and 3ML have κ values far above $1/\sqrt{2}$, their temperature dependences are qualitatively different. The smoother behavior of $\kappa(T)$ in 3ML suggests that the effective superconducting state reflected by $H_{c1}(T)$ and $H_{c2}(T)$ is more self-consistent over the measured range. In contrast, the abrupt change of $\kappa(T)$ in 2ML around 4.0 – 4.5 K indicates a possible crossover in the effective superconducting or vortex response.

This behavior is consistent with the $m(T)$ and $\chi(T)$ results discussed in Sec. 4.2.2 and Figure 4.5. The superconducting transition of 2ML is not a single sharp transition but a broader and segmented process. An onset-like superconducting response appears in the higher-temperature region, whereas a pronounced diamagnetic shielding response develops mainly at lower temperature. Therefore, the failure of a single full-range GL-like fit for 2ML is not merely a numerical problem. Instead, it reflects a temperature-dependent change in the effective superconducting state.

After $\kappa(T)$ is obtained, the effective penetration depth can be calculated using

$$\lambda_{\text{eff}}(T) = \kappa(T)\xi(T) \quad (4.11)$$



Here, the notation λ_{eff} is used instead of λ because H_{c1} in this work was defined from the deviation of the $m(H)$ -curve from the linear diamagnetic response. For thin-film and multilayer samples, this field may be affected by first flux penetration, demagnetization effects, geometrical barriers, and vortex pinning. Therefore, the penetration depth inferred from this procedure should be treated as an effective penetration depth rather than the intrinsic London penetration depth of bulk Nb.

Figure 4.13 shows $\lambda_{\text{eff}}(T)$ for 2ML and 3ML over the full measured temperature range. For 2ML, the GL model gives

$$\lambda_{\text{GL}}(0) = 590.20 \text{ nm}, \quad R^2 = -13.115,$$

whereas the extended model gives

$$\lambda_{\text{ext}}(0) = 821.57 \text{ nm}, \quad Q = 15.76, \quad P = -0.280, \quad R^2 = 0.785$$

For 3ML, the GL model gives

$$\lambda_{\text{GL}}(0) = 530.56 \text{ nm}, \quad R^2 = 0.888,$$

whereas the extended model gives

$$\lambda_{\text{ext}}(0) = 528.25 \text{ nm}, \quad Q = 5.738, \quad P = -0.853, \quad R^2 = 0.932$$

The negative R^2 of the GL fitting for 2ML indicates that the model does not describe the full-range $\lambda_{\text{eff}}(T)$ data. Since the fitting quality was evaluated using the coefficient of determination R^2 , defined as



$$R^2 = 1 - \frac{\sum_i (y_i - y_{\text{fit},i})^2}{\sum_i (y_i - \bar{y})^2} \quad (4.12)$$

A value of R^2 closer to 1 indicates better agreement between the model and the data. If $R^2 < 0$, the fitting residual is larger than the total variance relative to the mean value, indicating that the model is unsuitable for describing the data.

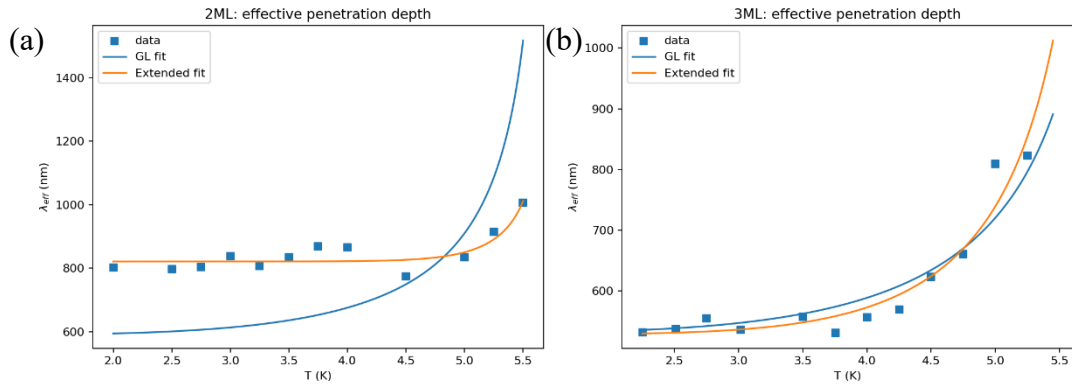


Figure 4.13: Temperature dependence of the effective penetration depth $\lambda_{\text{eff}}(T)$ for (a) 2ML and (b) 3ML over the full measured temperature range.

In Figure 4.13, $\lambda_{\text{eff}}(T)$ of 3ML increases with increasing temperature, and its overall trend can be reasonably described by both the GL and extended models, with the extended model giving a better fitting quality. This is consistent with the smoother temperature dependences observed in $\xi(T)$ and $\kappa(T)$ for 3ML.

In contrast, the full-range $\lambda_{\text{eff}}(T)$ of 2ML cannot be described by a single GL model. Since $\lambda_{\text{eff}}(T)$ is derived from H_{c1} , H_{c2} , and κ , it is particularly sensitive to the experimental definition of H_{c1} and to the vortex-entry condition. Therefore, the failure of the full-range $\lambda_{\text{eff}}(T)$ fitting in 2ML further supports the presence of different

effective superconducting or vortex states in different temperature regions.

To examine the segmented behavior of 2ML more directly, the data were further divided into low-temperature and high-temperature regions according to the changes observed in $\kappa(T)$, $m(T)$, and $\chi(T)$. In the low-temperature region, the fitting of $\xi(T)$ gives

$$\xi_{\text{GL}}(0) = 14.26 \text{ nm}, \quad R^2 = 0.938$$

for the GL model, and

$$\xi_{\text{ext}}(0) = 12.40 \text{ nm}, \quad p = 0.440, \quad R^2 = 0.945$$

for the extended model.

In the high-temperature region, the GL model gives

$$\xi_{\text{GL}}(0) = 13.73 \text{ nm}, \quad R^2 = -4.628,$$

whereas the extended model gives

$$\xi_{\text{ext}}(0) = 33.09 \text{ nm}, \quad p = 0.133, \quad R^2 = 0.876$$

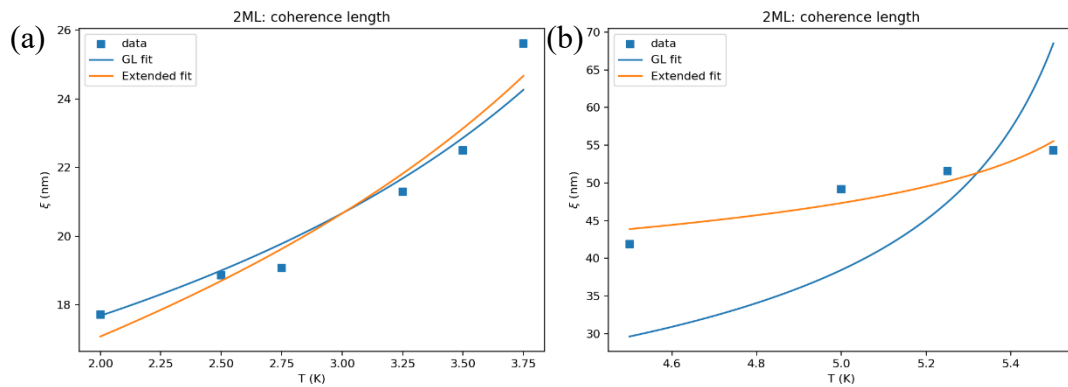


Figure 4.14: Fitting of $\xi(T)$ for 2ML at (a) Low-temperature and (b) High-temperature.

As shown in Figure 4.14(a), the low-temperature $\xi(T)$ data of 2ML can be well

described by both the GL and extended models, and both R^2 values are higher than those obtained from the full-range fitting. This suggests that the effective superconducting response of 2ML is more self-consistent in the low-temperature region and can be approximated by a single phenomenological branch.

In contrast, Figure 4.14(b) shows that the GL model fails in the high-temperature region, while the extended model still gives a moderate fitting quality. However, the fitted $\xi_{\text{ext}}(0)$ and p values are clearly different from those in the low-temperature region. This suggests that the high-temperature behavior is not a simple continuation of the low-temperature superconducting branch, but may correspond to a different effective superconducting response or weaker superconducting connectivity.

The same segmented analysis was also applied to $\lambda_{\text{eff}}(T)$ of 2ML. In the low-temperature region, the GL model gives

$$\lambda_{\text{GL}}(0) = 781.10 \text{ nm}, \quad R^2 = 0.833,$$

whereas the extended model gives

$$\lambda_{\text{ext}}(0) = 798.41 \text{ nm}, \quad Q = 9.657, \quad P = -5.000, \quad R^2 = 0.977$$

In the high-temperature region, the GL model gives

$$\lambda_{\text{GL}}(0) = 467.82 \text{ nm}, \quad R^2 = -1.891,$$

whereas the extended model gives

$$\lambda_{\text{ext}}(0) = 399.23 \text{ nm}, \quad Q = 0.050, \quad P = -0.150, \quad R^2 = 0.991$$

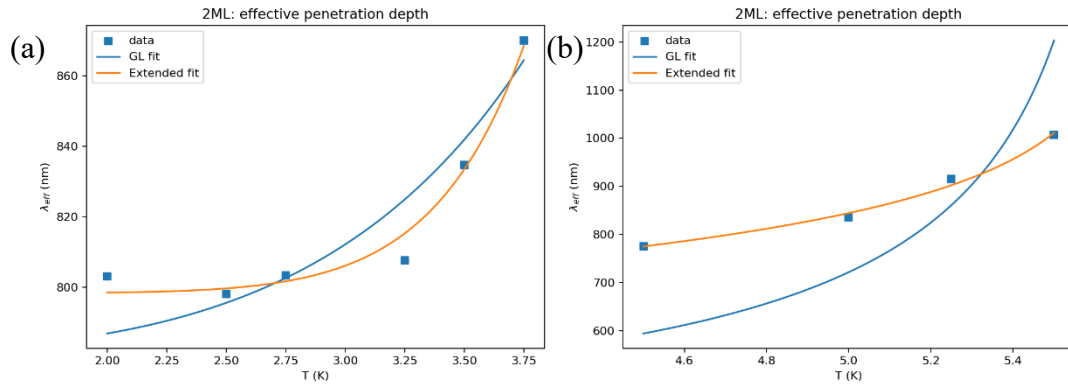
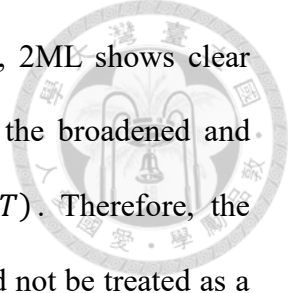


Figure 4.15: Fitting of $\lambda_{\text{eff}}(T)$ for 2ML at (a) Low-temperature and (b) High-temperature.

In Figure 4.15(a), the low-temperature $\lambda_{\text{eff}}(T)$ fitting is significantly improved compared with the full-range fitting, especially when the extended model is used. However, the fitted value $P = -5.000$ reaches the boundary of the fitting constraint. Therefore, this value should not be interpreted as a well-defined critical exponent. A more conservative interpretation is that the low-temperature $\lambda_{\text{eff}}(T)$ follows a more consistent effective temperature dependence, allowing the phenomenological model to describe the curve shape.

On the other hand, Figure 4.15(b) shows that although the high-temperature extended-model fitting gives a high R^2 , the fitted value $Q = 0.050$ reaches the lower fitting bound and the uncertainty of $\lambda(0)$ is extremely large. Therefore, the high-temperature extended-model fitting should be regarded as empirical curve fitting rather than a reliable extraction of intrinsic parameters.

Overall, 3ML exhibits smoother and more consistent temperature dependences in $\xi(T)$, $\lambda_{\text{eff}}(T)$, and $\kappa(T)$, suggesting that its effective superconducting response is closer



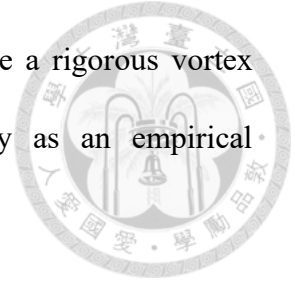
to a single continuous type-II superconducting branch. In contrast, 2ML shows clear segmented behavior in $\xi(T)$, $\lambda_{\text{eff}}(T)$, and $\kappa(T)$, consistent with the broadened and multi-step superconducting transition observed in $m(T)$ and $\chi(T)$. Therefore, the failure of the full-range GL or extended-model fitting in 2ML should not be treated as a simple fitting failure. Instead, it reflects possible temperature-dependent changes in superconducting volume fraction, interlayer connectivity, or vortex response. In the following discussion, the fitting parameters of 3ML are treated as more representative effective superconducting parameters, whereas the ξ , λ_{eff} , and κ values of 2ML are discussed as segmented effective parameters for semi-quantitative comparison.

4.3.4 Definition of H_g as an empirical diamagnetic maximum field

In addition to H_{c1} and H_{c2} , another characteristic field can be extracted from the background-subtracted $m(H)$ -curves. In this thesis, this field is denoted as H_g . The notation H_g is motivated by the vortex-matter phase diagram of type-II superconductors, where a glass-related field $H_g(T)$ is often used to separate a vortex-glass-like mixed state from a vortex-liquid-like mixed state between $H_{c1}(T)$ and $H_{c2}(T)$. In such a picture, the mixed state below H_g is associated with stronger vortex immobilization, whereas the region above H_g is associated with a more mobile vortex response [68].

However, it should be emphasized that the H_g used in this thesis is not determined from transport scaling or zero-resistance criteria. Previous studies have shown that identifying a true vortex-glass transition requires great caution, since apparent scaling behavior alone may not be sufficient to prove the existence of a thermodynamic phase

transition. Therefore, in the present work, H_g is not claimed to be a rigorous vortex glass–liquid transition field. Instead, it is defined operationally as an empirical characteristic field extracted from the $m(H)$ -curve.



As shown schematically in Figure 4.16, H_g is defined as the magnetic field at which the background-subtracted diamagnetic response reaches its maximum magnitude.

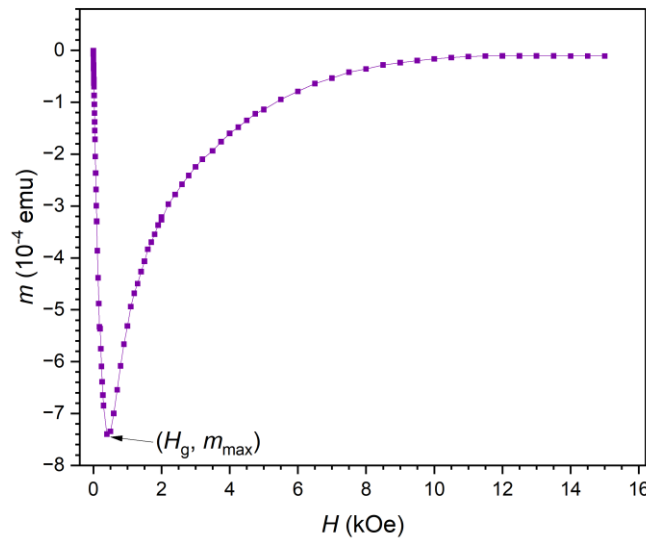


Figure 4.16: Definition of H_g in this work.

This definition gives H_g a phenomenological meaning in the present magnetic measurements. For $H < H_g$, the magnitude of the diamagnetic response increases with increasing magnetic field. For $H > H_g$, the diamagnetic contribution is progressively suppressed by the applied field and eventually approaches the high-field background near H_{c2} . Therefore, (H_g) is used here as an empirical crossover field separating the initial enhancement of the diamagnetic response from its field-induced suppression.

Figure 4.17 shows the temperature dependence of H_g for the 2ML and 3ML samples. For both samples, H_g decreases with increasing temperature, consistent with the weakening of the superconducting diamagnetic response near the superconducting transition. The 3ML sample generally exhibits a higher H_g than the 2ML sample over most of the measured temperature range, indicating that a larger applied field is required to reach the maximum diamagnetic response in the 3ML structure. This result suggests that the field-dependent superconducting magnetic response is more robust in the 3ML sample.

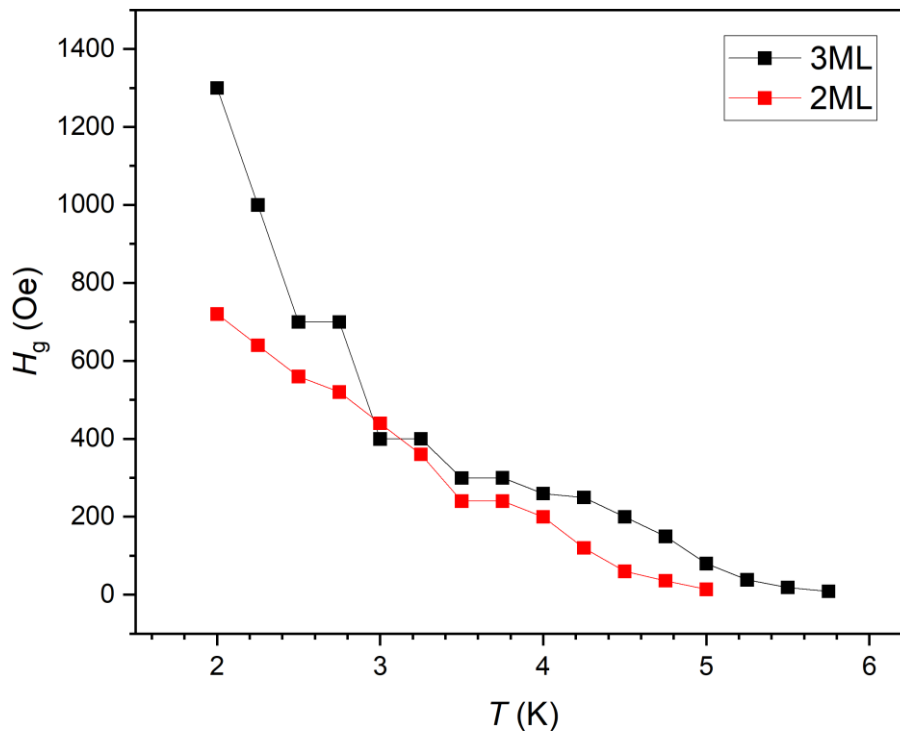


Figure 4.17: Temperature dependence of H_g for the 2ML and 3ML samples.

Together with H_g and H_{c2} , the empirical field H_g provides an intermediate field scale for describing the evolution of the $m(H)$ -curve. While H_{c1} characterizes the departure from the initial Meissner-like response and H_{c2} characterizes the suppression of the superconducting diamagnetic signal near the high-field region, H_g identifies the field where the diamagnetic response is strongest. These three characteristic fields establish the field-dependent magnetic-response framework before the hysteresis-loop-based vortex-pinning analysis is introduced in the following sections.

4.3.5 Temperature dependence of H_{c1} , H_g , and H_{c2}

After defining H_{c1} , H_g , and H_{c2} from the background-subtracted $m(H)$ -curves, their temperature dependences were summarized to construct a characteristic-field diagram for the Nb/Sb₂Te₃ multilayer samples. The purpose of this diagram is not to claim a complete thermodynamic vortex phase diagram, but to compare the characteristic magnetic-field scales obtained from the $m(H)$ -curve analysis.

Figure 4.18 shows a schematic vortex-matter phase diagram for a type-II superconductor adapted from Strachan et al. In the conventional picture, $H_{c1}(T)$ separates the Meissner state from the mixed state, while $H_{c2}(T)$ marks the boundary between the superconducting mixed state and the normal state. Within the mixed-state region, the vortex response may be further separated into vortex-glass-like and vortex-liquid-like regimes by a glass-related field $H_g(T)$ [68]. However, the existence of a true vortex-glass transition generally requires transport scaling or other dynamic evidence. Strachan et al. also emphasized that apparent scaling behavior alone is not sufficient to

prove a thermodynamic phase transition [68]. Therefore, in this thesis, the H_g extracted from $m(H)$ -curves is treated as an empirical diamagnetic maximum field rather than as a rigorously established vortex glass–liquid transition field.

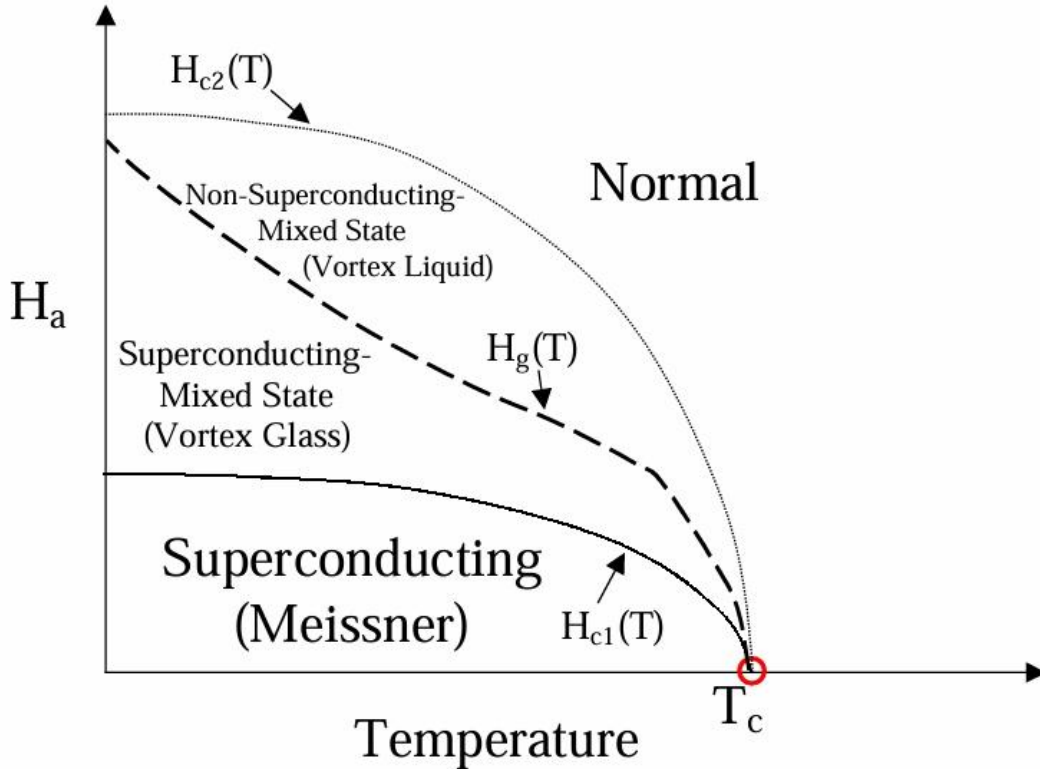
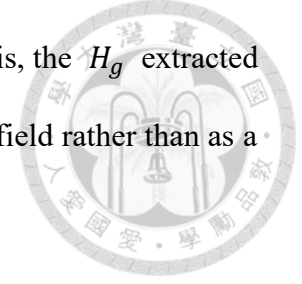


Figure 4.18: Schematic vortex-matter phase diagram for a type-II superconductor [68].

Figure 4.19(a) and Figure 4.19(b) show the temperature dependences of H_{c1} , H_g , and H_{c2} for the 2ML and 3ML samples, respectively. All three characteristic fields decrease with increasing temperature, consistent with the progressive weakening of the superconducting magnetic response as the system approaches the superconducting transition. This trend is consistent with the general expectation for type-II superconductors, in which the superconducting field scales shrink as thermal fluctuations

and field-induced suppression become stronger near T_c [26].

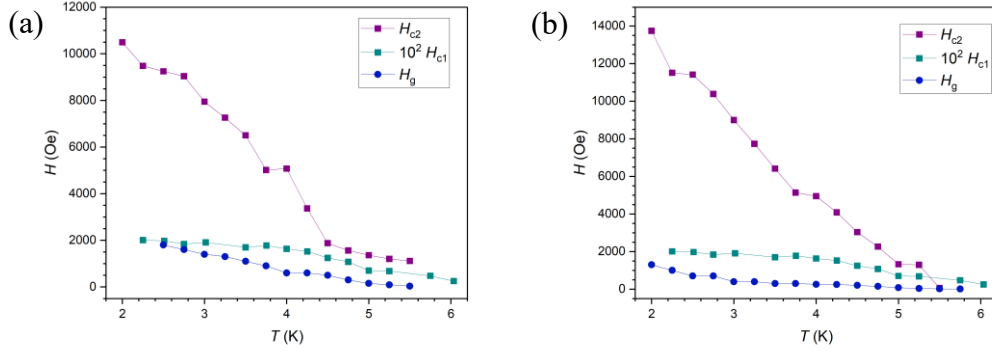
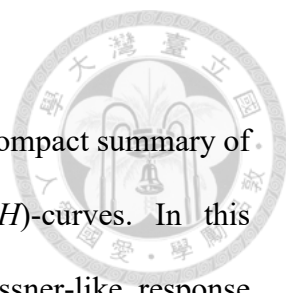


Figure 4.19: The phase diagram of (a)2ML and (b)3ML samples.

Because H_{c1} is much smaller than H_g and H_{c2} , H_{c1} is multiplied by 10^2 in the figure for clarity. This rescaling is only for graphical presentation and does not change the physical value of H_{c1} . The original H_{c1} values remain in the order of several to several tens of Oe, whereas H_g and H_{c2} are in the hundreds to thousands of Oe range. Therefore, the label $10^2 H_{c1}$ is explicitly shown in the legend to avoid confusion.

For the 2ML sample, H_{c2} decreases from the low-temperature region toward the superconducting transition, while H_g remains between H_{c1} and H_{c2} over the measured temperature range. This indicates that the maximum diamagnetic response occurs inside the mixed-state-like field range defined by the $m(H)$ -curve analysis. The same hierarchy is also observed in the 3ML sample, where H_g lies above H_{c1} and below H_{c2} . Compared with 2ML, the 3ML sample generally exhibits larger characteristic fields, especially at lower temperatures, suggesting a more robust field-dependent superconducting magnetic response.



The resulting characteristic-field diagrams therefore provide a compact summary of the superconducting magnetic response obtained from the $m(H)$ -curves. In this framework, H_{c1} represents the field scale where the initial Meissner-like response begins to deviate, H_g represents the field where the diamagnetic response reaches its maximum magnitude, and H_{c2} represents the field scale where the superconducting diamagnetic contribution is suppressed toward the high-field background. These quantities establish the magnetic-field hierarchy of the multilayer samples.

4.4 Magnetic hysteresis and irreversible vortex response

4.4.1 Temperature evolution of $m(H)$ -loops

After establishing the characteristic fields from the $m(H)$ -curves, magnetic hysteresis loop measurements were performed to examine the irreversible magnetic response of the Nb/Sb₂Te₃ multilayer samples. In contrast to the $m(H)$ -curves discussed in Sec. 4.3, the $M(H)$ -loops discussed here contain both increasing-field and decreasing-field branches. Therefore, they provide direct information on the hysteretic magnetization associated with vortex pinning and magnetic irreversibility in the superconducting state.

Figure 4.20(a) and Figure 4.20(b) show the raw $M(H)$ -loops of the 2ML and 3ML samples, respectively. In both samples, the raw loops exhibit a field-dependent background contribution, which becomes especially visible in the high-field region. This background may originate from the normal-state magnetic response of the multilayer film, substrate contribution, or instrumental offset. Since the superconducting hysteresis width

is the key quantity for the later calculation of the critical current density, the background contribution must be removed before extracting the loop width.

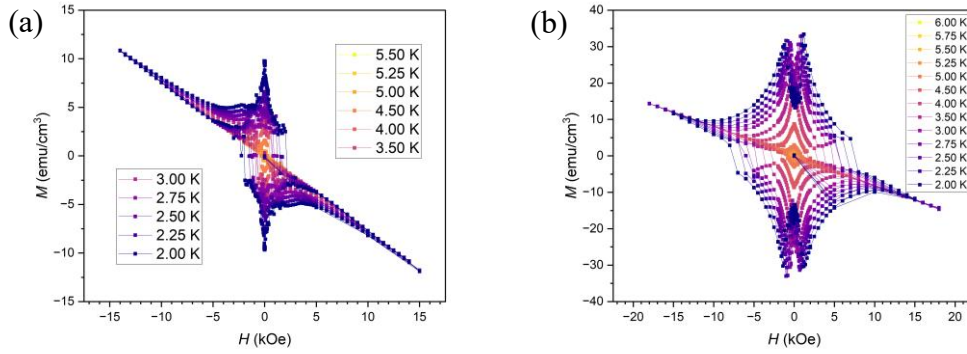


Figure 4.20: The raw $M(H)$ -loops of the (a)2ML and (b)3ML samples.

The background-subtracted $M(H)$ -loops of the 2ML and 3ML samples are shown in Figs. Figure 4.21(a) and Figure 4.21(b), respectively. The background subtraction was performed according to the procedure described in Sec. 3.5.3. After background subtraction, the superconducting hysteresis becomes more clearly resolved. Both samples exhibit closed hysteresis loops below the superconducting transition temperature, confirming the presence of irreversible magnetization. In type-II superconductors, such irreversible magnetization is generally associated with vortex pinning and the delayed response of flux motion during field sweeping [26].

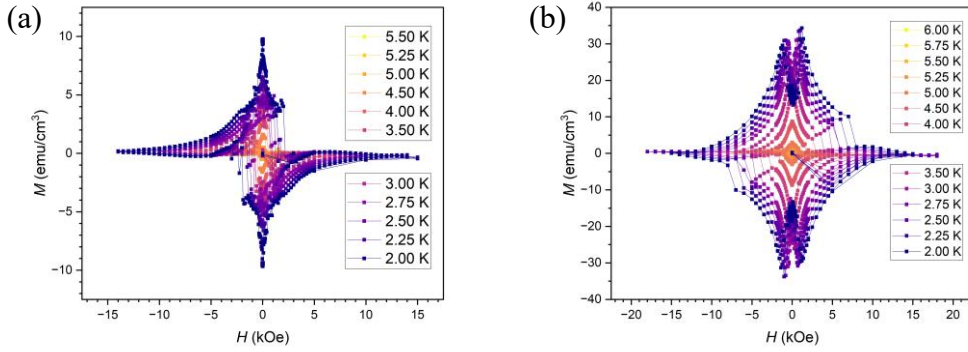


Figure 4.21: The background-subtracted $M(H)$ -loops of the (a)2ML and (b)3ML samples.

For both 2ML and 3ML, the loop width increases as temperature decreases. This behavior indicates that the irreversible magnetic response becomes stronger at lower temperatures, which is consistent with enhanced vortex pinning and reduced thermal activation of vortex motion. Near the superconducting transition temperature, the hysteresis loops become narrower, reflecting the weakening of superconductivity and the reduced ability of the sample to sustain irreversible vortex configurations.

A comparison between the two samples shows that the 3ML sample exhibits a more pronounced hysteretic response than the 2ML sample over the measured temperature range. In particular, the background-subtracted loops of 3ML remain broader and more strongly developed at low temperatures. This suggests that the multilayer configuration in 3ML leads to a stronger irreversible magnetic response than that in 2ML. However, this subsection focuses only on the qualitative temperature evolution of the loop morphology. The quantitative extraction of $J_c(H)$, H_{irr} , and $F_p(H)$ from these $M(H)$ -loops will be discussed in the following sections.

4.4.2 Temperature evolution of $J_c(H)$



After confirming the temperature-dependent hysteretic response in the background-subtracted $M(H)$ -loops, the critical current density $J_c(H)$ was calculated from the hysteresis width. In the Bean critical-state model, the irreversible magnetization of a type-II superconductor originates from persistent shielding currents sustained by vortex pinning. Therefore, the width of the magnetic hysteresis loop provides an estimate of the critical current density.

For each applied magnetic field, the loop width was obtained from the difference between the increasing-field and decreasing-field branches:

$$\Delta M(H) = |M_{\text{dec}}(H) - M_{\text{inc}}(H)| \quad (4.13)$$

where $M_{\text{dec}}(H)$ and $M_{\text{inc}}(H)$ denote the magnetization values on the decreasing-field and increasing-field branches, respectively. The critical current density was then calculated using the Bean-model expression for a rectangular sample,

$$J_c(H) = \frac{20\Delta M(H)}{a \left(1 - \frac{a}{3b}\right)} \quad (4.14)$$

where a and b are the sample dimensions in cm with $a < b$, ΔM is expressed in emu/cm³, and J_c is obtained in A/cm². The geometrical parameters used in this calculation follow the sample dimensions described in Chapter 3. Since the absolute value of J_c can be affected by sample geometry, demagnetization effects, and the assumptions of the Bean model, the following discussion focuses primarily on the relative temperature and field dependence of $J_c(H)$.

Figure 4.22(a) and Figure 4.22(b) show the field dependence of $J_c(H)$ for the 2ML and 3ML samples, respectively. For both samples, J_c increases significantly as the temperature decreases. This behavior is consistent with the strengthening of vortex pinning at lower temperatures, where thermal activation of vortex motion is reduced. Near the superconducting transition temperature, the hysteresis loop becomes narrower and the extracted J_c is strongly suppressed, reflecting the weakening of irreversible superconducting response.

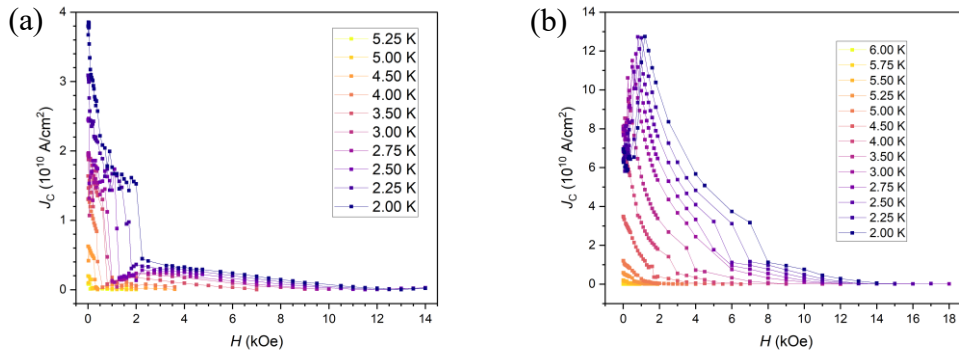


Figure 4.22: the field dependence of $J_c(H)$ for the (a)2ML and (b)3ML samples

For the 2ML sample, $J_c(H)$ is concentrated mainly in the low-field region. As the applied field increases, J_c rapidly decreases and approaches zero at high fields. At lower temperatures, a weak non-monotonic behavior appears in the intermediate-field region, suggesting that the irreversible vortex response is not governed by a simple monotonic field dependence.

In contrast, the 3ML sample exhibits a much larger J_c over a broader field range. At low temperatures, $J_c(H)$ not only reaches a higher magnitude but also remains finite up to higher magnetic fields. In addition, the $J_c(H)$ curves show a more pronounced

field-dependent enhancement in the low-to-intermediate field region. This result indicates that the 3ML multilayer structure supports a stronger and more robust irreversible current response than the 2ML structure.



The comparison between 2ML and 3ML demonstrates that the multilayer configuration strongly affects the field-dependent critical current density. The larger J_c and wider field range observed in 3ML are consistent with the stronger hysteresis loops discussed in Sec. 4.4.1. However, $J_c(H)$ alone mainly quantifies the irreversible current-carrying capability.

4.4.3 Temperature dependence of the zero-field critical current density and Griessen-type pinning analysis

To further examine the temperature-dependent pinning behavior, the critical current density at zero applied field, $J_c(H_{appl} = 0, T)$, was extracted from each $m(H)$ -loop. The decreasing-field and increasing-field branches were linearly interpolated to $H_{appl} = 0$, and the resulting loop width was converted into J_c using the Bean-model expression introduced in Sec. 4.4.2. It should be emphasized that $J_c(H_{appl} = 0, T)$ denotes the loop-derived critical current density at zero applied field, whereas $J_c(0)$ in the following equations represents the fitted zero-temperature scale.

Within the weak collective-pinning framework, spatial fluctuations of the local superconducting transition temperature correspond to δT_c pinning, whereas spatial fluctuations of the charge-carrier mean free path ℓ correspond to $\delta \ell$ pinning. These

two contributions exhibit distinct reduced-temperature dependences in the single-vortex-pinning regime [69]. Without repeating the microscopic derivation, the resulting Griessen-type expressions can be written as follows, where $t = T/T_c$:

$$J_c^{(\delta T_c)}(t) = J_c(0)(1 - t^2)^{(7/6)}(1 + t^2)^{(5/6)} \quad (4.15)$$

$$J_c^{(\delta \ell)}(t) = J_c(0)(1 - t^2)^{(5/2)}(1 + t^2)^{(-1/2)} \quad (4.16)$$

For each sample, T_c was fixed at the value obtained from the $m(T)$ analysis, namely 5.73 K for 2ML and 6.08 K for 3ML, while $J_c(0)$ was treated as the only fitting parameter. For 2ML, the $\delta \ell$ model gives $J_c(0) = 4.53 \times 10^6 \text{ A/cm}^2$, $R^2 = 0.903$, and $NRMSE = 0.104$. These values are substantially better than those obtained using the δT_c model, for which $R^2 = 0.509$ and $NRMSE = 0.235$. For 3ML, neither reference curve provides a satisfactory global description. The δT_c fit gives $J_c(0) = 1.10 \times 10^8 \text{ A/cm}^2$, $R^2 = 0.485$, and $NRMSE = 0.245$, whereas the $\delta \ell$ fit gives $J_c(0) = 2.10 \times 10^8 \text{ A/cm}^2$, $R^2 = 0.267$, and $NRMSE = 0.292$. Although the δT_c model yields a slightly smaller global fitting error for 3ML, the low R^2 and large $NRMSE$ values prevent a reliable classification using the complete Griessen model.

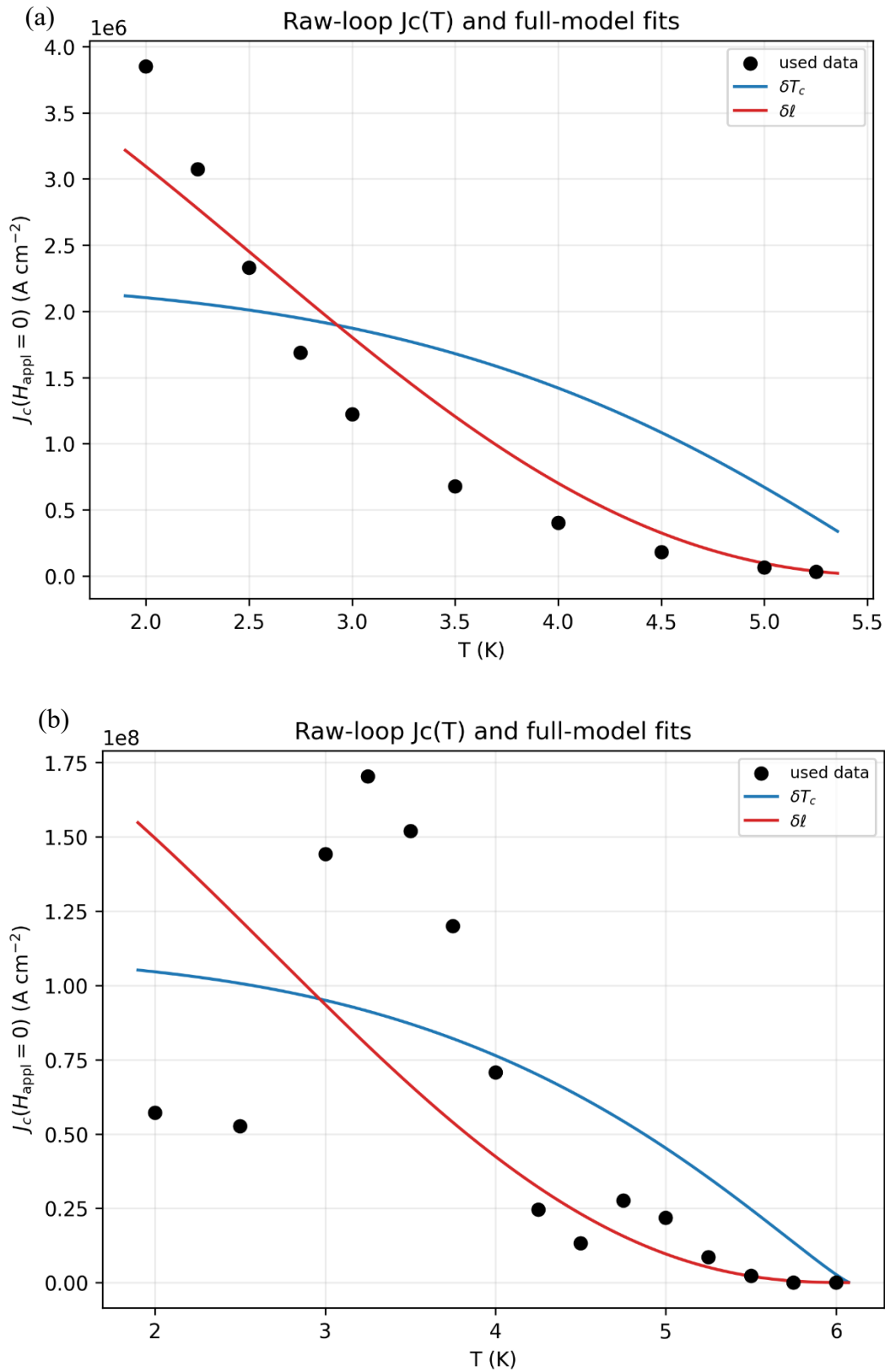
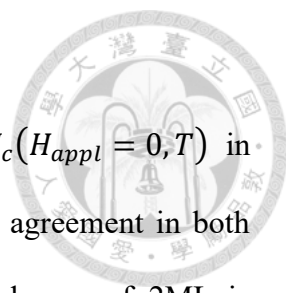


Figure 4.23: Zero-applied-field critical current density $J_c(H_{appl} = 0, T)$ for (a) 2ML and (b) 3ML samples.

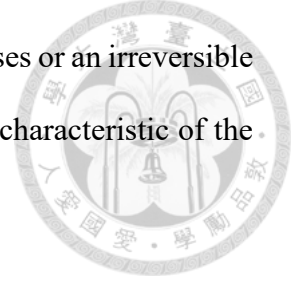


As shown in Figure 4.23(a), the temperature dependence of $J_c(H_{appl} = 0, T)$ in 2ML follows the $\delta\ell$ curve over most of the measured range. The agreement in both curvature and decay rate indicates that the effective pinning landscape of 2ML is consistent with a $\delta\ell$ -like temperature dependence. This result suggests that spatial variations of the charge-carrier mean free path constitute the predominant disorder-related pinning contribution in 2ML. In the present multilayer system, such variations may be associated with defect- and interface-induced carrier scattering. Nevertheless, this result should be regarded as a Griessen-type phenomenological classification rather than a unique identification of a particular microscopic defect.

In contrast, Figure 4.23(b) shows that the 3ML data exhibit a pronounced non-monotonic temperature dependence and cannot be represented by either single reference curve. A substantial excess in $J_c(H_{appl} = 0, T)$ develops between approximately 3.0 and 3.5 K. At 3.25 K, the measured J_c reaches $1.70 \times 10^8 A/cm^2$, corresponding to approximately 1.87 and 2.14 times the values predicted by the fitted δT_c and $\delta\ell$ curves, respectively. This temperature interval coincides with the anomalous pinning characteristics independently identified from the field-dependent analyses discussed later in Sec. 4.6.2.

Therefore, the unsuccessful global fit does not imply that the 3ML sample is invalid or degraded. Instead, the reproducible enhancement near 3 K indicates that the 3ML system contains an additional temperature-selective pinning or irreversible contribution that is not included in the single-component Griessen description. Because the loop width

near zero field may also be affected by abrupt low-field vortex processes or an irreversible magnetic background, the observed excess is treated as a consistent characteristic of the 3ML system without assigning it to a unique microscopic origin.



For comparison with the simplified power-law procedure adopted in previous work, the same data were also analyzed using

$$J_c(T) = J_c(0)[1 - (T/T_c)^2]^n \quad (4.17)$$

which can be expressed in linear form as

$$\ln J_c = \ln J_c(0) + n \ln[1 - (T/T_c)^2] \quad (4.18)$$

The corresponding effective exponents are $n = 2.80$ for 2ML, with $R_{log}^2 = 0.967$, and $n = 2.99$ for 3ML, with $R_{log}^2 = 0.894$. In this simplified diagnostic, $n = 7/6$ and $n = 5/2$ are used as the δT_c -like and $\delta \ell$ -like reference slopes, respectively.

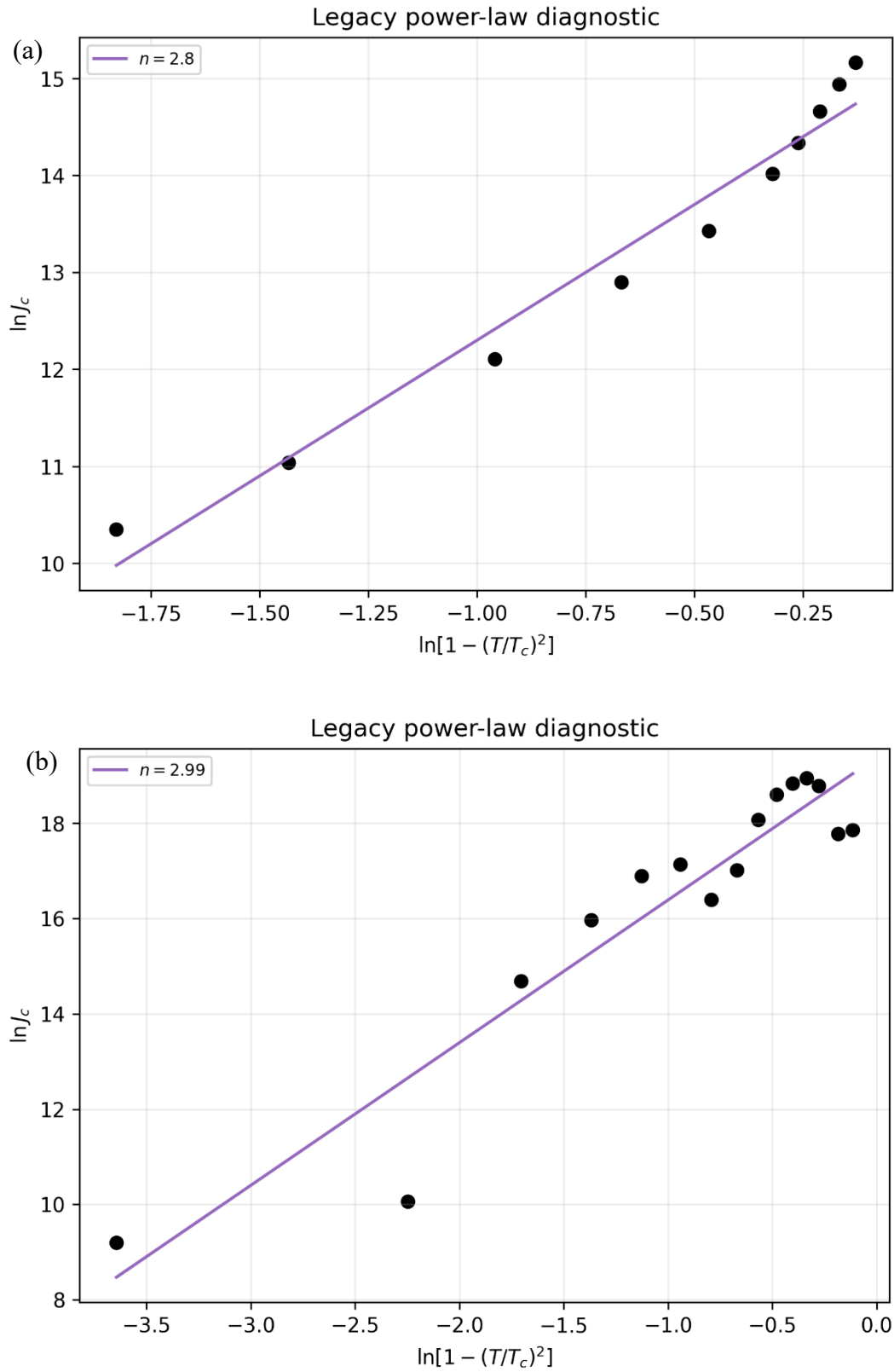
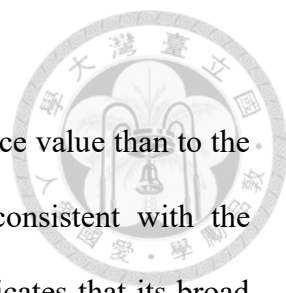


Figure 4.24: Simplified power-law analysis of $J_c(H_{appl} = 0, T)$ for (a) 2ML and (b) 3ML samples.



Both fitted exponents are considerably closer to the $\delta\ell$ reference value than to the δT_c reference value. For 2ML, the $\delta\ell$ -like effective slope is consistent with the successful complete-model fit. For 3ML, the exponent likewise indicates that its broad temperature-dependent baseline remains $\delta\ell$ -like. However, its lower R_{log}^2 and systematic deviation near 3 K retain the same limitation identified using the complete Griessen expressions.

Because the simplified power law omits the $(1 + t^2)$ factors contained in the complete Griessen expressions, the fitted exponent n should be interpreted only as an effective temperature-scaling parameter. Taken together, the two analyses support a predominantly $\delta\ell$ -like baseline in both samples, while the 3ML response requires an additional low-temperature pinning or irreversible contribution. The consistency of this deviation with the other anomalous features observed near 3 K therefore reflects the distinctive vortex response of the 3ML multilayer system rather than a failure of the sample.

4.4.4 Irreversibility field $H_{irr}(T)$ and extended phase diagram

In this work, H_{irr} is defined as the field at which J_c decreases to 100 A/cm² [58], [59], [60]. Therefore, H_{irr} represents the field scale above which the loop-derived irreversible current response becomes negligible within the present criterion.

Figure 4.25 shows the temperature dependence of H_{irr} for the 2ML and 3ML

samples. For both samples, H_{irr} generally decreases with increasing temperature, consistent with the weakening of vortex pinning and irreversible magnetization near the superconducting transition. Compared with 2ML, the 3ML sample maintains a finite H_{irr} over a wider temperature range and generally exhibits a higher H_{irr} in the intermediate-temperature region. This indicates that the irreversible vortex response is more robust in the 3ML multilayer structure.

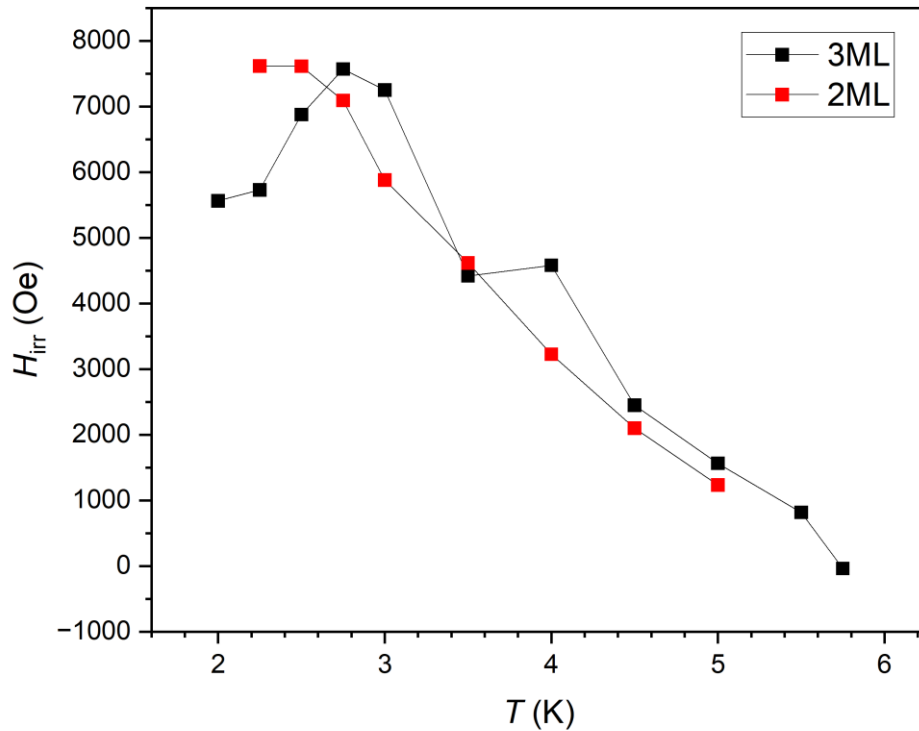


Figure 4.25: The $H_{irr}(T)$ for the 2ML and 3ML samples.

Figure 4.26(a) and Figure 4.26(b) show the extended phase diagrams of 2ML and 3ML, respectively, where H_{irr} is added to the previously established H_{c1} , H_g , and H_{c2} characteristic fields. Here, H_{c1} , H_g , and H_{c2} are obtained from the $m(H)$ -curves,

whereas H_{irr} is extracted from the loop-derived $J_c(H)$.

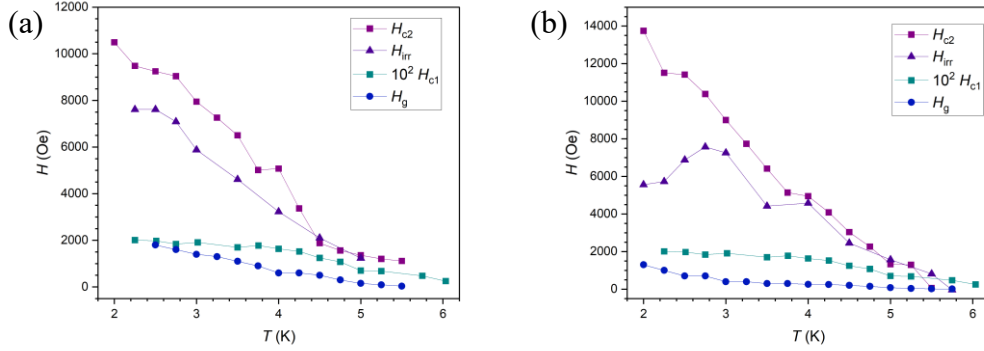


Figure 4.26: The extended phase diagrams of (a)2ML and (b)3ML.

The addition of H_{irr} provides a new physical layer to the field diagram. In both samples, H_{irr} is located between H_g and H_{c2} over most of the measured temperature range. This means that after the diamagnetic response reaches its maximum at H_g , the superconducting shielding signal begins to weaken with increasing field, but the vortex system can still sustain an irreversible current response until H_{irr} . Above H_{irr} , the loop-derived J_c falls below the threshold, while superconducting diamagnetism may still persist up to H_{c2} . Thus, H_{irr} separates the strongly irreversible vortex-response region from a more reversible high-field superconducting response.

4.5 Pinning force $F_p(H)$ and Field-dependent Pinning Regimes

4.5.1 Field dependence of $F_p(H)$ and Normalized pinning force $f_p(h)$

After obtaining $J_c(H)$ from the background-subtracted $M(H)$ -loops, the volume

pinning force density was calculated to further evaluate the vortex-pinning response. The pinning force density is generally expressed as

$$F_p(H, T) = |\mathbf{J}_c \times \mathbf{B}| \approx J_c(H, T)B(H) \quad (4.19)$$

where J_c is the critical current density and B is the magnetic flux density. In the present analysis, B is approximated by the applied magnetic field after unit conversion, and $F_p(H)$ is used to compare the field-dependent pinning strength of the 2ML and 3ML samples. This treatment follows the standard pinning-force analysis commonly used for type-II superconductors [26], [27].

Figure 4.27(a) and Figure 4.27(b) show the field dependence of $F_p(H)$ for the 2ML and 3ML samples, respectively. For both samples, F_p increases significantly as the temperature decreases, consistent with the enhancement of vortex pinning at lower temperatures. However, the most important feature is that $F_p(H)$ does not form a single smooth dome-like curve over the whole field range. Instead, the curves exhibit multiple field-dependent structures, including a low-field enhancement and a broader high-field contribution.

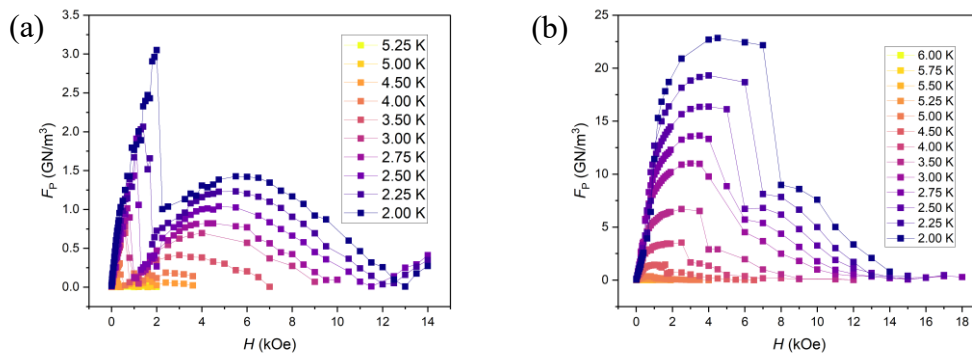


Figure 4.27: The field dependence of $F_p(H)$ for the (a)2ML and (b)3ML samples.



For the 2ML sample, the low-temperature $F_p(H)$ curves show a sharp enhancement in the low-field region, followed by a second broad contribution at higher fields. This indicates that the pinning response cannot be described by a single field scale. For the 3ML sample, $F_p(H)$ is much larger and extends to a wider magnetic-field range. The 3ML curves also show a stronger high-field pinning contribution, suggesting that the irreversible vortex response remains effective over a broader field interval.

To compare the pinning behavior at different temperatures, the pinning force was normalized as

$$f_p = \frac{F_p}{F_{p,\max}} \quad (4.20)$$

and the magnetic field was normalized by the irreversibility field,

$$h = \frac{H}{H_{\text{irr}}} \quad (4.21)$$

Figure 4.28(a) and Figure 4.28(b) show the normalized pinning force $f_p(h)$ for the 2ML and 3ML samples. If a single dominant pinning mechanism governed the entire field range, the normalized curves would be expected to collapse approximately onto one universal scaling curve. In contrast, the present data do not show a simple single-curve scaling behavior. The low-field and high-field regions evolve differently with temperature, implying that different vortex-pinning responses may dominate in different field ranges.

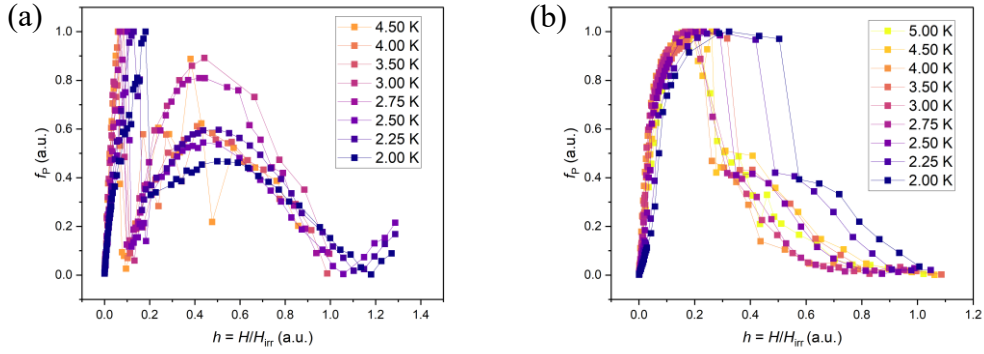


Figure 4.28: The normalized pinning force $f_p(h)$ for the (a)2ML and (b)3ML samples.

Therefore, the abnormal shape of $F_p(H)$ and the non-universal behavior of $f_p(h)$ provide the first direct evidence that the pinning response in the Nb/Sb₂Te₃ multilayers is not governed by a single conventional pinning regime. Rather, the data suggest the coexistence or crossover of multiple field-dependent pinning regimes. This observation motivates the Dew-Hughes-type analysis in the following subsection, where the low-field and high-field contributions are separately fitted to clarify their characteristic pinning behavior.

4.5.2 Dew-Hughes-type analysis of low-field and high-field regimes

To further examine the abnormal pinning-force behavior discussed in Sec. 4.5.1, a Dew-Hughes-type scaling analysis was performed on the normalized pinning force curves. In the conventional pinning-force scaling approach, the normalized pinning force is described by

$$f_p(h) = Ah^p(1 - h)^q \quad (4.22)$$

where $h = H/H_{irr}$, and p and q are fitting parameters related to the field dependence of

the pinning response. The peak position of the fitted curve is given by

$$h_{\max} = \frac{p}{p + q} \quad (4.23)$$



For a system governed by a single dominant pinning mechanism, $f_p(h)$ is expected to be approximately described by one smooth scaling curve. However, as shown in Sec. 4.5.1, the present $f_p(h)$ curves cannot be represented by a single continuous dome-like behavior over the full field range. Therefore, the data were separated into low-field and high-field regimes and fitted independently.

Figure 4.29(a) and Figure 4.29(b) show representative Dew-Hughes-type fitting results at $T = 2.5\text{ K}$ for the 2ML and 3ML samples, respectively. For the 2ML sample, the low-field regime is fitted with $p = 1.40$, $q = 10.00$, and $R^2 = 0.990$, corresponding to $h_{\max,L} \approx 0.123$. The high-field regime is fitted with $p = 1.6$, $q = 1.7$, and $R^2 = 0.957$, giving $h_{\max,H} \approx 0.48$. These two peak positions are clearly separated in reduced field, indicating that the low-field and high-field responses cannot be treated as the same pinning contribution.

For the 3ML sample, the separation between the two regimes is also evident. The low-field regime gives $p = 1.0$, $q = 4.4$, and $R^2 = 0.940$, corresponding to $h_{\max,L} \approx 0.19$, whereas the high-field regime gives $p = 3.4$, $q = 5.5$, and $R^2 = 0.998$, corresponding to $h_{\max,H} \approx 0.38$. Compared with 2ML, the 3ML sample shows a more strongly developed high-field fitting branch with a very high fitting quality. This result is consistent with the broader high-field $F_p(H)$ contribution observed in Figure 4.27.

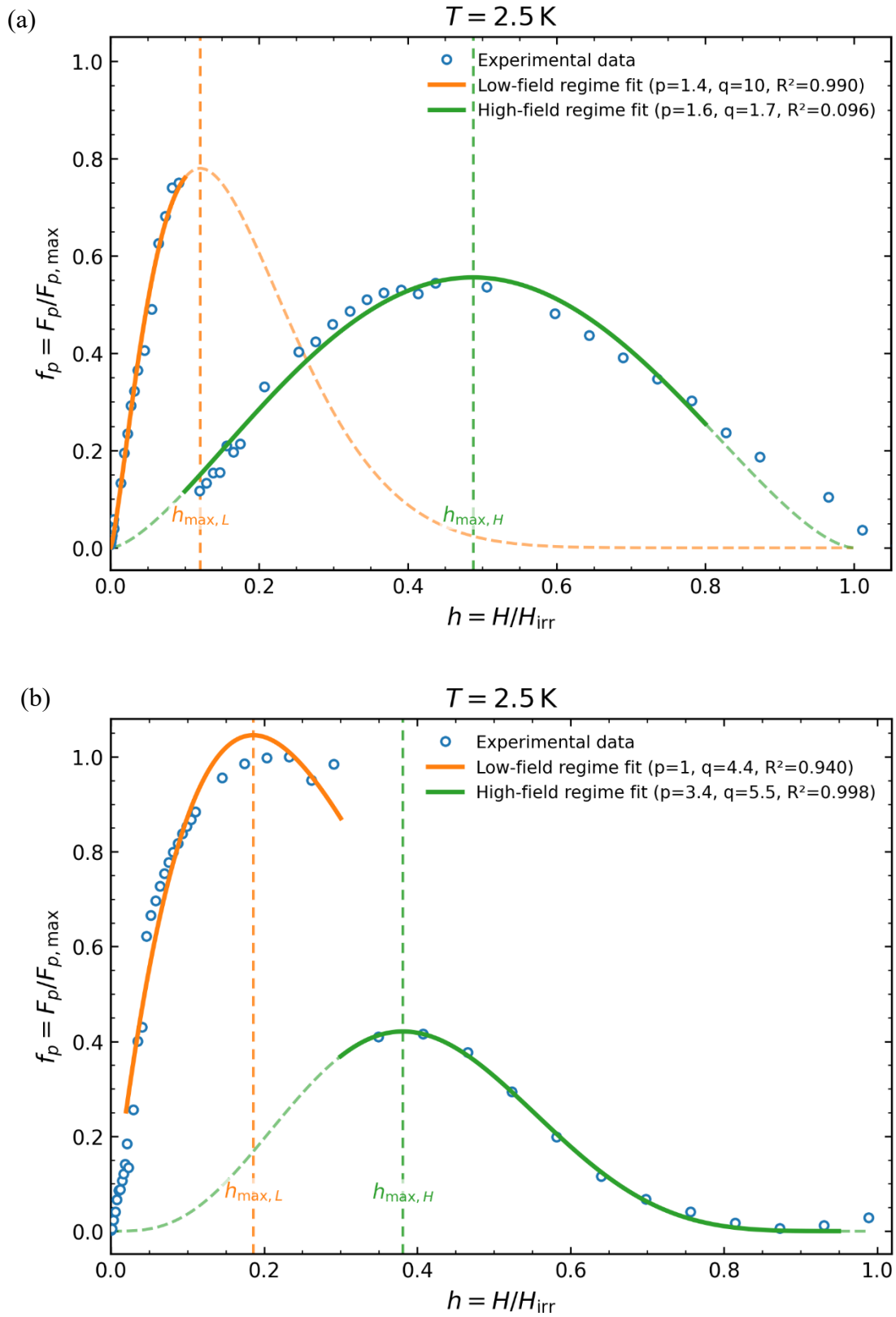
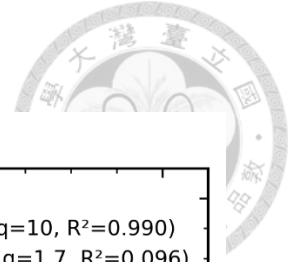


Figure 4.29: Dew-Hughes-type fitting at $T = 2.5 \text{ K}$ for the (a)2ML and (b)3ML

samples.



The important point of this analysis is not to assign each fitted branch to a unique microscopic pinning center. Instead, the fitting results show that the pinning response must be divided into at least two field-dependent regimes. The low-field branch describes the rapid initial enhancement of $f_p(h)$, while the high-field branch captures the broader pinning contribution that persists after the low-field response is suppressed. This supports the central interpretation that the Nb/Sb₂Te₃ multilayers exhibit competing pinning regimes rather than a single conventional pinning mechanism.

At the present stage, the Dew-Hughes-type fitting is used as a phenomenological decomposition of the pinning-force curves. A more complete microscopic assignment would require additional structural or dynamic evidence. Nevertheless, the distinct $h_{\max,L}$ and $h_{\max,H}$ values provide quantitative support for the existence of low-field and high-field vortex-pinning regimes. This result also motivates the following logarithmic analysis of $J_c(H)$, which provides an independent way to examine whether different field regions indeed exhibit distinct vortex-response behavior.

A further notable feature is observed in the low-temperature $f_p(h)$ curves of the 3ML sample. As shown in Figure 4.30, the normalized pinning force does not increase immediately with a single steep low-field slope. Instead, $f_p(h)$ first exhibits a weak and gradual increase in the very-low-field region, followed by a more rapid enhancement at higher reduced fields. Interestingly, the initial gradual increase appears closer to the extrapolated high-field fitting branch than to the low-field fitting curve. This suggests that the high-field-regime contribution may already influence the vortex response at very low

fields, while the low-field contribution becomes dominant only after a certain field scale is reached.

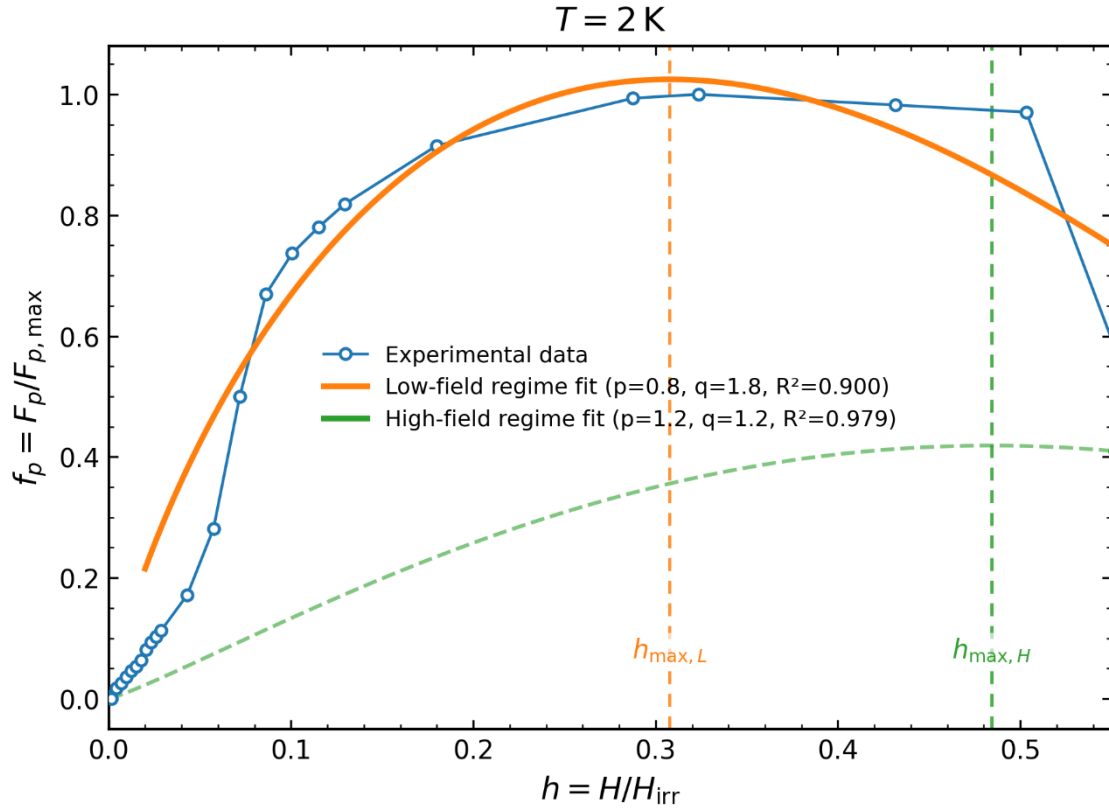


Figure 4.30: Dew–Hughes-type analysis for the 3ML sample at $T=2 \text{ K}$.

This behavior further supports the interpretation that the pinning response in 3ML is not governed by a single continuously evolving mechanism. Rather, the low-field and high-field contributions appear to coexist and compete over an overlapping field range. In this sense, the 3ML sample exhibits a more complex pinning landscape than 2ML. Considering that 3ML contains an additional Nb/Sb₂Te₃ spacer/interface structure compared with 2ML, the stronger competition between different pinning regimes may be associated with the more complicated multilayer architecture. Nevertheless, this interpretation should be regarded as phenomenological, because the present magnetic

data alone cannot uniquely identify the microscopic origin of each pinning contribution.



4.6 Evidence for competing vortex-pinning regimes

4.6.1 Logarithmic analysis of $J_c(H)$

To further examine the field-dependent vortex response, $J_c(H)$ was replotted in a double-logarithmic representation. Compared with the pinning-force analysis in Sec. 4.6, the $\log J_c - \log H$ plot does not involve the additional normalization by $F_{p,\max}$ or H_{irr} . Therefore, it provides a direct visualization of how the loop-derived critical current density evolves with magnetic field. This type of representation has been widely used to identify field-dependent J_c regimes, power-law decay, crossover behavior, and the possible involvement of different vortex-pinning mechanisms in superconducting films and coated conductors.

Figure 4.31(a) and Figure 4.31(b) show the $\log J_c - \log H$ plots of the 2ML and 3ML samples, respectively. For both samples, $J_c(H)$ cannot be described by a single monotonic power-law decay over the whole field range. Instead, several distinct field regions can be identified. In the low-field region, J_c remains relatively high and weakly field-dependent, forming a plateau-like response. Such a low-field plateau is commonly observed in superconducting thin films and can be influenced by self-field effects, sample geometry, and strong pinning at low vortex density. At intermediate fields, $J_c(H)$ changes more rapidly and exhibits shoulder-like or enhancement-like structures. At high fields, J_c is eventually suppressed as the system approaches the irreversibility boundary.

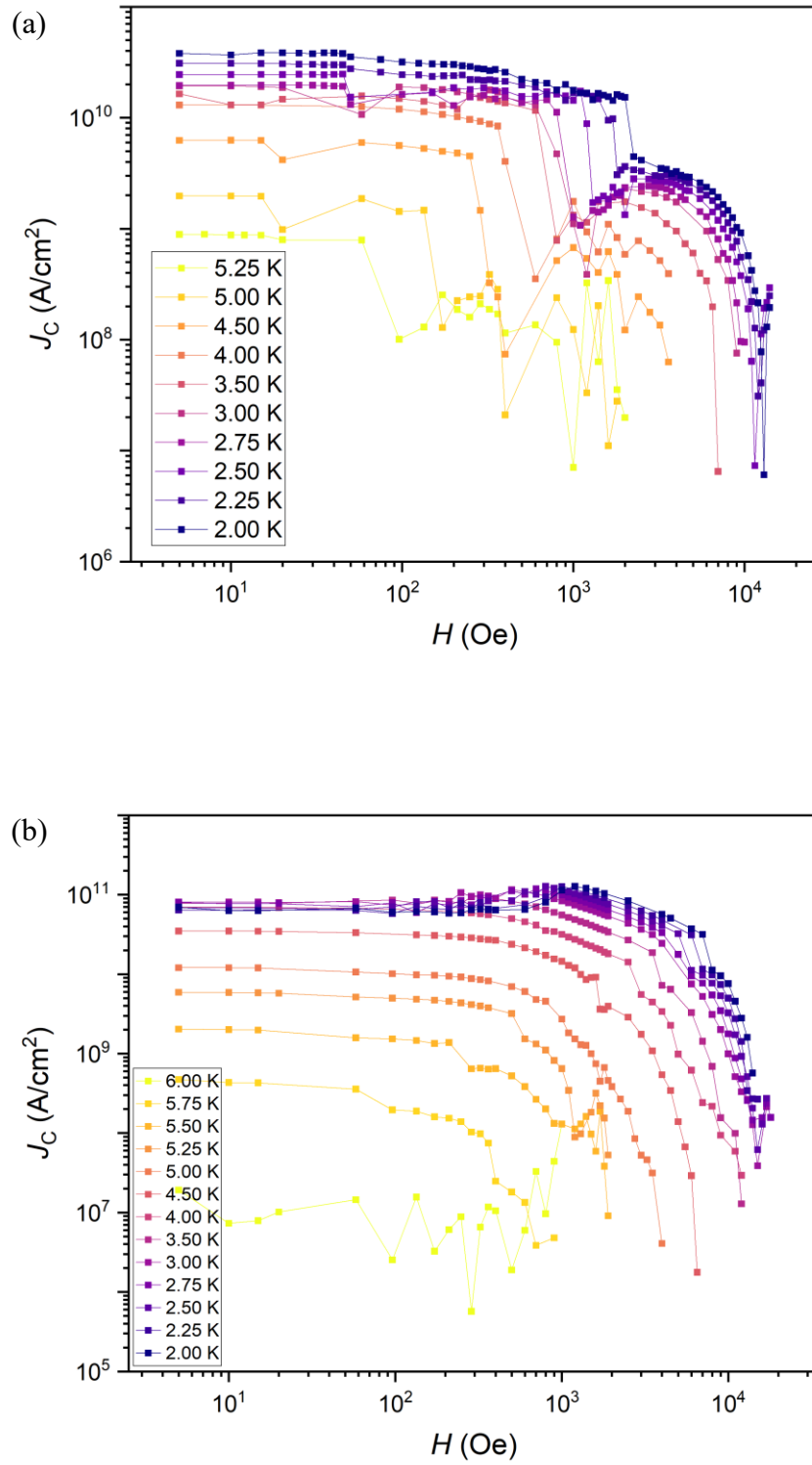
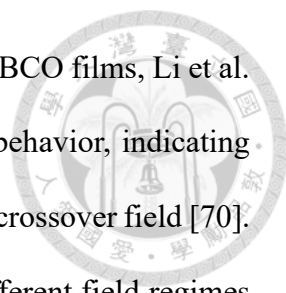


Figure 4.31: The logarithmic $J_c(H)$ plots of the (a)2ML and (b)3ML samples.



This segmented field dependence is physically meaningful. In YBCO films, Li et al. showed that $\log J_c - \log H$ curves can exhibit kink-like crossover behavior, indicating that $J_c(H)$ is controlled by different mechanisms above and below a crossover field [70]. Cai et al. also used the $\log J_c - \log H$ representation to compare different field regimes in REBCO films, where the intermediate-field behavior follows different power-law exponents depending on the pinning landscape [71]. These reports demonstrate that changes in the slope or morphology of $\log J_c - \log H$ plots can be used as evidence for changes in the dominant vortex-pinning response.

For the 2ML sample, the logarithmic plot shows that the high- J_c region is mainly concentrated at low fields, followed by a relatively abrupt suppression in the intermediate-to-high-field range. The limited field range of large J_c is consistent with the weaker high-field pinning contribution observed in the $F_p(H)$ and $f_p(h)$ analyses. In contrast, the 3ML sample exhibits a broader and more structured $J_c(H)$ behavior. A high- J_c plateau-like region persists over a wider field range, and an additional enhancement-like structure appears before the final high-field decay. This indicates that the vortex response in 3ML is not governed by a single field-dependent pinning contribution.

The logarithmic analysis therefore provides an independent confirmation of the interpretation developed from the pinning-force analysis. In Sec. 4.5.1, the abnormal $F_p(H)$ curves showed that the pinning force does not follow a single smooth dome-like behavior. In Sec. 4.5.2, the Dew-Hughes-type fitting further separated the response into low-field and high-field regimes with distinct $h_{\max,L}$ and $h_{\max,H}$. The present $\log J_c - \log H$ plots directly show that $J_c(H)$ itself contains multiple field-dependent segments

[27]. This consistency among $J_c(H)$, $F_p(H)$, $f_p(h)$, and Dew-Hughes-type fitting strongly supports the central conclusion that the Nb/Sb₂Te₃ multilayers exhibit competing vortex-pinning regimes rather than a single conventional pinning mechanism.

It should be noted that the present log-log analysis is used phenomenologically. The distinct slopes and field regions are not assigned to unique microscopic pinning centers in this thesis. Nevertheless, the segmented $J_c(H)$ behavior is consistent with the broader vortex-pinning literature, where changes in field dependence are commonly interpreted as signatures of crossover between different vortex-pinning or vortex-dynamics regimes. In particular, recent studies on disordered Nb thin films have also shown that Nb-based thin-film systems may exhibit more than one pinning regime, depending on disorder, thickness, and vortex dynamics. Therefore, the logarithmic $J_c(H)$ analysis provides the final and most direct evidence in this chapter for a complex pinning landscape in the Nb/Sb₂Te₃ multilayer system [26].

4.6.2 Temperature evolution of H_{SPE} , H_{collapse} , $h_{\text{max,L}}$ and $h_{\text{max,H}}$

In Section 4.6.1, the field dependence of the critical current density was analyzed using logarithmic $J_c(H)$ plots. Compared with a linear-scale representation, the logarithmic $J_c(H)$ plot makes the changes in slope more visible and is therefore useful for identifying the pinning enhancement associated with the second peak effect (SPE) and the rapid suppression of J_c in the high-field region. In this section, two characteristic fields, H_{SPE} and H_{collapse} , are defined from the logarithmic $J_c(H)$ curves and compared with the Dew-Hughes fitting parameters $h_{\text{max,L}}$ and $h_{\text{max,H}}$ obtained in

Section 4.5.2.



In this work, H_{SPE} is defined as the field at which the second-peak enhancement in $J_c(H)$ reaches its local maximum. Since $J_c(H)$ is calculated from the hysteresis width of the $m(H)$ -loop, H_{SPE} can also be identified directly from the second-peak feature in the corresponding $m(H)$ -loop. On the other hand, H_{collapse} is not used to indicate a complete microscopic disappearance of pinning centers. Instead, it is defined operationally as the crossover field where $J_c(H)$ enters the rapidly decreasing high-field region after the SPE. Therefore, H_{collapse} marks the field scale at which the effective pinning response loses stability in the high-field regime.

Figure 4.32 illustrates how H_{SPE} and H_{collapse} are defined from the logarithmic $J_c(H)$ curve. H_{SPE} corresponds to the local maximum of the intermediate-field enhancement, whereas H_{collapse} corresponds to the field where the post-SPE $J_c(H)$ curve begins to decrease rapidly.

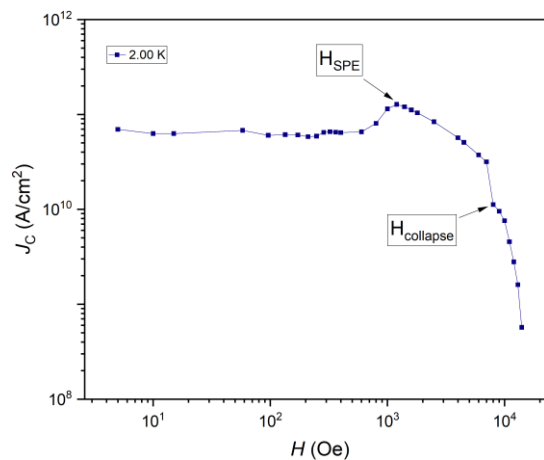


Figure 4.32: Definition of H_{SPE} and H_{collapse} from logarithmic $J_c(H)$.

In Figure 4.32, H_{SPE} and H_{collapse} are phenomenological characteristic fields rather than thermodynamic critical fields. Their role is to identify two distinct field regimes in the vortex response. H_{SPE} represents the field at which the intermediate-field pinning enhancement or vortex rearrangement is most pronounced, while H_{collapse} represents the onset of rapid suppression of the effective pinning response in the high-field region. Therefore, these two characteristic fields can be used to track the temperature evolution of the SPE-related vortex state.

Figure 4.33 shows the temperature dependence of H_{SPE} for 2ML and 3ML. Since H_{SPE} represents the characteristic field of the intermediate-field pinning enhancement, its temperature dependence provides information on how the SPE shifts with temperature in the two multilayer samples.

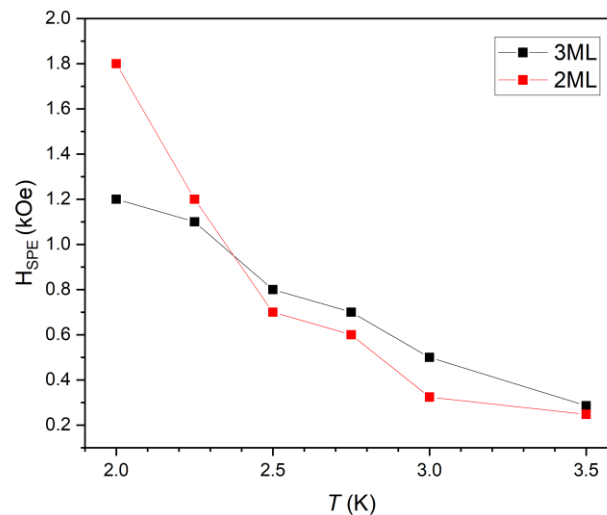


Figure 4.33: Temperature dependence of H_{SPE} for 2ML and 3ML.

In Figure 4.33, H_{SPE} shifts toward lower fields with increasing temperature, it suggests that the intermediate-field pinning enhancement is triggered at a lower magnetic field. This can be understood as a consequence of reduced vortex elasticity and enhanced thermal activation at higher temperature. In contrast, if H_{SPE} remains relatively stable over a certain temperature range, the SPE may be controlled by a more fixed geometrical or interfacial pinning scale. In the present analysis, H_{SPE} is not used alone to determine a specific microscopic pinning center. Instead, it is treated as a characteristic field of the anomalous intermediate-field pinning response and is compared with $h_{\text{max},L}$ and $h_{\text{max},H}$ below.

Figure 4.34 shows the temperature dependence of H_{collapse} for 2ML and 3ML. Compared with H_{SPE} , H_{collapse} is more closely related to the boundary where the high-field pinning response begins to break down.

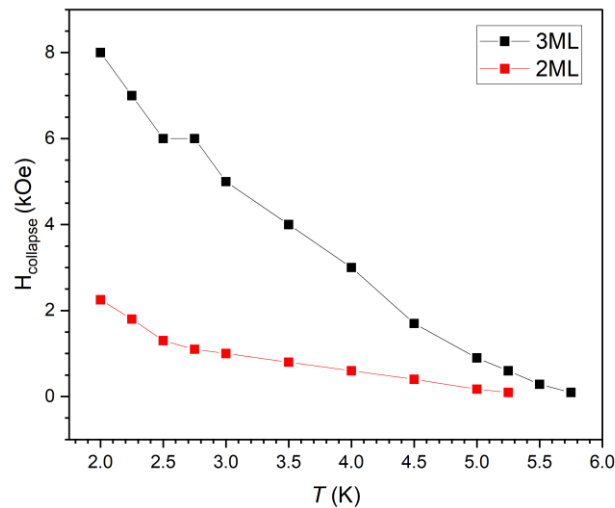


Figure 4.34: Temperature dependence of H_{collapse} for 2ML and 3ML.

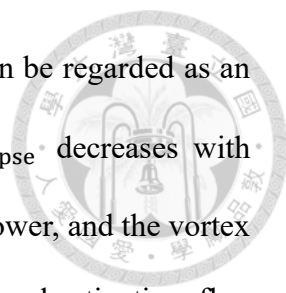


Figure 4.34 shows the temperature dependence of H_{collapse} can be regarded as an indicator of the stability of the high-field pinning regime. H_{collapse} decreases with increasing temperature, the pinning-dominated regime becomes narrower, and the vortex system more easily enters a rapidly decaying region governed by thermal activation, flux creep, or proximity to the irreversibility field. 3ML shows a more clearly traceable H_{collapse} over a wider temperature range than 2ML, it indicates that the SPE-related enhancement and the post-SPE high-field suppression are more clearly defined in 3ML. In contrast, a more discontinuous or irregular H_{collapse} in 2ML would suggest that its high-field vortex response is more strongly affected by multi-step superconducting transition or inhomogeneous effective superconducting volume.

In addition, the Dew-Hughes-type analysis introduced in Sec. 4.5.2 was extended to all measured temperatures using the same low-field and high-field fitting procedures. The central idea of the Dew-Hughes model is to use the peak position of the normalized pinning force, h_{max} , as a characteristic reduced-field scale of the pinning-force distribution, where $h = H/H_{\text{irr}}$. In general, a low $h_{\text{max}} \sim 0.2$ is commonly associated with surface-like or grain-boundary-like pinning, $h_{\text{max}} \sim 0.33$ is often related to point-like core pinning, and a higher h_{max} may indicate volume-like or order-parameter-fluctuation-related pinning contributions.

Since a single Dew-Hughes curve cannot fully describe the entire $f_p(h)$ response of the present multilayer samples, the data were fitted separately in the low-field and high-field regions in Sec. 4.5.2, yielding $h_{\text{max},L}$ and $h_{\text{max},H}$, respectively. Here, $h_{\text{max},L}$ characterizes the reduced-field peak position of the low-field or intermediate-field

pinning response, whereas $h_{\max,H}$ characterizes that of the high-field pinning response. These two quantities should not be interpreted as two completely independent pinning centers. Instead, they represent the effective pinning-force responses in different field regimes of the same multilayer system.

Figure 4.35 shows the temperature dependence of $h_{\max,L}$ and $h_{\max,H}$ for 2ML. This plot allows us to compare whether the low-field and high-field pinning-force peak positions follow similar or different temperature dependences. $h_{\max,L}$ is lower than $h_{\max,H}$, the maximum of the normalized pinning force in the low-field branch occurs at a smaller reduced field, while the high-field branch reaches its maximum at a larger reduced field.

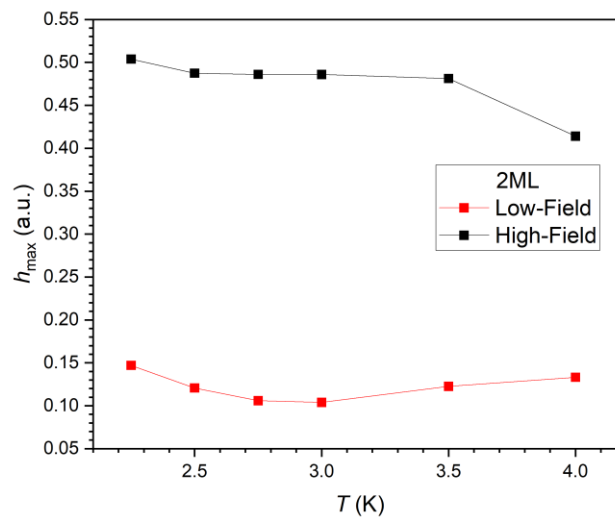


Figure 4.35: Temperature dependence of $h_{\max,L}$ and $h_{\max,H}$ for 2ML.

As shown in Figure 4.35, the temperature dependence of $h_{\max,L}$ and $h_{\max,H}$ in 2ML is relatively weak, suggesting that the Dew-Hughes normalized scaling does not

show a single strong monotonic trend. When $h_{\max,L}$ reaches a lower value around 3 K, this should not be described as “the strongest surface-like pinning.” A more precise description is that the low-field pinning-force maximum shifts toward a lower reduced field around this temperature, making its field dependence phenomenologically closer to surface-like or grain-boundary-like pinning. In other words, $h_{\max,L}$ describes the position of the normalized pinning-force peak, whereas the absolute pinning strength should be evaluated from $F_{p,\max}$ or J_c .

Figure 4.36 shows the temperature dependence of $h_{\max,L}$ and $h_{\max,H}$ for 3ML. Compared with 2ML, a more pronounced temperature dependence in 3ML would indicate that the low-field and high-field pinning-force distributions are more sensitive to temperature.

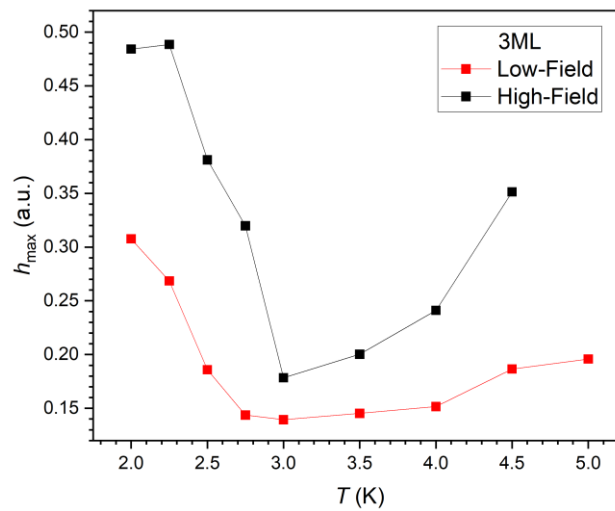
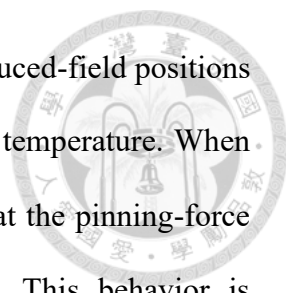


Figure 4.36: Temperature dependence of $h_{\max,L}$ and $h_{\max,H}$ for 3ML.

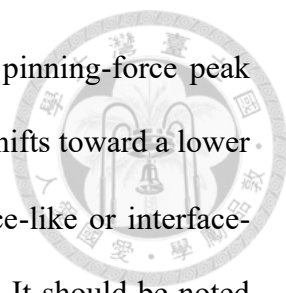
As shown in Figure 4.36, the $h_{\max,L}$ and $h_{\max,H}$ values of 3ML show a more



visible temperature dependence, indicating that the characteristic reduced-field positions of both the low-field and high-field pinning responses change with temperature. When both quantities shift toward lower $h_{\max}$ around 3 K, it suggests that the pinning-force peaks move toward lower reduced fields near this temperature. This behavior is phenomenologically consistent with an enhanced surface-like or interface-related pinning contribution. Nevertheless, $h_{\max}$ should not be directly equated with a single microscopic pinning center. A more careful interpretation is that the multilayer interfaces and vortex configuration jointly modify the distribution of $F_p(h)$, causing the peak positions of both low-field and high-field branches to shift.

Combining H_{SPE} , H_{collapse} , $h_{\max,L}$, and $h_{\max,H}$, a temperature-dependent picture of the SPE and the pinning-force distribution can be established. First, H_{SPE} and H_{collapse} are characteristic fields directly defined from $J_c(H)$ or the $m(H)$ -loop, marking the intermediate-field SPE enhancement and the rapid high-field suppression of J_c respectively. In contrast, $h_{\max,L}$ and $h_{\max,H}$ are obtained from the Dew-Hughes-type analysis and represent the peak positions of the normalized pinning force in the reduced-field scale. Therefore, these two sets of parameters describe different aspects of the vortex response: H_{SPE} and H_{collapse} trace the SPE evolution in the actual magnetic-field scale, whereas $h_{\max,L}$ and $h_{\max,H}$ describe the characteristic reduced-field scales of the low-field and high-field pinning-force distributions.

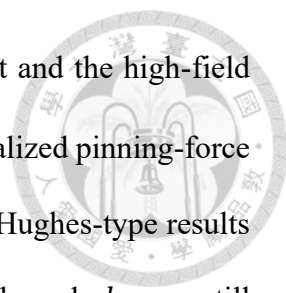
For 2ML, the Dew-Hughes-type analysis yields valid fitting results only in the $T \leq 4\text{ K}$ region. Therefore, $h_{\max,L}(T)$ and $h_{\max,H}(T)$ are used only to describe the low-temperature pinning-force distribution. Within this temperature range, $h_{\max,H}$ does not



show a drastic variation, indicating that the high-field normalized pinning-force peak position is relatively stable. However, the low-field branch $h_{\max,L}$ shifts toward a lower reduced field around 3 K, showing that 2ML also exhibits a surface-like or interface-related signature in the low-field pinning response, similar to 3ML. It should be noted that “surface-like” here does not mean that the absolute pinning strength reaches its maximum at 3 K. Rather, it means that the peak position of the normalized pinning force shifts toward a lower h , making the field dependence more consistent with the surface-like or grain-boundary-like pinning category in the Dew-Hughes classification.

For 3ML, $h_{\max,L}$ and $h_{\max,H}$ show a clearer temperature evolution over the measured temperature range, with a pronounced reduced-field peak shift near 3 K. This result indicates that both the low-field and high-field pinning-force distributions in 3ML are temperature dependent, with the most visible change occurring around 3 K. In the phenomenological classification of the Dew-Hughes model, a lower $h_{\max}$ corresponds to a field scale more characteristic of surface-like or interface-related pinning. Therefore, the change of $h_{\max}$ in 3ML near this temperature can be interpreted as evidence that the multilayer interfaces and vortex configuration jointly modify the distribution of $F_p(h)$. This does not imply that a single microscopic pinning center suddenly becomes dominant; rather, it indicates that the normalized pinning-force peak shifts toward a lower reduced field, making the field dependence more consistent with a surface-like or interface-related pinning signature.

Therefore, the central conclusion of this section is that the SPE and the pinning-force distribution in Nb/Sb₂Te₃ multilayers exhibit clear temperature dependence. H_{SPE} and



H_{collapse} reveal the actual magnetic-field scales of the enhancement and the high-field suppression of J_c , whereas $h_{\text{max},L}$ and $h_{\text{max},H}$ show how the normalized pinning-force peak positions evolve in the reduced-field scale. For 2ML, the Dew-Hughes-type results are mainly limited to the low-temperature region, but the low-field branch $h_{\text{max},L}$ still shifts toward a more surface-like or interface-related behavior around 3 K. For 3ML, both the low-field and high-field branches show a clearer temperature dependence. These results support the overall interpretation that the multilayer structure modifies not only the magnitude of $J_c(H)$, but also the vortex stability and the effective pinning form in different magnetic-field regimes.

4.7 Field-orientation dependence of vortex pinning in the 3ML sample

The preceding analyses revealed an anomalous enhancement of the irreversible vortex response when the magnetic field was applied in the in-plane (IP) direction. Because the IP field lies approximately within the crystallographic *ab* plane and parallel to the Nb/Sb₂Te₃ interfaces, an out-of-plane (OOP) measurement provides an orientation-control experiment for determining whether this enhancement is a general property of the 3ML sample or an anisotropic response associated with the *ab*-plane field configuration. In the OOP geometry, the applied field is perpendicular to the film plane and approximately parallel to the crystallographic *c* axis. The purpose of this section is therefore to establish the superconducting field range of the OOP response, identify the limitations introduced by the additional magnetic signal, and examine whether the

anomalous IP pinning enhancement is reproduced in the OOP direction.

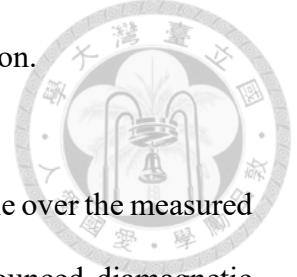


Figure 4.37 shows the raw OOP $m(H)$ -curves of the 3ML sample over the measured temperature range. At low temperatures, the curves exhibit a pronounced diamagnetic response that gradually approaches the high-field background as the applied field is increased. The OOP upper critical field H_{c2} was determined from the closure of this diamagnetic response using the same criterion adopted in Sec. 4.3.2. As shown in Figure 4.38, H_{c2} is approximately 13 kOe at 2 K and decreases monotonically with increasing temperature, approaching zero near the superconducting transition.

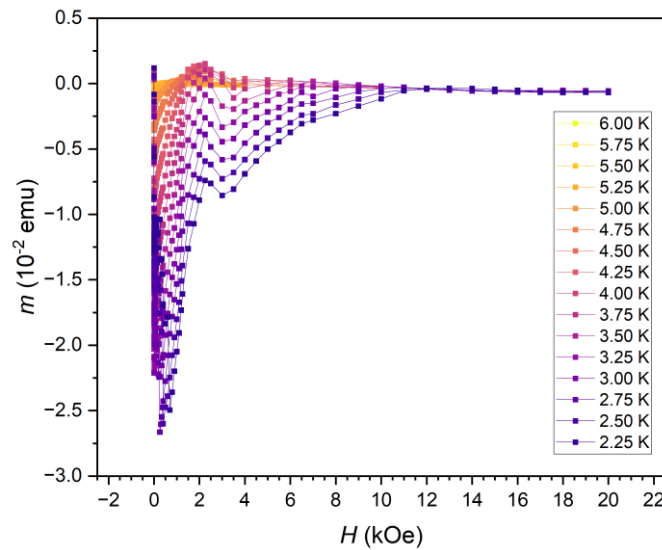


Figure 4.37: Raw OOP $m(H)$ -curves of the 3ML sample measured at different temperatures.

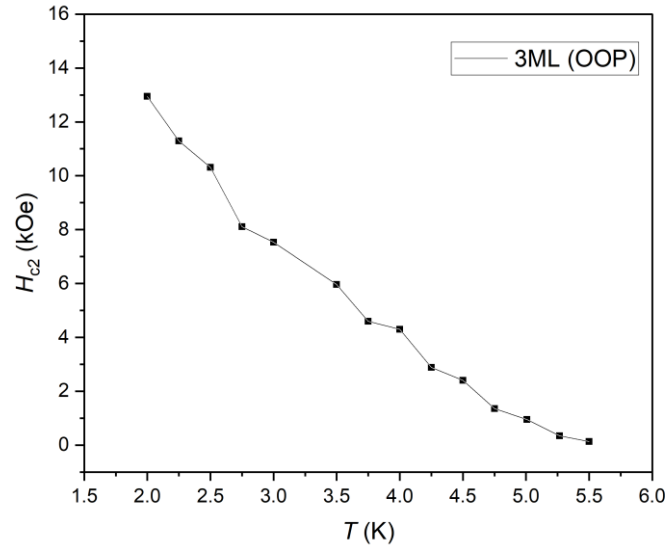


Figure 4.38: The upper critical field H_{c2} for 3ML sample at OOP-orientation.

The independently extracted $H_{c2}(T)$ provides an essential validity boundary for the subsequent $m(H)$ -loop analysis. Below H_{c2} , the irreversible loop width may contain a contribution from superconducting vortex pinning. Above H_{c2} , the diamagnetic response identified from the $m(H)$ -curve has already disappeared; consequently, any remaining magnetic hysteresis cannot originate from a superconducting critical state. This distinction is particularly important for the OOP data because a finite field-dependent magnetic signal remains visible beyond the superconducting field range.

The OOP $m(H)$ -loops of the 3ML sample were measured between 2.0 and 6.0 K over a magnetic-field range of approximately ± 20 kOe. As the temperature decreases, the hysteresis width increases and remains observable over a broader field range. This temperature dependence is also reflected in the corresponding $J_c(H)$ curves. At 2.0 K, the apparent maximum J_c reaches approximately $7 \times 10^8 \text{ A/m}^2$ near 2 kOe, while the

independently determined superconducting field range is bounded by $H_{c2} \approx 13$ kOe. With increasing temperature, both the magnitude of J_c and the magnetic-field range over which an irreversible response is observed decrease progressively. Nevertheless, the low-field data contain abrupt discontinuities and nonmonotonic variations, whereas a weak apparent J_c remains at some fields beyond the corresponding H_{c2} .

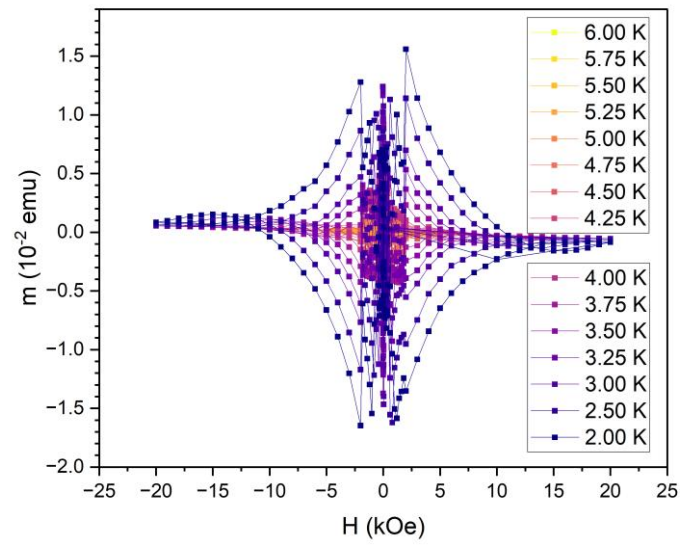


Figure 4.39: Raw OOP $m(H)$ -loops of the 3ML sample.

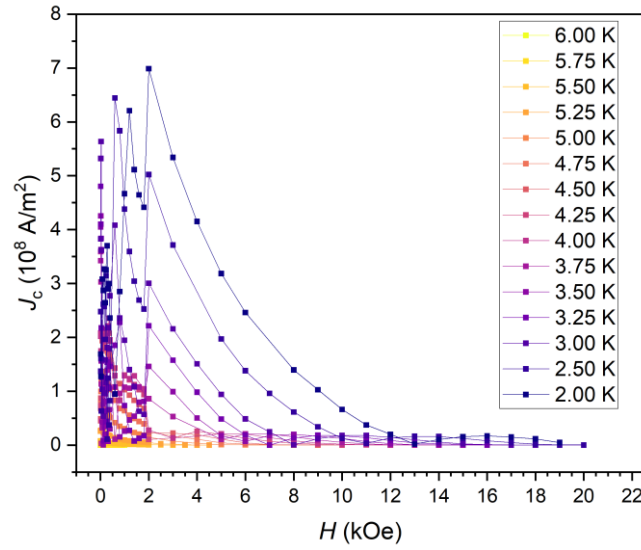


Figure 4.40: The apparent critical current density J_c for 3ML sample.

The overall increase in J_c with decreasing temperature is consistent with the strengthening of superconducting irreversibility at low temperature. However, not every feature in Figure 4.40 can be assigned to vortex pinning. In Figure 4.39, the low-field region exhibits abrupt oscillations and collapse-like reductions of the loop width. Because J_c is determined by the separation between the two branches of the $m(H)$ -loop, these irregularities produce discontinuities, nonmonotonic spikes, or locally indeterminate values in Figure 4.40. Consequently, the affected points cannot be regarded as evidence of an intrinsic enhancement of the critical current.

As discussed in Section 2.4.2, collapse-like hysteresis features may originate from competing or mutually compensating magnetic contributions rather than from a genuine disappearance or enhancement of vortex pinning [47]. A similar low-field anomaly was also reported in Figure 4.15 of Hong's thesis [48], supporting the reproducibility of this type of signal in related multilayer samples. Nevertheless, because the present Nb/Sb₂Te₃

multilayer does not contain the ferromagnetic layers considered in Ref. [47], the vortex-flipping mechanism proposed in that work cannot be transferred directly to the present system. The microscopic origin of the additional magnetic signal therefore remains unresolved.

A second limitation appears in the high-field region. Since the superconducting boundary has already been determined independently from the OOP $m(H)$ -curves, any apparent J_c remaining above the corresponding H_{c2} cannot originate from superconducting vortex pinning. Instead, it results from interpreting the residual magnetic hysteresis of the sample as a superconducting loop width. These high-field values are therefore magnetic artifacts and must be excluded from the intrinsic critical-current analysis.

After excluding the collapse-affected low-field intervals and the apparent response above H_{c2} , the valid OOP $J_c(H)$ data exhibit the expected enhancement with decreasing temperature but do not show a reproducible anomalous pinning-enhancement branch comparable to that observed in the IP configuration. Figure 4.39 and Figure 4.40 therefore support the interpretation that the anomalous pinning enhancement is associated specifically with the in-plane magnetic-field configuration.

To compare the OOP pinning-force behavior at different temperatures, the apparent pinning-force density $F_p(H)$ was evaluated between 2.0 and 6.0 K, as shown in Figure 4.41(a). At 2.0 K, the principal low-to-intermediate-field maximum reaches approximately 0.16 GN/m^3 near 4 kOe. Both the magnitude and the characteristic field of this maximum decrease with increasing temperature. In addition to the principal peak,

several curves retain a finite F_p or develop a broad secondary hump at approximately 14–18 kOe. For comparison of the reduced-field dependence, selected data between 2.0 and 3.75 K were normalized using

$$h = H/H_{c2}$$

and

$$f_p = F_p/F_{p,\max}$$

where H_{c2} was independently determined from the OOP $m(H)$ -curves in Figure 4.37. Within the superconducting interval $0 < h < 1$, the principal maxima of the normalized curves are distributed over approximately $h_{\max} = 0.30 - 0.45$, whereas the region below approximately $h = 0.25$ exhibits pronounced scattering and discontinuities.

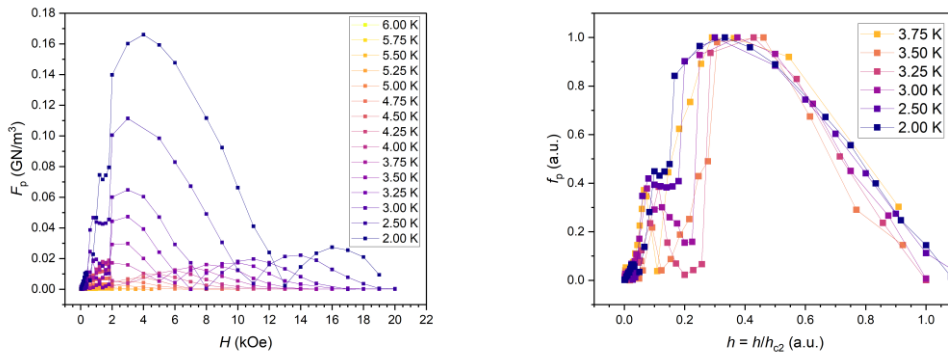


Figure 4.41: (a) Apparent pinning-force density F_p and (b) Normalized pinning-force curves of the 3ML sample.

Two field intervals in Figure 4.41 require separate consideration before the intrinsic pinning behavior can be discussed. First, the broad high-field contribution in Figure 4.41(a) occurs partially or entirely above the independently determined H_{c2} at the corresponding temperature. Because the superconducting diamagnetic response has

already closed in this region, this high-field hump cannot originate from vortex pinning. It is instead produced by converting the residual magnetic hysteresis identified in Figure 4.39 into an apparent J_c and subsequently into an apparent F_p . The resulting high-field structure is therefore a magnetic *pseudo* $-F_p$ and must be excluded from the superconducting pinning analysis.

Second, the irregular behavior below approximately $h = 0.25$ is inherited directly from the collapse-like low-field response of the OOP $m(H)$ -loops. In this interval, the loop width cannot always be determined uniquely, causing abrupt variations or discontinuities in the calculated J_c and F_p . Since the origin and influence of this additional magnetic contribution have already been established in the discussion of Figure 4.39, these low-field points are excluded without assigning them to a separate vortex-pinning mechanism.

After excluding the collapse-affected low-field interval and the magnetic *pseudo* $-F_p$ above H_{c2} , the valid normalized curves display broad principal maxima within $h_{\max} \approx 0.30 - 0.45$ and follow comparable decreasing trends on the high-field side. The limited variation of $h_{\max}$ indicates that the effective reduced-field scale of the dominant OOP pinning response remains broadly similar over the investigated temperature range. It does not, by itself, establish a unique microscopic pinning mechanism, but it shows that no distinct temperature-selective pinning contribution emerges in the OOP configuration.

Most importantly, the OOP $f_p(h)$ curves do not exhibit the reproducible anomalous

pinning-enhancement behavior observed near 3 K in the IP configuration. The absence of this feature in the OOP data, despite its consistent appearance in the IP measurements, supports the conclusion that the anomalous pinning enhancement is anisotropic and is specifically associated with the configuration in which the applied magnetic field is parallel to the ***ab*** plane.

Chapter 5 Conclusion and Future Work



5.1 Conclusion

This thesis investigated the magnetic response, irreversible vortex behavior, and flux-pinning characteristics of Nb/Sb₂Te₃ multilayer thin films. The main comparison was made between the 2ML and 3ML samples, in which the total Nb thickness was kept approximately constant while the number of Nb layers and Nb/Sb₂Te₃ interfaces was varied. This sample design allows the influence of multilayer stacking and interface density to be examined without simply changing the total superconducting Nb volume.

In Chapter 3, the experimental basis of this work was established. The Sb₂Te₃ precursor and target preparation, Nb/Sb₂Te₃ multilayer deposition, structural characterization, and SQUID-based magnetic measurements were standardized. The EDS and XRD results confirmed that the prepared Sb₂Te₃ bulk precursor was compositionally close to the stoichiometric ratio and structurally dominated by the Sb₂Te₃ phase. The XRD measurement of the multilayer film further confirmed the coexistence of crystalline Nb and Sb₂Te₃ contributions. In addition, Chapter 3 defined the extraction criteria for T_c , H_{c1} , H_g , H_{c2} , and H_{irr} . The automated analysis workflow was established to reduce subjective point selection and to keep the processing criteria consistent among different samples and temperatures.

In Chapter 4, the magnetic measurements revealed that both 2ML and 3ML retain superconductivity after the insertion of Sb₂Te₃ spacer layers. From the derivative analysis of the ZFC $m(T)$ curves, the superconducting transition temperatures were estimated to

be $T_c = 5.73$ K for 2ML and $T_c = 6.08$ K for 3ML. The temperature-dependent magnetization also showed anomalous paramagnetic-like features near the superconducting transition. These features were not assigned to a unique microscopic origin, but they indicate that the magnetic response of the Sb_2Te_3 multilayers is more complex than that of a simple single-layer Nb film.

The $m(H)$ -curve analysis provided the first field-dependent characterization of the superconducting state. After background subtraction, H_{c1} , H_g , and H_{c2} were extracted as characteristic field scales. The 3ML sample generally exhibited larger characteristic fields than 2ML, especially at lower temperatures. The field H_g was defined as an empirical diamagnetic maximum field rather than as a rigorous vortex glass–liquid transition field. Therefore, the $H_{c1} - H_g - H_{c2}$ diagram was interpreted as a phenomenological characteristic-field diagram, not as a strict thermodynamic phase diagram. This conservative interpretation is consistent with the caution that vortex-glass-related phase diagrams require more than a single magnetic criterion to establish a true phase transition [68].

The $m(H)$ -loop measurements further demonstrated the irreversible vortex response of the multilayer samples. Using the Bean critical-state model, the loop width was converted into $J_c(H)$ [25]. The 3ML sample exhibited a larger J_c and sustained irreversible current over a broader field range than 2ML. The irreversibility field H_{irr} , defined by the criterion $J_c = 100$ A/cm², was then added to the characteristic-field diagram. In most of the measured temperature range, H_{irr} was located between H_g and H_{c2} , indicating that the vortex system can still sustain irreversible current after the diamagnetic response has

started to weaken.

The central conclusion of this thesis is obtained from the pinning-force and logarithmic $J_c(H)$ analyses. The calculated $F_p(H)$ curves do not follow a single smooth dome-like behavior. Instead, both 2ML and 3ML show multiple field-dependent structures, including low-field enhancement and high-field contributions. The normalized pinning force $f_p(h)$ also fails to collapse into a single universal scaling curve, indicating that a single pinning regime is insufficient to describe the full field range.

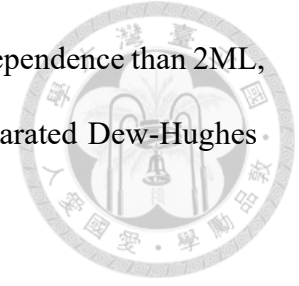
The Dew-Hughes-type analysis further quantified this behavior. By fitting the low-field and high-field regions separately using

$$f_p(h) = Ah^p(1 - h)^q \quad (5.1)$$

to distinct characteristic peak positions, $h_{\max,L}$ and $h_{\max,H}$, were obtained. The existence of two separated $h_{\max}$ values indicates that the low-field and high-field pinning responses cannot be treated as the same contribution. This analysis is used phenomenologically rather than as a unique identification of microscopic pinning centers, following the general use of pinning-force scaling in type-II superconductors [27].

Finally, the $\log J_c - \log H$ plots provide the most direct evidence for multiple field-dependent vortex responses. The $J_c(H)$ curves do not show a single monotonic power-law decay; instead, they contain multiple field regions with different slopes and morphologies. Such segmented behavior is consistent with previous studies where changes in $\log J_c - \log H$ morphology were used to identify crossover between different pinning regimes or field-dependent vortex responses [70], [71]. In this work, the 3ML

sample shows a broader high- J_c region and a more structured field dependence than 2ML, consistent with its larger $F_p(H)$, higher H_{irr} , and more clearly separated Dew-Hughes fitting branches.



Overall, this thesis concludes that Nb/Sb₂Te₃ multilayer thin films exhibit competing field-dependent vortex-pinning regimes rather than a single conventional pinning mechanism. The comparison between 2ML and 3ML shows that increasing the number of Nb/Sb₂Te₃ spacer/interface structures strengthens and complicates the irreversible vortex response. Since the total Nb thickness is kept approximately constant, the difference between the two samples is most reasonably associated with the multilayer architecture and the increased number of interfaces. Therefore, this work demonstrates that multilayer stacking is an effective parameter for modifying the vortex-pinning landscape in Nb/Sb₂Te₃-based superconducting thin films.

5.2 Future works

Owing to the lack of an effective Nb etching process, the present study was primarily based on magnetic measurements. To enable future electrical transport measurements, a thermally stable Au sacrificial-mask process was developed. After photolithographic patterning, approximately 20 nm of Sb₂Te₃ and several hundred nanometers of Au were sequentially deposited, followed by lift-off in acetone to form the Au pattern shown in Figure 5.1. The multilayer can then be deposited using the procedure described in Chapter 3, after which the Au mask can be removed using a KI/I₂ solution to obtain a patterned device. Transport measurements on such devices may provide further evidence for the

origin of the anomalous pinning enhancement.

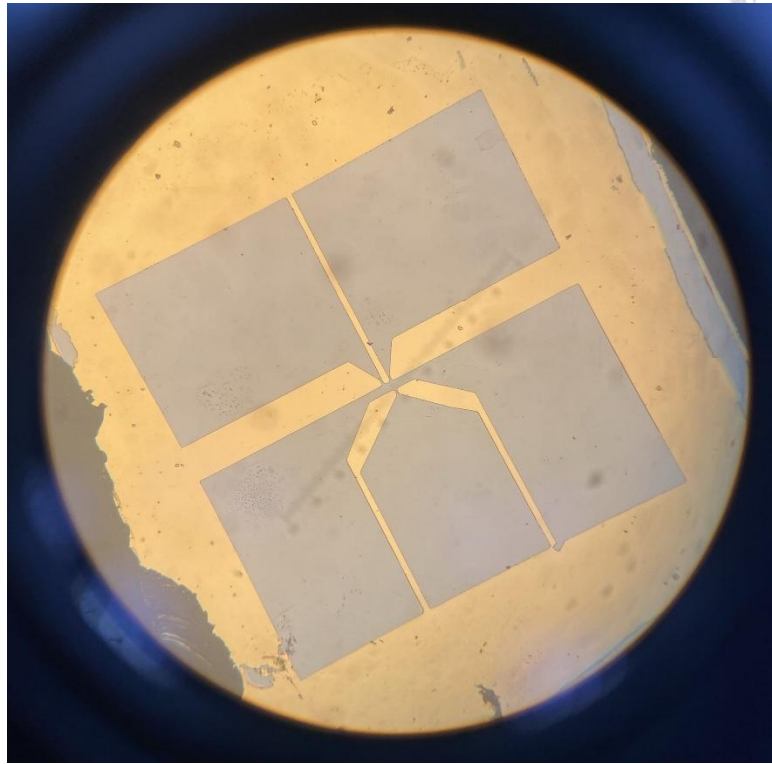


Figure 5.1: Optical image of the patterned Au sacrificial mask developed for future electrical-device fabrication.

References



- [1] D. Van Delft and P. Kes, “The discovery of superconductivity,” *Phys. Today*, vol. 63, no. 9, pp. 38–43, Sep. 2010, doi: 10.1063/1.3490499.
- [2] D. Goodstein and J. Goodstein, “Richard Feynman and the History of Superconductivity,” *Phys. Perspect.*, vol. 2, no. 1, pp. 30–47, Mar. 2000, doi: 10.1007/s000160050035.
- [3] J. E. Hirsch, “The Meissner effect in superconductors: emergence versus reductionism,” *J. Phys. Condens. Matter*, vol. 38, no. 7, p. 073001, Feb. 2026, doi: 10.1088/1361-648X/ae3cf2.
- [4] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, “Theory of Superconductivity,” *Phys. Rev.*, vol. 108, no. 5, pp. 1175–1204, Dec. 1957, doi: 10.1103/PhysRev.108.1175.
- [5] “ON THE THEORY OF SUPERCONDUCTIVITY,” in *Collected Papers of L.D. Landau*, Elsevier, 1965, pp. 546–568. doi: 10.1016/B978-0-08-010586-4.50078-X.
- [6] G. I. González-Pedrerros, J. A. Camargo-Martínez, and F. Mesa, “Cooper Pairs Distribution function for bcc Niobium under pressure from first-principles,” *Sci. Rep.*, vol. 11, no. 1, p. 7646, Apr. 2021, doi: 10.1038/s41598-021-87028-x.
- [7] D. Möckli and M. K. D. Azambuja, “Magnetic superconductivity,” *Rev. Bras. Ensino Fisica*, vol. 46, no. suppl 1, p. e20240094, 2024, doi: 10.1590/1806-9126-rbef-2024-0094.
- [8] G. I. González-Pedrerros, J. A. Camargo-Martínez, and F. Mesa, “Cooper Pairs Distribution function for bcc Niobium under pressure from first-principles,” *Sci. Rep.*, vol. 11, no. 1, p. 7646, Apr. 2021, doi: 10.1038/s41598-021-87028-x.
- [9] N. Schwartz, M. Gresh, and S. Karlik, “Niobium Solid Electrolytic Capacitors,” *J. Electrochem. Soc.*, vol. 108, no. 8, p. 750, 1961, doi: 10.1149/1.2428210.
- [10] M. Z. Hasan and C. L. Kane, “Colloquium : Topological insulators,” *Rev. Mod. Phys.*,

vol. 82, no. 4, pp. 3045–3067, Nov. 2010, doi: 10.1103/RevModPhys.82.3045.

[11] M. König *et al.*, “Quantum Spin Hall Insulator State in HgTe Quantum Wells,” *Science*, vol. 318, no. 5851, pp. 766–770, Nov. 2007, doi: 10.1126/science.1148047.

[12] D. Hsieh *et al.*, “A topological Dirac insulator in a quantum spin Hall phase,” *Nature*, vol. 452, no. 7190, pp. 970–974, Apr. 2008, doi: 10.1038/nature06843.

[13] H. Zhang, C.-X. Liu, X.-L. Qi, X. Dai, Z. Fang, and S.-C. Zhang, “Topological insulators in Bi₂Se₃, Bi₂Te₃ and Sb₂Te₃ with a single Dirac cone on the surface,” *Nat. Phys.*, vol. 5, no. 6, pp. 438–442, Jun. 2009, doi: 10.1038/nphys1270.

[14] D. Mondal, A. Bandyopadhyay, and D. Jana, “Su-Schrieffer-Heeger Model - From Fundamentals to Responses,” *Int. J. Theor. Phys.*, vol. 64, no. 5, p. 125, Apr. 2025, doi: 10.1007/s10773-025-05981-z.

[15] X. Zhang *et al.*, “Investigation on the electrical transport properties of highly (001)-textured Sb₂Te₃ films deposited by molecular beam epitaxy,” *J. Appl. Phys.*, vol. 115, no. 2, p. 024307, Jan. 2014, doi: 10.1063/1.4861394.

[16] Shama, R. K. Gopal, G. Sheet, and Y. Singh, “2D weak anti-localization in thin films of the topological semimetal Pd₃Bi₂SS₂,” *Sci. Rep.*, vol. 11, no. 1, p. 12618, Jun. 2021, doi: 10.1038/s41598-021-91930-9.

[17] S. Hikami, A. I. Larkin, and Y. Nagaoka, “Spin-Orbit Interaction and Magnetoresistance in the Two Dimensional Random System,” *Prog. Theor. Phys.*, vol. 63, no. 2, pp. 707–710, Feb. 1980, doi: 10.1143/PTP.63.707.

[18] L. Fu and C. L. Kane, “Superconducting Proximity Effect and Majorana Fermions at the Surface of a Topological Insulator,” *Phys. Rev. Lett.*, vol. 100, no. 9, p. 096407, Mar. 2008, doi: 10.1103/PhysRevLett.100.096407.

[19] W. Meissner and R. Ochsenfeld, “Ein neuer Effekt bei Eintritt der Supraleitfähigkeit,” *Naturwissenschaften*, vol. 21, no. 44, pp. 787–788, Nov. 1933, doi:

10.1007/BF01504252.

[20] F. London and H. London, “The electromagnetic equations of the supraconductor,” *Proc. R. Soc. Lond. Ser. - Math. Phys. Sci.*, vol. 149, no. 866, pp. 71–88, Mar. 1935, doi: 10.1098/rspa.1935.0048.

[21] Y. Zhang *et al.*, “High-temperature superconductivity with zero resistance and strange-metal behaviour in $\text{La}_3\text{Ni}_2\text{O}_7-\delta$,” *Nat. Phys.*, vol. 20, no. 8, pp. 1269–1273, Aug. 2024, doi: 10.1038/s41567-024-02515-y.

[22] L. D. Landau, “ON THE THEORY OF SUPERCONDUCTIVITY,” in *Collected Papers of L.D. Landau*, Elsevier, 1965, pp. 546–568. doi: 10.1016/B978-0-08-010586-4.50078-X.

[23] A. A. Abrikosov, “The magnetic properties of superconducting alloys,” *J. Phys. Chem. Solids*, vol. 2, no. 3, pp. 199–208, Jan. 1957, doi: 10.1016/0022-3697(57)90083-5.

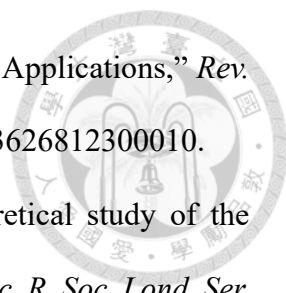
[24] J. M. Lock, “Critical Currents in Superconductors,” *Phys. Bull.*, vol. 24, no. 10, pp. 620–620, Oct. 1973, doi: 10.1088/0031-9112/24/10/041.

[25] C. P. Bean, “Magnetization of High-Field Superconductors,” *Rev. Mod. Phys.*, vol. 36, no. 1, pp. 31–39, Jan. 1964, doi: 10.1103/RevModPhys.36.31.

[26] G. Blatter, M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, “Vortices in high-temperature superconductors,” *Rev. Mod. Phys.*, vol. 66, no. 4, pp. 1125–1388, Oct. 1994, doi: 10.1103/RevModPhys.66.1125.

[27] D. Dew-Hughes, “Flux pinning mechanisms in type II superconductors,” *Philos. Mag. J. Theor. Exp. Appl. Phys.*, vol. 30, no. 2, pp. 293–305, Aug. 1974, doi: 10.1080/14786439808206556.

[28] M. Tinkham, *Introduction to superconductivity*, 2 ed. in Dover books on physics. Mineola, NY: Dover Publ, 2015.

- 
- [29] R. Flükiger, “Overview of Superconductivity and Challenges in Applications,” *Rev. Accel. Sci. Technol.*, vol. 05, pp. 1–23, Jan. 2012, doi: 10.1142/S1793626812300010.
- [30] A. B. Pippard and W. L. Bragg, “An experimental and theoretical study of the relation between magnetic field and current in a superconductor,” *Proc. R. Soc. Lond. Ser. Math. Phys. Sci.*, vol. 216, no. 1127, pp. 547–568, Feb. 1953, doi: 10.1098/rspa.1953.0040.
- [31] A. B. Pippard, “The surface impedance of superconductors and normal metals at high frequencies II. The anomalous skin effect in normal metals,” *Proc. R. Soc. Lond. Ser. Math. Phys. Sci.*, vol. 191, no. 1026, pp. 385–399, Nov. 1947, doi: 10.1098/rspa.1947.0122.
- [32] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, “Theory of Superconductivity,” *Phys. Rev.*, vol. 108, no. 5, pp. 1175–1204, Dec. 1957, doi: 10.1103/PhysRev.108.1175.
- [33] A. A. Abrikosov, “On the Magnetic Properties of Superconductors of the Second Group,” *Sov Phys JETP*, vol. 5, pp. 1174–1182, 1957.
- [34] A. I. Larkin and Yu. N. Ovchinnikov, “Pinning in type II superconductors,” *J. Low Temp. Phys.*, vol. 34, no. 3–4, pp. 409–428, Feb. 1979, doi: 10.1007/BF00117160.
- [35] P. W. Anderson and Y. B. Kim, “Hard Superconductivity: Theory of the Motion of Abrikosov Flux Lines,” *Rev. Mod. Phys.*, vol. 36, no. 1, pp. 39–43, Jan. 1964, doi: 10.1103/RevModPhys.36.39.
- [36] C. P. Bean, “Magnetization of Hard Superconductors,” *Phys. Rev. Lett.*, vol. 8, no. 6, pp. 250–253, Mar. 1962, doi: 10.1103/PhysRevLett.8.250.
- [37] E. Zeldov *et al.*, “Geometrical Barriers in High-Temperature Superconductors,” *Phys. Rev. Lett.*, vol. 73, no. 10, pp. 1428–1431, Sep. 1994, doi: 10.1103/PhysRevLett.73.1428.
- [38] E. H. Brandt, “Irreversible magnetization of pin-free type-II superconductors,” *Phys.*

Rev. B, vol. 60, no. 17, pp. 11939–11942, Nov. 1999, doi: 10.1103/PhysRevB.60.11939.

[39] D. M. Gokhfeld, “Secondary Peak on Asymmetric Magnetization Loop of Type-II Superconductors,” *J. Supercond. Nov. Magn.*, vol. 26, no. 2, pp. 281–283, Feb. 2013, doi: 10.1007/s10948-012-1741-8.

[40] T. H. Johansen, M. R. Koblishka, H. Bratsberg, and P. O. Hetland, “Critical-state model with a secondary high-field peak in $J_c(B)$,” *Phys. Rev. B*, vol. 56, no. 17, pp. 11273–11278, Nov. 1997, doi: 10.1103/PhysRevB.56.11273.

[41] W. Zhou, X. Xing, W. Wu, H. Zhao, and Z. Shi, “Second magnetization peak effect, vortex dynamics and flux pinning in 112-type superconductor $\text{Ca}_{0.8}\text{La}_{0.2}\text{Fe}_{1-x}\text{Co}_x\text{As}_2$,” *Sci. Rep.*, vol. 6, no. 1, p. 22278, Mar. 2016, doi: 10.1038/srep22278.

[42] G. P. Mikitik and E. H. Brandt, “Peak effect, vortex-lattice melting line, and order-disorder transition in conventional and high- T_c superconductors,” *Phys. Rev. B*, vol. 64, no. 18, p. 184514, Oct. 2001, doi: 10.1103/PhysRevB.64.184514.

[43] B. Rosenstein, B. Ya. Shapiro, I. Shapiro, Y. Bruckental, A. Shaulov, and Y. Yeshurun, “Peak effect and square-to-rhombic vortex lattice transition in $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$,” *Phys. Rev. B*, vol. 72, no. 14, p. 144512, Oct. 2005, doi: 10.1103/PhysRevB.72.144512.

[44] D. Stamopoulos and M. Pissas, “Hysteretic behavior of the vortex lattice at the onset of the second peak for the $\text{HgBa}_2\text{CuO}_{4+\delta}$ superconductor,” *Phys. Rev. B*, vol. 65, no. 13, p. 134524, Mar. 2002, doi: 10.1103/PhysRevB.65.134524.

[45] I. M. Babich, E. H. Brandt, G. P. Mikitik, and E. Zeldov, “Critical current in type-II superconductors near the order-disorder transition,” *Phys. Rev. B*, vol. 81, no. 5, p. 054517, Feb. 2010, doi: 10.1103/PhysRevB.81.054517.

[46] H. M. Cole, M. B. Venuti, B. Gorman, E. D. Bauer, M. K. Chan, and S. Eley, “Plastic vortex creep and dimensional crossovers in the highly anisotropic superconductor $\text{HgBa}_2\text{CuO}_{4+x}$,” *Phys. Rev. B*, vol. 107, no. 10, p. 104509, Mar. 2023, doi:

10.1103/PhysRevB.107.104509.

[47] E. J. Patiño, M. Aprili, M. G. Blamire, and Y. Maeno, “Vortex flipping in superconductor/ferromagnet spin-valve structures,” *Phys. Rev. B*, vol. 87, no. 21, p. 214514, Jun. 2013, doi: 10.1103/PhysRevB.87.214514.

[48] 洪浩哲, “磊晶碲化銻薄膜之成長與超導碲化銻/鈮多層膜之電磁傳輸特性之研究,” Master’s Thesis, 國立臺灣大學, 2024. doi: 10.6342/NTU202404181.

[49] R. Sultana, G. Gurjar, S. Patnaik, and V. P. S. Awana, “Crystal growth and characterization of bulk Sb_2Te_3 topological insulator,” *Mater. Res. Express*, vol. 5, no. 4, p. 046107, Apr. 2018, doi: 10.1088/2053-1591/aabc33.

[50] L. S. Brooks, “The Vapor Pressures of Tellurium and Selenium,” *J. Am. Chem. Soc.*, vol. 74, no. 1, pp. 227–229, Jan. 1952, doi: 10.1021/ja01121a059.

[51] M. J. Smith, R. J. Knight, and C. W. Spencer, “Properties of Bi_2Te_3 - Sb_2Te_3 Alloys,” *J. Appl. Phys.*, vol. 33, no. 7, pp. 2186–2190, Jul. 1962, doi: 10.1063/1.1728925.

[52] M. Tan, Y. Deng, and Y. Hao, “Enhanced thermoelectric properties and layered structure of Sb_2Te_3 films induced by special (0 01) crystal plane,” *Chem. Phys. Lett.*, vol. 584, pp. 159–164, Oct. 2013, doi: 10.1016/j.cplett.2013.08.084.

[53] S. Swann, “Magnetron sputtering,” *Phys. Technol.*, vol. 19, no. 2, pp. 67–75, Mar. 1988, doi: 10.1088/0305-4624/19/2/304.

[54] P. Gu, X. Zhu, H. Wu, and D. Yang, “Regulation of Substrate-Target Distance on the Microstructural, Optical and Electrical Properties of CdTe Films by Magnetron Sputtering,” *Materials*, vol. 11, no. 12, p. 2496, Dec. 2018, doi: 10.3390/ma11122496.

[55] G. Zhu, B. Xiao, G. Chen, and Z. Gan, “Study on the Deposition Uniformity of Triple-Target Magnetron Co-Sputtering System: Numerical Simulation and Experiment,” *Materials*, vol. 15, no. 21, p. 7770, Nov. 2022, doi: 10.3390/ma15217770.

[56] V. Strenzke *et al.*, “Challenges and solutions in RF sputtering of superconducting Nb

for nanostructuring processes,” *J. Appl. Phys.*, vol. 136, no. 20, p. 205301, Nov. 2024, doi: 10.1063/5.0239203.

[57] T. L. Anderson and H. B. Krause, “Refinement of the Sb_2Te_3 and $\text{Sb}_2\text{Te}_2\text{Se}$ structures and their relationship to nonstoichiometric $\text{Sb}_2\text{Te}_{3-y}\text{Se}_y$ compounds,” *Acta Crystallogr. B*, vol. 30, no. 5, pp. 1307–1310, May 1974, doi: 10.1107/S0567740874004729.

[58] V. Sandu, A. M. Ionescu, G. Aldica, M. A. Grigoroscuta, M. Burdusel, and P. Badica, “On the pinning force in high density MgB_2 samples,” *Sci. Rep.*, vol. 11, no. 1, p. 5951, Mar. 2021, doi: 10.1038/s41598-021-85209-2.

[59] N. M. Hapipi *et al.*, “Excess Mg in situ powder addition for enhancing critical current density of ex situ MgB_2 ,” *Appl. Phys. A*, vol. 128, no. 10, p. 913, Oct. 2022, doi: 10.1007/s00339-022-06060-4.

[60] A. Xu *et al.*, “Strongly enhanced vortex pinning from 4 to 77 K in magnetic fields up to 31 T in 15 mol.% Zr-added (Gd, Y)-Ba-Cu-O superconducting tapes,” *APL Mater.*, vol. 2, no. 4, p. 046111, Apr. 2014, doi: 10.1063/1.4872060.

[61] Y. Yeshurun, A. P. Malozemoff, and A. Shaulov, “Magnetic relaxation in high-temperature superconductors,” *Rev. Mod. Phys.*, vol. 68, no. 3, pp. 911–949, Jul. 1996, doi: 10.1103/RevModPhys.68.911.

[62] C. J. Van Der Beek *et al.*, “Flux pinning in $\text{PrFeAsO}_{0.9}$ and $\text{NdFeAsO}_{0.9}\text{F}_{0.1}$ superconducting crystals,” *Phys. Rev. B*, vol. 81, no. 17, p. 174517, May 2010, doi: 10.1103/PhysRevB.81.174517.

[63] M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, “Theory of collective flux creep,” *Phys. Rev. Lett.*, vol. 63, no. 20, pp. 2303–2306, Nov. 1989, doi: 10.1103/PhysRevLett.63.2303.

[64] W. Braunisch *et al.*, “Paramagnetic Meissner effect in Bi high-temperature

superconductors,” *Phys. Rev. Lett.*, vol. 68, no. 12, pp. 1908–1911, Mar. 1992, doi: 10.1103/PhysRevLett.68.1908.

[65] M. R. Koblishka, L. Püst, C.-S. Chang, T. Hauet, and A. Koblishka-Veneva, “The Paramagnetic Meissner Effect (PME) in Metallic Superconductors,” *Metals*, vol. 13, no. 6, p. 1140, Jun. 2023, doi: 10.3390/met13061140.

[66] A. E. Koshelev and A. I. Larkin, “Paramagnetic moment in field-cooled superconducting plates: Paramagnetic Meissner effect,” *Phys. Rev. B*, vol. 52, no. 18, pp. 13559–13562, Nov. 1995, doi: 10.1103/PhysRevB.52.13559.

[67] G. Lasher, “Series Solution of the Ginzburg-Landau Equations for the Abrikosov Mixed State,” *Phys. Rev.*, vol. 140, no. 2A, pp. A523–A528, Oct. 1965, doi: 10.1103/PhysRev.140.A523.

[68] D. R. Strachan, M. C. Sullivan, and C. J. Lobb, “Probing the limits of superconductivity,” presented at the International Symposium on Optical Science and Technology, I. Bozovic and D. Pavuna, Eds., Seattle, WA, Nov. 2002, p. 65. doi: 10.1117/12.452479.

[69] R. Griessen *et al.*, “Evidence for mean free path fluctuation induced pinning in YBa₂Cu₃O₇ and YBa₂Cu₄O₈ films,” *Phys. Rev. Lett.*, vol. 72, no. 12, pp. 1910–1913, Mar. 1994, doi: 10.1103/PhysRevLett.72.1910.

[70] Q. Li, M. Suenaga, Z. Ye, S. R. Foltyn, and H. Wang, “Crossover of thickness dependence of critical current density $J_c(T,H)$ in YBa₂Cu₃O_{7- δ} thick films,” *Appl. Phys. Lett.*, vol. 84, no. 18, pp. 3528–3530, May 2004, doi: 10.1063/1.1737067.

[71] C. Cai, B. Holzapfel, J. Hänisch, L. Fernández, and L. Schultz, “High critical current density and its field dependence in mixed rare earth (Nd,Eu,Gd)Ba₂Cu₃O_{7- δ} thin films,” *Appl. Phys. Lett.*, vol. 84, no. 3, pp. 377–379, Jan. 2004, doi: 10.1063/1.1640802.