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通匹配效應研究

Investigation of Flux Matching Effects Using Nb/
Antimony Telluride Bilayers with Artificial Pinning on
Anodic Aluminum Oxide Substrate

葉宇晟

Yu-Cheng Yeh

指導教授: 王立民 博士

Advisor: Li-Min Wang Ph.D.

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摘要

本實驗大致上可以分成兩個部分，首先為使用磁控濺鍍的方法製備 Nb 以及 Nb/Sb₂Te₃ 兩種薄膜樣品，傳統超導與拓撲材料的薄膜介面因鄰近效應成為非傳統超導材料-拓撲超導。在前人的努力下，我們已經掌握這兩種樣品在矽基板上的高品質鍍膜條件；Si(100) 基板上，Nb 在攝氏 540 度時有最好的品質，Sb₂Te₃ 則是需在攝氏 123 度。

但本實驗最重要的目的並非鎖定在製造拓撲超導，而是比較傳統超導以及拓撲超導產生的磁通量子匹配差異，與傳統超導的整數倍量子磁通不同，非傳統超導的量子磁通可能為半整數的性質，以鄰近效應造成之拓撲超導為例，自旋三重態會導致其量子磁通為半整數倍；透過將薄膜鍍在表面充滿高密度深孔的陽極氧化鋁基板，當樣品溫度低於超導溫度時，排磁性會使得每個孔洞成為一個人工的釘扎點，如此一來便可以將量子效應放大，穿過人工釘扎點的磁場強度基本單位必須為一個量子磁通量，其性質能容易被儀器所測量。

我們預期 Nb 薄膜樣品之量子磁通遵循 $H_n = nH_1$ ，以及 Nb/Sb₂Te₃ 雙層膜之量子磁通遵循 $H_n = (\frac{1}{2} + n)H_1$ ，其中因基板的釘扎結構使得 $H_1 \approx 100$ Oe。此週期性磁場可由統計方法運算並得出精確值。

關鍵字：超導體、磁通匹配、人工釘扎中心、鈮、碲化銻、陽極氧化鋁



Abstract

This experiment can be roughly divided into two parts. First, using the method of magnetron sputtering, we prepared two different thin film samples, Nb and Nb/Sb₂Te₃. Due to the proximity effect, the interface between conventional superconductor and topological material becomes unconventional superconductor, known as topological superconductor. Thanks to previous efforts, our senior have already optimized the deposition conditions for high-quality films of both materials on silicon substrate. On Si(100) substrates, Nb film shows high quality when deposited at 540°C, while Sb₂Te₃ requires a deposition temperature of 123°C.

However, the main goal of this experiment is not simply to fabricate a topological superconductor, but to compare the difference of flux matching effects between conventional and topological superconductors. Unlike conventional superconductors, which exhibit integer multiples of the magnetic flux quantum, unconventional superconductors may exhibit exotic quantization such as half-integer or one-third-integer flux quanta. In the case

of topological superconductors induced by the proximity effect, the spin-triplet pairing leads to half-integer flux quantization.



By depositing the films onto anodic aluminum oxide (AAO) substrates with a high-order nanopores, each pore becomes an artificial pinning center due to the Meissner effect when the sample is cooled below its superconducting critical temperature. This magnifies the quantum effect: the magnetic field penetrating the pinning sites must be quantized in units of the magnetic flux quantum, allowing for direct measurement using experimental instruments.

We expect that the flux quantization for the Nb film follows the relation $H_n = nH_1$, and for the Nb/Sb₂Te₃ bilayer it follows $H_n = (\frac{1}{2} + n)H_1$, where the artificial pinning array yields $H_1 \approx 100$ Oe. This periodic magnetic field structure can be computed and validated using statistical analysis methods.

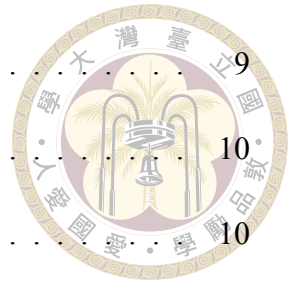
Keywords: Superconductors, Flux-matching effect, Artificial pinning center, Niobium, Antimony Telluride, Anodic Aluminum Oxide



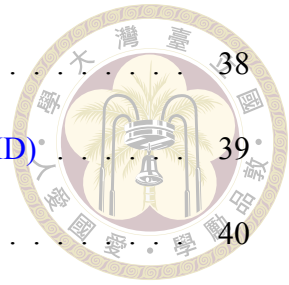
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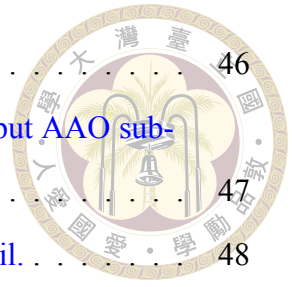




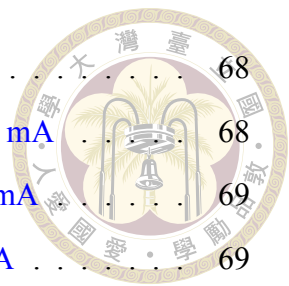
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Chapter 1 Introduction

1.1 Introduction of Superconductivity

1.1.1 History of Superconductor

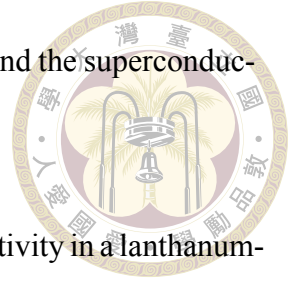
Superconductivity was first discovered by Dutch Heike Kamerlingh Onnes in 1911. While measuring the resistance of mercury at low temperatures, mercury surprisingly enters a state of zero resistance below 4.2 K [1].

In 1933, Walther Meissner and Robert Ochsenfeld discovered that superconductors not only exhibit zero resistance, but also expel magnetic fields from their interior—a phenomenon known as the Meissner effect. This showed that superconductivity is a thermodynamic phase rather than merely an ideal conductor [2].

Vitaly Ginzburg and Lev Landau proposed a theoretically introduction in 1950, describing superconductivity through order parameter. Their model will be discussed in detail in Chapter 2 [3].

In 1957, John Bardeen, Leon Cooper, and Robert Schrieffer formulated the microscopic theory of superconductivity, now known as BCS theory. It explained superconductivity in terms of Cooper pairs—pairs of electrons that move without scattering. This

marked a turning point in understanding the quantum mechanism behind the superconductivity [4].



In 1986, Georg Bednorz and Karl Müller discovered superconductivity in a lanthanum-based cuprate. The critical temperature amazingly increase to around 35 K. This sparked a global surge in research into high-temperature superconductors [5].

In the following year, 1987, a research team led by Paul Chu and Maw-Kuen Wu successfully synthesized a new cuprate compound, Yttrium Barium Copper Oxide (YBCO), with a critical temperature exceeding 90 K—well above the boiling point of liquid nitrogen (77 K) [6]. This breakthrough made liquid nitrogen-cooled superconductivity feasible and dramatically boosted practical applications and reduced the research costs. Figure 1.1 shows the development of superconductors. Nowadays, superconductors with a critical temperature above 77 K—the boiling point of liquid nitrogen—are referred to as high-temperature superconductors.

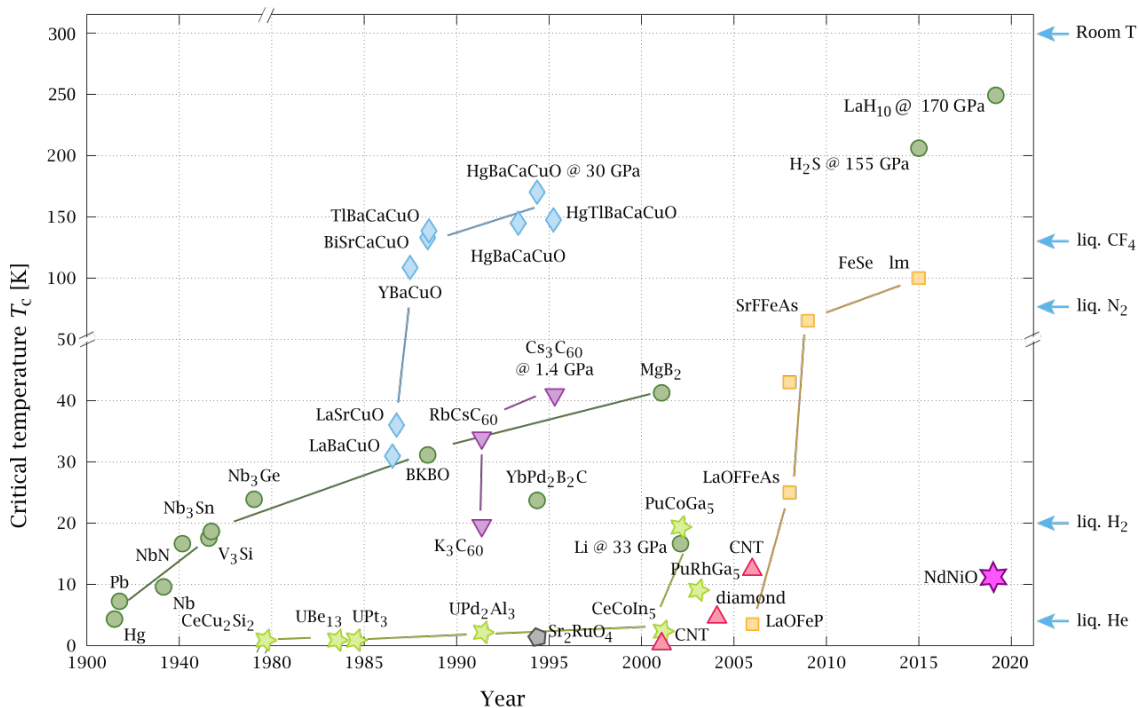


Figure 1.1: Timeline of superconductivity development.

Another breakthrough came in 2008 when iron-based superconductors were discovered after the significant progress of cuprate high-temperature superconductors. Unlike cuprates, which are based on copper-oxide planes, these materials are composed of layers of iron and pnictogen or chalcogen atoms. Despite containing magnetic iron atoms, these compounds exhibit critical temperatures up to 55 K, and their multi-band nature introduces new physics beyond conventional BCS theory [7].

1.1.2 Topological Insulators and Superconductors

Topological insulators (TIs) are quite special, while most of the material behaves as an insulator, the surface somehow behaves as conductors. The basic concept of this phenomenon can trace back to the discovery of the quantum Hall state in the 1980s. Then in 2005, Kane and Mele introduced a new idea: the quantum spin Hall effect (QSHE). Their research based on graphene, suggested that time-reversal symmetry could protect edge currents [8]. To make this precise, they introduced the $\mathbb{Z}_2$ topological invariant. When this number equals to one, the state corresponds to a topological insulator. In 2006, Bernevig, Hughes, and Zhang proposed that HgTe/CdTe quantum wells could support the QSHE [9]. The prediction was experimentally confirmed in 2007 by König et al. [10], marking the first direct observation of two-dimensional topological insulator. Soon after, in 2007, when researchers extended topological band theory to three-dimensional materials. The alloy $\text{Bi}_{1-x}\text{Sb}_x$ was discovered as the first 3D topological insulator [11, 12]. Then, in 2009, much more simpler materials like Bi_2Te_3 , Sb_2Te_3 were discovered to be 3D topological insulators [13].

Topological superconductors (TSCs) represent a new class of unconventional super-

conductors that host topologically protected surface or edge states. These surface states are often described as Majorana bound states. TSCs can arise intrinsically, as in some heavy-fermion compounds. Alternatively, TSCs can also be observed at the interface between a conventional superconductor and a topological insulator via the proximity effect.

In our research, we employ the interface between Sb_2Te_3 and Nb thin films to observe the topological superconducting state, which serves as a platform for investigating half-quantum magnetic fluxes.

1.2 Overview of the materials

1.2.1 Antimony telluride (Sb_2Te_3)

Due to its relatively low bulk conductivity and high surface mobility, Sb_2Te_3 has been extensively employed in heterostructures designed to realize topological superconductivity via the proximity effect [14]. When interfaced with usual superconductors like Nb, the induced superconductivity on its topological surface can mimic spinless p-wave pairing.

It crystallizes in a rhombohedral structure with a layered quintuple-layer unit, following the sequence Te-Sb-Te-Sb-Te. These QLs are held together by van der Waals forces. The lattice constant of Sb_2Te_3 $a = 4.262 \text{ \AA}$, $c = 30.435 \text{ \AA}$

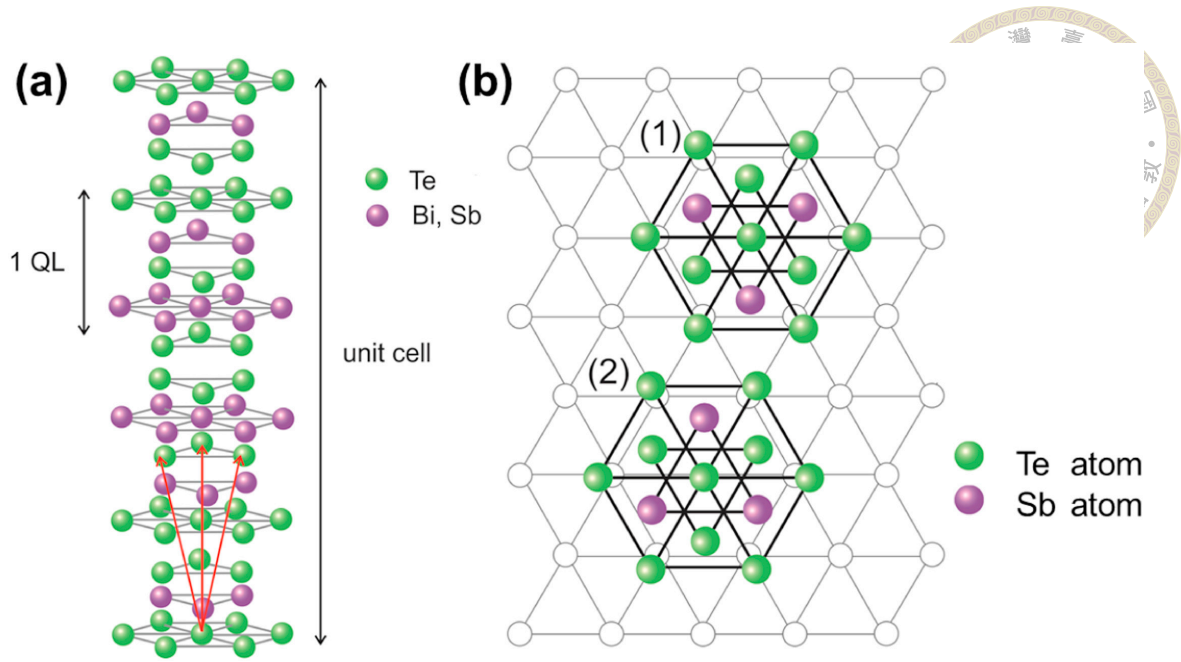


Figure 1.2: Schematic of Sb_2Te_3 structure.

1.2.2 Niobium (Nb)

Niobium (Nb) is a transition metal, It becomes superconducting below a critical temperature of approximately 9.2 K, making it one of the highest- T_c elemental superconductors. Nb is a type-II superconductor[15], widely used in superconducting devices and heterostructures. In proximity-effect-based systems, Nb often serves as a traditional s-wave superconductor. This characteristic makes Nb an ideal candidate for coupling with topological insulators such as Sb_2Te_3 .

Niobium crystallizes in a body-centered cubic (BCC) structure at room temperature, The lattice constant of niobium is approximately 3.30 Å.

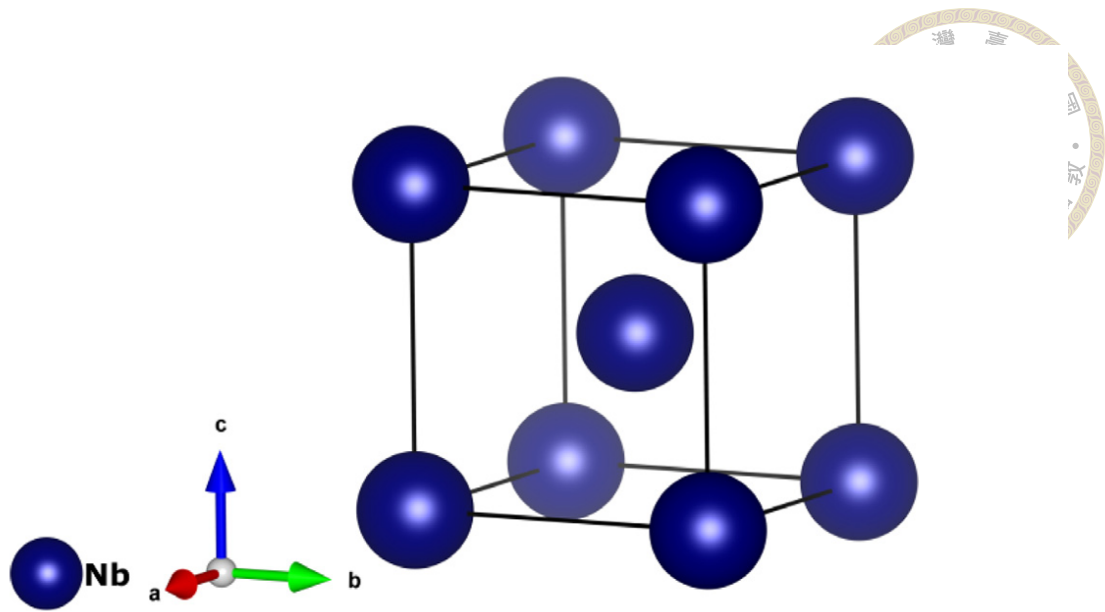


Figure 1.3: Schematic of Nb structure.

1.3 Electrical resistivity

Resistance of materials can be fundamentally explained by band theory, which accounts for the differences in electrical conductivity between metals, semiconductors, and insulators [16, 17].

In isolated atoms, electrons occupy discrete energy levels. However, when N atoms come together to form a solid, their atomic orbitals overlap, and each energy level splits into N closely spaced sub-levels. Due to the large number of atoms in a solid, these sub-levels form continuous energy bands. The two most important bands in this context are the valence band and the conduction band:

- The highest energy of valence band that is fully occupied by electrons at absolute zero.
- The conduction band lies above the valence band and contains energy states that electrons can occupy if they gain sufficient energy.

The energy gap between these two bands determines a material's conductivity. Metals typically have overlapping valence and conduction bands, or a partially filled conduction band, allowing electrons to move freely and resulting in low electrical resistance. Insulators possess a wide bandgap, making it extremely difficult for electrons to be thermally excited into the conduction band, hence exhibiting high resistance. Semiconductors have a moderate bandgap, allowing some electrons to be thermally excited to the conduction band, resulting in intermediate conductivity.

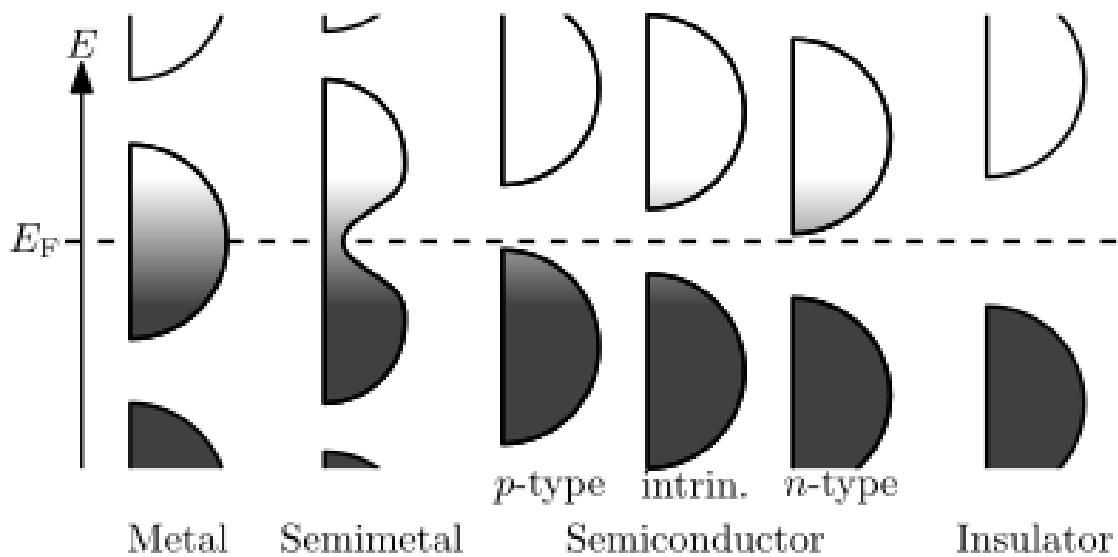


Figure 1.4: Band filling of different materials.

Beyond the band structure, At any temperature above absolute zero, the atoms in a crystal lattice vibrate, giving rise to phonons, which are quantized lattice vibrations. These phonons interact with conduction electrons, scattering them and impeding their motion. This electron-phonon scattering is a key source of resistance and increases with temperature due to the higher phonon population. Even at very low temperatures, metals do not exhibit zero resistance because of imperfections such as lattice defects, dislocations, and impurity atoms, which also scatter electrons. Unlike phonon scattering, this impurity scattering is largely temperature-independent [16, 17].



1.4 Propertes of superconductors

1.4.1 Meissner effect

The Meissner effect is a phenomenon of superconductivity, discovered in 1933 by Walther Meissner and Robert Ochsenfeld [2]. It refers to the expulsion of magnetic flux from the interior of a material when it transitions into the superconducting state below its critical temperature T_c . Unlike a perfect conductor, which merely prevents changes in magnetic flux, a superconductor actively excludes magnetic fields, surface currents are sgenerated near the surface of the superconductor. These currents circulate without resistance and produce a magnetic field that exactly cancels the applied magnetic field inside the material, ensuring that the internal magnetic field remains zero, resulting in a magnetic field $\vec{B} = 0$ inside the bulk. The Meissner effect is described quantitatively by the London equations [18], which predict an exponential decay of the magnetic field at the surface of the superconductor. We will discuss this in detail in Chapter 2.

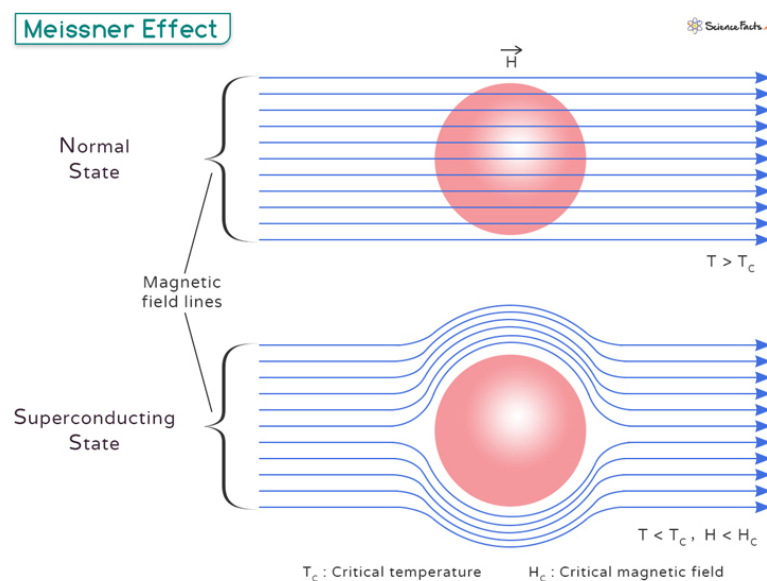


Figure 1.5: Schematic of Meissner effect, which shows the expulsion of magnetic flux



1.4.2 Different types of superconductors

Superconductors can be categorized into two main types according to how they respond to magnetic fields: type-I and type-II superconductors.

Type-I superconductors exhibit full Meissner effect, if the temperature and external magnetic field lower than critical temperature T_c and critical field H_c , it expel all the internal magnetic fields. When the external magnetic field larger than H_c , the superconducting state is destroyed, and the material reverts to its normal state. These superconductors generally have lower critical fields and are often elemental metals [19].

Magnetization, $-M$

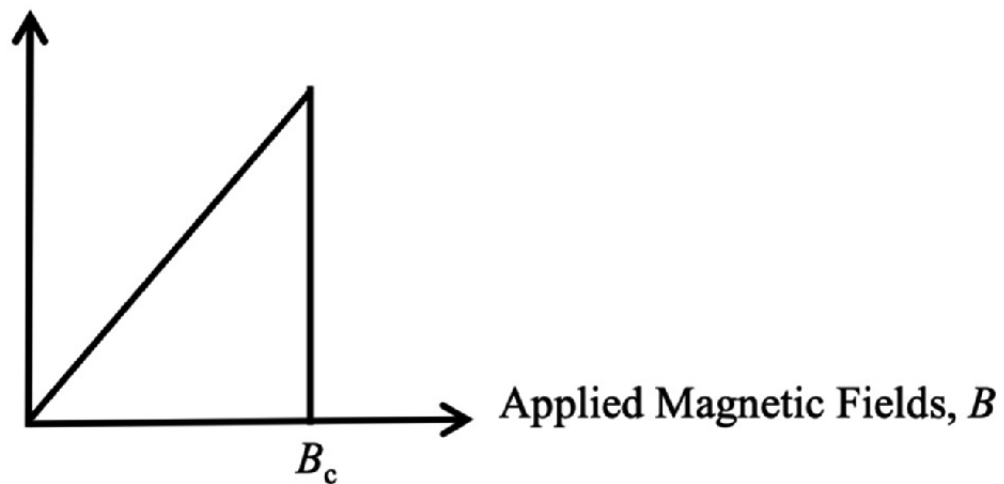


Figure 1.6: M-H diagram of type-I superconductor.

Type-II superconductors, on the other hand, shows the mixed state of the normal and superconducting state. They have two critical fields: H_{c1} and H_{c2} . When the external magnetic field lower than H_{c1} , the material performed just like Type-I superconductors, completely expels internal magnetic field. While the field between H_{c1} and H_{c2} , the material enters a mixed state. When the external magnetic field exceeds H_{c2} , the superconducting state is destroyed, superconductivity completely broken. Metallic alloys and all

high- T_c superconductors are typically type II superconductors [19].

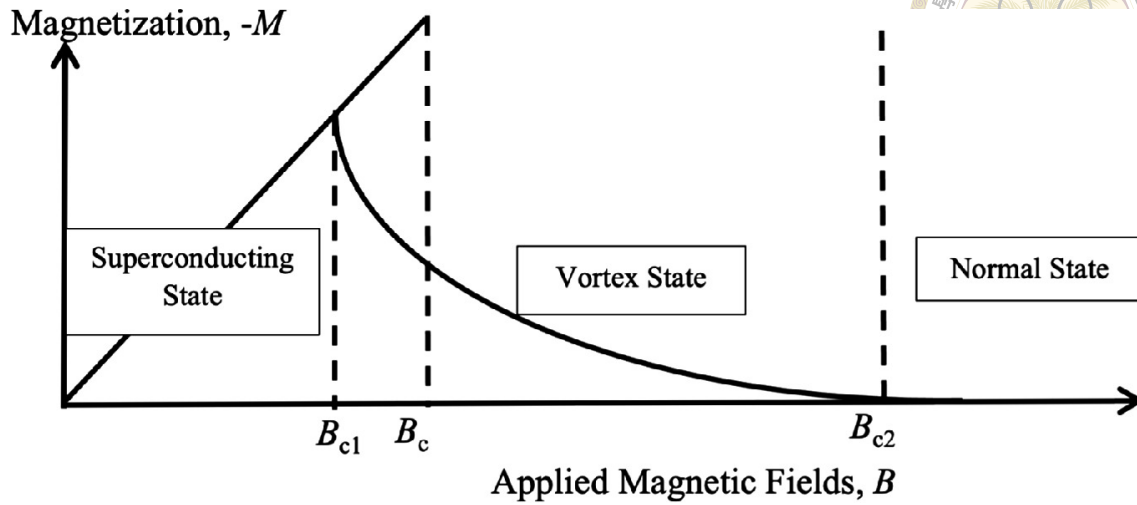


Figure 1.7: M-H diagram of type-II superconductor.

1.5 Study Background and Motivation

1.5.1 Literature Review

The quantization of magnetic flux in traditional superconductors has been established since the late 1990s. In particular, niobium (Nb) thin films—one of the main character in our experiment—have been fully studied under artificial pinning structures. In 1999, V. Metlushko et al. utilized anti-dot arrays fabricated on silicon substrates to demonstrate matching effects in Nb thin films [20]. Following this, Martin et al. (2000) use periodic arrays of Ni dots as artificial pinning centers (APCs) of Nb films [21]. Successfully measured clear pinning effects and their temperature dependence. In the same year, P. Prieto and Ivan K. Schuller compared magnetic (Ni) and non-magnetic (Au) materials as APCs [22], reveal how the magnetic properties and the dot size affect vortex pinning efficiency.

So far, these studies focused on square arrays of APCs as the basis for their experiments.

In 2001, Reichhardt et al. conducted numerical simulations to explore fractional matching effects in superconducting thin films with both square and triangular lattice configurations of pinning sites [23]. Our experiment using anodic aluminum oxide (AAO) substrate to obtain triangular lattice APCs, which naturally form a highly ordered hexagonal anti-dot lattice.

1.5.2 Fabrication of AAO Templates

AAO is a highly ordered nanoporous material formed by the electrochemical anodization of high-purity aluminum in acidic electrolytes. such electrolytes include selenic acid, sulfuric acid, oxalic acid, phosphoric acid, chromic acid, malonic acid, tartaric acid, citric acid, and malic acid.

The fabrication process of AAO can be roughly divided into several steps [24, 25]:

1. Initial Pore Formation

- A thin, compact barrier oxide layer first grows uniformly over the entire aluminum surface. maintain a limiting current j , and increase potential U linearly.
- Current j sharply drops to a minimum, then gradually increases due to the formation of electrolyte penetration paths.
- Further anodization leads to a gradual increase in j to a local maximum due to the ready diffusion of electrolyte.

- j reaches a steady value after passing through an overshoot. The current overshoot is related to the decrease of initial pore density as major pores grow and merge with adjacent ones.



2. Steady-State Pore Formation

- Larger pores continue to grow by merging with neighboring pores.
- During steady-state growth, the barrier layer thickness remains constant. This is due to a dynamic balance between oxide formation at the metal/oxide interface and oxide removal at the oxide/electrolyte interface.
- Structural parameters such as pore diameter, interpore distance, and barrier layer thickness can be precisely controlled by adjusting anodization conditions.

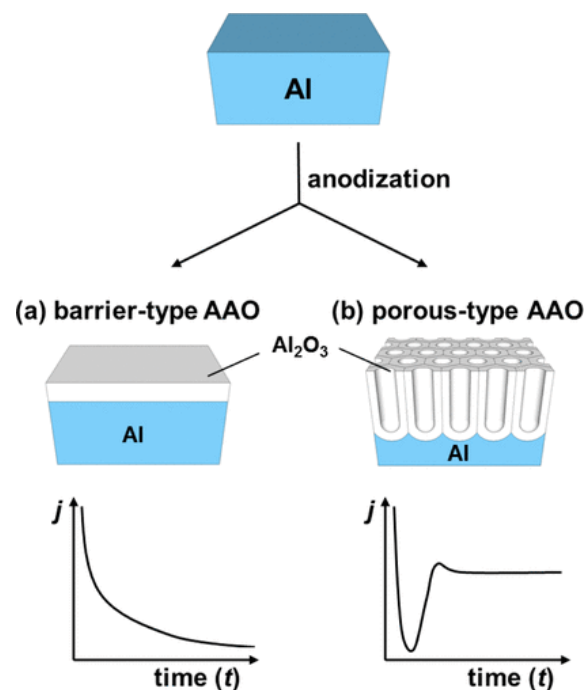


Figure 1.8: Control j to formed different type of AAO.

There's some notes of anodization conditions

- Potential and current density are the most important parameters to determine the AAO structure.
- When $U = \text{constant}$, j almost remains constant within a certain range, and the film thickness is linearly related to the anodization time.
- When $j = \text{constant}$, U increases linearly with time to remain the invariant of electric field.
- Temperature also affects pore diameter and interpore distance.



Figure 1.8 shows the schematic of AAO fabrication process.

1.5.3 Review of half-quantum flux effect

In 1987, Geshkenbein et al. theoretically predicted that the magnetic flux quantum in spin-triplet superconductors would exhibit a phase shift. Later in 1994, this concept was extended by C. C. Tsuei et al. to YBCO tricrystals with carefully engineered crystallographic boundaries [26]. In 2011, J. Jang et al. experimentally reported a distinct type of half-quantum flux effect in Sr_2RuO_4 , providing further evidence for unusual pairing symmetries [?].

Since the rise of interest in topological superconductors, they have been regarded as candidates for realizing spin-triplet superconductivity. In 2019, Chao Yang et al. utilized AAO substrates to fabricate YBCO thin films and investigate quantum phase transitions (Figure 1.9) [27]. In the same year, Yufan Li et al. demonstrated the presence of half-quantum magnetic flux in $-\text{Bi}_2\text{Pd}$ using a micro-ring silicon-based device [28]. Both works were published in Science, shows the growing significance and research potential

of this field. Our goal is to demonstrate the half-quantum flux effect by applying APCs

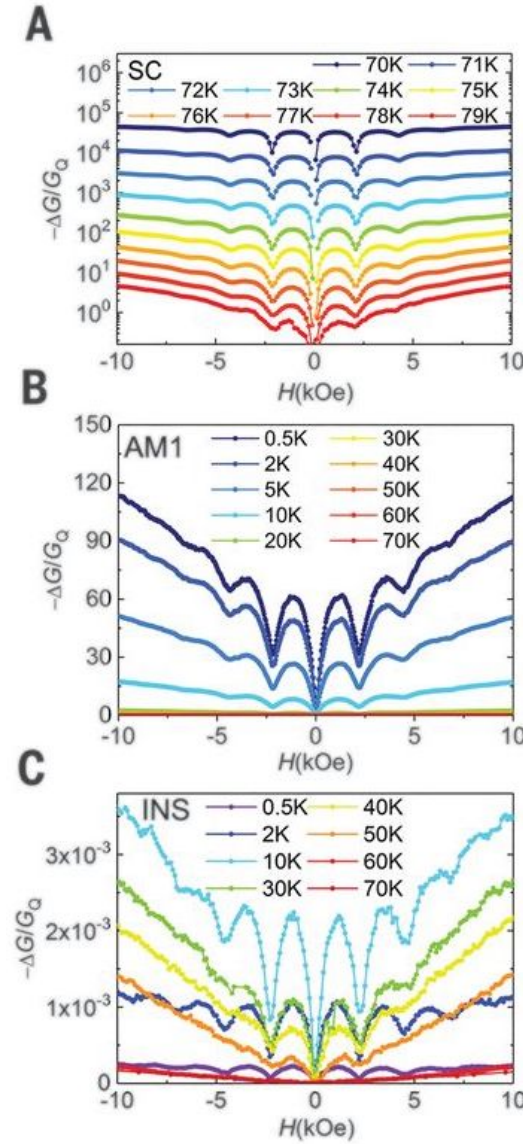


Figure 1.9: (A) Shows superconducting-state film, (B) Anomalous metallic-state film, (C) Insulating-state film.

in conjunction with topological superconductor. This is achieved by utilizing the proximity effect at the interface between niobium and antimony telluride to realize a S-P-S sandwich structure. The AAO substrate provides a well-ordered pores that function as APCs. Magnetic and electrical measurements are performed to characterize the resulting superconducting behavior.



Chapter 2 Principle and Theory

2.1 London penetration depth

To derive the London penetration depth, we begin with simple logic and the application of the equation of motion [29, 30]. If we give an electric field $\vec{E}$ to an electron without any resistance, it is trivial that the electron will accelerate indefinitely. the equation of motion is given by:

$$m \frac{d\vec{v}_s}{dt} = -e\vec{E} \quad (2.1)$$

where m is the mass of an electron, v_s is the velocity of superconducting electrons, and $e = 1.6 \times 10^{-19}$ C is the elementary charge. Next, we define the current density $\vec{J}_s$. Denote the number density of electrons as n_s , the current density is given by:

$$\vec{J}_s = -en_s\vec{v}_s \quad (2.2)$$

Diffenciate (2.2) to eliminate the velocity term, gives:

$$\frac{d\vec{J}_s}{dt} = -en_s \frac{d\vec{v}_s}{dt} \quad (2.3)$$

Substituting (2.1) into (2.3), we obtain:

$$\frac{d\vec{J}_s}{dt} = \frac{e^2 n_s}{m} \vec{E} \quad (2.4)$$



We call (2.4) the first London equation.

Using Faraday's law of induction,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (2.5)$$

We curl the (2.4)

$$\frac{d}{dt} (\nabla \times \vec{J}_s) = \frac{e^2 n_s}{m} \nabla \times \vec{E} \quad (2.6)$$

Using (2.5) in (2.6), which gives:

$$\frac{d}{dt} (\nabla \times \vec{J}_s) = -\frac{e^2 n_s}{m} \frac{\partial \vec{B}}{\partial t} \quad (2.7)$$

Now integral both sides of (2.7) with respect to time, we obtain:

$$\nabla \times \vec{J}_s = -\frac{e^2 n_s}{m} \vec{B} + C \quad (2.8)$$

Since the Meissner effect requires that the magnetic field inside a superconductor is zero, we set $C = 0$. Thus:

$$\nabla \times \vec{J}_s = -\frac{e^2 n_s}{m} \vec{B} \quad (2.9)$$

This is the second London equation.

To derive the London penetration depth, we need to relate the current density to the

magnetic field. Applying Ampère's law:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} \quad (2.10)$$



where μ_0 is the permeability of free space, I_{enc} is the total current enclosed by C . Using Stokes' theorem, we can rewrite (2.10) as:

$$\int_S (\nabla \times \vec{B}) \cdot d\vec{s} = \mu_0 I_{\text{enc}} \quad (2.11)$$

And the definition of the current density:

$$I_{\text{enc}} = \int_S \vec{J}_s \cdot d\vec{s} \quad (2.12)$$

Finally

$$\int_S (\nabla \times \vec{B}) \cdot d\vec{s} = \mu_0 \int_S \vec{J}_s \cdot d\vec{s} \quad (2.13)$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}_s \quad (2.14)$$

Our goal is to eliminate the current density from (2.9) and (2.13). First, curl the (2.14):

$$\nabla \times (\nabla \times \vec{B}) = \mu_0 (\nabla \times \vec{J}_s) \quad (2.15)$$

Using identities of vector calculus

$$\nabla \times (\nabla \times \vec{B}) = \underbrace{\nabla (\nabla \cdot \vec{B})}_{=0, \text{ Gauss's law}} - \nabla^2 \vec{B} \quad (2.16)$$

That is to say

$$\nabla \times (\nabla \times \vec{B}) = -\nabla^2 \vec{B} \quad (2.17)$$

(2.15) becomes:

$$-\nabla^2 \vec{B} = \mu_0 (\nabla \times \vec{J}_s) \quad (2.18)$$

(2.9) and (2.18) gives:

$$\nabla^2 \vec{B} = \frac{e^2 n_s}{m} \mu_0 \vec{B} \quad (2.19)$$

The London penetration depth, denoted as λ_L , is defined as the distance from the surface of a superconductor at which the intensity of an applied magnetic field has decreased to $\frac{1}{e}$ of its value at the surface. Which can be derived from (2.19):

$$\lambda_L = \sqrt{\frac{m}{\mu_0 e^2 n_s}} \quad (2.20)$$

where m is the mass of an electron, μ_0 is the permeability of free space, e is the elementary charge, and n_s is the number density of superconducting electrons. Rewrite (2.19) as

$$\nabla^2 \vec{B} = \frac{1}{\lambda_L^2} \vec{B} \quad (2.21)$$

Consider the solution of (2.21) in one dimension, we can assume the equation of the form:

$$\frac{\partial^2 B}{\partial x^2} = \frac{1}{\lambda_L^2} B \quad (2.22)$$

solve (2.22) gives:

$$B(x) = B_0 e^{-\frac{x}{\lambda_L}} \quad (2.23)$$

where B_0 is the magnetic field at the surface of the superconductor.



Note that the magnetic field inside a superconductor decays exponentially. Figure 2.1

[31]

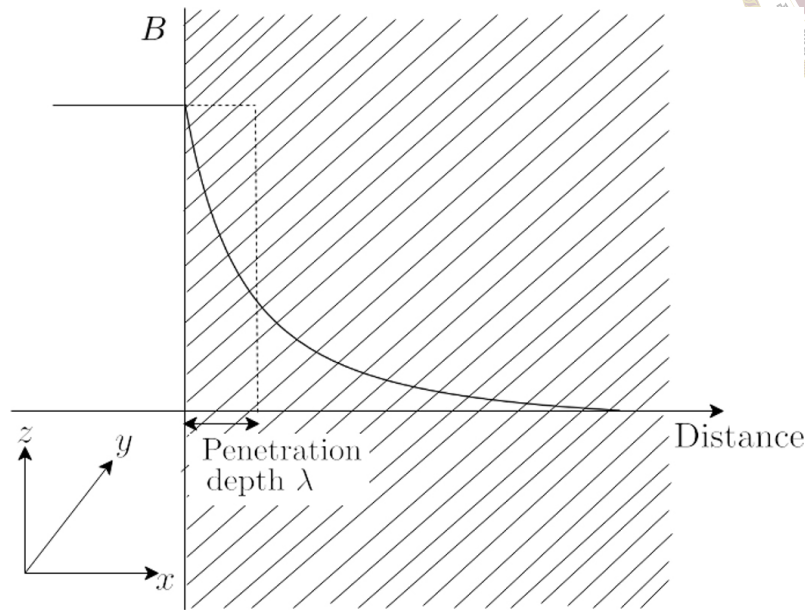


Figure 2.1: The schematic of London penetration depth, the magnetic field decays exponentially

2.2 Coherence Length

While the London theory describes how superconductors respond to electromagnetic fields, it assumes a perfectly rigid superconducting state. But how does the superconducting state itself change across space? We now turn to another important length scale in superconductivity: the coherence length. This quantity characterizes the size over which the superconducting order parameter (or wavefunction) can vary significantly in space. In other words, it describes how quickly the superconducting state can change over space.

Observe (2.9), the definition of magnetic vector potential $\vec{B} = \nabla \times \vec{A}$ allows us to

rewrite the second London equation as:

$$\nabla \times \vec{J}_s = -\frac{e^2 n_s}{m} (\nabla \times \vec{A}) \quad (2.24)$$

$$\vec{J}_s = -\frac{e^2 n_s}{m} \vec{A} \quad (2.25)$$

means ideally, the current reacts immediately to any change in the vector potential. However, in reality, the superconducting state cannot respond to changes infinitely fast. The behavior of electrons can be described using wave functions. If we try to force the wave function change too quickly, it will cost extra kinetic energy. This gives a constraint on how sharply the superconducting wave function can vary. That leads to the definition of the coherence length.

Starting from Schrodinger's equation for a superconducting wave function ψ :

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = E\psi \quad (2.26)$$

the kinetic energy term is given by:

$$T = \frac{p^2}{2m} = -\frac{\hbar^2}{2m} \nabla^2 \quad (2.27)$$

Consider a plane wave function of the form:

$$\psi(x) = e^{ikx} \quad (2.28)$$

Now, we introduce a small modulation: Superposing another plane wave with a slightly



different wavevector, resulting in a perturbed wave function of the form:

$$\psi(x) = \frac{1}{\sqrt{2}}(e^{ikx} + e^{i(k+a)x}) \quad (2.29)$$



combine (2.27) and (2.29), we have:

$$T = \frac{\hbar^2}{2m}k^2 \quad \text{for} \quad \psi = e^{ikx} \quad (2.30)$$

for $\psi(x) = \frac{1}{\sqrt{2}}(e^{ikx} + e^{i(k+a)x})$

$$T = \frac{\hbar^2}{2m} \frac{(2k^2 + 2ka + a^2)}{2} \approx \frac{\hbar^2}{2m}k^2 + \frac{\hbar^2}{2m}ka \quad (2.31)$$

If the increase in kinetic energy is larger than the energy gap E_g , it will break the superconducting state. Thus, we have the condition:

$$\frac{\hbar^2}{2m}ka_0 = E_g \quad (2.32)$$

define the coherence length $\xi_0 = \frac{1}{a_0}$

$$\xi_0 = \frac{\hbar^2 k}{2mE_g} = \frac{\hbar^2 v_F}{2E_g} \quad (2.33)$$

where v_F is the Fermi velocity. (2.33) gives a decent estimate of the coherence length. On BCS theory, the coherence length $\xi_0 = \frac{2\hbar^2 v_F}{\pi E_g}$. Figure 2.2 [32] shows the comparison of London penetration depth and coherence length.

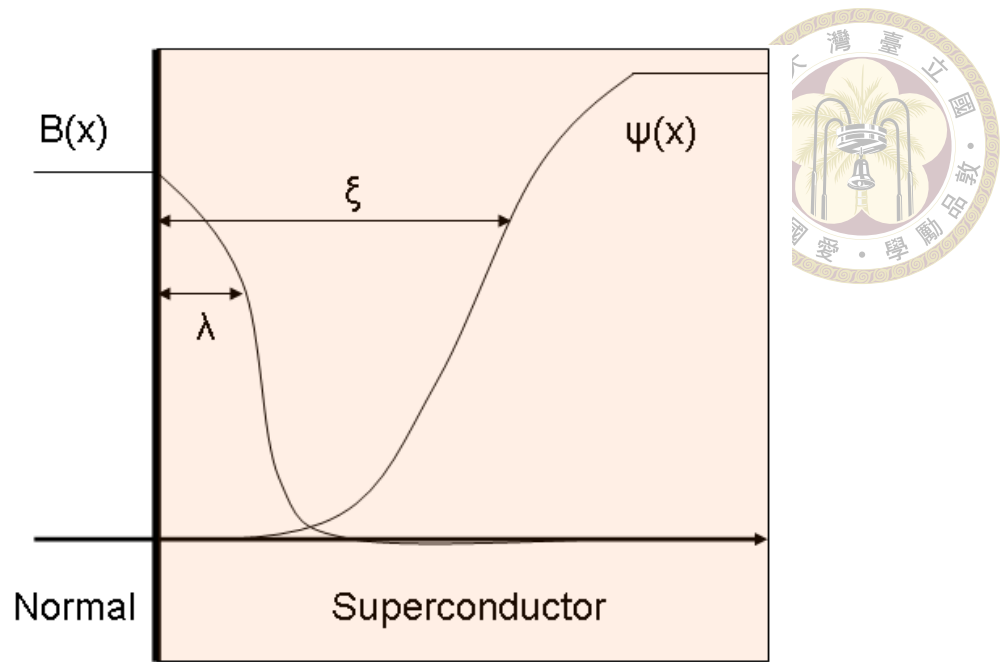


Figure 2.2: London penetration depth and coherence length

2.3 Ginzburg-Landau Theory

In order to describe superconductors in a more quantitative way, we need a theory that combines quantum effect, thermodynamics, and electromagnetism. This is exactly what the Ginzburg-Landau (GL) theory provides.

2.3.1 Landau's Theory of Second-Order Phase Transitions

A second-order phase transition describes a continuous transformation in which the system evolves from a disordered high-temperature phase to an ordered low-temperature phase with reduced symmetry [33]. The reduction in symmetry is spontaneous, a phenomenon known as spontaneous symmetry breaking. Although the resulting symmetry-broken states are macroscopically distinct, they may have the same free energy. To describe these ordered states, we introduce an order parameter that characterizes the properties of the broken symmetry.

Starting from total free energy:

$$F = \int f(\psi, T) d^3x \quad (2.34)$$



where $f(\psi, T)$ is the free energy density, ψ is the order parameter, and T is the temperature. Landau suggests that the free energy density can be expressed as a Taylor expansion in terms of the order parameter, and that the free energy must be invariant under the symmetries of the system. So we can only keep even terms in the expansion:

$$f(\psi, T) = f_0(T) + \alpha(T)|\psi|^2 + \frac{1}{2}\beta(T)|\psi|^4 + \mathcal{O}(|\psi|^6) + \gamma \nabla \psi \cdot \nabla \psi^* \quad (2.35)$$

since we are interested in the case around the critical temperature, we can ignore the higher-order terms.

2.3.2 Helmholtz free energy density

Now we are going to derive the Helmholtz free energy density [34]. When the system is placed in an electromagnetic field, the momentum operator must be modified to account for the vector potential. The momentum operator is given by:

$$\vec{p} \rightarrow \vec{p} - \frac{q}{c} \vec{A} \quad (2.36)$$

where $\vec{A}$ is the vector potential. For an electron, we take $q = e^*$, and by promoting the momentum to its quantum operator form, we obtain:

$$\vec{p} \rightarrow -i\hbar \nabla - \frac{e^*}{c} \vec{A} \quad (2.37)$$

So the last term in (2.35) becomes:

$$\gamma \nabla \psi \cdot \nabla \psi^* = \gamma \left(-i\hbar \nabla - \frac{e^*}{c} \vec{A} \right) \cdot \left(-i\hbar \nabla - \frac{e^*}{c} \vec{A} \right)^* \quad (2.38)$$

Denote $f(\psi, T) = f_s$, $f_0(T) = f_n$, choose $\gamma = \frac{1}{2m^*}$ for normalization, and adding the magnetic field energy term $\int \frac{\mu_0}{2} B^2 dr$ to the free energy density, we have:

$$f_s = f_0(T) + \alpha(T)|\psi|^2 + \frac{1}{2}\beta(T)|\psi|^4 + \frac{\hbar^2}{2m^*} \left| \left(\nabla - \frac{ie^*}{\hbar c} \vec{A} \right) \psi \right|^2 + \frac{B^2}{8\pi} \quad (2.39)$$

where m^* is the effective mass, f_n called normal state, and e^* is the effective charge (usually equals to $2e$ for Cooper pairs). The last term is equivalent to $\frac{1}{2\mu_0} B^2$ for MKS units.

2.3.3 Derivation of First Ginzburg-Landau Equation

Define the effective number density of superconducting electrons n_s as:

$$n_s^* = |\psi|^2 = \psi^*(r)\psi(r) \quad (2.40)$$

Ginzburg and Landau expressed the order parameter ψ as a wave function, which can be written as:

$$\psi(r) = |\psi(r)|e^{i\phi(r)} \quad (2.41)$$

where $|\psi(r)|$ is the amplitude of the wave function and $\phi(r)$ is the phase of the wave function.

Now we proceed to derive the Ginzburg-Landau (GL) equations. These equations are obtained by minimizing the Ginzburg-Landau free energy functional with respect to variations in both the order parameter ψ and the vector potential $\vec{A}$. To perform this mini-

mization, we apply the method of variational calculus.

We begin by taking the functional derivative of the free energy with respect to the order parameter ψ , treating ψ and ψ^* as independent variables. Let



$$\psi \rightarrow \psi + \delta\psi, F \rightarrow F + \delta F$$

This leads to the first Ginzburg-Landau equation:

$$\delta F = \int \delta f_s d^3x \quad (2.42)$$

when deriving the Ginzburg-Landau equations, we take the variation with respect to the order parameter complex conjugate ψ^* .

$$\delta F = \int \left[\frac{\partial f}{\partial \psi^*} \delta\psi^* + \frac{\partial f}{\partial \nabla \psi^*} \cdot (\nabla \delta\psi^*) \right] d^3x \quad (2.43)$$

Integrating by parts, we obtain:

$$\delta F = \int \left[\frac{\partial f}{\partial \psi^*} - \nabla \cdot \frac{\partial f}{\partial \nabla \psi^*} \right] \delta\psi^* + (\text{boundary terms}) d^3x \quad (2.44)$$

set $\delta F = 0$

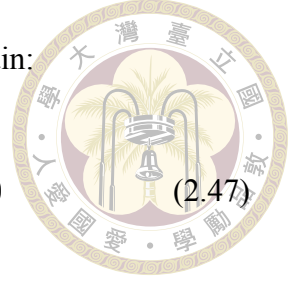
$$\frac{\partial f}{\partial \psi^*} - \nabla \cdot \frac{\partial f}{\partial \nabla \psi^*} = 0 \quad (2.45)$$

Recall that:

$$\left| \left(\nabla - \frac{ie^*}{\hbar c} \vec{A} \right) \psi \right|^2 = \left(\nabla \psi - \frac{ie^*}{\hbar c} \vec{A} \psi \right) \left(\nabla \psi^* + \frac{ie^*}{\hbar c} \vec{A} \psi^* \right) \quad (2.46)$$

Substituting (2.39) into (2.45), after computing the variation, we obtain:

$$\alpha(T)\psi + \beta(T)|\psi|^2\psi - \frac{\hbar^2}{2m^*} \left(\nabla - \frac{ie^*}{\hbar c} \vec{A} \right)^2 \psi = 0 \quad (2.47)$$



2.3.4 Derivation of Second Ginzburg-Landau Equation

To obtain the second Ginzburg-Landau equation, we minimize the GL free energy functional with respect to the vector potential $\vec{A}$. Let

$$\psi \rightarrow \psi + \delta\psi, F \rightarrow F + \delta F$$

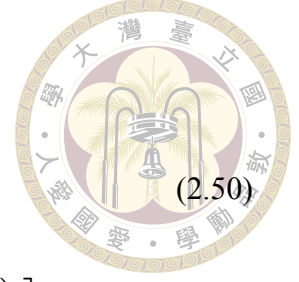
this time:

$$\delta F = \int \delta \left[\frac{\hbar^2}{2m^*} \left(\nabla\psi - \frac{ie^*}{\hbar c} \vec{A}\psi \right) \left(\nabla\psi^* + \frac{ie^*}{\hbar c} \vec{A}\psi^* \right) + \frac{(\nabla \times \vec{A})^2}{8\pi} \right] d^3x \quad (2.48)$$

for the last term, we have:

$$\delta \left(\frac{(\nabla \times \vec{A})^2}{8\pi} \right) = \frac{1}{4\pi} (\nabla \times \vec{A}) \cdot (\nabla \times \delta\vec{A}) \quad (2.49)$$

for the first term, we have:



$$\delta \left[\frac{\hbar^2}{2m^*} \left(\nabla \psi - \frac{ie^*}{\hbar c} \vec{A} \psi \right) \left(\nabla \psi^* + \frac{ie^*}{\hbar c} \vec{A} \psi^* \right) \right] \quad (2.50)$$

$$= \frac{\hbar^2}{2m^*} \left[-\frac{ie^*}{\hbar c} \psi \left(\nabla \psi^* + \frac{ie^*}{\hbar c} \vec{A} \psi^* \right) + \frac{ie^*}{\hbar c} \psi^* \left(\nabla \psi - \frac{ie^*}{\hbar c} \vec{A} \psi \right) \right] \delta \vec{A} \quad (2.51)$$

$$= \frac{ie^* \hbar}{2m^* c} \left[\psi^* \left(\nabla \psi - \frac{ie^*}{\hbar c} \vec{A} \psi \right) - \psi \left(\nabla \psi^* + \frac{ie^*}{\hbar c} \vec{A} \psi^* \right) \right] \cdot \delta \vec{A} \quad (2.52)$$

$$= \left[\frac{ie^* \hbar}{2m^* c} (\psi^* \nabla \psi - \psi \nabla \psi^*) + \frac{e^{*2}}{2mc} \vec{A} |\psi|^2 \right] \cdot \delta \vec{A} \quad (2.53)$$

Same, we set $\delta F = 0$ and combine (2.49), (2.53):

$$\int \left\{ \frac{\vec{B} \cdot (\nabla \times \delta \vec{A})}{4\pi} + \left[\frac{ie^* \hbar}{2m^* c} (\psi^* \nabla \psi - \psi \nabla \psi^*) + \frac{e^{*2}}{2mc} \vec{A} |\psi|^2 \right] \cdot \delta \vec{A} \right\} d^3x = 0 \quad (2.54)$$

use the vector calculus identity

$$\nabla \cdot (\vec{B} \times \delta \vec{A}) = \delta \vec{A} \cdot (\nabla \times \vec{B}) - \vec{B} \cdot (\nabla \times \delta \vec{A}) \quad (2.55)$$

Now, let us carefully examine each term within the integral. Note that

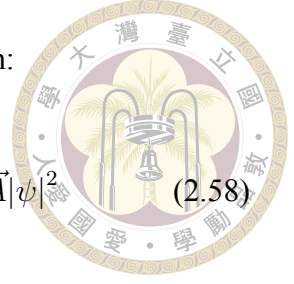
$$\int \nabla \cdot (\vec{B} \times \delta \vec{A}) d^3x = \int_S (\vec{B} \times \delta \vec{A}) \cdot d\vec{s} = 0 \quad (2.56)$$

Collecting the remaining terms, we obtain:

$$\left[\frac{(\nabla \times \vec{B})}{4\pi} + \frac{ie^* \hbar}{2m^* c} (\psi^* \nabla \psi - \psi \nabla \psi^*) + \frac{e^{*2}}{2mc} \vec{A} |\psi|^2 \right] \cdot \delta \vec{A} = 0 \quad (2.57)$$

Finally, by definition, we have the second Ginzburg–Landau equation:

$$\vec{J}_s = \frac{c}{4\pi} \nabla \times \vec{B} = -\frac{ie^*\hbar}{2m^*}(\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{e^{*2}}{2mc} \vec{A} |\psi|^2 \quad (2.58)$$



2.3.5 The Coherence Length in Ginzburg-Landau Theory

It's a simple application of the Ginzburg-Landau theory to derive the coherence length. Recall the first Ginzburg-Landau equation (2.47):

$$\alpha(T)\psi + \beta(T)|\psi|^2\psi - \frac{\hbar^2}{2m^*} \left(\nabla - \frac{ie^*}{\hbar c} \vec{A} \right)^2 \psi = 0 \quad (2.59)$$

Assume we have no extra magnetic field, $\vec{A} = 0$, and the order parameter is real

$$\alpha(T)\psi + \beta(T)|\psi|^2\psi - \frac{\hbar^2}{2m^*} \nabla^2 \psi = 0 \quad (2.60)$$

And consider a perturbation

$$\psi \rightarrow \psi_\infty + \delta\psi \quad (2.61)$$

Substituting (2.61) into (2.60), we have:

$$\alpha(T)\delta\psi + \beta(T)|\delta\psi|^2\delta\psi - \frac{\hbar^2}{2m^*} \nabla^2 \delta\psi = 0 \quad (2.62)$$

Assuming $\delta\psi$ is small, we can ignore the $|\delta\psi|^2$ term, and we have:

$$\alpha(T)\delta\psi - \frac{\hbar^2}{2m^*} \nabla^2 \delta\psi = 0 \quad (2.63)$$

The solution of (2.63) is given by:

$$\delta\psi = \delta\psi_0 e^{-\frac{x}{\xi}} \quad (2.64)$$



where $\delta\psi_0$ is a constant, and the coherence length ξ is defined as:

$$\xi = \sqrt{\frac{\hbar^2}{2m^*|\alpha(T)|}} \quad (2.65)$$

2.3.6 Other Considerations

We have explored the Ginzburg-Landau equations and the coherence length, it is worth to exam the temperature dependence of the expansion coefficients, especially $\alpha(T)$ and $\beta(T)$. Landou's theory suggests that make Taylor expansion of the coefficient around the critical temperature T_c :

$$\alpha(T) \approx a(T - T_c) = a'(t - 1), \quad t = \frac{T}{T_c} \quad (2.66)$$

where a is a positive constant. The coefficient $\beta(T) \propto \frac{1}{(1+t^2)^2} \approx 1$, which is independent of temperature. Actually, if we recall 2.35 from the very beginning, and try to minimizing the free energy density,

$$\frac{df_s}{d|\psi|} = 0 \Rightarrow \alpha(T) + \beta(T)|\psi|^2 = 0 \quad (2.67)$$

This gives us

$$|\psi|^2 = -\frac{\alpha(T)}{\beta(T)} \propto (1 - t) \quad (2.68)$$

Rewrite London penetration depth from (2.20). Now we have effective version:

$$\lambda_{\text{eff}} = \sqrt{\frac{m^*}{\mu_0 e^{*2} n_s^*}} = \sqrt{\frac{m^*}{\mu_0 e^{*2} |\psi|^2}} \propto (1 - t)^{-1/2} \quad (2.69)$$



The coherence length ξ is also temperature dependent:

$$\xi = \sqrt{\frac{\hbar^2}{2m^* |\alpha(T)|}} \propto (1 - t)^{-1/2} \quad (2.70)$$

Now we introduce Ginzburg–Landau parameter κ , defined as the ratio of the London penetration depth to the coherence length:

$$\kappa = \frac{\lambda_{\text{eff}}}{\xi} \quad (2.71)$$

since both λ_{eff} and ξ are proportional to $(1 - t)^{-1/2}$, the coefficient is independent of temperature.

The Ginzburg-Landau (GL) parameter involve the ratio of two length scales: the penetration depth λ , which describes how fast magnetic fields decay inside a superconductor. The other is the coherence length ξ , the typical distance over which the superconducting order parameter can change significantly. Superconductors with $\kappa < \frac{1}{\sqrt{2}}$ are known as Type I superconductors. In contrast, superconductors with $\kappa > \frac{1}{\sqrt{2}}$ are classified as Type II superconductors. Forming a so-called mixed state between two critical fields, H_{c1} and H_{c2} .

2.4 Flux Quantizing



Flux quantization refers to the phenomenon where the magnetic flux enclosed by a superconducting ring (or toroid) takes on only discrete values, quantized in units of the magnetic flux quantum Φ_0 . This arises naturally from both the London theory and the Ginzburg-Landau (GL) theory of superconductivity.

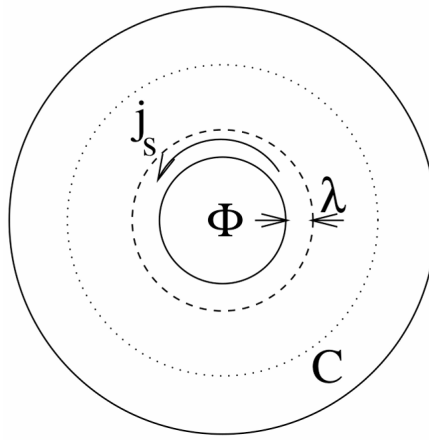


Figure 2.3: Quantization of flux

Consider a superconducting circle with a circulating supercurrent. (Figure 2.3) [30] The magnetic field is expelled from the bulk of the superconductor due to the Meissner effect and only penetrates a thin surface layer of thickness λ_L (the London penetration depth). We define a closed contour C inside the superconductor. According to GL theory, taking (2.41)

$$\vec{v}_s = \frac{1}{m^*} \left(\hbar \nabla \phi - \frac{e^*}{c} \vec{A} \right) \psi \quad (2.72)$$

the line integral over this closed path yields:

$$\oint_C \vec{v}_s \cdot d\vec{l} = \oint_C \frac{1}{m^*} \left(\hbar \nabla \phi - \frac{e^*}{c} \vec{A} \right) \cdot d\vec{l} = 0 \quad (2.73)$$

Because the wavefunction must be single-valued, the phase ϕ can only change by an inte-

ger multiple of 2π when going around the loop. This implies:

$$\oint_C \nabla \phi \cdot d\vec{l} = 2\pi n, \quad n \in \mathbb{Z} \quad (2.74)$$



Meanwhile, using Stokes' theorem:

$$\oint_C \vec{A} \cdot d\vec{l} = \int_S \nabla \times \vec{A} \cdot d\vec{s} = \int_S \vec{B} \cdot d\vec{s} \quad (2.75)$$

which is the total magnetic flux enclosed by the loop. So the (2.73) becomes:

$$\oint_C \vec{A} \cdot d\vec{l} = \frac{2\pi n m^*}{e^*} \quad (2.76)$$

2.5 Flux Matching Effect

In type-II superconductors, when an external magnetic field is applied above the lower critical field H_{c1} , magnetic flux penetrates the material in the form of quantized vortices. These vortices can be influenced by artificial pinning centers such as nanoholes or magnetic dots, which are often arranged in a regular lattice. The flux matching effect occurs when the density of vortices becomes commensurate with the density of pinning sites. Our research using the Anodic Aluminum Oxide (AAO) template, the substrate are arranged in a hexagonal pattern. When the density of vortices matches the density of pinning sites, each vortex can be trapped by a pinning center, leading to a highly ordered vortex lattice. Matching conditions can also occur at integer multiples or fractions of the fundamental matching field.

In the context of artificial pinning centers, there exists a concept known as the matching field. The matching field is defined as the minimum external magnetic field required for one magnetic flux quantum ϕ_0 to pass through each individual pinning site. The matching field can be approximated as:

$$H_1 = \frac{\phi_0}{A_p} \quad (2.77)$$

where A_p is the area of a single pinning site. We can also define the matching field for multiple flux quanta:

$$H_n = \frac{n\phi_0}{A_p} \quad n \in \mathbb{N} \quad (2.78)$$

It's important to note that the matching field is not a fixed value but rather depends on the pinning sites and the geometry of the sample.

2.6 Fractional Matching Effect

In topological insulator/superconductor hybrid structures, fractional matching effects have been reported in previous research. This behavior can be attributed to the interaction between superconductivity and topological surface states (Figure 2.4) [28] .In spin-triplet superconductors, half-quantum magnetic fluxes have been predicted by Geshkenbein, Larkin, and Barone (GLB) to be energetically more favorable than integer ones.

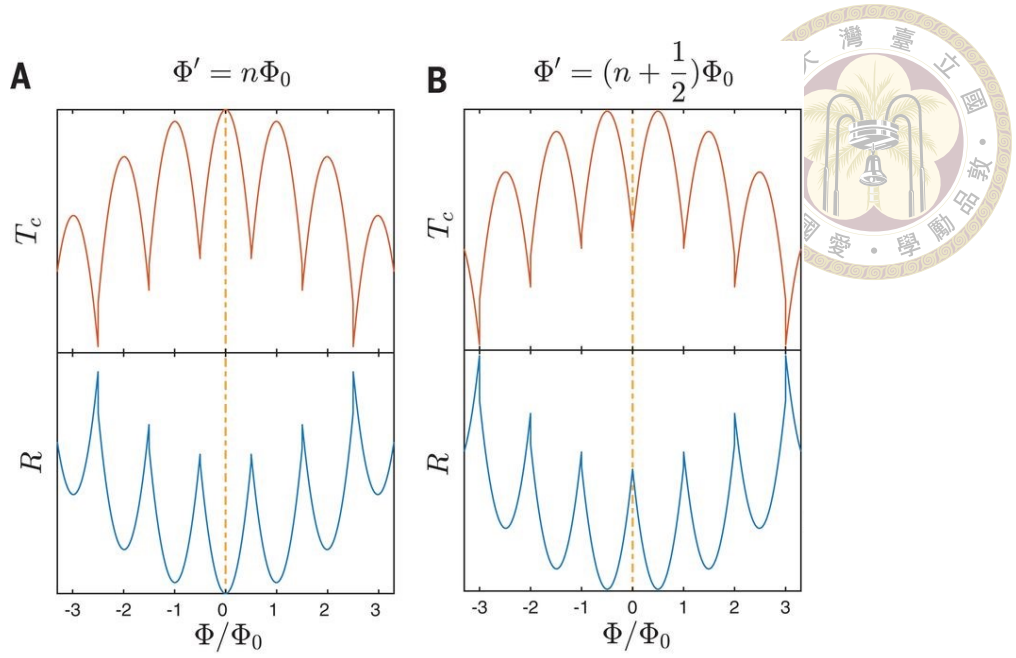


Figure 2.4: Integer and half-quantum flux.

2.6.1 Josephson Junctions

The Josephson effect describes the flow of supercurrent between two superconductors separated by a very thin insulating or non-superconducting barrier, forming a structure known as a Josephson junction. As Figure 2.5 [30] shows, despite the presence of the barrier, Cooper pairs can tunnel through it without resistance, giving rise to supercurrents. The supercurrent I_s across a Josephson junction is related to the phase difference $\Delta\phi$ between the wave functions of the two superconductors:

$$I_s = I_c \sin \Delta\phi \quad (2.79)$$

where I_c is the critical current, the maximum supercurrent that can flow through the junction without resistance.

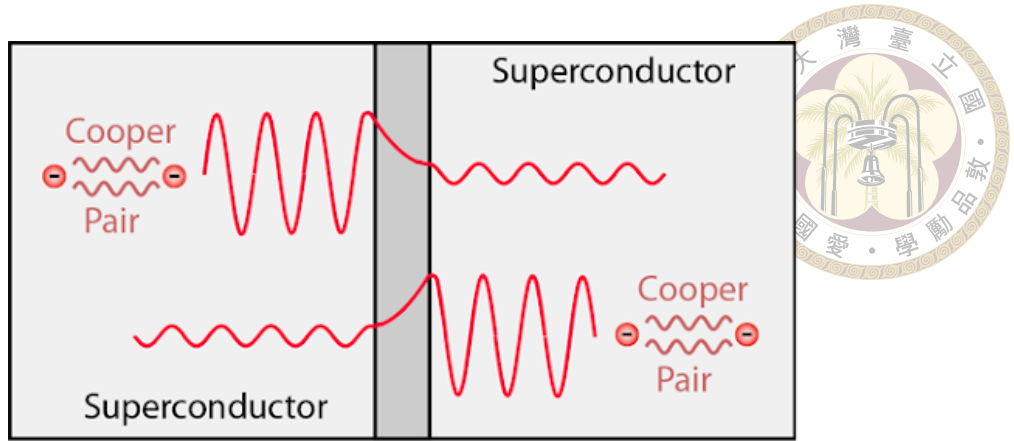


Figure 2.5: Cooper pair tunnel through Josephson junction

2.6.2 Half-quantum Magnetic Fluxes

To investigate unconventional superconductivity, GLB consider a Josephson junction composed of two usual superconductors (S) connected through an unusual superconductor (P), forming an S-P-S sandwich structure (Figure 2.6) [35]. The unusual superconducting

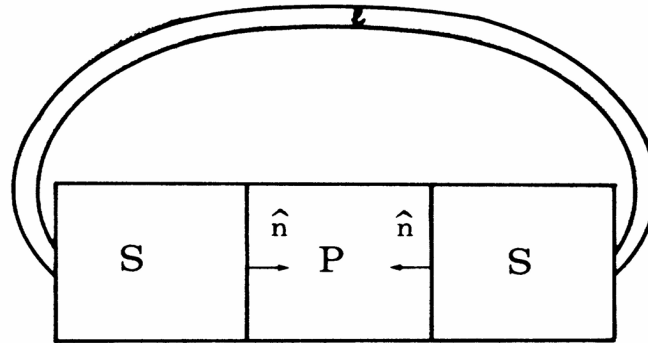


Figure 2.6: S-P-S sandwich structure

order parameter is much more complicated than the conventional one, and it can generally be expressed in tensor form as:

$$\Delta_{\alpha\beta} = \sum_i \eta^i \psi_{\alpha,\beta}^i(\vec{k}) \quad (2.80)$$

where η^i are the components of the order parameter without phases, and $\psi_{\alpha,\beta}^i(\vec{k})$

are the components of the wave function with phases. The case of order parameter of crystal symmetry in the one-dimensional representation of the crystal symmetry group. And a single crystal as the same form as conventional superconductor, which contains one single quantum flux. And they consider The total free energy of the system by integral order parameters, due to weak Josephson coupling across both S-P and P-S boundaries, can be written as:

$$F = A \operatorname{Re}\{\eta_1 \eta_2^*\} \quad (2.81)$$

The case (Figure 2.7 [36]) demonstrate a border of three crystal grains, the free energy becomes:

$$F = F_{12} + F_{23} + F_{31} \quad (2.82)$$

$$= \operatorname{Re}\{A_{12}\eta_1\eta_2^* + A_{23}\eta_2\eta_3^* + A_{31}\eta_3\eta_1^*\} \quad (2.83)$$

Suppose the energy minimum is appear on a boundary (for example, the 1-2), than the phase difference is π and the others becomes zero. That is

$$\Delta\phi = \Delta\phi_1 + \Delta\phi_2 + \Delta\phi_3 = \pi \quad (2.84)$$

Recall the (2.74), where the $\Delta\phi$ would shift by π phases, thus the contour there exists a half quantum flux.

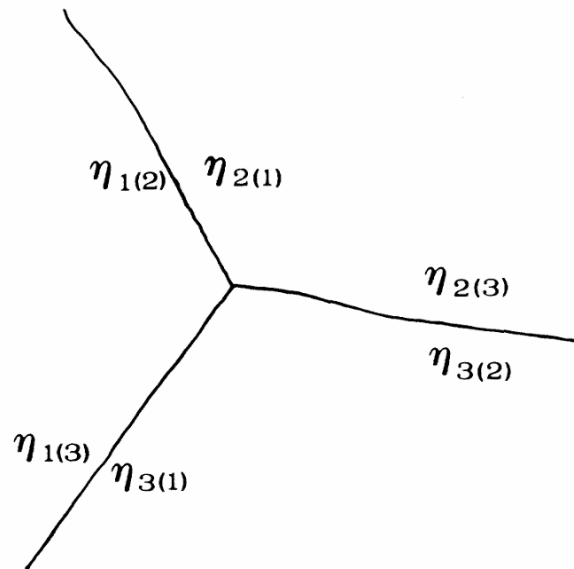


Figure 2.7: Order parameter of grains boundaries



Chapter 3 Experiment Methods

3.1 Setup of Systems

3.1.1 Magnetron Sputtering

Magnetron sputtering is a physical vapor deposition (PVD) technique commonly used for thin film fabrication. It relies on the ejection of atoms from a solid target material by bombarding it with energetic ions, typically from an inert gas like argon. In this method, a magnetic field is applied perpendicular to the electric field near the target surface. This configuration traps electrons in closed paths, which markedly raises the chance of ionizing collisions with the argon atoms.

In our setup, we utilized direct current (DC) and radio frequency (RF) magnetron sputtering techniques to deposit different types of thin films under high-vacuum conditions. DC magnetron sputtering for Nb and RF magnetron sputtering for Sb_2Te_3 . When we use metallic targets, it is conductive, allowing charge to move easily. However, insulating targets cannot conduct electricity. As a result, when argon ions bombard the surface during sputtering, positive charge gradually accumulates on the target surface and cannot be neutralized. This accumulation of charge generate the arcs, which is caused by sud-

denly discharges that can damage the film quality and the equipment. Moreover, once the surface charge reaches a certain threshold, it repels incoming argon ions and prevent further ion bombardment. Consequently, the sputtering process comes to a halt. Since Nb is conductive, DC sputtering is efficient and stable, allowing us to grow superconducting Nb thin films. And for the insulating target, which means Sb_2Te_3 , we use the RF magnetron sputtering. RF sputtering does not restrict the type of target material, making it suitable for both conductive and insulating targets. Because electrons are much more mobile than positive ions, the alternating electric field causes electrons to neutralize the positive surface charge during each cycle. This prevents excessive charge buildup. Additionally, a copper backing plate is often placed behind the target to serve as an electrode and help distribute the RF power uniformly. Through this oscillating charge balance, the sputtering of insulating materials can proceed continuously without arcing.

3.1.2 Superconducting Quantum Interference Device (SQUID)

The superconducting quantum interference device (SQUID) is one of the most sensitive instruments available for detecting extremely small magnetic fields. It relies on the Josephson effect and flux quantization. When an external magnetic field is applied to the SQUID loop (Figure 3.1 [30]), it changes the phase difference between the Josephson junctions. This phase modulation affects the total current through the device via quantum interference, resulting in a periodic voltage response as a function of the applied magnetic flux.

In this study, we use these two measurement platforms: MRMS2 (Figure 3.2) and MRMS3 (Figure 3.3). The MRMS3 system was used for magnetic property measurements,

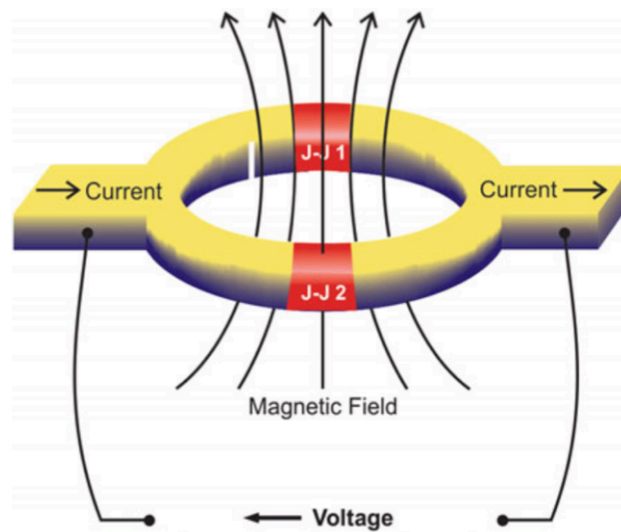


Figure 3.1: Design of SQUID

and the MRMS2 system was applied to electrical property measurements. Both systems use liquid helium as the cooling medium to maintain cryogenic condition, and are capable of reaching temperatures as low as 2K.

3.2 Experiment Procedure

Figure 3.4 shows the framework of our study.

3.2.1 Preparation Process of Antimony telluride Target

The fabrication process of the Sb_2Te_3 target follows the procedure of our senior [37].

1. Te powder with 99.5% purity and Sb powder with 98.5% purity were weighed and mixed uniformly in a mass ratio of 2:3.



Figure 3.2: Our MRMS 2



Figure 3.3: MRMS 3



Substrate Cutting

The AAO substrate is first diced into appropriately sized pieces (2.5 mm x 5 mm) suitable for deposition and measurement.



Target Preparation

The antimony telluride (Sb_2Te_3) target needs to be re-prepared every few months.



Substrate Processing

AAO substrate needs to be affixed onto Si substrate. Additionally, desired pattern must be pre-fabricated on the AAO surface



Magnetron Sputtering Deposition

DC sputtering is used for Nb, while RF sputtering is applied for Sb_2Te_3 .



Magnetic and Electrical Characterizations

After deposition, using MRMS 2 for electrical property measurements and MRMS 3 for magnetic property measurements.

Figure 3.4: Experiment Procedure

- 
2. The mixed powder was sealed into a quartz tube under vacuum to prevent oxidation during the heat treatment process.
 3. The sealed quartz tube was placed in a high-temperature furnace.
 4. The temperature was heated up to 850 °C at a rate of 2 °C per minute.
 5. Keep temperature at 850 °C for 12 hours to ensure the reaction.
 6. Decrease the furnace temperature to 650 °C at a rate of 2 °C per hour, then followed by natural cooling to room temperature.
 7. The obtained Sb_2Te_3 crystal was ground into fine powder. Added additional Te powder in a molar ratio of 1:1.
 8. Poured into a 2-inch mold and pressed into a target.
 9. The target was annealed in a high-temperature furnace at 300 °C for 12 hours to enhance mechanical strength and remove residual stresses.

3.2.2 Dealing with Anodic Aluminum Oxide (AAO) Substrate

The tradition method used to cut our substrates was using diamond wafering blades. After that, we would pattern them by using wet etching techniques. However, several challenges emerged during both the cutting and patterning processes on the AAO substrate. Due to its delicate surface nanostructure, the AAO cannot withstand significant mechanical stress. That makes conventional cutting methods difficult to apply without causing damage. Moreover, the surface roughness of AAO makes it difficult to align or apply shadow masks properly during deposition. Additionally, aluminum oxide dissolve in the

etching solutions typically used for niobium —hydrofluoric acid. Further increase the difficulty of the patterning process. As a practical solution, we manually use surgical scissors to cut the AAO substrates. Since the AAO substrate is relatively soft and thin, a sharp pair of fine scissors is an effective tool for controlling the stress applied to its surface. However, this technique introduces a new challenge: the shear force applied during cutting can cause the flexible substrate bending or deformation. To prevent this, we applied minimal stress near the substrate edges to preserve its flatness while avoiding damage to the delicate surface nanostructure. After several trials, we were finally able to cut the substrate consistently without damage.

The main challenge is to make patterns on the AAO substrate. Why do we care about these patterns? During our measurement of the electrical properties of the matching effect, the temperature must be kept very close to the critical temperature T_c , which causes the sample's resistance to become extremely low and the measurement signals to be very weak. In order to increase the resistance of our samples, it is necessary to reduce its cross-sectional area or increase the length of the measurement current path. However, as we mentioned above, the material constraints of AAO substrates make conventional patterning methods not applicable. Next, we attempted to attach soft materials —such as aluminum foil —as shadow masks on the sample surface. This attempt, however, faced a new limitation: thermal durability. Even high-temperature-resistant tapes can typically withstand up to 350 °C, far below the niobium deposition temperature, which is close to 500 °C.

Soon after, several methods were attempted to fabricate patterns without applying mechanical stress to the AAO surface. No matter which method is chosen, the initial

step always involves attaching the AAO substrate onto a properly sized Si substrate using silver paste (Figure 3.5). This setup serves not only as a stable platform for subsequent processing, but also addresses a practical issue: due to the extremely light weight of the AAO substrate, it can easily shift—or even be blown off—during vacuum evacuation. Initially, we attempted to fix several narrow strips of aluminum foil onto the Si substrate

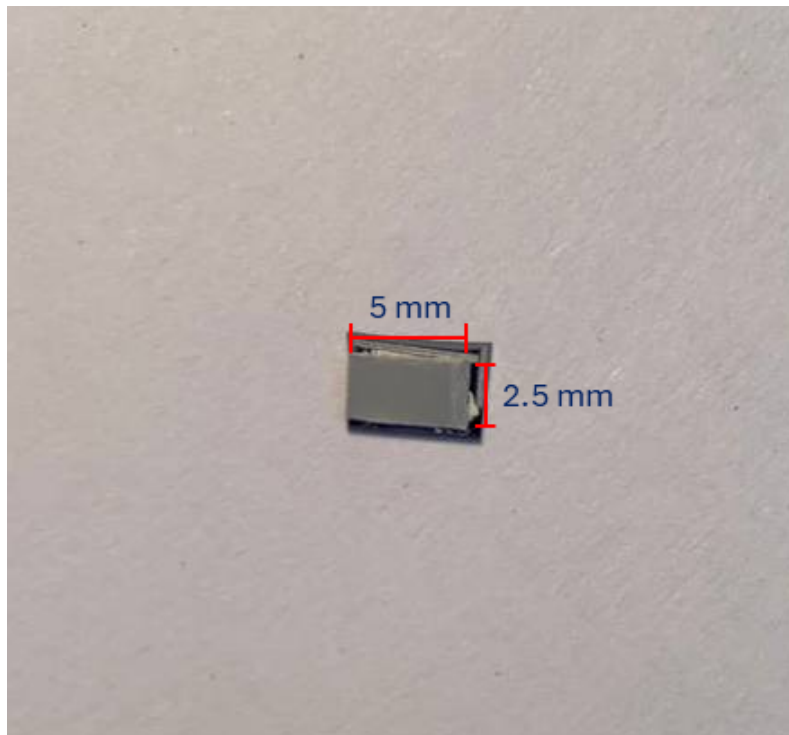


Figure 3.5: Image of AAO substrate paste on Si substrate

using silver paste, arranging them across the AAO substrate (Figure 3.6). However, this attempt presented two major drawbacks. First, it was difficult to achieve decent narrow patterns. Second, the aluminum foil could not fully conform to the AAO surface, resulting in edge diffusion during deposition.

After that, we explored an improved wet etching method. The idea was to use photoresist as a protective barrier by carefully coating all regions of the AAO that would be exposed to the etchant—including the extremely thin sides. This would allow us etching while preserving the rest of the substrate. However, the method proved to be technically



Figure 3.6: The aluminum foil stick across the Si substrate, so we can put AAO substrate between them.

challenging and unreliable, and was eventually abandoned.

Learning from the shortcomings of the initial attempts, we assumed that perfect contact between aluminum foil and the AAO surface was unrealistic. Instead, we placed two strips of aluminum foil along the top and bottom edges of the AAO substrate, leaving a narrow gap of approximately 0.5 to 0.2 mm between them (Figure 3.7). This setup successfully produced a long and thin sample. This method yielded a more decent pattern and enabled data collection, although it has not yet reached optimal performance.

3.2.3 Procedure of Deposit

Introduce the Sb_2Te_3 sputtering procedure:

1. Place the sample inside the sputtering chamber (Figure 3.8 3.9), activate the mechanical pump to evacuate the pressure to 1×10^{-2} Torr.

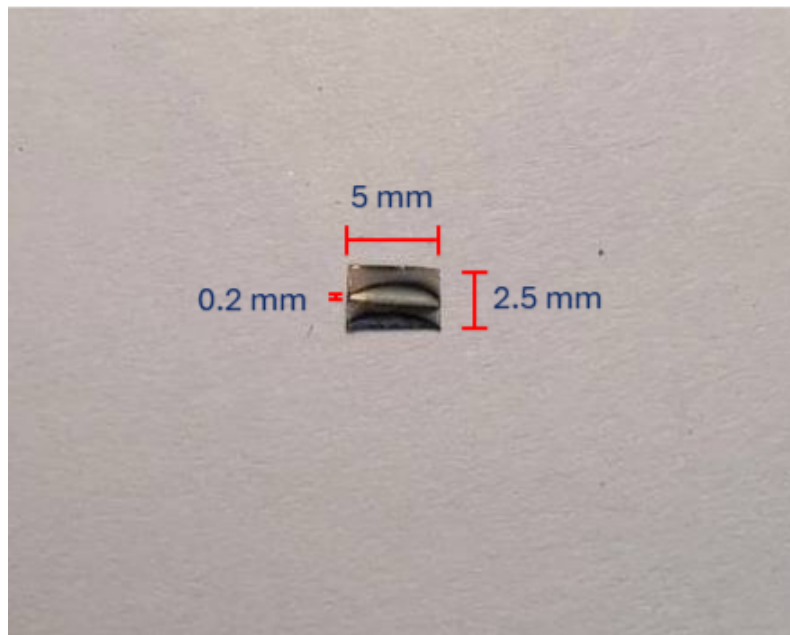


Figure 3.7: There is a extremely thin gap between the two aluminum foil.

2. Turn on the turbo pump to further evacuate the chamber, achieving a base pressure of around 2×10^{-5} Torr.
3. Heat the sample to 500 °C.
4. Introduce high-purity argon gas and stabilize the chamber pressure at approximately 6×10^{-3} Torr.
5. Use a DC power supply to perform magnetron sputtering of niobium at 80 W. Maintain deposition for 4 hours.
6. Allow the system to naturally cool to 123 °C, turn on the AC power supply to deposit Sb_2Te_3 at 20W. The deposition lasted for 3.5 min.
7. Finally, shut down the system and let the sample cool down naturally to room temperature.

Note that if only niobium is required, skip steps 6 and proceed directly from step 5 to step

7.



Figure 3.8: The sputtering chamber



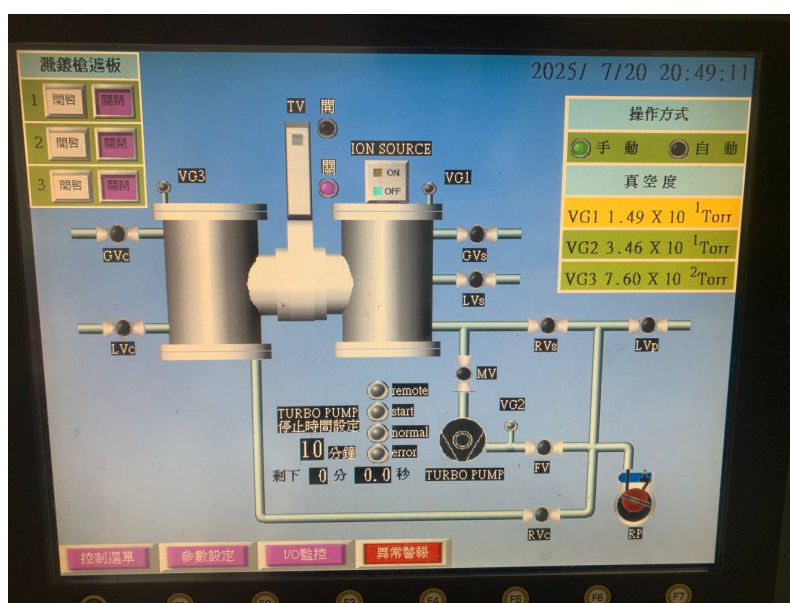


Figure 3.9: Operating interface of sputtering chamber



Chapter 4 Experiment Results

4.1 X-ray diffractometer (XRD)

Figure 4.1 shows the XRD diagram of niobium (Nb) single layer, Nb/Sb₂Te₃ (NbST) bilayer and pure anodic aluminum oxide (AAO) substrate samples.

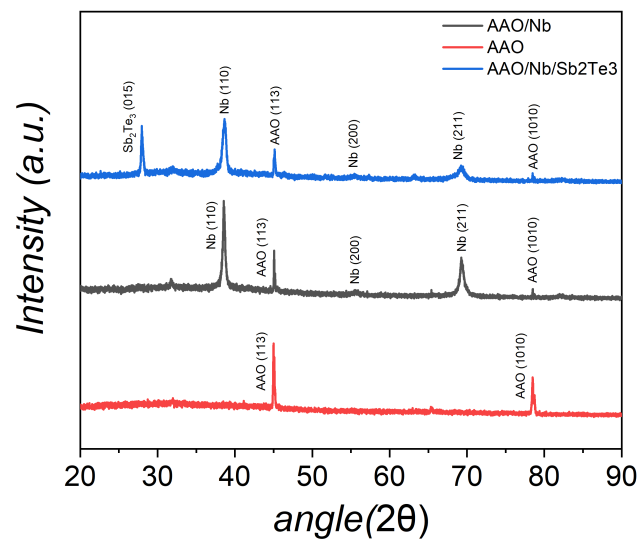


Figure 4.1: XRD diagram of samples.

use the Bragg's law

$$n\lambda = 2d \sin \theta \quad (4.1)$$

and the equation for interplanar spacing in a cubic crystal system:

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad (4.2)$$



where a is the lattice constant, and (hkl) are the Miller indices of the diffraction plane, we can calculate the lattice constant of niobium is

$$1.5406 \times \frac{\sqrt{2}}{2 \times \sin 19.3^\circ} \text{ \AA} \approx 3.30 \text{ \AA} \quad (4.3)$$

which is exactly the same as the bulk niobium lattice constant!

4.2 Moment and Temperature

Figure 4.2 and Figure 4.3 demonstrate the $M(T)$ -diagram of the Nb and NbST samples.

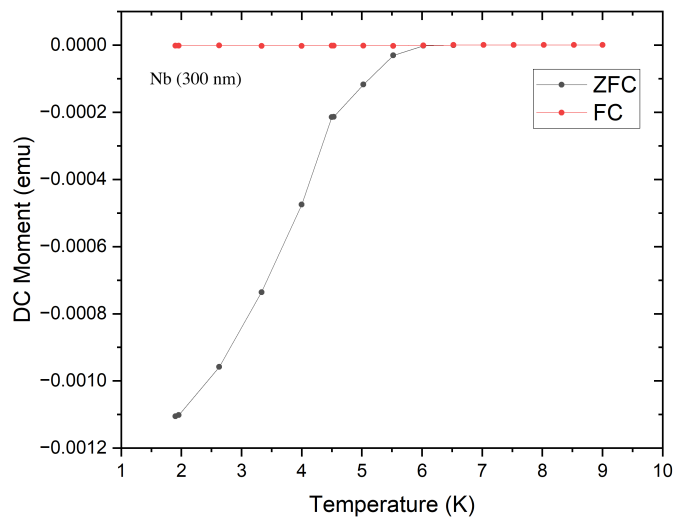


Figure 4.2: $M(T)$ -diagram of Nb thin film.

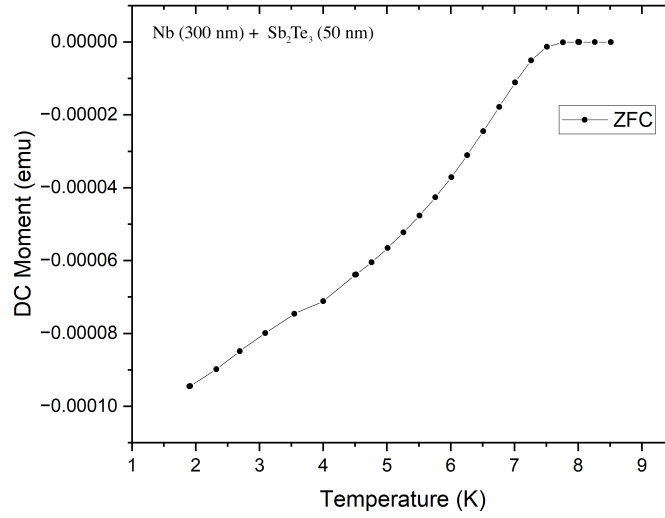


Figure 4.3: M(T)-diagram of NbST thin film.

We can estimate the critical temperature T_c by performing a linear fit on the first three points of the zero field cooled (ZFC) curve that deviate from the baseline. However, since the electrical measurements will provide R-T curves, the purpose of this magnetic measurement is to confirm the presence of superconductivity and roughly locate the quality of our samples.

The T_c of Nb thin film is 7.1 K, and the T_c of NbST bilayer is 6.97 K.

4.3 Resistance and Temperature

Figure 4.4 to Figure 4.6 present the R(T)-diagram of Nb film under different magnetic field. Choose the half of $R(9\text{ K})$ we get the T_c of Nb sample $\approx 8.17\text{ K}$.

In addition to identifying the superconductivity, our $R(T)$ measurements serve an important role in selecting optimal conditions for the following $R(H)$ measurement. The

observation of quantum flux matching effects is most effective near the superconducting transition temperature, where the applied magnetic field can drive the sample just across the superconducting-normal boundary. Since our $R(H)$ field range is target on 0-500 Oe, we focus on temperatures where the resistance changes most rapidly, as they are likely to provide the clearest flux matching signatures.

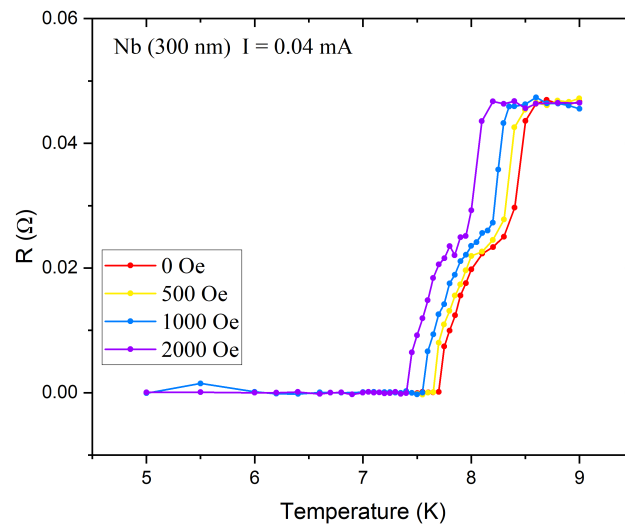


Figure 4.4: $R(T)$ -diagram of Nb thin film, measured with $I = 0.04$ mA

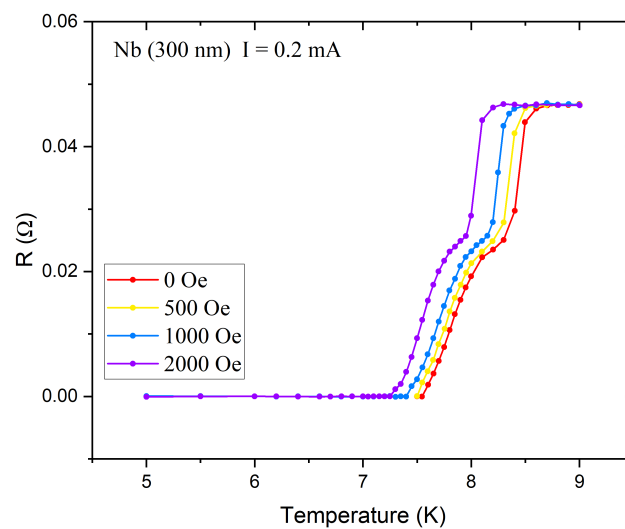


Figure 4.5: $R(T)$ -diagram of Nb thin film, measured with $I = 0.2$ mA

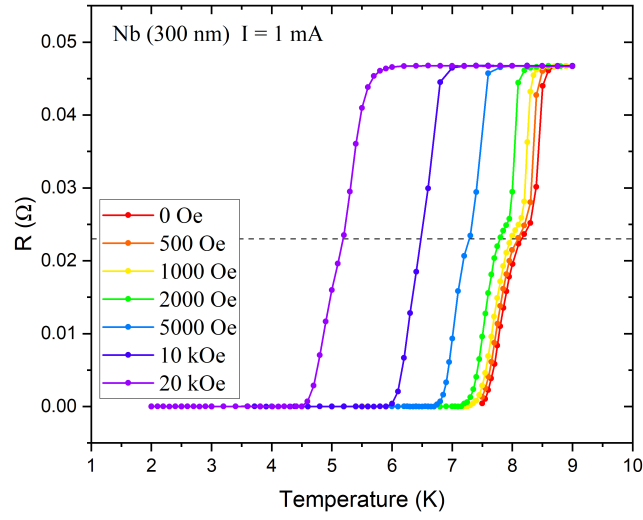


Figure 4.6: $R(T)$ -diagram of Nb thin film, measured with $I = 1$ mA

Figure 4.4 to Figure 4.6 shows the $R(T)$ -diagram of NbST film under different magnetic field. The T_c of NbST films is approximately 8.03 K, slightly lower than the pure Nb film due to the proximity effect. From the comparison of the Nb and NbST $R(T)$ of

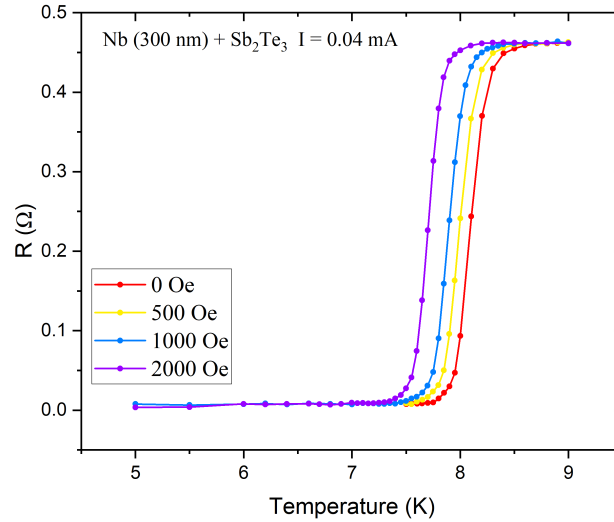


Figure 4.7: $R(T)$ -diagram of NbST sample, measured with $I = 0.04$ mA

0 and 500 Oe, we guess the decent condition of $R(H)$ measurement is around 7.7 – 8.4 K.

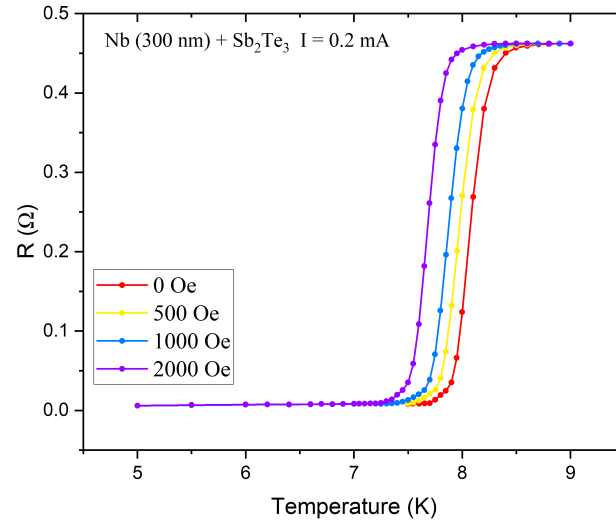


Figure 4.8: $R(T)$ -diagram of NbST sample, measured with $I = 0.2$ mA

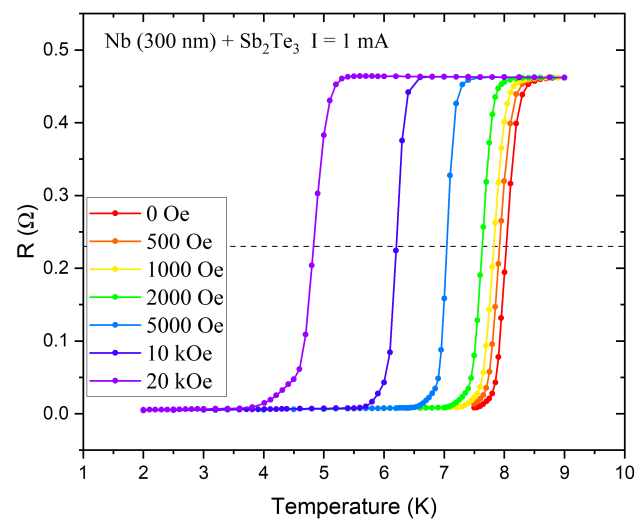


Figure 4.9: $R(T)$ -diagram of NbST sample, measured with $I = 1$ mA



4.4 Critical Field

For the 1 mA measurement, the $R(T)$ dataset is more complete. We extract the T_c and at each applied magnetic field, and define the corresponding magnetic field as the upper critical field H_{c2} . Now we construct the $H_{c2}(T)$ diagram (Figure 4.10). The curve

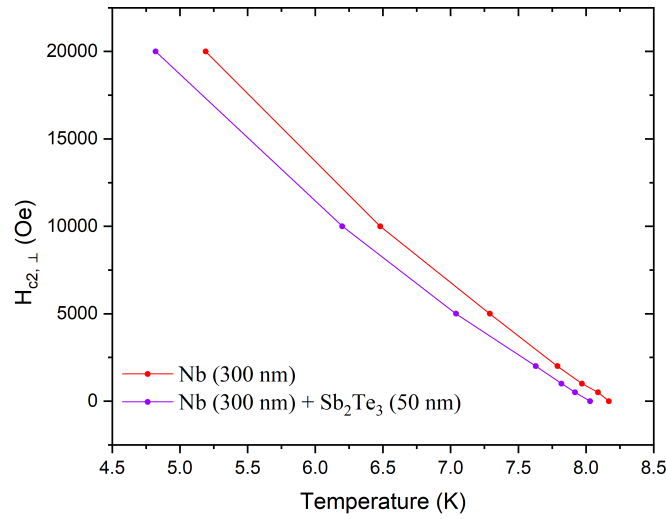


Figure 4.10: $H_{c2}(T)$ diagram of Nb and NbST films

consists of two nearly straight lines, which is not ideal, as a downturn is expected near T_c . We apply the Ginzburg-Landau (GL) theory ((4.4)) to extract the superconducting coherence length, and use the Werthamer-Helfand-Hohenberg (WHH) theory ((4.5)) to estimate the $H_{c2}(0)$.

$$H_{c2} = \frac{\Phi_0}{2\pi\xi^2(0)} \left(1 - \frac{T}{T_c}\right) \quad (4.4)$$

$$H_{c2}(0) = 0.69 \times T_c \left. \frac{dH_{c2}}{dT} \right|_{T_c} \quad (4.5)$$

the coherence length $\xi(0)$ of Nb ≈ 7.69 nm, of NbST ≈ 8.06 nm. The bulk of Nb $\xi(0) \approx 30$ nm. Compares to our seniors's research, their $\xi(0)$ of Nb ≈ 15.5 nm, and the $\xi(0)$ of

NbST ≈ 7.7 nm. We can observe that our coherence length of Nb is much more smaller than our senior's, but the NbST coherence length is slightly larger. For Nb, the reason for this difference is reasonable. Since our sample is deposited on an AAO substrate, the growth condition of the Nb film is worse than on a Si substrate, providing smaller grain sizes and increased electron scattering. But for NbST, the result is somewhat contradictory. A possible interpretation may be that the Nb layer acts as a buffer layer, so the influence of the AAO substrate is not significant.

The $H_{c2}(0)$ of Nb ≈ 37600 Oe, which is much more larger than bulk Nb (5000-8000 Oe), and the $H_{c2}(0)$ of NbST ≈ 34300 Oe. Same we compare to our seniors's research, their $H_{c2}(0)$ of Nb ≈ 16000 Oe, and the $H_{c2}(0)$ of NbST ≈ 29000 Oe. The reason of the increment is the same as the coherence length. From

$$\rho = \rho_0 e^{-\frac{U}{k_B T}} \quad (4.6)$$

where ρ is resistivity, U is activating energy, k_B is the Boltzmann constant. Take the logarithm

$$\ln \frac{\rho}{\rho_0} = -\frac{U}{k_B} \times \frac{1}{T} \quad (4.7)$$

since the rate of ρ remains the same, we can redraw our $R - T$ diagram to $\frac{R}{R_0} - \frac{1}{T}$, and the slope would be $\frac{U}{k_B}$.

Figure 4.11 shows the fitting lines meet a the temperature $T_m \approx 8.87$ K, a little bit higher than the critical temperature $T_c = 8.17$ K. Figure 4.12 presents the fitting lines are not intersect at a point, from our seniors research this behavior is expected to happen once the Sb_2Te_3 thickness reach about 50 nm.

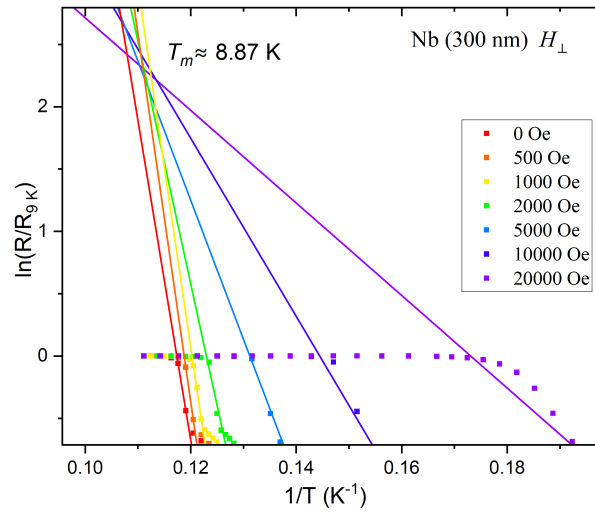


Figure 4.11: $\ln \frac{R}{R_{9\text{ K}}} \left(\frac{1}{T} \right)$ diagram of Nb sample

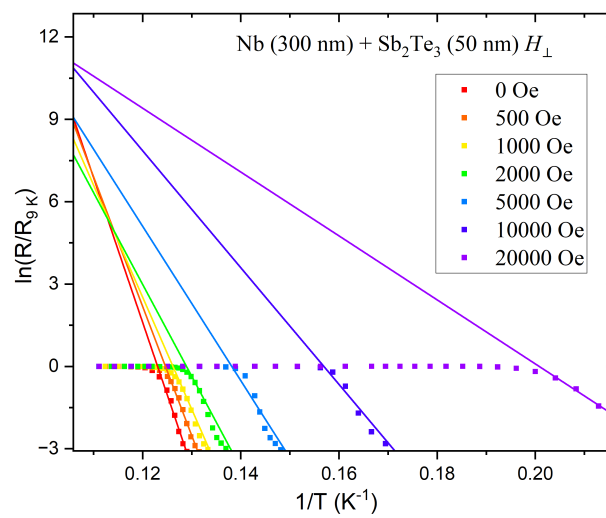


Figure 4.12: $\ln \frac{R}{R_{9\text{ K}}} \left(\frac{1}{T} \right)$ diagram of NbST sample

The activating energy U and magnetic field H have relationship:

$$U \propto \frac{1}{H^\alpha} \quad (4.8)$$



where α is a constant depends on properties of sample. Figure 4.13 and Figure 4.14 we demonstrate the $\frac{U}{k_B} - H$ diagram, the results indicate the activation energy and magnetic field has power-law and both sample have $\alpha \approx 0.3$.

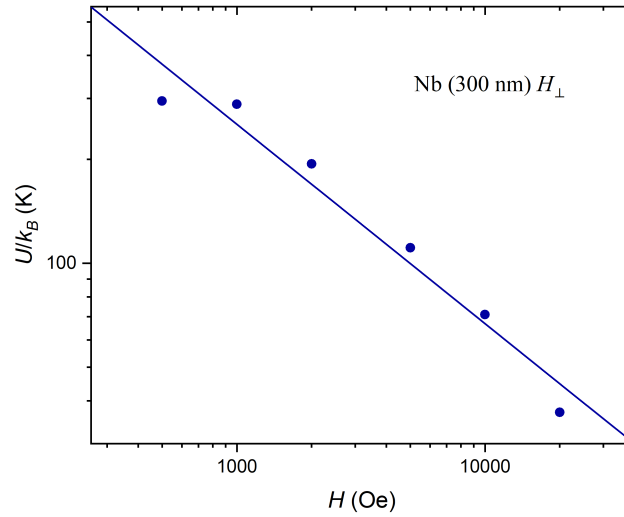


Figure 4.13: $\frac{U}{k_B}(H)$ diagram of Nb sample

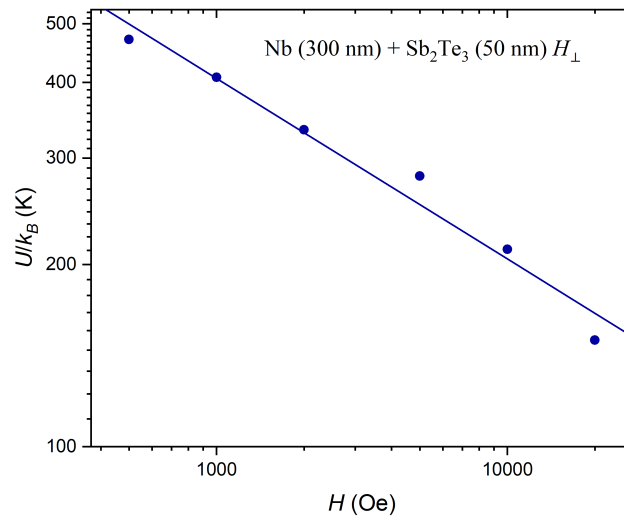


Figure 4.14: $\frac{U}{k_B}(H)$ diagram of NbST sample



4.5 Basic Information of AAO substrate

Table 4.1 shows the specifications Our AAO template has a hexagonal arrangement of pinning sites (Figure 4.15),

Table 4.1: Specifications of AAO-SF2 Substrate

Parameter	Value
Pore Diameter	200 nm
Pore Spacing	450 nm
Pore Depth	60 μm
Substrate Area	$20 \times 20 \text{ mm}^2$
Surface Appearance	Matte white structured surface

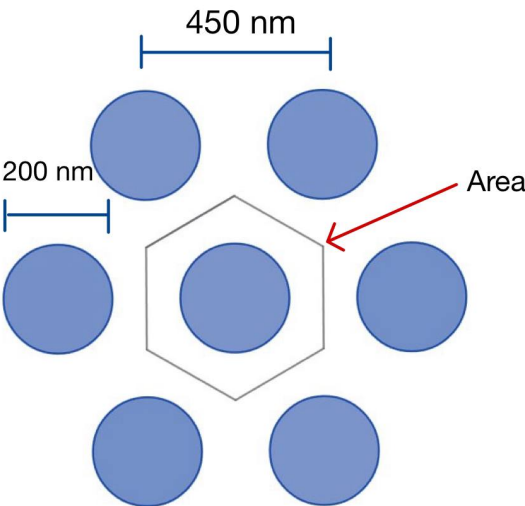
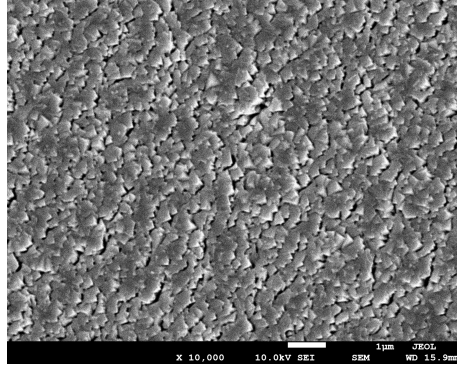


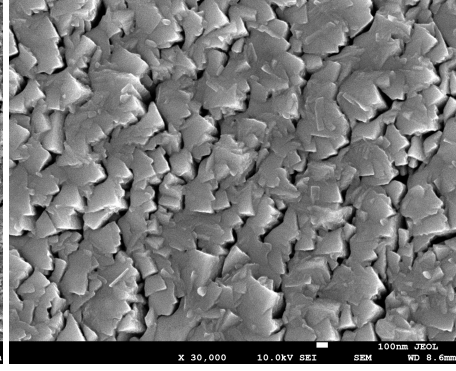
Figure 4.15: pores of AAO substrate

4.5.1 Scanning Electron Microscope (SEM)

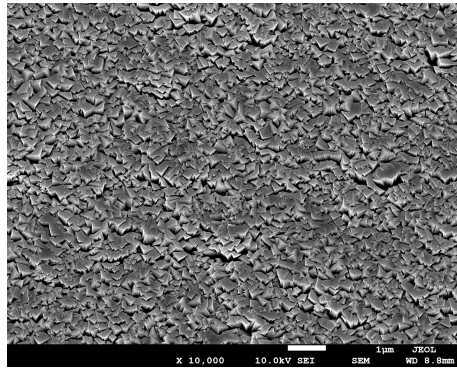
The AAO substrate is expected to exhibit a hexagonal array of pores with 200 nm diameter. Despite the coating reducing visibility, a regular surface pattern is still observable.



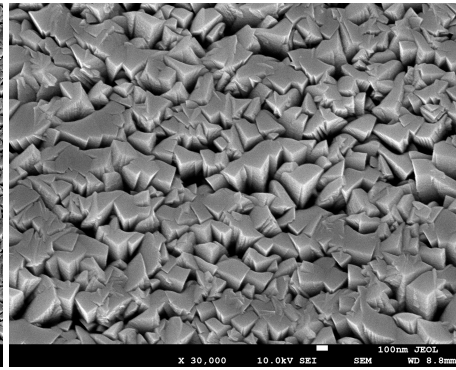
(a) Nb thin film at $\times 10000$.



(b) Same region at $\times 30000$.



(a) NbST sample with $\times 10000$.



(b) Same region at $\times 30000$.

4.6 Qualitative Analysis of Resistance and Magnetic Field

From the schematic diagram (Figure 4.15), we can calculate the area of a single pinning site,

$$A_p = \frac{\sqrt{3}}{2} d^2 \quad (4.9)$$

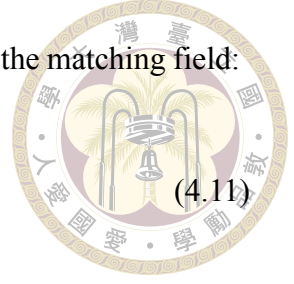
where d is the distance between two pinning sites, which is equal to 450 nm in our case.

Single quantum of magnetic flux is given by:

$$\phi_0 = \frac{h}{2e} \approx 2.07 \times 10^{-7} \text{ G} \cdot \text{cm}^2 \quad (4.10)$$

Substituting (4.9) and (4.10) into the formula below, we can calculate the matching field:

$$H_1 = \frac{\phi_0}{A_p} = \frac{2.07 \times 10^{-7}}{\frac{\sqrt{3}}{2}(4.5 \times 10^{-5})^2} \approx 118.3 \text{ G} \quad (4.11)$$



which suggests that the matching field will be around 118 Oe.

Therefore, we expect the $R(H)$ curve to exhibit a significant drop in resistance approximately every 118 Oe, corresponding to the matching of magnetic flux quanta with artificial pinning centers. However, due to the anodization process, the nanopore arrangement in AAO may exhibit a degree of disorder. This leads to a deviation between the theoretical and actual matching fields, as the effective pinning landscape is influenced by the real surface geometry. In our previous experiments, we consistently observed the first matching field H_1 to be approximately 100 Oe. For Nb films, we expect the matching fields following the integer sequence $H_n = nH_1$. For NbST films, the matching condition should be $H_n = (n + \frac{1}{2})H_1$.

This section aims to qualitatively examine the raw data to determine optimal conditions for further analysis.

From the Figure 4.18 to Figure 4.20, we can see that at the same temperature, using a larger current makes the resistance curve flatter —meaning the matching effect becomes less obvious. The clearest integer-flux quantum matching appears at 7.9 K in the Nb sample, which agrees with what we expected from the earlier $R(T)$ curve. The resistance shows a dramatically drop for every 100 Oe increment in the applied magnetic field, which is close to the expected matching field, indicating a clear flux matching effect.

As the Figure 4.21 to Figure 4.23 shows. At the same current level, we see the same

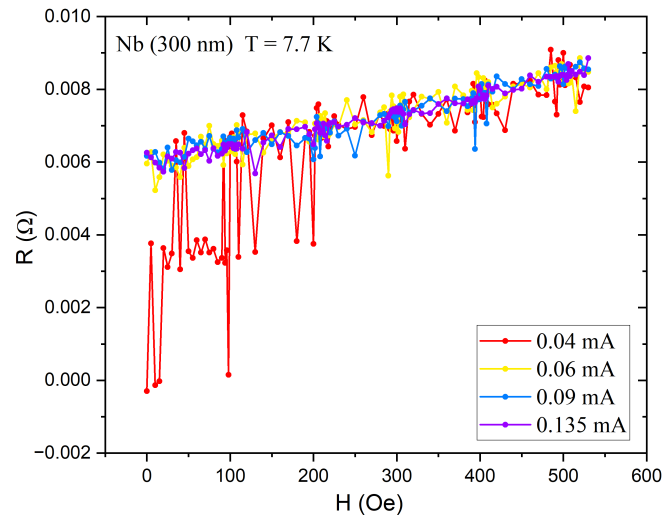


Figure 4.18: Nb $R - H$ diagram of different measure current on 7.7 K

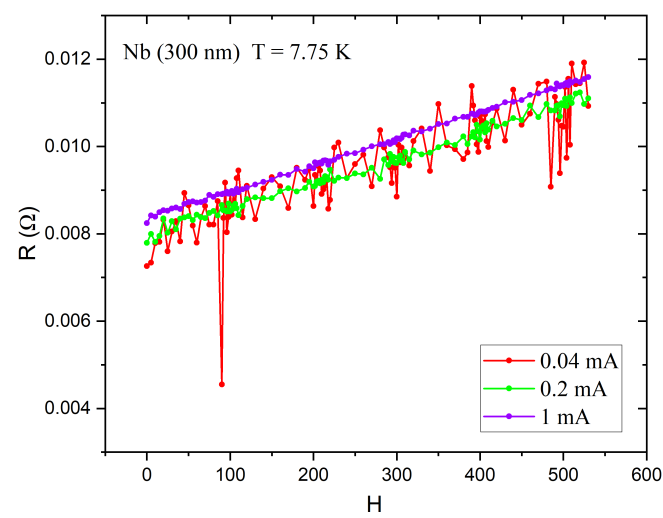


Figure 4.19: Nb $R - H$ diagram of different measure current on 7.75 K

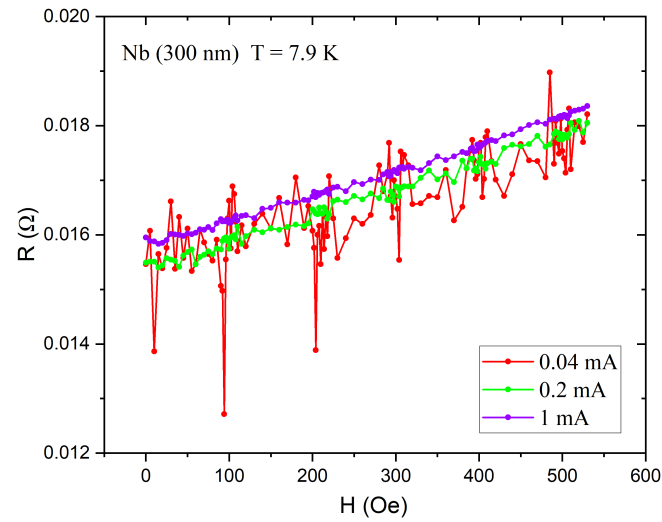


Figure 4.20: Nb $R - H$ diagram of different measure current on 7.9 K

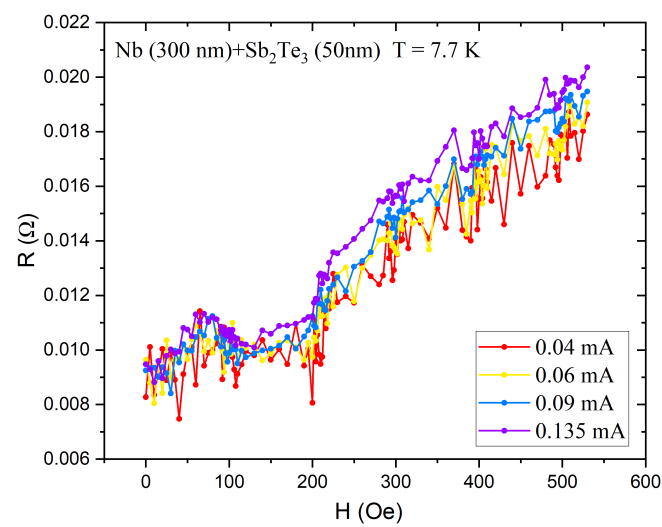


Figure 4.21: NbST $R - H$ diagram of different measure current on 7.7 K

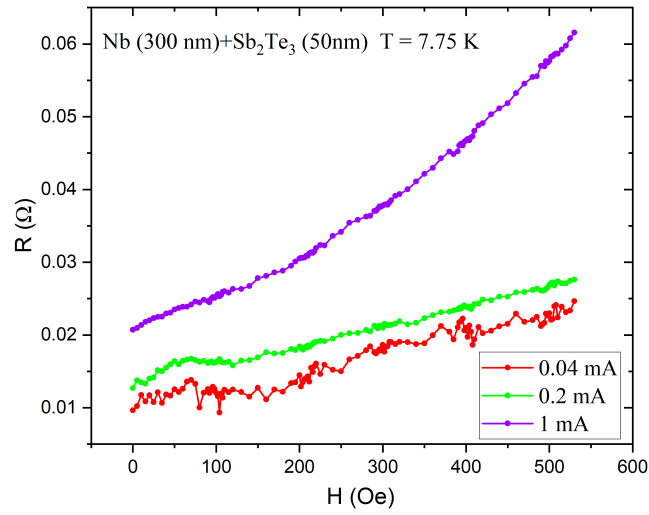


Figure 4.22: NbST $R - H$ diagram of different measure current on 7.75 K

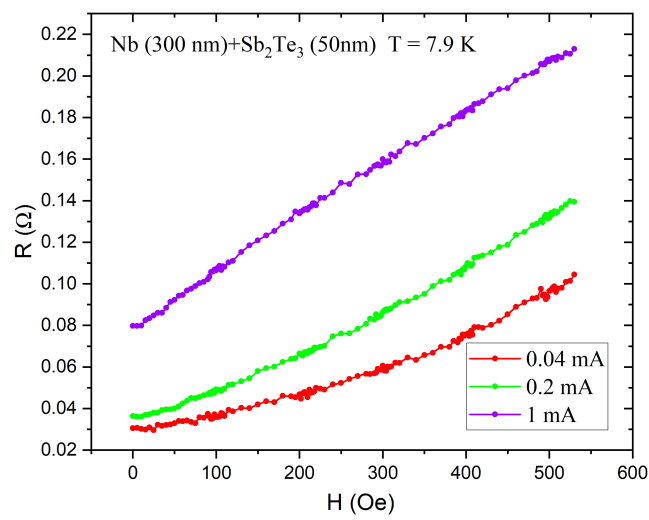


Figure 4.23: NbST $R - H$ diagram of different measure current on 7.9 K

trend: the larger the current, the less the effect. For Sb_2Te_3 , the most distinct matching appears at 7.7 K. Even at this optimal temperature, the half-quantum flux feature is only barely visible.

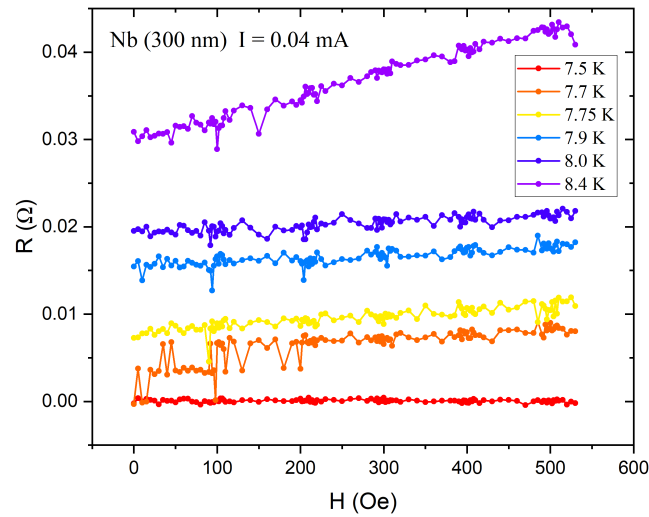


Figure 4.24: Nb $R - H$ diagram of different temperature on $I = 0.04$ mA

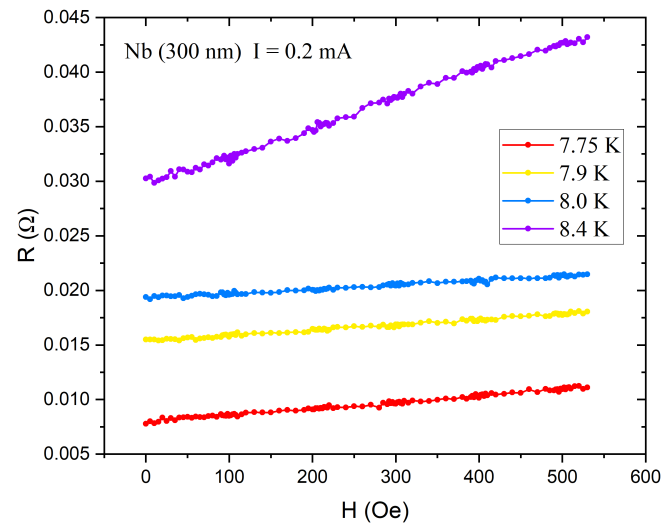


Figure 4.25: Nb $R - H$ diagram of different temperature on $I = 0.2$ mA

From Figure 4.24 to Figure 4.29 include the measurements that fix current with different temperatures. By measuring the $R-H$ curves under a fixed current at various tem-

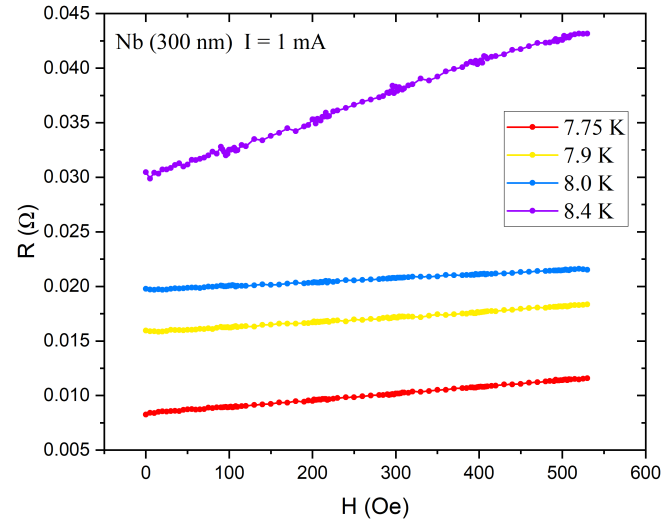


Figure 4.26: Nb $R - H$ diagram of different temperature on $I = 1$ mA

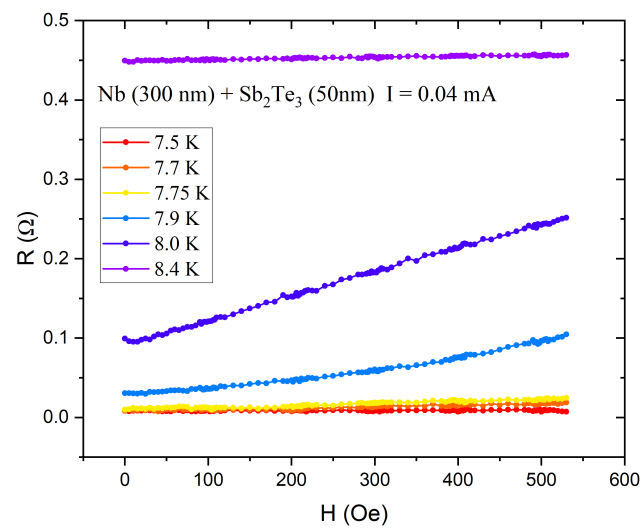


Figure 4.27: NbST $R - H$ diagram of different temperature on $I = 0.04$ mA

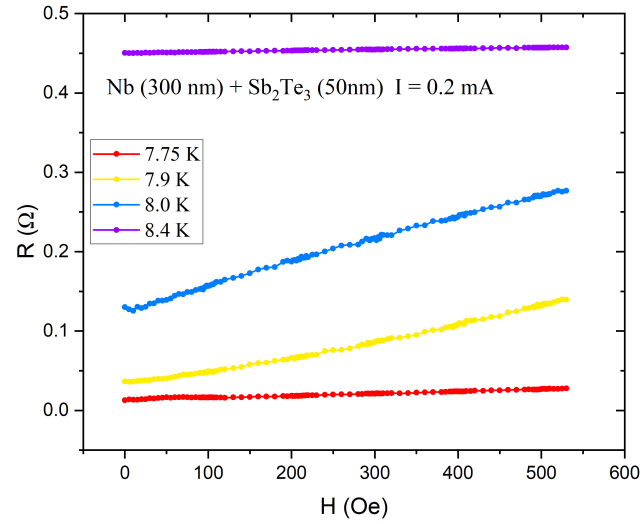


Figure 4.28: NbST $R - H$ diagram of different temperature on $I = 0.2$ mA

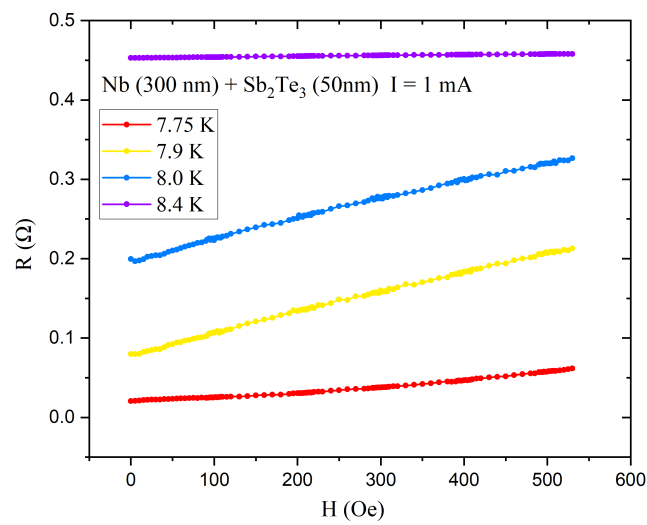
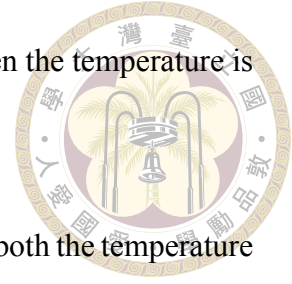


Figure 4.29: NbST $R - H$ diagram of different temperature on $I = 1$ mA

peratures, it is observed that flux matching effects are not visible when the temperature is too low or too high. But the half matching is too weak to measure.



Based on all the raw data plots and descriptions, it is evident that both the temperature distribution and the magnitude of the measurement current influence the experimental results. Regardless of the sample type, lower measurement currents tend to produce clearer effects. Therefore, we selected the 0.04 mA current for further analysis of the $R(H)$ at different temperatures.

4.7 Interpretation of $R(H)$

Our raw data resistance measurements contain noise due to temperature fluctuation or instrumental limitations, therefore, it is essential to smooth the data before conducting further analysis. We applied the Savitzky-Golay filter to our data using a window length of 8 and a polynomial order of 3.

To analyze the periodic variation in resistance with respect to magnetic field, several analytical tools can be employed:

1. Derivative Analysis:

Calculate the first derivative $\frac{dR}{dH}$, each matching field should results in a sharp drop.

By detecting local maxima or minima in the $\frac{dR}{dH}$ curve, we can determine the field positions H_n where these features occur.

2. Fast Fourier Transform (FFT):

To further investigate the periodic features in our data, we performed a FFT on the smoothed $R(H)$ data. FFT decomposes the signal into its frequency components, allowing us to identify periodicities corresponding to flux matching fields. If clear matching effects are present, we expect to observe a strong peak in the FFT spectrum at a frequency corresponding to the inverse of the expected matching period

$$f_1 = \frac{1}{H_1} \approx 0.01 \text{ Oe}^{-1} \quad (4.12)$$

3. Least-Squares Spectral (LSS) Analysis

Also known as the Lomb-Scargle Periodogram. LSSA is a technique for identifying periodic signals in unevenly spaced data. Unlike the traditional Fourier Transform, LSSA fits sinusoidal models to the data using a least-squares optimization. We will provide a more detailed introduction to the Least-Squares Spectral Analysis method when we apply it to our data.

4.7.1 Derivative Analysis

First, we examine the first derivative of the $R(H)$ data for the Nb sample across all measured temperatures (Figure 4.30). The x-axis has been rescaled to $n = \frac{H}{H_1}$, where H_1 represents the first matching field. The positions of the peaks are highly consistent, appearing nearly at integer multiples of H_1 .

Additionally, we highlight the data at 7.9 K, where the peaks are most clearly identified, and list the corresponding H of the peaks. (Figure 4.31)

We same applied the derivative analysis to the NbST sample (Figure 4.32). From the

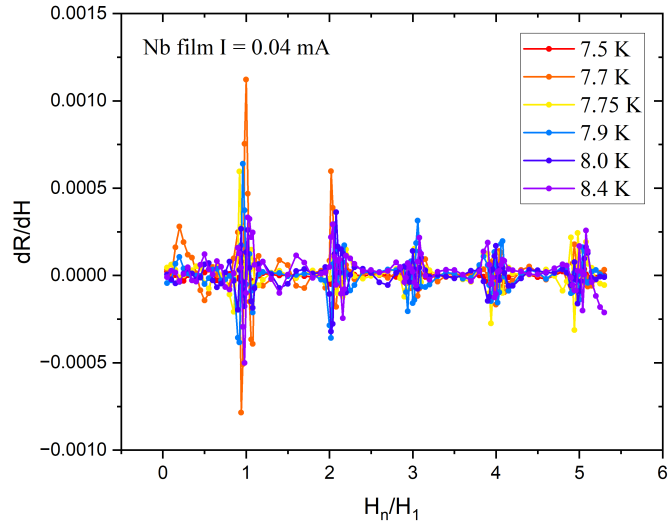


Figure 4.30: First derivative of Nb $R(H)$

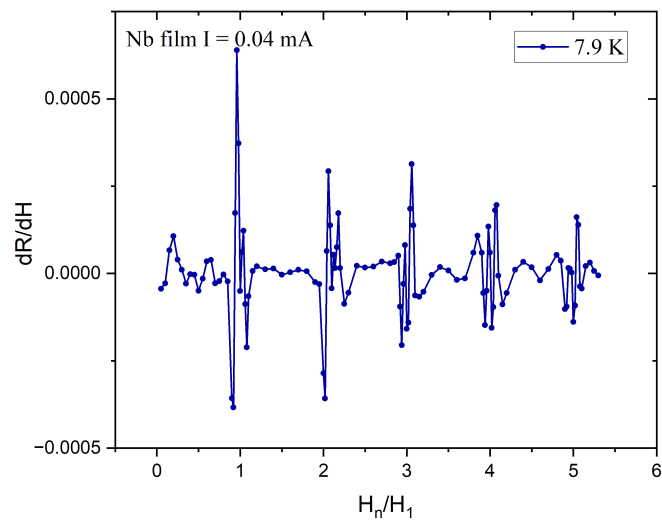


Figure 4.31: Focus on $\frac{dR}{dH}$ at $T = 7.9$ K

raw data, it is evident that the peaks in NbST are not as sharp as those observed in the Nb sample. After taking the first derivative, the peaks compare to Nb appear less well-defined, suggesting that the resistance changes more gradually. However, compared to the

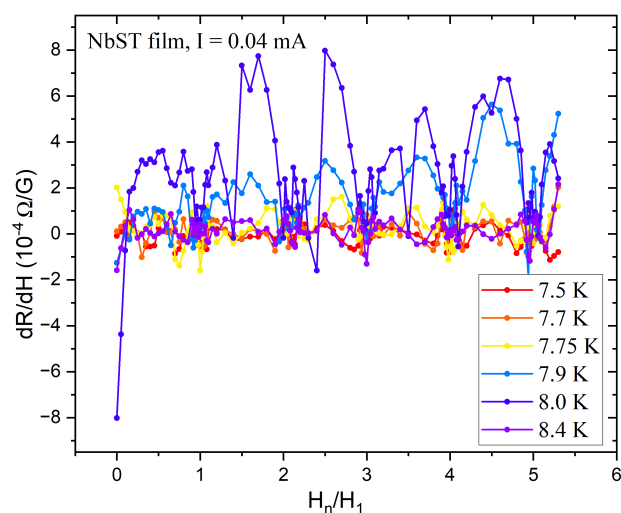
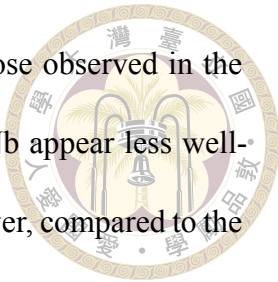


Figure 4.32: First derivative of NbST $R(H)$

raw data, the derivative plots reveal pronounced peaks around the half-integer multiples of H_1 especially at temperatures above 7.7 K. This indicates a notable response at fractional matching fields, consistent with the expected behavior of topological superconductors. We further plot the derivative results at 7.9 K, 8.0 K, and 8.4 K, and highlight the peaks H at 8.0 K for comparison with those of the Nb sample.

Table 4.2: Matching fields of sample

Sample	Matching Fields (Oe)					
Nb	94.0	204	306	398	496	
NbST	52	158	249	354	449	

4.7.2 Fast Fourier Transform



In the previous derivative analysis, our goal was primarily to identify the approximate locations of resistance peaks. To further confirm the periodicity of these features, we next apply fast fourier transform to the data.

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-i2\pi \frac{k}{N} n} \quad (4.13)$$

the term $\frac{k}{N}$ assume our data be equally spaced. The main challenge in using FFT for this analysis come from the non-uniform sampling of the magnetic field during measurements. Since we expect significant resistance changes near integer or half-integer multiples of the first matching field H_1 , the data measurement was made denser in those interval.

However, FFT requires the x-axis data (magnetic field H) to be uniformly spaced. To resolve this issue, we applied interpolation method to fill in missing data points, starting from the Nb sample. Once uniformly sampled, FFT was performed on resistance data across all measured temperatures. The result are shown below (Figure 4.33, Figure 4.34).

At first glance, the results appear reasonable. However, when we isolate the best-performing dataset at 8.0 K (Figure 4.35), a local maximum emerges at $x = 0.007$, which corresponds to a magnetic field of approximately 142 Oe. This result is different from the peak positions identified through qualitative inspection and derivative methods. Considering that we applied interpolation to correct for non-uniform sampling in the original data, it is important to check whether this process has provide distortions. We can introduce a null hypothesis to statistically verify whether the interpolation significantly affects the observed periodicity or peak positions.

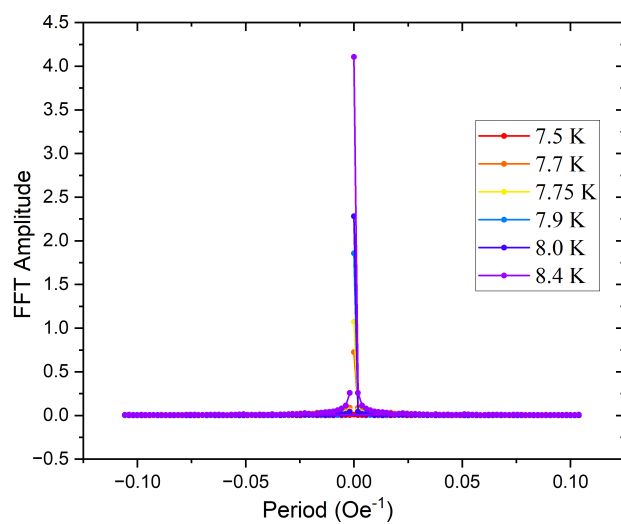


Figure 4.33: The FFT spectrum of the Nb sample

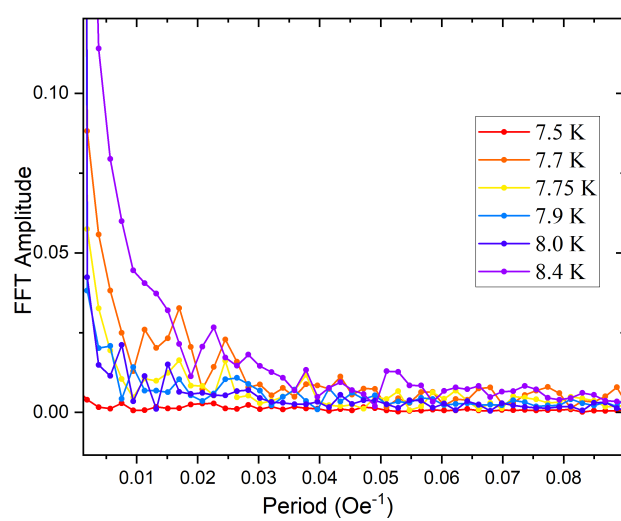


Figure 4.34: Zoom in the region near zero.

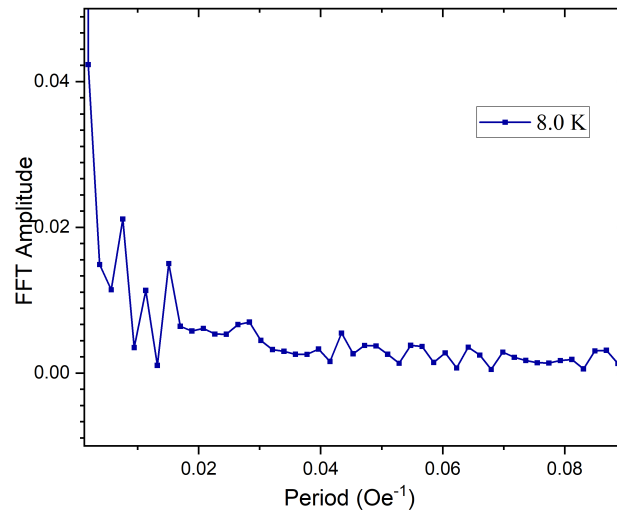


Figure 4.35: Isolate the $T = 8.0$ K data.

4.7.2.1 Statistical Verification of Interpolation Effects

To evaluate whether the interpolation process introduces significant distortions to the original data, we conducted a Z-test comparing the raw and interpolated datasets. The hypotheses were defined as follows:

- Null Hypothesis Z_0 : The original data and the interpolated data are statistically indistinguishable.
- Alternative Hypothesis Z_1 : There exists a statistically significant difference between the original and interpolated data.

First we define Z-score:

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \quad (4.14)$$

where $\bar{x}$ is sample mean, μ_0 is mean under the null hypothesis, σ is standard deviation and n is data size.

p -value is defined as the probability of obtaining a test statistic at least as extreme as the observed one under the null hypothesis. We should provide one tail test since we only care about the positive value of the spectrum. For one tail test, p -value defined:

$$p = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \quad (4.15)$$

this is the area under the standard normal curve beyond the test statistic.

A large p -value (typically > 0.05) means the observed data is consistent with what we'd expect under the null hypothesis, so we don't consider the result surprising enough to say something has changed.

Our $Z = -93.19$, and $p \approx 0$. A very small p -value indicates that the probability of observing such a result under the null hypothesis is extremely low. Since $p \approx 0$, which means that we have to reject the null hypothesis.

While the interpolated curve may appear smooth, the underlying structure—particularly the average behavior and local variation—has been statistically altered, meaning distortion is present. That is, the interpolation method is not a proper way to handle our data. Therefore, we introduce LSS as an alternative.

4.7.3 Least-Squares Spectral Analysis

The LSSA method models the observed signal $y(t)$ as a combination of sine and cosine terms:

$$y(t) = A \sin(\omega t) + B \cos(\omega t) \quad (4.16)$$

The coefficients A and B are determined by least-squares fitting to minimize the residual error between the model and data. By explicitly incorporating the x-axis values into the fitting process, LSSA inherently accounts for non-uniform sampling, thereby avoiding aliasing and improving frequency resolution. The method computes the “power” at each frequency as a measure of the goodness of the sinusoidal model, producing a power spectral density curve that indicates how much of the signal’s variance is explained by each frequency.

$$P_x(\omega) = \frac{1}{2} \left(\frac{\left[\sum_j X_j \cos \omega(t_j - \tau) \right]^2}{\sum_j \cos^2 \omega(t_j - \tau)} + \frac{\left[\sum_j X_j \sin \omega(t_j - \tau) \right]^2}{\sum_j \sin^2 \omega(t_j - \tau)} \right) \quad (4.17)$$

Figure 4.36 shows the result of Nb sample LSS analysis.

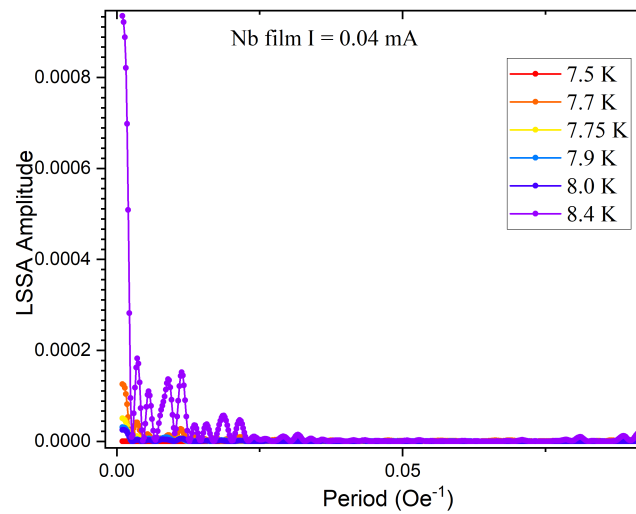


Figure 4.36: LSS analysis.

Learning from the lesson of interpolation method, we now apply a statistical tool to check the validity of our data after transformation before apply further analysis. Unlike the previous interpolation method, this time we fit the discrete data to a continuous function and then resample it. Since the transformation of our data involves function fitting

and resampling, the traditional Z-test is not appropriate. Instead, we apply resampling permutation test to evaluate the validity of our LSSA.



1. Compute the LSSA of the original resampled data and record the maximum spectral power P_{obs}
2. Generate N datasets by randomly shuffling the y -values while keeping x fixed.
3. For each new dataset, applied the LSSA and record its maximum spectral power.
4. Count the the number of shuffled datasets whose maximum $P \geq P_{\text{obs}}$, divided by the total number datasets N .
5. If $p < 0.05$, reject the null hypothesis.

Table 4.3 shows the result of permutation tests:

Table 4.3: p-values of Nb sample at different temperature.

Temperature (K)	Max Power	p-value
7.5	1.02×10^{-7}	0.193
7.7	1.26×10^{-4}	≈ 0
7.75	5.06×10^{-5}	≈ 0
7.9	3.13×10^{-5}	≈ 0
8.0	2.54×10^{-5}	≈ 0
8.4	9.35×10^{-4}	≈ 0

At 7.5 K, the permutation test yields a p-value of 0.193, there is no significant periodic structure exists in the data. In contrast, the p-values for all other temperatures are close to zero, appear that the results are significantly better than random noise. Therefore, we can confidently proceed with further analysis on these datasets.

Figure 4.37 zoom in the Figure 4.36 near zero. Extract the locations of the local

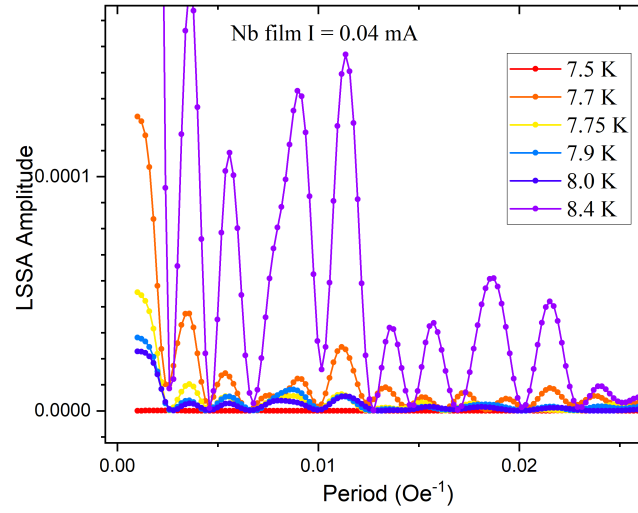


Figure 4.37: Zoom in the LSS result.

maxima, 0.00895 Oe^{-1} and 0.0114 Oe^{-1} , we can estimate the first matching field:

$$H_1 \approx \frac{2}{0.00895 + 0.0114} = 98.3 \text{ Oe} \quad (4.18)$$

it is in excellent agreement with our initial prediction!

Similarly, the result of NbST LSS analysis (Figure 4.38): We can get

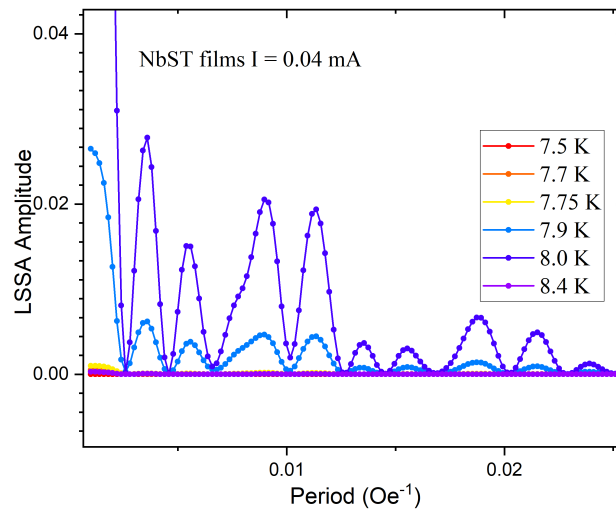


Figure 4.38: Zoom in the LSS result.

$$H_1 \approx \frac{2}{0.00900 + 0.0114} = 98.0 \text{ Oe} \quad (4.19)$$



Combining the derivative and least-squares spectral analyses, we can clearly identify the flux matching behavior in both Nb and NbST thin films. And the half quantum flux of NbST film.

$$H_n = nH_1 \quad H_1 = 98.3 \text{ Oe for Nb} \quad (4.20)$$

$$H_n = \left(n + \frac{1}{2}\right)H_1 \quad H_1 = 98.0 \text{ Oe for NbST} \quad (4.21)$$



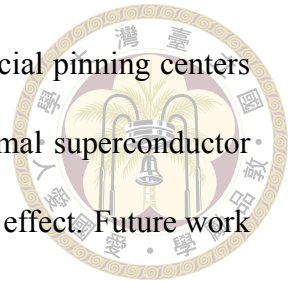
Chapter 5 Conclusion

In this study, we successfully deposited both monolayer Nb and bilayer Nb/Sb₂Te₃ films on AAO substrates. By applying derivative analysis and least-squares spectral analysis, we quantitatively determined the first matching field. The associated data transformations were also statistically convinced to ensure validation. However, from the measurements of $R(T)$, $H_{c2}(0)$ and $\xi(0)$, it is evident that the film quality is not optimal. This is also reflected in the $R(H)$ results, where only the Nb film exhibited a clear flux matching effect. Several factors may contribute to this outcome:

1. The deposition parameters were optimized based on Si substrates; significant adjustments, especially in deposition rate, may be required for AAO substrates.
2. The Nb target was near the end of its usable life, potentially affecting film growth quality.
3. The pattern resolution may be insufficient to induce a sensitive and observable matching effect.

In conclusion, while the flux matching effect was successfully observed in the Nb/AAO system, the half-quantum flux feature expected in the Nb/Sb₂Te₃ system was not distinctly captured. However, through detailed data analysis, we were able to identify

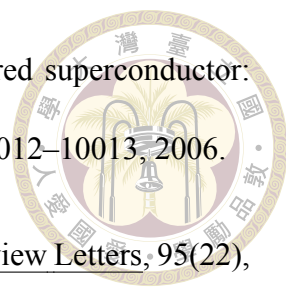
the underlying periodicity, thereby confirming that introducing artificial pinning centers and a system with spin-triplet topological material site into the normal superconductor can indeed transform the integer matching effect into a half matching effect. Future work should focus on improving film stability by refining deposition conditions, ensuring target quality, and employing higher-resolution patterning.



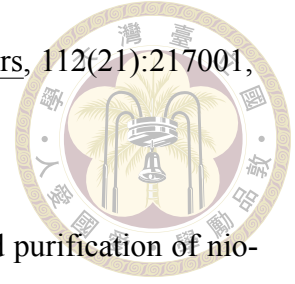


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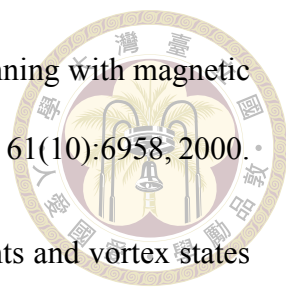
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