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一新適應性方法之重複控制器設計

A New Adaptive Approach to the Repetitive Controller Design

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A new Adaptive Approach to the Repetitive Controller Design

By

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Dissertation

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誌　謝

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99 年 8 月 22 日

摘　要

本論文提出一新的適應性前饋控制設計(AFC)技術來達成週期性追蹤和／或週期性干擾消除。與傳統的AFC相比較，主要的不同點是運用重新參數化的回歸式於適應性機制。這控制架構是包含一像干擾觀測器(DOB-like)的架構來產生一經系統濾波後的週期性訊號，和一訊號辨識器來對此訊號進行參數識別。因此，AFC控制系統的穩定問題不再受到受控體架構的影響。本論文更進一步利用此重新參數化的技術於新的干擾估測器設計，進而獲得此新AFC設計方法的一重複控制的通式。

不同於以往的適應性設計，本方法有以下的優點：第一點是它的增益可以任意選擇而不影響控制系統的穩定度。第二點是透過重新參數化過程，它不需反轉系統模式，可以適用於極小相與非極小相系統。第三點是它顯示週期性干擾輸入點與控制設計無關。第四點是本方法可運於線性系統有存在不確定性。最後一點是此新AFC控制與基於干擾觀測器的控制有一等價關係的表示，進而提供可應用彼此相關領域之控制知識的機會，因此，AFC的適應性增益可以藉由任何線性控制方法或適應性方法更有效率的選擇，如特徵值安置，卡曼濾波器和最小平方法等。另外，在控制設計方面，此控制技術提供工程師一非常友善及直覺的設計。

關鍵字：重複控制；適應性前饋消除；干擾觀測器；週期性干擾消除；內模式控制；週期性追蹤控制。

ABSTRACT

This dissertation proposes a new technique of adaptive feedforward control (AFC) that achieves periodic tracking and/or periodic disturbance rejection. The key difference compared with conventional AFC is a new re-parameterization regression form employed in adaptive mechanism. This new control structure is a combination of the disturbance-observer-like (DOB-like) structure and the disturbance identifier, where the DOB-like output generates a periodic disturbance which is filtered by the plant model, and the disturbance identifier is to identify the unknown parameter of the filtered disturbance. Consequently, the stabilizability problem is no longer subject to the plant structure. Utilizing the re-parameterization technique, the dissertation further proposes a general form of AFC control using repetitive control.

The proposed new control has several advantages over previous designs. First, its adaptation gain can be arbitrarily chosen without upsetting the system stability. Second, through re-parameterization process, the adaptive algorithm can be applied to minimum phase as well as non-minimum phase systems without using any approximations. Third, it is shown that the desired AFC control is independent of where the disturbance enters the system. Fourth the proposed control is proved to be robust with respect to system uncertainties. Finally and most importantly, the equivalent interpretation between the disturbance observer based control and the new AFC control provides an opportunity to apply knowledge to each other field. Therefore, AFC's adaptation gain can be efficiently chosen by any linear control methods or adaptive algorithms, such as eigenvalue assignment, Kalman filter, least-squares algorithm and so on. Besides, the control technique provides engineers with very friendly and intuitive design on the control performance.

keywords : repetitive control; adaptive feedforward cancellation; disturbance observer; periodic disturbance rejection; internal model control; periodic tracking control.

Contents

Abstract **I**

List of Figures **V**

List of Tables **IX**

1 Introduction **1**

1.1 Motivation 1

1.2 Literature Survey 2

1.3 Overview of the Dissertation 6

2 Review of Internal Model Based Repetitive Control **9**

2.1 Periodic Signal 10

2.2 Internal Model Principle 11

2.3 Time Delay Repetitive Control 12

2.4 Plug-in Time Delay Repetitive Control 15

3 AFC Control **19**

3.1 Review of Adaptive Algorithm 21

3.2 Problem Formulation 22

3.3 Linear Regression Form 23

3.4 Review of Adaptive Feedforward Control 29

3.5 New AFC Design for Open-Loop Stable System 35

3.5.1 Gradient Based AFC 35
3.5.2 LS Based AFC 41
3.6 New AFC Design for Open-Loop Unstable System 46
3.7 Robustness Analysis 50
3.8 Adaptive Disturbance Estimation 62

4 Disturbance Observer Based Control **71**
4.1 Review of Disturbance Observer Based Control 72
4.1.1 Disturbance Observer Based Control 72
4.1.2 Unknown Input Disturbance Observer Based Control 78
4.2 New Disturbance Observer Based Control 82
4.3 DOB-AFC design 84
4.4 Robustness Analysis 88

5 Conclusions **91**

Bibliography **92**

List of Figures

2.1 IMP structure for the periodic reference with single frequency 12
2.2 IMP structure for the periodic reference with multiple frequencies . . 12
2.3 Periodic signal generator 13
2.4 Time delay repetitive control system 13
2.5 A system equivalent to Figure 2.4 14
2.6 Modified time delay repetitive control system 14
2.7 Structure of plug-in repetitive control system 15
2.8 Plug-in time delay repetitive control system 16
2.9 A system equivalent to Figure 2.8 17

3.1 System 24
3.2 A system equivalent to Figure 3.1 27
3.3 Control system with Plug-in AFC controller 29
3.4 AFC control System 29
3.5 AFC control 31
3.6 IMP control which is equivalent to Figure 3.5 31
3.7 Trajectory of the output error $e(t)$ 34
3.8 Time history of the disturbance $d_1(t)$ and the estimate $\hat{d}_1(t)$ 34
3.9 AFC control system 36
3.10 Time history of periodic disturbance $d(t)$ 40
3.11 Trajectory of the reference signal $r(t)$ and the output $y(t)$ 40

3.12 Trajectory of the control input $u(t)$ 41
3.13 Output error $e(t)$ 43
3.14 Trajectory of the disturbance $d_1(t)$ and estimated disturbance $\hat{d}_1(t)$. 44
3.15 Trajectory of the parameter estimation $\hat{\theta}(t)$ 44
3.16 Output error $e(t)$ 45
3.17 Trajectory of the disturbance $d_1(t)$ and estimated disturbance $\hat{d}_1(t)$. 46
3.18 Trajectory of the reference signal $r(t)$ and the output $y(t)$ 49
3.19 Trajectory of the control input $u(t)$ 49
3.20 AFC control system under model with uncertainty 52
3.21 Feedback connection 54
3.22 LTI control system which is equivalent to Figure 3.20 57
3.23 Root-locus of $\Gamma_F(s)$ 59
3.24 Trajectory of $r(t)$ and $y(t)$ under model with uncertainty 61
3.25 Adaptive Disturbance Estimation 64
3.26 Time history of the disturbance $d(t)$ 65
3.27 Trajectory of the disturbance error $|d(t) - \hat{d}(t)|$ 66
3.28 Trajectory of the state $x(t)$ 68
3.29 Trajectory of the disturbance $d(t)$ and the estimated $\hat{d}(t)$ 68
3.30 Trajectory of the norm of state estimation error $\|x(t) - \hat{x}(t)\|$ 69

4.1 Disturbance observer based control 73
4.2 Trajectory of the the output error $e(t)$ on Case 1 76
4.3 Time history of the disturbance d_1 and the estimate $\hat{d}_1$ on Case 1 . . 76
4.4 Trajectory of the the output error $e(t)$ on Case 2 77
4.5 Time history of the disturbance d_1 and the estimate $\hat{d}_1$ on Case 2 . . 77
4.6 Unknown input disturbance observer 79
4.7 Trajectory of the the output error $e(t)$ 81
4.8 Trajectory of the periodic disturbance d_1 81

4.9 DOB control system 84
4.10 Trajectory of the the output error $e(t)$ 87
4.11 Trajectory of the periodic disturbance d_1 87
4.12 DOB control system under model with uncertainty 89

List of Tables

3.1 Closed-loop poles of $\Gamma_F(s)$ versus adaptation gain γ 60

Chapter 1

Introduction

1.1 Motivation

In many industrial applications, the control system is required to track or reject periodic exogenous signals (desired reference output or disturbance). Examples include periodic motion of robot manipulators [1], repeatable runout in disk drive [2], torque ripples in harmonic drives [3], periodic force disturbance in metal cutting, and so on. Clearly, these disturbances degrade the system performance. Hence, the basic requirements in control systems are that they have the ability to regulate the controlled variables to reference commands without a steady-state error against unknown and un-measurable disturbance inputs. Such control that can successfully drive the system to track or reject periodic signals is called the repetitive control. Besides, it may be desirable to estimate the unknown periodic disturbances acting on the system. In some cases, the purpose of disturbance estimation is to monitor the performance of systems for decision making. For examples, in the manufacturing processing, one may wish to estimate the cutting torque in drilling process [4] and the cutting force in CNC machine centers [5].

Although there are several approaches to cancel periodic disturbance, these approaches may cause original closed-loop system stability affected or these designs are excessive complexity. Therefore one of the objectives of the dissertation is to implement a simple plug-in type repetitive controller to cancel exogenous periodic signal. The other objective is to construct an adaptive disturbance/state observer to monitor the system performance for decision making. The last purpose is to construct a robust update law for repetitive control.

1.2 Literature Survey

Since many control systems are often subject to the disturbance, one of the fundamental research topics in control theory is to study the problem of disturbance rejection. The disturbances are mostly divided into un-deterministic and deterministic disturbance. In the case of un-deterministic disturbance, the robust disturbance attenuation control, such as H_∞ control [6] and variable structure control [7] which are high-gain controllers, has been investigated by many researchers. Although those are common methods for the improved performance of control systems, such high-gain controllers may not be applied in a mechanical system due to the reason of mechanical resonance. In contrast, from the 70s to the present, there have been many researchers who proposed various approaches to realize effective disturbance suppression without using high gains [8]-[13]. One of these methods employs that disturbance is estimated using an observer and cancelled out, and then the control design is reduced to nominal feedback control which generates just minimal control based on disturbance free assumption. Thus the disturbance rejection problem is transformed to the disturbance estimation design.

A recent survey paper on the disturbance estimation for linear systems can be found in [14], and extension to the disturbance estimation for nonlinear systems can be found in [15] and [16]. One approach for the disturbance estimation is the use of disturbance observer [17, 18], which does not need the dynamic model of the unknown disturbance. The disturbance observer estimates the equivalent disturbance which is the difference between the actual plant output and the output of the nominal model. The estimate is then inversely added at the input of the plant, so as to compensate for the disturbance effect on the output. Despite the simple structure, disturbance observer based (DOB) control as an effective add-on controller [19] is successful in enhancing disturbance attenuation capability. However, this approach relies on inverse the system dynamics, and hence can not be applied to non-minimum-phase systems (systems with unstable zeros). Even for minimum-phase systems, the obtained disturbance estimate may not asymptotically converge to true disturbance due to a Q-filter in the estimation process. Besides, in some systems, such as disk drive servo, the rotational speed is usually required as increasingly as possible for improving data transfer rate, so does the frequency of the periodic disturbance, which leads to a high loop gain in the track-following servo. However, such a design may not be feasible since the increase of the Q-filter cut-off frequency may cause an undesirable increase of the control bandwidth, which is established by a feedback controller.

The second approach for disturbance estimation applied to the unknown disturbance is generated by a known dynamic model. In this dissertation one considers the problem of rejecting periodic disturbances, whose magnitude and phase are unknown but frequency is known. In this case, the periodic disturbance model is augmented with the system model to form an expanded system. A Luenberger observer is then constructed to estimate not only the system state, but also the state of disturbance model. The disturbance estimated method was called unknown input disturbance observer [20] or Kalman disturbance observer [21]. Consequently, by using the reconstructed disturbance injecting into the plant input, disturbance rejection is accomplished. Note that, since the disturbance observer constructed by augmented system is only used to estimate the actual disturbance acting on the system, it does not control the plant. Therefore, for achieving the closed-loop system stability and performance, a normal feedback controller is still required.

In addition to the above methods using the disturbance estimation, repetitive control (RC) is a specialized control strategy designed for tracking a specific periodic command or rejecting a periodic disturbance. A recent survey paper on repetitive control designs can be found in [22] and [23]. These designs are roughly classified as being either internal model base or external model base. The internal model based repetitive control design is originally proposed in [24], which is based on the internal modelling principle [25]. In [24], a time delay internal model is placed inside the nominal stable feedback loop to guarantee asymptotic tracking or rejecting of the periodic signal. However, this approach may alter original closed-loop system stability and performance. Hence, it is often realized in a plug-in manner [22, 26]. The advantages of the internal model based repetitive control are that convergence is very rapid and that the controller is linear, making analysis easy. However, the internal model introduces an infinite number of open-loop poles on the stability boundary; making stabilization of the overall system difficult [27]. As a result, Hara et al. in [28] proposed a low-pass filter included in the repetitive controller to ensure closed-loop stability, that is Q-filter. However, it makes exact internal model lost and the system performance at the high frequency harmonics be sacrificed [29]. Another disadvantage of this approach is that robustness to noise and un-modelled dynamics is impaired by the time delay internal model [22]. Moon et al. in [30] proposed another repetitive controller design method on Q-filter, which is based on Nyquist plot technique, for the system with un-model dynamics. However, even under the ideal case, it can not reject the periodic disturbances asymptotically. When the disturbance frequencies are unknown, adaptive internal model is often used for disturbance rejection [31]-[34].

The other approach for repetitive control designs is the basis function approach, or often called adaptive feedforward cancellation (AFC) control, being a main method in the external model based repetitive control design. With this approach, the periodic exogenous signal is modelled as a linear combination of finite or infinite basis functions with unknown coefficients [22] [35]. An adaptive algorithm is proposed to estimate these unknown coefficients, and a feedforward control that cancels the disturbance efforts is then constructed [36][37]. The adaptive approach may be superior to the disturbance model based approach when the frequency can not be obtained but the angle can be measured by the sensor, or injected signal need to be disconnected temporarily. However, stability of the adaptive system is ensured only if the system is SPR (strictly positive real). When the system is not SPR, the adaptation gain must be constrained to be small in order to maintain stability. The equivalence between the AFC and the internal model based approach is established in [38]. A modified adaptive algorithm with an extra phase advance is proposed in [38][39] to expedite the algorithm's convergence. In [40], Ariyur and Krstić start with the sensitivity method but arrive at the same scheme. However, the adaptation gain is still constrained by the stability requirement. In [3], a different adaptive algorithm is proposed, whose adaptation gain can be arbitrarily chosen without disturbing the system stability. However, this adaptive algorithm is based on inversion of the system transfer function; hence, they can be applied to minimum-phase systems only. When the system is in a non-minimum phase, an approximation algorithm based on the zero-phase-error-tracking design may be used [22]. The other AFC approach called frequency adaptive control technique (FACT), which utilizes a collection of frequency sampling filters (FSF) to obtain the magnitude of individual frequency components of the truncated periodic signal and uses these individual components to do adaptive update again, is proposed in [2]. The feature of FACT design is able to cancel any unwanted harmonic signals without influencing the uncompensated ones but needs more computational cost and carefully chooses adaptation gain for the system stability.

The stability conditions for the general AFC controller design is analyzed in [41] depending on the available adaptation method used in AFC design. Bayard uses LTI representations of adaptive systems with sinusoidal regressors to do stability analysis. Under the plant model known exactly, he proved that adaptive algorithm using augmented error signal is completely phase-stabilized. In [42], Guo further shows that those AFC control algorithms on the time-varying frequency case are equivalent to linear time-varying compensators which is implemented by the IMP on the state space. It provides an opportunity to apply knowledge obtained from either adaptive control or linear control to the other field.

1.3 Overview of the Dissertation

Even though the repetitive control approach is very complete already, but AFC is preferred over other schemes because the AFC controller can easily freeze the parameter update when the output signal is not available during certain periods of time, and can be driven by the measuring frequency, making the control response more robust to variation in frequency. Furthermore, the adaptive implementation can adopt angular measurements directly. However, under non-minimum phase system, arbitrary update gain and controllable convergence rate, the current researches in AFC control have not obtained effective solution yet. In view of the tradeoff between system stability and disturbance rejection in the previous controller design, the goal of this dissertation is to propose a new AFC design technique to cancel exogenous periodic signal without altering the closed-loop stability. The key difference compared with conventional AFC is a new linear regression form employed in adaptive mechanism. This new control structure is similar to a typical DOB control, but the proposed AFC control uses a disturbance identifier instead of the low-pass filter $Q(s)$ in DOB control and does not need inverse plant model to obtain disturbance estimate. Consequently, the stabilizability problem is no longer subject to the plant structure. The proposed AFC control is just one of the special cases of [41] which called augmented error algorithm. Although, both control structures are the same, the proposed AFC control relies on an adaptive identifier through re-parameterization process to prove that AFC control system is nominally stable. Utilizing the re-parameterization technique, a general form of AFC control using repetitive control is proposed in advance.

The resultant new control has several advantages over previous designs. First, since the adaptation gain of the proposed AFC is independent of the state feedback gain, under exactly known plant model it can be arbitrarily chosen without affecting the system stability. It means that the proposed AFC adds into the nominal closed-loop system without affecting the performance. Second, through re-parameterization process, the adaptive algorithm can be applied to minimum phase as well as non-minimum phase systems without using any approximations. Third, the new design is only one estimation algorithm while previous indirect schemes need two estimation algorithms [2]. Fourth, this dissertation shows that the desired adaptive control remains the same no matter where the disturbance enters the system. This justifies many previous AFC designs in the literature in which the disturbance is "assumed" to come into the system at the input point even though in real situations it may not be the case. Finally, for promoting that the repetitive control performance has more design freedom on adaptive update law, we further propose DOB-AFC that is a general AFC form. The interpretation of AFC in terms of disturbance observer design can be implemented by any linear control methods, such as eigenvalue assignment, Kalman filter, least-squares algorithm and so on. Therefore, the control technique provides engineers with very friendly and intuitive design. Certainly, when the system model can not be exactly obtained, the control structure using LMI method will provide more robust performance.

A series of studies on disturbance rejection methods of control systems is organized as follows. Chapter 2 reviews internal model based repetitive control, which includes dynamics model of multiple frequencies disturbance and time-delay model. Chapter 3 firstly reviews the adaptive algorithm, formulates the problem, constructs a linear regression form through re-parameterization process, and introduces conventional adaptive feedforward control. And then one proposes the new AFC design and the adaptive disturbance estimation. In Chapter 4, one firstly reviews disturbance observer based control, which includes disturbance observer and unknown input disturbance observer for un-deterministic disturbance and deterministic disturbance respectively. And then, based on the linear regression form in Chapter 3, we propose a new disturbance observer design, which is different from previous designed, and makes use of the equivalence between the new AFC and disturbance observer to design a general form of AFC, called DOB-AFC. Chapter 5 gives the concluding remarks.

Chapter 2

Review of Internal Model Based Repetitive Control

In many industrial applications [1]-[5], the control system is required to track or reject exogenous periodic signals. When the system is subjected to periodic signal input, it is well known that the repetitive controller can work well. The conventional RC is often regarded as a simple learning control because the control input is calculated using the result of preceding periods to improve the current performance. One closely related study of repetitive control is iterative learning control (ILC) [44] which is achieved by iteration of the control action within finite duration. The difference between RC and ILC is the setting of the initial conditions for each trial. In the ILC, the same initial condition is assumed in every trial. Hence, the iterative action is discrete and it is enough to assure not only the stability but the convergence of the error. In the repetitive control, the repetitive process is continuously because the initial conditions are set to the final conditions of the previous trial. The difference in initial-condition resetting leads to different analysis techniques and results [45].

In this chapter, an internal model based repetitive control which is a typical one will be introduced. In Section 2.1, one firstly gives a brief review of a periodic signal for easy description on the latter sections and chapters. Section 2.2 reviews an internal model principle which states that a generating system model of the exogenous signal must be included in the feedback system in order to achieve perfect tracking at the steady state. Based on this internal model principle, the time delay internal model based repetitive control which includes all frequency modes of periodic signal in the closed-loop system is then introduced in Section 2.3. Finally, for keeping original feedback control performance and stability, a plug-in time delay repetitive control was presented In Section 2.4.

2.1 Periodic Signal

The objective of this dissertation is to construct a control that can reject or track an unknown periodic signal. Therefore, in the thesis, the periodic signal is assumed to satisfy the following assumptions.

Assumption A2.1. $d(t)$ is a periodic signal (in the thesis, it is taken as disturbance) that is $d(t) = d(t+T)$ for some known period T.

Assumption A2.2. $d(t)$ is continuous and has a piecewise continuous derivative.

The periodic signal has a *Fourier* series representation

$$d(t) = \theta_{0,c} + \sum_{i=1}^{\infty} \theta_{i,c} \cos(\omega_i t) + \sum_{i=1}^{\infty} \theta_{i,s} \sin(\omega_i t), \tag{2.1}$$

where $\omega_i = i \cdot 2\pi/T$ is the harmonic frequency in which $2\pi/T$ is the fundamental frequency, and $\theta_{0,c}, \theta_{i,c}$ and $\theta_{i,s}$ are constant coefficients. In practical applications, one uses a $(2N+1)$-term finite series approximation for the periodic signal,

$$d_N(t) = \theta_{0,c} + \sum_{i=1}^{N} \theta_{i,c} \cos(\omega_i t) + \sum_{i=1}^{N} \theta_{i,s} \sin(\omega_i t) = \phi^T(t)\theta_d, \tag{2.2}$$

where the regressor $\phi(t)$ is a bounded vector

$$\phi(t) = \begin{bmatrix} 1 & \cos(\omega_1 t) & \sin(\omega_1 t) & \ldots & \cos(\omega_N t) & \sin(\omega_N t) \end{bmatrix}^T \in R^{2N+1}, \tag{2.3}$$

and θ_d contains unknown parameters

$$\theta_d = \begin{bmatrix} \theta_{0,c} & \theta_{1,c} & \theta_{1,s} & \theta_{2,c} & \theta_{2,s} & \ldots & \theta_{N,c} & \theta_{N,s} \end{bmatrix}^T \in R^{2N+1}. \tag{2.4}$$

The theorem below suggests that under certain conditions, the finite series approximation is a "good" approximation of the periodic signal as long as N is large enough.

Theorem 2.1 [43] **:** Under Assumption A2.2, the finite series approximation $d_N(t)$ in (2.2) converges uniformly to the true signal $d(t)$ in (2.1) as N approaches infinity.

Because of Theorem 2.1, this dissertation will make no difference between $d_N(t)$ and $d(t)$ as long as N is sufficiently large. In fact, the low-pass properties of physical systems, at most a handful of harmonics needs to be considered in general. Hence, in the remainder of this thesis, one will write

$$d(t) = \phi^T(t)\theta_d. \tag{2.5}$$

2.2 Internal Model Principle

After reviewing the property of the periodic signal, one goes back to the internal model principle (IMP) design. The IMP was initially proposed by Francis and Wonham [25]. It means that the controlled output tracks a class of reference commands without a steady-state error if the generator for the references is included in the stable closed-loop system. Figure 2.1 shows the basic control structure of IMP, where $P(s)$ is a linear time-invariant plant, $C(s)$ is the controller, $y(t)$ is a controlled output, $e(t)$ is a tracking error, and $r(t)$ is a periodic reference signal which is expressed as the following form

$$r(t) = \theta_{1,c}\cos(\omega_1 t) + \theta_{1,s}\sin(\omega_1 t), \tag{2.6}$$

in which ω_1 is a known frequency and $\theta_{1,c}$ and $\theta_{1,s}$ are unknown constant coefficients. The compensator including an internal model $1/(s^2+\omega_1^2)$ is to provide a closed-loop transmission zero to cancel unstable poles of the periodic input so that it achieves perfect tracking. The design problem is to choose the remaining transfer function $C(s)$ so that the closed-loop transfer function is stable and has desire input-output properties.

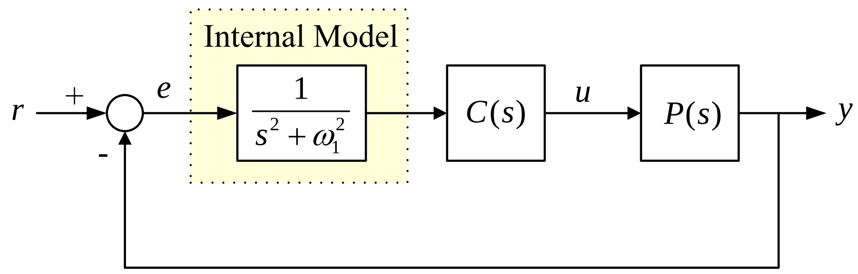

Figure 2.1: IMP structure for the periodic reference with single frequency

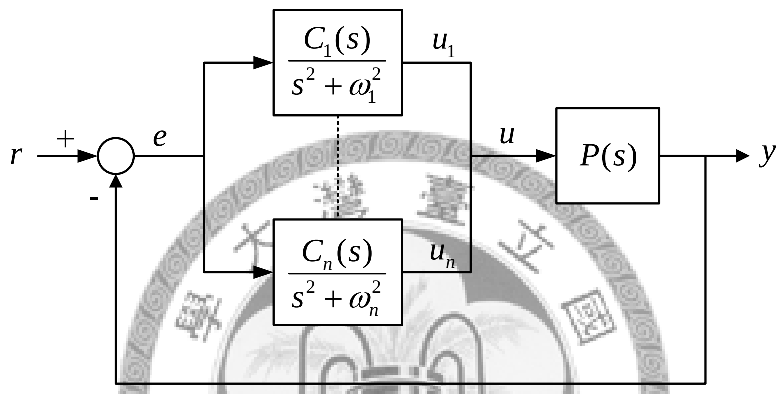

Figure 2.2: IMP structure for the periodic reference with multiple frequencies

The advantages of this type of controller are that it is linear, making analysis easier, and that convergence is very rapid. When the periodic exogenous signals is the sum of two or more sinusoids, the method is easily extended to the cases as shown in Figure 2.2. However, the stability problem becomes more and more difficult as poles are added on the jw-axis.

2.3 Time Delay Repetitive Control

In this section, our objective is based on the IMP to obtain a repetitive control with minimal system scheme that generates all periodic signals of period T. Based on the reason, Inoue et al. [24] originally employed a time delay system as shown in Figure 2.3 to serve as a periodic signal generator. It is readily seen that the delay element stores the function of the past one period and the system has infinitely many poles on the imaginary axis at $jk\omega$, where $\omega = 2\pi/T$. It is therefore expected from the IMP that the asymptotic tracking property for exogenous periodic signals may be achieved by implementing the model $1/(1-e^{-sT})$ into the closed-loop system. A controller including this model is said to be a repetitive controller and a system with such a controller is called a repetitive control system [46] as shown in Figure 2.4. In Figure 2.4, the feedback controller $C(s)$ is designed to stabilize the plant and has desire input-output properties.

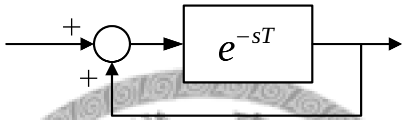

Figure 2.3: Periodic signal generator

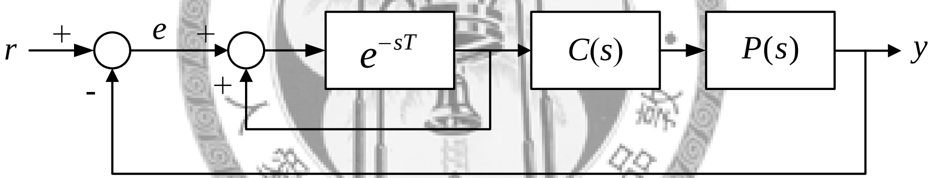

Figure 2.4: Time delay repetitive control system

Therefore, the transfer function from r to e is

$$W_{er}(s) = \frac{1-e^{-sT}}{1-(1-P(s)C(s))\,e^{-sT}}. \tag{2.7}$$

Consequently, $s = j2k\pi/T$ becomes the transmission zeros of $W_{er}(s)$. Therefore, the system asymptotically tracks the periodic signal of a fixed period T if the closed-loop system is stable. Let us start with some simple stability analysis. An easy loop transformation converts Figure 2.4 to Figure 2.5 [29]. Using the small gain theorem, the converted system is L_2 input/output stable if

$$\|1-P(s)C(s)\|_\infty < 1. \tag{2.8}$$

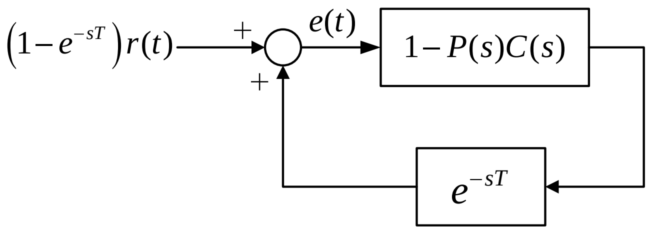

Figure 2.5: A system equivalent to Figure 2.4

Although the condition is only a sufficient condition, it is actually very close to necessity since the delay e^{-sT} introduces a large amount of phase shift especially in the high frequency range. It is clearly seen that the above condition can never be satisfied for a strictly proper $P(s)C(s)$. This restriction comes from the apparently unrealistic over specification of tracking in a very high frequency band. One way of handling this is to introduce a low-pass filter in front of the delay term, thereby replacing the delay element e^{-sT} by $Q(s)e^{-sT}$ for some strictly proper stable rational filter $Q(s)$ [28]. This, named finite dimensional repetitive control, relaxes the tracking requirement in the high frequency range, thereby relaxing the stability condition. The modified repetitive control system is shown in Figure 2.6. Then the stability condition becomes

$$\|Q(s)\,(1 - P(s)C(s))\|_\infty < 1. \tag{2.9}$$

Clearly, the high frequency band condition is relaxed here compared with (2.8), and the above condition can be satisfied with strictly proper $P(s)C(s)$. Although the stability robustness was improved, it was paid by the degradation of the steady-state tracking performance.

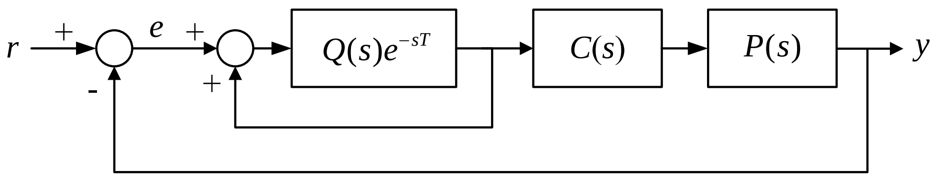

Figure 2.6: Modified time delay repetitive control system

The advantages of the time delay RC are that it is linear, making analysis easier, and that convergence is very rapid. The other advantage is that the repetitive compensator for any periodic signals is easily implemented by including the delay element. However, it alters the loop gain of the system and is not possible for selective harmonic cancellation. Besides, for the purpose of ensuring closed-loop stability, additional filtering is usually added to such schemes, but it sacrifices high frequency performance.

2.4 Plug-in Time Delay Repetitive Control

Normally, repetitive controller is realized in a plug-in manner, as shown in Figure 2.7. In Figure 2.7, the nominal controller is usually designed to stabilize the plant and reject a disturbance being across a broad frequency spectrum, and the repetitive controller is used to compensate periodic signals which have a known fundamental frequency.

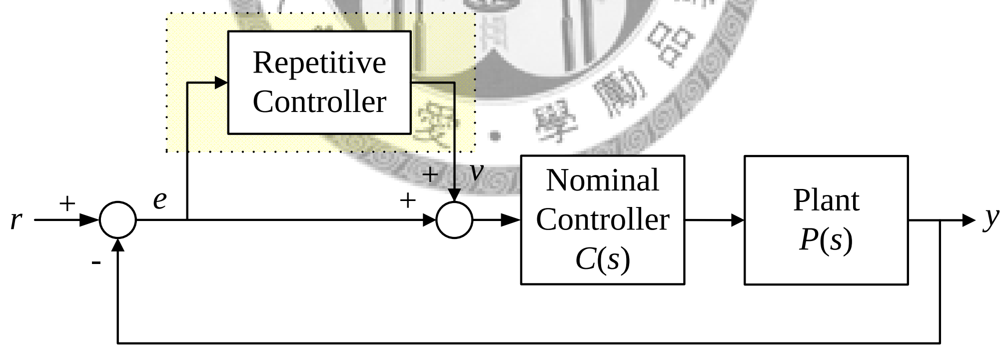

Figure 2.7: Structure of plug-in repetitive control system

In this section, our purpose is to make a description of a plug-in time delay repetitive controller design. Based on the stability analysis on the time delay repetitive control which has been introduced in the previous section, we know that perfect tracking for the periodic signal including higher order harmonic signal is the unrealistic. Therefore, for making stability condition be relaxed, a Q filter scheme must be also considered in the plug-in manner. Under this way, the RC could be added directly into the existing closed-loop system since it did not influence internal stability and system performance very much. Such plug-in manner was presented as shown in Figure 2.8.

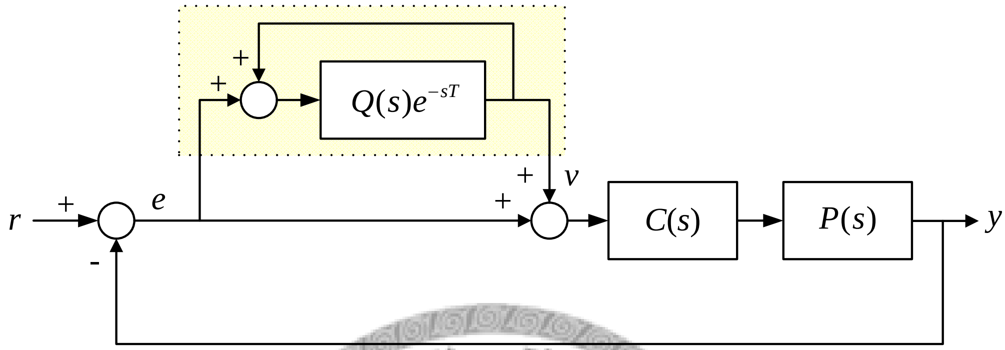

Figure 2.8: Plug-in time delay repetitive control system

According to the Figure 2.8, the transfer function from r to e is

$$W_{er}(s) = \frac{1 - Q(s)e^{-sT}}{1 + P(s)C(s) - Q(s)e^{-sT}}. \tag{2.10}$$

Define a sensitivity function as

$$S(s) = \frac{1}{1 + P(s)C(s)}, \tag{2.11}$$

where $S(s)$ is a stable sensitivity function since the nominal closed-loop system is stable. Then (2.10) becomes as

$$W_{er}(s) = \frac{\left(1 - Q(s)e^{-sT}\right) S(s)}{1 - S(s)Q(s)e^{-sT}}. \tag{2.12}$$

Note that the above equation shows the transmission zeros of $W_{er}(s)$ are no longer $s = j2k\pi/T$, but in the lower frequency range, they are still very approximate. It means that one must sacrifice the system performance at the high frequency harmonics in order to ensure closed-loop stability.

Let us start with some simple stability analysis. By an appropriate operating, Figure 2.8 can also be expressed equivalently as Figure 2.9. Therefore it exists a

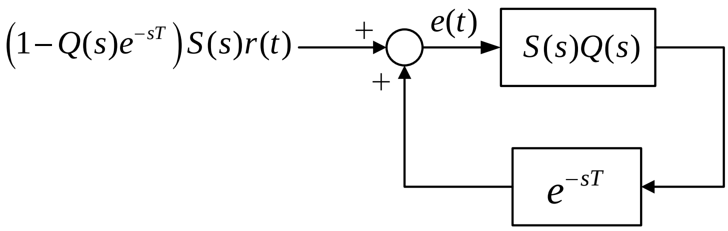

Figure 2.9: A system equivalent to Figure 2.8

stabilizing repetitive control when the following condition is satisfied,

$$\|Q(s)S(s)\|_\infty < 1. \tag{2.13}$$

Clearly, when (2.13) is satisfied, the plug-in repetitive control system is stable. Compared with time delay repetitive control, the main advantage of the design method is that it employs a plug-in manner to reduce changing nominal closed-loop system stability.

Chapter 3

AFC Control

As everyone knows, an internal model based repetitive control may cause large phase shift to make original controlled system change into narrow bandwidth, and hence makes the original system have poor transient response and be stabilized difficultly. An external model design where the model is adjusted adaptively to match the actual external signal and placed outside of the basic feedback loop, was then set up to take care of the problems. Among the external model based repetitive control designs, adaptive feedforward control (AFC) is a main method. In AFC design, it assumes that the unknown disturbance consists of the sum of sinusoids of known frequencies as the equation (2.1). The Fourier coefficients of the periodic disturbance with known frequency will be estimated adaptively by an adaptive algorithm in real-time. Since the output signal of repetitive controller is as being injected from outside of the feedback loop, it is more like feedforward and therefore is expected not to alter original closed-loop system stability and performance very much. This also implies that both repetitive controller and feedback controller designs are mutually independent.

Since synthesis of conventional repetitive control systems involves trade-off between robust stability and system performances, an optimized design method which can address the problem systematically is difficult to obtain. As a result, the goal of this chapter is to find an ideal control that achieves asymptotic tracking of the periodic reference $r(t)$ regardless of the periodic disturbance $d(t)$. Under this consideration, a modified AFC control will be presented. The key difference compared with conventional AFC is a new re-parameterization regression form employed in adaptive mechanism. Consequently, the stabilizability problem is no longer subject to the plant structure.

In the beginning of this chapter, one first gives a brief review of adaptive algorithm in order to describe it easily at the latter sections. In Section 3.2, the problem formulation in which we study will be discussed. In Section 3.3, a new re-parameterization regression form is proposed to employ in adaptive mechanism. Section 3.4 reviews conventional adaptive feedforward control (AFC). After reviewing conventional AFC, a new AFC design, which is based on the linear regression form of Section 3.3, is proposed to be independent of feedback control design. Furthermore, regarding that open-loop system is stable whether or not, the proposed control design takes different kinds of strategies in Section 3.5 and Section 3.6, respectively. The robustness of the proposed AFC with respect to un-modelled dynamics is studied in Section 3.7. Finally, Section 3.8 introduces an adaptive disturbance estimation algorithm for situations when it is desirable to track or monitor the unknown disturbance.

Note: In Chapter 3 and 4, notations in the time domain and frequency domain may be mixed in one expression; for example, $y(t) = W(s)u(t)$, where $W(s)$ is the transfer function from $u(t)$ to $y(t)$.

3.1 Review of Adaptive Algorithm

The adaptive algorithm is usually used in situations where one wishes to estimate an unknown constant vector $\theta \in R^p$, which characterizes either a signal or a dynamic system. The first step of the estimation process is to obtain, through a re-parameterization procedure, a linear regression form in θ,

$$w(t) = \phi^T(t)\theta, \tag{3.1}$$

where $w(t) \in R$ is an available signal, $\phi(t) \in R^p$ is a known bounded regressor, and $\theta \in R^p$ is the unknown constant vector to be estimated. Let $\hat{\theta}(t)$ be an estimate of θ. Based on the above linear regression form, there are two different kinds of identifier structures. One is the gradient algorithm, the other is the least-squares (LS) algorithm. The gradient algorithm suggests the following update law for $\hat{\theta}(t)$,

$$\dot{\hat{\theta}}(t) = \gamma\phi(t)(w(t) - \phi^T(t)\hat{\theta}(t)), \tag{3.2}$$

with a positive adaptation gain $\gamma > 0$, and an arbitrary initial guess $\hat{\theta}(0)$. Note that the regressor $\phi(t)$ in the linear regression form (3.1) needs to be uniformly bounded for the gradient algorithm (3.2). If one denotes the estimation error $\tilde{\theta}(t) = \theta - \hat{\theta}(t)$, the update law (3.2) results in a linear error dynamics

$$\dot{\tilde{\theta}}(t) = -\gamma\ \phi(t)\phi^T(t)\tilde{\theta}(t). \tag{3.3}$$

Theorem 3.1 [47] **:** If the regressor vector $\phi(t)$ is *persistently exciting* in the sense that for some finite interval length δ, the following matrix is positive definite for all $t > 0$,

$$\int_t^{t+\delta} \phi(\tau)\phi^T(\tau)d\tau > 0,$$

then the error dynamics (3.3) is exponentially stable, and $\hat{\theta}(t)$ in (3.2) converges to θ exponentially.

Based on the linear regression form (3.1), the LS algorithm suggests the following update law for $\hat{\theta}(t)$,

$$\dot{\hat{\theta}}(t) = \gamma\Omega(t)\phi(t)\left(w(t) - \phi^T\hat{\theta}(t)\right), \tag{3.4}$$

$$\dot{\Omega}(t) = -\gamma\left(-\eta\Omega(t) + \Omega(t)\phi(t)\phi^T(t)\Omega(t)\right), \tag{3.5}$$

where the adaptation gain $\gamma > 0$ is the design parameter which can be arbitrary chosen, the matrix $\Omega \in R^{pxp}$ is called covariance matrix and acts in the update law of $\hat{\theta}$ as a time-varying directional adaptation gain, and $\eta > 0$ being a forgetting factor prevents that Ω becomes arbitrarily small in some directions. The initial condition of the matrix Ω must be $\Omega(0) > 0$. From the textbook [47], one knows that it has the result which is similar to Theorem 3.1, that is, if the regressor $\phi(t)$ is persistently exciting, then the matrix $\Omega(t)$ in (3.5) is positive definite and $\hat{\theta}(t)$ in (3.4) converges to θ exponentially.

3.2 Problem Formulation

After reviewing the property of adaptive algorithm, one considers a linear time invariant (LTI) system subject to an unknown periodic disturbance:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Gd(t), \\ y(t) &= Cx(t) + Jd(t), \end{aligned} \tag{3.6}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R$ is the control input, $y(t) \in R$ is the system output, $d(t) \in R$ is an unknown periodic disturbance, and $A \in R^{n\times n}, B \in R^n, G \in R^n$, $C \in R^{1\times n}$, and $J \in R$ are known constant matrices. Note that the formulation of this thesis allows the disturbance $d(t)$ to enter the system at any place. The disturbance can enter the system at the input point ($G = B$ and $J = 0$), at the output point ($G = 0$ and $J \neq 0$) [37], or at any place in the system. One contribution of this dissertation is that the proposed control law remains the same no matter where the disturbance comes into the system; in other words, the proposed control law is independent of the matrices G and J.

Our objective in repetitive control design is to construct a control $u(t)$ for the system (3.6) that can drive the system output $y(t)$ to asymptotically track a periodic reference $r(t)$ despite the existence of unknown periodic disturbance $d(t)$. The disturbance $d(t)$ and the reference $r(t)$ are assumed to satisfy the following assumptions.

Assumption A3.1. $d(t)$ and $r(t)$ are of the same period; that is, $d(t) = d(t+T)$ and $r(t) = r(t+T)$ for some known period T.

Assumption A3.1 is only for easiness of presentation, the disturbance $d(t)$ and reference $r(t)$ are assumed to have the same period. The proposed control can be easily modified to allow $d(t)$ and $r(t)$ to have different periods.

Assumption A3.2. $d(t)$ and $r(t)$ are both continuous and have a piecewise continuous derivative.

Since the periodic signals $d(t)$ and $r(t)$ satisfy Assumption A3.1 and A3.2, the periodic disturbance thus has a *Fourier* series representation in (2.2), and the periodic reference signal also has has a finite series approximation

$$r(t) = \phi^T(t)\theta_r, \quad \theta_r \in R^{2N+1}, \tag{3.7}$$

where the harmonic regressor $\phi(t)$ was defined in (2.3), and θ_r is the unknown constant vector to be estimated.

3.3 Linear Regression Form

Since this thesis will adopt the AFC approach to deal with the repetitive control design problem, one thus needs transform the state space system (3.6) to the following

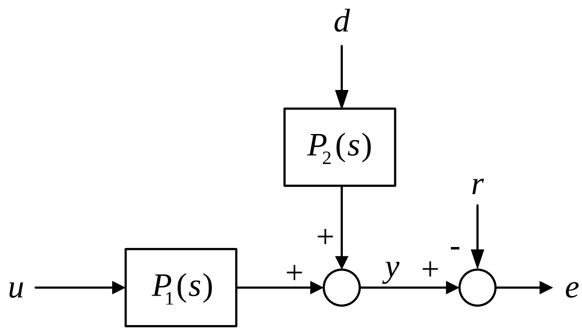

Figure 3.1: System

input-output description,

$$y(t) = P_1(s)u(t) + P_2(s)d(t), \tag{3.8}$$

where $P_1(s)$ and $P_2(s)$ are all stable transfer functions. The objective of repetitive control design is to construct a control $u(t)$ for the system (3.8) that can drive the system output $y(t)$ to asymptotically track a periodic reference $r(t)$.

Thus, setting a tracking error as

$$e(t) = y(t) - r(t), \tag{3.9}$$

and substituting (3.8) into (3.9), it becomes

$$e(t) = P_1(s)u(t) + P_2(s)d(t) - r(t). \tag{3.10}$$

Figure 3.1 shows the system block diagram. Re-arrange (3.10) into

$$e(t) - P_1(s)u(t) = P_2(s)d(t) - r(t). \tag{3.11}$$

It is important to note that on the right hand side of the above equation is still the periodic signal as a result of the periodic signal $d(t)$ passing into stable filter $P_2(s)$. Therefore, we guess that it has the following representation

$$P_2(s)d(t) - r(t) = P_1(s)d_1(t), \tag{3.12}$$

where $d_1(t)$ is a periodic signal with the period T. To prove the existence of such $d_1(t)$ in (3.12), one needs the following assumption.

Assumption A3.3. $P_1(j\omega_k) \neq 0$ for $k = 0, 1, \ldots, N$ where $\omega_k = k \cdot 2\pi/T$, T is the period of both $d(t)$ and $r(t)$, and $P_1(s)$ is the stable transfer function.

Remark 3.1 : Assumption A3.3 requires that the transfer function $P_1(s)$ has no system zero at $s = j\omega_k$. The reason is obvious: if $P_1(s)$ has a system zero at $s = j\omega_k$, due to the zero gain of $P_1(s)$ at $s = j\omega_k$, the equivalent periodic signal d_1 in (3.12) can not generate the k'th harmonic sinusoidal at the output point of $P_1(s)$. This assumption can be waived if the disturbance enters the system at the input point ($P_1(s) = P_2(s)$), and there is no tracking mission ($r(t) = 0$). However, as long as there is a periodic tracking mission ($r(t) \neq 0$), or the disturbance enters the system "not" at the input point, Assumption 3.3, which has been neglected by most previous literature, is necessary.

Lemma 3.2 : Given stable transfer function $P_1(s)$ and $P_2(s)$, and periodic reference $r(t)$ in (3.7) and periodic disturbance $d(t)$ in (2.5), if Assumption A3.3 holds, there exists an equivalent disturbance $d_1(t)$,

$$d_1(t) = \phi^T(t)\theta, \tag{3.13}$$

satisfying equation (3.12), with $\phi(t) \in R^{2N+1}$ as in (2.3), and $\theta \in R^{2N+1}$ some constant vector.

Proof: Since the steady state output of a stable system $P_1(s)$ subject to a sinusoid input is also a sinusoid but with different amplitude and phase, all subsequent analysis will assume that the filter output has reached a steady-state condition. Therefore, one has, for sufficiently large t,

$$P_1(s)[\phi(t)] \;=\; \begin{bmatrix} P_1(j0) \\ |P_1(j\omega_1)|\cos(\omega_1 t + \angle P_1(j\omega_1)) \\ |P_1(j\omega_1)|\sin(\omega_1 t + \angle P_1(j\omega_1)) \\ \vdots \\ |P_1(j\omega_N)|\cos(\omega_N t + \angle P_1(j\omega_N)) \\ |P_1(j\omega_N)|\sin(\omega_N t + \angle P_1(j\omega_N)) \end{bmatrix}$$

$$
\begin{aligned}
&= \begin{bmatrix} p_{0,c} \\ p_{1,c}\cos(\omega_1 t) - p_{1,s}\sin(\omega_1 t) \\ p_{1,s}\cos(\omega_1 t) + p_{1,c}\sin(\omega_1 t) \\ \vdots \\ p_{N,c}\cos(\omega_N t) - p_{N,s}\sin(\omega_N t) \\ p_{N,s}\cos(\omega_N t) + p_{N,c}\sin(\omega_N t) \end{bmatrix} \\
&= M_1\phi(t). \qquad (3.14)
\end{aligned}
$$

where

$$
\begin{aligned}
p_{k,c} &= |P_1(j\omega_k)|\cos(\angle P_1(j\omega_k)), \\
p_{k,s} &= |P_1(j\omega_k)|\sin(\angle P_1(j\omega_k)), \\
p_{0,c} &= P_1(j0), \qquad (3.15)
\end{aligned}
$$

and M_1 is a square matrix,

$$
M_1 = diag\left(p_{0,c}, \begin{bmatrix} p_{1,c} & -p_{1,s} \\ p_{1,s} & p_{1,c} \end{bmatrix}, \ldots, \begin{bmatrix} p_{N,c} & -p_{N,s} \\ p_{N,s} & p_{N,c} \end{bmatrix}\right). \qquad (3.16)
$$

Since M_1 is block diagonal, its determinant is

$$
\begin{aligned}
|M_1| &= p_{0,c}\prod_{k=1}^{N}(p_{k,c}^2 + p_{k,s}^2) \\
&= P_1(j0)\prod_{k=1}^{N}|P_1(j\omega_k)|^2. \qquad (3.17)
\end{aligned}
$$

From Assumption A3.3, one concludes that $|M_1| \neq 0$; hence, M_1 is invertible. Note also that

$$
\begin{aligned}
P_1(s)[\phi^T(t)] &= \{P_1(s)[\phi(t)]\}^T \\
&= [M_1\phi(t)]^T \\
&= \phi^T(t)M_1^T. \qquad (3.18)
\end{aligned}
$$

Substituting (2.5) and (3.7) into (3.12), and using a relationship similar to (3.14) on $P_2(s)$, equation (3.12) becomes

$$
\begin{aligned}
P_2(s)[\phi^T(t)\theta_d] - \phi^T(t)\theta_r &= \phi^T(t)[M_2^T\theta_d - \theta_r] \\
&= P_1(s)[d_1(t)]. \qquad (3.19)
\end{aligned}
$$

It is straightforward to check, using (3.14), that $d_1(t) = \phi^T(t)\theta$ with $\theta = M_1^{-T}[M_2^T\theta_d - \theta_r]$ satisfies the above equation. This proved the existence of $d_1(t)$. End of proof.

After identifying the existence of the equivalent periodic disturbance $d_1(t)$, substituting (3.12) into (3.11), one has

$$e(t) - P_1(s)u(t) = P_1(s)d_1(t). \tag{3.20}$$

Therefore, based on the equivalence of system, the system block in Figure 3.1 can be simplified to Figure 3.2. Note that equation is the same as those previous AFC

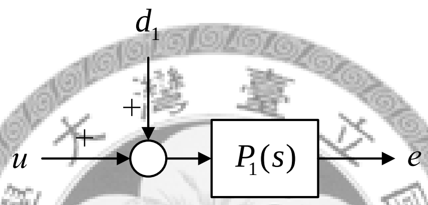

Figure 3.2: A system equivalent to Figure 3.1

systems when there is no tracking mission ($r(t) = 0$, $e(t) = y(t)$). The equation shows that an ideal control, which achieves asymptotic tracking of the periodic reference $r(t)$ regardless of the periodic disturbance $d(t)$, is $u = -d_1$. Therefore, the control design problem becomes the problem of estimating $d_1(t)$. According to (3.13) in Lemma 3.2, the estimation of the periodic disturbance $d_1(t)$ is further reduced to the estimation of the unknown constant vector θ in (3.13). In order to estimate θ, one needs a linear regression form in θ. This can be obtained by substituting (3.13) into (3.20),

$$e(t) - P_1(s)u(t) = P_1(s)[\phi^T(t)\theta] = P_1(s)[\phi^T(t)]\theta. \tag{3.21}$$

The above equation is in fact a linear regression form, but the regressor $P_1(s)\phi(t)$ increases many computational cost and analysis difficulties. In order to have a fine expression in the linear regression form (3.21), one further utilizes LTI system property, that is, when the periodic signal, $d_1(t)$, enters into an LTI system, $P_1(s)$, the system can be characterized by a superposition or sum of the zero-input response, $\epsilon(t)$, and the zero-state response, $\psi_1^T(t)\theta$. Therefore, (3.21) is rewritten as

$$e(t) - P_1(s)u(t) = \psi_1^T(t)\theta + \epsilon(t), \tag{3.22}$$

where $\epsilon(t)$ is the exponentially decaying term since the plant model is stable, and the regressor $\psi_1(t)$ is equivalent to $P_1(s)\phi(t)$ arriving in the steady state, that is $\psi_1(t) = P_1(s)\phi(t)|_{t\to\infty}$, and thus it has

$$\psi_1(t) = M_1\phi(t), \tag{3.23}$$

in which M_1 is the nonsingular block matrix as defined in (3.16). The regressor $\psi_1(t)$ is bounded since both M_1 and $\phi(t)$, defined in (3.16) and (2.3) respectively, are bounded. After some transient times, the exponentially decaying term $\epsilon(t)$ in equation (3.22) approaches to zero. Therefore, we will first neglect the presence of the $\epsilon(t)$ but latter show in Section 3.7 it does not affect the property of the identifier. In this case, the linear regression form in (3.22) is represented as

$$e(t) - P_1(s)u(t) = \psi_1^T(t)\theta. \tag{3.24}$$

Note that equation (3.24) shows that the linear regression form remains the same no matter where the disturbance enters the system. Therefore, if the system satisfies Assumption A3.3, one can assume that the periodic disturbance enter the system at input point.

Remark 3.2 : The key step in deriving the linear regression form (3.24) is to take the constant vector θ out of the square bracket of (3.21) after $P_1(s)$. Without this step, one must resort to model reference adaptive control scheme to estimate θ, as is done in many previous AFC designs, which have to enforce the minimum-phase assumption of $P_1(s)$ or small adaptation gain assumption.

3.4 Review of Adaptive Feedforward Control

Most of the AFC control systems are implemented in a plug-in manner as shown in Figure 3.3. In Figure 3.3, d, r, u, u_f, v, e and y are, respectively, the exogenous periodic disturbance, the reference input, the control input, the nominal feedback control, the feedforward control, the output error and the system output, $C(s)$ is the feedback controller and $P_1(s)$ is the transfer function of the plant. For the ease to analysis, it usually assumes that the control objective is to achieve the periodic tracking mission and the system plant P_1 is stable, and hence has $u_f = 0$. Consequently, according to Lemma 3.2, the control structure is simplified as Figure 3.4.

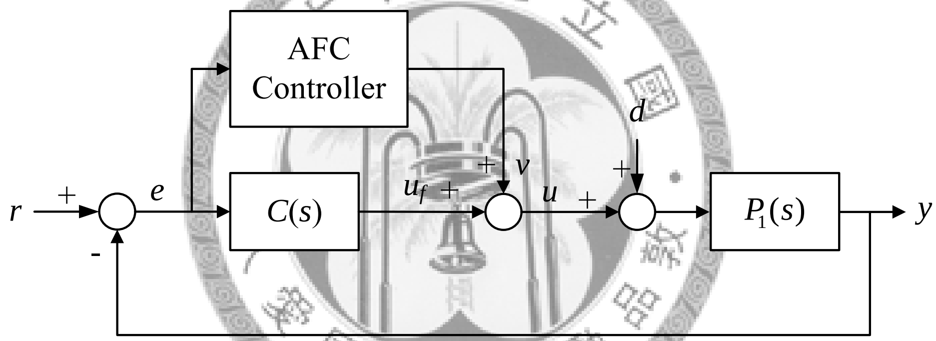

Figure 3.3: Control system with Plug-in AFC controller

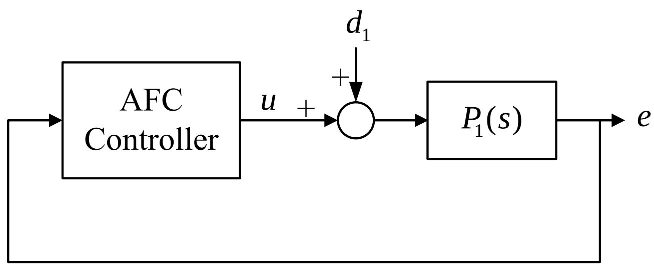

Figure 3.4: AFC control System

In Figure 3.4, the system output error is

$$e(t) = P_1(s)\left(u(t) + d_1(t)\right), \tag{3.25}$$

where the periodic disturbance d_1 has an expression form like (3.13). For being convenient to state, one assumes that d_1 is a single-tone harmonic signal

$$d_1(t) = \theta_{1,c}\cos(\omega_1 t) + \theta_{1,s}\sin(\omega_1 t), \tag{3.26}$$

in which ω_1 is a known frequency and $\theta_{1,c}$ and $\theta_{1,s}$ are unknown constant coefficients. Certainly, extended compensation for many sinusoids is straightforward. By adding the negative of disturbance at all time, the disturbance can be easily cancelled at the input of the plant. Hence, the feedforward control $u(t)$ is suggested as the negative of disturbance estimation. Consequently, the feedforward control has

$$u(t) = -\phi^T(t)\hat{\theta}(t). \tag{3.27}$$

Substituting (3.13) and (3.27) into (3.25), the plant output error is rewritten as

$$e(t) = P_1(s)[\phi^T(t)\theta - \phi^T(t)\hat{\theta}(t)]. \tag{3.28}$$

The problem is how to find an adjustment mechanism so that the parameter estimation $\hat{\theta}(t)$ converges to the nominal value θ and further the disturbance is cancelled exactly. Since this expression is similar to conventional *model reference identifiers* structure [47], the parameter vector $\hat{\theta}(t)$ has a possible update law which called the pseudo-gradient algorithm, that is

$$\dot{\hat{\theta}}(t) = \gamma\phi(t)e(t), \tag{3.29}$$

where $\gamma > 0$ is an adaptation gain. The AFC control diagram is shown in the Figure 3.5. According to the adaptive theory [47], if $P_1(s)$ is a strictly positive real (SPR), the system output $e(t)$ will converge to zero as $t \to \infty$. As a result of the SPR condition, stabilizing controller is only guaranteed on few physical systems.

Furthermore, based on the Laplace transform analysis, Messner and Bodson in [38] obtained an equivalent LTI representation. The resulting continuous-time transfer function from $e(t)$ to $u(t)$ is

$$\frac{u(t)}{e(t)} = -\gamma\frac{s}{s^2+\omega_1^2}. \tag{3.30}$$

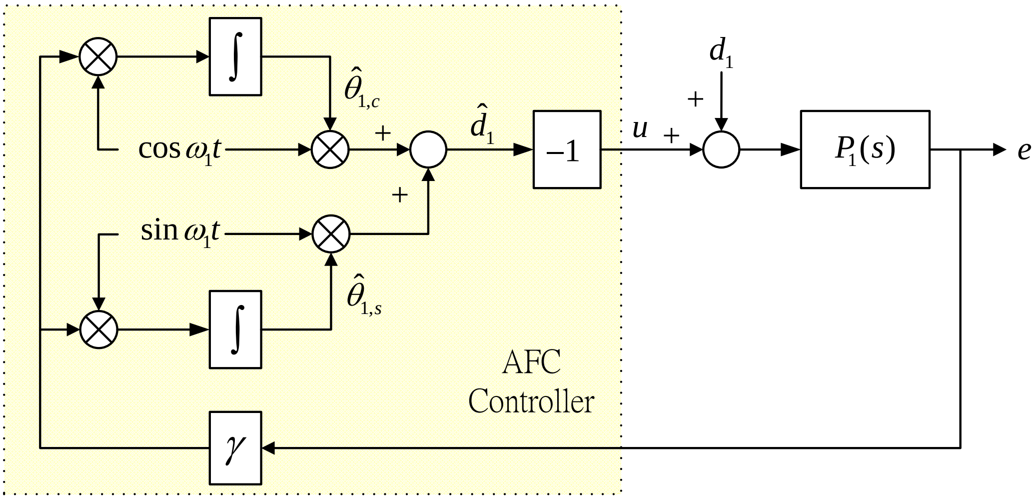

Figure 3.5: AFC control

Although it is derived from frequency domain, it can be derived from time domain more easily. Firstly, substituting the integration of (3.29) into (3.27), the control u can be written as

$$\begin{aligned} u(t) &= -\phi^T(t)\int_0^t \gamma\phi(\tau)e(\tau)d\tau \\ &= -\gamma\int_0^t \cos(\omega_1(t-\tau))e(\tau)d\tau, \end{aligned} \tag{3.31}$$

where the term of integral at the last equation expresses a convolution integral, that is $\cos(\omega_1 t) * e(t)$. Finally, taking the Laplace transform on $\cos(\omega_1 t)$, one can immediately obtain the result of (3.30).

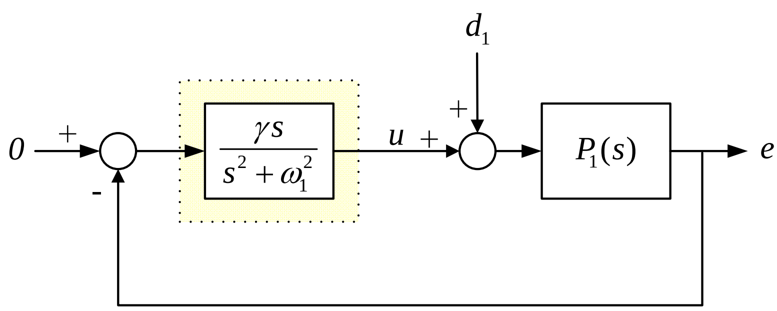

Figure 3.6: IMP control which is equivalent to Figure 3.5

It is obvious that the AFC scheme of Figure 3.5 and the IMP scheme of Figure

3.6 are functionally equivalence in principle. The result allows LTI analysis techniques to be used in adaptive system. Therefore, from the IMP controller design, the SPR condition can be relaxed if the adaptation gain γ is carefully chosen. Besides, Messner and Bodson also stated that the phase difference between the input disturbance and the measurement output might deteriorate the convergence property of the AFC system. The cause of the phase lag is the conventional AFC design without considering the plant model. In order to compensate for the phase lag, they add a phase shift α_1 into the regressor of (3.29). The modified regressor then becomes

$$\phi_\alpha(t) = \begin{bmatrix} \cos(\omega_1 t + \alpha_1) & \sin(\omega_1 t + \alpha_1) \end{bmatrix}^T,$$

and the update law (3.29) becomes

$$\dot{\hat{\theta}}(t) = \gamma \phi_\alpha(t) e(t).$$

Note that the feedforward control is still set as (3.27). Consequently, using similar operating process in (3.31), the resulting continuous-time transfer function from $e(t)$ to $u(t)$ becomes

$$\frac{u(t)}{e(t)} = -\gamma \frac{s\cos(\alpha_1) + \omega \sin(\alpha_1)}{s^2 + \omega_1^2}.$$

To achieve the fastest elimination of the periodic disturbance at low adaptive gain, Bodson and co-workers [39] suggested that the optimal regressor phase advance α_1 is the phase of the plant at the disturbance frequency.

The main advantages of the AFC have the following items. First, it can selectively remove harmonics from the frequency spectrum. Second, it is not necessary to acquire an exact plant model. Third, when the output signal is not available during certain periods of time, the AFC controller can simply freeze the parameter updates. On the contrary, the internal model based controller is not robust to this variation. Forth, it can be driven by the measuring frequency, making the control response more robust to variation in frequency. Finally, the adaptive implementation can adopt angular measurements directly. It needn't require the frequency to be computed from the angular measurements. However, the system must be SPR or the adaptation gain be small in order to maintain stability. Besides, from IMP equivalence perspective, when the controller introduces a number of sinusoidal signals, the stability problem becomes more and more difficult.

Example 3.1 : Consider an open-loop stable system with the input-output description as in (3.25), where the plant transfer function $P_1(s)$ is

$$P_1(s) = \frac{(s+3)(s+5)}{(s+2)(s+4)(s+6)},$$

and the periodic disturbance d_1 is

$$d_1(t) = 6\cos(\omega t) + \cos(2\omega t) + 0.5\sin(3\omega t),$$

in which the frequency $\omega = 1$.

In the adaptive estimation algorithm (3.29), one sets the adaptation gain $\gamma = 5$, and the regressor

$$\phi(t) = \left[\begin{array}{ccccccc} 1 & \cos(\omega t) & \sin(\omega t) & \cos(2\omega t) & \sin(2\omega t) & \cos(3\omega t) & \sin(3\omega t) \end{array}\right]^T,$$

Figure 3.7 shows the output error asymptotically converges to zero and the root mean square error at $70s \leq t \leq 100s$ is 7.1387×10^{-8}. Figure 3.8 shows the time history of the disturbance, where the true disturbance $d_1(t)$ and the disturbance estimation $\hat{d}_1(t)$ are shown by dotted line and solid line respectively. It shows the estimate asymptotically converges to the true disturbance. Therefore, the simulation verifies successfully that the AFC design has good performance under SPR system.

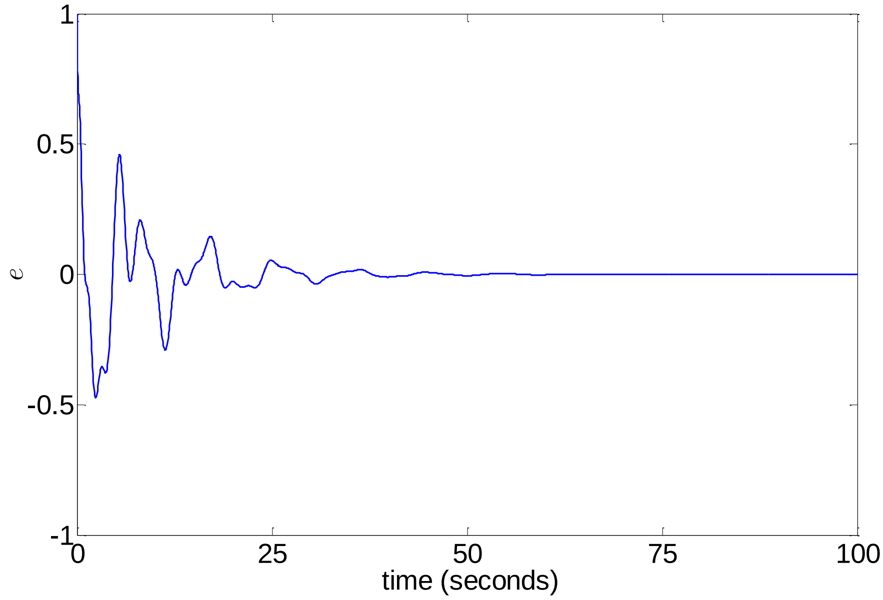

Figure 3.7: Trajectory of the output error $e(t)$

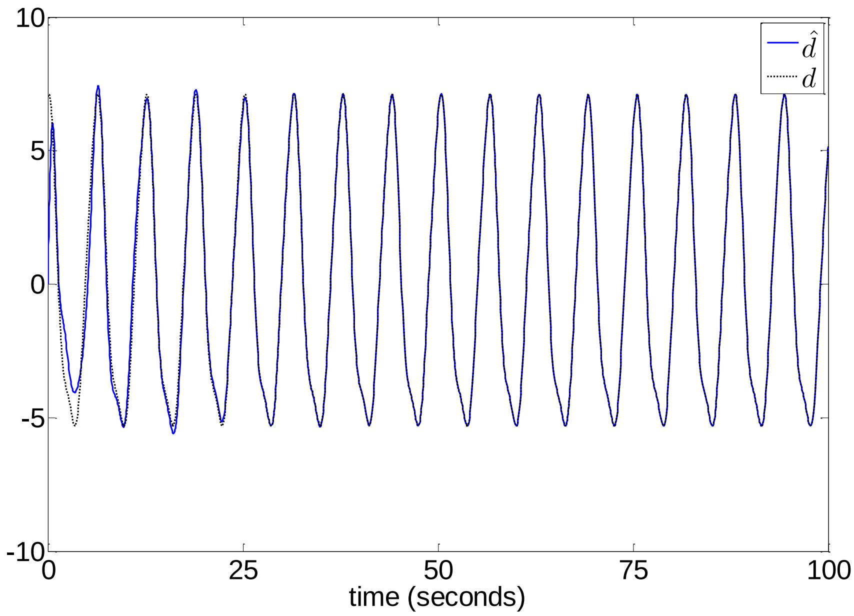

Figure 3.8: Time history of the disturbance $d_1(t)$ and the estimate $\hat{d}_1(t)$

3.5 New AFC Design for Open-Loop Stable System

The goal of this section is to construct a new AFC controller for open-loop stable systems subject to unknown periodic disturbances. One assumes that the system matrix A in (3.6) is stable. Denote

$$P_1(s) = C(sI - A)^{-1}B, \tag{3.32}$$

and

$$P_2(s) = C(sI - A)^{-1}G + J, \tag{3.33}$$

where $P_1(s)$ and $P_2(s)$ are stable transfer functions since the system matrix A is stable. Therefore, the state space system (3.6) has the input-output description as (3.8).

As a result of Lemma 3.2, the equivalent disturbance $d_1(t)$ is set up. In the thesis, one makes use of disturbance-observer-like method to obtain the filtered disturbance $P_1(s)d_1(t)$ and then do disturbance identification obtaining the estimated disturbance $\hat{d}_1$. Therefore, the AFC control u is set as $-\hat{d}_1$. So the identified mechanism for the disturbance is regarded as AFC controller by us. Figure 3.9 shows the control system structure. Our proposed AFC control designs in this section are all based on this one to set up.

3.5.1 Gradient Based AFC

Based on the new linear regression form (3.24), the gradient algorithm in (3.2) suggests the following update law for the estimated $\hat{\theta}(t)$,

$$\dot{\hat{\theta}}(t) = \gamma\psi_1(t)(e(t) - P_1(s)u(t) - \psi_1^T\hat{\theta}(t)), \quad \hat{\theta} \in R^{2N+1}, \tag{3.34}$$

where the adaptation gain $\gamma > 0$ can be arbitrarily chosen and plant transfer function P_1 is assumed to know exactly.

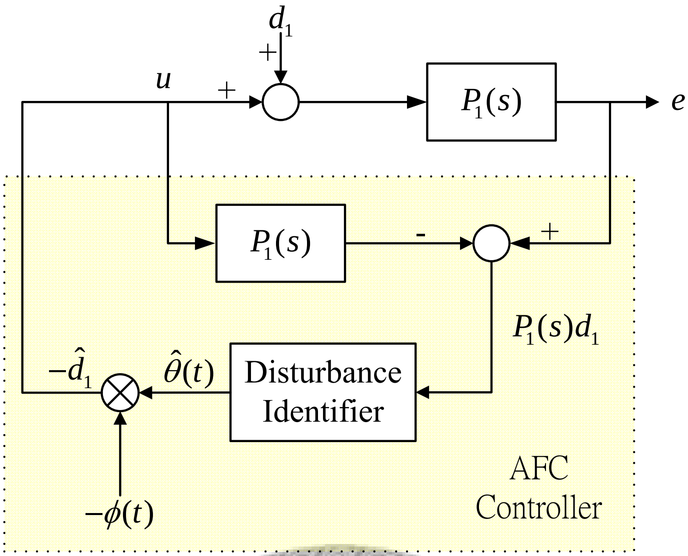

Figure 3.9: AFC control system

Lemma 3.3 : Under Assumption A3.3, $\psi_1(t)$ as defined in (3.23) is persistently exciting, and $\hat{\theta}(t)$ in (3.34) converges exponentially to θ.

Proof: One will prove Lemma 3.3 for $N = 2$ in (3.13); that is, $\phi(t) \in R^5$. To prove that $\psi_1(t)$ is persistently exciting, one can derive, by using (3.23),

$$\int_t^{t+T} \psi_1(\tau)\psi_1^T(\tau)d\tau = M_1 \int_t^{t+T} \phi(\tau)\phi^T(\tau)d\tau M_1^T. \tag{3.35}$$

Using equalities

$$\begin{aligned}
\int_t^{t+T} \sin(\omega_k\tau)\sin(\omega_m\tau)d\tau &= \begin{cases} 0, & k \neq m \\ T/2, & k = m \end{cases}, \\
\int_t^{t+T} \cos(\omega_k\tau)\cos(\omega_m\tau)d\tau &= \begin{cases} 0, & k \neq m \\ T/2, & k = m \neq 0 \\ T, & k = m = 0 \end{cases}, \\
\int_t^{t+T} sin(\omega_k\tau)\cos(\omega_m\tau)d\tau &= 0.
\end{aligned}$$

with m and k being integers, one can show that

$$\int_t^{t+T} \phi(\tau)\phi^T(\tau)d\tau$$

$$
= \int_t^{t+T} \begin{bmatrix} 1 & \cos(\omega_1\tau) & \sin(\omega_1\tau) \\ \cos(\omega_1\tau) & \cos^2(\omega_1\tau) & \cos(\omega_1\tau)\sin(\omega_1\tau) \\ \sin(\omega_1\tau) & \sin(\omega_1\tau)\cos(\omega_1\tau) & \sin(\omega_2\tau) \\ \cos(\omega_2\tau) & \cos(\omega_2\tau)\cos(\omega_1\tau) & \cos(\omega_2\tau)\sin(\omega_1\tau) \\ \sin(\omega_2\tau) & \sin(\omega_2\tau)\cos(\omega_1\tau) & \sin(\omega_2\tau)\sin(\omega_1\tau) \end{bmatrix.
$$

$$
\left. \begin{matrix} \cos(\omega_2\tau) & \sin(\omega_2\tau) \\ \cos(\omega_1\tau)\cos(\omega_2\tau) & \cos(\omega_1\tau)\sin(\omega_2\tau) \\ \sin(\omega_1\tau)\cos(\omega_2\tau) & \sin(\omega_1\tau)\sin(\omega_2\tau) \\ \cos^2(\omega_2\tau) & \cos(\omega_2\tau)\sin(\omega_2\tau) \\ \sin(\omega_2\tau)\cos(\omega_2\tau) & \sin^2(\omega_2\tau) \end{matrix} \right] d\tau
$$

$$
= \frac{T}{2} \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3.36}
$$

Substituting (3.36) and (3.16) into (3.35) leads to

$$
\int_t^{t+T} \psi_1(\tau)\psi_1^T(\tau)d\tau
$$

$$
= \frac{T}{2} \begin{bmatrix} 2|P_1(j0)|^2 & 0 & 0 & 0 & 0 \\ 0 & |P_1(j\omega_1)|^2 & 0 & 0 & 0 \\ 0 & 0 & |P_1(j\omega_1)|^2 & 0 & 0 \\ 0 & 0 & 0 & |P_1(j\omega_2)|^2 & 0 \\ 0 & 0 & 0 & 0 & |P_1(j\omega_2)|^2 \end{bmatrix}, \tag{3.37}
$$

which is positive definite since it is a diagonal matrix with positive elements on the diagonal. Hence, $\psi_1(t)$ is persistently exciting. Finally, quoting Theorem 3.1, one concludes that $\hat{\theta}(t)$ in (3.34) converges to θ exponentially. End of proof.

Remark 3.3 : It is interesting to know if it is possible to estimate all the coefficients in the infinite-term Fourier series of the reference input $d_1(t)$. Equation (3.37) in the proof of Lemma 3.3 gives a negative answer to this question. The reason is that $P_1(s) = C(sI - A)^{-1}B$ is strictly proper, and hence $P_1(j\omega_N)$ in (3.37) approaches zero as N approaches infinity. As a result, the diagonal matrix in (3.37) approaches singular as N approaches infinity, and hence the regressor $\psi_1(t)$ no longer satisfies the persistent excitation condition. Without the persistent excitation condition, the gradient algorithm can not guarantee convergence of $\hat{\theta}(t)$ to θ. For this reason, one "must" use the finite series approximation instead of the infinite Fourier series in the disturbance estimation process.

The proposed control $u(t)$ is then set to be

$$u(t) = -\hat{d}_1(t) = -\phi^T(t)\hat{\theta}(t). \tag{3.38}$$

Since $\hat{\theta}(t)$ converges to θ according to Lemma 3.3, one has

$$u(t) + d_1(t) = \phi^T(t)\left(-\hat{\theta}(t) + \theta(t)\right),$$

converges to zero which achieves tracking of the given reference $r(t)$ in the face of unknown disturbance $d(t)$. The result is summarized in the theorem below.

Theorem 3.4 : Under Assumptions A3.1, A3.2, and A3.3, the proposed control $u(t)$ in (3.38) and (3.34) drives the system output $y(t)$ to track exponentially the periodic reference $r(t)$ despite the existence of unknown periodic disturbance $d(t)$.

Remark 3.4 : Even though there is a small error $\tilde{d}_1(t) = d_1(t) - \phi^T(t)\theta$ between the finite series approximation $\phi^T(t)\theta$ and the infinite Fourier series $d_1(t)$, this approximation error $\tilde{d}_1(t)$ approaches zero as N approaches infinity according to Theorem 2.1. Therefore, the small error $\tilde{d}_1(t)$ will not create problems in the reference input estimation process in Theorem 3.4. The reason is as follows. Denote the estimation error $\tilde{\theta}(t) = \theta - \hat{\theta}(t)$. Using (3.24), the update law (3.34) results in an error dynamics

$$\dot{\tilde{\theta}}(t) = -\gamma\ \psi_1(t)\psi_1^T(t)\tilde{\theta}(t) - \gamma\ \psi_1(t)P_1(s)\tilde{d}_1(t).$$

Since the error dynamics (3.3) of the gradient algorithm is exponentially stable, small additive error $\tilde{d}_1(t)$ will only create small estimation bias. The final disturbance estimate $\hat{d}_1(t)$ will only be minutely biased from the equivalent disturbance $d_1(t)$ for sufficiently large N.

Notice again that the proposed adaptive feedfoward cancellation control in (3.38) and (3.34) is independent of the system matrices G and J in (3.6). In other words, no matter where the disturbance comes into the system, the proposed control remains the same. This is shown for the first time in the literature, and it justifies previous AFC designs in which the disturbances are mostly assumed to enter the system at the control input point, even though in reality it is not the case.

Example 3.2: Consider an open-loop stable system (3.6) with the input-output description as in (3.8), where the plant transfer function

$$P_1(s) = \frac{(s+6)(s-4)}{(s+2)(s+3)(s+4)}, \quad \text{and} \quad P_2(s) = \frac{(s+1)(s-1)}{(s+2)(s+3)(s+4)},$$

and the unknown periodic disturbance

$$d(t) = \begin{cases} \frac{8}{T}t, & 0 \leq t < \frac{T}{4} \\ 2, & \frac{T}{4} \leq t < \frac{3T}{4} \\ 8 - \frac{8}{T}t, & \frac{3T}{4} \leq t < T \end{cases}, \tag{3.39}$$

with a period $T = 10$ seconds as shown in Figure 3.10. Note that both transfer functions above are *non-minimum phase* (have unstable zeros). The system output $y(t)$ is required to track a periodic reference

$$r(t) = \begin{cases} -3 + \frac{12}{T}t, & 0 \leq t < \frac{T}{2} \\ 9 - \frac{12}{T}t, & \frac{T}{2} \leq t < T \end{cases}. \tag{3.40}$$

In the adaptive estimation algorithm (3.34), one sets $N = 4$; in other words, there are $2N + 1 = 9$ terms in the finite series approximation of the periodic $d_1(t)$. The adaptation gain is set to be $\gamma = 0.45$. Figure 3.11 shows the periodic reference $r(t)$ (dotted line) and the system output $y(t)$ (solid line). It shows that almost perfect tracking is achieved except those high frequency components which are unmodeling residual terms. The root mean square tracking error at $70s \leq t \leq 100s$ is 0.0697. The simulation study shows that if the number of terms $2N + 1$ in the finite series approximation of $d_1(t)$ is increased, the tracking error can be further reduced. Finally, Figure 3.12 shows the time history of control input $u(t)$, which remains uniformly bounded even though the system is non-minimum phase.

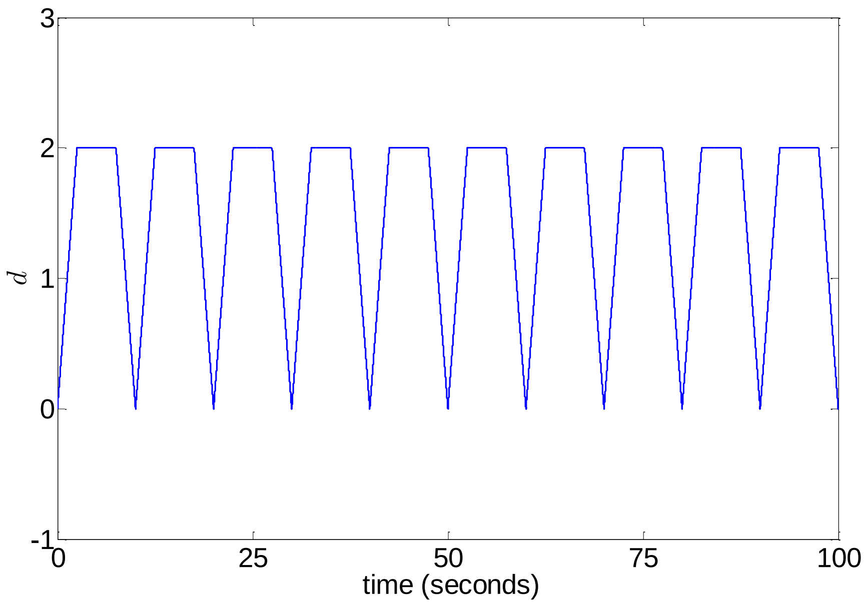

Figure 3.10: Time history of periodic disturbance $d(t)$

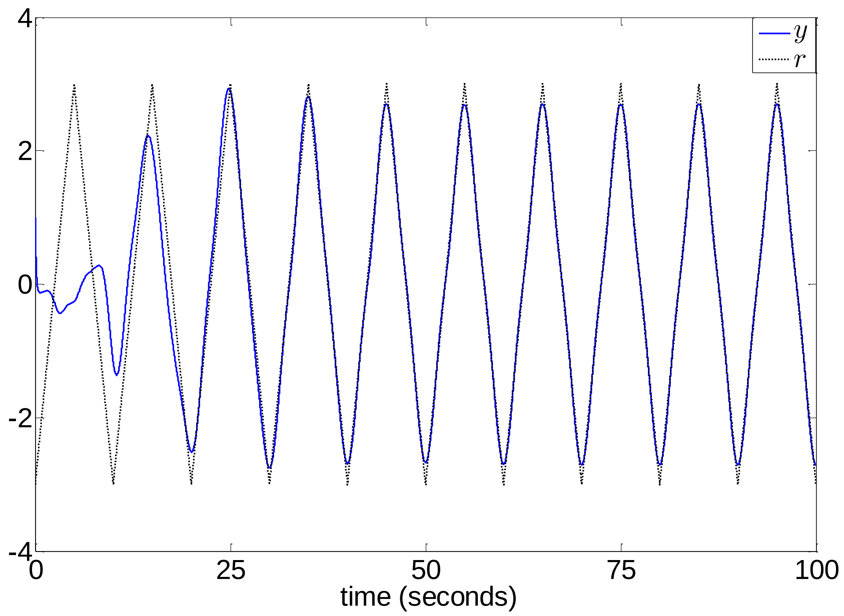

Figure 3.11: Trajectory of the reference signal $r(t)$ and the output $y(t)$

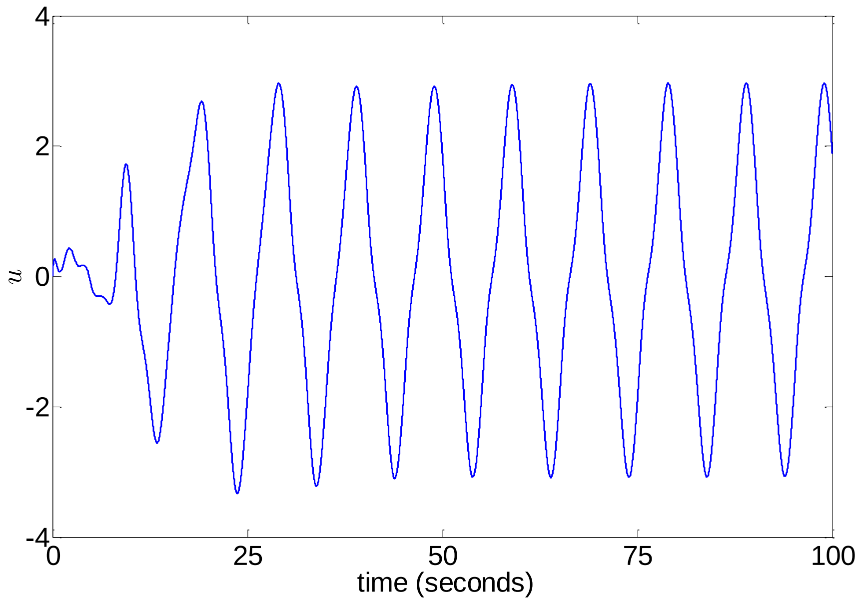

Figure 3.12: Trajectory of the control input $u(t)$

3.5.2 LS Based AFC

Although, in previous subsection, the gradient algorithm was used in our proposed control, any identification algorithms based on the new linear regression form (3.24) can also be done well. Therefore, we will introduce another kind of common method, that is least-squares (LS)[47]. Based on (3.24), the LS algorithm in (3.4) suggests the following update law for the estimated $\hat{\theta}(t)$,

$$\dot{\hat{\theta}}(t) = \gamma\Omega(t)\psi_1(t)\left(e(t) - P_1(s)u(t) - \psi_1^T\hat{\theta}(t)\right), \quad \hat{\theta} \in R^{2N+1}, \tag{3.41}$$

$$\dot{\Omega}(t) = -\gamma\left(-\eta\Omega(t) + \Omega(t)\psi_1(t)\psi_1^T(t)\Omega(t)\right), \quad \Omega \in R^{(2N+1)\times(2N+1)}, \tag{3.42}$$

where the adaptation gain $\gamma > 0$ is the design parameter which can be arbitrary chosen, the matrix Ω is called covariance matrix and acts in the update law of $\hat{\theta}$ as a time-varying directional adaptation gain, and $\eta > 0$ being a forgetting factor prevents that Ω becomes arbitrarily small in some directions. The initial condition of the matrix Ω must be $\Omega(0) > 0$.

Theorem 3.5 : Under Assumptions A3.1, A3.2, and A3.3, the proposed control $u(t)$ in (3.38) and (3.41) drives that the output error $e(t)$ in (3.20) converges to zero exponentially despite the existence of unknown periodic disturbance $d_1(t)$.

Proof: Denote the estimation error $\tilde{\theta}(t) = \theta - \hat{\theta}(t)$. Using (3.24), the update law (3.41) results in the parameter error dynamics

$$\dot{\tilde{\theta}}(t) = -\gamma\Omega(t)\psi_1(t)\psi_1^T(t)\tilde{\theta}(t).$$

Define a Lyapunov function $V = \tilde{\theta}^T\Omega^{-1}\tilde{\theta}$. The change rate of V along the above parameter error dynamics satisfies

$$\dot{V}(t) \;=\; 2\tilde{\theta}^T\Omega^{-1}\left(-\gamma\Omega(t)\psi_1(t)\psi_1^T(t)\tilde{\theta}(t)\right) + \tilde{\theta}^T\dot{\Omega}^{-1}\tilde{\theta}.$$

Using $\dot{\Omega}^{-1} = -\Omega^{-1}\dot{\Omega}\Omega^{-1}$ and (3.42), $\dot{V}$ becomes

$$\dot{V}(t) = -\gamma\left(\psi_1^T(t)\tilde{\theta}(t)\right)^2 - \gamma\eta V(t).$$

Due to $\dot{V} < 0$, this implies that $V(t)$ and hence $\tilde{\theta}(t)$ decay to zero exponentially. One thus proves that the system (3.20) is globally exponentially stable. End of proof.

It is important to note that the above theorem shows that the proposed LS based AFC can obtain an arbitrary fast convergence rate if γ is set sufficiently large. The following simulations mainly test the feasibility and convergence rate of the proposed LS based AFC control. Besides, when the direction of both disturbance and control are same, the following example also shows the proposed AFC can deal with the disturbance with time-varying frequency.

Example 3.3: In this example, a time-varying periodic disturbance rejection problem will be examined. Consider an open-loop stable system (3.20), where the plant transfer function

$$P_1(s) = \frac{(s+6)(s-4)}{(s+2)(s+3)(s+4)},$$

and the unknown periodic disturbance

$$
\begin{aligned}
d_1(t) &= 3\sin(\alpha(t)) - 2\cos(2\alpha(t)), \\
\alpha(t) &= t + \sin(0.5\pi t),
\end{aligned}
$$

in which α is an angle displacement with time-varying frequency.

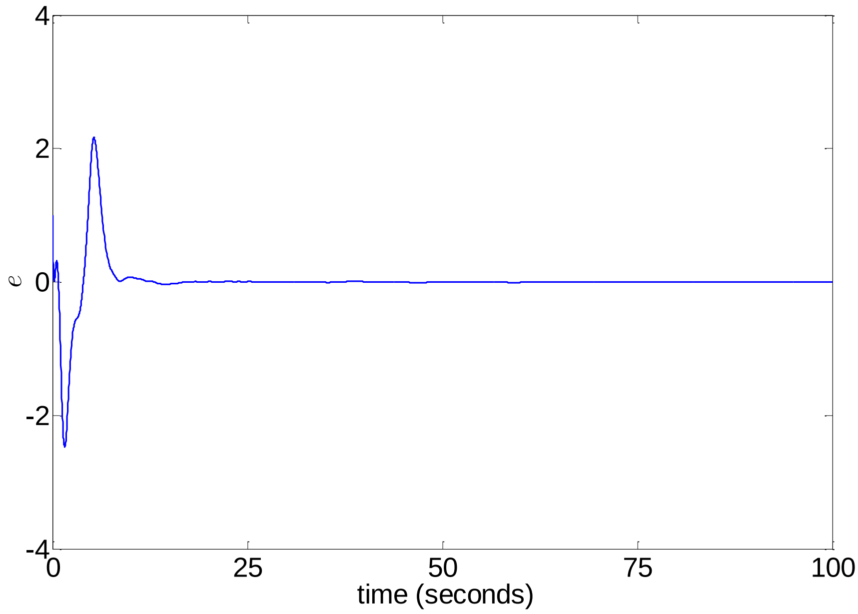

Figure 3.13: Output error $e(t)$

In the LS algorithm (3.41), one sets $N = 2$; in other words, there are $2N+1 = 5$ terms in the finite series approximation of the periodic $d_1(t)$. The adaptation gain, forgetting factor and initial covariance matrix are set to be $\gamma = 50$, $\eta = 10^{-5}$ and $\Omega(0) = I$ respectively. Figure 3.13 shows the output error $e(t)$. It is seen that time-varying periodic disturbance is almost rejected completely. The root mean square output error at $70s \leq t \leq 100s$ is 0.0015. Figure 3.14 shows the disturbance $d_1(t)$ (dotted line) and the estimated disturbance $\hat{d}_1(t)$ (solid line). Since both signals are almost overlapping, it shows that LS algorithm performs well. The root mean square estimation error of $d_1 - \hat{d}_1$ at $70s \leq t \leq 100s$ is 0.0022. Finally, Figure 3.15 shows the time history of the parameter estimation $\hat{\theta}$. It shows that the estimation approaches true parameter even though the frequency of disturbance is time-varying.

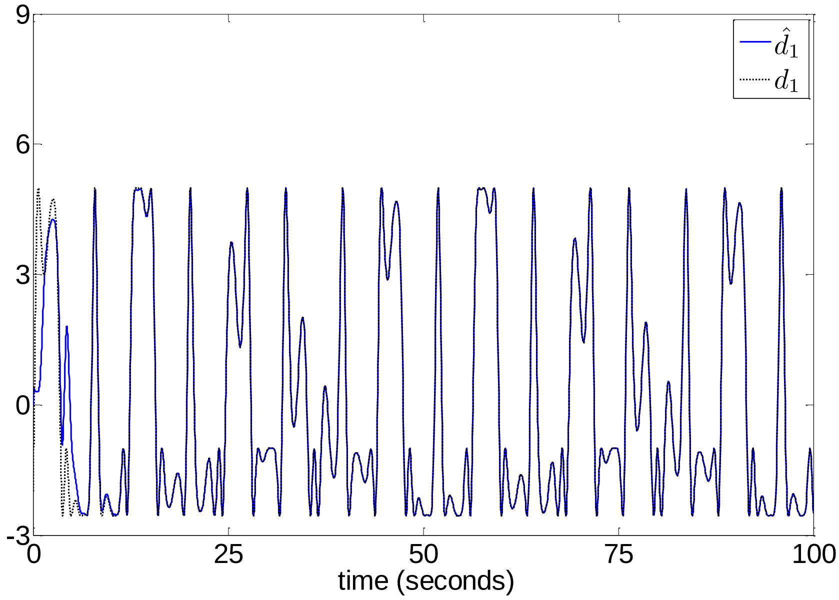

Figure 3.14: Trajectory of the disturbance $d_1(t)$ and estimated disturbance $\hat{d}_1(t)$

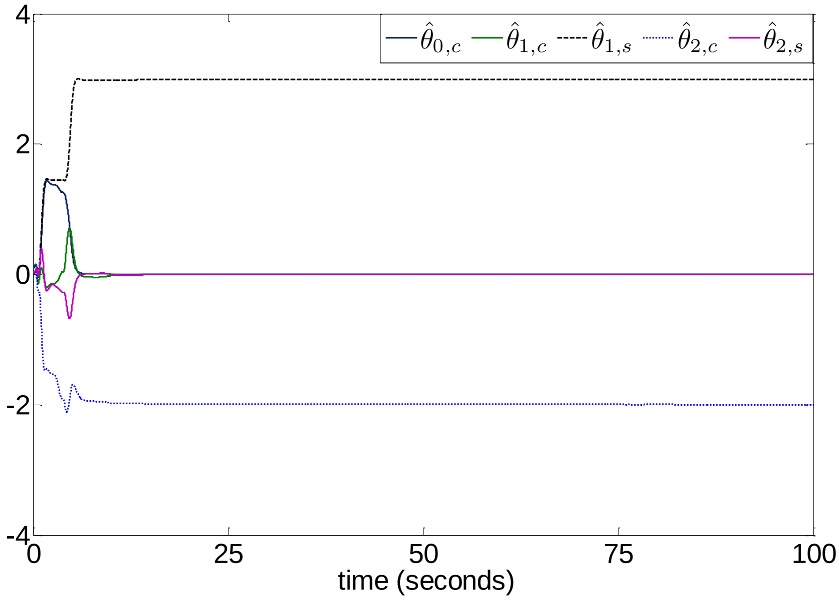

Figure 3.15: Trajectory of the parameter estimation $\hat{\theta}(t)$

Example 3.4: Consider the same system as in Example 3.3. The adaptive control law is as in Example 3.3 except the adaptation gain γ is chosen as $\gamma = 5000$.

Figure 3.16 shows the output error $e(t)$. The root mean square output error at $70s \leq t \leq 100s$ is 6.1904^{-5}. Figure 3.17 shows the disturbance $d_1(t)$ (dotted line) and the estimated disturbance $\hat{d}_1(t)$ (solid line). The root mean square estimation error of $d_1 - \hat{d}_1$ at $70s \leq t \leq 100s$ is 9.7019^{-5}. Obviously, the root mean square of e and $d_1 - \hat{d}_1$ is smaller than those in Example 3.3; and further, comparing with Figure 3.13 and Figure 3.14, Figure 3.16 and Figure 3.17 show more fast convergence rate. Consequently, according to the simulation result, one can conclude that the proposed LS based AFC has an arbitrary convergence rate.

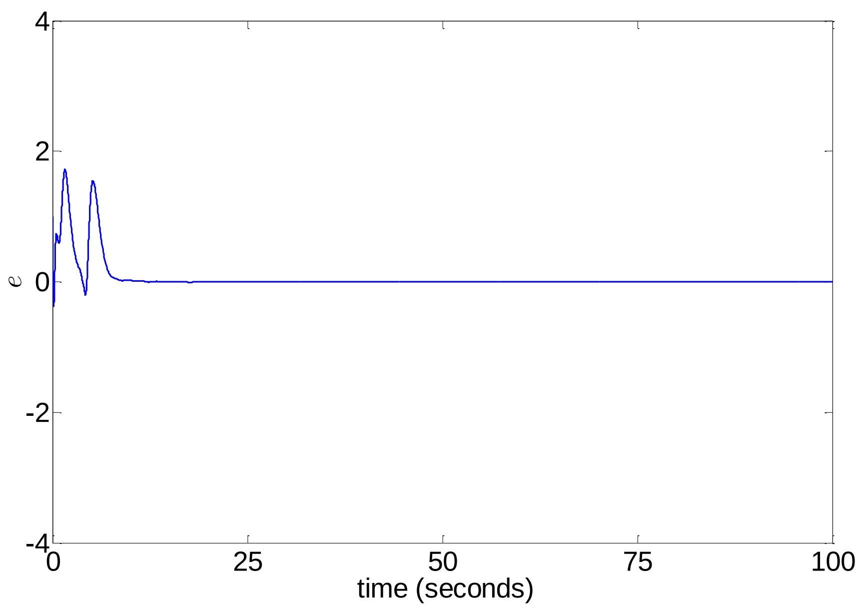

Figure 3.16: Output error $e(t)$

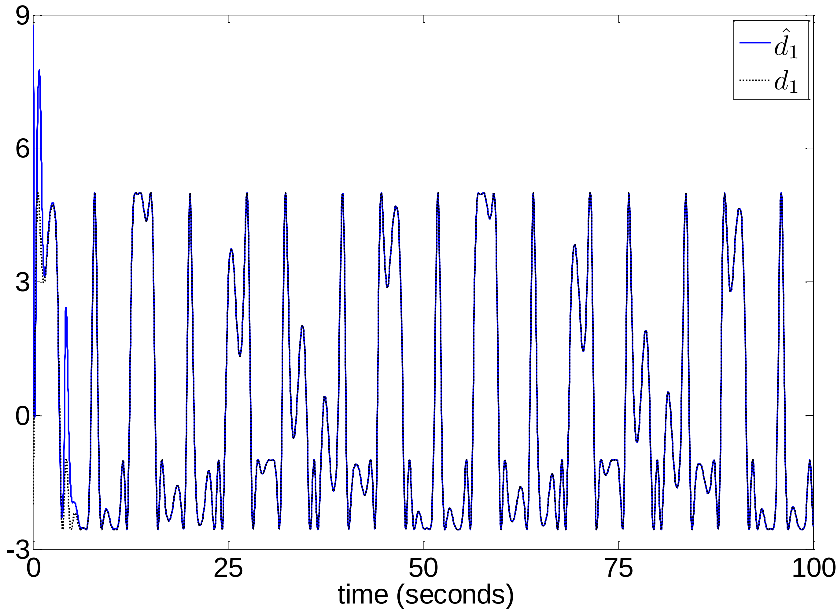

Figure 3.17: Trajectory of the disturbance $d_1(t)$ and estimated disturbance $\hat{d}_1(t)$

3.6 New AFC Design for Open-Loop Unstable System

When the open-loop system matrix A is unstable, the AFC controller introduced in the previous section no longer guarantees a bounded control. The reason is that $P_1(s)\phi(t)$ in (3.22) becomes unbounded since $P_1(s) = C(sI - A)^{-1}B$ is unstable in this case. In this section, the key difference for the AFC design is that one needs a stabilizing controller to stabilize the system firstly when the open-loop system is unstable.

Therefore, the control input $u(t)$ in (3.6) becomes

$$u(t) = v(t) - K\hat{x}(t), \tag{3.43}$$

where v is a feedforward control for cancelling the periodic disturbance $d(t)$ or/and tracking the periodic reference $r(t)$, and $-K\hat{x}(t)$ is a stabilizing control in which K is a feedback gain, and $\hat{x}$ is an estimated state obtained from the following state observer

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - C\hat{x}(t)), \tag{3.44}$$

in which L is an observer gain. Denote the state error $\tilde{x} = x - \hat{x}$. Using (3.6), the state observer (3.44) results in the state error dynamics

$$\dot{\tilde{x}}(t) = (A - LC)\tilde{x}(t) + Gd(t) - LJd(t). \tag{3.45}$$

The corresponding input-output description from d to $\tilde{x}$ is expressed as

$$\tilde{x}(t) = W_o(s)d(t) \tag{3.46}$$

where

$$W_o(s) = (sI - A + LC)^{-1}(G - LJ).$$

$W_o(s)$ is a stable transfer matrix since $A - LC$ is a stable matrix by design of L. Hence the state error is a bounded limit. Using (3.43), and (3.44), system equation (3.6) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= (A - BK)x(t) + BK\tilde{x}(t) + Bv(t) + Gd(t) \\ &= (A - BK)x(t) + (BKW_o(s) + G)\, d(t) + Bv(t). \end{aligned} \tag{3.47}$$

where the last equality results from (3.46). Denote

$$P_1(s) = C(sI - A + BK)^{-1}B, \tag{3.48}$$

and

$$P_2(s) = C(sI - A + BK)^{-1}(G + BKW_o(s)) + J, \tag{3.49}$$

where $P_1(s)$ and $P_2(s)$ are all stable transfer functions since $A-BK$ is a stable matrix by design of K. Furthermore, the system output $y(t)$ in (3.6) can be expressed as (3.8). Note that $P_1(s)$ is different from one which is in (3.34) since $P_1(s)$ in (3.48) contains feedback control. Using the similar to open-loop stable system operating in previous section, one has the same as the update law defined in (3.34) for the estimated $\hat{\theta}_u$, that is

$$\dot{\hat{\theta}}(t) = \gamma\psi_1(t)(e(t) - P_1(s)u(t) - \psi_1^T\hat{\theta}(t)), \quad \hat{\theta} \in R^{2N+1}. \tag{3.50}$$

Therefore, the proposed AFC control is also set to be

$$v(t) = -\phi^T(t)\hat{\theta}.$$

It is important to note that when the system matrix A is stable, the equation (3.48) and (3.49) are equivalent to (3.32) and (3.33) respectively due to the stabilizing control gain $K = 0$.

Example 3.5: Consider an open-loop unstable system (3.6) with system matrices

$$A = \begin{bmatrix} -2.9 & 1 & 0 \\ -1.7 & 0 & 1 \\ 0.2 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}, G = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, J = 0,$$

$x(0) = [1, 1, 1]^T$, and the unknown periodical disturbance is the same as (3.39) that was in Example 3.2. The system output $y(t)$ is required to track a periodic reference $r(t)$ in (3.40). The input-output description of the system is

$$y(t) = \frac{s-5}{(s+1)(s+2)(s-0.1)}u(t) + \frac{s-2}{(s+1)(s+2)(s-0.1)}d(t).$$

Note that the above transfer function from both u and d to y are unstable and non-minimum phase (has unstable zeros).

In the adaptive estimation algorithm (3.34), one also sets $N = 4$. The adaptation gain, observer gain and feedback gain, respectively, are set to be $\gamma = 5$, $L = [\ 3.1 \quad 9.3 \quad 6.2\ ]^T$ which makes the observer (3.44) have closed-loop poles $\lambda(A - LC) = \{-1, -2, -3\}$, $K = [\ 0.8855 \quad -1.1449 \quad -1.4490\ ]$ which makes the plant (3.48) have closed-loop poles $\lambda(A - BK) = \{-2, -3, -4\}$. Figure 3.18 shows the reference signal $r(t)$ (dotted line) and the system output $y(t)$ (solid line). It is

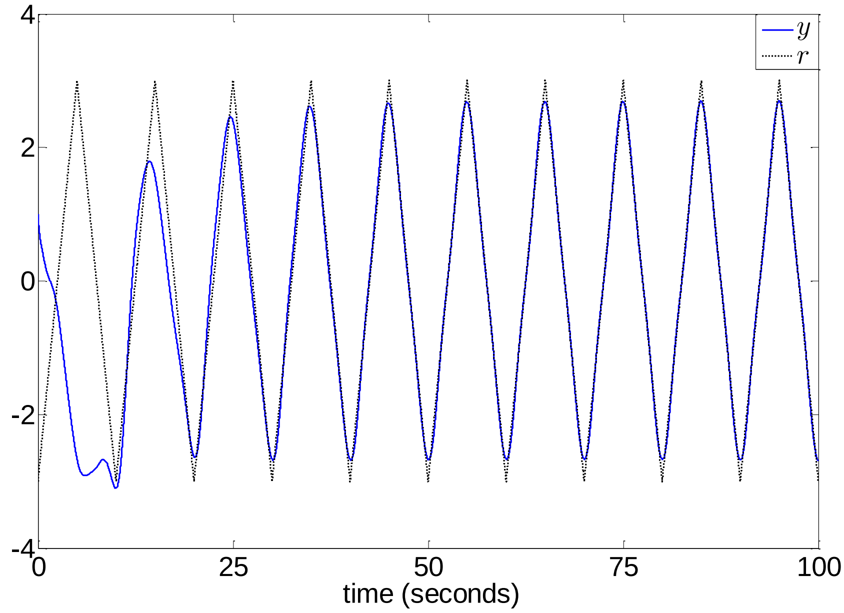

Figure 3.18: Trajectory of the reference signal $r(t)$ and the output $y(t)$

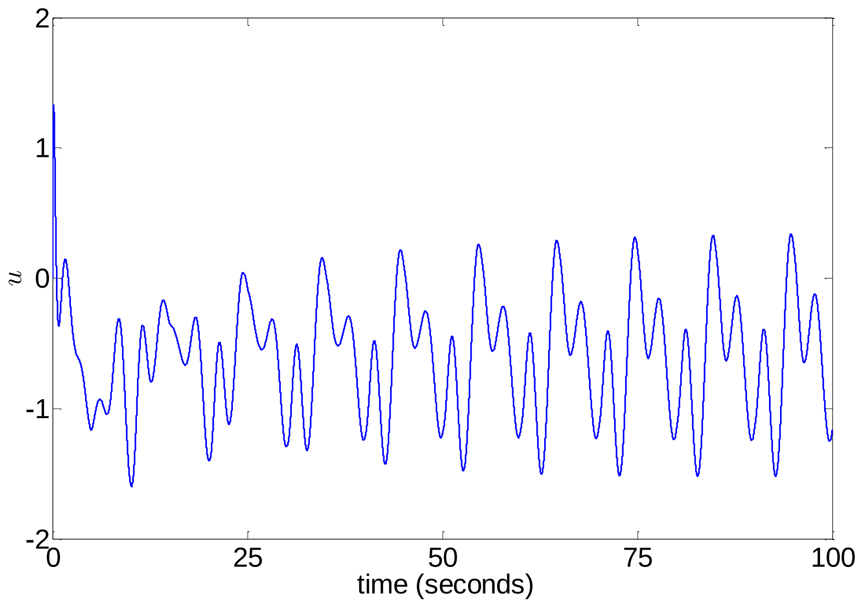

Figure 3.19: Trajectory of the control input $u(t)$

seen that almost perfect tracking is achieved except at high frequency region. The root mean square tracking error at $70s \le t \le 100s$ is 0.0680. Figure 3.19 shows the time history of control input $u(t)$, which remains uniformly bounded even though the system is unstable and non-minimum phase.

3.7 Robustness Analysis

It is interesting to study if the adaptive feedforward control proposed in the previous section is robust with respect to system uncertainties such as un-modelled dynamics. The purpose of this section is to show that the proposed adaptive feedforward control is indeed robust with respect to un-modelled dynamics if the adaptive gain γ in (3.34) is small.

To study robustness, the transfer function $P_1(s)$ in the system representation

$$e(t) = P_1(s)[u(t) + d_1(t)], \tag{3.51}$$

is assumed to have the form

$$P_1(s) = \bar{P}_1(s) + \Delta(s), \quad \text{and} \quad \|\Delta(s)\|_\infty < \beta, \tag{3.52}$$

where $\bar{P}_1(s)$ is the nominal system transfer function and $\Delta(s)$ is an additive un-modelled dynamics. One assumes that both $\bar{P}_1(s)$ and $\Delta(s)$ are stable. Besides, we also want to know an un-modelled residual term $\tilde{d}_1(t)$, measurement noise $\xi(t)$, the exponentially decaying term $\epsilon(t)$ in (3.22), and a bounded aperiodic input $u_f(t)$, which was concerned in [3], impact on the system stability. Therefore, we set

$$u(t) = u_f(t) - \phi^T(t)\hat{\theta}, \tag{3.53}$$

$$d_1(t) = \phi^T(t)\theta + \tilde{d}_1(t), \tag{3.54}$$

$$\bar{P}_1(s)\phi^T(t)\theta = \bar{\psi}_1^T(t)\theta + \epsilon(t), \tag{3.55}$$

where $\bar{\psi}_1(t)$ is equivalent to $\bar{P}_1(s)\phi(t)$ arriving at steady state, that is

$$\begin{aligned}
\bar{\psi}_1(t) = \bar{P}_1(s)\phi(t)\Big|_{t\to\infty} &= \begin{bmatrix} \bar{P}_1(j0) \\ |\bar{P}_1(j\omega_1)|\cos(\omega_1 t + \angle\bar{P}_1(j\omega_1)) \\ |\bar{P}_1(j\omega_1)|\sin(\omega_1 t + \angle\bar{P}_1(j\omega_1)) \\ \vdots \\ |\bar{P}_1(j\omega_N)|\cos(\omega_N t + \angle\bar{P}_1(j\omega_N)) \\ |\bar{P}_1(j\omega_N)|\sin(\omega_N t + \angle\bar{P}_1(j\omega_N)) \end{bmatrix} \\
&= \begin{bmatrix} \bar{p}_{0,c} \\ \bar{p}_{1,c}\cos(\omega_1 t) - \bar{p}_{1,s}\sin(\omega_1 t) \\ \bar{p}_{1,s}\cos(\omega_1 t) + \bar{p}_{1,c}\sin(\omega_1 t) \\ \vdots \\ \bar{p}_{N,c}\cos(\omega_N t) - \bar{p}_{N,s}\sin(\omega_N t) \\ \bar{p}_{N,s}\cos(\omega_N t) + \bar{p}_{N,c}\sin(\omega_N t) \end{bmatrix}, \qquad (3.56)
\end{aligned}$$

in which

$$\begin{aligned}
\bar{p}_{k,c} &= |\bar{P}_1(j\omega_k)|\cos(\angle P_1(j\omega_k)), \\
\bar{p}_{k,s} &= |\bar{P}_1(j\omega_k)|\sin(\angle P_1(j\omega_k)), \\
\bar{p}_{0,c} &= \bar{P}_1(j0).
\end{aligned}$$

Figure 3.20 shows the robust AFC control system structure.

The proposed adaptive law (3.34) is to estimate θ in the periodic disturbance $d_1(t)$ in (3.54). When there is un-modelled dynamics and measurement noise, one must add noise ξ and use the nominal transfer function $\bar{P}_1(s)$ instead of the practical transfer function $P_1(s)$ in the adaptive law. Hence, the adaptive law (3.34) becomes,

$$\dot{\hat{\theta}}(t) = \gamma\bar{\psi}_1(t)\left(e(t) - \xi(t) - \bar{P}_1(s)u(t) - \bar{\psi}_1^T(t)\hat{\theta}(t)\right). \qquad (3.57)$$

Using (3.51)—(3.54), the equation (3.57) can be written as

$$\begin{aligned}
\dot{\hat{\theta}}(t) &= \gamma\bar{\psi}_1(t)\Big(\bar{P}_1(s)(u(t) + \phi^T(t)\theta + \tilde{d}_1(t)) + \Delta(s)(u(t) + d_1(t)) - \\
&\quad \xi(t) - \bar{P}_1(s)u(t) - \bar{\psi}_1^T(t)\hat{\theta}(t)\Big) \\
&= \gamma\bar{\psi}_1(t)\Big(\bar{\psi}_1^T(t)(\theta - \hat{\theta}(t)) + \epsilon(t) + \bar{P}_1(s)\tilde{d}_1(t) + \\
&\quad \Delta(s)(u(t) + d_1(t)) - \xi(t)\Big), \qquad (3.58)
\end{aligned}$$

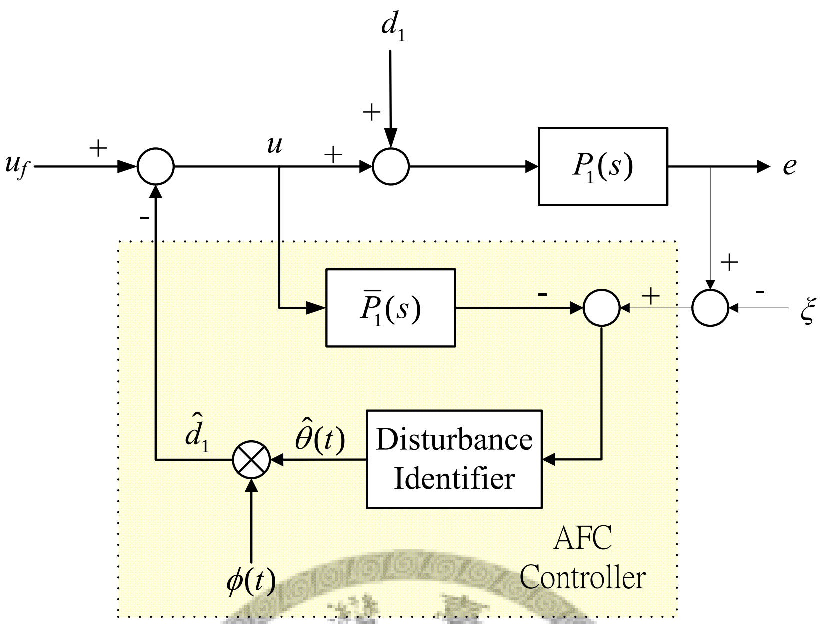

Figure 3.20: AFC control system under model with uncertainty

where the last equality results from (3.55). If one denotes $\tilde{\theta}(t) = \theta - \hat{\theta}(t)$,

$$\begin{aligned} \tilde{u}(t) &= u(t) + d_1(t) = u_f(t) + \tilde{d}_1(t) + \phi^T(t)\tilde{\theta}(t), & (3.59)\\ \tilde{u}_F(t) &= \bar{\psi}_1^T(t)\tilde{\theta}(t), & (3.60)\\ h(t) &= \tilde{u}_F(t) + \bar{P}_1(s)\tilde{d}_1(t) + \Delta(s)\tilde{u}(t) + \epsilon(t) - \xi(t), & (3.61) \end{aligned}$$

where the last equality in (3.59) results from (3.53) and (3.54), the adaptive law (3.58) results in an error dynamics

$$\dot{\tilde{\theta}}(t) = -\gamma\bar{\psi}_1(t)h(t). \tag{3.62}$$

Substituting the integration of (3.62) into (3.59), the control error $\tilde{u}$ can be written as

$$\tilde{u}(t) = u_f(t) + \tilde{d}_1(t) + \phi^T(t)\int_0^t -\gamma\bar{\psi}_1(\tau)h(\tau)d\tau$$

Substituting (3.56) into the above equation,

$$\tilde{u}(t) = u_f(t) + \tilde{d}_1(t) -$$

$$\begin{aligned}
&\gamma \int_0^t \left(\bar{p}_{0,c} + \sum_{i=1}^{N} \bar{p}_{i,c} \cos\omega_i(t-\tau) + \bar{p}_{i,s} \sin\omega_i(t-\tau) \right) h(\tau)d\tau \\
= \quad & u_f(t) + \tilde{d}_1(t) - \Gamma(s)h(t), \qquad (3.63)
\end{aligned}$$

where the term of integral expresses a convolution integral, that is

$$\left(\bar{p}_{0,c} + \sum_{i=1}^{N} \bar{p}_{i,c} \cos\omega_i t + \bar{p}_{i,s} \sin\omega_i t \right) * h(t),$$

and Γ is defined as

$$\begin{aligned}
\Gamma(s) &= \gamma \cdot \mathcal{L}\left\{ \bar{p}_{0,c} + \sum_{i=1}^{N} \bar{p}_{i,c} \cos\omega_i t + \bar{p}_{i,s} \sin\omega_i t \right\} \\
&= \frac{\gamma \bar{p}_{0,c}}{s} + \sum_{i=1}^{N} \gamma \frac{\bar{p}_{i,c} s + \bar{p}_{i,s}\omega_i}{s^2 + \omega_i^2}, \qquad (3.64)
\end{aligned}$$

in which the sign, $\mathcal{L}$, is denoted as Laplace operator. Using (3.56) and (3.62), equation (3.60) also has a convolution integral expression, that is

$$\begin{aligned}
\tilde{u}_F(t) &= -\gamma \int_0^t \left(\bar{P}_1^2(j0) + \sum_{i=1}^{N} \bar{P}_1^2(j\omega_i) \cos\omega_i(t-\tau) \right) h(\tau)d\tau \\
&= -\Gamma_F(s)h(t), \qquad (3.65)
\end{aligned}$$

where

$$\begin{aligned}
\Gamma_F(s) &= \gamma \cdot \mathcal{L}\left\{ \bar{P}_1^2(j0) + \sum_{i=1}^{N} \bar{P}_1^2(j\omega_i) \cos\omega_i t \right\} \\
&= \frac{\gamma \bar{P}_1^2(j0)}{s} + \sum_{i=1}^{N} \frac{\gamma \bar{P}_1^2(j\omega_i) s}{s^2 + \omega_i^2}. \qquad (3.66)
\end{aligned}$$

Finally, combining (3.61), (3.63) with (3.65), one obtains

$$\tilde{u} = \frac{1}{1 + K_\gamma(s)\Delta(s)} \left(u_f + (1 - K_\gamma(s)\bar{P}_1(s))\tilde{d}_1 + K_\gamma(s)(\xi - \epsilon) \right), \qquad (3.67)$$

where the internal model control $K_\gamma(s)$ is

$$K_\gamma(s) = \frac{\Gamma(s)}{1 + \Gamma_F(s)}. \qquad (3.68)$$

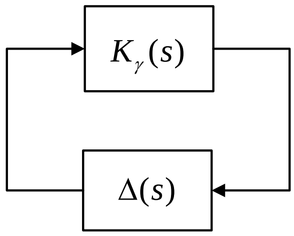

Figure 3.21: Feedback connection

The robust stability problem in (3.67) becomes that of proving $1+K_\gamma(s)\Delta(s) \neq 0$ for s in the right-half plane (RHP). The robustness study is in fact a stability problem of the feedback structure in Figure 3.21. In Figure 3.21, the feedback block is stable by hypothesis, and the feedforward block is stable if only $1/\left(1+\Gamma_F(s)\right)$ is stable (note that both $\Gamma(s)$ and $\Gamma_F(s)$ have the same denominator). The next lemma proves by mathematical induction that $1/\left(1+\Gamma_F(s)\right)$ is stable.

Lemma 3.6 : If the nominal frequency response $\bar{P}_1(j\omega_i) \neq 0$ for $i = 0, 1, \ldots, N$, the transfer function $S(s) = 1/\left(1+\Gamma_F(s)\right)$ is stable.

Proof: Proving that $S(s)$ is stable is equivalent of proving all zeros of $1+\Gamma_F(s)$ lie in the left-half complex plane (LHP). One will prove by mathematical induction that in fact all zeros of $1+\Gamma_F^k(s)$ lie in LHP for all $k = 0, 1, \cdots, N$, where

$$\Gamma_F^k(s) = \frac{\gamma_0'}{s} + \sum_{i=1}^{k} \frac{\gamma_i' s}{s^2+\omega_i^2} = \sum_{i=0}^{k} \frac{\gamma_i' s}{s^2+\omega_i^2} = \frac{B_k(s)}{A_k(s)}, \tag{3.69}$$

in which $\omega_0 = 0$, $\gamma_0' = \gamma|\bar{P}_1(j0)|^2$, $\gamma_i' = \gamma|\bar{P}_1(j\omega_i)|^2$, and the summation is from 0 to k. Note that $\Gamma_F(s) = \Gamma_F^N(s)$.

When one substitutes $s = j\Omega$ in A_k and B_k in (3.69), one obtains

$$A_k(j\Omega) = s\prod_{i=1}^{k}(s^2+\omega_i^2)\bigg|_{s=j\Omega} = jX_k(\Omega),$$

$$B_k(j\Omega) = \gamma_0'\prod_{i=1}^{k}(s^2+\omega_i^2) + \sum_{m=1}^{k}\gamma_m' s^2 \prod_{i\neq m}^{k}(s^2+\omega_i^2)\bigg|_{s=j\Omega} = Y_k(\Omega),$$

where both X_k and Y_k are real numbers. Since $1+\Gamma_F^k(s) = Q_k(s)/A_k(s)$, where the numerator is given by

$$Q_k(s) = A_k(s) + B_k(s),$$

the robust stability problem becomes that of proving $Q_k(s) \neq 0$ for s in the RHP. In the sequel, one will use mathematical induction to establish the proof.

When $k=0$, $A_0(s) = s$, $B_0(s) = \gamma_0'$, one has

$$Q_0(s) = A_0(s) + B_0(s) = s + \gamma_0'.$$

Obviously, the root of $Q_0(s)$ lies in the LHP ($\gamma_i' = \gamma|\bar{P}_1(j\omega_i)|^2 > 0$), hence $B_0(s)/Q_0(s)$ is analytic in the closed RHP. Quoting maximum modulus principle [48], the maximum of the modulus of $B_0(s)/Q_0(s)$ takes place on the imaginary axis as below

$$\left\|\frac{B_0(s)}{Q_0(s)}\right\|_\infty = \sup_\Omega \left|\frac{B_0(j\Omega)}{Q_0(j\Omega)}\right| = \sup_\Omega \left|\frac{Y_0(\Omega)}{\sqrt{X_0^2(\Omega)+Y_0^2(\Omega)}}\right| \begin{cases} < 1, & \Omega \neq \omega_0 \\ = 1, & \Omega = \omega_0 = 0 \end{cases} . \quad (3.70)$$

One can then follow the induction procedure to assume that (3.70) holds for $k = 0, 1, \cdots, N-1$. Hence, all roots of $Q_{N-1}(s)$ lie in the LHP and the following is true,

$$\left\|\frac{B_{N-1}(s)}{Q_{N-1}(s)}\right\| = \sup_\Omega \left|\frac{B_{N-1}(j\Omega)}{Q_{N-1}(j\Omega)}\right|_\infty \begin{cases} < 1, & \Omega \neq \omega_i, i = 0, 1, \ldots, N-1 \\ = 1, & \Omega = \omega_i, i = 0, 1, \ldots, N-1 \end{cases} . \quad (3.71)$$

Now, one needs to verify that (3.71) also holds for $k = N$. When $k = N$, using (3.69), one has

$$\frac{B_N(s)}{A_N(s)} = \frac{B_{N-1}(s)}{A_{N-1}(s)} + \frac{\gamma_N' s}{s^2+\omega_N^2} = \frac{B_{N-1}(s)\left(s^2+\omega_N^2\right) + A_{N-1}(s)\gamma_N' s}{A_{N-1}(s)\left(s^2+\omega_N^2\right)}.$$

Thus,

$$\begin{aligned} Q_N(s) &= A_N(s) + B_N(s) \\ &= A_{N-1}(s)\left(s^2+\omega_N^2\right) + B_{N-1}(s)\left(s^2+\omega_N^2\right) + A_{N-1}(s)\gamma_N' s \\ &= \left(A_{N-1}(s) + B_{N-1}(s)\right)\left(s^2+\gamma_N' s+\omega_N^2\right) - \gamma_N' s B_{N-1}(s) \\ &= Q_{N-1}(s)\left(s^2+\gamma_N' s+\omega_N^2\right)\left(1 - \frac{\gamma_N' s B_{N-1}(s)}{Q_{N-1}(s)\left(s^2+\gamma_N' s+\omega_N^2\right)}\right). \quad (3.72) \end{aligned}$$

Using the induction hypothesis, (3.71), and according to maximum modulus principle since both $1/Q_{N-1}$ and $1/(s^2+\gamma'_N s+\omega_N^2)$ are analytic over the closed RHP, one has

$$\left\|\frac{\gamma'_N s B_{N-1}(s)}{Q_{N-1}(s)\left(s^2+\gamma'_N s+\omega_N^2\right)}\right\|_\infty \leq \left\|\frac{B_{N-1}(s)}{Q_{N-1}(s)}\right\|_\infty \left\|\frac{\gamma'_N s}{\left(s^2+\gamma'_N s+\omega_N^2\right)}\right\|_\infty < 1, \quad (3.73)$$

where one has used (3.71), $\|\gamma'_N s/\left(s^2+\gamma'_N s+\omega_N^2\right)\|_\infty < 1$ for all $s \neq j\omega_N$ and $= 1$ for $s = j\omega_N$, and $\|B_{N-1}(\Omega)/Q_{N-1}(\Omega)\|_\infty$ and $\|\gamma'_N s/\left(s^2+\gamma'_N s+\omega_N^2\right)\|_\infty$ equal to 1 at different frequencies to conclude the inequality in (3.73) .

Since no roots of $Q_{N-1}(s)$ lie in the RHP from the induction hypothesis and so do the roots of $(s^2+\gamma'_N s+\omega_N^2)$, one can conclude from (3.72) and (3.73) that no roots of $Q_N(s)$ lie in the RHP. End of proof.

Therefore, using (3.67), one rewrites output error (3.51) as

$$\begin{aligned} e(t) &= P_1(s)\tilde{u}(t) \\ &= \frac{P_1(s)}{1+K_\gamma(s)\Delta(s)}\Big(u_f(t)+(1-K_\gamma(s)\bar{P}_1(s))\tilde{d}_1(t)+K_\gamma(s)(\xi(t)-\epsilon(t))\Big) \end{aligned} \quad (3.74)$$

In (3.74), we can replace $\tilde{d}_1$ with d_1 since the difference between d_1 and $\tilde{d}_1$, that is $\phi^T(t)\theta$, is eliminated by $1-K_\gamma(s)\bar{P}_1(s) = 0$ at the frequencies ω_i where $i = 0, 1, ..., N$. Figure 3.22 shows the LTI control system is equivalent to the AFC control system in Figure 3.20.

Recall the hypothesis that the feedback block in Figure 3.21 satisfies $\|\Delta(s)\|_\infty < \beta$. Note also that if the adaptive gain γ is sufficiently small, both $\Gamma(s)$ in (3.64) and $\Gamma_F(s)$ in (3.66) will have sufficiently small gains, and so does the feedfoward block transfer function $K_\gamma(s) = \Gamma(s)/(1+\Gamma_F(s))$. One can then quote the small gain theorem to conclude stability of the feedback structure in Figure 3.21.

Obviously, according to the transfer function from noise $\xi(t)$ to output error $e(t)$, that is $P_1(s)K_r(s)$ in (3.74), we can find it has low pass property and thus has an anti-noise ability. Besides, for the small additive error $\tilde{d}_1(t)$, the equation (3.74)

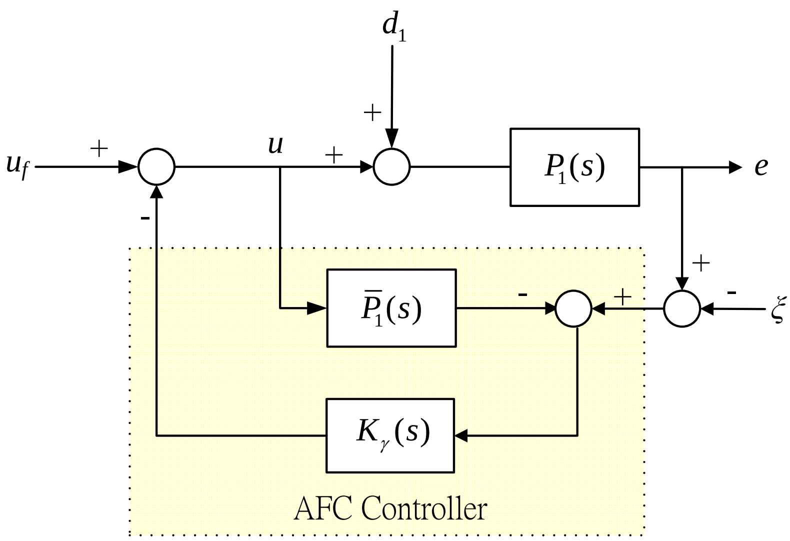

Figure 3.22: LTI control system which is equivalent to Figure 3.20

shows it will only create small output error $e(t)$. Fortunately, by using artificial taking sufficiently large N to approximate the periodic signal or the low-pass properties of physical systems, we can often make the additive error $\tilde{d}_1$ be small. This conclusion is a supplementary explanation in the result of Remark 3.4 which discussed in Section 3.5. Furthermore, the equation (3.74) also shows that the bounded aperiodic input u_f and the decaying term $\epsilon(t)$ do not influence the stability of the proposed AFC control. And the decaying term will exponentially converge to zero as $t \to \infty$. *So, the following theorem will no longer consider the bounded aperiodic input u_f, the decaying term ϵ, the measurement noise ξ and the disturbance modelling error $\tilde{d}_1$ influence on the output error.*

Theorem 3.7 : Given any stable additive un-modelled dynamics $\Delta(s)$ satisfying $\|\Delta(s)\|_\infty < \beta$, if the adaptive gain γ in the proposed adaptive law (3.57) is sufficiently small such that

$$\|K_\gamma(s)\|_\infty < \frac{1}{\beta},$$

then the proposed control $u(t)$ in (3.38) drives that the output error $e(t)$ in (3.51) goes to zero despite the existence of unknown periodic disturbance $d_1(t)$.

Proof: As explained before the theorem, the internal state in Figure 3.21 decays to zero exponentially according to the small gain theorem. Therefore, the estimation disturbance $\hat{d}_1(t)$ converges to the ideal disturbance $d_1(t)$ which achieves disturbance rejection in the face of unknown disturbance $d_1(t)$ and un-modelled dynamics $\Delta(s)$. End of proof.

Next, we need to check the internal state of the system is stable whether or not. Therefore, the update law (3.57) is re-written as

$$\dot{\hat{\theta}}(t) = -\gamma\bar{\psi}_1(t)\bar{\psi}_1^T(t)\hat{\theta}(t) + \gamma\bar{\psi}_1(t)\left(e(t) - \xi(t) - \bar{P}_1(s)u(t)\right). \tag{3.75}$$

Based on $\psi_1(t)$ being persistently exciting, Theorem 3.1 guarantees the unperturbed update law,

$$\dot{\hat{\theta}}(t) = -\gamma\bar{\psi}_1(t)\bar{\psi}_1^T(t)\hat{\theta}(t), \tag{3.76}$$

is an exponential convergence. Therefore, from converse theorem of Lyapunov (see [47], Theorem 1.5.1 and Theorem 5.3.1), equation (3.76) exists a function $V(t,\hat{\theta})$, and some strictly positive constants $\kappa_1, \kappa_2, \kappa_3$, and κ_4, such that

$$\kappa_1\|\hat{\theta}\|^2 \leq V(t,\hat{\theta}) \leq \kappa_2\|\hat{\theta}\|^2, \tag{3.77}$$

$$\left.\frac{dV(t,\hat{\theta})}{dt}\right|_{(3.76)} \leq -\kappa_3\|\hat{\theta}\|^2, \tag{3.78}$$

$$\left|\frac{\partial V(t,\hat{\theta})}{\partial\hat{\theta}}\right| \leq \kappa_4\|\hat{\theta}\|. \tag{3.79}$$

Consider the same function to study perturbed update law (3.75), inequalities (3.77) and (3.79) still hold, while (3.78) is modified, since the derivative is now taken along trajectories of (3.75) instead of (3.76). Therefore, one has

$$\begin{aligned}
\left.\frac{dV(t,\hat{\theta})}{dt}\right|_{(3.75)} &= \frac{\partial V(t,\hat{\theta})}{\partial t} + \frac{\partial V(t,\hat{\theta})}{\partial\hat{\theta}}\left.\dot{\hat{\theta}}\right|_{(3.75)} \\
&= \left.\frac{dV(t,\hat{\theta})}{dt}\right|_{(3.76)} - \frac{\partial V(t,\hat{\theta})}{\partial\hat{\theta}}\left.\dot{\hat{\theta}}\right|_{(3.76)} + \frac{\partial V(t,\hat{\theta})}{\partial\hat{\theta}}\left.\dot{\hat{\theta}}\right|_{(3.75)} \\
&= \left.\frac{dV(t,\hat{\theta})}{dt}\right|_{(3.76)} + \frac{\partial V(t,\hat{\theta})}{\partial\hat{\theta}}\gamma\bar{\psi}_1(t)\left(e(t) - \xi(t) - \bar{P}_1(s)u(t)\right)
\end{aligned}$$

$$\leq \quad -\kappa_3\|\hat{\theta}\|^2 + \kappa_4\|\hat{\theta}\| \left\| \gamma\bar{\psi}_1(t)\left(e(t) - \xi(t) - \bar{P}_1(s)u(t)\right) \right\|_\infty$$
$$\leq \quad -\kappa_3\|\hat{\theta}\| \left(\|\hat{\theta}\| - \mu \right), \tag{3.80}$$

where $\mu = \kappa_4 \left\| \gamma\bar{\psi}_1(t)e(t) - \xi(t) - \bar{P}_1(s)u(t) \right\|_\infty / \kappa_3$. According to Theorem 3.7, one knows that $e(t)$ in (3.74) and $\tilde{u}(t)$ in (3.67) are bounded. Therefore, one concludes from (3.80) that $\hat{\theta}$ is bounded, and further, the overall system is stable.

From equation (3.74), one knows that the system convergence rate is dependent on the transfer function $K_\gamma(s)$. Therefore, according to the loop gain, $\Gamma_F(s)$, one can use root locus technique to decide how large γ has better performance. Figure 3.23 shows the root-locus diagram of Γ_F that is AFC system in Example 3.2. And Table 3.1 shows its different combinations of closed-loop poles versus adaptation gain γ. Figure 3.23 and Table 3.1 show, under $\Delta = 0$, the proposed AFC is a stabilizing controller in the whole system, and has maximum convergence rate appearing at the neighbor area of $\gamma = 0.496$.

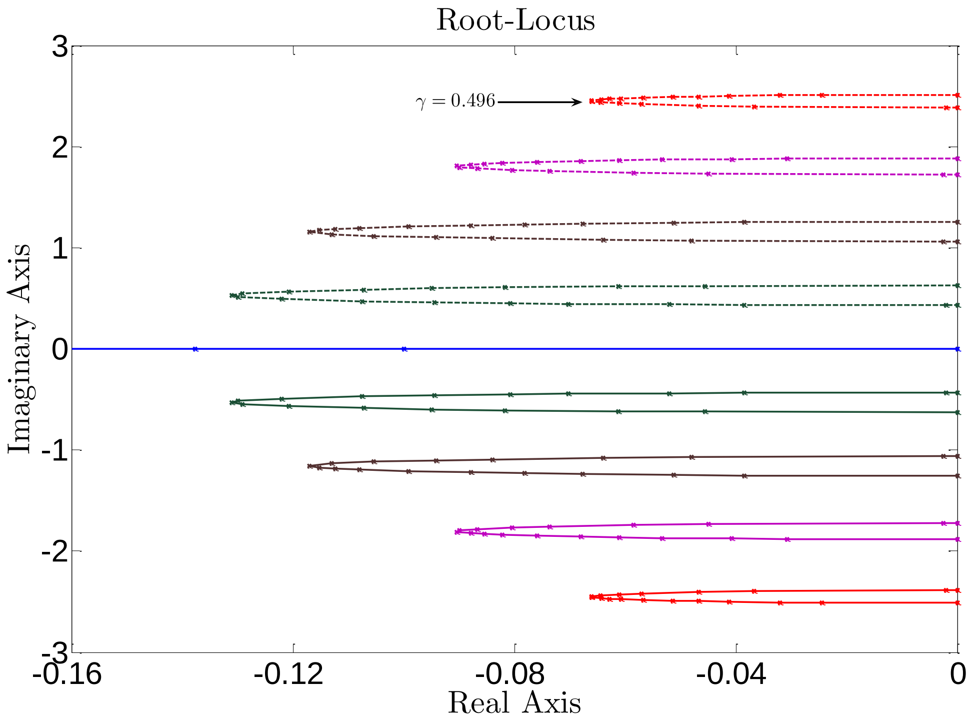

Figure 3.23: Root-locus of $\Gamma_F(s)$

γ	pole 1	pole 2	pole 3	pole 4	pole 5	pole 6	pole 7	pole 8	pole 9
0	0	0+2.51i	0-2.51i	0+1.88i	0-1.88i	0+1.26i	0-1.26i	0+0.63i	0-0.63i
0.0970	-0.1000	-0.0244+2.51i	-0.0244-2.51i	-0.0308+1.88i	-0.0308-1.88i	-0.0384+1.25i	-0.0384-1.25i	-0.0456+0.62i	-0.0456-0.62i
0.1302	-0.1377	-0.032+2.51i	-0.032-2.51i	-0.0408+1.88i	-0.0408-1.88i	-0.0512+1.25i	-0.0512-1.25i	-0.0612+0.62i	-0.0612-0.62i
0.1749	-0.1941	-0.0412+2.50i	-0.0412-2.5i	-0.0533+1.87i	-0.0533-1.87i	-0.0677+1.24i	-0.0677-1.24i	-0.0817+0.61i	-0.0817-0.61i
0.2048	-0.2375	-0.0466+2.50i	-0.0466-2.50i	-0.061+1.86i	-0.061-1.86i	-0.0781+1.23i	-0.0781-1.23i	-0.0949+0.60i	-0.0949-0.60i
0.2348	-0.2872	-0.0514+2.49i	-0.0514-2.49i	-0.0679+1.86i	-0.0679-1.86i	-0.0878+1.22i	-0.0878-1.22i	-0.1072+0.59i	-0.1072-0.59i
0.2750	-0.3676	-0.0567+2.48i	-0.0567-2.48i	-0.0759+1.85i	-0.0759-1.85i	-0.0991+1.21i	-0.0991-1.21i	-0.1208+0.57i	-0.1208-0.57i
0.3152	-0.4694	-0.0607+2.48i	-0.0607-2.48i	-0.0822+1.84i	-0.0822-1.84i	-0.1079+1.20i	-0.1079-1.20i	-0.1291+0.54i	-0.1291-0.54i
0.3422	-0.5518	-0.0628+2.47i	-0.0628-2.47i	-0.0854+1.83i	-0.0854-1.83i	-0.1123+1.19i	-0.1123-1.19i	-0.131+0.53i	-0.131-0.53i
0.3692	-0.6456	-0.0643+2.47i	-0.0643-2.47i	-0.0878+1.82i	-0.0878-1.82i	-0.1152+1.18i	-0.1152-1.18i	-0.1299+0.52i	-0.1299-0.52i
0.4232	-0.8599	-0.066+2.46i	-0.066-2.46i	-0.0903+1.81i	-0.0903-1.81i	-0.117+1.16i	-0.117-1.16i	-0.122+0.49i	-0.122-0.49i
0.4957	-1.1801	-0.0661+2.45i	-0.0661-2.45i	-0.0899+1.79i	-0.0899-1.79i	-0.113+1.13i	-0.113-1.13i	-0.1076+0.47i	-0.1076-0.47i
0.5682	-1.5137	-0.0645+2.44i	-0.0645-2.44i	-0.0867+1.78i	-0.0867-1.78i	-0.1054+1.12i	-0.1054-1.12i	-0.0945+0.46i	-0.0945-0.46i
0.6655	-1.9630	-0.061+2.43i	-0.061-2.43i	-0.0804+1.77i	-0.0804-1.77i	-0.0942+1.10i	-0.0942-1.10i	-0.0807+0.45i	-0.0807-0.45i
0.7628	-2.4054	-0.057+2.42i	-0.057-2.42i	-0.0737+1.76i	-0.0737-1.76i	-0.084+1.09i	-0.084-1.09i	-0.0702+0.45i	-0.0702-0.45i
1.0241	-3.5521	-0.0467+2.41i	-0.0467-2.41i	-0.0584+1.74i	-0.0584-1.74i	-0.064+1.08i	-0.064-1.08i	-0.052+0.44i	-0.052-0.44i
1.3750	-5.0265	-0.0367+2.40i	-0.0367-2.40i	-0.0448+1.74i	-0.0448-1.74i	-0.048+1.07i	-0.048-1.07i	-0.0385+0.44i	-0.0385-0.44i
25.776	-100.5126	-0.0021+2.39i	-0.0021-2.39i	-0.0025+1.72i	-0.0025-1.72i	-0.0026+1.06i	-0.0026-1.06i	-0.002+0.43i	-0.002-0.43i
Inf	Inf	0+2.39i	0-2.39i	0+1.72i	0-1.72i	0+1.06i	0-1.06i	0+0.43i	0-0.43i

Table 3.1: Closed-loop poles of $\Gamma_F(s)$ versus adaptation gain γ

Example 3.6: Consider the same system as in Example 3.2, except that the plant transfer function $P_1(s)$ contains an uncertainty $\Delta(s)$, where

$$\Delta(s) = \frac{-0.5}{s+1}\frac{(s+6)(s-4)}{(s+2)(s+3)(s+4)}.$$

The infinity norm of the uncertainty satisfies $\|\Delta\|_\infty < 0.5$. The adaptive control law is as in Example 3.2 except the adaptation gain γ is chosen as $\gamma = 0.15$ so that the robust stability condition in Theorem 3.7 is satisfied : $\|-\Gamma(s)/(1+\Gamma_F(s))\|_\infty = 1.9531 < 2$. Figure 3.24 shows that the output y achieves almost perfect tracking in spite of the existence of system uncertainty $\Delta(s)$. However, note that in order to satisfy the robust stability condition, the adaptation gain γ can not be too large. As a result, the convergence rate of $y(t)$ towards $r(t)$ becomes slow.

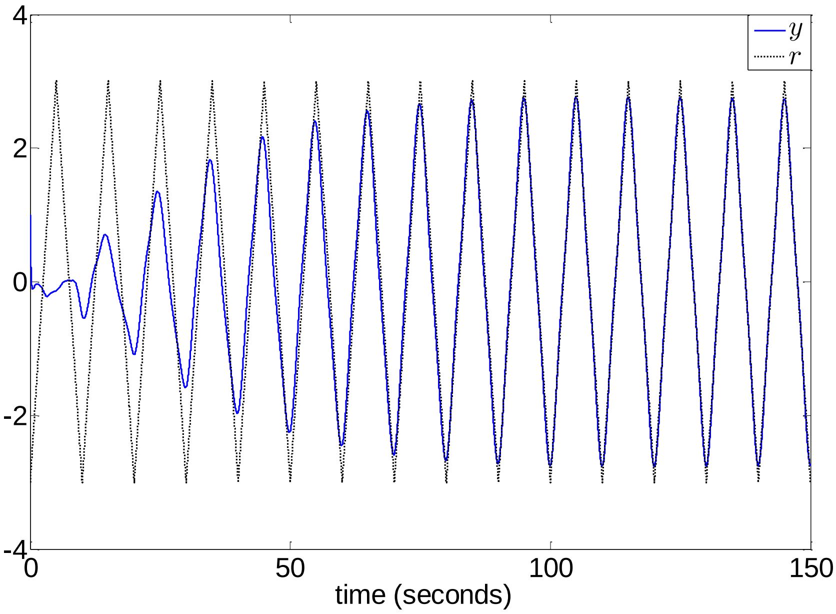

Figure 3.24: Trajectory of $r(t)$ and $y(t)$ under model with uncertainty

3.8 Adaptive Disturbance Estimation

In some applications, it may be desirable to monitor and track the time history of the unknown disturbance $d(t)$. Hence, the goal of this section is to construct an adaptive disturbance estimator for the system subjects to unknown periodic disturbances. The first step is to transform the system equation (3.6) to input-output description. Therefore, for obtaining a stable transfer function, rewrite the system equation (3.6) as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Gd(t) \pm Fy(t) \\ &= (A - FC)x(t) + Bu(t) + (G - FJ)d(t) + Fy, \end{aligned} \tag{3.81}$$

where $F \in R^n$ is any feedback gain that stabilizes the matrix $A - FC$. The observability of (A, C) ensures the existence of such F. Denote

$$\begin{aligned} W_1(s) &= C(sI - A + FC)^{-1}B, \\ W_2(s) &= C(sI - A + FC)^{-1}(G - FJ) + J, \\ W_3(s) &= C(sI - A + FC)^{-1}F, \end{aligned} \tag{3.82}$$

where $W_1(s), W_2(s)$ and $W_3(s)$ are all stable transfer functions since $A - FC$ is a stable matrix by design of F. The state space equation (3.81) has the following input-output representation,

$$y(t) - W_1(s)u(t) - W_3(s)y(t) = W_2(s)d(t). \tag{3.83}$$

The disturbance observer design in this section will be based on the above transfer function representation of the system. Estimation of the periodic disturbance d is equivalent to estimation of the unknown constant vector θ_d in (2.5). In order to estimate θ_d, one needs a linear regression form in θ_d. This is obtained by substituting (2.5) into (3.83),

$$y(t) - W_1(s)u(t) - W_3(s)y(t) \;=\; W_2(s)\left[\phi^T(t)\theta_d\right]$$

$$= W_2(s)\left[\phi^T(t)\right]\theta_d$$
$$= \psi_2^T(t)\theta_d + \epsilon_1(t), \tag{3.84}$$

where $\epsilon_1(t)$ is an exponentially decaying term and the regressor

$$\psi_2(t) = W_2(s)\phi(t)|_{t\to\infty}, \tag{3.85}$$

is bounded since $\phi(t)$ as defined in (2.3) is bounded and $W_2(s)$ is now a stable transfer function. Since ϵ_1 exponentially approaches to zero for sufficiently large t, it does not affect the property of the identifier. The following derivation will neglect it. Therefore, one represents (3.84) as

$$y(t) - W_1(s)u(t) - W_3(s)y(t) = \psi_2^T(t)\theta_d, \tag{3.86}$$

Based on the linear regression form (3.86), the gradient algorithm in Theorem 3.1 suggests the following update law for the estimated $\hat{\theta}_d(t)$,

$$\dot{\hat{\theta}}_d(t) = \gamma\psi_2(t)(y - W_1(s)u(t) - W_3(s)y(t) - \psi_2^T(t)\hat{\theta}_d(t)), \tag{3.87}$$

where $\gamma > 0$ is the adaptation gain. Therefore the structure of the adaptive disturbance observer can be expressed as Figure 3.25. To show that $\psi_2(t)$ as defined in (3.85) is persistently exciting, one needs another assumption.

Assumption A3.4 $W_2(j\omega_k) \neq 0$ for $k = 0, 1, \ldots, N$ where $\omega_k = k \cdot 2\pi/T$, T is the period of $d(t)$, and $W_2(s)$ is as given in (3.82).

The proof of the following lemma is omitted since it is the same as the proof of Lemma 3.3.

Lemma 3.8 : Under Assumption A3.4, $\psi_2(t)$ as defined in (3.85) is persistently exciting.

Theorem 3.9 : Under Assumptions A3.1, A3.2, and A3.4, the disturbance estimate

$$\hat{d}(t) = \phi^T(t)\hat{\theta}_d(t), \tag{3.88}$$

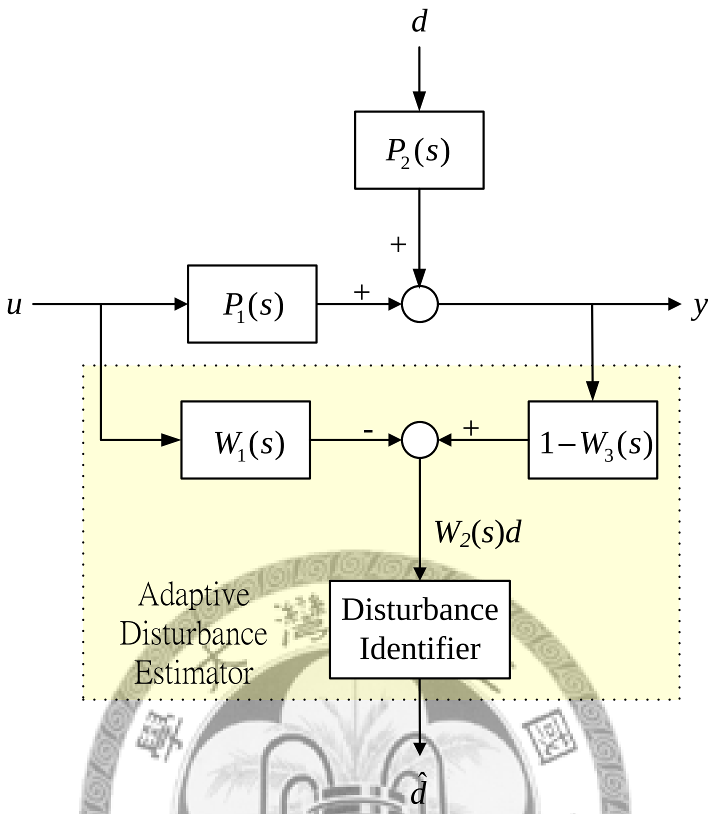

Figure 3.25: Adaptive Disturbance Estimation

where $\hat{\theta}_d(t)$ is from (3.87) and it converges to the true disturbance $d(t) = \phi^T(t)\theta_d$ exponentially.

Proof: Denote the estimation error $\tilde{\theta}_d(t) = \theta_d - \hat{\theta}_d(t)$. Using (3.86), the update law (3.87) results in an error dynamics

$$\dot{\tilde{\theta}}_d(t) = -\gamma\ \psi_2(t)\psi_2^T(t)\tilde{\theta}_d(t). \tag{3.89}$$

Since $\psi_2(t)$ is proved to be persistently exciting in Lemma 3.8, Theorem 3.1 says that the error dynamics (3.89) is exponentially stable. One then concludes that $\tilde{\theta}_d(t)$ converges to zero exponentially. In other words, the update law (3.87) guarantees that the estimated $\hat{\theta}_d(t)$ exponentially converges to the true θ_d, and hence $\hat{d}(t)$ in (3.88) exponentially tracks $d(t)$ in (2.5). End of proof.

Example 3.7: Consider an open-loop stable system (3.6) with system matrices

$$A = \begin{bmatrix} -6 & 1 & 0 \\ -11 & 0 & 1 \\ -6 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}, G = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, J = 0,$$

$x(0) = [1, 1, 1]^T$, and the unknown periodical disturbance

$$d(t) = 2 + 3\cos(0.4\pi t) + \sin(\pi t).$$

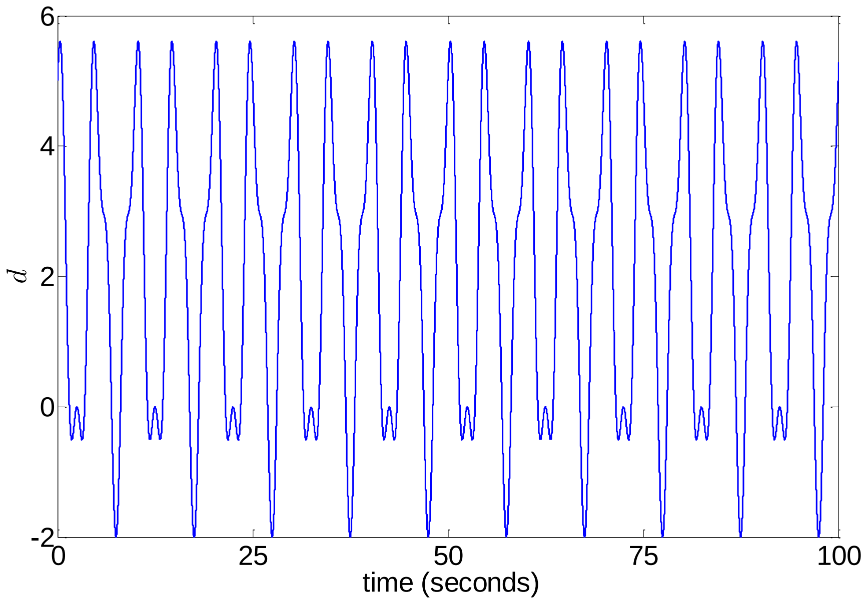

Figure 3.26: Time history of the disturbance $d(t)$

In this simulation, one assumes $u = 0$. Figure 3.26 shows the time histories of the true disturbance. For the adaptive disturbance observer design, one sets the regressor $\phi(t) = \begin{bmatrix} 1 & \cos(0.4\pi t) & \sin(0.4\pi t) & \cos(\pi t) & \sin(\pi t) \end{bmatrix}$, and the adaptation gain $\gamma = 30$ in the gradient algorithm (3.87). Since the matrix A is stable, one sets the feedback gain $F = 0$ and thus results the transfer function $W_3(s) = 0$ in (3.83). Figure 3.27 shows the disturbance estimation error $|d(t) - \hat{d}(t)|$. For showing its transient and steady state, the horizontal and vertical axis are plotted in the log scale. The disturbance estimation error is found to have a very small root mean square error, that is 1.9615×10^{-8} at $700s \leq t \leq 1000s$.

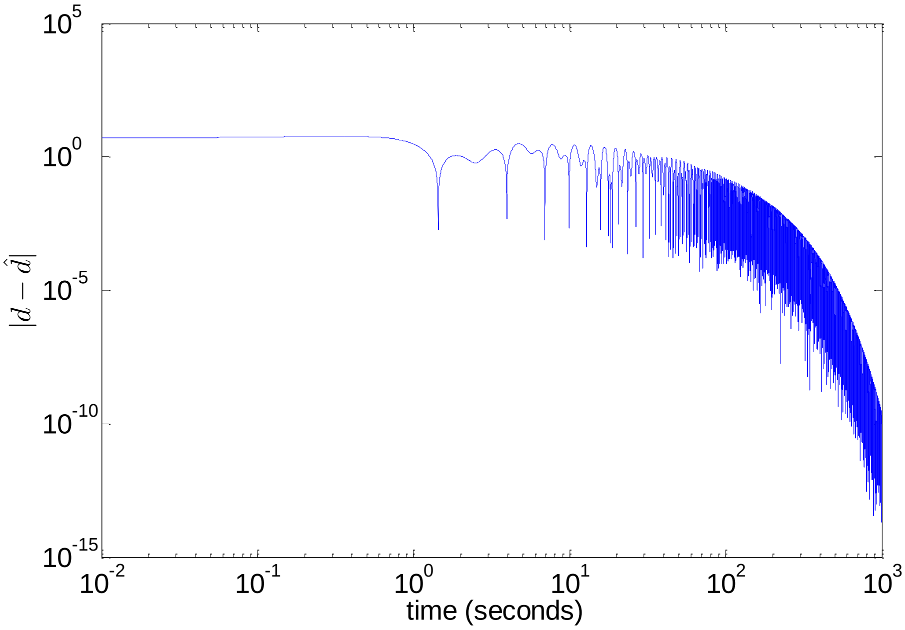

Figure 3.27: Trajectory of the disturbance error $|d(t) - \hat{d}(t)|$

Next, one proceeds to construct a robust observer that can accurately estimate the system state x in (3.6) despite the existence of periodic disturbance d. Such a robust observer can be easily obtained by employing the information of the disturbance estimate $\hat{d}(t)$ in the observer design. In other words, one combines the Luenberger observer with the adaptive disturbance estimator previously proposed to come up with the following robust observer,

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + G\hat{d}(t) + L(y(t) - C\hat{x}(t) - J\hat{d}(t)), \tag{3.90}$$

where $\hat{x}(t)$ is an estimate of $x(t)$, $L \in R^n$ is the observer gain chosen to stabilize the matrix $A - LC$, and $\hat{d}(t)$ is as given by Theorem 3.9. Since the disturbance estimate $\hat{d}(t)$ converges exponentially to the truce disturbance $d(t)$ according to Theorem 3.9, one can easily show that the above state estimate $\hat{x}(t)$ converges to the true system state $x(t)$ exponentially.

Theorem 3.10 : The state estimate $\hat{x}(t)$ from the robust observer (3.90) converges to the true state $x(t)$ exponentially.

Example 3.8: Consider an open-loop unstable system (3.6) with system matrices

$$A = \begin{bmatrix} -2.9 & 1 & 0 \\ -1.7 & 0 & 1 \\ 0.2 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}, G = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, J = 0,$$

$x(0) = [1, 1, 1]^T$, and the unknown periodical disturbance

$$d(t) = \begin{cases} \frac{8}{T}t, & 0 \le t < \frac{T}{4} \\ 2, & \frac{T}{4} \le t < \frac{3T}{4} \\ 8 - \frac{8}{T}t, & \frac{3T}{4} \le t < T \end{cases}.$$

The disturbance d has a period $T = 10$ seconds. The input-output description of the system is

$$y(t) = \frac{s+5}{(s+1)(s+2)(s-0.1)}u(t) + \frac{s-2}{(s+1)(s+2)(s-0.1)}d(t).$$

Note that the above transfer function from d to y is unstable and non-minimum phase (has unstable zeros).

In this simulation, one sets $u = 0$; hence, the system remains unstable and the system state x explodes to infinity exponentially. Figure 3.28 shows the time histories of the state. For the adaptive disturbance observer design, one chooses $N = 4$ in (2.2); in other words, there are $2N + 1 = 9$ terms in the finite series approximation of the periodic disturbance $d(t)$. One designs the feedback gain $F = [3.1, 9.3, 6.2]^T$ such that $\lambda(A - FC) = -1, -2, -3$ in (3.81), and the adaptation gain $\gamma = 3$ and in the gradient algorithm (3.87). For the robust observer design in (3.90), one chooses the observer gain $L = F$. Figure 3.29 shows the time histories of the true (dotted line) and estimated (solid line) disturbance. The disturbance estimation error $d(t) - \hat{d}(t)$ is found to have a very small root mean square error at $70s \le t \le 100s$ is 0.0358. Figure 3.30 shows the norm of state estimation error, $\|x(t) - \hat{x}(t)\|$, resulting from the proposed state observer (3.90). The root mean square state estimation error at $70s \le t \le 100s$ is around 0.0440.

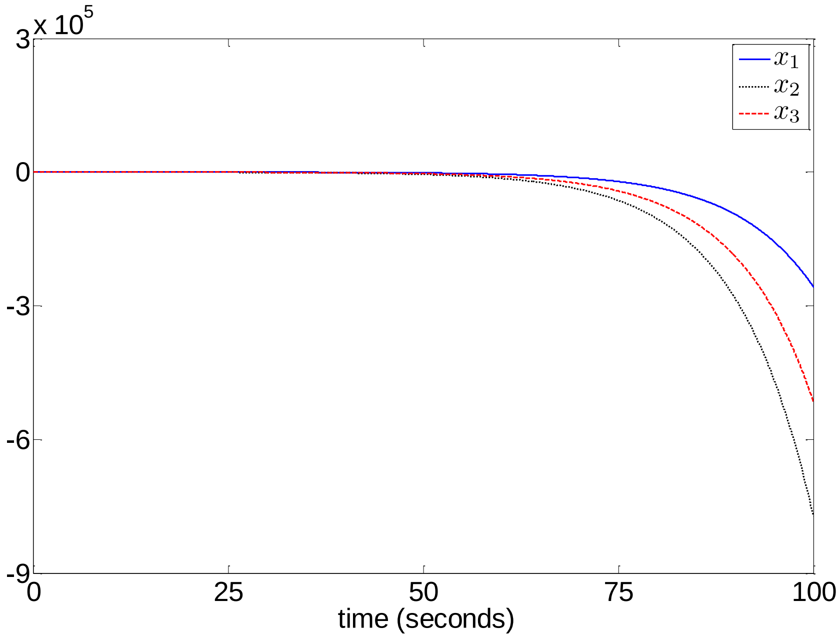

Figure 3.28: Trajectory of the state $x(t)$

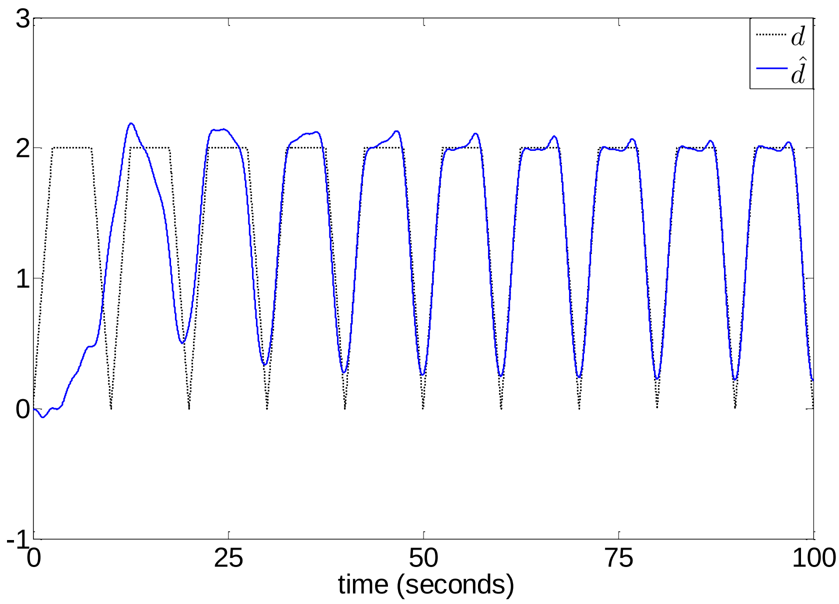

Figure 3.29: Trajectory of the disturbance $d(t)$ and the estimated $\hat{d}(t)$

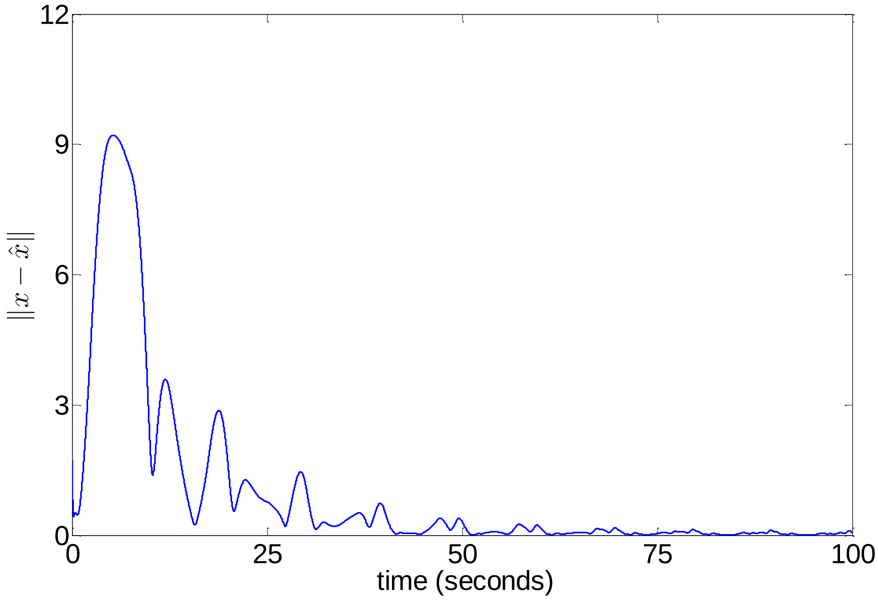

Figure 3.30: Trajectory of the norm of state estimation error $\|x(t) - \hat{x}(t)\|$

Chapter 4

Disturbance Observer Based Control

The goal of this dissertation is to develop a robust control to deal with exogenous periodic disturbances. Among control methods of disturbance rejection, control including the disturbance observer mechanism is an effective method for disturbance rejection. In the two-degree-of-freedom (2-dof) controller structure, 1-dof is used to design nominal feedback control for command input response, the other is designed to obtain disturbance rejection. The benefit of this control mechanism is that the disturbance observer design can be designed independently on the nominal feedback controller without affecting the closed-loop system performance very much.

Besides, in contrast with those designs of AFC which is in Chapter 3, we may want to ask how to speed up the convergence rate of AFC system. Although one knows that the convergence rate of gradient based AFC control system is correlated with adaptation gain γ, between of them are not proportional. So, we can not adjust directly by γ. The only method is to utilize root locus technique to try to get optimal gain γ. However, how to decide how large γ has better performance is still hard. Therefore, the other goal of this chapter is to propose a new method being different from AFC designs previously. The key difference compared with the AFC proposed in previous chapters is that adaptation gain of scalar becomes vector.

This chapter is constructed as follows. Firstly, Section 4.1 reviews disturbance observer based control, which includes a conventional disturbance observer based (DOB) control and an unknown input disturbance observer (UIDO). In Section 4.2, a non-adaptive control, which is a new disturbance observer based control using internal model principle, is established. Then, an expediting method of the AFC control introduced in previous chapter will be presented in Section 4.3. Finally, the robustness of the periodic disturbance observer based control with respect to un-modelled dynamics is studied in Section 4.4.

4.1 Review of Disturbance Observer Based Control

In this section, one reviews two kinds of disturbance observer designs which are the disturbance observer design and the unknown input disturbance observer design respectively. The two disturbance observers estimate the equivalent disturbance from the difference between the actual plant output and the nominal plant output. The estimate is then inversely added at the input of the plant in order to compensate for the disturbance effect on the output. Such designed control is called disturbance observer based control, which is the 2-dof control.

4.1.1 Disturbance Observer Based Control

The objective of this section is to construct a disturbance observer for the LTI system of (3.20). The disturbance observer structure that originated from the 2-dof control structure is depicted in Figure 4.1 [49, 19]. In Figure 4.1, $\bar{P}_1(s)$, ξ and $\hat{d}_1$ are, respectively, a nominal plant, a measurement noise and an estimate of the disturbance d_1, u_f is a nominal feedback control which is designed under disturbance free and $Q(s)$ is designed as a low pass filter in order to make the estimated disturbance observer be realized. Note that, in Figure 4.1, it has assumed

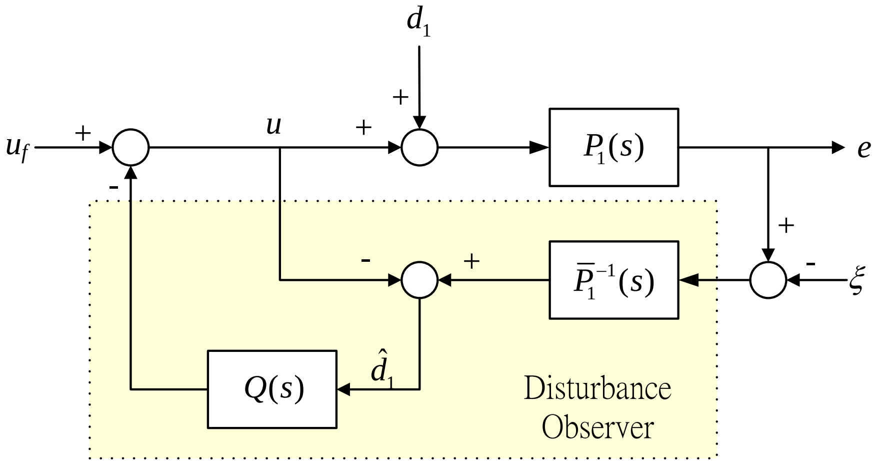

Figure 4.1: Disturbance observer based control

that the system is *minimum phase* and the plant output signal is located at low frequency range so that the disturbance observer can be constructed and can further obtain a satisfying *disturbance attenuation.*

From this figure, one shows the estimate of disturbance

$$\begin{aligned} \hat{d}_1(t) &= \bar{P_1}^{-1}(s)\left(e(t) - \xi(t)\right) - u(t) \\ &= \bar{P_1}^{-1}(s)\left[P_1(s)\left(u(t) + d_1(t)\right) - \xi(t)\right] - u(t). \end{aligned} \tag{4.1}$$

Obviously, if one knows the exactly plant model, i.e. $\bar{P}_1 = P_1$, and assumes measurement noise free, the above equation shows the disturbance estimate $\hat{d}_1$ equal to the true disturbance d_1. However, since the inverse nominal plant model, $\bar{P}_1^{-1}$, is non-proper, the disturbance estimate can not be realized. Hence, the low pass filter Q is introduced to make the disturbance observer be proper. Consequently, the output error of the disturbance observer can be derived as

$$e(t) = H_{eu_f}(s)u_f(t) + H_{ed_1}(s)d_1(t) + H_{e\xi}(s)\xi(t), \tag{4.2}$$

where the transfer functions $H_{eu_f}(s)$, $H_{ed_1}(s)$, and $H_{e\xi}(s)$, respectively, are

$$H_{eu_f}(s) = \frac{P_1(s)}{1 + Q(s)\bar{P}_1^{-1}(s)\Delta(s)}, \tag{4.3}$$

$$H_{ed_1}(s) = \frac{P_1(s)(1-Q(s))}{1+Q(s)\bar{P}_1^{-1}(s)\Delta(s)}, \tag{4.4}$$

$$H_{e\xi}(s) = \frac{P_1(s)\bar{P}_1^{-1}(s)Q(s)}{1+Q(s)\bar{P}_1^{-1}(s)\Delta(s)}, \tag{4.5}$$

in which $\Delta = P_1 - \bar{P}_1$ is an additive uncertainty. Assume that the nominal model of the plant is correct, i.e. $\bar{P}_1 = P_1$. Then the above transfer functions is simplified as

$$H_{eu_f}(s) = P_1(s), \tag{4.6}$$

$$H_{ed_1}(s) = P_1(s)(1-Q(s)), \tag{4.7}$$

$$H_{e\xi}(s) = Q(s). \tag{4.8}$$

Equation (4.6) shows the feedback control design is independent of the disturbance observer design. Equation (4.7) and (4.8) show that the ability of disturbance and noise attenuation are based on the design of $Q(s)$. Therefore, for satisfying proper condition and rejecting disturbance, $Q(s)$ at low frequency range is designed as $Q \approx 1$. Besides, for rejecting noise, $Q(s)$ at the high frequency range is designed as $Q \approx 0$.

The advantages of DOB control are that it is a 2-dof design and is a linear control, making analysis easier, and that convergence is very rapid. Moreover, the DOB control is robust to parameter uncertainty, it can deal with un-deterministic disturbance and it can generate a minimum control force to attenuate disturbance. However, the controlled system must be a minimum phase and can not easily cope with the disturbance of high frequency components. The other disadvantage is that DOB can only achieve disturbance attenuation even though the nominal model of the plant is correct.

In the following example, I want to state that typical DOB control can only achieve disturbance attenuation. The other objective is to discuss the influence of the unknown disturbance having high frequency component on DOB performance.

Example 4.1 : Consider an open-loop stable system with the input-output description as in (3.20), where the plant transfer function $P_1(s)$ is

$$P_1(s) = \frac{(s+3)(s+5)}{(s+2)(s+4)(s+6)},$$

and the periodic disturbance d_1 has following two kinds of disturbances, in which the frequency $\omega = 1$,

Case 1: periodic disturbance being all low frequency components

$$d_1(t) = 6\cos(\omega t) + \cos(2\omega t) + 0.5\sin(3\omega t),$$

Case 2: periodic disturbance including a high frequency component

$$d_1(t) = 6\cos(\omega t) + \cos(2\omega t) + 0.5\sin(3\omega t) + 3\sin(15\omega t).$$

In this example, one selects the filter $Q(s) = 8/s + 8$ to make $Q \approx 1$ at low frequency range and $Q \approx 0$ at the high frequency range. Although we can utilize loop shaping technique and H_∞ control to choose a better Q filter, it will increase the effort on design and computation. Figure 4.2 and Figure 4.4 show the time history of the output error $e(t)$ of Case 1 and Case 2 respectively. Figure 4.3 and Figure 4.5 show the time history of the disturbance of Case 1 and Case 2 respectively, where the true disturbance $d_1(t)$ and the disturbance estimation $\hat{d}_1(t)$ are shown by dotted line and solid line respectively. Obviously, the simulation result shows that the disturbance observer design which was used to cope with periodic disturbance only achieves disturbance attenuation and thus is not very good, disturbance with high frequency components especially. However, even though DOB control can not effectively reject deterministic disturbance, one must emphasize that it is a powerful method on un-deterministic disturbance attenuation since it does't need to know the information of disturbance exactly.

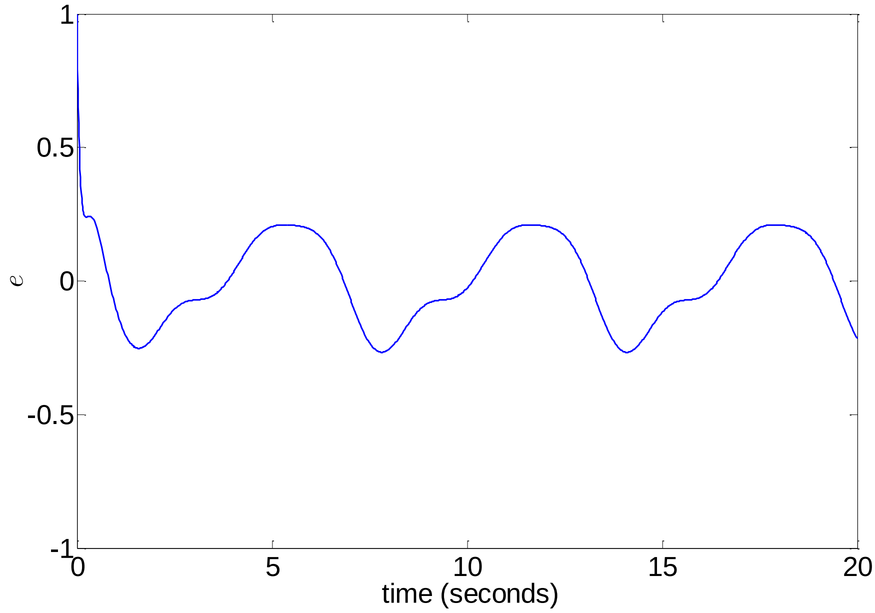

Figure 4.2: Trajectory of the the output error $e(t)$ on Case 1

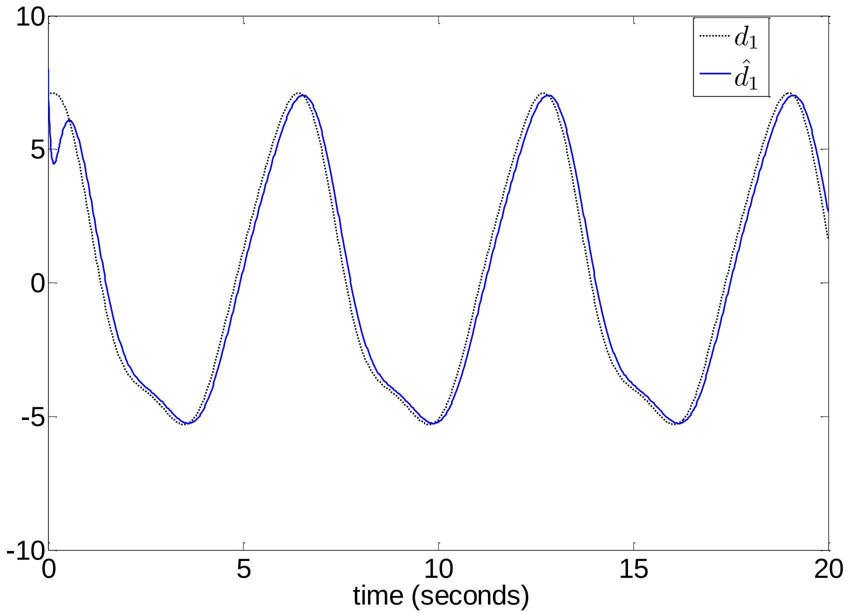

Figure 4.3: Time history of the disturbance d_1 and the estimate $\hat{d}_1$ on Case 1

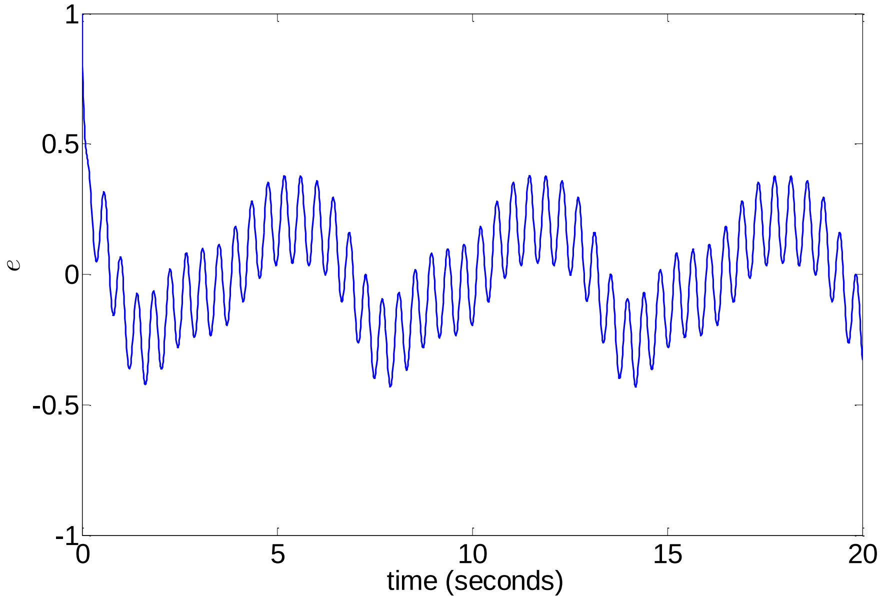

Figure 4.4: Trajectory of the the output error $e(t)$ on Case 2

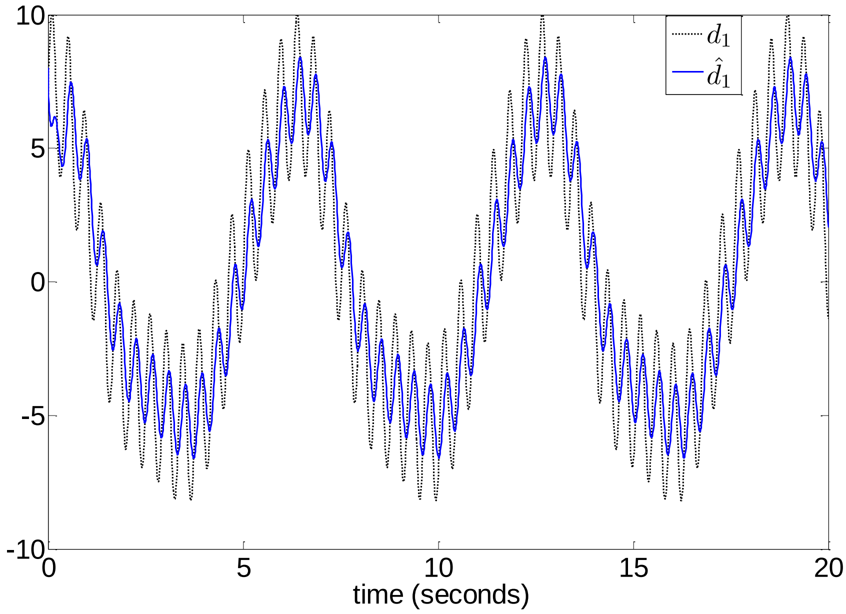

Figure 4.5: Time history of the disturbance d_1 and the estimate $\hat{d}_1$ on Case 2

4.1.2 Unknown Input Disturbance Observer Based Control

In this section one will review a called unknown input disturbance observer [11, 20]. It is an effective method for disturbance rejection control under the dynamic model of disturbance being known exactly. In this case, the periodic disturbance model is augmented with the system model to form an expanded system.

Considering an LTI system subject to periodic disturbance in equation (3.20), the plant model P_1 is assumed as a strictly proper and thus can be realized by the following equation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B\left(u(t) + d_1(t)\right), \\ e(t) &= Cx(t), \end{aligned} \tag{4.9}$$

where $x(t) \in R^n$ is the state vector, and $A \in R^{n\times n}$, $B \in R^n$, and $C \in R^{1\times n}$ are known constant matrices. Therefore, in (3.13), the disturbance

$$d_1(t) = \phi^T(t)\theta = \theta_{0,c} + \sum_{i=1}^{N} \cos\omega_i t \cdot \theta_{i,c} + \sin\omega_i t \cdot \theta_{i,s}, \tag{4.10}$$

by using internal model, can be represented as

$$\begin{aligned} \dot{x}_d(t) &= A_d x_d(t), \\ d_1(t) &= F_d x_d(t), \end{aligned} \tag{4.11}$$

where $x_d \in R^{2N+1}$ is the state vector of disturbance with the initial condition $x_d(0) = \theta = \begin{bmatrix} \theta_0 & \theta_{1,c} & \theta_{1,s} & \ldots & \theta_{N,c} & \theta_{N,s} \end{bmatrix}^T$, and

$$\begin{aligned} A_d &= diag\left(0, \begin{bmatrix} 0 & \omega_1 \\ -\omega_1 & 0 \end{bmatrix}, \ldots, \begin{bmatrix} 0 & \omega_N \\ -\omega_N & 0 \end{bmatrix}\right), \\ F_d &= \begin{bmatrix} 1 & 1 & 0 & \cdots & 1 & 0 \end{bmatrix}. \end{aligned} \tag{4.12}$$

Combining the plant (4.9) with the disturbance generator (4.11), the augmented plant is then constructed. Therefore, an augmented plant observer which is called an unknown input disturbance observer, is constructed as

$$\begin{aligned}\begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{x}}_d(t) \end{bmatrix} &= \begin{bmatrix} A & BF_d \\ 0 & A_d \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{x}_d(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \\ & L\left(e - \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} \hat{x}(t) \\ \hat{x}_d(t) \end{bmatrix}\right), \end{aligned} \tag{4.13}$$

where $\hat{x}(t)$ is an estimate of $x(t)$, $\hat{x}_d(t)$ is an estimate of $x_d(t)$, and $L \in R^{n+2N+1}$ is the augmented plant observer gain chosen to stabilize the following matrix

$$\begin{bmatrix} A & BF_d \\ 0 & A_d \end{bmatrix} - L \begin{bmatrix} C & 0 \end{bmatrix}. \tag{4.14}$$

Consequently, the disturbance observer based control is set as

$$u(t) = u_f(t) - \hat{d}_1(t),$$

where

$$\hat{d}_1(t) = F_d \hat{x}_d(t).$$

Figure 4.6 shows this augmented plant observer is in the block diagram of dashed line.

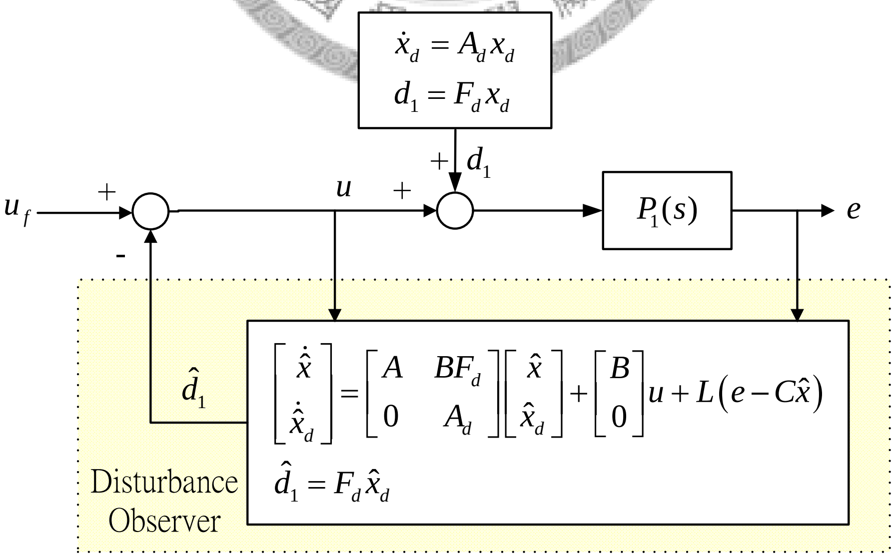

Figure 4.6: Unknown input disturbance observer

Note that, for achieving the 2-dof design, the disturbance observer constructed by augmented system is only used to estimate the actual disturbance acting on the system, but it does not control the plant. It means that another new state observer may be needed for output feedback controller design. The advantages of unknown input disturbance observer are that it can selectively remove harmonics from the frequency spectrum and is a linear control, making analysis easier, and that convergence is very rapid.

Example 4.2 : Consider an open-loop stable system with the input-output description as in (3.20), where the plant transfer function

$$P_1(s) = \frac{(s+6)(s-4)}{(s+2)(s+3)(s+4)},$$

and the unknown periodic disturbance

$$d_1(t) = 6\cos(\omega t) + \cos(2\omega t) + 0.5\sin(3\omega t),$$

in which ω is the fundamental frequency with a period $T = 10$ seconds. (Note that conventional AFC and DOB controller design can not be adopted here since the transfer function above is non-minimum phase)

In this example, one selects $N = 3$ and takes it into (4.11) and (4.12), and then the disturbance observer has matrices

$$\begin{aligned} A_d &= diag\left(0, \begin{bmatrix} 0 & \frac{2\pi\times 1}{10} \\ -\frac{2\pi\times 1}{10} & 0 \end{bmatrix}, \begin{bmatrix} 0 & \frac{2\pi\times 2}{10} \\ -\frac{2\pi\times 2}{10} & 0 \end{bmatrix}, \begin{bmatrix} 0 & \frac{2\pi\times 3}{10} \\ -\frac{2\pi\times 3}{10} & 0 \end{bmatrix}\right), \\ F_d &= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}. \end{aligned}$$

In the disturbance observer design (4.13), one chooses the disturbance observer gain

$$L = \begin{bmatrix} 4.3982 & 60.3980 & 216.5932 & -1.7453 & -3.5941 & 0.5483 \\ & & & -3.8680 & 0.5144 & -3.3161 & -0.9722 \end{bmatrix}^T,$$

which makes the disturbance observer (4.13) have closed-loop poles

$$\lambda(A_d - L_d C_d) = \{-2, -3, -4, -\omega, -\omega \pm j\omega, -\omega \pm j2\omega, -\omega \pm j3\omega\}.$$

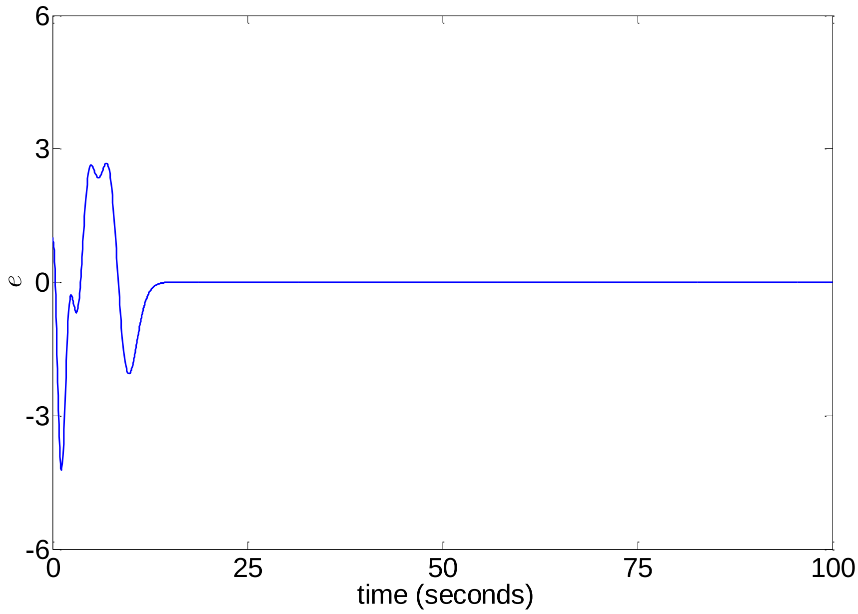

Figure 4.7: Trajectory of the the output error $e(t)$

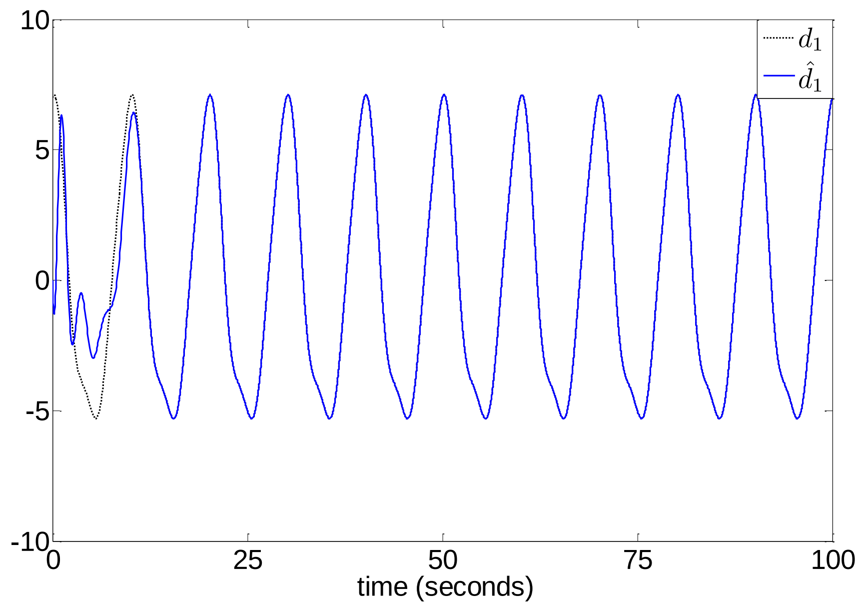

Figure 4.8: Trajectory of the periodic disturbance d_1

Figure 4.7 shows the time history of the output error $e(t)$. The output error of root mean square at $70s \leq t \leq 100s$ is 4.1609×10^{-9}. Figure 4.8 shows the time history of the disturbance, where the true disturbance $d_1(t)$ and the disturbance estimation $\hat{d}_1(t)$ are shown by dotted line and solid line respectively. It shows the estimate converges to the true disturbance quickly. Certainly, if we expect that the disturbance observer has faster convergence rate, we can design observer gain L to make the eigenvalues in (4.14) move far away the imaginary axis. The simulation result shows the unknown input disturbance observer design has good performance.

4.2 New Disturbance Observer Based Control

In this section, we want to propose a new disturbance observer design which utilizes a re-parameterization process to obtain an informal disturbance observer on the state space. For realizing the disturbance observer, one needs to make use of the linear regression form of (3.22), which is a re-parameterization result, to construct an output equation. Hence, $\psi_1^T(t)\theta$ in the right hand side of (3.22) denotes

$$\begin{aligned}\psi_1^T(t)\theta \quad = \quad & p_{0,c}\theta_{0,c} + \sum_{i=1}^{N} \left(p_{i,c}\cos\omega_i t - p_{i,s}\sin\omega_i t\right)\theta_{i,c} + \\ & \left(p_{i,s}\cos\omega_i t + p_{i,c}\sin\omega_i t\right)\theta_{i,s},\end{aligned} \tag{4.15}$$

where $p_{0,c}, p_{i,c}$ and $p_{i,s}$ are defined in (3.15), and the left hand side of (3.22) treats as the measurable signal. Besides, from (4.11), one has the state solution

$$x_d(t) = \Phi(t)x_d(0), \tag{4.16}$$

where $x_d(0) = \theta$ and $\Phi(t) \in R^{2N+1\times 2N+1}$ is the state-transition matrix

$$\Phi(t) = diag\left(1, \begin{bmatrix} \cos\omega_1 t & \sin\omega_1 t \\ -\sin\omega_1 t & \cos\omega_1 t \end{bmatrix}, \cdots, \begin{bmatrix} \cos\omega_N t & \sin\omega_N t \\ -\sin\omega_N t & \cos\omega_N t \end{bmatrix}\right). \tag{4.17}$$

Using (4.16) and (4.17), (4.15) can be represented as

$$\psi_1^T(t)\theta = C_d x_d(t), \tag{4.18}$$

where

$$C_d = \begin{bmatrix} p_{0,c} & p_{1,c} & p_{1,s} & \cdots & p_{N,c} & p_{N,s} \end{bmatrix} \in R^{2N+1}. \tag{4.19}$$

Substituting (4.18) into (3.22), one obtains an output equation

$$e(t) - P_1(s)u(t) = C_d x_d(t) + \epsilon(t). \tag{4.20}$$

According to the equation (4.11) and (4.20), a Luenberger observer is then constructed as

$$\begin{aligned} \dot{\hat{x}}_d(t) &= A_d\hat{x}_d(t) + L_d\left(e(t) - P_1(s)u(t) - C_d\hat{x}_d(t)\right), \\ \hat{d}_1 &= F_d\hat{x}_d(t), \end{aligned} \tag{4.21}$$

where $\hat{x}_d(t)$ is an estimate of $x_d(t)$, $L_d \in R^{2N+1}$ is the observer gain chosen to stabilize the matrix $A_d - L_d C_d$, and $\hat{d}_1$ is an estimate of $d_1(t)$. Denote $\tilde{x}_d = x_d - \hat{x}_d$. Substituting (4.20) into (4.21), one can show the error dynamics of $\tilde{x}_d(t)$, which converges to zero exponentially, as

$$\dot{\tilde{x}}_d(t) = (A_d - L_d C_d)\tilde{x}_d(t) - L_d\epsilon(t). \tag{4.22}$$

One then concludes that the estimated $\hat{d}_1$ in (4.21) exponentially approaches the true disturbance d_1 in (4.11). Consequently, the disturbance observer based (DOB) control is proposed as

$$u(t) = -\hat{d}_1(t) = -F_d\hat{x}_d(t). \tag{4.23}$$

Figure 4.9 shows the DOB control system structure.

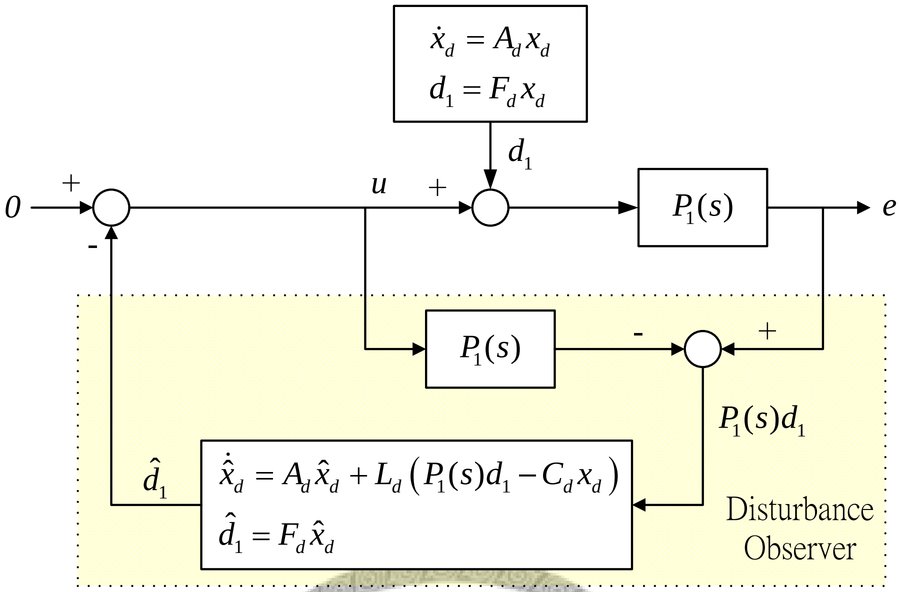

Figure 4.9: DOB control system

4.3 DOB-AFC design

In this section, our objective is to propose a new AFC control, DOB-AFC, which is based on the disturbance observer design in Section 4.2. According to (4.16), the parameter θ in (4.10) is

$$\theta = x_d(0) = \Phi^{-1}(t)x_d(t), \tag{4.24}$$

where

$$\Phi^{-1}(t) = \Phi^T(t). \tag{4.25}$$

Based on (4.24), the estimated parameter is set naturally as

$$\hat{\theta}(t) = \Phi^{-1}(t)\hat{x}_d(t), \tag{4.26}$$

where $\hat{x}_d$ is the state of disturbance observer that is defined in (4.21). Hence, the parameter update law of $\hat{\theta}$ along the closed-loop trajectory (4.21) satisfies

$$\dot{\hat{\theta}}(t) \quad = \quad -\Phi^{-1}(t)A_d\hat{x}_d(t) + \Phi^{-1}(t)\dot{\hat{x}}_d(t)$$

$$\begin{aligned} &= -\Phi^{-1}(t)A_d\hat{x}_d(t) + \Phi^{-1}(t)\left[A_d\hat{x}_d(t) + L_d\left(e(t) - P_1(s)u(t) - C_d\hat{x}_d(t)\right)\right] \\ &= \Phi^{-1}(t)L_d\left(e(t) - P_1(s)u(t) - C_d\hat{x}_d(t)\right). \end{aligned} \tag{4.27}$$

Substituting (4.18) into (4.27),

$$\dot{\hat{\theta}}(t) = \Phi^{-1}(t)L_d\left(e(t) - P_1(s)u(t) - \psi_1^T(t)\hat{\theta}(t)\right). \tag{4.28}$$

By the above analysis, one obtains the update law of DOB-AFC is a general form for adaptive algorithm. The adaptive update law has an arbitrary convergence rate based on the observer gain L_d resulting from disturbance observer design defined in (4.21).

Note that observer gain, L_d, can be obtained from eigenvalue assignment, Kalman filter or adaptive algorithm. Such as the gradient based AFC in Section 3.5 is a special case of (4.28) when one selects $L_d = \gamma C_d^T$. Similarly, when one sets L_d, a time-varying observer gain, into $L_d(t) = \gamma\Phi(t)\Omega(t)\psi_1(t)$, it immediately becomes LS based AFC defined in Section 3.5.

Example 4.3 : Consider the same system as in Example 4.2. Selecting $N = 3$ and taking it into (4.11), (4.12) and (4.19), the disturbance observer have matrices

$$\begin{aligned} A_d &= diag\left(0, \begin{bmatrix} 0 & \frac{2\pi\times 1}{10} \\ -\frac{2\pi\times 1}{10} & 0 \end{bmatrix}, \begin{bmatrix} 0 & \frac{2\pi\times 2}{10} \\ -\frac{2\pi\times 2}{10} & 0 \end{bmatrix}, \begin{bmatrix} 0 & \frac{2\pi\times 3}{10} \\ -\frac{2\pi\times 3}{10} & 0 \end{bmatrix}\right), \\ F_d &= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}, \\ C_d &= \begin{bmatrix} -1.0000 & -0.7070 & 0.6178 & -0.1670 & 0.7803 & 0.2046 & 0.6126 \end{bmatrix}. \end{aligned}$$

In this example, we want to make use of the simulation to show the performance of DOB-AFC is better than robust AFC in Section 3.5. Therefore, the following two kinds of design methods is discussed.

Case 1: Robust AFC based on the gradient algorithm

In the adaptive estimation algorithm (3.34), one sets $\gamma = 0.496$, that is the fast convergence rate derived from Figure 3.23 and Table 3.1. Since the gradient based AFC can be expressed as a DOB-AFC's special case, one thus has disturbance observer gain

$$L_d = \gamma C_d^T = \begin{bmatrix} -0.4960 & -0.3507 & 0.3064 & -0.0828 & 0.3870 & 0.1015 & 0.3039 \end{bmatrix}^T,$$

and DOB-AFC closed-loop poles

$$\lambda(A_d - L_d C_d) = \{-0.9619, -0.0579 \pm j1.8307, -0.1012 \pm j0.4884, -0.0880 \pm j1.1666\}.$$

Case 2: DOB-AFC based on eigenvalue assignment

In this case, we want to reveal that DOB-AFC using eigenvalue assignment on the design performance has more broad choice. One chooses the disturbance observer gain

$$L_d = \begin{bmatrix} -1.7453 & -3.5941 & 0.5483 & -3.8680 & 0.5144 & -3.3161 & -0.9722 \end{bmatrix}^T,$$

which makes the disturbance observer (4.21) have DOB-AFC closed-loop poles

$$\lambda(A_d - L_d C_d) = \{-\omega, -\omega \pm j\omega, -\omega \pm j2\omega, -\omega \pm j3\omega\}.$$

Figure 4.10 shows the time history of the output error $e(t)$, where $e(t)$ of both the gradient based AFC and DOB-AFC are shown by dashed line and solid line respectively. Figure 4.11 shows the time history of the disturbance, where the true disturbance $d_1(t)$, the gradient based AFC $\hat{d}_1(t)$ and DOB-AFC $\hat{d}_1(t)$ are shown by dotted line, dashed line and solid line respectively. Comparing with the gradient algorithm based AFC, the DOB-AFC control has faster response ability. Besides, the output error of root mean square at $70s \leq t \leq 100s$ is 1.9017×10^{-9} which is better than that gradient based AFC is 0.0020. Certainly, if we expect that the disturbance observer has faster convergence rate, we can also design observer gain L_d to make the eigenvalues in (4.21) move far away the imaginary axis.

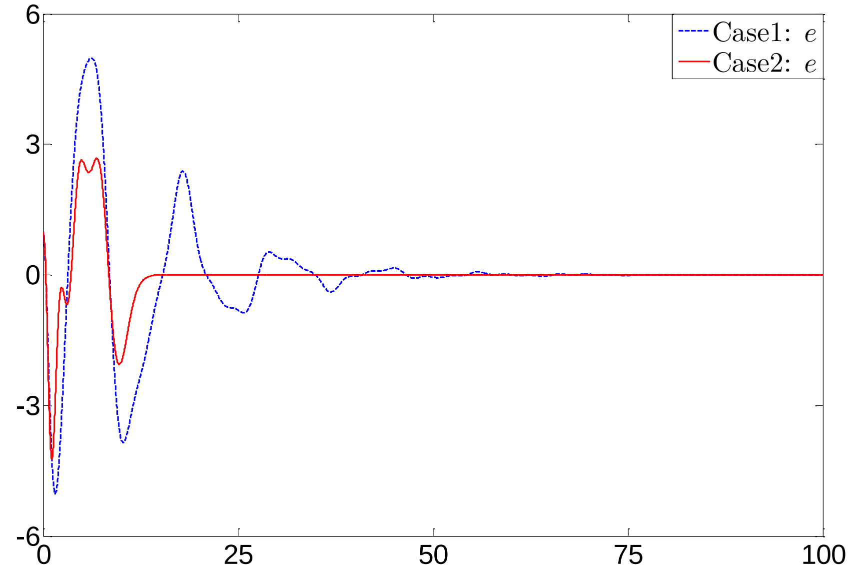

Figure 4.10: Trajectory of the the output error $e(t)$

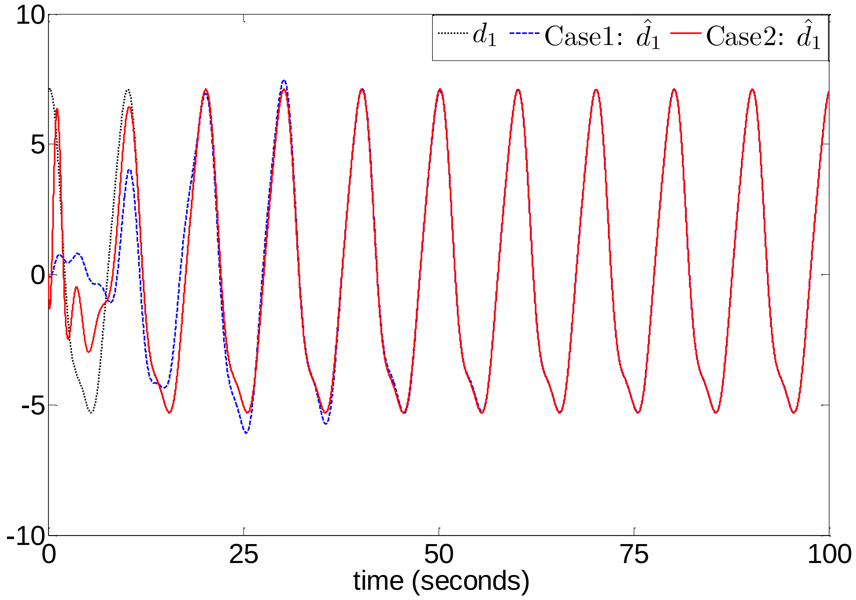

Figure 4.11: Trajectory of the periodic disturbance d_1

4.4 Robustness Analysis

In Section 4.2, our proposed disturbance observer based control needs to make use of an exact system model information. However, if a nominal plant can be got only, we may want to know how robust the proposed DOB control is. For comparing with the robustness of gradient based AFC control analysed in Section 3.7, we will analyse the DOB control design perturbed by un-modelled dynamics $\Delta(s)$ in (3.52), disturbance modelling error $\tilde{d}_1(t)$ in (3.54), and measurement noise $\xi(t)$.

Under considering un-modelled dynamics and measurement noise ξ, the disturbance observer in (4.21) becomes

$$\dot{\hat{x}}_d(t) = A_d\hat{x}_d(t) + L_d\left(e(t) - \xi(t) - \bar{P}_1(s)u(t) - C_d\hat{x}_d(t)\right), \tag{4.29}$$

where the control input is set as

$$u(t) = u_f(t) - F_d\hat{x}_d(t). \tag{4.30}$$

Then, $\hat{x}_d$ can be represented as

$$\hat{x}_d(t) = \left(sI - A_d + L_dC_d\right)^{-1} L_d\left(e(t) - \xi(t) - \bar{P}_1(s)u(t)\right). \tag{4.31}$$

Substituting (4.31) into (4.30),

$$u(t) = u_f(t) - F_d\left(sI - A_d + L_dC_d\right)^{-1} L_d\left(e(t) - \xi(t) - \bar{P}_1(s)u(t)\right). \tag{4.32}$$

Setting the DOB controller as

$$K_L(s) = F_d\left(sI - A_d + L_dC_d\right)^{-1} L_d, \tag{4.33}$$

and then substituting it into (4.32), one obtains

$$u(t) = \frac{1}{1 - K_L(s)\bar{P}_1(s)}\left(u_f(t) - K_L(s)e(t) + K_L(s)\xi(t)\right). \tag{4.34}$$

Therefore, substituting the equation into (3.51), the output error $e(t)$ is obtained as the following equation

$$e(t) = \frac{P_1(s)}{1 - K_L(s)\bar{P}_1(s)}\left(u_f(t) - K_L(s)e(t) + K_L(s)\xi(t)\right) + P_1(s)d_1(t). \tag{4.35}$$

By rearranging $e(t)$ and using (3.52), one obtains

$$e(t) = \frac{P_1(s)}{1 + K_L(s)\Delta(s)} \left(u_f(t) + \left(1 - K_L(s)\bar{P}_1(s)\right) d_1(t) + K_L(s)\xi(t)\right). \quad (4.36)$$

Obviously, $1 - K_L(s)\bar{P}_1(s)$ is a zero gain at the frequencies ω_i, where $i = 0, 1, ..., N$, since zero output error obtained from (4.22) under $\Delta(s) = 0$. Hence, one has $\left(1 - K_L(s)\bar{P}_1(s)\right) d_1(t) = \left(1 - K_L(s)\bar{P}_1(s)\right) \tilde{d}_1(t) - K_L(s)\epsilon(t)$. Consequently, the small un-modelled residual disturbance $\tilde{d}_1$ creates the small output error e which is $\left(1 - K_L(s)\bar{P}_1(s)\right) \tilde{d}_1(t)$. Furthermore, comparing with the controller $K_\gamma(s)$ of (3.68), the observer gain L_d in the DOB control is not a scalar, but vector. As a result, it has more large adjusting freedom to obtain better performance on repetitive system. Figure 4.12 shows the DOB control system structure.

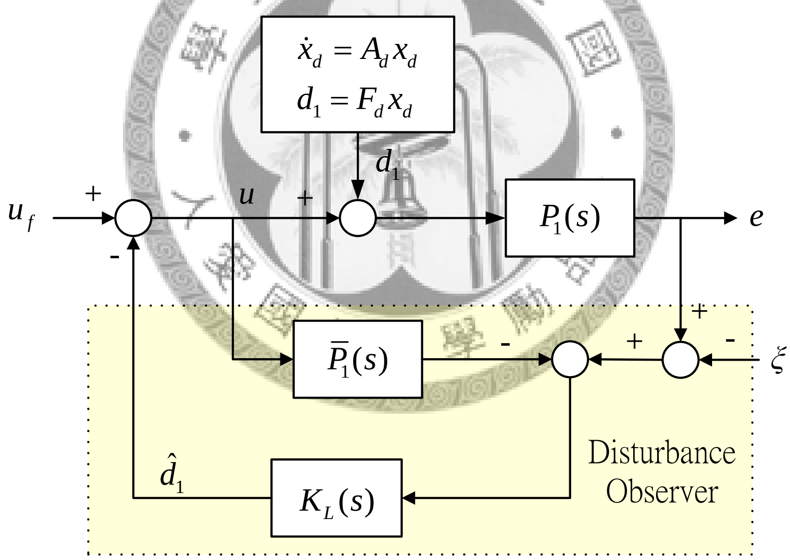

Figure 4.12: DOB control system under model with uncertainty

Comparing (4.36) with (3.74), the following corollary can be obtained directly from Theorem 3.7.

Corollary 4.1 : Given any stable additive un-modelled dynamics $\Delta(s)$ satisfying $\|\Delta(s)\|_\infty < \beta$, if the gain vector L_d in the proposed disturbance observer (4.29) is selected properly such that

$$\|K_L(s)\|_\infty < \frac{1}{\beta},$$

then the proposed control $u(t)$ in (3.38) drives that the output error $e(t)$ in (3.51) goes to zero despite the existence of unknown periodic disturbance $d_1(t)$.

Chapter 5

Conclusions

This dissertation presents a new AFC control to track and/or reject exogenous periodic signal in a linear time-invariant system. The new AFC design uses a re-parameterization process in the input-output description of the system in order to obtain a linear regression form. Based on the linear regression form, any mechanisms of system identification are regarded as AFC controller by us.

The new control has several advantages compared with previous designs. First, its adaptation gain can be arbitrarily chosen without disturbing the system stability. Second, it can be applied to non-minimum phase systems without using any approximation, while most previous AFC designs apply to minimum-phase systems only. Third, it is shown that the proposed control remains the same no matter where the disturbance comes into the system. This is shown for the first time in the literature, and it justifies previous AFC designs in which the disturbances are mostly assumed to enter the system at the control input point, even though in reality it is not the case. Forth, it shows the proposed control has good robustness. Hence, it can be applied on many engineering applications in the real world. Finally, for promoting the repetitive control performance, the thesis further proposes DOB-AFC, that is a general AFC form which has more design freedom on adaptive update law. The interpretation of AFC in terms of disturbance observer design can be implemented by any LTI control methods; making the proposed design very friendly and intuitive for engineers. Therefore, one can use eigenvalue assignment, Kalman filter, adaptive algorithm and so on to design the AFC. Certainly, when the system model can not be exactly obtained, the control structure using LMI method will provide more robust performance.

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