國立臺灣大學理學院物理學研究所

碩士論文

Department of Physics
College of Science
National Taiwan University

Master's Thesis

自發性同步：透過雙層振盪器探討群體動力學

Spontaneous Synchronization: Group dynamics approach via two-layer oscillators

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中文摘要

在音樂廳中，一位聽眾的拍手行為是由拍手那位聽眾的意志，來決定拍手動作的頻率與音量；但聽眾全體那同步化的拍手行為卻不是。而「聽眾全體同步地拍手」的行為，在大自然中被稱為「自發性同步」。自發性同步並非人類與動物的專利，而是在一個系統在受到局部效應後，如何讓系統整體逐步因應局部效應而變化的演示。

在我們的模擬中，我們會展示一個二層共振子系統在其中一層有類似水波那樣的點狀擾動時，對另外一層、甚至整個系統運動的影響。然而，觀察波動對一個完全沒有雜質的系統其實沒有什麼很大的意義。對此，我們會引入三種「雜質粒子」，障礙物、放大器與反射器，以觀察三種雜質粒子如何讓系統產生對應的變化、甚至如何對系統的自發性同步產生什麼影響。

關鍵字：自發性同步、群眾動力學、力學、共振子、相關函數

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ABSTRACT

In an auditorium, the applause of one audience member is driven by individual thought, but synchronized applause from all audience members is not. In nature, a phenomenon similar to synchronized applause is Spontaneous Synchronization (Sync). Sync is not unique to humans or animals but demonstrates how a local effect can cause an entire system to change in an orderly manner. In our simulation, we demonstrate Sync in a two-layer oscillator system, showing how an initial point perturbation in one of the layers leads to the simultaneous motion of both layers.

However, it's overly simplistic to study how a wave influences an oscillator system without any objects. To gain deeper insights, we introduce three types of objects—blocks, amplifiers, and reflectors—into the system to observe how replacing some particles with these objects alters the system's behavior. In our simulation, we will examine how these objects—blocks, amplifiers, and reflectors—impact the oscillator system and its ability to achieve synchronization.

Keywords: Spontaneous synchronization, Group dynamics, Mechanics, Oscillator, Correlation Function

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CONTENTS

doi:10.6342/NTU202404268

doi:10.6342/NTU202404268

LIST OF FIGURES

doi:10.6342/NTU202404268

doi:10.6342/NTU202404268

doi:10.6342/NTU202404268

doi:10.6342/NTU202404268

LIST OF TABLES

doi:10.6342/NTU202404268

Chapter 1 Introduction: What is Sync?

Spontaneous Synchronization (Sync) is a phenomenon in physics or biology, which a system with multiple members response the variety of one member of the system to regularly entire variety of the whole system without external force to the entire system. The concepts of sync are quite different from the concepts of entropy. The reason why the concepts are different is that the addition of entropy makes system be irregular, but sync makes system regular.

In system having sync, each member of the system behaves the rules below:

1. Local sensing: Each member can sense the varieties of the nearest few members.
2. Local aligning: Each member tends to the same state with the nearest few members.
3. Attracting but keeping a distance behavior: Each member attracts each other, but it keeps a distance between neighbor ones in order to reach an equilibrium inside the system.

From the rules we mentioned, the sync of the system is responding some local change of some members of the system step-by-step, to make the whole system vary regularly.

The simplest example of sync is audiences in auditorium. As an audience starts applauding, the nearby audiences will response the applaud to applaud gradually. Finally, all audiences applaud with same tempo. Another example of synchronization occurs when a noisy classroom of students suddenly falls silent without any apparent reason. This spontaneous synchronization is also seen in natural phenomena such as birds flying in flocks, fish swimming in schools, and the synchronized flashing of fireflies. [1]

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Chapter 2 Model Setup: Initial Condition and Force of each particle

In our approach, we consider a two-layer oscillator system on an $N \times N \times 2$ grid[2], where N is the number of particles along one dimension, with a unit spatial step $\Delta x = 1$. Unless otherwise specified, the mass of each particle is 1.

In our simulation, it takes approximately 274.5-unit times (rounded to 300) for the wave to propagate from $\left(\frac{N}{2}, \frac{N}{2}, 0\right)$ to the boundary. To satisfy the Courant-Friedrichs-Lewy (CFL) condition, we set the unit time step $\Delta t = 0.05$.

We assume the wave originates from an initial perturbation at $\left(\frac{N}{2}, \frac{N}{2}, 0\right)$ in layer 1, with an initial amplitude of 50.

Each particle in the oscillator system is bound to its neighbors with spring forces. The forces are described as follows and Fig.1:

Main direction force: The force between $(i \pm 1, j, k)$ (or $(i, j \pm 1, k)$, $(i, j, k \pm 1)$) and (i, j, k) in oscillators can be described as follows:

$$\begin{cases} f_{x,(i,j,k)} = -k \cdot \left[\left(\Delta r_{(i+1,j,k)} - \Delta r_{(i,j,k)}\right) - \left(\Delta r_{(i,j,k)} - \Delta r_{(i-1,j,k)}\right)\right] \\ f_{y,(i,j,k)} = -k \cdot \left[\left(\Delta r_{(i,j+1,k)} - \Delta r_{(i,j,k)}\right) - \left(\Delta r_{(i,j,k)} - \Delta r_{(i,j-1,k)}\right)\right] \\ f_{z,(i,j,k)} = -k_z \cdot \left(\Delta r_{(i,j,k+1)} - \Delta r_{(i,j,k)}\right) \quad (\text{for } z = 0) \\ f_{z,(i,j,k)} = -k_z \cdot \left(\Delta r_{(i,j,k)} - \Delta r_{(i,j,k-1)}\right) \quad (\text{for } z = 1) \\ f_{main,(i,j,k)} = f_{x,(i,j,k)} + f_{y,(i,j,k)} + f_{z,(i,j,k)} \end{cases}$$

where k is the spring coefficient for the x and y components, and k_z is the spring coefficient for the z component.

Cross direction force: The force between $(i \pm 1, j, k)$ and (i, j, k) in oscillators can be described as follows:

doi:10.6342/NTU202404268

$$\begin{cases} f_{\nearrow\swarrow,(i,j,k)} = -k_{cross} \cdot \left[\left(\Delta r_{(i+1,j+1,k)} - \Delta r_{(i,j,k)}\right) - \left(\Delta r_{(i,j,k)} - \Delta r_{(i-1,j-1,k)}\right)\right] \\ f_{\nwarrow\searrow,(i,j,k)} = -k_{cross} \cdot \left[\left(\Delta r_{(i-1,j+1,k)} - \Delta r_{(i,j,k)}\right) - \left(\Delta r_{(i,j,k)} - \Delta r_{(i+1,j-1,k)}\right)\right] \\ f_{cross,(i,j,k)} = f_{\nearrow\swarrow,(i,j,k)} + f_{\nwarrow\searrow,(i,j,k)} \end{cases}$$

where k_{cross} is the spring coefficient for cross directions.

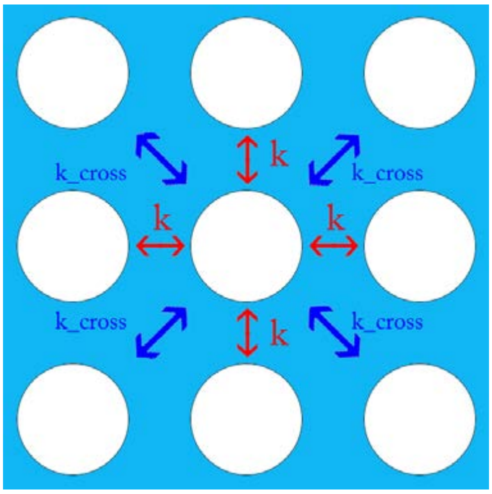

Fig. 1: The force description of each particle in xy plane.

Given the large value of N (in our simulation, N=1024), the system in one layer approximates an infinite plate oscillator. Thus, boundary conditions are necessary for the x and y components when describing the system's forces.

Combining the main direction force, cross direction force, and damping force, we get the total force:

$$f_{damping} = -\gamma v$$

where γ is the damping coefficient and v is the velocity of the particle in the system.

Thus, the total force on each particle in the system is:

$$F_{total} = f_{main,(i,j,k)} + f_{cross,(i,j,k)} - f_{damping}$$

However, in our simulation, we assume the damping force is zero.

doi:10.6342/NTU202404268

Chapter 3 Observing the tendency: Correlation and Power law fitting

In the two-layer oscillator system with an $N \times N$ scale, if some particles are replaced with 'objects' such as blocks (which do not move), amplifiers (which increase their speed), and reflectors (which reverse their speed), the correlation of the system will be influenced.

3.1 Introduction to System Correlation:

In our two-layer oscillator system with an $N \times N$ scale, correlation indicates the dependence between particles in the system. The correlation of interest is derived from the amplitude of the wave, which originates from an initial perturbation of a particle in the system.

We analyze the spatial correlations of amplitude in different layers ($z = 0$ and $z = 1$). These spatial correlations can be classified into the following types [3]:

3.1.1 Auto-Correlation

This function describes the relevance of amplitudes in the same layer but at different (x, y) positions. It can be described by the following formula:

$$C_{self}(\Delta r) = \int A(x, y)A^*(x, y)d^2r$$

where $A(x, y)$ is the amplitude function at (x, y) in the observed layer, Δr is the distance between two positions (x, y) and (x', y') ($\Delta r = \sqrt{(x' - x)^2 + (y' - y)^2}$), and $A^*(x, y)$ is complex conjugate of $A(x, y)$.

In numerical analysis, this formula can be expressed as:

doi:10.6342/NTU202404268

$$C_{self}(\Delta r) = \sum_k a_{\vec{k}} \cdot a_{\vec{k}}^* e^{-i(\vec{k}\cdot\Delta r)}$$

3.1.2 Cross-Correlation

This function describes the relevance of amplitudes at different (x, y) positions in different layers. It can be described by the following formula:

$$C_{cross}(\Delta r) = \int A(x,y)B^*(x,y)d^2r$$

In numerical analysis, this formula can be expressed as:

$$C_{cross}(\Delta r) = \sum_k a_{\vec{k}} \cdot b_{\vec{k}}^* e^{-i(\vec{k}\cdot\Delta r)}$$

In our simulation, we observe the amplitudes of the two layers at $t = 60, 120, 180, 240$ and 300time units. We then compute these to find the correlation functions.

After finding the correlation function, we take the ensemble average to observe the average behavior of the correlation function at a specific Δr. This allows us to plot $C(\Delta r)$ v.s. Δr to observe how $C(\Delta r)$ changes as Δr increases.

Finally, we apply Nonlinear Regression using the function

$$y = C_{fit} \cdot x^{-s}$$

to fit the portion of $C(\Delta r)$ within the interval $\Delta r = 10 \sim 100$. This allows us to observe how various objects influence the variation of the system's correlation.

By fitting this power law function, we can observe how different objects (blocks, amplifiers, and reflectors) affect the correlation behavior of the system. This analysis provides insights into how the presence and configuration of these objects influence the propagation and stability of waves within the two-layer oscillator system.

doi:10.6342/NTU202404268

We fit the interval $\Delta r = 10 \sim 100$ because most of the $C(\Delta r)$ v.s. Δr data points fall within this range.

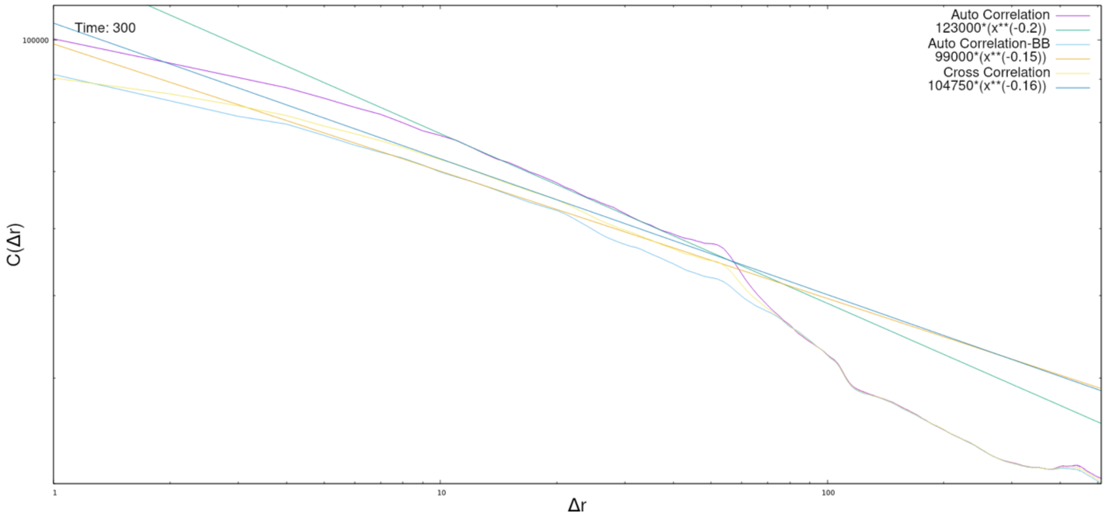

Fig. 2: the correlation functions and the power law fitting functions within the interval $\Delta r = 10 \sim 100$.

3.2 Sync in correlation:

In our simulation, 'Sync' refers to the condition where the waves propagating in both layer 1 and layer 2 have the same frequency and phase. This condition is indicated by the three correlations—Auto-correlation of layer 1, Auto-correlation of layer 2, and Cross-correlation—being nearly identical. In this thesis, we are going to demonstrate how to detect sync in our former discussion of using correlation function.

The power law coefficient, s, in the function $y = C_{fit} \cdot x^{-s}$ is almost identical for the three correlations in the interval we observe. On the other hand, amplitude coefficient C_{fit} is related to the amplitude of different layers. However, it does not affect the Sync.

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Chapter 4 Simulation Result: One Object

In our simulation, we will present the power law fitting results for scenarios involving 1 object, 2 objects, and 64 objects, each with different arrangements of their positions.

4.1 1 Object

In these cases, we will demonstrate how different coefficients of cross-direction force affect the system. The position of the object is at $(\frac{3}{4}n, \quad \frac{1}{2}n, \quad 0)$.

4.1.1 1 Block:

t	Auto correlation <AA>	Auto correlation <BB>	Cross correlation <AB>
60	0.43	0.12	0.115
120	0.17	0.13	0.035
180	0.09	0.148	0.01
240	0.07	0.132	0.012
300	0.081	0.1	0.012

Table 1. The power law coefficients of correlations of 1 block-case vs. time.

doi:10.6342/NTU202404268

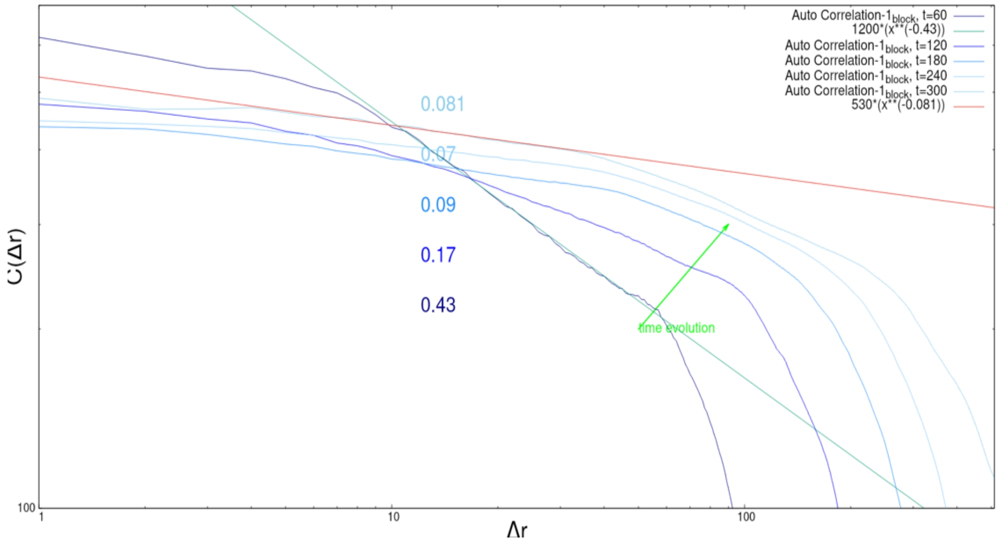

Fig. 3: The auto correlation of layer 1 of 1 block-case at different times.

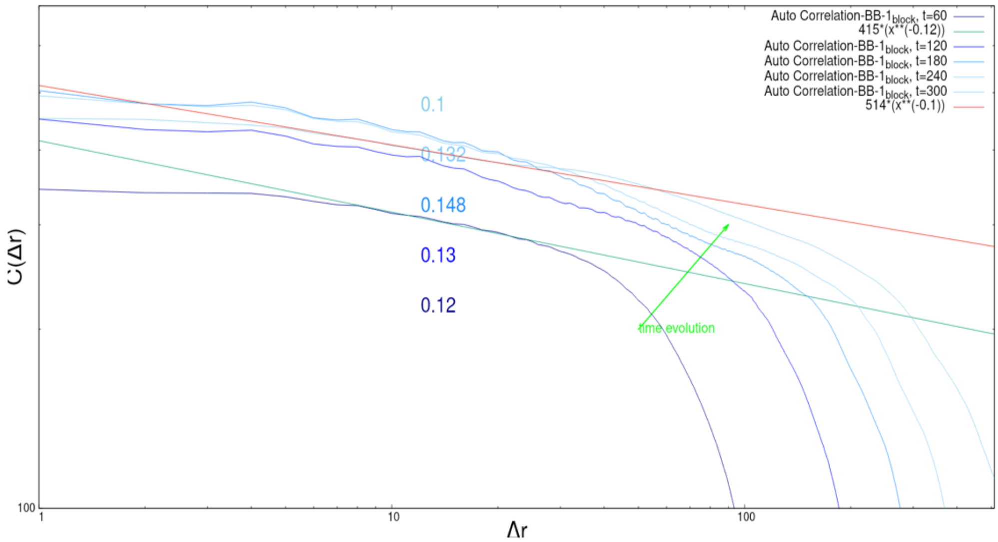

Fig. 4: The auto correlation of layer 2 of 1 block-case at different times.

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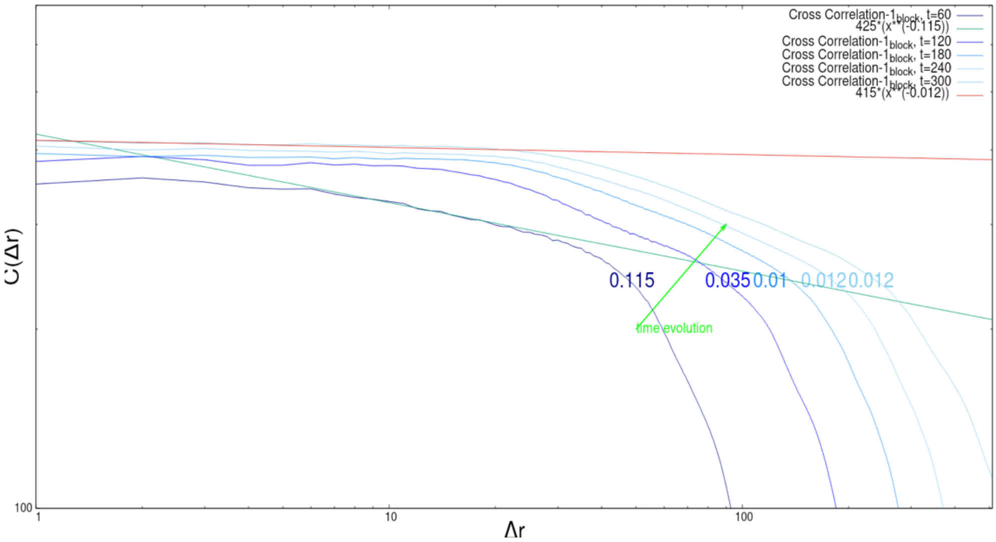

Fig. 5: The cross correlation of 1 block-case at different times.

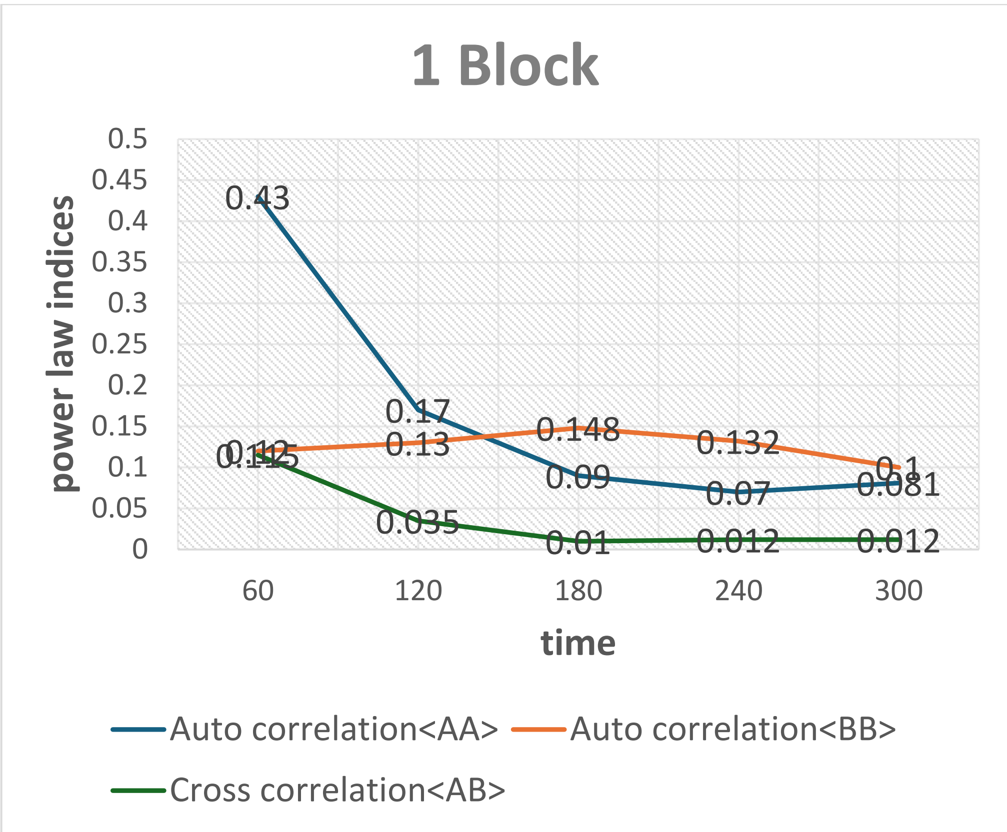

Fig. 6: The power law coefficients of correlations of 1 block-case vs. time chart.

doi:10.6342/NTU202404268

4.1.2 1 Amplifier:

t	Auto correlation <AA>	Auto correlation <BB>	Cross correlation <AB>
60	0.43	0.12	0.13
120	0.195	0.17	0.063
180	0.16	0.14	0.094
240	0.2	0.195	0.19
300	0.1	0.1	0.09

Table 2. The power law coefficients of correlations of 1 amplifier-case vs. time.

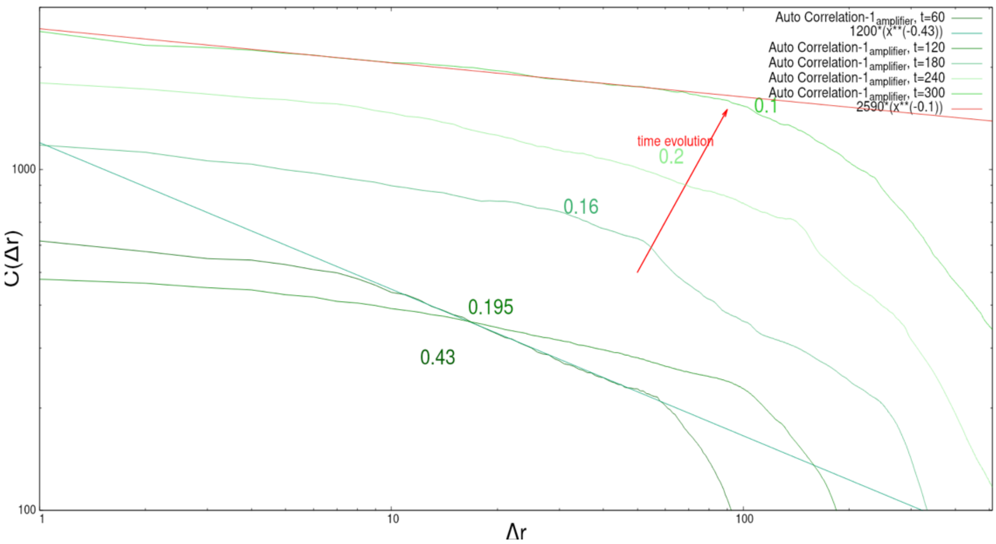

Fig. 7: The auto correlation of layer 1 of 1 amplifier-case at different times.

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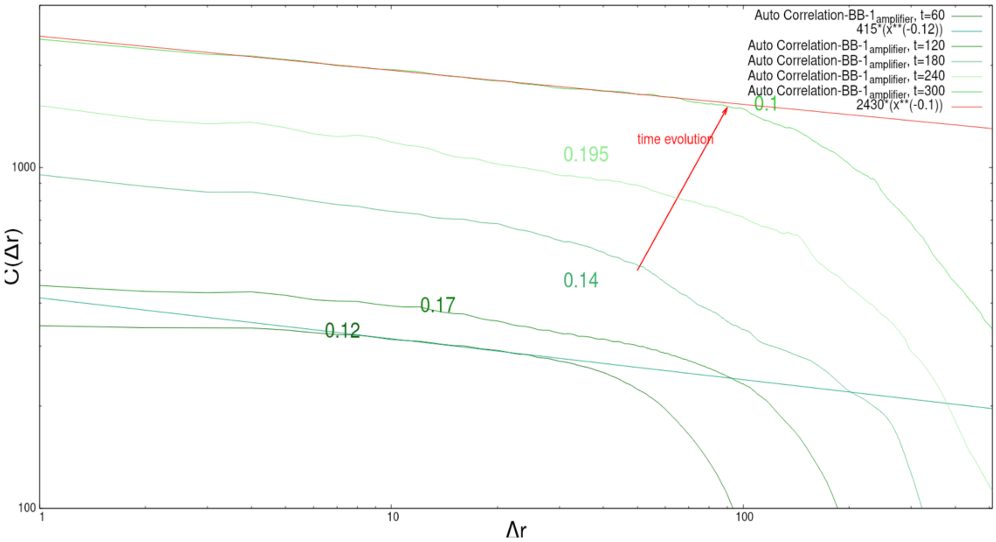

Fig. 8: The auto correlation of layer 2 of 1 amplifier-case at different times.

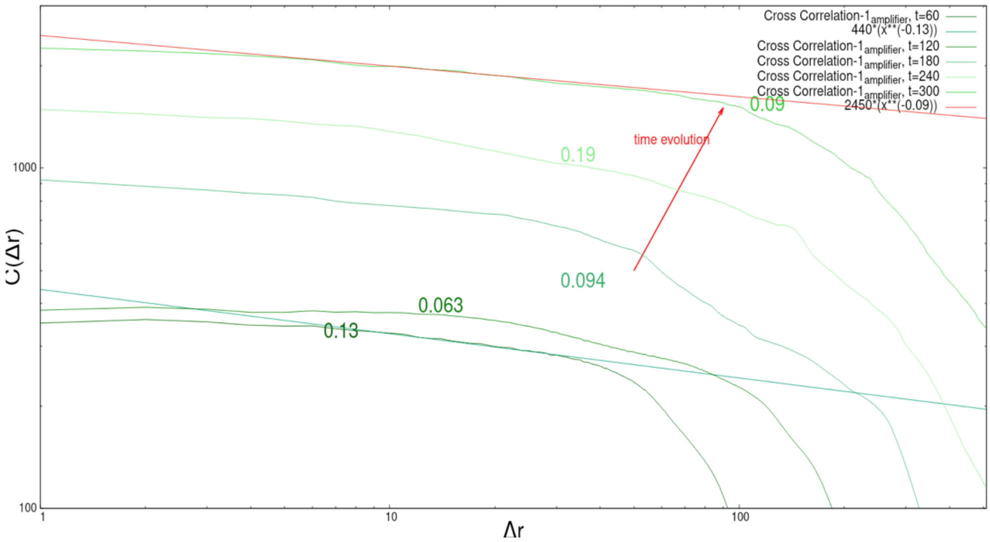

Fig. 9: The cross correlation of 1 amplifier-case at different times.

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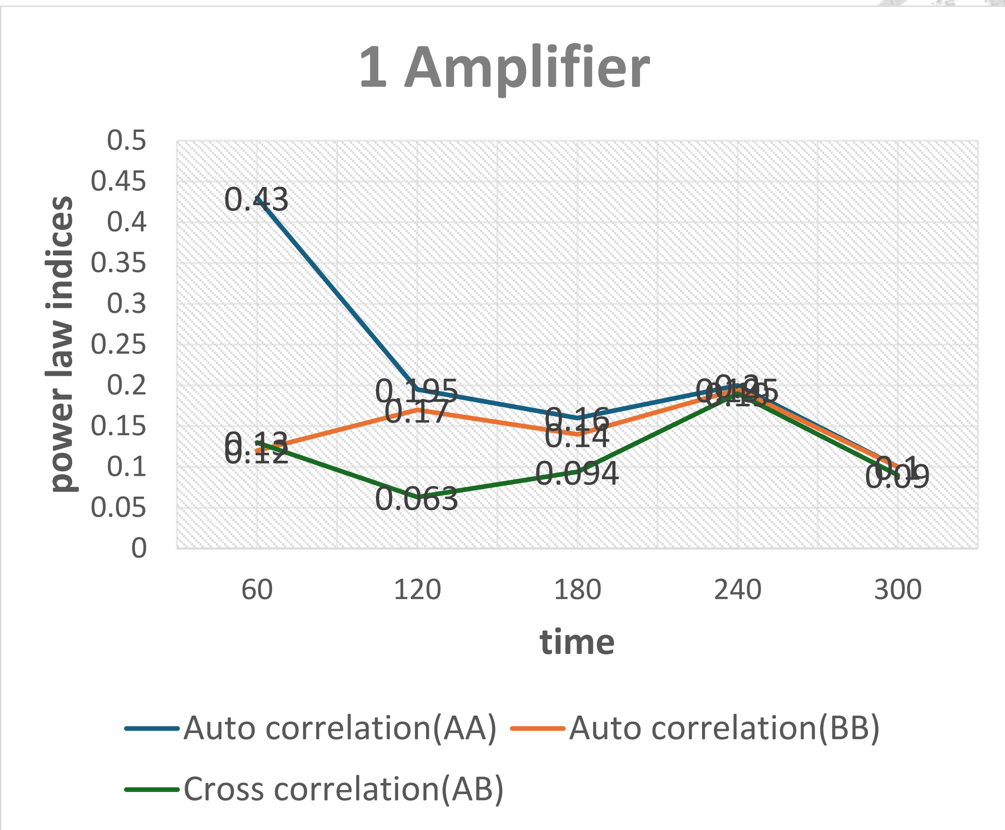

Fig. 10: The power law coefficients of correlations of 1 amplifier-case vs. time chart.

4.1.3 1 Reflector:

t	Auto correlation <AA>	Auto correlation <BB>	Cross correlation <AB>
60	0.18	0.12	0.09
120	0.14	0.01	0.25
180	0.09	0.13	0.01
240	0.46	0.48	0.5
300	0.15	0.15	0.15

Table 3. The power law coefficients of correlations of 1 reflector-case vs. time.

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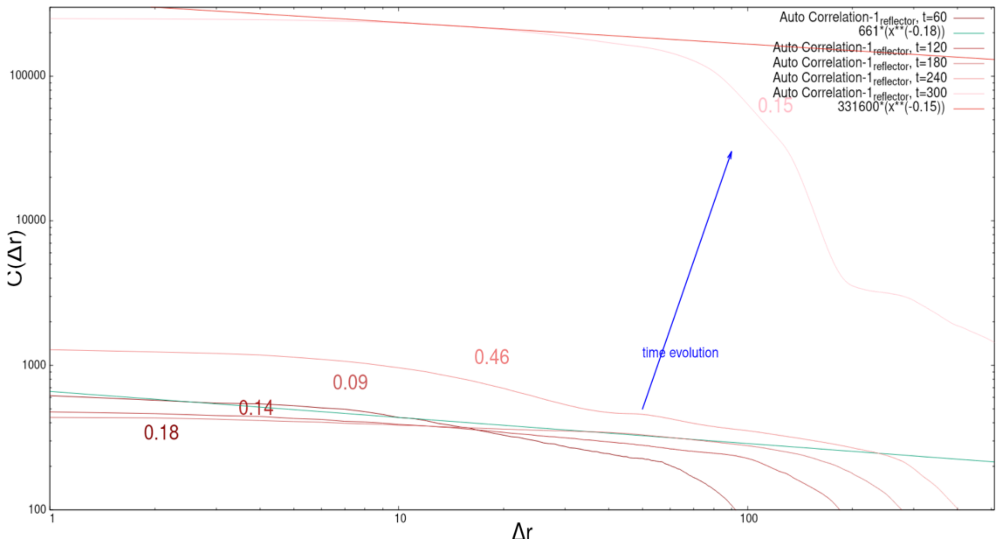

Fig. 11: The auto correlation of layer 1 of 1 reflector-case at different times.

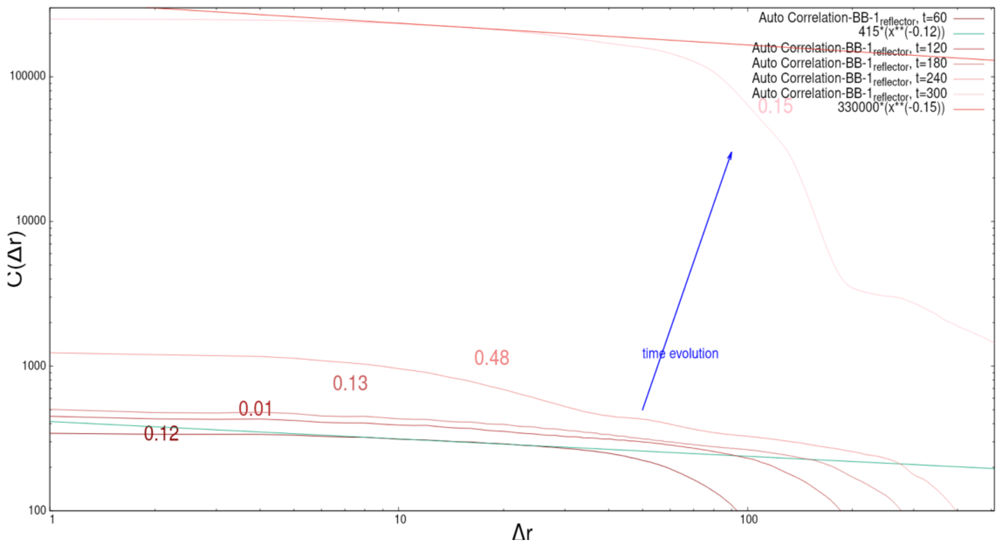

Fig. 12: The auto correlation of layer 2 of 1 reflector-case at different times.

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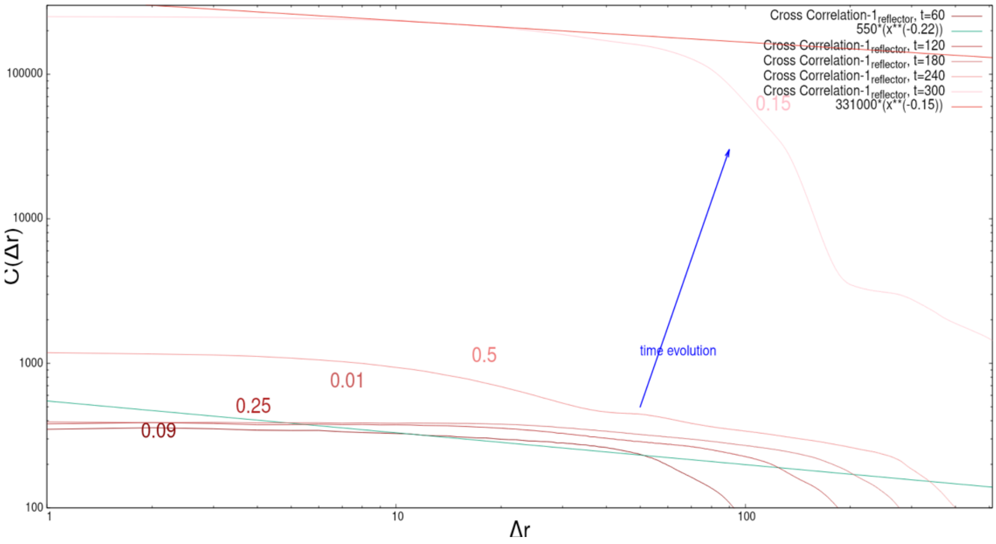

Fig. 13: The cross correlation of 1 reflector-case at different times.

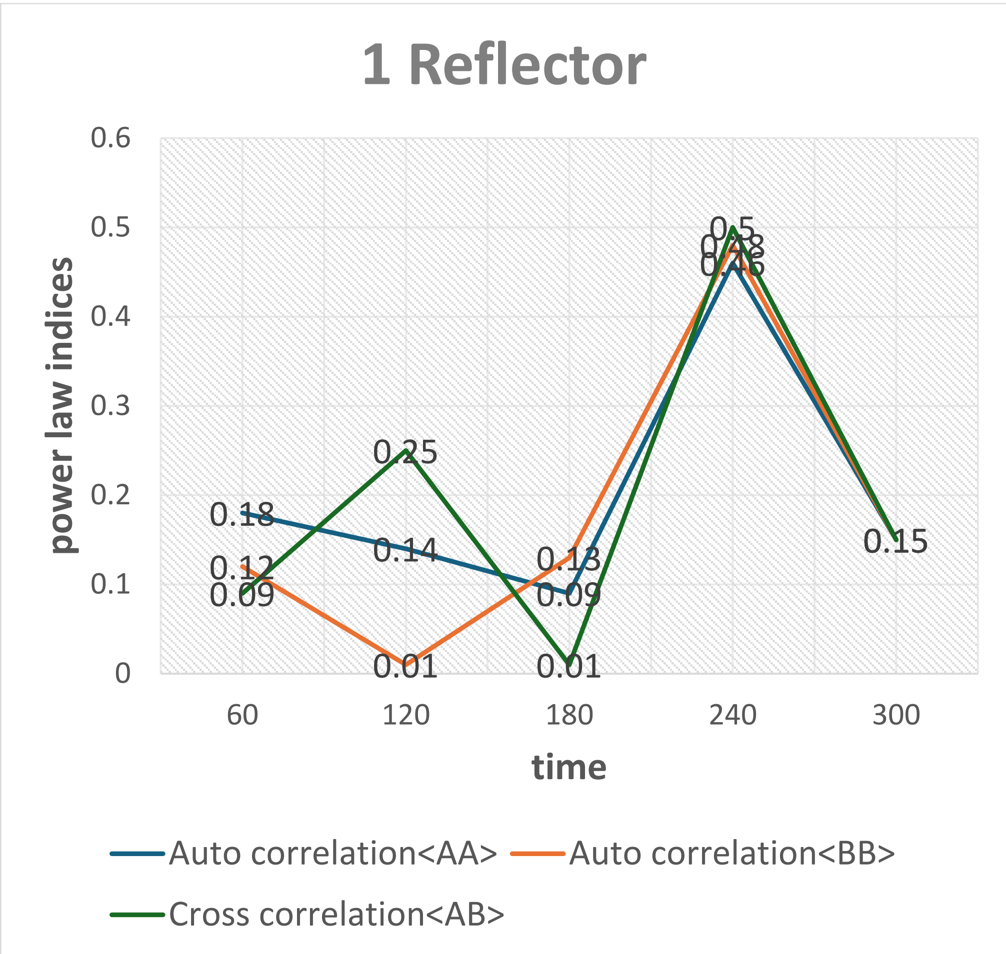

Fig. 14: The power law coefficients of correlations of 1 reflector-case vs. time chart.

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4.2 Multiple Objects:

We'll present the results for different random arrangements of objects.

4.2.1 Multiple Blocks:

Similar to the 1-block case

During the power law fitting in multiple block cases, we found that the $C(\Delta r)$ $v.s.$ Δr plots remain almost the same, regardless of the number of blocks in the system.

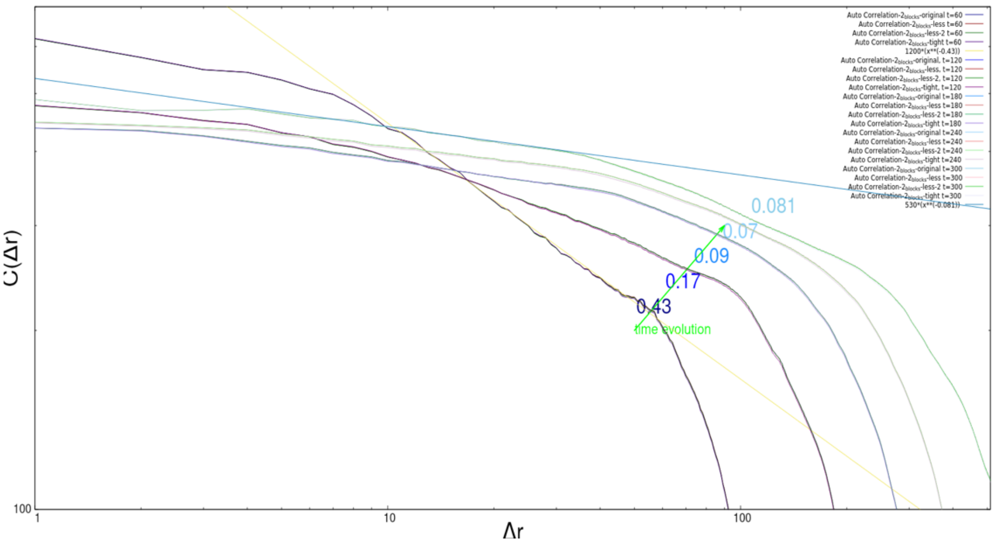

Fig. 15: The auto correlations of layer 1 of 2 blocks-case at different times.

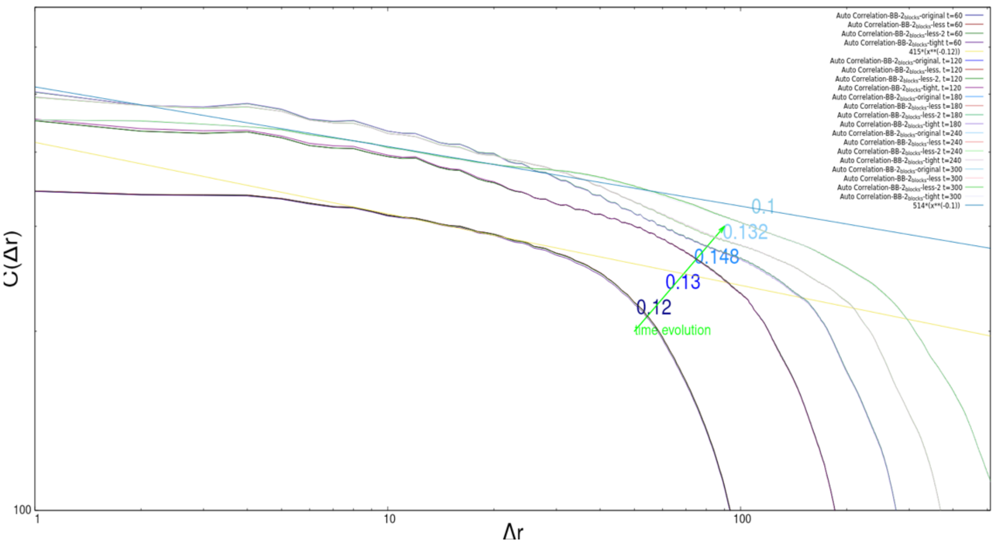

Fig. 16: The auto correlations of layer 2 of 2 blocks-case at different times.

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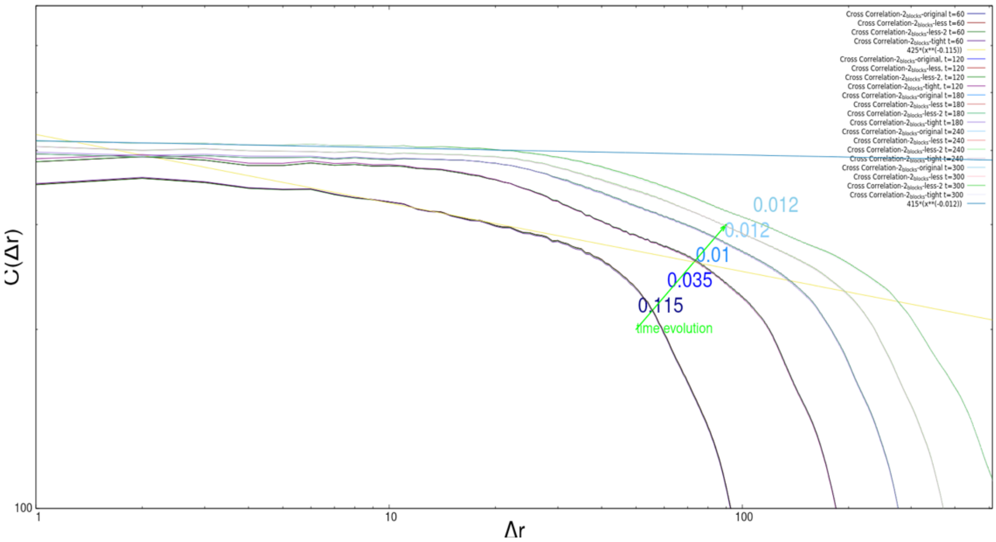

Fig. 17: The cross correlations of 1 block-case at different times.

In these 2 blocks case, blocks are distributed at different distances between them:

Original: The blocks are positioned at $\left(\left(\frac{1}{2}+\frac{1}{4\sqrt{2}}\right)n,\quad \left(\frac{1}{2}+\frac{1}{4\sqrt{2}}\right)n-10,\quad 0\right)$ and $\left(\left(\frac{1}{2}-\frac{1}{4\sqrt{2}}\right)n,\quad \left(\frac{1}{2}-\frac{1}{4\sqrt{2}}\right)n+10,\quad 0\right)$

Less distance: The blocks are positioned at $\left(\left(\frac{1}{2}+\frac{1}{10\sqrt{2}}\right)n,\quad \left(\frac{1}{2}+\frac{1}{10\sqrt{2}}\right)n,\quad 0\right)$ and$\left(\left(\frac{1}{2}-\frac{1}{10\sqrt{2}}\right)n,\quad \left(\frac{1}{2}-\frac{1}{10\sqrt{2}}\right)n,\quad 0\right)$

Less distance 2: The blocks are positioned at $(\left(\frac{1}{2}+\frac{1}{20\sqrt{2}}\right)n,\quad \left(\frac{1}{2}+\frac{1}{20\sqrt{2}}\right)n,\quad 0)$ and $(\left(\frac{1}{2}-\frac{1}{20\sqrt{2}}\right)n,\quad (\frac{1}{2}-\frac{1}{20\sqrt{2}})n\quad 0)$

Tight distance: The blocks are positioned at $\left(\frac{n}{2}+7,\quad \frac{n}{2}+7,\quad 0\right)$ and $\left(\frac{n}{2}-7,\quad \frac{n}{2}-7,\quad 0\right)$

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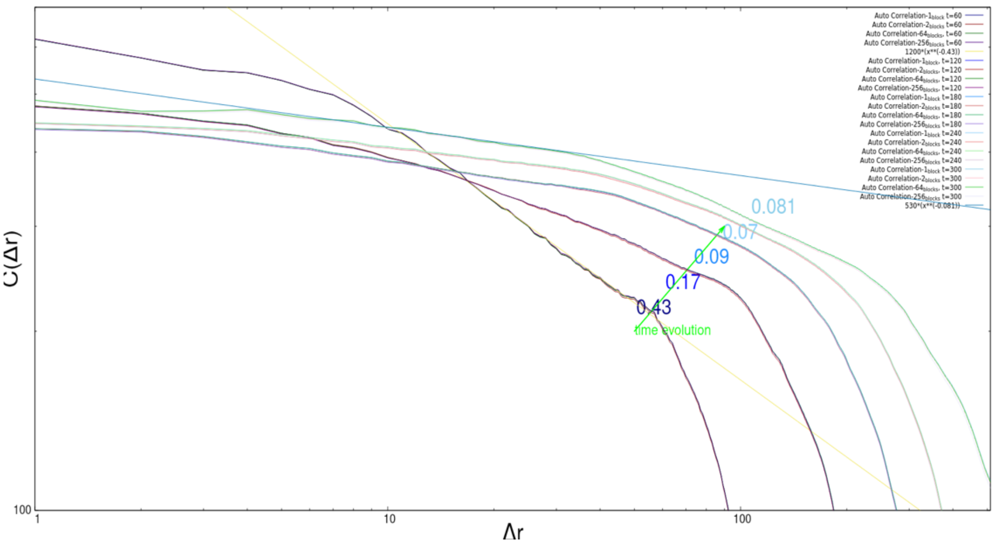

Fig. 18: The auto correlations of layer 1 for different quantities of blocks-case at different times. In the 2-blocks case, the blocks are positioned at $\left(\frac{n}{2}+7,\ \ \frac{n}{2}+7,\ \ 0\right)$ and $\left(\frac{n}{2}-7,\ \ \frac{n}{2}-7,\ \ 0\right)$

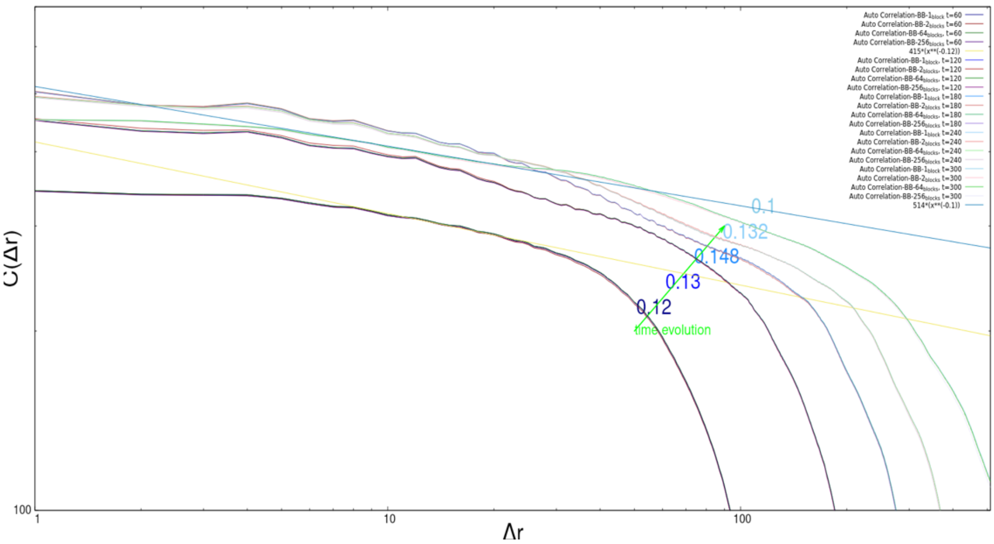

Fig. 19: The auto correlations of layer 2 for different quantities of blocks-case at different times. In the 2-blocks case, the blocks are positioned at $\left(\frac{n}{2}+7,\ \ \frac{n}{2}+7,\ \ 0\right)$ and $\left(\frac{n}{2}-7,\ \ \frac{n}{2}-7,\ \ 0\right)$

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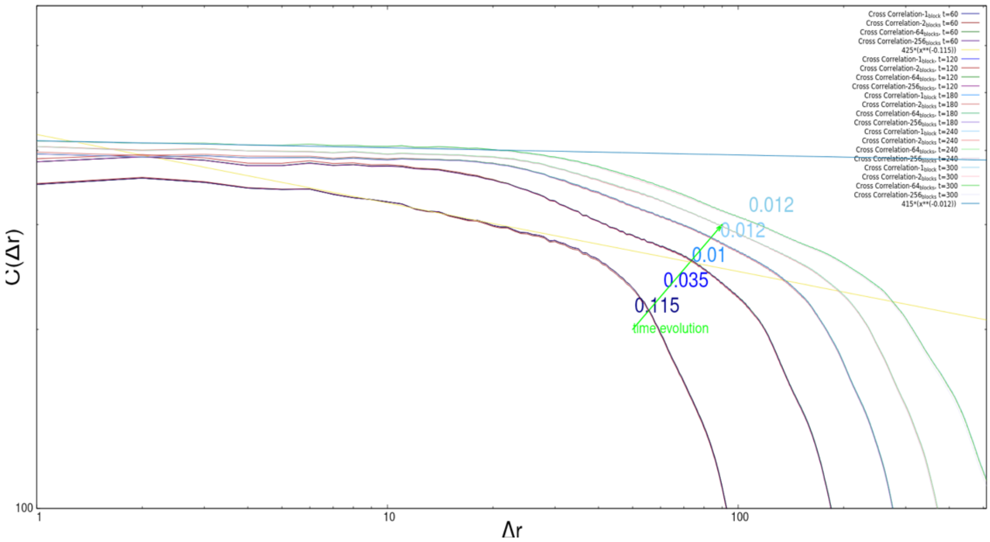

Fig. 20: The cross correlations for different quantities of blocks-case at different times. In the 2-blocks case, the blocks are positioned at $(\frac{n}{2}+7,\ \frac{n}{2}+7,\ 0)$ and $(\frac{n}{2}-7,\ \frac{n}{2}-7,\ 0)$

4.2.2 Multiple Amplifiers:

Amplifiers are positioned in both layers randomly:

t	Auto correlation <AA>	Auto correlation <BB>	Cross correlation <AB>
60	0.35	0.12	0.19
120	0.12	0.15	0.18
180	0.13	0.18	0.105
240	0.11	0.11	0.1
300	0.084	0.05	0.085

Table 4. The power law coefficients of correlations of 64 amplifiers in both layers case vs. time.

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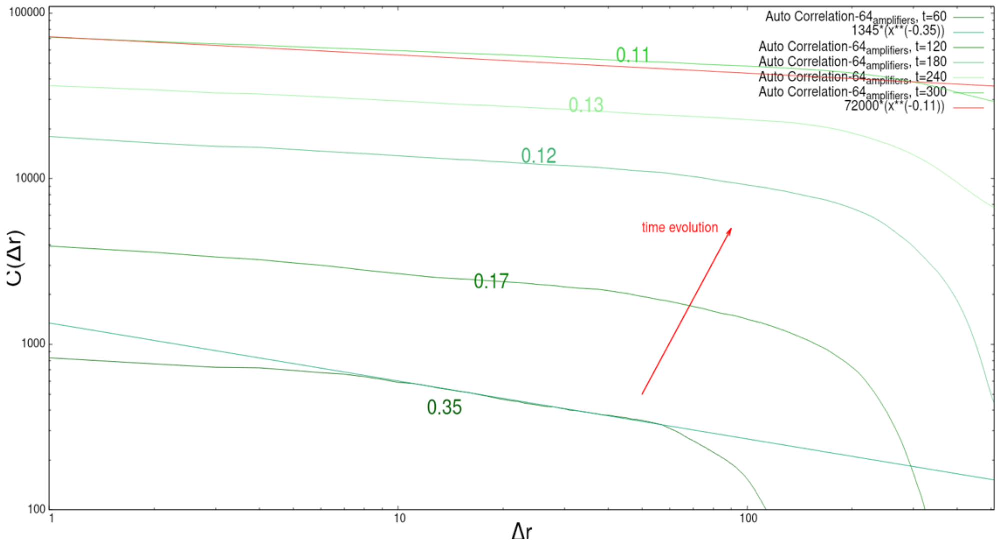

Fig. 21: The auto correlation of layer 1 of 64 amplifiers-case at different times.

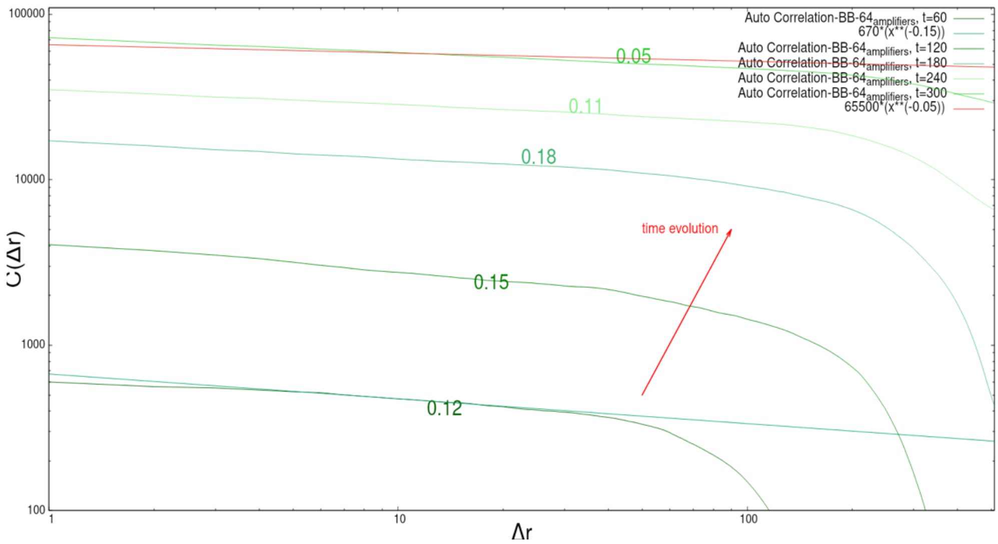

Fig. 22: The auto correlation of layer 2 of 64 amplifiers-case at different times.

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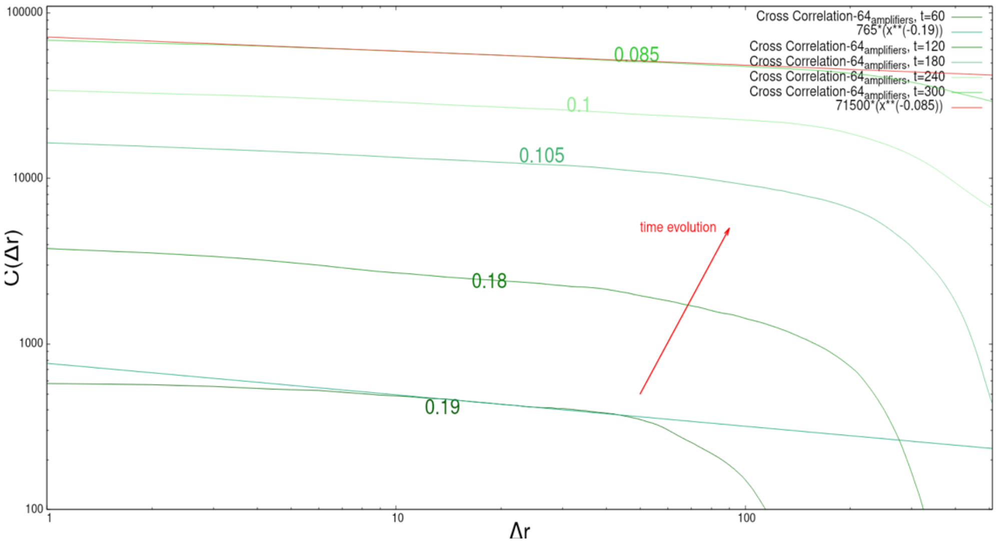

Fig. 23: The cross correlation of 64 amplifiers-case at different times.

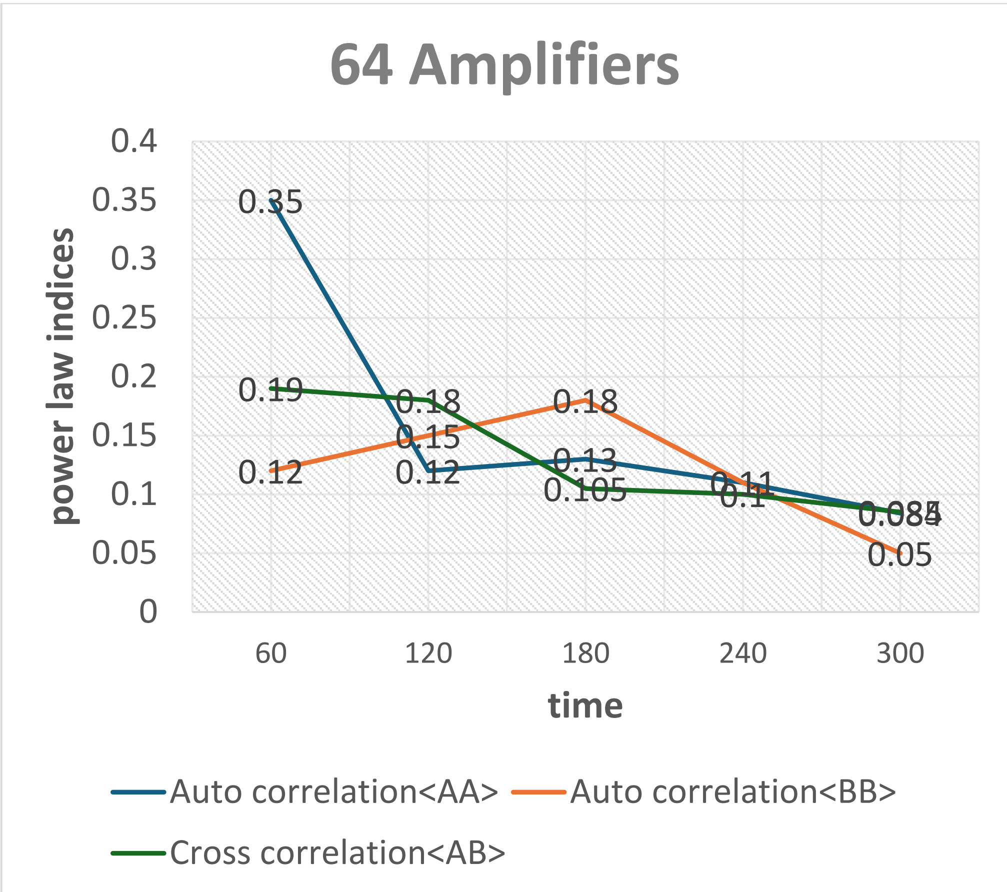

Fig. 24: The power law coefficients of correlations vs. time chart.

4.2.3 Multiples Reflectors:

doi:10.6342/NTU202404268

t	Auto correlation <AA>	Auto correlation <BB>	Cross correlation <AB>
60	0.43	0.11	0.11
120	0.2	0.18	0.12
180	0.13	0.15	0.13
240	0.15	0.15	0.15
300	0.1	0.09	0.1

Table 5. The power law coefficients of correlations of 64 reflectors in both layers case vs. time.

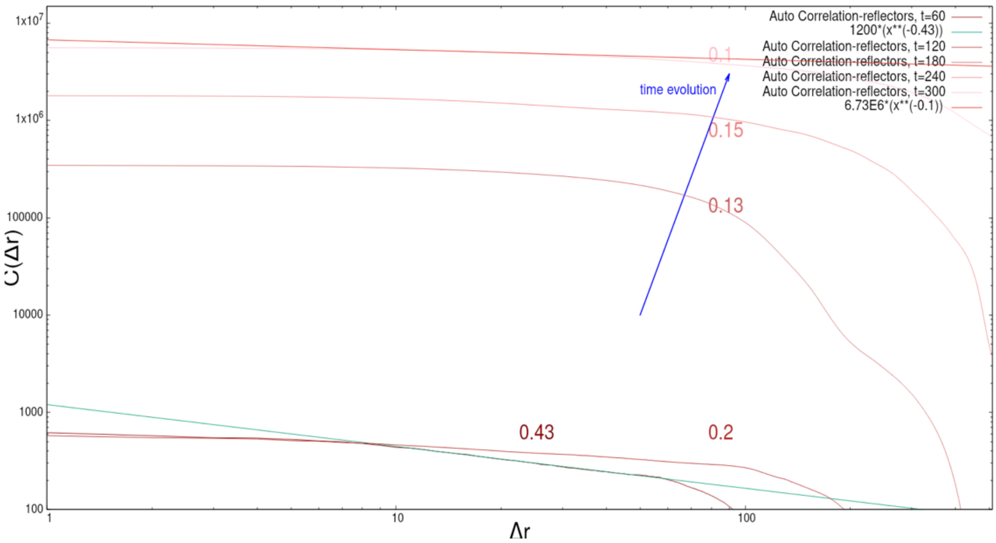

Fig. 25: The auto correlation of layer 1 of 64 reflectors-case at different times.

doi:10.6342/NTU202404268

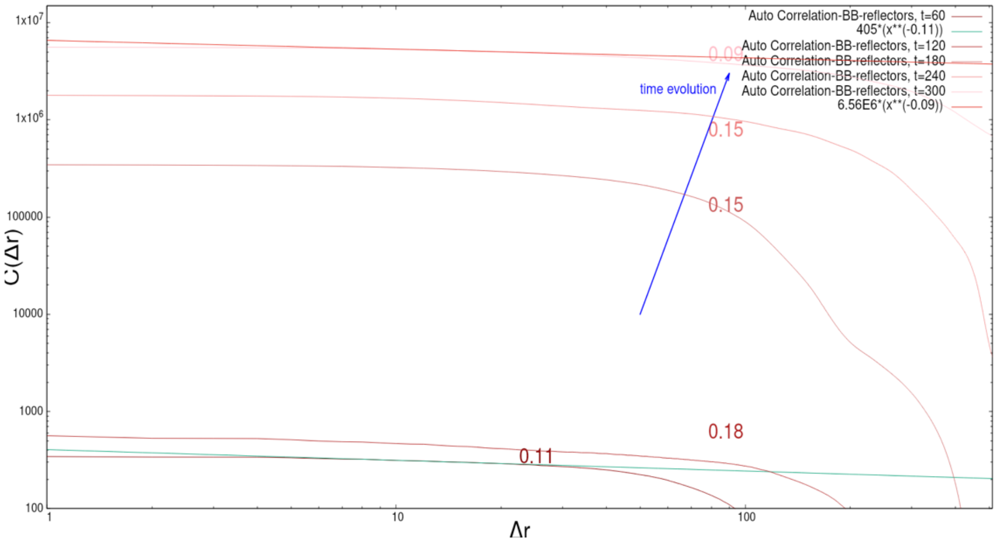

Fig. 26: The auto correlation of layer 2 of 64 reflectors-case at different times.

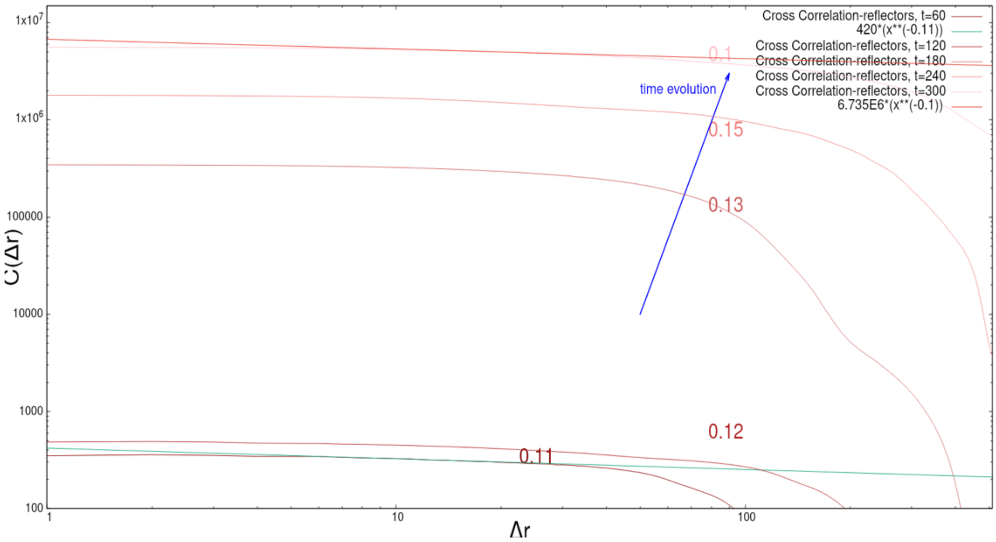

Fig. 27: The cross correlation of 64 reflectors-case at different times.

doi:10.6342/NTU202404268

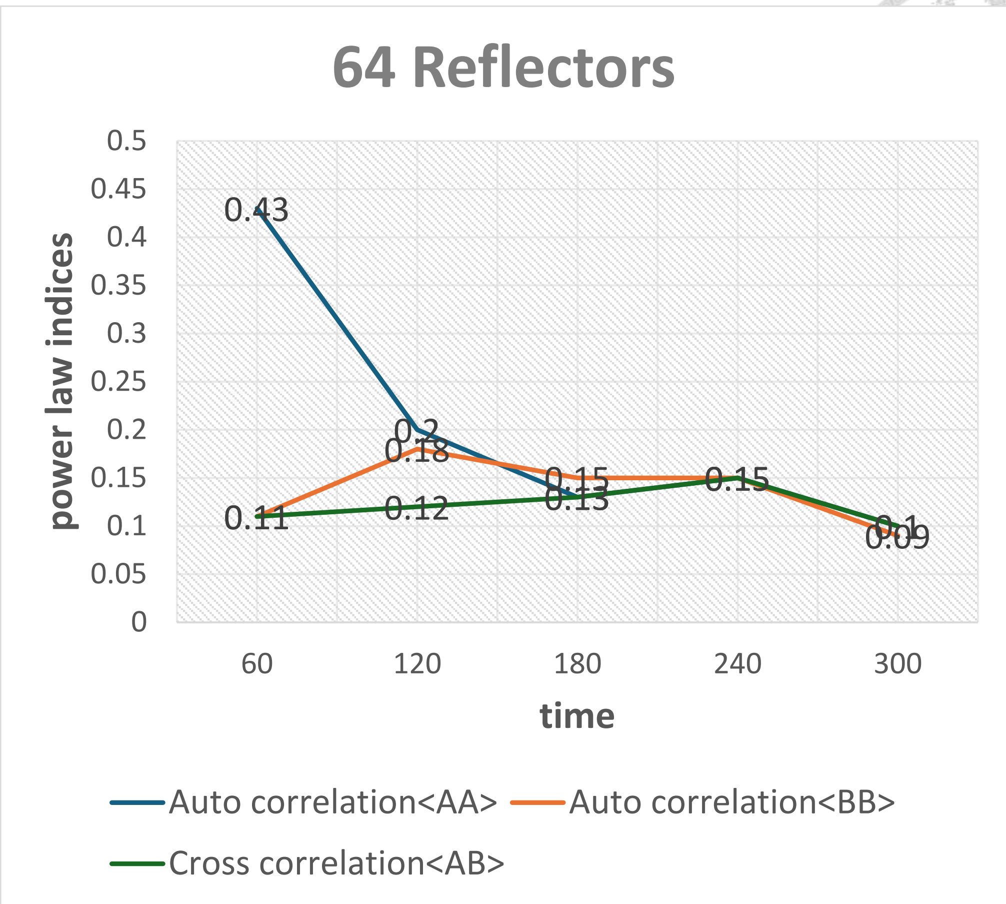

Fig. 28: The power law coefficients of correlations vs. time chart.

4.3 Characteristics of objects:

4.3.1 Blocks:

Due to their position-stable property, blocks can limit the motion of neighboring particles. This can reduce or even eliminate active interactions between blocks and their neighbors, leading to reduced correlations in the vicinity of the blocks.

By comparing $C(\Delta r)\ v.s.\Delta r$ plots for various numbers of blocks and observing wave propagation behavior near the blocks, we can see that blocks have a minimal effect on the correlations in the oscillator system. This minimal impact occurs because, when the wave reaches a block, the wave's amplitude is typically small, causing the block to have an insignificant effect that is not easily noticeable.

doi:10.6342/NTU202404268

4.3.2 Amplifiers:

The ability of amplifiers to increase speed results in stronger interactions with neighboring particles. When an amplifier increases its speed, the motion of particles nearby becomes more active. This can produce more active waves or instability, enhancing the correlation and nonlinear effects in the region near the amplifier. Consequently, the nonlinear effect from a single amplifier can lead to different correlation results depending on the arrangement of amplifiers.

In an amplifier-only system, the auto-correlation of layer 2 may change over time but remains within a certain interval. In the 1-amplifier case, although wave propagation between the two layers weakens over time, a high k_{cross} makes the wave in layer 2 stronger.

4.3.3 Reflectors:

Reflectors operate based on the fulfillment of the reflection condition, causing correlations in the system to decrease rapidly due to phase-reversing behavior, particularly in layer 1. However, over time, this reduction in correlations slows down. Eventually, the system's correlations stabilize at a low-value interval due to the reverse perturbations.

In comparison to amplifiers, reflectors exhibit quite different properties.

4.3.4 The Relation Between Objects and Configuration:

1. Although configurations of amplifiers or reflectors can’t affect whether the system achieves sync, they affect the procedure of formation of sync of the system.

doi:10.6342/NTU202404268

There are 4 examples of different configurations of amplifiers and reflectors, we can observe the second configuration of reflectors shows a keen action of the system at t=120.

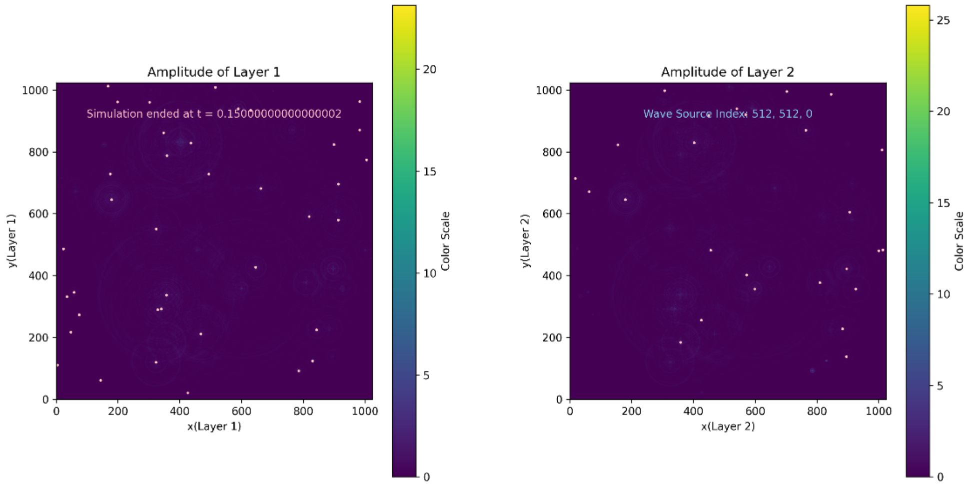

Fig. 29: The randomly-placed amplifiers are positioned in layer 1 and 2--1

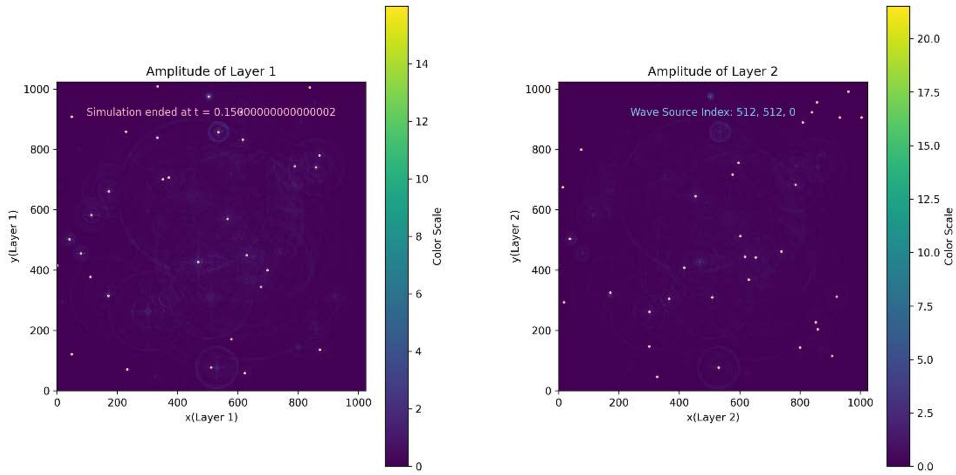

Fig. 30: The randomly-placed amplifiers are positioned in layer 1 and 2—2

doi:10.6342/NTU202404268

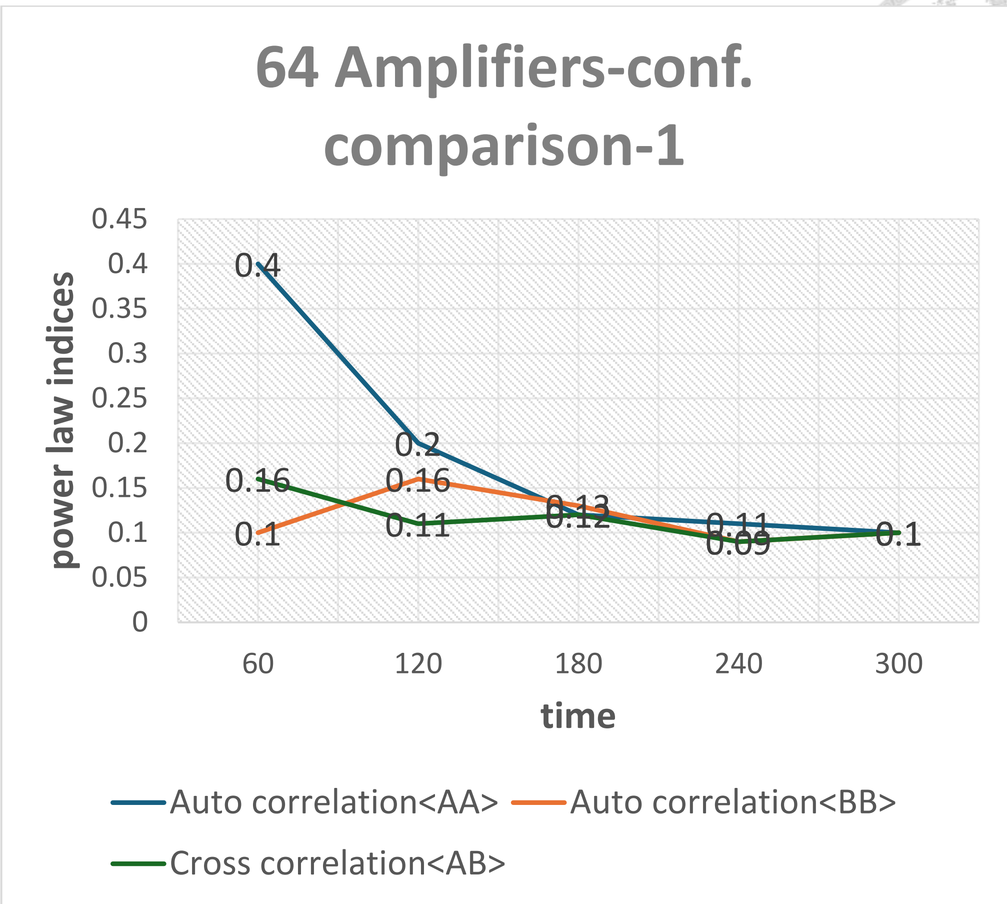

Fig. 31: The power law indices of correlation functions vs. time chart for different configurations of 64 amplifiers—1.

doi:10.6342/NTU202404268

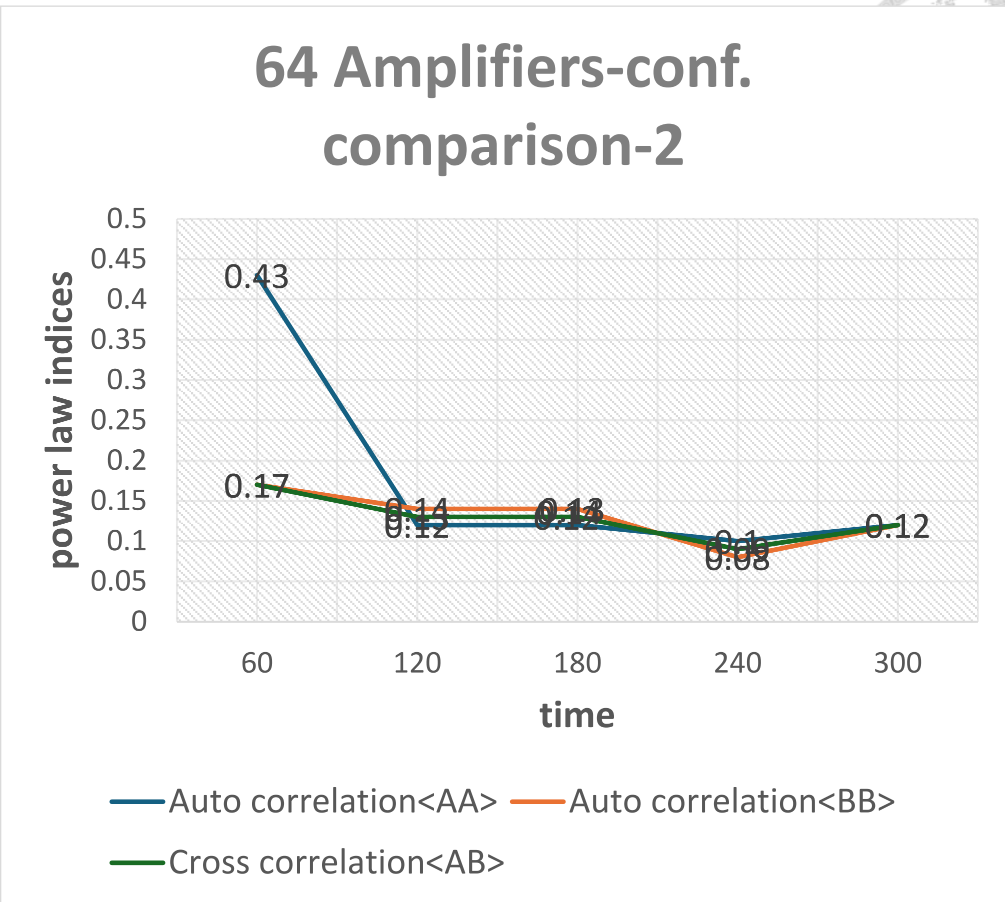

Fig. 32: The power law indices of correlation functions vs. time chart for different configurations of 64 amplifiers—2.

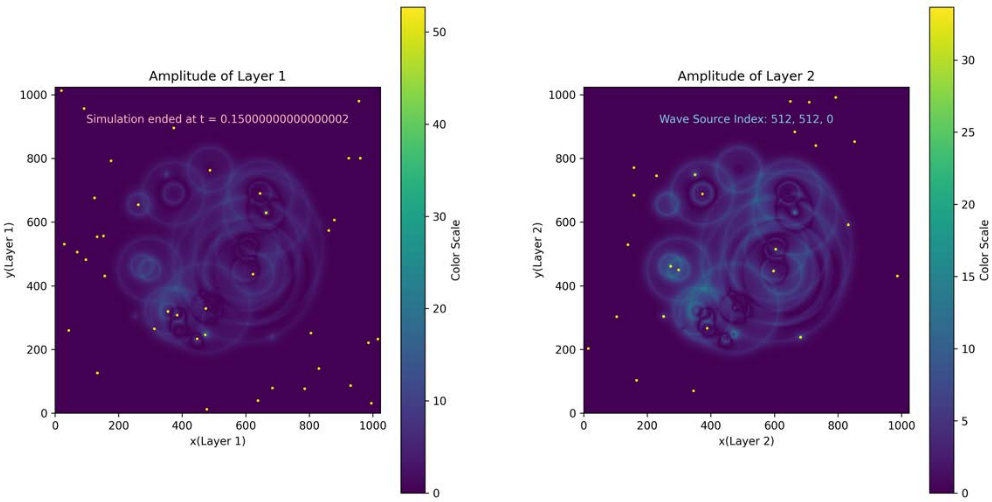

Fig. 33: The randomly-placed reflectors are positioned in layer 1 and 2--1

doi:10.6342/NTU202404268

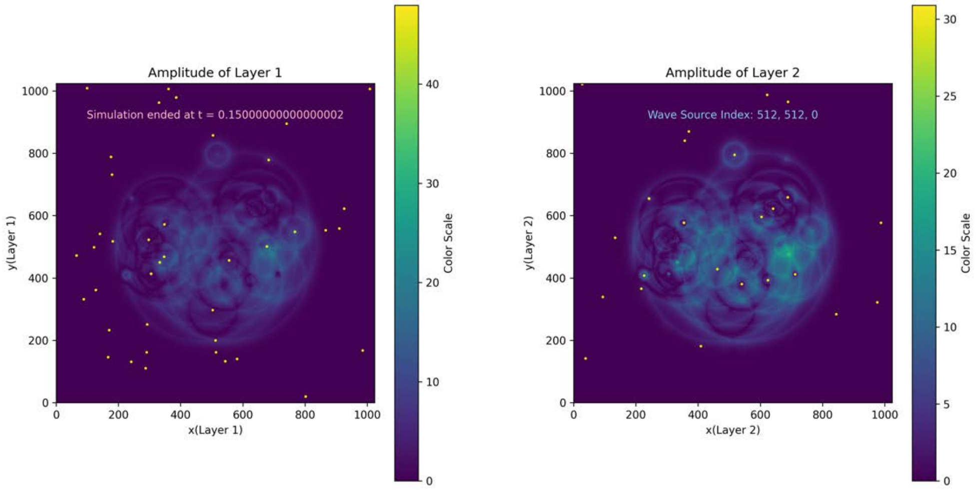

Fig. 34: The randomly-placed reflectors are positioned in layer 1 and 2—2

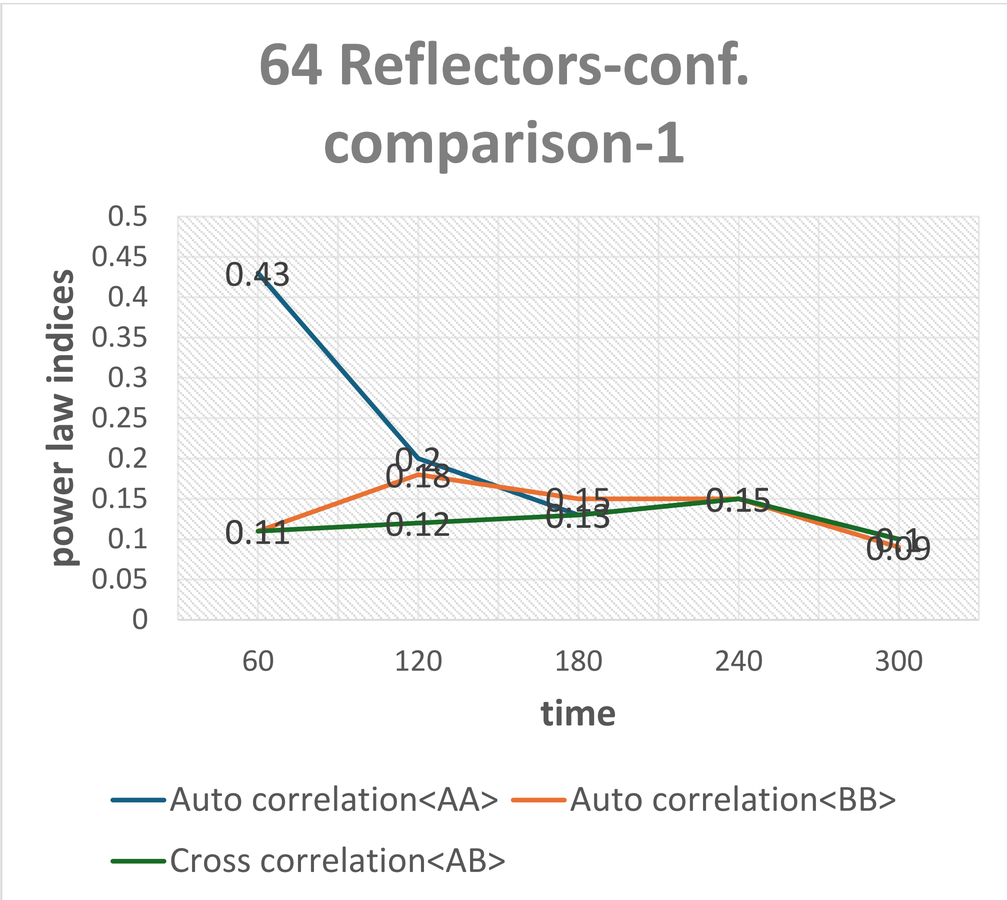

Fig. 35: The power law indices of correlation functions vs. time chart for different configurations of 64 reflectors—1.

doi:10.6342/NTU202404268

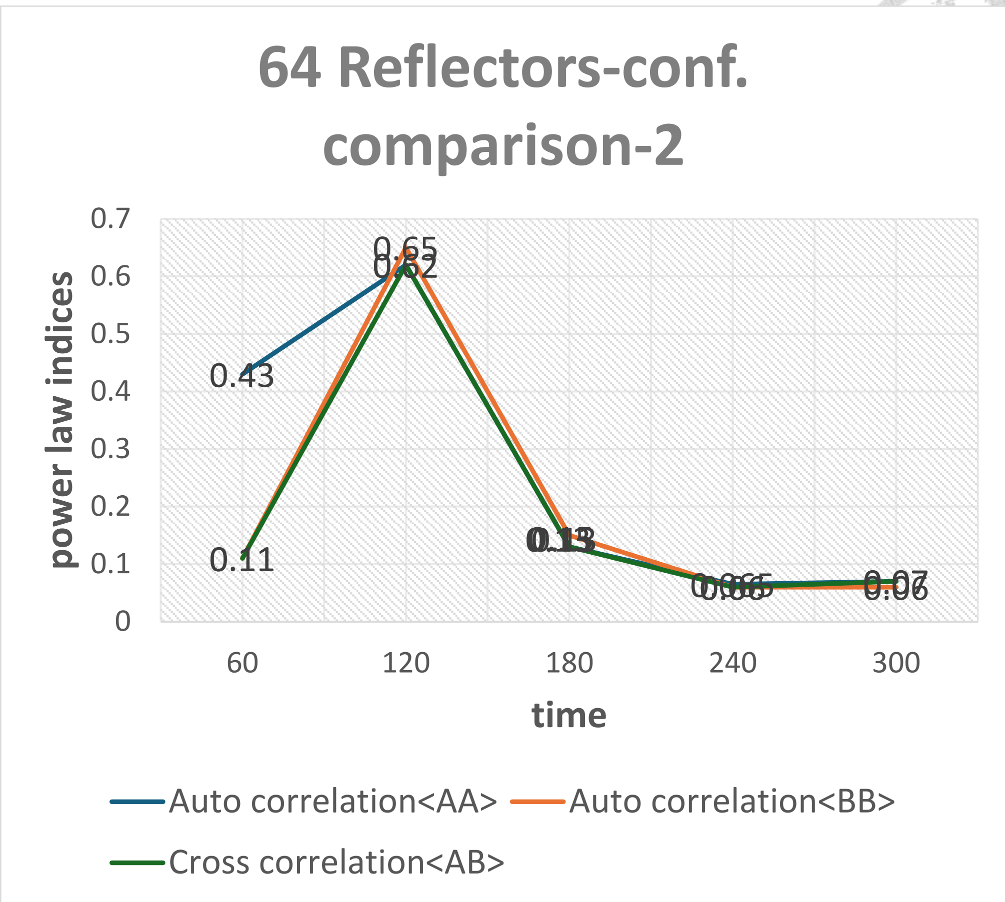

Fig. 36: The power law indices of correlation functions vs. time chart for different configurations of 64 reflectors—2.

doi:10.6342/NTU202404268

Chapter 5 Simulation Result: Combination of 2 objects

From the discussion of individual objects, we can understand the basic effects of each object on the system. Next, we will discuss how the system achieves Sync in conditions that multiple kinds of objects coexist in the system.

In our simulation, we consider the following conditions to observe how combinations of objects affect the system:

1. Amplifiers in layer 1
2. Amplifiers in layer 2

From the conditions which amplifiers are placed in one of the two layers, we acknowledge these conditions can't reach Sync. However, if we choose other objects to distribute in the other layer opposite the layer which amplifiers placed in, the system may achieve sync.

5.1 Amplifiers in layer 1：

5.1.1 Amplifiers are positioned in layer 1 randomly:

t	Auto correlation <AA>	Auto correlation <BB>	Cross correlation <AB>
60	0.05	0.177	0.2
120	0.14	0.08	0.09
180	0.12	0.18	0.16
240	0.12	0.145	0.12

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300	0.2	0.15	0.16

Table 6. The power law coefficients of correlations of 64 amplifiers in layer 1 case vs. time.

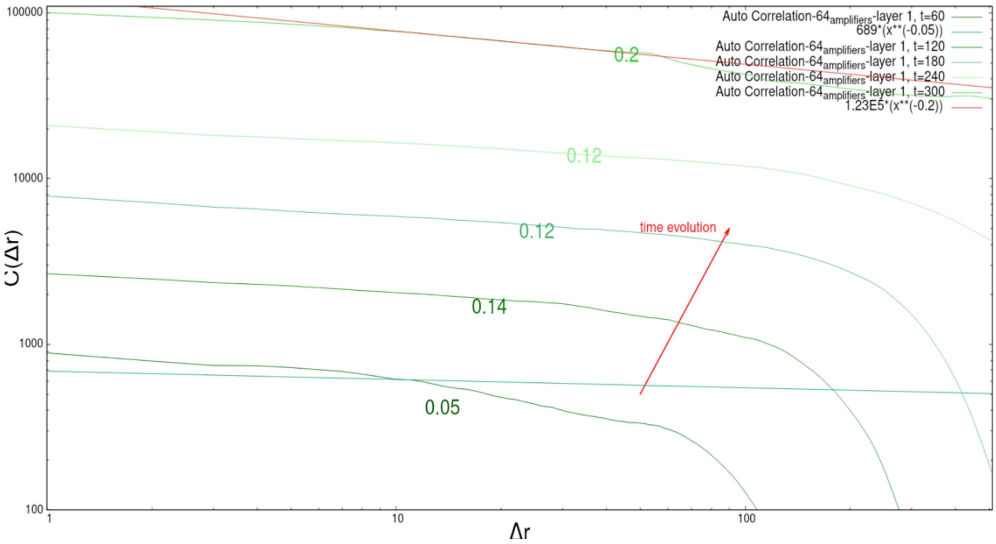

Fig. 37: The auto correlation of layer 1 of 64 amplifiers-in-layer-1-case at different times.

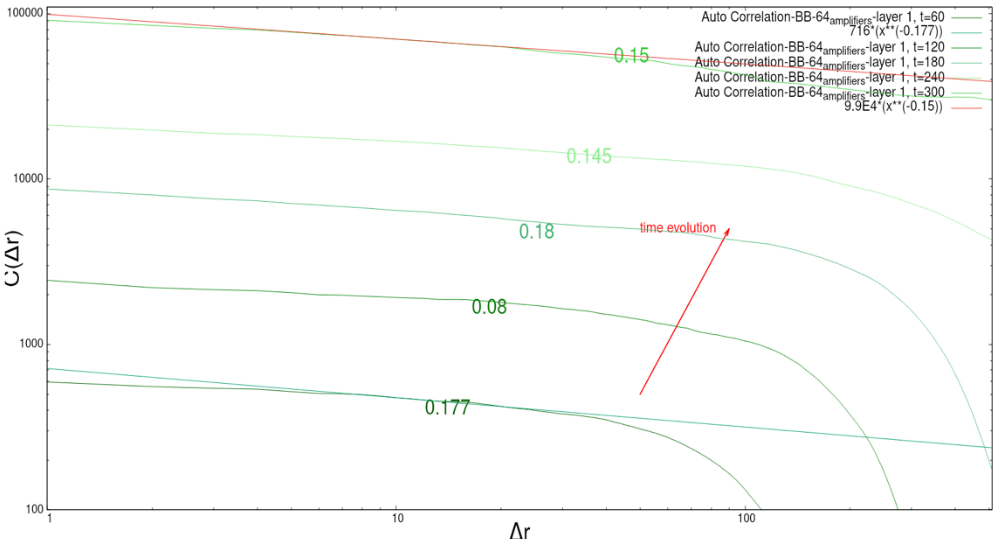

Fig. 38: The auto correlation of layer 2 of 64 amplifiers-in-layer-1-case at different times.

doi:10.6342/NTU202404268

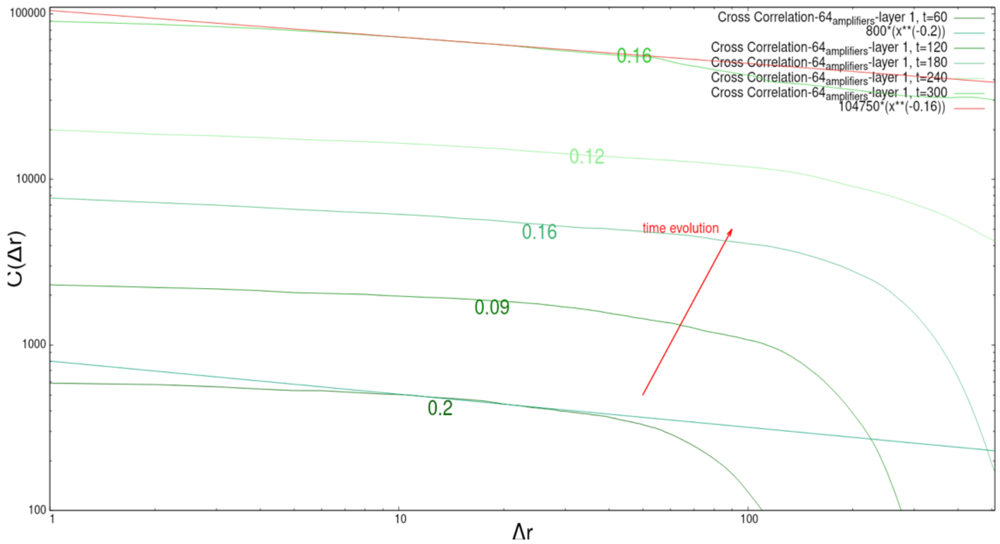

Fig. 39: The cross correlation of 64 amplifiers-in-layer-1-case at different times.

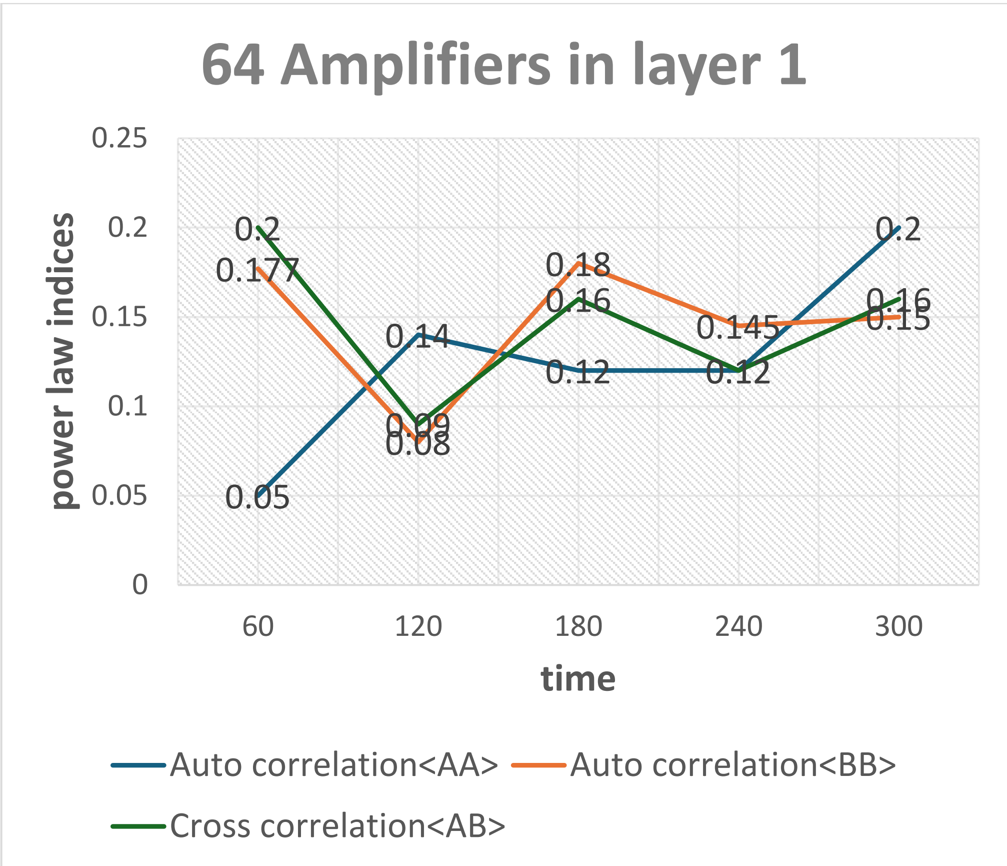

Fig. 40: The power law coefficients of correlations vs. time chart.

5.1.2 Amplifiers are in layer 1 and Blocks are in layer 2:

t	Auto correlation	Auto correlation	Cross correlation

doi:10.6342/NTU202404268

	<AA>	<BB>	<AB>
60	0.4	0.07	0.1
120	0.13	0.28	0.18
180	0.11	0.155	0.11
240	0.105	0.12	0.11
300	0.13	0.19	0.18

Table 7. The power law coefficients of correlations of amplifiers are in layer 1 and blocks are in layer 2 in both layers case vs. time.

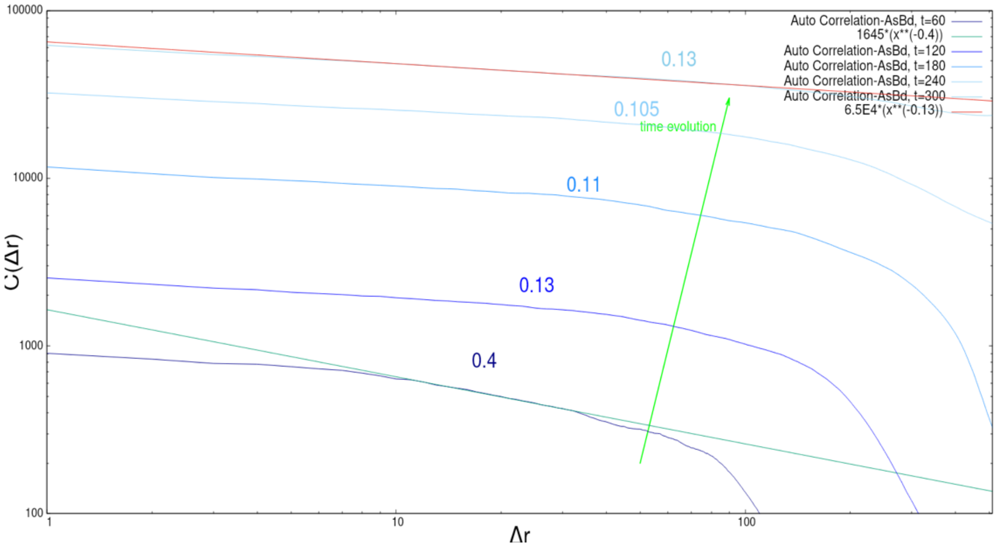

Fig. 41: The auto correlation of layer 1 of amplifiers-and-blocks-case at different times.

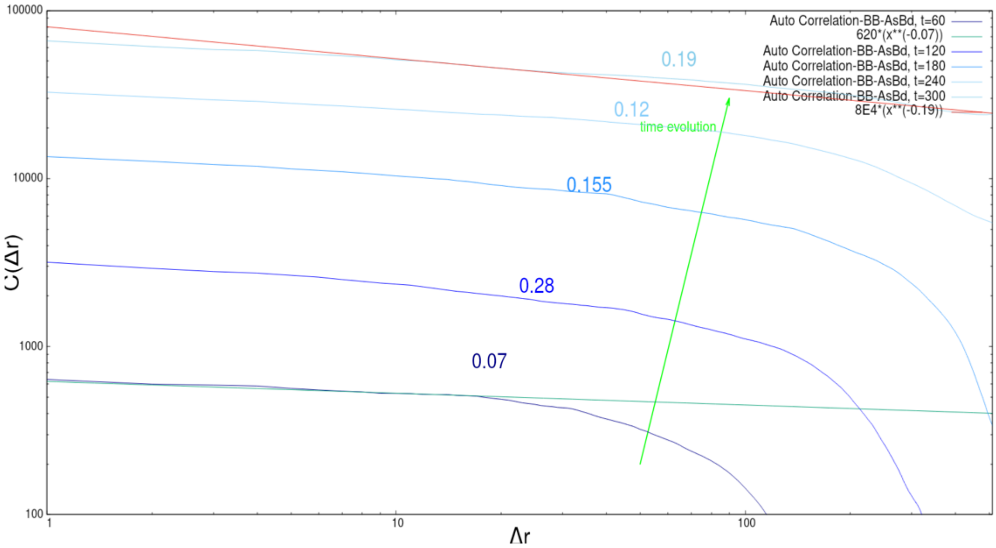

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Fig. 42: The auto correlation of layer 2 of amplifiers-and-blocks-case at different times.

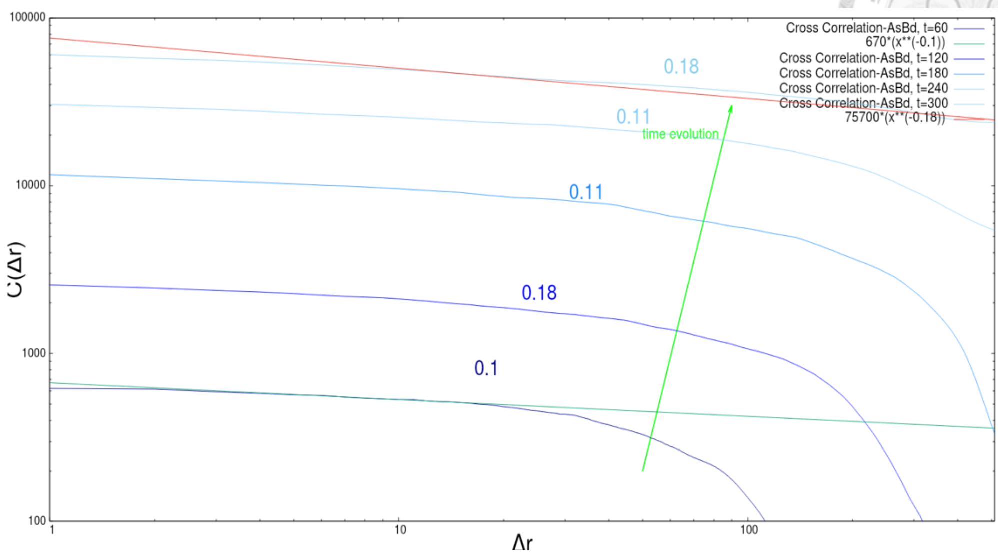

Fig. 43: The cross correlation of amplifiers-and-blocks-case at different times.

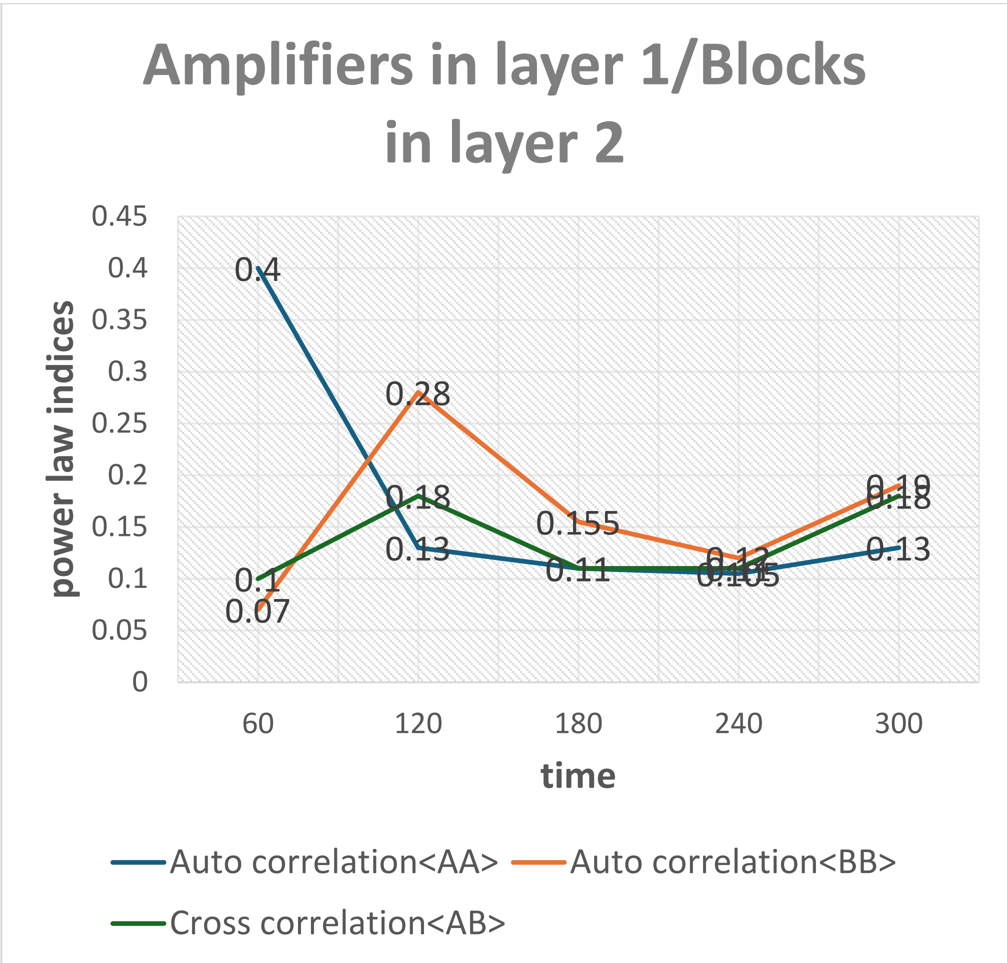

Fig. 44: The power law coefficients of correlations vs. time chart.

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5.1.3 Amplifiers are in layer 1 and Reflectors are in layer 2:

t	Auto correlation <AA>	Auto correlation <BB>	Cross correlation <AB>
60	0.38	0.11	0.13
120	0.2	0.2	0.2
180	0.15	0.15	0.15
240	0.16	0.16	0.16
300	0.15	0.15	0.15

Table 8. The power law coefficients of correlations of amplifiers are in layer 1 and reflectors are in layer 2 in both layers case vs. time.

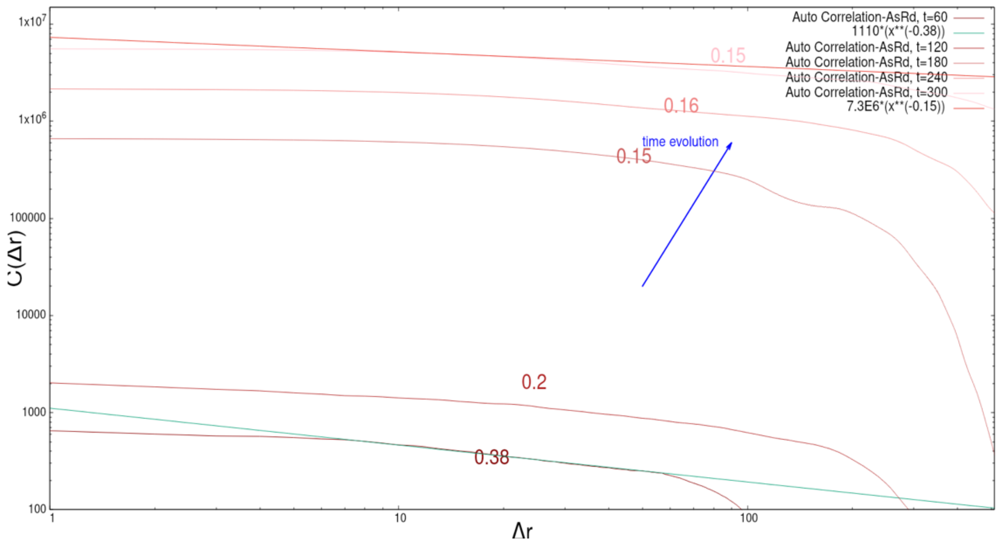

Fig. 45: The auto correlation of layer 1 of 64 amplifiers-and-reflectors-case at different times.

doi:10.6342/NTU202404268

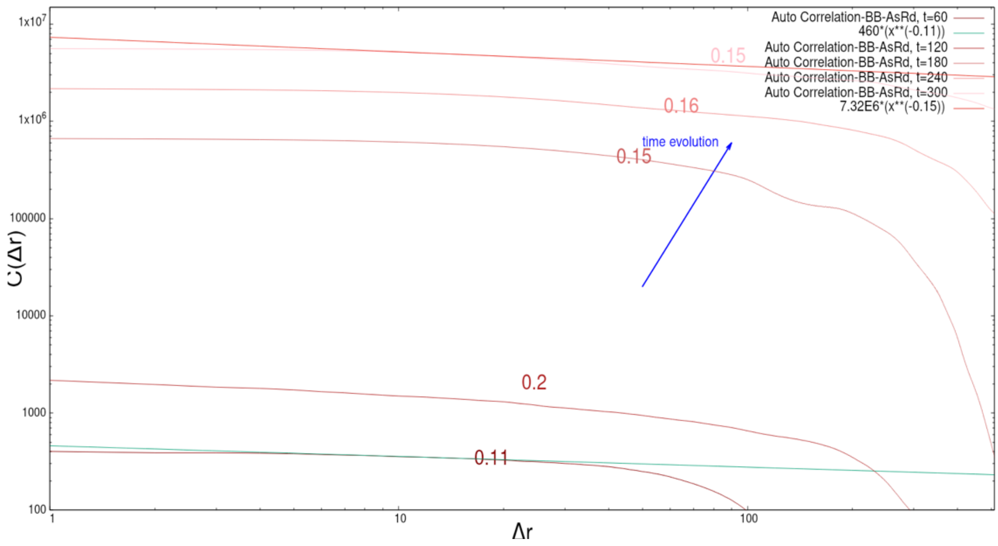

Fig. 46: The auto correlation of layer 2 of amplifiers-and-reflectors-case at different times.

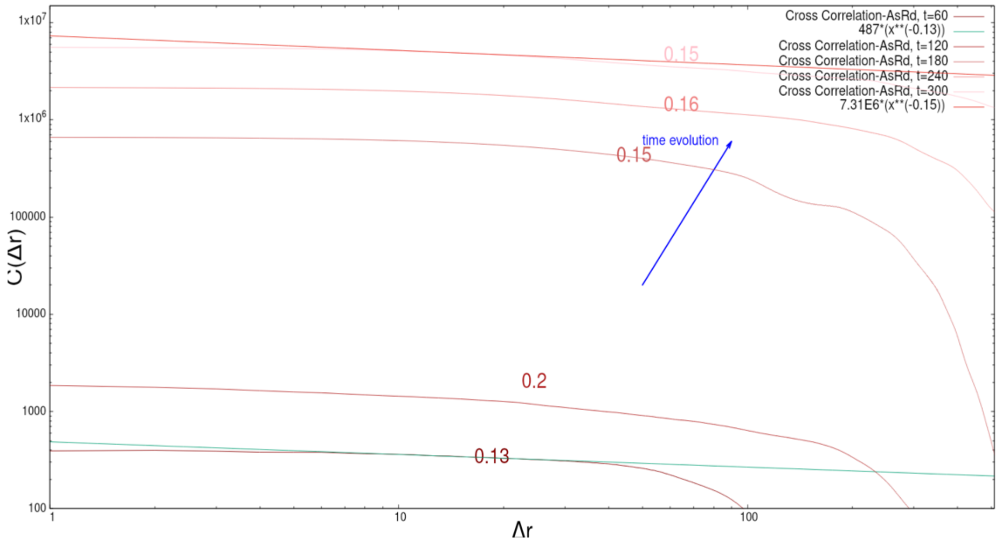

Fig. 47: The cross correlation of amplifiers-and-reflectors-case at different times.

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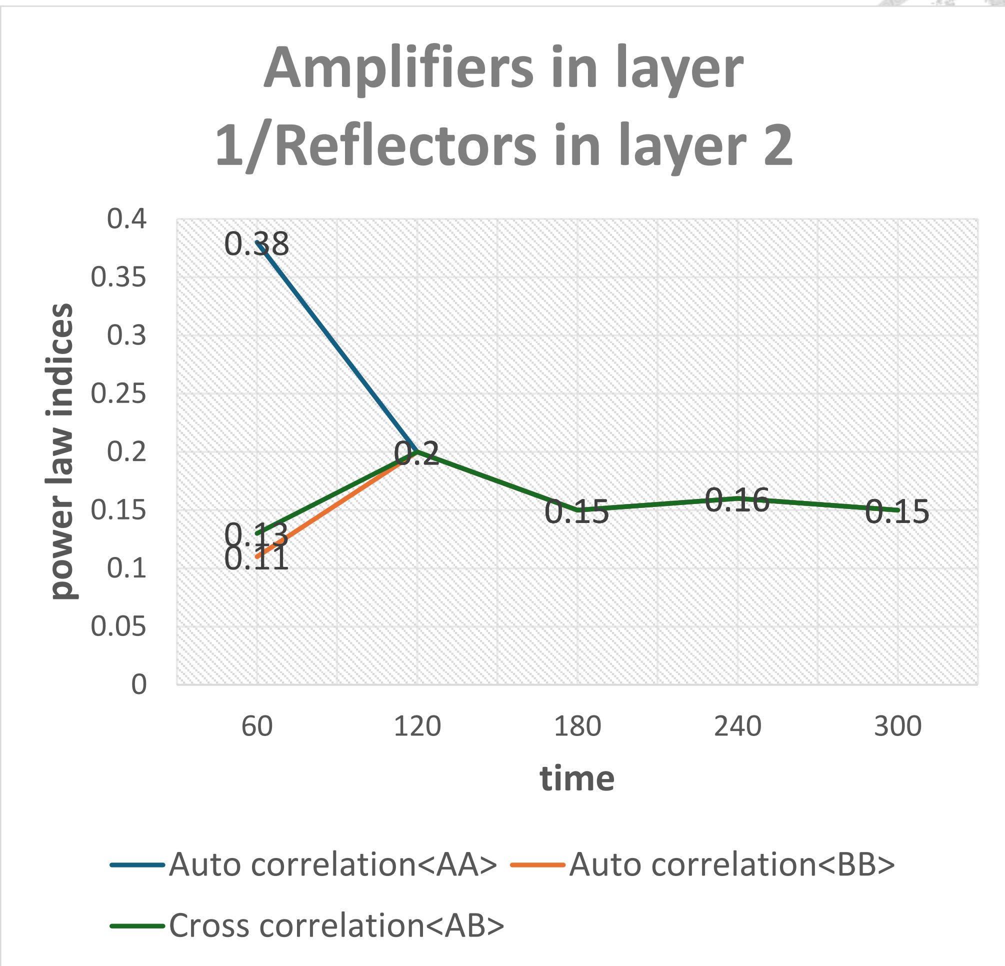

Fig. 48: The power law coefficients of correlations vs. time chart.

5.2 Amplifiers in layer 2：

5.2.1 Amplifiers are positioned in layer 2 randomly:

t	Auto correlation <AA>	Auto correlation <BB>	Cross correlation <AB>
60	0.29	0.14	0.25
120	0.21	0.27	0.2
180	0.16	0.13	0.14
240	0.19	0.11	0.115

doi:10.6342/NTU202404268

300	0.15	0.11	0.11

Table 9. The power law coefficients of correlations of 64 amplifiers in layer 2 case vs. time.

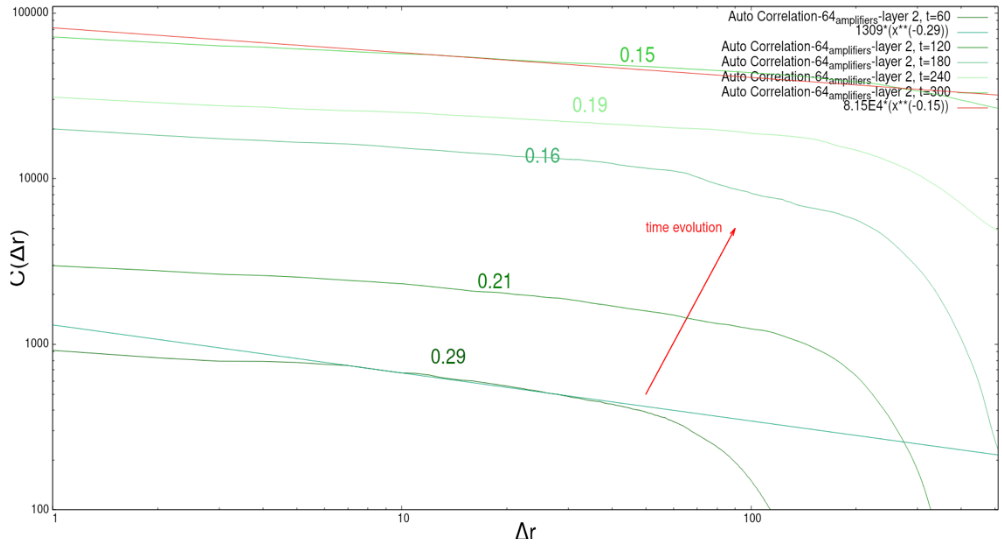

Fig. 49: The auto correlation of layer 1 of 64 amplifiers-in-layer-2-case at different times.

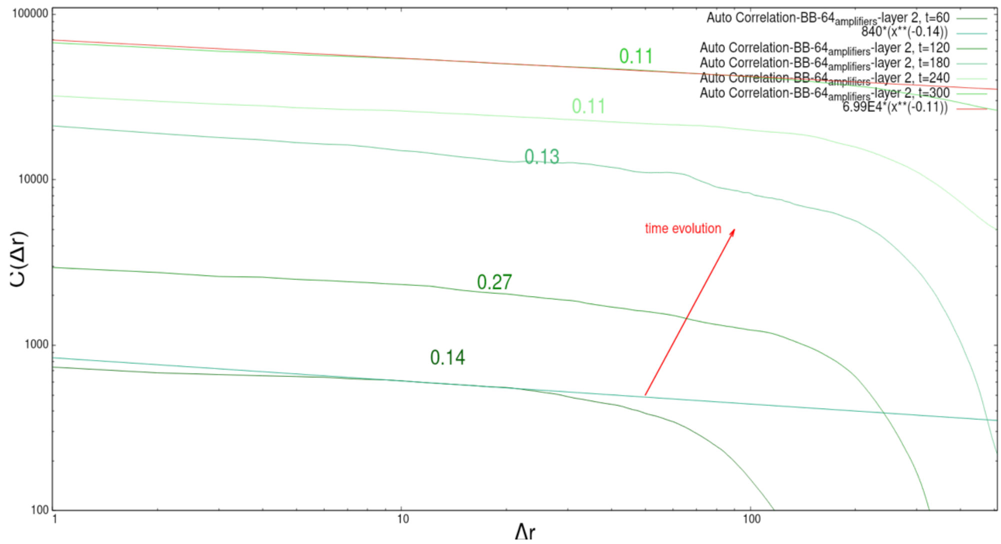

Fig. 50: The auto correlation of layer 2 of 64 amplifiers-in-layer-2-case at different times.

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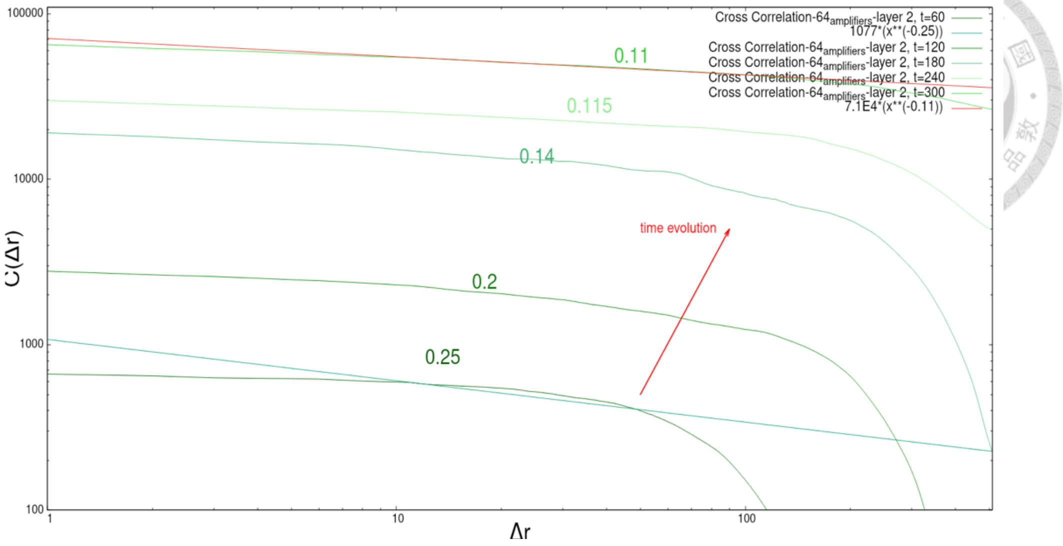


Fig. 51: The cross correlation of 64 amplifiers-in-layer-2-case at different times.

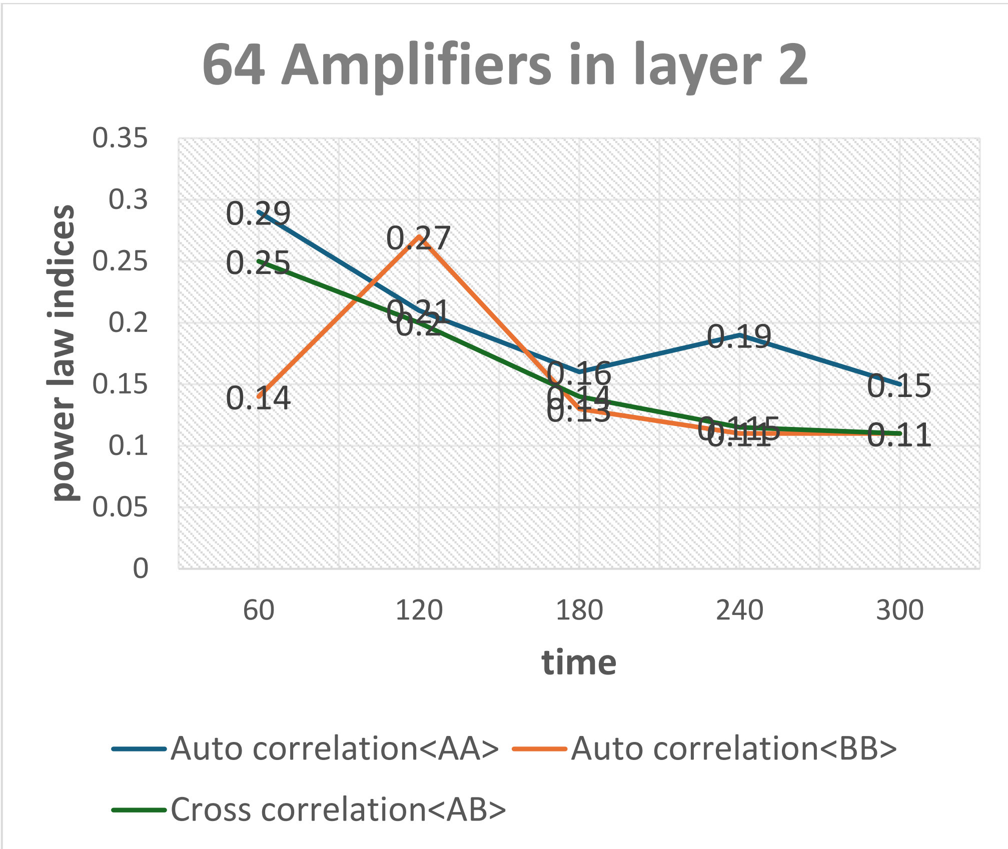


Fig. 52: The power law coefficients of correlations vs. time chart.

5.2.2 Blocks are in layer 1 and Amplifiers are in layer 2

t	Auto correlation	Auto correlation	Cross correlation

doi:10.6342/NTU202404268

	<AA>	<BB>	<AB>
60	0.28	0.08	0.125
120	0.17	0.35	0.19
180	0.18	0.18	0.14
240	0.23	0.2	0.22
300	0.13	0.13	0.13

Table 10. The power law coefficients of correlations of blocks are in layer 1 and amplifiers are in layer 2 in both layers case vs. time.

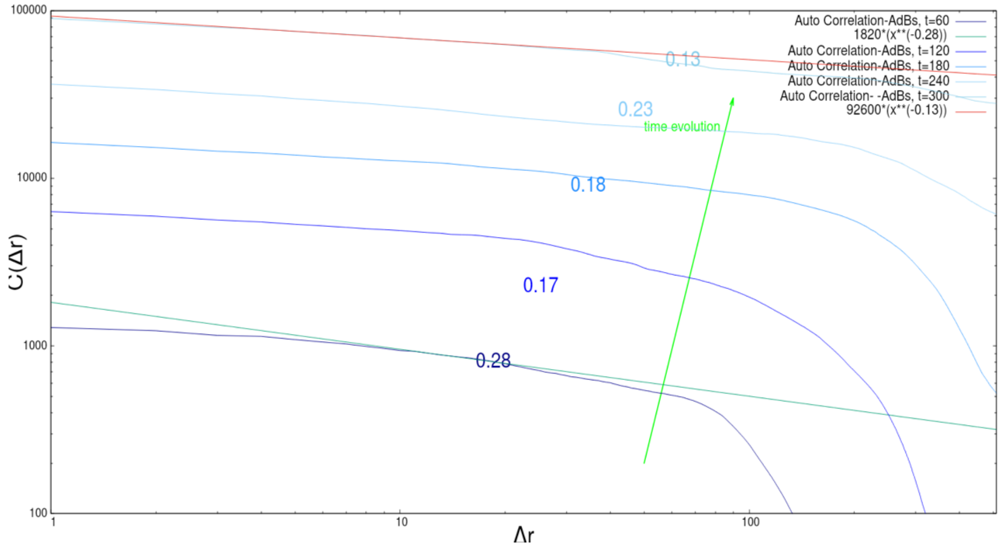
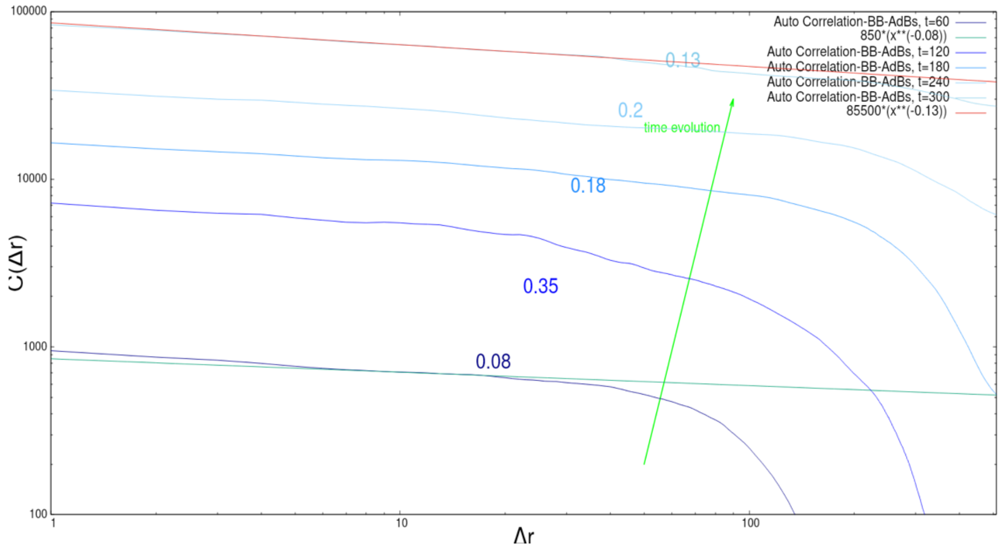


Fig. 53: The auto correlation of layer 1 of amplifiers-and-blocks-case at different times.



doi:10.6342/NTU202404268

Fig. 54: The auto correlation of layer 2 of amplifiers-and-blocks-case at different times.

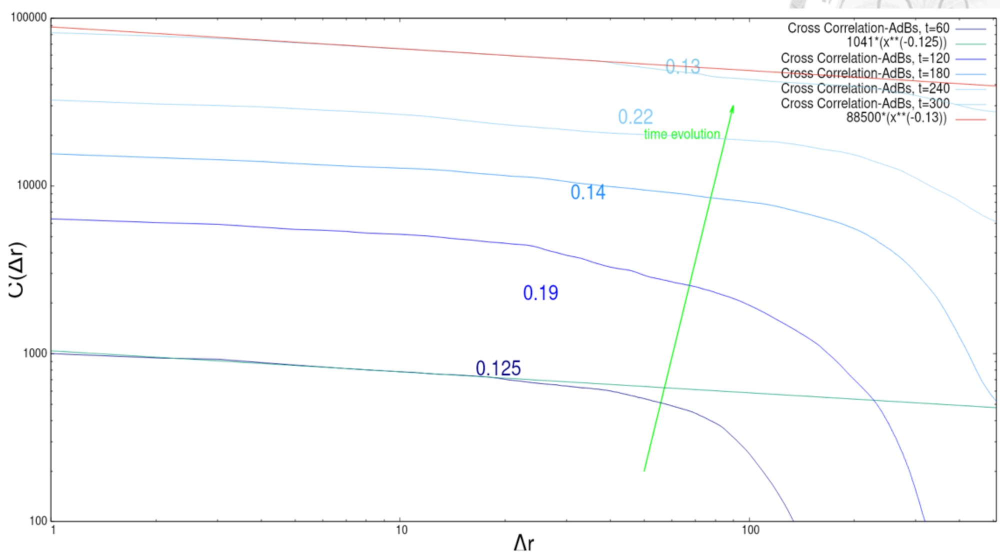


Fig. 55: The cross correlation of amplifiers-and-blocks-case at different times.

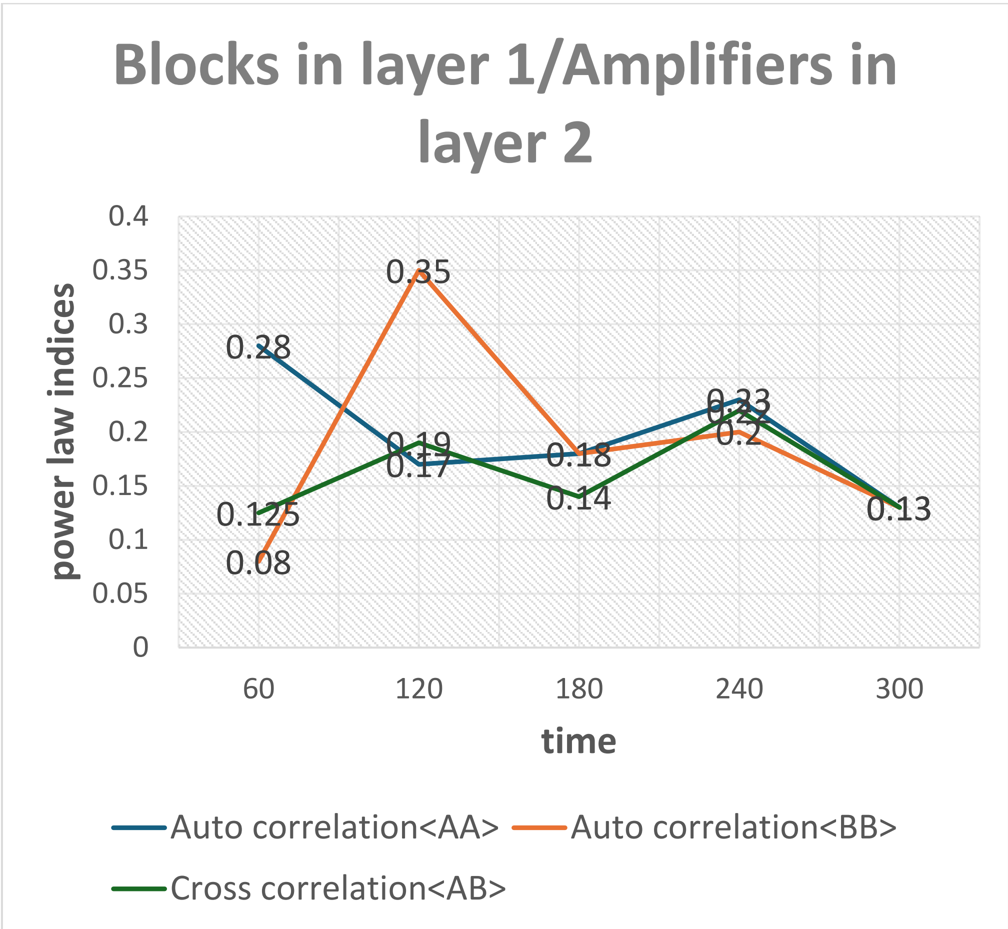


Fig. 56: The power law coefficients of correlations vs. time chart.

doi:10.6342/NTU202404268

5.2.3 Amplifiers are positioned in layer 1 randomly:

t	Auto correlation <AA>	Auto correlation <BB>	Cross correlation <AB>
60	0.37	0.13	0.13
120	0.15	0.19	0.19
180	0.07	0.1	0.1
240	0.09	0.13	0.13
300	0.07	0.07	0.07

Table 11. The power law coefficients of correlations of reflectors are in layer 1 and amplifiers are in layer 2 in both layers case vs. time.

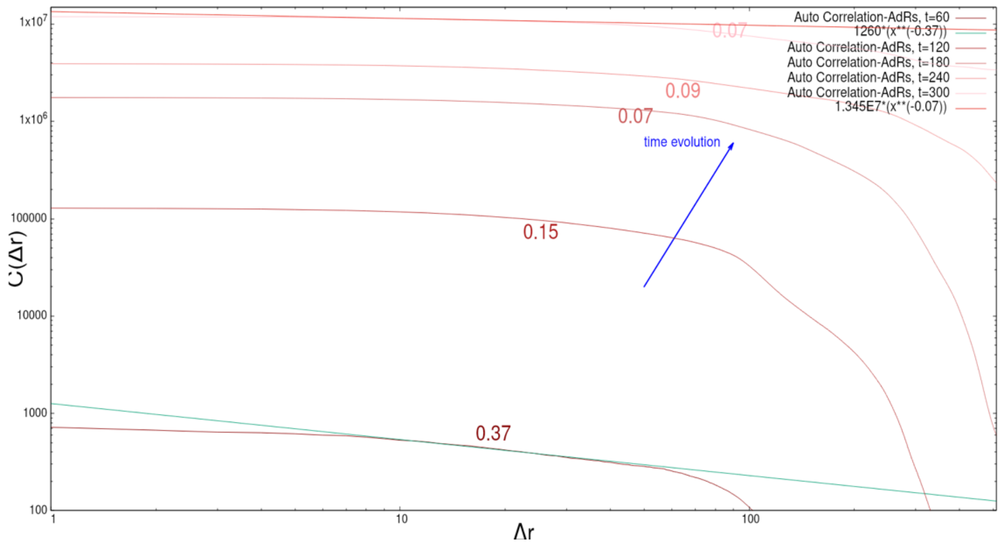


Fig. 57: The auto correlation of layer 1 of 64 amplifiers-and-reflectors-case at different times.

doi:10.6342/NTU202404268

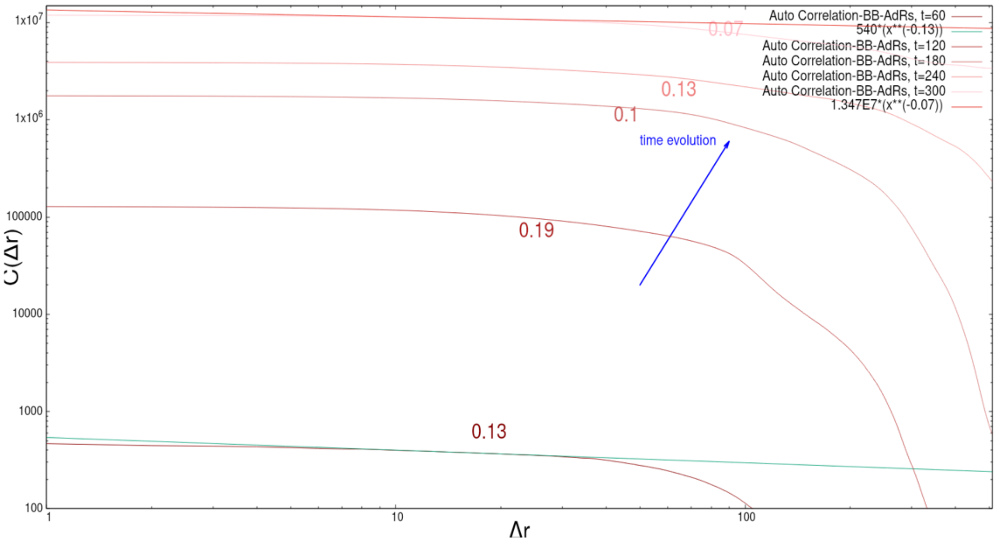


Fig. 58: The auto correlation of layer 2 of 64 amplifiers-and-reflectors-case at different times.

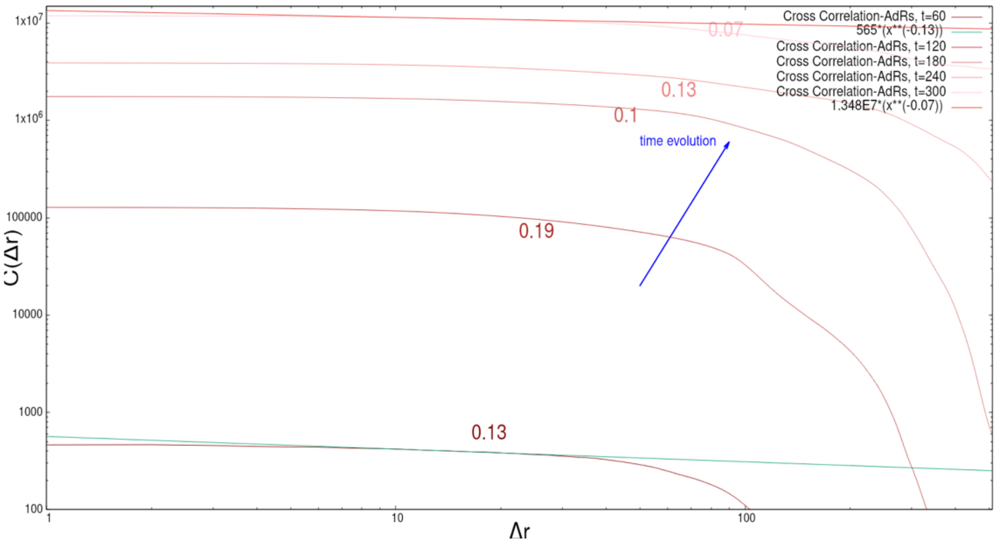


Fig. 59: The cross correlation of 64 amplifiers-and-reflectors-case at different times.

doi:10.6342/NTU202404268

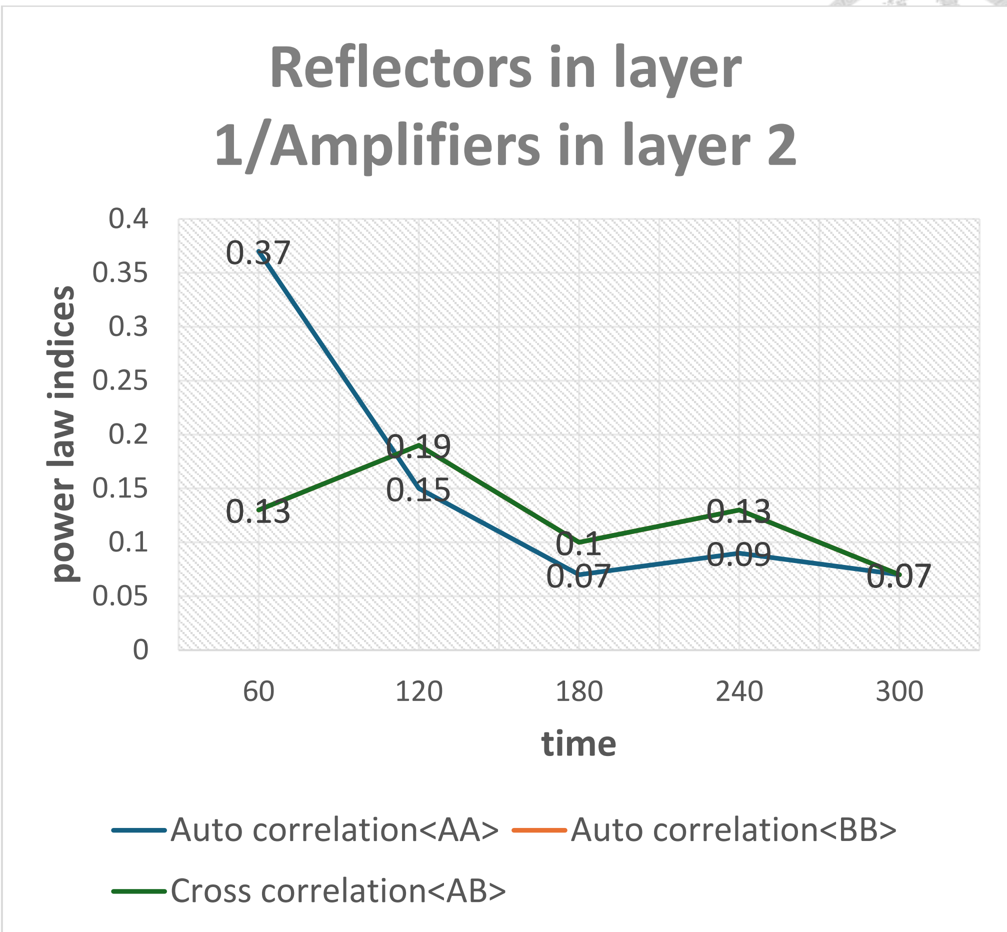


Fig. 60: The power law coefficients of correlations vs. time chart.

5.3 Discussion:

Amplifiers can enhance wave propagation in the early stages, but they may hinder the system from achieving synchronization. In cases where amplifiers are placed in a single layer, the system often fails to reach synchronization.

Blocks, on the other hand, can constrain wave movement while promoting stable wave propagation and synchronization. Reflectors stabilize the oscillation mode by reversing the wave phase, and their effectiveness in stabilizing the wave is greater than that of blocks.

When amplifiers are placed in one layer, they can disrupt the system’s synchronization.

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However, by adding blocks or reflectors to the opposite layer, the combination of amplifiers with blocks or reflectors ensures stable wave propagation. This stability is particularly evident in the auto-correlation of layer 2 and the cross-correlation between layers.

Since the wave originates in layer 1, we observe that systems with amplifiers in layer 1 are more resistant to achieving synchronization compared to systems with amplifiers in layer 2, especially in cases without reflectors. However, when reflectors are introduced, the system begins to show a tendency toward synchronization by t=120, and achieves full synchronization by t=300 in both configurations. Therefore, for optimal and stable wave effects and long-term synchronization, it is advisable to place reflectors in one layer while positioning other wave-enhancing objects in the opposite layer.

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Chapter 6 Summary:

From Chapter 4, we can determine that blocks can reduce the motion of particles and the diffusion of waves, thereby resisting the formation of spontaneous synchronization in the system. However, the influence of a single block is significantly smaller than that of a single amplifier or reflector.

Amplifiers can amplify the amplitude of the wave, thereby promoting wave diffusion in the system. If an amplifier is located near the origin of the wave, its effect is more pronounced. However, the influence of synchronization either positively or negatively, depends on the properties of the wave and the position of the amplifier. In some conditions, amplifiers may resist formation of sync.

Reflectors can reverse the phase of wave, thereby introducing interference to the system. Though the interference may affect wave diffution either positively or negatively, this can promote the synchronization of the system.

From Chapter 5, we conclude that blocks become increasingly important when multiple types of objects coexist in the system. In scenarios where amplifiers or reflectors are present, those objects located near blocks can strengthen the wave. When the wave encounters a block, the block's influence becomes more pronounced than in a system with blocks alone. Blocks can absorb additional wave energy, sustain interactions within the system, and enhance synchronization. However, their effectiveness is lower compared to reflectors.

While our simulation focuses on a strongly-interconnected oscillator system, the findings can also be applied to systems with weaker connections.

For instance, when broadcasting news to a crowd, you can strategically place key opinion

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leaders (KOLs) as amplifiers to disseminate the news directly. Simultaneously, positioning individuals who enjoy gossip (acting as reflectors) can help spread the news more subtly. However, it's essential to be mindful of those who remain unaffected by the news (acting as blocks). By carefully arranging KOLs, gossipers, and the indifferent individuals, you can effectively manage the news broadcast to achieve your desired outcome.

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