

國立臺灣大學電機資訊學院電信工程學研究所

碩士論文

Graduate Institute of Communication Engineering  
College of Electrical Engineering and Computer Science  
National Taiwan University

Master Thesis



自適應全雙工系統效能分析

Performance Analysis of Adaptive Full-Duplex System

黃云芯

Yun-Hsin Huang

指導教授：蘇炫榮 博士

Advisor: Hsuan-Jung Su, Ph.D.

中華民國 107 年 8 月

Aug. 2018



## 摘要

全雙工技術被視為有能力成為下一代無線網路的技術，因為全雙工技術有提升頻譜效率的能力。在這篇論文中，我們分析多用戶多細胞的無線系統效能，在這樣的系統下，每一個基地台都可以支援全雙工或半雙工技術，而使用者只支援半雙工技術。然而，只使用全雙工的基地台來傳輸資訊會對每一個傳輸鏈造成干擾，這會限制全雙工技術改善的能力。因此，我們提出一個自適應全雙工系統，此系統包含全雙工細胞和半雙工細胞。根據基地台和使用者的位置關係，我們分析此基地台做全雙工的機率，利用分析的機率結果，我們可以得到目前的環境中，此細胞適合的傳輸模式。在提出的自適應全雙工系統中，細胞會選擇較大的總傳輸率模式。因此根據此分析機率，我們可以自適應地調整細胞的傳輸模式。除此之外，我們將提出的系統總傳輸率和全部半雙工系統、全部全雙工的系統比較，因此得到自適應性全雙工系統的效能分析。

關鍵字 - 全雙工；多細胞系統；模式選擇；自適應系統；隨機幾何



## Abstract

Full-duplex (FD) is being considered as a potential technology for the next generation of wireless networks due to its ability to boost the spectral efficiency (SE). In the thesis, we analyze the performance of a multi-user multi-cell wireless system. In the system, each base station (BS) supports full-duplex operation or half-duplex (HD) operation and user equipment (UE) only supports HD operation. However, using only FD BS increases the aggregate interference on each communication link, which limits the capacity improvement. Therefore, we propose an adaptive FD system, consists of FD cells and HD cells. Based on the position of BS and UE, we analyze the probability that FD is preferred. From the analysis result, we can obtain what operation is suitable for the cell of current environment. According to the probability, we determine the mode of the cell adaptively. The adaptive FD system can select the better mode to maximize the system sum rate. By deriving the distribution, we investigate the sum rate of the system, and compare with all HD system and all FD system. Then, the adaptive performance of the proposed system is derived.

**Key Words** -Full-duplex; Multi-cell system; Mode selection; Adaptive system; Stochastic geometry.



# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Background . . . . .	1
1.2	Overview of Thesis . . . . .	4
1.3	Notations . . . . .	4
<b>2</b>	<b>System Model and Problem Formulation</b>	<b>5</b>
2.1	System Model . . . . .	5
2.2	Problem Description . . . . .	9
<b>3</b>	<b>Analysis of the Probability</b>	<b>12</b>
3.1	Probability Analysis . . . . .	12
3.1.1	The uplink probability . . . . .	19
3.1.2	The downlink probability . . . . .	22
3.2	The probability of FD BS . . . . .	25
3.2.1	The probability that FD is preferred . . . . .	25
<b>4</b>	<b>Simulation Results</b>	<b>27</b>
<b>5</b>	<b>Conclusion</b>	<b>32</b>
	<b>Bibliography</b>	<b>33</b>

**A PGFL of PPP**

**B Suburban environment**



**36**

**38**



# List of Figures

1.1	HD and FD multi-cell scenarios. . . . .	2
2.1	An adaptive multi-user multi-cell system model. . . . .	9
4.1	The relationship of symbols in a cell. . . . .	27
4.2	Simulation and analytical results of probability that FD is preferred versus different BS density. . . . .	29
4.3	Sum rate versus BS density with different system. . . . .	31



# List of Tables

4.1	The table of simulation parameters. . . . .	28
-----	---	----



# List of Abbreviations

**FD** Full-duplex

**HD** Half-duplex

**SE** Spectral Efficiency

**SI** Self Interference

**SIC** Self-interference Cancellation

**BS** Base Station

**UE** User Equipment

**CDF** Cumulative Distribution Function

**PDF** Probability Distribution Function

**SNR** Signal-to-Noise Ratio

**SINR** Signal-to-Interference-Noise Ratio

**PPP** Poisson Point Processes

**SPPP** Spatial Poisson Point Processes



**PGFL** Probability Generating Functional

**TDD** Time-division Duplex

**FDD** Frequency-division Duplex





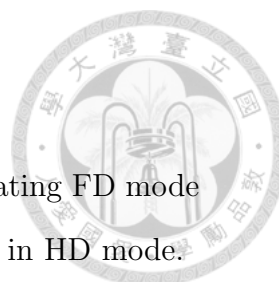
# Chapter 1

## Introduction

### 1.1 Background

Conventionally, radio transceivers are subject to half-duplex (HD) technology because of the receive and transmit chains. Current HD radios all use orthogonal signaling dimensions, i.e., time-division duplex (TDD) or frequency-division duplex (FDD), to avoid interference and achieve bidirectional communication.

Full-duplex (FD) enables downlink transmission and uplink transmission over the same frequency simultaneously [1]. Comparing with HD, FD has the potential of doubling the spectral efficiency (SE) due to its efficient bandwidth utilization. In the respect, FD has the potential of a significant breakthrough in the design of a novel 5th Generation (5G) radio. However, FD performance is affected by self interference (SI) caused by downlink transmission in FD radios. Recent advances in self-interference cancellation (SIC) allow suppressing this loopback interference by using digital-domain [2] [3], analog-domain [4] [5] and propagation-domain [6] [7] methods. To successfully achieve SIC, the FD circuit has a higher cost. Consider the high cost,



the practical implementation of FD transmission is only operating FD mode on the base station (BS). And user equipment (UE) operates in HD mode.

Although FD has the capability of improving SE, using only FD increases the aggregate interference on each communication link, which limits the capacity improvement. Fig. 1.1 shows HD mode and FD mode in multi-cell system. The figure reveals FD mode will bring more interference compares with HD mode. Because every downlink and uplink transmissions operate at same time and same frequency. Downlink user is not only affected interference of downlink transmissions. Uplink user is not only affected interference of uplink transmissions. In FD multi-cell scenario, they are affected interference from all neighbor cells.

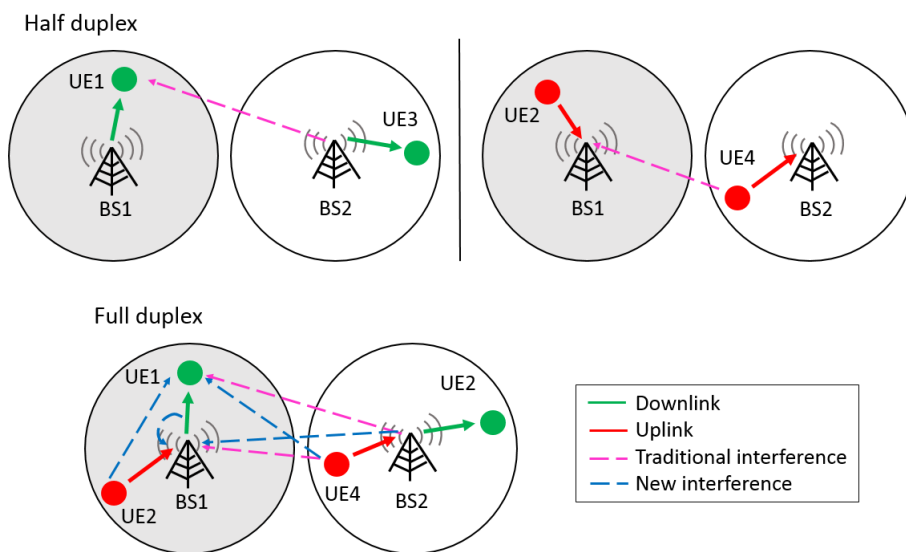
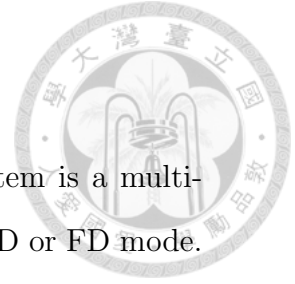


Figure 1.1: HD and FD multi-cell scenarios.



Therefore, we propose an adaptive FD system. The system is a multi-user multi-cell system. In the system, each BS can operate HD or FD mode. The UE only operates HD mode. Consider appropriate mode for a cell is affected by Rayleigh fading and interference [8], we propose a mode is selected adaptively based on the position of UE. Assume BS has fading information and the position of downlink UE and uplink UE, and we calculate sum rate by the information. The distance between nodes in wireless network can be evaluated by the received signal power [9] [10]. Based on the calculated sum rate, we analyze probability that FD is preferred. We define the probability that FD is preferred based on the condition FD rate is larger than HD rate. The condition ensures the cell has higher performance. According to the probability of FD, we determine what duplex is suitable for the BS. With different position of UE, the mode of BS could be allocated adaptively by the analytical probability. [11] proposed a mixed full and half duplex cell network, but it doesn't consider the probability that FD is preferred and it doesn't have the adaptive mode selection of BS. [12] and [13] proposed a single cell system which can operate FD or HD mode. Their mode selection are only based on channel fading. But in our proposed system, we not only consider the fading but also the position of UE. We also analyze the performance of the proposed system by the adaptive allocation. We calculate the sum rate of the adaptive FD system and compare with pure HD system and pure FD system. To the best of our knowledge, there is no research solving adaptive FD mode allocation in a multi-user multi-cell system.



## 1.2 Overview of Thesis

The thesis is organized as follows. Chapter 2 illustrates the system model and describes the probability of FD BS with the condition of FD rate is larger than HD rate. In Chapter 3, The probability is analyzed. In Chapter 3, we separate the probability to two parts and discuss them. The analysis is also compared with the simulation results in Chapter 4. At the end, we give the conclusion in Chapter 5.

## 1.3 Notations

$\mathbb{R}^n$  represents the  $n$ -dimensional real space.  $\mathcal{L}(\cdot)$  represents the Laplace transform.  $\mathcal{P}[\cdot]$  is the probability of an event. And the expectation operator is denoted by  $\mathbb{E}[\cdot]$ .  $\arctan(\cdot)$  represents the inverse tangent.



## Chapter 2

# System Model and Problem Formulation

### 2.1 System Model

We consider a multi-user multi-cell network system. Fig. 2.1 illustrates the network model. Each BS operates HD mode or FD mode and UE only can operate HD mode in this system. In the HD mode, the transmission can be either in downlink direction or in uplink direction. We assume BS has the information of position of UE by the received signal power. According to the information of position, we define the signal to interference and noise ratio (SINR) in the system. Through the SINR, we obtain the FD rate and HD rate. Based on the two rates and follow mode selected rule, the probability that FD is preferred can be derived.

We model the location of BSs and the locations of UEs are independent Spatial Poisson Point Processes (SPPP)  $\Phi_B$  and  $\Phi_U$  with density  $\lambda_B$  and  $\lambda_U$ , where  $\lambda_U > \lambda_B$  [11]. We assume each UE is associate with the closest BS and each BS has one downlink UE and one uplink UE at the same subframe. We define the probability of a BS to be FD mode, downlink HD mode and



uplink HD mode as  $\rho_F$ ,  $\rho_D$  and  $\rho_U$ , with  $\rho_F + \rho_D + \rho_U = 1$ . We also assume  $\rho_D = \rho_U = (1 - \rho_F)/2$ . By using a thinning process, the distribution of FD BSs, downlink HD BSs and uplink HD BSs also follow independent SPPP  $\Phi_B^F$ ,  $\Phi_B^D$  and  $\Phi_B^U$ , with densities  $\rho_F \lambda_B$ ,  $\rho_D \lambda_B$  and  $\rho_U \lambda_B$ , respectively.

The standard power loss propagation model is used with path loss exponent  $\alpha$ . We assume BS and UE experience only Rayleigh fading. Assume Rayleigh fading channels are subject to independent and identically distribution (i.i.d.). In this case, the received power at a typical node a distance  $r$  from its BS is  $hr^{-\alpha}$  where random variable  $h$  follows an exponential distribution with mean 1, which we denote as  $h \sim \exp(1)$ .

We present the SINR in our system. The SINR of FD mode downlink UE is given by

$$SINR_d^{FD} = \frac{P_{BS}hr^{-\alpha}}{N_0 + I_D^D + I_U^D + P_{UE}g_{u_0}D_{u_0}^{-\alpha}} \quad (2.1)$$

where  $P_{BS}$  is the power of BS,  $P_{UE}$  is the power of UE.  $h$  is Rayleigh fading for the BS to the downlink UE link,  $r$  is the distance between the downlink UE and its serving BS ( $b_0$ ),  $\alpha$  is the path loss exponent,  $N_0$  is Gaussian noise.  $g_{u_0}$  and  $D_{u_0}$  are the Rayleigh fading and distance between downlink UE and uplink UE ( $u_0$ ) in the cell, respectively.  $I_D^D$  is the total interference from the downlink transmissions.  $I_U^D$  is the total interference from the uplink transmissions except the uplink transmission from its own cell. The total interference from the downlink transmissions includes all FD cells ( $\Phi_B^F \setminus b_0$ ) and all HD downlink cells ( $\Phi_B^D$ ). The total interference from the downlink



transmissions can be defined as

$$I_D^D = P_{BS} \sum_{b \in \{\Phi_B^D \cup \Phi_B^F \setminus b_0\}} h_b R_b^{-\alpha} \quad (2.2)$$

where  $h_b$  is Rayleigh fading for another BS ( $b$ ) to the downlink UE link,  $R_b$  is the distance between the downlink UE and neighbor BS ( $b$ ). The total interference from the uplink transmissions except its own cell contains all FD cells ( $\Phi_B^F \setminus u_0$ ) and all HD uplink cells ( $\Phi_B^U$ ). The total interference from the uplink transmissions can be defined as

$$I_U^D = P_{UE} \sum_{u \in \{\Phi_B^U \cup \Phi_B^F \setminus u_0\}} g_u D_u^{-\alpha} \quad (2.3)$$

where  $g_u$  is Rayleigh fading for uplink UE ( $u$ ) which is in other cell to the downlink UE link,  $D_u$  is the distance between the downlink UE and uplink UE ( $u$ ) of other cells. The uplink interference from its own cell is written in  $P_{UE} g_{u_0} D_{u_0}^{-\alpha}$ .

The SINR of FD mode uplink UE is given by

$$SINR_u^{FD} = \frac{P_{UE} g d^{-\alpha}}{N_0 + I_D^U + I_U^U + I_{SI}} \quad (2.4)$$

where  $g$  is Rayleigh fading for the uplink UE to the BS link,  $d$  is the distance between the uplink UE and its serving BS ( $b_0$ ).  $I_D^U$  is the total interference from the downlink transmissions except its own cell which includes all FD cells ( $\Phi_B^F \setminus b_0$ ) and all HD downlink cells ( $\Phi_B^D$ ). The downlink interference from its own cell is calculated in  $I_{SI}$ .  $I_{SI}$  is the self-interference from the downlink transmission by its serving BS.  $I_U^U$  is the total interference from the uplink transmissions which includes all FD cells ( $\Phi_B^F \setminus u_0$ ) and all HD uplink cells ( $\Phi_B^U$ ). The total interference from the downlink transmissions except its





own cell can be defined as

$$I_D^U = P_{BS} \sum_{b \in \{\Phi_B^D \cup \Phi_B^F \setminus b_0\}} h'_b L_b^{-\alpha} \quad (2.5)$$

where  $h'_b$  is Rayleigh fading for another cell downlink transmission to the uplink UE link.  $L_b$  is the distance between the uplink UE and BS ( $b$ ) in other cell. The total interference from the uplink transmissions can be defined as

$$I_U^U = P_{UE} \sum_{u \in \{\Phi_B^U \cup \Phi_B^F \setminus u_0\}} g'_u X_u^{-\alpha} \quad (2.6)$$

where  $g'_u$  is Rayleigh fading for another cell uplink transmission to the uplink UE link.  $X_u$  is the distance between the uplink UE and other cell uplink UE ( $u$ ).

The SINR of HD mode is similar to SINR of FD mode. The SINR of HD downlink UE and uplink UE are given by

$$SINR_d^{HD} = \frac{P_{BS} h r^{-\alpha}}{N_0 + I_D^D + I_U^D} \quad (2.7)$$

$$SINR_u^{HD} = \frac{P_{UE} g d^{-\alpha}}{N_0 + I_D^U + I_U^U} \quad (2.8)$$

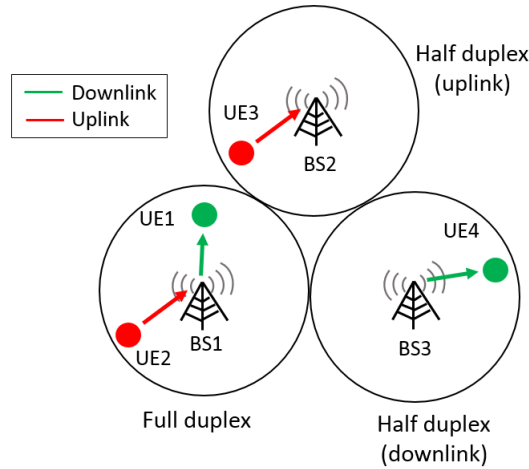


Figure 2.1: An adaptive multi-user multi-cell system model.

## 2.2 Problem Description

Most previous works show FD mode is a promising technique to improve the performance of wireless communication systems. However, using only FD BS increases the aggregate interference, which limits the capacity improvement. If the effect of interference is more than the improvement, the performance of FD mode might not be greater than HD mode.

Therefore, we propose an adaptive FD system, consists of FD cells and HD cells. We investigate the relation of duplex mode and distance between UE and BS. We assume the distance of downlink UE and its serving BS ( $r$ ), the distance of uplink UE and its serving BS ( $d$ ) and the distance between the downlink UE and uplink UE ( $D_{u_0}$ ) are known. Based on the known position of UE, we analyze the probability that FD is preferred. If the analytical probability is over 0.5, the cell operates FD mode. If the analytical probability is less than 0.5, the cell would operate HD mode. Through the



analysis of probability that FD is preferred, we choose suitable mode for the cell.

For analyzing the probability, we define the condition of BS operating FD mode is based on achieving larger capacity compares with HD mode. We assume the BS has been connected with one downlink UE and one uplink UE, which are a group. We calculate the sum rate of the group in FD mode and HD mode, respectively. We compare the two rates, if the rate of FD mode is larger than HD mode. Then, the BS operates FD mode. If the rate of HD mode is larger than FD mode, the BS would operate HD mode. According to the assumption, the condition of FD mode can be obtained as

$$\begin{aligned} \log_2(1 + SINR_d^{FD}) + \log_2(1 + SINR_u^{FD}) > \\ \frac{1}{2}(\log_2(1 + SINR_d^{HD}) + \log_2(1 + SINR_u^{HD})) \end{aligned} \quad (2.9)$$

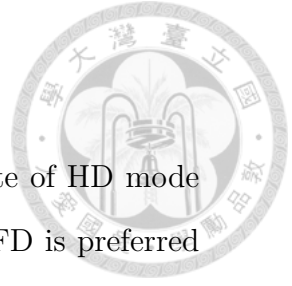
Based on the above condition, the probability of FD BS can be obtained as

$$\mathcal{P} \left[ \log(1 + SINR_d^{FD}) + \log(1 + SINR_u^{FD}) > \frac{1}{2} \log(1 + SINR_d^{HD}) + \frac{1}{2} \log(1 + SINR_u^{HD}) \right] \quad (2.10)$$

We take an exponential on both sides. Then, the probability of FD BS can be expressed as

$$\mathcal{P} \left[ (1 + SINR_d^{FD})(1 + SINR_u^{FD}) > \sqrt{(1 + SINR_d^{HD})(1 + SINR_u^{HD})} \right] \quad (2.11)$$

We investigate the probability that FD is preferred in a multi-user multi-cell system. Follow the condition, the performance of the cell can be guaranteed to be the best among FD mode or HD mode. Based on the condition,



if the interference effect by other cells is serious, the sum rate of HD mode would be larger than FD mode. Then, the probability that FD is preferred would be small. Vice versa.



## Chapter 3

# Analysis of the Probability

In the chapter, we analyze the probability that FD is preferred. The probability that FD is preferred follows the condition FD rate is larger than HD rate. Based on the previous defined SINR, we obtain the FD rate and HD rate, and the probability can be derived. In the process of analyzing the probability, we use some symbols and approximations to express the probability. Moreover, we separate the probability to two parts for analyzing easily. Final, combining the two parts and using iterative method to obtain the probability that FD is preferred.

### 3.1 Probability Analysis

With the SINR of different modes and the probability of FD BS are discussed in chapter 2, (2.11) can be rewritten as

$$\mathcal{P} \left[ \left(1 + \frac{P_B s h r^{-\alpha}}{N_0 + I_D^D + I_U^D + P_{UE} g_{u_0} D_{u_0}^{-\alpha}}\right) \left(1 + \frac{P_{UE} g d^{-\alpha}}{N_0 + I_D^U + I_U^U + I_{SI}}\right) > \sqrt{\left(1 + \frac{P_B s h r^{-\alpha}}{N_0 + I_D^D + I_U^D}\right) \left(1 + \frac{P_{UE} g d^{-\alpha}}{N_0 + I_D^U + I_U^U}\right)} \right] \quad (3.1)$$

The first step to analyze the probability of FD BS is fixing the variables



in the inequality. Except the known information ,  $r$ ,  $d$  and  $D_{u_0}$ , we fix other variables. Fixing the variables needs to product their probability distribution functions (PDFs) and integrate them. Then, the probability can be written as

$$\int \int \int \int \int \int \int \int \mathbb{1} \left[ \left( 1 + \frac{P_{BS}hr^{-\alpha}}{N_0 + I_D^D + I_U^D + P_{UE}g_{u_0}D_{u_0}^{-\alpha}} \right) \cdot \left( 1 + \frac{P_{UE}gd^{-\alpha}}{N_0 + I_D^U + I_U^U + I_{SI}} \right) > \sqrt{\left( 1 + \frac{P_{BS}hr^{-\alpha}}{N_0 + I_D^D + I_U^D} \right) \left( 1 + \frac{P_{UE}gd^{-\alpha}}{N_0 + I_D^U + I_U^U} \right)} \right] \cdot f_{g_{u_0}}f_hf_gf_{I_D^D}f_{I_U^D}f_{I_D^U}f_{I_U^U}d(I_D^D)d(I_U^D)d(I_D^U)d(I_U^U)d(g_{u_0})d(h)d(g) \quad (3.2)$$

Where  $g_{u_0}$ ,  $h$  and  $g$  are Rayleigh fading. Then,  $f_{g_{u_0}}$ ,  $f_h$  and  $f_g$  are all exponential distribution with mean 1.  $I_D^D$  and  $I_U^D$  are downlink interference and uplink interference at downlink UE.  $I_D^U$  and  $I_U^U$  are downlink interference and uplink interference at uplink UE. The PDF of interference ( $I$ ) with Rayleigh fading and suburban environment ( $\alpha = 4$ ) in Poisson network is [14]

$$f_I(x) = \frac{\lambda}{4} \left( \frac{\pi}{x} \right)^{3/2} \exp \left( -\frac{\pi^4 \lambda^2}{16x} \right) \quad (3.3)$$

Where  $\lambda$  is the density for the distances of the points of a two-dimensional Poisson Point Processes (PPP).  $f_{I_D^D}$ ,  $f_{I_U^D}$ ,  $f_{I_D^U}$  and  $f_{I_U^U}$  all follow the PDF.

The second step, we focus on the probability in the integral. To easier analyze, we use some symbols to express the integral. We replace  $P_{BS}hr^{-\alpha}$  and  $P_{UE}gd^{-\alpha}$  with  $A$  and  $B$ . Substitute  $x$  and  $y$  for  $N_0 + I_D^D + I_U^D$  and



$N_0 + I_D^U + I_U^U$ . Combine the above, the probability can be express as

$$\begin{aligned} & \mathcal{P} \left[ \left(1 + \frac{P_{BS}hr^{-\alpha}}{N_0 + I_D^D + I_U^D + P_{UE}g_{u_0}D_{u_0}^{-\alpha}}\right) \left(1 + \frac{P_{UE}gd^{-\alpha}}{N_0 + I_D^U + I_U^U + I_{SI}}\right) > \right. \\ & \quad \left. \sqrt{\left(1 + \frac{P_{BS}hr^{-\alpha}}{N_0 + I_D^D + I_U^D}\right) \left(1 + \frac{P_{UE}gd^{-\alpha}}{N_0 + I_D^U + I_U^U}\right)} \right] \\ & = \mathcal{P} \left[ \left(1 + \frac{A}{x + P_{UE}g_{u_0}D_{u_0}^{-\alpha}}\right) \left(1 + \frac{B}{y + I_{SI}}\right) > \sqrt{\left(1 + \frac{A}{x}\right) \left(1 + \frac{B}{y}\right)} \right] \quad (3.4) \end{aligned}$$

We assume the self-interference ( $I_{SI}$ ) is cancelled perfectly [15]. Then, the probability can be expressed as

$$\mathcal{P} \left[ \left(1 + \frac{A}{x + P_{UE}g_{u_0}D_{u_0}^{-\alpha}}\right) \left(1 + \frac{B}{y}\right) > \sqrt{\left(1 + \frac{A}{x}\right) \left(1 + \frac{B}{y}\right)} \right] \quad (3.5)$$

The next step is relaxing the root in right side of the inequality. By using arithmetic and geometric mean inequality of two positive numbers, the root in the inequality can be relaxed as

$$\sqrt{\left(1 + \frac{A}{x}\right) \left(1 + \frac{B}{y}\right)} \leq \frac{\left(1 + \frac{A}{x}\right) + \left(1 + \frac{B}{y}\right)}{2} \quad (3.6)$$

Based on the above, we get the relaxing probability is smaller than original probability. It can be written as

$$\begin{aligned} & \mathcal{P} \left[ \left(1 + \frac{A}{x + P_{UE}g_{u_0}D_{u_0}^{-\alpha}}\right) \left(1 + \frac{B}{y}\right) > \sqrt{\left(1 + \frac{A}{x}\right) \left(1 + \frac{B}{y}\right)} \right] \\ & \geq \mathcal{P} \left[ \left(1 + \frac{A}{x + P_{UE}g_{u_0}D_{u_0}^{-\alpha}}\right) \left(1 + \frac{B}{y}\right) > \frac{\left(1 + \frac{A}{x}\right) + \left(1 + \frac{B}{y}\right)}{2} \right] \quad (3.7) \end{aligned}$$



In order to obtain the probability easily, we analyze the approximate probability. The probability can be expressed as

$$\mathcal{P} \left[ \left(1 + \frac{A}{x + P_{UE}g_{u_0}D_{u_0}^{-\alpha}}\right)\left(1 + \frac{B}{y}\right) > \frac{\left(1 + \frac{A}{x}\right) + \left(1 + \frac{B}{y}\right)}{2} \right] \quad (3.8)$$

We arrange the probability. Then, the probability can be rewritten as

$$\mathcal{P} \left[ \frac{B}{y}(x + 2A + P_{UE}g_{u_0}D_{u_0}^{-\alpha}) > \frac{A}{x}(P_{UE}g_{u_0}D_{u_0}^{-\alpha} - x) \right] \quad (3.9)$$

We know  $B$  equals  $P_{UE}gd^{-\alpha}$  and  $y$  equals  $N_0 + I_D^U + I_U^U$ . Then, we get  $\frac{B}{y}$  is the SINR of uplink UE. As the same,  $A$  equals  $P_{BS}hr^{-\alpha}$  and  $x$  equals  $N_0 + I_D^D + I_U^D$ ,  $\frac{A}{x}$  is the SINR of downlink UE. We replace  $\frac{B}{y}$  with  $SINR_u$  and  $\frac{A}{x}$  with  $SINR_d$ . The probability can be written as

$$\mathcal{P} [SINR_u(x + 2A + P_{UE}g_{u_0}D_{u_0}^{-\alpha}) > SINR_d(P_{UE}g_{u_0}D_{u_0}^{-\alpha} - x)] \quad (3.10)$$

From the above definition,  $x$ ,  $A$  and  $P_{UE}g_{u_0}D_{u_0}^{-\alpha}$  are all related to downlink UE. So, we arrange the probability. Then, it can be rewritten as

$$\mathcal{P} \left[ SINR_u > SINR_d \frac{(P_{UE}g_{u_0}D_{u_0}^{-\alpha} - x)}{(x + 2A + P_{UE}g_{u_0}D_{u_0}^{-\alpha})} \right] \quad (3.11)$$

Replace  $x$  and  $A$  with  $(N_0 + I_D^D + I_U^D)$  and  $P_{BS}hr^{-\alpha}$ . The probability can be written as

$$\mathcal{P} \left[ SINR_u > SINR_d \frac{P_{UE}g_{u_0}D_{u_0}^{-\alpha} - (N_0 + I_D^D + I_U^D)}{(N_0 + I_D^D + I_U^D) + 2P_{BS}hr^{-\alpha} + P_{UE}g_{u_0}D_{u_0}^{-\alpha}} \right] \quad (3.12)$$

Now, we focus on the fraction of the above probability

$$\frac{P_{UE}g_{u_0}D_{u_0}^{-\alpha} - (N_0 + I_D^D + I_U^D)}{(N_0 + I_D^D + I_U^D) + 2P_{BS}hr^{-\alpha} + P_{UE}g_{u_0}D_{u_0}^{-\alpha}} \quad (3.13)$$





From the previous definition of  $I_D^D$  and  $I_U^D$ . They are

$$\begin{aligned} I_D^D &= P_{BS} \sum_{b \in \{\Phi_B^D \cup \Phi_B^F \setminus b_0\}} h_b R_b^{-\alpha} \\ I_U^D &= P_{UE} \sum_{u \in \{\Phi_B^U \cup \Phi_B^F \setminus u_0\}} g_u D_u^{-\alpha} \end{aligned} \quad (3.14)$$

The key factor of affecting (3.13) is path loss effect. There is relationship with distance ( $D_{u_0}$ ,  $R_b$ ,  $D_u$  and  $r$ ). We know that  $D_{u_0}$  is distance of downlink UE and uplink UE in the main cell.  $R_b$  is distance of downlink UE and neighbor cell BS.  $D_u$  is distance of downlink UE and neighbor cell uplink UE.  $r$  is distance of donlink UE and its serving BS. Because the distance of  $R_b$  and  $D_u$  are faraway, we approximate the fraction as

$$\frac{P_{UE} g_{u_0} D_{u_0}^{-\alpha} - (N_0 + I_D^D + I_U^D)}{(N_0 + I_D^D + I_U^D) + 2P_{BS} h r^{-\alpha} + P_{UE} g_{u_0} D_{u_0}^{-\alpha}} \approx \frac{P_{UE} g_{u_0} D_{u_0}^{-\alpha}}{2P_{BS} h r^{-\alpha} + P_{UE} g_{u_0} D_{u_0}^{-\alpha}} \quad (3.15)$$

We divide the numerator and denominator of approximate fraction with  $P_{UE} g_{u_0} D_{u_0}^{-\alpha}$ . It can be rewritten as

$$\frac{P_{UE} g_{u_0} D_{u_0}^{-\alpha}}{2P_{BS} h r^{-\alpha} + P_{UE} g_{u_0} D_{u_0}^{-\alpha}} = \frac{1}{\frac{2P_{BS} h r^{-\alpha}}{P_{UE} g_{u_0} D_{u_0}^{-\alpha}} + 1} \quad (3.16)$$

From the definition of  $r$  and  $D_{u_0}$ , the ranges of them are

$$\begin{aligned} 0 < r &\leq R \\ 0 < D_{u_0} &\leq 2R \end{aligned} \quad (3.17)$$

Where  $R$  is radius of cell. Based on the above ranges, the path loss effect of  $r$  is larger than  $D_{u_0}$ . We also assume the power of BS is large than UE ( $P_{BS} > P_{UE}$ ). To sum up,  $\frac{2P_{BS} h r^{-\alpha}}{P_{UE} g_{u_0} D_{u_0}^{-\alpha}}$  is larger than 1. We approximate the



fraction again. It can be written as

$$\begin{aligned} \frac{1}{\frac{2P_{BS}hr^{-\alpha}}{P_{UE}g_{u_0}D_{u_0}^{-\alpha}} + 1} &\approx \frac{1}{\frac{2P_{BS}hr^{-\alpha}}{P_{UE}g_{u_0}D_{u_0}^{-\alpha}}} \\ &= \frac{P_{UE}g_{u_0}D_{u_0}^{-\alpha}}{2P_{BS}hr^{-\alpha}} \end{aligned} \quad (3.18)$$

We take the approximate fraction into (3.12).

$$\mathcal{P} \left[ \text{SINR}^U > \text{SINR}^D \frac{P_{UE}g_{u_0}D_{u_0}^{-\alpha}}{2P_{BS}hr^{-\alpha}} \right] \quad (3.19)$$

Now, we replace  $\text{SINR}^U$  with  $\frac{P_{UE}gd^{-\alpha}}{N_0 + I_D^U + I_U^U}$  and replace  $\text{SINR}^D$  with  $\frac{P_{BS}hr^{-\alpha}}{N_0 + I_D^D + I_U^D}$ .

Then, the probability can be rewritten as

$$\mathcal{P} \left[ \frac{P_{UE}gd^{-\alpha}}{N_0 + I_D^U + I_U^U} > \frac{P_{BS}hr^{-\alpha}}{N_0 + I_D^D + I_U^D} \frac{P_{UE}g_{u_0}D_{u_0}^{-\alpha}}{2P_{BS}hr^{-\alpha}} \right] \quad (3.20)$$

Arrange the probability, it can be written as

$$\mathcal{P} \left[ \frac{gd^{-\alpha}}{N_0 + I_D^U + I_U^U} > \frac{1}{2} \frac{g_{u_0}D_{u_0}^{-\alpha}}{N_0 + I_D^D + I_U^D} \right] \quad (3.21)$$

Note that it is not easy to obtain the expression of the above probability directly. We follow [16], consider a simplification. We assume the interference from neighbor cell at downlink UE and at uplink UE are independent. The Rayleigh fading channels are all independent. According to the distance of uplink UE and its serving BS ( $d$ ) and distance of downlink UE and uplink UE ( $D_{u_0}$ ) are known, we use threshold  $t$  to separate the probability to two parts. Then, the probability can be expressed as

$$\int_0^{\infty} \mathcal{P} \left[ \frac{gd^{-\alpha}}{N_0 + I_D^U + I_U^U} > t \right] \mathcal{P} \left[ \frac{1}{2} \frac{g_{u_0}D_{u_0}^{-\alpha}}{N_0 + I_D^D + I_U^D} < t \right] dt \quad (3.22)$$



Next, we analyze the two parts of probability, the uplink probability and the downlink probability, respectively. The two parts of probability are given as

$$\mathcal{P} \left[ \frac{gd^{-\alpha}}{N_0 + I_D^U + I_U^U} > t \right] \quad (3.23)$$

$$\mathcal{P} \left[ \frac{1}{2} \frac{g_{u_0} D_{u_0}^{-\alpha}}{N_0 + I_D^D + I_U^D} < t \right] \quad (3.24)$$



### 3.1.1 The uplink probability

Now, we focus on the uplink part of probability. The uplink probability is

$$\mathcal{P} \left[ \frac{gd^{-\alpha}}{N_0 + I_D^U + I_U^U} > t \right] \quad (3.25)$$

We arrange the probability and follow that the propagation is affected by Rayleigh fading, which is exponentially distribution. From the fact  $g \sim \exp(1)$ . The probability can be expressed as

$$\Pr \left[ g > t \frac{N_0 + I_D^U + I_U^U}{d^{-\alpha}} \right] \quad (3.26)$$

$$= \exp \left( -\frac{t}{d^{-\alpha}} (N_0 + I_D^U + I_U^U) \right) \quad (3.27)$$

Next, we take Laplace transform of (3.27). Then, the exponential can be written as

$$\mathcal{L} \left( \exp \left( -\frac{t}{d^{-\alpha}} (N_0 + I_D^U + I_U^U) \right) \right) \quad (3.28)$$

$$= \exp \left( -\frac{tN_0}{d^{-\alpha}} \right) \mathcal{L}_{I_D^U + I_U^U}(s) \quad (3.29)$$

Where  $\mathcal{L}_{I_D^U + I_U^U}(s)$  is Laplace transform for the total interference at the uplink UE from all the downlink transmissions and from all the uplink transmissions.

$s$  is  $\frac{tN_0}{d^{-\alpha}}$ .



Now, we focus on the Laplace transform  $\mathcal{L}_{I_D^U+I_U^U}(s)$ . According to the previous definition of  $I_D^U$  and  $I_U^U$ , the Laplace transform can be written as

$$\begin{aligned} \mathcal{L}_{I_D^U+I_U^U}(s) = \mathbb{E}_{\Phi_B^F \cup \Phi_B^D \cup \Phi_B^U, h'_b, g'_u} [exp\{-s \sum_{b \in \Phi_B^D \cup \Phi_B^F \setminus b_0} P_{BS} h'_b L_b^{-\alpha}\} \cdot \\ exp\{-s \sum_{u \in \Phi_B^U \cup \Phi_B^F \setminus u_0} P_{UE} g'_u X_u^{-\alpha}\}] \end{aligned} \quad (3.30)$$

Follow from the independent of SPPP  $\Phi_B^F$ ,  $\Phi_B^D$  and  $\Phi_B^U$ . The above equation can be rewritten as

$$\begin{aligned} \mathcal{L}_{I_D^U+I_U^U}(s) = \mathbb{E}_{\Phi_B^F \cup \Phi_B^D, h'_b} [exp\{-s \sum_{b \in \Phi_B^D \cup \Phi_B^F \setminus b_0} P_{BS} h'_b L_b^{-\alpha}\}] \cdot \\ \mathbb{E}_{\Phi_B^F \cup \Phi_B^U, g'_u} [exp\{-s \sum_{u \in \Phi_B^U \cup \Phi_B^F \setminus u_0} P_{UE} g'_u X_u^{-\alpha}\}] \end{aligned} \quad (3.31)$$

We divide (3.31) into downlink interference part and uplink interference part.

Downlink interference part is given as

$$\mathbb{E}_{\Phi_B^F \cup \Phi_B^D, h'_b} [exp\{-s \sum_{b \in \Phi_B^D \cup \Phi_B^F \setminus b_0} P_{BS} h'_b L_b^{-\alpha}\}] \quad (3.32)$$

Uplink interference part is given as

$$\mathbb{E}_{\Phi_B^F \cup \Phi_B^U, g'_u} [exp\{-s \sum_{u \in \Phi_B^U \cup \Phi_B^F \setminus u_0} P_{UE} g'_u X_u^{-\alpha}\}] \quad (3.33)$$

Follow from the probability generating functional (PGFL) of PPP. The downlink interference part can be further written as

$$exp\{-2\pi\lambda_1 \int_0^\infty \frac{sP_{BS}v^{-\alpha}}{\mu + sP_{BS}v^{-\alpha}} v dv\} \quad (3.34)$$

*Proof.* See Appendix A □

Where  $\lambda_1$  is the density of all downlink BSs which includes FD mode downlink BSs and HD mode downlink BSs. From the previous definition,  $\lambda_1$  is  $\lambda_B(\rho_F + \rho_D)$ .



We consider the system environment is in suburban environment ( $\alpha = 4$ ). Then, the downlink interference part of  $\mathcal{L}_{I_D^U+I_U^U}(s)$  can be rewritten as

$$\exp\left\{-2\pi\lambda_1\frac{1}{2(sP_{BS})^{-1/2}}(\arctan(\infty) - \arctan(0))\right\} \quad (3.35)$$

*Proof.* See Appendix B □

Follow the same method, the uplink interference part of  $\mathcal{L}_{I_D^U+I_U^U}(s)$  can be rewritten as

$$\exp\left\{-2\pi\lambda_2\frac{1}{2(sP_{UE})^{-1/2}}(\arctan(\infty) - \arctan(0))\right\} \quad (3.36)$$

Where  $\lambda_2$  is  $\lambda_B(\rho_F + \rho_U)$  which is the density of all uplink BSs.

Combine (3.29) with (3.35) and (3.36). We obtain the first part of probability can be rewritten as

$$\exp\left(-\frac{tN_0}{d^{-4}}\right) \exp\left(-\pi(\arctan(\infty) - \arctan(0))\left(\lambda_1(sP_{BS})^{1/2} + \lambda_2(sP_{UE})^{1/2}\right)\right) \quad (3.37)$$

We know  $(\arctan(\infty) - \arctan(0))$  is  $\frac{\pi}{2}$ . Replace  $(\arctan(\infty) - \arctan(0))$  with  $\frac{\pi}{2}$  and take  $s$  into the probability. Then, the probability can be written as

$$\exp\left(-\frac{tN_0}{d^{-4}}\right) \exp\left(-\frac{\pi^2}{2}\left(\lambda_1\left(\frac{t}{d^{-4}}P_{BS}\right)^{1/2} + \lambda_2\left(\frac{t}{d^{-4}}P_{UE}\right)^{1/2}\right)\right) \quad (3.38)$$

$$= \exp\left(-\frac{tN_0}{d^{-4}}\right) \exp\left(-\frac{\pi^2}{2}d^2t^{1/2}\left(\lambda_1(P_{BS})^{1/2} + \lambda_2(P_{UE})^{1/2}\right)\right) \quad (3.39)$$



### 3.1.2 The downlink probability

Now, we focus on the downlink part of probability (3.24). It can be written as

$$\mathcal{P} \left[ \frac{1}{2} \frac{g_{u_0} D_{u_0}^{-\alpha}}{(N_0 + I_D^D + I_U^D)} < t \right] \quad (3.40)$$

We arrange the probability and follow  $g_{u_0} \sim \exp(1)$ . Then, the probability can be expressed as

$$\mathcal{P} \left[ g_{u_0} < \frac{2t}{D_{u_0}^{-\alpha}} (N_0 + I_D^D + I_U^D) \right] \quad (3.41)$$

$$= 1 - \exp \left( -\frac{2t}{D_{u_0}^{-\alpha}} (N_0 + I_D^D + I_U^D) \right) \quad (3.42)$$

Follow the same step, we take Laplace transform of (3.42). Then, the probability can be rewritten as

$$\mathcal{L} \left( 1 - \exp \left( -\frac{2t}{D_{u_0}^{-\alpha}} (N_0 + I_D^D + I_U^D) \right) \right) \quad (3.43)$$

$$= 1 - \exp \left( -\frac{2tN_0}{D_{u_0}^{-\alpha}} \right) \mathcal{L}_{I_D^D + I_U^D}(s') \quad (3.44)$$

Where  $\mathcal{L}_{I_D^D + I_U^D}(s')$  is the total received interference at the downlink UE from all transmissions.  $s'$  is  $\frac{2t}{D_{u_0}^{-\alpha}}$ . According to the previous definition of  $I_D^D$  and  $I_U^D$ , the Laplace transform can be written as

$$\begin{aligned} \mathcal{L}_{I_D^D + I_U^D}(s') &= \mathbb{E}_{\Phi_B^F \cup \Phi_B^D \cup \Phi_B^U, h_b, g_u} \left[ \exp \left\{ -s' \sum_{b \in \Phi_B^D \cup \Phi_B^F \setminus b_0} P_{BS} h_b R_b^{-\alpha} \right\} \right. \\ &\quad \left. \exp \left\{ -s' \sum_{u \in \Phi_B^U \cup \Phi_B^F \setminus u_0} P_{UE} g_u D_u^{-\alpha} \right\} \right] \quad (3.45) \end{aligned}$$



We follow the previous method, divide  $\mathcal{L}_{I_D^D+I_U^D}$  into downlink interference part and uplink interference part. The two parts are given as

$$\mathbb{E}_{\Phi_B^F \cup \Phi_B^D, h_b} [\exp\{-s' \sum_{b \in \Phi_B^D \cup \Phi_B^F \setminus b_0} P_{BS} h_b X_b^{-\alpha}\}] \quad (3.46)$$

$$\mathbb{E}_{\Phi_B^F \cup \Phi_B^U, g_u} [\exp\{-s' \sum_{u \in \Phi_B^U \cup \Phi_B^F \setminus u_0} P_{UE} g_u D_u^{-\alpha}\}] \quad (3.47)$$

Based on the PGFL of PPP, the downlink interference part can be further written as

$$\exp\{-2\pi\lambda_1 \int_r^\infty \frac{s' P_{BS} v^{-\alpha}}{\mu + s' P_{BS} v^{-\alpha}} v dv\} \quad (3.48)$$

Where  $\lambda_1$  is  $\lambda_B(\rho_F + \rho_D)$ . The integration limit is from  $r$  to  $\infty$  since the closest interferer BS or uplink UE is at least at a distance between downlink UE and its serving BS ( $r$ ). Follow previous assumption and consider suburban environment, the downlink interference part of  $\mathcal{L}_{I_D^D+I_U^D}$  can be written as

$$\exp\left(-\pi\lambda_1 (s' P_{BS})^{1/2} \left(\arctan(\infty) - \arctan\left(r^2 [2tD_{u_0}^4 P_{BS}]^{-1/2}\right)\right)\right) \quad (3.49)$$

Using the same way, the uplink interference part of  $\mathcal{L}_{I_D^D+I_U^D}$  can be obtained.

It is

$$\exp\left(-\pi\lambda_2 (s' P_{UE})^{1/2} \left(\arctan(\infty) - \arctan\left(r^2 [2tD_{u_0}^4 P_{UE}]^{-1/2}\right)\right)\right) \quad (3.50)$$

Where  $\lambda_2$  is  $\lambda_B(\rho_F + \rho_U)$ .

We take the Laplace transform of downlink and uplink interference into (3.44). Obtain the second part of probability. It can be written as

$$1 - \exp\left(-\frac{2tN_0}{D_{u_0}^{-4}}\right) \exp\left(-\pi\lambda_1 (s' P_{BS})^{1/2} \left(\arctan(\infty) - \arctan\left(r^2 [2tD_{u_0}^4 P_{BS}]^{-1/2}\right)\right)\right) \cdot \exp\left(-\pi\lambda_2 (s' P_{UE})^{1/2} \left(\arctan(\infty) - \arctan\left(r^2 [2tD_{u_0}^4 P_{UE}]^{-1/2}\right)\right)\right) \quad (3.51)$$





We replace  $\arctan(\infty)$  with  $\frac{\pi}{2}$  and take  $s'$  into the probability. Then, the probability can be written as

$$1 - \exp\left(-\frac{2tN_0}{D_{u_0}^{-4}}\right) \exp\left(-\pi\lambda_1(2tD_{u_0}^4 P_{BS})^{1/2} \left(\frac{\pi}{2} - \arctan\left(r^2[2tD_{u_0}^4 P_{BS}]^{-1/2}\right)\right)\right) \cdot \exp\left(-\pi\lambda_2(2tD_{u_0}^4 P_{UE})^{1/2} \left(\frac{\pi}{2} - \arctan\left(r^2[2tD_{u_0}^4 P_{UE}]^{-1/2}\right)\right)\right) \quad (3.52)$$



## 3.2 The probability of FD BS

### 3.2.1 The probability that FD is preferred

Now, we have the uplink part and downlink part probabilities. Combining (3.39) and (3.52), the original probability (3.8) can be express as

$$\begin{aligned} & \exp\left(-\frac{tN_0}{d^{-4}}\right) \exp\left(-\frac{\pi^2}{2}d^2t^{1/2}\left(\lambda_1(P_{BS})^{1/2} + \lambda_2(P_{UE})^{1/2}\right)\right) \left\{1 - \exp\left(-\frac{2tN_0}{D_{u_0}^{-4}}\right) \cdot \right. \\ & \quad \exp\left(-\pi\lambda_1(2tD_{u_0}^4P_{BS})^{1/2}\left(\frac{\pi}{2} - \arctan\left(r^2[2tD_{u_0}^4P_{BS}]^{-1/2}\right)\right)\right) \cdot \\ & \quad \left. \exp\left(-\pi\lambda_2(2tD_{u_0}^4P_{UE})^{1/2}\left(\frac{\pi}{2} - \arctan\left(r^2[2tD_{u_0}^4P_{UE}]^{-1/2}\right)\right)\right)\right\} \end{aligned} \quad (3.53)$$

We take (3.53) into (3.2). Then, the probability that FD is preferred can be rewritten as

$$\begin{aligned} & \int_0^\infty \exp\left(-\frac{tN_0}{d^{-4}}\right) \exp\left(-\frac{\pi^2}{2}d^2t^{1/2}\left(\lambda_1(P_{BS})^{1/2} + \lambda_2(P_{UE})^{1/2}\right)\right) \left\{1 - \exp\left(-\frac{2tN_0}{D_{u_0}^{-4}}\right) \cdot \right. \\ & \quad \exp\left(-\pi\lambda_1(2tD_{u_0}^4P_{BS})^{1/2}\left(\frac{\pi}{2} - \arctan\left(r^2[2tD_{u_0}^4P_{BS}]^{-1/2}\right)\right)\right) \\ & \quad \left. \exp\left(-\pi\lambda_2(2tD_{u_0}^4P_{UE})^{1/2}\left(\frac{\pi}{2} - \arctan\left(r^2[2tD_{u_0}^4P_{UE}]^{-1/2}\right)\right)\right)\right\} dt \end{aligned} \quad (3.54)$$

Where  $\lambda_1 = \lambda_B(\rho_F + \rho_D)$ .  $\lambda_2$  is  $\lambda_B(\rho_F + \rho_U)$ . From (3.54), the probability of FD BS ( $\rho_F$ ) can be derived. However, the  $\rho_F$  exists in the integral ( $\lambda_1$  and  $\lambda_2$ ). So, we use iterative method to obtain the probability. The iterative algorithm of the probability is shown in Alg 1.



---

**Algorithm 1** Framework of iterative method.

---

**Input:**

- The distance of BS and downlink UE,  $r$ ;
- The distance of BS and uplink UE,  $d$ ;
- The distance of downlink UE and uplink UE,  $D_{u_0}$ ;
- The initial FD probability is 0.5,  $\rho'_F = 0.5$ ;

**Output:**

- The FD probability,  $\rho_F$ ;
  - 1: Calculate the probability of FD by (3.54) and get analytical FD probability  $\rho_F^*$ ;
  - 2: Define  $g$  is  $(\rho_F^* - \rho'_F)$ ;
  - 3: **while**  $g \geq 0.05$  or  $g \leq -0.05$  **do**
  - 4:   **if**  $g > 0$  **then**
  - 5:      $\rho'_F = \rho'_F + 0.01$ ;
  - 6:   **else**
  - 7:      $\rho'_F = \rho'_F - 0.01$ ;
  - 8:   **end if**
  - 9:   Calculate the probability of FD by (3.54) and get  $\rho_F^*$ ;
  - 10:   Calculate  $g = \rho_F^* - \rho'_F$ ;
  - 11: **end while**
  - 12: Let  $\rho_F = \rho_F^*$ ;
  - 13: **return**  $\rho_F$ ;
-



# Chapter 4

## Simulation Results

In this chapter, simulation results are given to verify the analysis in the previous chapter. Fig. 4.1 shows the relationship of symbols in a cell.  $r$  is the distance of BS and downlink UE.  $d$  is the distance of BS and uplink UE.  $D_{u_0}$  is the distance between downlink UE and uplink UE.

The simulation parameters are as table 4.1. In the simulation, the BS location follows the PPP distribution with density  $\lambda_B$  (nodes/ $m^2$ ). Each BS connects with one downlink UE and one uplink UE.

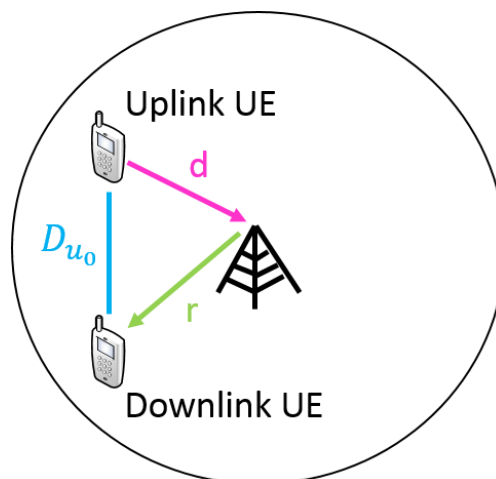


Figure 4.1: The relationship of symbols in a cell.



Parameter	Value
Bandwidth	160 MHz
Thermal Noise Density	-174 dBm/Hz
Path loss exponent ( $\alpha$ )	4
Simulation area	500000 $m^2$
Self-interference cancellation	-100 dB
BS power	24 dBm
UE power	23 dBm
Radius of cell	40 m
BS density ( $\lambda_B$ )	$[1, 1.5, 2, 2.5, 3] \cdot 10^{-5}$ nodes/ $m^2$

Table 4.1: The table of simulation parameters.

In the chapter 3, we analyze the probability that FD is preferred. In order to obtain the probability of a cell, we integral the equation (3.54) with  $r$  and  $d$ .

In fig. 4.2, we compare the analyzed probability with simulation probability. We calculate the analyzed probability that FD is preferred by (3.54). In the simulation case, we simulate the adaptive system and calculate statistical simulation probability. Fig. 4.2 reveals the trend of analyzed probability that FD is preferred is similar to the simulation probability. The figure also shows the analyzed probability that FD is preferred is smaller than simulation probability. In chapter 3, we relax original probability by using arithmetic and geometric mean inequality. To obtain the probability easily, we take the small probability to analyze. This is consistent with the approximation (3.7).

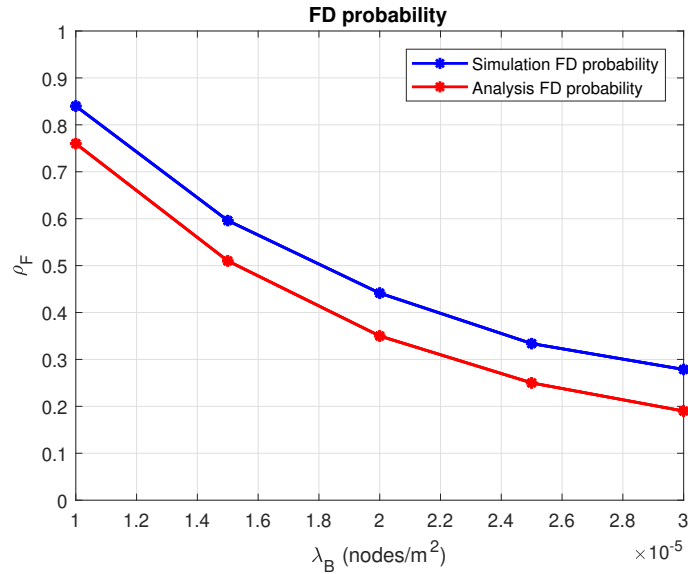


Figure 4.2: Simulation and analytical results of probability that FD is preferred versus different BS density.

Besides investigating the relation of probability that FD is preferred, we compare the performance of the adaptive system with a random selected system, a traditional synchronous TDD half duplex system (THD system) and an all FD system. In the simulation, the BS location follows the PPP distribution with density  $\lambda_B$ . Each BS has one downlink and uplink UE pair. Each simulation time, BSs and their UE pair would be different. We calculate the sum rate of cell with its duplex mode and compare the sum rate with different systems. Every system has different duplex mode, next paragraph has detailed description.

In the adaptive system, the mode of cell is decided adaptively by (3.54). Based on the current environment, we calculate the probability that FD is preferred. Use the probability to decide the mode of cell. Based on the simulation, we calculate the sum rate per hertz of the adaptive system and



compare the sum rate with random selected system, THD system and all FD system.

In the random selected system, it only the method of deciding mode is different from the adaptive system. The mode of cell is not rely on the analytical probability that FD is preferred. The mode is decided randomly.

In the THD system, all cells operate HD duplex, the number of time slots is divided equally between the uplink and downlink transmission. In this case, a downlink transmission receives interference only from the neighbor BSs and an uplink transmission receives interference only from the uplink transmissions of the neighbor cells.

In the all FD system, each cell operates FD mode. Downlink transmission and uplink transmission communicate at the same time and the same frequency.

Fig. 4.3 depicts the sum rate per hertz of the adaptive system versus the random selected system, the THD system and the all FD system with different BS density ( $\lambda_B$ ). The sum rate of the random selected system is close to the all FD system or the THD system, but it is still not greater than the all FD system. In the all FD system, with BS density increasing the sum rate of the system approaches to the THD system. Even the sum rate of the all FD system is smaller than the THD system when BS density is high. This is consistent with previous introduction.

According to the simulation result, the sum rate of all systems decreases with BS density increasing. However, the adaptive system still has better performance compares with the random selected system, the THD system and the all FD system.

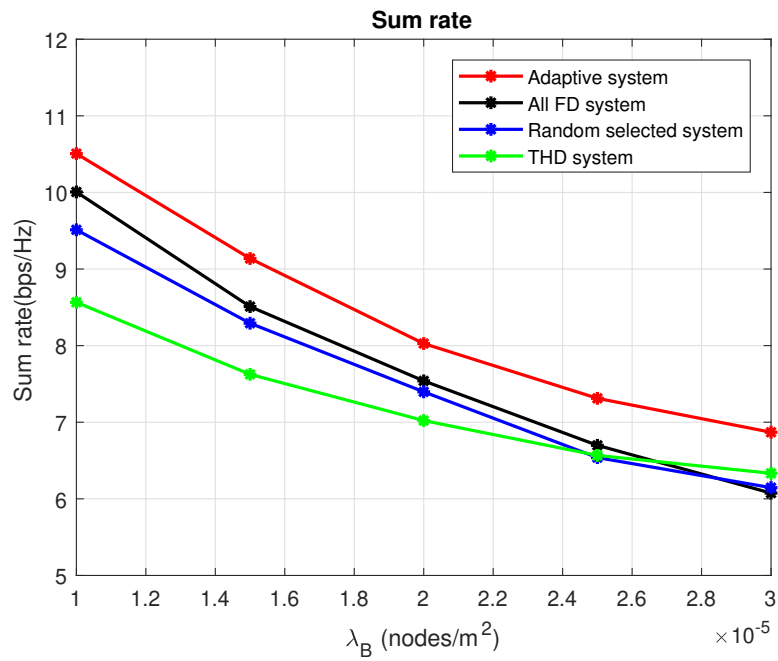


Figure 4.3: Sum rate versus BS density with different system.





## Chapter 5

# Conclusion

In the thesis, we investigate an adaptive multi-user multi-cell system. In the system, each BS can operate FD mode or HD mode. BSs are no longer restricted to operate the same mode. We propose a stochastic geometry model allows us to numerically assess the SINR and analyze the probability that FD is preferred. We derive the probability that FD is preferred in the system with the condition of FD rate is larger than HD rate. Based on the UE position we can calculate the probability. Then, we can allocate the mode of the cell adaptively by the probability that FD is preferred. Moreover, we also analyze the sum rate of the adaptive FD system. The simulation results are compared with the random selected system, the THD system and the all FD system. By adaptively switching between the FD and HD modes, the proposed system can achieve a better performance than the random selected system, pure HD system and pure FD system.



## Bibliography

- [1] J.I.Choiy, M.Jainy, K.Srinivasan, P.Levis, and S.Katti, "Achieving single channel, full duplex wireless communication," *ACM MobiCom, Chicago, IL, USA*, pp. 1–12, Sep. 2010.
- [2] M. Duarte and A. Sabharwal, "Full-duplex wireless communications using off-the-shelf radios: Feasibility and first results," *Asilomar Conf. Signals, Syst. Comput.*, pp. 1558–1562, Nov. 2010.
- [3] M. Duarte, C. Dick, and A. Sabharwal, "Experiment-driven characterization of full-duplex wireless systems," *IEEE Trans. on Wireless Communications*, vol. 11, no. 12, pp. 4296–4307, Dec. 2012.
- [4] E. Aryafar, M. Khojastepour, K. Sundaresan, S. Rangarajan, and M. Chiang, "MIDU: Enabling MIMO full duplex," *ACM MobiCom*, pp. 257–268, Aug. 2012.
- [5] M. Jain *et al.*, "Practical, real-time, full duplex wireless," *ACM MobiCom*, pp. 301–312, Sep. 2011.
- [6] T. Riihonen, S. Werner, and R. Wichman, "Mitigation of loopback self-interference in full-duplex MIMO relays," *IEEE Trans. on Signal Processing*, vol. 59, no. 12, pp. 5983–5993, Dec. 2011.



- [7] D. Senaratne and C. Tellambura, “Beamforming for space division duplexing,” *IEEE Int. Conf. on Commun. (ICC)*, pp. 1–5, Jun. 2011.
- [8] L. Haesoon, K. Dongkyu, and H. Daesik, “Mode Selection in Multi-user Full-Duplex Systems Considering Inter-user Interference,” *24th European Signal Processing Conference (EUSIPCO)*, pp. 769–772, 2016.
- [9] K. Benkic, M. Malajner, P. Planinsic, and Z. Cucej, “Using RSSI value for distance estimation in wireless sensor networks based on ZigBee.” *in Proc. Int. Conf. on Systems, Signals and Image Process.*, pp. 303–306, Jun. 2008.
- [10] P. Moravek, D. Komosny, M. Simek, M. Jelinek, D. Girbau, and A. Lazaro, “Signal Propagation and Distance Estimation in Wireless Sensor Networks,” *33rd International Conference on Telecommunications and Signal Processing Conference TSP2010*, Aug. 2010.
- [11] S. Goyal, C. Galiotto, N. Marchetti, and S. Panwar, “Throughput and Coverage for a Mixed Full and Half Duplex Small Cell Network,” *IEEE Int. Conf. on Commun. (ICC)*, May. 2016.
- [12] S. Li, M. Zhou, J. Wu, L. Song, Y. Li, and H. Li, “On the Performance of X-Duplex Relaying,” *IEEE Trans. on Wireless Communications*, vol. 16, no. 3, March 2017.
- [13] C. Yao, L. Song, and Y. Li, “X-Duplex Radios: Flexible Switching Between Full-Duplex and Half-Duplex,” *IEEE Wireless Communications Letters*, vol. 7, no. 1, Feb. 2018.



- [14] M. Haenggi and R. K. Ganti, "Interference in Large Wireless Networks." *NOW: Foundations and Trends in Networking*, 2009.
- [15] Z. Tong and M. Haenggi, "Throughput analysis for wireless networks with full-duplex radios," *IEEE WCNC, New Orleans, LA, USA*, pp. 717–722, Mar. 2015.
- [16] Y. Pan, M. Chen, Z. Yang, and H. Xu, "Performance Analysis for Full-Duplex Relaying D2D Communications in Cellular Networks," *9th International Conference on Wireless Communications and Signal Processing (WCSP)*, Oct. 2017.



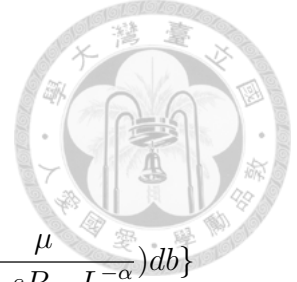
# Appendix A

## PGFL of PPP

$$\begin{aligned} & \mathbb{E}_{\Phi_B^F \cup \Phi_B^D, h'_b} [\exp\{-s \sum_{b \in \Phi_B^D \cup \Phi_B^F \setminus b_0} P_{BS} h'_b L_b^{-\alpha}\}] \\ &= \exp\{-2\pi\lambda_1 \int_0^\infty \frac{s P_{BS} v^{-\alpha}}{\mu + s P_{BS} v^{-\alpha}} v dv\} \end{aligned}$$

*Proof.*

$$\begin{aligned} & \mathbb{E}_{\Phi_B^F \cup \Phi_B^D, h'_b} [\exp\{-s \sum_{b \in \Phi_B^D \cup \Phi_B^F \setminus b_0} P_{BS} h'_b L_b^{-\alpha}\}] \\ &= \mathbb{E}_{\Phi_B^F, h'_b} [\exp\{-s \sum_{b \in \Phi_B^F \setminus b_0} P_{BS} h'_b L_b^{-\alpha}\}] \mathbb{E}_{\Phi_B^D, h'_b} [\exp\{-s \sum_{b \in \Phi_B^D} P_{BS} h'_b L_b^{-\alpha}\}] \\ &= \mathbb{E}_{\Phi_B^F, h'_b} [\prod_{b \in \Phi_B^F \setminus b_0} \exp\{-s P_{BS} h'_b L_b^{-\alpha}\}] \mathbb{E}_{\Phi_B^D, h'_b} [\prod_{b \in \Phi_B^D} \exp\{-s P_{BS} h'_b L_b^{-\alpha}\}] \\ &= \mathbb{E}_{\Phi_B^F} [\prod_{b \in \Phi_B^F \setminus b_0} \frac{\mu}{\mu + s P_{BS} L_b^{-\alpha}}] \mathbb{E}_{\Phi_B^D} [\prod_{b \in \Phi_B^D} \frac{\mu}{\mu + s P_{BS} L_b^{-\alpha}}] \end{aligned}$$



$$\begin{aligned} &= \exp\left\{-\lambda_F \int_{\mathbb{R}^2} \left(1 - \frac{\mu}{\mu + sP_{BS}L_b^{-\alpha}}\right) db\right\} \exp\left\{-\lambda_D \int_{\mathbb{R}^2} \left(1 - \frac{\mu}{\mu + sP_{BS}L_b^{-\alpha}}\right) db\right\} \\ &= \exp\left\{-2\pi\lambda_F \int_0^\infty \frac{sP_{BS}v^{-\alpha}}{\mu + sP_{BS}v^{-\alpha}} v dv\right\} \exp\left\{-2\pi\lambda_D \int_0^\infty \frac{sP_{BS}v^{-\alpha}}{\mu + sP_{BS}v^{-\alpha}} v dv\right\} \\ &= \exp\left\{-2\pi(\lambda_F + \lambda_D) \int_0^\infty \frac{sP_{BS}v^{-\alpha}}{\mu + sP_{BS}v^{-\alpha}} v dv\right\} \end{aligned}$$

Where  $\lambda_F = \lambda_B\rho_F$  and  $\lambda_D = \lambda_B\rho_D$ , it means  $\lambda_F + \lambda_D = \lambda_1$ .

Then,

$$\begin{aligned} &\exp\left\{-2\pi(\lambda_F + \lambda_D) \int_0^\infty \frac{sP_{BS}v^{-\alpha}}{\mu + sP_{BS}v^{-\alpha}} v dv\right\} \\ &= \exp\left\{-2\pi\lambda_1 \int_0^\infty \frac{sP_{BS}v^{-\alpha}}{\mu + sP_{BS}v^{-\alpha}} v dv\right\} \end{aligned}$$

□



## Appendix B

### Suburban environment

$$\begin{aligned} & \exp\left\{-2\pi\lambda_1 \int_0^\infty \frac{sP_{BS}v^{-\alpha}}{\mu + sP_{BS}v^{-\alpha}} v dv\right\} \\ &= \exp\left\{-2\pi\lambda_1 \frac{1}{2(sP_{BS})^{-1/2}} (\arctan(\infty) - \arctan(0))\right\} \end{aligned}$$

*Proof.*

We focus on the integral.

$$\int_0^\infty \frac{sP_{BS}v^{-\alpha}}{\mu + sP_{BS}v^{-\alpha}} v dv$$

Where  $\mu$  is 1. Then,

$$\begin{aligned} & \int_0^\infty \frac{sP_{BS}v^{-\alpha}}{\mu + sP_{BS}v^{-\alpha}} v dv \\ &= \int_0^\infty \frac{sP_{BS}v^{-\alpha}}{1 + sP_{BS}v^{-\alpha}} v dv \\ &= \int_{z_1}^{z_2} \frac{x^{-\alpha/2}}{1 + x^{-\alpha/2}} v \cdot \frac{dx}{2v(sP_{BS})^{-2/\alpha}} \end{aligned}$$



Where  $x = (sP_{BS})^{-2/\alpha}v^2$ .  $z_1 = 0$  and  $z_2 = \infty$ . Then,

$$\begin{aligned} & \int_{z_1}^{z_2} \frac{x^{-\alpha/2}}{1+x^{-\alpha/2}} v \cdot \frac{dx}{2v(sP_{BS})^{-2/\alpha}} \\ &= \frac{1}{2(sP_{BS})^{-2/\alpha}} \int_{z_1}^{z_2} \frac{x^{-\alpha/2}}{1+x^{-\alpha/2}} dx \\ &= \frac{1}{2(sP_{BS})^{-2/\alpha}} \int_0^{\infty} \frac{1}{1+x^{\alpha/2}} dx \end{aligned}$$

Where  $\alpha = 4$ . Then,

$$\begin{aligned} & \frac{1}{2(sP_{BS})^{-2/\alpha}} \int_0^{\infty} \frac{1}{1+x^{\alpha/2}} dx \\ &= \frac{1}{2(sP_{BS})^{-1/2}} [\arctan(\infty) - \arctan(0)] \end{aligned}$$

We obtain that

$$\begin{aligned} & \exp\left\{-2\pi\lambda_1 \int_0^{\infty} \frac{sP_{BS}v^{-\alpha}}{\mu + sP_{BS}v^{-\alpha}} v dv\right\} \\ &= \exp\left\{-2\pi\lambda_1 \frac{1}{2(sP_{BS})^{-1/2}} (\arctan(\infty) - \arctan(0))\right\} \end{aligned}$$

□