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金融中介與流動性

Essays on Financial Intermediaries and Liquidity

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本論文係林佳靜君（學號 D95323004）在國立臺灣大學經濟學系完成之博士學位論文，於民國 101 年 7 月 5 日承下列考試委員審查通過及口試及格，特此證明

口試委員：

（指導教授）

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念博士班的這六年來, 有時一天顯得孤獨而漫長, 有時一天流逝得太匆忙, 並非沒想過放棄, 幸而我遇見了我的指導教授, 李怡庭老師。感謝老師耐心地指導我寫論文, 更感謝老師在我不安、挫折時所給予的溫暖安慰。在老師的指導及鼓勵下, 我漸漸明白這條路背後的意義, 於是堅持了自己的步伐, 不再動搖。

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摘要

爲了解當資產或支付工具遭遇資訊不對稱問題時, 金融中介如何影響總體流動性與產出, 本論文的第二章提出一個銀行存款可作爲支付工具的貨幣搜尋模型, 探討道德危險與支付工具的關聯。銀行對僞造支票者的懲罰爲拒絕其未來的貸款, 僞造者必須持有足夠的貨幣以融通自身消費需求。因此, 僞造成本與銀行的懲罰機制內生決定出存款的流動性限制。此流動性限制受僞造成本、通貨膨脹率、交易障礙等因素影響。當僞造成本較低, 或交易障礙較低時, 存款流動性較差, 於是銀行支付較高的存款利息以彌補較差的流動性。較高的通貨膨脹提高了僞造者攜帶貨幣的自我融通成本, 降低了僞造的好處, 使人們較願意接受以存款作爲支付工具, 存款流動性因而提高, 因此人們有更高的存款意願, 使銀行可提供較多的可貸資金以融通消費需求, 造成總體流動性與產出增加。本文對於經濟體存在道德危險時, 銀行接受存款、提供貸款的雙重角色與流動性、總產出之間的關係提出了一個新的觀點。

人們經常需要流動性資產以融通預期外的消費、投資機會, 若經濟社會中的資產及支付工具的品質爲私人訊息, 則可能阻礙交易的進行, 導致低產出與福利水準。在此情況下, 即使不具資訊優勢的金融中介有可能增進總體流動性與福利嗎? 第三章假設消費者的稟賦爲實質資產, 根據其期末價值, 資產有好壞之分; 而資產的品質是持有者的私人訊息。銀行以接受存款發行股份的資金購買實質資產, 由於銀行提供的購買條件得以篩選資產品質, 使銀行不僅可以提供存款與股票等消費者可以辨識的資產作爲支付工具, 更可能可以解決實質資產在交易中的資訊不對稱問題。當銀行買進市場上所有的好資產, 如此一來, 即使賣方無法分辨實質資產的品質, 他們知道, 在非集中交易市場中作爲支付工具的實質資產都是壞

資產, 換言之, 銀行的投資策略完全消除了資產作爲支付工具的資訊不對稱問題。在此情況下, 實質資產作爲支付工具的接受度不受限制。若資產品質差異縮小, 銀行可能只買一部分的好資產與壞資產, 使得非集中交易市場上出現兩種實質資產作爲交易媒介。由於實質資產仍具有私人訊息的問題, 擁有好資產的消費者會選擇先用銀行存款及股票換取消費財; 當這兩種可辨識的資產使用完畢時, 才會以部分的實質資產支應消費支出, 這是因爲買方以保留一部分實質資產作爲傳訊機制。如此, 實質資產作爲融通消費的角色便受到限制, 也就是所謂的「流動性限制」。我們發現, 當銀行購買所有好資產可增進總體流動性並使經濟體達到最高的福利水準。

關鍵字: 流動性限制; 支付工具; 私人訊息; 金融中介; 福利水準

Abstract

This dissertation studies liquidity and financial intermediaries in economies with private information regarding means of payment. We first consider an economy with an explicit dual role of banks in providing credit and payment services. Agents can produce fraudulent checks at a positive cost, and sellers are not able to verify the authenticity of payments. Banks punish agents passing on fraudulent checks by not granting loans. Dishonest agents thus need to hold enough cash to insure themselves against the random consumption opportunities. The moral hazard problem results in an endogenous upper bound on the quantity of deposits that can be traded for consumption goods. Higher inflation can relax the endogenous liquidity constraint through raising the self-finance cost that prevents fraudulent activity. As the quantity of deposits that can be traded for consumption goods is raised by inflation, the aggregate liquidity and output rise. Our model offers new insights for the relationship between bank's dual role, aggregate liquidity and allocations under moral hazard.

In Chapter 3, we consider an economy with a risky real asset which can be used as a means of payment in the decentralized market. The real asset may turn out to be good or bad, depending on their dividend processes. The quality of an asset is private information to the asset holder. By investing in real assets and conducting asset transformation, banks provide deposits and bank equity that are riskless and fully recognizable to serve as means of payment. In some equilibria, banks buy all of one type of real assets, which eliminates the private information problem regarding means of payment. In other equilibria, there are good assets and bad real assets in the decentralized market, and the payment arrangement displays a pecking order: buyers use real assets to make payments only if their deposits and bank equity holdings are depleted. The existence of banks is helpful to improve aggregate liquidity and welfare, even if banks are not able to discern the quality of real assets. Moreover, when equilibria coexist, the one in which banks buy all good assets achieves the highest welfare.

Keywords: Means of payment; Liquidity constraints; Private information; Recognizability; Welfare; Financial intermediaries

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Chapter 1

Introduction

This dissertation addresses issues related to assets' recognizability and liquidity. It is observed that some assets are more acceptable as payments or collateral than others. Previous studies, e.g., Kiyotaki and Moore (2005), Lester at el. (2009), Telyukova and Wright (2008) and Lagos (2010), have used liquidity differences to explain some macroeconomic phenomena, such as asset prices, aggregate liquidity and allocations. However, some have imposed exogenous liquidity differences by assuming only a fraction of assets can be used to finance consumption or investment (e.g., Kiyotaki and Moore (2005) and Logas (2010)). In this dissertation we adopt a different approach; that is, we explain the liquidity differences across assets by resorting to their characteristics, such as recognizability. Moreover, we will show how monetary policy affect asset yields, aggregate liquidity, and output through the endogenously determined liquidity differences. In Chapter 2, we consider the moral hazard problem regarding the means of payment provided by banks: agents can produce fake checks in exchange for goods. We derive an endogenous liquidity constraint on deposits from the ease of counterfeiting and the dual role of banks – as a provider of credit and payment instruments. In Chapter 3, the quality of risky real assets is private information to the holders. Banks conducting asset transformation may remove the quality concern for means of payment. Agents' payment arrangements are involved with signaling which generates a liquidity constraint on good real assets.

The threat of fraudulent private money has been widespread, from the clipping of coins in ancient Rome to identity thefts associated with intangible means of payment nowadays. In Chapter 2, there are two assets that may be used as means of payments: fiat money that is perfectly recognizable; while checking deposits suffer from the moral hazard problem. Agents can produce fraudulent checks (or conduct fraudulent payments related to bank deposits) at a positive and fixed cost. We consider an economy, in which agents choose portfolios of fiat money and bank deposits, and whether or not to produce fraudulent checks before trading opportunities realize. Banks detect fake checks and punish counterfeiters by not granting loans, which are used to finance uncertain consumption needs. Dishonest agents thus need to hold enough money holdings to insure themselves against the random consumption opportunities.

We show that the moral hazard problem results in an endogenous upper bound on the quantity of deposits that can be traded for consumption goods. The upper bound of deposits works as a liquidity constraint on deposits, which is related to search frictions, costs of holding money, the counterfeiting cost and inflation. Unlike previous literature, even in the extreme case where the counterfeiting cost approaches to zero, deposits may still be accepted as a means of payment.

The reason is that banks' punishment works as a discipline that prevents the opportunistic behavior. If the counterfeiting cost is sufficiently high, the liquidity constraint on deposits is not binding; i.e., deposits are as liquid as fiat money, and they do not pay interest. If the counterfeiting cost is so low that makes the constraint binding, deposits dominate fait money in the rate of return. Deposits pay interest to compensate holders for its lower liquidity.

While the low counterfeiting cost impairs deposits' liquidity, higher inflation improve aggregate liquidity and allocations by relaxing the liquidity constraint on deposits. Counterfeiters, who cannot borrow money from banks, have to hold money against the random consumption. Higher inflation increases the cost of holding fiat money, that induces agents less willing to produce fraudulent checks. Consequently, the liquidity constraint on deposits is relaxed and people are more willing to make deposits. This, in turn, results in more loanable funds and lower loan rate which helps to finance the random consumption, and improve allocations.

Historical episodes reveal that the recognizability problem impairs assets' ability to serve as means of payment or collateral. In other words, people could become reluctant to accept assets as payments or collateral, if they cannot discern the authenticities of the assets or the true value of the assets. In Chapter 3, we provide a theory to spell out the relationship between assets' liquidity and their recognizability. We introduce banks to facilitate trades by removing private information problems, and analyze welfare-improving roles for financial intermediaries.

We consider an economy with a risky real asset which can be used as a means of payment in the decentralized market. The quality of real assets characterized by dividend states is private information to holders. Good assets realize high dividends with certainty, whereas bad assets yield high dividends with a probability. Banks issue deposits and bank equity to invest in the real assets; i.e., they convert risky investment into safer and recognizable assets that may be used as means of payment. There are types of equilibria sorted by banks' investing strategies. If banks buy all of one type of real assets in the asset market, the economy is free from private information problems regarding the means of payment in the decentralized market. Otherwise, both types of real assets may be used in the decentralized market, and so the means of payment are subject to private information problem. Under this situation, trades in the decentralized market are involved with signaling. In economies where banks eliminate private information problems, all assets are equally suitable to serve as means of payment. Higher aggregate liquidity thus entails a higher level of outputs as well as welfare. Moreover, asset prices reflect assets' usefulness as means of payment. Prices of bad assets are higher than a threshold if banks buy all of bad assets, and turn real assets into liabilities that secure bad asset holders higher marginal benefit from trade. Banks propose the higher price as compensation for consumption the holders lose in the future. On the other hand, there is an upper bound on prices of good assets, when good assets may be held not spent for signaling. Deposits and bank equity have identical returns since they enjoy the same liquidity.

In equilibria where banks do not remove the private information, agents' payment arrangements in the decentralized market display a pecking order theory: recognizable assets are preferably used to make payment, and buyers want to retain a proportion of good assets as signaling devices, even if they consume at a sufficiently low. By retaining a fraction of the asset holdings, good asset holders separate themselves from bad asset holders. The payment arrangements cause a liquidity constraint on good assets. We find that if the private information problem is not removed, economies are stuck with a lower aggregate liquidity, since only a fraction of good assets serve as means of payment for the purpose of signaling. Among equilibria, the one in which banks buy all good assets and eliminate the private information problem entails the highest welfare. The pricing of deposits and equity are based on people's belief on the returns from the investment. When banks buy only good assets, the returns on bank investment is the highest and with certainty. Therefore, people would assign the highest value on banks' liabilities, compared to other equilibria.

Chapter 2

Financial Intermediaries and Payment Instruments under Moral Hazard

2.1 Introduction

Most of private money is threatened by the fraudulent activities. For example, promissory note circulated among merchants in Europe around the sixteenth century,[1] but the use of promissory note as payments was obstructed by asymmetric information. The informed party tended to pass on notes issued by risky debtors and keep safer ones. Individuals used banknotes to make payments during the nineteenth century, however, sometimes they cannot verify the true value of banknotes which were determined by the risk exposure of issuing banks. Even today, people still expose to payment fraud when they are short of information about the authenticity of payments or the financial condition of the business partners. Checks, one of the most widely used noncash means of payment in the U.S., is the typical case: checks may be counterfeited,[2] or bounced due to insufficient funds

[1] Promissory notes are IOUs. They were used as a means of payment before banknotes or bills of exchange were introduced as media of change.

[2] 2010 AFP Payments Fraud and Control Survey of the United States: among the most widely used techniques to commit payments fraud were counterfeit checks using the orga-

in the issuer's accounts.[3] The 2010 AFP Payments Fraud and Control Survey of the United States reveals that most of the payment fraud takes the form of fraudulent checks.[4]

How does the moral hazard problem associated with the use of checks affect the acceptability of checking deposits as means of payment? What are the implications of monetary policy on aggregate liquidity and allocations, if the payment fraud regarding the deposit-based instruments is explicitly considered? To answer these questions we introduce banks into Li and Rocheteau (2009), in which agents can produce fraudulent checks (or conduct fraudulent payments related to bank deposits) at a positive cost, and agents are not able to verify the authenticity of payments. Banks detect counterfeiting checks and punish counterfeiters by denying the future credit.[5] Banks have the technology in recording individuals'

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nization's MICR line data (72 percent); alteration of payee names on checks issued by the organization (58 percent); alteration of dollar amount on checks issued (35 percent).

[3]Some moral hazard problems are due to the time lag between the point of sale and the availability of funds. In the United Sates, checks deposited from institutions located in the same state will generally take up to two business days to clear. Deposits made into an account located in a different state will be held longer. For nearby states, this is three to six business days. Therefore, merchants may turn away the customer who presents out-of-state checks.

[4]About 90 percentage of survey respondents experienced attempted or actual payments fraud in 2009 were victims of check fraud.

[5]In practice, if a person mishandles a checking account and repeatedly bounces checks, he will be put on a blacklists created by companies such as ChexSystems. The ChexSystems, Inc. network is comprised of member Financial Institutions that regularly contribute information

financial activities, and they can enforce the repayment of debts with no cost. We derive endogenously an upper bound on the quantity of deposits that can be traded for consumption goods; i.e., there is a liquidity constraint on deposits. We show that monetary policies influence macroeconomic outcomes through the channel of liquidity constraint.

The economy features dual roles of banks - as a provider of payment instruments and credit. Agents who make deposits at a bank can write checks to make payments and earn deposit interests. Banks issue loans to those who need liquidity to finance unanticipated consumption. There are two payment instruments: fiat money, that is perfectly recognizable, and checking deposits, that suffer from the moral hazard problem such as bouncing a check or handing over a fraudulent check. The credit arrangement is not feasible between individuals, so fiat money and deposits are used to exchange for goods. We construct a three-subperiod model, in which agents choose portfolios and whether or not to produce fraudulent checks before unexpected consumption opportunities realize. Because banks do not grant loans to counterfeiters, agents who produce fraudulent checks thus need to hold enough money to insure themselves against the random consumption opportunities. This self-finance cost is affected by the cost of holding money and,

on mishandled checking and savings accounts to a central location. ChexSystems shares this information among member institutions to help them assess the risk of opening new accounts. http://articles.moneycentral.msn.com/. These blacklists can prevent people from getting another bank account for five years.

therefore, by the monetary policy.

The main insight of our analysis is that the moral hazard problem related to checks generates an endogenous liquidity constraint on deposits. The liquidity constraint depends on the counterfeiting cost, the self-financing cost due to banks' punishment on counterfeiters by denying credit, savings in loan interest payments, inflation rate and search frictions. Lower search frictions encourage fraudulent activities, since counterfeiters hand over fake checks more easily. Moreover, as the counterfeiting cost becomes lower, agent's incentives to produce fraudulent checks increases, which makes the liquidity constraint on deposits more likely to bind. Agents become less willing to accept checking deposits as means of payment. So the deposit interest rate has to rise to compensate the lower acceptability of deposits. Unlike previous literature, even in the extreme case where the counterfeiting cost approaches to zero, deposits may still be accepted as a means of payment.[6] The reason is that banks' punishment works as a discipline that prevents the opportunistic behavior.

Although no counterfeiting takes place in equilibrium, the possibility of counterfeiting affects equilibrium outcomes. If the liquidity constraint does not bind, deposits are perfect substitutes for fiat money, and they do not pay interest. The quantity of goods traded using deposits as payments is independent of the coun-

[6]For instance, in Lagos (2007), Lester, Postlewaite, and Wright (2008), and Li and Rocheteau (2010), as the cost of producing fraudulent claims goes to zero, agents stop trading the asset in uninformed matches.

terfeiting cost. But when the liquidity constraint binds, higher inflation relaxes the constraint through raising the self-finance cost that discourages fraudulent activities. As the quantity of deposits that can be traded for consumption goods is raised by inflation, the aggregate liquidity and output rise, because banks provide more loanable funds at a lower interest rate to finance the random consumption. Our model offers new insights for the relationship between bank's dual role, aggregate liquidity and allocations of an economy with moral hazard.

2.1.1 Literature review

Liquidity matters for consumption and investment. The information frictions or limited enforcements have been used to motivate liquidity constraints. Lenders in Kiyotaki and Moore (1997) are threatened by moral hazard considerations regarding borrowers running away without repaying the debts, so loans need to be secured by collateral. In Holmstrom and Tirole (1998) debts are backed by investments, an entrepreneur may choose the inefficient technology to receive a private benefit. The moral hazard related to the entrepreneur's choice generates a borrowing constraint, which induces the entrepreneur to be diligent. Kiyotaki and Moore (2008) consider the quality of an asset is private information and introduce exogenous constraints on the resaleability of assets. Based on Kiyotaki and Moore (2008), Tomura (2010) endogenizes the resaleability constraint as agents choose not to sell a fraction of their real assets in the secondary market. Following the moral hazard caused by the imperfect recognizability of assets in Li and Rocheteau (2009), we derive an endogenous liquidity constraint of checking deposits. The distinction of our model is that, the endogenous liquidity constraint depends on the counterfeiting cost as well as the self-financing cost due to banks' punishment on fraudulent activities.

Imperfect information or properties of assets cause liquidity considerations, and induce the need for private money to finance unexpected consumptions. Banks in Gorton and Pennacchi (1990) provide information-insensitive riskless debt circulating among uninformed agents who avoid trading risky assets with informed agents. Williamson (1999) specify two types of banks, which specialize in good and bad project respectively. Agents receive claims on banks through depositing outputs in a certain bank, and use banks' claims to make payments. In Li (2011), people who use checks for payment have to incur a positive cost, but those who use currency do not. Hence, checks are used only in big transactions whereas cash is used in all transactions. In our paper, we also consider banks' role in providing loans, and monetary policies can improve aggregate liquidity and allocation by mitigating the moral hazard regarding bank liabilities.

2.2 The environment

Time is discrete, starts at $t = 0$, and continues forever. Each period is divided into three subperiods. Subperiod 1 is a decentralized market ($DM_{1,t}$) with no double coincidence of wants; in subperiod 2 trades occur in a Walrasian market ($CM_{2,t}$). All agents can both consume and produce general goods in a centralized market ($CM_{3,t}$). In each subperiod there is a perishable consumption good produced. There are two types of infinitely-lived agents: *buyers* and *sellers* which symbolizes their roles in $DM_{1,t}$ and $CM_{2,t}$. Buyers want to consume but can not produce, while sellers produce but do not want to consume. The measures of buyers and sellers are equal to 1.

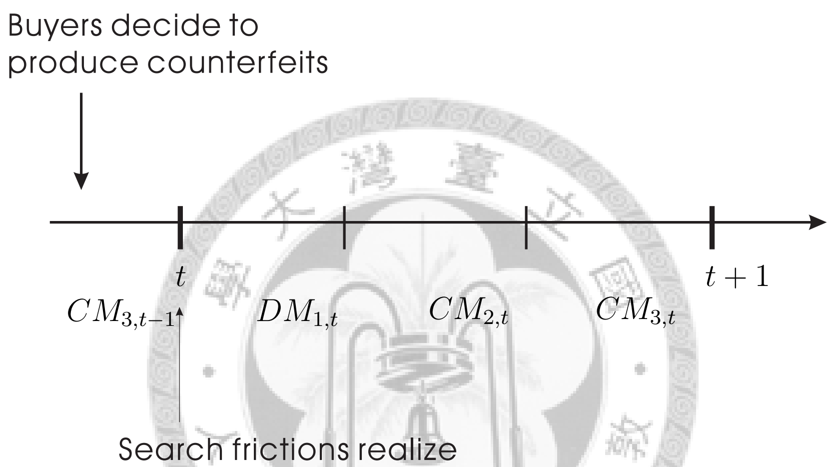

Figure 2.1. Time sequence

Trading frictions and the market structure of each subperiod. Before agents enter the $DM_{1,t}$ of period t, a trading shock realizes, and gives a fraction $\sigma \in (0, 1)$ of buyers and sellers a chance to trade in $DM_{1,t}$ and $CM_{2,t}$. The fraction σ of buyers are matched bilaterally and randomly with sellers in the $DM_{1,t}$. In each meeting, the buyer makes a take-it-or-leave-it offer, which contains the quantities of goods and the transfer of assets from the buyer to the seller. The seller decides to accepts the offer or not. Agents' portfolios are private information, so sellers' acceptance rule depends on the offer they receive. The fraction $(1-\sigma)$ of buyers and sellers do not have trading opportunities in $DM_{1,t}$ and $CM_{2,t}$.

Agents trading in the $CM_{2,t}$ are price takers. Buyers transfer assets to sellers for goods and get utility, sellers produce goods with disutility. If a buyer are not excluded from banks and have not enough assets to finance consumption opportunity, he can borrow money from banks. In the $CM_{3,t}$, all agents consume and produce general goods. Producing one unit of general goods needs one working hour and creates one unit of disutility. Since trading histories of agents are private information, and there is no commitment between agents, all trades are quid pro quo.

Banks and means of payments. Competitive banks take nominal deposits, issue loans, and provide payment services. In the $CM_{3,t}$, agents make deposits at nominal interest rate i_d, and repay loans at nominal interest rate i. Banks open before trades in the $CM_{2,t}$, that means, if agents have the needs for loans because of the unanticipated consumption opportunity, they borrow money from banks before they trade for goods. We confine our attention to the acceptability of deposit-based instruments under moral hazard, so for simplicity we assume banks have the ability to force borrowers to repay their debts; see Berentsen, Camera, and Waller (2007), and Li and Li (2010) for the discussions on credit constraints when default is possible.

Fiat money and checking deposits are two assets, which can be used as payments in this economy. Fiat money is supplied by government and perfectly recognizable. However, checking deposits suffer from the moral hazard problem such as bouncing a check or handing over a fraudulent check. Banks' payment service is defined by the payment system of checks. Buyers who deposit money in the $CM_{3,t}$ can write checks to execute trades in the $DM_{1,t+1}$. If sellers accept a check as payment in period of $t-1$, they present it to banks in the $CM_{2,t+1}$. After collecting all valid checks, banks clear funds between agents' accounts. The balance of the receiving seller's account is credited while that of the buyer who wrote a check is debited. Clearing checks takes time so those transferred funds realize in the $CM_{3,t+1}$. The banking system has a technology for record keeping on financial activities but not agents' trading histories.[7] This is for the essentiality of money and checks, otherwise banks can keep records that allow agents to settle payments with individuals' IOUs.

The counterfeiting technology and the financial punishment. Agents can produce fraudulent checks at a fixed cost, κ. The technology to produce counterfeits is available to buyers in the $CM_{3,t}$; but it would become obsolete in the next subperiod. Sellers can not recognize the authenticity of checks in

[7]We can image that there is a check clearing house, which collects checks from all banks and operates the check clearing process. After clearing all checks, the check clearing house will inform banks to record in books. Banks cannot identify the individual traders in the goods market according to this process.

the decentralized meeting. Banks confiscate fraudulent checks, and therefore counterfeits are valueless. Banks punish agents passing on fraudulent checks by not granting loans for one period, that leads the dishonest agent to bear the self-financing cost in order to buy goods in the $CM_{2,t+1}$.

Buyers enjoy utility $u_1(x_{1,t})$ and $u_2(x_{2,t})$ from consuming $x_{1,t}$ and $x_{2,t}$ in $DM_{1,t}$ and $CM_{2,t}$ respectively, and sellers suffer a disutility of producing, $c_1(x_{1,t})$ and $c_2(x_{2,t})$. All agents get utility $u_3(x_{3,t})$ from consuming $x_{3,t}$ and incur the disutility of working, where the disutility of working hours h^k is linear, $c_3(h^k) = h^k,\ k = s, b$. The lifetime expected utility of a buyer in period $t = 0$ is

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[u_1(x_{1,t}) + u_2(x_{2,t}) + u_3(x_{3,t}) - h^b\right], \tag{2.1}$$

where $u_j(0) = 0$, $u_j'(x_{j,t}) > 0$, and $u_j''(x_{j,t}) < 0$, $\beta \in (0,1)$ is a discount factor across periods.

The lifetime expected utility of a seller at $t = 0$ is

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[-c_1(x_{1,t}) - c_2(x_{2,t}) + u_3(x_{3,t}) - h^s\right], \tag{2.2}$$

The cost function $c_j(x_{j,t})$ is twice continuously differentiable, $c(0) = 0$, $c_j'(x_{j,t}) > 0$, and $c_j''(x_{j,t}) \geq 0$; Let x_j^* denote the solution to $u_j'(x_j^*) = c_j'(x_j^*), j = 1, 2$.

In the following part of this paper, to simplify notations, we drop the t, and write $DM_1 = DM_{1,t}$, $DM_{1,+1} = DM_{1,t+1}$; $x_1 = x_{1,t}$, $x_{1,+1} = x_{1,t+1}$, etc.

2.3 The counterfeiting game

Let ϕ be the real value of nominal assets at period t. Buyers in the CM_3 face a counterfeiting game similar to that described in Li and Rocheteau (2010). The game starts in the $CM_{3,-1}$ and ends in the CM_3. In the counterfeiting game buyers make offer first, and then decide to counterfeit or not.[8] Let (x_1, y_m, y_d) denote the offer made by the buyer, in which x_1 is the quantity of good traded, y_m and y_d represent the transfer of money and checking deposits, respectively. Let χ represent the buyer's strategy of counterfeiting, $\chi \in \{0, 1\}$. If $\chi = 1$, then the buyer produces a fraudulent check that is consistent with the value of y_d; $\chi = 0$ implies the buyer does not produce counterfeits. Let (m, d) be the portfolio of money and checking deposits that the buyer decides to hold. The game is solved by backward induction.

In this game, the sequences of the moves is as follows.

1. The buyer determines his DM_1 offer, (x_1, y_m, y_d) at the beginning of the game;

2. he chooses wether or not to counterfeit conditional on the offer (x_1, y_m, y_d);

[8] In the CM_3, the buyer writes down his offer and seals it before making any choice, and then he decide to counterfeit or not. This game has a solution to the game in which buyers make counterfeiting decision first, and then choose their genuine assets holdings, e.g. the original game described in Li and Rocheteau (2009).

3. he chooses the portfolio of genuine assets, (m, d), conditional on $((x_1, y_m, y_d), \chi)$;

4. the seller decides wether or not to accept the offer.

A behavioral strategy of the buyer in the game is a triple $\{\mathbb{F}, \eta(x_1, y_m, y_d), \mathbb{G}(x_1, y_m, y_d, \chi)\}$, where $\mathbb{F}$ is the distribution from which the buyer draws his offer, η is the probability that the buyer does not produce counterfeits conditional on the offer (x_1, y_m, y_d), $\mathbb{G}$ is the distribution for the choice of asset holding according to (x_1, y_m, y_d) and the decision of counterfeiting. We assume that the buyer's choice of his portfolios, (m, d), must be such that $m \geq y_m$ and $(1 + i_d)d \geq y_d$, if $\chi = 0$. While a buyer deposits d units of money in the bank, the interest on this account will be paid at redemption. So the feasible transfer of checking deposits is $(1 + i_d)d$. A pure strategy of the buyer in the counterfeiting game is a list $\{o, \chi(o), a(o, \chi)\}$ that specifies the choice of the offer, o, the decision to produce counterfeits conditional on the offer, χ, the holding of fiat money and checking deposits, a, as a function of the offer o and counterfeiting decisions χ. A pure strategy of sellers is the acceptance rule, μ. If $\mu = 1$, then the seller accepts the proposed offer; while if $\mu = 0$, the seller rejects it.

The Bernoulli payoff of the buyer in the counterfeiting game starting at the

CM_3 is

$$\begin{aligned}\Pi_t^b(m,d,x_1,y_m,y_d,x_2,\ell,\chi,\mu) &= -\kappa_c\mathbb{I}_{\{\chi=1\}} - \phi_{-1}(m+d) \\ &+ \beta\left\{\sigma[u_1(x_1) - \phi(y_m + y_d\mathbb{I}_{\{\chi=0\}})]\right\}\mathbb{I}_{\{\mu=1\}} \\ &+ \beta\sigma\left\{u_2(x_2) - \phi p x_2\right\} \\ &+ \beta\phi\{m + (1+i_d)d\} + \beta\sigma\phi i\ell\mathbb{I}_{\{\chi=0\}}, \qquad (2.3)\end{aligned}$$

where $\mathbb{I}_A$ is an indicator function equals to one if property A holds. If the buyer decides to counterfeit, i.e., $\chi = 1$, he incurs a cost $\kappa > 0$. The buyer has to produce $\phi_{-1}(m+d)$ units of the general goods in the $CM_{3,-1}$ to hold portfolio (m,d). In the DM_1 the buyer enjoys the utility $u_1(x_1)$ from consuming x_1, and transfers y_m units of money and y_d units of checking deposits to the seller. The buyer can borrow money from the bank to satisfy his consumption needs, x_2, at a nominal price p, and gets utility, $u_2(x_2)$. A buyer's portfolios of (m,d) are worth of $\phi[m + (1+i_d)d]$ units of general goods in the $CM_{3,+1}$. Let ℓ represent the quantities of loans, and therefore $\beta\sigma\phi i\ell$ is the present value of interest payment of a borrower.

The Bernoulli payoff of the seller is

$$\begin{aligned}\Pi_t^s(x_1,y_m,y_d,x_2,\chi,\mu) &= \beta\sigma\{[-c_1(x_1) + \phi(y_m + y_d\mathbb{I}_{\{\chi=0\}})]\mathbb{I}_{\{\mu=1\}} \\ &+ [-c_2(x_2) + \phi p x_2]\}. \qquad (2.4)\end{aligned}$$

Accepting the buyer's offer, $\mu = 1$, the seller incurs disutility $c_1(x_1)$ of producing x_1 and receives the transferred assets (y_m, y_d).

Lemma 2.1. *Assume that $\phi_{-1} \geq \beta\phi(1 + i_d)$. Given the offer, any optimal portfolio of the genuine buyer, $\chi = 0$, is (m, d) such that $m = y_m$ and $(1+i_d)d = y_d$.*

Under the assumption $\phi_{-1} \geq \beta\phi(1 + i_d)$ (i.e., when the rate of return of deposits is not larger than the discount rate) it is costly to hold either deposits or money. If a buyer deposits 1 dollar in the bank, he incurs costs ϕ_{-1} in terms of general goods in the $CM_{3,-1}$, and then redeems $(1 + i_d)$ dollars in the CM_3. The present value of his deposit is $\beta\phi(1+i_d)$ in terms of general goods. Obviously, to deposit more than y_d is not profitable. Further, $\phi_{-1} \geq \beta\phi(1+i_d)$ imply $\phi_{-1} \geq \beta\phi$: to hold money more than y_m is not profitable. Consequently, the buyer who does not counterfeit will choose the portfolio $(m, (1 + i_d)d) = (y_m, y_d)$.

To solve the game by backward induction, first, we take the offer (x_1, y_m, y_d) as given. According to these terms of trade, we look for a Nash equilibrium of the game where the buyer chooses to accumulate money and deposits or produce counterfeits to execute the offer he makes. The seller decides to accept the offer or not. Suppose a seller accepts the offer (x_1, y_m, y_d) with probability $\pi \in [0, 1]$ and a buyer makes deposit with probability $\eta \in [0, 1]$. Given η, the sellers' acceptance rule is described by

$$-c_1(x_1) + \phi(y_m + \eta y_d) \begin{matrix} > 0 \\ < 0 \\ = 0 \end{matrix} \implies \pi \begin{matrix} = 1 \\ = 0 \\ \in [0,1] \end{matrix} . \tag{2.5}$$

A seller's expected value of the transferred of asset is $\phi(y_m + \eta y_d)$, he accepts the offer with a positive probability, when $\phi(y_m + \eta y_d) \geq c_1(x_1)$.

To derive buyers' gains from trade, we specify sellers' belief regarding buyers' actions first. Given the offer (x_1, y_m, y_d), a seller constructs his belief regarding the buyer's action, χ. If the offer (x_1, y_m, y_d) satisfies the following belief system, it would be attributed to the genuine buyer and hence be accepted:

$$-\phi_{-1}(y_m + \tfrac{y_d}{1+i_d}) + \beta\sigma[u_1(x_1) - \phi(y_m + y_d)] + \beta\sigma[u_2(x_2) - \phi(px_2 + i\ell)] + \beta\phi(y_m + y_d)$$
$$> -k_c - \phi_{-1}(y_m + p\hat{x}_2) + \beta\sigma[u_1(x_1) - \phi y_m] + \beta\sigma[u_2(\hat{x}_2) - \phi p\hat{x}_2] + \beta\phi(y_m + p\hat{x}_2) \quad (2.6)$$

The left side of (2.6) is the expected payoff of a genuine buyer. To offer (x_1, y_m, y_d), the genuine buyer bears the cost of holding the portfolio $\phi_{-1}(y_m + \frac{y_d}{1+i_d})$. If trade occurs in following subperiods, his utility from consuming is $u_1(x_1)$ and the cost of transferring assets is $\phi(y_m + y_d)$. In the CM_2, his gains from trade is $u_2(x_2) - \phi(px_2 + i\ell)$. The terms of $(px_2 + i\ell)$ are the principal and the interest payment for the loan ℓ. Under conditions revealed by Lemma 2.1, honest buyers do not hold more money than y_m, even though they need to pay interests for borrowing money. If the genuine buyer has no chance to trade in the DM_1 and CM_2, he can sell (y_m, y_d) at the price ϕ. The right side of (2.6) is the expected payoff of a counterfeiter who makes the same offer (x_1, y_m, y_d). Producing fraudulent checks and having $y_m + p\hat{x}_2$ units of money cost the counterfeiter the counterfeiting cost, κ, and $\phi_1(y_m + p\hat{x}_2)$ units of general goods; the counterfeiter has no incentive to deposit such that $(\hat{m}, d) = (y_m + p\hat{x}_2, 0)$. That is, the counterfeiter must keep enough money for consumption needs in the CM_2. In the DM_1, he will enjoy utility, $u_1(x_1)$, and just transfer y_m units of money to his trading partner. The counterfeiter can sell his money holding, $y_m + p\hat{x}_2$, at the price ϕ.

Given π, the decision rule to produce counterfeits is

$$[\frac{\gamma - \beta(1+i)}{1+i_d} + \beta\sigma\pi]\phi y_d \begin{array}{c} < \\ > \\ = \end{array} k_c + B \Longrightarrow \eta \begin{array}{l} = 1 \\ = 0 \\ \in [0,1] \end{array} , \tag{2.7}$$

where $B \equiv (\gamma - \beta)\phi p\hat{x}_2 + \beta\sigma\{[u_2(x_2) - \phi p x_2] - [\hat{u}_2(\hat{x}_2) - \phi p\hat{x}_2]\} - \beta\sigma\phi i\ell$ and $\gamma \equiv \frac{\phi_{-1}}{\phi}$. The right side of equation (2.7) reveal terms caused by producing fraudulent checks: the counterfeiting cost, κ, and the financial punishment, B, that consists of the self-finance costs and the savings in loan interest payments, $\beta\sigma\phi i\ell$. The self-finance costs include two terms. Lemma 2.1 shows that holding more money than y_m is not profitable while $\gamma > \beta$, so the counterfeiter incurs the inflation cost $(\gamma - \beta)\phi p\hat{x}_2$. Second, the difference in consumptions between an genuine buyer and a counterfeiter, $\beta\sigma\{[u_2(x_2) - \phi p x_2] - [u_2(\hat{x}_2) - \phi p\hat{x}_2]\}$. Banks punish dishonest buyers by not granting loans, dishonest buyer thus face a CIA constraint in the CM_2. That is, the punishment binds his consumptions, $\hat{x}_2$, in his money holding when entering CM_2, whereas the genuine buyer's consumptions, x_2, are not bounded by any money holding or borrowing constraint. The left side of equation (2.7) are gains from counterfeiting. If a buyer does not make deposits, he avoids inflation costs of having checking deposits but cannot redeem deposits at the interest rate i_d. Therefore, the net gain from not depositing is $\frac{\gamma-\beta(1+i)}{1+i_d}\phi y_d$ in terms of general goods. The other gain is the saving in checking deposits, which should be exchanged for consumption goods, $\beta\sigma\phi y_d$, if the seller accepts (x_1, y_m, y_d) in the DM_1.

There are Nash equilibria where the buyer makes an offer such that $(\pi, \eta) \in (0,1)^2$, e.g., the offer is accepted potentially and the counterfeits may exist. From (2.5) and (2.7),

$$\begin{aligned} \eta &= \frac{c_1(x_1) - \phi y_m}{\phi y_d}, & (2.8) \\ \pi &= \frac{(k_c + B_c) - \left[\frac{\phi_{-1}}{1+i_d} - \beta\phi\right] y_d}{\beta\sigma\phi y_d}. & (2.9) \end{aligned}$$

The condition $\eta \in (0,1)$ implies $\phi_t y_d > c_1(x_1) - \phi_t y_m$. The condition $\pi \in (0,1)$ implies $\phi y_d \in \left(\frac{\phi(\kappa+B)}{\frac{\phi_{-1}}{1+i_d} - \beta\phi + \beta\sigma\phi}, \frac{\phi(\kappa+B)}{\frac{\phi_{-1}}{1+i_d} - \beta\phi} \right)$. According to (2.9), if the informed buyer attempts to maximize his gains from trade by any opportunistic behavior, such as trading large quantities, the seller would decrease the probability to accept. Therefore, the buyer is aware that it is not optimal to make an offer that may be rejected with a positive probability.

The offer made by the buyer at the beginning of the game solves

$$\begin{aligned} (x_1, y_m, y_d) \quad &\in \quad \arg\max \{ -(\kappa + B) \left[1 - \eta(x_1, y_m, y_d)\right] - \phi_{-1}[y_m + y_d \eta(x_1, y_m, y_d)] \\ &+ \quad \beta\phi[y_m + (1 + i_d) y_d \eta(x_1, y_m, y_d)] \\ &+ \quad \sigma\beta \left[u_1(x_1) - \phi(y_m + y_d \eta(x_1, y_m, y_d))\right] \pi(x_1, y_m, y_d) \}. \qquad (2.10) \end{aligned}$$

Following an offer (x_1, y_m, y_d) and a distribution of offers, $\mathbb{F}$, an equilibrium of the counterfeiting game is a list of $[\eta(x_1, y_m, y_d), \pi(x_1, y_m, y_d)]$, that satisfy (2.5), (2.7) and (2.10).

Proposition 2.1. ***(Endogenous liquidity constraints)***

The equilibrium offer solution to (2.10) is such that $\pi = 1$ and $\eta = 1$, and it satisfies

$$\max_{(x_1, y_m, y_d)} -(\frac{\gamma - \beta}{\beta})\phi y_m - (\frac{\frac{\gamma}{1+i_d} - \beta}{\beta})\phi y_d + \sigma\left[u(x_1) - \phi(y_m + y_d)\right] \tag{2.11}$$

$$s.t. \quad -c(x_1) + \phi(y_m + y_d) = 0, \tag{2.12}$$

$$\phi y_d \leq \frac{\kappa + B}{\frac{\gamma}{1+i_d} - \beta + \beta\sigma}. \tag{2.13}$$

Proposition 2.1 describes the buyer's optimization problem in the DM_1; the buyer chooses an offer to maximize his expected payoff in the DM_1 revealed by the objective function (2.11), and subject to condition (2.12), that illustrates the seller's participation condition, and condition (2.13), that is the endogenous upper bound on the transfer of checking deposits. Holding money and checking deposits costs the buyer $\frac{\gamma-\beta}{\beta}$ and $\frac{\frac{\gamma}{1+i_d}-\beta}{\beta}$, respectively. The buyer obtains gains from trade $[u(x_1) - \phi(y_m + y_d)]$ with probability σ. The seller has no gains from trade due to the buyer makes a take-it-or-leave-it offer such that condition (2.12) holds in equilibrium. From inequality (2.7) an equilibrium where buyers do not produce fraudulent checks, i.e., $\eta = 1$, requires a constraint of checking deposits, i.e., condition (2.13). The endogenous liquidity constraint depends on the counterfeiting cost, κ, the self-finance cost and the savings in loan interest payments, included in the present term B, inflation rate, γ, and search frictions, σ. The liquidity constraint shows that roles of financial intermediaries and policies also have impact on the acceptability of deposits as payments, besides the technology in producing fake checks and the ease of passing on that. The counterfeiting cost and the financial punishment from banks, B, are the key to derive constraint (2.13). Higher search frictions, i.e., σ is smaller, like the higher counterfeiting weakens buyers' incentive to produce counterfeits, that lifts the upper bound on the transfer of deposits. Inflation policies could induce agents to deposit by relaxing the liquidity constraint on checking deposits.

2.4 Equilibrium

We study stationary equilibria incorporating the counterfeiting game into the general equilibrium framework, in which the real value of asset holding is constant. In particular, $\phi_{-1}M_{-1} = \phi M$, which implies $\frac{\phi_{-1}}{\phi} = \gamma$; the inflation rate equals the growth rate of money. Let $W_j^k(m, d, \ell),\ k = s, b$ denote agents' value function in subperiod j of period t.

subperiod 1

There is a decentralized market where a fraction σ of buyers and sellers have trading opportunities. The buyer with money holdings, m, and checking deposits,

d, has the following expected lifetime utility for the offer (x_1, y_m, y_d).

$$\begin{aligned} W_1^b(m,d) &= \sigma\{u_1(x_1) + W_2^b[m - y_m, (1+i_d)d - y_d]\} \\ &+ (1-\sigma)W_3^b(m,d), \end{aligned} \tag{2.14}$$

The buyer trading with the seller enjoys utility $u_1(x_1)$ and brings his asset $m - y_m$ and $(1+i_d)d - y_d$ to the second subperiod. The non-trader neither consumes nor produces in the first two subperiods, and he can sell his assets, m and d in the last subperiod. The envelope conditions are:

$$W_{1,m}^b(m,d) = \sigma W_{2,m}^b[m - y_m, (1+i_d)d - y_d] + (1-\sigma)W_{3,m}^b(m,d), \tag{2.15}$$

$$W_{1,d}^b(m,d) = \sigma W_{2,d}^b[m - y_m, (1+i_d)d - y_d] + (1-\sigma)W_{3,d}^b(m,d). \tag{2.16}$$

subperiod 2

Let x_2^b and x_2^s denote the quantities consumed by a buyer and produced by a seller in the CM_2, respectively. p is the price of consumption goods. Here, trades occur in a Walrasian market, so all agents are price takers.

The seller's problem is

$$\max_{x_2^s} \quad -c_2(x_2^s) + W_3^s(y_m + px_2^s) \tag{2.17}$$

The first order condition is

$$-c_2'(x_2^s) + pW_{3,m}^s = 0. \tag{2.18}$$

As the buyer departs from the DM_1, he owns $m - y_m$ units of money and $(1+i_d)d - y_d$ units of checking deposits. In this subperiod, the buyer consumes and can borrow money, ℓ, from the bank to supplement his needs of consumption. The buyer's problem is

$$\max_{x_2^b,\ell} \quad u_2(x_2^b) + W_3^b[m - y_m + (1+i_d)d - y_d + \ell - px_2^b, \ell] \tag{2.19}$$

$$\text{s.t.} \quad px_2^b \leq m - y_m + (1+i_d)d - y_d + \ell. \tag{2.20}$$

The buyer face a budget constraint expressed in (2.20) such that his spending is no more than his money holdings plus borrowing. The first order conditions are

$$u_2'(x_2^b) - pW_{3,m}^b - \lambda p = 0, \tag{2.21}$$

$$W_m^b + W_{3,\ell}^b + \lambda = 0, \tag{2.22}$$

where λ is the multipliers on the buyer's budget constraint.

subperiod 3

From (2.19), the buyer's money holdings is $m - y_m + (1+i_d)d - y_d + \ell - px_2^b$ and debt is ℓ as he enters the CM_3. The buyer engages in various activities such as working, consuming, repaying loans and adjusting his portfolio, (m_{+1}, d_{+1}), for period $t+1$. Let h^b be working hours of the buyer and x_3 be the quantities of general goods. Let m_3 represent the buyer's money holding upon entering the CM_3, i.e., $m_3 \equiv m - y_m + (1+i_d)d - y_d + \ell - px_2^b$. The buyer's problem:

$$\max_{x_3,h_3,m_{+1},d_{+1}} u_3(x_3) - h^b + \beta W_{+1}^b(m_{+1}, d_{+1})$$

$$\text{s.t.} \quad x_3 = h^b + \phi[m_3 - (1+i)\ell] - \phi(m_{+1} + d_{+1}) + T.$$

Money holdings are worth ϕ units of general goods. If the buyer borrows ℓ units of money in the last subperiod, then he repays $\phi(1+i)\ell$ in terms of general goods.

Substituting h^b from the budget constraint into the objective function, we have

$$
\begin{aligned}
W_3^b(m_3,\ell) &= \max_{x_3,m_{+1},d_{+1}} \{u_3(x_3) - x_3 - \phi(m_{+1},d_{+1}) + \beta W_{+1}^b(m_{+1},d_{+1})\} \\
&+ \phi[m_3 - (1+i)\ell] + T, \qquad (2.23)
\end{aligned}
$$

The first order conditions are

$$u_3'(x_3) = 1 \qquad (2.24)$$

$$\beta W_{+1,m}^b(m_{+1},d_{+1}) = \phi, \qquad (2.25)$$

$$\beta W_{+1,d}^b(m_{+1},d_{+1}) = \phi. \qquad (2.26)$$

The envelope conditions are

$$W_{3,m}^b(m_3,\ell) = \phi, \qquad (2.27)$$

$$W_{3,\ell}^b(m_3,\ell) = -\phi(1+i). \qquad (2.28)$$

The seller has $y_m + px_2$ units of money and y_d in terms of checking deposits transferred from the buyer in this subperiod. So the expected utility of the seller with asset $(y_m + px_2, y_d)$ is

$$
\begin{aligned}
W_3^s(y_m + px_2, y_d) &= \max_{x_3,h^s} \; u_3(x_3) - h^s + \beta W_{+1}^s(0,0) \\
\text{s.t. } x_3 &= h^s + \phi[(y_m + px_2) + y_d] + T,
\end{aligned}
$$

where h^s is working hours of the seller and i_d is nominal deposit rate. Note that sellers bring neither money nor checking deposits across periods, so we have

$W^s_{+1}(0,0)$. According to Lemma 1, $y_d = (1+i_d)d$. The envelope conditions are

$$W^s_{3,m}(y_m + px_2, y_d) \;=\; \phi, \tag{2.29}$$

$$W^s_{3,d}(y_m + px_2, y_d) \;=\; \phi(1+i_d). \tag{2.30}$$

Substituting (2.29) into (2.18), we obtain

$$p = \frac{c'(x^s_2)}{\phi}. \tag{2.31}$$

Using equation (2.27), (2.28) and (2.31), (2.21) can be rewritten as

$$\frac{u'_2(x^b_2)}{c'_2(x^s_2)} \;=\; (1+i), \tag{2.32}$$

which implies that buyers borrow up to the point where the marginal benefit of borrowing an additional unit of money, $\frac{u'_2(x^b_2)}{c'_2(x^s_2)}$, equals the marginal cost of borrowing, $1+i$.

market clearing conditions

In equilibrium, the representative buyer deposits d units of money in the bank but he borrows ℓ units of money from the bank with probability σ. That means loan supply is d and loan demand is $\sigma\ell$ in the loan market. Hence, the loan market clearing condition is

$$\sigma\ell = d. \tag{2.33}$$

The zero-profit condition for competitive banks is

$$i = i_d. \tag{2.34}$$

The good market clearing condition in subperiod 2 is

$$x_2^b = x_2^s. \tag{2.35}$$

The money market clearing condition is

$$M_{-1} = m + d, \tag{2.36}$$

where M_{-1} is money supply in period $t-1$, m and d are the aggregate demand for fiat money and checking deposits, respectively.

Definition 2.1. *A stationary equilibrium is a list of individuals' choices* (x_2, x_3, m, d, ℓ), *terms of trade* (x_1, y_m, y_d), *the price* p, *the real value of money* ϕ *and nominal rates* i *and* i_d *that satisfy (2.32), (2.24), (2.15), (2.16), (2.28), (2.11)-(2.13) and market clearing conditions (2.33)-(2.36).*

There are potential equilibria characterized by whether or not the endogenous liquidity constraint binds. In equilibrium, the deposit interest rate should respond to the liquidity of checking deposits.

Proposition 2.2. ***(Nominal interest rate and the value of money)*** *Suppose* $\frac{u_1'(0)}{c_1'(0)} > \frac{\gamma-\beta+\beta\sigma}{\beta\sigma}$. *There exists a monetary equilibrium where the output traded in the* DM_1 *solves*

$$\frac{u_1'(x_1)}{c_1'(x_1)} = \frac{\gamma - \beta + \beta\sigma}{\beta\sigma}. \tag{2.37}$$

(i) If $\kappa \geq \frac{dc_1(x_1)[\gamma-\beta(1-\sigma)]}{M_{-1}} - B^{'}$,

$$i_d = 0, \tag{2.38}$$

$$\phi = \frac{c_1(x_1)}{M_{-1}}, \tag{2.39}$$

where $B^{'} \equiv (\gamma-\beta)\phi p \hat{x}_2 + \beta\sigma\left\{[u_2(x_2) - \phi p x_2] - [\hat{u}_2(\hat{x}_2) - \phi p \hat{x}_2]\right\}$.

(ii) If $\kappa \leq \frac{dc_1(x_1)\sigma(\gamma-\beta)}{M_{-1}} - B$,

$$i_d = \frac{\gamma-\beta}{\beta}, \tag{2.40}$$

$$\phi = \frac{c_1(x_1) - \frac{\kappa+B}{\beta\sigma}}{m}. \tag{2.41}$$

(iii) If $\frac{dc_1(x_1)\sigma(\gamma-\beta)}{M_{-1}} - B < \kappa < \frac{dc_1(x_1)[\gamma-\beta(1-\sigma)]}{M_{-1}} - B^{'}$,

$$i_d = \frac{\kappa + B^{'}}{\beta\sigma[c_1(x_1) + \sigma(u_2^{'} - c_2^{'})x_2]}\frac{M_{-1}}{d} - \frac{\gamma-\beta(1-\sigma)}{\beta\sigma}, \tag{2.42}$$

$$\phi = \frac{c_1(x_1) - (\frac{u_2^{'}}{c_2^{'}} - 1)d}{M_{-1}}. \tag{2.43}$$

Figure 2.2 illustrates the interest rate falls with the counterfeiting cost κ, and hits the lowest level, $i = 0$, while κ goes beyond the threshold $\frac{dc_1(x_1)[\gamma-\beta(1-\sigma)]}{M_{-1}} - B^{'}$, revealed by Proposition 2.2 (i). In such a case checking deposits are perfect substitutes for fiat money and do not pay interest; the value of money ϕ is irrelevant to the counterfeiting cost. Proposition 2.2 (ii) shows that very low counterfeiting cost exacerbates moral hazard problem regarding checking deposits, deposits pay interest to compensate agents for the time preference rate and inflations, $i = \frac{\gamma-\beta}{\beta}$, which is the upper bound of interest rate. Deposits have to pay the highest level if producing fraudulent checks costs lower than the threshold $\frac{dc_1(x_1)[(\gamma-\beta)\sigma]}{M_{-1}} - B$. The value of money falls with the counterfeiting cost; a higher counterfeiting cost implies deposits are more widely accepted, that decreases the contribution of money to trade, see (2.41) and figure 2.2. In general, checks are threatened by fraudulent activities, deposits pay a positive interest rate to compensate relatively lower liquidity. A higher counterfeiting cost secures an increasing acceptability of deposits through the liquidity constraint, the positive interest rate hence decreases.

Proposition 2.3. ***(Allocations)***

The Friedman rule achieves the first best if and only if $y_d \leq (1+i)d$ and

$$\kappa + B' \geq [c_1(x_1^*) - \phi m]\beta\sigma. \tag{2.44}$$

Friedman rule achieves the first best allocation if and only if the counterfeiting cost is large enough to hold the liquidity constraint unbound and buyers deposit more than they transfer in the DM_1. Since the liquidity constraint of deposits does not bind, money and deposits are perfect substitutes, deposits hence do not pay interest. So that banks are not essential under this case.

Proposition 2.4. *When liquidity constraint binds, $\frac{\partial i_d}{\partial \gamma} < 0$, $\frac{\partial x_2}{\partial \gamma} > 0$, $\frac{\partial \hat{x}_2}{\partial \gamma} < 0$, $\frac{\partial B}{\partial \gamma} > 0$, $\frac{\partial \phi y_d}{\partial \gamma} > 0$, if $|\frac{\partial i_d}{\partial x_2}| > |\frac{u_2''}{c_2'} - \frac{u_2'}{(c_2')^2}c_2''|$ and $\sigma < \bar{\sigma}(\kappa, \gamma, \beta)$.*

The proposition reveals how dual roles of banks and policies affect asset yields, liquidity and outputs through the liquidity constraint. The higher inflation re-

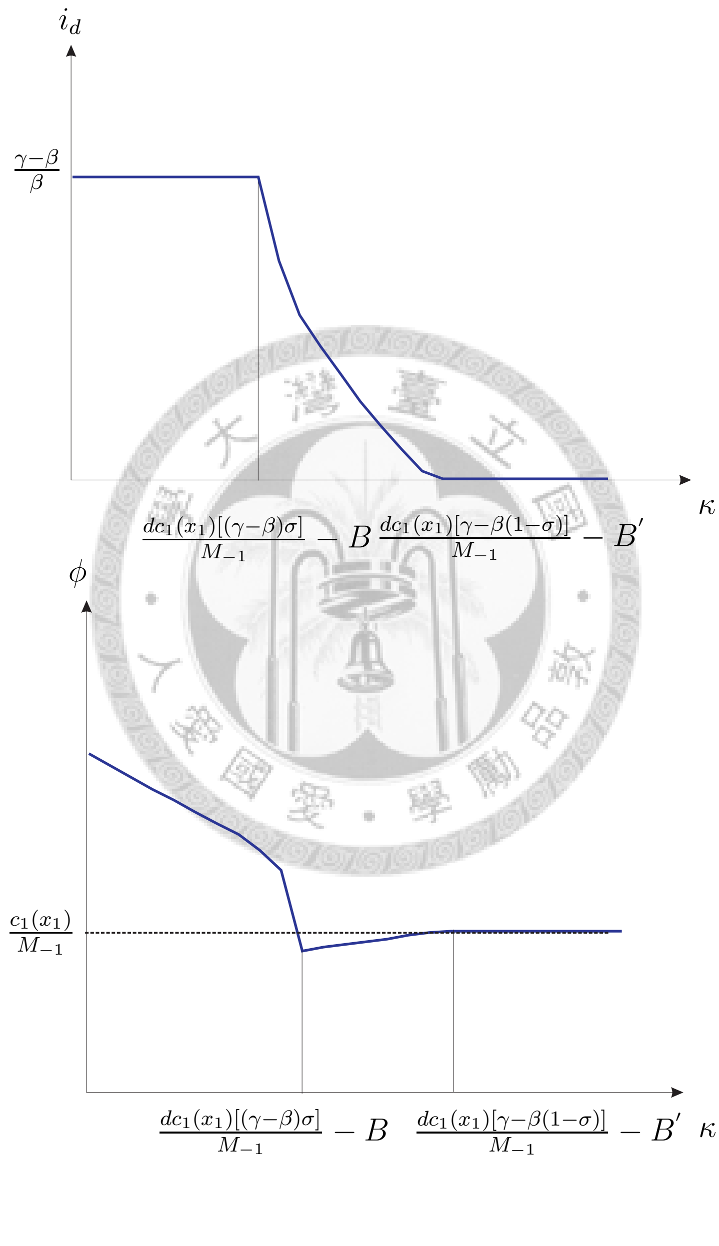

Figure 2.2. Interest rate and the value of fiat money

laxes the liquidity constraint by raising the self-finance cost, $\frac{\partial B}{\partial \gamma} > 0$, and therefore an agent enjoys less consumption if he counterfeits ($\frac{\partial \hat{x}_2}{\partial \gamma} < 0$). Agents would rather make deposits than produce fraudulent checks. The relaxed liquidity constraint raises the quantity of deposits that can be traded for consumption goods, i.e., $\frac{\partial \phi y_d}{\partial \gamma} > 0$. Banks thus provide more loanable funds at a lower interest rate, $\frac{\partial i_d}{\partial \gamma} < 0$. Such that a higher output level in the CM_2 is sustained by the lower borrowing costs, $\frac{\partial x_2}{\partial \gamma} > 0$.

2.5 Conclusion

We have shown that the positive counterfeiting cost and bank's punishment on fraudulent activity generate the endogenous liquidity constraint, which depends on the counterfeiting cost, self-finance costs, savings in loan interest payments, inflation rate and search frictions. Unlike previous studies where costless counterfeiting may prevent assets from being used in exchange, we find here that even if producing fraudulent checks is costless, the financial punishment guarantees the liquidity of checking deposits. In the United States, banks often punish the issuers of bounced checks by charging some fees such as insufficient funds fees. Some banks provide the overdraft check with bounce protection programs or payday loans, which charge mishandled checking accounts extra fees. If a check user bounces checks or mishandle his account several times, he may be prevented from getting another bank account for five years. That leads the user to obtain funds from non-mainstream financial channels, which are costly. The self-finance cost derived in our model captures this observation.

From considering the dual role of banks and potential payment frauds, we derive some policy implications. We find higher inflation can relax the endogenous liquidity constraint through raising the self-finance cost that prevents fraudulent activity. When deposits become more acceptable, agents will be more willing to make deposits in order to enjoy the means of payment services. This, in turn, results in more loanable funds to meet agents' need for liquidity to finance consumption. Monetary policy thus affects the aggregate output through the channel of endogenous liquidity constraints.

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2.5.1 Appendix A.

Proof of lemma 1 If the buyer offers (x_1, y_m, y_d) and decides to produce counterfeits, $\chi = 1$. Then his expected payoff if he chooses the portfolios (m_1, d_1) is

$$\begin{aligned}\Pi_t^b(m_1, d_1, x_1, y_m, y_d, x_2, \ell, \chi, \mu) &= -\kappa - \phi_{-1}(m_1 + d_1) \\ &+ \beta\left\{\sigma[u_1(x_1) - \phi(y_m + y_d)]\right\} \\ &+ \beta\sigma\left\{u_2(x_2) - \phi p x_2\right\} \\ &+ \beta\phi\{m_1 + (1 + i_d)d_1\}.\end{aligned}$$

Since $\phi_{-1} > \beta\phi(1 + i_d)$, the optimal portfolio is $m_1 = y_m, d_1 = 0$, that is, $\mathbb{G}((x_1, y_m, y_d), 1) = \delta_{\{y_m, o\}}$. If the buyer does not produce counterfeits and chooses the portfolio (m_1, d_1), his expected payoff is

$$\begin{aligned}\Pi_t^b(m_1, d_1, x_1, y_m, y_d, x_2, \ell, \chi, \mu) &= -\phi_{-1}(m_1 + d_1) \\ &+ \beta\left\{\sigma[u_1(x_1) - \phi(y_m + y_d)]\right\} \\ &+ \beta\sigma\left\{u_2(x_2) - \phi p x_2\right\} \\ &+ \beta\phi\{m_1 + (1 + i_d)d_1\} \\ &+ \beta\sigma\phi i\ell,\end{aligned}$$

Since $\phi_{-1} > \beta\phi(1+i_d)$, the optimal portfolio is $m_1 = y_m, d_1 = y_d$, i.e., $\mathbb{G}((x_1, y_m, y_d), 0) = \delta_{\{y_m, y_d\}}$.

Proof of proposition 2.1

Let U^* denote the maximum of (2.10) when $(\pi, \eta) = (1, 1)$. The offer made by the buyer at the beginning of the game solves the following problem

$$U^* = \max_{(x_1, y_m, y_d)} -\phi_{-1}(y_m + y_d) + \beta\phi[y_m + (1 + i_d)y_d] + \beta\sigma[u_1(x_1) - \phi(y_m + y_d)]$$

$$\text{s.t.} \quad c(x_1) - \phi y_m \le \phi y_d \le \frac{\phi(\kappa + B_c)}{\frac{\phi_{-1}}{1+i_d} - \beta\phi + \beta\phi\sigma}.$$

Now, we want to show that any other offer such that $(\pi, \eta) \neq (1, 1)$ generates a payoff that is less than U^*.

Suppose that the buyer makes an offer such that $(\pi, \eta) = (0, 1)^2$. From (2.5) and (2.7),

$$\eta = \frac{c_1(x_1) - \phi y_m}{\phi y_d}, \tag{2.45}$$

$$\pi = \frac{(\kappa + B_c) - (\frac{\phi_{-1}}{1+i_d} - \beta\phi)y_d}{\beta\sigma\phi y_d}. \tag{2.46}$$

The buyer's payoff is $-(\kappa+B_c)-(\phi_{-1}-\beta\phi)y_m+\beta\sigma[u_1(x_1)-\phi y_m]\frac{(\kappa+B_c)-(\frac{\phi_{-1}}{1+i_d}-\beta\phi)y_d}{\beta\sigma\phi y_d}$. Consider the offer such that $(\pi, \eta) = \{1\} \times (0, 1)$. Given (y_m, y_d), the buyer's payoff rises as x_1 increases; the solution corresponds to $c_1(x_1) = \phi(y_m + y_d)$ and $\eta = 1$; U^* is not achieved. Insert $\phi y_d = c_1(x_1) - \phi y_m$ in to the buyer's payoff:

$$-(\kappa + B_c) - (\phi_{-1} - \beta\phi)y_m + \beta\sigma[u_1(x_1) - \phi y_m]\frac{(\kappa + B_c) - (\frac{\phi_{-1}}{1+i_d} - \beta\phi)\frac{c_1(x_1)-\phi y_m}{\phi}}{\beta\sigma\frac{c_1(x_1)-\phi y_m}{\phi}}$$

rises when x_1 decreases. The condition $\pi \in (0, 1)$ implies $\phi y_d \in \left(\frac{\phi(k_c+B_c)}{\frac{\phi_{-1}}{1+i_d}-\beta\phi+\beta\sigma\phi}, \frac{\phi(k_c+B_c)}{\frac{\phi_{-1}}{1+i_d}-\beta\phi}\right)$. The solution corresponds to $c_1(x_1) = \phi y_m + \frac{\phi(k_c+B_c)}{\frac{\phi_{-1}}{1+i_d}-\beta\phi+\beta\sigma\phi}$ and $\pi = 1$; U^* is not achieved.

The case of $(\pi, \eta) = (0, 1) \times \{1\}$ has the similar proof.

2.5.2 Appendix B. (Proof of proposition 2.2)

$$\max_{(x_1,y_m,y_d,d)} -(\frac{\gamma-\beta}{\beta})\phi y_m - (\frac{\frac{\gamma}{1+i_d}-\beta}{\beta})\phi d + \sigma\left[u(x_1) - \phi(y_m+y_d)\right]$$

$$\begin{aligned} \text{s.t.} \quad & -c(x_1) + \phi(y_m+y_d) = 0, \\ & \phi y_d \leq \frac{\kappa + B}{\frac{\gamma}{1+i_d} - \beta + \beta\sigma}, \\ & y_d \leq (1+i_d)d. \end{aligned} \tag{2.47}$$

$$\begin{aligned} \mathcal{L} = & -\left(\frac{\gamma-\beta}{\beta}\right)\phi y_m - \left(\frac{\frac{\gamma}{1+i_d}-\beta}{\beta}\right)\phi d + \sigma\left\{u \circ c^{-1}\left[\phi(y_m+y_d)\right] - \phi\left(y_m+y_d\right)\right\} \\ & +\lambda_1\left(\frac{\kappa+B}{\frac{\gamma}{1+i_d}-\beta+\beta\sigma} - \phi y_d\right) + \lambda_2\phi[(1+i_d)d - y_d] \end{aligned}$$

where the Lagrange multiplier λ_1 is associated with the liquidity constraint and the Lagrange multiplier λ_2 with the feasibility constraint on the transfer of deposits.

1. w.r.t y_m:

$$-\frac{\gamma-\beta}{\beta} + \sigma\left(\frac{u'(x_1)}{c'(x_1)} - 1\right) = 0. \tag{2.48}$$

w.r.t y_d:

$$\sigma\left(\frac{u'(x_1)}{c'(x_1)} - 1\right) - \lambda_1 - \lambda_2 = 0. \tag{2.49}$$

w.r.t d:

$$-\frac{\frac{\gamma}{1+i_d}-\beta}{\beta} + \lambda_2(1+i_d) = 0. \tag{2.50}$$

From (2.49) and (2.50),

$$\lambda_1 = \frac{\frac{i_d}{1+i_d}\gamma + (\gamma - \beta)i_d}{(1+i_d)\beta}. \tag{2.51}$$

We consider the following three cases:

1. The constraint (2.13) is not binding ($\lambda_1 = 0$).

   From (2.51), $i_d = 0$. From (2.50), $\lambda_2 = \frac{\gamma-\beta}{\beta} > 0$ and hence $(1+i_d)d = y_d$. From (2.12), $\phi(m+d) = c(q)$ and hence (2.39). The constraint (2.13) is not binding if

   $$\frac{d}{M_{-1}} \leq \frac{\kappa + B}{c_1(x_1)\left[\gamma - \beta(1-\sigma)\right]}.$$

2. The constraint (2.13) binds ($\lambda_1 > 0$, i.e., $\phi y_d = \frac{\kappa+B}{\frac{\gamma}{1+i_d}-\beta+\beta\sigma}$) and the constraint (2.47) does not bind ($\lambda_2 = 0$, i.e., $(1+i_d)d \geq y_d$).

   From (2.50) and (2.51), $i_d = \frac{\gamma-\beta}{\beta}$ hence (2.40); From (2.12), $\phi(m+y_d) = c_1(x_1)$ which gives (2.41). The condition $(1+i_d)d \geq y_d$ implies $\frac{\kappa+B}{\sigma(\gamma-\beta)} \leq \frac{d}{M_{-1}}c_1(x_1)$.

3. $\phi y_d = \frac{\kappa+\Delta_c}{\frac{\gamma}{1+i_d}-\beta+\beta\sigma}$ and $\lambda_2 > 0$, i.e., $(1+i_d) = y_d$. The two binding constraints give (2.42) and (2.43). From (2.50) and (2.51), since $\lambda_1 > 0$ and $\lambda_2 > 0$, $0 < i_d < \frac{\gamma}{\gamma-\beta}$ which implies $\frac{\kappa+\Delta_c}{c_1(x_1)[\gamma-\beta(1-\sigma)]} < \frac{d}{m+d} < \frac{\kappa+\Delta_c}{c_1(x_1)\sigma(\gamma-\beta)}$.

Appendix B.1 (Proof of proposition 2.3)

1. Consider the Friedman rule holds, $\gamma = \beta$.

   From (2.48) and (2.49), $x_1 = x_1^*$. If $\gamma = \beta$ and (2.49) hold, then $\lambda_1 = \lambda_2 = 0$, that means the liquidity constraint does not bind and agents hold enough money in the DM_1.

2. Suppose neither (2.13) nor (2.47) binds, then $\lambda_1 = \lambda_2 = 0$.

   (2.49) holds, $x_1 = x_1^*$ and $\gamma = \beta$.

Since $\lambda_1 = \lambda_2 = 0$ and $\gamma = \beta$ which imply $\kappa + B' \geq [c_1(x_1^*) - \phi m]\beta\sigma$.

Chapter 3

Financial Intermediaries, Asset Transformation, and Liquidity

3.1 Introduction

Intrinsic properties of an asset, such as risk and recognizability, matter for its acceptability as means of payment or collateral. By the 1850s some banknotes ceased to circulate in the U.S., partly because they were threatened by counterfeits or suffered from the difficulty for the public to determine their value. Nowadays, some complex newly-innovated financial assets also suffer from the recognizability problem. During 2007-2008, it became hard to use asset-backed securities (ABS) as collateral in the repos market, since investors cannot verify these assets' true value. According to Akerlof's (1970) argument, when assets that are used as means of payment or collateral are subject to the lemon problem, the information friction can obstruct trading and lead to market failure. How does imperfect recognizability of an asset's authenticity of future value weaken its acceptability as a means of payment or collateral? Can financial intermediaries, even without the expertise to discern the quality of the asset, improve aggregate liquidity and welfare in an economy with private information?

To study these issues, we consider an economy with limited record keeping, enforcement and commitment, so that agents use assets to conduct transactions. In the decentralized market, agents bargain over the terms of trade, which include the quantity of goods exchanged, and the transfer of assets. Buyers are endowed with risky real assets which can turn out to be good or bad, depending on their dividend processes. Good assets yield high dividends with certainty, whereas bad assets yield high dividends with a probability, and the expected dividends are lower than that of good assets. The quality of the real asset is private information to the asset holder.

Banks raise funds from shareholders and depositors to invest in real assets; however, they do not have ability to discern the quality of assets. Banks offer schedules of quantity and price as a screening device to asset holders when buying real assets. We assume that the schedules of price and quantity offered by the bank in the asset market are public information. Hence, from observing bank's strategies agents can infer the quality of bank's portfolio; i.e., whether the portfolio consists of good assets only, bad assets only, or both good and bad assets. Therefore, bank equity can be priced fairly even though its dividend process may be uncertain. Deposits are the least risky assets in this economy because equity holders are residual claimants.

A main insight of our study is that banks, by converting risky assets into safer and more recognizable assets, may improve aggregate liquidity and welfare, even though banks have no expertise to discern the quality of assets. Equilibria are characterized by banks' investing strategies. If banks buy all of one type of real assets, agents know that the real asset used in the decentralized market must be of the other type, implying that the private information problem regarding the real asset is removed. Therefore, all assets used as means of payment, including deposits, bank equity, and real assets, are free from the private information problem. We find the relationship between assets' liquidity and prices. Deposits and bank equity enjoy the same liquidity, that results in deposits and bank equity have identical returns. Prices of good assets are lower than an upper bound if some good assets are held but not traded for goods. Banks may buy all of bad assets and convert real assets into liabilities, which secure bad asset holder higher marginal benefit from trade weighted by dividends. Under this case, banks propose prices higher than a threshold to bad asset holders to induce the holders to sell assets.

In another type of equilibria, banks buy a fraction of both types of assets, so good and bad assets may be used to make payments in the decentralized market. The bargaining is involved with the private information regarding means of payment. Good asset holders signal the quality of real assets by retaining a fraction of good assets; that is, the bargaining is proceeded as a signaling game. There is a pecking-order payment arrangement: good asset holders use bank deposits and equity first to finance consumption, and they use real assets to make payments only if their deposits and equity holdings are depleted. In case the first-best output is not affordable with the giving bank liabilities, good asset holders would also keep some real assets rather spend all of them. Good assets thus face an endogenous liquidity constraint caused by the private information problem regarding means of payment.

The welfare level depends on whether banks eliminate the private information problem regarding means of payment. If banks buy all of one type of assets, economies enjoy higher welfare than otherwise. If bank's strategies do not remove the private information problem concerning means of payment, a fraction of good assets is retained as a signaling device. Moreover, deposits and bank equity command lower returns. Consequently, aggregate liquidity is lower than the economy where the private information is eliminated, so is welfare. As equilibria coexist, the one in which banks buy all good assets and eliminate the private information problem entails the highest welfare. The pricing of deposits and equity are based on people's belief on the returns from the investment. When banks buy only good assets, the returns on bank investment is the highest and with certainty. Therefore, people would assign the highest value on banks' liabilities, compared to other equilibria.

3.1.1 Literature Review

Liquidity considerations help to explain macroeconomic phenomenon, such as asset pricing anomalies, the rate of return puzzle and the transmission mechanism of monetary policy. Also literature has discussed the liquidity constraint of assets from different standpoints. Kiyotaki and Moore (2005, 2008) motivate exogenous constraints due to the limited resaleability of assets by the lack of commitments. Lester, Postlewaite and Wright (2009) assume that the claims on capital can be counterfeited costlessly and is recognizable to a fraction of agents, so that claims on capital is less liquid than fully recognizable fiat money. Li and Rocheteau (2010) assume that counterfeiting incurs a positive cost to derive an endogenous upper bound on the quantity of assets that can be traded for consumption goods. Following Kiyotaki and Moore (2008), Tomura (2010) endogenizes the resaleability constraint as agents choose not to sell the undervalued fraction of their real asset in the secondary market. In our model, the imperfect recognizability of real assets is the underlying reason for the endogenous liquidity constraint of real assets, and also motivate the role of financial intermediaries to partly solve the private information problem.

Because of asymmetric information, trades rely on intermediaries for professional expertise in recognizing the quality of goods, for example, art, antiques and used cars. Li (1997) consider the moral hazard problem associated with goods, and middlemen emerge endogenously to mitigate the trading frictions caused by qualitative uncertainty. We introduce banks not only to solve information problem, but to provide recognizable assets to facilitate trades. Banks' asset transformation in our environment is related to studies in which banks' liabilities circulate to improve aggregate liquidity. But we focus on the private information problem regarding assets as the reason why banks are so special as a provider of means of payment. For example, in Williamson (1999), banks specialize in different types of projects and issue claims on projects which can be used to exchange for consumption goods. Gorton and Pennacchi (1990) study that banks provide information-insensitive riskless debt circulating among uninformed agents who avoid trading risky assets with informed agents.

Our model features multiple assets traded in the decentralized market, and derive endogenizes liquidity differences among assets due to their recognizability. In Lagos (2010), risk-free bonds and equity are both used as means of payment, and the use of equities is exogenously restricted in a fraction of meetings. Rocheteau (2009) uses the private information problem to show the riskier asset is partially illiquid, but he does not explore the role of financial intermediaries in solving the information problem.

3.2 The environment

Time is discrete, starts at $t = 0$, and continues forever, revealed in figure 3.1. Each period is divided into two subperiods: a decentralized market (DM) with no double coincidence of wants, followed by a competitive market (CM) in which banks operate. There are two types of infinitely-lived agents: *buyers* and *sellers* representing their roles in the DM. Buyers want to consume but cannot produce, and sellers produce only, while no one can do both in the DM. All agents are treated symmetrically in the CM: they can both consume and produce. The measures of buyers and sellers are equal to 1. Let $\mathfrak{b}$ denote the set of buyers, $\mathfrak{s}$ the set of sellers, and $\mathfrak{N} = \mathfrak{b} \cup \mathfrak{s}$.

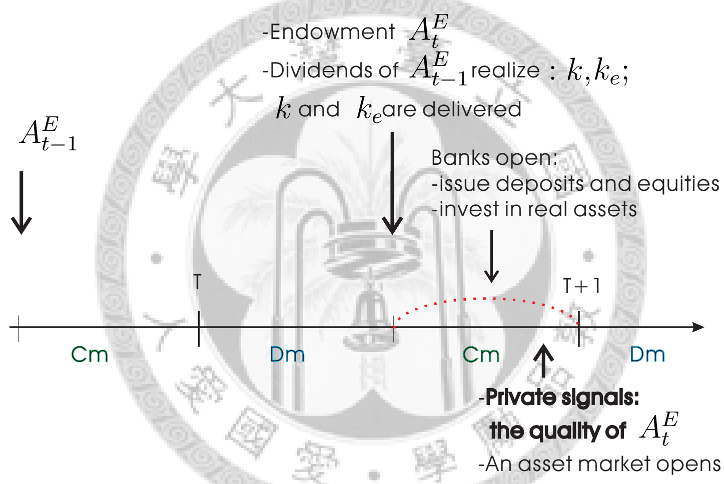

Figure 3.1. Time sequence

Let x_1 and x_2 be the perishable consumption goods produced in the DM and the CM, respectively. Buyers enjoy utility $u_1(x_{1,t})$ from consuming $x_{1,t}$, and sellers suffer a disutility of working $c_1(x_{1,t})$ in the DM. All agents get utility $u_2(x_{2,t})$ from consuming $x_{2,t}$ and incur the disutility of working, where the disutility of working hours h^n is linear, $c_2(h^n) = h^n$, $n = s, b$. Producing one unit of consumption goods demands one working hour that creates one unit of disutility.

The lifetime expected utility of a buyer in period $t = 0$ is

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[u_1(x_{1,t}) + u_2(x_{2,t}) - h^b \right], \tag{3.1}$$

where $u_m(0) = 0$, $u'_m(x_{m,t}) > 0$, and $u''_m(x_{m,t}) < 0$, $m = 1, 2$, $\beta \in (0, 1)$ is a discount factor across periods.

The lifetime expected utility of a seller at $t = 0$ is

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[-c_1(x_{1,t}) + u_2(x_{2,t}) - h^s \right], \tag{3.2}$$

The cost function $c_1(x_{1,t})$ is twice continuously differentiable, $c(0) = 0$, $c'_1(x_{1,t}) > 0$, and $c''_1(x_{1,t}) \geq 0$; let x_1^* denote the solution to $u'_m(x_m^*) = c'_m(x_m^*)$, $m = 1, 2$.

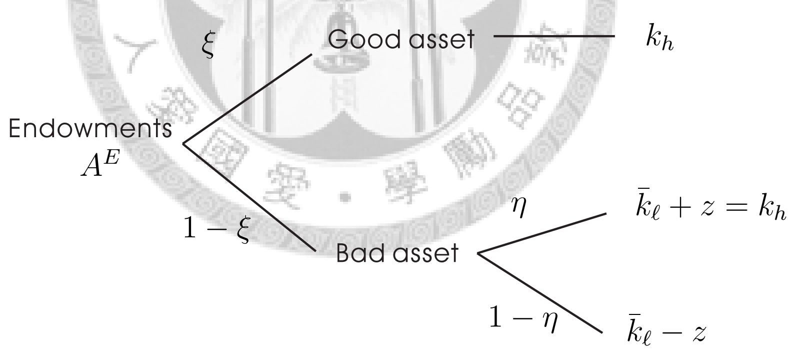

Figure 3.2. Dividend structure

Endowments and the dividend process of the real assets

Upon entering the CM, each buyer is endowed with $A^E > 0$ units of one-period-lived real assets. Because of the absence of wealth effects, it is irrelevant for the allocations who receives the endowment of asset. Each unit of the period-t real asset yields k_{t+1} units of CM goods as dividends at the beginning of the CM in $t+1$. The dividend of assets are subject to an iid shock: with probability ξ an asset turns out to be a good asset, and with probability $(1-\xi)$ it turns out to be a bad one. A good asset yields dividend $k_h > 0$ with certainty; while a bad asset yields dividend $\bar{k}_\ell + z = k_h$ with probability η, where $z = k_h - \bar{k}_\ell > 0$. The noise z is the mean-preserving spread due to which bad assets are risky assets. The expected dividend of bad assets is $k_\ell = \bar{k}_\ell - z(1-2\eta)$, where $\eta \in [0, 1/2]$. The real assets' dividend structure is illustrated in figure 3.2.

Asymmetric information and the asset market

An asset market opens at the end of the CM. Banks take deposits and issue equities before the asset market opens. Agents make deposits and buy bank equity first, and then adjust their portfolio for real assets while the asset market operates. As soon as asset holders enter the asset market they receive a perfect signal about the quality of their endowments, A^E, i.e., good or bad assets. The information regarding good assets or bad assets is private information to the asset holder. Sellers, in contrast, never learn any information about assets' quality. Because of the informational friction associated with the real assets' quality, agents in the asset market price all real assets at an identical price. That means good assets are undervalued. The market's undervaluation discourages good asset holders from selling their assets, that could result in a phenomenon of market failure caused by lemons problems.

Banks in the CM

Competitive banks open in the CM. Before the asset market open, banks take deposits and issue bank equities to collect funds to invest in real assets. In the asset market banks offer two price-quantity pairs to separate asset holders' type. Banks convert risky assets into less risky bank equities and deposits, and hence, engage in asset transformation. In the CM of period t, one unit of deposits and one unit of bank equities cost 1 and q_e units of current CM-goods, respectively. Agents making deposits at period t receive interest payments at the deposit rate i at period $t+1$. Likewise, bank equity holders receive one-period dividends k_e. Banks pay deposit interests and dividends with the CM-goods. Buyers potentially can use deposits (like writing checks) and bank equities to make payments in the DM. We assume the quality of a bank's asset holdings is public information but is subject to risk regarding bad assets' dividend state. Since there is no private information involved, bank equities can be priced fairly in the market.

Trades in the DM

In the DM, each seller is matched randomly and bilaterally with a buyer, and they bargain over the terms of trade. Buyers may bring real assets , deposits and bank equity to the DM. We assume in the bargaining game the buyer makes an take-it-or-leave-it offer to the seller who decides whether to accept it or not. The offer includes the quantity of DM-goods produced and the transfer of assets.[1]

[1] Assets used in the trade can be interpreted as a means of payment or collateral. For

Trading histories of agents are private information, and there is no commitment between agents. So all trades are quid pro quo and credit arrangements are not feasible.

Whether there is private information problem associated with the means of payment in the DM depends on banks' strategies in the asset market. If the quality of means of payment is subject to private information, then the bargaining is a signaling game: the buyer wants to signal the quality of real assets and prevent other type of buyers from imitating him. An equilibrium of the bargaining game consists of a portfolio of the buyer's offer, the seller's acceptance rule and belief about the quality of the real asset. In this paper, we use the Intuitive Criterion of Cho and Krep (1987) to refine the equilibrium concept. The refinement is described as following: a proposed equilibrium fails the Intuitive Criterion if an out-of-equilibrium offer makes one type buyer strictly better off, and the other type buyer strictly worse off as the out-of-equilibrium is accepted.

The role of bank capital

In reality, banks arrange a portfolio of loans with various degrees of default risks. Some financial institutions create claims such as collateralized debt obligations (CDOs) based on portfolio of bank risky assets. The CDO issuer classifies tranche according to the cash flows scheduled generated by the underlying loans. Investors of the residual tranche enjoy high return, but they absorb the loss from default.

example, in the repo market assets are used as collateral to secure a better trade.

Bank capital in this paper plays a similar role as the residual tranche in CDO. The deposit interest rate does not depend on banks' risk exposure, while if banks' portfolio consist of bad assets (which is like loans with default risk), equity holders absorb the risk shocks.

3.3 Equilibrium

In this economy banks' strategies affect the private information problem associated with payment arrangements, and the bargaining game. Banks' portfolios generate different payment arrangements. If banks buy all good assets or all bad assets, agent's trade and bargain in the DM is under symmetric information; otherwise, the bargaining game is a signaling game. To study agents' payment arrangement and payoffs in the DM, we derive some properties of the value function in the CM first.

An agent begins period t with a portfolio which contains a units of real assets, d units of deposits and e units of bank equity. Let $V_j^b(a, d, e)$ denote the value function of a buyer j entering with portfolio (a, d, e) after the private signal is realized. The subscript $j \in \{\ell, h\}$ indicates the type of buyers, determined by what type of assets the buyer is endowed. If $j = \ell$ (h), that means the buyer's endowments are bad (good) real assets, the expected dividend is k_ℓ (k_h), and we label the buyer ℓ (h) type in period t. Denote $W^b(a, d, e; k_j)$ the value function of the buyer with portfolio (a, d, e) entering the CM. Agents may carry real assets out of the DM, and then they receive dividends. So the expected life-time utility function is related to the expected dividend of real assets, $k_j \in \{k_\ell, k_h\}$.

Agents' problem in the CM

A buyer is endowed A^E real assets, produce h goods, consume x_2, and adjust asset holdings of real assets, deposits and bank equity. Denote q_a^j the price of one unit of the real asset j, that the bank offers in the asset market. To simplify notations we use superscript prime for variables corresponding to the next period. The value function of a buyer with portfolio (a, d, e) entering the CM of period t is

$$W^b(a, d, e; k_j) = \max_{x_2, h, a', d', s'} \{x_2 - h + \beta V^b_{j,+1}(a', d', e')\} \tag{3.3}$$

$$\text{s.t.} \quad x_2 + d' + q_e e' = h + k_j a + (1+i)d + k_e e + q_a^{j,+1}(A^E - a'), \quad j, +1 = h, \ell, \tag{3.4}$$

where $j, +1$ labels the buyer's type in the period $t+1$, which depends on the quality of endowments, A^E. Problem (3.3) reveals that the buyer chooses his net consumption, $x_2 - h$, and asset holdings to the next period, a', d' and e' to maximize his expected lifetime utility upon entering the CM subject to the budget constraint (3.4). To hold bank liabilities, (d', e'), the buyer deposits d' goods and spends $q_e e'$ units of goods for bank equity. Selling $(A^E - a')$ units of real assets to banks gives the buyer $q_a^{j,+1}(A^E - a')$ units of goods.

Substituting (3.4) into (3.3), we obtain

$$W^b(a, d, e; k_j) = k_j a + (1+i)d + k_e e + \max_{a', d', e'} \{q_a^{j,+1}(A^E - a') - d' - q_e e' + \beta V^b_{j,+1}(a', d', e')\} \tag{3.5}$$

The buyer's value function in the CM is linear in his wealth, and the portfolio choice is independent of the initial asset holdings. Under the quasilinear utility assumption, the distribution of bank liability holdings is degenerate at the beginning of a period.

Similarly, the seller's value function as entering the CM is:

$$W^s(a,d,e;k_j) = k_j a + (1+i)d + k_e e + \max_{d',e'}\{-d' - q_e e' + \beta V^s(d',e')\},$$

where $\beta V^s(d',e')$ is the value function of the seller upon entering the DM. In a decentralized meeting, the seller receives real assets a transferred from the matched buyer, and obtains dividends $k_j a$.

Banks' problem in the CM

In the CM competitive banks pay interests and dividends to depositors and shareholders, and invest in real assets. The zero profit condition of a representative bank is:

$$k_e E + (1+i)D + q_a^h \Omega_h' + q_a^\ell \Omega_\ell' = D' + q_e E' + (k_h \Omega_h + k_\ell \Omega_\ell). \tag{3.6}$$

The right side of equation (3.6) represents the source of funds which includes deposits, D', and equity issued outstanding, E', of which the value is $D' + q_e E'$; the bank's portfolios in real assets (Ω_h, Ω_ℓ) earn dividends $k_h \Omega_h + k_\ell \Omega_\ell$. These funds are used to finance real interest payments, $(1+i)D$, and dividend payments, $k_e E$ and a portfolio for next period $(\Omega_h', \Omega_\ell')$. The bank buys Ω_h' good assets and Ω_ℓ' bad assets at prices q_a^h and q_a^ℓ respectively.

In the asset market given the risk of market portfolio, captured by probabilities

ξ, η and z, the bank chooses a price-quantity schedule to maximize its expected surplus from investments, while separating assets' type. In this economy the bank's demand for real assets is bound to feasible constraints, A^E, the quantity of endowments buyers receive. The bank's problem is:

$$\max_{q_a^h, q_a^\ell, \omega_h, \omega_\ell} \{\xi[-q_a^h \omega_h + \beta k_h \omega_h] + (1-\xi)[-q_a^\ell \omega_\ell + \beta k_\ell \omega_\ell]\} \tag{3.7}$$

s.t.

$$q_a^h \omega_h + \beta V_h^b(A^E - \omega_h, d, e; k_h) \geq \beta V_h^b(A^E, d, e; k_h), \tag{3.8}$$

$$q_a^\ell \omega_\ell + \beta V_\ell^b(A^E - \omega_\ell, d, e; k_\ell) \geq \beta V_\ell^b(A^E, d, e; k_\ell); \tag{3.9}$$

$$q_a^h \omega_h + \beta V_h^b(A^E - \omega_h, d, e; k_h) \geq q_a^\ell \omega_\ell + \beta V_h^b(A^E - \omega_\ell, d, e; k_h), \tag{3.10}$$

$$q_a^\ell \omega_\ell + \beta V_\ell^b(A^E - \omega_\ell, d, e; k_\ell) \geq q_a^h \omega_h + \beta V_\ell^b(A^E - \omega_h, d, e; k_\ell); \tag{3.11}$$

$$q_a^h, q_a^\ell \geq 0, \tag{3.12}$$

$$\omega_h \leq A^E, \omega_\ell \leq A^E. \tag{3.13}$$

The bank chooses $\{q_a^h, q_a^\ell, \omega_h, \omega_\ell\}$ to maximize its expected profits subject to conditions (3.8)-(3.13). The objective function (3.7) illustrates that the bank expects he meets a good asset holder with probability ξ, the surplus from trade is $-q_a^h \omega_h + \beta k_h \omega_h$; and with probability $(1-\xi)$ he buys assets from a bad asset holder and obtains surplus $-q_a^\ell \omega_\ell + \beta k_\ell \omega_\ell$. The first set of constraints, (3.8)-(3.9), marks participation constraints for h and ℓ holders. For example, if a buyer h accepts the bank's proposal to sell ω_h units of good assets and get $q_a^h \omega_h$ units of goods, then he will bring the rest of assets, $(A^E - \omega_h)$, into the DM. For the asset holder to accept the bank's contract, these benefits should be no less than the discounted gains from bringing all endowed assets to trade in the DM, i.e.,$V_h^b(A^E, d, e)$. Condition (3.9) has a similar interpretation. Constraints (3.10)-(3.11) are incentive compatibility constraints, for h and ℓ holders, respectively. Consider constraint (3.10), a asset holder h's gains are $q_a^h \omega_h + \beta V_h^b(A^E - \omega_h, d, e)$ if he tells the truth; that is, his choice of the contract (ω_h, q_a^h) is consistent with his dividend state, k_h. If the buyer h chooses the contract (ω_ℓ, q_a^ℓ) to mimic the buyer ℓ, his gains are $q_a^\ell \omega_\ell + \beta V_h^b(A^E - \omega_\ell, d, e)$. In equilibrium the price-quantity schedule would induce the buyer h tells the truth, i.e., constraint (3.10) holds. Constraint (3.11) follows similarly. The remaining constraints represent feasibility. Before solving the bank's problem in the asset market, we derive some properties of agents' value functions in the DM.

Buyers' value functions in the DM

The expected lifetime utility of a buyer j entering the DM with a units of real assets, d units of deposits, e units of bank equity is

$$V_j^b(a, d, e) = u_1[x_1(y_a, y_d, y_e)] + W^b(a - y_a, d - y_d, e - y_e; k_j), \tag{3.14}$$

where x_1 is the quantities of DM goods produced by the seller, y_a is the transfer of real assets, y_d is the transfer of deposits, and y_e is the transfer of bank equity from the buyer to the seller. By the linearity of W^b, (3.14) becomes

$$V_j^b(a, d, e) = S_j(a, d, e) + k_j a + (1 + i)d + k_e e + W^b(0, 0, 0; k_j), j = h, \ell; \tag{3.15}$$

where $S_j(a,d,e)$ is the buyer's surplus in the DM if the dividend state is j, that is,

$$S_j(a,d,e) \equiv u_1[x_1(y_a,y_d,y_e)] - k_j y_a(a,d,e;k_j) - (1+i)y_d(a,d,e;k_j) - k_e y_e(a,d,e;k_j).$$

The buyer's payment arrangement (y_a, y_d, y_e) is function of asset holdings he brings into the DM, (a,d,e). We assume that the buyer and the seller's portfolios are not common knowledge in the match.[2] If the seller accepts the offer (x_1, y_a, y_d, y_e), the buyer j enjoys $u_1(x_1)$, but forgoes dividends and interest income paid by real assets, bank equities and deposits. The surplus from trade is the utility minus the future value of asset transfers.

Agents' portfolio choice in the CM

In the CM buyers adjust the balance of bank liabilities and real assets at different point of time, of which the information structure changes. Buyers choose their portfolio of bank liabilities first at the beginning of the CM before knowing the quality of endowments. This implies that every buyer will choose an identical portfolio of deposits and bank equity, and the choice based on the expected dividend of market portfolio, captured by ξ, η and z. The buyer's portfolio problem in the CM of period t is

$$\max_{d,e} -[\frac{1-(1+i)\beta}{\beta}]d - (\frac{q_e - k_e\beta}{\beta})e + \xi S_h(a,d,e) + (1-\xi)\{\eta[S_h(a,d,e) + (1-\eta)S_\ell(a,d,e)]\} \quad (3.16)$$

[2]Although we would show that the surplus functions in the DM are weakly monotone increasing in the agents' asset holdings, in this economy agents have no chance to show their portfolios in a pre-stage of the bargaining game.

Buyers choose bank liabilities to maximize their expected surplus net of costs of holding assets in the DM. A buyer spends one unit of goods for one unit of deposits, and redeems deposits at a interest rate i in the CM of period $t+1$. Therefore, $\frac{1}{\beta}-(1+i)$ represents the net cost of holding one unit of deposits. The term $\frac{q_e}{\beta}-k_e$ has similar interpretations.

Following the time sequence, the asset market open and private signals realize. Buyers enter the asset market to sell their endowments to banks. So a buyer's portfolio choice of real assets is

$$\max_{a}\{-(\frac{q_a^{j,+1}-k_{j,+1}\beta}{\beta})a+S_{j,+1}(a,d,e)\},\quad j,+1\in\{\ell,h\}. \tag{3.17}$$

For a buyer $j,+1$, carrying one unit of real assets out of the CM gains $k_{j,+1}$ dividends in the next period, but forgoes $q_a^{j,+1}$ units of goods paid by banks. The term $\left(\frac{q_a^{j,+1}}{\beta}-k_{j,+1}\right)$ hence represents the net cost of holding one unit of real assets.

Since buyers make take-it-or-leave-it offers, sellers obtain no surplus from the DM trades. Hence, a seller's portfolio problem is

$$\max_{d,e}\{-[\frac{1-(1+i)\beta}{\beta}]d-(\frac{q_e-k_e\beta}{\beta})e\}, \tag{3.18}$$

The following two lemmas illustrate some properties of agents' portfolio choice.

Lemma 3.1. *(Sellers' portfolio choices)*

There is a solution to problem (3.18) if and only if $1\geq(1+i)\beta$ *and* $q_e\geq k_e\beta$.

1. *If* $1>(1+i)\beta$, *then* $d=0$. *If* $1=(1+i)\beta$, *then* $d\in[0,\infty)$.

2. If $q_e > k_e\beta$, then $e = 0$. If $q_e = k_e\beta$, then $e \in [0, \infty)$.

For sellers, making deposits or buying bank equities depends purely on the difference between the price of assets and their returns, since sellers simply produce and they get no surplus from trades in the DM. They hold an asset if its price is equal to its fundamental value, i.e., $1 = (1+i)\beta$ or $q_e = k_e\beta$.

Let $S_{j,+1,1}(a,d,e)$, $S_{j,+1,2}(a,d,e)$ and $S_{j,+1,3}(a,d,e)$ be the partial derivatives of the buyer's surplus function, which represent the marginal contributions of real assets, deposits and bank equities, respectively, to the gains from trade for a buyer.

Lemma 3.2. *(Buyers' portfolio choices)*

If $q_a^{j,+1} \geq k_{j,+1}\beta$, $1 \geq (1+i)\beta$ and $q_e \geq k_e\beta$, there is a solution to problem (3.16) *and* (3.17). *The optimal portfolio choice must satisfy*

$$- \quad \frac{q_a^{j,+1} - k_{j,+1}\beta}{\beta} + S_{j,+1,1}(a,d,e) \leq 0, \quad \text{“}=\text{”} \quad if \quad a > 0 \tag{3.19}$$

$$- \quad \frac{1-(1+i)\beta}{\beta} + \xi S_{h,2}(a,d,e) + (1-\xi)[\eta S_{h,2}(a,d,e) + (1-\eta)S_{\ell,2}(a,d,e)] \leq 0, \quad \text{“}=\text{”} \ if \ d > 0 \tag{3.20}$$

$$- \quad \frac{q_e - k_e\beta}{\beta} + \xi S_{h,3}(a,d,e) + (1-\xi)[\eta S_{h,3}(a,d,e) + (1-\eta)S_{\ell,3}(a,d,e)] \leq 0, \quad \text{“}=\text{”} \ if \ e > 0 \tag{3.21}$$

The first term in the left side of (3.19) is the net cost of holding one unit of real assets, and the second term is the marginal benefit from using real assets to make payments in the DM. Condition (3.20) and (3.21) are related to deposits and bank equity choices, respectively. Lemma 3.2 reveals that buyers hold assets if the net cost is covered by the marginal benefit.

In this economy banks' balance of real assets generates different information structures of payment arrangements in the DM. For instance, if banks buy all good assets, then means of payment used in the DM trade are not threatened by private information. Let $\{(a_h, d, e), (a_\ell, d, e)\}$ represent the portfolio of the buyer h and the buyer ℓ, respectively. Equilibria are classified as follows:

I. $a_h < a_\ell$:

Equilibrium 1. $\{(a_h, d, e), (a_\ell, d, e)\} = \{(0, d, e), (A^E, d, e)\}$;

Equilibrium 2. $\{(a_h, d, e), (a_\ell, d, e)\} = \{(0, d, e), (a_\ell, d, e)\}$;

Equilibrium 3. $\{(a_h, d, e), (a_\ell, d, e)\} = \{(a_h, d, e), (a_\ell, d, e)\}$;

II. $a_h > a_\ell$:

Equilibrium 4. $\{(a_h, d, e), (a_\ell, d, e)\} = \{(A^E, d, e), (0, d, e)\}$;

Equilibrium 5. $\{(a_h, d, e), (a_\ell, d, e)\} = \{(a_h, d, e), (0, d, e)\}$;

Equilibrium 6. $\{(a_h, d, e), (a_\ell, d, e)\} = \{(a_h, d, e), (a_\ell, d, e)\}$;

III. $a_h = a_\ell$:

Equilibrium 7. $\{(a_h, d, e), (a_\ell, d, e)\} = \{(0, d, e), (0, d, e)\}$;

Equilibrium 8. $\{(a_h, d, e), (a_\ell, d, e)\} = \{(a, d, e), (a, d, e)\}$;

Equilibrium 9. $\{(a_h, d, e), (a_\ell, d, e)\} = \{(A^E, 0, 0), (A^E, 0, 0)\}$.

Banks in Equilibrium 1 buy all of good assets and no bad ones; Equilibrium 2. is the case in which banks buy all of good assets and some bad ones. The buyer h brings no real assets into the DM; agents know that real assets appearing in the DM are bad assets, and thus, they bargain over the terms of trade without private information problem. Similarly, banks solve the private information problem by buying all of bad assets and on or some good ones, revealed by Equilibria 4 and 5, respectively. When banks do not remove the private information problem, banks' portfolios could be several compositions as follows: Equilibrium 3 is the portfolio comprised of more good assets than bad assets; Equilibrium 6 is the one comprised of more bad assets than good assets; Equilibrium 8 represents the case banks buy the same quantity of good and bad assets, so $a_h = a_\ell$. Equilibrium 7 this is also an equilibrium without private information. Equilibrium 9 represents the economy without banks.

3.4 Payment arrangements without private information

We characterize equilibria in which banks buy all good assets (i.e., equilibria 1 and 2) or all bad assets (i.e. equilibria 4 and 5). The payment arrangement in the decentralized meeting thus is just subject only to the risk of bad assets, which is common knowledge. In bargaining over the terms of trade, the problem of a buyer holding real assets j is

$$\max_{x_1^j, y_a^j, y_d^j, y_e^j} [u_1(x_1) - k_j y_a - (1+i) y_d - \kappa_e y_e] \tag{3.22}$$

$$s.t. - c_1(x_1) + k_j y_a + (1+i) y_d + k_e y_e \geq 0, \tag{3.23}$$

$$y_a \leq a_j, \quad y_d \leq d, \quad y_e \leq e. \tag{3.24}$$

The buyer makes an offer (x_1, y_a, y_d, y_e) to maximize his expected surplus from trade subject to the seller's participation constraint (3.23), and the feasibility constrains (3.24). An equilibrium offer holds constraint (3.23) in equality, since the buyer makes a take-it-or-leave-it offer to the seller.

Any offer made by a type $-j$ buyer who sells all his endowments, i.e., $\omega_{-j} = A^E$, to the bank solve

$$\max_{x_1^{-j}, y_d^{-j}, y_e^{-j}} [u_1(x_1) - (1+i) y_d - k_e y_e] \tag{3.25}$$

$$s.t. - c_1(x_1) + (1+i) y_d + k_e y_e \geq 0, \tag{3.26}$$

$$y_d \leq d, \quad y_e \leq e, \tag{3.27}$$

Note from (3.25) that, unlike a buyer j who holds real assets, a buyer $-j$ uses only deposits and bank equity to make payment. Buyers make their complete information offers. The solution to (3.22)-(3.24) is

$$x_1^j = x_1^*,$$

$$c_1(x_1^*) = k_j y_a + (1+i) y_d + k_e y_e,$$

if $k_j a_j + (1+i)d + k_e e \geq c_1(x_1^*)$. If the buyer j's asset holdings are not enough to reach the efficient output, x_1^*, then

$$c_1(x_1^j) = k_j a_j + (1+i)d + k_e e,$$

and

$$y_a^j = a_j, \quad y_d^j = d, \quad y_e^j = e.$$

Similarly, if a type $-j$ buyer owns enough assets to consume x_1^*, he may not spend all his assets; i.e., $(1+i)d + k_e e \geq c_1(x_1^*)$. Otherwise, he spends all assets to consume $x_1^{-j} < x_1^*$.

Definition 3.1. *An equilibrium of the bargaining game under complete information is a list* $\{(a_j, d_j, e_j), (q_a^h, q_a^\ell, a_h, a_\ell), (x_1^j, y_a^j, y_d^j, y_e^j)\}$ *satisfying the following conditions:*

1. *agents' portfolio choices, conditions* (3.19)-(3.21)*;*
2. *banks' problem in the CM, i.e., conditions* (3.6) *and* (3.7)-(3.13)*;*
3. *agents' payment arrangement:* (3.22)-(3.27).

The next lemma describes the buyer's payoff from the bargaining game when he uses a risky asset to make payment.

Lemma 3.3. *(Gains from trade under risks)*

In an equilibrium where banks buy all good assets; i.e., $\{(0,d,e),(a_\ell,d,e)\}$ *is an equilibrium portfolio, a type* ℓ *buyer whose payments are exposed to an zero-mean* *risk z ($\eta = \frac{1}{2}$) will bargain a higher DM-output than a type h buyer, who use bank liabilities to make payments. That is, $x_1^h < x_1^\ell$.*

One may expect that buyers use bad assets (risky assets) to make payments in the bargaining may be worse than buyers who exchange safe assets for consumption, due to the risky payoff. However, we find that, on the contrary, the risky payoff from bargaining is involved with the full bargaining power for buyers – the take-it-or-leave-it offer, so using risky assets as payments would not lower buyers' payoff. White (2006, 2008) argue that if a player's utility function satisfies some properties, he will not become worse when his payoff is subject to uncertainty. We achieve the same conclusion with no need to impose the assumption on the properties of the utility function as in White (2008).[3]

Proposition 3.1. *(The liquidity-price relationship)*

When banks buy all one type of assets, deposits, bank equity and real assets have the same liquidity, and $\frac{k_e}{q_e} = 1 + i$.

1. *If banks buy all good assets, then $q_a^h > q_a^\ell$.*

[3]White (2008) finds that: as the expected surplus is risky, an extra surplus becomes more valuable, so a bargainer is more willing to hold out for an extra surplus in negotiation. Thus it is as a player behaves more patiently, i.e., the utility function satisfies $-\frac{U'''}{U''} > -\frac{U'}{U}$, and so he consumes more in equilibrium. We find in White (2006) if the player facing risks has full bargaining power and the risk is irrelevant to the expected surplus, then he does not become worse with the risky expected surplus.

2. If banks buy all bad assets and $\frac{\sigma_\ell k_\ell}{\sigma_h k_h} > 1$*, then*

$$q_a^\ell > q_a^h - \beta(k_h - k_\ell),$$

where $\sigma_j \equiv \frac{u_1'(x_1^j)}{c_1'(x_1^j)} - 1$*. Moreover, when* k_h *is large enough such that* $\frac{\sigma_\ell k_\ell}{\sigma_h k_h} < 1$*, then banks buy good assets at a higher price, i.e.,* $q_a^h > q_a^\ell$*.*

Proposition 3.1 reveals that, assets are valued for their usefulness as payments, besides their future yields. Bank deposits and equity enjoy the same liquidity, so they realize identical returns $\frac{k_e}{q_e} = 1 + i$. In case 1, banks buy all of good assets at a higher price $q_a^h > q_a^\ell$. The term σ_j represents the marginal benefit from trade of buyer j's portfolio. A set (k_h, k_ℓ) such that $\frac{\sigma_\ell k_\ell}{\sigma_h k_h} > 1$ in case 2 implies that banks deliver liabilities which secure bad asset holders more benefit from trade weighted by dividends. To compensate the holders' loss of future consumption, banks buy bad assets at a price higher than a certain threshold, i.e., $q_a^\ell > q_a^h - \beta(k_h - k_\ell)$. Otherwise, bad asset holders are not willing sell assets to banks, which can be traded for DM-output or yield expected dividends k_ℓ.

3.5 Payment arrangements under private information

When banks buy some, but not all, good assets, i.e., $\omega_h < A^E$, in each decentralized match the real asset transferred by the buyer could be good or bad. The seller cannot verify the quality of real assets, so a buyer h wants to separate himself from a buyer ℓ. We study equilibria incorporating the signaling game into the general equilibrium framework.

The signaling game

A strategy for the buyer draws an offer (x_1, y_a, y_d, y_e) from the distribution $\mathbb{F}$, and the transfer of assets is subject to the buyer's and seller's portfolios. The seller's strategy is an acceptance rule that specifies a set $\mathbb{A} \in \mathbb{F}$ of acceptable offers. Let $(a^n$, d^n, $e^n)$, $n \in \{b, s\}$, denote agents hold a^n units of real assets, d^n units of deposits and e^n units of bank equities, for buyers and sellers, respectively. The buyer's expected payoff with the dividend state k_j is

$$[u_1(x_1) + W^b(a^b - y_a, d^b - y_d, e^b - y_e; k_j)]\mathbb{I}_{\mathbb{A}}(x_1, y_a, y_d, y_e) + W^b(d^b, s^b, a^b; k_j)[1 - \mathbb{I}_{\mathbb{A}}(x_1, y_a, y_d, y_e)],$$

where $\mathbb{I}_{\mathbb{A}}(x_1, y_a, y_d, y_e)$ is an indicator function which is equal to one if the proposed offer $(x_1, y_a, y_d, y_e) \in \mathbb{A}$. If the offer is accepted, the buyer enjoys his utility of consumption in the DM, $u_1(x_1)$, but he forgoes y_d units of deposits, y_e units of bank equities and y_a units of real assets. The seller's expected payoff function is $\{-c_1(x_1) + W^s[y_a, d^s + y_d, s^s + y_e; k_j]\}\mathbb{I}_{\mathbb{A}}(x_1, y_a, y_d, y_e) + W^s(d^s, e^s)[1 - \mathbb{I}_{\mathbb{A}}(x_1, y_a, y_d, y_e)]$. The buyer's surplus from trade is $[u_1(x_1) - k_j y_a - (1+i)y_d - k_e y_e]\mathbb{I}_{\mathbb{A}}(x_1, y_a, y_d, y_e)$, and the seller's surplus is $[-c_1(x_1) + k_j y_a + (1+i)y_d + k_e y_e]\mathbb{I}_{\mathbb{A}}(x_1, y_a, y_d, y_e)$.

When the seller observes the offer made by the buyer, he constructs a belief system about the dividend state of real assets to decide whether to accept it. Let $\lambda(x_1, y_a, y_d, y_e) \in [0, 1]$ be the updated belief that in a match a seller believes the buyer holds high-dividend assets, conditional on the proposed offer (x_1, y_a, y_d, y_e). Then, $\mathbb{E}_\lambda = \lambda(x_1, y_a, y_d, y_e)k_h + [1 - \lambda(x_1, y_a, y_d, y_e)]k_\ell$.

Given a belief system, the set of acceptable offer for a seller is

$$\begin{aligned}\mathbb{A}(\lambda) &= \{(x_1, y_a, y_d, y_e) \in \mathbb{F} : -c_1(x_1) + \{\lambda(x_1, y_a, y_d, y_e)k_h \\ &+ [1 - \lambda(x_1, y_a, y_d, y_e)]k_\ell\}y_a + (1+i)y_d + k_e y_e \geq 0\}. \qquad (3.28)\end{aligned}$$

We adopt a tie-breaking rule by that a seller accepts any offer that makes him indifferent between accepting or rejecting a trade. The problem of a buyer holding an asset of dividend state k_j is then

$$\begin{aligned}&\max_{x_1, y_a, y_d, y_e} [u_1(x_1 - k_j y_a - (1+i)y_d - k_e y_e)]\mathbb{I}_{\mathbb{A}}(x_1, y_a, y_d, y_e) \\ &s.t. \;\; (x_1, y_a, y_d, y_e) \in \mathbb{R}_+ \times [0, a^b] \times [-d^s, d^b] \times [-e^s, e^b]. \qquad (3.29)\end{aligned}$$

An equilibrium of the bargaining game is a profile of strategies for the buyer and the seller, and a belief system λ. If an equilibrium offer (x_1, y_a, y_d, y_e) is made, then the seller's belief is derived from his prior belief according to Bayes's rule. In order to refine the equilibrium concept, we use the Intuitive Criterion of Cho and Krep (1987). Let U_h^b represent the surplus of a buyer with good assets and U_ℓ^b represent the surplus of a buyer with bad assets in a proposed equilibrium of the bargaining game. The Intuitive Criterion denies a proposed equilibrium if there is an out-of-equilibrium offer $(\hat{x}_1, \hat{y}_a, \hat{y}_d, \hat{y}_e) \in \mathbb{F}$ and the dividend state $j \in \{h, \ell\}$ such that the following is true:

$$u_1(\hat{x}_1) - k_j\hat{y}_a - (1+i)\hat{y}_d - k_e\hat{y}_e \;>\; U_j^b, \qquad (3.30)$$

$$u_1(\hat{x}_1) - k_{-j}\hat{y}_a - (1+i)\hat{y}_d - k_e\hat{y}_e \;<\; U_{-j}^b, \qquad (3.31)$$

$$-c_1(\hat{x}_1) + k_j\hat{y}_a + (1+i)\hat{y}_d + k_e\hat{y}_e \;\geq\; 0, \qquad (3.32)$$

where $\{-j\} = \{h, \ell\} \setminus \{j\}$. Inequality (3.30) reveal that the offer $(\hat{x}_1, \hat{y}_a, \hat{y}_d, \hat{y}_e)$ would make a buyer j strictly better off if it were accepted. The offer $(\hat{x}_1, \hat{y}_a, \hat{y}_d, \hat{y}_e)$ would make the buyer $-j$ strictly worse off by (3.31). The seller accepts the offer and believes it is made by a buyer j.

Definition 3.2. *An equilibrium of the bargaining game is a pair of strategies and a belief system, $\langle (x_1(k_j), y_a(k_j), y_d(k_j), y_e(k_j)), \mathbb{A}, \lambda \rangle$, such that: (i) $(x_1(k_j), y_a(k_j), y_d(k_j), y_e(k_j))$ is solution to (3.29) with $k_j \in \{k_h, k_\ell\}$; (ii) $\mathbb{A}$ is given by (3.28); (iii) $\lambda : \mathbb{F} \to [0, 1]$ satisfies Bayes's rule whenever possible and the Intuitive Criterion.*

Any offer made by a buyer holding bad assets is such that

$$\max_{x_1^\ell, y_a^\ell, y_d^\ell, y_e^\ell} [u_1(x_1) - k_\ell y_a - (1+i) y_d - k_e y_e] \tag{3.33}$$

$$s.t. - c_1(x_1) + k_\ell y_a + (1+i) y_d + k_e y_e \geq 0, \tag{3.34}$$

$$y_a \leq a_\ell, y_d \leq d, y_e \leq e. \tag{3.35}$$

Buyers with bad assets make the complete information offer, $y_a^\ell \geq 0$, that is always acceptable and irrespective of sellers' beliefs. If $c_1(x^*) \leq k_\ell a_\ell + (1+i)d + k_e e$, the solution to (3.33)-(3.35) is

$$\begin{aligned} x_1^\ell &= x_1^*, \\ c_1(x_1^*) &= k_\ell y_a + (1+i) y_d + k_e y_e. \end{aligned}$$

If $c_1(x^*) > k_\ell a_\ell + (1+i)d + k_e e$, the solution to (3.33)-(3.35) is

$$\begin{aligned} y_a^\ell &= a_\ell, \\ y_d^\ell &= d, \\ y_e^\ell &= e, \\ x_1^\ell &= c^{-1}[k_\ell y_a + (1+i)y_d + k_e y_e]. \end{aligned}$$

Any offer made by a buyer with good assets is such that

$$\max_{x_1^h, y_a^h, y_d^h, y_e^h} [u_1(x_1) - k_h y_a - (1+i)y_d - k_e y_e] \tag{3.36}$$

$$s.t. - c_1(x_1) + k_h y_a + (1+i)y_d + k_e y_e \geq 0, \tag{3.37}$$

$$u_1(x_1) - k_\ell y_a - (1+i)y_d - k_e y_e \leq u_1(x_1^\ell) - c_1(x_1^\ell), \tag{3.38}$$

$$y_a \leq a_h, y_d \leq d, y_e \leq e. \tag{3.39}$$

Buyers with good assets make offer $(x_1^h, y_a^h, y_d^h, y_e^h)$ to maximize their gains from trade. Constraint (3.37) shows that the buyer makes an offer so that the seller believes he is a good asset holder. The incentive-compatibility condition, revealed in (3.38), according to which a buyer with bad assets does not want to mimic the offer of a buyer with good assets.

Proposition 3.2. *There is a solution $(x_1^h, y_a^h, y_d^h, y_e^h)$ to (3.36)-(3.39), and it has the following properties:*

1. *If* $(1+i)d + k_e e \geq c_1(x_1^*)$*, then*

$$x_1^h = x_1^* \tag{3.40}$$

$$k_h y_a^h + (1+i)y_d^h + k_e y_e^h = c_1(x_1^*) \tag{3.41}$$

$$y_a^h = 0. \tag{3.42}$$

2. *If* $(1+i)d + k_e e < c_1(x_1^*)$*, then* $y_d^h = d, y_e^h = e$ *and* (x_1^h, y_a^h) *satisfies*

$$k_h y_a^h = c_1(x_1^h) - (1+i)d - k_e e, \tag{3.43}$$

$$u_1(x_1^\ell) - c_1(x_1^\ell) = u_1(x_1^h) - c_1(x_1^h) + (1 - \frac{k_\ell}{k_h})[c_1(x_1^h) - (1+i)d - k_e e], \tag{3.44}$$

where $x_1^\ell = min\{x_1^*, c_1^{-1}[k_\ell a_\ell + (1+i)d_\ell + k_e e_\ell]\}$*. Moreover, if* $a_h > 0$*, then* $x_1^h < x_1^\ell$ *and* $0 < y_a^h < a_h$*.*

Case 1 reveals that buyers exchange deposits and bank equity for consumption only, if the first-best consumption is affordable with the existing bank liabilities. On the other hand, if bank liabilities are not sufficient to support the first-best consumption, like case 2, buyers deplete all of the bank liabilities in their portfolios, then spend a fraction of good assets to reach their optimal consumption.

This proposition describes one type of equilibria where banks do not remove the private information problem regarding means of payment. Hence, trades in the decentralized market are hence involved with signaling. To separate themselves from bad asset holders, good asset holders prefer spending bank liabilities first, and keep some real assets. The intuition behind the asset retention is reminiscent of Dodd Frank regulation in the U.S. Good asset holders hint sellers:

“ We are willing to share risks with you if we keep some assets, so you should think the offer as though it is from good asset holders.” Good assets are not fully used in exchange for goods, i.e., there is a liquidity constraint on good assets, so the economy is stuck with a lower aggregate liquidity, and welfare is lower than economies in which the private information is removed.[4]

Proposition 3.3. *(The liquidity-price relationship)*
When banks do not remove private information problems, good assets are subject to liquidity constraints, and the asset prices are such that $q_a^h < q_a^\ell + \beta(k_h - k_\ell)$.

Participation conditions (3.8)-(3.9) imply asset holders accept banks' offer, if they gain more from asset trades than bringing all endowed assets to trade in the DM. Now, since payment arrangements are involved with private information, good assets may be held but not spent. The information friction impedes good assets' contribution to trade, and hence, the price which is lower than an upper bound; i.e., $q_a^h < q_a^\ell + \beta(k_h - k_\ell)$, is sufficient to attract good asset holders.

[4]Suppose we consider an economy in which there is no banks to provide perfect recognizable assets to illustrate the equilibrium allocation is inefficient due to the private information. If $k_\ell A^E \geq c_1(x_1^*)$, then the buyer ℓ consumes x_1^*, but the buyer h consumes $x_1^h < x_1^*$. If $k_\ell A^E < c_1(x_1^*)$, then DM outputs in all matches are inefficiently low, that is, $x_1^h < x_1^\ell < x_1^*$. Proposition 3.2 show that holding bank liabilities helps buyers overcome the inefficiency caused by the private information problem. If $(1+i)d + k_e e \geq c_1(x_1^*)$, buyers achieve the first best consumption, and the buyer h does not use real assets to make payment.

3.6 Financial intermediaries and welfare

In this section, we want to exam the welfare-improving role of banks when the economy has recognizable assets such that $(1+i)d + k_e e < c_1(x_1^*)$. The welfare measurement is defined as follows,

$$\mathbb{W} = \sum_{t \geq 0} \beta^t \int_{j \in h,\ell} [u_1(x_1^j) - c_1(x_1^j)]dj + \sum_{t \geq 0} \beta^t \int_{j \in h,\ell} [u_2(x_2^j) - h^j]dj.$$

The first integral on the right side corresponds to buyers' consumption net of sellers' disutility of production in random and bilateral matches in the DM. The second term is the consumption net of the disutility of labor inputs for buyers.

Our numerical examples show that when banks buy all of one type of assets, the economy enjoys higher welfare than otherwise (eqli.1 and eqli.2 in Figure 3). The reason is that, banks not only provide recognizable and safe assets to facilitate trades, but also eliminate the private information problem regarding the means of payment. When the quality difference of real assets becomes smaller, there also exists an equilibrium where the means of payment in the DM is subject to private information (eqli.3 in Figure 3). In this type of equilibrium, buyers need to retain a fraction of good assets as a signaling device, which reduces the aggregate liquidity and, therefore, welfare. The equilibrium (see eqli.1 in Figure 3) in which banks buy all of good assets entails the highest welfare. The reason is that, deposits and equity yield high dividends with certainty, and banks issue more liabilities in order to buy all of good assets of which the price is higher than

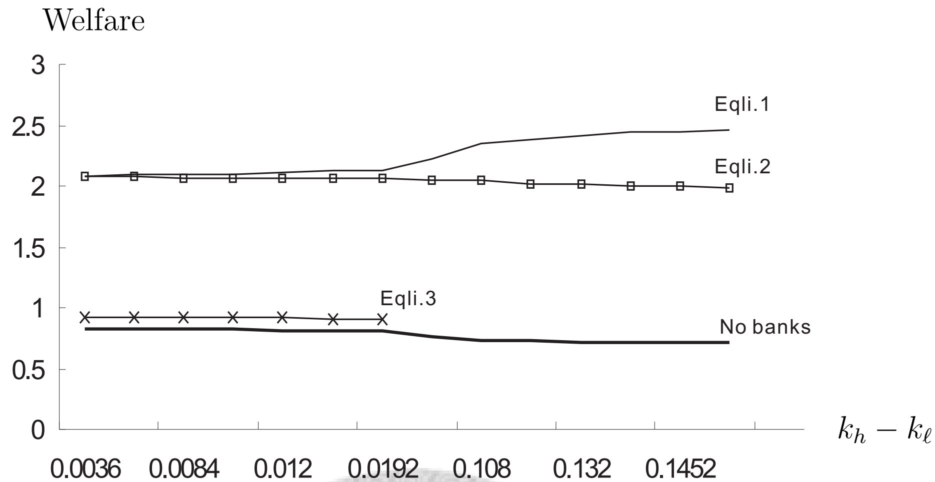

Figure 3.3. Welfare

$*$ We set up utility function $u_1(x_1) = \frac{x_1^{0.8}}{0.8}$, cost function $c_1(x_1) = 0.7x_1$; the parameter value for the benchmark are $A^E = 4.5, \bar{k}_\ell = 0.5, \eta = 0.4$, and $\xi = 0.5$.

$**$ The number of equilibrium is specified by banks' portfolios comprised of good assets and bad assets.

eqli.1: all good assets and no bad ones; eqli.2: all bad assets and no good ones; eqli.3: a fraction of real assets, and more bad assets than good ones.

that of bad assets. Banks' asset transformation thus creates liabilities with the highest total value, i.e., $k_e e + (1+i)d$. These assets are traded for output in the decentralized market, and therefore, the economy achieves the highest welfare.

3.7 Conclusion

This paper constructs a model of financial intermediaries based on recognizability concerning assets. Information frictions motivate the existence of banks, which do not have informational advantage over individuals, but provide recognizable assets to serve as means of payment. Equilibria are characterized by banks' investing strategies. When banks buy all of one type of assets, means of payment circulating in the decentralized market are not threatened by the private information problem. In some equilibria where banks buy a fraction of both types of assets, then a pecking-order payment arrangement derived from the signaling game: deposits and bank equity are preferred means of payment, whereas good assets are subject to a liquidity constraint. Good assets may be held but not spent for signaling purposes. It is found that asset prices reflect assets' liquidity. In case good assets face liquidity constraints, then prices of good assets are lower than an upper bound. Deposits and bank equity have same returns, because they are liquid equally. Banks buy bad assets at a price higher than a threshold, if bank liabilities lead higher marginal benefit from trade, which is weighted by dividends, for bad asset holders.

From the numerical analysis, we find the welfare level depends on whether banks eliminate the private information regarding means of payment. If banks remove the private information problem, economies enjoy higher welfare than otherwise. If the information problem is not eliminated, a fraction of good assets is retained as a signaling device; and deposits and bank equity command lower returns. As a result, aggregate liquidity is lower than economies where information frictions are removed, so is welfare. When banks buy all good assets and eliminate the private information problem entails the highest welfare. The reason is that, people would assign the highest value on banks' liabilities, compared to other equilibria.

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3.8 Appendix A.

3.8.1 Payment arrangement without private information

Nash bargaining solution

Any offer made by a buyer j is such that

$$\max_{x_1^j, y_a^j, y_d^j, y_e^j} [u_1(x_1) - k_j y_a - (1+i) y_d - \kappa_e y_e] \tag{3.45}$$

$$s.t. - c_1(x_1) + k_j y_a + (1+i) y_d + k_e y_e \geq 0, \tag{3.46}$$

$$y_a \leq (A^E - \omega_j), \quad y_d \leq d_j, \quad y_e \leq e_j, \tag{3.47}$$

The solution to (3.45) – (3.47):

if $k_j(A^E - \omega_j) + (1+i)d_j + k_e e_j \geq c_1(x_1^*)$,

$$x_1^j = x_1^*,$$

$$c_1(x_1^*) = k_j y_a + (1+i) y_d + k_e y_e,$$

if $k_j(A^E - \omega_j) + (1+i)d_j + k_e e_j < c_1(x_1^*)$,

$$c_1(x_1) = k_j(A^E - \omega_j) + (1+i)d_j + k_e e_j,$$

$$y_a^j = (A^E - \omega_j), \quad y_d^j = d_j, \quad y_e^j = e_j.$$

Buyers' value function in the DM

The expected lifetime utility of a buyer entering the DM with a units of real assets, d units of deposits, e units of bank equities and a private signal k_j, is

$$V_j^b(a, d, e; k_j) = u_1[x_1(y_a, y_d, y_e)] + W^b(a - y_a, d - y_d, e - y_e; k_j), \tag{3.48}$$

By the linearity of W^b, (3.48) becomes

$$V_j^b(a,d,e;k_j) = S_j(a,d,e) + k_j a + (1+i)d + k_e e + W^b(0,0,0;k_j),\ j = h,\ell \quad (3.49)$$

where $S_j(a,d,e)$ is the buyer's surplus in the DM if the dividend state is j, that is,

$$S_j(a,d,e) \equiv u_1[x_1(y_a,y_d,y_e)] - k_j y_a(a,d,e;k_j) - (1+i)y_d(a,d,e;k_j) - k_e y_e(a,d,e;k_j).$$

$$\begin{aligned} S_j(a,d,e) &= u_1 \circ c_1^{-1}[k_j a_j + (1+i)d_j + k_e e_j] - k_j a_j - (1+i)d_j - k_e e_j \quad if\ x_1^j < c_1(x_1^*) \\ &= u_1(x_1^*) - c_1(x_1^*) \quad otherwise. \end{aligned}$$

Let $S_{j,1}(a,d,e)$, $S_{j,2}(a,d,e)$ and $S_{j,3}(a,d,e)$ be the partial derivatives of the buyer's surplus function for $j \in \{h,\ell\}$, which represent the marginal contributions of real assets, deposits and bank equities, respectively, to the gains from trade for a buyer.

$$S_{j,1}(a,d,e) = [\frac{u_1'(x_1^j)}{c_1'(x_1^j)} - 1]k_j; \quad (3.50)$$

$$S_{j,2}(a,d,e) = [\frac{u_1'(x_1^j)}{c_1'(x_1^j)} - 1](1+i); \quad (3.51)$$

$$S_{j,3}(a,d,e) = [\frac{u_1'(x_1^j)}{c_1'(x_1^j)} - 1]k_e. \quad (3.52)$$

Buyers' portfolio choice (Proof of Lemma 3.2)

Using condition (3.50)-(3.52) to rewrite buyers' portfolio choice.

$$-\frac{q_a^h - k_h\beta}{\beta} + [\frac{u_1'(x_1^h)}{c_1'(x_1^h)} - 1]k_h \leq 0 \quad \text{``}=\text{''} \quad if \quad a_h > 0, \tag{3.53}$$

$$-\frac{q_e - k_e\beta}{\beta} + [\frac{u_1'(x_1^h)}{c_1'(x_1^h)} - 1]k_e \leq 0 \quad \text{``}=\text{''} \quad if \quad e_h > 0, \tag{3.54}$$

$$-\frac{q_d - (1+i)\beta}{\beta} + [\frac{u_1'(x_1^h)}{c_1'(x_1^h)} - 1](1+i) \leq 0 \quad \text{``}=\text{''} \quad if \quad d_h > 0, \tag{3.55}$$

$$-\frac{q_a^\ell - k_\ell\beta}{\beta} + [\frac{u_1'(x_1^\ell)}{c_1'(x_1^\ell)} - 1]k_\ell \leq 0 \quad \text{``}=\text{''} \quad if \quad a_\ell > 0, \tag{3.56}$$

$$-\frac{q_e - k_e\beta}{\beta} + [\frac{u_1'(x_1^\ell)}{c_1'(x_1^\ell)} - 1]k_e \leq 0 \quad \text{``}=\text{''} \quad if \quad e_\ell > 0, \tag{3.57}$$

$$-\frac{q_d - (1+i)\beta}{\beta} + [\frac{u_1'(x_1^\ell)}{c_1'(x_1^\ell)} - 1](1+i) \leq 0 \quad \text{``}=\text{''} \quad if \quad d_\ell > 0, \tag{3.58}$$

3.8.2 Payment arrangement under private information

The private information regarding means of payment comes along when two type buyers have the same portfolios, that is, $a_h = a_\ell = a$, $d_h = d_\ell = d$ and $e_h = e_\ell = e$.

Nash bargaining solution

Any offer made by a buyer holding bad assets is such that

$$\max_{x_1^\ell, y_a^\ell, y_d^\ell, y_e^\ell} [u_1(x_1) - k_\ell y_a - (1+i)y_d - k_e y_e] \tag{3.59}$$

$$s.t. \quad -c_1(x_1) + k_\ell y_a + (1+i)y_d + k_e y_e \geq 0, \tag{3.60}$$

$$y_a \leq a, \quad y_d \leq d, \quad y_e \leq e. \tag{3.61}$$

Buyers with bad assets make the complete information offer, $y_{a,b} \geq 0$, that is always acceptable and irrespective of sellers' beliefs. If $c_1(x^*) \leq k_\ell a + (1+i)d + k_e e$, the solution to (3.59)-(3.61) is

$$\begin{aligned} x_1^\ell &= x_1^*, \\ c_1(x_1^*) &= k_\ell y_a^\ell + (1+i)y_d^\ell + k_e y_e^\ell. \end{aligned}$$

If $c_1(x^*) > k_\ell a + (1+i)d + k_e e$, the solution to (3.59)-(3.61) is

$$\begin{aligned} y_a^\ell &= a, \\ y_d^\ell &= d, \\ y_e^\ell &= e, \\ x_1^\ell &= c^{-1}[k_\ell y_a^\ell + (1+i)y_d^\ell + k_e y_e^\ell]. \end{aligned}$$

Any offer made by a buyer with good assets is such that

$$\max_{x_1^h, y_a^h, y_d^h, y_e^h} [u_1(x_1) - k_h y_a - (1+i)y_d - k_e y_e] \tag{3.62}$$

$$s.t. \quad -c_1(x_1) + k_h y_a + (1+i)y_d + k_e y_e \geq 0, \tag{3.63}$$

$$u_1(x_1) - k_\ell y_a - (1+i)y_d - k_e y_e \leq u_1(x_1^\ell) - c_1(x_1^\ell), \tag{3.64}$$

$$y_a \leq a, \quad y_d \leq d, \quad y_e \leq e. \tag{3.65}$$

Buyers with good assets make offer $(x_1^h, y_a^h, y_d^h, y_e^h)$ to maximize their gains from trade. Constraint (3.63) shows that the buyer makes an offer so that the seller believes he is a good asset holder. The incentive-compatibility condition, revealed in (3.64), according to which a buyer with bad assets does not want to mimic the offer of a buyer with good assets.

There is a solution $(x_1^h, y_a^h, y_d^h, y_e^h)$ to (3.62)-(3.65), and it has the following properties:

1. If $(1+i)d + k_e e \geq c_1(x_1^*)$, then

$$\begin{aligned} x_1^h &= x_1^* && (3.66) \\ k_h y_a^h + (1+i)y_d^h + k_e y_e^h &= c_1(x_1^*) && (3.67) \\ y_a^h &= 0. && (3.68) \end{aligned}$$

2. If $(1+i)d + k_e e < c_1(x_1^*)$, then $y_d^h = d, y_e^h = e$ and (x_1^h, y_a^h) satisfies

$$\begin{aligned} k_h y_a^h &= c_1(x_1^h) - (1+i)d - k_e e, && (3.69) \\ u_1(x_1^\ell) - c_1(x_1^\ell) &= u_1(x_1^h) - c_1(x_1^h) + (1 - \frac{k_\ell}{k_h})[c_1(x_1^h) - (1+i)d - k_e e], && (3.70) \end{aligned}$$

   where $x_1^\ell = min\{x_1^*, c_1^{-1}[k_\ell a + (1+i)d + k_e e]\}$. Moreover, if $a > 0$, then $x_1^h < x_1^\ell$ and $y_a^h < a_h$.

Proof of proposition 3.2

The part 1 of the proposition.

- If (3.63) binds, but (3.64) does not, implying that $x_1^h = min[x_1^*, c_1^{-1}((1 + i)y_d + k_e y_e)] \geq x_1^\ell$. Rewriting condition (3.64),

$$u_1(x_1^h) - c_1(x_1^h) + y_a^h(k_h - k_\ell) \leq u_1(x_1^\ell) - c_1(x_1^\ell),$$

  if $y_a^h > 0$, then condition (3.64) is violated. If $(1+i)d + k_e e \geq c_1(x_1^*)$, then $x_1^h = x_1^\ell = x_1^*$, that impiles $y_a^h = 0$, as the case 1 of the proposition.

- Consider that (3.64) binds, but (3.63) does not. Rewriting the objective function,

$$U^h = \max_{y_a}[y_a(k_\ell - k_h) + U^\ell],$$

since $y_a^h = 0$, $U^h \leq \max_{y_a}[u_1(x_1) - (1+i)y_d - k_e y_e]$, subject to $-c_1(x_1) + (1+i)y_d + k_e y_e = 0$. Hence, if and only of $(1+i)d + k_e e \geq c_1(x_1^*)$, $U^h = u_1(x_1^\ell) - c_1(x_1^\ell)$. That means, $x_1^h = x_1^*$ and $(1+i)y_d + k_e y_e = c_1(x_1^*)$.

The part 2 of the proposition.

- Proof of that when $(1+i)d + k_e e < c_1(x_1^*)$, then $y_d = d$ and $y_e = e$.

  If $y_d \leq d$ and $y_e \leq e$, then $x_1^h = x_1^*$, $y_a^h = 0$ and $(1+i)y_d + k_e y_e + k_h y_a^h = c_1(x_1^*)$. It is a solution for the problem (3.62)-(3.65), but $(1+i)y_d + k_e y_e = c_1(x_1^*)$ is in contradiction with $(1+i)d + k_e e < c_1(x_1^*)$.

- Proof of that $y_a^h < a_h$.

  For all $x_1^h \in [0, x_1^\ell]$ the right side of (3.70) is strictly increasing. When $x_1^h = 0$, it is nonpositive; when $x_1^h = x_1^\ell$, it is larger than $u_1(x_1^\ell) - c_1(x_1^\ell)$, if $c_1(x_1^\ell) > (1+i)d + k_e e$. If $(1+i)d + k_e e < c_1(x_1^*)$, then $c_1(x_1^\ell) = min[c_1(x_1^*), k_\ell a_\ell + (1+i)d + k_e e]$. The objective function (3.62) is decreasing in x_{1^h} for any solution to (3.70). Hence, the unique solution in $(0, x_1^\ell)$ gives a maximum to the problem (3.62)-(3.65). Given a x_1^h, y_a^h is determined by (3.69). $c_1(x_1^h) = (1+i)d + k_e e + k_h y_a^h < c_1(x_1^\ell) = (1+i)d + k_e e + k_\ell y_a^\ell \leq (1+i)d + k_e e + k_\ell a_\ell$.

Buyers' surplus function in the DM

For the buyer ℓ, his surplus function in the DM is

$$
\begin{aligned}
S_\ell(a,d,e) &= u_1 \circ c_1^{-1}[k_\ell a + (1+i)d + k_e e] - k_\ell a - (1+i)d - k_e e \quad if\ x_1^\ell < c_1(x_1^*) \\
&= u_1(x_1^*) - c_1(x_1^*) \quad otherwise.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
S_{\ell,1}(a,d,e) &= [\frac{u_1'(x_1^\ell)}{c_1'(x_1^\ell)} - 1]k_\ell & (3.71) \\
S_{\ell,2}(a,d,e) &= [\frac{u_1'(x_1^\ell)}{c_1'(x_1^\ell)} - 1](1+i) & (3.72) \\
S_{\ell,3}(a,d,e) &= [\frac{u_1'(x_1^\ell)}{c_1'(x_1^\ell)} - 1]k_e & (3.73)
\end{aligned}
$$

From the bargaining solution for the buyer h, if $(1+i)d + k_e e < c_1(x_1^*)$, then x_1^h solves (3.70), i.e.,

$$
u_1(x_1^h) - c_1(x_1^h) + (1 - \frac{k_\ell}{k_h})[c_1(x_1^h) - (1+i)d - k_e e] = S_\ell(a,d,e).
$$

Totally differentiating the equation above,

$$
\begin{aligned}
[u_1'(x_1^h) - \tfrac{k_\ell}{k_h} c_1'(x_1^h)] \quad & \frac{dx_1^h}{da} = S_{\ell,1} \\
[u_1'(x_1^h) - \tfrac{k_\ell}{k_h} c_1'(x_1^h)] \quad & \frac{dx_1^h}{da} = (1 - \frac{k_\ell}{k_h})(1+i) + S_{\ell,2} \\
[u_1'(x_1^h) - \tfrac{k_\ell}{k_h} c_1'(x_1^h)] \quad & \frac{dx_1^h}{da} = (1 - \frac{k_\ell}{k_h})k_e + S_{\ell,3},
\end{aligned}
$$

for all $(1+i)d + k_e e < c_1(x_1^*)$, $\frac{dx_1^h}{dd} > 0$ and $\frac{dx_1^h}{de} > 0$; and for all (a,d,e) such that $k_\ell a + (1+i)d + k_e e < c_1(x_1^*)$, $\frac{dx_1^h}{da} > 0$. From sellers' participation constraint holds at equality so that $S_h(a,d,e) = u_1(x_1^h) - c_1(x_1^h)$. Hence,

$$\begin{aligned}
S_{h,1}(a,d,e) &= [u_1^{'}(x_1^h) - c_1^{'}(x_1^h)]\frac{dx_1^h}{da} = \Delta(x_1^h)S_{\ell,1} \\
S_{h,2}(a,d,e) &= [u_1^{'}(x_1^h) - c_1^{'}(x_1^h)]\frac{dx_1^h}{dd} = \Delta(x_1^h)[(1-\frac{k_\ell}{k_h})(1+i) + S_{\ell,2}] \\
S_{h,3}(a,d,e) &= [u_1^{'}(x_1^h) - c_1^{'}(x_1^h)]\frac{dx_1^h}{de} = \Delta(x_1^h)[(1-\frac{k_\ell}{k_h})k_e + S_{\ell,3}]
\end{aligned}$$

where

$$\Delta(x_1) \equiv \frac{u_1^{'}(x_1) - c_1^{'}(x_1)}{u_1^{'}(x_1) - \frac{k_\ell}{k_h}c_1^{'}(x_1)} = 1 - \frac{1-\frac{k_\ell}{k_h}}{\frac{u_1^{'}(x_1)}{c_1^{'}(x_1)} - \frac{k_\ell}{k_h}}.$$

3.9 Appendix B.

There are several possible asset holdings.

I. $a_h < a_\ell$:

1. $\{(0, d, e), (A^E, d, e)\}$;
2. $\{(0, d, e), (a_\ell, d, e)\}$;
3. $\{(a_h, d, e), (a_\ell, d, e)\}$;

II. $a_h > a_\ell$:

4. $\{(A^E, d, e), (0, d, e)\}$;
5. $\{(a_h, d, e), (0, d, e)\}$;
6. $\{(a_h, d, e), (a_\ell, d, e)\}$;

III. $a_h = a_\ell$:

7. $\{(0, d, e), (0, d, e)\}$;
8. $\{(a, d, e), (a, d, e)\}$;
9. $\{(A^E, d, e), (A^E, d, e)\}$.

We sort these possible equilibria into two type by the structure of information in the DM. If only good assets or bad assets appears in the DM, then the payment arrangement is not threatened by the private information, that is revealed by

Equilibrium 1, 2, 4, and 5. If banks do not buy all good assets or bad assets, then in a DM meeting the seller is matched with a buyer who may hold good assets or bad assets. As a result, there is private information regarding means of payment. Moreover, the bargaining game is a signaling game. Equilibrium 3, 6, 8 and 9 belong to this type equilibrium.

Equilibrium 1. & 2. (Proof of Lemma 3.3)

If the portfolio in equilibrium 1 is an equilibrium choice, then

$$\begin{aligned} x_1^h &= c_1^{-1}[(1+i)d + k_e e] \\ x_1^\ell &= c_1^{-1}[k_\ell A^E + (1+i)d + k_e e] \end{aligned}$$

$$\Rightarrow x_1^h < x_1^\ell.$$

The buyer h does not bring any real asset into the DM, that is, $a_h = 0$, i.e.,

$$\frac{q_a^h - \beta k_h}{\beta} > [\frac{u_1^{'}(x_1^h)}{c_1^{'}(x_1^h)} - 1]k_h,$$

the buyer ℓ brings real assets into the DM, so

$$\frac{q_a^\ell - \beta k_\ell}{\beta} = [\frac{u_1^{'}(x_1^\ell)}{c_1^{'}(x_1^\ell)} - 1]k_\ell.$$

Since $x_1^h < x_1^\ell$,

$$\frac{q_a^h - \beta k_h}{\beta} > [\frac{u_1^{'}(x_1^h)}{c_1^{'}(x_1^h)} - 1]k_h > [\frac{u_1^{'}(x_1^\ell)}{c_1^{'}(x_1^\ell)} - 1]k_\ell = \frac{q_a^\ell - \beta k_\ell}{\beta}.$$

Hence, we obtain $q_a^h > q_a^\ell$ under equilibria 1 and 2.

In the following step, we want to show buyers' equilibrium portfolio is consist with banks' portfolios, that means banks buy more good assets than bad assets, i.e., $\omega_h > \omega_\ell \Rightarrow a_h < a_\ell$. Recall banks' problem in the asset market, banks construct incentive constraints to prevent the buyer j from imitating the buyer $-j$. In equilibrium 1. the incentive constraints become

$$(a) \qquad q_a^h A^E + \beta V_h(0,d,e) \geq q_a^\ell 0 + \beta V_h(A^E,d,e),$$

$$(b) \qquad q_a^\ell 0 + \beta V_\ell(A^E,d,e) \geq q_a^h A^E + \beta V_\ell(0,d,e).$$

To prove condition (a) binds in the solution, suppose it does not, that is, suppose that $q_a^h A^E + \beta V_h(0,d,e) > q_a^\ell 0 + \beta V_h(A^E,d,e) > 0$. Under the hypothesis we have $q_a^h A^E > 0$. By choosing a smaller q_a^h, banks will increase the value of their objective function and still satisfy all the constraints. In particular, for small decrease in q_a^h the participation constraint of the buyer h is still satisfied. Because it holds with strict inequality under the hypothesis. Also the incentive constraint for the buyer h is satisfied and the incentive constraint for the buyer ℓ is relaxed. Therefore, a lower q_a^h is feasible and it is profitable for banks. From the incentive constraint for the buyer ℓ, we get that $\beta V_\ell(A^E,d,e) - \beta V_\ell(0,d,e) \geq q_a^h A^E > 0$ and hence $a_\ell > a_h$.[5]

On the other hand, suppose $q_a^h A^E + \beta V_h(0,d,e) > \beta V_h(A^E,d,e)$, by choosing a small increase in ω_ℓ such that $q_a^h A^E + \beta V_h(0,d,e) > q_a^\ell \omega_\ell + \beta V_h(a_\ell,d,e) > \beta V_h(A^E,d,e)$. This hypothesis illustrates that the incentive constraint for the buyer h does not bind in equilibrium 2. Similarly, by choosing a smaller q_a^h,

[5] $V_j(a,d,e) = S_j(a,d,e) + k_j a + (1+i)d + k_e e + W(0,0,0;k_j)$, we would show the surplus functions in the DM is weakly monotone increasing in the agents' asset holdings.

banks will increase the value of their objective function and still satisfy all the constraints. A lower q_a^h is feasible and it is profitable for banks. From the incentive constraint for the buyer ℓ, we get that $\beta V_\ell(a_\ell, d, e) - \beta V_\ell(0, d, e) \geq q_a^h A^E - q_a^\ell \omega_\ell > 0$ and hence $a_\ell > a_h$.

Equilibrium 4. & 5. (Proof of proposition 3.1)

Suppose equilibrium 4 is an equilibrium outcome, then

$$\begin{aligned} x_1^h &= c_1^{-1}[k_h a_h (1+i)d + k_e e] \\ x_1^\ell &= c_1^{-1}[(1+i)d + k_e e] \end{aligned}$$

$$\Rightarrow x_1^\ell < x_1^h.$$

The buyer h brings real assets into the DM, that is, $a_h > 0$, i.e.,

$$\frac{q_a^h - \beta k_h}{\beta} = [\frac{u_1'(x_1^h)}{c_1'(x_1^h)} - 1]k_h,$$

the buyer ℓ does not bring real assets into the DM, so

$$\frac{q_a^\ell - \beta k_\ell}{\beta} > [\frac{u_1'(x_1^\ell)}{c_1'(x_1^\ell)} - 1]k_\ell.$$

Price schedule can be:

1. if $\frac{\sigma_h k_h}{\sigma_\ell k_\ell} > 1$, then $q_a^h > q_a^\ell$, where $\sigma_j = \left(\frac{u_1'(x_1^j)}{c_1'(x_1^j)} - 1\right), \quad j = h, \ell.$

2. if $\frac{\sigma_h k_h}{\sigma_\ell k_\ell} < 1$, then $\frac{q_a^\ell - \beta k_\ell}{\beta} > \frac{q_a^h - \beta k_h}{\beta}$.

In equilibrium 4., the incentive constraints become

$$(a) \qquad q_a^h 0 + \beta V_h(A^E, d, e) \geq q_a^\ell A^E + \beta V_h(0, d, e),$$

$$(b) \qquad q_a^\ell A^E + \beta V_\ell(0,d,e) \geq q_a^h 0 + \beta V_\ell(A^E,d,e),$$

Equilibrium 4 has the symmetric solution to Equilibrium 1. Hence, the equilibrium portfolio $\{(A^E,d,e),(0,d,e)\}$, and prices q_a^h and q_a^ℓ satisfy

$$\begin{aligned} q_a^h 0 + \beta V_h(A^E,d,e) &\geq q_a^\ell A^E + \beta V_h(0,d,e), \\ q_a^\ell A^E + \beta V_\ell(0,d,e) &= q_a^h 0 + \beta V_\ell(A^E,d,e). \end{aligned}$$

We show that the incentive constraint for the buyer ℓ binds. Suppose that $q_a^\ell A^E + \beta V_\ell(0,d,e) > q_a^h 0 + \beta V_\ell(A^E,d,e)$. Similarly, a lower q_a^ℓ is feasible and it is profitable for banks. From the incentive constraint for the buyer h, we get that $\beta V_h(A^E,d,e) - \beta V_h(0,d,e) \geq q_a^\ell A^E > 0$ and hence $a_h > a_\ell$.

Equilibrium 3. & 6. (Proof of proposition 3.3)

Both good and bad real assets appear in the DM in equilibria 3 and 6. In order to signal the quality of the real assets, the buyer h will not transfer real assets more than a_ℓ.

$$\begin{aligned} x_1^h &= c_1^{-1}[k_h y_a^h + (1+i)d + k_e e] \\ x_1^\ell &= c_1^{-1}[k_\ell a_\ell + (1+i)d + k_e e] \end{aligned}$$

$$\Rightarrow x_1^h < x_1^\ell.$$

The buyer ℓ brings real asset into the DM, that is, $a_\ell > 0$, i.e.,

$$\frac{q_a^\ell - k_\ell \beta}{\beta} = [\frac{u_1'(x_1^\ell)}{c_1'(x_1^\ell)} - 1]k_\ell.$$

Also the buyer h brings real assets into the DM,

$$\begin{aligned}\frac{q_a^h - k_h\beta}{\beta} &= k_\ell\left(\frac{u_1'(x_1^h)}{c_1'(x_1^h)} - 1\right)\left(\frac{u_1'(x_1^\ell)/c_1'(x_1^\ell) - 1}{u_1'(x_1^h)/c_1'(x_1^h) - k_\ell/k_h}\right)\\ &= \left(\frac{\frac{u_1'(x_1^h)}{c_1'(x_1^h)} - 1}{\frac{u_1'(x_1^h)}{c_1'(x_1^h)} - \frac{k_\ell}{k_h}}\right)\frac{q_a^\ell - k_\ell\beta}{\beta},\end{aligned}$$

We show the net cost of holding bad assets is higher than holding good assets, i.e., $(q_a^\ell - k_\ell\beta) > (q_a^h - k_h\beta)$, since $\frac{\frac{u_1'(x_1^h)}{c_1'(x_1^h)}-1}{\frac{u_1'(x_1^h)}{c_1'(x_1^h)}-\frac{k_\ell}{k_h}} < 1$. The equilibrium portfolio $\{(a_h, d, e), (a_\ell, d, e)\}$ and prices q_{a^h} and q_a^ℓ under equilibrium 3 satisfy

$$\begin{aligned}q_a^h\omega_h + \beta V_h(a_h, d, e) &= q_a^\ell\omega_\ell + \beta V_h(a_\ell, d, e),\\ q_a^\ell\omega_\ell + \beta V_\ell(a_\ell, d, e) &\geq q_a^h\omega_h + \beta V_\ell(a_h, d, e).\end{aligned}$$

The logic of proving the incentive constraint for the buyer h is similar to that in equilibrium 1. From the incentive constraint for the buyer ℓ, $\beta V_\ell(a_\ell, d, e) - \beta V_\ell(a_h, d, e) \geq q_a^h\omega_h - q_a^\ell\omega_\ell > 0$, because $q_a^h\omega_h - q_a^\ell\omega_\ell = \beta V_h(a_\ell, d, e) - \beta V_h(a_h, d, e) > 0$. Hence, $a_\ell > a_h$

The equilibrium portfolio $\{(a_h, d, e), (a_\ell, d, e)\}$ and prices q_{a^h} and q_a^ℓ under equilibrium 6 satisfy

$$\begin{aligned}q_a^h\omega_h + \beta V_h(a_h, d, e) &\geq q_a^\ell\omega_\ell + \beta V_h(a_\ell, d, e),\\ q_a^\ell\omega_\ell + \beta V_\ell(a_\ell, d, e) &= q_a^h\omega_h + \beta V_\ell(a_h, d, e).\end{aligned}$$

The logic of proving the incentive constraint for the buyer ℓ is similar to that in equilibrium 1. From the incentive constraint for the buyer h, $\beta V_h(a_h, d, e) -$

$\beta V_h(a_\ell, d, e) \geq q_a^\ell \omega_\ell - q_a^h \omega_h > 0$, because $q_a^\ell \omega_\ell - q_a^h \omega_h = \beta V_\ell(a_h, d, e) - \beta V_\ell(a_\ell, d, e) > 0$. Hence, $a_h > a_\ell$.

Equilibrium 8. & 9.

Buyers carry the same portfolio into the DM, i.e., $(a_h, d, e) = (a_\ell, d, e) = (a, d, e)$ in equilibrium 8.

$$\begin{aligned} x_1^h &= c_1^{-1}[k_h y_a^h + (1+i)d + k_e e] \\ x_1^\ell &= c_1^{-1}[k_\ell a + (1+i)d + k_e e] \end{aligned}$$

to signal the quality of real assets, the buyer h will retain a part of his real assets, that is, $y_a^h < a$, so $x_1^h < x_1^\ell$. The buyer ℓ brings real asset into the DM, that is, $a_\ell > 0$, i.e.,

$$\frac{q_a^\ell - k_\ell \beta}{\beta} = [\frac{u_1'(x_1^\ell)}{c_1'(x_1^\ell)} - 1]k_\ell.$$

Also the buyer h brings real assets into the DM,

$$\begin{aligned} \frac{q_a^h - k_h \beta}{\beta} &= k_\ell \left(\frac{u_1'(x_1^h)}{c_1'(x_1^h)} - 1 \right) \left(\frac{u_1'(x_1^\ell)/c_1'(x_1^\ell) - 1}{u_1'(x_1^h)/c_1'(x_1^h) - k_\ell/k_h} \right) \\ &= \left(\frac{\frac{u_1'(x_1^h)}{c_1'(x_1^h)} - 1}{\frac{u_1'(x_1^h)}{c_1'(x_1^h)} - \frac{k_\ell}{k_h}} \right) \frac{q_a^\ell - k_\ell \beta}{\beta}, \end{aligned}$$

where $(q_a^\ell - k_\ell \beta) > (q_a^h - k_h \beta)$, since $\frac{\frac{u_1'(x_1^h)}{c_1'(x_1^h)} - 1}{\frac{u_1'(x_1^h)}{c_1'(x_1^h)} - \frac{k_\ell}{k_h}} < 1$. The equilibrium portfolio (a, d, e) and prices q_{a^h} and q_a^ℓ under equilibrium 8 satisfy

$$\begin{aligned} q_a^h \omega_h + \beta V_h(a_h, d, e) &= q_a^\ell \omega_\ell + \beta V_h(a_\ell, d, e), \\ q_a^\ell \omega_\ell + \beta V_\ell(a_\ell, d, e) &= q_a^h \omega_h + \beta V_\ell(a_h, d, e). \end{aligned}$$

Two binding incentive constraints show that $a_h = a_\ell$.

Chapter 4

Conclusion

In Chapter 2 we integrate the dual role of banks, as a provider of credit and payment instruments, in an economy with a moral hazard concern for checks. The moral hazard problem results in an upper bound on the quantity of deposits that can be traded for consumption goods. The upper bound works as a liquidity constraint on deposits related to the counterfeiting cost and costs of holding fiat money. This theme links banks' roles, aggregate liquidity and allocations of an economy with moral hazard. If the liquidity constraint does not bind, deposits are as liquid as fiat money, and they do not pay interest. Alternatively, if the constraint binds, deposits dominate fiat money in the rate of return. Also, the binding constraint offers the transmission mechanism of monetary policy to improve the aggregate liquidity and output. Higher inflation relaxes the constraint through raising costs of holding money, it induces agents to be more willing to make deposits rather than produce fraudulent checks. Banks thus provide more loanable funds at a lower interest rate. That leads to a higher aggregate liquidity and output.

In Chapter 3 we offer a theoretical model related to asset liquidity, in which the quality of real assets is private information to holders, and banks issue fully recognizable deposits and bank equity to serve as means of payment by asset transformation. The model allows us to spell out the relationship between assets' liquidity and their characteristics, and study welfare-improving roles for financial intermediaries. We obtain the following insights. First, the economy achieves higher welfare when banks buy all of one type of assets than otherwise. This is so because banks not only provide recognizable assets to facilitate trades, but also eliminate the private information problem regarding the means of payment. If banks do not eliminate the private information problem, some good assets are held for signaling purposes. Moreover, we find the economy reaches the highest welfare among equilibria, if banks buy all good assets and eliminate the private information.

In this dissertation, private information concerning means of payment provides roles for financial intermediaries to facilitate trades, and we illustrate channels through which financial intermediaries improve aggregate liquidity and output. Financial intermediaries themselves may work as discipline to secure assets' acceptability as payments, if they are able to detect fraudulent activities, like the case we consider in Chapter 2. Meanwhile, monetary policy can improve allocations through raising the cost of committing the fraudulent activity, which lifts the liquidity constraint on deposits. In Chapter 3, we show that financial intermediaries can improve aggregate liquidity and welfare by asset transformation and screening assets' quality, even if they cannot verify assets' authenticity or assets' true value.