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餐廳與第三方外送平台之最適合作策略

Optimal Cooperation Strategies of the Restaurant and the
Third Party Delivery Platform

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本論文係(歐怡君)君(學號：R09741031)在國立臺灣大學商學研究所完成之碩士學位論文，於民國112年4月7日承下列考試委員審查通過及口試及格，特此證明

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系所所長：







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摘要

隨著科技與網路平台的發展，人們對於餐飲的消費習慣也逐漸改變，更仰賴外送平台提供的服務，過去研究認為餐廳的利潤會因為外送平台而減少，但平台也可能為餐廳帶來好處，而在此基礎上，本研究探討餐廳如何與第三方外送平台合作。本研究建立一模型，假設市場上存在一家餐廳與兩家不同的第三方外送平台－高品質與低品質的外送平台，餐廳提供餐點給消費者，在權衡外送平台收取的分潤比例，並將一定比例的分潤成本轉嫁給消費者後，餐廳再決定是否與平台合作。最後，消費者會決定內用、使用任一外送平台或不消費。在我們假設的模型參數下，可以得到餐廳與平台的最適合作策略。我們發現，當兩種平台的服務品質皆遠不及內用時，餐廳只會採取內用策略，而當平台的服務品質提升後，餐廳才會與平台合作。當兩種平台的服務品質與內用相當，甚至超過內用時，餐廳會優先與高品質的平台合作，若低品質平台希望能與餐廳合作，需要以較低的分潤比例進入市場。

關鍵字：最適策略、食物外送平台、隨選服務、多通路營運、分潤





Abstract

With the rapid development of technology and platforms, people's consumption habits of food are changing, and they rely more on the delivery platforms services. Some of past research indicates that the delivery platforms destroy the restaurant's profitability due to the commissions, but platforms may also bring benefits to the restaurants. On this basis, our research explores how will restaurants cooperate with third-party delivery platforms. This thesis establishes a model, there is a restaurant and two kinds of third-party delivery platforms in the market: high-quality and low-quality platforms. The restaurant provides meals to customers, and the restaurant will decide whether to cooperate with the platform depending on the platform's commission rates, and also the restaurant's pass-through rate to customers. In the end, customers will decide to dine-in, use platform service or buy

nothing. Under the model parameters we assume, we can characterize the optimal strategies for the restaurant and platforms. We found that when the service quality of the two platforms is much inferior to dine-in, the restaurant will only adopt the dine-in strategy, and when the service quality of the platforms improves, the restaurant will cooperate with the platform. When the service quality of the two platforms is similar to or even better than dine-in, the restaurant will give priority to cooperating with the high-quality platform. If the low-quality platform expect to cooperate with the restaurant, it needs to the lower commission rate to enter the market.

Keywords: optimal strategy, food delivery platforms, on-demand services, omnichannel operations, commission rates

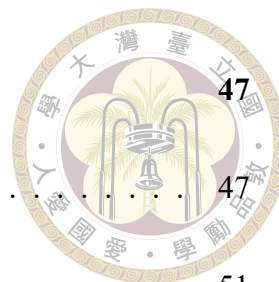




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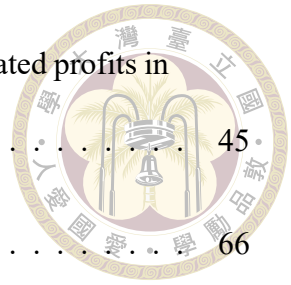




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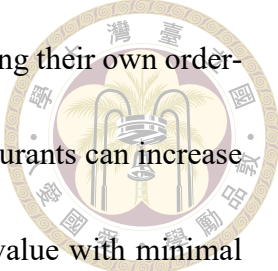




Chapter 1 Introduction

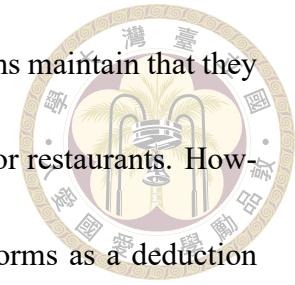
Food delivery platforms, such as Uber Eats, Foodpanda, and Grubhub, are an explosive growth industry, especially after the outbreak of the COVID-19 pandemic. In the US, the revenue of the food delivery market is predicted to exceed \$231.3 billion in 2023 (US Statista, 2022). The worldwide food delivery market is also expected to grow from USD \$167.54 billion in 2022 to USD \$432.32 billion by 2030 (The Brainy Insights, 2022).

In the past, customers usually have to visit the restaurant to get food physically. However, with the development and popularization of mobile and internet technology, food delivery platforms have gradually sprung up in all reaches of the earth. These food delivery services are third-party service platforms: usually through an easy-to-use online ordering system that connects customers and restaurants and has drivers pick up and delivers orders at customers' designated locations (Chen et al., 2022). The convenience of the delivery platforms will attract customers. They can wait at home and not have to go out when the weather is terrible. When the COVID-19 pandemic was severe, it further shaped customers' food delivery habits.



Restaurants can enter the food delivery market without establishing their own ordering system or investing in their own delivery drivers. Moreover, restaurants can increase the visibility of their brand as well as expand customers and create value with minimal investment or operating costs. Despite being favorable to restaurants, whether the food delivery platforms are beneficial is still controversial. Most platforms offer a standard contract that the revenue from each order is split between the platform and the restaurant with the accordingly pre-negotiated ratio. In practice, food delivery platforms typically charge restaurants approximately 20 – 35% commission (Hagiu and Wright. 2021) for each online order as a service fee, leaving the restaurant with only 65 – 80% of its average revenue on each order sold. Due to the already notoriously thin margins in the restaurant industry, which is approximately 5% or less (Biery. 2014), giving any share to third-party platforms could significantly reduce or eliminate the profitability of an order. In response to the reduction in profit margin, restaurants may raise prices on the platform menu to make up for approximately 7 – 44% (BBC News. 2021) compared to dine-in menu prices. However, restaurants may negotiate the commission rates with platforms based on the restaurant's scales. Some restaurants which value service quality may not want to cooperate with the platform for worrying about poor food taste or quality through the platform. Most are relatively high-end and famous restaurants, like Din Tai Fung (CommonWealth Magazine. 2020). The platforms usually want to cooperate with them for more exposure and revenue, so the restaurant has more bargaining power for the commission rates.

Platforms can be a double-edged sword for restaurants. Platforms maintain that they are helping restaurants and pitch themselves as demand generators for restaurants. However, restaurants feel differently. The restaurant saw delivery platforms as a deduction source of income (Isaac and Yaffe-Bellany. 2019).



We consider the relationships between a restaurant and two delivery platforms, where one is high quality and the other is low quality in the supply chain. The restaurant offers the meal to customers. The restaurant considers whether to cooperate with any delivery platform to gain more profits. Then the cooperated delivery platform(s) decides the commission rates, a fraction of the restaurant's revenue. To compensate for the partial loss, the restaurant charges a higher price to the customers who use the delivery platform service and decides this price through a pass-through rate. Then customers choose their dining choices: dine in, use a delivery platform service, or buy nothing. Each customer has a heterogeneous valuation of the meal. If the customers decide to dine in, they may have to wait for a meal in the restaurant. If the customers choose to use the delivery platform, they can stay for the food at home, and their valuation may increase or decrease due to the service quality. The increase or decrease of valuation depends on the delivery platforms' convenience, speed, and customer experience of meal ordering. Therefore, we will consider different scenarios: whether customers enjoy better or poorer dining experiences, and discuss the results.

Our research focuses on the restaurant's operation strategies, and we investigate the following:



- (a) How the restaurant changes its strategies under different platform quality?
- (b) Whether the restaurant cooperates with any delivery platforms, and the associated reasons?
- (c) How do the platforms change the commission rate decisions, and how does the restaurant change the pass-through rate decisions under different cooperation strategies?

We solve our model backward by first solving the restaurant's decision. Given the commission rates, we characterized the unique pass-through rate that maximizes the restaurant's profit that satisfies the first order condition. We also discuss the relationships between the commission rates and the associated pass-through rates, then substitute the equilibria pass-through rates into the delivery platforms' profit functions to derive the optimal commission rates of the delivery platforms. Because our problems are complicated, some optimal solutions cannot get by differential, and they lie in the boundary.

We conduct a numerical study for further managerial insights. Our results show that under Scenario 1 when the service quality levels of both delivery platforms are poorer than dine-in, the restaurant will not cooperate with any delivery platform. When the service quality level of the delivery platform improves, the restaurant will collaborate with both

delivery platforms and even will not adopt the dine-in strategy.



Under Scenario 2, the service quality levels of both delivery platforms are better than dine-in. The restaurant will only adopt a dine-in strategy when both platforms' service quality is low. When both platforms' service quality improves, the restaurant will cooperate with both platforms simultaneously. Furthermore, the restaurant will only cooperate with platform H when platform H has much better service quality than platform L . Unless platform L lower the commission rates, the restaurant will not cooperate with platform L .

Under Scenario 3, the service quality levels of delivery platform H are better than dine-in and platform L , and delivery platform L are poorer than dine-in. When platform L 's service quality is almost the same as platform H , the restaurant will cooperate with platform L with a lower commission rate. Otherwise, the restaurant will only cooperate with platform H .

This thesis is organized as follows. Chapter 2 discusses the survey of relevant literature. Chapter 3 describes the model in detail. The equilibria of the cases in that customers prefer buying from a restaurant that provides diverse consumption channels are in Chapter 4. For further managerial insights, we conduct a numerical study in Chapter 5. Chapter 6 concludes our research and extension.



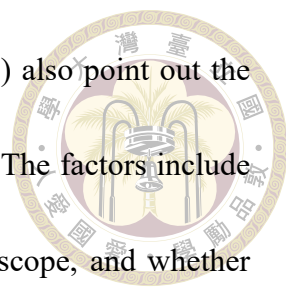


Chapter 2 Literature

With the rapid development of the online food delivery industry, the restaurant may consider providing meals through platform services. The related works of literature discuss several topics, and we will discuss these theses in the following parts:

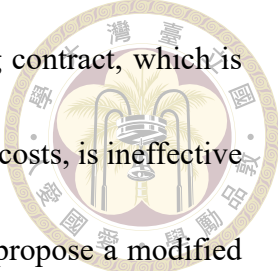
- (a) The motivations for the restaurant to cooperate with third-party food delivery platforms.
- (b) The contractual relationships between a restaurant and a third-party delivery platform.
- (c) The operation of omnichannel retailing and the interaction of online and offline stores.
- (d) The interactions between the restaurant and third-party delivery platforms.

Restaurants can build their food delivery platforms or cooperate with third-party food delivery platforms. Tao, Gou, and Zhang (2020) state that delivery cost and delivery service coverage are crucial to a merchant's decisions. When the delivery cost is high and delivery service coverage is vast, the merchant prefers a third-party delivery platform




rather than delivering by itself. Moreover, See-Kwong et al. (2017) also point out the motivations for merchants to cooperate with a third-party platform. The factors include the third-party platform's advertising effect, convenience, delivery scope, and whether the merchant can increase revenue. Du, Fan, and Gao (2021) consider more in detail. They conclude that when the advertising effect of the platform and customer benefits are both large, the merchant should choose the mode of third-party platform and self-delivery. Zhang et al. (2021) demonstrate that whether a manufacturer self-delivers or cooperates with third-party platforms depends on third-party platforms' commission rates, consumer inconvenience cost, and marginal cost. When the cost of inconvenience to consumers is low and the marginal cost is high, the manufacturer will build its platform; when the cost of inconvenience to consumers is high, and the marginal cost is low, the manufacturer will join a third-party platform. Niu et al. (2021) demonstrate that the restaurant prefers to use third-party platform logistics when the online market potential is low because this helps the restaurant reduce logistics investment costs. Unlike Du, Fan, and Gao (2021), we do not consider building a self-delivery fleet for the restaurant in our thesis. Still, we put more emphasis on cooperation way with third-party delivery platforms.

When the restaurant cooperates with a third-party delivery platform, they have to negotiate contractual relationships. Feldman, Frazelle, and Swinney (2022) and Chen, Hu, and Wang (2022) study the contractual relationships between restaurants and deliv-



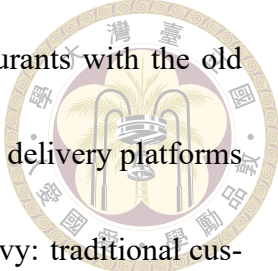
ery platforms. They consistently find that the single revenue sharing contract, which is the restaurant shares a partial revenue for the platform as commission costs, is ineffective in coordinating the system. Feldman, Frazelle, and Swinney (2022) propose a modified contract that the delivery platforms also share a fraction of platform revenue and a fixed fee to the restaurant for each meal order, which coordinates the restaurant and platform systems. There is more literature about revenue sharing contracts on supply chain contracts. Cachon and Lariviere (2005) demonstrate that contract coordination failures are common. A revenue-sharing contract can achieve coordination depending on the contract parameters, such as the retailer's share of the revenue for each unit. See Cachon (2003) and Lariviere (2015) for more reviews of contract literature. Our model adopts the single revenue-sharing contract because this is more consistent with the real-world situation.

Our thesis is also related to the operations of omnichannel retailing, that is, an integration of offline stores with online channels. The associated studies are thriving, including Gallino and Moreno (2014), Bell et al. (2018), Gao and Su (2017), and Baron, Chen, and Li (2022). Baron, Chen, and Li (2022) demonstrate a paradox that when both offline and online channels are used in equilibrium, online ordering can improve sales and reduce waiting times, but it may inadvertently reduce customers' personal utility and social welfare. The lower the cost of online orders and waiting, the more social welfare is reduced. Gallino and Moreno (2014) point out that omnichannel has already been a trend, and the




challenge for retailers is not deciding whether or not to do it but how to implement it most efficiently. Bell et al. (2018) indicate that online order technology will likely benefit customers, workers, and firms. Besides, it also shows that offline stores increase online and overall sales and improve operational efficiency. Hu et al. (2021) deliver that whether a retailer benefits from or harms an omnichannel strategy depends on the visiting cost to the store and the online waiting cost. Retailers can benefit from the omnichannel strategy if the costs are relatively low. Overall, omnichannel seems an inevitable trend with the rapid development of technology and platform economy. Most of the thesis only focuses on two roles, one merchant and one third-party channel, but in our thesis, we consider a multichannel strategy, including dine-in and two delivery platforms.

Our thesis puts focuses on the interaction between third-party delivery platforms and restaurants. Feldman, Frazelle, and Swinney (2022) especially consider customers choosing between delivery platforms or dine-in in a restaurant market. Customers would experience a waiting cost proportional to the platform's total volume and dine-in customers. The restaurant's profitability may decline when cooperating with the delivery platform. With the outbreak of the COVID-19 pandemic, Raj, Sundararajan, and You (2020) find evidence that small restaurants have increased their activity on the platform following the closure of the dine-in channel. They also find that platforms have a positive short-term effect on specific small restaurants. Raj and Eggers (2022) also demonstrate that the pen-



etration of platforms increases the likelihood of exit for those restaurants with the old business model. Chen, Hu, and Wang (2022) study the impact of food delivery platforms on a restaurant. They categorize customers as traditional and tech-savvy: traditional customers can only get a meal by walking into the restaurant, while tech-savvy customers can access a food delivery platform online. They demonstrate that the platform services may not bring extra demand for the restaurant but only change the composition of the customer group. They also show that when the food delivery system is adequately convenient, including enough tech-savvy customers and delivery workers, the restaurant can become a delivery-only kitchen and raise the food price, but this may hurt the profitability of the food delivery platforms. Restaurants can build their food delivery platforms or cooperate with third-party food delivery platforms. Tao, Gou, and Zhang (2020) state that delivery cost and delivery service coverage are crucial to a merchant's decisions. When the delivery cost is high and delivery service coverage is vast, the merchant prefers a third-party delivery platform rather than delivering by itself. Moreover, See-Kwong et al. (2017) also point out the motivations for merchants to cooperate with a third-party platform. The factors include the third-party platform's advertising effect, convenience, delivery scope, and whether the merchant can increase revenue. Du, Fan, and Gao (2021) consider more in detail. They conclude that when the advertising effect of the platform and customer benefits are both large, the merchant should choose the mode of third-party platform and self-delivery. Zhang et al. (2021) demonstrate that whether a manufacturer self-delivers



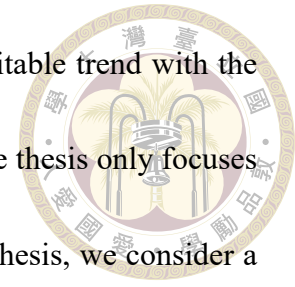
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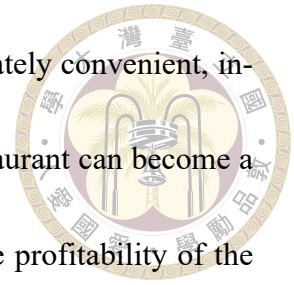
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group. They also show that when the food delivery system is adequately convenient, including enough tech-savvy customers and delivery workers, the restaurant can become a delivery-only kitchen and raise the food price, but this may hurt the profitability of the food delivery platforms.







Chapter 3 The Model

We consider a mathematical model in which the restaurant, upon serving meals to end customers, may cooperate with two external delivery platforms in the channel. To investigate the interactions between two different delivery platforms, in our model, one restaurant (R) offers the meal that charges at $p \in [0, 1]$, which is exogenously determined. The restaurant R decides how to sell meals to the customers and whether to cooperate with any delivery platforms. The choices include : (1) dine-in only, (2) cooperating with one or both delivery platforms with dine-in or (3) cooperating with one or two delivery platforms only without dine-in.

In the channel, there exist two external delivery platforms, H and L , whereas platform H possesses a higher quality than platform L perceived by the customers. This quality difference may vary due to delivery speed and customer experience of the meal order. Suppose restaurant R decides to cooperate with the delivery platforms. In that case, the restaurant R has to share a fraction of its revenue with the platform as a commission cost, decided by the delivery platforms. Let $c_i \in [0, 1]$ be the commission rates charged

by platform i while cooperating with restaurant R , where $i = H, L$. Note that such a commission rate can be as high as 20 – 35% in practice (Hagiu and Wright, 2021). In addition to the commission cost, the restaurant R charges a pass-through rate to each customer who orders the meal through the delivery platforms. Such pass-through rates can compensate for part of the loss of the restaurant due to the commission charged from the delivery platform. Let β_i be the pass-through rate the restaurant charges customers where $i = H, L$. Therefore, customers need to pay more if they order the meal through delivery platforms.

Customers' behaviors In the market, each customer has heterogeneous valuations toward the meal, V , which is uniformly distributed on a $[0,1]$ line. We normalize the customers' population to 1 without the loss of generality. If the customers decide to dine in, they may have to wait for a meal inside the restaurant and hence will incur a waiting cost, $q_I > 0$ (Buell, 2017). If the customers decide to order the meal through the platforms, they do not incur the waiting cost, and their valuation will increase or decrease due to the service provided by the delivery platforms. Define $0 < \gamma_L < \gamma_H < 2$ as the multipliers of service rewards for customers who use any delivery platform. That is, when customers order meals through the delivery platform i , their valuations toward the meal and the service become $V\gamma_i$. Also, ordering through platform H enhances a better experience compared to platform L . In the subsequent analysis, we will investigate three scenarios depending

on whether each γ_i is larger than 1. $\gamma_i > 1$ represents the case where the customers enjoy better dining experiences through the delivery platforms and thus increase their valuations. On the other hand, customers would downgrade their expectations when ordering through the platforms.



Based on the above settings, the expected utility of the dine-customer, U_I is the valuation V minus the food price p , and the waiting cost q_I : $U_I = V - p - q_I$. For customers who decide to order through the delivery platforms, the expected utility U_i is the valuation with the multiplier $R\gamma_i$ minus the food price p , and the part of the commission cost $p\beta_i c_i$: $U_i = V\gamma_i - p - p\beta_i c_i$. If customers decide not to consume, they receive zero utility. Note that in our model, we assume each customer is rational; hence, the customers will choose the one that gives the maximum and non-negative utility.

To facilitate the analysis more easily, let V_i be the indifferent valuation of having meals between dine-in, through delivery platforms, and no purchase, where $i = 1 \dots 6$, the following shows six indifferent valuations in detail:

(a) V_1 represents the indifferent valuation between dine-in and through delivery platform

H , thus, we have:

$$V_1 = \frac{q_I - p\beta_H c_H}{1 - \gamma_H}. \quad (1)$$

(b) V_2 represents the indifferent valuation between dine-in and through delivery platform

L , thus, we have

$$V_2 = \frac{q_I - p\beta_L c_L}{1 - \gamma_L}. \quad (2)$$

(c) V_3 represents the indifferent valuation between through delivery platform H and L ,

thus, we have

$$V_3 = \frac{p\beta_H c_H - p\beta_L c_L}{\gamma_H - \gamma_L}. \quad (3)$$

(d) V_4 represents the indifferent valuation between dine-in and no purchase; thus, we have

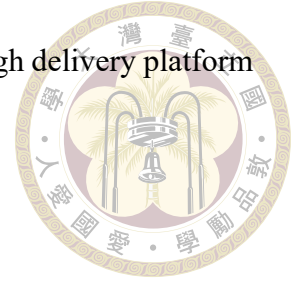
$$V_4 = p + q_I > 0. \quad (4)$$

(e) V_5 represents the indifferent valuation between through delivery platform H and no

purchase, thus, we have

$$V_5 = \frac{p(1 + \beta_H c_H)}{\gamma_H} > 0. \quad (5)$$

(f) V_6 represents the indifferent valuation between through delivery platform L and no



purchase, thus, we have

$$V_6 = \frac{p(1 + \beta_L c_L)}{\gamma_L} > 0.$$



In our model, we consider the three scenarios: (1) $0 < \gamma_L < \gamma_H < 1$, (2) $1 < \gamma_L < \gamma_H < 2$ and (3) $0 < \gamma_L < 1 < \gamma_H < 2$, to discuss the tradeoffs of the restaurant between the extra profit from cooperating with the delivery platforms and the commission costs the restaurant would incur. Our goal is to show how the delivery platforms decide the optimal commission rates and how the restaurant decides the pass-through rates for the customers to maximize their associated profits in the market under each scenario. The following is the summary of the sequence of the events. We illustrate the model parameters and the decision variables of each player in Table 3.1, 3.2.

The sequence of the events:

- (a) The restaurant decides how to sell the meal to the customers, dine-in only, cooperating with one or two delivery platforms with dine-in, or cooperating with one or two food delivery platforms without dine-in.
- (b) If the restaurant cooperates with any of the delivery platforms, the cooperated delivery platform(s) then sets the commission rate(s) c_i .
- (c) The restaurant sets the pass-through rate(s) β_i .

(d) Customers decide their dining choices: dine-in, use delivery platform service, or buy nothing.



(e) Customers' demands are realized, and restaurant's and delivery platforms' profits can be calculated.

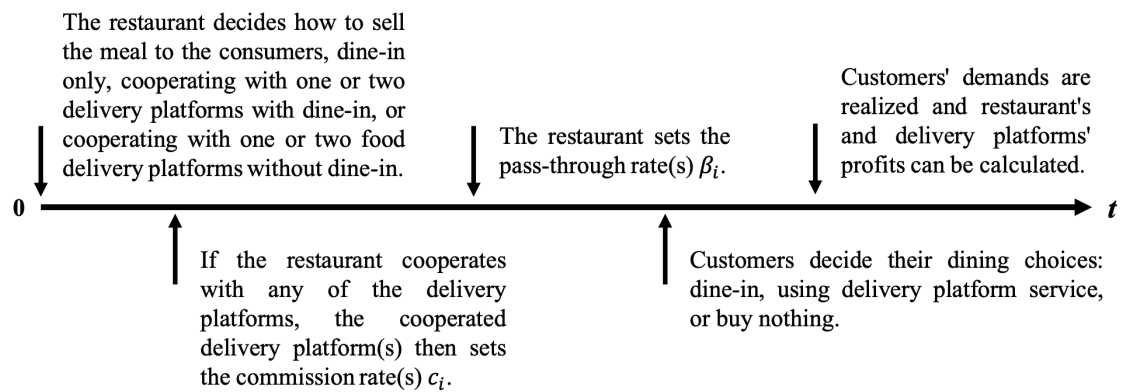


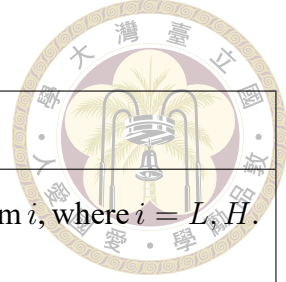
Figure 3.1: Sequence of events.

Table 3.1: The model parameters.

Model parameter	Explanation
p	The food price, $p > 0$.
V	The service reward. $V \in [0, 1]$.
γ_i	The multiplier of the service reward through delivery platform i where $i = L, H$ and $\gamma_L < \gamma_H$.
q_I	The waiting cost for dine-in, $q_I > 0$.

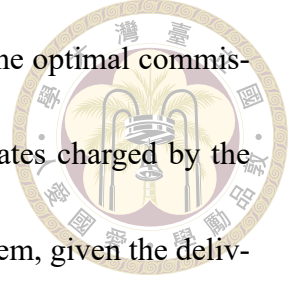
Table 3.2: The decision variables.

Decision variable	Explanation
c_i	The commission rate charged by delivery platform i , where $i = L, H$. $c_i > 0$.
β_i	The pass-through rate (proportion of commission rate) that customers need to afford through platform i , where $i = L, H$. $\beta_i > 0$.



Considering how the restaurant sells the meal to the customers, we represent all the possibilities depending on the relative locations of V_i where $i = 1 \dots 6$, and whether V_i is greater than one or less than zero. By removing unreasonable instances, we have 30 cases in each scenario, which is summarized in the appendix. We use some notations to represent the way a restaurant provides the meal: "X" represents no purchase, "I" represents dine-in, and "H" and "L" respectively represent higher-level and lower-level delivery platforms.

In our model, the profit margin that the restaurant earns from dine-in is the whole food price, p . As for the cooperation with delivery platforms, the profit margin is the food price, p minus the commission cost and plus the part of the commission cost charged to the customer, $p - pc_i + pc_i\beta_i$. On the other hand, the profit margin that the delivery platform earns is from the commission, pc_i . The demand structure is based on the preference of marginal customer who is indifferent between dine-in, through delivery platforms, and no purchase. We summarize the restaurant and platforms' profit functions in each scenario in Tables 3.3, 3.4, and 3.5.



In the following chapters, we use backward induction to solve the optimal commission rates charged by the delivery platforms and the pass-through rates charged by the restaurant. We first consider the restaurant's pass-through rates problem, given the delivery platforms' decisions. Then we solve the delivery platforms' problems to characterize the optimal commission rates decisions given the restaurant's best response. Finally, we discuss the optimal choices of the restaurant regarding whether to include delivery platforms in the business.

Table 3.3: The profit functions of Scenario 1.

Case	Restaurant's profit	Platform H 's profit	Platform L 's profit
(1) I	$p(1 - V_4)$	0	0
(2) $I + L$	$p(1 - c_L + c_L\beta_L)(V_2 - V_6) + p(1 - V_2)$	0	$pc_L(V_2 - V_6)$
(3) L	$p(1 - c_L + c_L\beta_L)(1 - V_6)$	0	$pc_L(1 - V_6)$
(4) $I + H$	$p(1 - c_H + c_H\beta_H)(V_1 - V_5)$	$pc_H(V_1 - V_5)$	0
(5) H	$p(1 - c_H + c_H\beta_H)(1 - V_5)$	$pc_H(1 - V_5)$	0
(6) $I + H + L$	$p(1 - c_L + c_L\beta_L)(V_3 - V_6)$ $+p(1 - c_H + c_H\beta_H)(V_1 - V_3)$ $+p(1 - V_1)$	$pc_H(V_1 - V_3)$	$pc_L(V_3 - V_6)$
(7) $H + L$	$p(1 - c_L + c_L\beta_L)(V_3 - V_6)$ $+p(1 - c_H + c_H\beta_H)(1 - V_3)$	$pc_H(1 - V_3)$	$pc_L(V_3 - V_6)$

Table 3.4: The profit functions of Scenario 2.

Case	Restaurant's profit	Platform H 's profit	Platform L 's profit
(1) I	$p(1 - V_4)$	0	0
(2) $I + L$	$p(1 - c_L + c_L\beta_L)(1 - V_2) + p(V_2 - V_4)$	0	$pc_L(1 - V_2)$
(3) L	$p(1 - c_L + c_L\beta_L)(1 - V_6)$	0	$pc_L(1 - V_6)$
(4) $I + H$	$p(1 - c_H + c_H\beta_H)(1 - V_1) + p(V_1 - V_4)$	$pc_H(1 - V_1)$	0
(5) H	$p(1 - c_H + c_H\beta_H)(1 - V_5)$	$pc_H(1 - V_5)$	0
(6) $I + H + L$	$p(1 - c_L + c_L\beta_L)(V_3 - V_2)$ $+p(1 - c_H + c_H\beta_H)(1 - V_3)$ $+p(V_2 - V_4)$	$pc_H(1 - V_3)$	$pc_L(V_3 - V_2)$
(7) $H + L$	$p(1 - c_L + c_L\beta_L)(V_3 - V_6)$ $+p(1 - c_H + c_H\beta_H)(1 - V_3)$	$pc_H(1 - V_3)$	$pc_L(V_3 - V_6)$

Table 3.5: The profit functions of Scenario 3.

Case	Restaurant's profit	Platform H 's profit	Platform L 's profit
(1) I	$p(1 - V_4)$	0	0
(2) $I + L$	$p(1 - c_L + c_L\beta_L)(V_2 - V_6) + p(1 - V_2)$	0	$pc_L(V_2 - V_6)$
(3) L	$p(1 - c_L + c_L\beta_L)(1 - V_6)$	0	$pc_L(1 - V_6)$
(4) $I + H$	$p(1 - c_H + c_H\beta_H)(1 - V_1) + p(V_1 - V_4)$	$pc_H(1 - V_1)$	0
(5) H	$p(1 - c_H + c_H\beta_H)(1 - V_5)$	$pc_H(1 - V_5)$	0
(6) $I + H + L$	$p(1 - c_L + c_L\beta_L)(V_2 - V_6)$ $+p(1 - c_H + c_H\beta_H)(1 - V_1)$ $+p(V_1 - V_2)$	$pc_H(1 - V_1)$	$pc_L(V_2 - V_6)$
(7) $H + L$	$p(1 - c_L + c_L\beta_L)(V_3 - V_6)$ $+p(1 - c_H + c_H\beta_H)(1 - V_3)$	$pc_H(1 - V_3)$	$pc_L(V_3 - V_6)$





Chapter 4 Analysis

In this chapter, we formulate the restaurant's and platforms' profit functions and then solve the restaurant's and platforms' problems by determining the optimal pass-through rates and commission rates. We consider three scenarios, and for each scenario, we discuss one instance to show the varied customers' behaviors. Due to the complexity of the analysis, we put the remaining analysis in the appendix.

4.1 Scenario 1: $0 < \gamma_L < \gamma_H < 1$

In this scenario, $0 < \gamma_L < \gamma_H < 1$ implies that both delivery platforms have poorer service quality than dine-in; hence, customers may not have better experiences through the platforms. Here, we demonstrate one case: the restaurant provides the meal through the dine-in and the two delivery platforms simultaneously and show the interplay between the restaurant and two delivery platforms. The remaining cases will be summarized in the appendix. We use *LHI* to represent the case where the restaurant provides the dine-

in and two delivery platforms. The restaurant decides the pass-through rates in order to maximize the profit after observing the commission rates decided by the platform(s).



Given the price, p , the waiting cost, q_I , the service reward multiplier through the delivery platform, γ_i , and commission cost, c_i , we can formulate the restaurant's profit function as π_R^{LHI} , where $i = H, L$:

$$\begin{aligned}
 \pi_R^{LHI} &= p(1 - c_L^{LHI} + c_L^{LHI} \beta_L^{LHI})(V_3 - V_6) + \\
 &\quad p(1 - c_H^{LHI} + c_H^{LHI} \beta_H^{LHI})(V_1 - V_3) + p(1 - V_1) \\
 &= p(1 - c_L^{LHI} + \beta_L^{LHI} c_L^{LHI}) \left(\frac{\beta_H^{LHI} c_H^{LHI} p - \beta_L^{LHI} c_L^{LHI} p}{\gamma_H - \gamma_L} - \frac{p + \beta_L^{LHI} c_L^{LHI} p}{\gamma_L} \right) + \\
 &\quad p(1 - c_H^{LHI} + \beta_H^{LHI} c_H^{LHI}) \left(\frac{-\beta_H^{LHI} c_H^{LHI} p + q_I}{1 - \gamma_H} + \frac{-\beta_H^{LHI} c_H^{LHI} p + \beta_L^{LHI} c_L^{LHI} p}{\gamma_H - \gamma_L} \right) + \\
 &\quad p \left(1 - \frac{-\beta_H^{LHI} c_H^{LHI} p + q_I}{1 - \gamma_H} \right). \tag{1}
 \end{aligned}$$

In equation (7), $p(1 - c_L^{LHI} + c_L^{LHI} \beta_L^{LHI})$ represents the restaurant's profit margin that cooperates with platform L , and $V_3 - V_6$ represents the demand of platform L ; $p(1 - c_H^{LHI} + c_H^{LHI} \beta_H^{LHI})$ represents the restaurant's profit margin that cooperates with platform H , and $V_1 - V_3$ represents the demand of platform H ; p represents the restaurant's profit margin from a dine-in customer, and $1 - V_1$ represents the demand of dine-in. We substitute V_i from equations (1) to (6) into (7), then we can obtain the second equation.

To obtain the optimal pass-through rates of the restaurant, we first check the Hessian matrix of the restaurant's profit function. Here we define H^{LHI} as the determinant of the

Hessian matrix of the restaurant's profit function. The maximum of the restaurant's profit will exist when $H^{LHI} > 0$. The Hessian matrix can be derived from equation (7):



$$H^{LHI} = \det \begin{bmatrix} \frac{\partial^2 \pi_R^{LHI}}{\partial \beta_H^{LHI^2}} & \frac{\partial^2 \pi_R^{LHI}}{\partial \beta_H^{LHI} \partial \beta_L^{LHI}} \\ \frac{\partial^2 \pi_R^{LHI}}{\partial \beta_L^{LHI} \partial \beta_H^{LHI}} & \frac{\partial^2 \pi_R^{LHI}}{\partial \beta_L^{LHI^2}} \end{bmatrix} = \frac{-4c_H^{LHI^2} c_L^{LHI^2} p^4}{(-1 + \gamma_H)(\gamma_H - \gamma_L)\gamma_L}.$$

With $\gamma_H > \gamma_L$ and $0 < \gamma_L < \gamma_H < 1$ in our model setting, we obtain $H^{LHI} > 0$, and hence, the optimal pass-through rate solutions uniquely exist. The pass-through rate solutions can be derived by the first-order conditions of π_R^{LHI} , that is, $\partial \pi_R^{LHI} / \partial \beta_H^{LHI} = 0$, and $\partial \pi_R^{LHI} / \partial \beta_L^{LHI} = 0$. We obtain:

$$\begin{aligned} \beta_H^{LHI} &= \frac{q_I \gamma_H + p(-2 + c_H^{LHI} + 2\gamma_H)}{2c_H^{LHI} p}, \\ \beta_L^{LHI} &= \frac{q_I \gamma_L + p(-2 + c_L^{LHI} + 2\gamma_L)}{2c_L^{LHI} p}. \end{aligned} \quad (2)$$

Given the restaurant's best response, the platforms decide the commission rates. We substitute the pass-through rate in equation (8) into the platforms' profit function and solve the platforms' problems. Let π_i^{LHI} be platform i 's profit function where $i = H, L$, we



have:

$$\begin{aligned}
 \pi_H^{LHI} &= pc_H^{LHI}(V_1 - V_3) \\
 &= \frac{-c_H^{LHI^2} \beta_H^{LHI} p^2(-1 + \gamma_L) + c_H^{LHI} p(\beta_L^{LHI} c_L^{LHI} p(-1 + \gamma_H) + q_I(-\gamma_H + \gamma_L))}{(\gamma_H - 1)(\gamma_H - \gamma_L)} \\
 &= \frac{c_H^{LHI^2} p(p - p\gamma_L) + c_H^{LHI} p(c_L^{LHI} p(-1 + \gamma_H) + q_I(-\gamma_H + \gamma_L))}{2(\gamma_H - 1)(\gamma_H - \gamma_L)}, \\
 \pi_L^{LHI} &= pc_L^{LHI}(V_3 - V_6) \\
 &= \frac{-p\beta_L^{LHI} \gamma_H c_L^{LHI^2} - p^2(\gamma_H - \gamma_L - \beta_H^{LHI} c_H^{LHI} \gamma_L) c_L^{LHI}}{\gamma_L(\gamma_H - \gamma_L)} \\
 &= \frac{-c_L^{LHI^2} p^2 \gamma_H + c_H^{LHI} c_L^{LHI} p^2 \gamma_L}{2\gamma_L(\gamma_H - \gamma_L)}. \tag{3}
 \end{aligned}$$

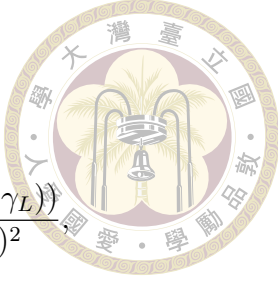
Note that it is not difficult to show that the platforms' profit functions in equation (9) are both quadratic functions. With $\gamma_H > \gamma_L$ and $0 < \gamma_L < \gamma_H < 1$ in our model setting, the quadratic coefficients of c_H^{LHI} and c_L^{LHI} are negative, and thus, π_i^{LHI} is parabolas open downward. That is, there exist unique commission rates based on the first-order conditions:

$$\begin{aligned}
 c_H^{LHI} &= \frac{2q_I \gamma_H (-\gamma_H + \gamma_L)}{p(3\gamma_H \gamma_L - 4\gamma_H + \gamma_L)}, \\
 c_L^{LHI} &= \frac{q_I \gamma_L (-\gamma_H + \gamma_L)}{p(3\gamma_H \gamma_L - 4\gamma_H + \gamma_L)}. \tag{4}
 \end{aligned}$$

Note that these solutions in equations (8) and (10) are optimal if they follow the valuation constraints summarized in Table A.4, or the solutions will lie in the corner point.

Based on equations (8) and (10), we obtain the restaurant's profit and the associated

pass-through rates:



$$\begin{aligned}\pi_R^{LHI} &= p - p^2 - pq_I - \frac{q_I^2 \gamma_H^2 (-1 + \gamma_L) (-5\gamma_L + \gamma_H (-4 + 9\gamma_L))}{4(-1 + \gamma_H)(\gamma_L + \gamma_H(-4 + 3\gamma_L))^2}, \\ \beta_H^{LHI} &= \frac{2q_I \gamma_H (\gamma_H - \gamma_L) - (2p\gamma_H - 2p + q_I \gamma_H)(3\gamma_H \gamma_L - 4\gamma_H + \gamma_L)}{4q_I \gamma_H (\gamma_H - \gamma_L)}, \\ \beta_L^{LHI} &= \frac{1}{2} - \frac{(2p(-1 + \gamma_L) + q_I \gamma_L)(\gamma_L + \gamma_H(-4 + 3\gamma_L))}{2q_I(\gamma_H - \gamma_L)\gamma_L}.\end{aligned}\quad (5)$$

We follow the same logic to discuss the remaining cases. For some cases, we can obtain the restaurant's optimal pass-through rates and profit, along with the platforms' commission rates and the associated platforms' profit, through the differential. For those optimal solutions that cannot get by differential, their solutions lie in the boundary. By removing the repeated and trivial cases, we can summarize one of the derivative results in the following lemmas.

Lemma 1. *Under Scenario 1, the restaurant's partial derivative results of pass-through rates and associated profits are in Tables 4.1 and 4.2.*

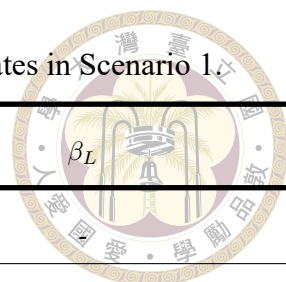


Table 4.1: Restaurant's partial derivative results of pass-through rates in Scenario 1.

Case	β_H	β_L
(1) I	-	-
(2) $I + L$	-	$\frac{3}{2} + \frac{2p(-1+\gamma_L)}{q_I\gamma_L}$
(3) L	-	$\frac{-4p+3\gamma_L}{2\gamma_L}$
(4) $I + H$	$\frac{3}{2} + \frac{2p(-1+\gamma_H)}{q_I\gamma_H}$	-
(5) H	$\frac{-4p+3\gamma_H}{2\gamma_H}$	-
(6) $I + H + L$	$\frac{2q_I\gamma_H(\gamma_H-\gamma_L)-(2p\gamma_H-2p+q_I\gamma_H)(3\gamma_H\gamma_L-4\gamma_H+\gamma_L)}{4q_I\gamma_H(\gamma_H-\gamma_L)}$	$\frac{1}{2} - \frac{(2p(-1+\gamma_L)+q_I\gamma_L)(\gamma_L+\gamma_H(-4+3\gamma_L))}{2q_I(\gamma_H-\gamma_L)\gamma_L}$
(7) $H + L$	$\frac{-8p\gamma_H+6\gamma_H^2+1p\gamma_L-3\gamma_H\gamma_L}{4\gamma_H^2-4\gamma_H\gamma_L}$	$\frac{-8p\gamma_H+2p\gamma_L+5\gamma_H\gamma_L-2\gamma_L^2}{2\gamma_H\gamma_L-2\gamma_L^2}$

Table 4.2: Restaurant's profits with associated partial derivative results in Scenario 1.

Case	π_R
(1) I	$p(-p - q_I + 1)$
(2) $I + L$	$p - p^2 - pq_I + \frac{q_I^2\gamma_L}{16-16\gamma_L}$
(3) L	$\frac{\gamma_L}{16}$
(4) $I + H$	$p - p^2 - pq_I + \frac{q_I^2\gamma_H}{16-16\gamma_H}$
(5) H	$\frac{\gamma_H}{16}$
(6) $I + H + L$	$p - p^2 - pq_I - \frac{q_I^2\gamma_H^2(-1+\gamma_L)(-5\gamma_L+\gamma_H(-4+9\gamma_L))}{4(-1+\gamma_H)(\gamma_L+\gamma_H(-4+3\gamma_L))^2}$
(7) $H + L$	$\frac{\gamma_H^2(4\gamma_H+5\gamma_L)}{4(-4\gamma_H+\gamma_L)^2}$

Lemma 2. Under Scenario 1, platforms' partial derivative results of commission rates and associated profits are in Table 4.3.

Table 4.3: Platforms' partial derivative results of commission rates and associated profits in Scenario 1.

Case	c_H	c_L	π_H	π_L
(1) I	-	-	-	-
(2) $I + L$	-	$\frac{q_I \gamma_L}{2p}$	-	$\frac{q_I^2 \gamma_L}{8-8\gamma_L}$
(3) L	-	$\frac{\gamma_L}{2p}$	-	$\frac{\gamma_L}{8}$
(4) $I + H$	$\frac{q_I \gamma_H}{2p}$	-	$\frac{q_I^2 \gamma_H}{8-8\gamma_H}$	-
(5) H	$\frac{-4p+3\gamma_H}{2\gamma_H}$	-	$\frac{\gamma_H}{8}$	-
(6) $I + H + L$	$\frac{2q_I \gamma_H (-\gamma_H + \gamma_L)}{p(3\gamma_H \gamma_L - 4\gamma_H + \gamma_L)}$	$\frac{q_I \gamma_L (-\gamma_H + \gamma_L)}{p(3\gamma_H \gamma_L - 4\gamma_H + \gamma_L)}$	$\frac{2q_I^2 \gamma_H^2 (\gamma_H - \gamma_L) (-1 + \gamma_L)}{(-1 + \gamma_H)(\gamma_L + \gamma_H (-4 + 3\gamma_L))^2}$	$\frac{q_I^2 \gamma_H (\gamma_H - \gamma_L) \gamma_L}{2(\gamma_L + \gamma_H (-4 + 3\gamma_L))^2}$
(7) $H + L$	$\frac{2\gamma_H (\gamma_H - \gamma_L)}{p(4\gamma_H - \gamma_L)}$	$\frac{\gamma_L (\gamma_H - \gamma_L)}{p(4\gamma_H - \gamma_L)}$	$\frac{2\gamma_H^2 (\gamma_H - \gamma_L)}{16\gamma_H^2 - 8\gamma_H \gamma_L + \gamma_L^2}$	$\frac{\gamma_H \gamma_L (\gamma_H - \gamma_L)}{2(4\gamma_H - \gamma_L)^2}$

4.2 Scenario 2: $1 < \gamma_L < \gamma_H < 2$

Scenario 2: $1 < \gamma_L < \gamma_H < 2$ implies both platforms have better service quality than the dine-in; customers may have better experiences through the platforms. Here, we demonstrate one case: the restaurant provides the meal through the dine-in and the two delivery platforms simultaneously and show the interplay between the restaurant and two delivery platforms. The remaining cases will be summarized in the appendix. We use ILH to represent the case where the restaurant provides the dine-in and two delivery platforms. The restaurant decides the pass-through rates in order to maximize the profit after observing the commission rates decided by the platform(s).

Given the price, p , the waiting cost, q_I , the service reward multiplier through the

delivery platform, γ_i , and commission cost, c_i , we can formulate the restaurant's profit function as π_R^{ILH} , where $i = H, L$:

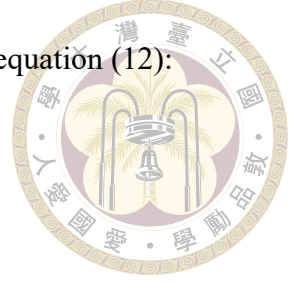


$$\begin{aligned}
 \pi_R^{ILH} &= p(V_2 - V_4) + p(1 - c_L^{ILH} + c_L^{ILH} \beta_L^{ILH})(V_3 - V_2) + p(1 - c_H^{ILH} + c_H^{ILH} \beta_H^{ILH})(1 - V_3) \\
 &= p(-p - q_I + \frac{-\beta_L^{ILH} c_L^{ILH} p + q_I}{1 - \gamma_L}) + \\
 &\quad p(\beta_L^{ILH} c_L^{ILH} - c_L^{ILH} + 1) (\frac{\beta_H^{ILH} c_H^{ILH} p - \beta_L^{ILH} c_L^{ILH} p}{\gamma_H - \gamma_L} - \frac{-\beta_L^{ILH} c_L^{ILH} p + q_I}{1 - \gamma_L}) + \\
 &\quad p(\beta_H^{ILH} c_H^{ILH} - c_H^{ILH} + 1) (1 - \frac{\beta_H^{ILH} c_H^{ILH} p - \beta_L^{ILH} c_L^{ILH} p}{\gamma_H - \gamma_L}). \tag{6}
 \end{aligned}$$

In equation (12), p represents the restaurant's profit margin from a dine-in customer, and $V_2 - V_4$ represents the demand for dine-in; $p(1 - c_L^{ILH} + c_L^{ILH} \beta_L^{ILH})$ represents the restaurant's profit margin that cooperates with platform L , and $V_3 - V_2$ represents the demand of platform L ; $p(1 - c_H^{ILH} + c_H^{ILH} \beta_H^{ILH})$ represents the restaurant's profit margin that cooperates with platform H , and $1 - V_3$ represents the demand of platform H . We substitute V_i from equations (1) to (6) into the first equation of (12), then we can obtain the second equation.

To obtain the optimal pass-through rate of the restaurant, we first check the Hessian matrix of the restaurant's profit function. Here we define H^{ILH} as the determinant of the Hessian matrix of the restaurant's profit function. The maximum of the restaurant's profit

will exist when $H^{ILH} > 0$. The Hessian matrix can be derived from equation (12):



$$H^{ILH} = \det \begin{bmatrix} \frac{\partial^2 \pi_R^{ILH}}{\partial \beta_H^{ILH^2}} & \frac{\partial^2 \pi_R^{ILH}}{\partial \beta_H^{ILH} \partial \beta_L^{ILH}} \\ \frac{\partial^2 \pi_R^{ILH}}{\partial \beta_H^{ILH} \partial \beta_L^{ILH}} & \frac{\partial^2 \pi_R^{ILH}}{\partial \beta_L^{ILH^2}} \end{bmatrix} = \frac{4c_H^{ILH^2} c_L^{ILH^2} p^4}{(-1 + \gamma_L)(\gamma_H - \gamma_L)}.$$

With $\gamma_H > \gamma_L$ and $1 < \gamma_L < \gamma_H < 2$ in our model setting, we obtain $H^{ILH} > 0$, and hence, the optimal pass-through rate solutions uniquely exist. The pass-through rate solutions can be derived by the first-order conditions of π_R^{ILH} , that is, $\partial \pi_R^{ILH} / \partial \beta_H = 0$, and $\partial \pi_R^{ILH} / \partial \beta_L^{ILH} = 0$. We obtain:

$$\begin{aligned} \beta_H^{ILH} &= \frac{c_H^{ILH} p + q_I + \gamma_H - 1}{2c_H^{ILH} p}, \\ \beta_L^{ILH} &= \frac{c_L^{ILH} p + q_I + \gamma_L - 1}{2c_L^{ILH} p}. \end{aligned} \quad (7)$$

Given the restaurant's best response, the platforms decide the commission rates. We substitute the pass-through rate in equation (13) into the platforms' profit function and solve the platforms' problem. Let π_i^{ILH} be platform i 's profit function, where $i = H, L$,



we have:

$$\begin{aligned}
 \pi_H^{ILH} &= pc_H^{ILH}(1 - V_3) \\
 &= \frac{c_H^{ILH} p(-\beta_H^{ILH} c_H^{ILH} p + \beta_L^{ILH} c_L^{ILH} p + \gamma_H - \gamma_L)}{\gamma_H - \gamma_L} \\
 &= -\frac{c_H^{ILH^2} p^2}{2(\gamma_H - \gamma_L)} + \frac{c_H^{ILH} p(c_L^{ILH} p + \gamma_H - \gamma_L)}{2(\gamma_H - \gamma_L)}, \\
 \pi_L^{ILH} &= pc_L^{ILH}(V_3 - V_2) \\
 &= \frac{c_L^{ILH} p(p(\gamma_L - 1)(\beta_H^{ILH} c_H^{ILH} - \beta_L^{ILH} c_L^{ILH}) - (\gamma_H - \gamma_L)(\beta_L^{ILH} c_L^{ILH} p - q_I))}{(\gamma_H - \gamma_L)(\gamma_L - 1)} \\
 &= -\frac{c_L^{ILH^2} p^2(-1 + \gamma_H)}{2(\gamma_H - \gamma_L)(-1 + \gamma_L)} - \frac{c_L^{ILH} p(q_I(-\gamma_H + \gamma_L) + c_H^{ILH}(p - p\gamma_L))}{2(\gamma_H - \gamma_L)(-1 + \gamma_L)}. \tag{8}
 \end{aligned}$$

Note that it is not difficult to show that the platforms' profit functions in equation (14) are both quadratic functions. With $\gamma_H > \gamma_L$ and $1 < \gamma_L < \gamma_H < 2$ in our model setting, the quadratic coefficients of c_H^{ILH} and c_L^{ILH} are negative, and thus, π_i^{ILH} is parabolic open downward. That is, there exist unique commission rates based on the first-order conditions:

$$\begin{aligned}
 c_H^{ILH} &= \frac{-2q_I\gamma_H + 2q_I\gamma_L - \gamma_H\gamma_L + \gamma_H + \gamma_L^2 - \gamma_L}{p(-4\gamma_H + \gamma_L + 3)}, \\
 c_L^{ILH} &= \frac{q_I\gamma_H - q_I\gamma_L + 2\gamma_H^2 - 2\gamma_H\gamma_L - 2\gamma_H + 2\gamma_L}{p(4\gamma_H - \gamma_L - 3)}. \tag{9}
 \end{aligned}$$

Note that the solutions in equations (13) and (15) are optimal if they follow the valuation constraints summarized in Table A.5, or the solutions will lie at the corner point.

Based on equations (13) and (15), we obtain the restaurant's profit and associated

pass-through rates:



$$\begin{aligned}
 \pi_R^{ILH} &= p - p^2 + pq_I + \\
 &\quad \frac{(-1 + \gamma_H)(2q_I(-1 + \gamma_L)(-9 + 8\gamma_H + \gamma_L)) + q_I^2(-9 + 4\gamma_H + 5\gamma_L)}{4(-1 + \gamma_L)(3 - 4\gamma_H + \gamma_L)^2} + \\
 &\quad \frac{(-1 + \gamma_H)(-1 + \gamma_L)(-9 + 4\gamma_H + 5\gamma_L)}{4(-1 + \gamma_L)(3 - 4\gamma_H + \gamma_L)^2}, \\
 \beta_H^{ILH} &= \frac{3 - 3q_I - 9\gamma_H + 5q_I\gamma_H + 6\gamma_H^2 + 3\gamma_L - 2q_I\gamma_L - 3\gamma_H\gamma_L}{2(-2 + q_I + 2\gamma_H)(\gamma_H - \gamma_L)}, \\
 \beta_L^{ILH} &= \frac{3 - 3q_I - 5\gamma_H + 6q_I\gamma_H - \gamma_L - 3q_I\gamma_L + 5\gamma_H\gamma_L - 2\gamma_L^2}{2(\gamma_H - \gamma_L)(-1 + 2q_I + \gamma_L)}. \tag{10}
 \end{aligned}$$

We follow the same logic to discuss the remaining cases. For some cases, we can obtain the restaurant's pass-through rates and profit, along with the platforms' commission rates and the associated platforms' profit, through the differential. For some optimal solutions that cannot obtain by differential, their solutions lie in the boundary. By removing the repeated and trivial cases, we can summarize one of the derivative results in the following lemmas.

Lemma 3. *Under Scenario 2, the restaurant's partial derivative of pass-through rates and associated profits are in Table 4.4 and 4.5.*

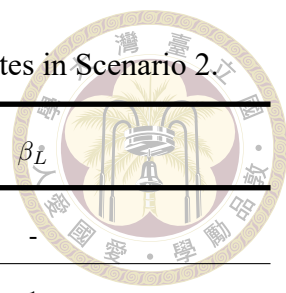


Table 4.4: Restaurant's partial derivative results of pass-through rates in Scenario 2.

Case	β_H	β_L
(1) I	-	-
(2) $I + L$	-	1
(3) L	-	$\frac{-4p+3\gamma_L}{2\gamma_L}$
(4) $I + H$	1	-
(5) H	$\frac{-4p+3\gamma_H}{2\gamma_H}$	-
(6) $I + L + H$	$\frac{3-3q_I-9\gamma_H+5q_I\gamma_H+6\gamma_H^2+3\gamma_L-2q_I\gamma_L-3\gamma_H\gamma_L}{2(-2+q_I+2\gamma_H)(\gamma_H-\gamma_L)}$	$\frac{3-3q_I-5\gamma_H+6q_I\gamma_H-\gamma_L-3q_I\gamma_L+5\gamma_H\gamma_L-2\gamma_L^2}{2(\gamma_H-\gamma_L)(-1+2q_I+\gamma_L)}$
(7) $H + L$	$\frac{-8p\gamma_H+6\gamma_H^2+1p\gamma_L-3\gamma_H\gamma_L}{4\gamma_H^2-4\gamma_H\gamma_L}$	$\frac{-8p\gamma_H+2p\gamma_L+5\gamma_H\gamma_L-2\gamma_L^2}{2\gamma_H\gamma_L-2\gamma_L^2}$

Table 4.5: Restaurant's profits by associated partial derivative results in Scenario 2.

Case	π_R
(1) I	$p(-p - q_I + 1)$
(2) $I + L$	$p - p^2 - pq_I + \frac{(-1+q_I+\gamma_L)^2}{16(-1+\gamma_L)}$
(3) L	$\frac{\gamma_L}{16}$
(4) $I + H$	$p - p^2 - pq_I + \frac{(-1+q_I+\gamma_H)^2}{16(-1+\gamma_H)}$
(5) H	$\frac{\gamma_H}{16}$
(6) $I + L + H$	$p - p^2 + pq_I + \frac{(-1+\gamma_H)(2q_I(-1+\gamma_L)(-9+8\gamma_H+\gamma_L))+q_I^2(-9+4\gamma_H+5\gamma_L)+(-1+\gamma_H)(-1+\gamma_L)(-9+4\gamma_H+5\gamma_L)}{4(-1+\gamma_L)(3-4\gamma_H+\gamma_L)^2}$
(7) $H + L$	$\frac{\gamma_H^2(4\gamma_H+5\gamma_L)}{4(-4\gamma_H+\gamma_L)^2}$

Lemma 4. Under Scenario 2, the platforms' partial derivative results of commission rates and associated profits are in Table 4.6 and 4.7.

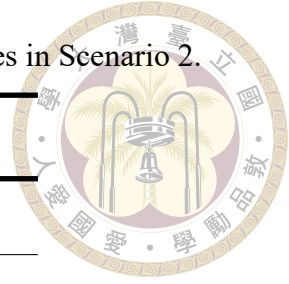


Table 4.6: Platforms' partial derivative results of commission rates in Scenario 2.

Case	c_H	c_L
(1) I	-	-
(2) $I + L$	-	$\frac{q_I + \gamma_L - 1}{2p}$
(3) L	-	$\frac{\gamma_L}{2p}$
(4) $I + H$	$\frac{q_I + \gamma_H - 1}{2p}$	-
(5) H	$\frac{-4p + 3\gamma_H}{2\gamma_H}$	-
(6) $I + H + L$	$\frac{(-2 + q_I + 2\gamma_H)(\gamma_H - \gamma_L)}{p(-3 + 4\gamma_H - \gamma_L)}$	$\frac{(-1 + 2q_I + \gamma_L)(\gamma_H - \gamma_L)}{p(-3 + 4\gamma_H - \gamma_L)}$
(7) $H + L$	$\frac{2\gamma_H(\gamma_H - \gamma_L)}{p(4\gamma_H - \gamma_L)}$	$\frac{\gamma_L(\gamma_H - \gamma_L)}{p(4\gamma_H - \gamma_L)}$

Table 4.7: Platforms' partial derivative results of profits in Scenario 2.

Case	π_H	π_L
(1) I	-	-
(2) $I + L$	-	$\frac{(-1 + q_I + \gamma_L)^2}{8(-1 + \gamma_L)}$
(3) L	-	$\frac{\gamma_L}{8}$
(4) $I + H$	$\frac{(-1 + q_I + \gamma_H)^2}{8(-1 + \gamma_H)}$	-
(5) H	$\frac{\gamma_H}{8}$	-
(6) $I + H + L$	$\frac{(-2 + q_I + 2\gamma_H)^2(\gamma_H - \gamma_L)}{2(3 - 4\gamma_H + \gamma_L)^2}$	$\frac{(-1 + \gamma_H)(\gamma_H - \gamma_L)(-1 + 2q_I + \gamma_L)^2}{2(-1 + \gamma_L)(3 - 4\gamma_H + \gamma_L)^2}$
(7) $H + L$	$\frac{2\gamma_H^2(\gamma_H - \gamma_L)}{(-4\gamma_H + \gamma_L)^2}$	$\frac{\gamma_H\gamma_L(\gamma_H - \gamma_L)}{2(-4\gamma_H + \gamma_L)^2}$

4.3 Scenario 3: $0 < \gamma_L < 1 < \gamma_H < 2$

Scenario 3: $0 < \gamma_L < 1 < \gamma_H < 2$, implies platform H has better service quality than the dine-in, but platform L has poorer service quality than dine-in. That is, customers have better experiences through the delivery platform H and may not meet the expectation

of food experiences through the platform L . Here, we demonstrate one case: the restaurant provides the meal through the dine-in and the two delivery platforms simultaneously and show the interplay between the restaurant and two delivery platforms. The remaining cases will be summarized in Table A.3 in the appendix. We use LIH to represent the case where the restaurant provides the dine-in and two delivery platforms. The restaurant decides the pass-through rates in order to maximize the profit after observing the commission rates decided by the platform(s).



Given the price, p , the waiting cost, q_I , the service reward multiplier through the delivery platform, γ_i , and commission cost, c_i , we can formulate the restaurant's profit function as π_R^{LIH} , where $i = H, L$:

$$\begin{aligned}
 \pi_R^{LIH} &= p(1 - c_L^{LIH} + c_L^{LIH} \beta_L^{LIH})(V_2 - V_6) + p(V_1 - V_2) + \\
 &\quad p(1 - c_H^{LIH} + c_H^{LIH} \beta_H^{LIH})(1 - V_1) \\
 &= p(\beta_L^{LIH} c_L^{LIH} - c_L^{LIH} + 1) \left(\frac{p \beta_L^{LIH} c_L^{LIH} - p}{\gamma_L} + \frac{-\beta_L^{LIH} c_L^{LIH} p + q_I}{1 - \gamma_L} \right) + \\
 &\quad p(\beta_H^{LIH} c_H^{LIH} - c_H^{LIH} + 1) \left(1 - \frac{1 + \beta_H^{LIH} c_H^{LIH} p - q_I}{1 - \gamma_H} \right) + \\
 &\quad p \left(\frac{\beta_L^{LIH} c_L^{LIH} p - q_I}{1 - \gamma_L} + \frac{-\beta_H^{LIH} c_H^{LIH} p + q_I}{1 - \gamma_H} \right). \tag{11}
 \end{aligned}$$

In equation (17), $p(1 - c_L^{LIH} + c_L^{LIH} \beta_L^{LIH})$ represents the restaurant's profit margin that

cooperates with platform L , and $V_2 - V_6$ represents the demand of platform L ; p represents the restaurant's profit margin from a dine-in customer, and $V_1 - V_2$ represents the demand of dine-in; $p(1 - c_H^{LIH} + c_H^{LIH} \beta_H^{LIH})$ represents the restaurant's profit margin that cooperates with platform H , and $1 - V_1$ represents the demand of platform H . We substitute V_i from equations (1) to (6) into (17)'s first equation, then we can obtain the second equation.



To obtain the optimal pass-through rate of the restaurant, we first check the Hessian matrix of the restaurant's profit function. Here we define H^{LIH} as the determinant of the Hessian matrix of the restaurant's profit function. The maximum of the restaurant's profit will exist when $H^{LIH} > 0$. The Hessian matrix can be derived from equation (17):

$$H^{LIH} = \det \begin{bmatrix} \frac{\partial^2 \pi_R^{LIH}}{\partial \beta_H^2} & \frac{\partial^2 \pi_R^{LIH}}{\partial \beta_H \partial \beta_L} \\ \frac{\partial^2 \pi_R^{LIH}}{\partial \beta_L \partial \beta_H} & \frac{\partial^2 \pi_R^{LIH}}{\partial \beta_L^2} \end{bmatrix} = \frac{4c_H^2 c_L^2 p^4}{(-1 + \gamma_L)(-1 + \gamma_H)\gamma_L}.$$

With $\gamma_H > \gamma_L$ and $0 < \gamma_L < 1 < \gamma_H < 2$ in our model setting, we obtain $H^{LIH} > 0$, and hence, the optimal pass-through rate solutions uniquely exist. The pass-through rate solutions can be derived by the first-order conditions of π_R^{LIH} , that is, $\partial \pi_R^{LIH} / \partial \beta_H^{LIH} = 0$,

and $\partial\pi_R^{LIH}/\partial\beta_L^{LIH} = 0$. We obtain:

$$\begin{aligned}\beta_H^{LIH} &= \frac{c_H^{LIH}p + q_I + \gamma_H - 1}{2c_H^{LIH}p}, \\ \beta_L^{LIH} &= \frac{q_I\gamma_L + p(-2 + c_L^{LIH} + 2\gamma_L)}{2c_L^{LIH}p}.\end{aligned}\tag{12}$$



Given the restaurant's best response, the platforms decide the commission rates. We substitute the pass-through rate in equation (18) into the platforms' profit function and solve the platforms' problems. Let π_i^{LIH} be platform i 's profit function, where $i = H, L$, we have:

$$\begin{aligned}\pi_H^{LIH} &= pc_H^{LIH}(1 - V_1) \\ &= \frac{c_H^{LIH}p(-\beta_H^{LIH}c_H^{LIH}p + q_I + \gamma_H - 1)}{\gamma_H - 1} \\ &= -\frac{3p^2c_H^{LIH^2}}{2(-1 + \gamma_H)} + \frac{c_H^{LIH}p(-1 + q_I + \gamma_H)}{-1 + \gamma_H}, \\ \pi_L^{LIH} &= pc_L^{LIH}(V_2 - V_6) \\ &= \frac{c_L^{LIH}p(\beta_L^{LIH}c_L^{LIH}p - p\gamma_L + p - q_I\gamma_L)}{\gamma_L(\gamma_L - 1)} \\ &= -\frac{c_L^{LIH^2}p(4p - \gamma_Lp - 3q_I\gamma_L)}{2q_I\gamma_L^2(-1 + \gamma_L)} - \frac{c_L^{LIH}p(-p + p\gamma_L + q_I\gamma_L)}{\gamma_L(-1 + \gamma_L)}.\end{aligned}\tag{13}$$

Note that it is not difficult to show that the platforms' profit functions in equation (19) are both quadratic functions. With $\gamma_H > \gamma_L$ and $0 < \gamma_L < 1 < \gamma_H < 2$ in our model setting, the quadratic coefficients of c_H^{LIH} and c_L^{LIH} are negative, and thus, π_i^{LIH}

is parabolas open downward. That is, there exist unique commission rates based on the first-order conditions:



$$\begin{aligned} c_H^{LIH} &= \frac{q_I + \gamma_H - 1}{2p}, \\ c_L^{LIH} &= \frac{q_I \gamma_H}{2p}. \end{aligned} \quad (14)$$

Note that the solutions in equations (18) and (20) are optimal if they follow the valuation constraints summarized in Table A.6, or the solutions will lie at the corner point.

Based on equations (18) and (20), we obtain the restaurant's profit and associated pass-through rates:

$$\begin{aligned} \pi_R^{LIH} &= \frac{-1 - 16p^2 - 16p(-1 + q_I) + 2q_I + \gamma_H}{16} - \frac{q_I^2(1 + \gamma_L(-2 + \gamma_H))}{(-1 + \gamma_H)(-1 + \gamma_L)}, \\ \beta_H^{LIH} &= \frac{3}{2}, \\ \beta_L^{LIH} &= \frac{3}{2} + \frac{2p(-1 + \gamma_L)}{q_I \gamma_L}. \end{aligned} \quad (15)$$

Note that $\beta_i > 0$ in our model setting, but $\beta_H^{LIH} = \frac{3}{2}$ doesn't lie in our feasible solution. Therefore, the β_H^{LIH} solution will be at the boundary.

We follow the same logic to discuss the remaining cases. For some cases, we can obtain the restaurant's pass-through rates and profit, along with the platforms' commission rates and the associated platforms' profit, through the differential. For those optimal solu-

tions that cannot obtain by differential, their solutions are in the boundary. After removing repetitions and trivial cases, we can summarize one of the partial derivative results in the following lemma.



Lemma 5. *Under Scenario 3, the restaurant's partial derivative results of pass-through rates and associated profits are in the Table 4.8 and 4.9.*

Table 4.8: Restaurant's partial derivative of pass-through rates and associated profits in Scenario 3.

Case	β_H	β_L
(1) I	-	-
(2) $L + I$	-	$\frac{3}{2} + \frac{2p(-1+\gamma_L)}{q_I\gamma_L}$
(3) L	-	$\frac{-4p+3\gamma_L}{2\gamma_L}$
(4) $I + H$	$\frac{3}{2}$	-
(5) H	$\frac{-4p+3\gamma_H}{2\gamma_H}$	-
(6) $L + I + H$	$\frac{3}{2}$	$\frac{3}{2} + \frac{2p(-1+\gamma_L)}{q_I\gamma_L}$
(7) $H + L$	$-\frac{8p\gamma_H-6\gamma_H^2-2p\gamma_L+3\gamma_H\gamma_L}{4\gamma_H(\gamma_H-\gamma_L)}$	$-\frac{8p\gamma_H-2p\gamma_L-5\gamma_H\gamma_L+2\gamma_L^2}{2\gamma_L(\gamma_H-\gamma_L)}$

Table 4.9: Restaurant's partial derivative of pass-through rates and associated profits in Scenario 3.

Case	π_R
(1) I	$p(-p - q_I + 1)$
(2) $L + I$	$p - p^2 - pq_I + \frac{q_I^2\gamma_L}{16-16\gamma_L}$
(3) L	$\frac{\gamma_L}{16}$
(4) $I + H$	$p - p^2 - pq_I + \frac{(-1+q_I+\gamma_H)^2}{16(\gamma_H-1)}$
(5) H	$\frac{\gamma_H}{16}$
(6) $L + I + H$	$\frac{-1-16p^2-16p(-1+q_I)+2q_I+\gamma_H}{16} - \frac{q_I^2(1+\gamma_L)(-2+\gamma_H)}{(-1+\gamma_H)(-1+\gamma_L)}$
(7) $H + L$	$\frac{\gamma_H^2(4\gamma_H+5\gamma_L)}{4(-4\gamma_H+\gamma_L)^2}$

Lemma 6. *Under Scenario 3, the platforms' partial derivative of commission rates and associated profits are in Table 4.10.*



Table 4.10: Platforms' partial derivative of commission rates and associated profits in Scenario 3.

Case	c_H	c_L	π_H	π_L
(1) I	-	-	-	-
(2) $I + L$	-	$\frac{q_I \gamma_L}{2p}$	-	$-\frac{(q_I \gamma_L)^2}{8(-1+\gamma_L)}$
(3) L	-	$\frac{\gamma_L}{2p}$	-	$\frac{\gamma_L}{8}$
(4) $I + H$	$\frac{q_I + \gamma_H - 1}{2p}$	-	$\frac{(-1 + q_I + \gamma_H)^2}{8(-1 + \gamma_H)}$	-
(5) H	$\frac{\gamma_H}{2p}$	-	$\frac{\gamma_H}{8}$	-
(6) $I + H + L$	$\frac{-1 + q_I + \gamma_H}{2p}$	$\frac{q_I \gamma_L}{2p}$	$\frac{(-1 + q_I + \gamma_H)^2}{8(-1 + \gamma_H)}$	$-\frac{(q_I \gamma_L)^2}{8(-1 + \gamma_L)}$
(7) $H + L$	$\frac{2\gamma_H(\gamma_H - \gamma_L)}{p(4\gamma_H - \gamma_L)}$	$\frac{\gamma_L(\gamma_H - \gamma_L)}{2p}$	$\frac{2\gamma_H^2(\gamma_H - \gamma_L)}{(-4\gamma_H + \gamma_L)^2}$	$\frac{\gamma_H \gamma_L(\gamma_H - \gamma_L)}{2(-4\gamma_H + \gamma_L)^2}$





Chapter 5 Numerical Study

We conduct a numerical study to investigate further managerial insights. In this chapter, we focus our analysis on the relationships between the optimal choice of the restaurant's operation choice as well as the model parameters, which are untraceable through theoretical discussions in the previous chapter. In each scenario, we consider several different parameter value combinations to investigate the changes in the restaurant's optimal strategy.

5.1 Restaurant's operation choice under Scenario 1

Here we discuss the restaurant's optimal operation decision in Scenario 1 under the situation that the customers spend less waiting cost, which given $q_I = 0.1$. When the price, p , changes, we still have similar patterns of strategy changes. Here, we show one of the results in Figure 5.1.

Observation 1. *When we fix $p = 0.1$, $q_I = 0.1$, (a) if γ_H is low, the restaurant will only*

adopts strategy I as γ_L increases. (b) if γ_H is medium, the restaurant will adopt strategy $H + I$, then switches to $L + H + I$ as γ_L increases. (c) if γ_H is high, the restaurant will adopt strategy H , then switches to $L + H$ as γ_L increases.



When we fix γ_H , if γ_H is low ($\gamma_H < 0.5$), both platforms provide much poorer service quality than dine-in. There are extra commission costs with the platform service, and customers' surplus by the platform will be negative, so the customers won't buy food, which can infer that the restaurant only provides meals through dine-in. If the platforms' service quality is getting better ($\gamma_H \geq 0.5$), the restaurant will cooperate with the platforms. If platforms' service quality is almost the same as dine-in ($\gamma_H > 0.95$, $\gamma_L > 0.7$), dine-in will not even exist in the market and be replaced by only platforms.

Observation 2. When we fix $p = 0.1$, $q_I = 0.1$, (a) if γ_L is low, the restaurant adopts strategy I , then switches to $H + I$, then switches to H as γ_H increases. (b) if γ_L is medium, the restaurant adopts strategy $L + H + I$, then switches to $H + I$, then switches to H as γ_H increases. (c) if γ_L is high, the restaurant adopts strategy $L + H + I$, then switches to $L + H$ as γ_H increases.

When we fix γ_L , if γ_L is low ($\gamma_L < 0.5$), the restaurant won't cooperate with platform L ; instead, the restaurant collaborates with platform H as $\gamma_H > 0.5$ because platform H can provide better service than platform L . If γ_L is medium or high ($\gamma_L \geq 0.5$), there's a chance that the restaurant cooperates with platform L . We fix γ_H to observe and explain

the reasons in detail.

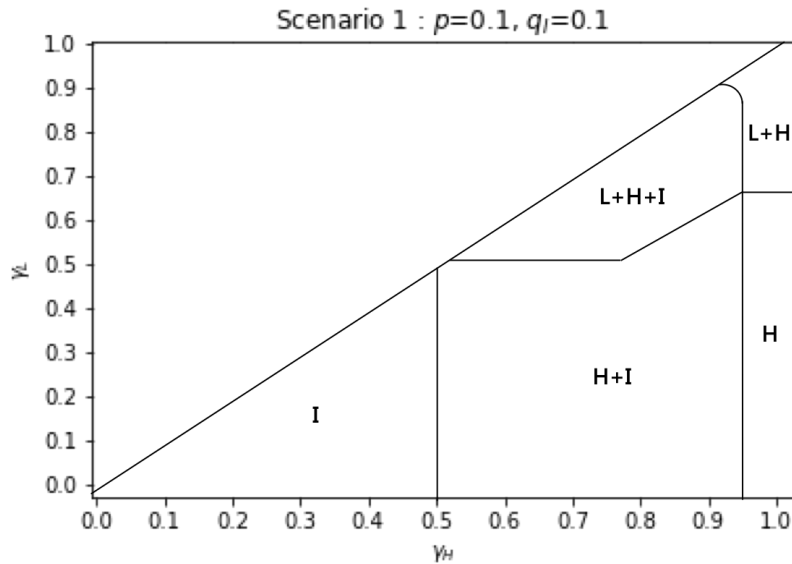


Figure 5.1: The restaurant's optimal strategy when $p = 0.1$, $q_I = 0.1$ in Scenario 1.

Let's look further at Figure 5.2. We fix $\gamma_H = 0.64$ and observe the changes in the commission rates and pass-through rates. We can observe that the strategy switches from $H + I$ to $L + H + I$ by platform L sets a lower commission rate, c_L . For example, when $\gamma_L = 0.55$, $c_L^* = 0.1$. Although platform H maintains a high commission rate, it provides better service quality than platform L , so the restaurant will always cooperate with platform H , which can also show that strategy $L + H + I$ is better than strategy $L + I$ or L . There's no strategy $L + I$ or L in the market.

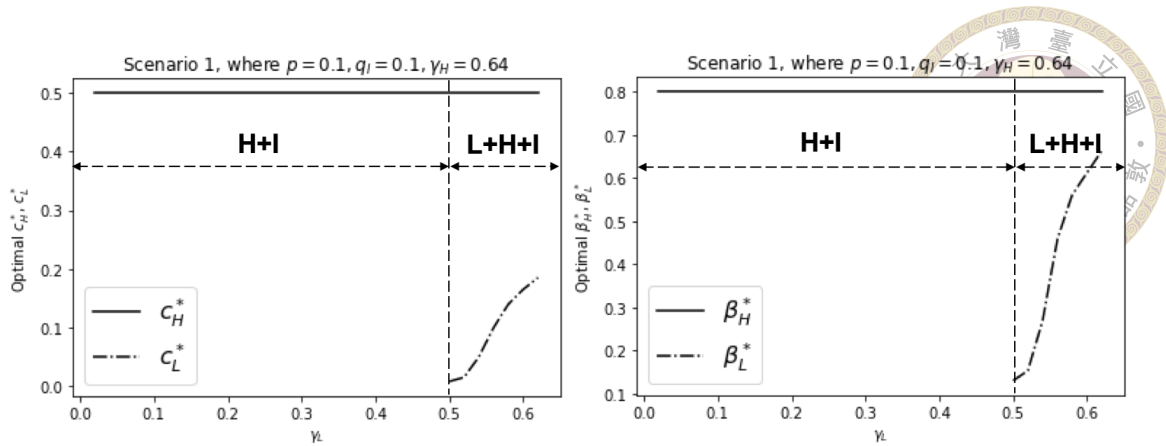


Figure 5.2: The changes in the commission rates and pass-through rates in Scenario 1. Here, $p = 0.1, q_I = 0.1, \gamma_H = 0.64$.

We also fix $\gamma_H = 0.96$ to see the changes in commission rates and pass-through rates in Figure 5.3. We can observe that platform L also enters the market by lower the commission rates, c_L . For example, when $\gamma_L = 0.7, c_L^* = 0.13$. Moreover, platform H can also exist in the market by lowering the commission rates, c_H . For example, when $\gamma_L = 0.92, c_H^* = 0.39$. Because platform H has better service quality than platform L , platform H has more flexibility to lower the commission rates. This can also infer why strategy $H + L$ is better than strategy L and strategy $H + L$ can exist in the market.

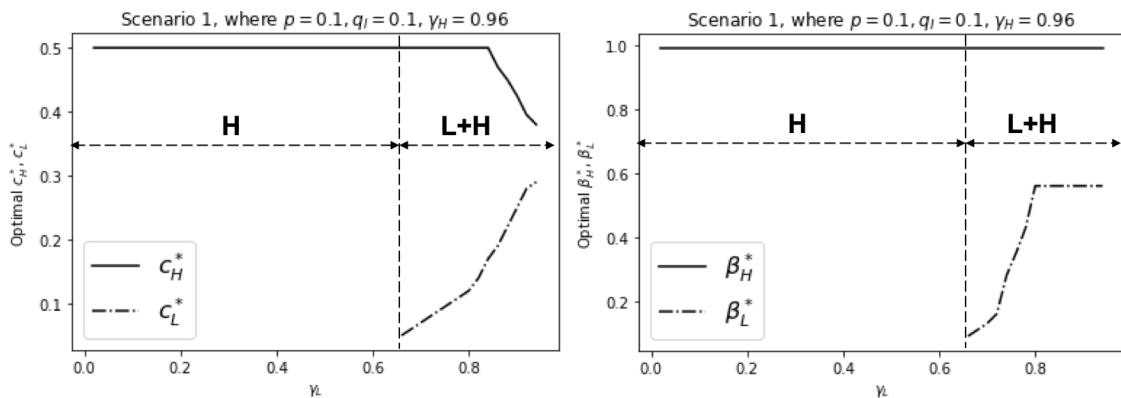


Figure 5.3: The changes in the commission rates and pass-through rates in Scenario 1. Here, $p = 0.1, q_I = 0.1, \gamma_H = 0.96$.



5.2 Restaurant's operation choice under Scenario 2

Here we discuss the restaurant's optimal operation decision in Scenario 2 under the situation that the customers spend less waiting cost, which given $q_I = 0.02$. When the price, p , changes, we still have similar patterns of the changes in strategies. Here, we show one of the results in Figure 5.4.

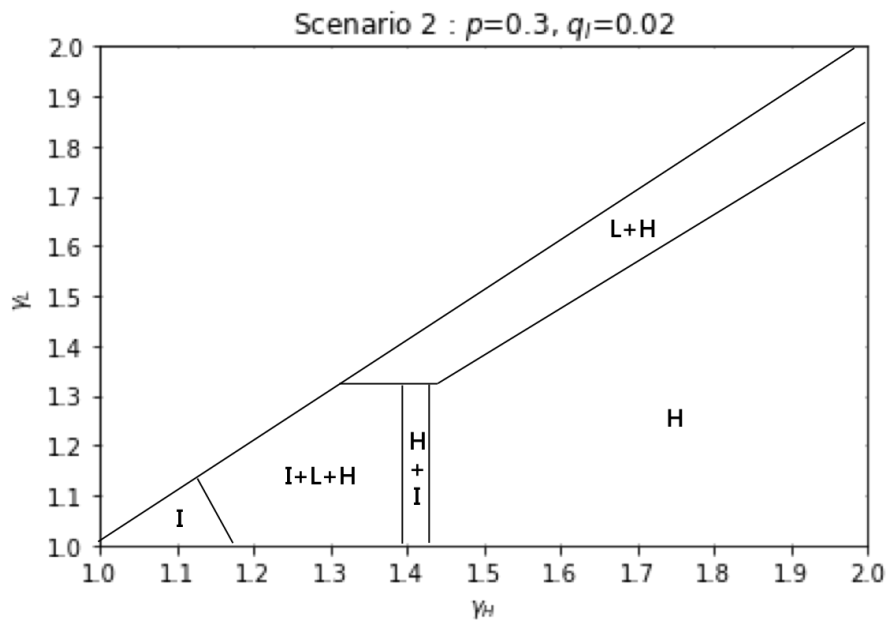
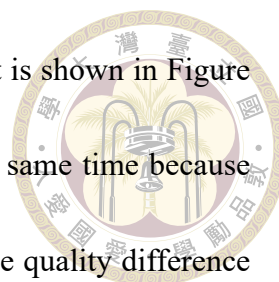


Figure 5.4: The figure illustrates the restaurant's operation choice in Scenario 2. Here, $\gamma_H \in [1, 2]$, $\gamma_L \in [1, 2]$, $p = 0.3$, $q_I = 0.02$, where $\gamma_H > \gamma_L$.

Observation 3. When we fix $p = 0.3$, $q_I = 0.02$, (a) if γ_L is low, the restaurant adopts strategy I , then switches to $I + L + H$, then switches to $I + H$, and finally switches to H as γ_H increases. (b) if γ_L is medium, the restaurant adopts strategy $I + L + H$, then switches to $I + H$, and finally switches to H as γ_H increases. (c) if γ_L is high, the restaurant adopts strategy $L + H$, then switches to H as γ_H increases.



We fix $\gamma_L = 1.02$ for more detailed observations, and the result is shown in Figure 5.5. When γ_H is low, platform L and H will join the market at the same time because they both set similar commission rates, and there is only little service quality difference between platform H and L . The restaurant is willing to afford the commission cost from platform L by itself, and platform L can join the market. For example, when $\gamma_H = 1.2$, $\beta_L^* = 0.32$. However, as γ_H keeps increasing, γ_L is still the same without improvement, so the restaurant no longer cooperates with platform L . Finally, the restaurant will not adopt the dine-in strategy because platform H can provide twice better service quality than dine-in, customers can gain more surplus from platform H , and dine-in will lose the competitive edge.

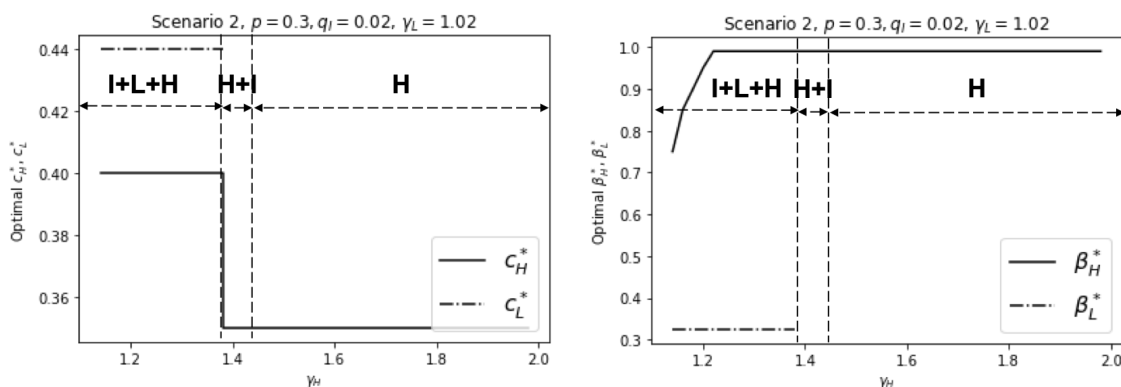


Figure 5.5: The changes in the commission rates and pass-through rates in Scenario 2. Here, $p = 0.3, q_I = 0.02, \gamma_L = 1.02$.

Let's take a further look at Figure 5.6. We fix $\gamma_H = 1.5$ and observe the changes in the commission and pass-through rates. We can observe that the strategy switch from H to $L + H$, and platform L enters the market by setting a low commission rate. For example, when $\gamma_L = 1.4, c_L^* = 0.21$. But, there's no strategy L alone in the market because platform



H also lowers the commission rates, c_H , to join the market. Due to platform H having better service quality than platform L , platform H has more flexibility than platform L to lower the commission rates. For example, when $\gamma_L = 1.44$, $c_L^* = 0.22$. For the restaurant, it's better to cooperate with the platform H .

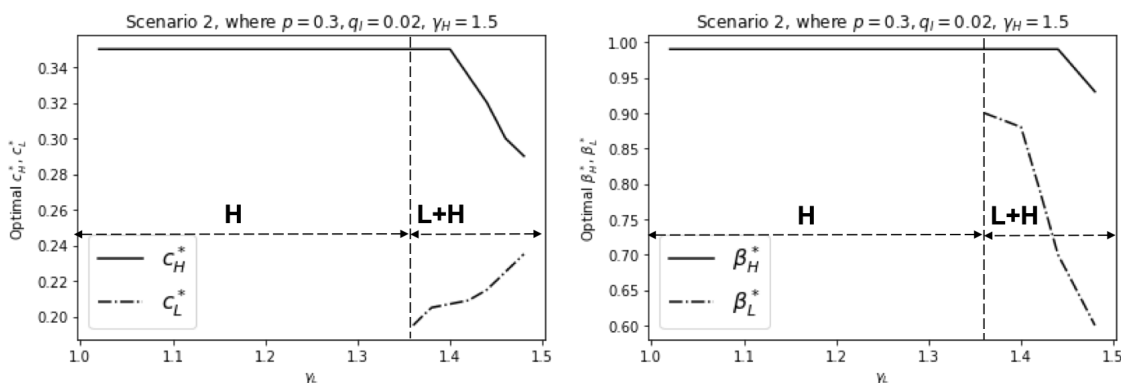
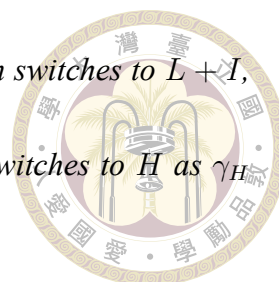


Figure 5.6: The changes in the commission rates and pass-through rates in Scenario 2. Here, $p = 0.3, q_I = 0.02, \gamma_H = 1.5$.

5.3 Restaurant's operation choice under Scenario 3

Here we discuss the restaurant's optimal operation decision in Scenario 3 under the situation that the customers spend less waiting cost, which given $q_I = 0.04$. When the price, p changes, we still have similar patterns of the changes in strategies. Here, we show one of the results in Figure 5.7.

Observation 4. When we fix $p = 0.2, q_I = 0.04$ (a) if γ_L is low, the restaurant adopts strategy I , then switches to H as γ_H increases. (b) if γ_L is medium, the restaurant adopts strategy $L + I$, then switches to H as γ_H increases or the restaurant adopts strategy $L + I$, then switches to $I + L + H$, then switches to $L + H$, and finally switches to H as



γ_H increases. (c) if γ_L is high, the restaurant adopts strategy L , then switches to $L+I$, then switches to $I+L+H$, then switches to $L+H$, and finally switches to H as γ_H increases.

When γ_L is low ($\gamma_L \leq 0.84$), the restaurant won't cooperate with platform L for low service quality. Only γ_L is medium or high ($\gamma_L > 0.84$) will the restaurant consider cooperating with platform L . In the following, we fix γ_L to observe the reasons in detail.

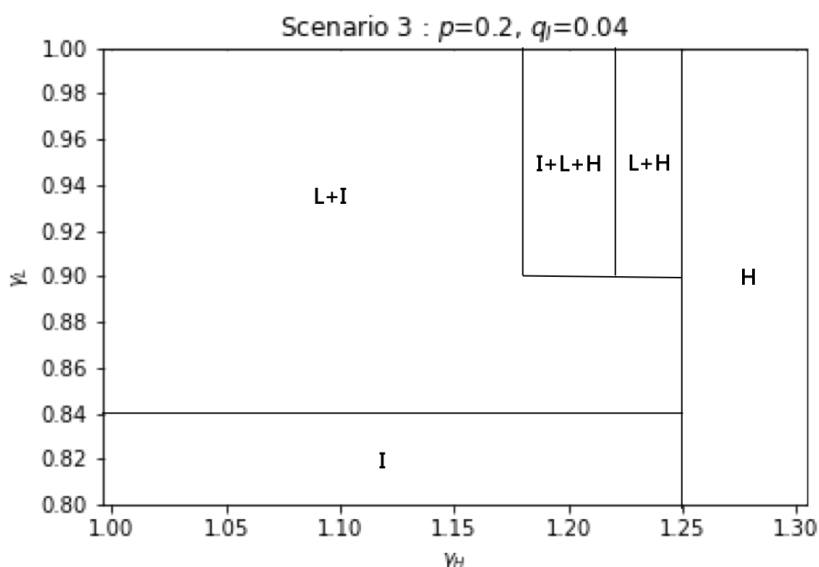
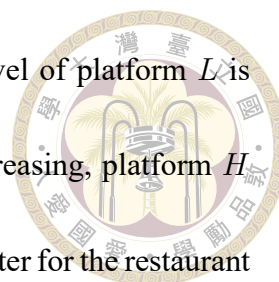


Figure 5.7: The figure illustrates the restaurant's operation choice in Scenario 3. Here, $p = 0.2, q_I = 0.04$.

In Figure 5.8, we fix $\gamma_L = 0.92$ and observe the changes in the commission rates and pass-through rates. Since the service level of platform H is much higher than that of L , when the service level of L is almost equal to that of H , platform L replaces H with a low commission rate. For example, when $\gamma_H = 1.1, c_L^* = 0.1$. Besides, platform H 's commission rates have always been high, so the restaurant adopts strategy $L+I$. However,



as the service level of the platform H improves, and the service level of platform L is fixed, platform H will be added to the market. When γ_H keeps increasing, platform H can provide much better service than dine-in and platform L , so it's better for the restaurant to cooperate with platform H only, dine-in and L strategy will not exist in the market.

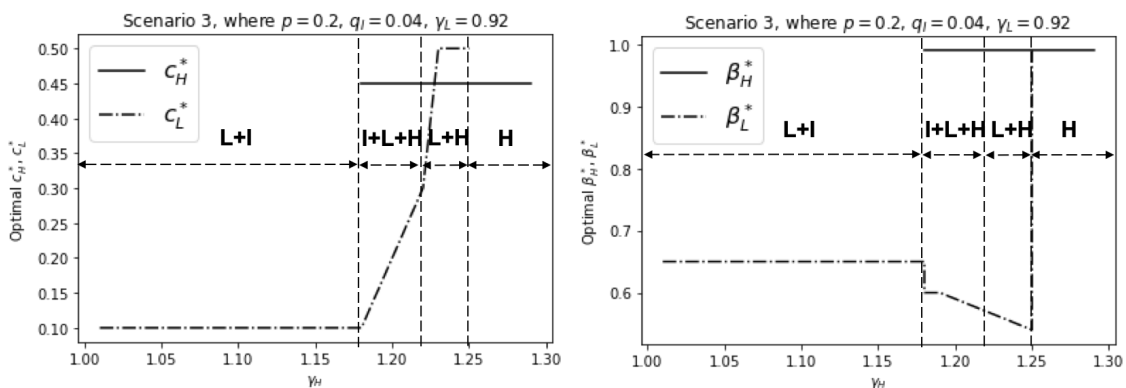


Figure 5.8: The changes in the commission rates and pass-through rates in Scenario 3. Here, $p = 0.2$, $q_I = 0.04$, $\gamma_L = 0.92$, $\gamma_H \in [1, 1.3]$.

From the above result, we can infer some rules. The restaurant acts as the first mover in the model, and it can first decide whether to cooperate with platforms depending on the platforms' service quality and commission rates. When both platforms' service quality is much poorer than dine-in and the waiting time is short enough, the restaurant will adopt the dine-in strategy. When both platforms' service quality improves or is even better than dine-in, platform H will join the market and always exist because platform H always provides better service than platform L . On the other hand, platform L has to lower the commission rates to enter the market; otherwise, it cannot compete with platform H .





Chapter 6 Conclusion

This thesis studies a model consisting of a restaurant and two outside delivery platforms, one of which is of high quality and the other of which is of low quality. The restaurant provides the meal to the customers and considers whether to cooperate with any one of the delivery platforms to gain more profit while the delivery platforms will charge commission costs. We use three scenarios, based on the differences of service quality of the platforms, to discuss the restaurant's tradeoffs between extra profit and commission costs. In each scenario, we individually discuss one of the most complicated cases: the restaurant simultaneously provides the meal through dine-in and two delivery platforms. The cooperated delivery platforms shall decide the commission rates to charge the restaurant for providing service. To compensate for part of the loss of the restaurant's profit margin, the restaurant decides on the pass-through rates to charge the customers who buy the meal through the delivery platform. Then the customers decide their dining choices: buy the meal through a dine-in or delivery platform or buy nothing.

Our model characterized the optimal food offering strategies and pass-through rates

of the restaurant as well as the commission rates of the delivery platforms. Since there are four decision variables in each case, the problem may be too complicated, thus, we may not have close-form solutions for the optimal commission rates or pass-through rates.



Therefore, we conduct a numerical study for further managerial insights, and we find some interesting results.

For numerical study, we observe the changes of strategies patterns in price, p with the same waiting costs, q_I in a different scenario, and we find that the changes of patterns are similar. We show one of the most complicated cases in each scenario. In Scenario 1, when both platforms have much poorer service quality than dine-in, and using platforms services ask for extra commission costs, customers will not purchase with a negative surplus and the restaurant will adopt the dine-in strategy. Only when the platforms' service improves, and they can join the market. In Scenario 2, even if both platforms have better service quality than dine-in, the restaurant will adopt the dine-in strategy because customers will suffer a negative surplus with platform service. When the platforms' service improves, platform L will lower the commission rates to enter the market. Platform H will always provide better service than platform L , so the restaurant will always cooperate with platform H , and even platform H can take over the whole market by keep lowering its commission rates. In Scenario 3, platform L has poorer service quality than dine-in and platform H , but platform H has better service quality than dine-in and platform L .

When both platforms' service quality is near 1, the restaurant will cooperate with platform L by platform L providing a low commission rate, but otherwise, platform H can always provide the service in the market.



Overall, the optimal strategies will change depending on the model parameters. The restaurant will adopt the dine-in strategy when the platform's service quality is quite bad. Otherwise, the restaurant will cooperate with platform H because of better service quality and cooperate with platform L if platform L can provide a lower commission rate.

There are some limitations in our research. First, when we consider the customers' waiting time in the restaurant, we use a model parameter, q_I , to represent customers' waiting costs. In reality, each customer's waiting cost may differ depending on the restaurant's business, but we do not use the queuing theory to consider the details. Furthermore, if the restaurant cooperates with any one of the platforms, the restaurant's operation of serving the meals will be more complicated because the customers will come from different sources. Whenever the restaurant cooperates with a platform, the restaurant may need to modify its internal ordering system to have a better connection with the platform system. Second, in the real world, delivery workers play an essential role in platform operation. We do not focus on the number of delivery workers or the quality of delivery workers. Third, platforms usually use promotions to attract more consumers, which can increase customer surplus. These are the extra situation that we ignore, which are also the direc-

tion for future research.





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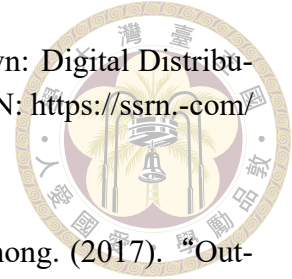
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Appendix A —

A.1 Summary of cases in each Scenario

All the cases in each Scenario are individually summarized in Table A.1, A.2 and

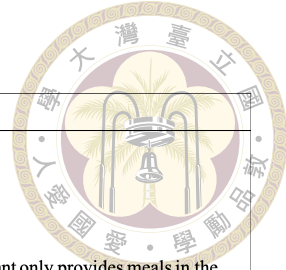
A.3.

Table A.1: The 30 cases in Scenario 1.



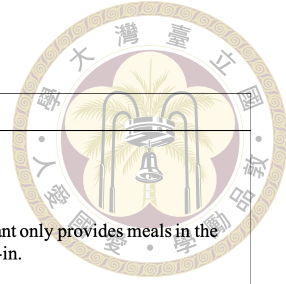
Case		Explanation
(1) I	$\begin{array}{c} \text{X} \quad \quad \quad \text{I} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_1 \quad v_2 \quad v_3 \quad v_4 \quad v_5 \quad v_6 \quad 1 \end{array}$	The restaurant only provides meals in the way of dine-in.
	$\begin{array}{c} \text{X} \quad \quad \quad \text{I} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_1 \quad v_2 \quad v_4 \quad v_6 \quad v_5 \quad v_3 \quad 1 \end{array}$	
(2) L+I	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{I} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_1 \quad v_6 \quad v_4 \quad v_2 \quad v_5 \quad v_3 \quad 1 \end{array}$	The restaurant provides meals through dine-in and the delivery platform L .
	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{I} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_1 \quad v_6 \quad v_4 \quad v_5 \quad v_2 \quad v_3 \quad 1 \end{array}$	
	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{I} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_6 \quad v_5 \quad v_4 \quad v_1 \quad v_2 \quad v_3 \quad 1 \end{array}$	
(3) L	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_1 \quad v_6 \quad v_4 \quad 1 \quad v_2 \quad v_5 \quad v_3 \end{array}$	The restaurant only provides meals through the delivery platform L .
	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_1 \quad v_6 \quad v_4 \quad v_5 \quad 1 \quad v_2 \quad v_3 \end{array}$	
	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_6 \quad v_1 \quad v_4 \quad 1 \quad v_2 \quad v_5 \quad v_3 \end{array}$	
	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_6 \quad v_5 \quad v_4 \quad v_1 \quad 1 \quad v_2 \quad v_3 \end{array}$	
(4) I+H	$\begin{array}{c} \text{X} \quad \quad \quad \text{H} \quad \quad \quad \text{I} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_3 \quad v_2 \quad v_5 \quad v_4 \quad v_1 \quad v_6 \quad 1 \end{array}$	The restaurant provides meals through dine-in and the delivery platform H .
	$\begin{array}{c} \text{X} \quad \quad \quad \text{H} \quad \quad \quad \text{I} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_3 \quad v_2 \quad v_5 \quad v_4 \quad v_6 \quad v_1 \quad 1 \end{array}$	
	$\begin{array}{c} \text{X} \quad \quad \quad \text{H} \quad \quad \quad \text{I} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_3 \quad v_5 \quad v_2 \quad v_4 \quad v_1 \quad v_6 \quad 1 \end{array}$	
(5) H	$\begin{array}{c} \text{X} \quad \quad \quad \text{H} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_3 \quad v_2 \quad v_5 \quad v_4 \quad 1 \quad v_1 \quad v_6 \end{array}$	The restaurant only provides meals through the delivery platform H .
	$\begin{array}{c} \text{X} \quad \quad \quad \text{H} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_3 \quad v_2 \quad v_5 \quad v_4 \quad v_6 \quad 1 \quad v_1 \end{array}$	
	$\begin{array}{c} \text{X} \quad \quad \quad \text{H} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_3 \quad v_5 \quad v_2 \quad v_4 \quad 1 \quad v_1 \quad v_6 \end{array}$	
(6) L+I+H	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{H} \quad \quad \quad \text{I} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_6 \quad v_5 \quad v_3 \quad v_4 \quad v_2 \quad v_1 \quad 1 \end{array}$	The restaurant provides meals through dine-in, delivery platform H and L .
(7) L+H	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{H} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \\ 0 \quad v_6 \quad v_5 \quad v_3 \quad v_4 \quad v_2 \quad 1 \quad v_1 \end{array}$	The restaurant provides meals through the delivery platform H and L .

Table A.2: The 30 cases in Scenario 2.



Case		Explanation
(1) I	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^I \quad \overbrace{\quad\quad\quad}^X \\ 0 \quad v_3 \quad v_4 \quad v_5 \quad 1 \quad v_1 \quad v_6 \quad v_2 \end{array}$	The restaurant only provides meals in the way of dine-in.
	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^I \quad \overbrace{\quad\quad\quad}^X \\ 0 \quad v_3 \quad v_4 \quad v_5 \quad v_6 \quad 1 \quad v_1 \quad v_2 \end{array}$	
	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^I \quad \overbrace{\quad\quad\quad}^X \\ 0 \quad v_4 \quad v_3 \quad v_5 \quad 1 \quad v_1 \quad v_6 \quad v_2 \end{array}$	
	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^I \quad \overbrace{\quad\quad\quad}^X \\ 0 \quad v_4 \quad v_3 \quad v_5 \quad v_6 \quad 1 \quad v_1 \quad v_2 \end{array}$	
(2) L+I	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^I \quad \overbrace{\quad\quad\quad}^L \quad \overbrace{\quad\quad\quad}^X \\ 0 \quad v_4 \quad v_6 \quad v_2 \quad v_5 \quad v_1 \quad 1 \quad v_3 \end{array}$	The restaurant provides meals through dine-in and the delivery platform L .
	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^I \quad \overbrace{\quad\quad\quad}^L \quad \overbrace{\quad\quad\quad}^X \\ 0 \quad v_4 \quad v_6 \quad v_5 \quad v_2 \quad v_1 \quad 1 \quad v_3 \end{array}$	
(3) L	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^L \quad \overbrace{\quad\quad\quad}^X \\ 0 \quad v_2 \quad v_1 \quad v_6 \quad v_5 \quad 1 \quad v_3 \quad v_4 \end{array}$	The restaurant only provides meals through the delivery platform L .
	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^L \quad \overbrace{\quad\quad\quad}^X \\ 0 \quad v_2 \quad v_6 \quad v_1 \quad v_5 \quad v_4 \quad 1 \quad v_3 \end{array}$	
	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^L \quad \overbrace{\quad\quad\quad}^X \\ 0 \quad v_2 \quad v_6 \quad v_1 \quad v_5 \quad 1 \quad v_3 \quad v_4 \end{array}$	
(4) I+H	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^I \quad \overbrace{\quad\quad\quad}^H \\ 0 \quad v_3 \quad v_4 \quad v_5 \quad v_1 \quad v_6 \quad v_2 \quad 1 \end{array}$	The restaurant provides meals through dine-in and the delivery platform H .
	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^I \quad \overbrace{\quad\quad\quad}^H \\ 0 \quad v_3 \quad v_4 \quad v_5 \quad v_6 \quad v_1 \quad v_2 \quad 1 \end{array}$	
	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^I \quad \overbrace{\quad\quad\quad}^H \\ 0 \quad v_4 \quad v_3 \quad v_5 \quad v_1 \quad v_6 \quad v_2 \quad 1 \end{array}$	
(5) H	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^H \\ 0 \quad v_2 \quad v_1 \quad v_3 \quad v_5 \quad v_6 \quad v_4 \quad 1 \end{array}$	The restaurant only provides meals through the delivery platform H .
	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^H \\ 0 \quad v_3 \quad v_1 \quad v_2 \quad v_5 \quad v_6 \quad v_4 \quad 1 \end{array}$	
(6) L+I+H	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^I \quad \overbrace{\quad\quad\quad}^L \quad \overbrace{\quad\quad\quad}^H \\ 0 \quad v_4 \quad v_6 \quad v_2 \quad v_5 \quad v_1 \quad v_3 \quad 1 \end{array}$	The restaurant provides meals through dine-in, delivery platform H and L .
(7) L+H	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^L \quad \overbrace{\quad\quad\quad}^H \\ 0 \quad v_2 \quad v_1 \quad v_6 \quad v_5 \quad v_3 \quad v_4 \quad 1 \end{array}$	The restaurant provides meals through the delivery platform H and L .
	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^L \quad \overbrace{\quad\quad\quad}^H \\ 0 \quad v_2 \quad v_6 \quad v_1 \quad v_5 \quad v_4 \quad v_3 \quad 1 \end{array}$	
	$\begin{array}{c} \overbrace{\quad\quad\quad}^X \quad \overbrace{\quad\quad\quad}^L \quad \overbrace{\quad\quad\quad}^H \\ 0 \quad v_2 \quad v_6 \quad v_1 \quad v_5 \quad v_3 \quad v_4 \quad 1 \end{array}$	

Table A.3: The 30 cases in Scenario 3.



Case		Explanation
(1) I	$\begin{array}{c} \text{X} \quad \quad \quad \text{I} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_2 \quad v_3 \quad v_4 \quad v_5 \quad 1 \quad v_1 \quad v_6 \end{array}$	The restaurant only provides meals in the way of dine-in.
	$\begin{array}{c} \text{X} \quad \quad \quad \text{I} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_2 \quad v_3 \quad v_4 \quad v_5 \quad v_6 \quad 1 \quad v_1 \end{array}$	
	$\begin{array}{c} \text{X} \quad \quad \quad \text{I} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_2 \quad v_4 \quad v_3 \quad v_5 \quad 1 \quad v_1 \quad v_6 \end{array}$	
(2) L+I	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{I} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_6 \quad v_4 \quad v_2 \quad v_5 \quad v_3 \quad 1 \quad v_1 \end{array}$	The restaurant provides meals through dine-in and the delivery platform L .
	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{I} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_6 \quad v_4 \quad v_5 \quad v_2 \quad v_3 \quad 1 \quad v_1 \end{array}$	
(3) L	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_6 \quad v_1 \quad v_5 \quad v_4 \quad 1 \quad v_3 \quad v_2 \end{array}$	The restaurant only provides meals through the delivery platform L .
	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_6 \quad v_4 \quad v_5 \quad 1 \quad v_2 \quad v_3 \quad v_1 \end{array}$	
	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_6 \quad v_4 \quad v_5 \quad v_1 \quad 1 \quad v_3 \quad v_2 \end{array}$	
	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{X} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_1 \quad v_6 \quad v_5 \quad 1 \quad v_3 \quad v_4 \quad v_2 \end{array}$	
(4) I+H	$\begin{array}{c} \text{X} \quad \quad \quad \text{I} \quad \quad \quad \text{H} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_2 \quad v_3 \quad v_4 \quad v_5 \quad v_1 \quad v_6 \quad 1 \end{array}$	The restaurant provides meals through dine-in and the delivery platform H .
	$\begin{array}{c} \text{X} \quad \quad \quad \text{I} \quad \quad \quad \text{H} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_2 \quad v_4 \quad v_6 \quad v_5 \quad v_3 \quad v_1 \quad 1 \end{array}$	
	$\begin{array}{c} \text{X} \quad \quad \quad \text{I} \quad \quad \quad \text{H} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_2 \quad v_4 \quad v_3 \quad v_5 \quad v_1 \quad v_6 \quad 1 \end{array}$	
(5) H	$\begin{array}{c} \text{X} \quad \quad \quad \text{H} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_1 \quad v_3 \quad v_2 \quad v_5 \quad v_4 \quad v_6 \quad 1 \end{array}$	The restaurant only provides meals through the delivery platform H .
	$\begin{array}{c} \text{X} \quad \quad \quad \text{H} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_2 \quad v_3 \quad v_1 \quad v_5 \quad v_4 \quad v_6 \quad 1 \end{array}$	
(6) L+I+H	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{I} \quad \quad \quad \text{H} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_6 \quad v_4 \quad v_2 \quad v_5 \quad v_3 \quad v_1 \quad 1 \end{array}$	The restaurant provides meals through dine-in, delivery platform H and L .
	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{I} \quad \quad \quad \text{H} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_1 \quad v_6 \quad v_5 \quad v_3 \quad v_4 \quad v_2 \quad 1 \end{array}$	
(7) L+H	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{H} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_6 \quad v_1 \quad v_5 \quad v_4 \quad v_3 \quad v_2 \quad 1 \end{array}$	The restaurant provides meals through the delivery platform H and L .
	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{H} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_1 \quad v_6 \quad v_5 \quad v_4 \quad v_3 \quad v_2 \quad 1 \end{array}$	
	$\begin{array}{c} \text{X} \quad \quad \quad \text{L} \quad \quad \quad \text{H} \\ \quad \quad \quad \quad \quad \quad \quad \\ 0 \quad v_6 \quad v_4 \quad v_5 \quad v_1 \quad v_3 \quad v_2 \quad 1 \end{array}$	



A.2 Explanation for the optimal solutions

In each scenario, we have 30 cases. Take one case in Scenario 1 for example, and the case shows in Figure A.1. This case represents the restaurant adopts the strategy of dine-in and cooperating with two platforms.

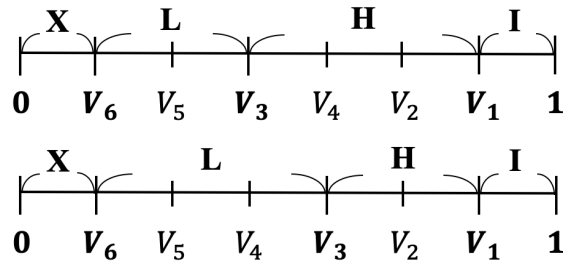


Figure A.1: One of cases in Scenario 1.

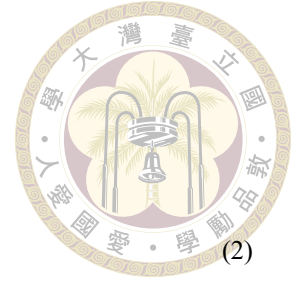
In this case, we can formulate the restaurant's profit function as π_R^{LHI} , where $i = H, L$. $p(1 - c_L^{LHI} + c_L^{LHI} \beta_L^{LHI})$, $p(1 - c_H^{LHI} + c_H^{LHI} \beta_H^{LHI})$ and p are individually represented the profit margin of the restaurant while the customers using platform L , H and dine-in; $V_3 - V_6$, $V_1 - V_3$ and $1 - V_1$ are individually represent the demand for platform L , H and dine-in:

$$\pi_R^{LHI} = p(1 - c_L^{LHI} + c_L^{LHI} \beta_L^{LHI})(V_3 - V_6) + p(1 - c_H^{LHI} + c_H^{LHI} \beta_H^{LHI})(V_1 - V_3) + p(1 - V_1). \quad (1)$$

The platforms' profits can formulate as π_i^{LHI} , where $i = H, L$. pc_i^{LHI} represents the

platform's profit margin:

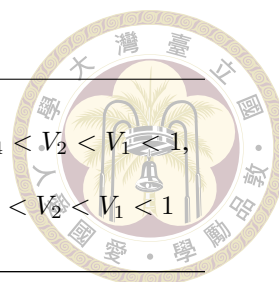
$$\begin{aligned}\pi_H^{LHI} &= pc_H^{LHI}(V_1 - V_3), \\ \pi_L^{LHI} &= pc_L^{LHI}(V_3 - V_6).\end{aligned}\tag{2}$$



We can obtain the optimal solutions of commission rates and pass-through rates through the derivative if the solutions follow the order of valuation constraints. For example, in this case, if the solutions follow $0 < V_6 < V_5 < V_3 < V_4 < V_2 < V_1 < 1$ and $0 < V_6 < V_5 < V_4 < V_3 < V_2 < V_1 < 1$, then we can obtain the optimal solution through the derivative, or the solutions will lie at the corner points. All the optimal decision variables in 30 cases in each Scenario can be obtained in this way, and the other constraints in each scenario are summarized in Table A.4, A.5 and A.6.

Table A.4: The constraints of optimal solutions in Scenario 1.

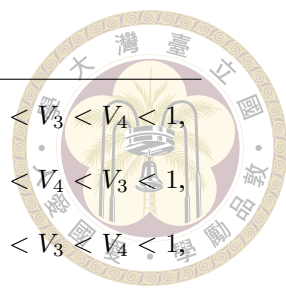
Case	Restaurant's profit	Constraints
(1) I	$p(1 - V_4)$	$0 < V_1 < V_2 < V_3 < V_4 < V_5 < V_6 < 1,$ $0 < V_1 < V_2 < V_4 < V_6 < V_5 < V_3 < 1,$ $0 < V_1 < V_2 < V_4 < V_3 < V_5 < V_6 < 1,$ $0 < V_3 < V_2 < V_1 < V_4 < V_5 < V_6 < 1$
(2) $I + L$	$p(1 - c_L + c_L\beta_L)(V_2 - V_6)$ $+p(1 - V_2)$	$0 < V_1 < V_6 < V_4 < V_2 < V_5 < V_3 < 1,$ $0 < V_1 < V_6 < V_4 < V_5 < V_2 < V_3 < 1,$ $0 < V_6 < V_1 < V_4 < V_2 < V_5 < V_3 < 1,$ $0 < V_6 < V_5 < V_4 < 1 < V_3 < V_2 < V_1,$ $0 < V_6 < V_5 < V_4 < V_1 < V_2 < V_3 < 1$
(3) L	$p(1 - c_L + c_L\beta_L)(1 - V_6)$	$0 < V_1 < V_6 < V_4 < 1 < V_2 < V_5 < V_3,$ $0 < V_1 < V_6 < V_4 < V_5 < 1 < V_2 < V_3,$ $0 < V_6 < V_1 < V_4 < 1 < V_2 < V_5 < V_3,$ $0 < V_6 < V_1 < V_4 < V_5 < 1 < V_2 < V_3,$ $0 < V_6 < V_1 < V_4 < V_5 < V_2 < V_3 < 1,$ $0 < V_6 < V_5 < V_4 < V_1 < 1 < V_2 < V_3,$ $0 < V_6 < V_5 < 1 < V_3 < V_4 < V_2 < V_1$
(4) $I + H$	$p(1 - c_H + c_H\beta_H)(V_1 - V_5)$ $+p(1 - V_1)$	$0 < V_3 < V_2 < V_5 < V_4 < V_1 < V_6 < 1,$ $0 < V_3 < V_2 < V_5 < V_4 < V_6 < V_1 < 1,$ $0 < V_3 < V_5 < V_2 < V_4 < V_1 < V_6 < 1,$ $0 < V_3 < V_5 < V_2 < V_4 < V_6 < V_1 < 1,$ $0 < V_3 < V_5 < V_6 < V_4 < V_2 < V_1 < 1$
(5) H	$p(1 - c_H + c_H\beta_H)(1 - V_5)$	$0 < V_3 < V_2 < V_5 < V_4 < 1 < V_1 < V_6,$ $0 < V_3 < V_2 < V_5 < V_4 < V_6 < 1 < V_6,$ $0 < V_3 < V_5 < V_2 < V_4 < 1 < V_1 < V_6,$ $0 < V_3 < V_5 < V_2 < V_4 < V_6 < 1 < V_1,$ $0 < V_3 < V_5 < V_6 < V_4 < V_2 < 1 < V_1$



(6) $I + H + L$	$p(1 - c_L + c_L\beta_L)(V_3 - V_6)$ $+p(1 - c_H + c_H\beta_H)(V_1 - V_3)$ $+p(1 - V_1)$	$0 < V_6 < V_5 < V_3 < V_4 < V_2 < V_1 < 1,$ $0 < V_6 < V_5 < V_4 < V_3 < V_2 < V_1 < 1$
(7) $H + L$	$p(1 - c_L + c_L\beta_L)(V_3 - V_6)$ $+p(1 - c_H + c_H\beta_H)(1 - V_3)$	$0 < V_6 < V_5 < V_3 < V_4 < V_2 < 1 < V_1,$ $0 < V_6 < V_5 < V_4 < V_3 < V_2 < 1 < V_1$

Table A.5: The constraints of optimal solutions in Scenario 2.

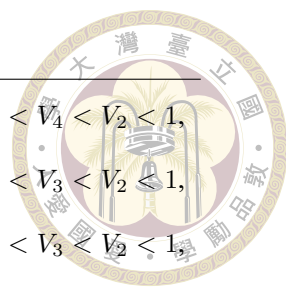
Case	Restaurant's profit	Constraints
(1) I	$p(1 - V_4)$	$0 < V_3 < V_4 < V_5 < 1 < V_1 < V_6 < V_2,$ $0 < V_3 < V_4 < V_5 < V_6 < 1 < V_1 < V_2,$ $0 < V_4 < V_3 < V_5 < 1 < V_1 < V_6 < V_2,$ $0 < V_4 < V_3 < V_5 < V_6 < 1 < V_1 < V_2,$ $0 < V_4 < V_6 < V_5 < V_3 < 1 < V_1 < V_2,$ $0 < V_4 < V_6 < 1 < V_2 < V_5 < V_1 < V_3,$ $0 < V_4 < V_6 < V_5 < 1 < V_2 < V_1 < V_3$
(2) $I + L$	$p(V_2 - V_4)$ $+p(1 - c_L + c_L\beta_L)(1 - V_2)$	$0 < V_4 < V_6 < V_2 < V_5 < V_1 < 1 < V_3,$ $0 < V_4 < V_6 < V_5 < V_2 < V_1 < 1 < V_3$
(3) L	$p(1 - c_L + c_L\beta_L)(1 - V_6)$	$0 < V_2 < V_1 < V_6 < V_5 < 1 < V_3 < V_4,$ $0 < V_2 < V_1 < V_6 < V_5 < V_4 < 1 < V_3,$ $0 < V_2 < V_6 < V_1 < V_5 < 1 < V_3 < V_4,$ $0 < V_2 < V_6 < V_1 < V_5 < V_4 < 1 < V_3,$ $0 < V_2 < V_6 < V_4 < V_5 < V_1 < 1 < V_3$
(4) $I + H$	$p(V_1 - V_4)$ $+p(1 - c_H + c_H\beta_H)(1 - V_1)$	$0 < V_3 < V_4 < V_5 < V_1 < V_6 < V_2 < 1,$ $0 < V_3 < V_4 < V_5 < V_6 < V_1 < V_2 < 1,$ $0 < V_4 < V_3 < V_5 < V_1 < V_6 < V_2 < 1,$ $0 < V_4 < V_3 < V_5 < V_6 < V_1 < V_2 < 1,$ $0 < V_4 < V_6 < V_5 < V_3 < V_1 < V_2 < 1$
(5) H	$p(1 - c_H + c_H\beta_H)(1 - V_5)$	$0 < V_2 < V_1 < V_3 < V_5 < V_6 < V_4 < 1,$ $0 < V_3 < V_1 < V_2 < V_5 < V_6 < V_4 < 1,$ $0 < V_3 < V_1 < V_5 < V_2 < V_6 < V_4 < 1,$ $0 < V_3 < V_1 < V_5 < V_4 < V_6 < V_2 < 1$
(6) $I + H + L$	$p(V_2 - V_4)$ $+p(1 - c_L + c_L\beta_L)(V_3 - V_2)$ $+p(1 - c_H + c_H\beta_H)(1 - V_3)$	$0 < V_4 < V_6 < V_2 < V_5 < V_1 < V_3 < 1,$ $0 < V_4 < V_6 < V_5 < V_2 < V_1 < V_3 < 1$



<p>(7) $H + L$</p>	$p(1 - c_L + c_L\beta_L)(V_3 - V_6)$ $+p(1 - c_H + c_H\beta_H)(1 - V_3)$	$0 < V_2 < V_1 < V_6 < V_5 < V_3 < V_4 < 1,$ $0 < V_2 < V_1 < V_6 < V_5 < V_4 < V_3 < 1,$ $0 < V_2 < V_6 < V_1 < V_5 < V_3 < V_4 < 1,$ $0 < V_2 < V_6 < V_1 < V_5 < V_4 < V_3 < 1,$ $0 < V_2 < V_6 < V_4 < V_5 < V_1 < V_3 < 1$
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Table A.6: The constraints of optimal solutions in Scenario 3.

Case	Restaurant's profit	Constraints
(1) I	$p(1 - V_4)$	$0 < V_2 < V_3 < V_4 < V_5 < 1 < V_6,$ $0 < V_2 < V_3 < V_4 < V_5 < V_6 < 1 < V_1,$ $0 < V_2 < V_4 < V_3 < V_5 < 1 < V_6,$ $0 < V_2 < V_4 < V_3 < V_5 < V_6 < 1 < V_1,$ $0 < V_2 < V_4 < V_6 < V_5 < V_3 < 1 < V_1$
(2) $I + L$	$p(1 - c_L + c_L\beta_L)(V_2 - V_6)$ $+p(1 - V_2)$	$0 < V_6 < V_4 < V_2 < V_5 < V_3 < 1 < V_1,$ $0 < V_6 < V_4 < V_5 < V_2 < V_3 < 1 < V_1$
(3) L	$p(1 - c_L + c_L\beta_L)(1 - V_6)$	$0 < V_6 < V_1 < V_5 < 1 < V_3 < V_4 < V_2,$ $0 < V_6 < V_1 < V_5 < V_4 < 1 < V_3 < V_2,$ $0 < V_6 < V_4 < V_5 < V_1 < 1 < V_3 < V_2,$ $0 < V_1 < V_6 < V_5 < 1 < V_3 < V_4 < V_2,$ $0 < V_1 < V_6 < V_5 < V_4 < 1 < V_3 < V_2,$ $0 < V_6 < V_4 < V_5 < 1 < V_2 < V_3 < V_1,$ $0 < V_6 < V_4 < 1 < V_2 < V_5 < V_3 < V_1$
(4) $I + H$	$p(V_1 - V_4)$ $+p(1 - c_H + c_H\beta_H)(1 - V_1)$	$0 < V_2 < V_3 < V_4 < V_5 < V_1 < V_6 < 1,$ $0 < V_2 < V_3 < V_4 < V_5 < V_6 < V_1 < 1,$ $0 < V_2 < V_4 < V_3 < V_5 < V_1 < V_6 < 1,$ $0 < V_2 < V_4 < V_3 < V_5 < V_6 < V_1 < 1,$ $0 < V_2 < V_4 < V_6 < V_5 < V_3 < V_1 < 1$
(5) H	$p(1 - c_H + c_H\beta_H)(1 - V_5)$	$0 < V_1 < V_3 < V_2 < V_5 < V_4 < V_6 < 1,$ $0 < V_1 < V_3 < V_5 < V_2 < V_4 < V_6 < 1,$ $0 < V_1 < V_3 < V_5 < V_6 < V_4 < V_2 < 1,$ $0 < V_2 < V_3 < V_1 < V_5 < V_4 < V_6 < 1$
(6) $I + H + L$	$p(1 - c_L + c_L\beta_L)(V_2 - V_6)$ $+p(V_1 - V_2)$ $+p(1 - c_H + c_H\beta_H)(1 - V_1)$	$0 < V_6 < V_4 < V_2 < V_5 < V_3 < V_1 < 1,$ $0 < V_6 < V_4 < V_5 < V_2 < V_3 < V_1 < 1$



<p>(7) $H + L$</p>	$p(1 - c_L + c_L\beta_L)(V_3 - V_6)$ $+p(1 - c_H + c_H\beta_H)(1 - V_3)$	$0 < V_6 < V_1 < V_5 < V_3 < V_4 < V_2 < 1,$ $0 < V_6 < V_1 < V_5 < V_4 < V_3 < V_2 < 1,$ $0 < V_6 < V_4 < V_5 < V_1 < V_3 < V_2 < 1,$ $0 < V_1 < V_6 < V_5 < V_3 < V_4 < V_2 < 1,$ $0 < V_1 < V_6 < V_5 < V_4 < V_3 < V_2 < 1$
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