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風險及情感認知對員工認股權價值及履約決策之影響

The Impact of Risk and Sentiment on Executive Stock Options
and Exercise Decision



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本論文係陳麗君君(D93723007)在國立臺灣大學財務金融學系、所完成之博士學位論文，於民國一百年六月二十八日承下列考試委員審查通過及口試及格，特此證明

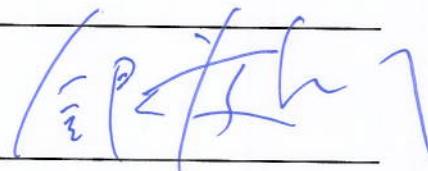
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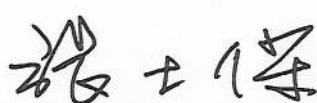

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中文摘要

在考量違約風險、員工風險趨避程度及限制持股比例的條件下，本研究提供一套評價公式，得到美式員工認股權的主觀價值。進一步地，我們探討不同風險對員工認股權主觀價值的影響。研究發現，員工的風險趨避程度及限制持股比例愈大，則主觀價值愈小。然而，情感認知 (sentiment) 如過度自信 (over-confidence) 或私有訊息 (private information) 可以抵銷限制持股比例造成的價格貶值影響。利用 Compustat 經理人報酬資料庫 1992 至 2004 年員工認股權及報酬資料，我們發現經理人評價其所得之員工認股權高於 Black-Scholes 價值的 48%，此可由 12% 年超額報酬的情感認知水準解釋，隱含經理人有高度的過度自信或私有訊息。

本文發現：員工認股權主觀價值與限制持股比例有負向的關係，而與情感認知有正向的關係；情感認知高者願意延後履約時間，然而無論是限制持股比例、系統性或非系統性風險增加皆誘使員工提早履約。由傳統選擇權定價理論可得，當標的資產總風險愈高時，選擇權的價值也會愈高。但本研究發現增加股價波動性 (volatility) 除非大部分來自於系統性風險，否則對員工認股權的主觀價值很可能帶來負面的影響，進而影響員工投資決策。

關鍵詞：員工認股權、履約界限、跳躍擴散模型、情感認知、主觀價值、限制持股

Abstract

This study provides an analytic approximation for finite horizon American employee stock options (ESOs) and a closed form solution for perpetual American ESOs, which take into account risk aversion, stock holding constraint and default risk. Accounting for stock holding constraint, option pricing models generally imply a discount to market value. In contrast, our model further considers the role of sentiment, which offsets the impact of stock holding constraint. Using executive stock options and compensation data paid between 1992 and 2004 for firms covered by Compustat Executive Compensation Database, we find that executives value ESOs at a 48% premium to Black-Scholes value and ESO premia are explained by a sentiment level of 12% in risk-adjusted, annualized excess return, suggesting a high level of executive over-confidence.

Subjective value is positively related to sentiment, and negatively related to stock holding constraint and idiosyncratic risk in all specifications, consistent with the offsetting roles of sentiment and risk aversion. Based on our proposed model, we can observe that exercise boundary decreases with stock holding constraint and idiosyncratic risk, while employee with high sentiment will postpone the exercise timing. Moreover, ESOs may not generate the sort of risk-taking behavior implied by more traditional options pricing formulae owing to the restriction of the employee's holdings. Full diversification is impossible, hence, as idiosyncratic risk increases, the risk-premium associated with holding the asset likewise increases.

Keywords: Employee stock options, exercise boundary, jump diffusion model, sentiment, subjective value, stock holding constraint

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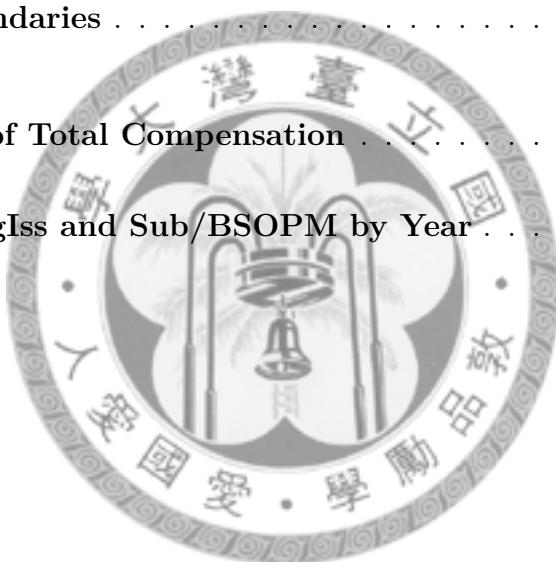
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Chapter 1

Introduction



In a world where diversification is relatively low cost or where diversified assets are tradable, risk-averse investors require returns as compensation for risks associated with illiquidity. For those with investments in illiquid assets, however, illiquidity costs may be offset by positive private information or confidence in future prospects where one believes future returns will outpace the market's expectations and hence provide the necessary risk compensation. One product for which this tradeoff can be explicitly modeled is employee stock options (ESOs). The use of stock option programs for employees has attracted considerable attention both in corporate governance and finance research. In the knowledge-based economy, the most important factor in determining enterprise success may be talent. Enterprises and employees may seek a joint perspective on shared future benefits through an employee stock option plan. Indeed, small and medium-sized enterprises often cannot attract or retain talent based on salary compensation alone, so clever applications of ESOs provide a realizable future capital gain possibility to employees that they may find attractive.¹

¹ESOs can potentially help firms to retain talent and reduce agency costs (Jensen and Meckling, 1976) and mitigate risk-related incentive problems (Agrawal and Mandelker, 1987; Hemmer et al., 2000) as well as attract highly motivated and able potential employees (Core and Guay, 2001; Oyer and Schaefer, 2005).

Because of the illiquid nature of ESOs, the value perceived by employees (henceforth “subjective value”) may be quite different from the cost of issuance (the market or “objective” value). Academia has put forth a number of approaches and modeling techniques to account for this difference, virtually all concluding that the subjective value of ESOs should be less than the usual Black-Scholes value. Empirical evidence of the same, however, has been elusive owing to the lack of a clear closed-form solution and appropriate data to apply. Indeed, if ESOs are generally worth less to an employee than its market value, why do employees continue to covet options as part of total compensation when doing so implies less cash compensation? One sensible explanation is that employees believe the market to have undervalued the options either because they possess positive private information and/or suffer from behavioral over-confidence regarding the future risk-adjusted returns of the firm (henceforth termed “sentiment”). If employees believe that the firm will generate positive risk-adjusted returns over and above that which is priced into the options, even as undiversification of employees owing to stock holding constraint tends to generate a discount, sentiment effects may make ESOs as desirable as, or even more desirable than, the equivalent market value in cash. “How to value ESOs? How risk and sentiment affect the ESO values and the exercise decision?” This study seeks to illuminate these issues.

Applying a comprehensive set of executive options and compensation data, this thesis explicitly tests these notions and prices the impact of stock holding constraint and sentiment. The dataset used includes 13 years of executive options issuances in the US and nearly 82,000 observations. The application of executive options data in particular is noteworthy as options issued to executives are particularly illiquid, are generally a larger portion of total income than those offered to rank-and-file employees, and are most closely monitored by regulatory officials.² In addition, executives are most likely to believe themselves to have private information. Each of these characteristics will tend to generate relatively pronounced effects for this subset of assets. The specificity of the data allows

²Much of the literature in the study of subjective value and sentiment, including that of Oyer and Schaefer (2005) and Bergman and Jenter (2007) studies rank and file employees.

us to compute the proportion of total income that is attributed to options each year for each executive. Indeed, our data also include information for each executive's title, rank, and industry, allowing for relatively specific parsing of the data. These data are necessary as we then split the data into groups by title, year, and industry. We further controls for the size of each firm measured by firm's total market value, number of employees in each firm, non-option compensation, and the immediate exercise value of the option using the K-means approach for hierarchical clustering to split executives into comparative groups. Then, by assuming that all executives within the same cluster receive the same total compensation, the implied subjective value each employee places on his/her options compared to the average compensation in his employment cluster can be calculated, a notion described in detail later. It is this subjective value that we relate to key variables, including investor sentiment, in our main tests. In contrast, Bergman and Jenter (2007) analyzes options issuance, positing without the aid of a model that optimism should coincide with more issuances. Related work by Oyer and Schaefer (2005) also limits its investigation to issuance policy, not pricing. As it shown in this thesis, issuance behavior and subjective value are not closely related, and while the former as fluctuated a great deal over our period of study, the later has been relatively stable.

A number of papers address the valuation of options where value is not the typical Black-Scholes result. Mozes (1995), Hull and White (2004), Carpenter (1998), and Bettis et al. (2005) study early exercise and its impact on standard American option pricing models. Huddart and Lang (1996), Hemmer et al. (1996), and Core and Guay (2001) further link early exercise behavior to under-diversification of employees, but do not explicitly price the premium associated with under-diversification. Lambert et al. (1991) and Hall and Murphy (2002) estimate the subjective value of employee stock options through a certainty equivalent approach, showing it to be lower than market value owing to exogenously constrained fixed holdings in the underlying stock. Ingersoll (2006) also considers the effects of fixed holdings, presenting a constrained portfolio problem where employees allocate wealth between the company's stock, a market portfolio, and a risk-free security. Each paper,

however, presents different modeling limitations on the underlying stock diffusion process and, none of them models the role of employee sentiment.

Our model extends Chang et al. (2008) which considers default jump and European ESOs in a world where an employee allocates his wealth among the company's stock, the market portfolio, and a risk-free security with constrained fixed holding in his company's stock. Different from Chang et al. (2008), this study employs a double exponential jump diffusion model which captures the leptokurtic feature of the return distribution and the volatility smile observed in options prices and admits the jump has a recovery proportion (Kou, 2002). Besides, our option contract is American type. Hemmer et al. (1996), Huddart and Lang (1996), and Bettis et al. (2005) show that early exercise is a pervasive phenomenon owing to risk aversion and undiversification of employees. Importantly, early exercise effect is critical in valuation of ESOs, especially for employees that are more risk averse and when there are more restrictions on the stock holding. A proper calculation must recognize that the decision to exercise is endogenous. We extend the method developed in Gukhal (2001), with a modification to include that an agent faces a constrained portfolio problem, and derive the exercise policies endogenously. In fact, employee exercise decisions and American ESO values are closely related: if an employee exercises his options, he values it less than or equal to its realizable intrinsic value at the exercise date. Conversely, if an employee does not exercise his options, he deems the option value exceeds the intrinsic value he can realize by exercising. Thus, factors affecting the employees' exercise policies will directly influence the valuation of ESOs.

For simple use of the proposed model, this study attempts to extend the analytical tractability of Black-Scholes analysis as in Ingersoll (2006). We first give an analytic approximation for finite horizon American ESOs, and then provide a closed form solution for perpetual American ESOs, which are simply like that of the market values with altered parameters. Numerical simulations are also given for illustration.

In addition, our model is applied to executive compensation data, and it is able to empirically and explicitly price both the subjective value discount created by illiquidity (stock holding constraint) and the risk-adjusted excess returns necessary for employees to accept options in lieu of equivalent cash compensation, i.e. the sentiment effect. This study finds that subjective value is in all but one sector significantly higher than Black-Scholes value, suggesting a substantial role for sentiment. Indeed, we find that executives on average value ESOs at a premium of nearly 48%, indicating extremely high levels of sentiment. Although Hodge et al. (2009) similarly finds in a survey of mid and entry-level managers that subjective values exceed Black-Scholes values, virtually all options pricing models conclude that subjective value should be lower than market value.³ The inclusion of a sentiment variable, however, resolves this puzzle as our finding that the average executive prices 12% risk-adjusted excess return over the expected return of the stock into the ESO value. In other words, they believe the firm will significant outperform the market's expectations and hence value the options more highly than the market, even despite the illiquidity discount.

Also, this thesis shows that subjective value is positively related to sentiment level and negatively related to the proportion of total wealth held in illiquid firm specific holdings, even after controlling for key options pricing variables such as money-ness, time to maturity, volatility, and dividend payout. These results are in accord with the most unique predictions of our model, are statistically significant, and suggest that, while risk aversion generates a discount in subjective value, positive sentiment offsets it. As a proxy of sentiment, previous year CAPM alpha, this study finds that it is positively related to subjective value, implying higher sentiment levels generated by stronger prior year performance. Importantly, we separate our data into “insider” and “true sentiment” groups based on whether the sign of sentiment is the same as that of the resulting returns. If

³While Hodge et al. (2009) uses mid and entry-level managers, we investigate executives. Interestingly, that paper finds that risk aversion and stock volatility do not significantly impact subjective values, possibly because of the relatively small proportion of total income that options constitute for lower level managers.

they are the same, these executives are considered as “informed” rather than behaviorally biased as per traditional sentiment-based arguments. The results show that sentiment is positively related to subjective value in both subsets, indicating sentiment increases subjective value even if subsequent returns are not in concert with ex-ante sentiment. However, the effect is more pronounced for insiders than for true sentiment. Finally, we also apply Fortune Magazine’s list of Top 100 firms to work for as a proxy for sentiment under the assumption that employees of such firms are generally more optimistic about their work environment and prospects. Again, these firms enjoy substantially higher subjective values, though generally lower same-year returns. These results hold despite numerous variable re-specifications, controls for outliers, and controls for industry effects. The jump specification used also does not significantly impact results.

Interestingly, subjective value may be either positively or negatively related to volatility. The former result can be explained by the convex payout of the option. DeFusco et al. (1991), Nohel and Todd (2005), and Ryan and Wiggins (2001) indeed find that options values increase with risk. The latter, however, arises because, as risk increases, the risk premium related to the under-diversification caused by stock holding constraint also increases. The theoretical construct presented in Chang et al. (2008) is capable of capturing this result, and Carpenter (2000) and Ross (2004) present examples where convex incentive structures do not imply that the manager is more willing to take risks. This study shows that, depending on the parameterization, this relation may either be positive or negative, an important departure from the traditional Black-Scholes, moral hazard result. We find specifically that there is a strong negative relation between subjective value and idiosyncratic risk.

The remainder of this thesis is organized as follows. Chapter 2 introduces some relative literature. Chapter 3 develops our model and derives the pricing formulae for finite horizon and perpetual American ESOs. Chapter 4 presents the simulation results. Chapter 5 proposes an empirical methodology and discusses results. Conclusions and future work are

presented in Chapter 6. Justifications of our formulae are deferred to the Appendix.



Chapter 2

Literature Review

2.1 Valuation of Employee Stock Options

Standard methods for valuing options are difficult to apply in these ESOs. Unlike the traditional options, ESOs usually have a vesting period during which they cannot be exercised and employees are not permitted to sell their ESOs. Due to the restriction of ESOs, many employees have undiversified portfolios with large stock options for their own firms. A number of papers address the valuation of options where value is not the typical Black-Scholes result. Lambert et al. (1991) and Hall and Murphy (2002) estimate the subjective value of ESOs through a certainty equivalent approach, showing it to be less than its cost to the issuing firm. They point out that employees discount the value of option because of the additional illiquidity risk they are exposed to. Option values are generally lower for employees that are more risk averse and have more of their wealth invested in company stock. Ingersoll (2006) considers the effects of fixed holdings, presenting a constrained portfolio problem where employees allocate wealth between the company's stock, a market portfolio, and a risk-free security. Each paper, however, presents different modeling limitations on the underlying stock diffusion process and, none of them models

the role of employee sentiment.

Along these lines, Chang et al. (2008) considers default jump, sentiment effect and European ESOs in a world where an employee allocates his wealth among the company's stock, the market portfolio, and a risk-free security with constrained fixed holding in his company's stock. Different from Chang et al. (2008), our study admits the jump has a recovery proportion and employs a double exponential jump diffusion model which captures the leptokurtic feature of the return distribution and the volatility smile observed in options prices (Kou, 2002). Besides, the option contract that we use is American type. Importantly, early exercise effect is critical in valuation of ESOs, especially for employees that are more risk averse and when there are more restrictions on the stock holding.

2.2 Exercise Pattern

The valuation of employee stock options and individual exercise decisions are closely related. Two general approaches to estimate exercise patterns. One approach is modeling exercise by maximizing expected utility subject to hedging restrictions. The other approach models exercise as a random, exogenous event that arrives with some fixed probability. Carpenter (1998) shows that her calibrated extended American option model with random, exogenous exercises and forfeitures predicts actual exercise times just as well as an elaborate utility-maximizing model that accounts for the nontransferability of options. Bettis et al. (2005) estimates the time to maturity by simply using the expected time until exercise in place of the actual time to maturity. The expected time until exercise is estimated from past experience. However, Ingersoll (2006) mentions that even using an unbiased estimate of the expected time until exercise will not give a correct estimate of the option's value.

A proper calculation must recognize that the decision to exercise is endogenous. Liao and Lyuu (2009) incorporates the exercise pattern instead of using the expected time until

exercise technique in valuation of ESOs, to which the exercise patterns are under Chi-square distribution assumption and not derived endogenously. Hull and White (2004) and Ingersoll (2006) derive the exercise boundaries endogenously, while the exercise policies are restricted constant in time. We extend the method developed in Gukhal (2001), with a modification to include that an agent faces a constrained portfolio problem, and derive the time varying exercise policies endogenously.

Huddart and Lang (1996), Hemmer et al. (1996), Core and Guay (2001) and Bettis et al. (2005) show that early exercise is a pervasive phenomenon owing to risk aversion and undiversification of employees, but do not explicitly price the premium associated with under-diversification. Huddart and Lang (1996) finds that exercise is negatively related to the time to maturity and positively correlated with the market-to-strike ratio and with the stock price volatility. Hemmer et al. (1996) and Bettis et al. (2005) find that stock price volatility has a significant effect on exercise decisions. In high volatility firms, employees exercise options much earlier than in low volatility firms.

2.3 Sentiment Issue

Optimism or sentiment is an attracting issue in behavior finance. Often the manager awarded an incentive option may have different beliefs about the company's prospects than the public investor. The employee believes that he possesses private information and can benefit from it. Or he has behavioral over-confidence regarding future risk-adjusted return of his firm and believes ESOs are valuable. Hodge et al. (2009) uses mid and entry-level managers and provides survey evidence that managers subjectively value stock options greater than their Black-Scholes values. Zhang (2002) regards ESO programs as an indirect mechanism for firms selling overvalued equity. This paper assumes that share prices exceed its fundamental value owing to inside managers and outside investors have different perspectives about future profitability of firm. Managers grant at-the-money ESOs

to their employees. After the vesting period, employees exercise their options, and the firm receives the cash. Finally, employees sell their overvalued shares to optimistic investors. In this paper, the author assumes that employees and managers have identical perspectives about future profitability of firm, and employees don't mind buying overvalued stocks by exercising their options.

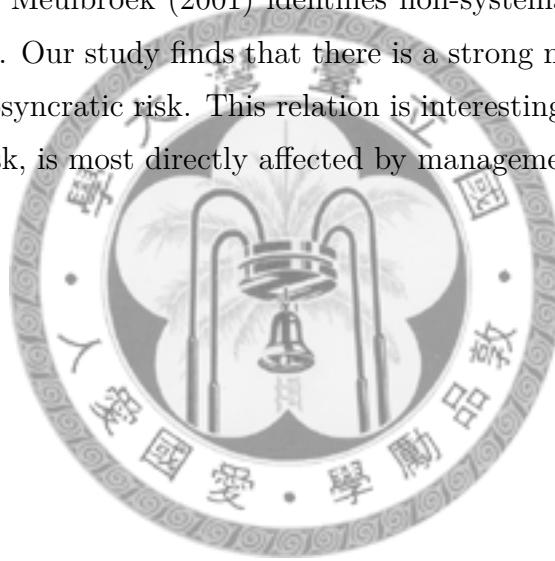
Loosen the assumption, Bergman and Jenter (2007) analyzes whether the popularity of option compensation for rank and file employees may be driven by employee optimism. They construct a model of optimal compensation policy for a firm faced with employees that exhibit sentiment and test their employee optimism assertion, empirically. The theoretical results show that any behavioral explanation for equity compensation based on employee optimism requires that employees need be over-optimistic about firm value, and firms must be able to extract part of the rents created by the overvaluation. In addition, the empirical evidence also supports the sentiment hypothesis. Before Bergman and Jenter (2007), Oyer and Schaefer (2005) calibrates optimism effect about future returns on employees' relative valuations of stock and options by considering a model that employees have different perspectives regarding the firm's prospects. Given this assumption, the firm can benefit by using stock options to attract the optimistic employees. Both focus on the relation between sentiment and the number of options granted. Importantly, our study focuses the sentiment effect on the price of ESO and empirically tests the relation.

2.4 Risk Effect

Subjective value may be positively related to volatility. The result can be explained by the convex payout of the option. DeFusco et al. (1991) shows that ESO plan changes induce increased risk taking. The variance of earnings increases subsequent to the adoption of such plans. Nohel and Todd (2005) proposes that ESOs help to overcome managerial conservatism. Ryan and Wiggins (2001) presents that risky investment is positively related

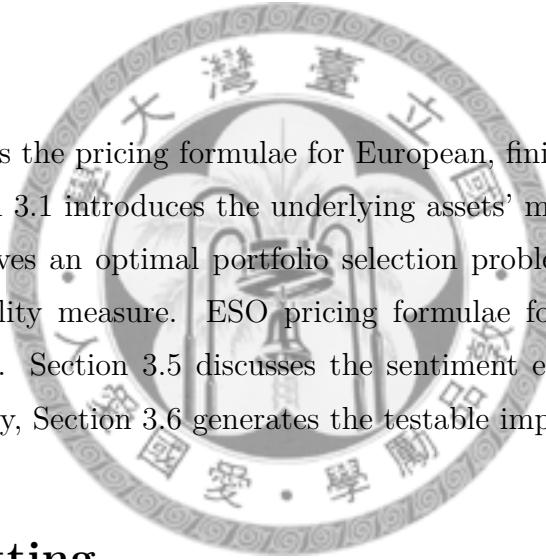
to options, suggesting that firms use options to encourage managers to take risks.

Subjective value may be negatively related to volatility, however, arises because, as risk increases, the risk premium related to the under-diversification caused by stock holding constraint also increases. The theoretical construct presented in Chang et al. (2008) is capable of capturing this result. Carpenter (2000) and Ross (2004) present examples where convex incentive structures do not imply that the manager is more willing to take risks. To properly align incentives using equity-linked compensation, the firm's managers must be exposed to firm-specific risks, but this concentrated exposure prevents optimal portfolio diversification. Meulbroek (2001) identifies non-systematic, firm-specific risk as more costly to managers. Our study finds that there is a strong negative relation between subjective value and idiosyncratic risk. This relation is interesting since idiosyncratic risk, as opposed to market risk, is most directly affected by management.



Chapter 3

Employee Stock Option Valuation



This chapter presents the pricing formulae for European, finite horizon and perpetual American ESOs. Section 3.1 introduces the underlying assets' model. To derive the ESO formulae Section 3.2 solves an optimal portfolio selection problem and Section 3.3 finds the risk-neutral probability measure. ESO pricing formulae for different contracts are presented in Section 3.4. Section 3.5 discusses the sentiment effect on ESO values and exercise decisions. Finally, Section 3.6 generates the testable implications in this study.

3.1 Model Setting

Consider an economy that the employee allocates his wealth among three assets: the company stock S , the market portfolio M , and the risk-free bond B where stock price follows a jump-diffusion process:

$$\begin{cases} \frac{dS}{S} = (\mu_s - d - \lambda k)dt + \sigma_s dW_m + \nu dW_s + d \sum_{i=0}^{N_t} (Y_i - 1), \\ \frac{dM}{M} = (\mu_m - d_m)dt + \sigma_m dW_m, \\ \frac{dB}{B} = rdt, \end{cases} \quad (3.1)$$

where μ_s, μ_m, r are instantaneous expected rates of return for the stock, market portfolio and risk-free bond, respectively. d and d_m are dividends for the stock and market portfolio, respectively. The Brownian motion process W_m represents the Normal systematic risk of the market portfolio. The Brownian motion process W_s and jump process N_t are the idiosyncratic risk of the company stock, where N_t captures the jump risk of company stock and follows a Poisson distribution with average frequency λ . $Y_i - 1$ represents the percentage of stock variation when i th jump occurs. Denote $E(Y_i - 1) = k$ and $E(Y_i - 1)^2 = k_2$ for all i . σ_s and σ_m are the Normal systematic portions of total volatility for the stock and the market portfolio, respectively, while ν is the Normal unsystematic volatility of the stock. The two Brownian motions and jump process are presumed independent.

For simplicity, we assume that CAPM holds so that the efficient portfolio is the market. The vector of cum-dividend expected returns and the covariance matrix of the two risky assets are:

$$\mu = \begin{pmatrix} r + \beta(\mu_m - r) \\ \mu_m \end{pmatrix}, \quad \text{and} \quad \Omega = \begin{pmatrix} \sigma_s^2 + \nu^2 + \lambda k_2 & \sigma_s \sigma_m \\ \sigma_s \sigma_m & \sigma_m^2 \end{pmatrix}, \quad (3.2)$$

where $\beta = \sigma_s / \sigma_m$ is the standard beta.

3.2 Optimal Portfolio Problem

Let W and C be the wealth and consumption processes, then the optimal portfolio selection problem becomes

$$\begin{cases} J[W(t), t] = \max_{\{C, w_s, w_m, w_b\}} E_t \int_t^T e^{-\rho s} U(C(s)) ds + B[W(T), T], \\ \text{s.t. } J[W(T), T] = B[W(T), T], \\ w_s + w_m + w_b = 1, \end{cases} \quad (3.3)$$

where $J[W(t), t]$ is the employee's total utility at time t . The employee's utility function $U(\cdot)$ is set as $U(C) = \frac{C^\gamma}{\gamma}$ with a coefficient of relative risk aversion $R(C) = -\frac{CU''(C)}{U'(C)} =$

$1 - \gamma > 0$. $B[W(T), T]$ is the bequest function at the date of termination T . Optimal consumption and portfolio choices are then the solution to:

$$0 = \max_{\{C, w_s, w_m, w_b\}} e^{-\rho t} U(C(t)) + \frac{1}{2} \mathbf{w}' \Omega \mathbf{w} W^2 J_{WW} + ([r + \mathbf{w}'(\mu - r\mathbf{1})]W(t) - C(t))J_W + J_t$$

where $\mathbf{w} = (w_s \ w_m)'$ and $\mathbf{1} = (1, 1)'$. Due to the restriction of ESO, the employee is usually constrained to allocate a fixed fraction α of his wealth to company stock (via some form of ESO), i.e. $w_s \geq \alpha$. Then, the optimal consumption and portfolio weights become (see Appendix 1):

$$C^* = [b(t)]^{\frac{-1}{1-\gamma}} W, \quad w_s^* = \alpha, \quad w_m^* = \frac{\mu_m - r}{(1-\gamma)\sigma_m^2} - \alpha\beta, \quad w_b^* = 1 - \frac{\mu_m - r}{(1-\gamma)\sigma_m^2} - \alpha(1-\beta),$$

where

$$\begin{aligned} b(t) &= \left\{ \frac{1 + (H[b(T)]^{\frac{1}{1-\gamma}} - 1)e^{H(t-T)}}{H} \right\}^{1-\gamma}, \\ H &= \frac{\gamma}{1-\gamma} \left[\frac{\rho}{\gamma} - r - \frac{1}{2} \frac{(\mu_m - r)^2}{(1-\gamma)\sigma_m^2} + \frac{1}{2}(1-\gamma)(\nu^2 + \lambda k_2)\alpha^2 \right]. \end{aligned}$$

The total utility function is $J[W(t), t] = b(t)e^{-\rho t \frac{W^\gamma}{\gamma}}$. Particularly, when employees do not face the restricted stock holding problem the optimal consumption and portfolio can be similarly derived as follows:

$$\tilde{C} = [\tilde{b}(t)]^{\frac{-1}{1-\gamma}} W, \quad \tilde{w}_s = 0, \quad \tilde{w}_m = \frac{\mu_m - r}{(1-\gamma)\sigma_m^2}, \quad \tilde{w}_b = 1 - \frac{\mu_m - r}{(1-\gamma)\sigma_m^2},$$

where

$$\tilde{b}(t) = \left\{ \frac{1 + (\tilde{H}[b(T)]^{\frac{1}{1-\gamma}} - 1)e^{\tilde{H}(t-T)}}{\tilde{H}} \right\}^{1-\gamma}, \quad \tilde{H} = \frac{\gamma}{1-\gamma} \left[\frac{\rho}{\gamma} - r - \frac{1}{2} \frac{(\mu_m - r)^2}{(1-\gamma)\sigma_m^2} \right].$$

The total utility function is $J[W(t), t] = \tilde{b}(t)e^{-\rho t \frac{W^\gamma}{\gamma}}$. Employees with no portfolio restrictions allocate their wealth only in the market portfolio and risk-free asset. If $\beta > 0$, then $w_m^* < \tilde{w}_m$ and restricted employees have incentive to reduce risk by investing less in the market portfolio. If $\beta > 1$, then $w_b^* > \tilde{w}_b$ and restricted employees invest more in the risk-free asset. If risk aversion is larger than 1 (i.e. $\gamma < 0$), then $H < \tilde{H}$, and $b > \tilde{b}$. Optimal consumption and utility for the restricted employee are also lower than that of the unrestricted.

3.3 Derivation of Risk-Neutral Probability P^*

To easily calculate the ESO values, it is necessary to find a probability measure P^* . This section derives the pricing kernel and then obtains the risk-neutral measure. Here we give a brief summary and define necessary notations.

By Ito's formula for jump processes and the evolution of wealth, the process of employee's marginal utility or the pricing kernel can be derived as (See Appendix 2):

$$\frac{dJ_W}{J_W} = -\hat{r}dt - \hat{\sigma}dW_m - (1 - \gamma)\alpha\nu dW_s + d \sum_{i=0}^{N_t} \{[\alpha(Y_i - 1) + 1]^{\gamma-1} - 1\}, \quad (3.4)$$

where $J_W = \frac{\partial J_W[W(t), t]}{\partial W(t)}$ is the marginal utility, $\hat{r} = r - (1 - \gamma)(\alpha^2\nu^2 + \frac{1}{2}\alpha^2\gamma\lambda + \alpha\lambda k)$, and $\hat{\sigma} = \frac{\mu_m - r}{\sigma_m}$. To find the risk-neutral probability P^* , let $B(t, T)$ be the price of a zero coupon bond at time t with maturity date T . Then the bond yield

$$r^* := -\frac{1}{T-t} \ln B(t, T) = r - (1 - \gamma)(\alpha\lambda k + \frac{1}{2}\gamma\lambda k_2\alpha^2 + \alpha^2\nu^2) - \lambda(\xi - 1),$$

where $\xi = E[\alpha(Y_i - 1) + 1]^{\gamma-1}$. Define $Z(t) = e^{r^*t} J_W[W(t), t]$, hence, the marginal rate of substitution $\frac{J_W[W(T), T]}{J_W[W(t), t]} = e^{-r^*(T-t)} \frac{Z(T)}{Z(t)}$. The rational equilibrium value of the ESO $F(S, t)$ satisfies the Euler equation

$$F(S, t) = \frac{E_t\{J_W[W(T), T]F(S, T)\}}{J_W[W(t), t]} = e^{-r^*(T-t)} E_t^*[F(S, T)],$$

where $\frac{dP^*}{dP} = \frac{Z(T)}{Z(t)}$, $F(S, T)$ is the payoff at the maturity T and E_t^* is the expectation under P^* and information at time t . Under P^* , the stock process can be expressed as

$$\frac{dS}{S} = [r^* - d^* - \lambda^*(\xi^* - 1)]dt + \sigma_N dW_t^* + d \sum_{i=0}^{N_t} (Y_i - 1),$$

where

$$\begin{aligned} d^* &= d - (1 - \gamma)[\alpha\lambda k + \frac{1}{2}\gamma\lambda k_2\alpha^2 - (1 - \alpha)\alpha\nu^2] - \lambda(\xi - 1) + \lambda k - \lambda^*(\xi^* - 1), \\ \sigma_N^2 &= \sigma_s^2 + \nu^2, \quad \lambda^* = \lambda\xi, \quad \xi^* = \frac{1}{\xi} E\{Y_i[\alpha(Y_i - 1) + 1]^{\gamma-1}\}, \end{aligned}$$

W_t^* is the standard Brownian motion and N_t is a Poisson process with rate λ^* . By using the probability measure P^* , the derived ESO formula is simply like that of the market values with altered parameters.

3.4 Valuation of Employee Stock Options

The pricing formulae for European, finite horizon and perpetual American ESOs are derived in this section.

3.4.1 European ESO

First, we consider the simple ESO contract, European ESO. The price formula is presented in Theorem 3.4.1.

Theorem 3.4.1 *The value of the European ESO with strike price K and time to maturity τ , written on the jump-diffusion process in (3.1) is as follows*

$$C_E(S_t, \tau) = \sum_{j=0}^{\infty} \frac{(\lambda^* \tau)^j e^{-\lambda^* \tau}}{j!} \left\{ S_t e^{-[d^* + \lambda^*(\xi^* - 1)]\tau} E^* \left[\prod_{i=0}^j Y_i \Phi(d_1^*) \right] - K e^{-r^* \tau} E^* [\Phi(d_2^*)] \right\} \quad (3.5)$$

where

$$\begin{aligned} d_1^* &= \frac{\ln[S_t \prod_{i=0}^j Y_i / K] + [r^* - d^* - \lambda^*(\xi^* - 1) + \frac{1}{2}\sigma_N^2]\tau}{\sigma_N \sqrt{\tau}}, \quad d_2^* = d_1^* - \sigma_N \sqrt{\tau}, \\ r^* &= r - (1 - \gamma)(\alpha \lambda k + \frac{1}{2}\gamma \lambda k_2 \alpha^2 + \alpha^2 \nu^2) - \lambda(\xi - 1), \quad \sigma_N^2 = \sigma_s^2 + \nu^2, \\ d^* &= d - (1 - \gamma)[\alpha \lambda k + \frac{1}{2}\gamma \lambda k_2 \alpha^2 - (1 - \alpha)\alpha \nu^2] - \lambda(\xi - 1) + \lambda k - \lambda^*(\xi^* - 1), \\ \xi &= E[\alpha(Y_i - 1) + 1]^{\gamma-1}, \quad \lambda^* = \lambda \xi, \quad \xi^* = \frac{1}{\xi} E\{Y_i[\alpha(Y_i - 1) + 1]^{\gamma-1}\}. \end{aligned}$$

The proof of Theorem 3.4.1 is in Appendix 3.

3.4.2 Finite Horizon American ESO

Suppose that the option can be exercised at n time instants. These time instants are assumed to be regularly spaced at intervals of Δt , and denoted by t_i , $0 \leq i \leq n$, where $t_0 = 0$, $t_n = T$, and $t_{i+1} - t_i = \Delta t$ for all i . Denote C_A as the value of American call option, C_E as the value of European call option, K as the strike price, and $S_i = S_{t_i}$. The critical price at these time points is denoted by S_i^* , $0 \leq i \leq n$, and is the price at which the agent is indifferent between holding the option and exercising. Denote E_i^* as the expectation under P^* and information at time t_i .

Theorem 3.4.2 *The value of the American ESO exercisable at n time instants, when the ESO is not exercised, written on the jump-diffusion process in (3.1) is as follows*

$$\begin{aligned} & C_A(S_0, T) \\ &= C_E(S_0, T) + \sum_{\ell=1}^{n-1} e^{-r^* \ell \Delta t} E_0^* \{ [S_\ell (1 - e^{-d^* \Delta t}) - K (1 - e^{-r^* \Delta t})] I_{\{S_\ell \geq S_\ell^*\}} \} \\ & \quad - \sum_{j=2}^n e^{-r^* j \Delta t} E_0^* \{ [C_A(S_j, (n-j) \Delta t) - (S_j - K)] I_{\{S_{j-1} \geq S_{j-1}^*\}} I_{\{S_j < S_j^*\}} \}. \end{aligned} \quad (3.6)$$

The critical price S_i^* at time t_i for $i = 1, \dots, n$ is defined as the solution to the following equation

$$\begin{aligned} & S_i^* - K \\ &= C_E(S_i^*, (n-i) \Delta t) + \sum_{\ell=1}^{n-i-1} e^{-r^* \ell \Delta t} E_i^* \{ [S_{i+\ell} (1 - e^{-d^* \Delta t}) - K (1 - e^{-r^* \Delta t})] I_{\{S_{i+\ell} \geq S_{i+\ell}^*\}} \} \\ & \quad - \sum_{j=2}^{n-i} e^{-r^* j \Delta t} E_i^* \{ [C_A(S_{i+j}, (n-i-j) \Delta t) - (S_{i+j} - K)] I_{\{S_{i+j-1} \geq S_{i+j-1}^*\}} I_{\{S_{i+j} < S_{i+j}^*\}} \}, \end{aligned}$$

where $C_E(S_0, T)$ and $C_E(S_i^*, (n-i) \Delta t)$ are calculated in Theorem 3.4.1.

The proof of Theorem 3.4.2 is in Appendix 4.

The value of American call option, when exercise is allowed at any time before maturity, is obtained by taking the limit as Δt tends to zero in equation (3.6).

3.4.3 Perpetual American ESO

Perpetual American options are interesting because they serve as simple examples to illustrate finance theory. Furthermore they have some applications in studying real options, and the solution of the infinite horizon problems can lead to an approximation for the value of finite horizon American options (Kou and Wang, 2004). In the ESO context, under a *double exponential jump diffusion model* we will derive a closed form solution for the perpetual American options. In fact, under such model, Kou (2002) shows that the rational-expectations equilibrium price of an option is given by the expectation of the discounted option payoff under a risk-neutral probability measure P^* when using a HARA type utility function for a representative agent. Under P^* , the return process of stock price S_t , $X_t := \ln(S_t/S_0)$, is given by

$$X_t = [r^* - d^* - \frac{1}{2}\sigma_N^2 - \lambda^*(\xi^* - 1)]t + \sigma W_t^* + \sum_{i=0}^{N_t} U_i, \quad X_0 = 0,$$

where W_t^* is the standard Brownian motion, N_t is a Poisson process with rate λ^* and U_i are i.i.d. jumps with double exponential distribution ($U_i \sim \text{Douexp}(p, \eta_1, \eta_2)$)

$$f_U^*(u) = p\eta_1 e^{-\eta_1 u} I_{\{u \geq 0\}} + q\eta_2 e^{\eta_2 u} I_{\{u < 0\}}, \quad \eta_1 > 1, \quad \eta_2 > 0.$$

Denote $G(x) = x\mu^* + \frac{1}{2}x^2\sigma_N^2 + \lambda^*(\frac{p\eta_1}{\eta_1 - x} + \frac{q\eta_2}{\eta_2 + x} - 1)$, with $\mu^* = r^* - d^* - \frac{1}{2}\sigma_N^2 - \lambda^*(\xi^* - 1)$. The moment generating function of X_t is $E^*(e^{\theta X_t}) = \exp[G(\theta)t]$. Kou and Wang (2003) shows that for $a > 0$, the equation $G(x) = a$ has exactly four roots: $\beta_{1,a}, \beta_{2,a}, -\beta_{3,a}, -\beta_{4,a}$, where $0 < \beta_{1,a} < \eta_1 < \beta_{2,a} < \infty$ and $0 < \beta_{3,a} < \eta_2 < \beta_{4,a} < \infty$.

Theorem 3.4.3 Assume that

$$r^* + \lambda^* q \frac{\beta_{1,r^*} \beta_{2,r^*} (\eta_1 + \eta_2)}{\eta_1 (\eta_2 + 1) (\beta_{1,r^*} + \eta_2) (\beta_{2,r^*} + \eta_2)} - d^* \frac{(\eta_1 - 1) \beta_{1,r^*} \beta_{2,r^*}}{\eta_1 (\beta_{1,r^*} - 1) (\beta_{2,r^*} - 1)} < 0. \quad (3.7)$$

The value of the perpetual American ESO, written on the jump-diffusion process in (3.1), is given by $V(S_t)$, where the value function is given by

$$V(v) = \begin{cases} v - K, & v \geq v_0, \\ Av^{\beta_{1,r^*}} + Bv^{\beta_{2,r^*}}, & v < v_0, \end{cases} \quad (3.8)$$

with the optimal exercise boundary ¹

$$v_0 = K \frac{\eta_1 - 1}{\eta_1} \frac{\beta_{1,r^*}}{\beta_{1,r^*} - 1} \frac{\beta_{2,r^*}}{\beta_{2,r^*} - 1}, \quad (3.9)$$

and the coefficients

$$\begin{aligned} A &= v_0^{-\beta_{1,r^*}} \frac{\beta_{2,r^*} - 1}{\beta_{2,r^*} - \beta_{1,r^*}} (v_0 - \frac{\beta_{2,r^*}}{\beta_{2,r^*} - 1} K) > 0, \\ B &= v_0^{-\beta_{2,r^*}} \frac{\beta_{1,r^*} - 1}{\beta_{2,r^*} - \beta_{1,r^*}} (\frac{\beta_{1,r^*}}{\beta_{1,r^*} - 1} K - v_0) > 0. \end{aligned}$$

Furthermore, the optimal stopping time is given by $\tau^* = \inf\{t \geq 0 : S_t \geq v_0\}$.

The proof of Theorem 3.4.3 is given in Appendix 5.

An employee does not exercise his ESOs early when he has no constrained stock holding ($\alpha = 0$) and no dividend paying ($d = 0$). However, the assumption in Theorem 3.4.3, equation (3.7), ensures the possibility of early exercise. Note that equation (3.7) is satisfied in general parameters setting.

In the case of no jump part, the diffusion processes for three assets are considered as follows:

$$\begin{cases} \frac{dS}{S} = (\mu_s - d)dt + \sigma_s dW_m + \nu dW_s, \\ \frac{dM}{M} = (\mu_m - d_m)dt + \sigma_m dW_m, \\ \frac{dB}{B} = rdt, \end{cases} \quad (3.10)$$

with all parameters defined as equation (3.1).

¹It is obvious that the exercise boundary is proportional to strike price from formula (3.9).

Corollary 1 *The value of the perpetual American ESO with $\tilde{d} > 0$, written on the diffusion process in (3.10) is given by $V(S_t)$, where the value function is given by*

$$V(v) = \begin{cases} v - K, & v \geq L, \\ \tilde{A}v^h, & v < L, \end{cases} \quad (3.11)$$

with the optimal exercise boundary and the coefficients

$$\begin{aligned} L &= \frac{h}{h-1}K; \quad \tilde{A} = (L-K)L^{-h}, \quad h = \frac{1}{\sigma_N^2}[\sqrt{\tilde{\mu}^2 + 2\tilde{r}\sigma_N^2} - \tilde{\mu}], \\ \tilde{\mu} &= \tilde{r} - \tilde{d} - \frac{1}{2}\sigma_N^2, \quad \tilde{r} = r - (1-\gamma)\alpha^2v^2, \quad \tilde{d} = d + (1-\gamma)\alpha(1-\alpha)v^2. \end{aligned}$$

Moreover, the optimal stopping time is given by $\tilde{\tau} = \inf\{t \geq 0 : S_t \geq L\}$.

Note that the value of jump-diffusion perpetual American ESO reduces to the diffusion's case by taking $\lambda^* = 0$ and $\eta_1 \rightarrow \infty$ in Theorem 3.4.3.

3.5 ESO Value with Sentiment

Often the manager awarded an incentive option may have different beliefs about the company's prospects than the investing public does. The manager believes that he possesses private information and can benefit from it. Or he has behavioral over-confidence regarding future risk-adjusted return of his firm and believes ESOs are valuable. Now, we consider the impact of sentiment on ESO values and the exercise decision. Define the processes for the three assets as follows:

$$\begin{cases} \frac{dS}{S} = (\mu_s + s - d - \lambda k)dt + \sigma_s dW_m + \nu dW_s + d \sum_{i=0}^{N_t} (Y_i - 1), \\ \frac{dM}{M} = (\mu_m - d_m)dt + \sigma_m dW_m, \\ \frac{dB}{B} = rdt, \end{cases} \quad (3.12)$$

Here, sentiment level be denoted by s . In other words, the employee over-estimates or rationally adjusts the risk-adjusted return of the company owing to inside information by

s , then the same analysis in Theorem 3.4.1 is valid with a simple adjustment in parameters. The adjusted interest rate and dividend yield used in pricing are

$$\begin{aligned} r^* &= r + \alpha s - (1 - \gamma)(\alpha \lambda k + \frac{1}{2} \gamma \lambda k_2 \alpha^2 + \alpha^2 \nu^2) - \lambda(\xi - 1), \\ d^* &= d - (1 - \alpha)s - (1 - \gamma)[\alpha \lambda k + \frac{1}{2} \gamma \lambda k_2 \alpha^2 - (1 - \alpha)\alpha \nu^2] - \lambda(\xi - 1) + \lambda k - \lambda^*(\xi^* - 1). \end{aligned}$$

3.6 Testable Implications

We now turn our attention to the relationships between ESO value and the individual variables that determine it. Taking partial derivatives, we generate the testable predictions of this model and test empirically in the later chapter. Due to the difficulty of calculating partial derivatives for American ESO this section only presents the partial derivatives for European ESO. The effect on American ESO are discussed by simulation in the following chapter.

Denote F be the European ESO value, and then the partial derivatives are evaluated:

$$\begin{aligned} \frac{\partial F}{\partial S} &= \sum_{j=0}^{\infty} \frac{(\lambda^* \tau)^j e^{-\lambda^* \tau}}{j!} e^{-[d^* + \lambda^*(\xi^* - 1)]\tau} E^* \left[\prod_{i=0}^j Y_i \Phi(d_1^*) \right] > 0, \\ \frac{\partial F}{\partial K} &= - \sum_{j=0}^{\infty} \frac{(\lambda^* \tau)^j e^{-\lambda^* \tau}}{j!} e^{-r^* \tau} E^* \left[\Phi(d_2^*) \right] < 0, \\ \frac{\partial F}{\partial d} &= - \sum_{j=0}^{\infty} \frac{(\lambda^* \tau)^j e^{-\lambda^* \tau}}{j!} S_t e^{-[d^* + \lambda^*(\xi^* - 1)]\tau} E^* \left[\prod_{i=0}^j Y_i \Phi(d_1^*) \right] < 0, \\ \frac{\partial F}{\partial \alpha} &= -(1 - \gamma) \tau \nu^2 \sum_{j=0}^{\infty} \frac{(\lambda^* \tau)^j e^{-\lambda^* \tau}}{j!} \left\{ S_t e^{-[d^* + \lambda^*(\xi^* - 1)]\tau} E^* \left[\prod_{i=0}^j Y_i \Phi(d_1^*) \right] \right\} \\ &\quad + [(1 - \gamma)(\lambda k + \gamma \lambda k_2 \alpha + 2 \alpha \nu^2) - s] \tau F, \end{aligned}$$

$$\begin{aligned}
\frac{\partial F}{\partial s} &= \sum_{j=0}^{\infty} \frac{(\lambda^* \tau)^j e^{-\lambda^* \tau}}{j!} \tau \left\{ S_t (1 - \alpha) e^{-[d^* + \lambda^*(\xi^* - 1)]\tau} E^* \left[\prod_{i=0}^j Y_i \Phi(d_1^*) \right] \right. \\
&\quad \left. + \alpha K e^{-r^* \tau} E^* \left[\Phi(d_2^*) \right] \right\} > 0, \\
\frac{\partial F}{\partial \tau} &= \sum_{j=0}^{\infty} \frac{(\lambda^* \tau)^j e^{-\lambda^* \tau}}{j!} \left\{ \frac{\sigma_N}{2\sqrt{\tau}} S_t e^{-[d^* + \lambda^*(\xi^* - 1)]\tau} E^* \left[\prod_{i=0}^j Y_i \Phi'(d_1^*) \right] + r^* K e^{-r^* \tau} E^* \left[\Phi(d_2^*) \right] \right. \\
&\quad \left. - [d^* + \lambda^*(\xi^* - 1)] S_t e^{-[d^* + \lambda^*(\xi^* - 1)]\tau} E^* \left[\prod_{i=0}^j Y_i \Phi(d_1^*) \right] \right\}.
\end{aligned}$$

Subjective value relates positively to stock price but negatively to dividend payout and strike price. These relationships hold in general for the Black-Scholes value of options as well, and are not surprising. The critical new variables evaluated in these formulas are sentiment and α , which is defined as the proportion of total wealth that is held in illiquid firm specific holdings. In this case, α is the illiquid suboptimal holding that the investor holds by accepting ESOs as a part of his compensation package. s should be positively related to ESO value, a fact that is clear by inspection. However, the relationship to α is less straightforward and can be either positive or negative, depending on the level of sentiment. Simulation results show, though, that the relationship is only positive in knife-edge cases and only when sentiment is substantially negative, implying that employees are severely pessimistic regarding the outlook of the firm. For normal parameterizations, the relationship is negative. That is, the higher the proportion of one's wealth tied into illiquid holdings, the higher the risk impact and hence the larger the discount to value.

Interestingly, the sensitivity of value to time to maturity τ can be either positive or negative, despite being generally positive in the Black-Scholes setup. The usual intuition that longer time to maturity translates into larger time value attributed to the option is offset by the larger risk premium associated with holding a suboptimal holding for a longer period of time. Along the same lines, it is also not necessarily the case that subjective value is positively related to risk, as is the traditional moral hazard result. Consider the

following partial derivatives where total variance of the stock price $\sigma^2 = \sigma_s^2 + \nu^2 + \lambda k_2$.

$$\begin{aligned}
\frac{\partial F}{\partial \sigma} &= \frac{\sigma}{\sigma_N} \cdot \sum_{j=0}^{\infty} \frac{(\lambda^* \tau)^j e^{-\lambda^* \tau}}{j!} S_t e^{-[d^* + \lambda^*(\xi^* - 1)]\tau} E^* \left[\prod_{i=0}^j Y_i \Phi'(d_1^*) \sqrt{\tau} \right] > 0, \\
\frac{\partial F}{\partial \nu} &= -2\tau(1 - \gamma)\alpha\nu \sum_{j=0}^{\infty} \frac{(\lambda^* \tau)^j e^{-\lambda^* \tau}}{j!} \left\{ (1 - \alpha) S_t e^{-[d^* + \lambda^*(\xi^* - 1)]\tau} E^* \left[\prod_{i=0}^j Y_i \Phi(d_1^*) \right] \right. \\
&\quad \left. + \alpha K e^{-r^* \tau} E^* \left[\Phi(d_2^*) \right] \right\} < 0, \\
\frac{\partial F}{\partial \sqrt{\lambda}} &= \tau \left[(1 - \gamma)(2\alpha\sqrt{\lambda}k + \gamma\sqrt{\lambda}k_2\alpha^2) - 2\sqrt{\lambda}(k + 1) \right] F \\
&\quad - 2\sqrt{\lambda}k\tau \sum_{j=0}^{\infty} \frac{(\lambda^* \tau)^j e^{-\lambda^* \tau}}{j!} K e^{-r^* \tau} E^* \left[\Phi(d_2^*) \right] \\
&\quad + \frac{2}{\sqrt{\lambda}} \sum_{j=0}^{\infty} \frac{(\lambda^* \tau)^j e^{-\lambda^* \tau}}{j!} \tau \left\{ S_t e^{-[d^* + \lambda^*(\xi^* - 1)]\tau} E^* \left[\prod_{i=0}^j Y_i \Phi(d_1^*) \right] - K e^{-r^* \tau} E^* \left[\Phi(d_2^*) \right] \right\}, \\
\frac{\partial F}{\partial \sqrt{k_2}} &= (1 - \gamma)\gamma\lambda\alpha^2\tau\sqrt{k_2}F < 0.
\end{aligned}$$

With respect to total risk, the partial is positive, i.e. greater risk, great options value owing to its convex payout. On the contrast, with respect to idiosyncratic risk, the partial is negative. While this result may seem counterintuitive, it is consistent with the role of risk aversion that lies at the foundation of subjective value's illiquidity discount.

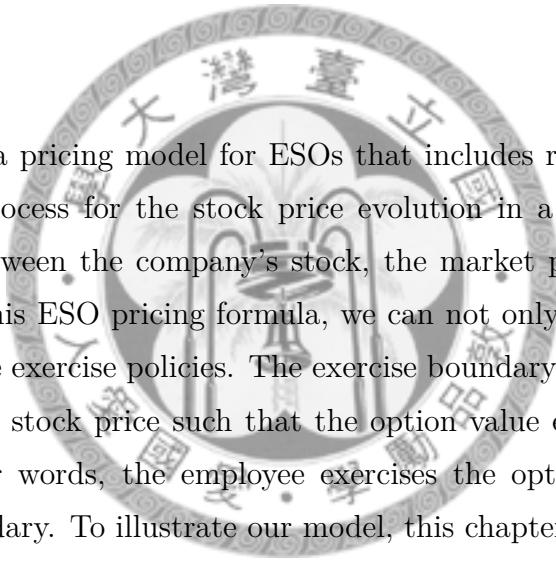
In general, when employees do not face portfolio restrictions, they allocate wealth between the market portfolio and risk-free asset and do not make additional investments in individual firm stock. Idiosyncratic risk is diversified away and hence does not affect options value. However, in our employee stock option model, some portion of the idiosyncratic risk cannot be diversified owing to the restriction of the employee's holdings. Hence, as idiosyncratic risk increases, the risk-premium associated with holding the asset likewise increases and subjective value decreases. This important finding suggests that increasing firm-specific risk may in fact reduce the value of the ESO. This may act to reduce the effort and value-creation incentives intended by options issuance. On the other hand, the convex

payout of options may also lead to excessive risk-taking and moral hazard, which would likewise be discouraged in our model. Moreover, this relation is particularly interesting since idiosyncratic risk, as opposed to market risk, is most directly affected by management. Indeed, Meulbroek (2001) identifies non-systematic, firm-specific risk as more costly to managers. These relations are tested empirically in detail in our main findings. Finally, regarding jump risk, the partial with respect to jump frequency is indeterminate in sign while that related to jump size is negative.



Chapter 4

Simulation Results



Chapter 3 provides a pricing model for ESOs that includes restriction of the options and a jump diffusion process for the stock price evolution in a world where employees balance their wealth between the company's stock, the market portfolio, and a risk-free asset. Moreover, from this ESO pricing formula, we can not only estimate the subjective values but also study the exercise policies. The exercise boundary is endogenously derived by finding the minimum stock price such that the option value equals its intrinsic value for each time. In other words, the employee exercises the option when stock price is above the exercise boundary. To illustrate our model, this chapter discusses factors which affect ESO values and exercise decisions including: stock holding constraint, level of risk aversion, moneyness, dividend, time to maturity, total volatility and normal unsystematic volatility. A comparison between perpetual and finite horizon American ESOs and default risk analysis are also given for illustration.

According to the collected data from Compustat, the model parameters stock price S , strike price K , total volatility σ , dividend yield d , interest free rate r , time to maturity τ are set to 25, 25, 0.3, 2%, 5%, 10, respectively. Normal unsystematic volatility ν is two-thirds of the total volatility following calibrations applied by Bettis et al. (2005) and

Ingersoll (2006). We employ the common parameterization for the coefficient of relative risk aversion $R = 1 - \gamma = 2$ and two jump size models: double exponential and $Y=0$ (no residual value).¹ Additionally, default intensity $\lambda = 0.01$, following Duffee (1999) and Fruhwirth and Sogner (2006) which use US and German bond data, respectively.

4.1 Exercise Behavior

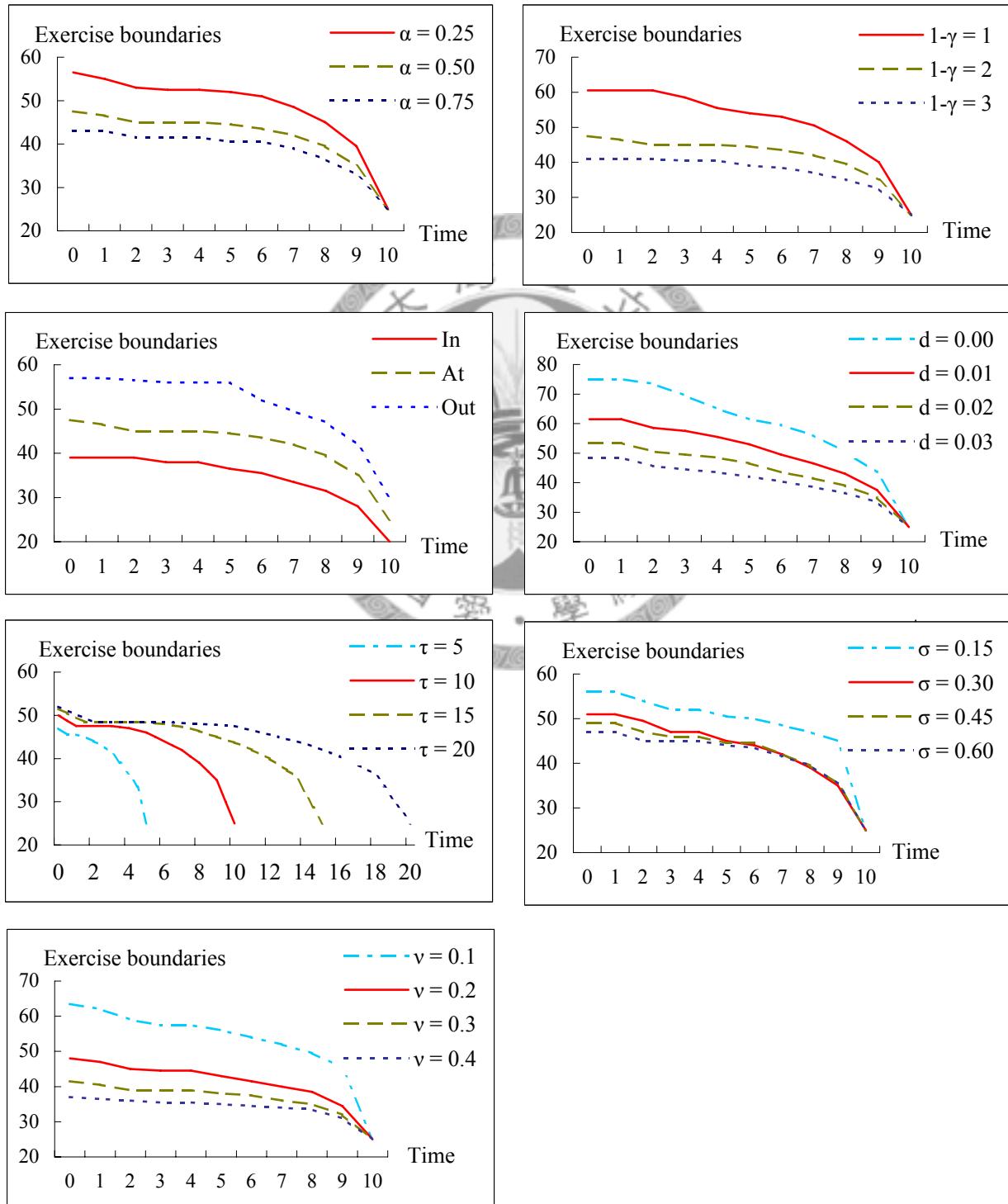
Employees exercising their ESOs earlier are pervasive phenomena. Considering the exercise policies is necessary for studying American ESOs. This is an essential departure from Chang et al. (2008) which considers European type ESOs. A number of papers link early exercise behavior to under-diversification of employees (Hemmer et al., 1996; Core and Guay, 2001; Bettis et al., 2005). The problem of valuing ESOs with early exercise is often approximated in practice by simply using the expected time until exercise in place of the actual time to maturity (Hull and White, 2004; Bettis et al., 2005). The expected time until exercise is estimated from past experience. However, Ingersoll (2006) mentions that even using an unbiased estimate of the expected time until exercise will not give a correct estimate of the option's value. And this method cannot be used to determine the subjective value since it will be smaller due to the extra discounting required to compensate the lack of diversification.

A proper calculation must recognize that the decision to exercise is endogenous. Liao and Lyuu (2009) incorporates the exercise pattern instead of using the expected time until exercise technique in valuation of ESOs, to which the exercise patterns are under Chi-square distribution assumption and not derived endogenously. Ingersoll (2006) derives the exercise boundaries endogenously, while the exercise policies are restricted constant in

¹The parameters of double exponential are estimated by daily return data from 1992 to 2004. A jump occurs if return goes beyond $\pm 10\%$ which relates to an approximately 3-standard-deviation daily return during this period.

Figure 4.1: Exercise Boundaries

This figure presents the exercise boundaries according to stock holding constraint α , level of risk aversion $1 - \gamma$, moneyness In: $K = 20$, At: $K = 25$, Out: $K = 30$, where K is exercise price, dividend yield d , time to maturity τ , total volatility σ and idiosyncratic risk ν , respectively.



time. We extend the method developed in Gukhal (2001), with a modification to include that an agent faces a constrained portfolio problem, and derive the time varying exercise policies endogenously.

Which factors cause employees to exercise their options early? Figure 4.1 compares the exercise boundaries for some factors. Note that exercise boundaries are decreasing function of time in all cases, which are different from the constant exercise policies in Ingersoll (2006). The more restrictions on the stock holding or the more risk averse the employee, the lower the exercise boundary. In other words, because of the impossibility of full diversification employees who are more restricted on the stock holding or more risk averse prefer early exercise their options. The employees who receive the in the money type options also tend to early exercise. Besides, larger dividends induce employees to exercise their options sooner. Options with shorter lifetime are quicker exercised. Employees do not have much time value in these options and tend to exercise their options earlier. Employees early exercise volatile options to balance their portfolio risk especially for idiosyncratic risk increasing. Indeed, our model findings are consistent with several empirical studies. For instance, Hemmer et al. (1996), Huddart and Lang (1996), and Bettis et al. (2005) show that early exercise is a pervasive phenomenon owing to risk aversion and undiversification of employees. Huddart and Lang (1996) find that exercise is negatively related to the time to maturity and positively correlated with the market-to-strike ratio and with the stock price volatility. Hemmer et al. (1996) and Bettis et al. (2005) also find that stock price volatility has a significant effect on exercise decisions. In high volatility firms, employees exercise options much earlier than in low volatility firms.

4.2 Factors Effect on ESOs and the Exercise Decision

Understanding the factors which affect ESO values and the exercise decision is important for firm to design the stock option programs. As we mentioned before, ESO values

and exercise decisions are closely related. Factors affecting the employees exercise policies will directly influence the valuation of ESOs. This section discusses the impact of factors on American ESOs and the exercise decision. The results are shown in Table 4.1 and 4.2, which present the studying factors effect on ESO value, discount ratio, and early exercise premium, where ESO value is calculated by formula (3.6), discount ratio is defined as one minus the ratio of subjective to market value, and early exercise premium is the difference between American and European ESO value.

4.2.1 Stock Holding Constraint, Level of Risk Aversion, Money-ness

Unlike traditional options, ESOs usually have a vesting period during which they can not be exercised and employees are not permitted to sell their ESOs. In this situation, employees receive the ESOs in a very illiquid market. Table 4.1 shows that subjective values ($\alpha \neq 0$) are uniformly smaller than the market values ($\alpha = 0$). These results are consistent with Lambert et al. (1991) and Hall and Murphy (2002) that the subjective value is lower than market value due to the constrained fixed holding in the underlying stock. The more risk averse the employee (more positive $1 - \gamma$) and more restrictions on the stock holding (larger α), lean to depreciate the option values and incur the higher early exercise premium. Note that early exercise effect on ESO values can not be ignored in these situations.

Because of the restriction of ESOs, many employees have undiversified portfolios with large stock options for their own firms. Therefore, a risk averse employee discounts the ESO values. Discount ratios increase with stock holding constraint and the degree of risk aversion. In other words, employees who are more risk averse and more restricted on the stock holdings need to compensate more risk premium. In the money options have higher values, lower discount and higher early exercise premium. Interesting, even in the money

options having less discount than out of the money, employees still more tend to early exercise in the money options to diversify their wealth portfolio risk.²

Further we examine the effect of vesting on subjective value. Panel D compares the ESOs that vest immediately, after two, and four years, respectively. Vesting obviously reduces the ESO values since it restricts the exercise timing. Discount ratio increasing with vesting period implies that market value is affected less than the subjective value. Because the constrained ESOs are usually exercised much earlier than unconstrained ESOs and more tend to fall afoul of the vesting rule. While vesting has negative effect on the American ESOs, it has no effect on the European ESOs, therefore, early exercise premium decreases with vesting.

Table 4.1: Stock Holding Constraint, Risk Aversion and Moneyness Effect

| Panel A: ESO values | | | | | | | | | |
|---------------------|---------------------|---------|---------|---------------------|--------|--------|-------------------------|--------|--------|
| | In the money (K=20) | | | At the money (K=25) | | | Out of the money (K=30) | | |
| | R = 1 | R = 2 | R = 3 | R = 1 | R = 2 | R = 3 | R = 1 | R = 2 | R = 3 |
| $\alpha = 0.00$ | 11.2694 | 11.2694 | 11.2694 | 9.6487 | 9.6487 | 9.6487 | 8.3902 | 8.3902 | 8.3902 |
| $\alpha = 0.25$ | 10.2323 | 9.3061 | 8.6979 | 8.5661 | 7.5859 | 6.9702 | 7.3532 | 6.4228 | 5.7020 |
| $\alpha = 0.50$ | 9.6120 | 8.2193 | 7.3964 | 7.8852 | 6.4628 | 5.5200 | 6.6944 | 5.3040 | 4.2714 |
| $\alpha = 0.75$ | 9.2649 | 7.6015 | 6.6195 | 7.4822 | 5.7706 | 4.6858 | 6.2934 | 4.6129 | 3.4367 |

| Panel B: Early Exercise Premiums | | | | | | | | | |
|----------------------------------|---------------------|--------|--------|---------------------|--------|--------|-------------------------|--------|--------|
| | In the money (K=20) | | | At the money (K=25) | | | Out of the money (K=30) | | |
| | R = 1 | R = 2 | R = 3 | R = 1 | R = 2 | R = 3 | R = 1 | R = 2 | R = 3 |
| $\alpha = 0.00$ | 0.3440 | 0.3440 | 0.3440 | 0.1839 | 0.1839 | 0.1839 | 0.1405 | 0.1405 | 0.1405 |
| $\alpha = 0.25$ | 0.6990 | 1.0268 | 1.5489 | 0.4010 | 0.5796 | 0.9958 | 0.3089 | 0.4413 | 0.6562 |
| $\alpha = 0.50$ | 0.9080 | 1.4265 | 2.2173 | 0.5191 | 0.8592 | 1.3642 | 0.4070 | 0.6271 | 0.8844 |
| $\alpha = 0.75$ | 0.9520 | 1.5777 | 2.4918 | 0.5345 | 0.9366 | 1.5192 | 0.4285 | 0.6747 | 0.9545 |

²While Panel A, B, and C show the results of options that vest immediately, we can also consider the vesting effect and there is no significant qualitative differences.

| Panel C: Discount Ratios | | | | | | | | | |
|--------------------------|---------------------|---------|---------|---------------------|---------|---------|-------------------------|---------|---------|
| | In the money (K=20) | | | At the money (K=25) | | | Out of the money (K=30) | | |
| | $R = 1$ | $R = 2$ | $R = 3$ | $R = 1$ | $R = 2$ | $R = 3$ | $R = 1$ | $R = 2$ | $R = 3$ |
| $\alpha = 0.25$ | 0.0920 | 0.1733 | 0.2302 | 0.1122 | 0.2110 | 0.2811 | 0.1236 | 0.2321 | 0.3209 |
| $\alpha = 0.50$ | 0.1471 | 0.2698 | 0.3454 | 0.1828 | 0.3278 | 0.4306 | 0.2021 | 0.3658 | 0.4913 |
| $\alpha = 0.75$ | 0.1779 | 0.3247 | 0.4142 | 0.2245 | 0.3998 | 0.5167 | 0.2499 | 0.4484 | 0.5907 |

| Panel D: Vesting Effect | | | | | | | | | |
|-------------------------|--------|--------|--------|--------|--------|--------|---------|--------|--------|
| | CA | | | CD | | | Premium | | |
| | VP = 0 | VP = 2 | VP = 4 | VP = 0 | VP = 2 | VP = 4 | VP = 0 | VP = 2 | VP = 4 |
| $\alpha = 0.25$ | 7.5859 | 7.5660 | 7.4923 | 0.2110 | 0.2125 | 0.2200 | 0.5796 | 0.5651 | 0.4898 |
| $\alpha = 0.50$ | 6.4628 | 6.4029 | 6.2639 | 0.3278 | 0.3335 | 0.3479 | 0.8592 | 0.8041 | 0.6635 |
| $\alpha = 0.75$ | 5.7706 | 5.6867 | 5.5299 | 0.3998 | 0.4081 | 0.4243 | 0.9366 | 0.8570 | 0.6989 |

Note: This table presents the impact of factors on employee stock options (ESOs) and the exercise decision. The results of ESO values, early exercise premiums, and discount ratios are shown in Panels A, B and C, respectively. Panel D shows the result of vesting effect. α , K , and R represent stock holding constraint, exercise price and level of risk aversion. In Panel D, CA, CD, Premium and VP are the ESO value, discount ratio (1-subjective/market), early exercise premium and vesting period, respectively.

4.2.2 Dividend, Time to Maturity, Volatility Risk

Larger dividends depreciate the option values and induce employees to exercise their options sooner even they have lower discount ratios. More interestingly, the early exercise premium is not zero when no dividends paid. This is a departure from traditional option theory, while it is consistent with the phenomenon that ESOs are exercised substantially before maturity date even ESOs not paying dividends because of the lack of diversification. Options with longer lifetime have more values, at the same time, they have higher discount ratios and early exercise premiums. Although not reported in the table, the lifetime of option may be negatively related to European ESO value. This is due to the longer one has to wait and then the more the risk caused by undiversification affects the ESO value.

While options may provide incentives for employees to work harder, they can also

Table 4.2: Factors Effect on Employee Stock Options and the Exercise Decision

| Panel A: ESO Values & Discount Ratios & Early Exercise Premiums | | | | | | | |
|---|--------|--------|---------|-------------|--------|--------|---------|
| | CA | CD | Premium | | CA | CD | Premium |
| $d = 0.00$ | 8.4788 | 0.3557 | 0.3372 | $\tau = 5$ | 5.4222 | 0.2545 | 0.4084 |
| $d = 0.01$ | 7.5724 | 0.3260 | 0.8031 | $\tau = 10$ | 6.7636 | 0.3018 | 1.1608 |
| $d = 0.02$ | 6.7704 | 0.3054 | 1.1729 | $\tau = 15$ | 7.4605 | 0.3257 | 1.9932 |
| $d = 0.03$ | 6.0673 | 0.2941 | 1.4674 | $\tau = 20$ | 7.8992 | 0.3363 | 2.8481 |
| $\sigma = 0.15$ | 5.4571 | 0.1800 | 0.0677 | $\nu = 0.1$ | 7.1077 | 0.2607 | 0.2400 |
| $\sigma = 0.30$ | 6.5440 | 0.3216 | 0.9413 | $\nu = 0.2$ | 6.2785 | 0.3470 | 1.0146 |
| $\sigma = 0.45$ | 7.0701 | 0.4378 | 2.1005 | $\nu = 0.3$ | 5.6321 | 0.4142 | 2.1113 |
| $\sigma = 0.60$ | 7.2776 | 0.5184 | 3.4397 | $\nu = 0.4$ | 5.1440 | 0.4650 | 3.0637 |

| Panel B: Summary of Factors Effect | | | | | | | |
|------------------------------------|----------|---|-----|---|--------|----------|-------|
| | α | R | sok | d | τ | σ | ν |
| CA | - | - | + | - | + | + | - |
| CD | + | + | - | - | + | + | + |
| Premium | + | + | + | + | + | + | + |
| Exercise | + | + | + | + | - | + | + |

Note: This table presents the impact of factors on employee stock options (ESOs) and the exercise decision. d , τ , σ , ν , α , R, sok are dividend yield, time to maturity, total volatility, normal unsystematic volatility, stock holding constraint, level of risk aversion and the ratio of stock price to exercise price, respectively. In Panel A, CA, CD and Premium are the ESO value, discount ratio (1-subjective/market) and early exercise premium, respectively. Panel B shows the relationship between the factor and the item listed in the left column. The last item Exercise means early exercise. Symbols "+" and "-" represent positive and negative relationship.

induce suboptimal risk-taking behavior. General option pricing results show that value should increase with risk while employees need to compensate more risk premium at the same time. It is not necessarily that subjective value is positive related to risk, as is the traditional result.³ We have usual finding that total volatility increases the option value, however, with respect to normal unsystematic volatility, the subjective value decreases

³Nohel and Todd (2005), Ryan and Wiggins (2001), and others show that option values increase with risk, however, they do not study the impact of increased idiosyncratic risk. Carpenter (2000) presents examples where convex incentive structures do not imply that the manager is more willing to take risks. The model used in Chang et al. (2008) is able to capture this result.

with it, oppositely. In Black-Scholes framework, this risk is eliminated under risk-neutral measure. However, in our model, the employee has an illiquid holding and full diversification is impossible. Hence, a risk averse employee depreciates the ESO values. The discount and early exercise premium increasing with the volatility risk also can be found in Table 4.2. This is intuitive, since the more volatile stock price, the higher is the opportunity cost of not being able to exercise. Therefore, employees have more incentives to early exercise volatile options. All factors effect are summarized in Panel B.

4.3 Perpetual American Options

The perpetual American ESO results are shown in Table 4.3 and 4.4. Table 4.3 presents the perpetual ESO values and optimal exercise boundaries by formula (3.8) and (3.9) in Theorem 3.4.3. Absolute and relative differences between perpetual and finite horizon American ESOs are discussed in Table 4.4. Note that for values and optimal exercise boundaries, perpetual American ESOs have the same patterns as those for finite horizon American ESOs. That is, subjective values are uniformly smaller than the market values; the more risk averse the employee and more stock holding restrictions lean to depreciate the option values and decline the exercise boundaries; for moneyness, in the money options have higher option values and lower exercise boundaries.

Table 4.3: Perpetual ESO values and Optimal Exercise Boundaries

| | Panel A: Perpetual ESO values | | | | | | | | | |
|-----------------|-------------------------------|---------|---------|---------------------|---------|---------|-------------------------|---------|---------|--|
| | In the money (K=20) | | | At the money (K=25) | | | Out of the money (K=30) | | | |
| | $R = 1$ | $R = 2$ | $R = 3$ | $R = 1$ | $R = 2$ | $R = 3$ | $R = 1$ | $R = 2$ | $R = 3$ | |
| $\alpha = 0.00$ | 14.5067 | 14.5067 | 14.5067 | 13.7747 | 13.7747 | 13.7747 | 13.2042 | 13.2042 | 13.2042 | |
| $\alpha = 0.25$ | 12.6407 | 11.1791 | 10.0119 | 11.7198 | 10.0940 | 8.7818 | 11.0174 | 9.2861 | 7.8898 | |
| $\alpha = 0.50$ | 11.7755 | 9.7389 | 8.2450 | 10.7594 | 8.4725 | 6.7572 | 9.9947 | 7.5611 | 5.7433 | |
| $\alpha = 0.75$ | 11.5666 | 9.1089 | 7.3460 | 10.5268 | 7.7546 | 5.6954 | 9.7469 | 6.7989 | 4.6262 | |

| Panel B: Optimal Exercise Boundaries | | | | | | | | | |
|--------------------------------------|---------------------|---------|---------|---------------------|---------|---------|-------------------------|---------|---------|
| | In the money (K=20) | | | At the money (K=25) | | | Out of the money (K=30) | | |
| | $R = 1$ | $R = 2$ | $R = 3$ | $R = 1$ | $R = 2$ | $R = 3$ | $R = 1$ | $R = 2$ | $R = 3$ |
| $\alpha = 0.00$ | 106.165 | 106.165 | 106.165 | 132.706 | 132.706 | 132.706 | 159.248 | 159.248 | 159.248 |
| $\alpha = 0.25$ | 78.973 | 63.687 | 54.023 | 98.716 | 79.609 | 67.529 | 118.459 | 95.531 | 81.035 |
| $\alpha = 0.50$ | 69.432 | 52.019 | 42.411 | 86.790 | 65.023 | 53.014 | 104.148 | 78.028 | 63.616 |
| $\alpha = 0.75$ | 67.350 | 47.706 | 37.520 | 84.187 | 59.632 | 46.900 | 101.024 | 71.559 | 56.281 |

Note: This table studies perpetual employee stock options (ESOs). The results of perpetual ESO values and optimal exercise boundaries are shown in Panels A and B, respectively. α , K , and R represent stock holding constraint, exercise price and level of risk aversion.

Interestingly, the differences between perpetual and finite horizon American ESOs are related to factors that affect exercise behavior. Specifically, the differences are reduced when employees face large restricted holding, are more risk averse and receive in the money type options. In these situations, the employees tend to exercise early. The relative difference, which is defined as the ratio of difference between perpetual and finite horizon American ESO to finite horizon American ESO, also has the same phenomenon. In other words, perpetual American ESO approximates finite horizon American ESO better when an agent with large restricted holding, more risk averse and receiving in the money type options. These phenomena can be explained as that the time values of perpetual options are reduced in these situations. Note that from our simulation studies, the same phenomenon holds when there is no jump occurs.

Table 4.4: Differences Between Perpetual and Finite Horizon ESO Values

| Panel A: Absolute Differences Between Perpetual and Finite Horizon ESO Values | | | | | | | | | |
|---|---------------------|---------|---------|---------------------|---------|---------|-------------------------|---------|---------|
| | In the money (K=20) | | | At the money (K=25) | | | Out of the money (K=30) | | |
| | $R = 1$ | $R = 2$ | $R = 3$ | $R = 1$ | $R = 2$ | $R = 3$ | $R = 1$ | $R = 2$ | $R = 3$ |
| $\alpha = 0.00$ | 3.2373 | 3.2373 | 3.2373 | 4.1261 | 4.1261 | 4.1261 | 4.8140 | 4.8140 | 4.8140 |
| $\alpha = 0.25$ | 2.4084 | 1.8730 | 1.3140 | 3.1536 | 2.5081 | 1.8116 | 3.6642 | 2.8634 | 2.1877 |
| $\alpha = 0.50$ | 2.1634 | 1.5196 | 0.8486 | 2.8742 | 2.0098 | 1.2372 | 3.3002 | 2.2571 | 1.4719 |
| $\alpha = 0.75$ | 2.3017 | 1.5074 | 0.7265 | 3.0446 | 1.9840 | 1.0096 | 3.4536 | 2.1860 | 1.1895 |

| Panel B: Relative Differences Between Perpetual and Finite Horizon ESO Values | | | | | | | | | |
|---|---------------------|---------|---------|---------------------|---------|---------|-------------------------|---------|---------|
| | In the money (K=20) | | | At the money (K=25) | | | Out of the money (K=30) | | |
| | $R = 1$ | $R = 2$ | $R = 3$ | $R = 1$ | $R = 2$ | $R = 3$ | $R = 1$ | $R = 2$ | $R = 3$ |
| $\alpha = 0.00$ | 0.2873 | 0.2873 | 0.2873 | 0.4276 | 0.4276 | 0.4276 | 0.5738 | 0.5738 | 0.5738 |
| $\alpha = 0.25$ | 0.2354 | 0.2013 | 0.1511 | 0.3682 | 0.3306 | 0.2599 | 0.4983 | 0.4458 | 0.3837 |
| $\alpha = 0.50$ | 0.2251 | 0.1849 | 0.1147 | 0.3645 | 0.3110 | 0.2241 | 0.4930 | 0.4255 | 0.3446 |
| $\alpha = 0.75$ | 0.2484 | 0.1983 | 0.1098 | 0.4069 | 0.3438 | 0.2155 | 0.5488 | 0.4739 | 0.3461 |

Note: This table studies perpetual employee stock options (ESOs). Panel A and B exhibit the absolute and relative differences between perpetual and finite horizon American ESO values. α , K , and R represent stock holding constraint, exercise price and level of risk aversion.

4.4 Default Risk

| Table 4.5: Default Risk Analysis | | | | | | |
|----------------------------------|--------------|----------|----------|----------|----------|----------|
| Panel A: ESO values | | | | | | |
| | Default Jump | | | No Jump | | |
| | $K = 20$ | $K = 25$ | $K = 30$ | $K = 20$ | $K = 25$ | $K = 30$ |
| $\alpha = 0.00$ | 11.8111 | 10.3038 | 9.0617 | 11.2495 | 9.7201 | 8.3836 |
| $\alpha = 0.25$ | 9.4634 | 8.0239 | 6.7112 | 9.3837 | 7.8436 | 6.4794 |
| $\alpha = 0.50$ | 8.2630 | 6.5258 | 5.0974 | 8.3920 | 6.7768 | 5.3815 |
| $\alpha = 0.75$ | 7.9499 | 5.3414 | 4.4200 | 7.8234 | 6.1118 | 4.6976 |

| Panel B: Discount Ratios | | | | | | |
|--------------------------|--------------|----------|----------|----------|----------|----------|
| | Default Jump | | | No Jump | | |
| | $K = 20$ | $K = 25$ | $K = 30$ | $K = 20$ | $K = 25$ | $K = 30$ |
| $\alpha = 0.25$ | 0.1988 | 0.2213 | 0.2594 | 0.1659 | 0.1931 | 0.2271 |
| $\alpha = 0.50$ | 0.3004 | 0.3527 | 0.4375 | 0.2540 | 0.3028 | 0.3581 |
| $\alpha = 0.75$ | 0.3269 | 0.4957 | 0.5122 | 0.3046 | 0.3712 | 0.4397 |

This section studies the impact of default risk on ESO values. Table 4.5 compares two cases: stock having no residual value if jump occurs (default jump) and stock following diffusion process (no jump). When employees face less restricted holding ($\alpha = 0, 0.25$), the values of options with default risk are larger than the options if the underlying stocks

| | Panel C: Early Exercise Premiums | | | | | |
|-----------------|----------------------------------|----------|----------|----------|----------|----------|
| | Default Jump | | | No Jump | | |
| | $K = 20$ | $K = 25$ | $K = 30$ | $K = 20$ | $K = 25$ | $K = 30$ |
| $\alpha = 0.00$ | 0.2659 | 0.1967 | 0.1614 | 0.3406 | 0.2734 | 0.1460 |
| $\alpha = 0.25$ | 1.1163 | 0.8778 | 0.5432 | 1.1209 | 0.8539 | 0.5104 |
| $\alpha = 0.50$ | 1.8060 | 1.1317 | 0.5417 | 1.6181 | 1.1912 | 0.7180 |
| $\alpha = 0.75$ | 2.6161 | 1.0025 | 0.8403 | 1.8243 | 1.2999 | 0.7760 |

Note: This table compares the ESO results for stock having no residual value if jump occurs (default jump) with stock following diffusion process (no jump). The results of ESO values, discount ratios and early exercise premiums are shown in Panels A, B and C, respectively. α and K represent stock holding constraint and exercise price.

follow diffusion processes. Interestingly, unlike the traditional option theory, we have the opposite results when employees are confronted by large restricted holding of company stock ($\alpha = 0.5, 0.75$). In other words, when employees encounter large restricted holding, the option values with default risk are no longer larger than the options if the underlying stock processes are continuous. Panel B shows that options with default risk have higher discount ratios. Again, in this situation, employees need to compensate more risk premium. However, from Panel C, there are no obvious patterns for early exercise premiums.

4.5 Sentiment Analysis

The level of sentiment is estimated from two perspectives. First, we consider the sentiment effect on ESO value (SenV), and then the estimated sentiment level can be calculated whereby subjective value with sentiment is equal to market value. Secondly, we estimate sentiment level from the early exercise perspective (SenE), i.e., what value of sentiment such that employees exercise their options at the time that unconstrained investors do. The sentiment level of European ESOs (SenVE) is also calculated. Due to the limitation of European options, they are not allowed to early exercise, the sentiment level can only be estimated from value perspective.

Table 4.6: Sentiment Analysis

| | Panel A: Sentiment Level | | | | | | | | | |
|-----------------|--------------------------|---------|---------|---------|---------|---------|---------|---------|---------|--|
| | SenV | | | SenVE | | | SenE | | | |
| | $R = 1$ | $R = 2$ | $R = 3$ | $R = 1$ | $R = 2$ | $R = 3$ | $R = 1$ | $R = 2$ | $R = 3$ | |
| $\alpha = 0.25$ | 0.0100 | 0.0200 | 0.0300 | 0.0100 | 0.0200 | 0.0300 | 0.0097 | 0.0180 | 0.0291 | |
| $\alpha = 0.50$ | 0.0199 | 0.0399 | 0.0599 | 0.0200 | 0.0400 | 0.0599 | 0.0194 | 0.0369 | 0.0585 | |
| $\alpha = 0.75$ | 0.0298 | 0.0598 | 0.0896 | 0.0299 | 0.0598 | 0.0897 | 0.0289 | 0.0579 | 0.0866 | |

| Panel B: Sentiment Effect on ESO Values and the Exercise Decision | | | | | | | | | | |
|---|--------|---------|---------------------|-------------|-------------|-------------|--|--|--|--|
| | | | Exercise boundaries | | | | | | | |
| | | | $s = -0.005$ | $s = 0.000$ | $s = 0.005$ | $s = 0.010$ | | | | |
| CA | CD | Premium | | | | | | | | |
| $s = -0.005$ | 6.1495 | 0.3120 | 0.9385 | | | | | | | |
| $s = 0.000$ | 6.4611 | 0.3279 | 0.8592 | | | | | | | |
| $s = 0.005$ | 6.7632 | 0.3469 | 0.7504 | | | | | | | |
| $s = 0.010$ | 7.1101 | 0.3655 | 0.6661 | | | | | | | |

Note: This table presents the sentiment levels necessary to offset the employee stock option (ESO) risk premium and the impact of sentiment on ESO values and the exercise decision. The estimated sentiment levels are listed in Panel A. Sentiment levels SenV and SenVE are calculated while the subjective value with sentiment is equal to market value for American and European options, respectively. SenE is the value of sentiment such that an employee exercises his options at the time that unconstrained investors do. α and R represent the stock holding constraint and level of risk aversion. The results of sentiment effect are shown in Panel B. s , CA, CD and Premium are the sentiment level, ESO value, discount ratio and early exercise premium, respectively.

Sentiment results are shown in Table 4.6. Here, we only list the estimated sentiment level of at the money option since there is no obvious relationship between sentiment level and moneyness. Table 4.6 shows that sentiment estimated from American and European ESO formulae have similar patterns. The more risk averse the employee and more restricted on the stock holding, the higher the sentiment level is needed. SenVE is slight higher than SenV because of the more restrictions in European contract. Employee sentiment enhances the option value and reduces the early exercise premium. Options with high sentiment

having higher discount implies the option value declining sharply when employees face undiversification problem. Employee with high sentiment will postpone the exercise timing due to the brightening prospect of the company.



Chapter 5

Empirical Study

Applying a comprehensive set of executive options and compensation data, this study empirically prices both the subjective value discount created by stock holding constraint and the risk-adjusted excess returns necessary for employees to offset the ESO risk premium, i.e. the sentiment effect. While our modeling specification is similar to Chang et al. (2008), this study is the first to test such a model to empirically price the impact of stock holding constraint and sentiment. Besides, the compensation data are first used to calculate subjective value, and by applying our model sentiment levels can be estimated.



5.1 Data and Preliminary Results

Data for this study are collected from the Compustat Executive Compensation (Execucomp) database. From this database, all executive stock options issued between 1992 and 2004 are collected with stock price at issuance S , strike price K , maturity date T , implied volatility Vol , and dividend yield Div . In addition to options data, we collect total compensation data from Execucomp database, which includes salary, bonus, restricted stock, option, long-term incentive pay and other income earned by executives each year.

Using these data, we calculate a number of variables. The money-ness of each option S_{ok} is the stock price at issuance divided by strike price. If the option is in (out of) the money, S_{ok} is greater (less) than 1. The time-to-maturity is denoted τ and normal unsystematic volatility is calculated as two-thirds of implied volatility, following calibrations applied by Compustat and the majority of papers in the area.¹

Following Dittmann and Maug (2007), we further define the net cash inflow ($NCash$) for each year as follows:

$$\begin{aligned}
 NCash = & \text{ Fixed salary (after tax)} \\
 & + \text{ Dividend income from shares held in own company (after tax)} \\
 & + \text{ Value of restricted stock granted} \\
 & - \text{ Personal taxes on restricted stock that vest during the year} \\
 & + \text{ Net value realized from exercising options (after tax)} \\
 & - \text{ Cash paid for purchasing additional stock}
 \end{aligned}$$

Fixed salary is the sum of five Compustat data types: Salary, Bonus, Other Annual, All Other Total, and long-term incentive pay (LTIP).² Denote the year when the executive enters the database by t_E . The executive's cumulative wealth for year t is then

$$W_t = NCash_t + \sum_{\ell=t_E}^{t-1} NCash_{\ell} \prod_{s=\ell+1}^t (1 + r_f^s).$$

In other words, assume that the executive has no wealth before entering the firm, all $NCash_t$ are realized at the end of the fiscal year and invested at the risk-free rate r_f^{t+1} during the next fiscal year. Then, α is the sum of all illiquid firm-specific holdings, including unvested restricted stocks and options, divided by total cumulative wealth. Alternate approaches to calculate α are addressed in robustness tests, including an iterated approach

¹See Bettis et al. (2005), Aggarwal and Samwick (2003), Ingersoll (2006), and Bryan et al. (2000).

²For cash paid for purchasing additional stock, where direct data is unavailable, we use the change in stock holdings times the year-end stock price to calculate this value.

that synchronizes α and subjective value simultaneously. Qualitative findings with respect to sentiment are identical.

Table 5.1: ESO summary statistics

| | <i>Sok</i> | τ | <i>Vol</i> | <i>Div</i> | α |
|---------|------------|--------|------------|------------|----------|
| Mean | 1.012 | 9.308 | 0.431 | 1.37% | 0.353 |
| Median | 1.000 | 9.668 | 0.370 | 0.62% | 0.307 |
| Std Dev | 0.434 | 1.728 | 0.243 | 1.77% | 0.227 |
| Max | 37.50 | 25.51 | 4.120 | 20.39% | 1 |
| Min | 0.230 | 0.100 | 0.102 | 0.00% | < .001 |

Note: This table presents summary statistics for ESOs used in this study. *Sok*, τ , *Vol*, *Div*, and α are the ratio of stock price to exercise price, time to maturity, implied volatility, dividend yield and the proportion of total wealth held in illiquid firm specific holdings, respectively.

Summary statistics for each of these variables are shown in Table 5.1. While the median option is issued at the money, the mean is in the money (*Sok* = 1.012). Note that virtually all options are issued at the money (*Sok* = 1). Indeed this is true for about 90% of our dataset. As a robustness check, we also try removing *Sok* as a variable, and find no qualitative differences. Average time to maturity is about 9.3 years³ and α is about 35%, implying that the illiquid firm specific holdings account for more than one-third of executive total wealth.⁴ Median values of other model parameters are *Vol* = 0.37, *Div* = 0.62% and the risk-free rate r = 5.3%. The default intensity λ = 0.01 and the coefficient of relative risk aversion $1 - \gamma$ = 2 are following Duffee (1999) and Fruhwirth and Sogner (2006) and common parameterization. Throughout our regression analysis, outliers are excluded by using a standardized residuals approach, removing those with residuals greater than 3 or less than -3. In all, about 0.05% of our sample is removed.

³For some issues for which there is no time stamp, we assume an issuance date of July 1 since this would be the middle of the fiscal year for the vast majority of firms.

⁴Holland and Elder (2006) find that rank-and-file employees exhibit an α close to 10% and concur that subjective value is decreasing in α because of risk aversion and under-diversification.

5.2 Compensation-based Approach to Subjective Value

The subjective value of ESOs implied by total compensation packages is calculated by using a K-means hierarchical clustering methodology to split executives into like groups based upon industry, rank, year, the firm market value,⁵ non-option compensation, and the immediate exercise value of the options. The number of groups is decided by a cubic clustering criterion and the average total compensation is calculated. Then, assuming that all executives within the same cluster receive the same total compensation, for each executive in this cluster, the implied subjective value is derived by comparing the difference between non-option compensation and the average compensation. We then set all negative implied ESO values equal to zero and recalculate average compensation in each cluster with these subjective values, repeating until the relative sum of changes in subjective values in a given cluster is less than 0.01. This eliminates some negative subjective values such that the final number of negative or zero values is about 5.7% of our dataset. Worth noting is the observation that, even in the first iteration of the process, after grouping, only about 7.9% of our data has options with a negative or zero value, lending credence to the stability of our groupings.

To illustrate the intuition, presume that all executives within the same cluster receive the same compensation on average, where any differences in salaries, bonuses, and other income should be accounted for by options. If CEOs average total annual compensation in a given year of \$2,000,000, a particular CEO who receives \$1,500,000 in non-option compensation must then value options awarded to her at \$500,000 in order to agree to continued employment. Importantly, it may be the case that the market value of these options is only \$100,000, but the CEO subjectively values them at \$500,000 because she believes the market to have undervalued the options. While this method of calculation is

⁵Gabaix and Landier (2008) finds that total market value as a proxy for firm size has the strongest predictive power on compensation. We, however, re-do all tests using number of employees as the size proxy and find qualitatively identical results.

clearly not perfectly precise, numerous robustness checks using different grouping criteria are provided, all of which arrive at qualitatively identical results. Included in these checks, we control for potentially systematic differences in compensation level related to α (the percent of total wealth held in illiquid firm specific holdings). Some intangible sources of value such as training, learning opportunities, and advantageous work environments are not controlled here but may be relatively unimportant given that this is an executive database of listed firms.

Table 5.2: Compensation Summary Statistics

| | Aggregate | | | Mean By Title | | | |
|------------------|-----------|--------|---------|---------------|------|------|-------|
| | Mean | Median | Std Dev | B&C | B&NC | NB&C | NB&NC |
| Salary | 365 | 300 | 234 | 556 | 481 | 335 | 286 |
| Bonus | 336 | 151 | 816 | 650 | 479 | 289 | 222 |
| Other Annual | 24 | 0 | 179 | 44 | 35 | 22 | 16 |
| All Other Total | 70 | 11 | 540 | 94 | 131 | 50 | 45 |
| LTIP | 77 | 0 | 442 | 127 | 128 | 72 | 48 |
| Restricted Stock | 163 | 0 | 803 | 366 | 220 | 184 | 101 |
| Options | 1178 | 378 | 3407 | 2382 | 1683 | 1264 | 748 |
| Total | 2214 | 1074 | 4262 | 4219 | 3158 | 2217 | 1465 |

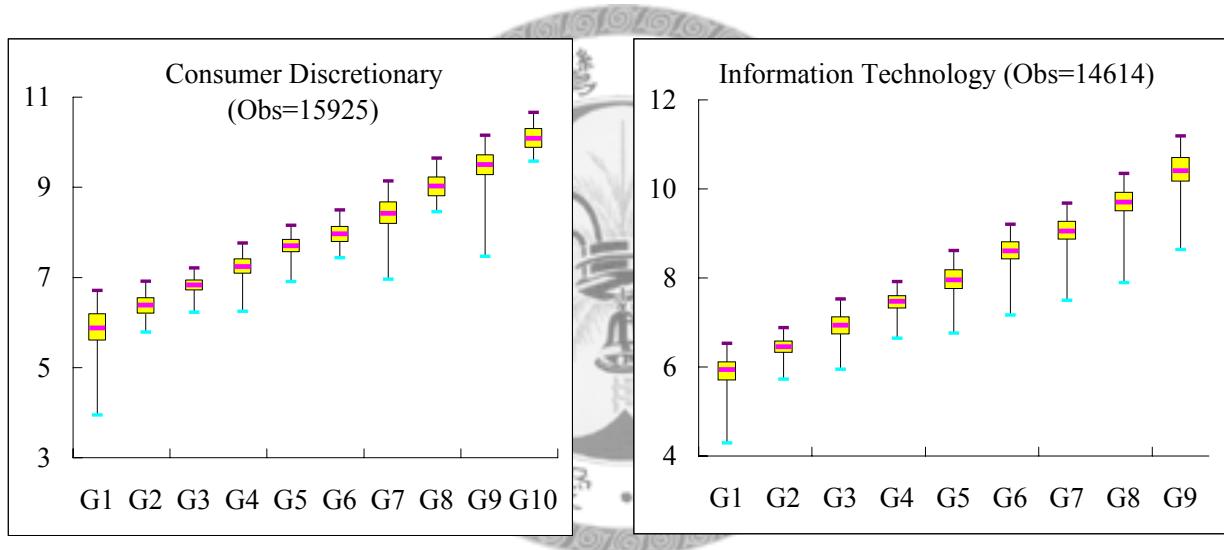
Note: This table presents summary statistics for compensation data for four categories of executives: board & CEO (B&C), board & not CEO (B&NC), not board & CEO (NB&C), and not board & not CEO (NB&NC). Numbers are reported in 1000's and LTIP represents the long-term incentive pay.

As can be seen in Table 5.2, the mean total annual compensation for executives in this dataset is a bit over \$2 million with a median of just over \$1 million. The mean and median ESO compensation numbers are roughly \$1.2 and \$0.4 million, respectively. Not surprisingly, chief executives who were also board members received the highest compensation (\$4.2 million), but options are a substantial portion of that compensation (\$2.4 million). Indeed, options compensation generally substantially outweighs all other forms of compensation.

The following figures present box plots of the natural log of total compensation for the

two largest industries in our sample: Consumer Discretionary and Information Technology. With the exception of some outliers, which are subsequently removed in our main tests, the boxed areas generally do not overlap from cluster to cluster, demonstrating the relative homogeneity of firms within each cluster and generally distinctly separated from other clusters. As a result, we believe that compensation characteristics within each cluster should be quite comparable, lending a measure of credence to our method of calculating subjective value.

Figure 5.1: Natural Log of Total Compensation



The box plots show the natural log of total compensation for the two largest industries in our sample. Executives are grouped according to position, the firm's total market value, non-option compensation, and the immediate exercise value of the options for each industry by hierarchical clustering using a K-Means approach.

5.3 Preliminary findings

Substituting the subjective value implied by compensation data into our model along with the options variables given in our dataset, we are able to back out sentiment levels Sen . Results are presented in Table 5.3. There are about 105,000 options issued by each firm (AvgIss) over the test period with a total of nearly 2700 firms and 82000 total observations accounted for. Industry breakdowns, while exhibiting some fluctuations in point estimates, show that results across industries are qualitatively similar. While the mean Black-Scholes value of options $BSOPM$ is about \$13.09 with some variation across industries, the mean subjective value Sub is more than \$19.38, reflecting a 48% premium. That is, although virtually all of the theoretical literature implies a subjective value discount, empirical data show that executives generally value ESOs more highly than their Black-Scholes values. Though not reported in the table, t-tests show that subjective values are statistically significantly higher than Black-Scholes values at the 1% level for almost all industries and in aggregate. The only exception is the others industry, where Sen is still significantly positive but Sub is about equal to $BSOPM$ owing to a particularly high α in this industry.

Given the large proportion of executive income that is attributed to illiquid, firm-specific options holdings, this finding suggests substantial over-confidence or positive inside-information regarding their firm's future prospects. Indeed, the data show that the average executives prices ESOs such that the firm should outperform the market's expectations by an average of 12% per annum (Sen). T-tests show that these values are significantly different from zero at the 1% level in all industries and in aggregate.

Table 5.4 shows the mean and median values of R_t and Sub in each subsample, where R_t is the CAPM alpha. Top is a dummy variable taking value 1 if the executive works for a firm listed in Fortune Magazine's top 100 companies for which to work. Results show that firms with higher previous-year return tend to have significantly higher subjective values. This is true of both the mean and median value. Interestingly, subsequent return momentum

Table 5.3: Summary Statistics for Subjective Value and Sentiment by Industry

| Sector | | <i>BSOPM</i> | <i>Sub</i> | <i>Sen</i> | AvgCom | AvgIss | Obs |
|--------|-------------|--------------|------------|------------|---------|--------|-------|
| 10 | Energy | 12.589 | 16.239 | 0.089 | 1774.08 | 78.46 | 4307 |
| 15 | Materials | 10.822 | 17.613 | 0.076 | 1399.90 | 61.89 | 6412 |
| 20 | Industrials | 12.569 | 20.305 | 0.115 | 1591.53 | 70.64 | 12134 |
| 25 | Con. Dis. | 12.177 | 19.799 | 0.106 | 2030.63 | 98.64 | 15925 |
| 30 | Con. Sta. | 12.053 | 18.121 | 0.076 | 2405.29 | 111.40 | 4347 |
| 35 | Health Care | 16.290 | 21.098 | 0.115 | 2338.68 | 105.61 | 8883 |
| 40 | Financials | 13.440 | 21.770 | 0.066 | 2892.28 | 99.37 | 10441 |
| 45 | Inf. Tec. | 15.853 | 18.499 | 0.229 | 2748.47 | 164.53 | 14614 |
| 50 | Tel. Ser. | 12.768 | 22.941 | 0.162 | 5310.29 | 272.27 | 1324 |
| 55 | Utilities | 5.475 | 14.633 | 0.063 | 1370.11 | 67.66 | 3931 |
| | Others | 9.487 | 9.583 | 0.208 | 1360.34 | 111.24 | 56 |
| | Total | 13.088 | 19.385 | 0.120 | 2213.73 | 105.13 | 82374 |

Note: This table presents, by industry: Black-Scholes value *BSOPM*, subjective value *Sub*, sentiment level *Sen*, average total compensation AvgCom, number of options issued AvgIss, and number of observations by individual Obs. AvgCom and AvgIss are reported in thousands. *Sen* is calculated using the European ESO formula (3.5) where the distribution of jump size follows $y=0$. Con. Dis., Con. Sta., Inf. Tec., and Tel. Ser. refer to Consumer Discretionary, Consumer Staples, Information Technology and Telecommunication Services, respectively.

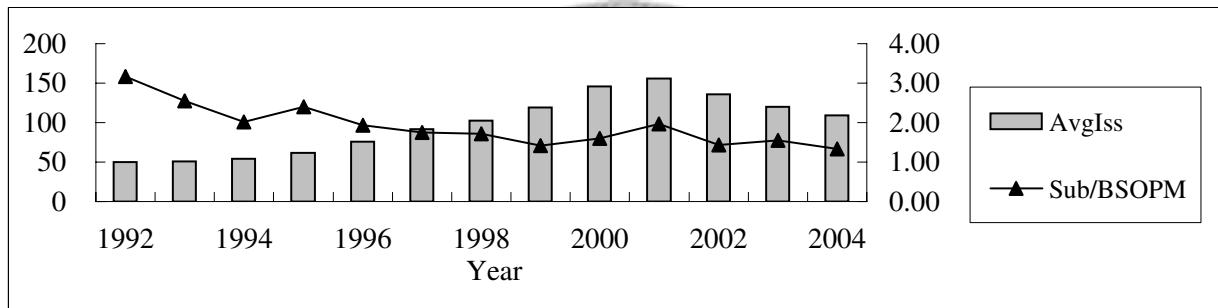
is not consistently present in this data, at least as regards mean values. Firms listed in the top 100 in fact make significantly lower risk adjusted returns in the year in which they are so listed. However, they enjoy substantially higher subjective value. This indicates that sentiment may generally be independent of performance but does significantly affect subjective value.

The accompanying time-series figure shows that relative subjective values are greater than one but relatively stable over time. In contrast, the number of issuances generally increases. The industry with the second highest subjective values (Financials) has a below average number of issuances. These observations highlight the importance of looking at pricing, rather than issuance alone, as high subjective values do not imply that ESOs will be a more popular financing tool.

Table 5.4: Difference Tests for Subjective Value and Sentiment

| | $R_{t-1} > 0$ | $R_{t-1} < 0$ | P-value | $Top = 1$ | $Top = 0$ | P-value |
|-----------------|---------------|---------------|---------|-----------|-----------|---------|
| mean(R_t) | 0.00042 | 0.00041 | 0.4493 | 0.00038 | 0.00059 | < .0001 |
| median(R_t) | 0.00031 | 0.00028 | 0.0176 | 0.00030 | 0.00044 | < .0001 |
| mean(Sub) | 21.4131 | 15.6336 | < .0001 | 24.7420 | 17.6169 | 0.0003 |
| median(Sub) | 14.1350 | 10.9487 | < .0001 | 17.4147 | 10.9794 | < .0001 |

Note: This table shows the mean and median values of R_t and Sub in each subsample where R_t is the CAPM alpha at time t. Top equals 1 if the firm is listed as a top 100 firm by Fortune magazine in a given year. The p-values measure the significance of difference tests.

Figure 5.2: Summary AvgIss and Sub/BSOPM by Year

AvgIss and Sub/BSOPM for each year are graphed in this figure. AvgIss, BSOPM and Sub are number of options issued, Black-Scholes value and subjective value, respectively. The y-axis of the histogram is on the left and that of the line chart is on the right.

5.4 Main Results

5.4.1 Regression results and variable sensitivities implications

We now shift our attention to the testable implications of our model, namely confirming the relations between key options variables and subjective value. Specifically, we apply the following regression equation:

$$Sub = Int + \beta_{\alpha}\alpha + \beta_{Sen}Sen + \beta_{Sok}Sok + \beta_{\tau}\tau + \beta_{Vol}Vol + \beta_{Div}Div + \varepsilon, \quad (5.1)$$

where Int is the intercept term and all variables are defined as before. Note that for all results presented here the calculation of significance is via clustered standard errors by firm, though OLS results are nearly identical.

First, we apply gross subjective value Sub as the dependent variable. The first three tests in Table 5.5 use CAPM risk-adjusted alpha from the year prior to option issuance R_{t-1} as a proxy for sentiment under the conjecture that those stocks which performed better in the previous year generate more positive sentiment prior to options being issued. Note that our model implies that only the risk-adjusted excess return should be priced since the market portion of the firm's return is eliminated via the risk-neutral measure. Bergman and Jenter (2007), in contrast, test the gross prior year return. Since a year's worth of data is required to calculate these alphas, the dataset is reduced to about 57,000 observations. We find that α is significantly negatively related to subjective value. This matches our intuition that, the larger the proportion of one's portfolio held in options, the less diversified the portfolio, and the less valuable the ESO. Sen , on the other hand, is positively related, significantly so. In other words, positive sentiment is associated with higher subjective value. Note that these results control for the usual options pricing factors. While Div is significantly negative related as expected, Sok and τ are not consistently significantly related, and Vol is negatively related. As explored more fully later, this last negative relation is quite telling and is consistent with our model as the sensitivity of subjective value to idiosyncratic risk is negative.

Further the data are split into two groups according to the sign of the product of R_t and Sen . A positive (negative) sign implies that the positive sentiment measure is (not) accompanied by strong performance. The positive case (insider), then, can be explained by non-sentiment related factors. The executive may have private inside information and hence be able to forecast future returns. He also has the ability to affect future returns so that optimism may be self-fulfilling. The negative case (true sentiment), on the other hand, has not such concern since it would imply that positive (negative) sentiment is

Table 5.5: Regression Results for Subjective Value

| | <i>Int</i> | α | <i>Sen</i> | <i>Sok</i> | τ | <i>Vol</i> | <i>Div</i> | Obs |
|-------------------------|------------|-----------|------------|------------|----------|------------|------------|-------|
| <i>Sen</i> = R_{t-1} | | | | | | | | |
| Coefficient | 1.8988 | -0.2126 | 0.0356 | 0.0361 | -0.2813 | -0.3983 | -0.0783 | 56602 |
| (p-value) | (< .0001) | (< .0001) | (< .0001) | (0.2476) | (0.0489) | (< .0001) | (< .0001) | |
| Insider | | | | | | | | |
| Coefficient | 1.8650 | -0.3001 | 0.0628 | 0.0860 | -0.2883 | -0.3409 | -0.0844 | 23826 |
| (p-value) | (< .0001) | (< .0001) | (< .0001) | (0.0401) | (0.1712) | (0.0004) | (0.0017) | |
| True Sentiment | | | | | | | | |
| Coefficient | 1.9071 | -0.1252 | 0.0100 | -0.0039 | -0.2631 | -0.4344 | -0.0904 | 21333 |
| (p-value) | (< .0001) | (< .0001) | (< .0001) | (0.8583) | (0.2858) | (< .0001) | (< .0001) | |
| <i>Sen</i> = <i>Top</i> | | | | | | | | |
| Coefficient | 1.5014 | -0.2244 | 0.0114 | 0.0802 | -0.0560 | -0.2318 | -0.0807 | 49090 |
| (p-value) | (< .0001) | (< .0001) | (0.0169) | (0.6641) | (0.7216) | (< .0001) | (< .0001) | |
| $Y = 0$ | | | | | | | | |
| Coefficient | 2.2013 | -0.4364 | 0.0076 | 0.0424 | -0.3265 | -0.3872 | -0.1012 | 82374 |
| (p-value) | (< .0001) | (< .0001) | (< .0001) | (0.1194) | (0.3041) | (< .0001) | (< .0001) | |
| Double Exp | | | | | | | | |
| Coefficient | 2.1671 | -0.4270 | 0.0017 | 0.0429 | -0.3056 | -0.3786 | -0.1005 | 82374 |
| (p-value) | (< .0001) | (< .0001) | (< .0001) | (0.1155) | (0.3310) | (< .0001) | (< .0001) | |
| Bivariate Con | | | | | | | | |
| Coefficient | 2.1799 | -0.4267 | 0.0021 | 0.0428 | -0.3075 | -0.3883 | -0.1023 | 82374 |
| (p-value) | (< .0001) | (< .0001) | (< .0001) | (0.1162) | (0.3284) | (< .0001) | (< .0001) | |

Note: This table presents the estimated coefficients from the following regressions:

$$Sub = Int + \beta_\alpha \alpha + \beta_{Sen} Sen + \beta_{Sok} Sok + \beta_\tau \tau + \beta_{Vol} Vol + \beta_{Div} Div + \varepsilon$$

where *Sub*, *Int*, α , *Sen*, *Sok*, τ , *Vol* and *Div* refer to the subjective value, intercept term, proportion of total wealth held in illiquid firm specific holdings, sentiment, ratio of stock price to exercise price, time to maturity, implied volatility, and dividend payout, respectively. In the first three tests, *Sen* = R_{t-1} , the CAPM alpha. We split the data into two groups according to the sign of the product of R_t and *Sen*. When *Sen* correctly forecasts the sign of the CAPM alpha for a given year, this is denoted as an "insider." When *Sen* and R_t do not match in sign, we denote this "true sentiment." In the fourth test, *Sen* is a dummy variable that takes value 1 if the firm is in Fortune's top 100 and 0 otherwise. In the next three tests, *Sen* is calculated from European ESO formula (3.5) with the distribution of jump size following $y=0$, a double exponential, and a bivariate constant jump model, respectively.

followed by poor (good) performance. As it turns out, similar results are obtained in both cases: sentiment is positively related to subjective value while α is negatively related, both significantly so. As a result, it is not likely that insider information explains whole sentiment effect on subjective value.

Next, the *Top* dummy is selected as a proxy for sentiment. Once again, sentiment is significantly positively related to subjective value while α is significantly negatively related. All other relations are as above.

We also back *Sen* out of European ESO formula (3.5) under the aforementioned three different jump size assumptions.⁶ Since our model itself determines the relation between subjective value and *Sen*, the purpose of these tests is simply to observe the other variable relations as well as the stability of the model to the specification of the jump process. Results are quite consistent across the three processes tested here. All other coefficients remain qualitatively as before with the coefficient of α , importantly, remaining significantly negative in all cases.

Finally, in order to more clearly test the difference in impact of sentiment for insider vs true sentiment events, we interact the event identification dummy with our sentiment proxy as follows:

$$Sub = Int + \beta_\alpha \alpha + \beta_{InSen} D_{In} Sen + \beta_{TSen} D_T Sen + \beta_{Sok} Sok + \beta_\tau \tau + \beta_{Vol} Vol + \beta_{Div} Div + \varepsilon. \quad (5.2)$$

All variables are defined as before and *Sen* is the previous-period CAPM alpha, also as before. D_{In} is a dummy variable that takes value 1 if the event is insider and 0 otherwise. By analogy, D_T takes value 1 if the event is true sentiment and 0 otherwise. The results appear as in Table 5.6. Note that, while sentiment increases subjectively value significantly

⁶Here, sentiment is estimated from European option formula. It can also be calculated from American option formula but more exhaustively computations. As we mentioned before, sentiment estimated from European and American ESO formulae have similar patterns. It may not affect the regression results much.

in both cases, the impact of sentiment when the event is likely to be an insider event is much larger. In other words, when strong prior performance reveals real information regarding future performance that may be known to managers, the impact on subjective value is strong. When prior performance proves not to be informative, the impact on subjective value is small. However, the impact is positive and significant in both cases.

Table 5.6: Regression Results for Insider vs True Sentiment Events

| | <i>Int</i> | α | $D_{In}Sen$ | D_TSen | <i>Sok</i> | τ | <i>Vol</i> | <i>Div</i> |
|------------------------|------------|-----------|-------------|-----------|------------|----------|------------|------------|
| <i>Sen</i> = R_{t-1} | | | | | | | | |
| Coefficient | 1.9070 | -0.2214 | 0.0372 | 0.0044 | 0.0343 | -0.2866 | -0.3872 | -0.0876 |
| (p-value) | (< .0001) | (< .0001) | (< .0001) | (< .0001) | (0.3769) | (0.0876) | (< .0001) | (< .0001) |

Note: This table presents regression results for insider vs true sentiment events where D_{In} is a dummy variable taking value 1 if the event is insider and 0 otherwise, D_T takes value 1 if it is true sentiment and 0 otherwise, and *Sen* is again defined as the CAPM alpha.

5.4.2 Normalized results

As a normalization, we re-run all tests using the quotient subjective value divided by Black-Scholes value. Results presented in the first three tests in Table 5.7 confirm key findings. Relative subjective value is increasing in *Sen* and decreasing in α , significantly so in both cases. That is, the more positive the sentiment the higher the ESO value, while the larger the illiquid holding, the lower the ESO value. However, while the direction of relations remains consistent for both insider and true sentiment subsets, statistical significance is weaker now in the case of the insider subset. *Vol* and *Div* are no longer reliably negatively related to subjective value, perhaps because the Black-Scholes value now appears in the quotient, negating effects. Interestingly, τ is significantly negatively related to subjective value since the longer one has to wait, the more the risk caused by under-diversification affects subjective valuation. *Sok* remains insignificant as before.

Main results are unchanged when *Top* is used as a proxy for sentiment, and results

Table 5.7: Regression Results for Normalized Subjective Value

| | <i>Int</i> | α | <i>Sen</i> | <i>Sok</i> | τ | <i>Vol</i> | <i>Div</i> | Obs |
|-------------------------|------------|-----------|------------|------------|-----------|------------|------------|-------|
| <i>Sen</i> = R_{t-1} | | | | | | | | |
| Coefficient | 2.4945 | -0.6085 | 0.0312 | 0.0183 | -1.1156 | 0.0542 | 0.1258 | 56602 |
| (p-value) | (< .0001) | (< .0001) | (0.0305) | (0.8771) | (0.0005) | (0.6842) | (0.0151) | |
| Insider | | | | | | | | |
| Coefficient | 2.4584 | -0.8531 | 0.0716 | 0.0469 | -1.0446 | 0.2450 | 0.0758 | 23826 |
| (p-value) | (0.0018) | (< .0001) | (0.1511) | (0.8566) | (0.1023) | (0.3911) | (0.4775) | |
| True Sentiment | | | | | | | | |
| Coefficient | 2.3315 | -0.3640 | 0.0014 | -0.0028 | -1.1155 | -0.0386 | 0.1880 | 21333 |
| (p-value) | (< .0001) | (< .0001) | (0.0668) | (0.9027) | (< .0001) | (0.1974) | (< .0001) | |
| <i>Sen</i> = <i>Top</i> | | | | | | | | |
| Coefficient | 2.4547 | -0.4581 | 0.0035 | -0.2289 | -1.0200 | 0.0983 | 0.1505 | 49066 |
| (p-value) | (< .0001) | (< .0001) | (0.0392) | (0.1010) | (< .0001) | (< .0001) | (< .0001) | |
| $Y = 0$ | | | | | | | | |
| Coefficient | 2.7797 | -0.7190 | 0.0132 | 0.0252 | -1.2932 | -0.0627 | 0.2569 | 82364 |
| (p-value) | (< .0001) | (< .0001) | (< .0001) | (0.3946) | (< .0001) | (0.0142) | (< .0001) | |
| Double Exp | | | | | | | | |
| Coefficient | 2.6516 | -0.6902 | 0.0026 | 0.0229 | -1.2425 | -0.0536 | 0.3091 | 82362 |
| (p-value) | (< .0001) | (< .0001) | (< .0001) | (0.3735) | (< .0001) | (0.0159) | (< .0001) | |
| Bivariate Con | | | | | | | | |
| Coefficient | 2.6719 | -0.6889 | 0.0033 | 0.0228 | -1.2461 | -0.0693 | 0.3062 | 82362 |
| (p-value) | (< .0001) | (< .0001) | (< .0001) | (0.3768) | (< .0001) | (0.0019) | (< .0001) | |

Note: This table presents the estimated coefficients from the following regressions:

$$Sub/BSOPM = Int + \beta_\alpha \alpha + \beta_{Sen} Sen + \beta_{Sok} Sok + \beta_\tau \tau + \beta_{Vol} Vol + \beta_{Div} Div + \varepsilon$$

where *Sub*, *BSOPM*, *Int*, α , *Sen*, *Sok*, τ , *Vol* and *Div* refer to the subjective value, Black-Scholes value, intercept term, proportion of total wealth held in illiquid firm specific holdings, sentiment, ratio of stock price to exercise price, time to maturity, implied volatility, and dividend payout, respectively. In the first three tests, *Sen* = R_{t-1} , the CAPM alpha. We split the data into two groups according to the sign of the product of R_t and *Sen*. When *Sen* correctly forecasts the sign of the CAPM alpha for a given year, this is denoted as an “insider.” When *Sen* and R_t do not match in sign, we denote this “true sentiment.” In the fourth test, *Sen* is a dummy variable that takes value 1 if the firm is in Fortune’s top 100 and 0 otherwise. In the next three tests, *Sen* is calculated from European ESO formula (3.5) with the distribution of jump size following $y=0$, a double exponential, and a bivariate constant jump model, respectively.

are not significantly impacted by change in jump model used, all findings qualitatively the same as without the normalization.

Finally, when testing the impact of insider vs true sentiment events, sentiment remains positively related to subjective value. However, in true sentiment events, the coefficient is indistinguishable from zero. Again, the implication is that insider events dominate the effect.

Table 5.8: Regression Results for Insider vs True Sentiment Events

| | <i>Int</i> | α | $D_{In}Sen$ | $D_T Sen$ | <i>Sok</i> | τ | <i>Vol</i> | <i>Div</i> |
|------------------------|------------|-----------|-------------|-----------|------------|----------|------------|------------|
| <i>Sen</i> = R_{t-1} | | | | | | | | |
| Coefficient | 2.4612 | -0.6541 | 0.0550 | 0.0004 | 0.0204 | -1.1028 | 0.1018 | 0.1181 |
| (p-value) | (< .0001) | (< .0001) | (0.0071) | (0.9062) | (0.8813) | (0.0048) | (0.5359) | (0.0730) |

Note: This table presents relative regression results for insider vs true sentiment events where D_{In} is a dummy variable taking value 1 if the event is insider and 0 otherwise, D_T takes value 1 if it is true sentiment and 0 otherwise, and Sen is again defined as the CAPM alpha.

5.4.3 Subjective value and risk

We now turn our attention to the sensitivity of subjective value to risk. While we note that our model implies a positive relation between total risk and subjective value, it further dictates that the sensitivity of subjective value to idiosyncratic risk is negative, a notion supported by our empirical findings. This indicates that increased levels of risk may negatively affect subjective value owing to the inability of executives to fully diversify their holdings. In contrast, the Black-Scholes as well as the majority of options pricing models prescribe no role to idiosyncratic risk, i.e. the sensitivity should be zero, and are generally not be able to capture the empirical finding that subjective value is negatively related to risk.

In applying the empirical data to the formulas for the sensitivities of subjective value to various forms of risk, our model does indeed generate a negative relation between firm

specific risk and subjective value, a finding that is consistent also with the empirical observations of Meulbroek (2001). This finding is particularly important as managers can easily affect the firm's idiosyncratic risk level through various moral hazard-related activities.

In Table 5.9, risk sensitivities are calculated, vegas, for all options issues in our dataset assuming there are no illiquid holdings (UV), i.e. $\alpha = 0$, and using our default value for α (V), with and without consideration of sentiment. The first two columns find as expected that the sensitivity with respect to total risk is positive, for both UV and V, regardless of whether sentiment is considered or not. This is true of all jump specifications. In every case, the sensitivity is higher when sentiment is not considered. Looking at the vegas with respect to jump frequency, UV(freq) can be either positive or negative depending on the jump specification, while V(freq) is always negative. Interestingly, UV is positive for the constant jump model but negative for the other two models, pointing out the importance of jump specification when stock holding constraint is not also considered. The magnitude of UV is always smaller than that of V.

Perhaps the most interesting factor affecting our subjective value in our model is idiosyncratic risk, for which the estimate is always negative and is significantly larger in magnitude than the other vegas. While the jump size vega also plays a role and is likewise always negative, the magnitude of this effect is much smaller. This finding highlights the role of idiosyncratic risk in our model and explains why the empirical sensitivity of subjective value to volatility is found to be negative, contrary to generally accepted moral hazard models which dictate that option compensation encourages risk taking. If agents are sufficiently under-diversified, the risk premium from taking on excess idiosyncratic risk offsets gains from convexity and discourages risk-taking behavior. The corresponding UVs for idiosyncratic and jump size risk are both zero as these do not play a role in determining market value when there are no under-diversified holdings. Also, the Vs are substantially more negative when sentiment is introduced, pointing out the sharply offsetting effects of positive sentiment and risk aversion in this model. Which piece dominates then depends

on the risk aversion parameter and α of the employee.

Table 5.9: Summary Statistics for Vega

| | Panel A: Y=0 | | | | | |
|--|--------------|----------|----------|---------|---------|---------|
| | UV(total) | V(total) | UV(freq) | V(freq) | V(idio) | V(size) |
| Without sentiment | | | | | | |
| Mean | 11.237 | 13.715 | 12.787 | -0.739 | -24.212 | -0.162 |
| Median | 9.173 | 11.684 | 10.159 | -0.143 | -19.611 | -0.077 |
| With sentiment | | | | | | |
| Mean | 5.968 | 7.646 | 16.486 | -11.222 | -41.638 | -0.415 |
| Median | 3.007 | 4.906 | 13.191 | -2.857 | -31.063 | -0.144 |
| Panel B: Double exponential jump model | | | | | | |
| | UV(total) | V(total) | UV(freq) | V(freq) | V(idio) | V(size) |
| Without sentiment | | | | | | |
| Mean | 13.456 | 16.791 | -1.039 | -0.564 | -25.877 | -0.017 |
| Median | 10.875 | 14.129 | -0.871 | -0.449 | -20.396 | -0.008 |
| With sentiment | | | | | | |
| Mean | 7.397 | 10.063 | -14.217 | -1.213 | -44.233 | -0.042 |
| Median | 4.173 | 7.041 | -1.823 | -0.806 | -32.466 | -0.014 |
| Panel C: Bivariate constant jump model | | | | | | |
| | UV(total) | V(total) | UV(freq) | V(freq) | V(idio) | V(size) |
| Without sentiment | | | | | | |
| Mean | 13.366 | 16.792 | -1.194 | -0.677 | -25.860 | -0.017 |
| Median | 10.879 | 14.131 | -1.015 | -0.542 | -20.382 | -0.008 |
| With sentiment | | | | | | |
| Mean | 7.396 | 10.060 | -15.984 | -1.407 | -44.231 | -0.042 |
| Median | 4.173 | 7.037 | -2.103 | -0.957 | -32.468 | -0.014 |

Note: This table presents test results for vega. In Panel A, B and C, the distribution of jump sizes are zero jump, double exponential jump and bivariate constant jump, respectively. UV(total) and UV(freq) are total risk and jump frequency risk vegas under our model when all holdings are liquid. V(total), V(idio), V(freq) and V(size) refer to total risk vega, idiosyncratic risk vega, jump frequency risk vega and jump size risk vega, respectively.

5.5 Robustness Checks

Numerous robustness checks are executed in this study. Unless otherwise noted, none yield appreciable differences, and our conclusions are unaffected. Numerical results and testing specifics are available upon request.

5.5.1 Estimation of subjective values

To aggregate executives with similar compensation characteristics, we apply a K-means clustering method. However, all tests are re-done using a simpler, split-sample methodology, determining groups simply controlling for industry, rank, year, size, immediate exercise value, and α . Then the calculation of subjective value is based on these groupings, and there is no significant qualitative differences.

While it is intuitively clear that one should never value an option at less than 0, a small number of negative implied values are implied in our estimation process. We try re-running all tests allowing for negative implied subjective values assuming that α is simply $\alpha = BSOPM / (Salary + Bonus + Other + BSOPM)$ where $BSOPM$ is the Black-Scholes value of the options and $Other$ is the value of other annual compensation. Whether negative implied values are equated to zero (the default calibration), allowed to be negative, or entirely removed from the dataset, none of our findings are affected.

We also try estimating subjective values using an iterated method, solving for a fixed-point α^* that uses subjective value as an input to α and vice versa. Specifically, first calculate:

$$\alpha^* = \frac{Option}{Salary + Bonus + Other + Option}$$

with $Option$ initialized as the Black-Scholes value. Then, calculate subjective value by

our ESO formula using α^* . Then, re-calculate of α^* using this candidate subjective value and iterate until the differences between α 's and subjective values are both less than 10^{-5} . Using these new subjective values, but with α removed, the regression results are as before that Sen is always significantly positively related to subjective value.

5.5.2 Sub-sample tests and outlier controls

As a control for outliers, standardized residuals approach is used to remove outliers from our dataset and re-run all tests. Alternatively, to account for differing variable magnitudes, we also try normalizing each option pricing variable by its sample mean (centering all variables about 1). There is no significant qualitative differences in either case. While point estimates vary, Sen is always positively related to subjective value and α is negatively related, and significantly so. Again, positive sentiment increases ESO value while having a large illiquid holding decreases it.

All tests are also re-run with both dependent and independent variables normalized by industry average. To be even more thorough with regard to industry effects, we also redo all tests separately for each industry. Again, no qualitative differences are noted. In every industry, positive sentiment increases value while increase α decreases it. We conclude that industry effects are minimal.

As aforementioned, results requiring the calculation of previous year's CAPM alpha utilize a smaller sample. All results are re-run by using only this same reduced sample. All results are qualitatively identical to those found when utilizing the full dataset.

Finally, we also test the sub-sample for positive Sen . When calculated using our model, Sen is predominantly positive (more than 80% of data points). When applying prior year returns, that number is only about 60%. In all cases, α remains negatively related to subjective value and Sen is positively related to subjective value.

5.5.3 Test and model re-specifications

In our main tests, $Sub/BSOPM$ is calculated as a normalized subjective value. We repeat all tests using the arithmetic difference $Sub - BSOPM$. This method lacks magnitude normalization but allows for positive or negative subjective values. Results, however, are qualitatively identical.

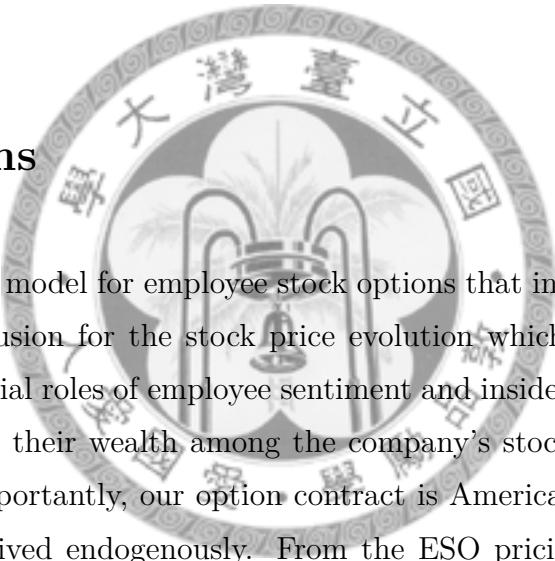
Also, our model can be amended to include jumps in the market portfolio. The resulting valuation formula and partials do not change the intuition discussed in this study, though solutions are decidedly more complicated. Derivations results are available upon request.



Chapter 6

Conclusions and Future Work

6.1 Conclusions



This study extends a model for employee stock options that incorporates restriction of the options, a jump diffusion for the stock price evolution which includes various jump processes, and the potential roles of employee sentiment and insider information in a world where employees balance their wealth among the company's stock, the market portfolio, and a risk-free asset. Importantly, our option contract is American type and the optimal exercise boundary is derived endogenously. From the ESO pricing formula, we can not only estimate the subjective values but also study the exercise policies.

It is the first study to apply empirical data to calculate the subjective value placed on ESOs implied by compensation data. Specifically, using data provided by Compustat, executives are grouped by using a K-means hierarchical method based on a number of firm and individual criteria. By assuming that all executives in the same cluster receive the same total compensation, a notion that relies on the existence of competitive labor markets, we then back out the valuation placed by each executive on his respective ESO. These groups include consideration of non-option compensation, rank, industry, year, firm

size, and immediate exercise value. Though the extant literature predicts that employees should discount the value of their options, we find that executives in fact value their options more highly than implied by Black-Scholes, applying an average premium of 48%. As such, the cost of issuance for the firm is vastly lower than the benefit perceived by employees, suggesting that ESO compensation should be an even larger part of executive compensation.

We then relate subjective value to sentiment levels and generate the novel finding that executives must expect their firm's risk-adjusted returns to outpace that predicted by the market by 12% in order to justify the subjective value placed on ESOs. This expectation may be the result of private information regarding the growth prospects of the firm. Moreover, in controlling for the sign of sentiment and resulting returns, even when the former does not match the latter, subjective value is positive related to ex-ante sentiment. Also, given the magnitude of return and the observation that options account for an enormous part of total compensation, it is unlikely that executives project such a large sentiment premium for signaling purposes alone.

Testing subjective value and its relation to pertinent variables, subjective value is negatively related to the proportion of wealth held in illiquid firm specific holdings and positively related sentiment. In other words, the larger the illiquid ESO position is, the larger, the discount risk aversion prescribes and the lower the subjective value implied in the compensation package. On the other hand, the more positive the employee's view of future risk-adjusted returns, the more valuable the ESO. This is robust regardless of if this view is likely to be generated by inside information or pure sentiment. Though both factors are significantly and positively related to subjective value, the impact of the former appears to quite a bit larger. In addition to previous year's CAPM alpha as a proxy for positive sentiment, this study also considers inclusion on Fortune's list of 100 best firms and finds the same results. We confirm that specification of the jump model does not affect results.

Interestingly, subjective value may be negatively related to risk as the inability of executives to fully diversify their holdings may lead to risk premia that outweigh the value placed on risk by the convexity of options payouts. Note that this relation is particularly negative with regard to idiosyncratic risk and are empirically also negative for risk associated with both jump frequency and size. Since these aspects of return are precisely those that may be most directly controlled by executives, traditional moral hazard arguments relating solely to the convexity of the options payout may not hold.

We conclude that employee sentiment is a necessary consideration when issuing options and that executives may be substantially over-valuing ESOs because of it. Firms that have performed well should issue more options, and firms should place effort and attention into maintaining positive sentiment within a firm, especially when offering ESO compensation. Moreover, ESOs may not generate the sort of risk-taking behavior implied by more traditional options pricing formulas owing to stock holding constraint. The more illiquid the ESO, the larger the proportion of total wealth ESOs represent, the less likely employees are to engage in risk-taking behavior. On the one hand, options are meant to incentive effort and value-creating risk taking. However, managers holding illiquid ESOs are discouraged from taking firm-specific risk as it may erode ESO value. On the other hand, options may also lead to moral hazard since equity holders are insulated from downside risk in the case of bankruptcy. Once, the illiquid nature of ESOs discourages value-destroying idiosyncratic risk taking, acting as a protection against moral hazard.

All in all, this study provides evidence that the subjective value of an ESO departs significantly from the Black-Scholes value, and offers a framework with which to investigate these concerns and opportunities.

6.2 Future Work

Subjective ESO values and exercise decisions are closely related. While subjective ESO values are difficult to observe, it can be perceived from exercise behavior of each employee. In this study, the exercise boundary is endogenously derived by simulation. More specifically, it is the minimum stock price such that the option value equals its intrinsic value for each time. In the future, we want to empirically test the relationship between the exercise decision and considering factors, especially for risk and sentiment.

Firms increasingly grant nontraditional employee stock options to link stock price performance and managerial wealth and provide greater incentives to employees. While this study focuses on the traditional employee stock option, the main intuition can be involved in nontraditional ESOs. Premium stock option, performance-vested stock option, repriceable stock option, purchased stock option, reload stock option and index stock option are the objects of future study. We plan to derive the option formulae and compare the value, incentive effect and cost per unit of subjective incentive across the nontraditional ESOs and the traditional ones. This future study provides firm a proper compensation vehicle according to its firm characteristics.

Appendix 1: Optimal Consumption and Portfolio Weights

Let W and C be the wealth and consumption processes, then the optimal portfolio selection problem becomes

$$\begin{cases} J[W(t), t] = \underset{\{C, w_s, w_m, w_b\}}{\text{Max}} E_t \int_t^T e^{-\rho s} U(C(s)) ds + B[W(T), T], \\ \text{s.t. } J[W(T), T] = B[W(T), T], \\ w_s \geq \alpha, w_s + w_m + w_b = 1, \end{cases} \quad (\text{A.1})$$

where $J[W(t), t]$ is the employee's total utility at time t , the employee's utility function $U(C) = \frac{C^\gamma}{\gamma}$ and $B[W(T), T]$ is the bequest function at the date of termination T .

The derived utility function and the optimal consumption and portfolio choices are the solution to

$$\begin{aligned} 0 &= \underset{\{C, w_s, w_m, w_b\}}{\max} e^{-\rho t} U(C(t)) + \frac{1}{2} \mathbf{w}' \Omega \mathbf{w} W^2 J_{WW} + ([r + \mathbf{w}'(\mu - r\mathbf{1})]W(t) - C(t))J_W + J_t \\ &\equiv \underset{\{C, w_s, w_m, w_b\}}{\max} \phi \end{aligned} \quad (\text{A.2})$$

where $\mathbf{w} = (w_s \ w_m)'$, $\mathbf{1} = (1, 1)'$ and

$$\begin{aligned} \phi &= \frac{1}{\gamma} e^{-\rho t} C^\gamma + \frac{1}{2} [(\beta^2 \sigma_m^2 + \nu^2 + \lambda k_2) w_s^2 + 2\beta \sigma_m^2 w_s w_m + \sigma_m^2 w_m^2] W^2 J_{WW} \\ &\quad + \{[r + w_s(\mu_s - r) + w_m(\mu_m - r)]W - C\} J_W + J_t. \end{aligned}$$

By using the Kuhn-Tucker method, the necessary conditions are:

$$e^{-\rho t} C^{\gamma-1} - J_W = 0,$$

$$(\beta \sigma_m^2 w_s + \sigma_m^2 w_m) W^2 J_{WW} + (\mu_m - r) W J_W = 0,$$

$$\{(\beta^2 \sigma_m^2 + \nu^2 + \lambda k_2) w_s + \beta \sigma_m^2 w_m\} W^2 J_{WW} + (\mu_s - r) W J_W + \tilde{\lambda} = 0,$$

$$\tilde{\lambda} \geq 0, \quad w_s \geq \alpha, \quad \tilde{\lambda}(w_s - \alpha) = 0.$$

Implying

$$\begin{aligned}
C^* &= (e^{\rho t} J_W)^{\frac{1}{\gamma-1}}, \\
w_m^* &= -\frac{\mu_m - r}{\sigma_m^2 W} \frac{J_W}{J_{WW}} - \beta w_s^*, \\
w_m^* &= -\frac{\mu_m - r}{\sigma_m^2 W} \frac{J_W}{J_{WW}} - \frac{\beta^2 \sigma_m^2 + \nu^2 + \lambda k_2}{\beta \sigma_m^2} w_s^* - \frac{\tilde{\lambda}}{\beta \sigma_m^2 W^2 J_{WW}}, \\
w_s &\geq \alpha, \quad \tilde{\lambda} \geq 0, \quad \tilde{\lambda}(w_s - \alpha) = 0.
\end{aligned}$$

Hence

$$w_s^* = \alpha, \quad w_m^* = -\frac{\mu_m - r}{\sigma_m^2 W} \frac{J_W}{J_{WW}} - \beta w_s^*, \quad C^* = (e^{\rho t} J_W)^{\frac{1}{\gamma-1}}.$$

The trivial solution $J[W(t), t] = b(t)e^{-\rho t \frac{W\gamma}{\gamma}}$, satisfies equation (A.2). From $\phi(C^*, w_s^*, w_m^*) = 0$, we get

$$b(t) = \left\{ \frac{1 + (H[b(T)]^{\frac{1}{1-\gamma}} - 1)e^{H(t-T)}}{H} \right\}^{1-\gamma},$$

where

$$H = \frac{\gamma}{1-\gamma} \left[\frac{\rho}{\gamma} - r - \frac{1}{2} \frac{(\mu_m - r)^2}{(1-\gamma)\sigma_m^2} + \frac{1}{2}(1-\gamma)(\nu^2 + \lambda k_2)\alpha^2 \right].$$

Therefore

$$C^* = [b(t)]^{\frac{-1}{1-\gamma}} W, \quad w_s^* = \alpha, \quad w_m^* = \frac{\mu_m - r}{(1-\gamma)\sigma_m^2} - \alpha\beta, \quad w_b^* = 1 - \frac{\mu_m - r}{(1-\gamma)\sigma_m^2} - \alpha(1-\beta).$$

□

Appendix 2: Derivation of Risk-Neutral Probability P^*

The employee's wealth process is defined as

$$dW = (w_s \frac{dS}{S} + w_m \frac{dM}{M} + w_b \frac{dB}{B})[W(t-) - C(t-)dt] - C(t-)dt.$$

Then the evolution of the wealth process is

$$\begin{aligned}\frac{dW}{W} &= [r + w_s(\mu_s - r - \lambda k) + w_m(\mu_m - r) - [b(t)]^{\frac{-1}{1-\gamma}}]dt \\ &\quad + (w_s\sigma_s + w_m\sigma_m)dW_m + w_s\nu dW_s + w_s(Y-1)dN_t.\end{aligned}$$

By Ito's formula for jump process, we have

$$\begin{aligned}dJ_W[W(t), t] &= J_{WW}dW^c + J_{Wt}dt + \frac{1}{2}J_{WWW}(dW^c)^2 + \{J_W[W(t), t] - J_W[W(t-), t-]\}dN_t \\ &= (\gamma - 1)J_W \frac{dW^c}{W} + \left[\frac{b'(t)}{b(t)} - \rho \right] J_W dt + \frac{1}{2}(\gamma - 1)(\gamma - 2)J_W \left(\frac{dW^c}{W} \right)^2 \\ &\quad + \{J_W[W(t), t] - J_W[W(t-), t-]\}dN_t.\end{aligned}$$

Where

$$\begin{aligned}J[W(t), t] &= b(t)e^{-\rho t} \frac{W^\gamma}{\gamma}, \\ J_W &= b(t)e^{-\rho t} W^{\gamma-1}, \\ \frac{b'(t)}{b(t)} - \rho &= (1 - \gamma) \left\{ r + \frac{(\mu_m - r)^2}{2(1 - \gamma)\sigma_m^2} + \frac{1}{2}\gamma(\nu^2 + \lambda k_2)\alpha^2 - [b(t)]^{\frac{1}{\gamma-1}} \right\} - r - \frac{(\mu_m - r)^2}{2(1 - \gamma)\sigma_m^2}, \\ \frac{dW^c}{W} &= [r + w_s(\mu_s - r - \lambda k) + w_m(\mu_m - r) - [b(t)]^{\frac{-1}{1-\gamma}}]dt \\ &\quad + (w_s\sigma_s + w_m\sigma_m)dW_m + w_s\nu dW_s.\end{aligned}$$

If jump occurs at time t

$$\begin{aligned}W(t) &= [\alpha(Y-1) + 1]W(t-), \\ J_W[W(t), t] &= [\alpha(Y-1) + 1]^{\gamma-1}J_W[W(t-), t-].\end{aligned}$$

Therefore

$$\frac{dJ_W}{J_W} = -\hat{r}dt + \hat{\sigma}dW_m + \hat{\nu}dW_s + \{\hat{Y} - 1\}dN_t,$$

where

$$\begin{aligned}\hat{r} &= r - (1 - \gamma)[\alpha^2\nu^2 + \frac{1}{2}\alpha^2\gamma\lambda k_2 + \alpha\lambda k], \\ \hat{\sigma} &= -\frac{\mu_m - r}{\sigma_m}, \\ \hat{\nu} &= -(1 - \gamma)\alpha\nu, \\ \hat{Y} &= [\alpha(Y - 1) + 1]^{\gamma-1},\end{aligned}$$

then

$$J_W(t) = \exp\{-[\hat{r} - \frac{1}{2}\hat{\sigma}^2 - \frac{1}{2}\hat{\nu}^2]t + \hat{\sigma}W_m(t) + \hat{\nu}W_s(t)\} \prod_{i=0}^{N(t)} \hat{Y}_i.$$

Let $B(t, T)$ be the price of a zero coupon bond with maturity T , then

$$\begin{aligned}B(t, T) &= E\left\{ \frac{J_W[W(T), T]}{J_W[W(t), t]} B(T, T) | \mathcal{F}_t \right\} \\ &= E\left\{ \exp\{-[\hat{r} - \frac{1}{2}\hat{\sigma}^2 - \frac{1}{2}\hat{\nu}^2]\tau + \hat{\sigma}W_m(\tau) + \hat{\nu}W_s(\tau)\} \prod_{i=0}^{N(\tau)} \hat{Y}_i \right\} \\ &= \exp\{-[\hat{r} + \lambda(\xi - 1)]\tau\},\end{aligned}$$

where

$$\tau = T - t, \xi = E(\hat{Y}) = E\{[\alpha(Y - 1) + 1]^{\gamma-1}\}.$$

The bond yield

$$\begin{aligned}r^* &\equiv -\frac{1}{T-t} \ln B(t, T) \\ &= r - (1 - \gamma)[\alpha\lambda k + \frac{1}{2}\gamma\lambda k_2\alpha^2 + \alpha^2\nu^2] - \lambda(\xi - 1).\end{aligned}$$

Let

$$\begin{aligned}Z(t) &\equiv e^{r^*t} J_W \\ &= \exp\{-[\frac{1}{2}\hat{\sigma}^2 - \frac{1}{2}\hat{\nu}^2 - \lambda(\xi - 1)]t + \hat{\sigma}W_m(t) + \hat{\nu}W_s(t)\} \prod_{i=0}^{N(t)} \hat{Y}_i,\end{aligned}$$

then $Z(t)$ is a martingale under P , and we have

$$\frac{J_W[W(T), T]}{J_W[W(t), t]} = e^{-r^*(T-t)} \frac{Z(T)}{Z(t)}.$$

The rational equilibrium value of the ESO $F(S, t)$ satisfies the Euler equation,

$$F(S, t) = E_t \left\{ \frac{J_W[W(T), T]}{J_W[W(t), t]} F(S, T) \right\} = e^{-r^*(T-t)} E_t^*[F(S, T)],$$

where $\frac{dP^*}{dP} = \frac{Z(t)}{Z(0)}$, and E_t^* is the expectation under P^* and information at time t . Under the probability measure P^* , the processes $W_m^* = W_m - \hat{\sigma}t$ and $W_s^* = W_s - \hat{\nu}t$ are Brownian motions, N_t is a Poisson process with intensity $\lambda^* = \lambda\xi$ and the jump sizes follow density $f_Y^*(y)$,

$$f_Y^*(y) = \frac{1}{\xi} [\alpha(y-1) + 1]^{\gamma-1} f_Y(y).$$

Therefore

$$\begin{aligned} \frac{dS}{S} &= (\mu_s - d - \lambda k)dt + \sigma_s dW_m + \nu dW_s + (Y-1)dN_t \\ &= [r - d - (1-\gamma)\alpha\nu^2 - \lambda k]dt + \sigma_s dW_m^* + \nu dW_s^* + (Y-1)dN_t \\ &\equiv [r^* - d^* - \lambda^*(\xi^* - 1)]dt + \sigma_N dW_t^* + (Y-1)dN_t, \end{aligned}$$

where

$$\begin{aligned} r^* &= r - (1-\gamma)[\alpha\lambda k + \frac{1}{2}\gamma\lambda k_2\alpha^2 + \alpha^2\nu^2] - \lambda(\xi-1), \\ d^* &= d - (1-\gamma)[\alpha\lambda k + \frac{1}{2}\gamma\lambda k_2\alpha^2 - (1-\alpha)\alpha\nu^2] - \lambda(\xi-1) + \lambda k - \lambda^*(\xi^* - 1), \\ \sigma_N^2 &= \sigma_s^2 + \nu^2, \\ \sigma_N W_t^* &= \sigma_s W_m^* + \nu W_s^*. \end{aligned}$$

In other words,

$$S_t = S_0 \exp\{[r^* - d^* - \lambda^*(\xi^* - 1) - \frac{1}{2}\sigma_N^2]t + \sigma_N W_t^*\} \prod_{i=0}^{N_t} Y_i.$$

□

Appendix 3: Valuation of European ESO

The option price at time t is

$$\begin{aligned}
F(S, t) &= e^{-r^*(T-t)} E_t^* \{[S_T - K]^+\} \\
&= e^{-r^*\tau} E_t^* \{ \{S_t \exp[(r^* - d^* - \lambda^*(\xi^* - 1) - \frac{1}{2}\sigma_N^2)\tau + \sigma_N(W_T^* - W_t^*)] \\
&\quad \times \prod_{i=N_t}^{N_T} Y_i - K\}^+ \} \\
&= S_t E^* \{ \exp[-d^* - \lambda^*(\xi^* - 1) - \frac{1}{2}\sigma_N^2\tau + \sigma_N W_\tau^*] \times \prod_{i=0}^{N_\tau} Y_i I_{(W_\tau^* \geq a_1)} \} \\
&\quad - K e^{-r^*\tau} E^* \{ I_{(W_\tau^* \geq a_1)} \} \\
&= S_t e^{-[d^* + \lambda^*(\xi^* - 1)]\tau} E^* \{ E^* \{ \exp[-\frac{1}{2}\sigma_N^2\tau + \sigma_N W_\tau^*] \prod_{i=0}^{N_\tau} Y_i I_{(W_\tau^* \geq a_1)} | \prod_{i=0}^{N_\tau} Y_i \} \} \\
&\quad - K e^{-r^*\tau} E^* \{ E^* \{ I_{(W_\tau^* \geq a_1)} | \prod_{i=0}^{N_\tau} Y_i \} \} \\
&= S_t e^{-[d^* + \lambda^*(\xi^* - 1)]\tau} E^* \{ \prod_{i=0}^{N_\tau} Y_i \Phi(-\frac{a_1 - \sigma_N \tau}{\sqrt{\tau}}) \} - K e^{-r^*\tau} E^* \{ \Phi(-\frac{a_1}{\sqrt{\tau}}) \} \\
&= \sum_{j=0}^{\infty} \frac{(\lambda \xi \tau)^j e^{-\lambda \xi \tau}}{j!} \left\{ S_t e^{-[d^* + \lambda^*(\xi^* - 1)]\tau} E^* \left[\prod_{i=0}^j Y_i \Phi(d_1^*) \right] - K e^{-r^*\tau} E^* [\Phi(d_2^*)] \right\}
\end{aligned}$$

where

$$d_1^* = \frac{\ln[(S_t \prod_{i=0}^j Y_i)/K] + [r^* - d^* - \lambda^*(\xi^* - 1) + \frac{1}{2}\sigma_N^2]\tau}{\sigma_N \sqrt{\tau}}, d_2^* = d_1^* - \sigma_N \sqrt{\tau}, a_1 = -d_2^* \sqrt{\tau}.$$

□

Appendix 4: Valuation of Finite Horizon American ESO

We will derive the valuation formula for the American call ESO exercisable at n time

instants by backward induction. At time t_{n-1} , $C_A(S_{n-1}, \Delta t) = C_E(S_{n-1}, \Delta t)$. The exercise boundary is S_{n-1}^* such that $S_{n-1}^* - K = C_A(S_{n-1}^*, \Delta t)$. At time t_{n-2} ,

$$\begin{aligned} & C_A(S_{n-2}, 2\Delta t) \\ &= e^{-r^*\Delta t} E_{t_{n-2}}^* \{(S_{n-1} - K) I_{\{S_{n-1} \geq S_{n-1}^*\}}\} + e^{-r^*\Delta t} E_{t_{n-2}}^* \{C_A(S_{n-1}, \Delta t) I_{\{S_{n-1} < S_{n-1}^*\}}\} \\ &= C_E(S_{n-2}, 2\Delta t) + e^{-r^*\Delta t} E_{t_{n-2}}^* \{[S_{n-1}(1 - e^{-d^*\Delta t}) - K(1 - e^{-r^*\Delta t})] I_{\{S_{n-1} \geq S_{n-1}^*\}}\} \\ &\quad - e^{-r^*2\Delta t} E_{t_{n-2}}^* \{[C_A(S_T, 0) - (S_T - K)] I_{\{S_{n-1} \geq S_{n-1}^*\}} I_{\{S_T < K\}}\}. \end{aligned}$$

The exercise boundary is S_{n-2}^* such that $S_{n-2}^* - K = C_A(S_{n-2}^*, 2\Delta t)$.

Suppose that the value of the American ESO at time t_m , for $m < n - 2$, can be expressed as

$$\begin{aligned} & C_A(S_m, (n - m)\Delta t) \\ &= C_E(S_m, (n - m)\Delta t) + \sum_{\ell=1}^{n-m-1} e^{-r^*\ell\Delta t} E_{t_m}^* \{[S_{m+\ell}(1 - e^{-d^*\Delta t}) - K(1 - e^{-r^*\Delta t})] I_{\{S_{m+\ell} \geq S_{m+\ell}^*\}}\} \\ &\quad - \sum_{j=2}^{n-m} e^{-r^*j\Delta t} E_{t_m}^* \{[C_A(S_{m+j}, (n - m - j)\Delta t) - (S_{m+j} - K)] I_{\{S_{m+j-1} \geq S_{m+j-1}^*\}} I_{\{S_{m+j} < S_{m+j}^*\}}\}. \end{aligned}$$

By induction, we consider the case at time t_{m-1} ,

$$\begin{aligned} & C_A(S_{m-1}, (n - m + 1)\Delta t) \\ &= e^{-r^*\Delta t} E_{t_{m-1}}^* \{(S_m - K) I_{\{S_m \geq S_m^*\}}\} + e^{-r^*\Delta t} E_{t_{m-1}}^* \{C_A(S_m, (n - m)\Delta t) I_{\{S_m < S_m^*\}}\}. \end{aligned} \tag{A.3}$$

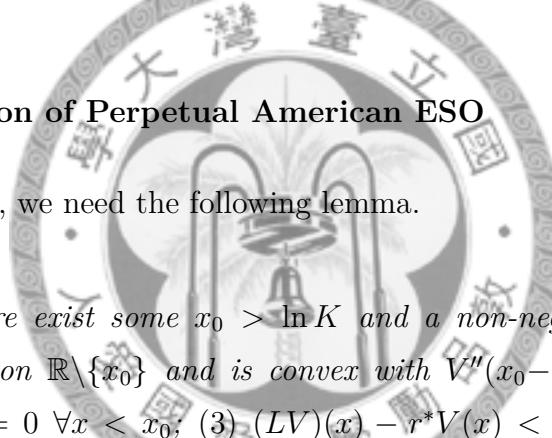
Note that the first term in (A.3)

$$\begin{aligned} & e^{-r^*\Delta t} E_{t_{m-1}}^* \{(S_m - K) I_{\{S_m \geq S_m^*\}}\} \\ &= e^{-r^*(n-m+1)\Delta t} E_{t_{m-1}}^* \{(S_T - K) I_{\{S_T \geq S_T^*\}} I_{\{S_m \geq S_m^*\}}\} \\ &\quad + \sum_{\ell=1}^{n-m} e^{-r^*\ell\Delta t} E_{t_{m-1}}^* \{[S_{m-1+\ell}(1 - e^{-d^*\Delta t}) - K(1 - e^{-r^*\Delta t})] I_{\{S_{m-1+\ell} \geq S_{m-1+\ell}^*\}} I_{\{S_m \geq S_m^*\}}\} \\ &\quad - \sum_{j=2}^{n-m+1} e^{-r^*j\Delta t} E_{t_{m-1}}^* \{[C_A(S_{m-1+j}, (n - m - j + 1)\Delta t) - (S_{m-1+j} - K)] \\ &\quad \times I_{\{S_{m-1+j-1} \geq S_{m-1+j-1}^*\}} I_{\{S_{m-1+j} < S_{m-1+j}^*\}} I_{\{S_m \geq S_m^*\}}\}. \end{aligned}$$

By induction, the second term in (A.3) is

$$\begin{aligned}
& e^{-r^* \Delta t} E_{t_{m-1}}^* \{C_A(S_m, (n-m)\Delta t) I_{\{S_m < S_m^*\}}\} \\
= & e^{-r^* \Delta t} E_{t_{m-1}}^* \{C_E(S_m, (n-m)\Delta t) I_{\{S_m < S_m^*\}}\} \\
& + \sum_{\ell=1}^{n-m-1} e^{-r^*(\ell+1)\Delta t} E_{t_{m-1}}^* \{[S_{m+\ell}(1 - e^{-d^* \Delta t}) - K(1 - e^{-r^* \Delta t})] I_{\{S_{m+\ell} \geq S_{m+\ell}^*\}} I_{\{S_m < S_m^*\}}\} \\
& - \sum_{j=2}^{n-m} e^{-r^*(j+1)\Delta t} E_{t_{m-1}}^* \{[C_A(S_{m+j}, (n-m-j)\Delta t) - (S_{m+j} - K)] \\
& \quad \times I_{\{S_{m+j-1} \geq S_{m+j-1}^*\}} I_{\{S_{m+j} < S_{m+j}^*\}} I_{\{S_m < S_m^*\}}\}.
\end{aligned}$$

Hence, we prove that the result holds for $t = t_{m-1}$, and complete the whole proof. \square



Appendix 5: Valuation of Perpetual American ESO

To prove Theorem 2, we need the following lemma.

Lemma 1 Suppose there exist some $x_0 > \ln K$ and a non-negative C^1 function $V(x)$ such that (1) V is C^2 on $\mathbb{R} \setminus \{x_0\}$ and is convex with $V''(x_0-)$ and $V''(x_0+)$ existing; (2) $(LV)(x) - r^*V(x) = 0 \forall x < x_0$; (3) $(LV)(x) - r^*V(x) < 0 \forall x > x_0$; (4) $V(x) > (e^x - K)^+ \forall x < x_0$; (5) $V(x) = (e^x - K)^+ \forall x \geq x_0$; (6) there exists a random variable Z with $E^*(Z) < \infty$ such that $e^{-r(t \wedge \tau \wedge \tau^*)} V(X_{t \wedge \tau \wedge \tau^*} + x) \leq Z$, for any $t \geq 0, x$ and any stopping time τ . Then the option price $\psi(S_0) = V(\ln(S_0))$ and the optimal stopping time is given by $\tau^* = \inf\{t \geq 0 : S_t \geq e^{x_0}\}$. Here the infinitesimal generator L is defined as

$$(LV)(x) := \frac{1}{2}\sigma^2 V''(x) + [r^* - d^* - \frac{1}{2}\sigma^2 - \lambda^*(\xi^* - 1)]V'(x) + \lambda^* \int_{-\infty}^{\infty} [V(x+u) - V(u)]f_U^*(u)du.$$

Since the proof follows an argument similar to that in Mordecki (1999) and Kou and Wang (2004), it is omitted.

Let $x = \ln v$, $x_0 = \ln v_0$, then

$$V(x) = \begin{cases} e^x - K, & x \geq x_0, \\ Ae^{\beta_{1,r^*}x} + Be^{\beta_{2,r^*}x}, & x < x_0. \end{cases}$$

To prove Theorem 2, we only need to check conditions in Lemma 1 hold. Conditions, 1, 4, and 5 are easily to verify. Condition 6 follows from Mordecki (1999). Therefore, we only need to check conditions 2 and 3 hold. For notation simplicity, we shall write $\beta_1 = \beta_{1,r^*}$, and $\beta_2 = \beta_{2,r^*}$.

For $x < x_0$,

$$\begin{aligned} & \int_{-\infty}^{\infty} V(x+u) dF_U^*(u) \\ &= \int_{-\infty}^0 [Ae^{\beta_1(x+u)} + Be^{\beta_2(x+u)}] q\eta_2 e^{\eta_2 u} du \\ & \quad + \int_0^{x_0-x} [Ae^{\beta_1(x+u)} + Be^{\beta_2(x+u)}] p\eta_1 e^{-\eta_1 u} du + \int_{x_0-x}^{\infty} (e^{x+u} - K) p\eta_1 e^{-\eta_1 u} du \\ &= pe^{-\eta_1(x_0-x)} \left(\frac{\eta_1 e^{x_0}}{\eta_1 - 1} - K \right) + \frac{p\eta_1 A}{\eta_1 - \beta_1} [e^{\beta_1 x} - e^{-(x_0-x)\eta_1 + \beta_1 x_0}] \\ & \quad + \frac{p\eta_1 B}{\beta_2 - \eta_1} [e^{-\eta_1(x_0-x) + \beta_2 x_0} - e^{\beta_2 x}] + A \frac{q\eta_2 e^{\beta_1 x}}{\beta_1 + \eta_2} + B \frac{q\eta_2 e^{\beta_2 x}}{\beta_2 + \eta_2}. \end{aligned}$$

Then

$$\begin{aligned} & (LV)(x) - r^* V(x) \\ &= Ae^{\beta_1 x} \left\{ \frac{1}{2} \sigma^2 \beta_1^2 + \mu^* \beta_1 + \lambda^* \left(\frac{p\eta_1}{\eta_1 - \beta_1} + \frac{q\eta_2}{\eta_2 + \beta_1} - 1 \right) - r^* \right\} \\ & \quad + Be^{\beta_2 x} \left\{ \frac{1}{2} \sigma^2 \beta_2^2 + \mu^* \beta_2 + \lambda^* \left(\frac{p\eta_1}{\eta_1 - \beta_2} + \frac{q\eta_2}{\eta_2 + \beta_2} - 1 \right) - r^* \right\} \\ & \quad + \lambda^* pe^{-\eta_1(x_0-x)} \left\{ \frac{\eta_1 - 1}{\eta_1 - 1} e^{x_0} - \frac{\eta_1 A}{\eta_1 - \beta_1} e^{\beta_1 x_0} + \frac{\eta_1 B}{\beta_2 - \eta_1} e^{\beta_2 x_0} - K \right\}. \end{aligned}$$

By using the definitions of β_1 and β_2 , and

$$\begin{aligned} & \frac{\eta - 1}{\eta_1 - 1} v_0 - \frac{\eta_1 A}{\eta_1 - \beta_1} v_0^{\beta_1} + \frac{\eta_1 B}{\beta_2 - \eta_1} v_0^{\beta_2} \\ &= \left\{ \frac{\eta_1 \beta_2}{(\eta_1 - \beta_1)(\beta_2 - \beta_1)} + \frac{\eta_1 \beta_1}{(\beta_2 - \eta_1)(\beta_2 - \beta_1)} \right\} K \\ & \quad - \left\{ \frac{\eta_1(\beta_2 - 1)}{(\eta_1 - \beta_1)(\beta_2 - \beta_1)} + \frac{\eta_1(\beta_1 - 1)}{(\beta_2 - \eta_1)(\beta_2 - \beta_1)} - \frac{\eta_1}{\eta_1 - 1} \right\} v_0 = K, \end{aligned}$$

condition 2 follows.

For $x > x_0$,

$$\begin{aligned}
& \int_{-\infty}^{\infty} V(x+u) dF_U^*(u) \\
&= \int_{-\infty}^{x_0-x} [Ae^{\beta_1(x+u)} + Be^{\beta_2(x+u)}] q\eta_2 e^{\eta_2 u} du \\
&\quad + \int_{x_0-x}^0 (e^{x+u} - K) q\eta_2 e^{\eta_2 u} du + \int_0^{\infty} (e^{x+u} - K) p\eta_1 e^{-\eta_1 u} du \\
&= e^x \left(\frac{q\eta_2}{\eta_2 + 1} + \frac{p\eta_1}{\eta_1 - 1} \right) + qe^{\eta_2(x_0-x)} \left(K - \frac{\eta_2 e^{x_0}}{\eta_2 + 1} + \frac{A\eta_2 e^{\beta_1 x_0}}{\beta_1 + \eta_2} + \frac{B\eta_2 e^{\beta_2 x_0}}{\beta_2 + \eta_2} \right) - K.
\end{aligned}$$

Then

$$\begin{aligned}
& (LV)(x) - r^*V(x) \\
&= \frac{1}{2}\sigma^2 e^x + [r^* - d^* - \frac{1}{2}\sigma_2\lambda^*(\xi^* - 1)]e^x - (r^* + \lambda^*)(e^x - K) \\
&\quad + \lambda^* \left\{ e^x \left(\frac{q\eta_2}{\eta_2 + 1} + \frac{p\eta_1}{\eta_1 - 1} \right) + qe^{\eta_2(x_0-x)} \left(K - \frac{\eta_2 e^{x_0}}{\eta_2 + 1} + \frac{A\eta_2 e^{\beta_1 x_0}}{\beta_1 + \eta_2} + \frac{B\eta_2 e^{\beta_2 x_0}}{\beta_2 + \eta_2} \right) - K \right\} \\
&= r^*K - d^*e^x + \lambda^*qe^{\eta_2(x_0-x)} \left(K - \frac{\eta_2 e^{x_0}}{\eta_2 + 1} + \frac{A\eta_2 e^{\beta_1 x_0}}{\beta_1 + \eta_2} + \frac{B\eta_2 e^{\beta_2 x_0}}{\beta_2 + \eta_2} \right) \\
&= r^*K - d^*e^x + \lambda^*qe^{\eta_2(x_0-x)} \frac{\eta_2\beta_1\beta_2(\eta_1 + \eta_2)}{\eta_1(\eta_2 + 1)(\beta_1 + \eta_2)(\beta_2 + \eta_2)} K.
\end{aligned}$$

Since $LV(x) - r^*V(x)$ is a decreasing function, to show $LV(x) - r^*V(x) < 0$, for all $x > x_0$, it suffices to show $(LV - r^*V)(x_0+) < 0$. Under condition (3.6),

$$(LV - r^*V)(x_0+) = \left\{ r^* + \lambda^*q \frac{\beta_1\beta_2(\eta_1 + \eta_2)}{\eta_1(\eta_2 + 1)(\beta_1 + \eta_2)(\beta_2 + \eta_2)} - d^* \frac{(\eta_1 - 1)\beta_1\beta_2}{\eta_1(\beta_1 - 1)(\beta_2 - 1)} \right\} K < 0.$$

The proof is completed. \square

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