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浮動還是固定？零利率之下的匯率政策

Floating or Fixed? Exchange Rate Flexibility in Liquidity Traps

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本論文係林奕君君（學號 R05323011）在國立臺灣大學經濟學系完成之碩士學位論文，於民國 107 年 5 月 30 日承下列考試委員審查通過及口試及格，特此證明

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陳南光

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摘要

當一國的利率低至零利率下限時，稱此國掉入流動性陷阱。本文針對一個具有金融摩擦與流動性陷阱的經濟體，比較浮動匯率與固定匯率之差異。我們分別檢視以下三種情況：一般情形的浮動匯率制度、流動性陷阱中的浮動匯率制度、固定匯率制度。結果顯示，固定匯率比掉入流動性陷阱的浮動匯率更能穩定經濟體；而當一定程度的金融摩擦存在時，一般情形的浮動匯率制度表現則劣於可實行負利率的固定匯率制度。

關鍵詞：流動性陷阱，零利率下限，浮動匯率，固定匯率，金融摩擦

JEL 分類：F41, F42, E31

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Abstract

The liquidity trap refers to the situation where countries have their interest rates near the zero lower bound. This paper compares whether a floating exchange rate or a fixed exchange rate is more preferable under liquidity traps for the world economy with financial frictions. We specifically examine three cases: a floating-rate system in normal times, a floating-rate system in liquidity traps, and a fixed-rate system. Our results indicate that a fixed exchange rate performs better than a floating one in liquidity traps, and that the floating-rate system in a normal situation may be inferior to the fixed-rate system with negative interest rates if some financial frictions exist.

Keywords: liquidity trap, zero lower bound, floating exchange rate, fixed exchange rate, financial frictions

JEL Classification: F41, F42, E31

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1 Introduction

The liquidity trap is not purely an object of economic research, but a problem that many advanced industrialized countries encounter to combat a recession.[1] For example, the global financial crisis in 2008 led the United States Federal Reserve to lower the federal funds rate to zero in December 2008, while the Bank of England cut the Bank Rate to an effective lower bound near zero in early 2009. In recent years, countries like Japan, Switzerland, and Sweden, as well as the euro area operate under extremely low interest rates or even negative interest rates, as shown in *Figure* 1.[2] The issue with a steadily low interest rate lies in that it leaves no room for a cyclical decline in the policy rate to boost the economy. What happens when the main instrument of monetary policy, the short-term interest rate, is not effective anymore? This paper aims to answer this question from the perspective of exchange rate flexibility. Specifically, we want to know whether a floating exchange rate or a fixed exchange rate is more desirable for countries that fall into liquidity traps.

We construct a New Keynesian model with two countries: "home" and "foreign". Supposing that there is a negative demand shock in the home country, we observe the way this shock spills over to affect the foreign country, and then compare the welfare of the economy under three situations: (1) a floating exchange rate where both countries implement regular monetary policies, (2) a floating exchange rate with both countries in liquidity traps, and (3) a fixed exchange rate. Moreover, we allow for different degrees of trade openness between countries. For a country that is isolated from trade, any exogenous shock outside the country has a small influence upon itself because there is no need for relative price adjustments. On the other

[1] The idea of liquidity trap was introduced by Hicks (1937) along with the IS-LM model. It describes a situation whereby the monetary policy loses its effectiveness because the nominal interest rate hits the zero lower bound.

[2] Although the negative rate is not the topic of this paper, there is a growing number of studies about unencumbering the zero lower bound. For example, see Buiter (2009), Goodfriend (2016), Bech and Malkhozov (2016).

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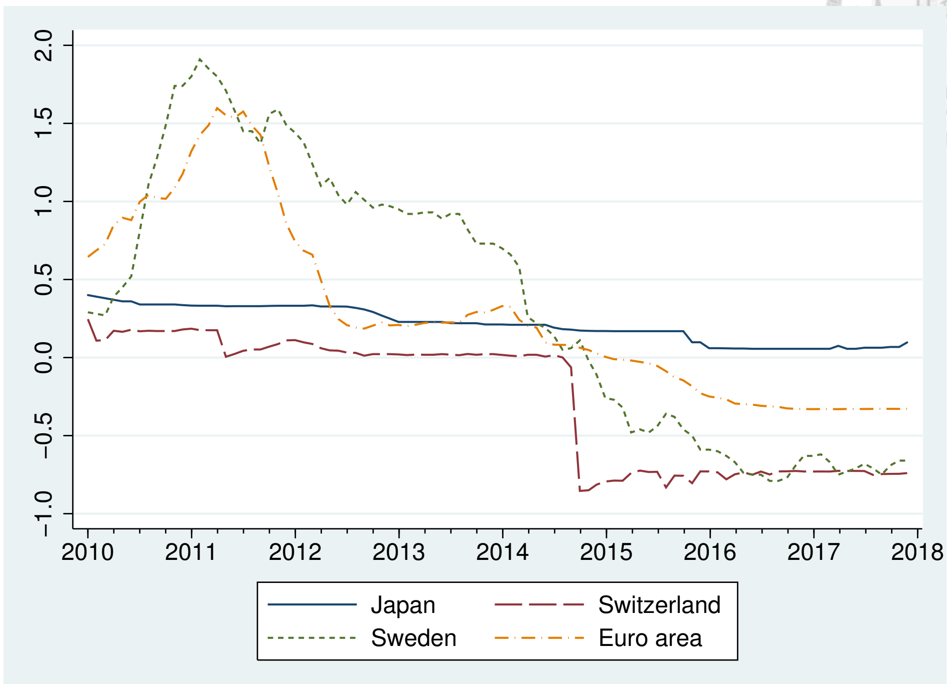
Figure 1: Short-term interest rates (Apr 2010 - Mar 2018)



Short-term interest rates are the rates at which short-term borrowings are effected between financial institutions or the rate at which short-term government paper is issued or traded in the market, measured in percentage per annum.
Data source: OECD (2018), Short-term interest rates (indicator). doi: 10.1787/2cc37d77-en (Accessed on 08 May 2018)

hand, if a country relies heavily on trade, a negative demand shock to its trading partner may force it to lower the selling price in order to boost demand. Hence, trade integration is important in our analysis of shock transmissions.

Our study is highly related to two strands of the literature, which are the topic of liquidity trap and the comparison between floating and fixed exchange rates. The first line of research originates from Krugman et al. (1998), who identify the problem of liquidity trap in Japan in the 1990s. Eggertsson (2003), Jung et al. (2005), and Auerbach and Obstfeld (2005) provide optimal monetary policies to save the economy from near-zero interest rates. Nakajima (2008) and Fujiwara et al. (2010) extend the problem of liquidity trap to open economies. The importance of fiscal policies such as government spending expansions and tax reductions in the zero

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lower bound is emphasized in Eggertsson (2011), and Christiano et al. (2011).

More recent studies on this topic introduce financial frictions to the model, and so does our paper. When financial markets are complete, households and firms have full access to international borrowing and lending to insure against risks, such as preference shocks, productivity shocks, or any abrupt change of policies. Financial assets are traded to ensure that the marginal utility value of a unit of currency is equalized across country borders, which is called perfect risk-sharing. However, in reality, there are financial frictions such as the transaction fee for stocks and bonds, the securities exchange tax, and capital controls by governments. The literature has shown how to introduce financial frictions to the model extensively, but we use the method in Devereux and Yetman (2014), where the distortion to financial markets lies in the wedge between the returns of securities for two countries.[3]

The second line of literature that is related to our paper is about the exchange rate flexibility. As Friedman (1953) argues, if nominal prices adjust quickly, the choice of exchange rate system would be irrelevant since price adjustments occur internally. However, in reality, internal prices are highly inflexible and relative prices adjust sluggishly, and so the exchange rate system varies.[4] Dornbusch et al. (1976) states that monetary policies should focus on targeting output gaps and inflation, but let the exchange rate float freely. Obstfeld and Rogoff (1995) analyze the costs and practicality of pegging the exchange rate. In Clarida et al. (2002), the exchange rate adjustment is important for a producer-currency pricing economy, whereas Devereux and Engel (2003) claim that a fixed exchange rate facilitates a local-currency pricing economy. We adopt producer-currency pricing in our model.

By combining the two strands of the literature, we compare the welfare of floating-rate and fixed-rate systems when monetary policies fail to stimulate aggregate de-

[3]For example, in Benigno (2009), an extra cost is introduced for undertaking positions in the asset market. In Corsetti et al. (2013), only non-state-contingent bonds are allowed for trading.
[4]See Bils and Klenow (2004) and Kehoe and Midrigan (2015) for empirical evidence.

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mand. Our main results are as follows. When both countries fall into liquidity traps, a fixed exchange rate outperforms a floating one in terms of welfare, regardless of the degree of financial integration. The perverse response of relative prices in a floating-rate system under liquidity traps is the key factor that causes a fixed exchange rate to override a floating exchange rate. The fixed-rate system provides a cushion to the economy when a negative shock occurs by having a smaller change in relative prices. Therefore, the output gap and inflation are both milder. If we ignore the zero lower bound and allow for negative interest rates, then a floating exchange rate dominates a fixed one mostly, while with some degree of capital controls the fixed-rate system may outweigh. This is because in the fixed-rate system, capital controls may help the economy stabilize the trade balance. In contrast, relative price adjustments in the floating-rate system are faster and more efficient, so extra interference in financial markets such as capital controls is welfare-reducing.

This paper is closely related to studies that include exchange rate comparison and liquidity traps. Erceg and Lindé (2012) compare the effects of a government spending cut in either a single currency union or a zero lower bound. Benigno and Romei (2014) study the role of monetary policy and exchange rate regimes in mitigating the cost of debt deleveraging. Cook and Devereux (2016) state that a fixed exchange rate is more desirable than a floating one in liquidity traps, but Corsetti et al. (2017) claim that a floating-rate system is better in the zero lower bound if demand shocks originate from outside of the country. It is concluded that the source of the shock as well as the policy instrument is the key factor that determines whether a country should adopt a floating or fixed exchange rate.

Our study differs from them in that we allow for different degrees of financial integration that will affect the transmission of shocks. The primary result of Cook and Devereux (2016) is unchanged with financial imperfections, but in the presence of negative interest rates, a floating-rate system can be inferior under some financial

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frictions. In sum, we provide a comparison between floating and fixed exchange rates in an economy with restraints on monetary policy and financial imperfections.

The paper proceeds as follows. Section 2 sets up the two-country model. Section 3 examines the impacts of a negative demand shock to different exchange rate regimes. Section 4 provides numerical simulation of the welfare, and Section 5 concludes.

2 A two-country model

We introduce financial frictions into a world economy as developed by Cook and Devereux (2016). There are two countries, "home" and "foreign", in the world economy. Both countries are of the same size with their population normalized to unity. Households consume and work given both prices and wages. Financial markets are complete within countries, but cross-country financial completeness varies from fully integrated financial markets to financial autarky. Firms produce differentiated goods with sticky prices.[5] The government subsidizes firms by lump-sum taxation. All foreign variables are marked with asterisks.

2.1 Households

A representative household in the home country can live infinitely with the utility evaluated from time 0 in the form of:

$$\mathcal{U}_t = E_0 \sum_{t=0}^{\infty} \beta^t (U(C_t, \xi_t) - V(N_t)), \tag{1}$$

where $\beta \in (0,1)$ is the time discount factor, U is the utility of the composite consumption C_t, and V is the disutility of labor supply N_t. The variable ξ_t is the demand shock, by which a positive value means a rise in the preference to consume

[5] The equilibrium under fully flexible prices is derived in Appendix A.

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today, and a negative value means a rise in the preference to consume in the future, so savings increase today. The function U is differentiable in C_t and concave, while V is differentiable in N_t and convex. Moreover, suppose that $U_{C\xi} > 0$ because an increase in ξ_t raises the marginal utility of consumption at time t.[6]

Composite consumption can be illustrated as a basket of home and foreign produced goods, formally defined as:

$$C_t = (\frac{\nu}{2} C_{H_t})^{\frac{\nu}{2}}((1-\frac{\nu}{2})C_{F_t})^{1-\frac{\nu}{2}},\ \nu \in [1,2],$$

where C_{H_t} is the home produced composite good, and C_{F_t} is the foreign produced composite good. If $\nu = 1$, households prefer home and foreign goods equally; if $\nu > 1$, there is a home bias for domestic goods; if $\nu = 2$, home households do not consume foreign goods at all, which represents no trade in goods markets between two countries. Similarly, foreign households have composite consumption (denoted in foreign currency) of the form:

$$C_t^* = (\frac{\nu}{2} C_{F_t}^*)^{\frac{\nu}{2}}((1-\frac{\nu}{2})C_{H_t}^*)^{1-\frac{\nu}{2}},\ \nu \in [1,2].$$

Both C_{H_t} and C_{F_t} (with prices P_{H_t} and P_{F_t}, respectively) are CES aggregates over a continuum of differentiated goods, and $\theta > 1$ is the elasticity of substitution between intermediate goods.

$$C_{H_t} = [\int_0^1 C_{H_t}(i)^{1-\frac{1}{\theta}} di]^{\frac{1}{1-\frac{1}{\theta}}},\ C_{F_t} = [\int_0^1 C_{F_t}(i)^{1-\frac{1}{\theta}} di]^{\frac{1}{1-\frac{1}{\theta}}},$$
$$P_{H_t} = [\int_0^1 P_{H_t}(i)^{1-\theta} di]^{\frac{1}{1-\theta}},\ P_{F_t} = [\int_0^1 P_{F_t}(i)^{1-\theta} di]^{\frac{1}{1-\theta}},$$

where differentiated goods are produced by a continuum of firms i whose produc-

[6]The notation $U_{C\xi}$ indicates the partial derivative of U with respect to C_t, then with respect to ξ_t.

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tion function is determined later. The demand for each differentiated good is:

$$\frac{C_{j_t}(i)}{C_{j_t}} = (\frac{P_{j_t}(i)}{P_{j_t}})^{-\theta},\ j = H, F.$$

We assume that the law of one price holds for both home and foreign goods, so $P_{H_t}(i) = S_t P^*_{H_t}(i)$ and $P_{F_t}(i) = S_t P^*_{F_t}(i)$, where S_t is the nominal exchange rate measured as the home price of foreign currency. From the home country's perspective, $P_{H_t}(i)$ is the price firm i charges at home and $P^*_{H_t}(i)$ is the price firm i charges abroad in foreign currency. Therefore, a rise in S_t represents a nominal depreciation of home currency. The aggregate (CPI) price indices are:

$$P_t = P_{H_t}^{\frac{\nu}{2}} P_{F_t}^{1-\frac{\nu}{2}},$$
$$P_t^* = P_{F_t}^{*\ \frac{\nu}{2}} P_{H_t}^{*\ 1-\frac{\nu}{2}}.$$

For aggregate price indices, the law of one price does not hold because the consumption baskets for home and foreign households have different weights due to home bias. The real exchange rate is $\frac{S_t P_t^*}{P_t} = (\frac{P_{F_t}}{P_{H_t}})^{\nu-1}$, which equals to 1 only when there is no home bias ($\nu = 1$).

Given price P_t and nominal wage W_t, home households decide how much labor to supply according to:

$$U_C(C_t, \xi_t) W_t = P_t V_N(N_t), \tag{2}$$

where U_C is the first derivative of U with respect to the variable C_t, and V_N is the first derivative of V with respect to the variable N_t.

As for the financial market, there is a full set of state-contingent securities traded between countries. To model the degree of financial integration, we introduce a wedge in risk-sharing by allowing the government to tax the returns (or to subsidize the deficits) on securities, which means that the returns to home and foreign

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households can be different.[7] The tax revenue (or the subsidy cost) is financed by lump-sum transfers (taxes). Optimal risk-sharing across countries implies:

$$U_C(C_t, \xi_t) = U_{C^*}(C_t^*, \xi_t^*) \frac{P_t}{S_t P_t^*}(1 + \chi_t) = U_{C^*}(C_t^*, \xi_t^*) T_t^{1-\nu}(1 + \chi_t), \tag{3}$$

where U_{C^*} is the first derivative of U with respect to the variable C_t^*, and $T_t = \frac{P_{F_t}}{P_{H_t}}$ is the home country terms of trade, defined as the ratio of foreign goods price to home goods price. A rise in T_t marks a depreciation of the home's terms of trade because home goods become relatively cheaper.

The state-contingent tax χ_t has the form:

$$(1 + \chi_t) = (\frac{P_t C_t}{P_{H_t} Y_t})^{\frac{\lambda}{1-\lambda}}, \tag{4}$$

where Y_t is the home country GDP, and the parameter $\lambda \in [0, 1]$ governs the degree of financial integration. For the home government, whether to tax or subsidize depends on the trade balance. If $\frac{P_t C_t}{P_{H_t} Y_t} = 1$, the trade balance is zero, so there is no tax or subsidy. If $\frac{P_t C_t}{P_{H_t} Y_t} > 1$, there is a trade deficit and a positive tax on returns of securities to curb domestic consumption. If $\frac{P_t C_t}{P_{H_t} Y_t} < 1$, there is a trade surplus and a subsidy on securities to boost domestic consumption. The amount of tax and subsidy is then determined by λ. When $\lambda = 0$, financial markets are complete with unlimited securities trade, meaning that no tax or subsidy will be collected, regardless of the trade balance. When $\lambda = 1$, financial autarky indicates zero securities trade, so there is no cross-country risk-sharing through financial markets.

Households also possess domestic nominal government bonds that pay interest rate R_t. The Euler equation is:

$$\frac{U_C(C_t, \xi_t)}{P_t} = \beta R_t E_t \frac{U_C(C_{t+1}, \xi_{t+1})}{P_{t+1}}. \tag{5}$$

[7]This method follows Devereux and Yetman (2014).

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A representative foreign household's preference and decision rules are defined symmetrically. Only the wedge in risk-sharing is created by the home government. In other words, the foreign government does not tax or subsidize security returns.

2.2 Firms

The model exhibits a producer-currency pricing economy where firms set prices in their own country's currency and face a Calvo pricing technology. Each firm i produces with only labor as input. The production function takes the form:

$$Y_t(i) = N_t(i).$$

Profits are $\Pi_t(i) = P_{H_t}(i)Y_t(i) - W_t N_t(i)(1-\frac{1}{\theta})$, where $\frac{1}{\theta}$ represents a wage subsidy to all home firms by the government, financed by lump-sum taxation. This optimal subsidy removes the markup distortion due to the monopoly pricing of intermediate goods with elasticity of substitution θ, so that the steady state level of production is efficient. From time $t-1$ to t, a fraction κ of firms are unable to change their prices $P_{H_{t-1}}$, while the other $1-\kappa$ of firms can readjust their prices flexibly to $\tilde{P}_{H_t}$ to maximize the present value of profits, $E_t \sum_{j=0} m_{t+j}\kappa^j \Pi_t(i)$, using the stochastic discount factor $m_{t+j} = \beta \frac{U_C(C_{t+j},\xi_{t+j})}{P_{t+j}} / \frac{U_C(C_t,\xi_t)}{P_t}$. We state the adjusted price as:

$$\tilde{P}_{H_t}(i) = \frac{E_t \sum_{j=0} m_{t+j}\kappa^j W_{t+j} Y_{t+j}(i)}{E_t \sum_{j=0} m_{t+j}\kappa^j Y_{t+j}(i)}. \tag{6}$$

In the aggregate, the price index of home goods becomes:

$$P_{H_t} = [(1-\kappa)\tilde{P}_{H_t}^{1-\theta} + \kappa P_{H_{t-1}}^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{7}$$

Foreign firms behave similarly.

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2.3 Market clearing

Home firms supply home produced goods facing demands from both home and foreign households; foreign firms supply foreign produced goods facing demands from foreign (or domestic, from their point of view) and home households. The market equilibrium for home good i is:

$$Y_t(i) = (\frac{P_{H_t}(i)}{P_{H_t}})^{-\theta}[\frac{\nu}{2}\frac{P_t}{P_{H_t}}C_t + (1-\frac{\nu}{2})\frac{S_t P_t^*}{P_{H_t}}C_t^*].$$

Aggregate market clearing conditions are:

$$Y_t = \frac{\nu}{2}\frac{P_t}{P_{H_t}}C_t + (1-\frac{\nu}{2})\frac{S_t P_t^*}{P_{H_t}}C_t^*, \quad (8)$$

$$Y_t^* = \frac{\nu}{2}\frac{P_t^*}{P_{F_t}^*}C_t^* + (1-\frac{\nu}{2})\frac{P_t}{S_t P_{F_t}^*}C_t, \quad (9)$$

where $Y_t = Z_t^{-1}\int_0^1 Y_t(i)di$ is the aggregate home output, and $Z_t = \int_0^1 (\frac{P_{H_t}(i)}{P_{H_t}})^{-\theta}di$. Home labor demand is $N_t = \int_0^1 N(i)di = Y_t Z_t$. Foreign variables are defined similarly.

We solve the world equilibrium by equations (2),(3),(5),(6),(7),(8),(9) as well as the interest rate rule (to be described later). For given values of Z_t and Z_t^*, the sequence of $C_t, C_t^*, N_t, N_t^*, W_t, W_t^*, P_{H_t}, \tilde{P}_{H_t}, P_{F_t}^*, \tilde{P}_{F_t}^*, S_t, R_t, R_t^*$ can be determined.

3 The effects of a negative demand shock

We now suppose that there is a negative demand shock to the home country. In other words, out of some exogenous reasons, agents in the home country suddenly prefer to defer their consumption to the future, and increase their savings today. For simplicity, there is no demand shock in the foreign country. The demand shock in the home country returns to zero with probability $1-\mu$ next period, meaning that there is only $\mu \in (0,1)$ probability that the shock persists. Given this shock, we

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examine which type of exchange rate system is more preferable in terms of welfare.

For any variable X, we define $x = \ln X$, and the term $\tilde{x} = x - \bar{x}$ is the gap between the log of a variable under sticky prices and its efficient value under flexible prices. The inverse of the elasticity of intertemporal substitution in consumption is $\sigma > 1$, and the elasticity of the marginal disutility of working hours is ϕ, and $\varepsilon_t = \frac{U_{C\xi}}{U_C} \ln \xi_t$ measures a demand shock of the home country.

Since we are interested in the world economy as well as the reaction of each country when facing a demand shock, we define $x^W = \frac{x+x^*}{2}$ to be the world average value and $x^R = \frac{x-x^*}{2}$ to be the relative value for variables x and x^*. Moreover, define $D = \sigma\nu(2-\nu) + (1-\nu)^2$ and a function of the parameter λ that governs the financial completeness as:

$$\omega(\lambda) = \frac{\lambda(2-\nu)[2-\nu+\sigma(\nu-1)+\phi]}{2(1-\lambda)(\phi D+\sigma)+\lambda(2-\nu)[2-\nu+\sigma(\nu-1)+\phi]},$$

where $\omega(0) = 0,\ \omega(1) = 1$, and $\omega'(\lambda) > 0$ for $\lambda \in [0,1]$.

We analyze the model with forward looking inflation equations and open economy IS relations. For the world average variables, we have:

$$\pi_t^W = k(\phi+\sigma)\tilde{y}_t^W + \beta E_t \pi_{t+1}^W, \tag{10}$$

$$\sigma E_t(\tilde{y}_{t+1}^W - \tilde{y}_t^W) = r_t^W - \bar{r}_t^W - E_t \pi_{t+1}^W. \tag{11}$$

The relative variables are as follows:

$$\pi_t^R = k[\phi + \sigma_D\omega_1 + (1-\omega_1)(2-\nu+\sigma(\nu-1))]\tilde{y}_t^R + \beta E_t \pi_{t+1}^R, \tag{12}$$

$$E_t[\sigma_D\omega_1 + (1-\omega_1)(2-\nu+\sigma(\nu-1))](\tilde{y}_{t+1}^R - \tilde{y}_t^R) = r_t^R - \bar{r}_t^R - E_t\pi_{t+1}^R, \tag{13}$$

where $\sigma_D = \frac{\sigma}{D}$, and $\omega_1(\lambda) = \frac{2D(1-\lambda)}{2D(1-\lambda)+\lambda(2-\nu)}$. The degree of price stickiness is determined by $k = \frac{(1-\beta\kappa)(1-\kappa)}{\kappa}$. Equations (10) and (12) are the forward looking inflation

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equations stating the positive relation between the output gap and the inflation. Equations (11) and (13) are the IS equations that relate nominal interest rates to output gaps. Natural interest rates $\bar{r}_t^W$ and $\bar{r}_t^R$ are defined as the rates that sustain the flexible price equilibrium while controlling for zero inflation.[8]

By solving equations (10)-(13), we can obtain the responses of $\tilde{y}_t^W$, π_t^W, $\tilde{y}_t^R$, and π_t^R to a negative demand shock. The solutions for these variables depend on the monetary rules r_t^W and r_t^R determined by both countries. We discuss the following three cases: (1) a floating exchange rate where both countries use a Taylor rule to set nominal interest rates, (2) a floating exchange rate with both countries in liquidity traps, and (3) a fixed exchange rate where the home country uses a Taylor rule and the foreign country pegs its interest rate to the home's interest rate.

3.1 Floating exchange rate under a Taylor rule

In a floating-rate system, there is no restriction to the nominal exchange rate between two countries. Home and foreign both have their own monetary autonomy to set the nominal interest rate in order to control for the domestic output gap and inflation. We assume that both countries adopt a simple Taylor rule to set their interest rate, by which the monetary policy targets producer price index (PPI) inflation. The home interest rate and the foreign interest rate are:

$$r_t = \rho + \gamma \pi_t, \tag{14}$$

$$r_t^* = \rho + \gamma \pi_t^*, \tag{15}$$

where ρ is the steady state value of the natural interest rate, and γ is the parameter for Taylor rule. The world average and relative values are written as $r_t^W = \rho + \gamma \pi_t^W$ and $r_t^R = \gamma \pi_t^R$. The solutions to equations (10)-(13) are thus determined by the interest rate rules in equations (14) and (15).

[8]For a full derivation of natural interest rates, see Appendix B.

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The real exchange rate can be represented by the terms of trade, $\frac{S_t P_t^*}{P_t} = T_t^{\nu-1}$, because given a degree of trade openness between two countries, a rise in the terms of trade indicates a real depreciation of the home currency. Therefore, the nominal exchange rate has its relation with the real exchange rate governed by:

$$s_t - s_{t-1} = \pi_t^R + \tau_t - \tau_{t-1}, \tag{16}$$

where $\tau_t = \ln T_t$.

We then derive the relationship between the relative inflation and the relative output gap. Since there is no state variable in the floating exchange rate model, every variable follows the same stochastic process as ε_t, which is $E_t \varepsilon_{t+1} = \mu \varepsilon_t$.[9] By substituting $E_t \pi_{t+1}^R = \mu \pi_t^R$ in equation (12), we obtain:

$$\pi_t^R = \frac{k}{1-\beta\mu}[\phi + \sigma_D \omega_1 + (1-\omega_1)(2-\nu+\sigma(\nu-1))]\tilde{y}_t^R. \tag{17}$$

On a $\pi_t^R - \tilde{y}_t^R$ diagram, it is an upward sloping line, indicating that a rise in the relative output gap leads to a rise in the relative inflation. As for equation (13), the Taylor rule implies $r_t^R = \gamma \pi_t^R$ (assume $\gamma > \mu$),[10] and $\bar{r}_t^R$ is the relative natural interest rate; thus, we have:

$$\begin{aligned}\pi_t^R = &-\frac{1-\mu}{\gamma-\mu}[\sigma_D \omega_1 + (1-\omega_1)(2-\nu+\sigma(\nu-1))]\tilde{y}_t^R \\ &+ \frac{1-\mu}{\gamma-\mu}[\frac{(1-\omega)\phi(\nu-1)}{\sigma+\phi D} + \frac{\omega\phi}{2-\nu+\sigma(\nu-1)+\phi}]\varepsilon_t^R,\end{aligned} \tag{18}$$

which is a downward sloping line on the $\pi_t^R - \tilde{y}_t^R$ diagram. This is an analogue of the traditional credit demand curve. By equations (17) and (18), we can solve the equilibrium of relative inflation and output gap. *Figure* 2 shows that when facing a

[9] This rule does not apply to the fixed-rate system, which will be described later.

[10] This assumption is reasonable because γ is usually estimated as 1.5 in a standard Taylor rule (see Hofmann and Bogdanova (2012) for example), and $\mu \in (0,1)$ is a probability measure.

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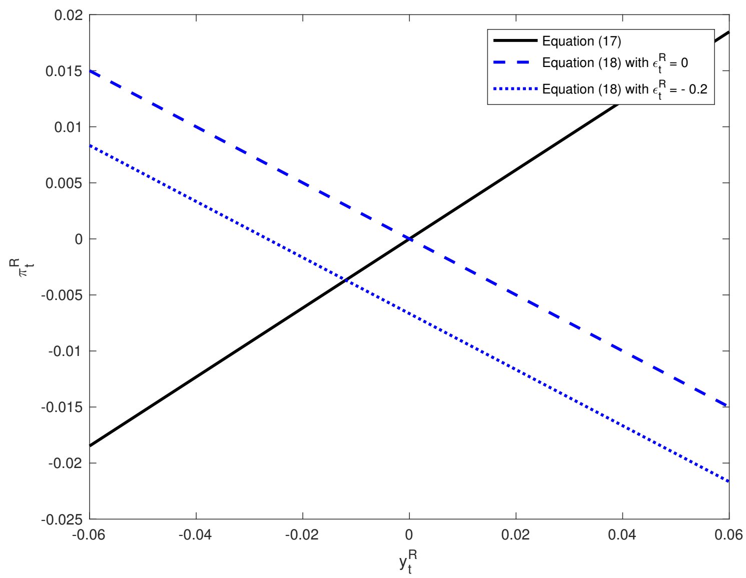
Figure 2: The $\pi_t^R - \tilde{y}_t^R$ diagram under a Taylor rule



negative demand shock ($\varepsilon_t^R < 0$), the whole line of equation (18) moves to the left, so both the equilibrium π_t^R and $\tilde{y}_t^R$ decrease.[11] Notice that there is no effect under the case of $\lambda = 0,\ \nu = 1$ because outputs and interest rates stay the same if consumers have no home bias.

With monetary autonomy, the government can set its own nominal interest rate following a Taylor rule. By the uncovered interest rate parity, the relative interest rate equals to the expected change of the nominal exchange rate. We can therefore derive the relationship between the relative inflation and the terms of trade:

$$r_t^R = \gamma \pi_t^R = E_t(\pi_{t+1}^R + \tau_{t+1} - \tau_t), \tag{19}$$

[11]The parameters in *Figure* 2 are $\nu = 1.5,\ \lambda = 0.5,\ \beta = 0.99,\ k = 0.05,\ \sigma = 2,\ \phi = 1,\ \rho = 0.01,\ \gamma = 3,$ and $\mu = 0.6$. The relative demand shock ε_t^R goes from 0 to -0.2.

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so the terms of trade becomes:

$$\tau_t = -\frac{\gamma - \mu}{1 - \mu}\pi_t^R.$$

When a negative demand shock lowers the relative inflation, the terms of trade deteriorates.[12] Home goods are now relatively cheaper than foreign goods, so the impact of a negative demand shock at the home country can be mitigated. As for the nominal exchange rate, the home currency depreciates:

$$s_t - s_{t-1} = -\frac{\gamma - 1}{1 - \mu}\pi_t^R.$$

3.2 Floating exchange rate in liquidity traps

Now we turn to the case where both countries are in liquidity traps. Different from the previous case, if the negative demand shock pushes the natural interest rate down below zero, the government can no longer use an effective monetary policy to stabilize the economy.

We assume that nominal interest rates in equations (14) and (15) hit the zero lower bound, $r_t = r_t^* = 0$, so that both countries are stuck in liquidity traps. Equation (13) becomes:

$$\begin{aligned}\pi_t^R =& \frac{1-\mu}{\mu}[\sigma_D\omega_1 + (1-\omega_1)(2-\nu+\sigma(\nu-1))]\tilde{y}_t^R \\ &- \frac{1-\mu}{\mu}[\frac{(1-\omega)\phi(\nu-1)}{\sigma+\phi D} + \frac{\omega\phi}{2-\nu+\sigma(\nu-1)+\phi}]\varepsilon_t^R,\end{aligned} \tag{20}$$

which is an upward sloping line on the $\pi_t^R - \tilde{y}_t^R$ diagram. The reversing slope is caused by the nominal interest rate binding at zero, so a high inflation refers to a low real interest rate. Thus, agents in the loan market tend to borrow more, re-

[12] We will see in the next section that the terms of trade goes in the different direction under liquidity traps, rendering ineffective outcomes.

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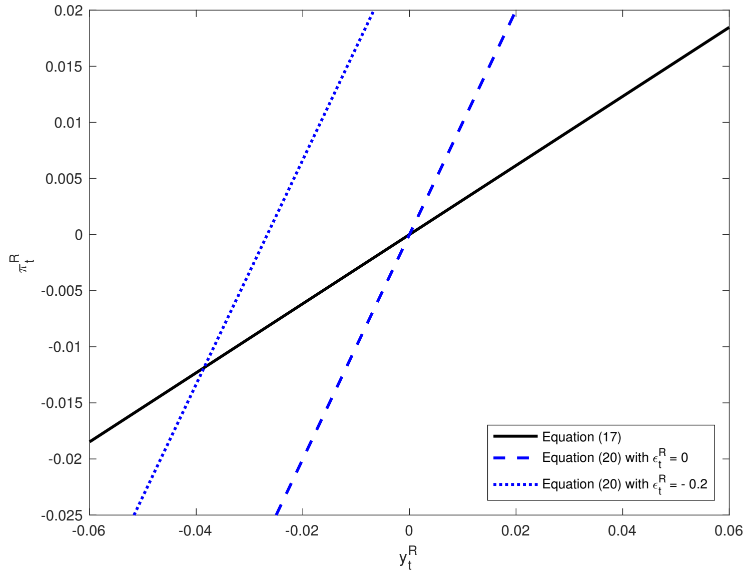
Figure 3: The $\pi_t^R - \tilde{y}_t^R$ diagram in liquidity traps



sulting in a positive credit demand curve. In this scenario, the equilibrium relative inflation and output gap in the zero lower bound are solved by equations (17) and (20). We also assume that equation (20) has a bigger slope.[13] When facing a negative demand shock ($\varepsilon_t^R < 0$), the line of equation (20) moves to the left, which is demonstrated in *Figure* 3.[14] The equilibrium π_t^R and $\tilde{y}_t^R$ decrease on a larger scale versus the case under a Taylor rule. This is because when the nominal interest rate is stuck at zero, the fall in relative inflation raises the real interest rate, which pushes down the relative demand further, creating a larger decrease in the equilibrium output.

In liquidity traps, the uncovered interest rate parity in equation (19) has zero on the left-hand side, so the terms of trade becomes:

$$\tau_t = \frac{\mu}{1-\mu}\pi_t^R.$$

[13]The parameters satisfy: $\frac{1-\mu}{\mu}[\sigma_D\omega_1+(1-\omega_1)(2-\nu+\sigma(\nu-1))] > \frac{k}{1-\beta\mu}[\phi+\sigma_D\omega_1+(1-\omega_1)(2-\nu+\sigma(\nu-1))]$.
[14]The parameters are the same as *Figure* 2.

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One key difference in the zero-lower-bound scenario is that the terms of trade moves in the "wrong" direction. When the negative demand shock pushes down the relative inflation, the terms of trade appreciates instead. Again, when the nominal interest rate is stuck at zero, a decrease in inflation raises the real interest rate, according to the Fisher equation. Thus, home goods prices increase, and so the terms of trade appreciates. The nominal exchange rate also rises as the home currency becomes stronger:

$$s_t - s_{t-1} = \frac{1}{1-\mu}\pi_t^R.$$

3.3 Fixed exchange rate

In a traditional single currency area, countries pegging a nominal exchange rate must have the same nominal interest rate, given complete financial markets. If not, investors may invest more in the country with higher returns, and thus capital inflows may appreciate the nation's currency. The exchange rate cannot be fixed. This is the trilemma of an open economy. Therefore, two countries must have the same interest rate if they want to maintain the stability of exchange rates as well as zero capital control.

However, we allow for financial frictions in our model, so nominal interest rates in different countries do not have to be the same in a fixed-rate system. Instead, we adopt an alternative method in which the home country can adjust its interest rate according to a Taylor rule, while the foreign country pegs its interest rate to that of the home country, with a deviation of the nominal exchange rate. Nominal interest rates in the two countries are connected by the following rule:[15]

$$r_t^* = r_t - \delta \hat{s}_t, \ \delta > 0,$$

[15] This type of a fixed-rate system follows Benigno and Benigno (2008).

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where $\hat{s}_t = \ln \frac{S_t}{S^\star}$ and $S^\star = 1$ is the exchange rate target we set. If foreign currency tends to appreciate, which means an increase in the nominal exchange rate S_t, then the foreign interest rate will be adjusted lower, and vise versa if foreign currency depreciates. The world average and relative interest rates are $r_t^W = \rho + \gamma \pi_t^W$ and $r_t^R = \frac{\delta \hat{s}_t}{2}$.

Another aspect whereby a fixed exchange rate differs from a floating one is the constraint on the changes in the nominal exchange rate. With a fixed exchange rate, the left-hand side of equation (16) is zero, meaning that the nominal exchange rate is stable. The relative inflation of a fixed-rate system is then determined by the dynamics of the terms of trade, which follows:

$$\pi_t^R = \tau_{t-1} - \tau_t. \tag{21}$$

Notice that $E_t \varepsilon_{t+1} = \mu \varepsilon_t$ does not hold for the terms of trade in a fixed-rate system because with the constraint on the nominal exchange rate, the terms of trade in the previous period becomes the state variable in the current period.

In order to determine the relative inflation, we therefore have to know how the terms of trade evolves. It can be expressed in "gap" terms as:

$$\begin{aligned} \tau_t = & \frac{2\sigma(1-\lambda)+\lambda(2-\nu)}{(\nu-1)\triangle}\tilde{y}_t^R - \frac{2(1-\lambda)}{\triangle}\varepsilon_t^R \\ & + \frac{2\sigma(1-\lambda)+\lambda(2-\nu)}{(\nu-1)\triangle}[(1-\omega)(\frac{\nu-1}{\sigma+\phi D}) + \frac{\omega}{2-\nu+\sigma(\nu-1)+\phi}]\varepsilon_t^R, \end{aligned} \tag{22}$$

where $\triangle \equiv \frac{(1-\lambda)D-\lambda(\frac{\nu}{2}-1)}{\nu-1}$.[16] When financial markets are complete, we have $\lambda = 0$, $\omega = 0$, and $\triangle = \frac{D}{\nu-1}$, so the terms of trade becomes:

$$\tau_t = 2\sigma_D \tilde{y}_t^R - \frac{2\phi(\nu-1)}{\sigma+\phi D}\varepsilon_t^R.$$

[16] Equation (22) is derived from Appendix A: equations (A.8), (A.9), and (A.10).

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A rise in the relative output gap deteriorates the terms of trade, while a negative relative demand shock does the same if $\nu > 1$, thus having no effect if $\nu = 1$.

In the situation of financial autarky, $\lambda = 1$, $\omega = 1$, and $\triangle = \frac{1-\frac{\nu}{2}}{\nu-1}$, the terms of trade is:

$$\tau_t = 2\tilde{y}_t^R + \frac{2}{2-\nu+\sigma(\nu-1)+\phi}\varepsilon_t^R.$$

This indicates that if financial markets are completely closed, a positive relative output gap deteriorates the terms of trade, while a negative relative demand shock appreciates it. Similar to the floating-rate system in liquidity traps, there is also a perverse response of the terms of trade. The main difference is that the nominal exchange rate does not change in a fixed-rate system, so it is the rise of the relative inflation that drives the appreciation in the terms of trade. We exclude the case where $\nu = 2$ because there is no trade at all.

Combining equations (21) and (22), we obtain the relation between the relative inflation and the relative output gap in a fixed-rate system:

$$\begin{aligned}\pi_t^R = &\frac{2\sigma(1-\lambda)+\lambda(2-\nu)}{(\nu-1)\triangle}(\tilde{y}_{t-1}^R - \tilde{y}_t^R) - \frac{2(1-\lambda)}{\triangle}(\varepsilon_{t-1}^R - \varepsilon_t^R) \\ &+ \frac{2\sigma(1-\lambda)+\lambda(2-\nu)}{(\nu-1)\triangle}[(1-\omega)(\frac{\nu-1}{\sigma+\phi D}) + \frac{\omega}{2-\nu+\sigma(\nu-1)+\phi}](\varepsilon_{t-1}^R - \varepsilon_t^R).\end{aligned} \tag{23}$$

This equation is an analogue of equations (18) and (20) under a floating exchange rate. It is a dynamic equation by which the equilibrium can be solved along with equation (17), but it cannot be presented on a $\pi_t^R - \tilde{y}_t^R$ diagram directly due to the existence of state variables $\tilde{y}_{t-1}^R$ and ε_{t-1}^R. The numerical simulation is described in Section 4.

4 Numerical simulation

To compare the impact of a negative demand shock to the world economy under three situations discussed above, which are the floating exchange rate under a Taylor

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rule, the floating exchange rate in the zero lower bound, and the fixed exchange rate, we conduct a numerical simulation to explicitly demonstrate the difference. In order to compare our results with Cook and Devereux (2016), we follow their parameter settings: $\beta = 0.99$, $k = 0.05$, $\sigma = 2$, $\phi = 1$, $\rho = 0.01$, $\gamma = 3$. The negative demand shock $\varepsilon = -0.5$ occurs at $t = 0$ and persists with probability $\mu = 0.6$. This shock is strong enough to push both countries into liquidity traps.

Figure 4 presents the responses of variables $\tilde{y}_t^R$, $\tilde{y}_t^W$, π_t^R, π_t^W, and τ_t to a negative demand shock, fixing the parameter $\nu = 1.5$ and $\lambda = 0$. From the perspective of a home agent, the consumption basket is composed of $\frac{3}{4}$ home produced goods and $\frac{1}{4}$ foreign produced goods, and vice versa for a foreign agent. The financial markets are fully integrated. We can see that the relative output gap and inflation fall under all exchange rate regimes given a negative demand shock. The floating exchange rate in normal times, when the Taylor rule applies, renders a smaller decrease in relative output gap and inflation because of the depreciation in the terms of trade. If the price of home goods is relatively cheaper, the impact of the negative demand shock in the home country will be milder. As for the fixed exchange rate, the terms of trade also depreciates, but adjusts slowly due to the constraint on the nominal exchange rate. The relative output gap and inflation in the fixed-rate system fall more than those in the floating-rate system in normal times, but to a less extent versus the zero-lower-bound case.

When both countries fall into liquidity traps, which means that nominal interest rates are stuck at zero, the stabilization of a floating exchange rate disappears. Monetary policies are no longer effective, and a negative demand shock appreciates the terms of trade, making home produced goods even more expensive. Thus, the relative output gap and inflation severely decrease, resulting in a greater loss than in the fixed-rate system. Notice that the world output and world inflation under a normal floating and a fixed exchange rate are the same because they are both solved from

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equations (10) and (11), using the same world interest rate. A floating exchange rate in the zero lower bound has the world interest rate set to zero in equation (11).

The main reason why a floating exchange rate is less preferable in liquidity traps is that the terms of trade reacts in the "wrong" direction, appreciating instead of depreciating when a negative shock occurs. If home goods become relatively more expensive when a negative demand shock occurs, there is going to be an even larger fall in demand and the relative output. On the contrary, a fixed exchange rate cushions the impact of a negative shock to relative prices, enabling the terms of trade to adjust slowly to the market demands. This is the scenario of having complete financial markets. However, the results are different under financial autarky.

Figure 5 presents the impacts of a negative demand shock in financial autarky. All parameters are equal as in *Figure* 4, except that the parameter governing financial integration is $\lambda = 1$. First, let us focus on the floating exchange rate. When financial markets are shut down, the direction that each variable moves is the same as that in complete financial markets, with only differences in scale. The terms of trade has a larger jump in financial autarky because without any trade of state-contingent securities, exogenous shocks can only be absorbed by the goods markets, and thus reflected on goods prices. A larger change in goods prices also explains a wider relative output gap and a more severe inflation in financial autarky.

When it comes to the fixed exchange rate, the relative output gap and inflation as well as the terms of trade react differently in *Figure* 4 and *Figure* 5. This is because a fixed exchange rate also makes the terms of trade go in the "wrong" direction, like the situation in the zero-lower-bound floating exchange rate. Instead of decreasing the relative price of home goods, the fixed exchange rate makes the home goods price higher, appreciating the terms of trade and enlarging the inflation and output gap. The relative output gap and inflation being positive in a fixed-rate system indicates that in the absence of financial activities, the relatively high price for home goods

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leads to over-production in home factories. Home firms produce above the efficient level because given a fixed nominal exchange rate, they are unable to make up for the loss in profits at home by adjusting the price of goods selling abroad. They can only produce more and try to compensate for the profit loss by raising the quantity.

So far we have discussed in details about the impacts of a negative demand shock to different exchange rate systems, but which type of exchange rate regime is better in terms of welfare? To answer this question, we derive a second-order approximation of the loss function according to Engel (2011) to compare the welfare of different exchange rate systems. The social welfare of the economy is measured jointly by both countries. Specifically, it is the quadratic combination of the world average output gap, the relative output gap, the home inflation, and the foreign inflation terms. The loss function in each period has the following form:[17]

$$\begin{aligned}\tilde{v}_t = &-(\sigma+\phi)(\tilde{y}_t^W)^2 - [(1+\phi)-(1-\sigma)(\frac{\nu-1}{D})^2](\tilde{y}_t^R)^2 - \frac{\theta}{2k}(\pi_t^2+\pi_t^{*2}) \\ &-\frac{\nu}{2}(1-\frac{\nu}{2})[(1-\frac{1}{\sigma})\frac{2\sigma(1-\lambda)+\lambda(2-\nu)}{\triangle}]^2(\tilde{y}_t^R)^2.\end{aligned} \tag{24}$$

Notice that the function produces a negative value because any output gap or inflation caused by the exogenous shock is deemed as a loss in welfare. In other words, a perfect economy with no output gap and zero inflation will have the highest welfare under this measure.

Figure 6 presents the discounted sum of welfare loss for each scenario under different degree of financial integration.[18] As the upper panel shows, the welfare of a floating exchange rate under normal situations is always higher than that under liquidity traps, without a doubt. What we are interested in is the performance of the fixed exchange rate. A fixed exchange rate outperforms a floating one in liquid-

[17]The loss function is approximated around a first-best undistorted steady state. See Appendix C for a detailed derivation.

[18]*Figure* 6 uses the same parameters as *Figure* 4. The horizontal axis is the degree of financial integration λ ranging from 0 to 1. We sum up the discounted loss in the period $t = 0 \sim 10$.

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ity traps regardless of the degree of financial integration. Comparing the fixed-rate regime to the floating-rate one in normal times, we can see that the latter always outweighs the former.

However, if negative interest rates can be implemented, as the lower panel shows, a fixed-rate system may outplay a floating-rate system with a certain degree of financial frictions across countries. From our numerical simulation, it is approximately within the range of $\lambda = 0.4 \sim 0.8$ that a fixed-rate system surpasses a floating-rate system in normal times. It is conceivable that a certain degree of financial frictions is good for an economy with a fixed exchange rate. Financial frictions such as capital controls have the property of stabilizing trade balances. When facing risk premium shocks, optimal capital controls can mitigate capital inflow surges or capital flights, neither of which can be easily dismissed if the country is bounded by a fixed exchange rate. Nevertheless, capital controls should not be too extreme as the financial autarky scenario where the terms of trade will appreciate and firms over produce. As for a floating exchange rate system, capital controls do not play an important role since prices in financial markets adjust more flexibly. Financial restrictions can thus be unnecessary and welfare-reducing.

5 Conclusion

This study presents a comparison of floating and fixed exchange rates in an economy with financial frictions and liquidity traps. Our analysis shows that a fixed exchange rate outperforms a floating one in liquidity traps, regardless of the degree of financial integration. The perverse response of relative prices in a floating-rate system under liquidity traps is the key factor that causes a fixed exchange rate to dominate. If we ignore the zero lower bound and allow for negative interest rates, a fixed-rate system can perform better under some financial frictions. For a floating-rate system, imperfect financial integration is welfare-reducing.

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We conclude by providing three possible extensions of our paper. First, it is applicable for countries to set their monetary policies cooperatively in a world economy. In our analysis, we simply assume that both countries use a Taylor rule for setting domestic interest rates. Second, we introduce financial frictions by taxing securities, but frictions can arise in other channels such as transaction costs, liquidity constraints of capital, and so on. Different sources of financial imperfections may result in other possibilities. Finally, since the source of the shock is the key factor that determines the exchange rate policy, it is interesting to analyze the productivity shock or other shock that originates from the foreign country.

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Figure 4: Impacts of a negative demand shock in complete financial markets

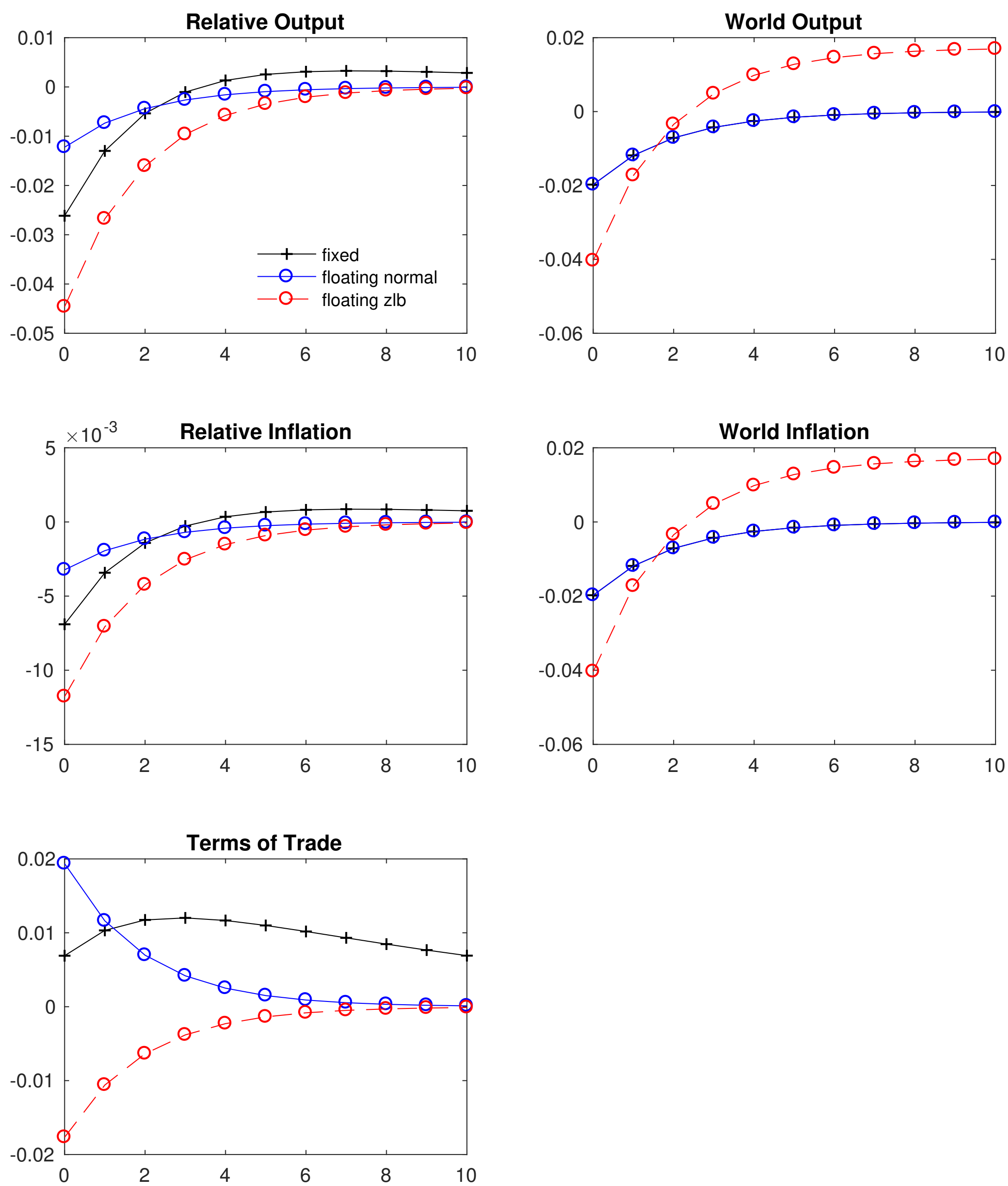

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Figure 5: Impacts of a negative demand shock in financial autarky

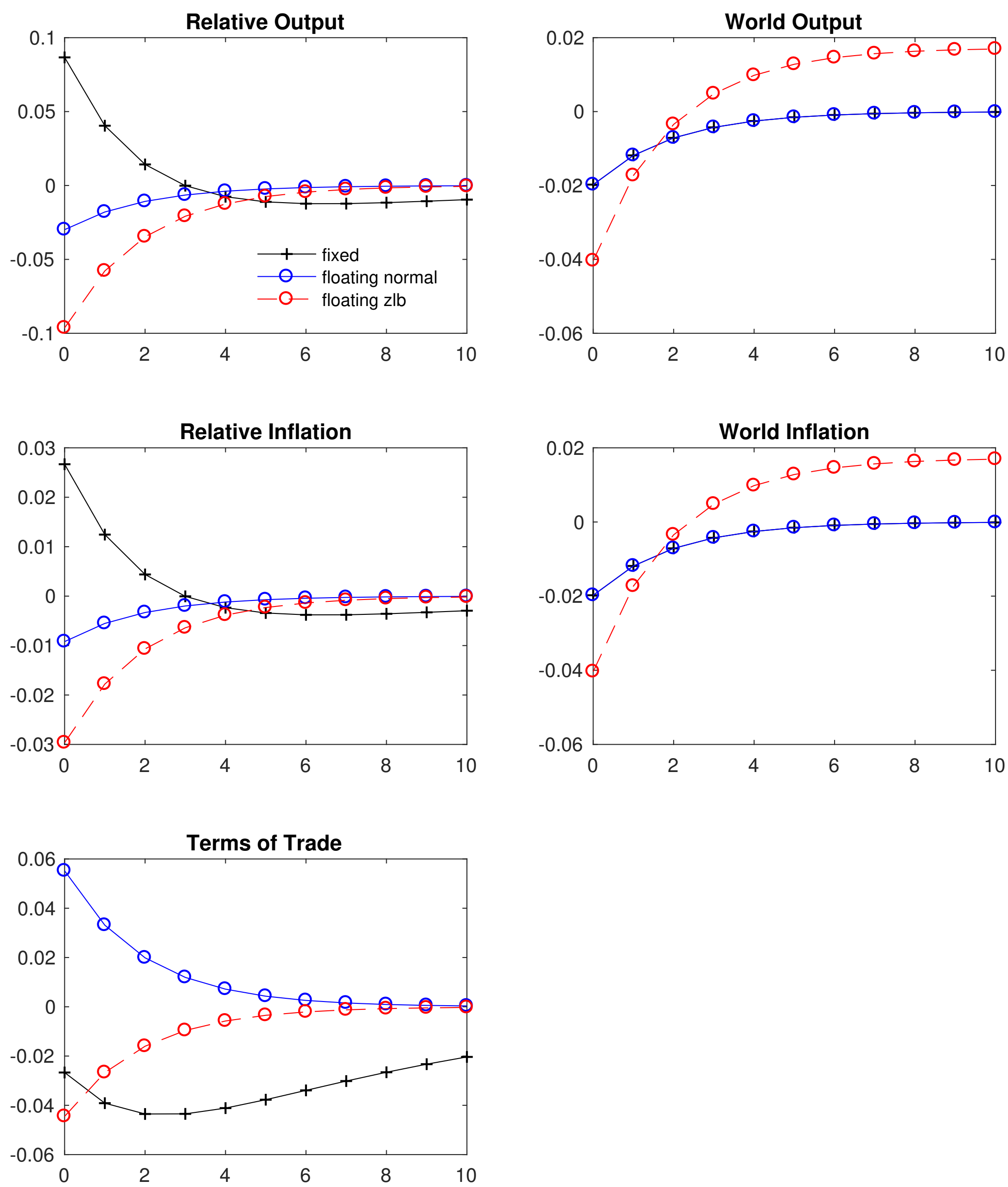

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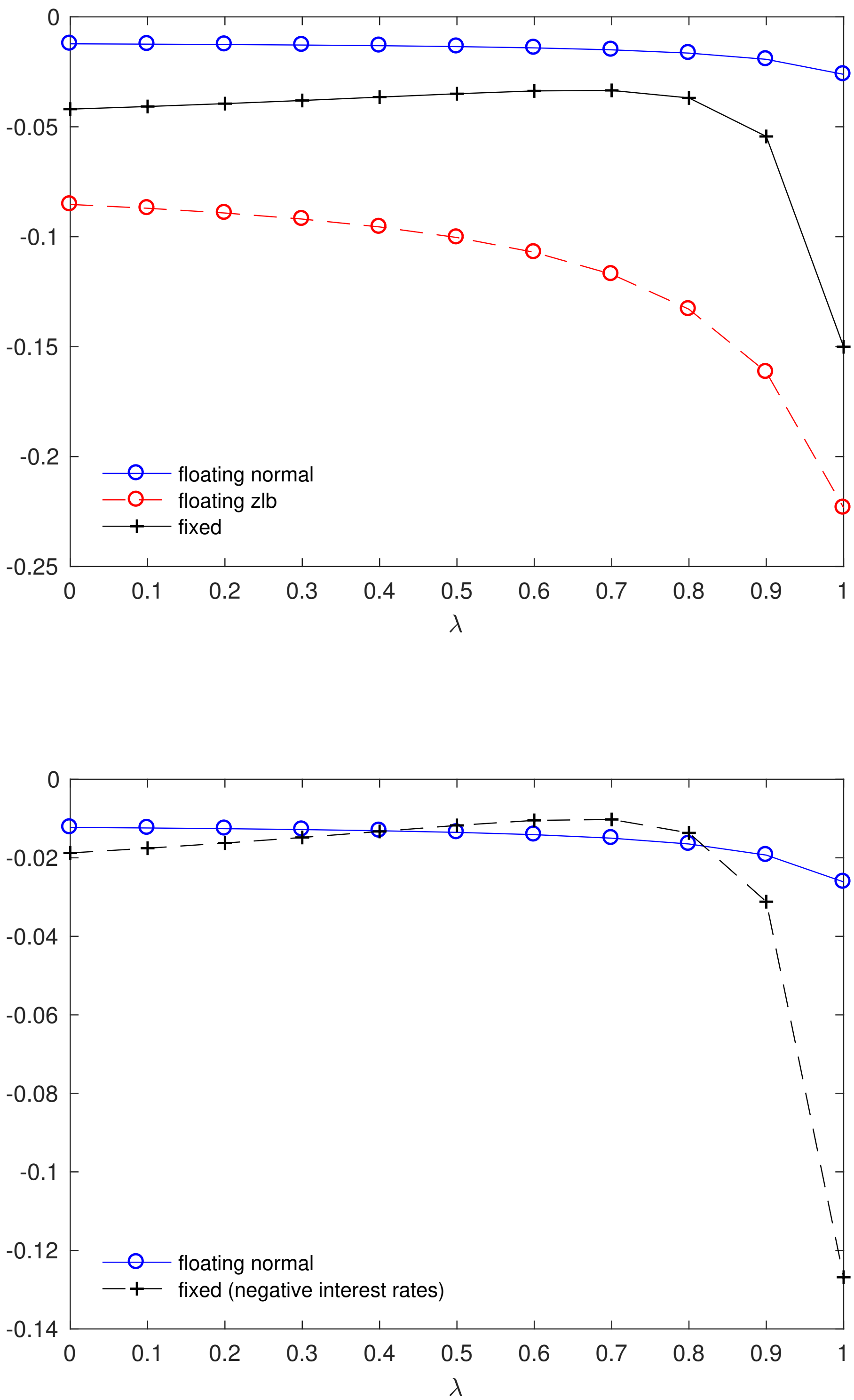
Figure 6: Welfare comparison of exchange rate regimes



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Appendix A Equilibrium under flexible prices

When prices are fully flexible ($\kappa = 0$), the economy allows all firms to reset their prices given any shock in the previous period. We have $P_{H_t}(i) = P_{H_t}$, $P_{F_t}(i) = P_{F_t}$, and $Z_t = Z_t^* = 1$. With optimal subsidy, we derive $P_{H_t} = W_t$, $P^*_{F_t} = W_t^*$ from the profit maximization of firms. Each variable in the flexible price equilibrium is denoted by a bar. Equation (2) and its foreign counterpart become:

$$U_C(\bar{C}_t, \xi_t) = \bar{T}_t^{1-\frac{\nu}{2}} V_N(\bar{N}_t), \tag{A.1}$$

$$U_{C^*}(\bar{C}_t^*, \xi_t^*) = \bar{T}_t^{\frac{\nu}{2}-1} V_{N^*}(\bar{N}_t^*). \tag{A.2}$$

The risk-sharing condition in equation (3) can be rearranged as:

$$U_C(\bar{C}_t, \xi_t)\bar{T}_t^{\nu-1}\left(\frac{\bar{T}_t^{\frac{\nu}{2}-1}\bar{Y}_t}{\bar{C}_t}\right)^{\frac{\lambda}{1-\lambda}} = U_{C^*}(\bar{C}_t^*, \xi_t^*). \tag{A.3}$$

Market clearing conditions in equations (8) and (9) are:

$$\bar{Y}_t = \frac{\nu}{2}\bar{T}_t^{1-\frac{\nu}{2}}\bar{C}_t + (1-\frac{\nu}{2})\bar{T}_t^{\frac{\nu}{2}}\bar{C}_t^*, \tag{A.4}$$

$$\bar{Y}_t^* = \frac{\nu}{2}\bar{T}_t^{\frac{\nu}{2}-1}\bar{C}_t^* + (1-\frac{\nu}{2})\bar{T}_t^{-\frac{\nu}{2}}\bar{C}_t. \tag{A.5}$$

For any variable $\bar{X}$, we define $\bar{x} = \ln\frac{\bar{X}}{\bar{X}^\star}$ to be its log-deviation from the non-stochastic steady state level $\bar{X}^\star$, except for the variable $\bar{r}_t$, which is the level of nominal interest rate. Notice that $\bar{T}^\star = 1$ because the model is symmetric. Moreover, we define the inverse of the elasticity of intertemporal substitution in consumption to be $\sigma = -\frac{U_{CC}\bar{C}^\star}{U_C} > 1$. The elasticity of the marginal disutility of working hours is $\phi = \frac{V_{NN}\bar{N}^\star}{V_N}$, and $\varepsilon_t = \frac{U_{C\xi}}{U_C}\ln\xi_t$ measures a demand shock of the home country.[19]

[19]The notation U_{CC} indicates the second derivative of U with respect to C_t, and V_{NN} indicates the second derivative of V with respect to N_t. The notation $U_{C\xi}$ indicates the partial derivative of U with respect to C_t, then with respect to ξ_t.

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The linear approximation of equations (A.1)-(A.5) around the steady state are:

$$\sigma\bar{c}_t - \varepsilon_t + \phi\bar{y}_t + (1-\frac{\nu}{2})\bar{\tau}_t = 0, \tag{A.6}$$
$$\sigma\bar{c}_t^* - \varepsilon_t^* + \phi\bar{y}_t^* - (1-\frac{\nu}{2})\bar{\tau}_t = 0, \tag{A.7}$$
$$(1-\lambda)[\sigma(\bar{c}_t - \bar{c}_t^*) - (\varepsilon_t - \varepsilon_t^*) - (\nu-1)\bar{\tau}_t] = \lambda[(\frac{\nu}{2}-1)\bar{\tau}_t + \bar{y}_t - \bar{c}_t], \tag{A.8}$$
$$\bar{y}_t = \frac{\nu}{2}\bar{c}_t + (1-\frac{\nu}{2})\bar{c}_t^* + \nu(1-\frac{\nu}{2})\bar{\tau}_t, \tag{A.9}$$
$$\bar{y}_t^* = \frac{\nu}{2}\bar{c}_t^* + (1-\frac{\nu}{2})\bar{c}_t - \nu(1-\frac{\nu}{2})\bar{\tau}_t. \tag{A.10}$$

By solving the linear system of equations (A.6)-(A.10), we can obtain the first-order solutions for consumption, output, and the terms of trade when facing demand shocks in the economy.

To simplify the following solutions, we define $x^W = \frac{x+x^*}{2}$ to be the world average value and $x^R = \frac{x-x^*}{2}$ to be the relative value for variables x and x^*. Define $D = \sigma\nu(2-\nu)+(1-\nu)^2$ and a function of the parameter λ that governs the financial completeness:

$$\omega(\lambda) = \frac{\lambda(2-\nu)[2-\nu+\sigma(\nu-1)+\phi]}{2(1-\lambda)(\phi D+\sigma)+\lambda(2-\nu)[2-\nu+\sigma(\nu-1)+\phi]},$$

where $\omega(0) = 0,\ \omega(1) = 1$.

As a result, the home and foreign consumption can be written as:

$$\bar{c}_t = \frac{1}{\phi+\sigma}\varepsilon_t^W + [(1-\omega)(\frac{1+\phi\nu(2-\nu)}{\sigma+\phi D}) + \omega(\frac{\nu-1}{2-\nu+\sigma(\nu-1)+\phi})]\varepsilon_t^R, \tag{A.11}$$
$$\bar{c}_t^* = \frac{1}{\phi+\sigma}\varepsilon_t^W - [(1-\omega)(\frac{1+\phi\nu(2-\nu)}{\sigma+\phi D}) + \omega(\frac{\nu-1}{2-\nu+\sigma(\nu-1)+\phi})]\varepsilon_t^R. \tag{A.12}$$

A rise in the world demand ($\varepsilon_t^W > 0$) raises consumption in both countries, but a rise in the relative demand ($\varepsilon_t^R > 0$) has different influences to each country according to the degrees of trade integration and financial completeness. When financial markets are complete ($\lambda = 0, \omega = 0$), a positive relative demand shock increases home

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consumption and decreases foreign consumption whether consumers bias for home produced goods or not. In financial autarky ($\lambda = 1, \omega = 1$), there is no risk-sharing by trading securities, then a rise in the relative demand shock raises home consumption and lowers foreign consumption only when $\nu > 1$. In other words, if consumers have no bias for local goods ($\nu = 1$), the relative demand shock has no impact for both countries in financial autarky.

The output levels with flexible prices can be expressed as:

$$\bar{y}_t = \frac{1}{\phi+\sigma}\varepsilon_t^W + [(1-\omega)(\frac{\nu-1}{\sigma+\phi D}) + \omega(\frac{1}{2-\nu+\sigma(\nu-1)+\phi})]\varepsilon_t^R, \tag{A.13}$$

$$\bar{y}_t^* = \frac{1}{\phi+\sigma}\varepsilon_t^W - [(1-\omega)(\frac{\nu-1}{\sigma+\phi D}) + \omega(\frac{1}{2-\nu+\sigma(\nu-1)+\phi})]\varepsilon_t^R. \tag{A.14}$$

The impacts of a demand shock to output levels also vary with different degrees of trade and financial integration. When the world demand increases ($\varepsilon_t^W > 0$), home and foreign output levels rise. With complete financial markets ($\omega = 0$), the relative demand shock has opposite impacts on two country's output levels only if $\nu > 1$. When financial markets totally shut down ($\omega = 1$), a positive relative demand shock raises home output and reduces foreign output regardless of the value of ν.

As for the terms of trade, only the relative demand shock influences its value. It is intuitive because the terms of trade is defined as the relative price of foreign goods to home goods, so a world demand shock may have the same impact on both prices thus does not alter the terms of trade. We only have to focus on the impact of a relative demand shock. The flexible price terms of trade in response of a demand shock can be written as:

$$\frac{\bar{\tau}_t}{2} = [-\frac{(1-\omega)\phi(\nu-1)}{\sigma+\phi D} + \frac{\omega}{2-\nu+\sigma(\nu-1)+\phi}]\varepsilon_t^R. \tag{A.15}$$

For $\varepsilon_t^R > 0$, the terms of trade appreciates ($\bar{\tau}_t$ decreases) when financial markets are complete and home bias exists ($\omega = 0, \nu > 1$). A joint increase in home goods

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consumption and home goods production raises the price of domestic goods. If there is financial autarky ($\omega = 1$), the terms of trade depreciates ($\bar{\tau}_t$ increases) instead. In this situation, home goods become relatively cheaper because home firms over produce when there is zero securities trade.

Appendix B Derivation of natural interest rates

The concept of natural interest rates was introduced by German economist Knut Wicksell in 1898. It is defined as the interest rate that would sustain the flexible price equilibrium, controlling for zero inflation. Denote the home natural interest rate as $\bar{r}_t$ and foreign natural interest rate as $\bar{r}_t^*$, and ρ is the steady state value of the natural interest rate. By log-linear approximation, equation (5) and its foreign counterpart become:

$$\bar{r}_t = \rho + \sigma E_t(\bar{c}_{t+1} - \bar{c}_t) - E_t(\varepsilon_{t+1} - \varepsilon_t) + E_t \pi_{H_{t+1}} + (1 - \frac{\nu}{2}) E_t(\bar{\tau}_{t+1} - \bar{\tau}_t), \tag{B.1}$$

$$\bar{r}_t^* = \rho + \sigma E_t(\bar{c}_{t+1}^* - \bar{c}_t^*) - E_t(\varepsilon_{t+1}^* - \varepsilon_t^*) + E_t \pi_{F_{t+1}} - (1 - \frac{\nu}{2}) E_t(\bar{\tau}_{t+1} - \bar{\tau}_t). \tag{B.2}$$

Since natural interest rates control for zero inflation, the expected terms of inflation can be eliminated. In addition, suppose that the demand shock returns to zero with probability $1 - \mu$ next period, meaning that there is only μ probability that the shock persists. All variables in the economy have the same persistence in expectation because there is no state variable in the model. Therefore, for any variable x_t, we have $E_t(x_{t+1}) = \mu x_t$. By substituting equations (A.11),(A.12),(A.15) into equations (B.1) and (B.2), the home and foreign natural interest rates become:

$$\bar{r}_t = \rho + [\frac{\phi}{\phi + \sigma} \varepsilon_t^W + (\frac{(1-\omega)\phi(\nu - 1)}{\sigma + \phi D} + \frac{\omega \phi}{2 - \nu + \sigma(\nu - 1) + \phi}) \varepsilon_t^R](1 - \mu), \tag{B.3}$$

$$\bar{r}_t^* = \rho + [\frac{\phi}{\phi + \sigma} \varepsilon_t^W - (\frac{(1-\omega)\phi(\nu - 1)}{\sigma + \phi D} + \frac{\omega \phi}{2 - \nu + \sigma(\nu - 1) + \phi}) \varepsilon_t^R](1 - \mu). \tag{B.4}$$

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If written in world and relative terms,

$$\bar{r}_t^W = \rho + \frac{(1-\mu)\phi}{\phi+\sigma}\varepsilon_t^W, \tag{B.5}$$

$$\bar{r}_t^R = (1-\mu)[\frac{(1-\omega)\phi(\nu-1)}{\sigma+\phi D} + \frac{\omega\phi}{2-\nu+\sigma(\nu-1)+\phi}]\varepsilon_t^R. \tag{B.6}$$

From equations (B.3) and (B.4), we can observe that given any demand shock, natural interest rates are determined by the state of the economy: the degree of trade openness ν and financial integration λ. For a fix value of λ, a positive world demand shock ($\varepsilon_t^W > 0$) raises interest rates in both countries. A positive relative demand shock ($\varepsilon_t^R > 0$) affects natural interest rates in different directions when financial markets are complete ($\omega = 0$) only if consumers are biased toward domestic goods ($\nu > 1$). If consumers have no particular preference toward local or foreign produced goods ($\nu = 1$), the relative demand shock will have zero effect on the interest rate under complete financial markets. On the contrary, if financial markets are banned ($\omega = 1$), the opposite impacts on each country's natural interest rate still exist.

Figure 7 compares the home and foreign natural interest rates under different values of ν and λ.[20] When a negative demand shock hits the home country, natural interest rates in both countries fall under zero if $\nu = 0,\ \lambda = 0$. In other words, the liquidity trap spreads immediately from home to foreign country when trade and financial markets are fully integrated. As either one of the parameter increases, trade for goods or assets are restricted, the comovements of natural interest rates decrease. In the extreme case where both countries are completely closed for trade, the shock in home country will not affect the foreign country at all. To make a brief summary, Table 1 lists out the change of each variable when facing a positive relative demand shock under different degrees of trade and financial integration.

[20] *Figure* 7 uses the same parameters as *Figure* 4. We omit the case $\nu = 2,\ \lambda = 1$ where there is no trade and no financial assets at all.

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Figure 7: Natural interest rates

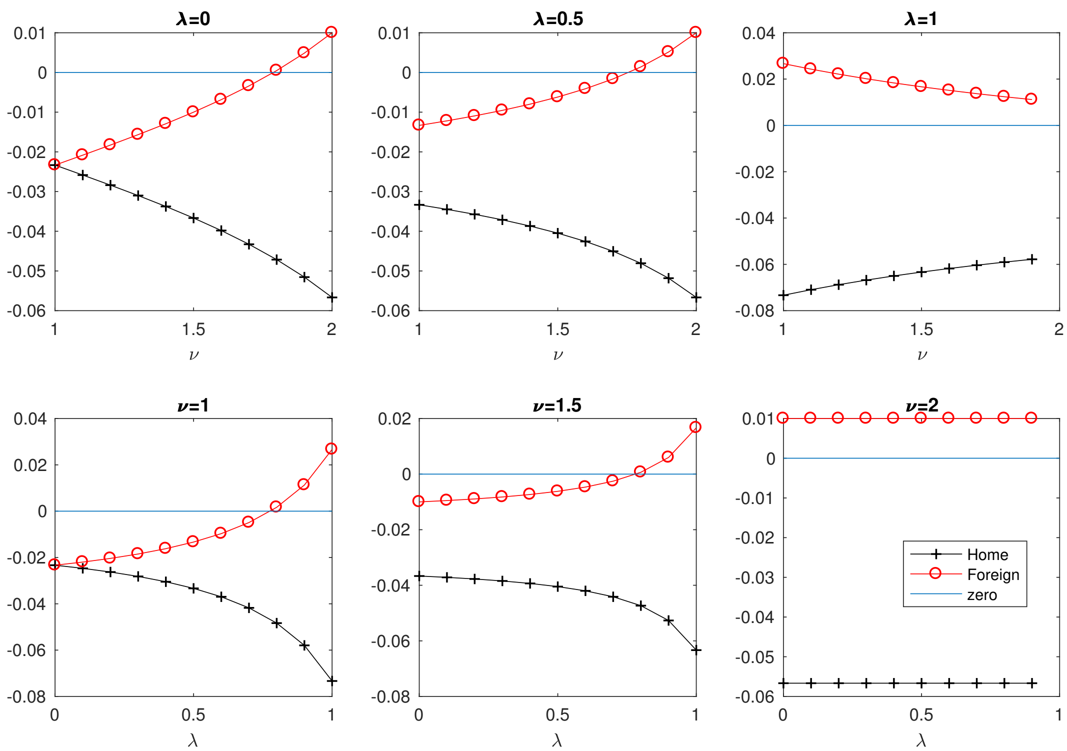

Table 1: Impacts of a positive relative demand shock ($\varepsilon_t^R > 0$) under flexible prices

	Complete financial markets $\lambda = 0,\ \omega = 0$	Financial autarky $\lambda = 1,\ \omega = 1$
$\nu > 1$	$c_t \uparrow\ c_t^* \downarrow$ $y_t \uparrow\ y_t^* \downarrow$ $\tau_t \downarrow$ $r_t \uparrow\ r_t^* \downarrow$	$c_t \uparrow\ c_t^* \downarrow$ $y_t \uparrow\ y_t^* \downarrow$ $\tau_t \uparrow$ $r_t \uparrow\ r_t^* \downarrow$
$\nu = 1$	$c_t \uparrow\ c_t^* \downarrow$ y_t, y_t^* unchanged τ_t unchanged r_t, r_t^* unchanged	c_t, c_t^* unchanged $y_t \uparrow\ y_t^* \downarrow$ $\tau_t \uparrow$ $r_t \uparrow\ r_t^* \downarrow$

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Appendix C Derivation of the loss function

The loss function is defined as the difference between the total welfare of the economy and its maximum efficient level, which occurs when consumption and employment take on their efficient values. We apply the second-order approximation method in Engel (2011) to derive the joint welfare function of home and foreign households. Notice that the notation $||o||^2$ indicates that there are second-order and higher terms left out, and $||o||^3$ leaves out third-order and higher terms.

Suppose that the period utility of the planner takes the form:

$$\mathcal{V}_t = \frac{C_t^{1-\sigma} + C_t^{*1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi} + N_t^{*1+\phi}}{1+\phi}, \tag{C.1}$$

where the term ξ_t is omitted as it is exogenous. We assume that allocations are efficient at steady state, so we have $C^{\star 1-\sigma} = N^{\star 1+\phi}$ for both countries by market clearing conditions and the unity value of marginal rate of substitution between consumption and work. We take a second-order log approximation around the non-stochastic steady state and obtain:

$$\begin{aligned} v_t = 2(\frac{1}{1-\sigma} - \frac{1}{1+\phi})c^{\star 1-\sigma} + c^{\star 1-\sigma}(c_t + c_t^*) + \frac{1-\sigma}{2}c^{\star 1-\sigma}(c_t^2 + c_t^{*2}) \\ -c^{\star 1-\sigma}(n_t + n_t^*) - \frac{1+\phi}{2}c^{\star 1-\sigma}(n_t^2 + n_t^{*2}) + ||o||^3. \end{aligned} \tag{C.2}$$

It is equivalent as maximizing the linear transformation of equation (C.2):

$$v_t = c_t + c_t^* - n_t - n_t^* + \frac{1-\sigma}{2}(c_t^2 + c_t^{*2}) - \frac{1+\phi}{2}(n_t^2 + n_t^{*2}) + ||o||^3. \tag{C.3}$$

The utility is maximized by consumption and employment taking efficient values,

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denoted by a bar:

$$v_t^{max} = \bar{c}_t + \bar{c}_t^* - \bar{n}_t - \bar{n}_t^* + \frac{1-\sigma}{2}(\bar{c}_t^2 + \bar{c}_t^{*2}) - \frac{1+\phi}{2}(\bar{n}_t^2 + \bar{n}_t^{*2}) + \|o\|^3. \tag{C.4}$$

Recall that we define the term $\tilde{y} = y - \bar{y}$ to be the gap between the log of an variable under sticky prices and its efficient value under flexible prices. The loss function is thus:

$$\begin{aligned}\tilde{v}_t = v_t - v_t^{max} = 2\tilde{c}_t^W - 2\tilde{n}_t^W + (1-\sigma)[(\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2] - (1+\phi)[(\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2] \\ +2(1-\sigma)(\bar{c}_t^R\tilde{c}_t^R + \bar{c}_t^W\tilde{c}_t^W) - 2(1+\phi)(\bar{n}_t^R\tilde{n}_t^R + \bar{n}_t^W\tilde{n}_t^W) + \|o\|^3.\end{aligned} \tag{C.5}$$

Our goal is to rewrite equation (C.5) in terms of output gaps and inflation. From equations (A.8), (A.9) and (A.10), the relative gap terms of consumption and employment can be derived as:

$$\tilde{c}_t^R = \frac{\nu-1}{D}\tilde{y}_t^R + \|o\|^2, \tag{C.6}$$

$$\tilde{n}_t^R = \tilde{y}_t^R + \|o\|^2. \tag{C.7}$$

The terms of trade τ_t can also be represented by:

$$\tau_t = \frac{2\sigma(1-\lambda) + \lambda(2-\nu)}{(\nu-1)\triangle}\tilde{y}_t^R + \|o\|^2, \tag{C.8}$$

where $\triangle \equiv \frac{(1-\lambda)D - \lambda(\frac{\nu}{2}-1)}{\nu-1}$.

A second-order approximation of c_t, c_t^* gives us:

$$c_t = y_t - (1-\frac{\nu}{2})(\nu + \frac{1-\nu}{\sigma})\tau_t - \frac{\nu}{4}(1-\frac{\nu}{2})(\nu-1)^2(1-\frac{1}{\sigma})^2\tau_t^2 + \|o\|^3, \tag{C.9}$$

$$c_t^* = y_t^* + (1-\frac{\nu}{2})(\nu + \frac{1-\nu}{\sigma})\tau_t - \frac{\nu}{4}(1-\frac{\nu}{2})(\nu-1)^2(1-\frac{1}{\sigma})^2\tau_t^2 + \|o\|^3. \tag{C.10}$$

Substituting equation (C.8) into (C.9) and (C.10), and subtracting the efficient levels,

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we obtain the average gap term of consumption:

$$\tilde{c}_t^W = \tilde{y}_t^W - \frac{\nu}{4}(1-\frac{\nu}{2})[(1-\frac{1}{\sigma})\frac{2\sigma(1-\lambda)+\lambda(2-\nu)}{\triangle}]^2[(\tilde{y}_t^R)^2 + 2\tilde{y}_t^R \bar{y}_t^R] + \|o\|^3. \quad \text{(C.11)}$$

As for the average gap term of employment, a second-order approximation for $\tilde{n}_t$, $\tilde{n}_t^*$ renders:

$$\tilde{n}_t = \tilde{y}_t + \frac{\theta}{2} var(P_{H_t}) + \|o\|^3, \quad \text{(C.12)}$$

$$\tilde{n}_t^* = \tilde{y}_t^* + \frac{\theta}{2} var(P_{F_t}^*) + \|o\|^3, \quad \text{(C.13)}$$

where $var(\cdot)$ represents the variance of the given variable. When prices are adjusted according to equation (7) in a producer-currency pricing model, the variance of the price has a relation with the inflation, which is:

$$\sum_{j=0}^{\infty} \beta^j var(P_{H_{t+j}}) = \frac{1}{k} \sum_{j=0}^{\infty} \beta^j \pi_{t+j}^2, \quad \text{(C.14)}$$

where $k = \frac{(1-\beta\kappa)(1-\kappa)}{\kappa}$ is the degree of price stickiness. Therefore, we can write the average gap term as:

$$\tilde{n}_t^W = \tilde{y}_t^W + \frac{\theta}{4k}(\pi_t^2 + \pi_t^{*2}) + \|o\|^3. \quad \text{(C.15)}$$

Combining equations (C.6),(C.7),(C.11),(C.15), the loss function in equation (C.5) becomes:

$$\begin{aligned} \tilde{v}_t = &-(\sigma+\phi)(\tilde{y}_t^W)^2 - [(1+\phi)-(1-\sigma)(\frac{\nu-1}{D})^2](\tilde{y}_t^R)^2 - \frac{\theta}{2k}(\pi_t^2 + \pi_t^{*2}) \\ &-\frac{\nu}{2}(1-\frac{\nu}{2})[(1-\frac{1}{\sigma})\frac{2\sigma(1-\lambda)+\lambda(2-\nu)}{\triangle}]^2(\tilde{y}_t^R)^2 + \|o\|^3, \end{aligned} \quad \text{(C.16)}$$

which is equation (24) in Section 4.

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