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股價資訊的改變點偵測:實證研究

Change Point Detection from Stock Data：An Empirical Study

研究生：白斯宇

Advisee: SZU-YU PAI

指導教授：傅承德博士;張森林博士

Advisors： Cheng-Der Fuh, Ph.D.;

San-Lin Chung, Ph.D.

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Abstract

In this paper, we discuss the problems of change point detection. There are some classical methods for change point detection, such as the cumulative sum (CUSUM) procedure. However, when utilizing CUSUM, we must be sure about the model of the data before detecting.

We here introduce a new method to detect the change points by using Hilbert-Huang Transformation (HHT) to devise a new algorithm. This new method (called the HHT test in this paper) has the advantage that no model assumptions are required. Moreover, in some cases the HHT test performs better than the CUSUM test, and has better simulation results. In the end, an empirical study of the volatility change based on S&P 500 is also given for illustration.

KEY WORDS: Arbitrage detection, change point detection, Hilbert-Huang transformation, volatility, stock prices.

摘要

本篇論文中，我們討論了改變點偵測的問題，傳統上，改變點偵測的問題已有一個廣為人知的方法， the cumulative sum (CUSUM) procedure。然而，在使用 CUSUM 時，我們必須知道時間序列改變前與改變後的機率密度函數，這使得我們在偵測前必須對資料做出很多假設。

在此，我們提供一個新的方法可用於改變點偵測，並且不須對資料做出過多假設。新的方法使用到了希爾伯特-黃轉換 (HHT) ，這個新方法(我們稱之為 the HHT test)，不需要對資料做出模型假設，並且，在某些情況下表現的比傳統的 CUSUM 出色。最後，我們並提供一些實證研究作為參考，我們針對次級房貸與網路泡沫化時的 S&P 500 指數做 the HHT test，並且也有不錯的結果。

關鍵字： 套利偵測，改變點偵測，希爾伯特-黃轉換，波動率，股價

Contents

1 Introduction

1.1 Background

In stock market, people always pursue the goal of finding change points promptly. These changes may come from the alteration of company policies or from the recession of the economy. We hope to detect the changes as soon as they appear. There is already a term to describe these problems: change points detection or arbitrage detection.

Many results, such as the CUSUM test, exist in the change point detection in previous studies. The first result can be found in Page (1954) who constructed classical CUSUM test. This test provides a widely accepted procedure to detect change points.

However, there are still some problems in the classical procedure, the CUSUM test. For example, it is necessary to ascertain the distribution of the data. Without the distribution of the data, the CUSUM test can not be implemented. Therefore we introduce a new method on the change point detection: the Hilbert-Huang Transformation test (the HHT test), which had already been used extensively in other fields. For example, there are many studies in geophysics, structural safety, and operating research. However, the HHT is rarely applied in finance. We here try to use the HHT on some finance problems, because many financial data are nonlinear, non-stationary time series with unknown models, which is exactly what the HHT can analyze.

This paper is organized as follows. In chapter 1, we introduce a new method (called the HHT test in this paper) to detect the change points. [In chapter 2, we describe some cases the HHT test performs better than the CUSUM test, and has better simulation results. In chapter 3, we find some weakness of the HHT test in the change of the mean. In chapter 4, an empirical study of the volatility change based on S&P 500 is also given for illustration.

1.2 Previous Studies in Change Point Detection

Consider an infinite sequence of observation: x_1, x_2, x_3, …, x_i, ….
These variables represent the stock price at time i. At some unknown time v, either the company altered its policy or it was the beginning of a period of economic recession, like the dot-com bubble in 2000. These kind of events have a common characteristic: transferring the whole structure of the market. In other words, the distribution of the time series changes parameters, like the mean or the variance. Therefore, what we seek is a stopping rule T which detects the change promptly.

When mentioning change point detection, the CUSUM test cannot be left unnoticed. We will use the above assumption to explain the idea of CUSUM. First, x_1, x_2, x_3,…, x_{v-1} are independent and identically distributed with the probability density function f_0, whereas x_v, x_{v+1},.... are independent and identically distributed with the probability density function f_1, for some $v>1$.

Let P_i denote the probability as the change from f_0 to f_1 occurs at the ith observation; $E_i(T)$ denotes the expectation of a stopping rule T when the change occurs at time i. If $i=0$, there is no change. A stopping rule τ can be described as:

Minimize

$$\sup_{v\geq 1} E_v(\tau - v + 1 \mid \tau \geq v)$$

subject to

$$E_0\tau \geq B$$

for some given (large) constant B.

A special method to solve the above problem is the following.

Assume x_1, … x_n have been observed.

Consider for $1 \leq v \leq n$

$$H_v : x_1, \ldots, x_{v-1} \sim f_0 \ ; \ x_v, x_{v+1}, \ldots, x_n \sim f_1$$

against

$$H_0 : x_1, \ldots, x_n \sim f_0 .$$

Then the log likelihood ratio statistic can be written down:

$$\max_{0 \le k \le n} (s_n - s_k) = s_n - \min_{0 \le k \le n} s_k$$

where

$$S_n = \sum_{j=1}^{n} \log[\frac{f_1(x_j)}{f_0(x_j)}] .$$

Then we can get a stopping rule :

$$\tau = \inf \left\{ n : (s_n - \min_{0 \le k \le n} s_k) \ge b \right\} .$$

1.3 The Use of Hilbert-Huang on Transformation on Change Point Detection

The HHT was used to analyze data in two ways. The first one is using the whole time-frequency-amplitude plot. This is the classical way to utilize the HHT. Its advantage is that we can collect the information from all of the data; however, the disadvantage of the classical method is its being less sensitive than method two.

The second way is using a part of the time-frequency-amplitude plot. This method is a new application of the HHT. In this way, we focus on the high frequency part while detecting volatility change, since high frequency means short period. Hence the long period part, which is useless in detecting volatility change, can be omitted.

By using the above methods we have several simulation studies. The first simulation study analyzes the time series from normal distribution which represents the basic model. Following, we analyze the time series from Brownian Motion Models and Geometric Brownian Motion Models which are close to real financial data. The last simulation study analyzes the time series from Markov Switch Models. Unlike the previous models, in this one we can not know when the distribution of the data will change. Therefore it is sure that the HHT test can detect the change points even though the change timing is unknown.

1.3.1 Hilbert-Huang Transformation

A new instrument on change point detection, the Hilbert-Huang Transformation (HHT), has already been extensively used in engineering. Here are some reasons why we choose the HHT instead of other spectrum analysis methods, such as Fourier Transformation and Wavelet Transformation.

First, the HHT can be utilized on nonlinear, non-stationary time series. Although Fourier Transformation has a wide application on spectrum analysis, it only can be used on the stationary time data; Wavelet Transformation can be applied on non-stationary data, but it is impotent on nonlinear data. However, financial data are usually nonlinear and non-stationary time series. Therefore, the HHT becomes the first choice.

Second, although Fourier Transformation is useful on spectrum analysis, it can only transform a function from time domain into frequency domain. In other words, we can not know the frequency at the specific timing.

	Fourier Transformation	***Hilbert-Huang Transformation***
Symbol	$F(\bullet)$	$H(\bullet)$
Transformation of Domain	*From Time domain to frequency domain*	*From time domain to time and frequency domain*
Function	$F(X(t)) = A(\omega)$	$H(X(t)) = A(t,\omega)$

Table 1-1

Third, compared to Wavelet Transformation, the HHT is more precise. In Huang (1996), the time-frequency-amplitude plots from Hilbert-Huang Transformation can show more details than Wavelet Transformation.

1.3.2 The Process of Hilbert-Huang Transformation

I. Empirical Mode Decomposition (EMD)

1. Let $X(t)$ denote a time series, identify local maxima and local minima of $X(t)$, and then connect all the local maxima by a cubic spline line named the upper envelope. Repeat the above process for local minima to produce the lower envelope. $\max(t)$ denotes the upper envelope and $\min(t)$ denotes the lower envelope. Let their mean be

$$M_1(t) = \frac{[\max(t) + \min(t)]}{2},$$

and the difference between the data and $M_1(t)$ is

$$X(t) - M_1(t) = H_1(t).$$

2. Repeat step 1 to obtain $M_j(t)$, $H_j(t)$:

$$H_1(t) - M_2(t) = H_2(t),$$
$$\vdots$$
$$H_{k-1}(t) - M_k(t) = H_k(t),$$

and let $H_k(t) = C_1(t)$,

when $SD_k = \sum_{t=1}^{T} \frac{[H_{k-1}(t) - H_k(t)]^2}{H^2_{k-1}(t)}$ is smaller than a predetermined value (about 0.2~0.3 in Huang (1998)).

3. Let $X(t) - C_1(t) = R_1(t)$, and do the above procedure again. We can obtain $C_2(t)$, $C_3(t)$, $\cdots$etc.

4. Let $C_j(t)$ be the Intrinsic Mode Functions (IMFs).

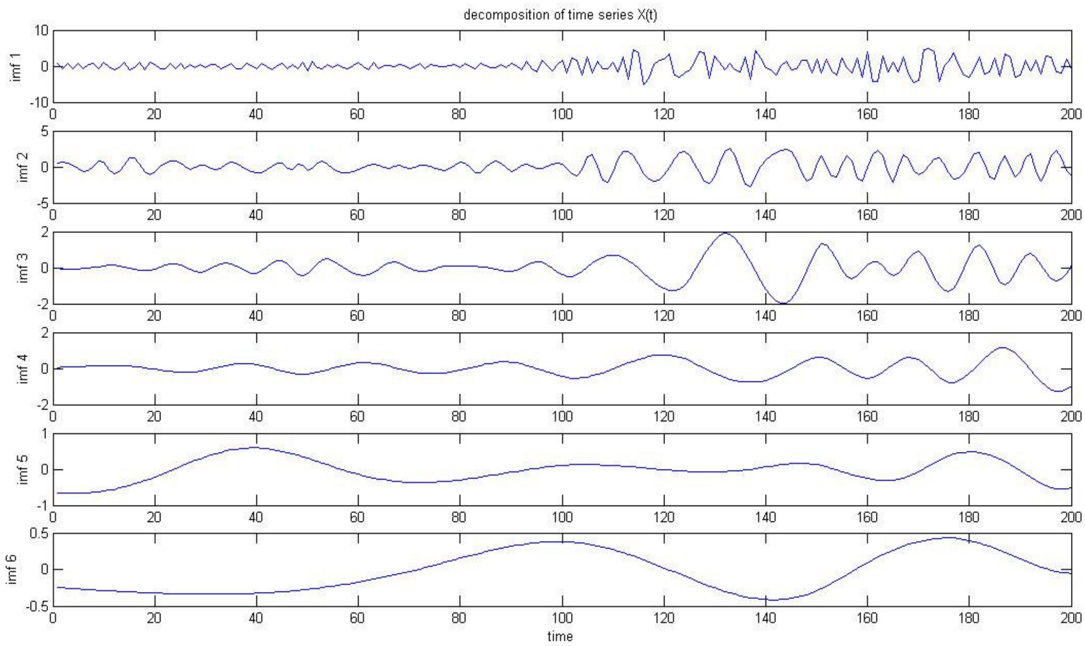

Figure 1-1: An example of IMFs

II. Hilbert Transformation

1. Apply the Hilbert Transformation to each IMFs

Hilbert Transformation:

$$Y_j(t) = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{C_j(s)}{t-s} ds \, .$$

Here, PV indicates the principal value of the singular integral.

2. The analytic signal is defined as

$$Z_j(t) = C_j(t) + iY_j(t) = A_j(t) e^{i\theta_j(t)} ,$$

where

$$A_j(t) = \sqrt{[C_j^{\,2}(t) + Y_j^{\,2}(t)]} ,$$

and

$$\theta_j(t) = \arctan\left(\frac{Y_j(t)}{C_j(t)}\right) .$$

Here, $A_j(t)$ is the instantaneous amplitude, and $\theta_j(t)$ is the phase function, so the instantaneous frequency is

$$\omega_j(t) = \frac{d\theta_j(t)}{dt} .$$

3. The original time series can be expressed as following,

$$X(t) = \Re\left\{ \sum_{j=1}^{n} A_j(t) \exp\left[i \int \omega_j(t) dt \right] \right\} ,$$

where $\Re$ denotes the real part of the number.

4. The above equation represents the amplitudes to be contoured on the frequency-time plane. This frequency-time distribution of the amplitude is designated as "Hilbert amplitude spectrum", $A(t,\omega)$, or simply "Hilbert spectrum."

2 Change Point detection: Volatility Change

2.1 Volatility Change in Normal Distribution Models

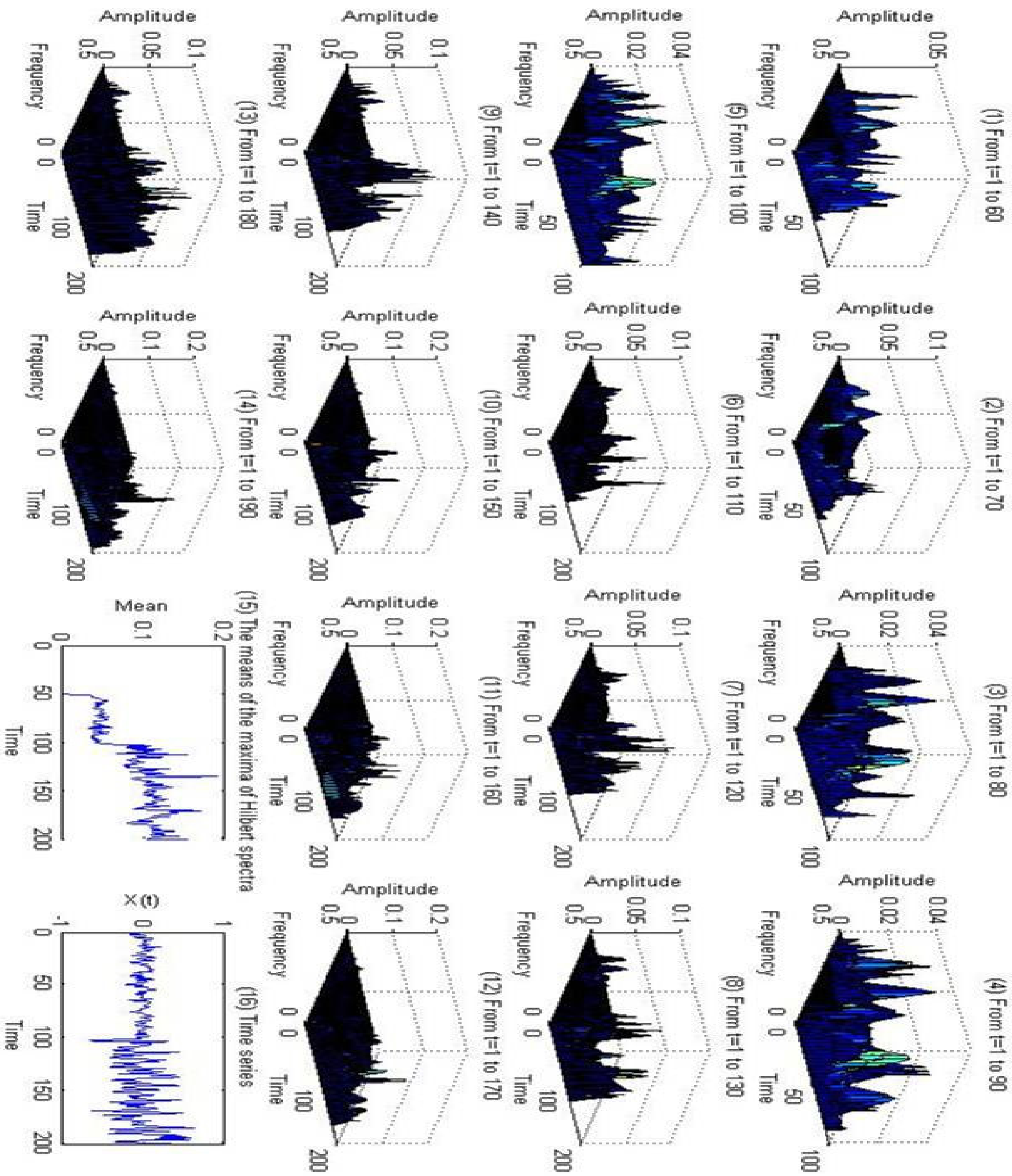

Figure 2-1: Figure 2-1 (1) to Figure 2-1 (14) are Hilbert spectra. Figure 2-1 (15) is the plot of mean of maxima in Hilbert spectra. Figure 2-1(16) is the original data.

The essential model is described as the following.

Our model:

$$x_1,\ \ x_2,\ \ x_3\ \ \ldots x_{100} \sim N(0,\sigma_1{}^2)$$
$$x_{101},\ \ x_{102},\ \ x_{103}\ \ \ldots x_n \sim N(0,\sigma_2{}^2)$$

After executing the HHT, the difference between “time from 1 to 100” and “From t=1 to 110” can be observed in Figure 2-1. There is a “peak” in the plot of “From t=1 to 110.” The appearance of the peak is rational, because of the change of the magnified volatility. In engineering, the volatility change can be treated as amplitude change. Therefore, if the maximum amplitudes of the plots are “large enough,” then the change is called to be detected. Now the problem is how to define “large enough.” Here we adopt the idea of change point detection problem. In this idea, the null hypothesis is that $x_i \sim N(0,\sigma_1{}^2)$ for all i, and the expectation of stopping rule in our detection method can be as large as B in CUSUM. In practice, we (can) find that B is approximately 800. In other words, we can define our stopping rule as following:

$$\tau' = \inf\{M > b\}$$

Where M is the mean of the three maximum amplitudes in the Hilbert spectra (to avoid the extreme value), subject to

$$E_0[\tau'] > B .$$

Compared with CUSUM, we obtain the following result.

Change of volatility	Stopping time of the Cusum test	Stopping time of the HHT test

2.2 Volatility Change in Geometric Brownian Motion Models

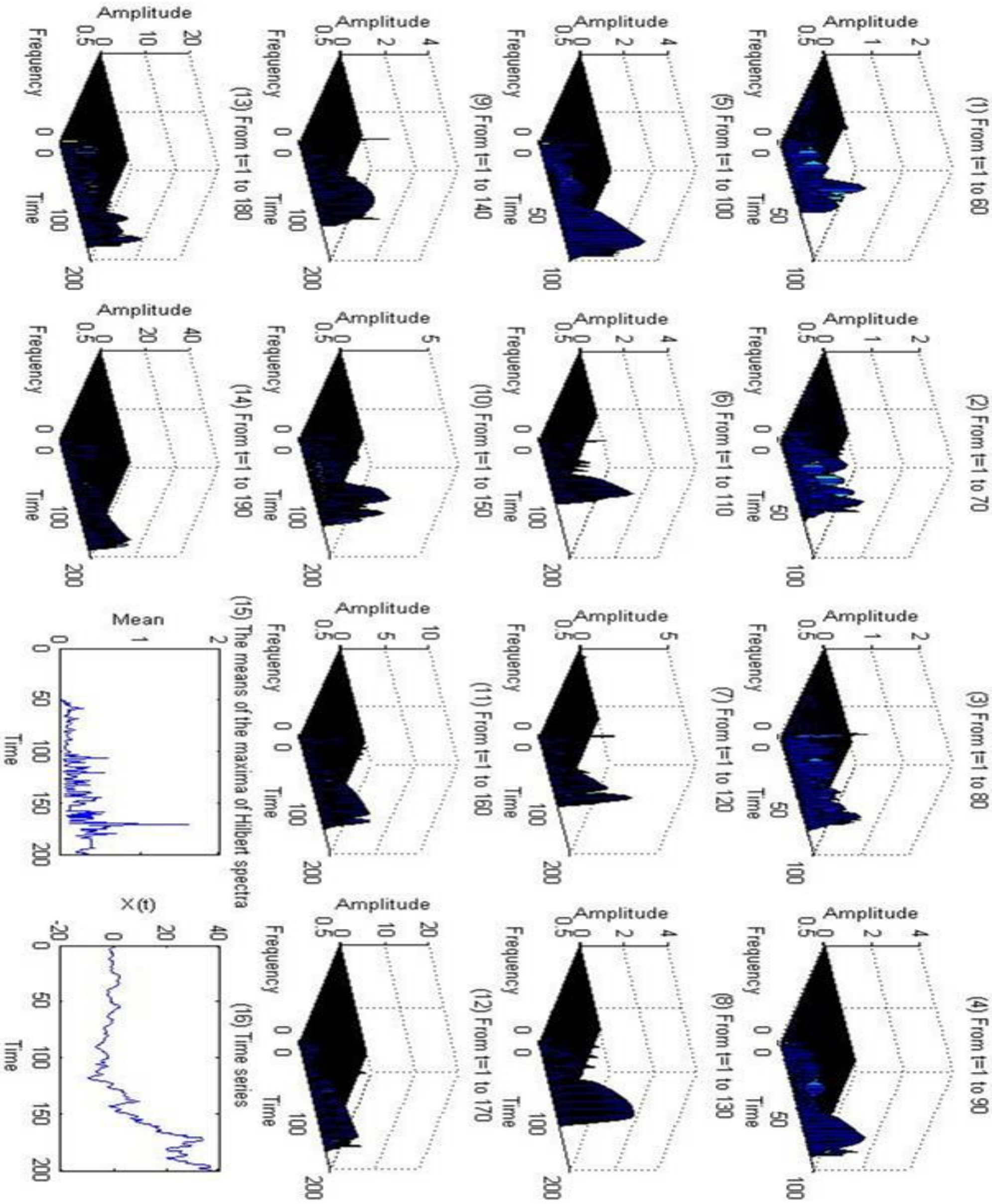

Figure 2-2: Figure 2-2 (1) to Figure 2-2 (14) are Hilbert spectra. Figure 2-12(15) is the means of maxima in Hilbert spectra. Figure 2-1(16) is the original data.

After detecting change points in normal distribution, we apply the HHT test to Geometric Brownian Motion Models (GBM), which is always used to describe stock prices. In option pricing problems, modeling stock prices by Geometric Brownian Motion Models has a well-known problem: the implied volatility is not a constant. In practice, stock prices also have the volatility clustering property. Here we do not seek to reduce the inaccuracy. On the contrary, we pursue to detect the volatility clustering.

First of all, assume

$$x_1,\ x_2,\ x_3\ \ldots x_{100} \sim GBM(r=0.08, \sigma=\sigma_1)$$
$$x_{101},\ x_{102},\ x_{103}\ \ldots x_n \sim GBM(r=0.08, \sigma=\sigma_2)$$

In other words,

$$x_i = x_{i-1} e^{(r-0.5\sigma^2)\Delta t + \sigma W(\Delta t)}$$

Where $W(\Delta t)$ is Standard Brownian Motion. Let x_1=100, and then we can use the above model to generate x_1, x_2, x_3 …, etc. After generating all x_i, we let $y_i = x_i - 100$ for all i to reduce the influence of the start point. In this model, we could focus on high frequency parts of time-frequency-amplitude plots, because high frequency means short period. The larger σ implies the larger amplitude of the short period part. Therefore we focus on the frequency from 0.375 to 0.5, in other words, in the period from 2 to 2.7 days. In Figure 2-2, we can not see the difference directly from the Hilbert amplitude spectrum, but in Figure 2-2 (15) we can see the mean of maximum amplitude touch 0.5 after t>100. We use the same idea to make a criterion, and we have:

Change of volatility	Stopping time of the Cusum test	Stopping time of the HHT test
sigma from 0.1 to 0.3	3.59	30.67
sigma from 0.1 to 0.2	10.51	158.75
sigma from 0.1 to 0.15	39	340.59
sigma from 0.1 to 0.125	214.32	591.21
sigma from 0.1 to 0.11	642.7	663.24
sigma from 0.1 to 0.105	766.38	702.8
sigma from 0.1 to 0.1	853.46	840.55

Table 2-2

The result is similar to section 2-1. the HHT performs better than CUSUM, when σ changes slightly.

2.3 Volatility Change in Markov Switch Models

The simulation study here analyzes the time series from Markov Switch Models. Unlike the previous models, in this one we can not know when the distribution of the data will change. Therefore it is sure that the HHT test can detect the change points even though the change timing is unknown.

The Markov Switch model can be described as:

$$Y_t = \alpha_{S_t} Y_{t-1} + \varepsilon_t ,$$

and

$$\varepsilon_t \sim N(u = 0, \sigma_{S_t}{}^2) ,$$

where

$$S_t \in \{1, 2\} ,$$

represent two states. In state 1, let

$$\alpha_1 = 0.1 ,$$

$$\sigma_1 = 0.1 .$$

In state 2, let

$$\alpha_2 = 0.1 ,$$

$$\sigma_2 = \sigma .$$

Let the transition matrix be

$$p = \begin{bmatrix} 0.99 & 0.01 \\ 0.001 & 0.999 \end{bmatrix}$$

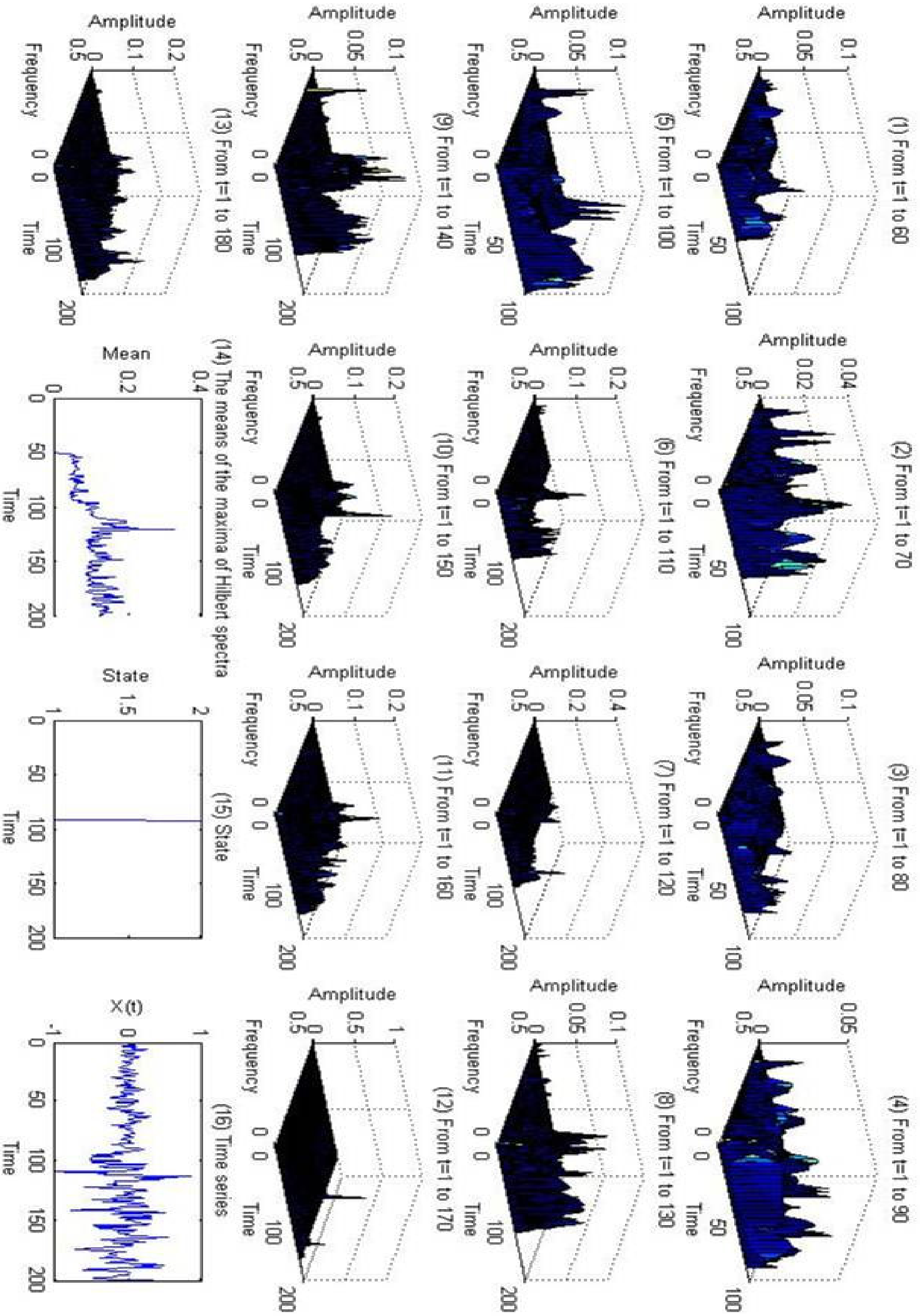

Figure 2-3: Figure 2-3 (1) to Figure 2-3 (14) are Hilbert spectra. Figure 2-3 (14) is the means of maxima in Hilbert spectra. Figure 2-3 (15) is the state of the data. Figure 2-3 (16) is the original data.

The result of Markov Switch Models is the following:

Change of volatility	Stopping time of the HHT test
sigma from 0.1 to 0.3	38.8
sigma from 0.1 to 0.2	180.4
sigma from 0.1 to 0.15	431.0
sigma from 0.1 to 0.125	549.7
sigma from 0.1 to 0.11	666.3
sigma from 0.1 to 0.105	770.1
sigma from 0.1 to 0.1	889.5

Table 2-3

From the above table we can still find the good characteristic of the HHT test. Although we can not detect fast in large change, the increasing of stopping times is getting slow.

2.4 A Brief Summary

In volatility change, we can find a detection rule from the ideas of the HHT. The HHT is a method to decompose a time series, and can produce Hilbert spectra which are time-frequency-amplitude plots. The plots can be used to determine whether the volatility changes or not. In detecting volatility change, what we focus on is the amplitude change. The stopping rule is that the maxima of the amplitude are larger than a constant b subject to small type 1 error. In other words, the expectation of stopping time must be larger than a constant B in null hypothesis. Here we adopt B=800 which is usually utilized in practice. Then the criterion b can be produced subject to B=800.

In the result of the HHT test and the CUSUM test, we can find that as the change becomes minor, both the stopping time of the HHT test and the CUSUM test increase. However the increasing speed is different between the two tests. The stopping time of CUSUM test increases faster than that of the HHT test.

Therefore, the HHT test performs better than the CUSUM test when the change is slight.

3 Change Point detection: Mean Change

3.1 Mean Change in Normal Distribution Models

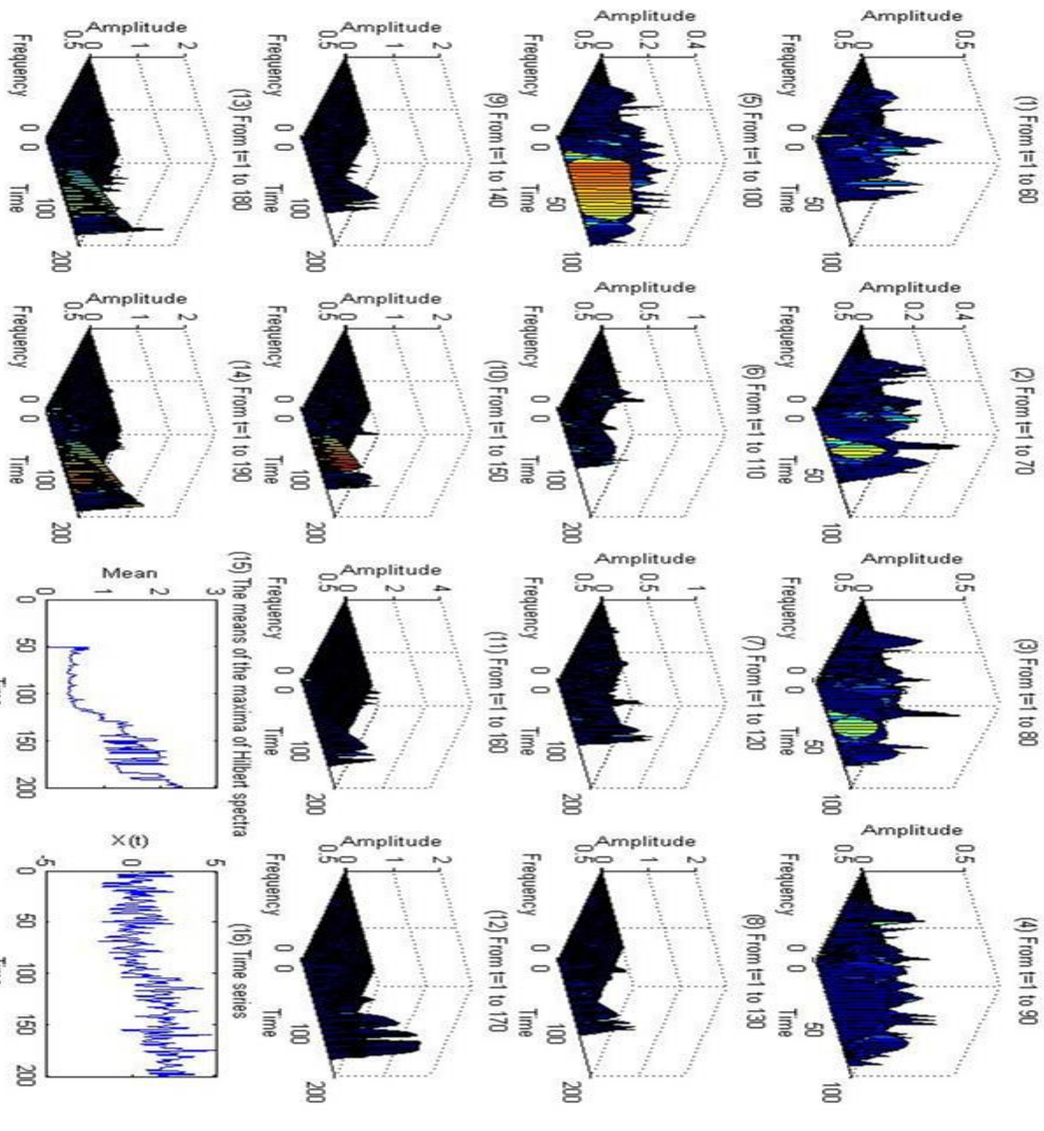

Figure 3-1: Figure 3-1 (1) to Figure 3-1 (14) are Hilbert spectra. Figure 3-1 (15) is the means of maxima in Hilbert spectra. Figure 3-1 (16) is the original data.

The change of the mean is an important topic in finance. It represents the trend of the stock prices. Let us start from the essential model: the normal distribution.

Let

$$x_1,\ x_2,\ x_3\ \ldots x_{100} \sim N(u = u_1, \sigma^2 = 1),$$
$$x_{101},\ x_{102},\ x_{103}\ \ldots x_n \sim N(u = u_2, \sigma^2 = 1).$$

Change of mean	Stopping time of the HHT test	Stopping time of CUSUM test
mean from 0 to 2	14.7	3.81
mean from 0 to 1	38.7	12.54
mean from 0 to 0.5	136.8	45.69
mean from 0 to 0.25	589.7	174.25

Table 3-1

The stopping time of the HHT test is increasing faster than that of the CUSUM test. This means that the HHT test is weak in detecting the change of the mean. Reviewing the procedure of the HHT may explain the reason. It treats the low frequent IMFs as unimportant components. However, in the change of the mean, low frequent IMFs represent the trend of the data. Therefore, the results of the HHT test in the change of the mean can not satisfy us.

3.2 Mean Change in Brownian Motion Models

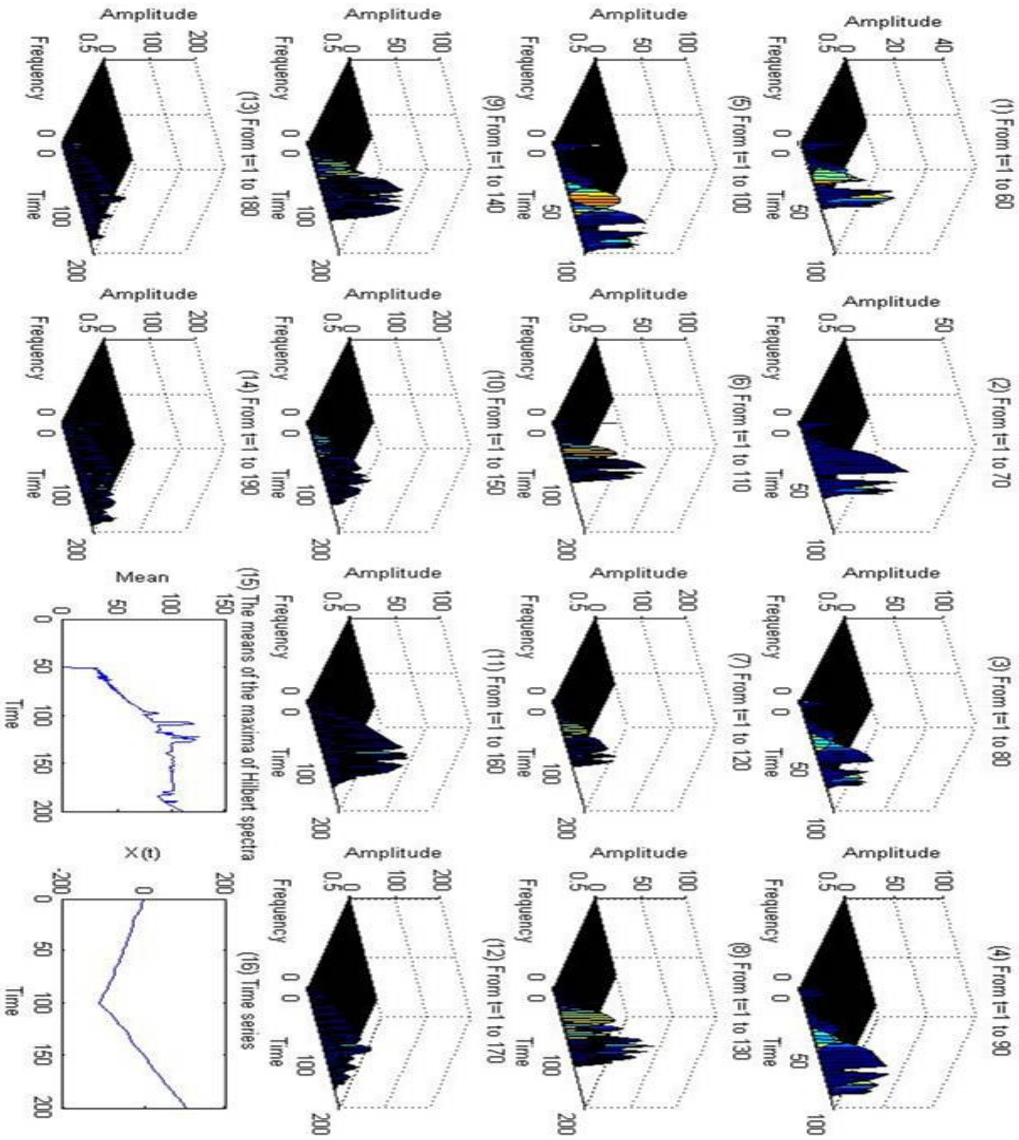

Figure 3-2: Figure 3-2 (1) to Figure 3-2 (14) are Hilbert spectra. Figure 3-2 (15) is the means of the maxima in Hilbert spectra. Figure 3-2 (16) is the original data.

After the unsatisfying results in section 3.1, we try another model, the Brownian Motion Mode, which cumulates the value of u.

The model we use is:

$$x_1,\ \ x_2,\ \ x_3\ \ \ldots x_{100} \sim BM(u = u_1, \sigma^2 = 1)$$
$$x_{101},\ \ x_{102},\ \ x_{103}\ \ \ldots x_n \sim BM(u = u_2, \sigma^2 = 1)$$

In Figure 3-2 (15), we find that when the value of the time series decreases, the amplitude of mean of maximum will increase; when the time series start to increase (the change point), the amplitude of mean of maximum will stop changing. We can not find a good criterion for the phenomenon, so we can detect visually.

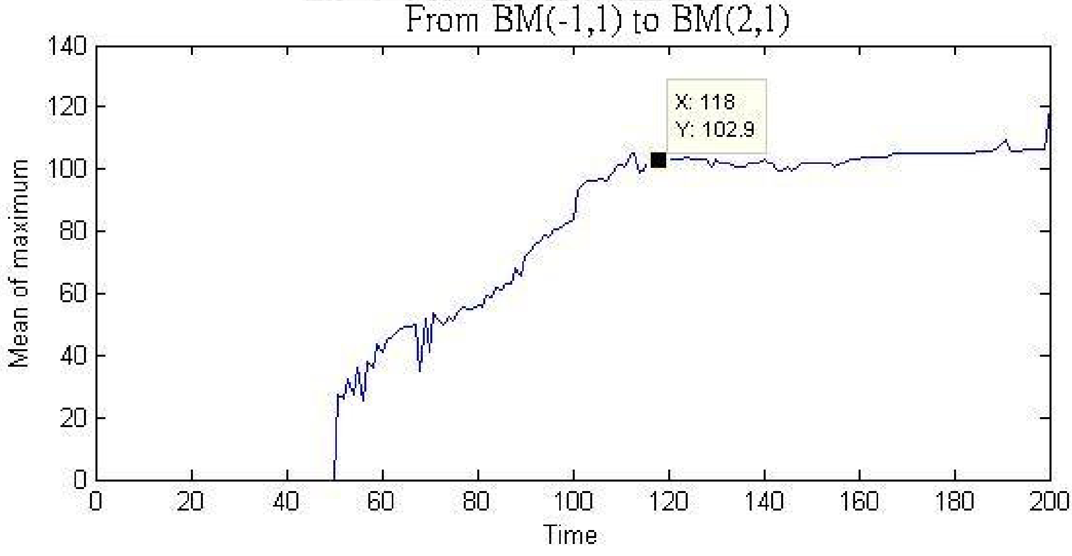

Figure 3-3

In Figure 3-3, we can observe that after around t=110 the value stops increasing. Conservatively, we choose t=118 to be the stopping time.

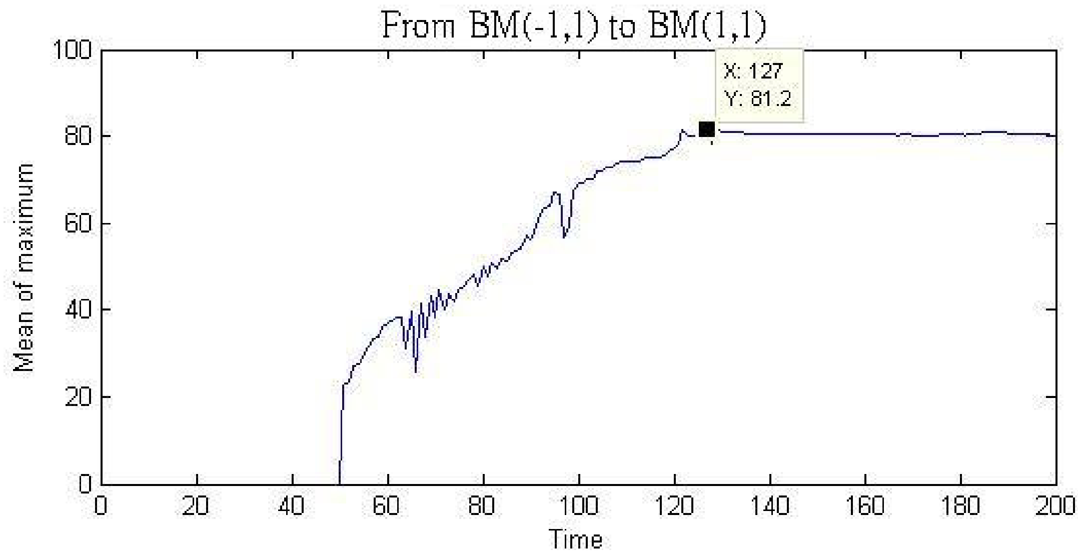

Figure 3-4

In Figure 3-4, we find the same phenomenon, and we detect the change on t=127.

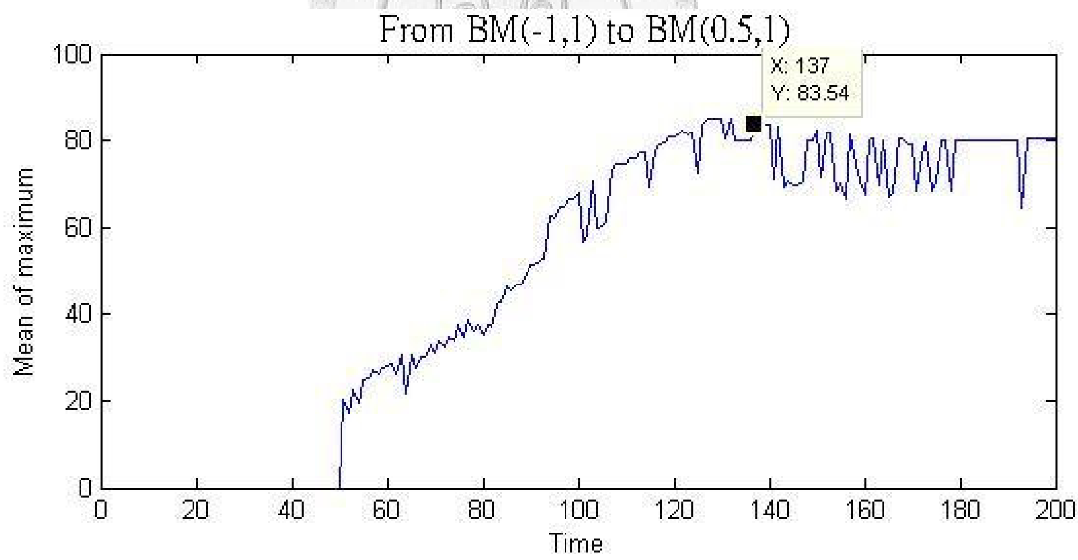

Figure 3-5

In Figure 3-5, we can detect the change at t=137, and we find another special phenomenon, which is when u_2 becomes smaller, the "shake" after the change point becomes larger.

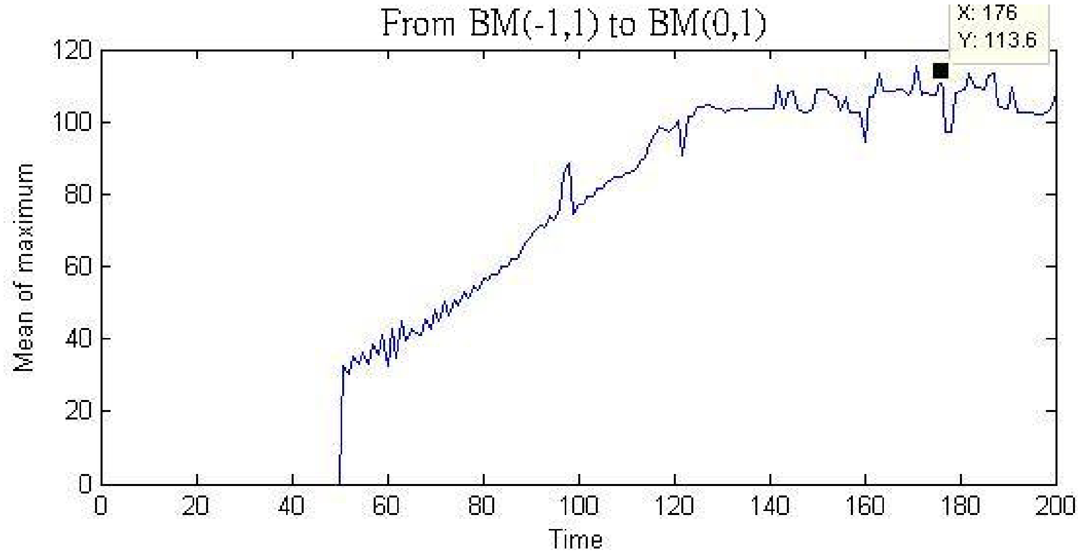

Figure 3-6

In Figure 3-6, we can detect the change at t=176, and the range of the special phenomenon, the “shake”, increases from 70~80 to 90~120.

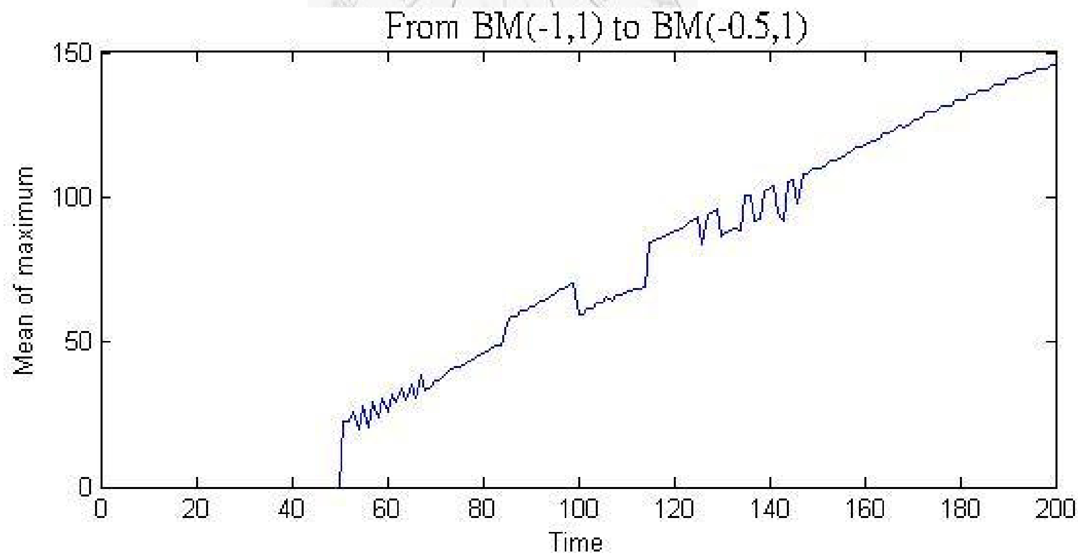

Figure 3-7

In Figure 3-7, we can not detect by eyes any more. In the case of $u_1 \cdot u_2 > 0$, the means of maximum continue increasing. We should analyze the data $x_i - x_{i-1}$ instead in this case.

Change of mean	Stopping time of the HHT test
u from -1 to 2	18
u from -1 to 1	27
u from -1 to 0.5	37
u from -1 to 0	76
u from -1 to -0.5	can not detect

Table 3-2

In the above table we can see the HHT test performs well in the case when $u_1 \cdot u_2 \leq 0$. However, It is useless in the case when $u_1 \cdot u_2 > 0$.

3.3 A Brief Summary

In this section, we discuss the problem of the change of the mean. Unfortunately, the HHT test does not have any good performance in this case.

In normal distribution models, the HHT test loses its good characteristic mentioned in chapter 2, which is that the stopping time increases slowly when the change is slight. Therefore, the HHT test can not perform better than the CUSUM test under such condition.

In Brownian motion models, it is hard to find a good criterion to detect the change, but we still can find some phenomena in plots of means of maxima. When time series starts to change, the value of the mean of maxima will stop at the same level; when u_2 is small, the "shake" of the amplitude of mean of maximum will increase. However, this detection rule is useless while $u_1 \cdot u_2 > 0$.

The reason that explains the weakness of the HHT test about detecting the change of the mean may be the procedure of the HHT, which focuses on high frequent parts but neglects the change of the trend.

4 Empirical Studies

This section discusses the empirical studies of the HHT test. The CUSUM test can not be applied here, because we do not have any model assumptions of the S&P 500 index. On the contrary, when utilizing the HHT test, we do not need any assumptions, and the empirical studies of the HHT test lead us to a good conclusion.

4.1 Volatility Change Data (Subprime Mortgage Crisis in 2007)

4.1.1 Using Stock Prices Directly

Let us use our new method on empirical studies. In 2007, the global market faced a serious crisis, and suffered an unprecedented credit risk. At the same time, stock prices underwent acute vibration. This event is a good example for us to test our new method. First, extracting from the S&P 500 index from 1/3/2006 to 4/9/2008, we have 570 daily data. Second, we use the data of VIX index, which is the implied volatility form S&P 500 to check our result. When we apply our new method to detect change points, is it efficient? If a change does exit, how quickly can we detect it?

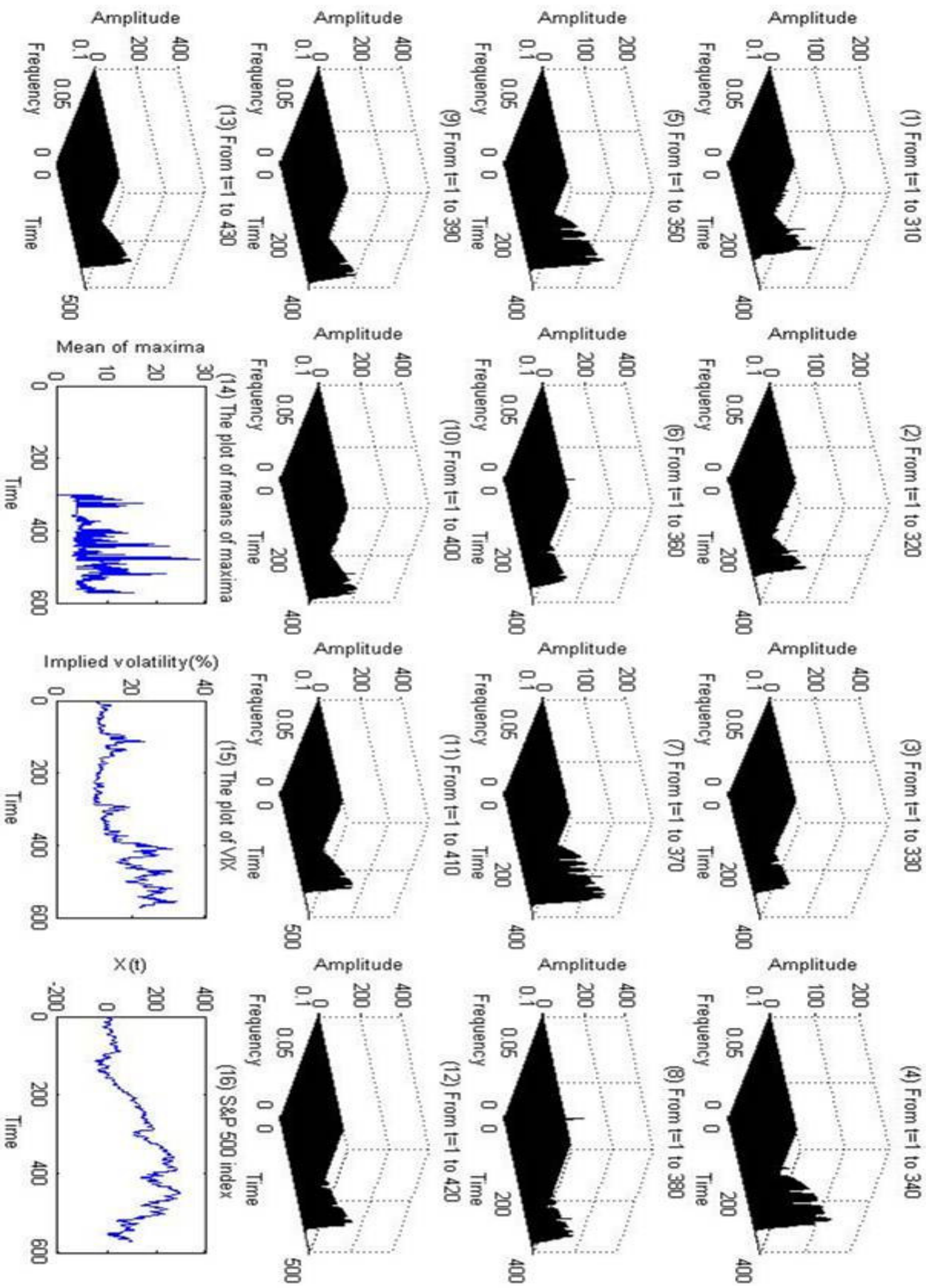

Figure 4-1: Figure 4-1 (1) to Figure 4-1 (13) are Hilbert spectra. Figure 4-1 (14) is the means of the maxima in Hilbert spectra. Figure 4-1 (15) is the vix index. Figure 4-1 (16) is the S&P 500 index.

In Figure 4-1 (16), S&P500 index has an acute vibration after t=400. The same phenomenon can be found in Figure 4-1 (15), the VIX index. After t=400, VIX index are all approximately larger than 20%

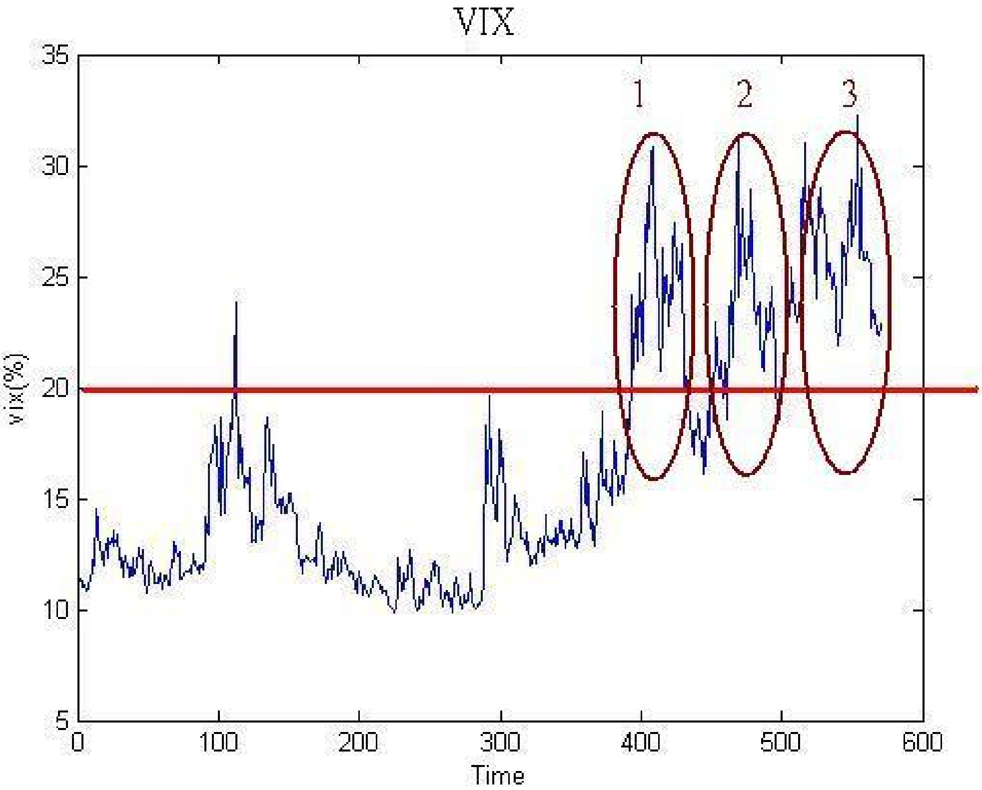

Figure 4-1 (15)

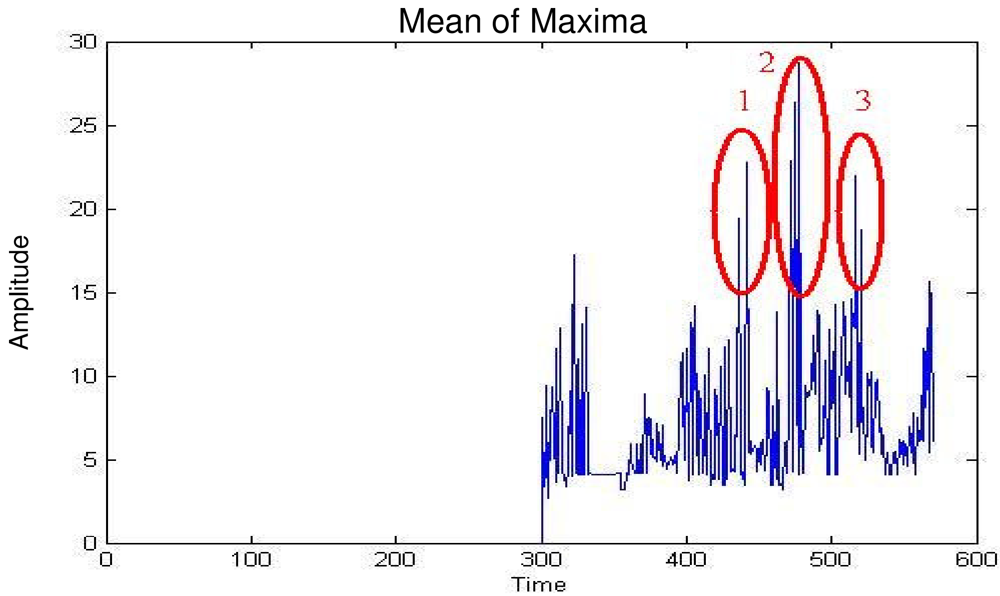

Figure 4-1 (14)

Volatility change event	Start date	End date	Date of the HHT test detection	Stopping time of the HHT test
Event 1	393	430	436	43
Event 2	465	495	475	10
Event 3	501	570	517	16

Table 4-1

Table 4-1 tells us that the change points can be detected efficiently exclusive of Event 1. After detecting, the changes of volatility in Event 2 and Event 3 still continue for 20 days and 53 days respectively.

Only a minority of indexes has its own implied volatility index. The S&P 500 index is one of them, and the implied volatility index is called VIX. Some markets do not have big enough option trading volume. Therefore the new method can be utilized on these markets. We can regard trading options as trading the volatility. Accordingly, in the market with low option trading volume, when we detect the increase of the volatility, the prices of options are probably undervalued. Contrarily, when the volatility decreases, the prices of options are possibly overvalued.

4.1.2 Using Log Return of Stock Prices

In this section, the log returns of stock prices are analyzed. The use of the log returns of stock prices can make us focus on the volatility change without the influence of the trend. Therefore, analyzing the log returns of data yields better results than analyzing the original data does. Figure 4-2 (16) is the log return of S&P500 index from 1/3/2006 to 4/9/2008.

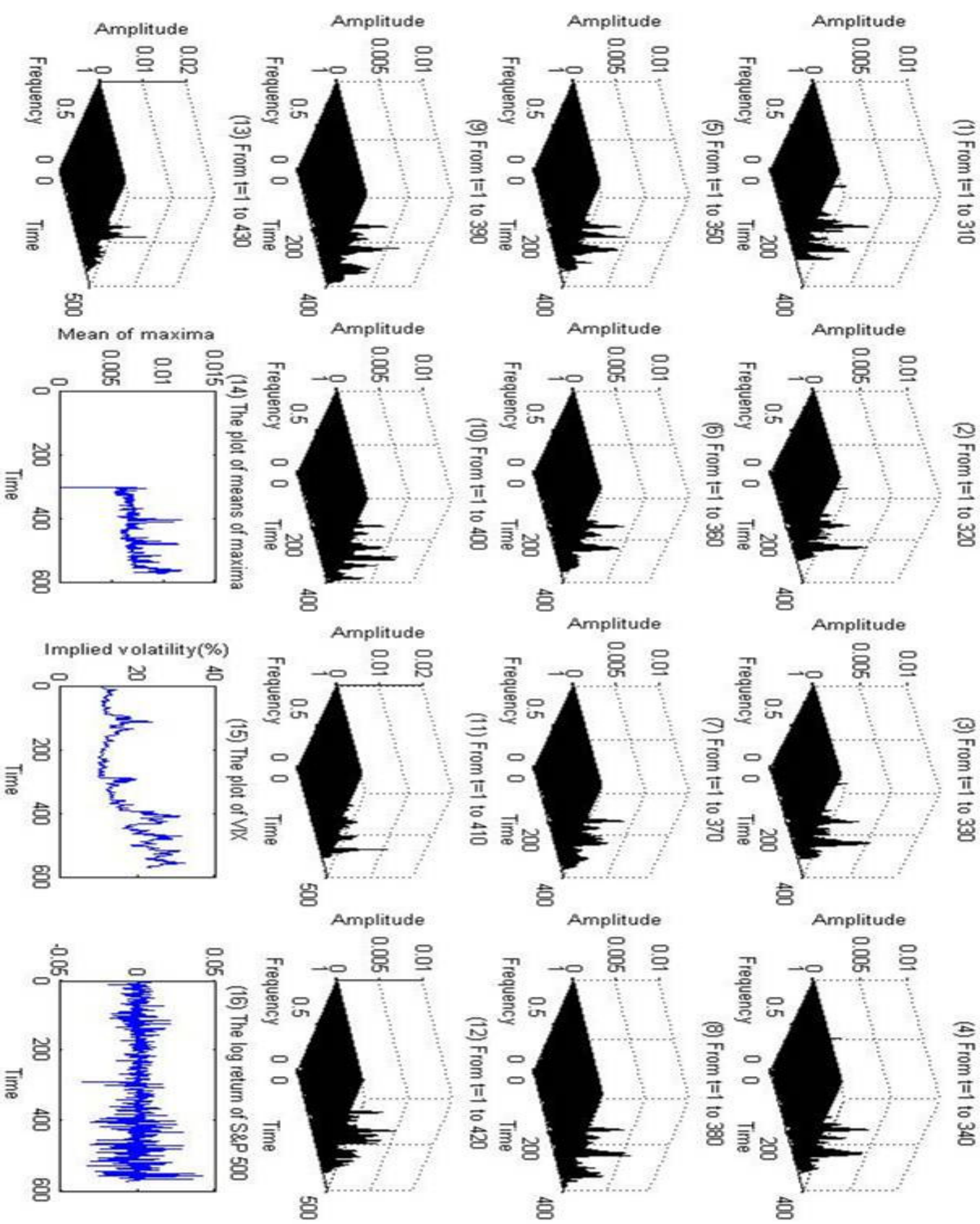

Figure 4-2: Figure 4-2 (1) to Figure 4-2 (13) are Hilbert spectra. Figure 4-2 (14) is the means of the maxima in Hilbert spectra. Figure 4-2 (15) is the VIX index. Figure 4-2 (16) is the log return of S&P 500 index.

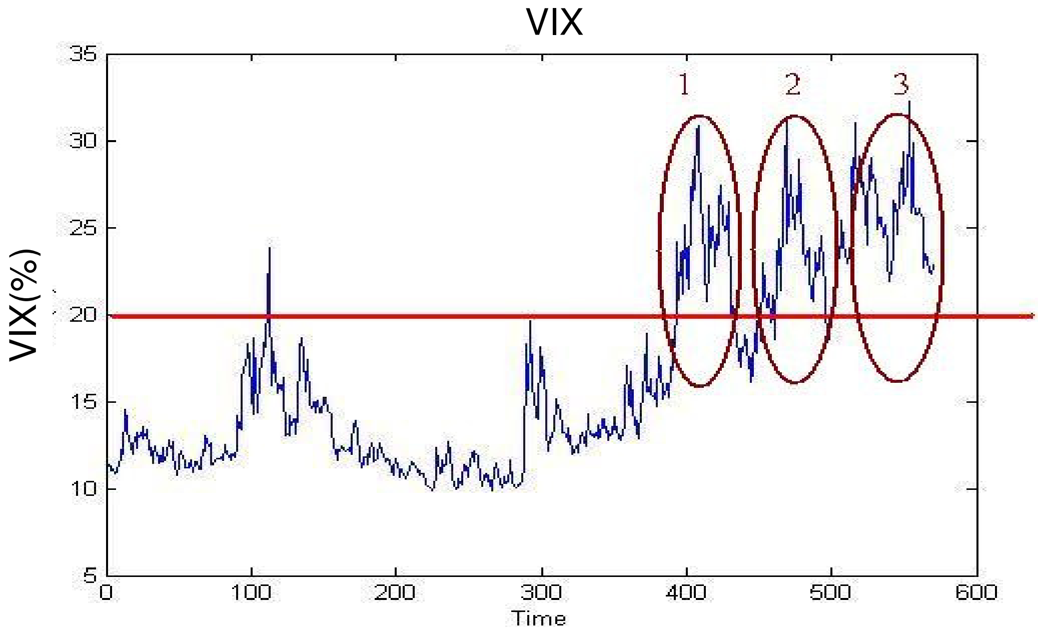

Figure 4-2(15)

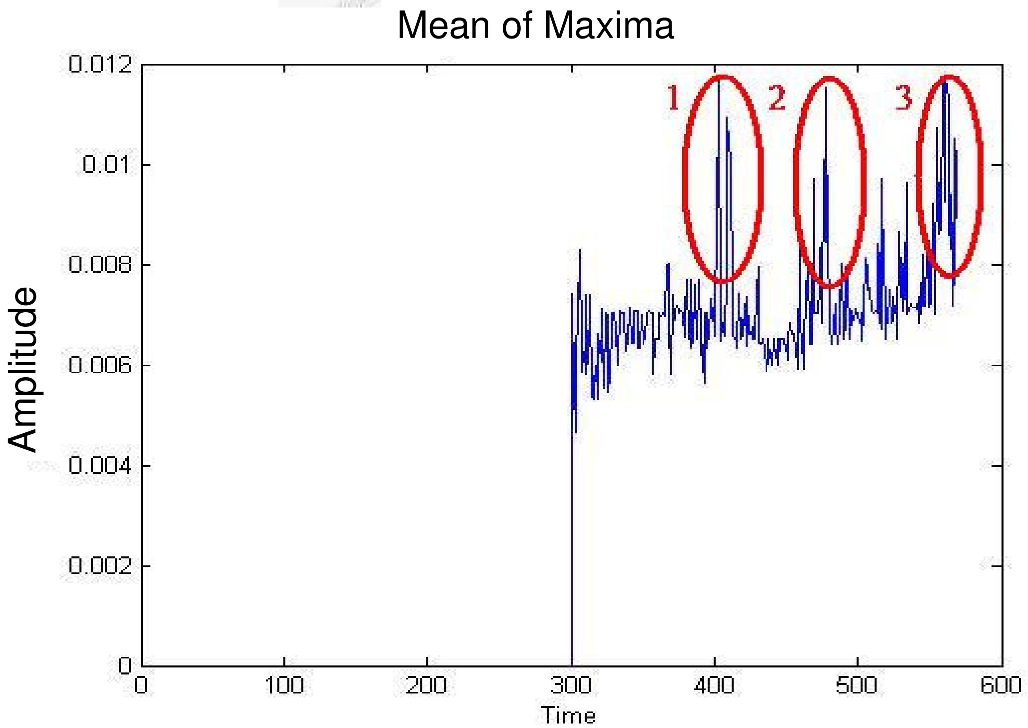

Figure 4-2(14)

Volatility change event	Start date	End date	Date of the HHT test detection	Stopping time of the HHT test
event 1	393	430	402	9
event 2	465	495	469	4
event 3	501	570	516	15

Table 4-2

From Figure 4-2 (14) and Table 4-2 the results of the detection show that the method in 4.1.2 can detect the changes faster then in 4.1.1. Therefore we can make investment decisions more promptly by applying the results in 4.1.2.

All the three results of events in this section are better than in 4.1.1. In event 1, stopping time = 43 when using the data of S&P500 index directly, however, we have stopping time = 9 here. In event 2, stopping time = 10 in 4.1.1 contrasts with stopping time = 4 here. Only in event 3 the stopping time=16 in 4.1.1 is almost the same as stopping time=15 in this section. It means that by using our detection results we can react more quickly to the change of the volatility.

4.2 Volatility Change Data (Dot-com Bubble in 2000)

Another economic recession in 2000 is considered here. In 1998~2000 stock prices of dot-com companies rose fast. However, most dot-com companies had not even made a profit yet. Hence, after irrational investors spent all the wealth they had buying shares, the stock prices of dot-com companies began to fall , and were accompanied by the rise of the volatility.

We adopt the data of S&P 500 index from 7/3/2000 to 12/31/2001, including 374 daily prices. Because of the better results when analyzing log return of data in 4.1, we prefer log return of data here.

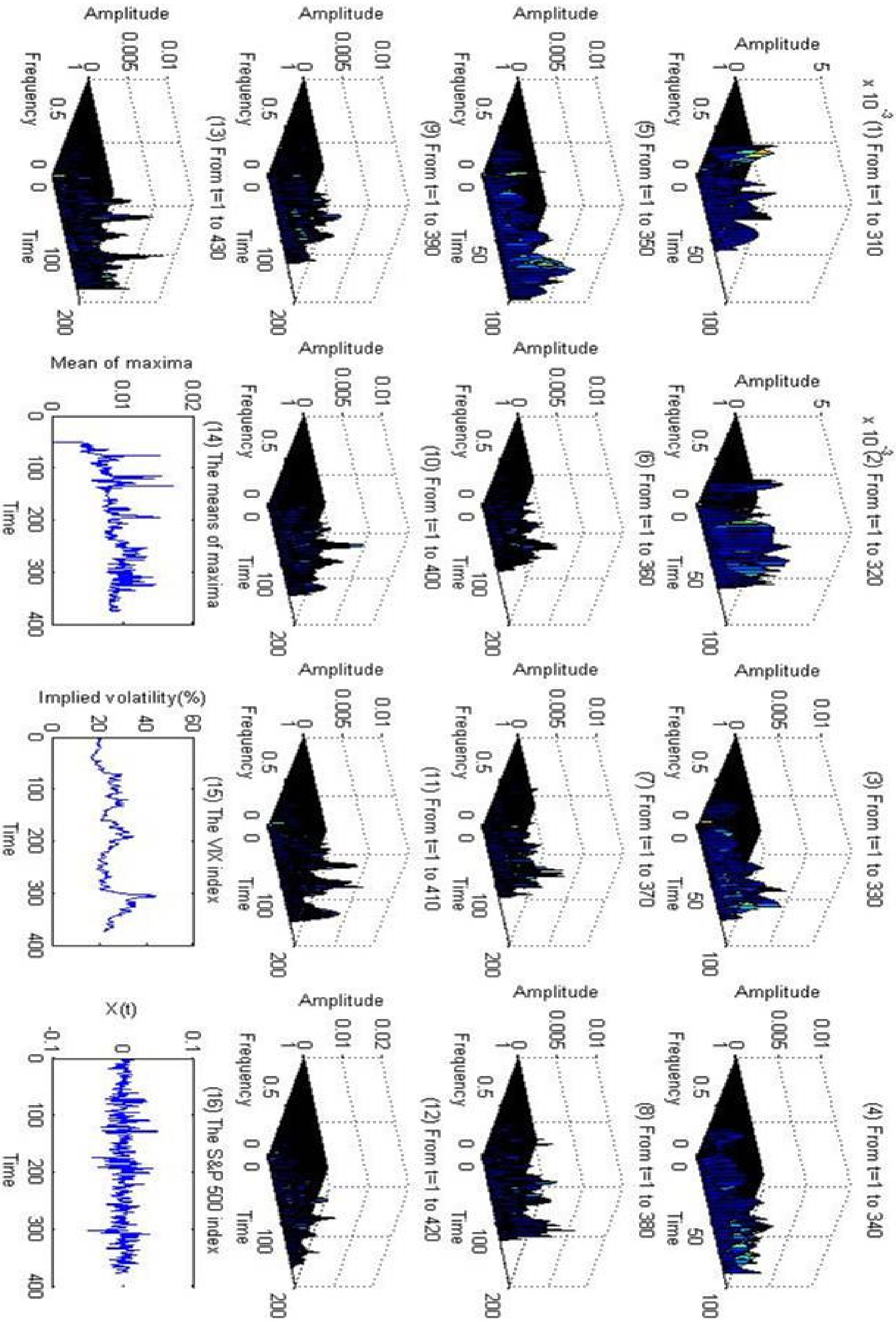

Figure 4-3: Figure 4-3 (1) to Figure 4-3 (13) are Hilbert spectra. Figure 4-3 (14) is the means of the maxima in Hilbert spectra. Figure 4-3 (15) is the VIX index. Figure 4-3 (16) is the log return of S&P 500 index.

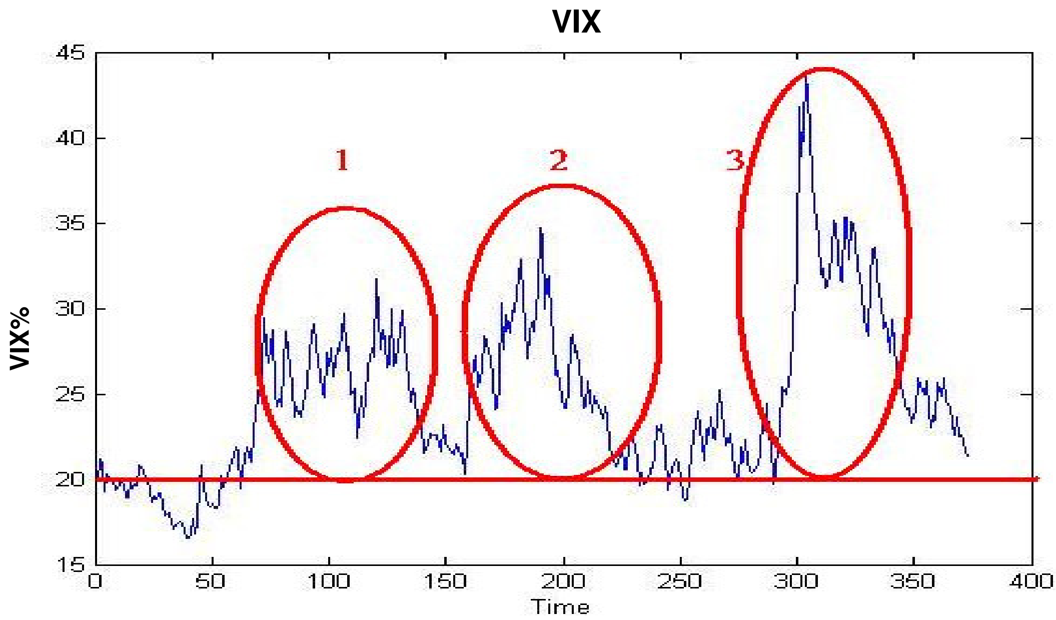

Figure 4-3 (15)

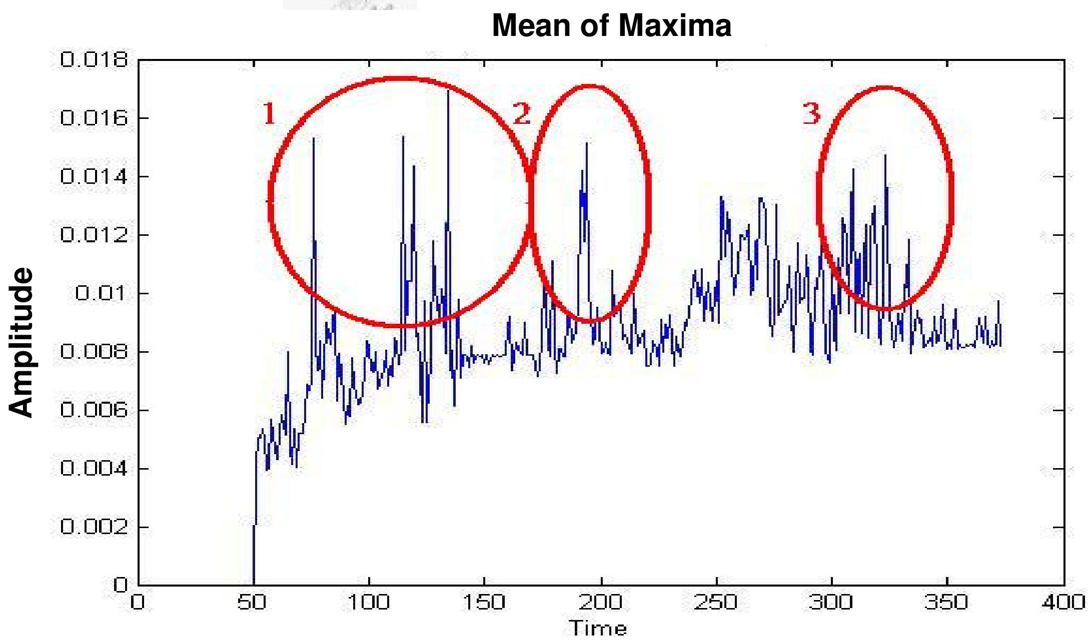

Figure 4-3 (14)

Here we try to find the change points by sight. The conclusion is Table 4-3.

Volatility change event	Start date	End date	Date of the HHT test detection	Stopping time of the HHT test
Event 1	63	158	76	13
Event 2	159	226	194	35
Event 3	291	373	309	18

Table 4-3: The stopping times of the HHT test in the dot-com bubble.

In Event 1, because the change is from around 16% to 30%, the stopping time is short. In Event 2 and 3, the change is from around 20% to 30%, so the stopping time is longer than in event 1, but still quite small when compared to the length of duration of the change. In other words, the HHT test can detect the change points promptly so we can make investment decisions before the changes finish in the Dot-com bubble case.

4.3 A Brief Summary

In empirical study, we get two conclusions. First of all, the new method, the HHT test, is useful on empirical data. In both the subprime mortgage crisis and the dot-com bubble, we can detect the change points correctly and quickly. In subprime mortgage crisis, we can find three events in which the volatility changes form under 20% to above 20%. We can detect all the changes promptly in subprime mortgage crisis. In dot-com bubble, there exist four events and all of them can be detected quickly, too. The results we get in both subprime mortgage crisis and dot-com bubble are great.

Second, because of shorter stopping time, using the log return of data is better than using the original data. The reason is that in log return of data, we can focus on the volatility change without the influence of the trend.

5 Conclusions and Further Researches

5.1 Conclusions

In this paper we obtain four conclusions.

First of all, we introduce a new method to change point detection. This method can be utilized without any model assumptions.

The Hilbert-Huang Transformation (HHT) can produce Hilbert spectra which represents frequency-time distribution of the amplitude. By utilizing high frequency parts of Hilbert spectra, we can devise the HHT test. Moreover, this test can be applied without model assumptions. In the classic method of change point detection, the CUSUM test, we need to know the distribution of the data before and after change to compute the log likelihood ratio statistic. The above information is needless in the HHT test.

Second, the HHT test performs better than the CUSUM test in some cases.

In chapter 2, although the CUSUM test performs well in large change problems, the HHT test is a good choice to handle those cases with slight change. Therefore, if facing the problem of slight change, we can consider the HHT test first.

Third, in practice, we still obtain good results.

In chapter 4, we review the data of dot-com bubble in 2000 and subprime mortgage crisis in 2007 respectively. Both of the two financial crises are well-known and extensively influential. The HHT test can detect these two crises promptly and efficiently, and we can even make a profit from the successful detection.

Fourth, the advantage and disadvantage of the HHT test.

In chapter 3, we can learn the characteristics of the HHT test. When detecting the change of the mean, it yields a mediocre result. Although the HHT test is sensitive to volatility change, it is insensitive to the change of the mean.

In this paper, we know some features of the new method, the HHT test. It is a good method to deal with the volatility change problems, but has some baffles when facing the changes of the mean. These imperfect parts of the HHT test remain to be solved.

5.2 Further Researches

The evidence presented above indicates that there are some persuasive reasons for preferring the HHT test in some cases. Therefore, we indicate some open problems here.

First, some strict proof of the HHT test surpassing the CUSUM needs to be provided. Because the HHT does not have theoretical bases, the proof may be the hardest part of the further research.

Second, why does the HHT test yield better results of detection of volatility change? Why is it insensitive when dealing with the change of the mean? All of above problems need to be solved

Third, can we utilize the HHT test in other fields? We have a good conclusion in finance, so the next step should be doing some research on other kinds of data.

Fourth, how can we settle the weakness of the HHT test in mean change? The HHT test may have some limits congenitally in mean change case. However, if there are some means that can overcome the obstacles of this weakness, then the HHT test could become a comprehensive method for change point detection.

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