國立臺灣大學理學院物理學系

博士論文

Department of Physics

College of Science

National Taiwan University

Doctor Dissertation

前沿方法: 多分步量子漫步在量子態準備與金融市場模擬中的應用

An Application of Multi-Split-Step Quantum Walks to Quantum State Preparation and Financial Market Simulations : A Frontier Approach

張晏瑞

Yen-Jui Chang

指導教授: 張慶瑞 教授

Advisor: Prof. Ching-Ray Chang

中華民國 113 年 6 月

June, 2024

doi:10.6342/NTU202401141

國立臺灣大學博士學位論文
口試委員會審定書

PhD DISSERTATION ACCEPTANCE CERTIFICATE
NATIONAL TAIWAN UNIVERSITY

前沿方法: 多階段量子行走在量子態準備與金融市場模擬中的應用

An Application of Multi-Split-Step Quantum Walks to Quantum State Preparation and Financial Market Simulations : A Frontier Approach

本論文係__張晏瑞__(姓名)_D09222012__（學號）在國立臺灣大學__物理所__(系/所/學位學程)完成之博士學位論文，於民國_113_年_06_月_07_日承下列考試委員審查通過及口試及格，特此證明。

The undersigned, appointed by the Department / Institute of __Physics__ on _07_(date)_06_(month)_2024_(year) have examined a PhD dissertation entitled above presented by__Yen Jui Chang__(name)__D09222012__(student ID) candidate and hereby certify that it is worthy of acceptance.

口試委員 Oral examination committee:

(指導教授 Advisor)

doi:10.6342/NTU202401141

Acknowledgements

在此，我必須表達我對指導教授張慶瑞教授的深深感激。在我台大四年的學習及研究旅程中，張教授不僅在學術上提供深入的指導，也在生活上給予我巨大的支持和關懷，更為我打開了與國際學術界及產業界交流的大門，讓我得以與許多卓越的學者和專業人士相識。

我同時要感謝王韋廷與所有實驗室同伴們，在後疫情時代充滿不確定的時刻，他們持續地支持我的研究工作，對我的學術生涯產生了深遠的影響。

此外，我衷心感謝廖世偉教授及于濂波博士在我的博士研究期間提供的無價建議和慷慨幫助。他們的專業引導對我的研究成果貢獻良多。

最後，我必須對我的太太張祐寧表達最深的感謝。在這漫長的學術旅程中，她的愛和支持一直是我不可或缺的力量，使我能夠克服各種挑戰，達成我的學術目標。

doi:10.6342/NTU202401141

摘要

本博士論文探討量子計算在金融領域的創新應用，專注於為應用及開發量子演算法於複雜金融模擬。透過運用量子力學的原理—疊加、纏結及干涉—本研究將量子物理與計算金融結合，創造了一個開創性的量子演算法。本研究的核心，多分步量子漫步（multi-SSQW），在傳統量子漫步的框架上進行擴展，融入多代理決策過程。此整合使得能夠精細模擬反映真實世界金融市場複雜性的複雜金融分佈和場景。該演算法的顯著靈活性、可靠的收斂特性和快速計算能力，將其定位為金融分析和戰略決策的變革工具。本研究展示了實際應用，為未來的金融模型和風險評估設定了新的先例。

關鍵字：量子漫步、量子演算法、量子態製備、量子金融、量子計算

doi:10.6342/NTU202401141

Abstract

This dissertation delves into the innovative application of quantum computing within the finance sector, focusing on developing and implementing specialized quantum algorithms designed for complex financial simulations. By harnessing the principles of quantum mechanics—superposition, entanglement, and interference—this research synergizes quantum physics with computational finance to create a pioneering quantum algorithm. The centerpiece of our research, the multi-split-steps quantum walk (multi-SSQW), expands on traditional quantum walk frameworks by integrating multi-investors decision-making processes. This integration allows for the sophisticated modeling of complex financial distributions and scenarios, reflecting real-world financial market complexities. The algorithm's remarkable adaptability, consistent convergence, and capacity for swift calculations establish it as a revolutionary resource for financial analysis and strategic decision-making. Beyond theoretical implications, this research demonstrates practical applications, providing a significant computational edge over traditional methods and set-

doi:10.6342/NTU202401141

ting a new precedent for the future of financial modeling and risk assessment.

Keywords: Quantum Walks, Quantum Algorithm, Preparing Quantum State, Quantum Finance, Quantum Computing

doi:10.6342/NTU202401141

Contents

doi:10.6342/NTU202401141

doi:10.6342/NTU202401141

List of Figures

doi:10.6342/NTU202401141

doi:10.6342/NTU202401141

doi:10.6342/NTU202401141

doi:10.6342/NTU202401141

doi:10.6342/NTU202401141

Chapter 1 Introduction

1.1 Quantitative Finance

The advancement of computing technology has played a crucial role in the development of modern finance, particularly in option pricing and portfolio management. As computers became more powerful, faster, and more accessible, they enabled financial professionals to perform complex calculations, analyze vast amounts of data, and make informed decisions in real time. In the early days of option pricing, the Black-Scholes-Merton model required significant computational power to solve the complex partial differential equations that underpin the model [1]. The availability of faster computers in the 1970s and 1980s made it possible for traders and analysts to quickly price options and other derivatives, paving the way for the rapid growth of the derivatives market [2]. Similarly, the increasing computational power in portfolio management allowed for the practical implementation of Markowitz's modern portfolio theory [3]. At the core of modern portfolio theory, the mean-variance optimization technique requires the computation of large covariance matrices and the solution of quadratic optimization problems [4]. As computers became more powerful, portfolio managers could construct and rebalance portfolios more efficiently, considering a wider range of assets and constraints. The 1960s and 1970s saw the emergence of the Capital Asset Pricing Model (CAPM) and the Arbitrage

doi:10.6342/NTU202401141

Pricing Theory (APT), which further advanced the understanding of risk and return in financial markets [5, 6]. These models and the increasing availability of financial data and computing power enabled the development of more sophisticated portfolio management strategies.

The advent of high-performance computing and parallel processing in the 1990s and 2000s further revolutionized the fields of option pricing and portfolio management[7]. These technologies enabled the development of more sophisticated pricing models, such as the Heston stochastic volatility model and the Bates jump-diffusion model, which could better capture the complexities of real-world financial markets [8, 9]. In portfolio management, high-performance computing enabled more advanced optimization techniques, such as resampling and robust optimization, which could handle larger datasets and incorporate more realistic assumptions [10]. In recent years, the explosion of big data and the advancements in machine learning and artificial intelligence have opened up new frontiers in option pricing and portfolio management [11]. Machine learning techniques, such as neural networks and support vector machines, are being used to develop more accurate pricing models and to identify new pricing factors [12]. In portfolio management, machine learning algorithms are employed to identify financial data patterns, optimize asset allocations, and develop new investment strategies [13]. The evolution of computing technology has not only made existing financial models and techniques more efficient but has also enabled the development of entirely new approaches to option pricing and portfolio management. As computing power continues to grow and new technologies emerge, it is likely that the fields of finance and computer science will become even more intertwined, driving further innovation and shaping the future of the financial industry.

doi:10.6342/NTU202401141

1.2 Quantum Computing

Since the 1950s, digital computing has significantly enhanced our ability to solve complex problems with quantitative methods. This has led to a significant transformation in fields such as finance, where modern quantitative methods have revolutionized financial analysis and decision-making.

Similarly, quantum mechanics has transformed our understanding of particle physics by introducing principles like superposition, entanglement, and interference. These principles highlight the intricate probabilities of quantum states that go beyond the precise predictions of classical mechanics.

In the 1980s, theoretical physicists Richard Feynman and David Deutsch proposed the revolutionary idea of a quantum computer, a machine capable of simulating phenomena that classical computers could not, leveraging the inherent uncertainty and probabilistic nature of quantum mechanics, which allow qubits to exist in multiple states at once and to be interconnected in ways that enable rapid data processing. This differs fundamentally from classical computers, which represent data as binary bits—either 0 or 1. This proposition marked a pivotal moment, suggesting that quantum computers could perform tasks like factorization and simulation with unprecedented efficiency. Feynman's seminal paper on simulating quantum systems [14] laid the groundwork for the concept of quantum computing. In this work, Feynman introduced the revolutionary idea that a quantum computer could simulate systems that are infeasible for classical computers, fundamentally changing our approach to computational physics and beyond. Shor's algorithm for factorization [15] demonstrated quantum computing's potential to break classical cryptographic protocols, marking a pivotal moment in integrating quantum mechanics with computer science. In

doi:10.6342/NTU202401141

his influential paper, Peter Shor introduced an algorithm that proved quantum computers could factor large numbers exponentially faster than the best-known classical algorithms, highlighting a significant computational advantage of quantum technology. While not as foundational as earlier works, the article by Trabesinger [16] provides a comprehensive overview of quantum computing developments up to 2012. This piece examines both theoretical advancements and experimental implementations, serving as a crucial bridge between the initial theoretical proposals and their practical realizations. Trabesinger's review captures the evolving landscape of quantum computing, highlighting key milestones and the emerging possibilities enabled by this cutting-edge technology. Further cementing the reality of these advancements, the work of F. Arute et al. [17], which was published in Nature in 2019, reports the achievement of quantum supremacy using a programmable superconducting processor, showcasing the practical capabilities of quantum computers to perform a task—sampling output from a quantum circuit—that is beyond the reach of today's most powerful supercomputers.

Advancements in quantum algorithms have shown significant promise across various computational domains. For instance, Harrow, Hassidim, and Lloyd's algorithm for linear systems of equations [18] could revolutionize computational speed, while Lloyd, Mohseni, and Rebentrost's work on quantum principal component analysis [19] has implications for more efficient data processing. Havlíček et al.'s exploration of quantum-enhanced feature spaces in supervised learning [20] suggests innovative approaches to machine learning. Further literature [21, 22] underscores quantum computing's impact, highlighting the breadth of its applications and suggesting a future where quantum solutions redefine the efficiency of solving complex problems. These contributions illustrate the rapid evolution of quantum computing and its capacity to transform industries by of-

doi:10.6342/NTU202401141

fering new computational paradigms.

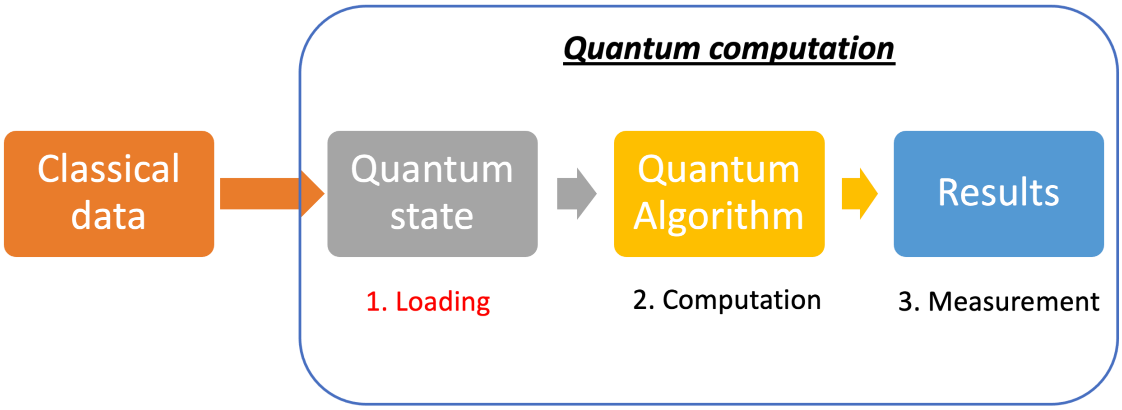


Figure 1.1: Illustration of the three essential phases in the quantum computing process.

Achieving this advantage involves three critical stages: encoding classical data into a quantum state, running quantum computations through sophisticated algorithms, and finally, measuring the results to interpret the outcome, which is shown in Fig.1.1. These stages harness quantum computing's potential to address problems at speed beyond the capability of classical algorithms, assuming efficient data-to-quantum-state conversion. The generation of specific distributions of probabilities using quantum randomness is among the exciting applications being explored, for example in finance, where it enables the assignment of probability amplitudes to a vast number of market scenarios represented by a quantum system's superposition states.

The evolution of quantum computing of IBM presents a transformative landscape for various sectors, including finance. Moving beyond the quantum supremacy milestone Google AI and NASA achieved in 2019, we see a focused trajectory toward creating scalable, fault-tolerant quantum systems. IBM's roadmap through 2023 and beyond envisions the enhancement of quantum execution speed and the scaling of quantum systems up to thousands of qubits, indicative of quantum computing's trajectory toward tackling incredibly complex problems unmanageable by classical computers.

doi:10.6342/NTU202401141

Error correction and fault tolerance are cornerstones of IBM's quantum development strategy. By 2023, IBM aims to enhance quantum execution with error mitigation techniques, setting the stage for even more sophisticated quantum algorithms that can handle intricate calculations involved in financial simulations. The roadmap further anticipates the improvement of quantum circuit quality to allow 1,000+ qubits systems by 2029, suggesting an era where quantum computing may become integral to financial analysis, capable of managing the voluminous, complex data and the computation-intensive tasks that the financial industry demands.

Advancements in quantum algorithms will play a pivotal role in finance, particularly in risk analysis, asset pricing, and investment portfolio optimization. Quantum algorithms can potentially speed up calculations for options pricing models dramatically, Monte Carlo simulations, and other stochastic modeling techniques central to financial analysis and decision-making.

1.3 Quantum Walks

Quantum walks (QWs) are employed as a methodological tool, incorporating the intricacies and probabilities inherent in quantum systems to reflect the market' s behavior accurately. This quantum-based model offers a novel perspective, allowing for a more detailed and nuanced prediction of financial market movements and decision-making processes.

QWs, as a pivotal development in quantum computing, trace their conceptual origins to the work of physicists such as Aharonov et al., who in 1993 laid the theoretical foundations for quantum random walks, the quantum analog of classical random walks[23].

doi:10.6342/NTU202401141

This new framework introduced unique characteristics of quantum mechanics, such as superposition and entanglement, into algorithms, enhancing their computational capabilities beyond classical constraints.

Figure 1.2 illustrates the concepts of QW and classical random walk (CRW). The left side of the figure depicts the CRW, where the particle moves step by step, flipping a coin to decide its direction, resulting in a binomial distributio. The right side of the figure represents the QW, where the particle can be in a superposition of states, allowing it to move along multiple paths simultaneously. This leads to a more complex probability distribution compared to the CRW. The graphs below each walk show the respective probability distributions after several steps, highlighting the differences between quantum and classical approaches. Differences and Similarities Between Quantum Walk and Classical

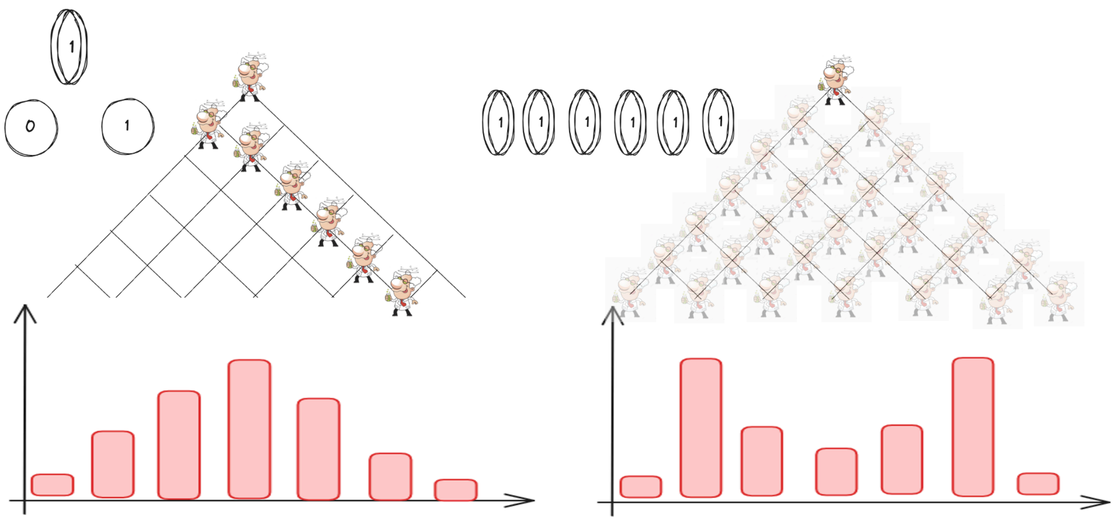


Figure 1.2: Illustration of the concepts of classical random walk (CRW) and quantum walk (QW).

Random Walk:

Similarities:

1. **Basic Concept**: Both simulate the movement of particles in space.

doi:10.6342/NTU202401141

2. **Probabilistic Nature**: In both quantum walk and classical random walk, a particle has multiple possible directions to move at each step, and these directions are determined by a certain probability distribution.

Differences:

1. **Superposition**:
    - **Quantum Walk**: A quantum bit can be in multiple states simultaneously, allowing the quantum particle to move along multiple paths at the same time.
    - **Classical Random Walk**: A particle can only choose one path at each step.
2. **Entanglement**:
    - **Quantum Walk**: Quantum particles can establish entanglement, meaning that changing the state of one particle will immediately affect another particle, even if they are far apart.
    - **Classical Random Walk**: There is no such immediate correlation between particles.
3. **Interference**:
    - **Quantum Walk**: The paths taken by the particles can interfere with each other, which can be used to enhance the probability of the correct path and reduce the probability of incorrect paths, thus improving computational accuracy.
    - **Classical Random Walk**: There is no interference phenomenon between the paths of particle movement.

doi:10.6342/NTU202401141

QWs, the quantum analogs of classical random walks, represent a sophisticated tool for developing quantum algorithms. Pioneering contributions from Andrew M. Childs and colleagues have characterized significant strides in using QWs for quantum computing. Their work, including developing algorithms for universal computation through QWs, has profoundly impacted the ability to perform complex quantum operations [24–26]. Additionally, the integration of Dirac cellular automata into QWs by Mallick and Chandrashekar [27] further illustrates the adaptability and power of QWs in modeling quantum mechanical phenomena. Moreover, the innovative application of machine learning techniques to optimize quantum walk parameters by Rajauria and Chawla [28] demonstrates quantum walk strategies' ongoing evolution and refinement. These advancements not only enhance our understanding of QWs from both theoretical and practical perspectives but also showcase the dynamic, ever-expanding field of quantum computing. QWs have become a solid field of research in quantum computation, attracting attention from physicists, computer scientists, and engineers alike. This interest is driven by QWs' potential to simulate natural processes and perform complex computational tasks more efficiently than classical approaches.

The Discrete Time Quantum Walk(DTQW) was further explored by Aharonov, Ambainis, Kempe, and Vazirani in 2001, who demonstrated its applications in algorithm development, particularly in quantum search algorithms [29]. These initial studies underscored the DTQW's potential for significant algorithmic speedups, offering a robust platform for solving complex computational tasks more efficiently than classical approaches. On the other hand, continuous-time quantum walks (CTQW) were introduced by Farhi and Gutmann in 1998 as a means to perform quantum computation [30]. CTQW has been instrumental in modeling physical processes, such as energy transfer in photosynthesis,

doi:10.6342/NTU202401141

showcasing the practical applications of QWs in simulating natural systems.

QWs enable the simulation of various quantum-mechanical phenomena by adjusting parameters and evolution coin operators specific to the quantum walk. In this discussion, we focus on the one-dimensional DTQW. Unlike classical random walks that require only a position Hilbert space, a DTQW also incorporates a coin Hilbert space to describe its dynamics fully. This coin space encapsulates the internal state of the walker, essential for directing its controlled dynamics [23].

The Hilbert space for QWs is defined by:

$$\mathcal{H} = \mathcal{H}_c \otimes \mathcal{H}_p,$$

where $\mathcal{H}_c$ and and $\mathcal{H}_p$ represent the coin and position Hilbert space. In a one-dimensional DTQW, $\{|\uparrow\rangle and|\downarrow\rangle\}$ are the basis states of the coin space, and the position space is represented by the states $|x\rangle$ where $x \in \mathbb{Z}$. The probability amplitude for the quantum state at position x is represented as:

$$\Psi(x,t)\rangle = \begin{pmatrix} \Psi^{\uparrow}(x,t) \\ \Psi^{\downarrow}(x,t) \end{pmatrix},$$

with two internal degrees of freedom indicating the quantum state's dynamics.

The DTQW evolution is driven by two unitary operators: the coin and shift operators. The coin operator manipulates the coin Hilbert space affecting the amplitude distribution in the position space:

$$\hat{C}(\theta,\phi,\lambda) = \begin{pmatrix} \cos(\frac{\theta}{2}) & -e^{i\lambda}\sin(\frac{\theta}{2}) \\ e^{i\phi}\sin(\frac{\theta}{2}) & e^{i(\lambda+\phi)}\cos(\frac{\theta}{2}) \end{pmatrix},$$

doi:10.6342/NTU202401141

where θ, ϕ, and λ are parameters defining the unitary coin operation. Correctly setting these parameters is critical for effectively employing the DTQW in quantum simulations and for researching realistic dynamics [24, 27].

The shift operator defines the walker’ s movement in position space:

$$\hat{S} = |\downarrow\rangle\langle\downarrow| \otimes \sum_x |x-1\rangle\langle x| + |\uparrow\rangle\langle\uparrow| \otimes \sum_x |x+1\rangle\langle x|.$$

This operator shifts the walker’s position left or right based on its internal state, dictated by the coin operator.

The system’ s initial state often involves a superposition of position states influenced by the coin state:

$$|\Psi_0\rangle = (\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes |x=0\rangle.$$

Each timestep involves applying the shift operator after the coin operator to the internal state, describing the system’ s evolution through the operator:

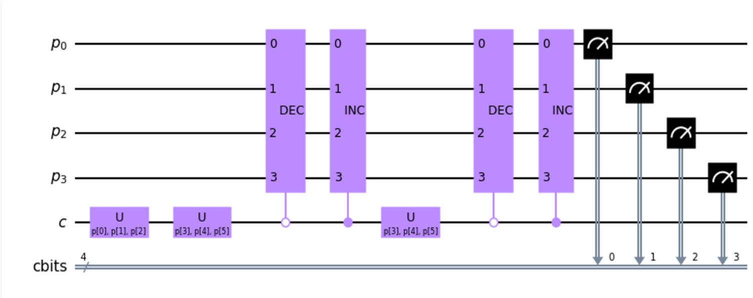
$$\Psi(x,t) = [\hat{S}(\hat{I} \otimes \hat{C})]^t|\Psi_0\rangle = \hat{W}^t|\Psi_0\rangle,$$

where $\hat{I}$ is the identity operator in position space [31].

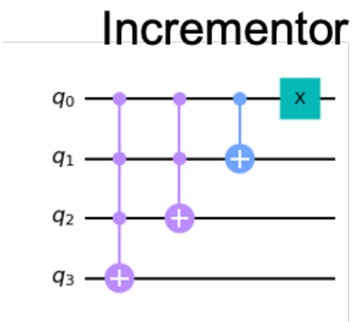
Figure1.3 illustrates a quantum circuit designed for a quantum walk algorithm. The circuit comprises multiple qubits qubits representing the position space labeled p_0 to p_3, with an additional coin qubit labeled c. The operations applied to the qubits include unitary operations U and controlled increment (INC) and decrement (DEC) operators. These gates manipulate the qubits’ states, facilitating the quantum walk’s progress. The unitary operations U are applied to specific qubits at different stages of the circuit, while the INC

doi:10.6342/NTU202401141

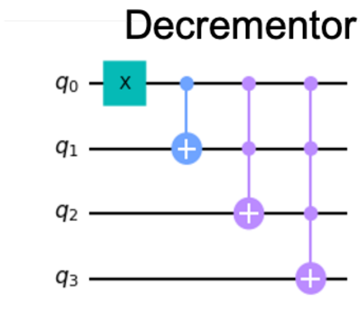
and DEC operators control the increment and decrement of the quantum states in the position space. Finally, measurements are taken, which measure the position space of the qubits and output the results to classical bits (cbits). This configuration represents a step-by-step process in a DTQW, showcasing the iterative and controlled evolution of quantum states within the algorithm.



(a)



(b)



(c)

Figure 1.3: Quantum circuit illustrating a DTQW. The main circuit (a) shows the overall setup for a multi-split-step quantum walk, while (b) and (c) illustrate the increment and decrement operations, respectively, which manage the state transitions in the quantum walk.

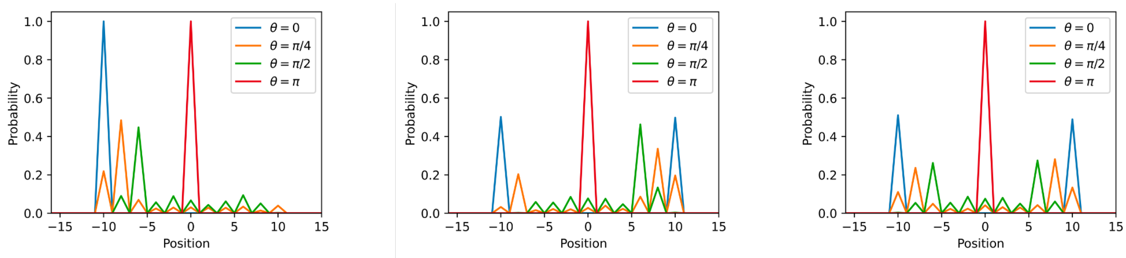
Figure 1.4 illustrates the use of the rotation gate $RY(\theta)$ as a coin operator in a quantum walk. The rotation matrix $RY(\theta)$ is given by:

$$RY(\theta) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

This matrix shows how the rotation gate operates on the quantum state, with elements

doi:10.6342/NTU202401141

expressed in terms of cos and sin functions of θ. The Bloch sphere representation of a qubit state $|\psi\rangle$ indicates how the rotation affects the state vector. Additionally, the slide presents probability distributions after t steps for different initial states and rotation angles θ. The probability distributions are shown for three different initial states: $|\Psi(0)\rangle = |\downarrow\rangle \otimes |x = 0\rangle$, $|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle + |\uparrow\rangle) \otimes |x = 0\rangle$, and $|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle + i|\uparrow\rangle) \otimes |x = 0\rangle$. The plots illustrate how the probability distribution spreads over the interval $[-t\cos(\theta), t\cos(\theta)]$ for different values of θ (0, $\pi/4$, $\pi/2$, π). These plots demonstrate the impact of the rotation angle on the dynamics of the quantum walk, showcasing how different values of θ affect the spread of the probability distribution. This experiment shows that the angle of the R_y-gate controls the spread speed of the walk, while the initial state influences the direction of the distribution.



(a) (b) (c)

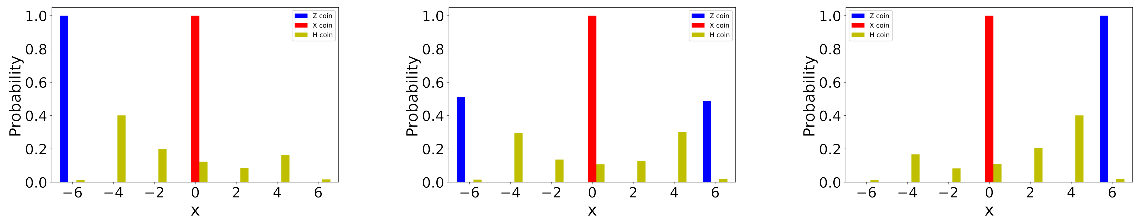
Figure 1.4: Position space probability distributions after t steps using a rotation gate ($RY(\theta)$) as the coin operator with various initial states: (a) $|\Psi(0)\rangle = |\downarrow\rangle \otimes |x = 0\rangle$ (b) $|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle\rangle + |\uparrow\rangle) \otimes |x = 0\rangle$ (c) $|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle\rangle + i|\uparrow\rangle) \otimes |x = 0\rangle$.

Figure (1.5) vividly illustrates the distributions of probabilities in position space resulting from DTQW using different coin operators. Each subfigure represents a unique set of initial conditions and operator configurations. Each figure underlines the distinct dynamics induced by different quantum gates in a quantum walk, offering insights into how initial conditions and coin choices critically affect the evolution of quantum states across a discrete lattice.

By adjusting the gate operator, rotation angle, and initial states, we can observe sig-

doi:10.6342/NTU202401141

nificant variations in the behavior of the quantum walk. This adjustment highlights the versatility of quantum walks in modeling complex systems. These visualizations serve as a compelling demonstration of the versatility and depth of QWs in exploring quantum mechanical phenomena and their potential applications in various quantum computing and simulation tasks.



(a) (b) (c)

Figure 1.5: Visualization of the distributions of probabilities across different positions using the DTQW with different coin operators.: (a) $\Psi_0 = |\downarrow\rangle \otimes |x=0\rangle$, (b) $\Psi_0 = \frac{1}{\sqrt{2}}(|\downarrow\rangle + i|\uparrow\rangle) \otimes |x=0\rangle$, (c) $\Psi_0 = |\uparrow\rangle \otimes |x=0\rangle$.

QWs have profound implications in various fields, including algorithm development, cryptography, and complex system simulations, making them a pivotal concept in theoretical and applied quantum computing[31]. The ongoing development and study of QWs continue to be an affluent area of research, significantly impacting the theoretical underpinnings and practical applications of quantum computing. The field has expanded to include various adaptations and innovations, such as the split-step quantum walk, which have broadened the scope of problems addressable by QWs [32]. Each step in the evolution of QWs advances our understanding of quantum computational principles and paves the way for novel quantum technologies.

1.4 Quantum Financial Simulation

The inherent uncertainty in financial markets poses significant challenges in predicting investor decision-making processes and understanding the impact of macroeconomic

doi:10.6342/NTU202401141

conditions and structural dynamics on asset valuation. This uncertainty is similar to the principles of quantum theory, highlighting the potential of quantum computing to revolutionize the financial sector. Quantum algorithms have the potential to effectively tackle a wide range of financial problems. Our aim is to create quantum states that capture the inherent uncertainties in financial markets, allowing these states to be processed by quantum computers. Moreover, we aim to develop quantum algorithms that can effectively simulate the dynamic characteristics of financial systems, thus improving our capacity to analyze and predict market behaviors. QWs, with their ability to explore multiple possibilities simultaneously due to superposition, entanglement and interference present an advanced method for simulating the intricacies of financial markets. These quantum analogs to classical random walks can potentially encapsulate the market's multifaceted nature more precisely by simulating various market conditions in parallel. Employing QWs in financial models allows for a thorough analysis of potential market scenarios and outcomes. In such simulations, the quantum walk initializes a superposition of quantum states, each mapping to a possible financial scenario. This approach effectively mirrors the probabilistic distribution of market outcomes. It enhances the preparation of quantum states —a foundational step in quantum computing that enables simultaneous processing of multiple information strands. The employment of specific quantum algorithms promises substantial computational speed over traditional methods.

In traditional finance, the classical random walk theory is a a widely accepted model. This popular theory, proposed by Louis Bachelier[33], suggests that the path of stock prices is fundamentally unpredictable. This implies that historical stock prices are insufficient to accurately forecast future prices. The Efficient Market Hypothesis [34] extends this concept, asserting that a stock's present value fully reflects all relevant information,

doi:10.6342/NTU202401141

and only unexpected occurrences can cause price fluctuations. Stock prices are established through the uninterrupted interplay of buying and selling transactions conducted by all market participants[35]. This dynamic process reflects the aggregate sentiments, beliefs, and actions of these market participants[36]. As a result, the random walk theory of stock prices does not imply that prices are erratic. Rather, they develop based on the cumulative choices of market participants, frequently in reaction to fresh information [37].

Quantum mechanics and finance both embody inherent uncertainty, making the application of quantum principles to financial market simulations a compelling area of research. The spectrum of quantum computing's applications within finance is broad and diverse, with cutting-edge research focused on refining algorithms for tasks such as amplitude estimation and Monte Carlo simulations, which are pivotal for pricing financial derivatives. The work of Egger et al. [38] and Herman et al. [39] surveys the landscape, providing a holistic view of the intersections between quantum computing and finance.

Brassard et al.'s algorithm for amplitude estimation [40], founded upon Grover's quantum search technique, enhances the probability of pinpointing preferred outcomes, presenting a breakthrough in outcome prediction within the financial sector. Rebentrost et al. [41] put forward a quantum algorithm that utilizes the fundamental concepts of superposition and quantum circuits for the Monte Carlo pricing of financial derivatives.

Moreover, the method outlined by Stamatopoulos et al. [42] utilizes quantum amplitude estimation to achieve a quadratic speedup over conventional Monte Carlo methods for option pricing, indicating a quantum leap in the financial modeling realm. Additional studies by Zoufal et al. [43], Stamatopoulos et al. [44], Dong An et al. [45], and Chakrabarti et al. [46] delve into various facets of quantum computational finance. These range from

doi:10.6342/NTU202401141

enhancing option pricing models to capitalizing on the quantum advantage for market risk assessment and resolving stochastic differential equations, each contributing invaluable insights into the transformative potential of quantum algorithms in finance.

By harnessing quantum mechanics principles, we propose an innovative framework for analyzing financial markets. This approach allows us to encode various potential financial scenarios within a quantum structure, yielding a sophisticated grasp of market behavior.

We draw an analogy between financial market outcomes and quantum states through the principle of superposition, as represented by Equation 1.1:

$$|\psi\rangle = \sum_i \sqrt{p_i}\, |i\rangle \tag{1.1}$$

where, $|\psi\rangle$ is the state vector representing a superposition of all possible market states, $|i\rangle$ symbolizes a specific market outcome, and p_i is the probability amplitude for that outcome. Such a representation captures the diversity of potential future market states in a single quantum state, facilitating using quantum probability to make financial forecasts.

Quantum state preparation is pivotal in utilizing quantum circuits across various applications, including simulating complex distributions of probabilities. This process hinges on practical algorithms capable of managing the randomness inherent in quantum systems, which mirrors the behavior of idealized Haar-random distributions, a key concept in realizing versatile quantum computations [47, 48]. Significant advancements in quantum state preparation have been demonstrated by Sanders et al., who introduced a black-

doi:10.6342/NTU202401141

box quantum state preparation method that operates without conventional arithmetic, enhancing efficiency [49]. Further, Rocchetto et al. have explored the application of variational autoencoders in encoding probability distributions, which helps prepare quantum states and assess the performance of quantum circuits [50]. Additionally, Dasgupta and Paine have developed methods for loading symmetric and asymmetric probability distributions into quantum circuits, enhancing the scope of quantum simulations [51]. Similarly, Choi et al. have shown the feasibility of preparing random states and benchmarking them against many-body quantum chaos, underscoring the randomness of these systems [52]. These methodologies collectively advance the field of quantum computing, providing robust frameworks for preparing quantum states and executing complex quantum algorithms efficiently.

We apply quantum evolution principles akin to those governing quantum systems to model the dynamic and evolving nature of financial markets. This is encapsulated in Equation 1.2:

$$U(t)\,|\psi(0)\rangle = |\psi(t)\rangle, \tag{1.2}$$

In this equation, $U(t)$ is a time-dependent unitary operator that evolves the initial quantum state $|\psi(0)\rangle$ to a new state $|\psi(t)\rangle$ at a later time t, analogous to the evolution of market conditions over time. By employing quantum mechanics to model financial markets, we introduce an innovative approach that uses multiple walkers represents diverse investors and underlines its potential for modeling and simulating dynamic financial systems. This novel perspective is poised to augment our understanding of market dynamics significantly and could redefine how financial predictions are made.

doi:10.6342/NTU202401141

Grover and Rudolph's innovative approach utilizes ancilla registers for controlled rotations to generate specific distributions of probabilities, marking a significant step in quantum state preparation [53]. This method simplifies the generation of superpositions of quantum states, which is crucial for the effective use of quantum circuits:

$$\sqrt{p_i}\,|i\rangle \rightarrow \sqrt{p_i}\,|i\rangle \otimes (\cos\theta_i\,|0\rangle + \sin\theta_i|1\rangle). \tag{1.3}$$

This approach allows for the creation of specific distributions of probabilities using auxiliary qubits. It represents a significant advancement in the field and is central to our innovative approach to financial simulation and preparation.

The distributions of probabilities shown in position space from DTQW [27, 28, 31]do not mirror everyday probability distributions, shown in Fig.(1.4) and (1.5), such as those observed in financial markets. These QWs utilize various coin operators—like Z, X, and H gates—to demonstrate how different initial states influence the probability distribution of positions, thereby offering a quantum-based perspective to understand the complex dynamics of financial markets. Such simulations provide valuable insights into the underlying mechanisms of market sentiment and its impact on stock prices, reflecting how quantum technologies can offer novel approaches to modeling economic behaviors.

In the financial world, market prices are primarily influenced by the interactions of buyers and sellers, where sentiment plays a crucial role. Sentiment-driven buying and selling is a significant determinant of stock price fluctuations, shaping market dynamics in short-term financial settings [54, 55]. Investor sentiment, broadly categorized into optimism and pessimism, drives market behaviors significantly. Optimistic investors tend to invest proactively, pushing stock prices upwards, whereas pessimistic investors are

doi:10.6342/NTU202401141

likely to sell off their stocks, leading to a decline in stock prices. This interaction of market forces can be likened to the controlled dynamics seen in split-step quantum walks (SSQW)[56, 57], which have been proposed as a model for simulating financial markets.

In this research, we present an innovative method that builds upon Eq. 1.2, expanding the single-split-step quantum walk (SSQW) into a multi-split-step quantum walk (multi-SSQW). This technique is employed in preparing quantum states and simulating financial systems. The multi-SSQW model integrates an ancilla qubit, inspired by the groundbreaking methodology introduced by Grover and Rudolph [53].This approach utilizes the ancilla qubit in the coin space to control the position space, allowing for accurate depiction of desired quantum states. The structure of the paper is organized as follows:

Initially, we revisit the theoretical foundations of QWs, emphasizing their significance in simulating financial pricing mechanisms and quantum state preparation. Subsequently, we develop the multi-SSQW methodology and apply it to various test scenarios, including simulations of daily returns distribution of financial assets, binomial distributions, and log-normal using the quantum simulator available through IBM Quantum Platform. Finally, we illustrate how the multi-SSQW approach can be leveraged to achieve quantum advantages in pricing financial derivatives.

I focus on developing and applying quantum algorithms for complex financial simulations, leveraging the principles of quantum mechanics to achieve this goal.

doi:10.6342/NTU202401141

Chapter 2 Methodology

2.1 Architecture of Multi-Split-Step Quantum Walk (Multi-SSQW)

The multi-SSQW represents an expansion of conventional QWs, extending its capabilities and potential applications. The SSQW[56, 57] is an advanced variation of the discrete-time quantum walk (DTQW) that adds complexity by dividing the quantum evolution into two distinct steps, allowing for intricate control over the system dynamics. This dual-step process is described by the evolution operator $\hat{W}$, which is structured as follows:

$$\hat{W} = \hat{S}_{-}\hat{C}_{\theta_2}\hat{S}_{+}\hat{C}_{\theta_1}, \tag{2.1}$$

where, $\hat{C}_{\theta_k}$ represents the universal coin operator, and $\hat{S}_{\pm}$ are the conditional shift operators defined by:

$$\begin{aligned}\hat{S}_{+} &= \sum_x [|\uparrow\rangle\langle\uparrow| \otimes |x+1\rangle\langle x| + |\downarrow\rangle\langle\downarrow| \otimes |x\rangle\langle x|],\\ \hat{S}_{-} &= \sum_x [|\uparrow\rangle\langle\uparrow| \otimes |x\rangle\langle x| + |\downarrow\rangle\langle\downarrow| \otimes |x-1\rangle\langle x|].\end{aligned} \tag{2.2}$$

The coin operation is split into two components, $C_{\theta1}$ and $C_{\theta2}$, which allows for greater

doi:10.6342/NTU202401141

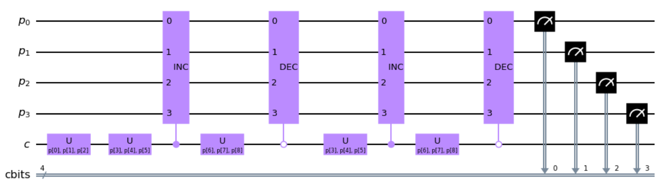
control over the particle's path. The conditional shift operators $\hat{S}_+$ and $\hat{S}_-$ respectively direct the walker to either move to the right ($|\uparrow\rangle$) or stay in place ($|\downarrow\rangle$), and to either move to the left ($|\downarrow\rangle$) or stay in place ($|\uparrow\rangle$).

(a) (b)

Figure 2.1: Illustration of the SSQW and DTQW Transition Mechanisms.

The conceptual diagrams in Fig. 2.1 contrast DTQW and SSQW, essential components in quantum computing. Figure2.1.a illustrates the SSQW, where the coin operation is subdivided into $\hat{C}_{\theta_1}$ and $\hat{C}_{\theta_2}$, enhancing control over the particle's trajectory, thus facilitating a more detailed simulation of quantum systems. These configurations underscore the adaptability of QWs in simulating complex quantum phenomena and contributing to quantum computational advancements. Figure2.1.b depicts the DTQW, where a particle on a line moves to adjacent positions at discrete time intervals, directed by the quantum coin operation C that determines its movement.

Figure 2.2 illustrates the quantum circuit for the SSQW. The operations applied to the qubits include unitary operations U and controlled increment (INC) and decrement (DEC) operators. These gates manipulate the qubits' states, facilitating the quantum walk's progression. This quantum circuit represents the iterative and controlled evolution of quantum states in an SSQW.



(a)

Figure 2.2: Quantum circuit illustrating a SSQW

doi:10.6342/NTU202401141

In the application of the SSQW to investor behavior, the two sub-steps are used as quantum analogs to represent investor sentiment. In the first sub-step, the $\hat{C}_{\theta_1}$ influences the coin state that controls the increment operator and the increment operator conditionally shift to the right , much like pushing the price up. The operator $\hat{C}_{\theta_1}$ is designed to reflect the investor's optimism. Conversely, in the second sub-step, the $\hat{C}_{\theta_2}$ influences the coin state that controls the decrement operator, and the decrement operator conditionally shifts to the left, much like pushing the price down. $\hat{C}_{\theta_2}$ captures investor' s anxiety. It is much like investor behavior. They may make a decision based on their sentiments. They might feel great and decide to buy, pushing the price up, or might hold off, maintaining the current price. Conversely, they may feel anxious and decide to sell, causing the price to drop or hold off. This decision-making process is effectively modeled with SSQW, where the each sub-step represents investor sentiments that influence the probability amplitudes for different market conditions.

This approach underscores the role of investor sentiment as a fundamental component of financial market analysis, akin to the principle of superposition in quantum mechanics. With diverse investor sentiments and perspectives simultaneously influencing the market, they shape trading behaviors and mold the market's overall perception of risk and valuation.

The complexity of the financial market is shaped by diverse investor profiles, each with distinct attitudes and investment strategies. To effectively model this diversity, we have developed the multi-SSQW approach. This quantum model simulates varying investor sentiments, enabling nuanced predictions of short-term financial price distributions. The multi-SSQW setup, depicted in Figure (2.3), implements a dynamic system that captures the intricate behaviors of market participants.

doi:10.6342/NTU202401141

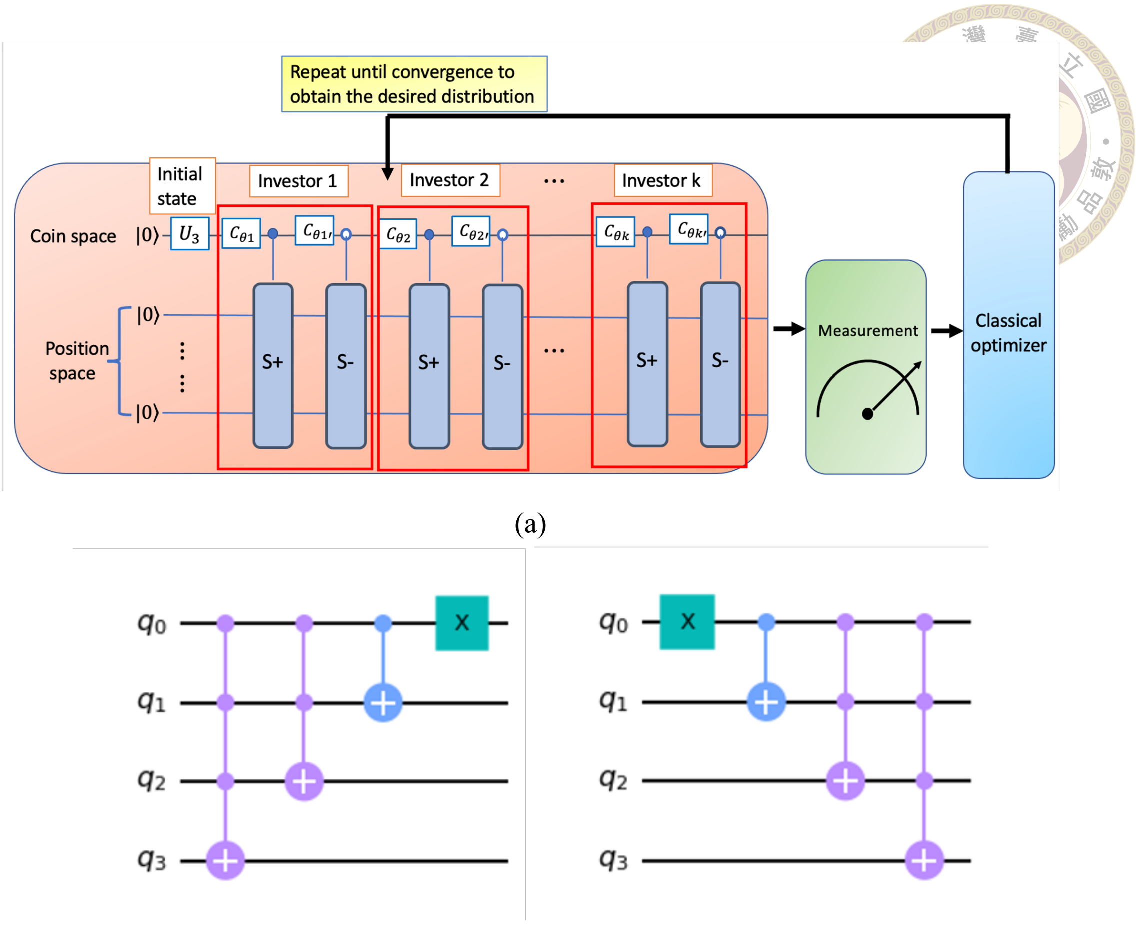
The U3 gate initializes the quantum state, representing the aggregate market sentiment. This state is manipulated by subsequent unitary operations (two U3 gates per investor), which encode the biases of individual investors towards specific financial assets. The evolution operator, $\hat{W}$, is segmented into actions: first applying $\hat{C}_{\theta_1}$ to simulate positive sentiment (incrementing with $\hat{S}_+$), followed by $\hat{C}_{\theta_2}$ for negative sentiment (decrementing with $\hat{S}_-$). These operations, together with the conditional shift operators $\hat{S}_\pm$, control the quantum walker's trajectory, reflecting investor sentiment dynamics within the financial market.

Figure 2.3introduces a novel methodology for analyzing the quantum dynamics of financial markets. The process starts with initializing the quantum state, followed by the application of the SSQW with coin operators $C_{\theta 1}, C_{\theta 2}, \ldots, C_{\theta k}$ for each investor. The quantum state evolves through split-step operations (S+ and S-) representing market dynamics. Measurement of the quantum state provides data for the classical optimizer, which analyzes the distribution and iterates the process until convergence to the desired distribution is achieved. This approach demonstrates the practical application of quantum mechanics to financial market analysis, leveraging the inherent uncertainties to replicate real-world stock market distributions more accurately.

2.2 Solution Architecture

The multi-SSQW framework integrates a dual-domain computational strategy that combines a parameterized quantum circuit (PQC) with a classical optimizer. The PQC, consisting of n+1 qubits—where one qubit is allocated for the coin space and the rest for position space—is utilized to model and simulate the distribution of short-term financial

doi:10.6342/NTU202401141



(a)

(b) (c)

Figure 2.3: The multi-SSQW Setup: (a)Schematic of the multi-SSQW setup. (b) Quantum circuit for $\hat{S}_{+}$, which manages the incrementation of the quantum state in position space. (c) Quantum circuit for $\hat{S}_{-}$, which manages the decrementation of the quantum state.

prices. To align these simulations with empirical data, a classical optimizer utilizing the Constrained Optimization By Linear Approximations (COBYLA) algorithm fine-tunes the parameters to minimize the discrepancy between the simulated and target distributions. This optimization process uses mean-square error (MSE) and Kullback-Leibler (KL) divergence as metrics to fine-tune the fit to the desired distribution. This method facilitates financial simulations and enables the effective encoding of probability data into quantum states, merging traditional financial models with quantum computational approaches.

The coin space in our multi-SSQW setup, which orchestrates the movement of the quantum walker across the position space, operates similarly to an ancilla qubit undergoing controlled rotations as described in Eq.(1.3). The objective is to optimize the coin

doi:10.6342/NTU202401141

parameters within our multi-SSQW to refine the probability distribution across the position space. This is achieved using parameterized quantum circuits (PQC). The detailed steps involved are outlined below:

- Initiate with a target dataset $p = \{p_0, \ldots, p_{2^N-1}\}$ sampled from a distribution representing short-term financial prices.
- Implement the multi-SSQW with an auxiliary qubit for the coin space and N qubits for the position space to represent 2^N potential outcomes.
- Apply quantum operators $\hat{W}_1, \hat{W}_2, \ldots, \hat{W}_k$ on the circuit and repeat for t steps to evolve the system.
- Measure the amplitude probabilities of the position space and calculate the resulting distribution.
- Employ the classical optimizer to adjust the coin parameters using MSE and KL divergence as measures to assess and minimize the deviation from the targeted distribution.
- Iterate this process n times until the system's output converges to the desired distribution.

This approach demonstrates the application of a quantum computational framework to simulate and analyze financial market dynamics. This methodology exemplifies how quantum technology can enhance traditional financial models by providing a new layer of depth and accuracy in simulating market behaviors.

doi:10.6342/NTU202401141

Chapter 3 Results

3.1 Performance Analysis of Daily Return Distributions for Stocks

In this study, we have constructed a multi-investor simulation framework termed the multi-SSQW and conducted a detailed simulation to evaluate the efficacy of the multi-SSQW framework in modeling the daily return distributions of various financial assets. These distributions, which provide a statistical analysis of the day-to-day percentage changes in value for assets like stocks or commodities, are not just theoretical concepts. They are crucial for investors, enabling them to assess the risks and expected returns of their investments. Typically visualized as histograms, the daily return distributions plot the frequency of return values against their respective percentages on the x-axis, making them a practical and applicable tool in financial analysis.

To generate these distributions, we initially collected daily stock data from Yahoo Finance for the period spanning January to March 2022. We classified the daily returns into 16 separate categories according to return percentages. These categories were subsequently depicted as a frequency distribution bar chart, illustrating the return variability. We employed the multi-SSQW approach to simulate these financial scenarios, to obtain a

doi:10.6342/NTU202401141

financial probability distribution of market behavior. Our simulation study encompassed daily return distributions over a quarter for various stocks or indices, conducting 100 trial runs for each scenario. We explored the impact of varying the number of walkers (denoted as **num**) from 1 to 10 while maintaining a fixed step size (**step**=1). This setup enabled us to perform statistical analysis of errors, observe convergence patterns, and measure the computational time required for each scenario.

Furthermore, we incorporated random parameters to conduct optimization simulations. These simulations were designed to produce outcome data and assess error convergence, thus validating the efficacy of our multi-SSQW methodology. This study offers an in-depth examination of the daily return distributions for a diverse range of financial assets, highlighting the capacity of quantum computation to improve financial modeling.

First, we employed the multi-SSQW method to simulate the daily return distributions of significant stock indices, specifically the S&P 500 and Nasdaq 100. The S&P 500 serves as a fundamental indicator of U.S. equities, encompassing the performance of 500 large companies across diverse sectors. Similarly, the Nasdaq 100 includes 100 of the largest non-financial companies listed on the Nasdaq stock exchange and is heavily weighted towards technology. By applying the multi-SSQW approach to these indices, we aim to capture the complex dynamics and variations in daily returns that characterize these key stock market segments.

In Figure 3.1, we present the results of simulating the daily return distribution for the S&P 500 index. The S&P 500 is a crucial gauge of the stock performance of 500 leading companies listed on U.S. exchanges and spans multiple sectors. These companies, noted for their stable operations, solid corporate governance, precise strategic planning, and

doi:10.6342/NTU202401141

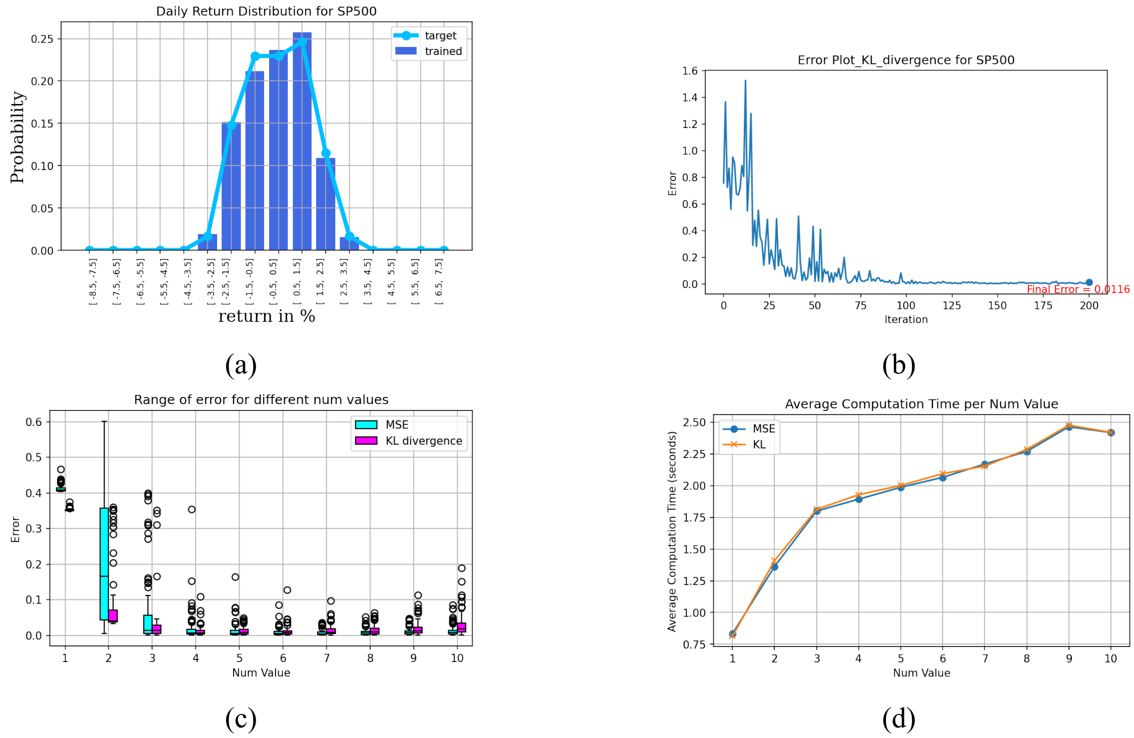


Figure 3.1: Analysis of S&P 500 using the multi-SSQW method across four key dimensions: (a) **Daily Returns Distribution**: The graph presents the simulated distribution of daily returns for the S&P 500 juxtaposed with actual market data. Utilizing four quantum walkers, our model effectively mirrored real-world market fluctuations, underscoring the multi-SSQW's capability in capturing intricate market dynamics. (b) **Error Progression**: This is depicted using the KL divergence metric, showcasing a significant initial reduction in error which eventually stabilizes at a minimal value. This demonstrates the method's efficacy in enhancing the precision of financial market forecasts through iterative refinement. (c) **Error Variability**: A boxplot shows error rates across varying numbers of quantum walkers (ranging from 1 to 10), interpreting the impact of walker quantity on the simulation's accuracy. The results indicate that a higher number of walkers generally increases the fidelity of the simulation, up to an optimal point. (d) **Computational Time Analysis**: This graph outlines the mean processing time required for each simulation, emphasizing the method's scalability and operational efficiency.

doi:10.6342/NTU202401141

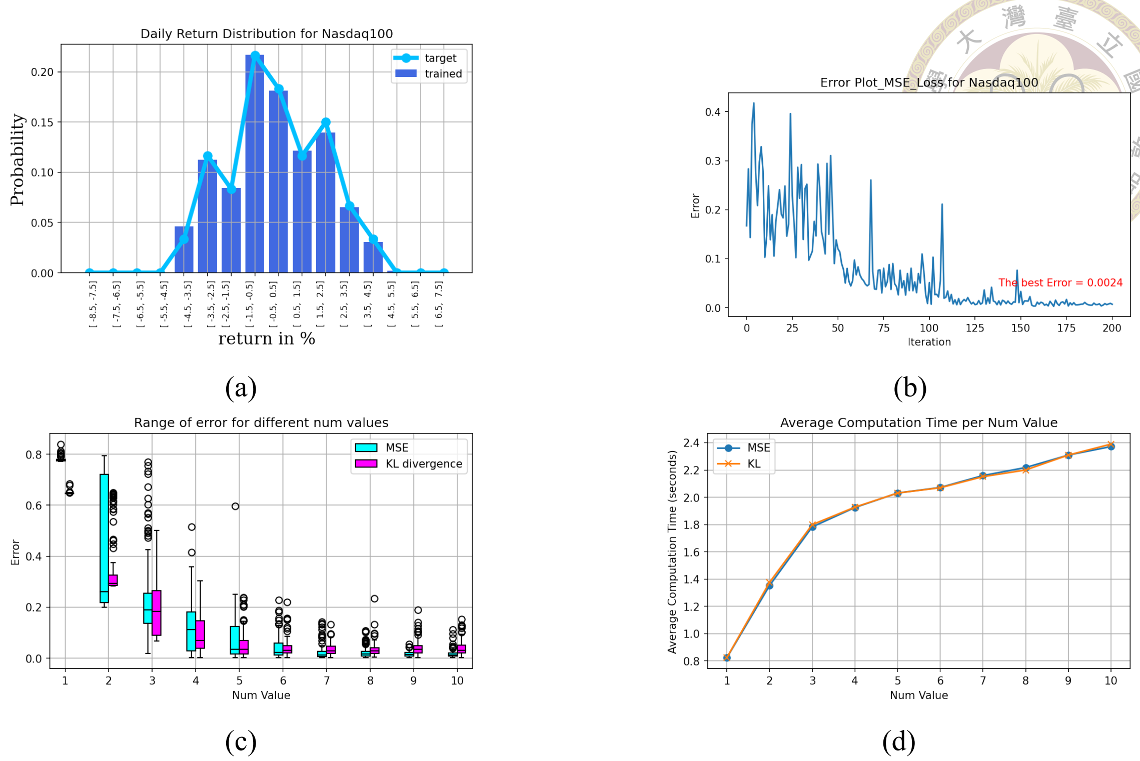


Figure 3.2: Analysis of Nasdaq100 using the multi-SSQW methods: (a) the daily return distribution for the Nasdaq100, which we simulated and compared against actual market data. (b) illustrates the error progression over iterations using MSE as a metric. (c) provides a boxplot of errors across different numbers of quantum walkers (from 1 to 10).(d) charts the mean processing time required per simulation.

robust performance, naturally attract long-term investment. The homogeneous nature of this investor base typically requires fewer iterations for quantum simulations to converge accurately to realistic market outcomes, reflecting the predictability and consistency in their stock performance.

In Figure 3.2, our quantum simulation captures this dynamic in Nasdaq100, suggesting more sentiment-driven investors in tech. The simulated return distributions' fluctuations underscore investor behavior's impact, particularly in sectors like technology, where various sentiments can heavily influence stock prices.

Figure 3.3 , 3.5 and 3.4 detail the outcomes of implementing the multi-SSQW approach on Johnson & Johnson (JNJ), JPMorgan Chase & Co. (JPM), and P&G(PG) respectively, showcasing the model's capability in simulating daily return distributions

doi:10.6342/NTU202401141

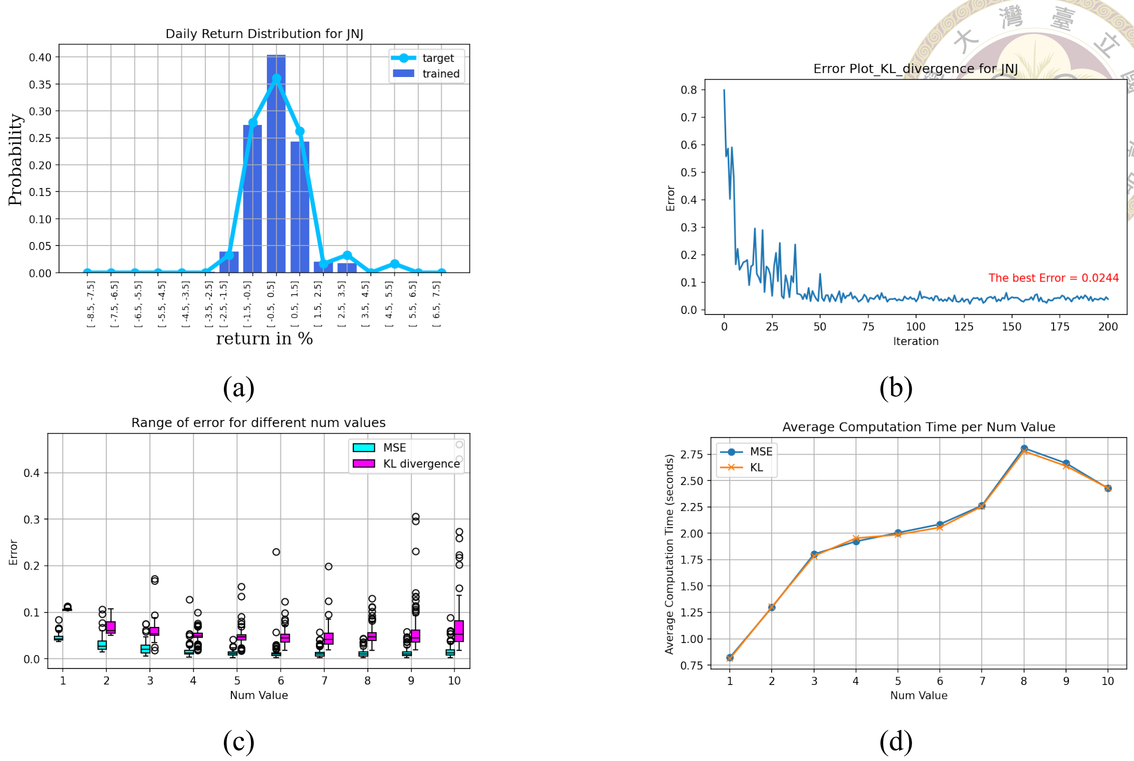
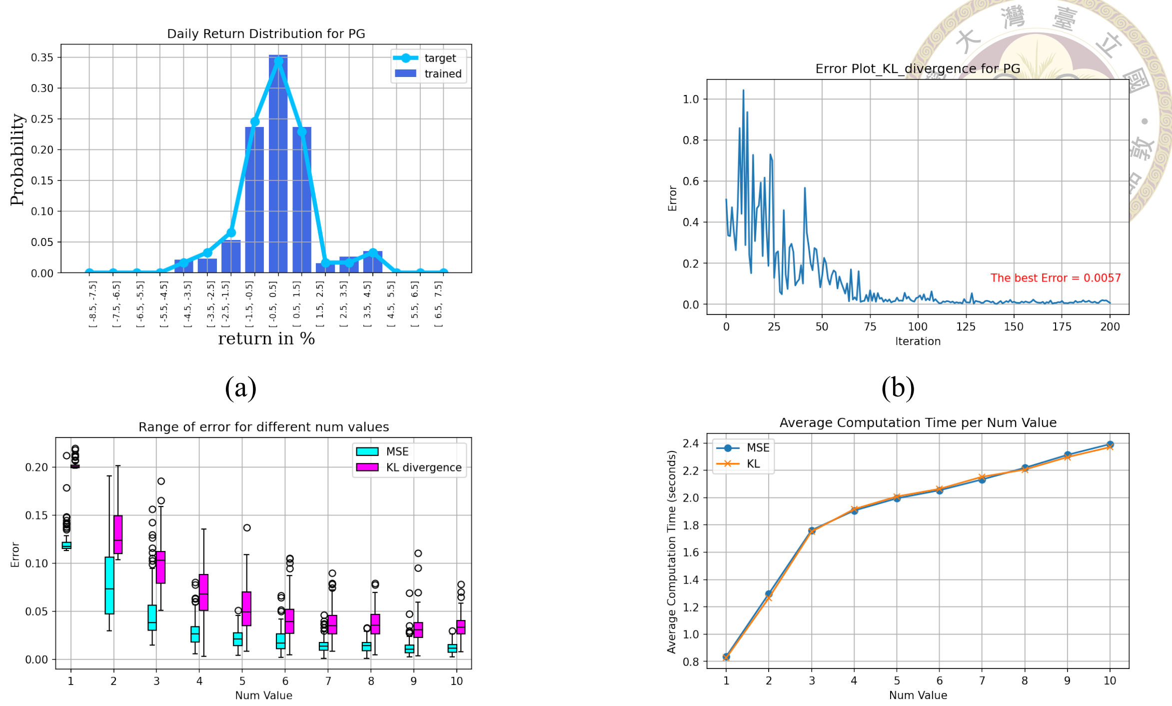


Figure 3.3: Analysis of Johnson & Johnson (JNJ) using the multi-SSQW method: (a) depicts the daily return distribution simulated with four quantum walkers, closely mirroring actual market data, (b) shows the progression of the KL divergence error reduction over multiple iterations. (c) provides a boxplot of errors across different numbers of quantum walkers (from 1 to 10). (d) charts the mean processing time required per simulation.

effectively. For J&J, using four quantum walkers, the analysis demonstrated a strong alignment between the simulated and actual daily return distributions over a quarter, highlighting the model's accuracy. Similarly, for JPM and PG, employing five walkers, the simulations reflected a precise mapping of daily returns, confirmed by the error trends and computational efficiency displayed across various metrics.

J&J and P&G, both multinational giants in their respective domains of pharmaceuticals and consumer goods, are often favored by conservative investors for their robust positions within the defensive sector. This sector typically remains stable during economic downturns due to the consistent demand for healthcare and consumer staple products. J&J, with its strong foothold in healthcare, offers diversified product lines and sustained financial performance, attracting long-term investors. Similarly, P&G, known for its wide

doi:10.6342/NTU202401141



(a) (b) (c) (d)

Figure 3.4: Analysis of P&G (PG) using the multi-SSQW method: (a) depicts the daily return distribution simulated with five quantum walkers. (b) shows the progression of the KL divergence error reduction over multiple iterations.(c) provides a boxplot of errors across different numbers of quantum walkers (from 1 to 10).(d) plots the mean processing time required per simulation.

range of household and personal care products, also maintains a stable market presence with its broad consumer base.

Our simulation effectively mirrored this market stability for both companies. The strong alignment of the simulated and actual distributions with just four walkers for J&J and five for P&G indicates a predominance of sentiment-stable investors. This result reflects predictable market behaviors, making J&J and P&G attractive stocks for investors seeking reliable returns. The simulation underscores the resilience and appeal of these companies in the defensive sector, highlighting their suitability for conservative investment strategies.

JPMorgan Chase & Co. (JPM) is a prominent global financial services bank with significant market impact. This banking giant operates in a highly scrutinized regulatory

doi:10.6342/NTU202401141

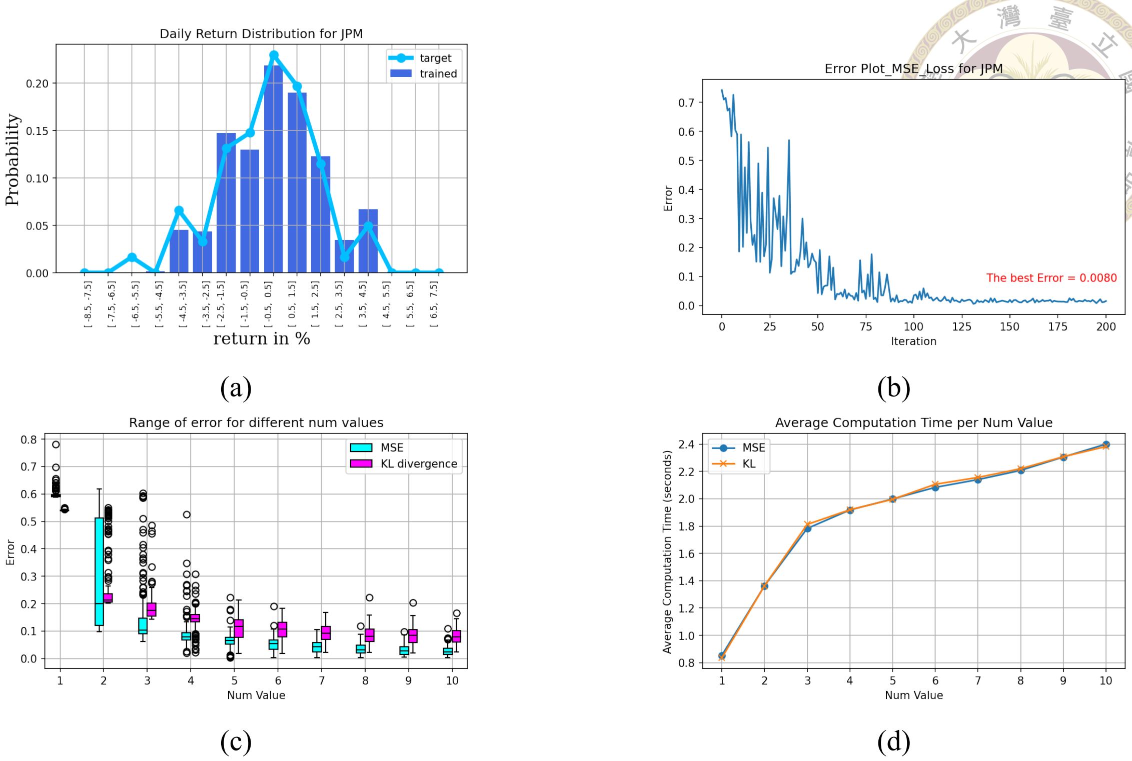


Figure 3.5: Analysis of JPMorgan Chase & Co. (JPM) using the multi-SSQW method: (a) the simulation of the daily return distribution with five walkers, (b) shows the progression of the error reduction over multiple iterations, (c) provides a boxplot of errors across different numbers of quantum walkers (from 1 to 10). (d) plots the mean processing time required per simulation.

environment and is pivotal in the financial markets due to its size, operational scope, and systemic importance. Financial analysts often view JPM as a bellwether for the banking industry and broader economic health because its performance can reflect underlying economic trends and banking sector conditions. From a financial perspective, JPM's stock is typically favored by investors looking for stability and consistent returns, thanks to its diversified revenue streams and strong balance sheet. The firm's involvement in various aspects of banking, from retail to investment services, and its global reach allows it to manage risks more effectively than smaller institutions with more concentrated exposures.

In terms of investment, analysts might recommend JPM as a solid holding due to its ability to leverage its size and resources to navigate financial cycles better than its peers. It often attracts conservative investors who value the bank's ability to pay regular dividends

doi:10.6342/NTU202401141

and its reputation for prudent risk management. Moreover, a component of major stock indices like the S&P 500 adds to its appeal as it reflects steady performance and robust governance.

Stability of Non-Tech Sectors and Investor Demographics: Observing the results of the multi-SSQW model, the return distributions of different stocks or indices have unique characteristics, which are also reflected in the optimal number of walkers required for accurate simulation. We can see that stocks with lower tech components, such as S&P 500, J&J, P&G, and JPMorgan Chase & Co., can accurately simulate probability distributions with fewer walkers. This suggests that the types of investors in these stocks are relatively simple and few, likely comprising long-term stable investors who are typically more risk-averse, seeking steady dividends and long-term growth rather than speculative gains.

J&J, P&G, and JPMorgan Chase & Co. (JPM) are quintessential examples of industry giants in non-tech, respectively. These companies epitomize stability and resilience, attracting a more stable investor base, less susceptible to the rapid shifts often seen in the tech industry. This relative stability is a key factor in why fewer quantum walkers (four for J&J and five for P&G and JPM) are sufficient for effective multi-SSQW simulation.

Therefore, to validate this hypothesis, we will compare it with other major tech companies to verify our assumption.

In the technology sector, the landscape is quite different. Technological innovation drives rapid changes, making the prospects of tech companies volatile and unpredictable. Breakthroughs can suddenly disrupt market dynamics, influencing stock prices significantly. This environment attracts noise investors who invest without substantial informa-

doi:10.6342/NTU202401141

tion and often make decisions based on market sentiments rather than fundamentals. Such behavior can lead to significant price volatility in tech stocks, which are highly susceptible to shifts in investor sentiment.

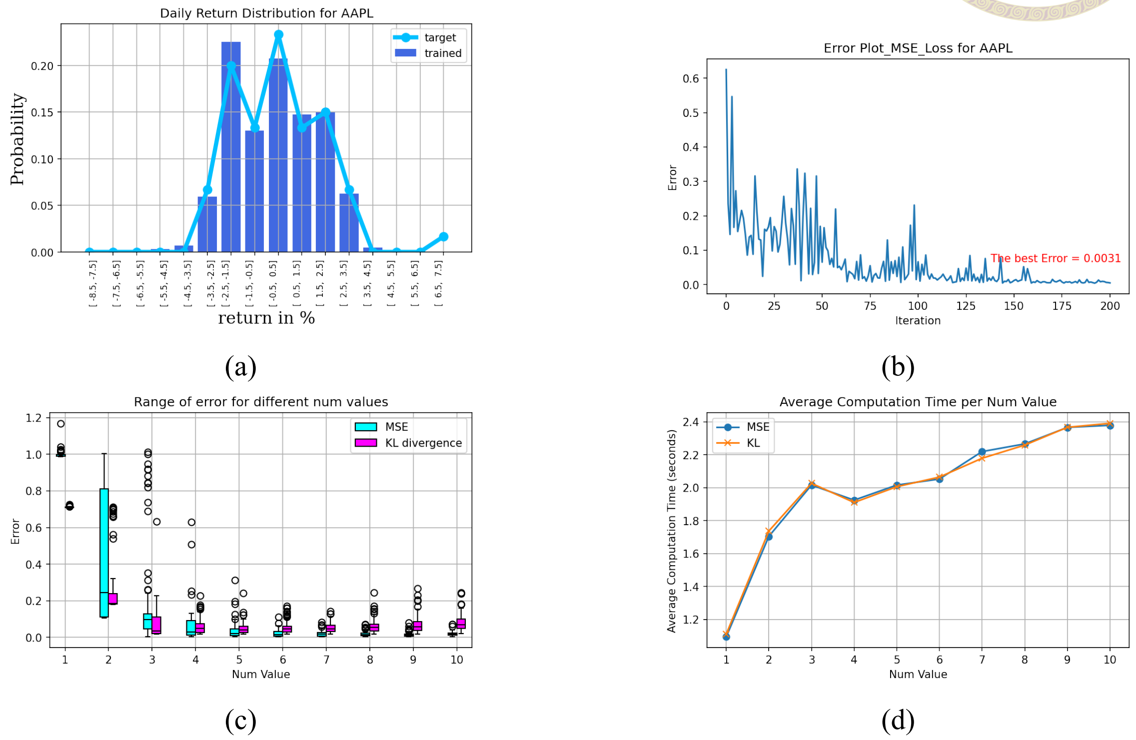


Figure 3.6: Analysis of Apple (AAPL) using the multi-SSQW method: (a) the simulation of the daily return distribution with seven walkers, (b) shows the progression of the error reduction over multiple iterations, (c) provides a boxplot of errors across different numbers of quantum walkers (from 1 to 10). (d) plots the mean processing time required per simulation.

Applying the multi-SSQW approach to tech companies such as Apple, Microsoft, Google, Nvidia, and Tesla to the inherent volatility and rapid innovation within the technology sector demands a more complex simulation model. The daily return distributions of these companies exhibit significant fluctuations influenced by market sentiment, technological breakthroughs, and investor reactions to industry news. As illustrated in the figures for Apple, Microsoft, Google, Nvidia, and Tesla (Fig.3.6 to 3.10), the multi-SSQW method necessitated using seven walkers to represent these companies' dynamics accurately.

doi:10.6342/NTU202401141

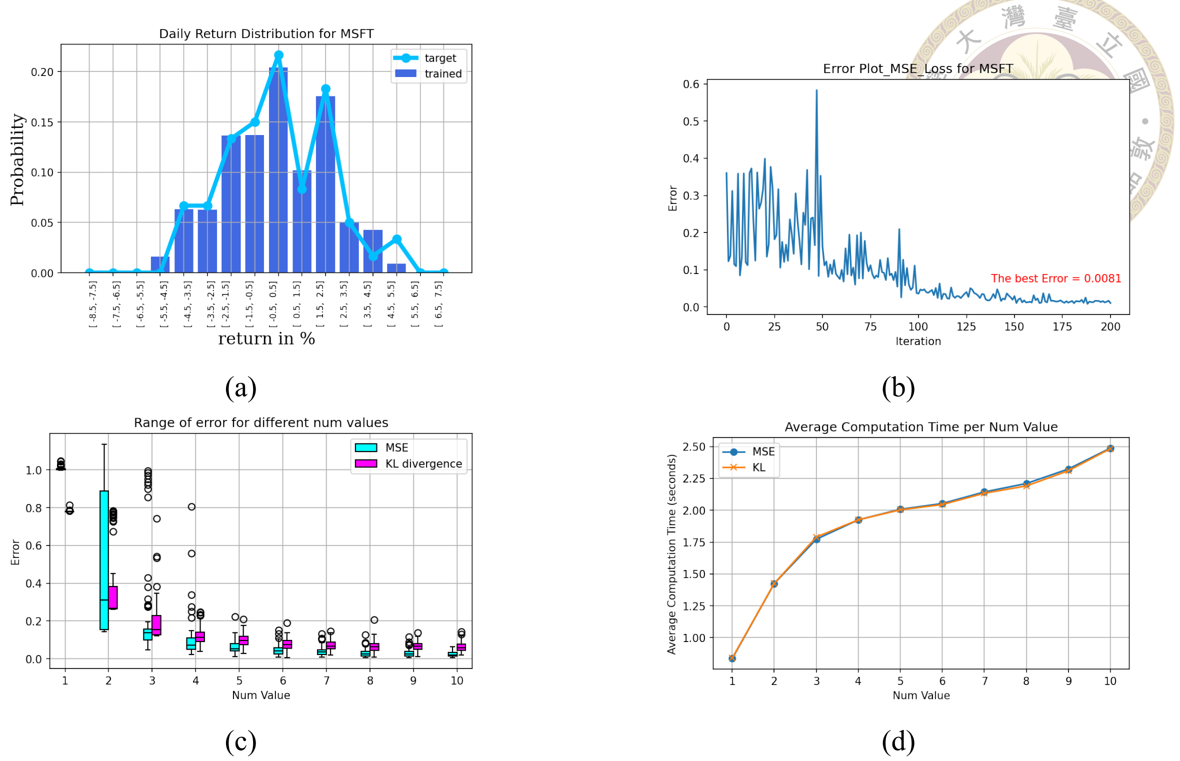


Figure 3.7: Analysis of Microsoft (MSFT) using the multi-SSQW method: (a) the simulation of the daily return distribution with seven walkers, (b) shows the progression of the error reduction over multiple iterations, (c) provides a boxplot of errors across different numbers of quantum walkers (from 1 to 10). (d) plots the mean processing time required per simulation.

This higher number of quantum walkers is necessary to effectively model the erratic movements and the broader range of outcomes that characterize tech stocks. Such companies are susceptible to market news and technological advancements, leading to more pronounced stock price swings than stable sectors. Consequently, a more nuanced approach, as enabled by the multi-SSQW with a higher number of walkers, proves essential for providing a realistic simulation of the daily return distributions in the tech industry. This enhanced model offers a deeper insight into the probabilistic nature of tech stocks, reflecting their complex market behaviors and the more extensive range of investor sentiments that influence them. These insights are crucial for investors and analysts focusing on tech stocks, offering a nuanced understanding of risk and potential strategies to mitigate undue exposure to market volatility driven by erratic investor sentiment.

doi:10.6342/NTU202401141

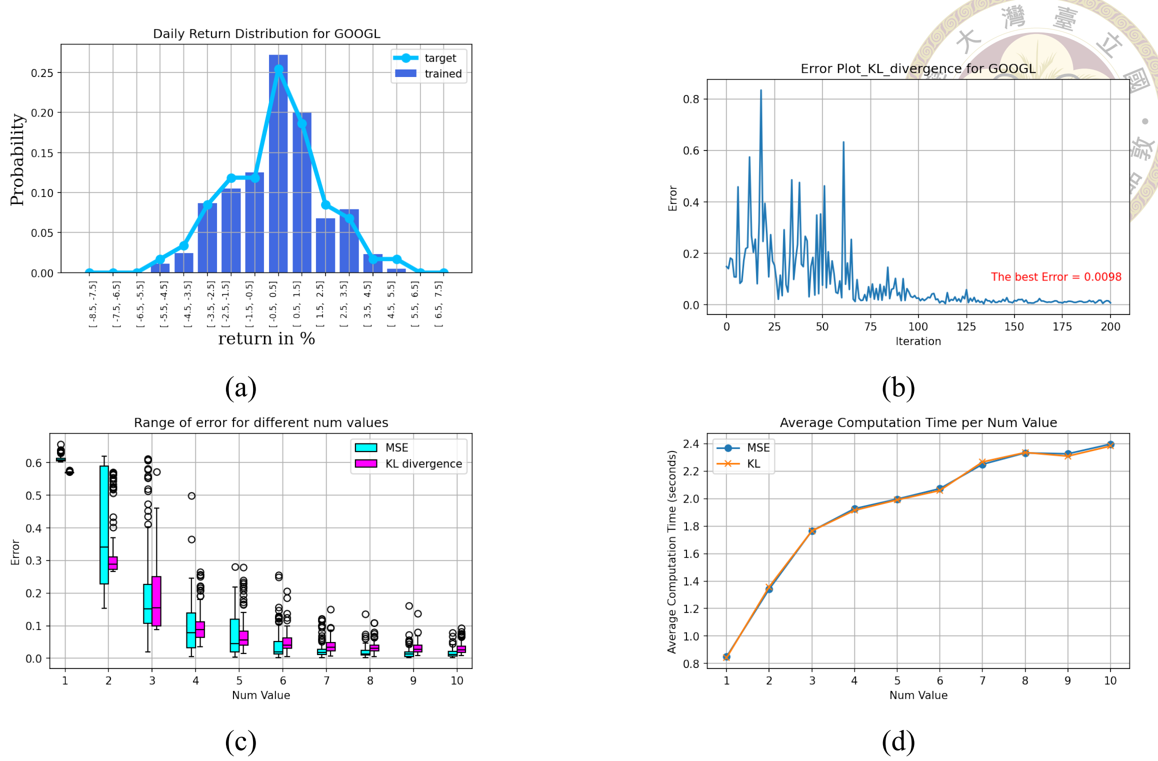
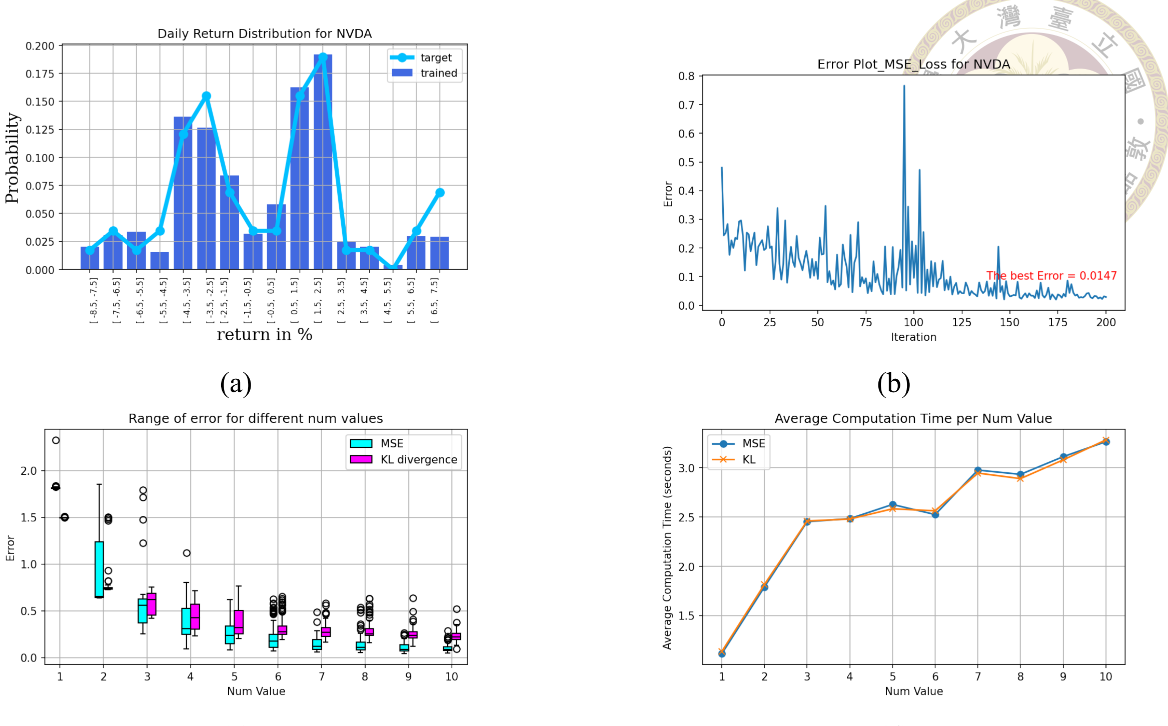


Figure 3.8: Analysis of Google (GOOGL) using the multi-SSQW method: (a) the simulation of the daily return distribution with seven walkers, (b)shows the progression of the error reduction over multiple iterations, (c) provides a boxplot of errors across different numbers of quantum walkers (from 1 to 10). (d) plots the mean processing time required per simulation.

This section illustrates the efficacy of the multi-SSQW as a robust financial simulator. This method excels in capturing the complexities of financial systems and delivering dependable simulations. A key strength of multi-SSQW is its ability to swiftly achieve convergence, establishing it as an invaluable financial analytics and modeling asset. The boxplots generated in our simulations provide insight into the accuracy of multi-SSQW in reflecting financial market behaviors. These visual representations illustrate that by increasing the number of quantum walkers, which symbolize market participants, the precision of the simulations is enhanced, as evidenced by the reduction in MSE and KL divergence scores. These results highlight the multi-SSQW's resilience in capturing the intricate dynamics of financial markets, solidifying its position as a vital analyst instrument. Additionally, the method's dependable convergence verifies its value in generating spe-

doi:10.6342/NTU202401141



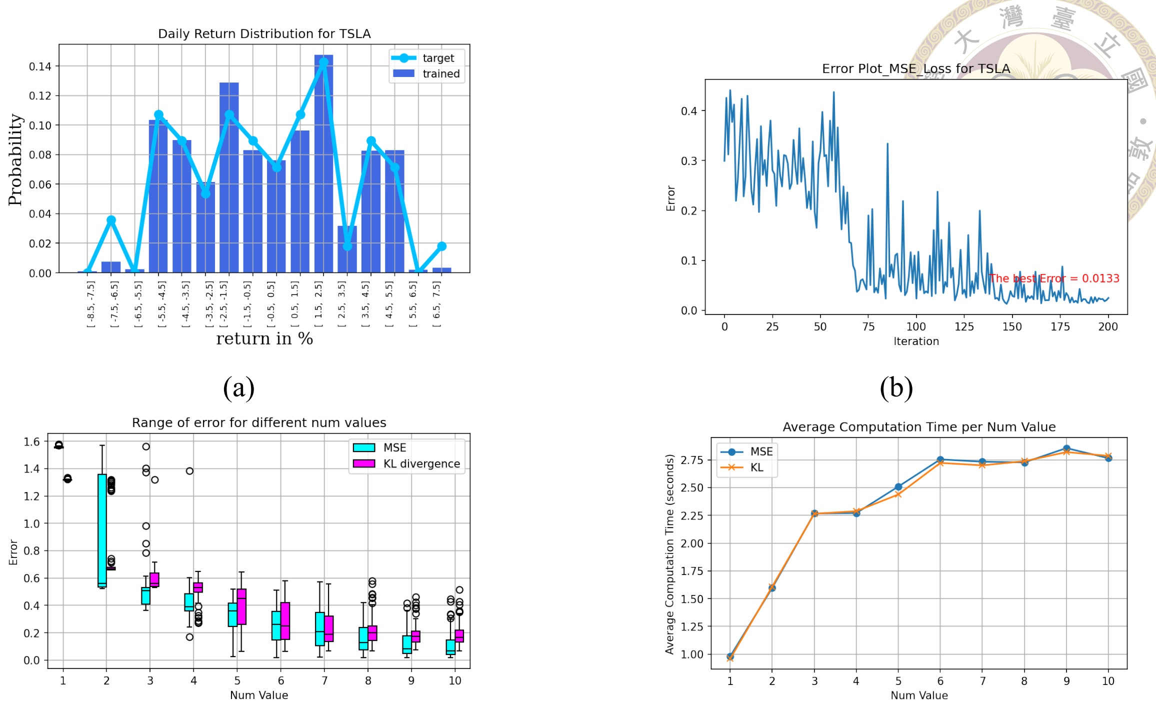
(c) (d)

Figure 3.9: Analysis of Nvidia (NVDA) using the multi-SSQW method: (a) the simulation of the daily return distribution with ten walkers, (b) shows the progression of the error reduction over multiple iterations throughout the simulation, (c) provides a boxplot of errors across different numbers of quantum walkers (from 1 to 10), and (d) plots the mean processing time required per simulation.

cific market simulations, strengthening its application in sophisticated financial analysis and forecasting. Similar to machine learning, determining the ideal number of parameters in the multi-SSQW method is essential for precise simulations, underscoring the significance of fine-tuning to improve model performance.

The concept of the multi-SSQW can be seen as a superposition of multiple investors' sentiments on the value of a particular asset. Each quantum walker in this model represents an individual investor's decision-making process or viewpoint. By superposing these walkers, the model aggregates their diverse opinions to reflect the overall market valuation of the asset. This approach enables a detailed simulation of varied market sentiments and investor behaviors, effectively capturing the complex dynamics of opinions that drive asset prices in financial markets. Adjusting the number of walkers and their parame-

doi:10.6342/NTU202401141



(a) (b) (c) (d)

Figure 3.10: Analysis of TESLA (TSLA) using the multi-SSQW method: (a) the simulation of daily return distributions with ten walkers, (b) shows the progression of the error reduction throughout the simulation, (c) provides a boxplot of errors across different numbers of quantum walkers and (d) plots the mean processing time required per simulation.

ters within the multi-SSQW allows for an adept representation of the intricate interactions within market dynamics, providing valuable forecasts and insights.

The application of the multi-SSQW approach demonstrates its effectiveness in simulating the daily return distributions of both tech and non-tech companies, highlighting the distinct characteristics of these sectors.

- Tech stocks require a higher number of quantum walkers in the multi-SSQW model to account for their volatility and diverse investor sentiments.
- Non-tech stocks can be accurately modeled with fewer walkers due to their stable and predictable investment patterns.
- The multi-SSQW approach provides a nuanced understanding of risk and potential

doi:10.6342/NTU202401141

strategies for both tech and non-tech sectors, offering valuable insights for investors and analysts.

These conclusions underscore the versatility and efficacy of the multi-SSQW approach in simulating financial markets, tailored to the unique dynamics of different sectors.

3.2 Analysis of Binomial Distribution Performance

In this study, we present the efficacy of the multi-SSQW model for simulating binomial distributions, a fundamental concept in probability theory [58, 59]. Binomial distributions are widely employed to characterize the number of successes in a fixed sequence of independent Bernoulli trials, each with an identical probability of success. This includes scenarios like coin tosses, where each toss is an independent event with two possible outcomes: success or failure. Our application of the multi-SSQW model aimed to simulate the probability distribution of such binomial outcomes, manipulating variables such as the coin operator coefficient and the number of quantum walkers and steps to reflect the success probability of each trial.

We present simulations of three different binomial distributions, each with a success probability of $p = 0.3$ across $n = 31, 63$, and 127 trials, respectively. These simulations are depicted in Figures 3.11 to 3.13. Each setup involved two quantum walkers and utilized six parameters adjusted to mirror the success probability, employing varying numbers of control steps to align closely with the characteristics of the theoretical binomial distribution.

Each figure illustrates the multi-SSQW method's capability in effectively replicating

doi:10.6342/NTU202401141

the binomial distribution, detailing aspects such as the distribution of outcomes, the convergence of error rates, and the computational efficiency. Error metrics used include the MSE and the KL divergence, both indicating the fidelity of the simulation relative to the expected theoretical distribution. These results highlight the multi-SSQW model's potential for complex probabilistic simulations, providing a robust tool for statistical analysis and predictive modeling in fields reliant on understanding and predicting discrete outcomes.

The following figures and descriptions elaborate on the methodology and results:

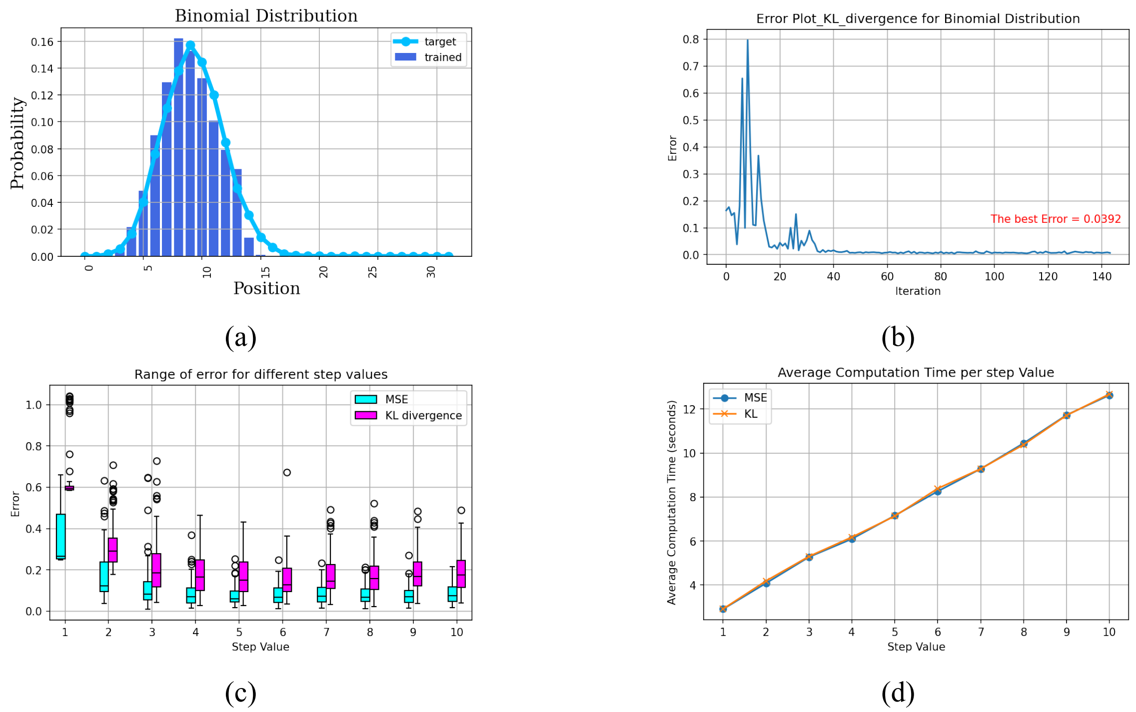


Figure 3.11: This series of figures illustrates the application of the multi-SSQW method to simulate a binomial distribution with $p = 0.3$ and $n = 31$ trials: (a) Shows the simulated distribution overlaid with the target distribution, demonstrating the accuracy of the model. (b) Details the error progression during simulations, highlighting the method's ability to refine its accuracy over iterations. (c) Presents a boxplot of error metrics, including MSE and KL divergence, across different step values, indicating variability and optimization potential. (d) Displays the mean processing time for each simulation step, showcasing the efficiency of the multi-SSQW method.

doi:10.6342/NTU202401141

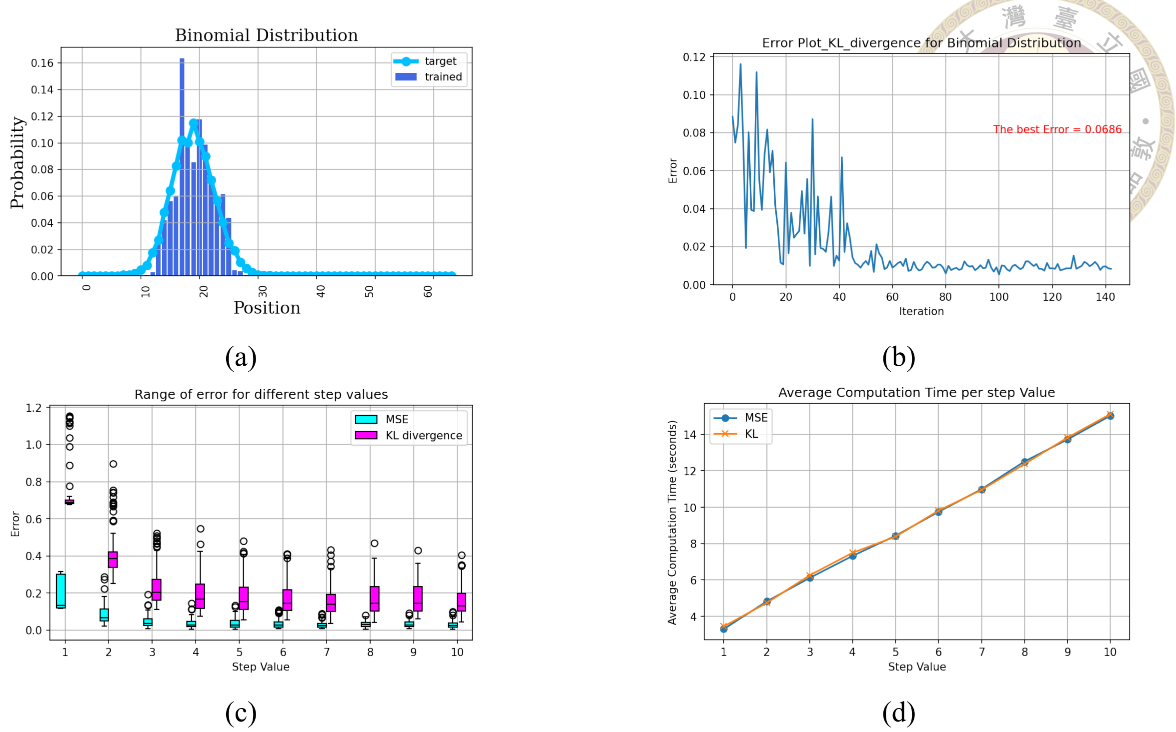


Figure 3.12: This series of figures illustrates the application of the multi-SSQW to simulate a binomial distribution with $p = 0.3$ and $n = 63$ trials: (a) Shows the simulated distribution overlaid with the target distribution. (b) Details the error progression during simulations. (c) Presents a boxplot of error metrics, including MSE and KL divergence, across different step values. (d) Displays the mean processing time for each simulation step.

3.3 Application: Pricing European Call Options

The modeling of financial instruments such as European call options demonstrates the potential of quantum computing to enhance financial analysis and forecasting. Quantum algorithms for amplitude estimation and quantum-based Monte Carlo simulations are promising areas for future research in the pricing of financial derivatives [40–43].

The log-normal distribution characterizes the probability distribution of a random variable for which the logarithm follows a normal distribution. This distribution is right-skewed, featuring heavier tails than a normal distribution, which makes it suitable for modeling financial variables that exhibit significant fluctuations, such as stock prices. It is beneficial for pricing options based on the assumption that the underlying asset's price

doi:10.6342/NTU202401141

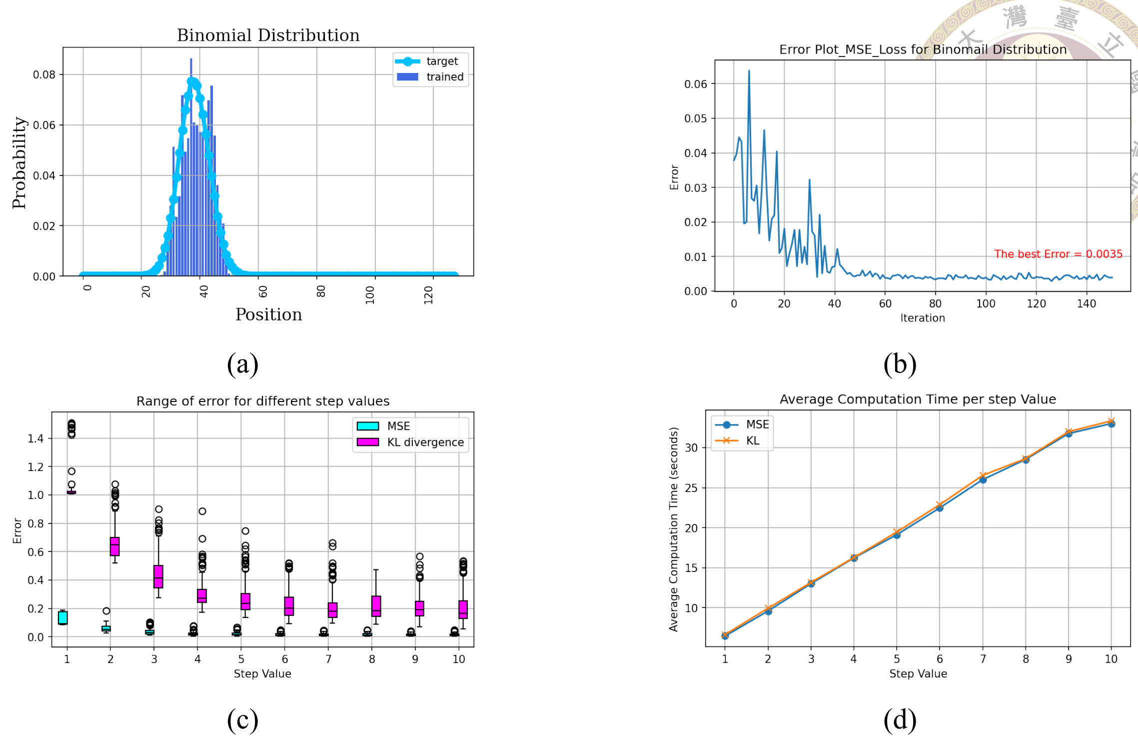


Figure 3.13: This series of figures illustrates the application of the multi-SSQW to simulate a binomial distribution with $p = 0.3$ and $n = 127$ trials: (a) Shows the simulated distribution overlaid with the target distribution. (b) Details the error progression during simulations. (c) Presents a boxplot of error metrics, including MSE and KL divergence, across different step values. (d) Displays the mean processing time for each simulation step.

attaches to a geometric Brownian motion. This suggests that the logarithm of the asset's price is typically distributed, a crucial assumption that forms the foundation of the widely used Black-Scholes (BS) model for option pricing [1].

The BS model [1] computes the theoretical value of options by integrating several key parameters: strike price (K), the price of the underlying asset (S), time to expiration (T), risk-free interest rate (r) and volatility (σ). It assumes that the price follows a geometric Brownian motion, allowing for the derivation of the asset's price distribution at expiration. The steps to derive parameters for the log-normal distribution, essential for using the BS model, include:

1. Collecting historical price data of the underlying asset.

doi:10.6342/NTU202401141

2. Calculating the mean and variance of the logarithm of the asset's prices.

3. Estimating the volatility (σ) and drift (α) using the calculated mean and variance, where:

$$\sigma = vol \cdot \sqrt{T},$$

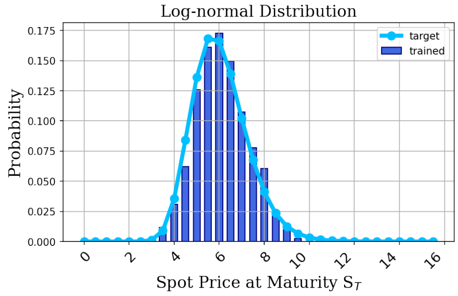
$$\alpha = \ln(S_0) + (\mu - r - \frac{\sigma^2}{2}) \cdot T.$$

4. Utilizing these parameters to determine the expected option payoff at expiration, $E[\max(S_T - K, 0)]$, where S_T is the price at maturity.

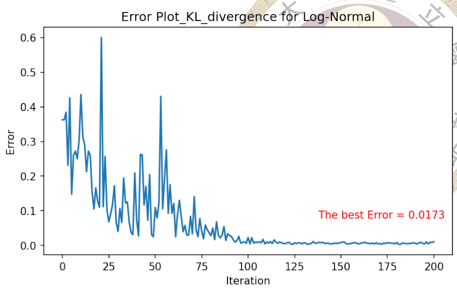
We employ the multi-SSQW scheme to simulate this log-normal distribution, adjusting parameters like the number of quantum walkers (num) and control steps ($step$) to capture the characteristics of the distribution accurately. Figures 3.14 illustrate the performance of the log-normal distribution using the multi-SSQW, showcasing the approximation of the trained probability distribution against the targeted one for parameters $S_0 = 6$, $K = 7$, $vol = 0.4$, $r = 0.04$, and $T = 90$ days. The analysis indicates that an accurate trained probability distribution is crucial for determining a more precise expected payoff, which is 0.1739 using the targeted distribution and 0.1460 with the trained. Although there is a slight discrepancy between the expected payoffs, the values are reasonably close, demonstrating that the multi-SSQW approach can approximate the expected payoff with a high degree of accuracy. This highlights the potential for quantum algorithms to provide reliable financial predictions.

This methodology highlights the practical benefits of incorporating advanced quantum simulation techniques into real-world financial scenarios, effectively narrowing the gap between traditional financial models and the evolving field of quantum technologies.

doi:10.6342/NTU202401141



(a) The log-normal distribution modeled with multi-SSQW.



(b) Progression of error in modeling the distribution.

Figure 3.14: Performance analysis of the log-normal distribution for European call option pricing using parameters $num = 3$ and $step = 4$.

The improved precision of the trained distributions directly correlates with more favorable outcomes, emphasizing the potential for the multi-SSQW approaches to enhance financial analysis and decision-making processes.

doi:10.6342/NTU202401141

Chapter 4 Discussion

Our study outlines the theoretical foundations and practical implementations of the multi-SSQW, demonstrating its effectiveness in quantum financial simulations and quantum state preparation. Leveraging the unique capabilities of quantum computation, the multi-SSQW provides a sophisticated means of modeling complex financial distributions and scenarios, offering valuable new perspectives and tools for financial analysis and decision-making.

Here are the key benefits of using the multi-SSQW in our research:

Flexibility in Modeling: The multi-SSQW exhibits remarkable versatility in modeling intricate systems, especially within the financial domain. It offers the flexibility to modify variables such as the number of quantum walkers and steps and fine-tune parameters in the coin space. This adaptability empowers the multi-SSQW to represent a wide range of complex systems precisely, solidifying its position as a potent tool for financial simulations and other applications.

Stable Convergence: Stability in convergence is a critical feature of multi-SSQW, essential for its role as a robust financial simulator. This stability ensures that QWs consistently reach a steady state, enhancing the accuracy of simulations. For instance, our experiments with binomial and log-normal distributions showed that increasing the num-

doi:10.6342/NTU202401141

ber of walkers could effectively minimize simulation errors.

Efficient Computation: The multi-SSQW algorithm capitalizes on navigating complex quantum state spaces efficiently. It enables rapid exploration and convergence to the desired states or solutions by advancing multiple walkers simultaneously. Quantum superposition and entanglement further boost this process, allowing for parallel processing and significantly faster computations than traditional methods. This efficiency makes the multi-SSQW particularly effective for quickly addressing complex problems.

Additionally, optimization of the multi-SSQW involves careful tuning of the number of walkers, steps, and coin parameters to enhance its operational efficiency. However, this can be challenging because it requires an in-depth understanding of system dynamics and meticulous parameter adjustments.

When employing Variational Quantum Algorithms (VQA), it is essential to consider the interplay between the intricacy of quantum circuits and the optimizer's efficiency. Increasing the depth and complexity of quantum circuits can lead to a broader exploration of the state space, potentially uncovering superior solutions. However, this expansion may also introduce complications in the optimization landscape, which could impede the optimization process. Acknowledging and navigating this trade-off is pivotal for harnessing the full potential of VQA and attaining practical, significant outcomes.

The design of the multi-SSQW circuit is specifically employed to efficiently navigate complex quantum state spaces, aiming for swift exploration and convergence. This approach is ideal for simulating financial markets, where rapid modeling and prediction are essential due to these environments' fast-paced and volatile nature. The advantages of multi-SSQW, including its modeling speed and predictive accuracy, make it a precious

doi:10.6342/NTU202401141

tool for financial simulations.

In the realm of financial analysis, the integration of quantum computing techniques, the multi-SSQW presents groundbreaking potential and is beginning to recognize the substantial benefits this advanced quantum simulation technique can bring to real-world financial scenarios:

1. Implications for Financial Modeling: The capability of the multi-SSQW method to closely approximate traditional financial models suggests that quantum walks can effectively model complex financial systems. This is particularly important for assets with high volatility, where traditional models might struggle to capture rapid changes in market conditions.

2. Enhanced Market Predictability: The multi-SSQW, as a quantum-based financial simulation tool, offers precise and effective probability assessments. This technology enhances the ability to predict daily return distributions, including those for major indices like the S&P 500. With this improved predictability, investors can more accurately assess potential returns and related risks, vital for making informed decisions in volatile market environments.

3. Advanced Risk Assessment: By simulating detailed risk scenarios and the distribution of returns under various conditions, the multi-SSQW aids financial institutions in developing sophisticated risk management strategies that anticipate a range of market volatilities.

doi:10.6342/NTU202401141

Chapter 5 Conclusions and Further Work

In conclusion, this thesis has successfully demonstrated the practical application and theoretical underpinnings of the multi-SSQW in the field of financial simulations. Through rigorous experimentation and detailed analysis, we have showcased the capability of quantum computing to transform financial modeling and analysis. The multi-SSQW method has proven to be an invaluable tool in understanding complex financial systems, providing insights that are unattainable through traditional computational methods. As we stand on the brink of a new era in quantum computing, the work presented herein not only contributes to the ongoing development of quantum technologies but also paves the way for future innovations in financial analysis. The potential for quantum computing to revolutionize industry practices is immense, and it is hoped that the foundational research conducted in this thesis will inspire further studies and continued advancement in integrating quantum computing within the financial sector.

As we continue to explore this promising frontier, future work will focus on:

2.Enhanced Parameter Prediction with Machine Learning: We plan to integrate classical machine learning techniques, such as deep neural networks, to predict the parameters for our quantum algorithms. By leveraging economic indicators, events, and reports,

doi:10.6342/NTU202401141

we can refine the input parameters for our multi-SSQW model, developing powerful predictive models that enable us to understand market behaviors better.

3. Simulating Joint Probability Distributions : One of our next steps is to incorporate quantum entanglement into our simulations to model the joint probability distributions of multiple assets. This will allow us to effectively capture the correlations between different financial instruments, providing a more comprehensive view of the market dynamics and interactions.

4. Application Areas: Applying the insights gained from the quantum simulation to various financial strategies.

- **Risk Management**: Assessing and managing the risk associated with financial assets and portfolios.
- **Hedging**: Developing strategies to offset potential investment losses.
- **Risk Management**: Optimizing the allocation of assets in a portfolio to achieve the best possible return for a given level of risk.

5. Scalability and Computational Efficiency: We will continue to explore ways to scale our quantum algorithms to handle larger datasets and more complex financial models. Improving computational efficiency will be crucial for practical applications, especially in high-frequency trading and real-time market analysis.

doi:10.6342/NTU202401141

References

[1] Fischer Black and Myron Scholes. The pricing of options and corporate liabilities. Journal of Political Economy, 81:637–654, 1973.

[2] Robert C Merton. Theory of rational option pricing. The Bell Journal of Economics and Management Science, pages 141–183, 1973.

[3] Harry Markowitz. Portfolio selection. The Journal of Finance, 7(1):77–91, 1952.

[4] Harry M Markowitz. Portfolio selection: Efficient diversification of investments. John Wiley Sons, 1959.

[5] William F Sharpe. Capital asset prices: A theory of market equilibrium under conditions of risk. The Journal of Finance, 19(3):425–442, 1964.

[6] Stephen A Ross. The arbitrage theory of capital asset pricing. Journal of Economic Theory, 13(3):341–360, 1976.

[7] Stavros A Zenios. High-performance computing in finance: The last 10 years and the next. Parallel Computing, 25(13-14):2149–2175, 1999.

[8] Steven L Heston. A closed-form solution for options with stochastic volatility with applications to bond and currency options. The Review of Financial Studies, 6(2):327–343, 1993.

doi:10.6342/NTU202401141

[9] David S Bates. Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options. The Review of Financial Studies, 9(1):69–107, 1996.

[10] Frank J Fabozzi, Petter N Kolm, Dessislava A Pachamanova, and Sergio M Focardi. Robust portfolio optimization and management. John Wiley Sons, 2007.

[11] Ahmed Oussous, Fatima-Zahra Benjelloun, Ayoub Ait Lahcen, and Samir Belfkih. Big data technologies: A survey. Journal of King Saud University-Computer and Information Sciences, 30(4):431–448, 2018.

[12] James M Hutchinson, Andrew W Lo, and Tomaso Poggio. A nonparametric approach to pricing and hedging derivative securities via learning networks. The Journal of Finance, 49(3):851–889, 1994.

[13] JB Heaton, NG Polson, and Jan Hendrik Witte. Deep learning for finance: deep portfolios. Applied Stochastic Models in Business and Industry, 33(1):3–12, 2017.

[14] Richard P. Feynman. Simulating physics with computers. International Journal of Theoretical Physics, 21(6/7):467–488, 1982.

[15] Peter W. Shor. Algorithms for quantum computation: discrete logarithms and factoring. In Proceedings 35th Annual Symposium on Foundations of Computer Science, pages 124–134, 1994.

[16] Andreas Trabesinger. Quantum computing: Towards reality. Nature Physics, 8:263, 2012.

[17] F. Arute et al. Quantum supremacy using a programmable superconducting processor. Nature, 574:505–510, 2019.

doi:10.6342/NTU202401141

[18] A. W. Harrow, A. Hassidim, and S. Lloyd. Quantum algorithm for linear systems of equations. Phys. Rev. Lett., 103:150502, 2009.

[19] S. Lloyd, M. Mohseni, and P. Rebentrost. Quantum principal component analysis. Nature Physics, 10:631–633, 2014.

[20] V. Havlíček, A.D. Córcoles, K. Temme, et al. Supervised learning with quantum-enhanced feature spaces. Nature, 567:209–212, 2019.

[21] Chien-Hung Cho, Chih-Yu Chen, Kuo-Chin Chein, et al. Quantum computation: Algorithms and applications. Chinese Journal of Physics, 72:248–269, 2021.

[22] A. Montanaro. Quantum algorithms: an overview. npj Quantum Information, 2:15023, 2016.

[23] Y. Aharonov, L. Davidovich, and N. Zagury. Quantum random walks. Phys. Rev. A, 48:1687–1690, 1993.

[24] Andrew M. Childs. Universal computation by quantum walk. Phys. Rev. Lett., 102:180501, 2009.

[25] Andrew M. Childs and Jeffrey Goldstone. Spatial search by quantum walk. Phys. Rev. A, 70:022314, 2004.

[26] Andrew M. Childs, Richard Cleve, Enrico Deotto, et al. Exponential algorithmic speedup by a quantum walk. In Proceedings of the thirty-fifth annual ACM symposium on Theory of computing, pages 59–68, 2003.

[27] Anirban Mallick and C. M. Chandrashekar. Dirac cellular automaton from split-step quantum walk. Scientific Reports, 6:25779, 2016.

doi:10.6342/NTU202401141

[28] Parth Rajauria, Prateek Chawla, and C. M. Chandrashekar. Estimation of one-dimensional discrete-time quantum walk parameters by using machine learning algorithms. 2020.

[29] Dorit Aharonov, Andris Ambainis, Julia Kempe, and Umesh Vazirani. Quantum walks on graphs. In Proceedings of the thirty-third annual ACM symposium on Theory of computing. ACM, 2001.

[30] Edward Farhi and Sam Gutmann. Quantum computation and decision trees. Physical Review A, 58:915–928, 1998.

[31] Salvador Elías Venegas-Andraca. Quantum walks: a comprehensive review. Quantum Information Processing, 11:1015–1106, 2012.

[32] Takuya Kitagawa, Mark S. Rudner, Erez Berg, and Eugene Demler. Exploring topological phases with quantum walks. Physical Review A, 82:033429, 2010.

[33] Louis Bachelier. Theory of speculation. Ann. Sci. École Norm. Sup., 3:21–86, 1900. In French.

[34] Eugene F. Fama. Efficient capital markets: A review of theory and empirical work. The Journal of Finance, 25(2):383–417, 1970.

[35] A. W. Lo and A. C. MacKinlay. A Non-Random Walk Down Wall Street. Princeton University Press, 1999.

[36] R. J. Shiller. Market Volatility. MIT Press, 1992.

[37] R. C. Merton. A simple model of capital market equilibrium with incomplete information. The Journal of Finance, 42(3):483–510, 1987.

doi:10.6342/NTU202401141

[38] Daniel J. Egger and et al. Quantum computing for finance: Overview and prospects. IEEE Transactions on Quantum Engineering, 1, 2020.

[39] Dylan Herman, Cody Googin, Xiaoyuan Liu, and et al. A survey of quantum computing for finance. arXiv preprint arXiv:2201.02773, 2022.

[40] Gilles Brassard, Peter Hoyer, Michele Mosca, and Alain Tapp. Quantum amplitude amplification and estimation. In Quantum Computation and Quantum Information, volume 305 of AMS Contemporary Mathematics, pages 53–74, 2002.

[41] Patrick Rebentrost, Brajesh Gupt, and Thomas R. Bromley. Quantum computational finance: Monte carlo pricing of financial derivatives. Phys. Rev. A, 98:022321, 2018.

[42] Nikitas Stamatopoulos and et al. Option pricing using quantum computers. Quantum, 4:291, 2020.

[43] Christa Zoufal, Aurelien Lucchi, and Stefan Woerner. Quantum generative adversarial networks for learning and loading random distributions. npj Quantum Information, 5:103, 2019.

[44] Nikitas Stamatopoulos and et al. Towards quantum advantage in financial market risk using quantum gradient algorithms. Quantum, 6:770, 2022.

[45] Dong An and et al. Quantum-accelerated multilevel monte carlo methods for stochastic differential equations in mathematical finance. Quantum, 5:481, 2021.

[46] Shouvanik Chakrabarti and et al. A threshold for quantum advantage in derivative pricing. Quantum, 5:463, 2021.

doi:10.6342/NTU202401141

[47] M. A. Nielsen and I. L. Chuang. Quantum Computation and Quantum Information. Cambridge University Press, 2010.

[48] Hubert de Guise, Olivia Di Matteo, and Luis L. Sánchez-Soto. Simple factorization of unitary transformations. Phys. Rev. A, 97:022328, 2018.

[49] Yuval R. Sanders, Guang Hao Low, Artur Scherer, and Dominic W. Berry. Black-box quantum state preparation without arithmetic. Phys. Rev. Lett., 122:020502, 2019.

[50] A. Rocchetto, E. Grant, S. Strelchuk, et al. Learning hard quantum distributions with variational autoencoders. npj Quantum Information, 4:28, 2018.

[51] Kalyan Dasgupta and Binoy Paine. Loading probability distributions in a quantum circuit, 2022.

[52] J. Choi, A.L. Shaw, I.S. Madjarov, et al. Preparing random states and benchmarking with many-body quantum chaos. Nature, 613:468–473, 2023.

[53] L. Grover and T. Rudolph. Creating superpositions that correspond to efficiently integrable probability distributions, 2002.

[54] Alfred R. Oxenfeldt. A decision-making structure for price decisions. Journal of Marketing, 37(1):48–51, 1973.

[55] Bouteska Ahmed. Understanding the impact of investor sentiment on the price formation process: A review of the conduct of american stock markets. The Journal of Economic Asymmetries, 22:e00172, 2020.

[56] Yutaka Matsuzawa. An index theorem for split-step quantum walks. Quantum Information Processing, 19:227, 2020.

doi:10.6342/NTU202401141

[57] Akihiro Narimatsu, Hiromichi Ohno, and Kazuyuki Wada. Unitary equivalence classes of split-step quantum walks, 2023.

[58] Daniel W. Stroock. Probability Theory: An Analytic View. Cambridge University Press, 2010.

[59] Olav Kallenberg. Foundations of Modern Probability. Springer-Verlag, New York, 2002.

doi:10.6342/NTU202401141