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以賽局理論分析能量收集無線感測網路之隨機接取

控制

Random Access Control in Energy-Harvesting
Wireless Sensor Networks: A Game-Theoretical
Approach

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本論文係左傳勻君(學號 R989821039)在國立臺灣大學電機工程學系完成之碩士學位論文，於民國 100 年 10 月 21 日承下列考試委員審查通過及口試及格，特此證明。

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中文摘要

傳統無線感測網路以電池作為電源供應，然而一旦電池用盡，網路將陷入無法使用的困境，直至更新電池為止。但在某些無線感測網路應用當中，更換電池幾乎不可行，譬如建築監測感測系統。為了解決這問題，人們轉向使用可從環境汲取能源之能量收集無線感測網路。在這論文裡，我們建立了能量收集無線感測網路之隨機接取控制理論模型，探討眾多無線感測裝置爭取有限傳輸資源產生之問題。考慮無線感測裝置會自私地最大化自己的效用，所有裝置將會不顧系統整體效能選擇傳輸，使得系統陷入最糟情況。我們提出兩種激勵機制：收費機制和干擾機制，用以防止系統陷入最糟狀況。這兩種激勵機制可以誘使無線感測裝置選擇對系統而言最佳的策略，使系統達到社會最適(Social Optimal)、或是比例公平(Proportional Fair)的分配。在論文最後，我們深入探討無線感測裝置可選擇留存能源之延伸模型。我們發現，無線感測裝置會根據每段時間的能量收集機率，決定是否將能源留存至未來。在系統達成平衡之後，無線感測裝置會在能量收集機率較高的期間，選擇較高的傳輸機率。

關鍵字: 賽局理論、納許均衡、能量收集無線感測網路、激勵機制、社會最適、比例公平

Abstract

Traditional wireless sensor networks (WSN) are powered by batteries. Once the batteries run out, the devices become useless until they are replenished. However, for some kinds of applications, such as building structure monitoring, it is nearly impossible to replenish the batteries of devices. To overcome this problem, people turn to the energy-harvesting (EH) WSNs which can harvest energy from the environment. In this work, we construct theoretic models where devices are competing for limited transmission resource. Since the devices are selfish, they all choose to transmit regardless of others' strategy, which leads to the severe network congestion. We propose two incentive mechanisms, a pricing scheme and an intervention scheme, that prohibit the system outcome from the worst case. The incentive scheme can induce the desired optimal outcomes which maximize the social welfare or the proportional fairness. In the last part, we also build an extension model in which the energy can be stored for the future. We show that it is more likely that the device chooses to save some energy for the period when the energy harvesting probability is comparatively low. On the other hand, the devices will choose a higher transmission probability at the period when the energy harvesting probability is comparatively high.

Keywords: game theory, Nash equilibrium, energy-harvesting WSNs, incentive mechanism, social optimal, proportional fairness

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Chapter 1

Introduction

Due to the advancements in the technology for miniaturization of electronic devices, wireless sensor networks (WSNs) have drawn increasing attention recently. There are two types of WSNs according to how the sensors are powered up. The first is the battery-powered WSNs, where the energy of the battery can only deplete with time. The second type are energy-harvesting WSNs, where the sensor makes use of renewable energy to maintain its operation and the battery energy usually remains at low level to avoid energy leakage [1]. In both types of WSNs, proper energy management is essential to maximize each sensor's utility, such as transmission success rate. As the energy level typically fluctuates at low level (Fig. 1.1), modeling of energy-harvesting sensors require the knowledge of instantaneous harvesting probability and the transmission policy.

In a model for energy-harvesting WSNs, the transmission policy of energy harvesting

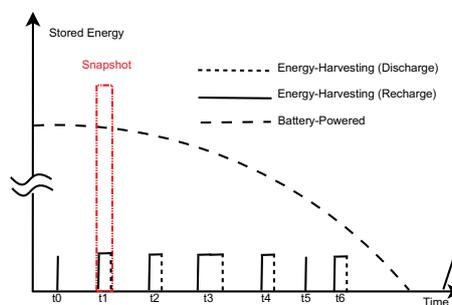


Figure 1.1: The comparison between battery-powered WSNs and WSN-HEAP.

device should take the harvesting probability into consideration. Take Fig. 1.1 as an example. At time t_0 , the device successfully harvests a unit of energy and transmit information packets at the same time slot. (Assume that a unit of energy can afford the number of packets that bring a unit of information.) At time t_1 , the device harvests a unit of energy but transmit at the next time slot $t_1 + 1$. In general, the device should choose a transmission policy that maximizes the probability of successful transmission, given the probability of harvesting energy. In this work, we use game theory to build models for energy-harvesting WSNs. The transmission resources are limited so that the devices have to compete with each other. In the first model, we consider a battery-based WSN model where devices are not aware of harvesting energy. The devices choose the transmission policy to maximize its own probability of successful transmission. In the second model, we propose our main model for energy-harvesting WSNs where devices harvest energy from the environment. The devices maximize its own probability of successful transmission based on the knowledge of energy-harvesting probabilities. In the third model, we extend the energy-harvesting model into the multi-stage one. The devices are able to store energy for future and consider the future benefit of transmission.

Energy-harvesting technique can be implemented on most electric devices. However, we focus on WSNs in this work because of their well-known problem: sensors are easily depleted and difficult to renew, which is exactly the problem that the energy-harvesting technique aims to solve. Besides, sensors are typically small in size and equipped with small energy capacity, which matches the setting in our models. Therefore, to stay focused at the energy-harvesting issue, we only discuss energy-harvesting WSNs throughout this work.

Chapter 2

Related Works

2.1 Energy Issue in WSNs

Energy management plays an important role in wireless sensor networks (WSNs). Pantazis *et al.* provide a survey on power control issue in WSNs [2]. The power conservation mechanisms are classified into two categories: passive and active. Passive power conservation mechanisms reduce energy consumption of sensor node by turning off its transceiver when there is no transmission, whereas active power conservation mechanisms count on concept of improving the node's operation instead of turning off the radio module into power-saving mode.

Compared with the traditional WSNs, the energy harvesting WSNs start to arise much attention in these years. The energy harvesting devices have some fundamental differences from the traditional ones [1]. First, the energy source is different: traditional devices aim to utilize the battery energy efficiently, while energy-harvesting devices aim to use the harvested energy smartly. Second, the energy capacity is different: typically, traditional devices have a large-capacity battery to reduce the replenishment cost, while energy-harvesting devices are equipped small-capacity storage to minimize the size of device.

Kansal *et al.* propose an analytic model that characterizes the power management of energy harvesting WSNs [3][4]. The authors propose the energy-neutral operation where

the energy used never exceeds the energy harvested. Given the energy-neutral condition is achieved, they also seek the optimal network performance.

Seyedi and Sikdar propose a Markov model for energy harvesting nodes and derive the closed form for the loss probability and the average time to run out energy [5]. Such analytic results provide a good guideline for engineers to design protocols for energy harvesting WSNs. Niyato *et al.* develop a multidimensional discrete-time Markov chain to model the channel, the solar radiation, and the packet arrival [6]. The authors then use Nash bargaining solution to obtain the optimal sleep-or-wake-up strategy. Lei *et al.* propose a generic model for energy harvesting by using Markov chain [7]. They derive the optimal transmission policy for the node to decide whether to transmit or not. The node would transmit if the value of transmission exceeds a threshold, which depends on its current energy state. Susu *et al.* present a stochastic framework for energy harvesting WSN nodes [8], which enables designers to assess statistical system performance such as operation time or lifetime. Ho *et al.* propose a generalized Markovian (GM) model that introduces an additional parameter to capture the non-stationary properties of energy-harvesting circumstance [9]. Based on their empirical experiments, the GM model is better than the stationary Markovian model.

Sharma *et al.* study the optimal policy for energy harvesting nodes. The generated data bits and replenished energy are independently independent and identically distributed random variable respectively. After deriving the necessary condition for stability, the authors construct the throughput-optimal policy and the delay-optimal policy.

However, the energy conservation is not always beneficial since energy storage units have limited capacity and are leakage-prone. The more energy store in the storage unit, the more energy leak away. Zhu *et al.* formulate the leakage problem and implement leakage-aware feedback control techniques to utilize energy that could leak away effectively[10].

2.2 Medium Access Control Game

Akkarajitsakul *et al.* provide a broad survey for multiple access game-theoretic models [11]. Yang *et al.* construct a non-cooperative game for CSMA/CA networks and design an adaptive price setting to achieve the desired outcome [12]. Cui *et al.* consider multiple contention measure signals in the random access game model [13]. Their work also studies the dynamics for random access game, including best-response, gradient-play, and Jacobi-play.

Chen *et al.* propose a dynamic game model for contention control [14]. The node can choose its own transmission probability to maximize its own utility. Chowdhury *et al.* present a game-theoretic model for contention control in IEEE 802.16/WiMax networks [15]. The game considers a saturated network where nodes always have packets to transmit. To ensure the unique existence of Nash equilibrium, the author design a special form of utility function.

Since the equilibrium is not necessarily the best outcome, incentive schemes are used to achieve the best outcome. Park *et al.* have designed a series of new incentive schemes based on intervention to induce the target outcome [16]. The pricing scheme, the repeated interaction, and the intervention schemes are well-known for providing incentives to the users. However, to charge price requires a secure and reliable process between the manager and the user, which creates burden on both sides. On the other hand, the repeated interaction is hard to implement since users change frequently in mobile networks and fixed interactions are hard to sustain. The intervention schemes directly impose intervention on the users, which exclude the problems of pricing schemes and repeated interactions. Compared with the general intervention schemes in [16], the authors provide an example by applying intervention schemes in medium access control game in [17].

2.3 Contribution

In this work, we construct game-theoretic models for energy harvesting WSNs where several devices compete for limited transmission resources (e.g., wireless channels). The models are generalized from the previous models which consider only single transmission resource. The models can apply to many promising applications. For example, for machine type communication, the wireless machines contend for random access channels (RACH) for dedicated transmission channel [18]. Typically, the number of RACH resources is fewer than the number of machine. Only the machines receiving grants from the base station (BS) can start transmission. However, if more than two machines are granted for transmission, both machines encounter collision and the transmission fails. Besides collisions, the machine also have to consider the energy harvesting probability when determining the transmission policy. The similar problem arises in many other wireless applications.

We generalize the medium access control (MAC) game model proposed in [17] and the one in [12]. Compared with their model, our models further consider the issue of energy harvesting and more than one transmission resource that the devices can request for. We derive the proportionally fair outcome and the social optimal outcome, and adopt two incentive schemes to achieve them. Finally, we extend the model to the multiple-period model where the energy can be stored for the future use.

Chapter 3

One-Shot Models

In this chapter, we discuss one-shot models where devices make decisions simultaneously. The devices have a chance to harvest energy for transmission. The transmission may collide with other devices if two devices choose the same transmission channel. To emphasize the importance of energy harvesting issue, we first build a traditional WSN model where sensors cannot harvest energy. Then we construct our main model, the energy-harvesting WSN model in the second section.

The first section considers a typical model where the devices have unlimited energy source and need not to take energy issue into consideration. This is a generalized form of traditional median access control (MAC) model discussed in [14] [16], which consider only single unit of transmission resource.

The second section considers the main model where the devices harvest energy from the environment for transmission. In this model, the device harvests a unit of energy with a fixed probability. The transmission cannot success if the device fails to harvest the energy. That is, the transmission success rate is bounded by the harvesting probability.

3.1 The Random Access Control (RAC) Model

We consider a WSN where N devices asking for transmission resources from the base station (BS). There are totally M transmission resource (channel) to allocate. Device

$i \in \mathcal{N}$ chooses a probability $s_i \in [0, 1]$ to request a transmission channel from these M channels. The set of possible strategies is denoted by \mathcal{S}_i for device i and $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2 \times \dots \times \mathcal{S}_N$ for all devices.

The device requesting for channel sends the request directly through that channel. If two devices send the request at the same time, both requests collide and fail. Although collisions are not favorite, there is no way for device to avoid collision before sending the request since there is no global information about the channel occupation. A generally used method for the device to request the channel is to choose randomly (with uniform distribution). The model is therefore called random access control (RAC) model.

Given that every device randomly chooses a channel to request, the probability that a device collides with the other is $\frac{1}{M}$. On the contrary, the probability that a device does not collide with the other is $\frac{M-1}{M}$. Then the value of device as the transmission success rate is:

$$v_i(\mathbf{s}) = s_i \prod_{j \neq i} \left((1 - s_j) + s_j \frac{M-1}{M} \right) = s_i \prod_{j \neq i} \left(1 - \frac{s_j}{M} \right)$$

The first term in the product is the probability that device j does not request for a channel, and the second term is the probability that device j requests a different channel from the one that device i gets. The summation of these two terms is the probability that device j does not collide with device i . Then finally we know that the value v_i is the probability that device i chooses to request and other devices do not collide with device i .

For example, consider a three-device system where the devices choose $s_1 = 0.3, s_2 = 0.5, s_3 = 0.7$. The number of transmission channel is $M = 1$. Then the value of first device will be

$$v_1 = 0.3 * (1 - 0.5) * (1 - 0.7) = 0.045$$

and similarly $v_2 = 0.105$ and $v_3 = 0.245$. Due to unavoidable collisions, the transmission success rate of each device is smaller than the transmission probability it chooses. Besides, the value of third device is highest since it chooses the highest transmission probability.

Generally speaking, the device choosing the highest transmission probability obtains the highest value.

Considering a special case $M = 1$, we obtain the previous models like [12] and [17] which consider only single one transmission channel, with the definition of utility function

$$v_i(\mathbf{s}) = s_i \prod_{j \neq i} (1 - s_j).$$

Since our model considers a more general case, the results derived in the following sections can apply to these previous models.

3.2 The RAC Game: Definition and Solution Concept

After defining necessary notations, we now introduce the mathematical tool to analyze the system. Due to the selfish nature of wireless devices who cares its own transmission, we adopt several concepts in the game theory to analyze the model, such as the potential game and the Nash equilibrium. Our game model is formulated as a RAC game:

$$\Gamma^{RAC} = \langle \mathcal{N}, (\mathcal{S}_i)_{i \in \mathcal{N}}, (v_i)_{i \in \mathcal{N}} \rangle$$

We introduce the most basic equilibrium concept called Nash equilibrium.

Definition 3.1. (*Nash Equilibrium*) A strategy profile $\mathbf{s}^* = (s_1, s_2, \dots, s_N)$ is a Nash equilibrium if no unilateral deviation in strategy is profitable for any single device, that is,

$$u_i(s_i^*, \mathbf{s}_{-i}^*) > u_i(s_i, \mathbf{s}_{-i}^*), \forall s_i \in \mathcal{S}_i, \forall i,$$

where $\mathbf{s}_{-i} = (s_1, s_2, \dots, s_{i-1}, s_{i+1}, \dots, s_N)$ is the strategy profile except for device i .

To derive Nash equilibrium, we have to solve the optimization problems with multiple objective functions, which has no standard method to solve. However, we find that the RAC game Γ^{RAC} is a (exact) potential game whose equilibrium can be derived in an easier

way. First, we have to introduce the potential game.

Definition 3.2. (*Potential Game*) A game is an (exact) potential game if and only if there exists a function $P : \mathcal{S} \rightarrow \mathcal{R}$ such that

$$P(s'_i, s_{-i}) - P(s_i, s_{-i}) = u_i(s'_i, s_{-i}) - u_i(s_i, s_{-i}), \forall i, s_i, s'_i.$$

If u_i is a continuous function with respect to s_i , the condition is equivalent to

$$\frac{\partial P(\mathbf{s})}{\partial s_i} = \frac{\partial u_i(\mathbf{s})}{\partial s_i} \quad \forall i. \quad (3.1)$$

Potential games form a precious subset of games. In potential games, all players can be thought of as optimizing a joint objective function, i.e., the potential function P . If the deviation in strategy can increase some players utility, then it can also increase the value of potential function. Therefore, the strategy maximizing the joint objective function P coordinates the Nash equilibrium. Then we can derive the Nash equilibrium by solving single maximization problem.

3.3 Nash Equilibrium: A Potential Game Approach

In this section, we use the potential function to derive the Nash equilibrium. To prove that the RAC game is a potential game, we need the following lemma.

Lemma 3.1. Suppose u_i are twice continuously differentiable. The game is a potential game if and only if

$$\frac{\partial^2 u_i}{\partial s_i \partial s_j} = \frac{\partial^2 u_j}{\partial s_i \partial s_j}, \forall i, j.$$

The detail explanation refers to Monderer and Shapleys' work [19].

Theorem 3.2. The RAC game Γ^{RAC} is a potential game with the potential function

$$P(\mathbf{s}) = -M \prod_i \left(1 - \frac{s_i}{M}\right) \quad (3.2)$$

Proof. To prove that RAC game is a potential game, we must show the condition in Lemma 3.1 is satisfied.

$$\frac{\partial^2 u_i}{\partial s_i \partial s_j} = \frac{1}{M} \prod_{k \neq i, j} \left(1 - \frac{s_k}{M}\right) = \frac{\partial^2 u_j}{\partial s_i \partial s_j}, \forall i, j.$$

In addition, the derived potential function must satisfy the condition in Definition 3.2.

$$\frac{\partial P}{\partial s_i} = \prod_{j \neq i} \left(1 - \frac{s_j}{M}\right) = \frac{\partial u_i}{\partial s_i}$$

□

Theorem 3.3. *The unique Nash equilibrium strategy in Γ^{RAC} is $s_i^* = 1$ for all i .*

Proof. Since the RAC game is a potential games, the strategy maximizing P is the equilibrium strategy s^* .

$$\max_s P = -M \prod_i \left(1 - \frac{s_i}{M}\right) \quad (3.3a)$$

subject to

$$s_i \prod_{j \neq i} \left(1 - \frac{s_j}{M}\right) \geq 0 \quad (\text{Individual Rationality}) \quad (3.3b)$$

$$s_i \in [0, 1] \quad (\text{Strategy Space}) \quad (3.3c)$$

Obviously, the objective function (3.3a) is linearly increasing with s_i since the first-order derivative

$$\frac{\partial P}{\partial s_i} = \prod_{j \neq i} \left(1 - \frac{s_j}{M}\right) > 0$$

is a positive constant. Therefore, the unique solution for this maximization problem is $s_i^* = 1$ for all i . □

The equilibrium $s_i^* = 1$ for all i leads to an unwanted result where every device requests for the transmission resource with probability one. The system will become full

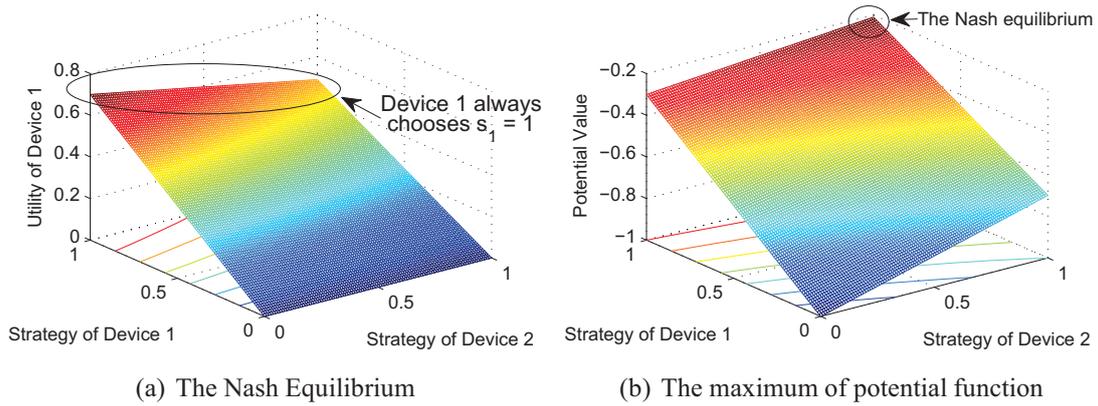


Figure 3.1: The Nash equilibrium is the point that maximizes the potential function.

of aggressive devices requesting for limited transmission resource. To avoid this result, we will propose several incentive schemes to induce better results later.

We visually show the Nash equilibrium in Fig. 3.1. In Fig. 3.1(a), we show why the device always chooses the strategy $s_i^* = 1$. This is a two-device system. Whatever strategy device 2 chooses, device 1 always chooses $s_1^* = 1$, and so does device 2, $s_2^* = 1$. Besides, we find that the utility of device linearly increases with its strategy.

In Fig. 3.1(b), we show that the potential value with respect to all possible strategy profile (s_1, s_2) . Recall that the strategy profile $(s_1^*, s_2^*) = (1, 1)$ that maximizes the value of potential function is the Nash equilibrium, which coincides with the result in Fig. 3.1(a). Notice that the potential value is always negative as we derive in previous section.

3.4 The Energy-Harvesting RAC Game

In the previous section, we have constructed the RAC game and derived the Nash equilibrium by adopting the potential game concept. In this section, we discuss the energy-harvesting device, which has to harvest energy for transmission. The energy harvesting devices must take the energy issue into consideration since the transmission may fail due to the absence of harvested energy.

The harvesting probability is assumed to be a constant [4] [5] [7]. In [4], Kansal *et*

al. use a function $P(t)$ to formulate the probability of harvesting energy at time t . In [5], Seyedi *et al.* set the probability a constant ρ_a . In [7], Lei *et al.* assume that the battery is replenished with probability α , and is recharged with probability β . In our work, we denote the probability that device i harvests one unit of energy by p_i . A unit of energy is enough to send a request and start a transmission. Combining with the energy harvesting issue, we can build our main model as follows. (With a slight abuse of notation, we adopt the notations of energy harvesting RAC game rather than RAC game in the remaining paper.)

$$v_i(\mathbf{s}) = s_i p_i \prod_{j \neq i} \left((1 - s_j) + s_j (1 - p_j) + s_j p_j \frac{M-1}{M} \right) = s_i p_j \prod_{j \neq i} \left(1 - \frac{s_j p_j}{M} \right) \quad (3.4)$$

The first term in the product is the probability that device j does not request for a channel, the second term is the probability that device j sends a request but has not harvested energy, and the third term is the probability that device j has harvested energy but requests a different channel from the one that device i gets. The summation of these three terms is the probability that device j does not collide with device i . Then the value v_i is the probability that device i chooses to request and other devices do not collide with device i . The energy-harvesting RAC game is formulated as

$$\Gamma^{EH} = \langle \mathcal{N}, (\mathcal{S}_i)_{i \in \mathcal{N}}, (v_i)_{i \in \mathcal{N}} \rangle$$

3.5 Nash Equilibrium in Energy Harvesting RAC Game

Similarly, we prove that the energy harvesting RAC game is also a potential game and then derive the Nash equilibrium.

Theorem 3.4. *The energy harvesting RAC game Γ^{EH} is a potential game with the potential*

function

$$P(\mathbf{s}) = -M \prod_i \left(1 - \frac{s_i p_i}{M}\right). \quad (3.5)$$

Proof. To prove Γ^{EH} is a potential game, we show that the condition of Lemma 3.1 is satisfied.

$$\frac{\partial^2 u_i}{\partial s_i \partial s_j} = \frac{p_i p_j}{M} \prod_{k \neq i, j} \left(1 - \frac{s_k p_k}{M}\right) = \frac{\partial^2 u_j}{\partial s_i \partial s_j}, \forall i, j$$

The potential function must satisfy the condition (3.1).

$$\frac{\partial P}{\partial s_i} = p_i \prod_{j \neq i} \left(1 - \frac{s_j p_j}{M}\right) = \frac{\partial u_i}{\partial s_i}$$

□

Theorem 3.5. *The unique Nash equilibrium strategy in the energy harvesting RAC game Γ^{EH} is $s_i^* = 1$ for all i .*

Proof. To derive the equilibrium strategy, we have to solve the following maximization problem.

$$\max_{\mathbf{s}} P = -M \prod_i \left(1 - \frac{s_i p_i}{M}\right) \quad (3.6a)$$

subject to

$$s_i p_i \prod_{j \neq i} \left(1 - \frac{s_j p_j}{M}\right) \geq 0 \quad (\text{Individual Rationality}) \quad (3.6b)$$

$$s_i \in [0, 1] \quad (\text{Strategy Space}) \quad (3.6c)$$

Obviously, the objective function (3.6a) is linearly increasing with s_i since

$$\frac{\partial P}{\partial s_i} = p_i \prod_{j \neq i} \left(1 - \frac{s_j p_j}{M}\right) > 0$$

is a positive constant. Therefore, the unique solution for this maximization problem is $s_i^* = 1$ for all i . □

Same as the RAC game, the EH-RAC game has only one Nash equilibrium $s_i^* = 1$ for all device i . The equilibrium that every device requests a channel is unwanted outcome. In the next chapter, we introduce two incentive schemes that can induce the target outcome. By designing incentive schemes which provide appropriate incentive to devices, we can induce the devices to choose the target strategy.



Chapter 4

Incentive Mechanisms

In the previous chapter, we have proposed the energy harvesting RAC game and derived the Nash equilibrium. However, the Nash equilibrium where every device chooses to transmit with probability one leads to the severe collision problem. To deal with the problem, we propose two incentive mechanisms to induce the desired outcome in this chapter. With any of these two incentive mechanisms, we prove that the outcome where every device transmits is no longer the Nash equilibrium. Moreover, the target outcome becomes the Nash equilibrium. The target outcome is any outcome the system manager want to implement. With help of the incentive schemes, we prevent the system from falling into the worst situation and induce it to the desired outcome.

4.1 Pricing Scheme

The pricing scheme is the most direct mechanism that can provide the incentive to reduce the devices' transmission probability. The system manager can charge the devices according to the strategy that the device chooses and the probability of harvesting energy. Rather than proposing a specific pricing function, we choose to define a generalized price function $c_i(s_i, p_i)$ of device i . Any pricing function that satisfy the following conditions can be used to induce the target outcome. Under the pricing scheme, the utility of device i

becomes

$$u_i^P = v_i(\mathbf{s}) - c_i(s_i, p_i) = s_i p_j \prod_{j \neq i} \left(1 - \frac{s_j p_j}{M}\right) - c_i(s_i, p_i) \quad (4.1)$$

The sequence of events can be listed as follows.

1. The manager chooses a pricing rule $c_i(s_i, p_i)$ that charges the devices according to the requesting probability and the harvesting probability.
2. Knowing the pricing rule, the device chooses a strategy s_i^* that maximizes its own utility u_i^P .
3. The system reaches the Nash equilibrium \mathbf{s}^* .

And the EH-RAC game under the pricing scheme can be formulated as follows.

$$\Gamma^{EH-P} = \langle \mathcal{N}, (\mathcal{S}_i)_{i \in \mathcal{N}}, (u_i^P)_{i \in \mathcal{N}} \rangle,$$

which is also a potential game.

The first condition is: the pricing function $c_i(s_i, p_i)$ must be twice differentiable and increasing marginal with respect to s_i , that is,

$$\frac{\partial^2 c_i(s_i, p_i)}{\partial s_i^2} > 0.$$

That is, the charging fee has to increase faster than the linear function of s_i . With this condition, we can derive the Nash equilibrium in the following theorem.

Theorem 4.1. *Given the target strategy $\tilde{\mathbf{s}} = (\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_N)$, the pricing functions solving the following simultaneous equations can induce the target Nash equilibrium in Γ^{EH-P} .*

$$\frac{\partial c_i(s_i, p_i)}{\partial s_i} \Big|_{s_i = \tilde{s}_i} = p_i \left(1 - \frac{\tilde{s}_i p_i}{M}\right)^{N-1}, \quad \forall i \quad (4.2)$$

Proof. First, we prove the game is a potential game.

$$\frac{\partial^2 u_i}{\partial s_i \partial s_j} = \frac{p_i p_j}{M} \prod_{k \neq i, j} \left(1 - \frac{s_k p_k}{M}\right) = \frac{\partial^2 u_j}{\partial s_i \partial s_j}, \forall i, j$$

Second, we prove

$$P = -M \prod_i \left(1 - \frac{s_i p_i}{M}\right) - \sum_i c(s_i, p_i). \quad (4.3)$$

is a potential function by showing that

$$\frac{\partial P}{\partial s_i} = p_i \prod_{j \neq i} \left(1 - \frac{s_j p_j}{M}\right) - \frac{\partial c_i(s_i, p_i)}{\partial s_i} = \frac{\partial u_i}{\partial s_i}.$$

Lastly, we derive the maximal point of the potential by applying the KKT condition.

In this way, we have to solve these simultaneous functions $\frac{\partial P}{\partial s_i} = 0, \forall i$, that is,

$$\frac{\partial P}{\partial s_i} = p_i \prod_{j \neq i} \left(1 - \frac{s_j p_j}{M}\right) - \frac{\partial c_i(s_i, p_i)}{\partial s_i} = 0$$

Rewrite the equation we get

$$\frac{\partial c_i(s_i, p_i)}{\partial s_i} \frac{1}{p_i} = \prod_{j \neq i} \left(1 - \frac{s_j p_j}{M}\right).$$

Multiple both sides from $i = 1$ to $i = N$, we get

$$\prod_i \frac{\partial c_i(s_i, p_i)}{\partial s_i} \frac{1}{p_i} = \prod_i \prod_{j \neq i} \left(1 - \frac{s_j p_j}{M}\right) = \prod_i \left(1 - \frac{s_i p_i}{M}\right)^{N-1}.$$

One of the solution is

$$\frac{\partial c_i(s_i, p_i)}{\partial s_i} \frac{1}{p_i} = \left(1 - \frac{s_i p_i}{M}\right)^{N-1}.$$

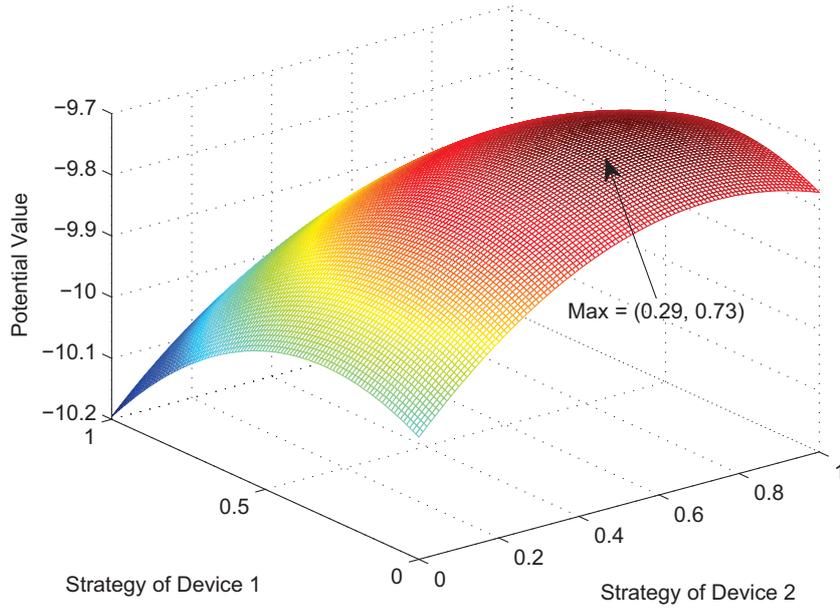


Figure 4.1: The equilibrium under the pricing scheme is nearly identical to the target one.

To prove that the solution is a maximum point, we show that

$$\frac{\partial^2 P}{\partial s_i^2} = -\frac{\partial^2 c_i(s_i, p_i)}{\partial s_i^2} < 0$$

by the prior condition. The potential function is a concave function. Hence we complete the proof.

□

The system manager can achieve the target $\tilde{s} = (\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_N)$ by designing an appropriate pricing rule $c_i(s_i, p_i)$. Since that the outcome that every device chooses to transmit with probability one is not favorite, the system manager now can adopt the pricing scheme to induce the target outcome as it wishes.

In Fig. (4.1), we use the pricing scheme to achieve the target outcome (0.3,0.7). The maximal point of the potential function (0.3,0.7) is the Nash equilibrium.

4.2 Intervention Function

Besides the pricing scheme, we adopt another incentive scheme that can induce the target outcome. Jaeok Park and Mihaela van der Schaar have designed an intervention function that can achieve the desired equilibrium [16][17]. In the intervention scheme, the system manager imposes a certain level of intervention on devices' transmission according to the strategy \mathbf{s} of devices. Formally, the intervene function can be expressed by $g : \mathcal{S} \rightarrow [0, 1]$. The intervention level is not tailored to devices as the pricing scheme, but identical for every device.

The sequence of events can be listed as follows.

1. The manager chooses an intervention rule g .
2. The devices i choose a strategy s_i^* that maximizes its own utility.
3. The system manager imposes an intervention on all the devices according to their chosen strategy.
4. The system reaches a Stackelberg equilibrium.

Because of the participation of the system manager, the EH-RAC game under the intervention scheme is neither a typical non-cooperative game as previous one nor a potential game. In game theory, this kind of game is called *Stackelberg Game* where a ruler decides the game rule before other players make decision [17]. After the system manager imposes the intervention, the utility of device i becomes

$$u_i^I(g, \mathbf{s}) = s_i p_j (1 - g(\mathbf{s})) \prod_{j \neq i} \left(1 - \frac{s_j p_j}{M}\right).$$

The utility is discount by a factor $(1 - g(\mathbf{s}))$ which is inversely proportional to the level of intervention imposed by the system manager. And since the system manager aim to induce the target outcome without imposing much intervention, the utility of the system manager is

$$u_0(g, \mathbf{s}) = \begin{cases} 1 - g(\mathbf{s}), & \text{if } \mathbf{s} = \tilde{\mathbf{s}} \\ 0, & \text{otherwise} \end{cases}$$

where $\tilde{\mathbf{s}}$ is the target outcome that the system manager aims to induce.

The manager gets zero utility if the target equilibrium is not achieved. On the other hand, if the target equilibrium is achieved, the manager aims to reduce the level of intervention. The EH-RAC game under the intervention function is formulated as follows.

$$\Gamma_g^{EH-I} = \langle \mathcal{N}, (\mathcal{S}_i)_{i \in \mathcal{N}_0}, (u_i^I(g, \cdot))_{i \in \mathcal{N}_0} \rangle$$

where $\mathcal{N}_0 = \mathcal{N} \cup \{0\}$ is the player set including the system manager.

According to [17], we construct the intervention function as follows.

$$g^*(\mathbf{s}) = \left[\sum_{i=1}^N \frac{s_i - \tilde{s}_i}{\tilde{s}_i} \right]_0^1 \quad (4.4)$$

where the operator $[x]_a^b = \min\{\max\{x, a\}, b\}$ is used to trim the value between 0 and 1. The level of intervention g increases if any device deviates from the target outcome. Moreover, when the level of intervention reaches the upper bound (i.e., $g = 1$), the utility of all devices will be zero.

We then prove that the intervention function g^* and the target outcome $\tilde{\mathbf{s}} = (\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_N)$ constitute a Stackelberg equilibrium.

Definition 4.1. (*Stackelberg Equilibrium*) An intervention rule g^* and a strategy \mathbf{s}^* constitute a Stackelberg equilibrium if \mathbf{s}^* is a Nash equilibrium of the game under the intervention scheme and

$$g^* \in \arg \max_{g \in \mathcal{G}} u_0(g, \mathbf{s}^*)$$

where \mathcal{G} is the set of all possible intervention schemes.

Theorem 4.2. The intervention function g^* defined in Eq. (4.4) and the target outcome $\tilde{\mathbf{s}} = (\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_N)$ constitute an Stackelberg equilibrium in the energy-harvesting RAC

game under the intervention scheme Γ_g^{EH-I} .

Proof. We prove the target outcome constitutes a Stackelberg equilibrium by showing that it constitutes a Nash equilibrium for every device. Then we show that the utility of the system manager is also maximized.

Let the strategy of devices except for i choose the strategy profile \tilde{s}_{-i} . We aim to prove the strategy leading to the highest utility to device i is $s_i = \tilde{s}_i$. Therefore, no device will deviate and $\tilde{s} = (\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_N)$ constitutes a Nash equilibrium.

The utility of device i under the intervention g^* is

$$u_i(g^*, \mathbf{s}) = s_i p_j (1 - g^*(\mathbf{s})) \prod_{j \neq i} (1 - \frac{s_j p_j}{M}).$$

$$= \begin{cases} 0, & \text{if } s_i > 2\tilde{s}_i \\ s_i p_i (2 - \frac{s_i}{\tilde{s}_i}) \prod_{j \neq i} (1 - \frac{s_j p_j}{M}), & \text{if } \tilde{s}_i \leq s_i \leq 2\tilde{s}_i \\ s_i p_i \prod_{j \neq i} (1 - \frac{s_j p_j}{M}), & \text{if } s_i < \tilde{s}_i \end{cases}$$

In the first case $s_i > 2\tilde{s}_i$, the level of intervention is $g^*(s_i, \tilde{s}_{-i}) = 1$ from Eq. (4.4) and every device obtains zero utility. In the second case $\tilde{s}_i \leq s_i \leq 2\tilde{s}_i$, the level of intervention is $g^*(s_i, \tilde{s}_{-i}) = \frac{s_i - \tilde{s}_i}{\tilde{s}_i}$. In the third case $s_i < \tilde{s}_i$, the level of intervention is $g^*(s_i, \tilde{s}_{-i}) = 0$.

The utility of device i increases on $s_i < \tilde{s}_i$, reaches the maximum at $s_i = \tilde{s}_i$, then decreases on $\tilde{s}_i \leq s_i \leq 2\tilde{s}_i$, and stays at 0 on $s_i > 2\tilde{s}_i$. Therefore, the strategy $s_i = \tilde{s}_i$ is the only strategy that maximizes the utility.

Finally, when every device chooses the target outcome, the intervention function becomes zero and the system manager obtains the highest utility $u_0(g^*, \tilde{s}) = 1 - g^*(\tilde{s}) = 1$. Therefore, we prove that the intervention function g^* and the strategy profile \tilde{s} constitute a Stackelberg equilibrium. \square

In Fig. 4.2, we show the Nash equilibrium under the intervention scheme. The target outcome (0.3, 0.7) is exactly the Nash equilibrium. Note that the energy-harvesting RAC game under the intervention scheme is not a potential game since the condition (3.1) is

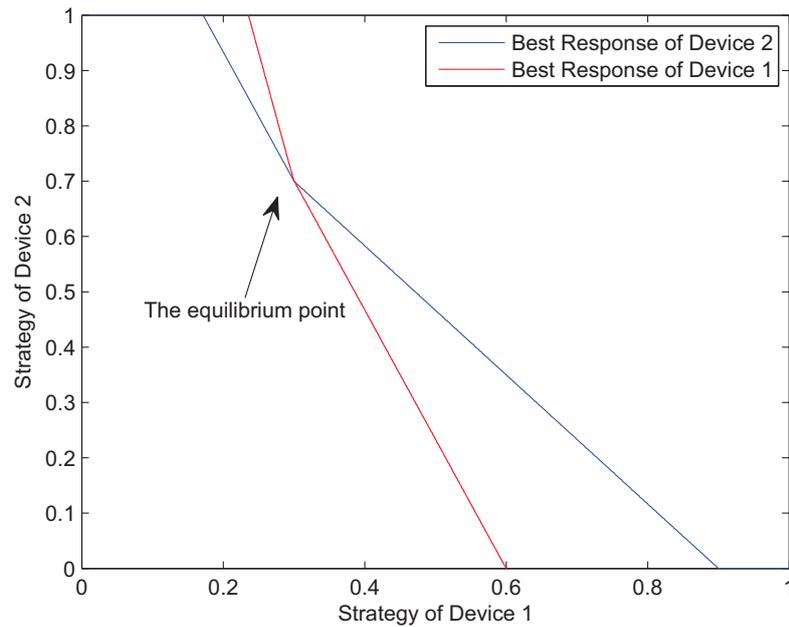


Figure 4.2: The equilibrium under the intervention scheme is exactly identical to the desired one.

not satisfied. Therefore, we cannot use the potential function. Instead, we derive the Nash equilibrium by deriving the intersection of best response functions of two devices.

So far we have proposed two methods for achieving the target outcome. In the following section, we will discuss two target outcomes that we aim to achieve: the proportionally fair outcome, and the social optimal outcome.

Chapter 5

The Desired Outcome

In the previous chapter, we have proposed two incentive schemes to induce the system to the target outcome. With either the pricing scheme or the intervention scheme, the system manager can achieve any feasible outcome it desire. In this chapter, we are going to investigate the outcomes that the system manager desire.

We always want to fulfill the demand of users in wireless systems. However, this is a dream that rarely happens in reality due to the lack of system resource. In a system where limited resource cannot suffice users' demand, we have to make an *optimal allocation*. The definition of *optimal allocation* varies with the purpose of the system applications. We adopt two most common kinds of optimal allocations: the proportionally fair outcome, and the social-optimal outcome.

5.1 The Proportionally Fair Outcome

First, we introduce the proportionally fairness. The proportionally fair outcome maximizes the overall utility while at the same time allow all devices at least the minimal level of service.

Definition 5.1. A strategy profile \mathbf{s}^* is proportionally fair if $s_i \geq 0$ for all i and

$$\sum_i \frac{u_i(\mathbf{s}) - u_i(\mathbf{s}^*)}{u_i(\mathbf{s}^*)} \leq 0, \forall i, \quad (5.1)$$

for any other $\mathbf{s} \in \mathcal{S}$ [20], or equivalently, \mathbf{s}^* is the solution for

$$\max_{\mathbf{s}} \sum_i \log(u_i(\mathbf{s})). \quad (5.2)$$

The condition 5.1 indicates that the deviation in strategy from the proportionally fair outcome \mathbf{s}^* increases some utilities and decreases some other utilities, but the summation of these difference is negative. Moreover, the player whose utility is originally small has larger weight factor in the summation, so it guarantees the fairness of the system.

The conditions in Definition 5.1 are equivalent since \mathbf{s}^* is the solution for (5.2) if and only if $\log(u_i)$ decreases around the point \mathbf{s} , that is,

$$\sum_i (u_i(\mathbf{s}) - u_i(\mathbf{s}^*)) \frac{\partial \log(u_i(\mathbf{s}))}{\partial \mathbf{s}} \Big|_{\mathbf{s}=\mathbf{s}^*} = \sum_i \frac{u_i(\mathbf{s}) - u_i(\mathbf{s}^*)}{u_i(\mathbf{s}^*)} \leq 0,$$

which coordinates with the condition (5.1).

Theorem 5.1. The proportionally fair outcome (the solution to the (5.3a)) is $s_i^* = \frac{M}{Np_i}$.

Proof. To find the proportionally fair outcome, we have to solve the following maximization problem.

$$\max_{\mathbf{s}} \sum_i \log(s_i p_i \prod_j (1 - \frac{s_j p_j}{M})) \quad (5.3a)$$

subject to

$$s_i \in [0, 1] \quad (\text{Strategy Space}) \quad (5.3b)$$

First, we rephrase the objective function (5.3a)

$$\begin{aligned}
& \max_{\mathbf{s}} \sum_i \log \left(s_i p_i \prod_{j \neq i} \left(1 - \frac{s_j p_j}{M} \right) \right) \\
&= \max_{\mathbf{s}} \sum_i \log(s_i p_i) + \sum_i \sum_{j \neq i} \log \left(1 - \frac{s_j p_j}{M} \right) \\
&= \max_{\mathbf{s}} \sum_i \log(s_i p_i) + (N-1) \sum_i \log \left(1 - \frac{s_i p_i}{M} \right) \\
&= \max_{\mathbf{s}} \sum_i \log \left(s_i p_i \left(1 - \frac{s_i p_i}{M} \right)^{N-1} \right) \\
&= \max_{s_i} s_i p_i \left(1 - \frac{s_i p_i}{M} \right)^{N-1},
\end{aligned}$$

which becomes a maximization problem with single variable.

We use the first-order condition

$$\frac{\partial s_i p_i \left(1 - \frac{s_i p_i}{M} \right)^{N-1}}{\partial s_i} = 0$$

to derive the maximum point

$$s_i^* = \frac{M}{N p_i}.$$

To verify this is a maximum point, we use the second-order differentiation:

$$\frac{\partial^2 s_i p_i \left(1 - \frac{s_i p_i}{M} \right)^{N-1}}{\partial s_i^2} \Big|_{s_i=s_i^*} = \frac{-p_i^2 N}{M} \left(1 - \frac{1}{N} \right)^{N-2} < 0$$

Therefore, $s_i^* = \frac{M}{N p_i}$ is a solution for (5.3a) and \mathbf{s}^* is the proportionally fair outcome. \square

From this theorem, the proportional fair strategy profile enables the device with low p_i to choose a higher transmission probability, and, on the contrary, enables the device with high p_i to choose a lower transmission probability. The strategy is also proportional to the number of transmission resource and inversely proportional to the number of device. The devices have to choose lower transmission probabilities when the number of devices are too many, while they can choose higher transmission probabilities when the number

of transmission resource is abundant.

Denote the *fairness index* by the value of

$$\sum_i \log(u_i(\mathbf{s})).$$

We can further prove that the value of fairness index is independent of the distribution of harvesting probability $\mathbf{p} = (p_1, p_2, \dots, p_N)$.

Theorem 5.2. *The value of fairness index is a function of the number of device N and the number of transmission resource M , that is,*

$$\sum_i \log(u_i(\mathbf{s})) = N(N-1) \log(M(N-1)) - N^2 \log(N),$$

for given N, M .

By definition, we have

Proof.

$$\begin{aligned} \sum_i \log(u_i(\mathbf{s})) &= \sum_i \log\left(\frac{M}{N} \prod_j \left(1 - \frac{1}{N}\right)\right) \\ &= \sum_i \log\left(\frac{M}{N} \left(\frac{N-1}{N}\right)^{N-1}\right) \\ &= \sum_i \log\left(\frac{M(N-1)^{N-1}}{N^N}\right) \\ &= N \log\left(\frac{M(N-1)^{N-1}}{N^N}\right) \\ &= N(N-1) \log(M(N-1)) - N^2 \log(N) \end{aligned}$$

and complete the proof. □

This theorem shows that the fairness index is independent of the distribution of energy harvesting probability of the devices in the system. Even if the devices have higher (or lower) chance to harvest energy, the fairness remains the same.

5.2 The Social Optimal Outcome

In this section, we derive another optimal outcome that maximizes the overall utility of the system. The outcome is called *social optimal* since the overall utility represents the social welfare. To derive the social optimal outcome, we have to solve the maximization problem as follows.

$$\begin{aligned} \max_s \sum_i u_i(s) & \quad \text{subject to} \\ s_i \in [0, 1] & \quad \text{(Strategy Space)} \end{aligned} \quad (5.4a)$$

Theorem 5.3. *Without loss of generality, we let $p_1 > p_2 > \dots > p_N$. Then the strategy profile $s_1 = s_2 = \dots = s_k = 1$ and $s_{k+1} = \dots = s_N = 0$ with the maximal positive integer k such that*

$$\prod_{i \leq k} \left(1 - \frac{p_i}{M}\right) - \frac{1}{M} \sum_{i \leq k} p_i \prod_{\substack{j \neq i \\ j \leq k}} \left(1 - \frac{p_j}{M}\right) > 0 \quad (5.5)$$

is the social optimal strategy set.

Proof. First, we prove that in the social strategy set, each device's strategy is either $s_i = 1$ or $s_i = 0$. Secondly, we prove that the social optimal strategy consists of consecutive devices from 1 to k . Thirdly, we prove that the value k is the maximal positive integer such that Eq. (5.5) holds.

First, to prove that each device's strategy is either $s_i = 1$ or $s_i = 0$, we check the first-order derivative of the objective function (5.4a)

$$\frac{\partial \sum_i s_i p_i \prod_j \left(1 - \frac{s_j p_j}{M}\right)}{\partial s_i} = p_i \prod_j \left(1 - \frac{s_j p_j}{M}\right) - \frac{p_i}{M} \sum_{j \neq i} s_j p_j \prod_{k \neq i, j} \left(1 - \frac{s_k p_k}{M}\right),$$

which is independent with s_i . Since the first-order derivative is a constant, the optimal

point is located at the boundary. If the first-order derivative is positive, the optimal point is $s_i^* = 1$. On the contrary, if the first-order derivative is negative, the optimal point is $s_i^* = 0$.

For notational convenience, we denote the set containing all devices whose $s_i = 1$ by Ω . On the contrary, the complementary set $\mathbf{N} \setminus \Omega$ consists of all devices whose $s_i = 0$. Therefore, for every possible strategy set $s = \{s_1, s_2, \dots, s_N\}$, there must exist a corresponding one-to-one set Ω . Then the strategy profile in the statement of theorem can be rephrased to that the device set $\Omega = \{1, 2, \dots, k\}$ is the social optimal solution.

After that, now we can define $\Delta(\Omega)$ as the social welfare given the set Ω as follows.

$$\Delta(\Omega) \equiv \sum_{i \in \Omega} p_i \prod_{j \neq i} \left(1 - \frac{p_j}{M}\right)$$

Note that only devices in the set Ω choose $s_i = 1$. Besides, if a new device i is added, the new social welfare becomes

$$\Delta(\Omega \cup \{i\}) = \Delta(\Omega) \left(1 - \frac{p_i}{M}\right) + p_i \prod_{j \in \Omega} \left(1 - \frac{p_j}{M}\right). \quad (5.6)$$

The first term on the right hand side is the social welfare deduction (within those original devices except for device i) since the probability of collision increases if device i joins in the set Ω , and the second term is the social welfare addition since the utility of device i also contributes to the social welfare.

Secondly, we use the contradiction to prove that the social optimal strategy consists of consecutive number of devices from 1 to k . Let k be the largest number in the set Ω . If Ω does not contain consecutive number of devices, there exists a value $h < k$ such that $h \notin \Omega$. Construct two new sets $\Omega' = \Omega \setminus k$ and $\Omega'' = \Omega' \cup h$. We show that the social welfare of new set $\Delta(\Omega'')$ is higher than $\Delta(\Omega)$ by comparing

$$\Delta(\Omega) = \Delta(\Omega' \cup \{k\}) = \Delta(\Omega') \left(1 - \frac{p_k}{M}\right) + p_k \prod_{j \in \Omega'} \left(1 - \frac{p_j}{M}\right)$$

and

$$\Delta(\Omega'') = \Delta(\Omega' \cup \{h\}) = \Delta(\Omega')\left(1 - \frac{p_h}{M}\right) + p_h \prod_{j \in \Omega'} \left(1 - \frac{p_j}{M}\right).$$

We can find that

$$\Delta(\Omega'') = \Delta(\Omega') + p_h \left(\prod_{j \in \Omega'} \left(1 - \frac{p_j}{M}\right) - \frac{\Delta(\Omega')}{M} \right) > \Delta(\Omega') + p_k \left(\prod_{j \in \Omega'} \left(1 - \frac{p_j}{M}\right) - \frac{\Delta(\Omega')}{M} \right) = \Delta(\Omega).$$

since $p_h > p_k$.

In other word, if Ω does not contain consecutive number of devices, we can remove the device with the largest number and add one with smaller number to increase the social welfare. Therefore, the set Ω cannot be the social optimal solution. For any set Ω that does not contain consecutive number of devices, there exists a new set Ω'' that contains consecutive number of devices bring higher social welfare. A social optimal strategy set must contain consecutive number of devices.

Lastly, from Eq. (5.6), we derive the difference between the social welfare before and after device i is added.

$$\Delta(\Omega \cup \{i\}) - \Delta(\Omega) = p_i \left(\prod_{j \in \Omega} \left(1 - \frac{p_j}{M}\right) - \frac{\Delta(\Omega)}{M} \right) = p_i \left(\prod_i \left(1 - \frac{s_i p_i}{M}\right) - \frac{\Delta(\Omega)}{M} \right).$$

Device i would be included in the social optimal outcome if and only if the difference remains positive, or equivalently, Eq. (5.5) is positive. We complete the proof. □

Different from the proportionally fair strategy profile, the social optimal strategy profile only enables the device with higher harvesting probability to choose the highest transmission probability, while prohibits others from transmitting. In view of maximizing the system performance, this is the best way to allocate the limited resource to the better devices. The theorem also indicates what the number of device in the social optimal strategy profile is. To express in a practical form, we design an algorithm to implement the social optimal outcome as follows. The algorithm is used in the simulation.

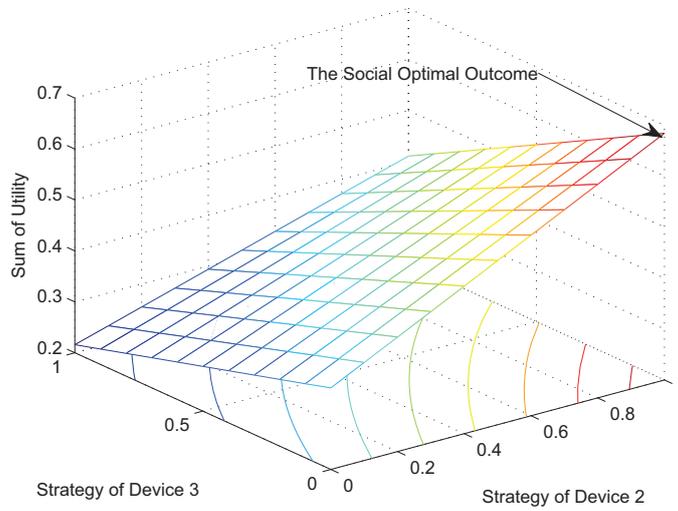


Figure 5.1: An example of social optimal outcome: device 2 has to choose $s_2 = 1$ and device 1 has to choose $s_1 = 0$.

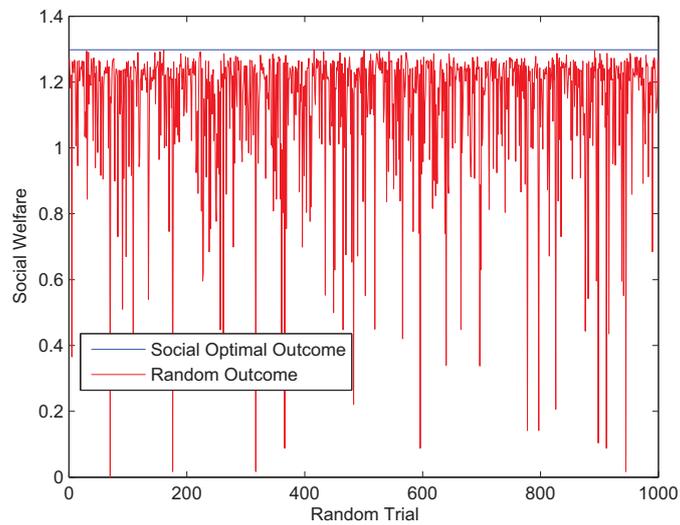
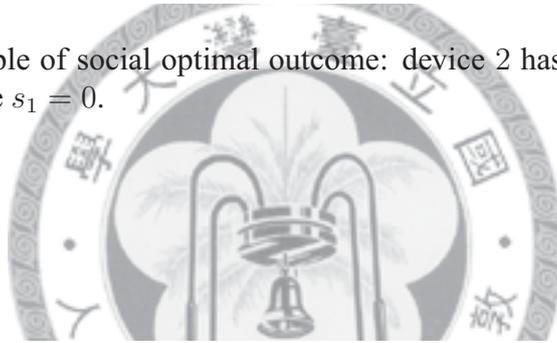


Figure 5.2: The social optimal outcome leads to the highest social welfare, compared with other outcomes that are randomly generated.

Algorithm I: SOCIALOPTIMALALGORITHM(p, N, M)

$p \leftarrow \text{DESCENTSORT}(p)$

for $k \leftarrow 2$ **to** N

do $\left\{ \begin{array}{l} \text{if } \prod_{i \leq k} (1 - \frac{p_i}{M}) - \frac{1}{M} \sum_{i \leq k} p_i \prod_{\substack{j \neq i \\ j \leq k}} (1 - \frac{p_j}{M}) < 0 \\ \text{then break} \end{array} \right.$

return (k)

The algorithm first sorts the devices with respect to their energy harvesting probability in descent order. Then it adds the device with the largest p_i in the list, and then the second largest one, and so one. The process terminates if the condition cannot be satisfied.

In Fig. 5.1, we use a simple system with four devices to show the social optimal outcome. We calculate that the social optimal outcome is $(s_1, s_2, s_3, s_4) = (1, 1, 0, 0)$. Because of the constraint of figure dimension, Fig. 5.1 illustrates the effect of strategy of device 2 and device 3 on the social welfare. Apparently, to maximize the social welfare, device 2 has to choose $s_2 = 1$ and device 3 has to choose $s_3 = 0$. On the other hand, Fig. 5.2 shows that the outcome derived by Algorithm I indeed leads to an outcome whose social welfare is maximized.

5.3 Adopting the Incentive Schemes to Achieve the Optimal Outcomes

After proposing two incentive schemes (the pricing scheme and the intervention scheme) and deriving two optimal outcomes (the proportionally fair outcome and the social optimal outcome), we start to adopt the incentive schemes to find the optimal outcomes in this section. There are four kinds of combination.

Theorem 5.4. *To achieve the proportionally fair outcome by using the pricing scheme,*

we have to set the price function $c_i(s_i, p_i)$ which satisfies

$$\frac{\partial c_i(s_i, p_i)}{\partial s_i} \Big|_{s_i = \frac{M}{Np_i}} = p_i \left(\frac{N-1}{N} \right)^{N-1}, \quad \forall i$$

Proof. Adopting $s_i^* = \frac{M}{Np_i}$ in Theorem 4.1, we complete the proof. \square

Theorem 5.5. To achieve the social optimal outcome by using the pricing scheme, we have to set the price function $c_i(s_i, p_i)$ which satisfies

$$\frac{\partial c_i(s_i, p_i)}{\partial s_i} \Big|_{s_i=1} = p_i \left(1 - \frac{p_i}{M} \right)^{N-1}, \quad \forall i \leq k$$

and

$$\frac{\partial c_i(s_i, p_i)}{\partial s_i} \Big|_{s_i=0} = p_i, \quad \forall i > k$$

where the value k is determined by Theorem 5.3.

Proof. Adopting $s_i^* = 1$ for all $i \leq k$ and $s_i^* = 0$ for all $i > k$ in Theorem 4.1, we complete the proof. \square

Theorem 5.6. To achieve the proportionally fair outcome by using the intervention scheme, we have to set the intervention function

$$g^*(\mathbf{s}) = \left[\sum_{i=1}^N \frac{s_i - \frac{N}{Mp_i}}{\frac{N}{Mp_i}} \right]_0^1$$

Proof. Adopting $s_i^* = \frac{M}{Np_i}$ in Theorem 4.2, we complete the proof. \square

Here we have presented the way to implement proportionally fair outcome and social optimal outcome by using the pricing scheme and the intervention scheme. Note that theoretically the social optimal outcome cannot be implemented by the intervention scheme, since the intervention function is not well-defined if the element of target outcome s_i is zero. However, one can still implement the social optimal outcome by choosing infinitesimal strategy that is extremely close to zero. That is, with an arbitrarily chosen $\delta \rightarrow 0$, we can

construct a sub-optimal strategy $s_i = 1$ for $i \leq k$ and $s_i = \delta$ for $i > k$. From Theorem 4.2, we know the sub-optimal strategy constitutes a NE, and leads to social welfare equal to

$$SW^\delta = \sum_{i=1}^k p_i \prod_{\substack{j \neq i \\ j \leq k}} (1 - \frac{p_j}{M}) \prod_{\substack{j \neq i \\ j > k}} (1 - \frac{\delta p_j}{M}) + \sum_{i=k+1}^N \delta p_i \prod_{\substack{j \neq i \\ j \leq k}} (1 - \frac{p_j}{M}) \prod_{\substack{j \neq i \\ j > k}} (1 - \frac{\delta p_j}{M}).$$

On the other hand, the optimal strategy leads to social welfare equal to

$$SW^0 = \sum_{i=1}^k p_i \prod_{\substack{j \neq i \\ j \leq k}} (1 - \frac{p_j}{M}).$$

Since $\delta \rightarrow 0$, the ratio $\frac{SW^\delta}{SW^0}$ can be approximated as

$$\frac{SW^\delta}{SW^0} \approx \frac{\sum_{i=1}^k p_i \prod_{\substack{j \neq i \\ j \leq k}} (1 - \frac{p_j}{M}) \prod_{\substack{j \neq i \\ j > k}} (1 - \frac{\delta p_j}{M})}{\sum_{i=1}^k p_i \prod_{\substack{j \neq i \\ j \leq k}} (1 - \frac{p_j}{M})},$$

which is bounded by

$$(1 - \frac{\delta}{M})^{N-k} \leq \frac{SW^\delta}{SW^0} \leq 1.$$

Even in the worst case, the social welfare of sub-optimal strategy is $(1 - \frac{\delta}{M})^{N-k}$ of optimal strategy. We can conclude that, by using infinitesimal strategy to achieve the social optimal under the intervention scheme, the social welfare decreases by a factor of $(1 - \frac{\delta}{M})^{N-k}$ at most. The larger δ we choose, the more social welfare we lose.

In implementing these two incentives, we have to bear in mind that these two schemes have a fundamental difference. The pricing scheme charges different pricing to the devices. The BS has to design proper pricing functions to achieve the target outcome. On the contrary, the intervention scheme imposes the same level of intervention on the devices, which can be implemented by dropping a certain percentage of packets that the BS receives. Therefore, to implement the pricing scheme, we have to install the pricing program on the device side. On the other hand, to implement the intervention scheme,

we have to install the intervention program on the BS side. It is more recommended to implement the intervention scheme since the change on the BS side is usually easier.



Chapter 6

Extension: Multi-Period Model

In the previous section, we construct an one-shot game model and propose two incentive schemes to achieve the target equilibrium. In this section, we start to construct an extension model that considers multi-period energy-harvesting WSN model. Since the energy can be stored for the future usage, the devices face new trade-off in the multi-period energy-harvesting WSN.

6.1 Model Setting

Adopting the notations from the previous model, there are N devices competing for M transmission channel. The collision occurs if there is more than two device requesting for the same channel.

We consider the slotted-time model. There are T periods in each of which the energy arrives with different probability. Since the value of information increases with the number of transmission success, the devices aim to maximize the number of transmission success during these T periods.

At the period 0, the device i chooses a transmission probability s_i which cannot be changed during these T period. The probability of harvesting energy varies with the time, which is p_i^t at the period t .

The energy capacity is also another important issue. However, to keep the result

tractable, we consider a simple scenarios where the devices can store at most one unit of energy.

6.2 The Probability of Harvesting Energy

Since the energy can be stored for future usage, the probability of harvesting energy at period t depends on the strategy of previous periods. To clarify, we denote the harvesting probability that the device harvests a unit of energy at the period t by p_i^t , while the powered-up probability that the device has enough energy to transmit at the period t by q_i^t . Notice that $q_i^t > p_i^t$ since if the device has enough energy to transmit (i.e., the device is powered-up), the energy sources from either the harvested energy at period t , or the energy stored in the previous periods.

For example, at the period 1

$$q_i^1 = p_i^1$$

and at the period 2

$$q_i^2 = 1 - (1 - p_i^2)(1 - q_i^1(1 - s_i)).$$

The device has one unit of energy at period 2 if it harvests a unit of energy at period 2 or it has harvested energy in the past and did not use it. Therefore, the general form of q_i^t at period t is

$$q_i^t = 1 - (1 - p_i^t)(1 - q_i^{t-1}(1 - s_i)). \quad (6.1)$$

We find that q_i^t is a recursive function of q_i^{t-1} . By rearranging the recursive function (Eq. (6.1)), we can derive q_i^t in form of $(p_i^1, p_i^2, \dots, p_i^t)$ and s_i .

Lemma 6.1. *The probability that the device has enough energy to transmit at period t can be expressed as*

$$q_i^t = \sum_{n=1}^t \left(p_i^n \prod_{m=n+1}^t (1 - p_i^m)(1 - s_i) \right) \quad (6.2)$$

Proof. First, we rephrase Eq. (6.1) to

$$q_i^t = p_i^t + (1 - p_i^t)(1 - s_i)q_i^{t-1}$$

We define the summation factor

$$b^t \equiv (1 - p_i^t)(1 - s_i)$$

and divide $\prod_{m=2}^t b^m$ from the both sides.

$$\frac{q_i^t}{\prod_{m=2}^t b^m} = \frac{q_i^{t-1}}{\prod_{m=2}^{t-1} b^m} + \frac{p_i^t}{\prod_{m=2}^t b^m}$$

After summing up both sides of equation, we have

$$\frac{q_i^t}{\prod_{m=2}^t b^m} = q_i^1 + \sum_{n=2}^t \frac{p_i^n}{\prod_{m=2}^n b^m}$$

or equivalently,

$$q_i^t = q_i^1 \prod_{m=2}^t b^m + \sum_{n=2}^t \left(p_i^n \prod_{m=n+1}^t b^m \right) = \sum_{n=1}^t \left(p_i^n \prod_{m=n+1}^t b^m \right).$$

Replacing b^t with $(1 - p_i^t)(1 - s_i)$, we derive the solution.

$$q_i^t = \sum_{n=1}^t \left(p_i^n \prod_{m=n+1}^t (1 - p_i^m)(1 - s_i) \right)$$

□

From this lemma, we can determine q_i^t from the previous probability of energy harvesting $(p_i^1, p_i^2, \dots, p_i^t)$ and the strategies s_i

6.3 Nash equilibrium: Intersection of Best Response

In this section, we provide a mathematical solution for the Nash equilibrium of multi-period model. We first introduce the best response, that is, the strategy that produces the most favorite outcome for a device, taking others' strategies as given. Therefore, if all the devices select the best response to others' players' strategies, it is a Nash equilibrium.

From Lemma 6.2, we have

$$q_i^t = \sum_{n=1}^t \left(p_i^n (1 - s_i)^{t-n} \prod_{m=n+1}^t (1 - p_i^m) \right)$$

and the multi-period game can be formulated as following.

$$\Gamma^{MP} = \langle \mathcal{N}, (\mathcal{S}_i)_{i \in \mathcal{N}}, (u_i^M)_{i \in \mathcal{N}} \rangle$$

where the utility is the summation of probability of successful transmission

$$u_i^M = \sum_{t=1}^T s_i q_i^t \prod_{j \neq i} \left(1 - \frac{s_j q_j^t}{M} \right).$$

First, we prove that the Nash equilibrium exists in the game Γ^{MP} .

Lemma 6.2. *The pure-strategy Nash equilibrium exists if the strategy sets \mathcal{S}_i are nonempty compact convex subsets of a Euclidean space and the utility functions u_i are continuous in s_i and quasi-concave in s_i . [21]*

Theorem 6.3. *In the multi-period game Γ^{MP} , the pure-strategy Nash equilibrium exists.*

Proof. First, the strategy set \mathcal{S}_i in the multi-period game Γ^{MP} are nonempty compact convex subsets of a Euclidean space. And the utility functions u_i are continuous in s_i .

Second, we prove the utility function is a concave function with respect to the strategy s_i . Replacing q_i^t with Eq. (6.2) and $s_i^t = s_i$ for all period t , we find the utility

$$u_i^M = \sum_{t=1}^T s_i \left(\sum_{n=1}^t \left(p_i^n (1 - s_i)^{t-n} \prod_{m=n+1}^t (1 - p_i^m) \right) \right) \prod_{j \neq i} \left(1 - \frac{s_j q_j^t}{M} \right)$$

is a polynomial function of variable s_i which can be rephrased as

$$u_i^M = s_i \sum_{m=0}^{T-1} C_{m,i} (1 - s_i)^m$$

where the constant

$$C_{m,i} = \prod_{j \neq i} \left(1 - \frac{s_j q_j^{m+1}}{M}\right) \sum_{t=1}^{T-m} p_i^t \prod_{n=1}^m (1 - p_i^{t+n})$$

is independent with the strategy s_i . Since $s_i \in [0, 1]$, we verify the dual function

$$\bar{u}_i^M = (1 - s_i) \sum_{m=0}^{T-1} C_{m,i} s_i^m,$$

which has the same concavity (or convexity) with u_i^M , to prove \bar{u}_i^M is concave by using the second-order derivative

$$\begin{aligned} \frac{\partial^2 \bar{u}_i^M}{\partial s_i^2} &= -2 \sum_{m=1}^{T-1} m C_{m,i} s_i^{m-1} + (1 - s_i) \sum_{m=2}^{T-1} C_{m,i} m(m-1) s_i^{m-2} \\ &< \sum_{m=2}^{T-1} (-2s_i + (m-1)(1-s_i)) C_{m,i} m s_i^{m-2} \\ &= \sum_{m=2}^{T-1} ((m-1) - (1+m)s_i) C_{m,i} m s_i^{m-2} \\ &= 2(1-3s_i)C_2 - T(T-1)C_{T-1} s_i^{T-2} + \sum_{m=3}^{T-1} ((m-1) - (1+m-1)) C_{m,i} m s_i^{m-2} \\ &< \sum_{m=3}^{T-1} ((m-1) - (1+m-1)) C_{m,i} m s_i^{m-2} \\ &= - \sum_{m=3}^{T-1} C_{m,i} m s_i^{m-2} \\ &< 0 \end{aligned} \tag{6.3}$$

That is, both dual function \bar{u}_i and the utility function u_i are concave functions in s_i .

Therefore, the multi-period game Γ^{MP} has a pure-strategy Nash equilibrium. \square

Theorem 6.4. *The strategy profile $\mathbf{s}^* = (s_1^*, s_2^*, \dots, s_N^*)$ such that $s_i = [\bar{s}_i^*]_0^1$ constitutes a Nash equilibrium in Γ^{MP} , where $(\bar{s}_1^*, \bar{s}_2^*, \dots, \bar{s}_N^*)$ solves the following simultaneous equations.*

$$\sum_{m=0}^{T-1} (1 - \bar{s}_i^*)^m C_{m,i}^* - \sum_{m=0}^{T-2} \bar{s}_i^* (m+1) (1 - \bar{s}_i^*)^m C_{m+1,i}^* = 0, \forall i \quad (6.4)$$

where

$$C_{m,i}^* = \prod_{j \neq i} \left(1 - \frac{\bar{s}_j^* q_j^{m+1}}{M}\right) \sum_{t=1}^{T-m} p_i^t \prod_{n=1}^m (1 - p_i^{t+n}) \quad (6.5)$$

Proof. Since the utility function

$$u_i^M = s_i \sum_{m=0}^{T-1} C_{m,i} (1 - s_i)^m$$

is concave in s_i , we can use the first-order derivative to derive the best response

$$\frac{\partial u_i^M}{\partial s_i} = \sum_{m=0}^{T-1} C_{m,i} (1 - s_i)^m - s_i \sum_{m=1}^{T-1} m C_{m,i} (1 - s_i)^{m-1}$$

We show that the first order condition (the best response function) is

$$\sum_{m=0}^{T-1} C_{m,i}^* (1 - s_i^*)^m - s_i^* \sum_{m=1}^{T-1} m C_{m,i}^* (1 - s_i^*)^{m-1} = 0, \forall i.$$

Denote the solution of the first order conditions by $(\bar{s}_1^*, \bar{s}_2^*, \dots, \bar{s}_N^*)$, which may violate the constraint of strategy space $[0, 1]$. However, due to the convexity of utility function, the maximal value locates at the boundary of strategy space (i.e., $s_i = 0, 1$). Therefore, the best response is the transacted value of the solution above, that is, $s_i = [\bar{s}_i^*]_0^1$. \square

6.4 Case Study

In this section, we study a special case where $T = 2$ and $N = 2$. Denote the devices by 1 and 2. Then we can rephrase Eq. (6.4) as

$$C_{0,i} + (1 - s_i)C_{1,i} = s_i C_{1,i}$$

or equivalently,

$$(2s_i - 1)C_{1,i} = C_{0,i}$$

for $i = 1, 2$.

Replacing $C_{m,i}$ with Eq. (6.5), we have

$$(2s_i - 1)p_i^1(1 - p_i^2)\left(1 - \frac{s_j q_j^2}{M}\right) = (p_i^1 + p_i^2)\left(1 - \frac{s_j q_j^1}{M}\right)$$

and then replace $q_j^1 = p_j^1$ and

$$q_j^2 = p_j^1(1 - p_j^2)(1 - s_j) + p_j^2,$$

we have the following two simultaneous equations to solve

$$\begin{cases} (2s_1 - 1)p_1^1(1 - p_1^2)\left(1 - \frac{s_2}{M}(p_2^1(1 - p_2^2)(1 - s_2) + p_2^2)\right) = (p_1^1 + p_1^2)\left(1 - \frac{s_2 p_2^1}{M}\right) \\ (2s_2 - 1)p_2^1(1 - p_2^2)\left(1 - \frac{s_1}{M}(p_1^1(1 - p_1^2)(1 - s_1) + p_1^2)\right) = (p_2^1 + p_2^2)\left(1 - \frac{s_1 p_1^1}{M}\right) \end{cases}$$

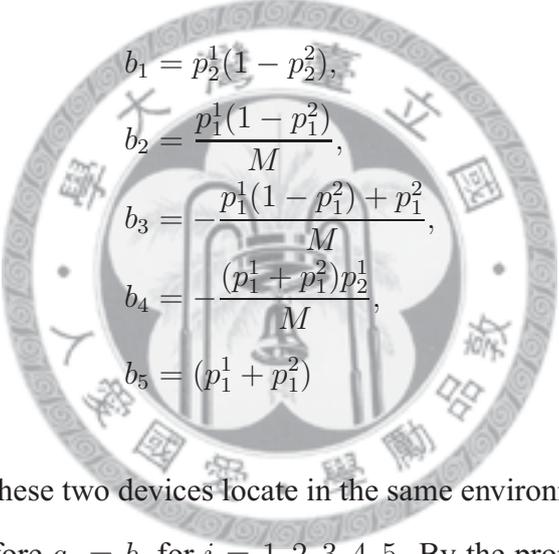
which can be rephrased to

$$\begin{cases} (2s_1 - 1)a_1(1 + a_2 s_2^2 + a_3 s_2) = a_4 s_2 + a_5 \\ (2s_2 - 1)b_1(1 + b_2 s_1^2 + b_3 s_1) = b_4 s_1 + b_5 \end{cases}$$

where

$$\begin{aligned}
 a_1 &= p_1^1(1 - p_1^2), \\
 a_2 &= \frac{p_2^1(1 - p_2^2)}{M}, \\
 a_3 &= -\frac{p_2^1(1 - p_2^2) + p_2^2}{M}, \\
 a_4 &= -\frac{(p_2^1 + p_2^2)p_1^1}{M}, \\
 a_5 &= (p_2^1 + p_2^2)
 \end{aligned}$$

and



$$\begin{aligned}
 b_1 &= p_2^1(1 - p_2^2), \\
 b_2 &= \frac{p_1^1(1 - p_1^2)}{M}, \\
 b_3 &= -\frac{p_1^1(1 - p_1^2) + p_1^2}{M}, \\
 b_4 &= -\frac{(p_1^1 + p_1^2)p_2^1}{M}, \\
 b_5 &= (p_1^1 + p_1^2)
 \end{aligned}$$

By assuming that these two devices locate in the same environment, we have $p_1^t = p_2^t$ for $t = 1, 2$ and therefore $a_i = b_i$ for $i = 1, 2, 3, 4, 5$. By the property of symmetry, we know that the solution $s_1 = s_2 = s$. Then we can simplify the problem into

$$(2s - 1)b_1(1 + b_2s^2 + b_3s) = b_4s + b_5$$

or equivalently,

$$2b_1b_2s^3 + b_1(2b_3 - b_2)s^2 + (b_1(2 - b_3) - b_4)s - b_1 - b_5 = 0,$$

which is a cubic function that can be solved by the general formula.

Chapter 7

Numerical Results

In this chapter, we build EH-RAC systems and examine the effect of parameter change on the system performance (i.e., social welfare and fairness index). Besides, we also numerically verify the case study in the multi-period EH-RAC game. By the simulation, we can gain more insight about the case study.

7.1 System Performance With Different Parameters

We discuss how the device parameters affect the system performance. In Fig. 7.1(a) and Fig. 7.1(b), we build a system with $N = 100$ devices and $M = 5$ transmission channels. There are 9 groups of device whose average harvesting probability is 0.1, 0.2, ..., 0.9 respectively. The harvesting probability within each group follows the normal distribution $N(0.1, 0.1)$, $N(0.2, 0.1)$, ..., $N(0.9, 0.1)$ respectively.

In Fig. 7.1(a), the social welfare of social optimal outcome is superior to that of a random outcome. Any random outcome decreases in social welfare since the mutual interference increases when the average energy harvesting probability increases. The social welfare of social optimal outcome also increases with the average energy harvesting probability of group, since only some specific devices are allowed to transmit in the social optimal algorithm. When the energy harvesting probabilities of those devices increase, their transmission success rate increases, which improves social welfare. On the other

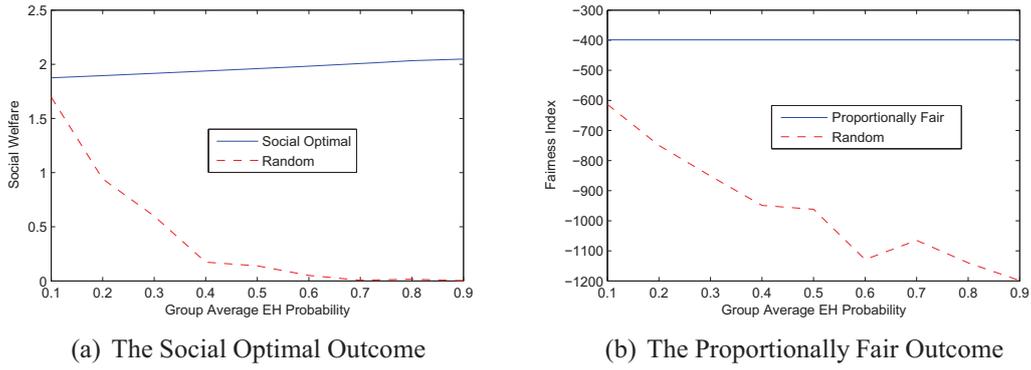


Figure 7.1: The social optimal outcome and the proportionally fair outcome under different choices of parameters.

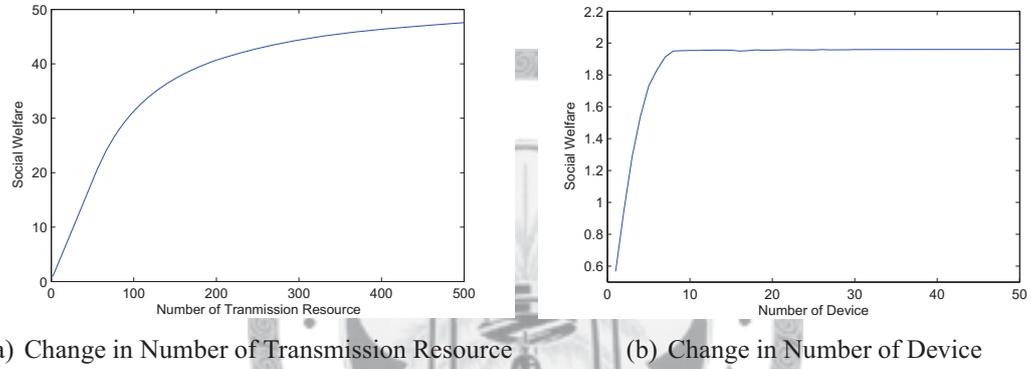


Figure 7.2: The value of social welfare changes with the number of transmission resource and the number of resource.

hand, in Fig. 7.1(b), the fairness index of proportionally fair outcome is superior to that of a random outcome, and it also remains constant for each group. As we prove in Theorem 5.2, the value of fairness index is a function of N and M . Since each group has the same parameters N and M , the fairness indexes are identical.

In Fig. 7.2(a), the value of social welfare increases with the number of transmission resource. When the number of transmission resource is below the device number, social welfare increases sharply mainly because the number of transmitting device is increasing. Later, when the number of transmission resource is above the device number, the number of transmitting device remains the same, so social welfare fare increases only because the average energy harvesting probability increases.

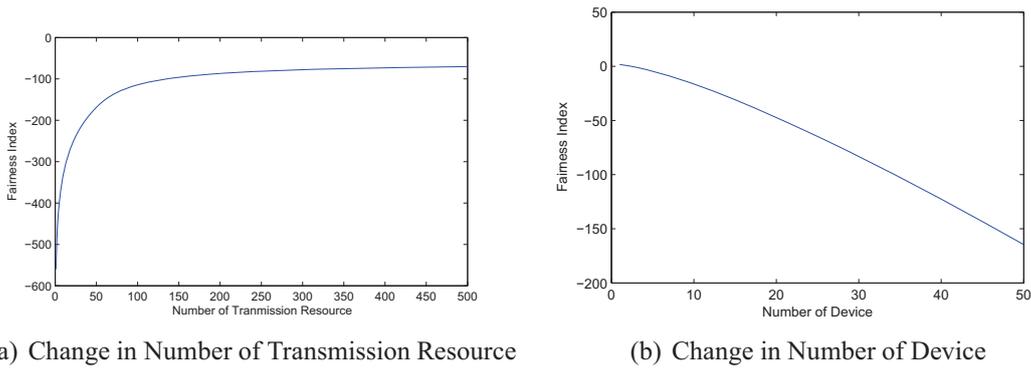


Figure 7.3: The value of fairness index changes with the number of transmission resource and the number of resource.

In Fig. 7.2(b), the value of social welfare increases with the number of device, but saturates when the number of device is too large. In the increasing region, the system still has capacity to accommodate more devices and the social welfare still has room to increase. However, in the saturation region, the transmission resource is fully utilized and the system cannot accommodate any more device. Therefore, the social welfare stop increasing.

In Fig. 7.3(a), the value of fairness index increases with the number of transmission resource. If the system has more transmission resource, then the contention among devices can be reduced and the outcome can be more fair. The reason is similar to the previous one. On the other hand, in Fig. 7.3(b), the value of fairness index decreases with the number of device. If there are more devices in the system, then the problem of lack of transmission resource would become more severe, and the fairness becomes lower.

7.2 Simulation of The 2-Period 2-Device EH-RAC Game

We take advantage of computer simulation to observe the case discussed in the case study of multi-period game. We choose $T = 2$, $N = 2$, and $M = 1$. The harvesting probabilities are $(p_1^1, p_1^2) = (p_2^1, p_2^2) = (0.2, 0.5)$. The simulation algorithm is shown in Algorithm I.

In Fig. 7.4(a), the utility function of device 2 increases along the strategy of device 1 but decreases along the strategy of device 2. The utility function is a concave function and has maximal point at $s_2^* = 1$ for any s_1 . Since the concavity of utility function is not obvious in Fig. 7.4(a), we derive its first-order differentiation (Fig. 7.4(b)) and find that the value strictly decreases along the strategy of device 2. Remember that if the first-order differentiation is strictly decreasing, then the function is concave.

Moreover, from the previous analysis, we have $b_1 = b_2 = 0.1, b_3 = -0.6, b_4 = -0.14, b_5 = 0.7$. Then the solution of

$$0.02s^3 - 0.13s^2 + 0.4s - 0.8 = 0,$$

is $s = 4$. Since the utility function u_1 is concave in s_1 , u_1 is monotonically increasing for $s \in [0, 4]$. Due to the constraint of strategy space, the device strategy will choose $s_i = 1$ for $i = 1, 2$, that is, the strategy profile $(s_1, s_2) = (1, 1)$ is the Nash equilibrium, as shown in 7.5.

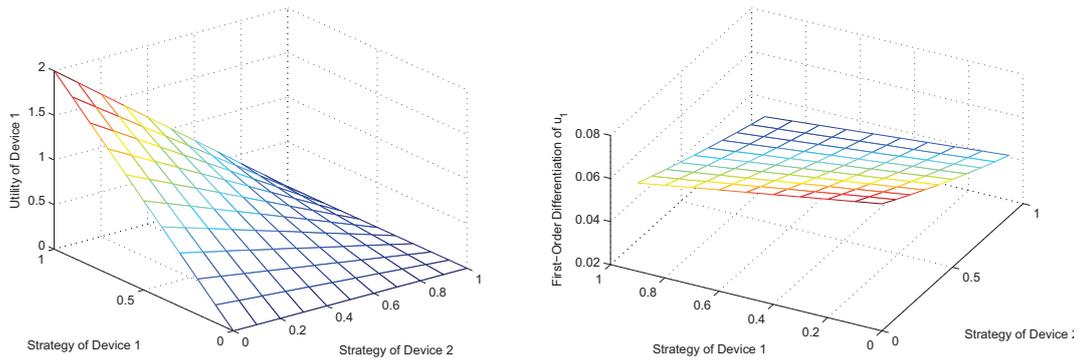
Algorithm I: TWO-PERIOD TWO-PLAYER NASH EQUILIBRIUM(p, M, T)

for $i \leftarrow 0$ **to** 100

do $\left\{ \begin{array}{l} s_i \leftarrow \frac{i}{100} \\ \textbf{for } j \leftarrow 0 \textbf{ to } 100 \\ \textbf{do} \left\{ \begin{array}{l} s_j \leftarrow \frac{j}{100} \\ \textbf{for } t \leftarrow 1 \textbf{ to } T \\ \textbf{do} \left\{ \begin{array}{l} q_i^t = \sum_{n=1}^t p_i^n (1 - s_i)^{t-n} \prod_{m=n+1}^t (1 - p_i^m) \\ q_j^t = \sum_{n=1}^t p_j^n (1 - s_j)^{t-n} \prod_{m=n+1}^t (1 - p_j^m) \\ u_i^M(s_i, s_j) = \sum_{t=1}^T s_i q_i^t (1 - \frac{s_j q_j^t}{M}) \\ u_j^M(s_i, s_j) = \sum_{t=1}^T s_j q_j^t (1 - \frac{s_i q_i^t}{M}) \end{array} \right. \end{array} \right. \end{array} \right.$

$BR_i(s_j) = \arg \max_{s_i} u_i^M(s_i, s_j)$

$BR_j(s_i) = \arg \max_{s_j} u_j^M(s_i, s_j)$



(a) The utility function

(b) The first-order differentiation

Figure 7.4: The utility function u_1 is a concave function increasing along the strategy s_1 .

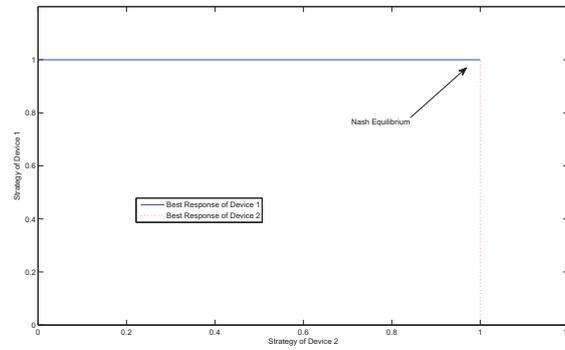


Figure 7.5: The Nash equilibrium in the multi-period model

7.3 Extended Simulations of Multi-period Multi-device EH-RAC Game

To seek the insight of Nash equilibrium in multi-period energy-harvesting RAC game, we use the computer simulation to show how the energy harvesting probabilities affect

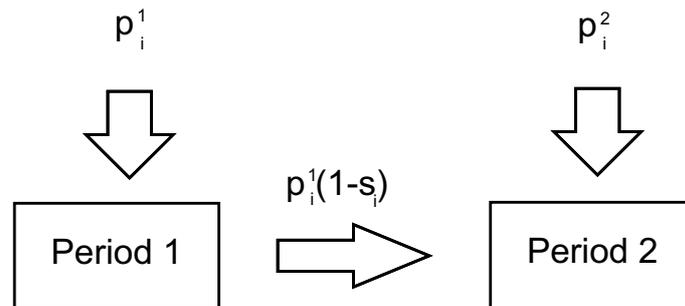


Figure 7.6: The energy relation of two-period model

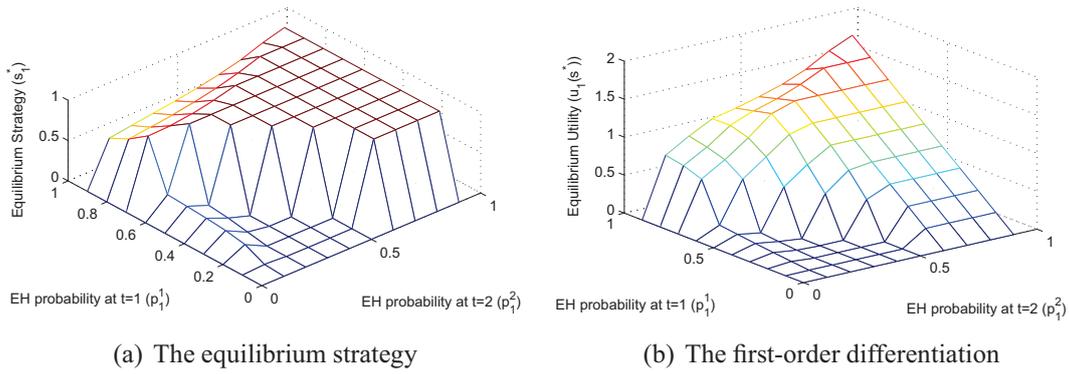


Figure 7.7: The difference of equilibrium strategy and utility of device i with respect to the harvesting probabilities.

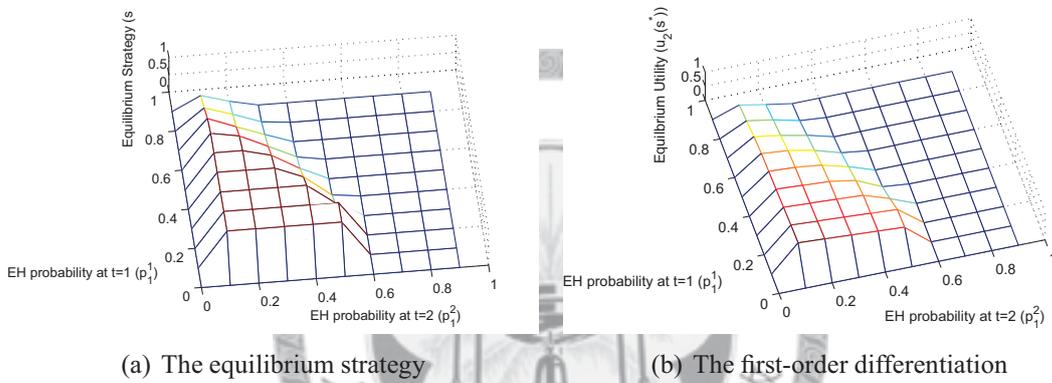


Figure 7.8: The difference of equilibrium strategy and utility of device j with respect to the harvesting probabilities.

the Nash equilibrium in this section. To make the results tractable, we consider only a two-period-multi-device system. For all devices, the energy can be stored from period 1 to period 2. We use Fig. 7.6 to illustrate the simulation model. The device has the probability p_i^1 of harvesting a unit of energy at period 1 and has the probability p_i^2 of harvesting a unit of energy at period 2. Once the device chooses the strategy s_i , the harvested energy will be stored to period 2 with probability $p_i^1(1 - s_i)$. Note that we have assumed that the energy capacity is one unit. If the device has the energy at period 2, it can be either stored from period 1 or harvested at period 2.

In Fig. 7.7(a) and Fig. 7.7(b), we manipulate the harvesting probabilities of device 1 at period 1 and 2 while fix the harvesting probabilities of device 2 as constants $p_2^1 = p_2^2 =$

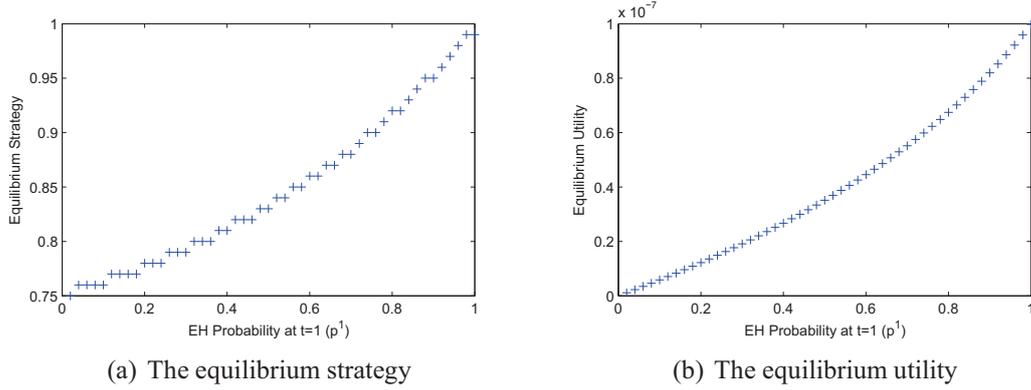


Figure 7.9: The difference of equilibrium strategy and utility of device j with respect to the number of device.

0.5. The number of devices is $N = 50$ and the transmission resource is $M = 1$. In general, no matter which harvesting probability increases, the equilibrium strategy increases at the same time. However, there is a trade-off when the harvesting probability p_1^1 is high. The equilibrium strategy of device 1 decreases from $s_1^* = 1$ since the probability that the device i has the energy at period 2 would decrease if the harvesting probability at is too high. Therefore, the device i must decrease its equilibrium strategy to maintain the utility at period 2. On the other hand, we read from Fig. 7.7(b) that the equilibrium utility increases along both harvesting probabilities p_1^1 and p_1^2 . When $p_1^1 = p_1^2 = 1$, the utility of device 1 reaches the optimum.

In Fig. 7.8(a) and Fig. 7.8(b), we want to observe how the harvesting probabilities of device 1 influence the equilibrium strategy and equilibrium utility of the other device (device 2). When the harvesting probabilities of device 1 are both low, the equilibrium strategy is to choose a high transmission probability; meanwhile, the utility of device 2 reaches the optimum.

In Fig. 7.9(a) and Fig. 7.9(b), we build a system composed of devices with different energy harvesting probabilities. To obtain the insight of system, we need to include the devices with all possible harvesting probability. Therefore, we set the harvesting probability $p_i^1 = i/N$ and $p_i^2 = i/2N$. The harvesting probability at period 2 is discounted by 50% so the devices will focus on the period 1.

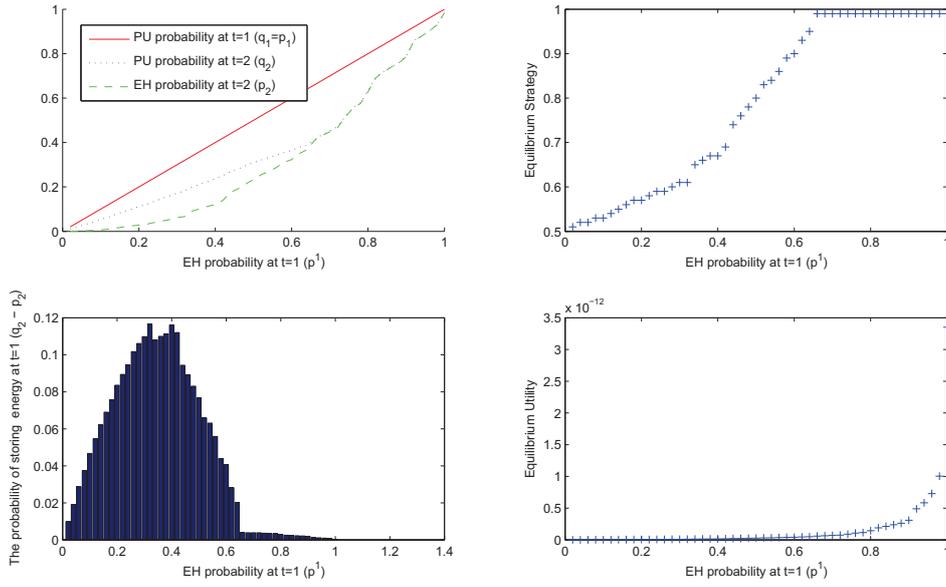


Figure 7.10: If the EH probability ratio is increasing, the devices with higher EH probability at period 1 will choose a higher transmission probability.

We observe that the device with higher harvesting probability will choose higher equilibrium strategy since the difference between the harvesting probability at period 1 and one at period 2 is larger. Moreover, the utility of devices with higher harvesting probability is higher since these devices have more chance to send transmission successfully.

In Fig. 7.10 and Fig. 7.11, we show that the devices with higher energy-harvesting (EH) probability ratio (i.e., p_i^2/p_i^1) will choose higher transmission probability and are less likely to leave energy from period 1 to period 2. First, we construct a system with parameter $N = 50, M = 1$. The devices' EH probability $q_i^1 = p_i^1$ are equally distributed between 0 to 1 and p_i^2 is set to satisfy the increasing (or decreasing) order. Formally, in Fig. 7.10, the EH probability ratio p_i^2/p_i^1 is increasing, while p_i^2/p_i^1 is decreasing in Fig. 7.10.

In both Fig. 7.10 and Fig. 7.11, we present four figures. The left-top figure shows the EH probabilities q_1, p_1, q_2, p_2 : q_1 is the EH probability at period 1 that equals to the powered-up (PU) probability p_1 , q_2 is the EH probability at period 2 that takes account of the stored energy from period 1 and the PU probability at period 2, and p_2 is the EH

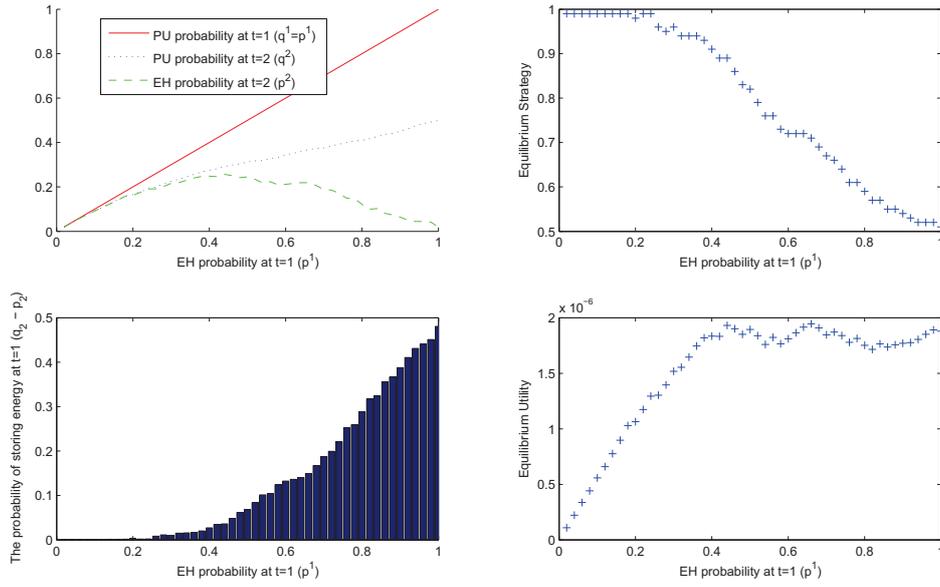


Figure 7.11: If the EH probability ratio is decreasing, the devices with lower EH probability at period 1 will choose a higher transmission probability.

probability that excludes the stored energy from period 1. The left-bottom figure shows the probability that the device stores energy at the period 1, that is, the difference between $q_2 - p_2$. The right-top figure is the equilibrium strategy that the device chooses in the Nash equilibrium, while the right-bottom figure is the equilibrium utility.

In Fig. 7.10, the EH probability ratio p_i^2/p_i^1 is increasing. That is, the devices with higher EH probability at period 1 is have higher EH probability at period 2. The simulation shows that, in the Nash equilibrium, the device with higher EH probability ratio will choose the higher transmission probability, which decreases the probability of storing the harvested energy from period 1 to period 2. Moreover, the utility of devices with higher EH probability is also higher. Therefore, we conclude that it is less likely that the devices with higher EH probability ratio will store energy from period 1. Fig. 7.11 also supports the same argument, where the EH probability ratio p_i^2/p_i^1 is decreasing.

Chapter 8

Conclusion

In this work, we first construct the RAC game and the energy-harvesting RAC game. To derive the Nash equilibrium, we adopt the potential solution concept and prove both games are potential games. However, the equilibria of both games are not efficient. Therefore, we propose the pricing scheme and the intervention scheme to achieve the target outcome and prevent the inefficient outcome from occurring. We mathematically prove that the target outcome is a unique equilibrium when the pricing scheme or the intervention scheme is used. In other word, the inefficient outcome is no longer an equilibrium.

Next we derive two target outcomes: the proportionally fair outcome and the social optimal outcome. The proportionally fair outcome maximizes the overall probability of successful transmission while at the same time allow all devices at least the minimal level of service. The social optimal outcome maximizes the overall utility of the system, that is, the sum of probability of successful transmission. Besides, we show how to use the pricing scheme and intervention scheme to achieve these two target outcomes.

In addition to the one-shot game models, we also formulate a multi-period energy-harvesting RAC game. We consider the scenarios where the devices store one unit of energy. The device can choose a specific strategy during totally T periods in which the harvesting probabilities can be different. Therefore, if the device chooses a large

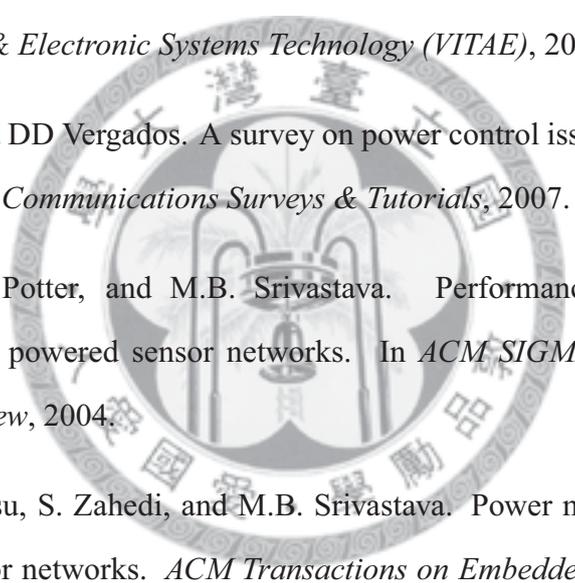
transmission probability, then it would acquire benefit from the transmission in the current period but lose benefit from leaving energy to the future periods, and vice versa. We derive the harvesting probabilities in form of the harvesting probabilities and the previous strategies.

Different from the one-shot games, the multi-period game is composed of convex utility functions. Therefore, even without incentive schemes, in the equilibrium the devices may choose transmission probability other than $s_i = 1$. We prove that the pure-strategy Nash equilibrium exists in this game. The Nash equilibrium strategy profile can be derived by solving the simultaneous equations given in Theorem 6.4.

Finally, we use the simulations to show how the systems parameters affect the equilibrium. The figures can improve the comprehension of the previous mathematical models and results.



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