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Belle II 實驗中 Level-1 觸發系統之三維追蹤方法的實作與

$\bar{B}^0 \rightarrow J/\psi \Lambda \bar{n}$ 衰變的蒙地卡羅研究

Implementation of the 3D tracking method for the Level-1 trigger
system of the Belle II experiment, and Monte Carlo Study of

$\bar{B}^0 \rightarrow J/\psi \Lambda \bar{n}$

林傑澄

Jie-Cheng Lin

指導教授：王名儒 博士

Advisor: Min-Zu Wang, Ph.D.

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The undersigned, appointed by the Department / Institute of Physics
on 11 (date) 7 (month) 2025 (year), have examined a PhD dissertation entitled above presented
by Jie-Cheng Lin (name) d06222009 (student ID) candidate and hereby certify
that it is worthy of acceptance.

口試委員 Oral examination committee:

王名偉
(指導教授 Advisor)
徐靜戈

胡貝穎
張賢棟

張敏娟



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中文摘要

Belle II實驗位於SuperKEKB電子對撞機，是一項高亮度的B工廠實驗，旨在精確研究B介子的罕見衰變；其一級（L1）觸發系統需要即時從大量背景事件中快速篩選出物理感興趣的碰撞事例。我們為該觸發系統研發了一種新穎的三維軌跡重建（3D tracker）算法，可在中央漂移室內對帶電粒子的軌跡進行三維重建，並將處理延遲控制在亞微秒以內。該3D重建算法利用漂移室軸向和立體層的探測訊號，通過幾何變換和線性回歸擬合計算每條軌跡的縱向頂點 z （即交點位置）和極角 θ 。模擬結果表明，幾何算法（geo-finder）的3D追蹤相較於原有的2D追蹤，可將束流引起的背景觸發率降低約 38%，同時在對 $|z_0| < 35$ cm 的軌跡施加篩選時仍保留約 97% 的真實信號事例。此外，我們實現了一種基於查找表的算法（LUT-finder），透過遍歷所有立體層擊中組合以最小化軌跡擬合的 χ^2 ，進一步提高了 z_0 重建精度並增強了背景抑制能力。

在物理研究方面，我們利用蒙地卡羅（Monte Carlo）模擬分析了一種罕見的重子型B介子衰變模式 $\overline{B}^0 \rightarrow J/\psi \Lambda^0 \bar{n}$ （其中 $J/\psi \rightarrow e^+e^-$ 或 $\mu^+\mu^-$ ， $\Lambda^0 \rightarrow p\pi^-$ ）。該衰變末態包含一個粲偶素態（ J/ψ ）和一個中性反重子（反中子），屬於顏色抑制的重子衰變過程，為研究重味物理中的強子產生動力學提供了獨特的檢驗機會。我們對模擬事件採用了專為Belle II電磁量能器（ECL）開發的多變量分析（MVA）反中子粒子鑑別算法，從電磁量能器的簇射訊號中識別反中子候選體。基於相當於積分亮度 200 fb^{-1} 的模擬樣本（假設該衰變的分支比約為 10^{-4} ），我們預測了此衰變模式的信號產額，並將其與通過擬合B介子質心系動量分佈（ P_B^* ）提取得到的信號產額進行比較。結果顯示，擬合得到的信號事件數（約67件）與預期值（約63件）在統計誤差範圍內一致，驗證了反中子鑑別方法的有效性；同時也表明，在當前Belle II數據下觀測該衰變是可行的。

關鍵詞

Belle II 實驗／L1 觸發系統／三維追蹤器／重子衰變／反中子鑑別

Abstract

Belle II at the SuperKEKB collider is a high-luminosity e^+e^- experiment designed to study rare B -meson decays; its Level-1 (L1) trigger system must rapidly distinguish signal events from background in real time. To meet this challenge, we developed a novel three-dimensional (3D) track reconstruction algorithm for the Belle II L1 trigger that operates within sub-microsecond latency and uses geometric transformations with linear regression fits to determine each track's longitudinal z -vertex and polar angle θ . By combining the drift chamber's axial and stereo wires information, the 3D tracker greatly improves background rejection over the existing 2D trigger. In simulation, a geometry-based 3D tracking algorithm ("geo-finder") reduced beam-induced background triggers by 38% relative to the standard 2D tracking, while retaining 97% of genuine signal tracks for vertices within $|z_0| < 35$ cm. Additionally, a lookup-table algorithm ("LUT-finder") that scans all stereo hit combinations to minimize track-fit χ^2 further improves z_0 resolution and background suppression.

For physics analysis, we studied the rare baryonic B -meson decay $\overline{B}^0 \rightarrow J/\psi \Lambda^0 \overline{n}$, where $J/\psi \rightarrow e^+e^-$ or $\mu^+\mu^-$ and $\Lambda^0 \rightarrow p\pi^-$. This decay mode, featuring a charmonium state (J/ψ) and a neutral anti-baryon ($\overline{n}$) in the final state, is a color-suppressed channel providing a unique probe of baryonic decay dynamics. We applied a multivariate analysis (MVA) based anti-neutron particle identification (PID) algorithm, developed for the Belle II electromagnetic calorimeter (ECL), to identify anti-neutron-candidate clusters in simulated events. Using samples corresponding to an integrated luminosity of 200 fb^{-1} (assuming a $\sim 10^{-4}$ branching fraction), we estimated the expected signal yield for this decay and compare it to the yield extracted from fits to the B -meson's center-of-mass momentum distribution (P_B^*). The fitted yield (67 events) agreed with the expected yield (63 events) within uncertainties, demonstrating the anti-neutron PID method's effectiveness; this agreement suggests that this decay mode can be observed with the current Belle II dataset.

Keywords: Belle II experiment, L1 trigger system, 3D tracker, baryonic decay, antineutron identification

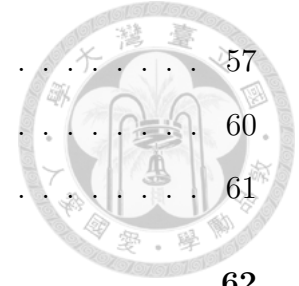


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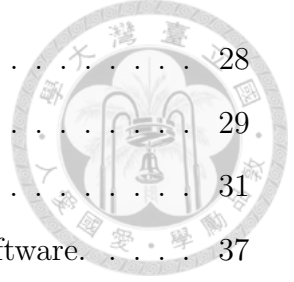




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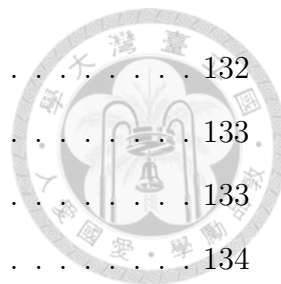


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Chapter 1

Introduction

This chapter provides the theoretical background and motivation for the research presented in this thesis. It begins with an overview of the Standard Model (SM) of particle physics and the structure of flavor physics, proceeds to the mechanism of quark mixing through the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and discusses the origin of charge-parity (CP) violation and its implications for the matter-antimatter asymmetry observed in the Universe. The motivation for using B mesons as a probe to study CP violation is then explained, followed by a summary of the research objectives and structure of the thesis.

1.1 Standard Model and Flavor Physics

The SM of particle physics is the prevailing theoretical framework that describes the electromagnetic, weak, and strong interactions among fundamental particles. It organizes fermions into three generations of quarks and leptons and incorporates the gauge symmetry group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Through decades of experimental validation, the SM has successfully explained a wide range of phenomena, from the dynamics of elementary particles to the mechanisms of fundamental forces.

In flavor physics, the SM accounts for the replication of quark families and their transition patterns, offering a mathematical formalism to describe changes in quark flavors through weak interactions. Understanding the decay processes of hadrons

containing bottom (b) or charm (c) quarks is a key motivation for developing a mathematical formalism to describe quark flavor transitions through weak interactions within the SM.

Fig. 1.1 shows the elementary particles as described by the SM, including the three generations of fermions and the fundamental bosons that mediate interactions.

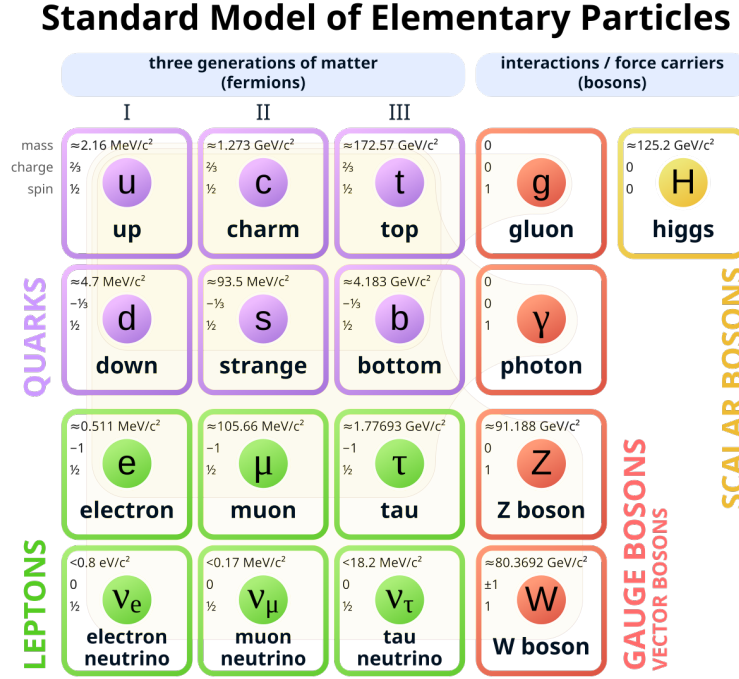


Figure 1.1: Elementary particles of the SM (reproduced from Wikipedia).

1.2 Quark Mixing and Cabibbo-Kobayashi-Maskawa Matrix

In the SM, the weak interaction eigenstates of quarks differ from their mass eigenstates. This misalignment leads to quark flavor transitions via the charged weak current, mediated by the W bosons. The transformation between these two bases is encapsulated in the CKM matrix, a 3×3 unitary matrix that encodes the coupling strengths between different quark generations.

The CKM matrix introduces a complex phase, which is the only known source of CP violation in the quark sector within the SM. Precise determination of the CKM

matrix elements and their phases is essential for testing the internal consistency of the SM and searching for signs of new physics.



1.3 Charge-Parity Violation and Matter–Antimatter Asymmetry

CP violation refers to the violation of the combined symmetry of charge conjugation (C), which swaps particles with antiparticles, and parity (P), which inverts spatial coordinates. In the SM, CP violation originates from the complex phase in the CKM matrix. Although experimentally observed, the magnitude of CP violation predicted by the SM is insufficient to explain the observed matter–antimatter asymmetry in the Universe.

This discrepancy suggests the existence of additional sources of CP violation, possibly from physics beyond the SM. As a result, studying CP-violating processes in various systems, particularly those involving B mesons, has become a central pursuit in experimental particle physics.

1.4 B Physics as a Laboratory for Charge-Parity Violation

B mesons, which contain a bottom quark and a lighter quark (up, down, strange, or charm), provide an ideal system for studying CP violation. Their relatively long lifetimes, well-understood decay channels, and sensitivity to interference effects make them an excellent platform for precise tests of the SM.

The development of dedicated "B factories" such as KEKB/Belle and its upgraded version SuperKEKB/Belle II have enabled the production of vast numbers of B mesons. Operating at the $\Upsilon(4S)$ resonance, these experiments provide a controlled environment with clean backgrounds and high event rates, especially when compared to hadron collider environments such as the LHC, where background levels

are significantly higher [1, 2].

The Belle II experiment, situated at the SuperKEKB collider in Tsukuba, Japan, was the next-generation B-factory experiment at that time, succeeding Belle, designed to extend its physics reach with improved detector technologies and much higher luminosity. First beam-background measurements at SuperKEKB were reported by R. Lewis et al. (2019) [3] providing initial characterization of background sources, detector occupancies, and radiation doses during early commissioning. These studies informed detector protection strategies and operational tuning prior to full-luminosity running. With its significantly improved luminosity and detector capabilities, Belle II aims to conduct precision studies of CP-violating phenomena and search for deviations from SM predictions.

1.5 Research Objectives and Thesis Structure

The primary objective of this thesis is the development and performance evaluation of a 3D tracking algorithm for the Level-1 (L1) trigger system in the Belle II experiment. Inherited from the High Energy Physics group at Korea University, the 3D tracker is designed to reconstruct the z -vertex and polar angle (θ) of charged particle tracks using geometric transformations and linear regression techniques. By improving vertex resolution, the tracker enhances the ability to reject beam-induced background events, thus increasing the purity of triggered events.

The 3D tracker algorithms implemented in this study include both a geometry-based approach and a lookup-table (LUT) based χ^2 -minimization method. Their performance is assessed through both simulation and experimental data studies.

In addition, this thesis explores a baryonic B meson decay channel: $\overline{B}^0 \rightarrow J/\psi (J/\psi \rightarrow e^+e^-, \mu^+\mu^-) \Lambda^0 (\Lambda^0 \rightarrow p^+\pi^-) \bar{n}$. A Monte Carlo (MC) study is conducted to apply a multivariate analysis (MVA) based antineutron particle identification (PID) technique developed in Belle II. The simulation results are examined to evaluate the feasibility and effectiveness of the method.

The remainder of the thesis is structured as follows:

Chapter 2 introduces the Belle II experiment, including the SuperKEKB accelerator, detector components, trigger systems, and data handling infrastructure.

Chapters 3 and 4 detail the design, implementation, and validation of the two-dimensional (2D) and three-dimensional (3D) trackers, respectively, with particular focus on firmware development and simulation-based performance evaluation.

Chapter 5 presents MC studies of specific baryonic B meson decays, showcasing applications of PID techniques and vertex fitting.

Chapter 6 summarizes the key findings and discusses possible directions for future work.



Chapter 2

Belle II Experiment

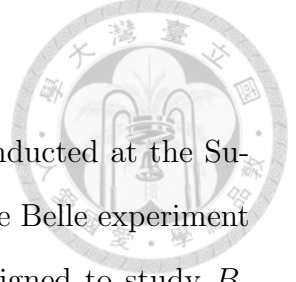
This chapter introduces the experimental apparatus and data processing infrastructure of the Belle II experiment, which serves as the foundation for the physics analyses and algorithm developments presented in this thesis. Located at the SuperKEKB electron–positron collider in Tsukuba, Japan, Belle II was a next-generation B-factory experiment at that time designed to operate at unprecedented luminosities and precision. The chapter begins with an overview of the SuperKEKB accelerator and its running modes, including on-resonance and off-resonance operations, followed by a detailed description of the Belle II detector subsystems such as the vertex detector (VXD), central drift chamber (CDC), calorimeters, and PID systems. It also covers the online data acquisition and real-time trigger systems, which play a crucial role in reducing the data rate and preserving events of physical interest. Furthermore, this chapter outlines the types of data samples used in physics analyses, the structure of MC simulations, and the event reconstruction framework based on the Belle Analysis Framework 2 (basf2) software platform. Collectively, these components constitute the experimental environment in which real-time tracking algorithms are developed, deployed, and evaluated.

2.1 Overview

The Belle II experiment is a high-energy physics experiment conducted at the SuperKEKB accelerator in Tsukuba, Japan. As the successor to the Belle experiment at KEKB, Belle II is a high-precision B-factory experiment designed to study B -meson decays at the $\Upsilon(4S)$ resonance. Its primary scientific goals are to test the SM and explore new physics such as CP violation and potential dark matter candidates. In particular, it focuses on fundamental questions including the asymmetry between matter and antimatter in the Universe and rare processes that may hint at physics beyond the SM.

To achieve these goals, the Belle II experiment operates in conjunction with SuperKEKB, which is currently the world's highest-luminosity e^+e^- collider. This enables the collection of massive datasets with clean experimental conditions, essential for precision measurements and rare decay studies. The Belle II detector is a large-scale general-purpose detector with multiple specialized subsystems arranged in concentric layers around the beam interaction point (IP). These subsystems include high-resolution VXD, a CDC for tracking, Cherenkov detectors for PID, an Electromagnetic Calorimeter (ECL) for energy measurements, and a muon/ K_L^0 detector embedded in the magnet yoke. Each of these subsystems contributes to the reconstruction and identification of particles produced in the e^+e^- collisions.

Fig. 2.1 provides a schematic view of the Belle II experiment, illustrating the location of the detector within the SuperKEKB facility. The cylindrical structure at the center represents the Belle II detector, which surrounds the IP where electron and positron beams collide. The accelerator components form a storage ring that circulates high-intensity beams at asymmetric energies to enhance the laboratory-frame separation of decay products from B mesons. The integration of detector and accelerator systems is critical for maximizing the physics potential of the experiment.



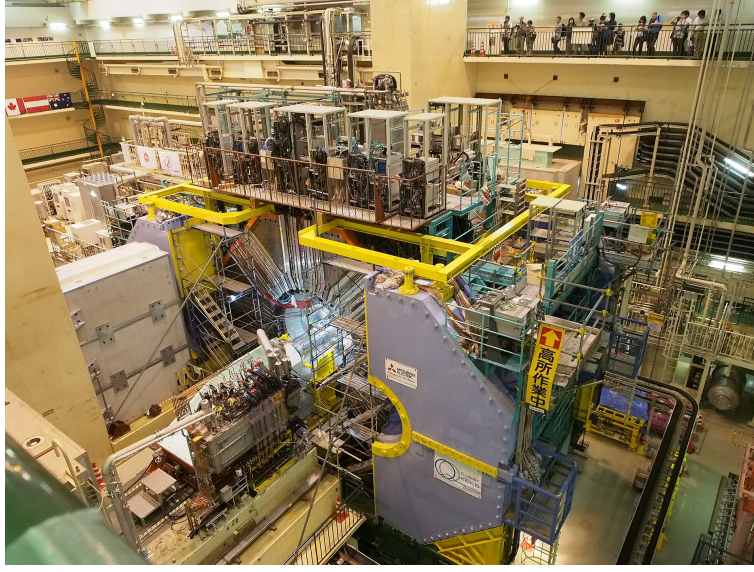


Figure 2.1: Overview of the Belle II experiment at SuperKEKB (reproduced from Wikipedia). The central detector is situated at the interaction region of the e^+e^- beams, surrounded by various subdetector layers. SuperKEKB is an asymmetric-energy collider designed for high-luminosity operation, enabling the production of large numbers of B meson pairs for precision flavor physics studies.

2.2 SuperKEKB Accelerator

SuperKEKB is a state-of-the-art electron–positron collider operated by KEK (High Energy Accelerator Research Organization) in Tsukuba, Japan. As the upgraded successor of the KEKB accelerator, SuperKEKB is a core facility of the Belle II experiment, delivering unprecedented luminosity at the $\Upsilon(4S)$ resonance for the exploration of rare B meson decays, precision flavor physics, CP violation, and potential signs of physics beyond the SM.

SuperKEKB is designed to achieve a peak luminosity of $8 \times 10^{35} \text{ cm}^{-2}\text{s}^{-1}$, approximately 40 times greater than that of KEKB. This is made possible through a combination of high beam currents and the implementation of the nano-beam scheme, which reduces the vertical beam size at the IP to enhance the collision rate. As of 2024, it has achieved a peak luminosity of about $5.1 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$, which is roughly 2.5 times higher than the maximum luminosity reached by KEKB, and continues to improve toward its design goal. The collider consists of two separate storage rings: the High Energy Ring (HER) for 7 GeV electrons and the Low Energy

Ring (LER) for 4 GeV positrons. The beams circulate in opposite directions and are brought into collision at the IP, which is situated at the center of the Belle II detector.

A schematic overview of the SuperKEKB accelerator is shown in Fig. 2.2, illustrating the separate beamlines, collision region, and layout of the storage rings. The asymmetric beam energies provide a boost in the laboratory frame, allowing better spatial separation of the decay vertices from the two B mesons, which is essential for time-dependent CP violation studies.

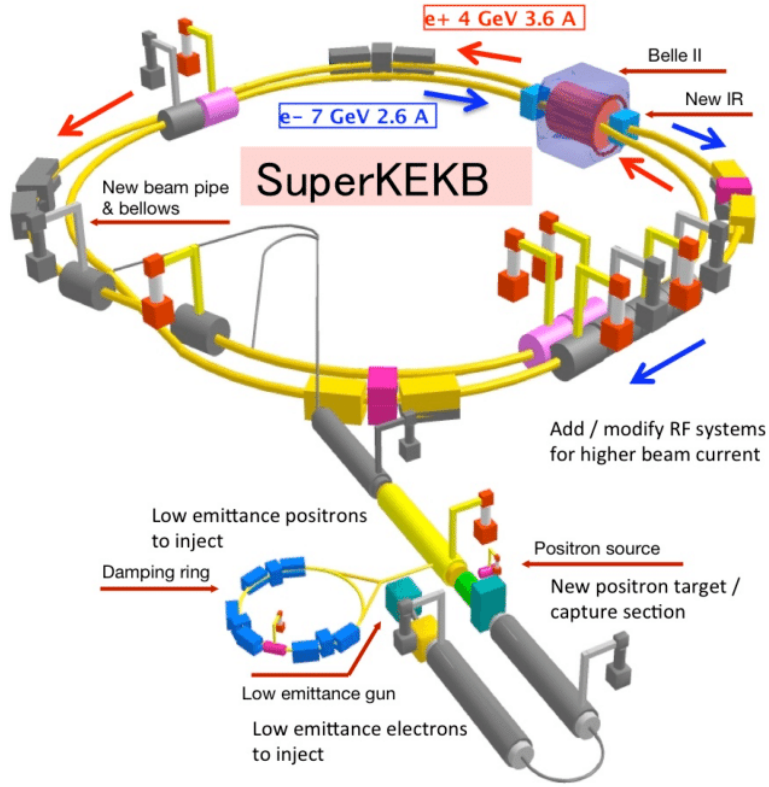


Figure 2.2: The collider consists of two separate storage rings for electrons (HER) and positrons (LER), which intersect at the Belle II IP. The nano-beam scheme and high beam currents are key innovations enabling unprecedented luminosity.

To meet the needs of various physics goals, SuperKEKB operates under multiple data-taking modes, each corresponding to specific beam energy settings and experimental objectives:

- **On-resonance:** The beam energy is tuned to the center-of-mass (CM) energy of the $\Upsilon(4S)$ resonance ($\sim 10.58 \text{ GeV}$), which predominantly decays into B

meson pairs. This is the primary mode used for B physics studies, enabling the precise measurement of CP violation and rare decays.

- **Off-resonance:** The beam energy is lowered by approximately 60 MeV from the $\Upsilon(4S)$ peak. This energy is chosen to suppress $\Upsilon(4S)$ resonance production while remaining close enough in energy to ensure comparable initial-state kinematics. This mode allows the collection of continuum background events such as $e^+e^- \rightarrow q^+q^-$, which are crucial for background subtraction and systematic studies in B meson analyses.
- **Non-4S:** SuperKEKB can be operated at different CM energies to access other bottomonium resonances, such as $\Upsilon(1S)$ through $\Upsilon(6S)$. These runs expand the experiment's scope to include spectroscopy, exotic hadrons, and dark sector searches.
- **Cosmic Run:** In periods without beam injection, the detector records cosmic-ray muons passing through it. These events are used for calibration and alignment of tracking detectors such as the Pixel Detector (PXD), Silicon Vertex Detector (SVD), and CDC. Cosmic runs are typically conducted at the start and end of each run period or during long accelerator downtimes.
- **Beam Run:** During beam runs, electrons and positrons circulate in the rings without colliding. These runs are used to study beam-induced backgrounds, particularly in the inner subdetectors, and to optimize beam optics without the influence of collision products.
- **Energy Scan:** An energy scan involves varying the beam energy in small steps to map out the resonance line shape or search for new resonant states. These scans are conducted over short periods and are useful for studies in charmonium and exotic spectroscopy.

These operating modes are essential for both the calibration of the Belle II detector and the broad physics program pursued by the collaboration.

2.3 Belle II Detector

The Belle II detector is a large-scale, general-purpose spectrometer optimized for the reconstruction and identification of particles produced in high-luminosity electron-positron collisions at SuperKEKB. It is designed to provide precise measurements of B meson decays and rare processes that are sensitive to new physics. Surrounding the IP, the detector consists of multiple subsystems arranged in a layered, cylindrical geometry, each specialized for different aspects of particle tracking, energy measurement, and identification.

A schematic view of the Belle II detector and its subcomponents is shown in Fig. 2.3. From the beam pipe outward, the system includes the VXD, CDC, PID devices—Time-Of-Propagation (TOP) counter and Aerogel Ring-Imaging Cherenkov (ARICH) detector—, and ECL, all enclosed in a 1.5 T superconducting solenoid magnet. The outermost K-long and muon detector (KLM) is located outside the magnet, embedded in the iron return yoke.

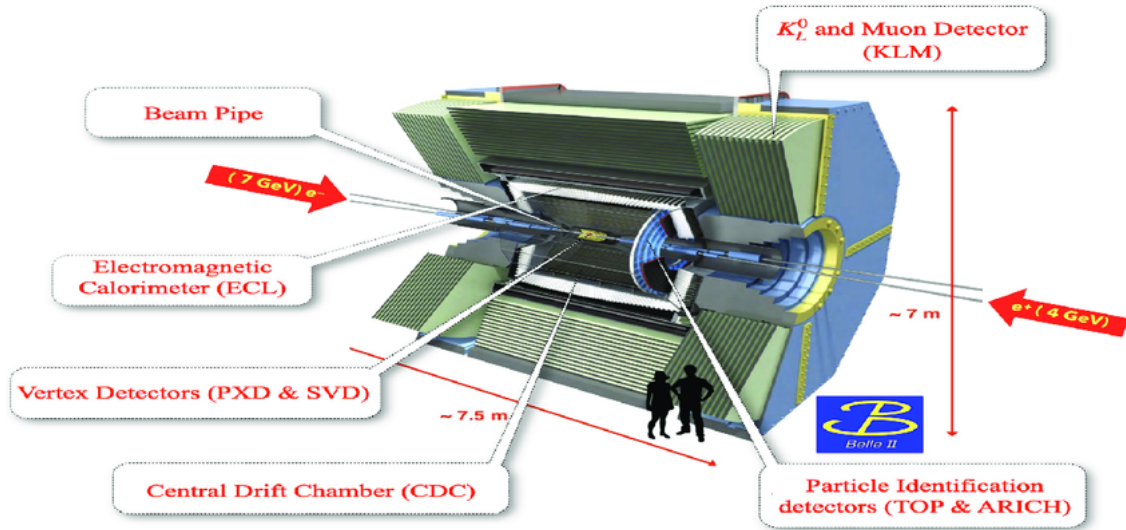


Figure 2.3: Schematic layout of the Belle II detector and highlights of the subdetectors. The subdetectors are arranged cylindrically around the IP to provide full angular coverage for tracking, calorimetry, and PID.

Each component plays a distinct role in event reconstruction:

- **Beam Pipe:** Located at the very center of the detector, the beam pipe provides a vacuum enclosure for the colliding beams. It is constructed from thin

beryllium to minimize multiple scattering and material interactions for particles produced at the IP.

- **VXD:** The VXD consists of two subsystems: the PXD and SVD. These layers offer high spatial resolution for reconstructing decay vertices and impact parameters of short-lived particles such as B and D mesons. The configuration is shown in Fig. 2.4. Positioned closest to the beam pipe, the PXD employs DEPFET (Depleted P-channel Field-Effect Transistor) technology to achieve excellent 2D spatial resolution, enabling sub-100 μm precision in vertex measurements. See Fig. 2.5 for its structural diagram. Surrounding the PXD, the SVD provides additional tracking layers using double-sided silicon strip sensors. It improves the efficiency and accuracy of track reconstruction when combined with the PXD. The structure is illustrated in Fig. 2.6

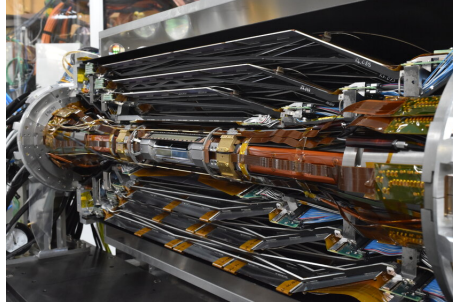


Figure 2.4: Cross-sectional diagram of the Belle II VXD, composed of the innermost PXD layers and the outer SVD layers for precise vertex reconstruction.

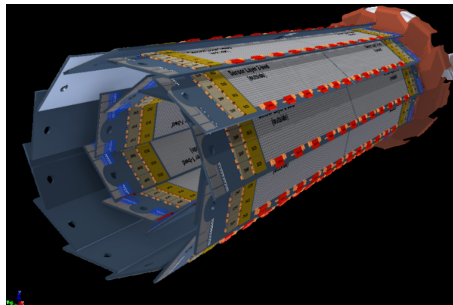


Figure 2.5: Layout of the Belle II PXD, composed of two layers of DEPFET-based sensors surrounding the beam pipe.

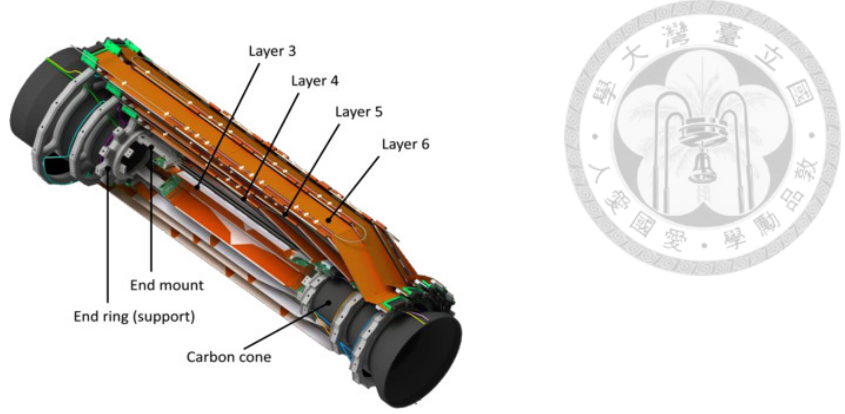


Figure 2.6: Configuration of the Belle II SVD, showing the four layers of silicon strip sensors.

- **CDC:** The Central Drift Chamber (CDC), described by Naoki Taniguchi in 2017 [4], is a large cylindrical gas-filled detector that plays a central role in reconstructing charged particle trajectories and measuring their momenta by analyzing their curvature in the 1.5 T solenoidal magnetic field. The CDC detects charged particles using sense wires in a helium-based gas mixture (50% He, 50% C₂H₆), which becomes ionized as charged particles traverse it. The resulting electrons drift toward the sense wires under the influence of an electric field, and the measured drift time is used to determine hit positions with high spatial precision. Structurally, the CDC consists of nine concentric superlayers (SLs) of wires, which include five axial layers aligned with the beam axis and four stereo layers tilted at the end plates in the azimuthal direction. The stereo layers are arranged with alternating tilts in the $+\theta$ (U-layers) and $-\theta$ (V-layers) directions, following an AUAVAUAVA pattern. This specific arrangement enables full 3D reconstruction of charged particle tracks, including the determination of the longitudinal (z) coordinate. In addition to trajectory reconstruction, the CDC contributes to PID by measuring the ionization energy loss (dE/dx) of charged particles as they traverse the chamber. Furthermore, the CDC readout electronics transmit both hit and drift-time information to the L1 trigger system via high-speed optical links, enabling real-time track-based triggering and low-latency processing. The geometric configuration and design evolution of the CDC are illustrated in Fig. 2.7–2.9,

which compare the Belle and Belle II CDC layouts. The readout electronics for the CDC are described in Ref. [5, 6]

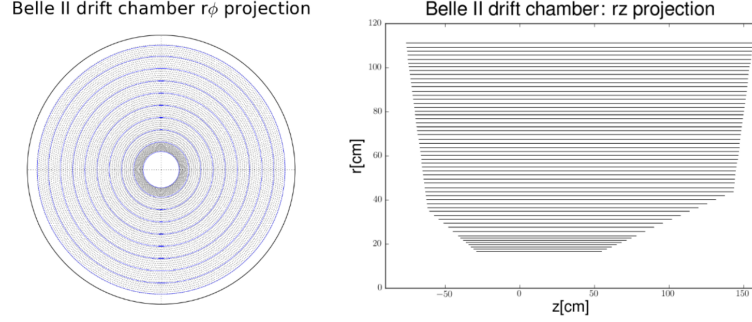


Figure 2.7: Belle II CDC geometry. Left: r - ϕ view. Right: r - z view of SL structure.

SL type and ID	Number of layers	Signal cells per layer	radius (mm)	Stereo angle (mrad)
Axial 0	8	160	168.0 – 238.0	0.
Stereo U 1	6	160	257.0 – 348.0	45.4 – 45.8
Axial 2	6	192	365.2 – 455.7	0.
Stereo V 3	6	224	476.9 – 566.9	-55.3 – -64.3
Axial 4	6	256	584.1 – 674.1	0.
Stereo U 5	6	288	695.3 – 785.3	63.1 – 70.0
Axial 6	6	320	802.5 – 892.5	0.
Stereo V 7	6	352	913.7 – 1003.7	-68.5 – -74.0
Axial 8	6	384	1020.9 – 1111.4	0.

Figure 2.8: Belle II CDC sense wire configuration. ID of 9 SL in is used in later chapter about the CDCTRG system.

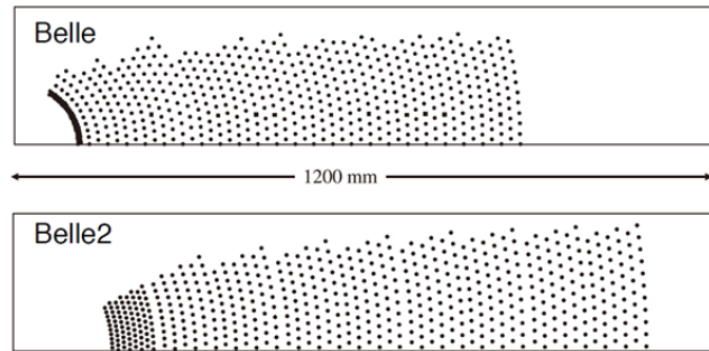


Figure 2.9: Sense wire configuration comparison between Belle and Belle II CDC.

- **TOP:** Installed in the barrel region, the TOP counter identifies particles via Cherenkov light and precise timing measurements. It is crucial for distinguishing kaons from pions over a broad momentum range. See Fig. 2.10.

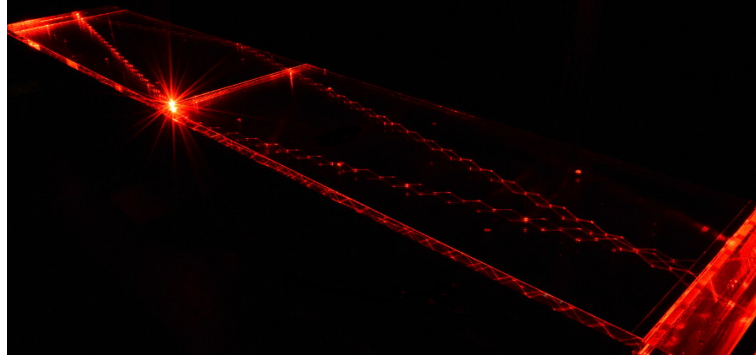


Figure 2.10: The Belle II TOP counter.

- **ARICH:** Located in the forward endcap, the ARICH complements the TOP for PID by detecting Cherenkov photons produced in aerogel radiators. Depicted in Fig. 2.11. The use of two different PID detectors—TOP in the barrel and ARICH in the endcap—stems from their respective geometrical and kinematic requirements. The TOP counter is optimized for near-perpendicular tracks in the barrel region using precise timing of Cherenkov photons, while the ARICH provides efficient PID for forward-going tracks by detecting Cherenkov photons in aerogel.

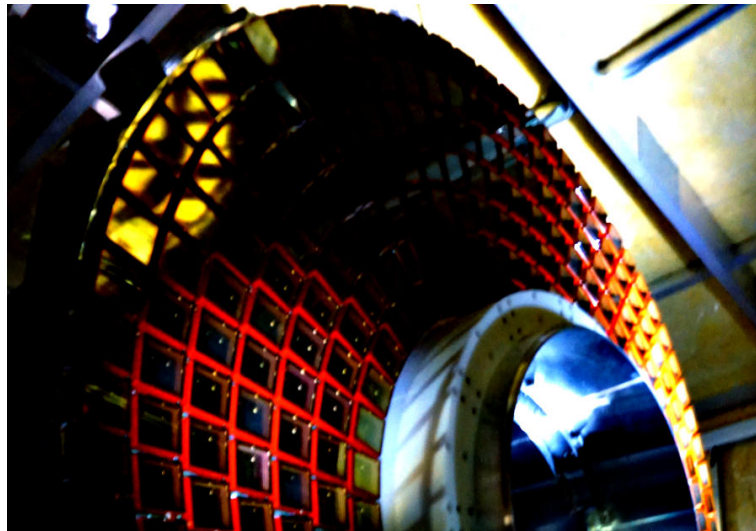


Figure 2.11: The Belle II ARICH detector.

- **ECL:** ECL is composed of thallium-doped cesium iodide crystals $[\text{CsI}(\text{Tl})]$ and measures the energy and position of electrons and neutral particles. It plays a key role in detecting neutral particles and performing Bremsstrahlung recovery.

Its geometry is shown in Fig. 2.12. A convolutional neural network (CNN)-based energy correction using deposited-energy images has shown improved root-mean-square (RMS) and resolution in simulations [7].

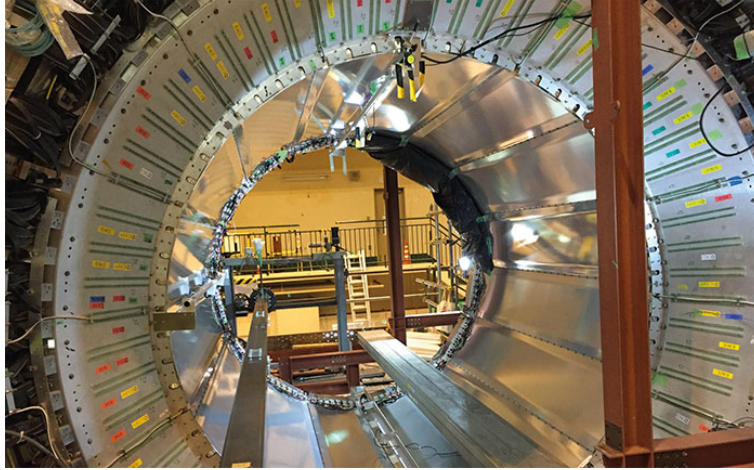


Figure 2.12: The Belle II ECL.

- **KLM:** the KLM detector sits outside the solenoid, embedded in the iron return yoke, and identifies muons and long-lived neutral kaons (K_L^0). Its technology is region-dependent: the barrel uses resistive plate chambers (RPCs) interleaved with absorber plates, whereas the endcaps use plastic scintillator strip modules with wavelength-shifting fibers and SiPM readout to withstand high neutron backgrounds and high rates. This configuration preserves timing and position resolution while ensuring reliable performance under SuperKEKB's high-luminosity conditions. KLM information contributes to muon identification and K_L^0 reconstruction and can be matched with CDC tracks at L1 to suppress backgrounds. See Fig. 2.13.

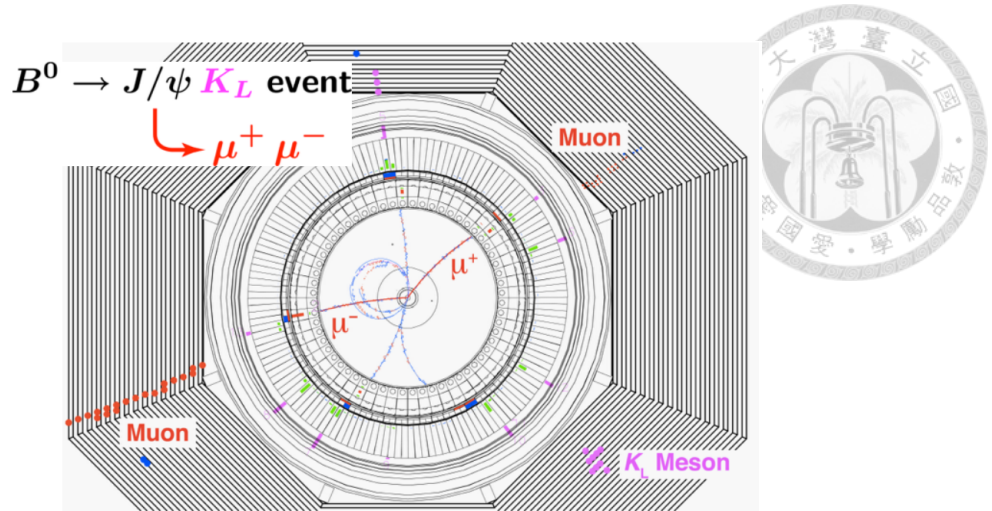


Figure 2.13: The Belle II KLM detector.

2.4 Online System

2.4.1 Overview

The online system of the Belle II experiment is responsible for the real-time acquisition, selection, and processing of events generated by electron–positron collisions at SuperKEKB. Given the high interaction rate enabled by SuperKEKB’s extreme luminosity, an efficient and scalable data handling system is essential to reduce the raw data throughput from the front-end electronics to manageable levels for storage and offline analysis.

The online system is composed of three major components:

- Data Acquisition (DAQ) System
- L1 Trigger
- High-Level Trigger (HLT)

These components are designed to work in coordination to perform real-time event filtering, synchronization, and classification.

An overview of the full Belle II online data flow is shown in Fig. 2.14, which illustrates the relationship between detector front-end electronics, trigger systems,

and data recording/storage infrastructure. A simplified diagram of the data processing path, from detector to storage, is shown in Fig. 2.15. The data flow and HLT of the Belle II DAQ system are described in Ref. [8].

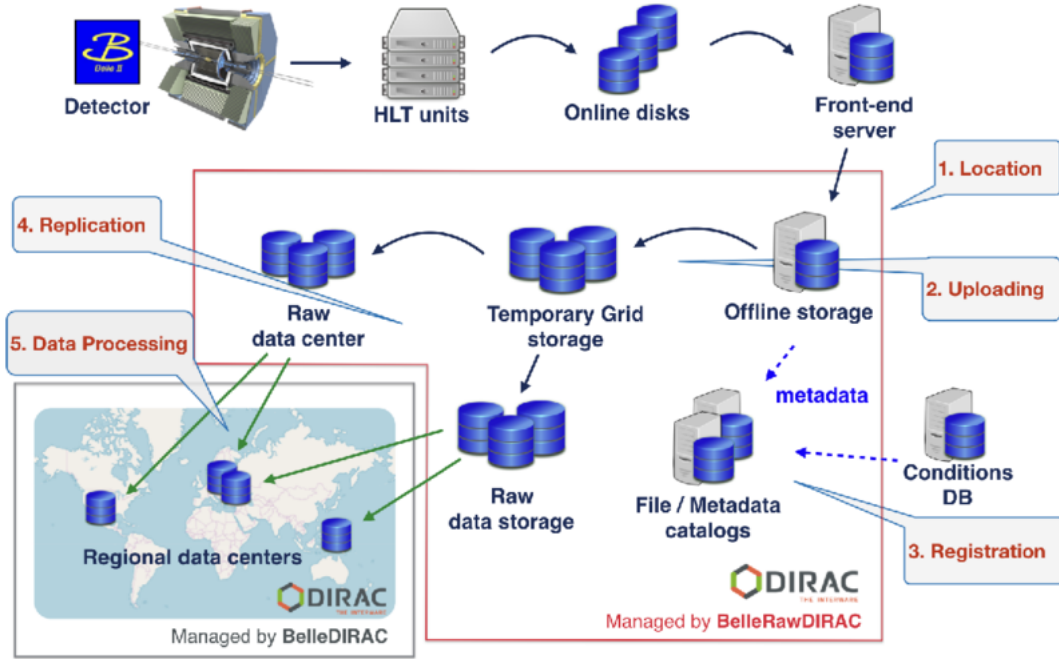


Figure 2.14: Structure of the Belle II online system. Subdetector signals are read out through front-end electronics and processed by the L1 trigger system, followed by event selection by the HLT, which stores events that pass predefined criteria.

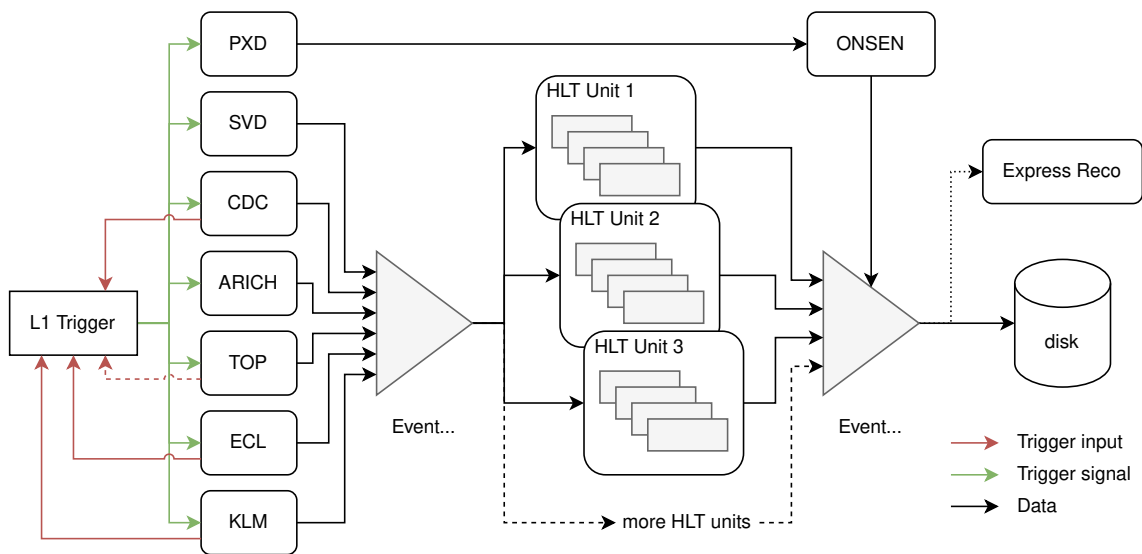


Figure 2.15: Simplified flow of Belle II data acquisition. Signals from subdetectors are digitized and buffered, then selected through the trigger hierarchy before being written to storage.

2.4.2 Level-1 Trigger

The Belle II L1 trigger system is a hardware-based real-time event selection mechanism designed to handle the enormous data flow generated at the SuperKEKB accelerator, reducing the 30 kHz collision rate to 10 kHz by selecting events with high-energy deposits or specific track patterns. Operating with a fixed latency of 5 microseconds, it utilizes Field-Programmable Gate Arrays (FPGAs) and Application-Specific Integrated Circuits (ASICs) to process signals from multiple subdetectors, including the CDC, TOP counter, ECL, and the KLM detector. Although the ARICH detector provides PID capabilities in the forward endcap region, its readout timing and data processing characteristics are not compatible with the low-latency requirements of the L1 trigger system. As such, ARICH information is not used at the L1 stage and is instead incorporated at higher levels of the data processing chain, such as the HLT and offline reconstruction.

Y.-T. Lai et al. (2020, 2025) [9, 10] presented the FPGA-based design and beam-operation performance of the Belle II CDC L1 track trigger and its Global Reconstruction Logic, which reconstruct charged-particle trajectories in real time, performs short-track finding and cross-detector matching, and delivers consolidated trigger conditions with high efficiency and reduced false-trigger rates.

Fig. 2.16 is overview of the Belle II L1 trigger system. The sub-trigger systems include CDC, ECL, BPID (Barrel PID Detector, implemented with the TOP counter), and KLM. The global trigger logic summarizes information from these sub-triggers and issues triggers for various physics events to the DAQ system. Due to the fixed $5\mu\text{s}$ latency, each component is using FPGAs to process the logic flow with high clock rate. All the trigger algorithms are implemented as digitized FPGA firmware, and data are transmitted with high speed optical serial links, as described in detail in Refs. [11, 12]. Each FPGA chip in the whole system is driven by a general 127 MHz clock source, which is called system clock.



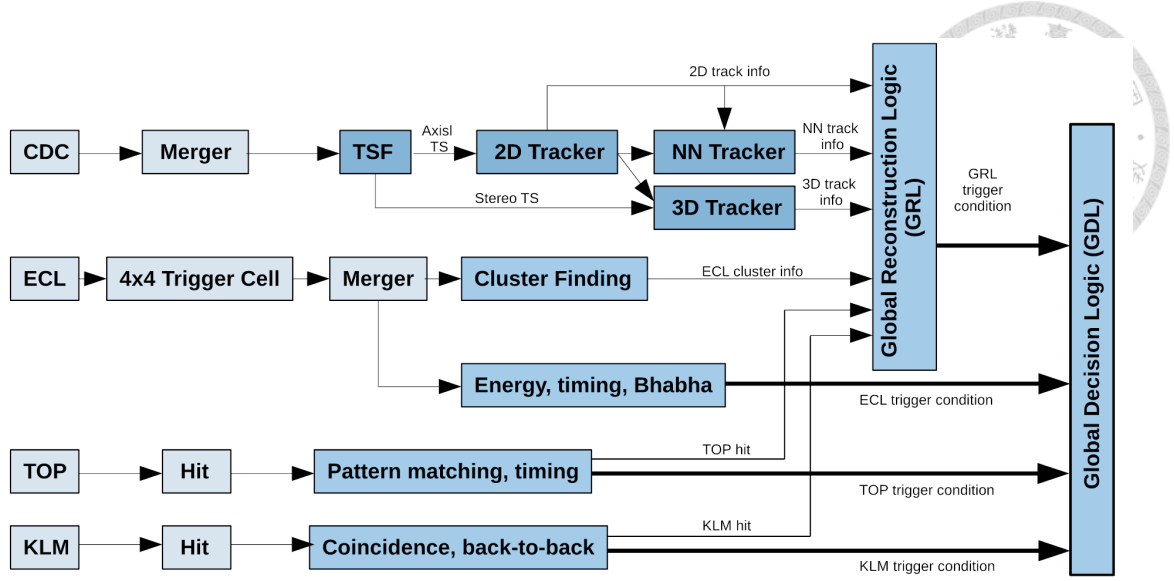


Figure 2.16: Schematic of the Belle II L1 trigger system, illustrating its key components and data flow (adapted from Ref. [10]).

The functions of each sub-trigger system are listed as follows:

1. **TOPTRG:** TOPTRG can provide timing information with a good resolution of under a ns.
2. **KLMTRG:** Identifies muon candidates primarily based on hits in the KLM detector, and its performance can be enhanced by matching with CDC tracks.
3. **GRL (Global Reconstruction Logic):** The information requiring input from all sub-triggers, like track-cluster matching, is performed at the GRL.
4. **GDL (Global Decision Logic):** Integrates information from all sub-trigger systems and determines whether to issue an L1 trigger.
5. **ECLTRG:** Identifies high-energy electromagnetic clusters from the ECL and plays a key role in detecting low-multiplicity final states such as e^+e^- , u^+u^- , $\gamma^+\gamma^-$, and $\tau^+\tau^-$. Although the ECL does not detect charged tracks, cluster-track matching with the CDC and the E/P ratio are useful for identifying electrons at the L1 trigger level. Photons, lacking associated tracks, are instead identified based on the energy and spatial characteristics of isolated ECL clusters.

6. **CDCTRG:** The CDC Trigger (CDCTRG) is responsible for fast online track reconstruction using wire hit data from the CDC. It calculates key tracking parameters such as transverse momentum (p_T), azimuthal angle (ϕ), polar angle (θ), longitudinal impact parameter (z_0), and track multiplicity.

The CDCTRG architecture includes the following main stages:

- **CDC Front-End Electronics (CDCFE):** There are 292 CDCFE boards, each connected to 48 sense wires. These boards digitize wire hit signals with 1 ns timing resolution and output hit data.
- **Merger Boards:** 73 Merger boards aggregate and convert hit into full Track Segment (TS) data, which represent clusters of spatially and temporally correlated hits.
- **Track Segment Finder (TSF):** The TSF detects hit clusters called TSs in each CDC SL and sends up to 15 TSs per SL every 32 nano-seconds to the 2D, 3D, and Neural Network trackers. Nine TSFs (one per CDC SL) identify valid TS candidates using a lookup table (LUT) derived from simulations. A valid TS requires 4 or 5 hits in a time window of 512 ns). The TSF also encodes timing and left/right track orientation to improve resolution, as described in Ref. [13].
- **Event Timing Finder (ETF):** A dedicated board estimates the global event time using early TS data, essential for synchronization in downstream modules.
- **2D Tracker:** Four 2D tracker boards receive TS hits from the 5 axial SLs and perform Hough-transform-based tracking in the r - ϕ plane, identifying circular trajectories and estimates track and vertex proximity.
- **3D Tracker and Neural Network (NN):** Four 3D trackers combine 2D tracks with stereo TSs from U/V SLs to estimate z_0 and θ via fitting. Neural-network-based z -vertex estimation (NN trigger) serves as an alternative path trained on simulated samples.

These modules process all incoming hits in under $5\mu\text{s}$ using clock frequencies of 31.75 MHz (data clock) and 127 MHz (system clock). The 2D and 3D tracking modules play a key role in rejecting beam background by identifying tracks with large radial or longitudinal impact parameters.

Fig 2.17 shows the TS configuration in different CDC SLs, and Fig 2.18 presents the bitmap of the TSF output, illustrating the encoded data format. The 9-bit **data clock counter timestamp** (TDC bin size: 32 ns) indicates the global timestamp when the TSF outputs a batch of TS hits. This timestamp is derived from the front-end free-running clock counter shared across the entire CDC trigger system, ensuring synchronization among all modules. The 9-bit **priority time** (TDC bin size: 2 ns) assigned to each TS hit has two variants:

$$\text{priority time} = \begin{cases} \text{TDC}_{\text{wire}} & (\text{raw TDC, legacy definition}) \\ \text{TDC}_{\text{wire}} - \min(\text{TDC}_{\text{priority wires}}) & (\text{relative TDC, current implementation}) \end{cases}$$

In this study, the priority time follows the legacy definition based on the raw TDC value.

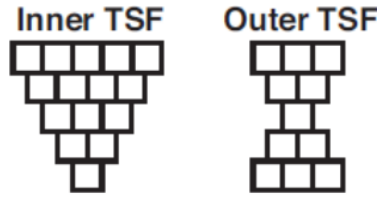


Figure 2.17: TS configuration in different CDC SLs.

c.c : 9 bits clock counter

TSHits : $8(\text{TSID}) + 9(\text{priority time}) + 2(\text{L/R}) + 2(\text{priority hit position})$

L/R

priority hit position

00 - no hit

00 - no hit

01 - right

01 - right cell of 2nd priority layer

10 - left

10 - left cell of 2nd priority layer

11 - unknown

11 - 1st priority layer

170bits/lane

lane 1

c.c										TSHit00										0x99
TShit00										TSHit01										0x88
TSHit01																				0x77
TSHit02																				0x66
TSHit02										TShit03										0x55
TSHit03										TSHit04										0x44
TSHit04										TSHit05										0x33
TSHit05															TSHit06					0x22
TSHit06																				0x11
TSHit06										TSHit07										0x00
TSHit07										TSHit08										0x99
TSHit08										TSHit09										0x88
TSHit09															TSHit10					0x77
TSHit10																				0x66
TSHit10										TSHit11										0x55
TSHit11										TSHit12										0x44
TSHit12										TSHit13										0x33
TSHit13															TSHit14					0x22
TSHit14																				0x11
TSHit14	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	0x00		

lane 0

Figure 2.18: Bitmap of the TSF output, illustrating the encoded data format.

Fig. 2.19 shows the acceptance range of the first 2D/3D/NN tracker. The full ϕ (tangent at the origin) acceptance range (0–360 degrees) is divided into four sectors, each covering approximately 90 degrees and processed by an FPGA board: 45°–135° for Board0; 135°–225° for Board1; 225°–315° for Board2; 315°–45° for Board3, respectively. The actual boundaries are slightly overlapped to avoid track loss at the edges.

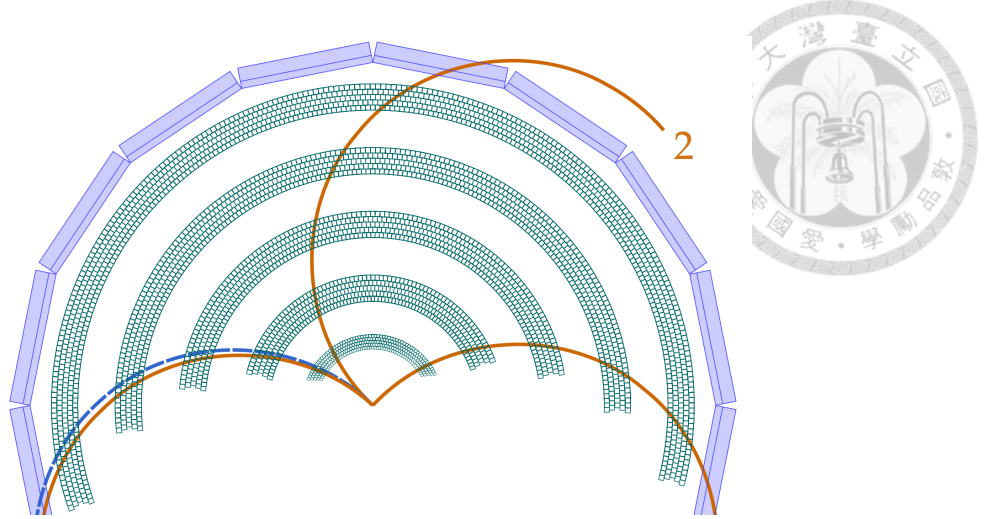


Figure 2.19: Acceptance range of the first 2D/3D/NN tracker (reproduced from Refs. [14]).

In the Belle II CDC, drift-time correction is applied in track fitting to improve the accuracy of azimuthal hit position reconstruction (ϕ_0), while it is omitted in track finding to efficiently handle a large number of TS hits. The drift length is calculated based on the calibrated x - t relation $x = f(t_{raw} - t_{evt})$, where t_{raw} is the raw TDC time and t_{evt} is the event timing as illustrated in Fig. 2.20. For each TS, the drift length toward the sense wire at the segment's priority position is retrieved from a pre-computed lookup table. This length is then added to or subtracted from the wire coordinate based on the left/right (L/R) bit of the segment, producing a fine correction to the ϕ_0 coordinate. If the L/R bit is undetermined, no correction is applied. This approach enables the L1 trigger firmware to achieve optimal ϕ_0 resolution within the limited granularity imposed by the finite number of sense wires in each CDC SL.

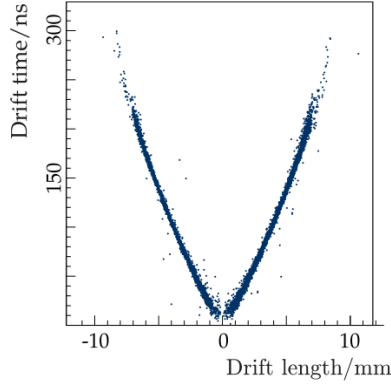


Figure 2.20: Measured drift time vs. drift length in the CDC (reproduced from Refs. [14]).

The self-timing system in the Belle II CDC trigger is designed to improve the efficiency and precision of track-based triggering by dynamically adjusting to event timing. It derives event time from the timing of related TS hits, providing an alternative to the event time from the ETF for drift-time calibration. This approach enhances real-time event selection, reduces timing uncertainties, and improves trigger performance under high-luminosity conditions. In 2D, the self-time is defined as the earliest priority time among all related TSs within a data clock, with a fixed offset x subtracted: $t_{\text{self}} = t_{\text{fastest}} - x$. This value, shared by all 2D tracks within the same data clock, is updated only when a new valid track appears, as later tracks (clone tracks) are often derived from it and do not improve timing accuracy. In 3D, the self-time is the minimum of the 2D self-time and the priority times of related 3D TSs: $t_{\text{self}}^{3D} = \min(t_{2D\text{-self}}, t_{3D\text{-TSs}}) - x$. Each 3D track uses its own self-time.

To improve the precision of self-timing, the original 9-bit TDC priority time (2 ns/bin) is extended to 13 bits by combining it with the coarse timestamp from the data clock counter (32 ns/bin). Since the TSF outputs are always later than the TDC hit, adjustment is required:

$$\text{TDC}_{13} = \begin{cases} \text{CC}[8:5] \& \text{TDC}[8:0], & \text{if } X \geq Y, \\ (\text{CC}[8:5] - 1) \& \text{TDC}[8:0], & \text{if } X < Y. \end{cases}$$

Here $X = \text{CC}[4:0]$, which is **the 5 LSBs of the 9-bit data clock counter**;

$Y = \text{TDC}[8 : 4]$, which is the 5 MSBs of the 9-bit **TDC priority time**; & denotes bitwise concatenation. An example is given below. We compare the data clock counter timestamp with the TDC priority time. Since the red segment is greater than the black segment, we use AAAA as the MSBs of the 13-bit TDC; otherwise, AAAA - 1 is used.

Example: Since $01100 > 01010$ ($X \geq Y$), $\text{TDC}_{13} = \text{AAAA01100BBBB}$.

- AAAA01100 (data clock counter timestamp)
- 01010BBBB (TDC priority time)

Fig. 2.21 illustrates the schematic of TSF data persistence and timing alignment. To extend the 9-bit **TDC priority time** (2 ns/bin) to 13 bits, a coarse **clock-counter** (CC) reference is required. Two options are available:

- **Persisted CC (Original)**: Each TS carries its own CC. It's accurate but FPGA-heavy; fails to synthesize after adding new logic.
- **Present CC (Modified)**: all TS hits in a data-clock share the module's current CC. It synthesizes more easily, but since each 3D TS hit is designed to persist for 1024 ns, a late **Present CC** may differ from the hit's TDC priority time by one coarse cycle (≈ 1024 ns), causing a one-bit overflow that must be corrected. **Overflow mitigation:**

1. **CC - 4**: subtract 4 data-clocks (128 ns) for empirical alignment.
2. **Overflow Correction**: if $(\text{drift-time} - 1024 \text{ ns}) \in [-100 \text{ ns}, 512 \text{ ns}]$ (that is, if the corrected drift-time falls within the valid window) use it; else keep original.

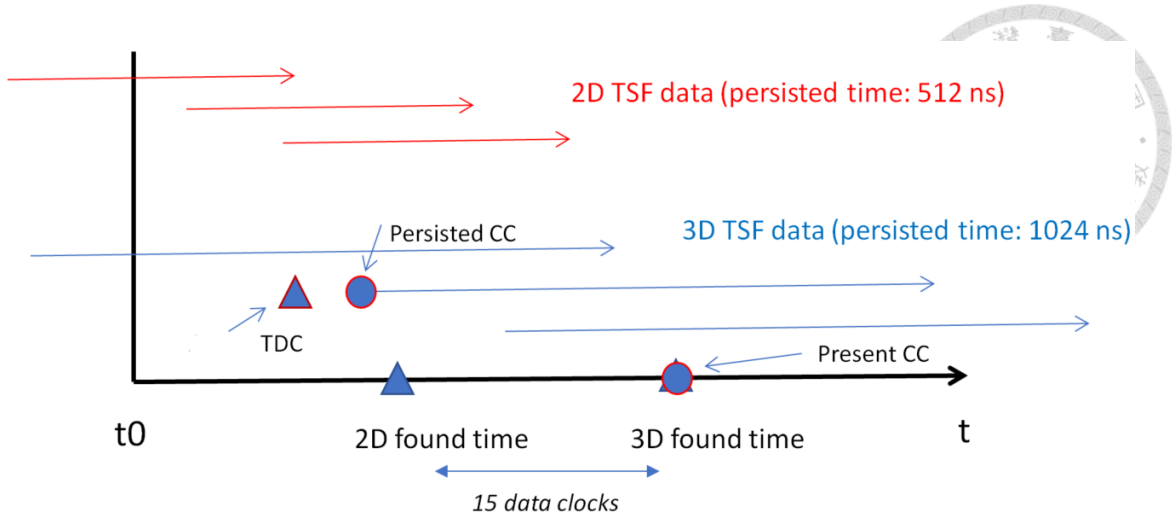


Figure 2.21: Schematic of TSF data persistence and timing alignment. Red arrows represent 2D TSF data with a persistence time of 512 ns, and blue arrows represent 3D TSF data with a longer persistence of 1024 ns. The actual TDC time precedes the persisted coarse CC, which is used for aligning and extending self-timing resolution.

2.4.3 High Level Trigger

The HLT is a software-based event processing system implemented on a CPU cluster ($\sim 10,000$ cores) located on-site. It performs full event reconstruction to further reduce the event rate to about 1 kHz for storage using the same basf2 software as in offline analysis, enabling advanced event classification in real time. The HLT reduces the event rate by approximately 60%, storing only those events that pass detailed selection criteria. The HLT system can operate in two modes:

- **Filtering mode:** Events that fail all HLT selection lines are discarded immediately.
- **Monitoring mode:** All events are retained, and HLT decisions are logged for later analysis.

In addition to classification, the HLT performs HLT skimming, tagging events into broad categories (e.g., hadronic, tau, Bhabha, dimuon) for calibration and early-stage physics analyses. The structure and role of the HLT system are illustrated in Fig. 2.22.

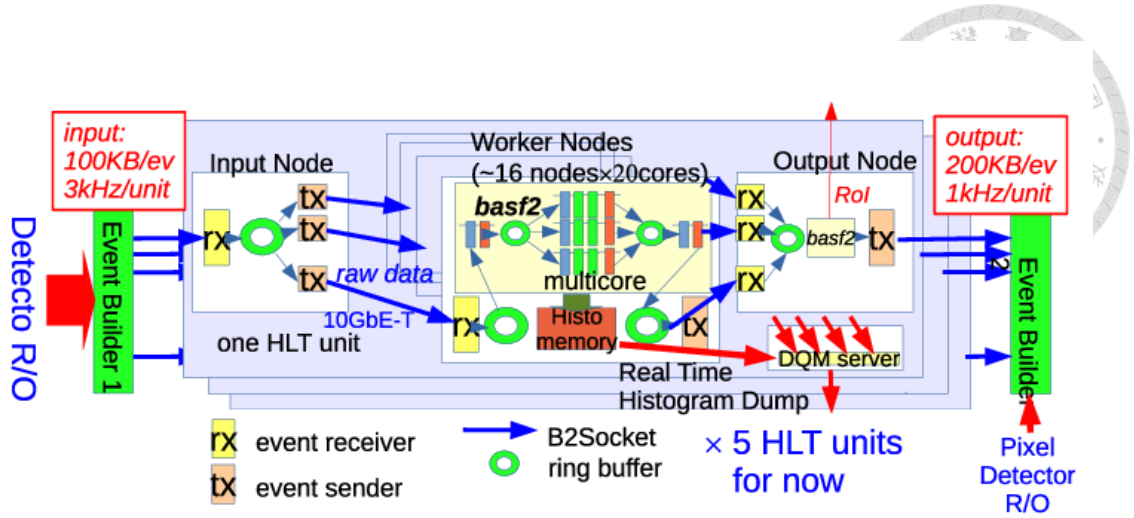


Figure 2.22: Belle II HLT system. After passing the L1 trigger, full event data is reconstructed and analyzed in real time on a computing cluster to determine final storage decisions.

2.4.4 Data Acquisition System

The DAQ system orchestrates the synchronized delivery of trigger signals, manages precise timing and high-speed readout for all subdetectors, and assembles and forwards complete events from the front-end electronics to the HLT and storage. It consists of:

- Timing distribution and trigger signaling via Front-End Timing Switch (FTSW) modules;
- High-speed optical data links based on Xilinx RocketIO technology (up to 2.54 Gbps or 3.125 Gbps, depending on module) using the Belle2link protocol, employed for data transmission from all subdetectors—including CDC, SVD, PID (TOP and ARICH), ECL, and KLM—to the central COMmon Pipelined Platform for Electronics Readout (COPPER) or PCIe40 readout systems (PXD is excluded).
- Readout and buffer control through the COPPER.

The global system clock (127 MHz) is distributed from the SuperKEKB facility, and synchronized trigger signals ensure consistent event building across all detectors.

Trigger latency and readout dead time are minimized to handle the extremely high event rate.

A block diagram of the full DAQ system is shown in Fig. 2.23, including timing distribution, trigger logic, readout electronics, and data transfer pathways to the HLT farm. This architecture enables synchronized data acquisition across more than 1,000 front-end readout systems.

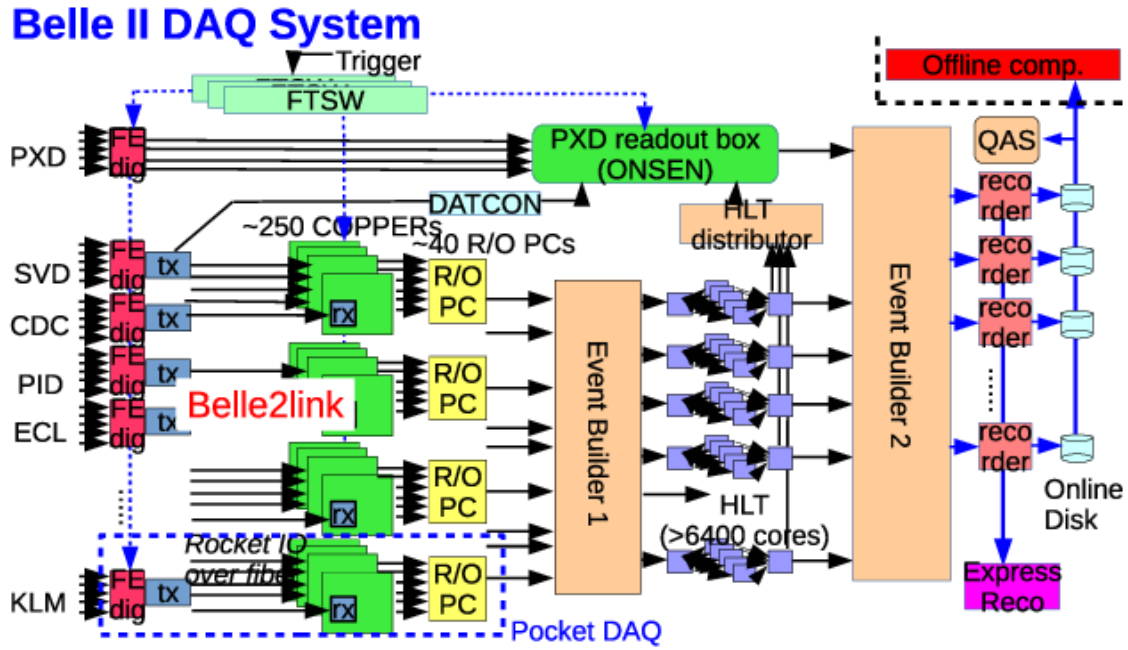


Figure 2.23: Architecture of the Belle II Data Acquisition DAQ system. Trigger and timing signals are distributed via FTSW modules to front-end electronics. Detector signals are transmitted to COPPER boards via high-speed optical links and assembled for processing by the HLT.

2.5 Data Samples

While the HLT performs online event selection and partial reconstruction to reduce the event rate in real time, the prompt reconstruction step is conducted offline a few days after data-taking. It uses improved calibration constants and alignment parameters available at that time to produce a more complete and refined reconstruction. Unlike HLT output—which is optimized for speed and event filtering—prompt data provide early access to physics-quality event content and serve as a valuable intermediate step before full reprocessing. To support a broad range of physics analyses,

the Belle II experiment collects and processes large volumes of data through various acquisition modes. The collected data are classified based on their processing stage and usage purpose. The main categories include prompt data, reprocessed data, and MC simulation samples (discussed in Section 2.6). These datasets are essential for performing event reconstruction, detector calibration, background estimation, and physics measurements:

- **Prompt Dataset:** After data are recorded by the online system, an initial prompt processing step is performed within a few days. The resulting prompt datasets are reconstructed using the best-known calibration constants and detector alignment parameters available at the time of data-taking. While not fully optimized, prompt data provide early access to events for detector monitoring, fast calibration studies, and preliminary physics analysis. They are also useful for validating the performance of reconstruction software and assessing data quality. Although not intended for final physics publications, prompt datasets play a vital role in enabling timely feedback and guiding further improvements in detector operations and reconstruction algorithms.
- **Reprocessed Dataset:** Once improved calibration constants, alignment parameters, and reconstruction software become available, the data are subjected to a full reprocessing procedure. Reprocessed datasets incorporate all available detector updates, leading to enhanced vertexing, tracking, and PID performance. These datasets ensure consistent reconstruction quality across different data-taking periods and are used as the official input for physics analyses and publications. Each reprocessing campaign is labeled with a unique version tag to ensure traceability and reproducibility within the collaboration.

The data processing workflow, including the transition from raw data to prompt and reprocessed datasets, is illustrated in Fig. 2.24.

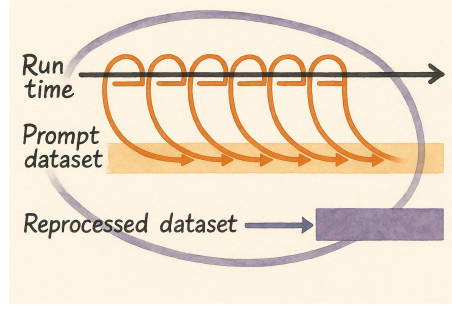


Figure 2.24: Overview of Belle II data collection and processing. Events recorded during runtime are first processed to generate prompt datasets and later undergo full reprocessing with updated calibration and alignment constants.

2.6 Monte Carlo Simulation

MC simulations play a crucial role in the Belle II experiment. They are extensively used for evaluating detector performance, developing and validating reconstruction algorithms, estimating background contributions, and extracting physics observables such as efficiencies and systematic uncertainties. MC samples are designed to closely mirror real experimental conditions, including beam backgrounds, detector geometry, and time-dependent effects.

The generation of MC events begins with physics-level event generation using tools such as EvtGen, KKMC, PYTHIA, and PHOTOS, depending on the physics process of interest. These generators simulate particle decays, quantum electrodynamics (QED) radiation, and fragmentation processes according to theoretical models and input parameters (such as form factors or branching fractions).

The generated particles are then passed to the Geant4-based full detector simulation framework, which models their interactions with the Belle II detector material. This includes energy loss, multiple scattering, secondary particle production, and detector-specific responses such as scintillation, ionization, and Cherenkov light emission.

To match realistic conditions, simulated events are overlaid with beam-induced background events collected from random triggers during real data-taking. This procedure, known as background mixing, ensures that MC samples reflect the same

noise, pileup, and occupancy as the corresponding data periods.

Reconstructed MC events are processed through the same software pipeline as real data, using the basf2 framework. The output includes raw detector signals, reconstructed objects (tracks, clusters, PID), and MC truth information for performance evaluation.

Multiple categories of MC samples are produced:

- **Generic MC:** Inclusive samples covering the full $B\bar{B}$, $c\bar{c}$ and continuum processes.
- **Signal MC:** Exclusive samples for specific signal decay modes studied in individual analyses.
- **Calibration MC:** Samples used for validating detector response, such as Bhabha events for tracking or dimuon samples for muon identification.

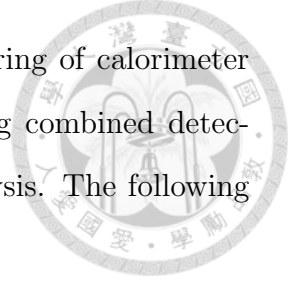
Each MC production campaign is carefully synchronized with data-taking periods to ensure matching run conditions, beam energies, and detector configurations. These campaigns are identified by tags that indicate the software version, geometry, and conditions database snapshot used during simulation.

Through large-scale MC production, the Belle II collaboration ensures that all physics analyses are supported by reliable, high-statistics simulated datasets tailored to the needs of precision flavor physics and new physics searches.

2.7 Reconstruction

The reconstruction process in the Belle II experiment refers to the conversion of raw detector signals into high-level physics objects such as tracks, clusters, vertices, and identified particles. This is performed by the modular and flexible software framework basf2, which is used for both real and simulated data. The reconstruction chain provides the essential inputs for event selection, background rejection, and final physics measurements.

The reconstruction proceeds in several logical stages: clustering of calorimeter and PID signals, tracking using the VXD and CDC, PID using combined detector information, and vertex reconstruction for decay chain analysis. The following subsections describe these components in detail.



2.7.1 Clustering

Clustering refers to the grouping of detector signals into localized energy deposits or hit patterns, which represent the interaction of a particle with a subdetector. In the ECL, clustering algorithms aggregate adjacent crystals with significant energy deposits to form electromagnetic showers. These clusters are then used to identify and measure photons and electrons, and to reconstruct neutral pions (π^0) via their $\gamma\gamma$ decay.

In the KLM system, clusters are reconstructed from Resistive Plate Chamber (RPC) or scintillator hits. These are used to identify penetrating particles like muons or K_L^0 mesons. Meanwhile, in the TOP and ARICH detectors, patterns of Cherenkov photons and timing information are collected into hit clusters for subsequent use in PID algorithms.

Each subdetector applies its own clustering logic optimized for signal geometry, timing, and noise characteristics.

2.7.2 Tracking

Tracking involves reconstructing the trajectories of charged particles by combining hits from the VXD and the CDC. It begins with standalone track finding in the CDC using pattern recognition techniques such as Hough transform and cellular automaton methods. These preliminary tracks are then extrapolated and matched to hits in the VXD to improve resolution and vertex information.

Standalone tracking in the VXD is also performed to recover low- p_T or displaced tracks not fully visible in the CDC. Final track candidates are fitted using a Kalman filter or other iterative methods to extract precise parameters such as momentum,

charge, and impact parameters (d_0, z_0)

Reconstructed tracks serve as the basis for downstream reconstruction steps including PID, vertex fitting, and composite particle reconstruction.



2.7.3 Particle Identification

PID combines measurements from multiple subdetectors to determine the most probable particle hypothesis for each track. PID inputs include:

- Specific ionization (dE/dx) in the CDC: Different particle types lose energy at different rates as they pass through the gas, depending on their mass and velocity.
- Time-of-propagation and Cherenkov angle in the TOP counter: The velocity of a particle affects the emission angle and arrival time of Cherenkov photons, allowing mass discrimination through precise timing and optical path reconstruction.
- Ring image size and pattern in the ARICH: Particles with different velocities produce Cherenkov rings of different radii in the aerogel radiator, which can be used to separate π , K , and p .
- Energy deposition and shower shape in the ECL: Electrons produce electromagnetic showers with characteristic shapes and full energy deposition, while hadrons and muons deposit less energy and show different profiles.
- Penetration depth and hit pattern in the KLM: Muons penetrate deeply and leave hits in multiple layers, while hadrons either stop earlier or produce hadronic showers.

These inputs are processed using likelihood-based algorithms that compare the measured variables to expected distributions for different mass hypotheses (e.g., e , μ , π , K , p). The output is a set of PID likelihoods or likelihood ratios, which are then

used for event selection, background suppression, and reconstruction of specific decay modes.

Good PID performance is essential for many analyses, especially those involving flavor tagging, rare decays, or lepton universality tests.



2.7.4 Data Formats

The outputs of the reconstruction chain are stored using the ROOT-based Belle II event data model. Several data formats are defined to support different purposes across calibration, validation, and physics analyses:

- **Raw data:** Contains the digitized output directly from the detector front-end electronics. This format preserves all low-level detector information and is mainly used for calibration, debugging, and reprocessing. It is not suitable for physics analyses due to its volume and complexity.
- **DST (Data Summary Tape):** Includes full reconstruction output, such as hits, tracks, clusters, and detector-specific information. DST files are primarily used for software validation and algorithm development, rather than user-level physics analyses.
- **mDST (mini Data Summary Tape):** The primary format for physics analyses. It includes high-level reconstructed objects such as tracks, neutral clusters, PID likelihoods, and composite particles. Low-level detector signals are excluded to reduce size and improve performance.
- **cDST (calibration DST):** A subset of DST tailored for calibration and alignment tasks. It contains specific information relevant to time-dependent detector calibration, such as alignment hits and monitoring histograms. These files are used during prompt and reprocessing stages to generate calibration constants.
- **uDST (user DST):** A reduced data format derived from mDST, created by users or physics groups for specific analyses. It typically contains skimmed

subsets of events, meaning only events that satisfy certain selection criteria are retained. In addition, users may keep only a custom set of variables relevant to their analysis to improve I/O performance and reduce analysis time. uDSTs are not centrally produced and vary depending on the analysis requirements.

Each data file is tagged with metadata indicating the processing campaign, software version, and conditions database snapshot used, ensuring full reproducibility and consistency.

2.8 Belle Analysis Framework 2

The basf2 serves as the primary software framework for the Belle II experiment, supporting a full spectrum of tasks ranging from detector simulation to data reconstruction and physics analysis. Designed with modularity and scalability in mind, basf2 enables researchers to construct and execute customized data processing pipelines with high efficiency and flexibility.

The development of the Belle II software framework was first undertaken by Soo-hyung Lee in 2010 [15], establishing the modular and scalable design principles that underpin the present basf2 system. The framework combines high-performance C++ code for backend processing with a Python-based steering interface, allowing users to define and control the processing chain without direct modification of the compiled code. Its modular architecture ensures consistency, reproducibility, and maintainability across large-scale collaborative workflows.

2.8.1 Overview

At its core, basf2 is built upon the concept of a data processing path composed of discrete functional modules. Each module is responsible for a specific operation, such as data input, event simulation, reconstruction, or analysis. These modules are chained together through a user-defined Python script, which determines the order and configuration of processing.

The Data Store, a shared memory structure, mediates the communication between modules by storing intermediate data objects that can be read and written as needed. This design facilitates modular development and simplifies debugging and data validation.

Fig. 2.25 illustrates the data flow in basf2, depicting how modules interact with the Data Store and how the entire pipeline is orchestrated via a Python script.

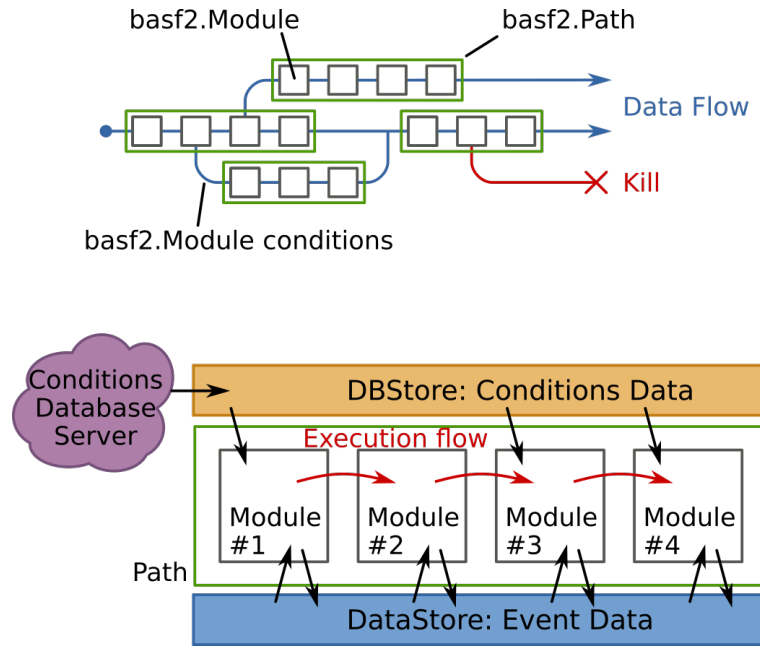
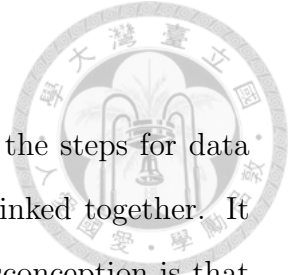


Figure 2.25: Schematic view of the processing flow in the Belle II Software. This diagram shows the modular design of basf2, where each rectangular block represents a software module (e.g., simulation, reconstruction, or analysis). The arrows denote the data flow through the shared Data Store. The user defines the sequence and configuration of modules using a Python steering script, enabling flexible construction of complex processing paths.

2.8.2 Modules

Each basf2 module encapsulates a specific task in the event processing pipeline. Modules are written in C++ for performance and are dynamically loaded at runtime. They are registered with the framework along with metadata such as input/output types, configurable parameters, and descriptions.



2.8.3 Path

In basf2, a path is an ordered sequence of modules that defines the steps for data processing. You can think of it as a chain where modules are linked together. It is represented in Python as a `basf2.Path` object. A common misconception is that adding modules to a path means the data is already being processed. In reality, adding modules only prepares the path for processing.

2.8.4 Package

In basf2, a package is a logical grouping of code. A typical package contains multiple modules along with Python scripts that set up paths for performing common tasks. These packages are usually given meaningful names—such as tracking or reconstruction—or are named after the subdetectors they are associated with, like `ecl`, `klm`, `cdc`, or `top`.

2.8.5 Steering

A steering file, or steering script, is a Python program that sets up an analysis or data-processing task in basf2. Typically, a steering file creates a `basf2.Path`, configures various basf2 modules, and adds them to the path. It then calls `basf2.process` to begin the actual processing, and may include commands to print status or summary information. The term “steering” is used because the Python code does not perform the data processing itself—it simply directs how the processing should be carried out.

2.8.6 Databases

basf2 integrates several types of databases for accessing runtime information, ensuring consistency across different data-taking periods and software versions:

- The conditions database stores time-dependent calibration constants, alignment parameters, and detector configurations.

- The payload database handles static resources such as magnetic field maps, detector masks, and correction tables.

Modules query these databases automatically based on the run and event number. This synchronization mechanism guarantees that the most appropriate constants are used during event processing.

Database access is abstracted via Application Programming Interfaces (APIs), allowing seamless use in both local and distributed computing environments.

2.8.7 Trigger Simulation

The Trigger Simulation (TSIM) system in the **basf2** framework provides a software-based emulation of the hardware-level CDC trigger system used in the Belle II L1 trigger. It reproduces the core logic of FPGA-based firmware, enabling detailed offline validation, debugging, and development of trigger algorithms. TSIM is fully integrated into **basf2**, allowing users to simulate the behavior of trigger modules using standard Python steering scripts and reconstructed detector hits.

As illustrated in Fig 2.26, the simulation begins by generating single-track muons (μ^+ and μ^-) using a Particle Gun, with inverse transverse momentum distributions and uniformly sampled angular directions. The muons are then propagated through the detector using a full physics simulation (**FullSim**), which includes the Beam Pipe, PXD, SVD, and the CDC in a uniform magnetic field. The CDC response is digitized by the **CDCDigitizer**, producing wire hit information similar to what the actual hardware receives.

TSIM is handled via the **TRGCDC** module, which encapsulates the entire CDC trigger logic chain—TSF, ETF, 2D Tracker, 3D Tracker, and neural network-based z -vertex estimation. These components are executed in sequence to generate trigger-level quantities such as track multiplicity, transverse momentum, polar angle, and z -vertex position. The results are written to ROOT files for further analysis or comparison with hardware-level outputs.

To facilitate integration, a Python function `add_cdc_trigger()` is provided in the

module `add_cdc_trigger.py`. This function offers a unified interface to insert the CDC TSIM into a `basf2` processing path, with options for either fast simulation or full firmware emulation using `TRGCDC`. It also allows customization of track selection criteria, including minimum momentum and length thresholds.

Early versions of the simulation used the script `trgcdc.py`, which directly invoked TRGCDC for full trigger emulation. A more flexible alternative, `CDCtrigger.py`, loads TRGCDC only when full simulation is requested. Both scripts are located in the `/trg/cdc/example/` directory and serve as templates for implementing CDC TSIM within user-defined workflows.

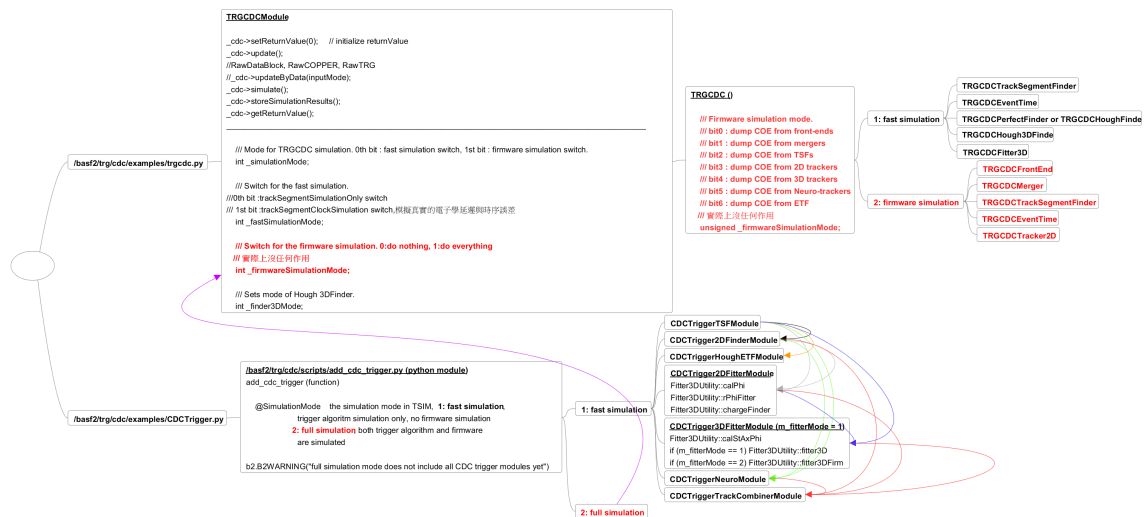


Figure 2.26: Schematics of the TSIM System in basf2. This figure illustrates the flow of the TSIM. Muons generated by a Particle Gun are propagated through detector components using `FullSim`, and CDC signals are digitized by `CDCDigitizer`. The trigger algorithm—executed by `TRGCDC`—simulates firmware logic, reconstructs track-level quantities such as z_0 and θ , and outputs them for validation against hardware or offline reconstruction results.



Chapter 3

2D Tracker

3.1 Introduction

The L1 track trigger in the Belle II experiment plays a critical role in reducing the event rate to a manageable level by rapidly identifying charged particle tracks in the CDC. Among its key components, the 2D tracker is responsible for reconstructing transverse track parameters within strict latency constraints using FPGA-based logic. The development of this system spans over a decade, integrating a wide range of algorithmic innovations and hardware implementations.

A number of significant contributions have shaped the current 2D tracker. Initial simulation-based studies by Cheng-Heng Huang (2011) [16] investigated 2D track reconstruction using the Hough transform, laying the groundwork for later designs. Hsin-Po Chiu (2013) [17] proposed a three-stage algorithm—Hough voting, clustering, and center finding—which was implemented in both TSIM and firmware. Yen-Fu Kuo (2014) [18] enhanced the Hough and peak-finding algorithm for improved reliability. Kai-Yu Chen (2015) [19] focused on optimizing the TSIM software to reduce FPGA resource usage and improve cluster localization. Zheng-Xian Chen (2016) [20] developed firmware modules such as the finder and selector, introducing Hough voting, clustering, and TS association logic with Chipscope verification. Sara Pohl (2018) [21] advanced the system by optimizing the Hough transform and integrating neural networks to improve z -vertex resolution and efficiency. Tzu-An Sheng

(2018) [14] implemented the full 2D finder and fitter in firmware with high efficiency and low latency, and improved the optical transmission system. Most recently, Ping Ni (2022) [22] developed the “Full-Hit” 2D tracking algorithm, achieving a reduction in fake trigger rates from over 30 kHz to under 1 kHz and enabling physics operations readiness for 2023.

While the architecture of the 2D tracking system has been established and integrated into the universal trigger board, the fitter module—responsible for computing track parameters from Hough candidates—has not yet undergone full debugging or validation in the installed system. Its correct functioning is essential to ensure reliable trigger decisions under realistic conditions.

The aim of this study is to complete the implementation, debugging, and validation of the 2D fitter module. We performed detailed performance studies and ensure that the firmware behaves robustly under both cosmic-ray and beam data inputs within the L1 trigger.

The 2D tracking system has been constructed upon an inherited and well-tested framework. By validating the fitter with real data, our work consolidates the 2D tracking chain, providing a solid foundation for future upgrades such as 3D tracking and high-luminosity applications.

3.2 Overview

The 2D tracker is a key component of the Belle II L1 trigger system, designed for fast and accurate reconstruction of charged particle trajectories in the transverse plane. It operates under strict latency constraints and significantly reduces the computational burden on the subsequent 3D tracker and HLT. To understand the structure and data flow of the tracking system, this section provides an overview of its main functional components. The following sections will examine each module in greater detail, starting with the 2D Hitmap Maker.

This tracking system consists of three main modules that work sequentially, as illustrated in Fig. 3.1:

- **The 2D Hitmap Maker:** Generates a 2D time-correlated spatial hitmap in Hough space where each cell accumulates votes based on TS hit positions;
- **The 2D Finder:** Uses Hough transformation on axial TS hits to identify circular trajectories by locating intersections in parameter space;
- **The 2D Fitter:** Applies drift-time-corrected circular fitting to TS hits associated with each 2D track candidate to refine transverse parameters.

Together, these modules provide a real-time estimate of track multiplicity, transverse momentum, and azimuthal angle, which are essential for triggering on high-momentum physics events and suppressing beam-induced backgrounds. The outputs of the 2D tracker are forwarded to the 3D tracker for z -vertex reconstruction and to the GDL for final event selection.

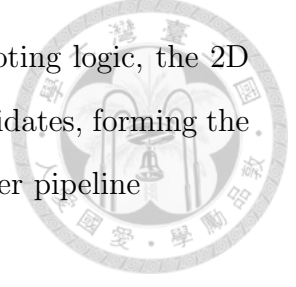


Figure 3.1: Schematic of 2D tracker.

3.3 2D Hitmap Maker

The 2D hitmap maker is a core component of the L1 trigger system in the Belle II experiment, responsible for real-time pattern recognition of charged particle tracks in the transverse (r - ϕ) plane using data from the CDC. Upon receiving TS hits from the five axial superlayers, the system performs a series of transformations to detect track patterns. Each TS hit is first mapped into a conformal space, which linearizes circular trajectories, and then further transformed via a Hough transformation into a parameter space defined by the track curvature (ω) and initial azimuthal angle (ϕ_0). In this space, tracks manifest as clusters or peaks on a 2D Hough map. The system identifies these peaks, representing candidate tracks, and determines their parameters. To accommodate FPGA hardware constraints, the clustering and peak-finding processes are optimized for low-latency implementation using simplified logic

operations. By encoding hits into a binary map and applying voting logic, the 2D hitmap maker enables rapid and efficient extraction of track candidates, forming the foundation for subsequent filtering and fitting stages in the trigger pipeline



3.4 2D Finder

The 2D track finder is responsible for identifying track candidates from the hitmaps generated by the previous stage. It uses a combination of spatial clustering and pattern recognition techniques to detect track-like structures among the CDC hits.

The core algorithm is based on the Hough Transform, which maps spatial hit coordinates into a parameter space where circular or linear patterns become clusters or intersections. This approach is robust to noise and allows fast, parallelizable implementation in FPGA logic. Prior to the Hough Transform, a conformal transformation is applied to convert curved track segments into straight lines, simplifying the pattern recognition stage. Fig. 3.2 shows the schematic view of the conformal transformation.

Despite its effectiveness, the 2D track finder faces challenges such as high occupancy, overlapping trajectories, and time jitter. To mitigate these issues, optimization techniques including adaptive thresholds and real-time filtering are employed. The finder's outputs are passed to the fitter for parameter refinement.

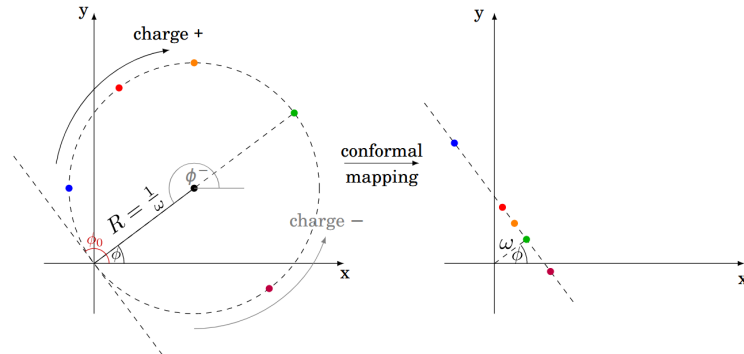


Figure 3.2: Schematic view of the 2D Tracker transformation procedure (reproduced from Refs. [21]). (a) shows particle tracks in the CDC transverse (x - y) plane as arcs due to the magnetic field. (b) shows the corresponding conformal-transformed space, where circular arcs become straight lines, allowing efficient detection via Hough transformation.

3.5 2D Fitter

The 2D track fitter receives candidate trajectories from the finder and refines their parameters—such as transverse momentum and azimuthal angle—through least-squares regression. This module incorporates drift-time information from the priority TS, enabling a more precise estimate of the actual track position within a wire cell.

Accurate 2D fitting is essential for reliable downstream processing, especially for the 3D tracker, which relies on transverse parameters to compute the z -vertex of each track. Thus, the performance of the 2D fitter directly impacts the overall vertex resolution and trigger efficiency of the system.

To ensure optimal resolution within the L1 latency constraints, the fitter uses pipelined arithmetic and fixed-point logic for real-time execution in FPGA. Systematic studies have shown that including drift-time corrections improves the precision and robustness of transverse track parameter reconstruction.

The main relation that the 2D fitter relies on is obtained from the CDC geometry in the r - ϕ plane $2\rho\cos(\varphi - \varphi_i) = R_i$, where $\rho = 1/\omega$ is the radius of curvature of the track, R_i the radius of each axial SL from the IP, φ the azimuthal angle to the center of the track circle, and φ_i the azimuthal angle of a TS found by the 2D finder. If the drift time information is available, φ_i would be replaced by the fine φ_i . The best fit of the track from the positions of associate TSs is found by minimizing

$$\chi^2 = \sum_{i=0}^4 \frac{[2\rho\cos(\varphi_0 - \varphi_i) - R_i]^2}{\sigma_i^2},$$

where φ_0 is angle to the center of the track (signed; range from $-\pi$ to $+\pi$) ; φ_i is **fine** (drift-time corrected) azimuthal angle of the i -th TS; R_i : radius of each axial SL from the IP; σ_i : uncertainty of the corresponding TS hits. Fig. 3.3 shows the geometric definition of 2D tracking parameters.

After using the method of Lagrange multipliers, we obtain the fitted track pa-

rameters φ_0 and ρ as:

$$\tan \varphi_0 = \frac{S}{C}, \quad \rho = \frac{\sqrt{C^2 + S^2}}{\sqrt{2}B},$$

where $C_i = \frac{\cos \varphi_i}{\sigma_i}$, $S_i = \frac{\sin \varphi_i}{\sigma_i}$ and

$$S = \sum_{i=0}^4 C_i^2 \sum_{i=0}^4 R_i S_i - \sum_{i=0}^4 C_i S_i \sum_{i=0}^4 R_i C_i, \quad (3.1)$$

$$C = \sum_{i=0}^4 S_i^2 \sum_{i=0}^4 R_i C_i - \sum_{i=0}^4 C_i S_i \sum_{i=0}^4 R_i S_i, \quad (3.2)$$

$$B = \sum_{i=0}^4 C_i^2 \sum_{i=0}^4 S_i^2 - \left(\sum_{i=0}^4 C_i S_i \right)^2. \quad (3.3)$$

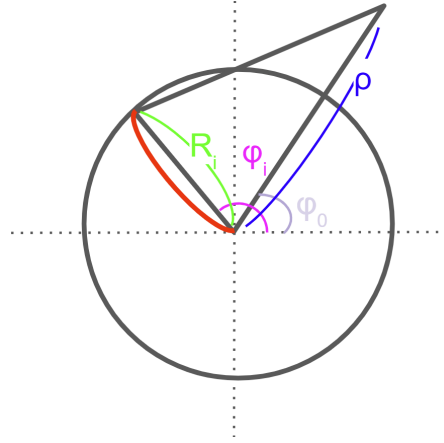


Figure 3.3: Geometric definition of 2D tracking parameters such as ϕ_0 and ω in the transverse plane.

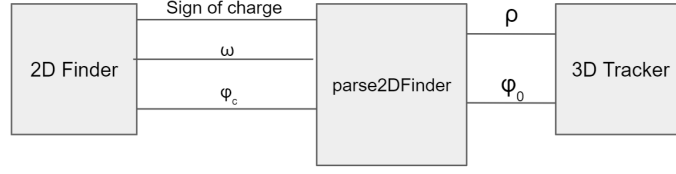
3.6 Implementation

The 3D tracker can operate without the 2D fitter by connecting to the 2D finder through a `parse2DFinder` converter, as shown in Fig. 3.4, where ω is the curvature of the 2D track (signed; same sign as the electric charge); φ_c is the incident angle of the track (unsigned; range from 0 to π); ρ is radius of curvature of the track (unsigned); φ_0 is the angle to the center of the track (signed; range from $-\pi$ to $+\pi$). Note that In basf2, φ_0 denotes the angle defined by the transverse momentum at the point of closest approach (POCA: the smallest distance to the z -axis and with the



smallest absolute z value) and the x axis, ranging from $-\pi$ to $+\pi$, which is different from the definition in this thesis.

Firmware w/o 2D Fitter



Firmware with 2D Fitter

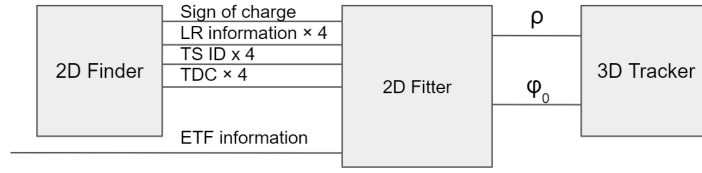


Figure 3.4: I/O interfaces of the 2D fitter and `parse2DFinder`.

The bitmap of the 2D fitter is presented in Fig. 3.5, and the step-by-step calculation of the 2-dimensional track parameters fitting is illustrated in Fig.3.6. The pipeline stages are marked in the diagram. Most registers are omitted. Firstly, the drift time of a TS is obtained by subtracting the event time from the priority time of the TS. The angular displacement of a TS due to the drift time is calculated by an LUT. The angular position of the priority wire in the TS is the ID multiplied by the angular interval in the specific SL. Depending on the left/right information, the drift angle is added to or subtracted from the wire angular position to obtain a more precise estimation of the track's hit position in the SL. Then, the sine and cosine of the fine angle are calculated. Five combination of sine and cosines are calculated, then summed up using 4 adders. The products of these five terms form another six terms, which give the track parameters $\rho = 1/\omega$ and ϕ_0 , as described in the rest of the figure. Each row in the diagram corresponds to a clock cycle. The latency for an LUT function is four clock cycles. One clock cycle for reading the input signal to each branch of the composite LUT, two for reading from the block RAM, and one for choosing a branch and filling to the output signal.

track	content	type	length	position (full speed)	position (half speed)	description
	CC	unsigned	9	746:738	504:496	front-end clock counter
0	flag	logic	1	737	495	old track:1, new track:0
1	flag	logic	1	736	494	ditto
2	flag	logic	1	735	493	
3	flag	logic	1	734	492	
4	flag	logic	1	733	491	reserved
5	flag	logic	1	732	490	reserved
0	trigger	logic	1	731	489	found:1, not found:0
1	trigger	logic	1	730	488	ditto
2	trigger	logic	1	729	487	
3	trigger	logic	1	728	486	
4	trigger	logic	1	727	485	reserved
5	trigger	logic	1	726	484	reserved
0	charge	logic	2	725:724	483:482	+:01, -:10, ?:11
	ω^1	signed ²	7	723:717	481:475	$[-33, 33] \rightarrow [-3.2, 3.2]\text{GeV}^{-1}$
	ϕ_0^3	unsigned	7	716:710	474:468	$[0, 82] \rightarrow [46.125^\circ, 138.375^\circ]$
	TSF0	logic	21	709:689	467:447	same as TSF output
	TSF2	logic	21	688:668	446:426	ditto
	TSF4	logic	21	667:647	425:405	
	TSF6	logic	21	646:626	404:384	
	TSF8 ⁴	logic	21	625:605	383:363	
1	etc.		121	604:484	362:242	
2			121	483:363	241:121	
3			121	362:242	120:0	
4			121	241:121	N/A	
5			121	120:0	N/A	

¹The track parameters ω and ϕ_0 corresponds to the indices of the Hough fine map (the grids of the cluster centers). $\omega = 0$ corresponds to the central row (charge undetermined).

$p_T = 0.3 \cdot 34 / |\text{firmware } \omega| \text{ (GeV)}.$

²Signed binary is encoded in 2's complement.

³The index 0 of ϕ_0 on the Hough fine map starts from the first possible center position, 46.125° . For 2D0,

$\phi_0 = 45^\circ + 90^\circ / 80 \cdot (\text{firmware } \phi_0 + 1).$

⁴The TS ID given by TSF8 is larger than the actual ID by 16, so that continuous unsigned binary encoding can be achieved. $[370, 383] + [0, 225] \rightarrow [2, 241]$

Figure 3.5: Bitmap of 2D fitter (reproduced from Refs. [14]).

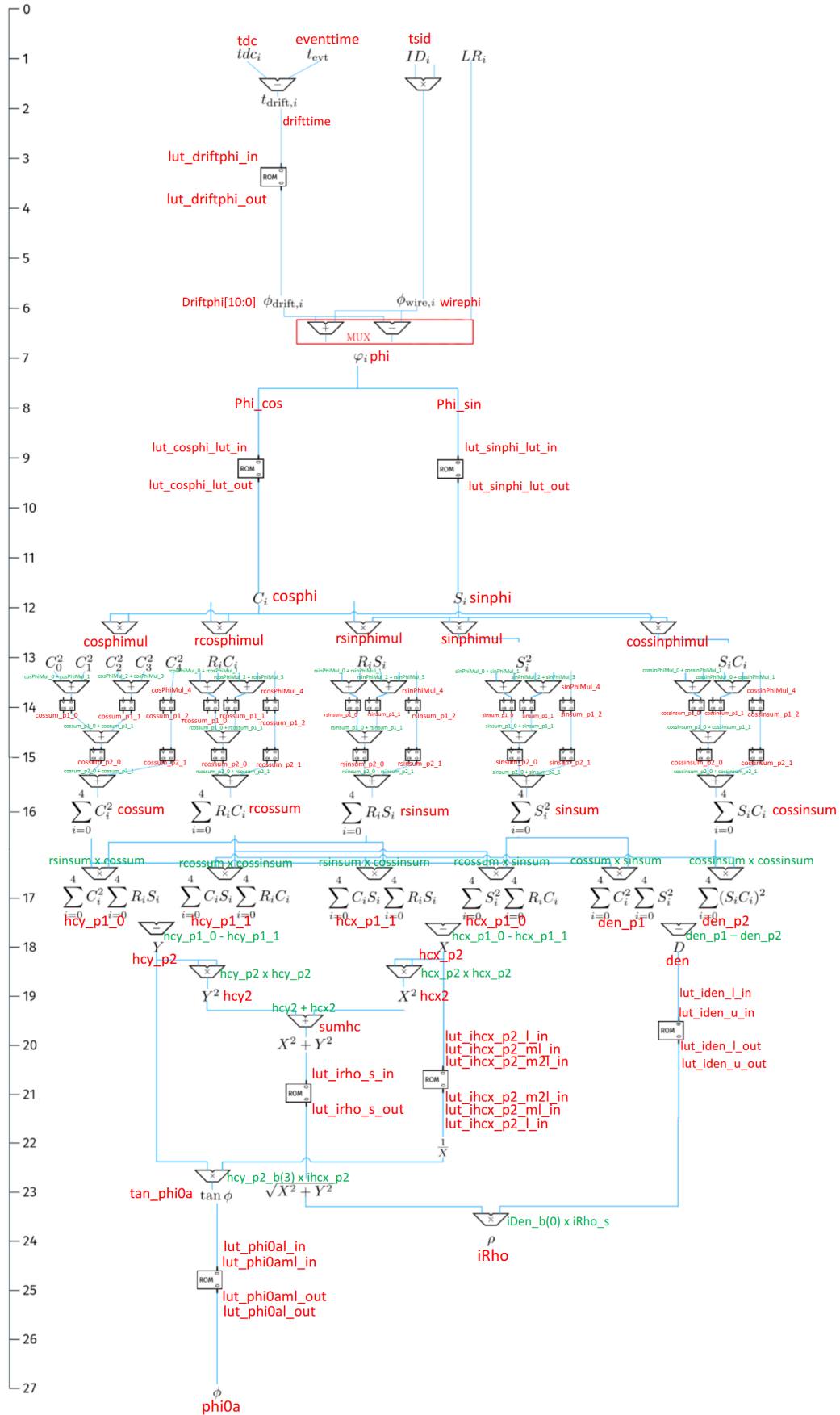
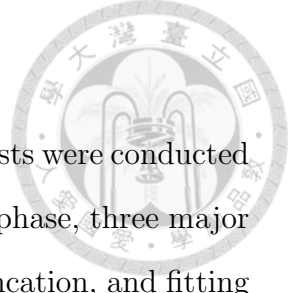


Figure 3.6: Implementation of 2D fitter. The vertical axis stands for the latency (pipeline stage) of the signal (adapted from Ref. [14]).



3.7 Debugging and Validation

Prior to the acquisition of cosmic-ray data, a series of functional tests were conducted on the 2D track fitter to verify its basic correctness. During this phase, three major issues were identified—relating to signal synchronization, TS truncation, and fitting instability under specific configurations. These problems were subsequently fixed through iterative debugging and firmware updates.

Once the fitter achieved stable behavior in controlled test conditions, the firmware was deployed to the trigger system at KEK, where it was used to process real cosmic-ray data. This allowed a more comprehensive evaluation of the fitter’s performance under realistic detector conditions.

The following subsections present the debugging process in detail, including the nature of two distinct types of issues encountered (addressed in Section 3.7.1 and Section 3.7.2, respectively), their impact on track-fitting performance, and the validation of the final results through offline comparisons of reconstructed track parameters (Section 3.7.3).

3.7.1 Signal Desynchronization and Truncation

During debugging and validation, we observed and fixed signal desynchronization and incorrect truncation due to resizing as shown in Fig. 3.7 and Fig. 3.8.

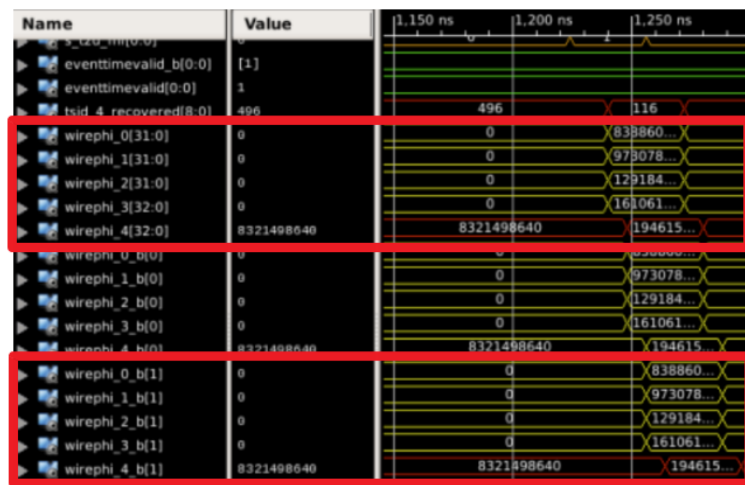


Figure 3.7: Signal desynchronization of 2D fitter.

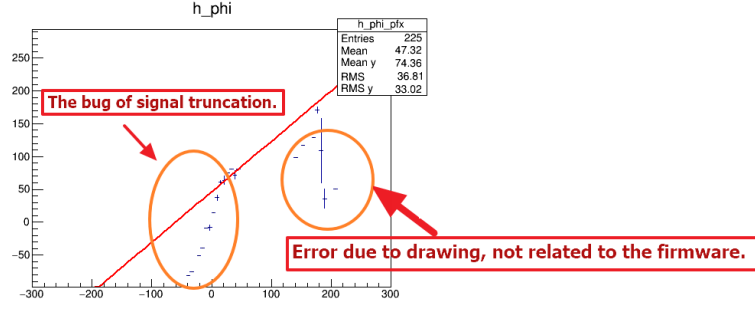


Figure 3.8: Signal truncation of 2D fitter.

3.7.2 Track Fitting Failure at Coincidence Equals Four

The 2D track finding design requires each reconstructed track to be associated with at least four TSs; however, we found that track fitting failed when the **coincidence** (i.e., **the number of valid TSs associated with the track**) was four, as observed in the beam data from Board0. Fig. 3.9 shows the example of a 2D track with coincidence = 5.

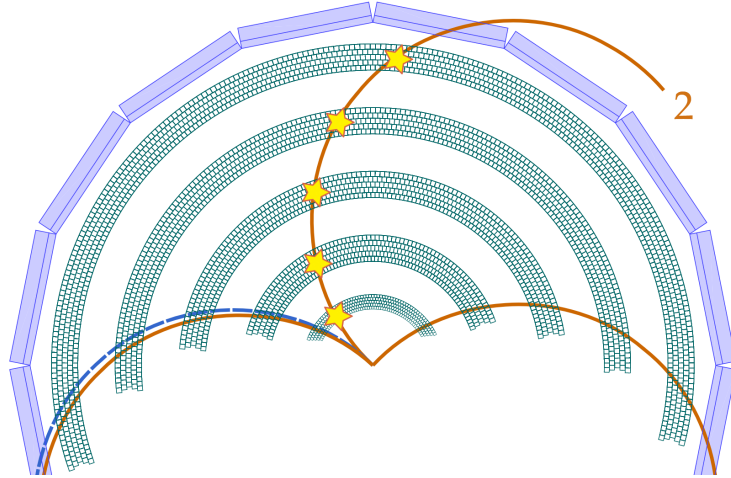


Figure 3.9: Example of 2D track with coincidence = 5. The track (orange) is associated with five TSs (yellow stars) across five SLs.

The 2D track finding design requires each reconstructed track to be associated with at least four TSs; however, we found that track fitting failed when the coincidence was four, as observed in the beam data. As shown in Fig. 3.10, Fig. 3.11, and Fig. 3.12, the track parameter distributions from the 2D fitter and the 2D finder are different, they are also not linearly correlated, and their differences do not follow a Gaussian distribution, respectively.

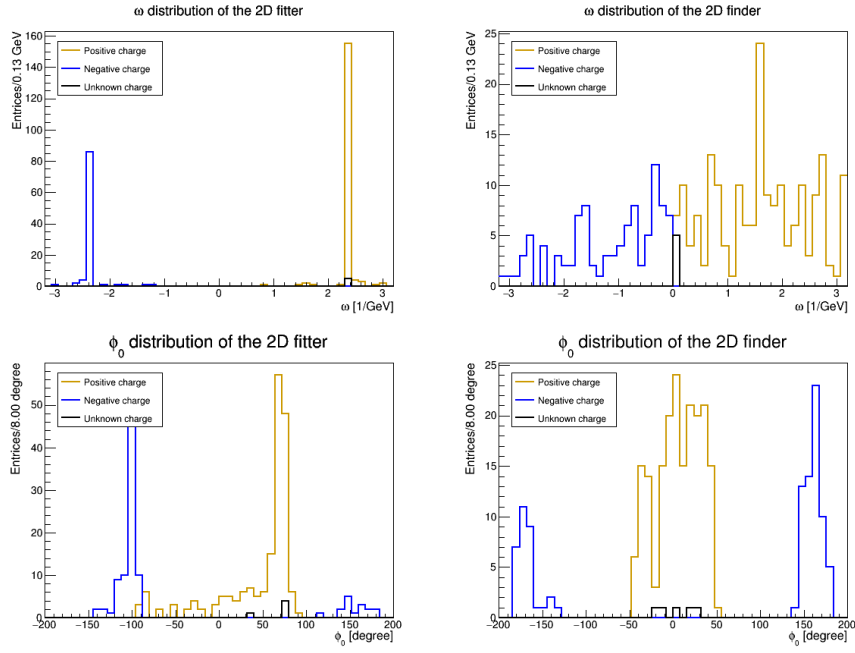


Figure 3.10: Distributions of ω and ϕ_0 (bottom) for the 2D fitter (left) and the 2D finder (right), categorized by charge. Positive, negative, and unknown charges are shown in orange, blue, and black, respectively. All events are selected under the condition coincidence = 4 from run #2725 of experiment 3.

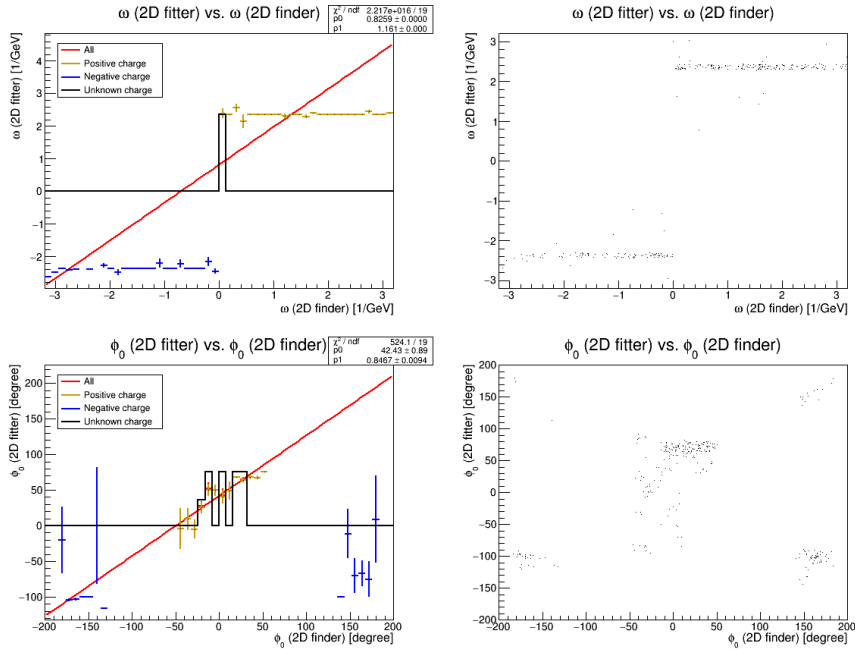


Figure 3.11: Correlation between the 2D fitter and 2D finder for the fitted track parameters ω (top) and ϕ_0 (bottom). The left plots show mean values and standard deviations for events categorized by charge: orange (positive), blue (negative), and black (unknown). The right plots show the corresponding 2D scatter distributions. All events are selected under the condition coincidence = 4 from run #2725 of experiment 3.

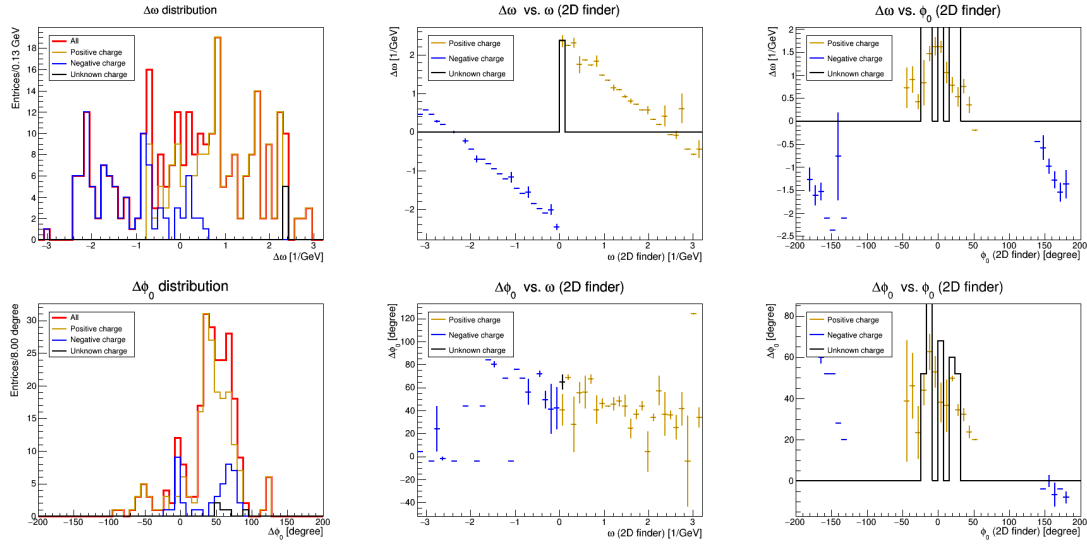


Figure 3.12: Distributions and correlations of the fitting resolution for ω (top row) and ϕ_0 (bottom row), defined as the difference between the 2D fitter and 2D finder results: $\Delta\omega = \omega_{fitter} - \omega_{finder}$, and $\Delta\phi_0 = \phi_{0,fitter} - \phi_{0,finder}$. The left column shows the overall distributions with Gaussian fits, while the middle and right columns show the resolution as a function of the 2D finder output. Events are categorized by track charge: orange (positive), blue (negative), black (unknown). All plots correspond to events with coincidence = 4 from run #2725 of experiment 3.

The reason is that the Coincidence = 4 case was not handled and was instead treated as Coincidence = 5 by substituting default TSs for missing hits. These defaults, being **nonphysical** and **outside the acceptance** range, caused invalid inputs and numerical **overflow**. The solution is that when no valid TS hit is present, $\cos \varphi_i$ and $\sin \varphi_i$ are set to zero. By doing so, the corresponding terms in the parameter calculation formulas are effectively canceled, ensuring that the default or missing information does not contribute to the final result. This prevents faulty inputs from propagating through the computation and restores the stability of the fitting process. These enhancements allow the 2D fitter to correctly handle four-hit scenarios and incomplete input patterns, improving its robustness and extending its applicability to edge-case detector conditions. Fig. 3.13 shows a schematic illustrating the default spatial distribution of TSs associated with the 2D fitter for Board0.

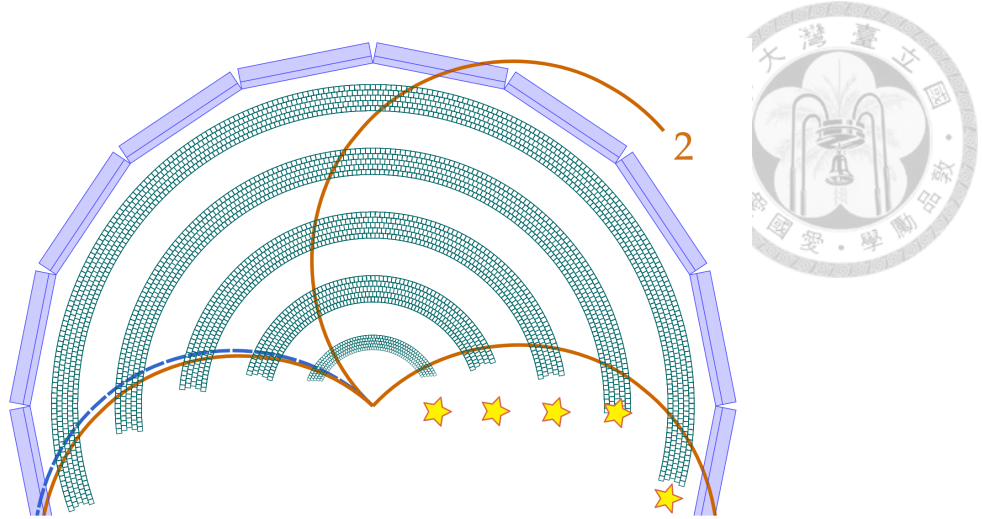


Figure 3.13: Schematic illustrating the default spatial distribution of TSs associated with the 2D fitter (Board0).

3.7.3 Validation of 2D Fitter

We validated the 2D fitter by directly comparing measured and simulated outputs using the Xilinx Integrated Software Environment (ISE) simulation, with events satisfying coincidence greater than or equal to four as input, as observed in the cosmic-ray data from Board0. The primary objective was to assess the distributions and resolutions of reconstructed track parameters—such as curvature and azimuthal angle—and to validate the consistency between real data and hardware emulation. Raw data were used as input to the ISE Design Suite (Xilinx) for firmware simulation, allowing direct comparison between measured and simulated outputs under identical conditions. This approach provided a detailed understanding of the 2D fitter’s resolution and robustness in a noise-suppressed environment.

As shown in Fig. 3.14 and Fig. 3.16, the distributions are the same. The data and simulation results are also linearly correlated as demonstrated in Fig. 3.15 and Fig. 3.17. These results confirm that the 2D fitter has been successfully validated—demonstrating both agreement in distributions and consistency in numerical values when compared to simulation and 2D finder outputs.

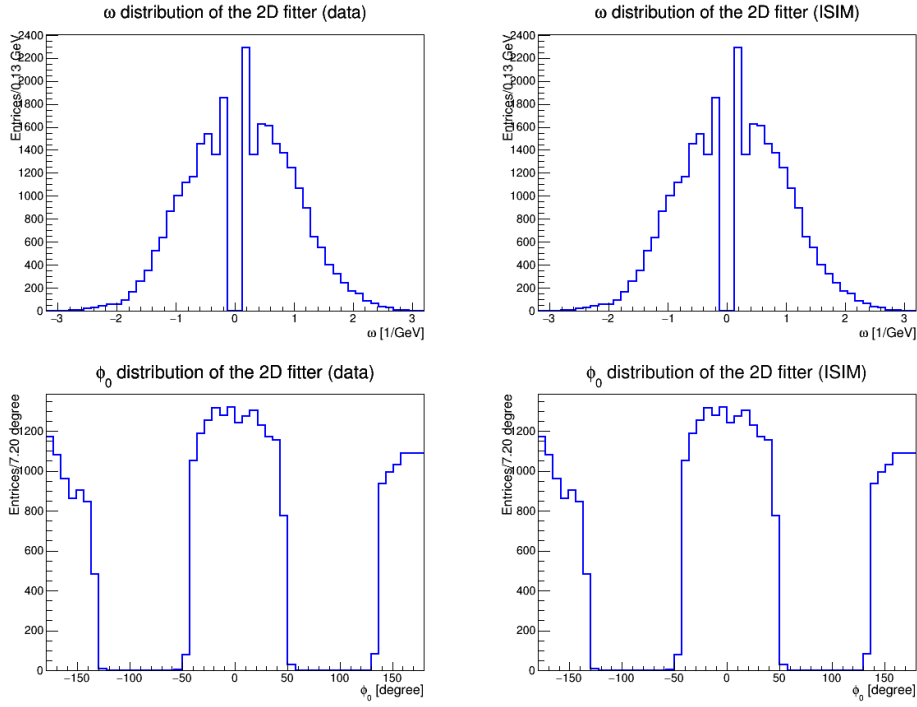


Figure 3.14: Distributions of ω and ϕ_0 (bottom) for the 2D fitter (left) and the 2D finder (right) without charge separation applied. All events are selected under the condition coincidence ≥ 4 from run #1709 of experiment 3.

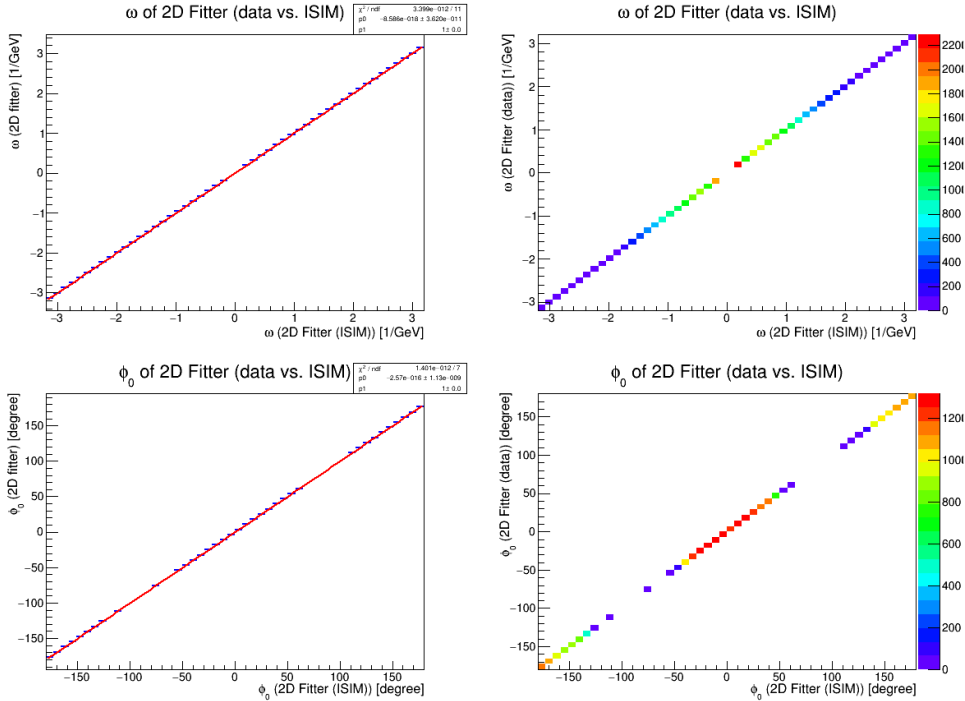


Figure 3.15: Correlation between the 2D fitter and 2D finder for the fitted track parameters ω (top) and ϕ_0 (bottom). The left plots show mean values and standard deviations for events without charge separation applied. The right plots show the corresponding 2D scatter distributions. All events are selected under the condition coincidence ≥ 4 from run #1709 of experiment 3.

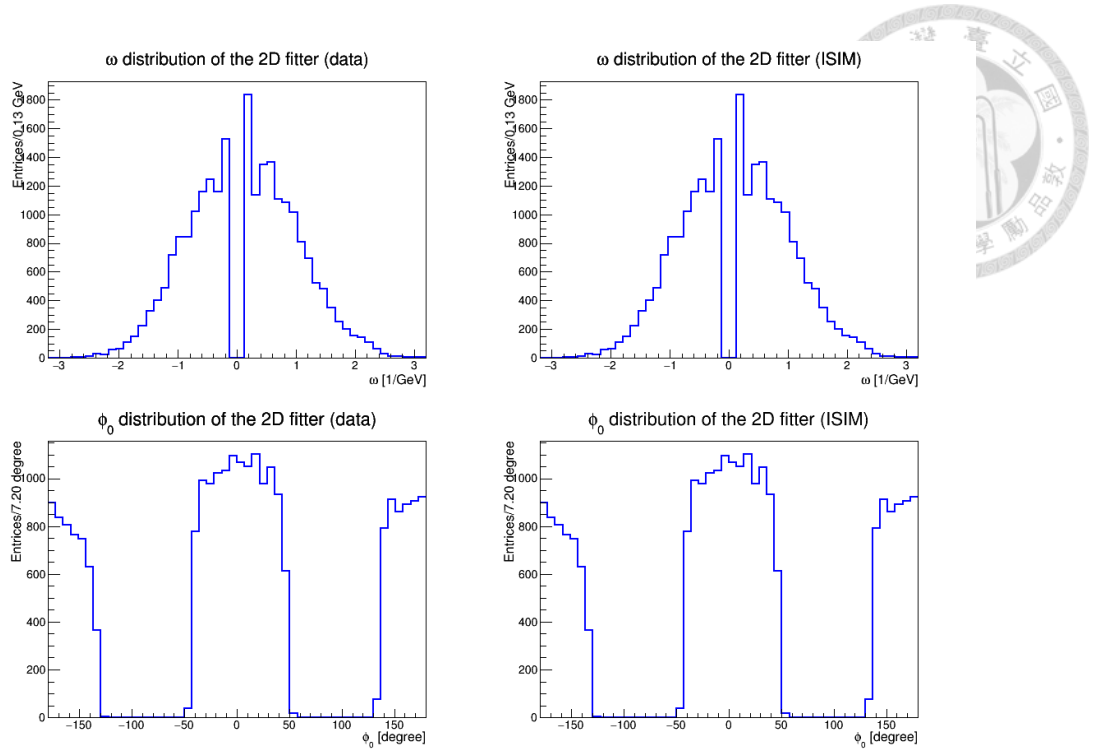


Figure 3.16: Distributions of ω and ϕ_0 (bottom) for the 2D fitter (left) and the 2D finder (right) without charge separation applied. All events are selected under the condition coincidence ≥ 4 from run #1918 of experiment 3.

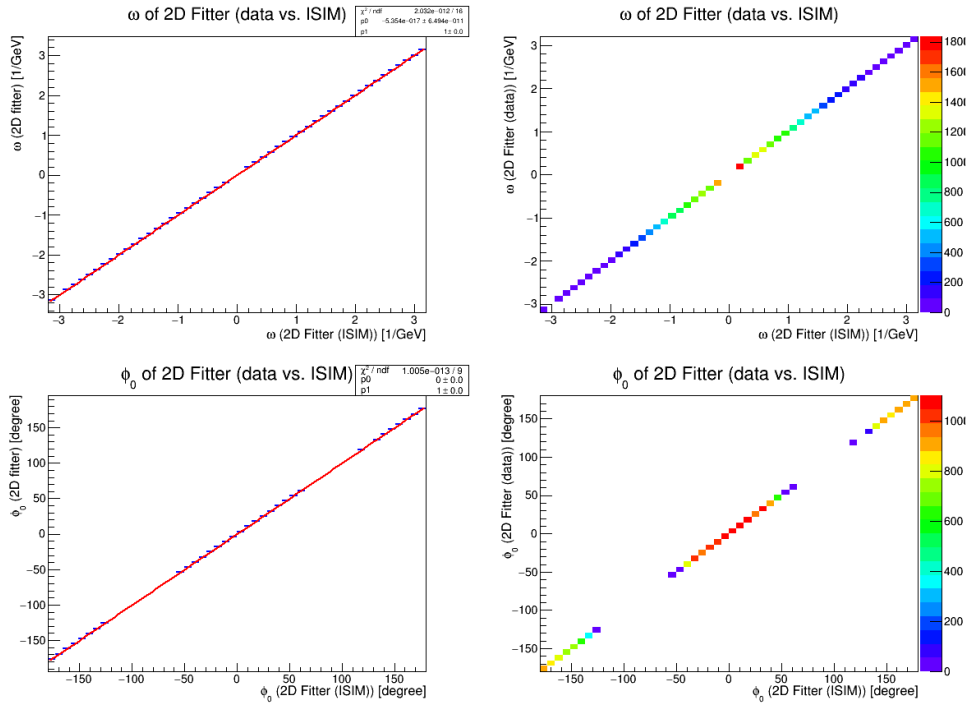


Figure 3.17: Correlation between the 2D fitter and 2D finder for the fitted track parameters ω (top) and ϕ_0 (bottom). The left plots show mean values and standard deviations for events without charge separation is applied. The right plots show the corresponding 2D scatter distributions. All events are selected under the condition coincidence ≥ 4 from run #1918 of experiment 3.

3.8 Performance

Following the validation, a dedicated performance study was conducted using beam data collected by Board0 during run #2725 of experiment 3, recorded on June 1, 2018. This dataset reflects real operating conditions at SuperKEKB and provides valuable insights into the 2D tracker's behavior under high-luminosity collision environments.

For coincidence equals four, as you shown in Fig. 3.18, Fig. 3.19, and Fig. 3.20, the track parameter distributions from the 2D fitter and the 2D finder are the same, they are also linearly correlated, and their differences follow a Gaussian distribution., respectively. The yellow region corresponds to the acceptance ranges for positively charged tracks, and the blue region corresponds to the acceptance ranges for negatively charged tracks. For coincidence equals five, the results are even better, as shown in Fig. 3.21, Fig. 3.22, and Fig. 3.23.

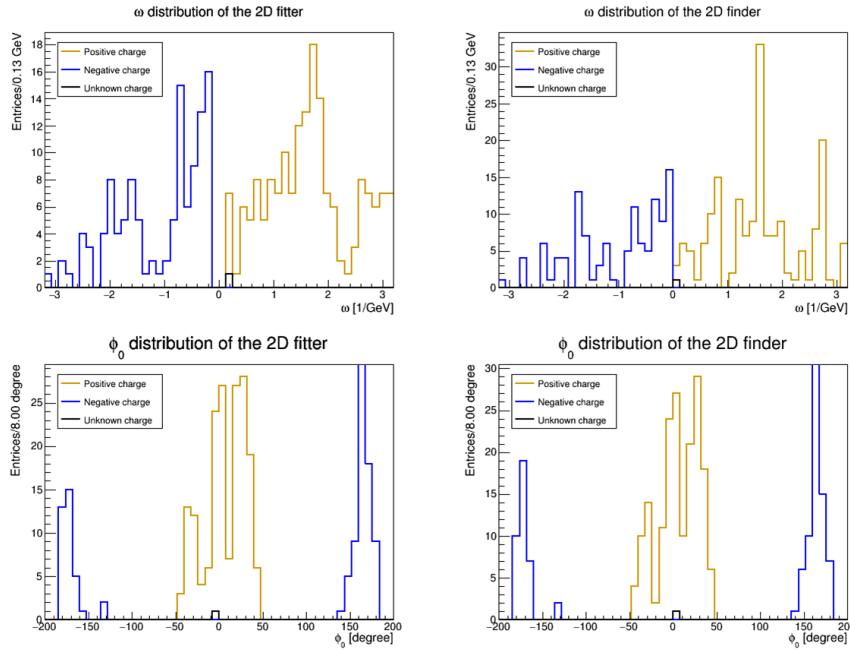


Figure 3.18: Distributions of ω and ϕ_0 (bottom) for the 2D fitter (left) and the 2D finder (right), categorized by charge. Positive, negative, and unknown charges are shown in orange, blue, and black, respectively. All events are selected under the condition coincidence = 4 from run #2725 of experiment 3.

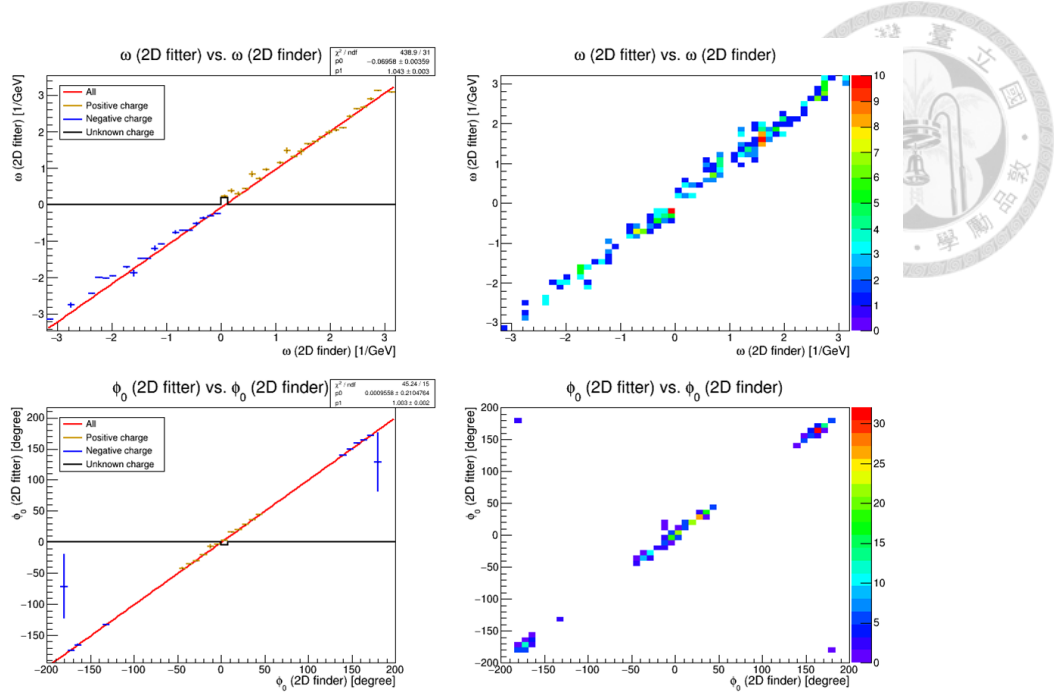


Figure 3.19: Correlation between the 2D fitter and 2D finder for the fitted track parameters ω (top) and ϕ_0 (bottom). The left plots show mean values and standard deviations for events categorized by charge: orange (positive), blue (negative), and black (unknown). The right plots show the corresponding 2D scatter distributions. All events are selected under the condition coincidence = 4 from run #2725 of experiment 3.

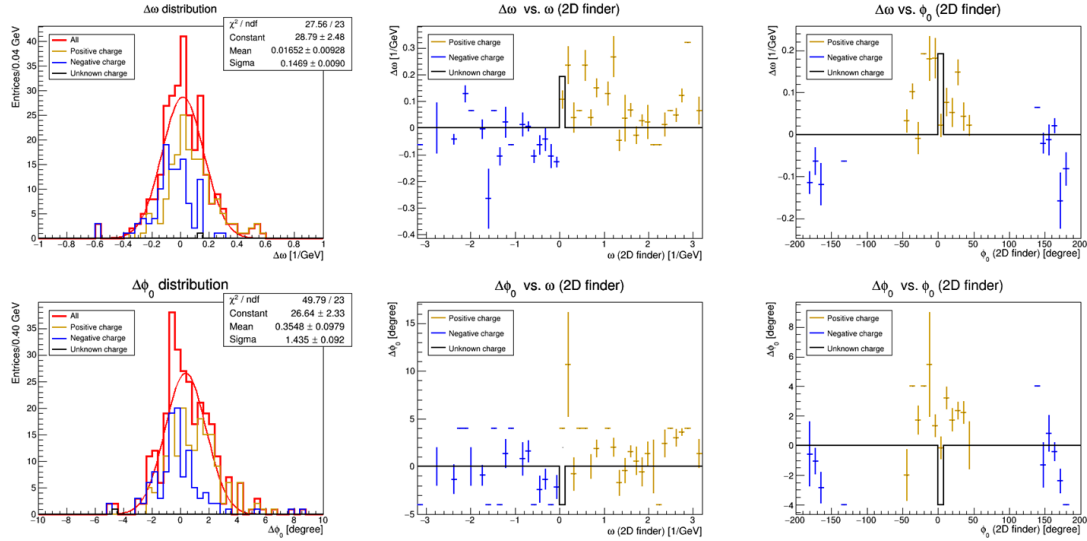


Figure 3.20: Distributions and correlations of the fitting resolution for ω (top row) and ϕ_0 (bottom row), defined as the difference between the 2D fitter and 2D finder results: $\Delta\omega = \omega_{fitter} - \omega_{finder}$, and $\Delta\phi_0 = \phi_{0,fitter} - \phi_{0,finder}$. The left column shows the overall distributions with Gaussian fits, while the middle and right columns show the resolution as a function of the 2D finder output. Events are categorized by track charge: orange (positive), blue (negative), black (unknown). All plots correspond to events with coincidence = 4 from run #2725 of experiment 3.

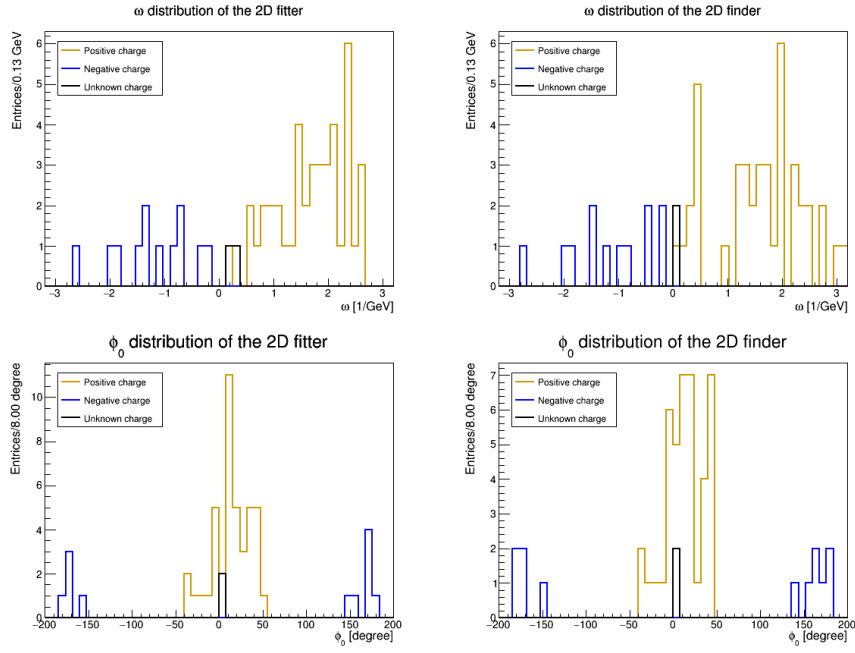


Figure 3.21: Distributions of ω and ϕ_0 (bottom) for the 2D fitter (left) and the 2D finder (right), categorized by charge. Positive, negative, and unknown charges are shown in orange, blue, and black, respectively. All events are selected under the condition coincidence = 5 from run #2725 of experiment 3.

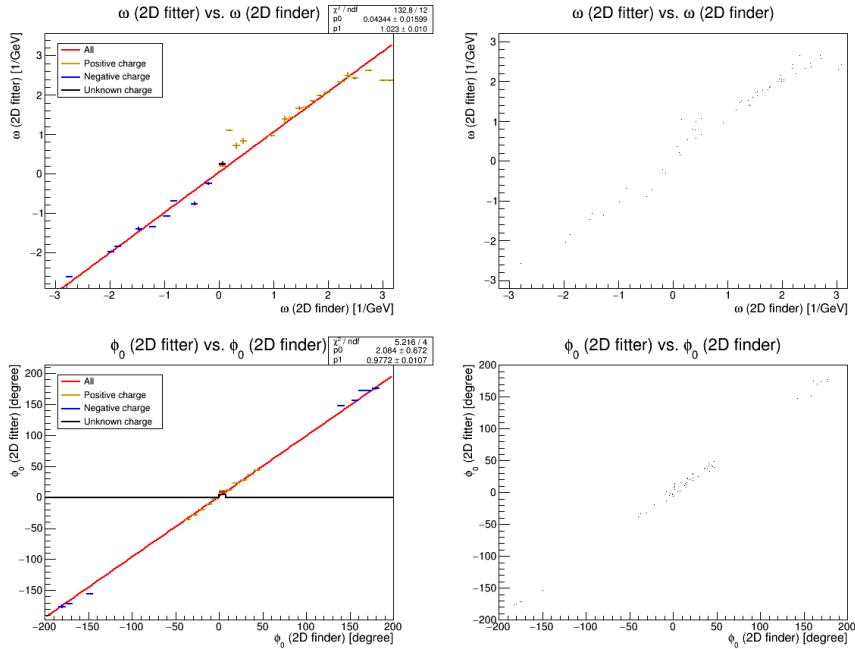


Figure 3.22: Correlation between the 2D fitter and 2D finder for the fitted track parameters ω (top) and ϕ_0 (bottom). The left plots show mean values and standard deviations for events categorized by charge: orange (positive), blue (negative), and black (unknown). The right plots show the corresponding 2D scatter distributions. All events are selected under the condition coincidence = 5 from run #2725 of experiment 3.

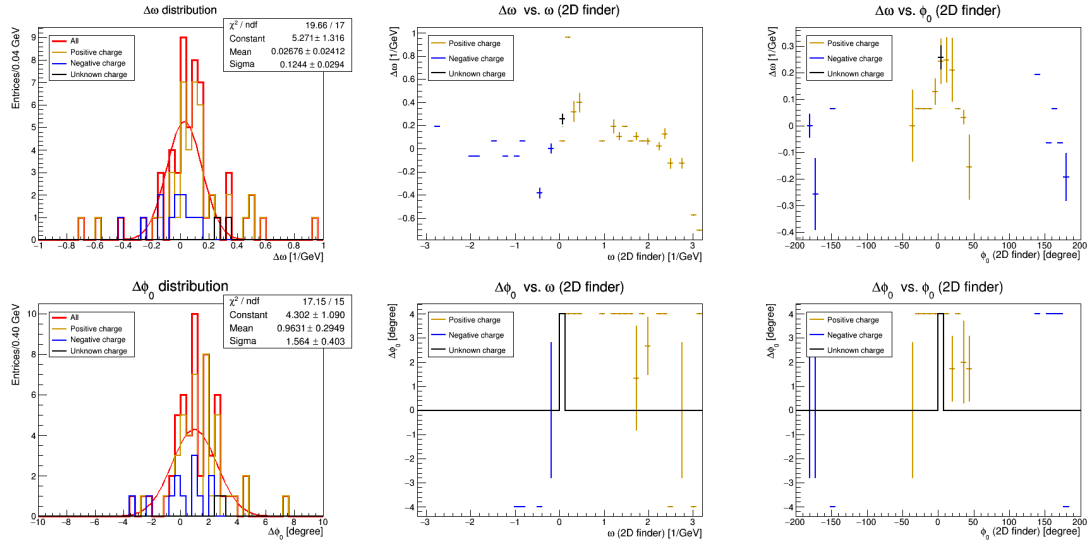


Figure 3.23: Distributions and correlations of the fitting resolution for ω (top row) and ϕ_0 (bottom row), defined as the difference between the 2D fitter and 2D finder results: $\Delta\omega = \omega_{\text{fitter}} - \omega_{\text{finder}}$, and $\Delta\phi_0 = \phi_{0,\text{fitter}} - \phi_{0,\text{finder}}$. The left column shows the overall distributions with Gaussian fits, while the middle and right columns show the resolution as a function of the 2D finder output. Events are categorized by track charge: orange (positive), blue (negative), black (unknown). All plots correspond to events with coincidence = 5 from run #2725 of experiment 3.

3.9 Discussion

The 2D track fitter has demonstrated robust and reliable performance throughout simulation and validation studies. It achieves precise reconstruction of track parameters in the transverse plane, including the azimuthal angle ϕ_0 and the curvature ω , with high resolution. The fitting algorithm, based on a least-squares method with drift-time correction, effectively reduces uncertainties arising from wire hit timing and left-right ambiguities.

In particular, the 2D fitter performs well in high-occupancy environments, maintaining a low ghost rate and stable fitting efficiency across various coincidence levels. Its output parameters show strong correlation with the corresponding values from the 2D track finder and match the MC truth with minimal bias. These results confirm that the fitter not only refines track candidates efficiently but also provides high-quality inputs for the 3D tracker and for z -vertex estimation at the trigger level.

However, certain limitations remain. For example, the fitting performance slightly degrades for low transverse momentum tracks with small curvature radii, where hit alignment becomes less ideal. Additionally, fitting failures may occur under edge-case coincidence configurations or in the presence of noise-induced TS patterns. These limitations suggest potential areas for future improvement, such as incorporating more sophisticated drift-time models or adaptive fitting strategies.

Overall, the 2D fitter fulfills its role as a critical component of the L1 trigger tracking system, providing accurate and timely track reconstruction under real-time constraints.

3.10 Summary

The 2D tracker system for the Belle II L1 trigger consists of three key modules: the 2D hitmap maker, the 2D track finder, and the 2D track fitter. These components operate sequentially to process CDC wire hits, identify candidate tracks in the transverse plane, and extract relevant parameters such as curvature and azimuthal angle with high efficiency.

The 2D fitting system is developed based on an inherited framework and has been fully implemented in both firmware and software. Its performance has been validated using both cosmic-ray and beam data. The hitmap maker provides a fast and reliable representation of detector activity, while the track finder robustly locates track candidates using pattern recognition techniques. The 2D track fitter further refines these candidates, achieving high accuracy and low ghost rates, even under high-background or high-occupancy conditions.

Overall, the 2D fitter demonstrates stable performance and fulfills its role in real-time event selection. Its outputs provide essential inputs for subsequent z -vertex reconstruction in the 3D tracker, and contribute directly to the GDL in the L1 trigger system. The successful operation of the 2D tracker thus establishes a solid foundation for efficient and accurate triggering in high-luminosity environments.



Chapter 4

3D Tracker

4.1 Introduction

This section introduces the motivation and background for the development of the 3D tracking system, building upon the 2D tracker discussed in the previous chapter. It also outlines the specific challenges posed by 3D reconstruction in the Belle II environment and the goals of this work.

The L1 trigger system in the Belle II experiment plays a vital role in suppressing background events in real time under high-luminosity conditions. One of its key components is the 3D tracker, which estimates the z -vertex of charged particles using stereo wire information from the CDC. This additional spatial information helps discriminate beam-induced background from genuine physics events with displaced vertices.

The concept of fast 3D tracking in the L1 trigger was first introduced by Jae-Bak Kim in 2012 [23]. His work proposed a real-time 3D track fitting algorithm that estimates the z -vertex using stereo wire hits, combined with transverse track parameters from a 2D finder based on fixed-point arithmetic, optimized for FPGA implementation within the CDC-based L1 trigger system. This design laid the groundwork for hardware-level 3D reconstruction at the trigger stage. This approach was further developed into a complete firmware system by Kim and Won in 2018 [24], which included a TS map maker, stereo TS finder, and z_0 fitter, all of which enable

accurate z_0 reconstruction and effective beam background rejection. They also introduced a general-purpose arithmetic framework for FPGA trigger algorithms [25]. This framework provides a C++-based interface for defining arithmetic pipelines and supports automatic VHDL code generation, facilitating consistency between simulation and firmware. While our implementation was developed independently using hand-optimized Verilog, the framework offers a useful conceptual reference for arithmetic block structuring in FPGA-based tracking systems.

Despite the solid architectural foundation laid by previous efforts, the 3D tracking system remained incomplete, lacking thorough debugging, real-beam validation, and a sophisticated segment-finding algorithm. To address these limitations, our team resumed development with the aim of enhancing firmware robustness, algorithmic precision, and real-time operability.

We implemented a complete 3D tracking pipeline tailored for FPGA deployment, featuring a pipelined dataflow design with a LUT-based segment finder and stereo linear fitter. Both the geo-finder and the newly developed LUT-finder algorithms were incorporated to improve track reconstruction accuracy and background rejection efficiency. The system was rigorously validated using simulation and cosmic-ray data, resolving critical timing and synchronization issues. It was subsequently integrated into the operational Belle II L1 trigger system, marking the first full realization of a functional 3D tracker within the firmware. With this robust framework now in place, future work will focus on optimizing the LUT-finder algorithm and refining firmware logic to further improve performance under hardware constraints.

Subsequent to this development, Hiroto Sudo continued and extended the work in 2024 [26]. He introduced a Hough-based z -vertex selection algorithm for the upgraded UT4 board, which significantly improves background suppression while maintaining signal efficiency under high-luminosity conditions. His study is based on and inherits the firmware structure developed in our earlier implementation. The following sections describe the architecture, algorithmic components, and performance of the 3D tracker, emphasizing its integration into the L1 trigger and its

critical role in enhancing the spatial resolution of trigger decisions.



4.2 Overview

To provide a high-level understanding of the 3D tracking system, this section presents the architecture, component interactions, and data flow. Each of the system's core modules is introduced and their roles briefly described.

The 3D tracker is a crucial module in the CDC trigger system of the Belle II experiment's L1 Trigger. Designed for high-luminosity environments, the 3D tracker reconstructs charged particle trajectories in three dimensions with sub-microsecond latency, leveraging geometric transformations and linear regression techniques. Implemented on FPGA hardware for rapid computation, it identifies high-momentum tracks and suppresses background noise, thereby enhancing the trigger's ability to select events of physics interest.

To achieve this, the 3D tracker integrates several components that operate in sequence, as illustrated in Fig. 4.1:

- **The 3D Hitmap Maker:** This module receives TSF outputs and persists TS hits for about $1\mu\text{s}$. It generates a structured hitmap, indexed by TS ID, which enables downstream modules to reference time-persistent stereo hits during reconstruction.
- **The 3D Finder:** Finds the stereo related TSs using the 2D information. These stereo segments provide the necessary longitudinal information for full 3D track reconstruction.
- **The 3D Fitter:** Applies drift-time-corrected linear regression fitting using 2D information and stereo TS hits to determine the z -origin and polar angle.
- **The 2D Fitter:** This module is integrated into the 3D tracker firmware to provide transverse track parameters as input for 3D reconstruction.

Together, these modules allow the 3D tracker to deliver precise 3D track parameters in real time, playing a vital role in the Belle II experiment's fast event selection pipeline.

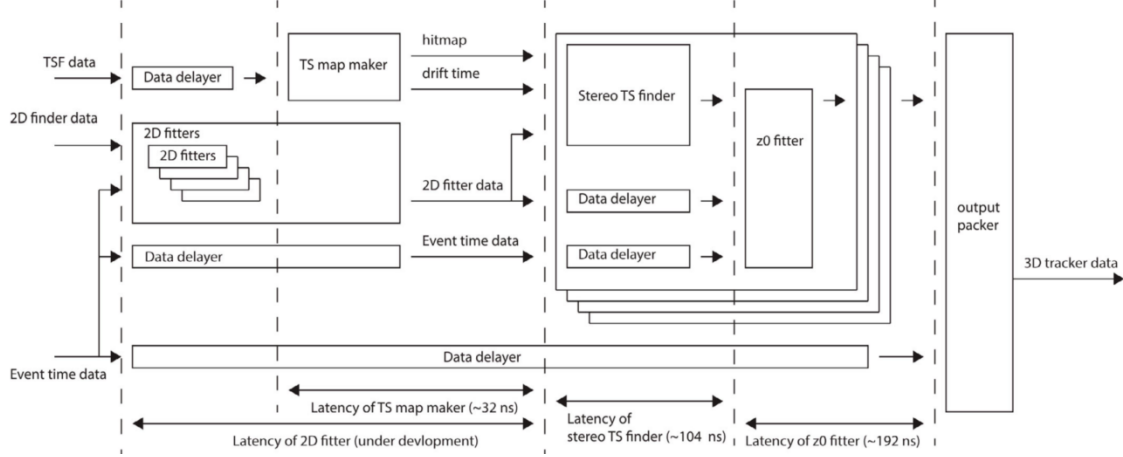


Figure 4.1: Schematics of 3D tracker (reproduced from Ref. [24]).

4.3 3D Hitmap Maker

Unlike the 2D hitmap maker, which provides a 2D representation of hits, the 3D tracker combines spatial and timing information to reconstruct full particle trajectories. As charged particles pass through the CDC, ionization signals from stereo layers (U/V) are recorded and assigned time slot IDs. The 3D hitmap maker focuses solely on these stereo-layer hits and ignores axial-layer data, which is already handled by the 2D tracking system. Each stereo TS ID is marked as active if a hit is detected, and its validity is extended over a predefined time window (time extension). The result is a time-extended list of active TS IDs from stereo layers, without explicit coordinate reconstruction. This 3D hitmap serves as the input for the 3D track finder, which uses it in combination with 2D tracking results to identify valid 3D track candidates through real-time pattern matching.

4.4 3D Finder

The 3D finder serves as the first stage of the 3D tracking pipeline, responsible for identifying relevant stereo TSs that are geometrically consistent with the 2D track candidate found by the 2D tracker. While the 2D tracker reconstructs the transverse momentum and azimuthal angle of charged particles using axial layers of the CDC, the 3D finder extends this information by associating stereo layer hits that can later be used to estimate the z -vertex.

Due to the helical motion of charged particles in a magnetic field, this process requires precise geometrical consistency checks. The 3D finder scans the stereo SLs for hit candidates that lie within a predicted window around the extrapolated 2D track trajectory and identifies those likely to be associated stereo TSs.

By selecting a small number of likely stereo hits and rejecting incompatible ones early on, the 3D finder significantly reduces the combinatorial load of the downstream stereo fitter. The performance of this module directly affects the overall resolution and efficiency of the 3D tracker. In the following sections, we describe the algorithmic structure, firmware implementation, and validation results of the 3D finder.

4.4.1 Principles

In the ideal case without wire tilting, the stereo wire corresponding to a track can be directly inferred from 2D track information. However, since the stereo wires are tilted, the actual hit position in the azimuthal plane is shifted from the 2D prediction, causing azimuthal displacement of the TS hits from the true track position. The typical shift is about ten wires depending on the SL. Based on this, an azimuthal window is defined around the predicted position for each SL, and only the TS hits within that window are selected. The z position is then determined by using the azimuthal displacement and trigonometry. Fig. 4.2 shows the illustration of azimuthal displacement caused by tilted stereo wires.

To handle the ambiguity caused by wire tilting and to optimize track finding

performance, we developed and evaluated several algorithms that implement different strategies for selecting the best combination of stereo TS hits. These algorithms differ in how they prioritize candidate hits, resolve overlapping windows, and minimize the impact of noise or missing hits. By comparing their accuracy, efficiency, and hardware feasibility, we identified the most effective approach for use in the 3D finder.

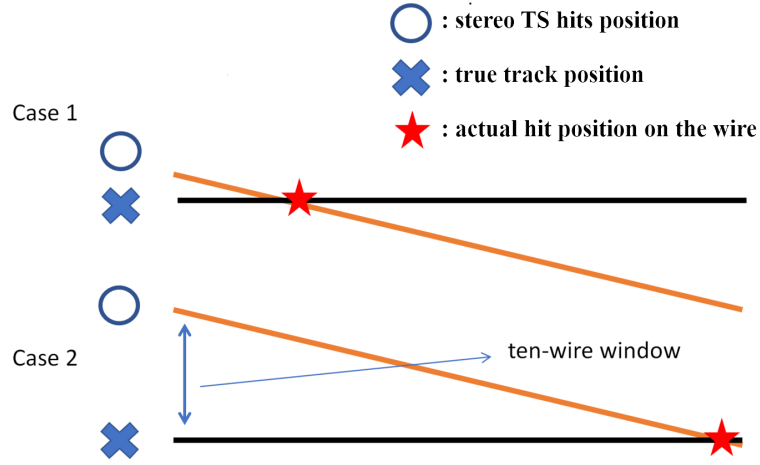


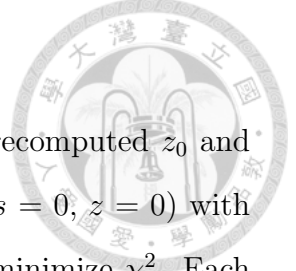
Figure 4.2: Illustration of azimuthal displacement caused by tilted stereo wires.

4.4.2 Geo-Finder and Snake-Finder

Both the geo-finder (original finder) and the snake-finder adopt a favored TS strategy, selecting the stereo TS hit closest to a predefined favored TS among those falling within the azimuthal window. For the geo-finder, in each SL, the Favored TS is defined independently as the stereo TS hit closest to the center of the azimuthal window. For the snake-finder, the 4-th candidate in SL1 is used as the initial favored TS. In subsequent SLs, the favored TS is determined by propagating from the TS found in the previous layer. Table 4.1 summarizes the preferred TS strategies in the geo-finder and the snake-finder for each SL.

	Geo-finder	Snake-finder
SL1	4-th candidate	4-th candidate
SL3	4-th candidate	related TS found by SL1
SL5	4-th candidate	related TS found by SL3
SL7	4-th candidate	related TS found by SL5

Table 4.1: Favored TS in geo-finder and snake-finder for each SL.



4.4.3 LUT-Finder

The LUT-finder performs an IP-constrained fit by comparing precomputed z_0 and χ^2 values stored in a lookup table, assuming a fixed vertex at ($s = 0, z = 0$) with $\sigma = 1$ cm, cm, and looping through all TS hit combinations to minimize χ^2 . Each stereo SL includes one "no-hit" case, resulting in up to $11 \times 11 \times 11 \times 12 = 15,972$ possible hit configurations. To simplify logic, all layers are treated as having 12 options, expanding the scan space to $12^4 = 20,736$ combinations. Fig. 4.3 presents the schematic showing the concept of LUT-finder.

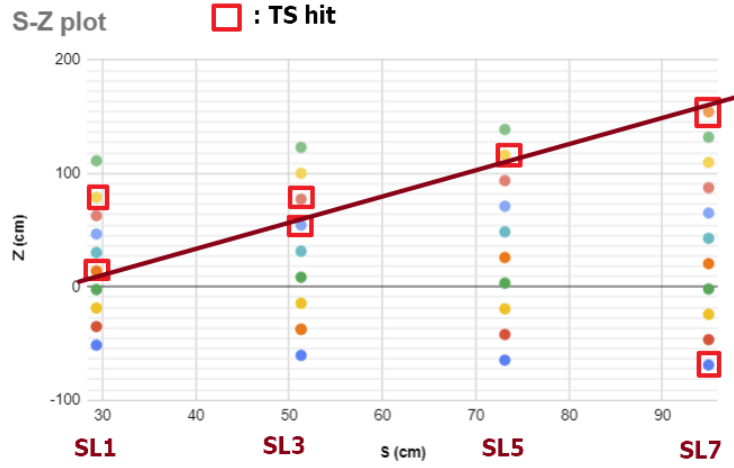


Figure 4.3: Schematic showing the concept of LUT-finder.

4.5 3D Fitter

The 3D fitter is responsible for reconstructing the longitudinal track parameters, specifically the z -vertex and the polar angle θ , using stereo layer information from the CDC. After the 3D finder identifies the relevant stereo TSs associated with a 2D track, the fitter performs a fast estimation of the track's trajectory in the r - z plane. This reconstruction process is based on a linear fitting method, assuming the charged particle follows a straight-line path through the stereo layers. To achieve reliable fitting results, it is essential to account for the drift-time behavior of the CDC and the geometrical configuration of the wires. The algorithm is designed with hardware constraints in mind, enabling real-time execution within the L1 trigger

latency budget.

As introduced in the previous subsection, the z position of each stereo hit is inferred from its azimuthal displacement using trigonometric relations. With these z positions determined, the 3D fitter performs a linear regression fitting to obtain z_0 and $\cot \theta$ using the arc length s of the 2D track and the corresponding z values. The fitting minimizes the following chi-squared function, where σ_i is the error of the z position:

$$\chi^2 = \sum_{i=0}^4 \frac{[z_i - (\cot \theta \times s_i + z_0)]^2}{\sigma_i^2}.$$

The $\cot \theta$ and the z_0 that minimize the χ^2 can be found analytically as below:

$$\cot \theta = \frac{\sum_{i=0}^4 \left(\frac{1}{\sigma_i^2}\right) \sum_{i=0}^4 \left(\frac{s_i z_i}{\sigma_i^2}\right) - \sum_{i=0}^4 \left(\frac{s_i}{\sigma_i^2}\right) \sum_{i=0}^4 \left(\frac{z_i}{\sigma_i^2}\right)}{\sum_{i=0}^4 \left(\frac{1}{\sigma_i^2}\right) \sum_{i=0}^4 \left(\frac{s_i^2}{\sigma_i^2}\right) - \sum_{i=0}^4 \left(\frac{s_i}{\sigma_i^2}\right) \sum_{i=0}^4 \left(\frac{s_i}{\sigma_i^2}\right)},$$

$$z_0 = \frac{-\sum_{i=0}^4 \left(\frac{s_i}{\sigma_i^2}\right) \sum_{i=0}^4 \left(\frac{s_i z_i}{\sigma_i^2}\right) + \sum_{i=0}^4 \left(\frac{s_i^2}{\sigma_i^2}\right) \sum_{i=0}^4 \left(\frac{z_i}{\sigma_i^2}\right)}{\sum_{i=0}^4 \left(\frac{1}{\sigma_i^2}\right) \sum_{i=0}^4 \left(\frac{s_i^2}{\sigma_i^2}\right) - \sum_{i=0}^4 \left(\frac{s_i}{\sigma_i^2}\right) \sum_{i=0}^4 \left(\frac{s_i}{\sigma_i^2}\right)}.$$

Fig. 4.4 depicts the linear regression of track arc length s versus z to extract z_0 and $\cot \theta$.

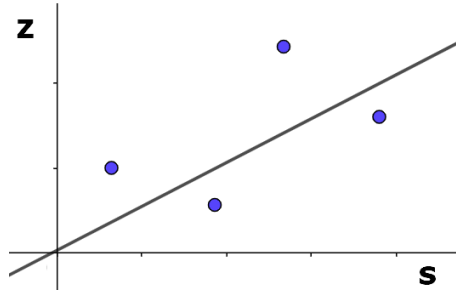


Figure 4.4: Linear regression of track arc length s vs. z is to extract z_0 and $\cot \theta$.

4.6 Implementation

This section describes the firmware and software implementation strategies developed to realize the 3D tracker within the constraints of the Belle II L1 trigger system. Due to the limited logic resources available on the FPGA and the need to maintain the 32 ns data throughput, several architectural and algorithmic solutions were

adopted.

In the firmware, a time-multiplexing approach is used to serialize and deserialize track data, allowing a single 3D finder and fitter pair to process multiple track candidates without exceeding hardware limits (Section 4.6.1). A dedicated LUT-based pattern recognition module, the LUT-finder, is implemented with optimized resource usage and tiered selection logic to identify the best track candidate among possible hit combinations (Section 4.6.2). For debugging and verification, the system includes a bitmap mechanism that records selected stereo hit or track-level data, enabling inspection of intermediate or final results (Section 4.6.3).

In parallel, several improvements were made to the software side to support offline validation and analysis. These include data structure changes, unit corrections, consistency fixes across boards, and enhancements to filtering and matching logic. Together, these modifications significantly reduced execution time and improved overall robustness (Section 4.6.4).

After extensive debugging and feature development, the universal 3D firmware has been updated, as shown in Fig. 4.5. It now includes many new functionalities and configurable options to support broader use cases and validation needs. These new features will be detailed in the following sections.

```
entity UT3_3D is
  generic (
    -- switch between full version or skeleton
    OPTION : switchtype := full; -- choose between 'full' and 'skeleton'

    -- Input of 3D finder. PERFECT2D is only for simulation!
    OPTION_INPUT : switchtype_input := FINDER2D; -- choose between (FINDER2D, FITTER2D, PERFECT2D).

    -- Algorithm of 3D finder.
    OPTION_ALGORITHM : switchtype_algorithm := MIDDLE; -- choose between (LUT, MIDDLE, SNAKE).

    -- Reference of 13-bit TDC. Persisted CC is only for simulation, because it makes the routing too complicated!
    OPTION_CC : switchtype_cc := PRESENT; -- choose between (PRESENT, PERSISTED).
    BW_DRIFTINFO : integer := NB_RAWDTINFO; -- choose between (NB_RAWDTINFO, NB_RAWDTINFO + NB_CC).
    --Bit size of canddriftmap_type_l2e should also be changed (Tracker3D_pkg.vhd)

    --Event time for 3D fitter
    OPTION_EVT : switchtype_evt := FASTEST3D; -- choose between (ETF, FASTEST2D, MEAN2D, MEDIAN2D, FASTEST3D, MEAN3D, MEDIAN3D).
  );
end entity;
```

Figure 4.5: Universal 3D firmware.

4.6.1 Serializer and Deserializer in Firmware

Belle II's CDC trigger provides four parallel track data streams every 32 ns. Processing each track independently would require instantiating four separate sets of the 3D finder and 3D fitter modules, significantly exceeding available FPGA hardware

resources.

To address this hardware limitation, we adopted a serialization strategy, enabling a single 3D finder and a single 3D fitter to handle all four track streams in a time-sharing manner. This method effectively reduces the hardware resource usage while maintaining the system's required throughput.

The detailed approach involves three main steps:

- **Serialize:** Within each 32 ns data window, we extract an 8 ns slice (one-fourth of the total duration) from each of the four parallel tracks and concatenate these slices into a single, sequential 32 ns serial data stream.
- **Process:** The serialized data stream is processed sequentially at the higher-speed 8 ns system clock rate by the single 3D finder and 3D fitter modules. Each module sequentially handles the four data slices within each 32 ns interval.
- **Deserialize:** After processing, the serial data stream is demultiplexed to restore the original parallel format. The processed results are distributed back into four parallel 32 ns streams, corresponding precisely to the original track indices.

Through serialization and deserialization, we successfully ensure that the FPGA hardware resource constraints are respected, while maintaining the trigger system's essential requirement of processing four parallel tracks every 32 ns.

4.6.2 LUT-Finder in Firmware

To implement the LUT-finder in firmware, track candidates are first grouped by their degrees of freedom (2-hit, 3-hit, 4-hit). Within each group, the LUT-finder uses an Olympic-style divide-and-conquer scan to pick the best (a, b, c, d) out of 12^4 combinations. Four tournament stages (four clocks) iteratively reduce the group to a single winner. After the three group winners are known, the final choice is made with tuned χ^2/N thresholds:



If the best 4-hit track has $\frac{\chi^2}{N} < Cut_1$,

→ Select the best 4-hit track.

else if the best 3-hit track has $\frac{\chi^2}{N} < Cut_2$,

→ Select the best 3-hit track.

else

→ Select the best 2-hit track.

- **N**: degree of freedom of fitting, the IP constraint ($s = 0, z = 0$) is included.
- **Best n -hit track**: the candidate with the smallest $\frac{\chi^2}{N}$ among all n -hit tracks.

To ensure the 3D LUT-finder compiles on UT3, we removed **persisted-CC** timing and reduced the **azimuthal window** (target $10 \times 10 \times 10 \times 11 \rightarrow$ achievable $7 \times 7 \times 9 \times 10$), accepting the resulting performance trade-off, as shown in Fig. 4.6.

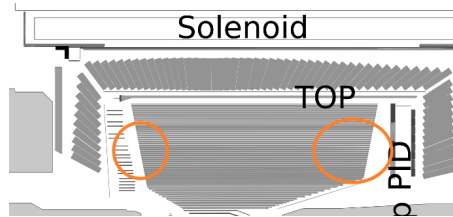


Figure 4.6: Circled areas indicate the edge wires are trimmed to meet the 3D LUT-finder resource limits.

4.6.3 Bitmap in Firmware

Fig. 4.7 shows the bitmap of the 3D tracker in firmware. The bitmap was originally designed to store the stereo TS input data—each including the TS ID, left/right information, and TDC count—for up to four track candidates, primarily for debugging purposes. In this particular case, however, the raw z_0 values of track0 and track3 were stored instead, in order to examine a wider z_0 range during validation.

B2link Bitmap for 3D Tracker (15 TS)					
Jie-Cheng Lin 20211206					
For monitoring the stability of the system					
Name	width	position (MSB)	position (LSB)	description	
	header1	16	2623	2608	x"dddd"
	header2	5	2607	2603	"00000"
	revoclk	11	2602	2592	
	cntr125M	16	2591	2576	
	cntrtrg	32	2575	2544	debug the bit shift
data_trg	rd_req	1	2303	2303	
	tsf_sl1_hit	30	2302	2273	priority hit position of 15 TSs for SL1
	tsf_sl3_hit	30	2272	2243	priority hit position of 15 TSs for SL3
	tsf_sl5_hit	30	2242	2213	priority hit position of 15 TSs for SL5
	tsf_sl7_hit	30	2212	2183	priority hit position of 15 TSs for SL7
	finder2d (489 downto 0)	6	2182	2177	2D finder data (6 bits trigger)
	ETF_IN (0 downto 0)	1	2176	2176	ETF data (LSB)
data_debug_ila	ETF_IN (31 downto 2)	9	2175	2167	ETF data
data_ila	finder2d (495 downto 0)	6	2166	2161	2D finder data (6 bits flag)
	fitter3d_debug	z0_raw_delay	9	2160	2152 ??
			1	2151	2151 ??
			1	2150	2150
			11	2149	2139
			13	2138	2126
		tsId_0 & lr_0 & tdc_0	19	2125	2107 8 + 2 + 9 = 19
		tsId_1 & lr_1 & tdc_1	19	2106	2088 8 + 2 + 9 = 19
		tsId_2 & lr_2 & tdc_2	20	2087	2068 9 + 2 + 9 = 20
		tsId_3 & lr_3 & tdc_3	20	2067	2048 9 + 2 + 9 = 20
		z0_raw_delay	9	2047	2039 ??
			1	2038	2038 ??
			1	2037	2037
			11	2036	2026
			13	2025	2013
		tsId_0 & lr_0 & tdc_0	19	2012	1994 8 + 2 + 9 = 19
		tsId_1 & lr_1 & tdc_1	19	1993	1975 8 + 2 + 9 = 19
		tsId_2 & lr_2 & tdc_2	20	1974	1955 9 + 2 + 9 = 20
		tsId_3 & lr_3 & tdc_3	20	1954	1935 9 + 2 + 9 = 20
	fitter3d	phase3d	2	1934	1933 ?? An out port of Tracker3DMaster
		validts	4	1932	1929 ?? An out port of Tracker3DMaster
					6 bits 2d_fnf (original) => 3d_fnf (2d_fnf & validts)
			110	1928	1819 11bits Z0 + 11bits COT + 4bits ZCHI2 for Track1
					11bits Z0 + 11bits COT + 4bits ZCHI2 for Track2
					11bits Z0 + 11bits COT + 4bits ZCHI2 for Track3
					11bits Z0 + 11bits COT + 4bits ZCHI2 for Track4
		rd_req	1	1818	1818
		tsf_sl1	324	1817	1494 TSF data for SL1 (9 bits CC + 21 bit TS hits × 15 TSs)
		tsf_sl3	324	1493	1170 TSF data for SL3 (9 bits CC + 21 bit TS hits × 15 TSs)
		tsf_sl5	324	1169	846 TSF data for SL5 (9 bits CC + 21 bit TS hits × 15 TSs)
		tsf_sl7	324	845	522 TSF data for SL7 (9 bits CC + 21 bit TS hits × 15 TSs)
		finder2d (504 downto 0)	9	521	513 2D finder data (9 clock counter)
		finder2d (489 downto 0)	490	512	23 2D finder data (489 downto 0)
		ETF_IN (22 downto 0)	23	22	0 ETF data

Figure 4.7: Bitmap of 3D tracker in firmware.

4.6.4 Improvement of Software

The key modifications of the software are listed below:

- Replaced the C++ `vector` with `multi_array` to better support multi-dimensional data structures.
- Correct the segment ID of TS hits — the zero points of raw segment ID for



four boards are different. The old version did not consider this.

- Correct the ϕ_0 of 2D/3D tracks — the zero points of raw ϕ_0 for four boards are different. The old version did not consider this.
- Unify the unit of the output of track parameters — ω : the curvature of the 2D track; ϕ_c : the incident angle of the track.
- Change the condition of the filter — from choosing only valid new tracks to choosing all valid tracks.
- Add relations from the 3D tracks to the 2D tracks.

After the modifications of software, the execution time was significantly reduced, as shown in Fig. 4.8.

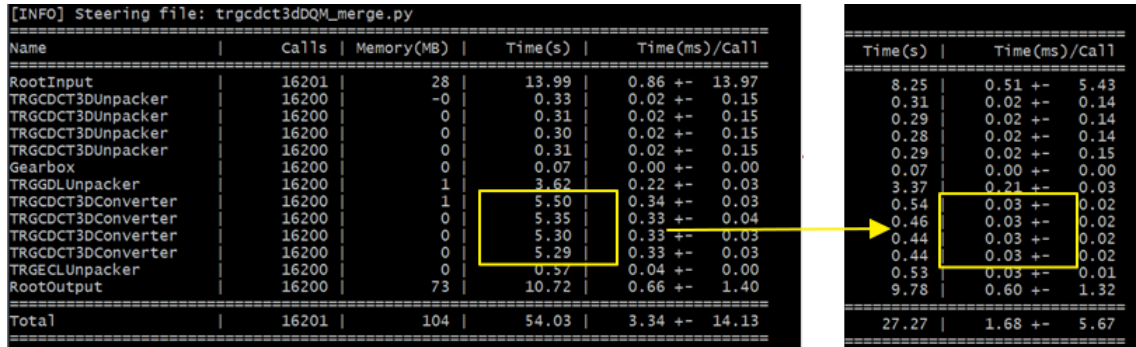
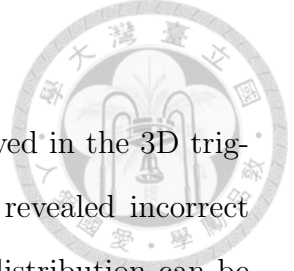


Figure 4.8: Reduction in execution time of the 3D tracker software after optimization.

4.7 Debugging and Validation

This section presents the debugging and validation processes of the 3D tracking firmware. It begins with the simulation framework used to reproduce the firmware behavior and then describes the input preparation, including event matching between offline-reconstructed (offline-reco) and trigger tracks. The validation compares the firmware output with software predictions using both simulation and beam data, ensuring functional correctness under realistic conditions.



4.7.1 LUT Underflow

During debugging and validation, some irregularities were observed in the 3D trigger output, as illustrated in Fig. 4.9. The previous 3D plots revealed incorrect track information. Specifically, the left and right peaks in the distribution can be attributed to the firmware’s lower and upper output bounds and are therefore considered expected behavior rather than bugs. However, the central peak of z_0 appeared unusually sharp. A comparison with offline reconstruction results indicated that most of the tracks contributing to this central peak were fake.

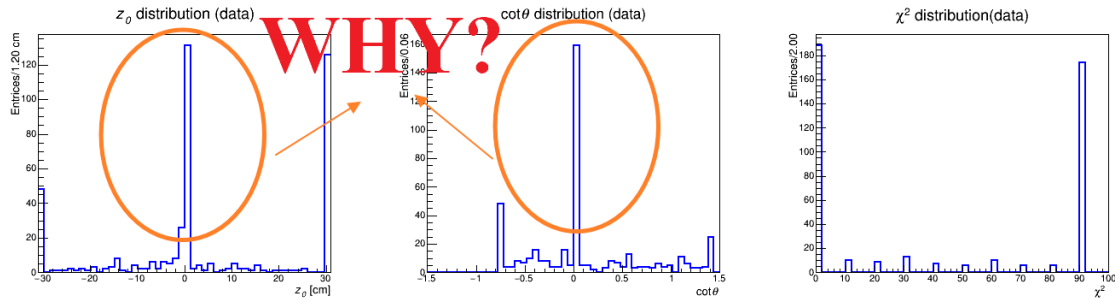
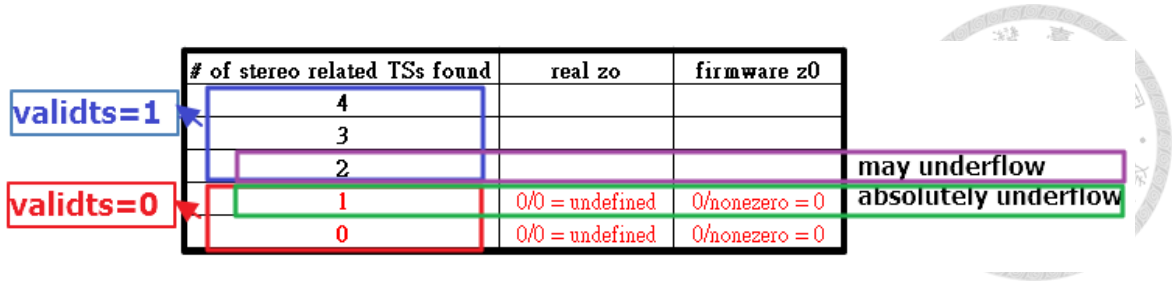


Figure 4.9: Distributions of z_0 for 3D trigger tracks.

The root cause of this behavior was identified and is explained as follows. Originally, a valid 3D track required hits in at least two stereo TSs. However, under certain conditions—missing TS hits, tracks with very short radii, or significantly negative drift times—the corresponding uncertainty values (σ_i) were set to infinity, thus effectively removing their contribution from the summation. This sometimes led to LUT inputs below the lower bound and underflow to zero, producing extra invalid tracks. To resolve this issue, the definition was revised to require at least two stereo layers with finite uncertainty values, effectively excluding all invalid tracks due to LUT underflow. Fig. 4.9 and Fig. 4.9 illustrate the original and revised criteria for valid 3D tracks, and show the LUT underflow effect when inputs drop below the valid range, respectively, clearly demonstrating both the cause of and the solution to the problem.



	# of stereo related TSs found	real z_0	firmware z_0	
validts=1	4			
	3			
	2			may underflow
validts=0	1	0/0 = undefined	0/nonzero = 0	absolutely underflow
	0	0/0 = undefined	0/nonzero = 0	

Figure 4.10: Original and revised criteria for valid 3D tracks.

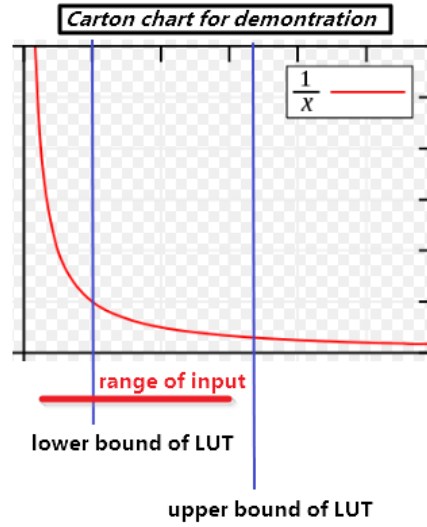
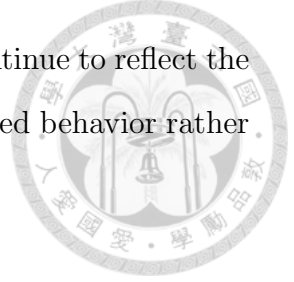


Figure 4.11: Illustration of LUT underflow when input drops below the valid range.

Additional implementation details include:

- The `2d.fnf` flag, indicating whether a corresponding 2D track was found, was previously not synchronized correctly with the 3D outputs (z_0 , $\cot \theta$, and χ^2), making it impossible to properly relate 3D tracking information to the corresponding 2D tracks.
- The `validts` flag, indicating whether the stereo TS information was valid, was previously insufficiently defined. To address these issues, a new flag, `3d.fnf`, was defined as the logical AND of the corrected `validts` and synchronized `2d.fnf`. The criterion for a valid TS was updated from "at least two stereo-related TSs" to "at least two stereo layers with finite σ_i ".

After implementing these corrections, the previously observed sharp central peak in the z_0 distribution has been successfully removed, as demonstrated by comparing



previous and new DQM plots. The remaining left/right peaks continue to reflect the firmware’s lower/upper output bounds and are considered expected behavior rather than errors.

4.7.2 Signal Desynchronization and Drift-Time Miscalculation

By comparing TS hits with wire hits, we identified missing TS hits caused by signal desynchronization, resulting in efficiency loss, as illustrated in Fig. 4.12, and this issue has since been resolved. We also fixed an error caused by wrap-around, a condition where a cyclic counter resets to zero after reaching its maximum value.

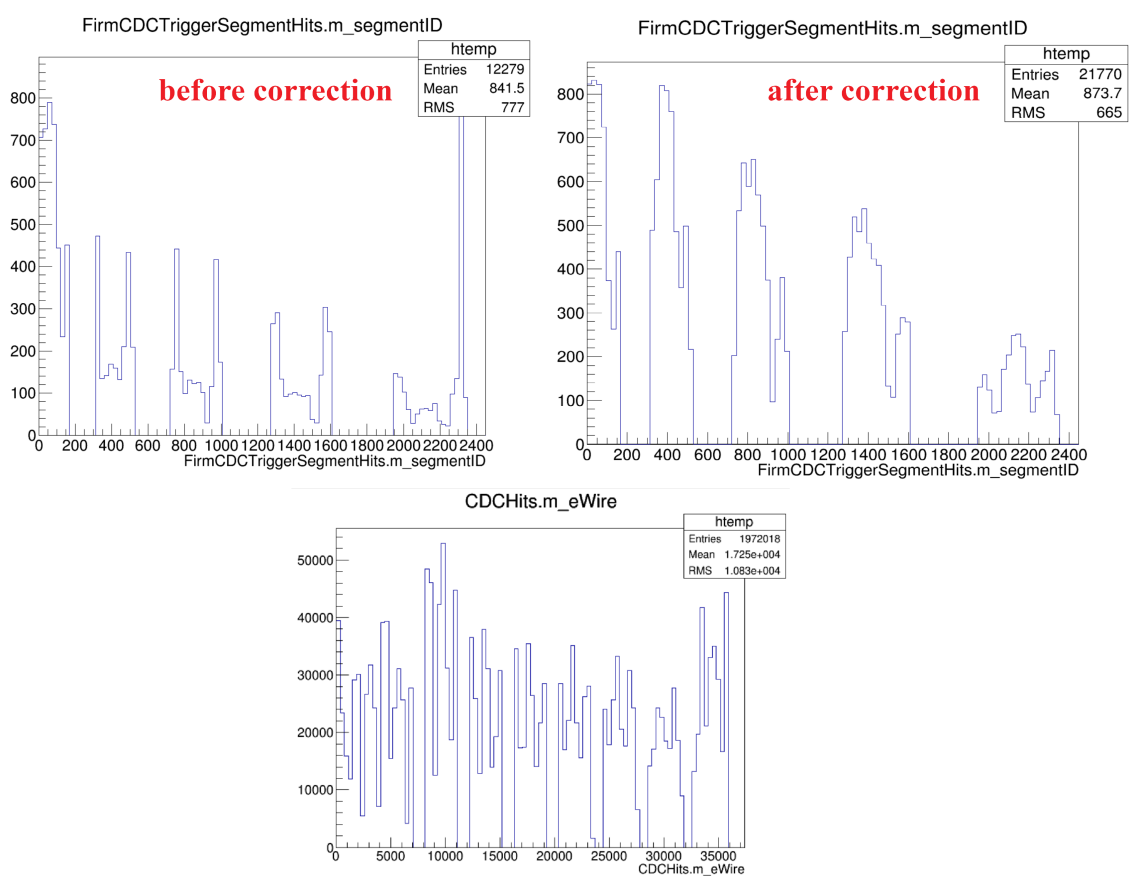
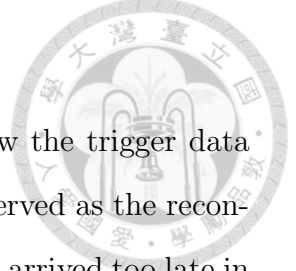


Figure 4.12: TS hits before (top left) and after correction (top right), compared to wire hits (bottom).



4.7.3 Readout Window Misalignment

As exhibited in Fig. 4.13, the timing distributions illustrate how the trigger data propagate across subsystems, with progressively later timing observed as the reconstruction proceeds. Notably, we observed that the valid 3D tracks arrived too late in the read-out window, so we adjusted the latency to shift the signals into the correct window and restore efficiency.

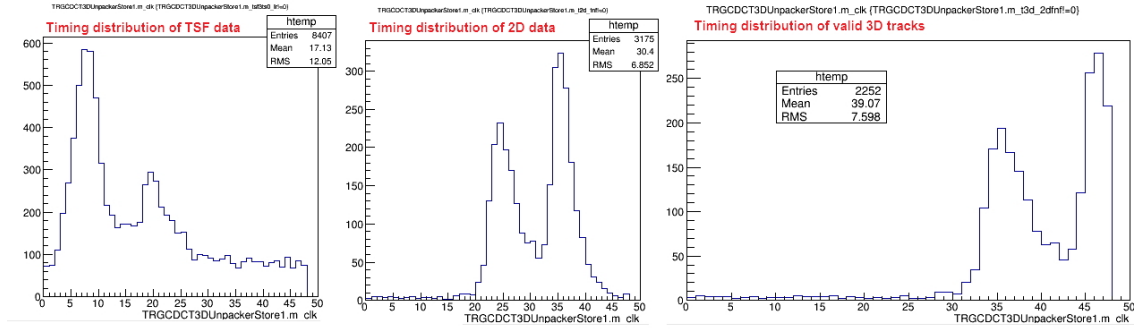


Figure 4.13: Timing distributions of TSF, 2D, and valid 3D data within the readout window.

We found that the timing distribution of valid 3D tracks lay too close to the end of the read-out window, so a late fraction of the data was truncated and about 29 % of the tracks were lost. The window has now been returned, eliminating this timing cut-off and restoring the efficiency.

Additional implementation details and performance evaluations are provided below. Before and after resolving the LUT underflow issue, the estimated survival rate of valid 3D tracks improved from $\sim 50\%$ to $\sim 72\%$, assuming 100% 2D track efficiency. The 72% loss can be decomposed as follows: an initial 18% loss due to readout window misalignment, followed by a further 86% reduction among the surviving tracks, calculated as:

$$\frac{1168 \text{ valid 3D tracks}}{1357 \text{ 2D tracks}} \approx 86\%.$$

To obtain a more accurate measurement of efficiency at this firmware development stage, we evaluated the system performance after resolving the aforementioned issues. In this test setup, the persisted CC was used as a coarse timing reference for the 13-bit TDC priority time.

Table 4.2 and Table 4.3 show the efficiency of the 3D tracker before and after correcting the readout window misalignment, using data from run #5609 of experiment 10 and run #1397 of experiment 12, respectively. The application of the p -value constraint on offline-reco tracks had no significant impact on the tracking efficiency for run #5609 of Experiment 10. In contrast, a substantial improvement in efficiency was observed for run #1397 of experiment 12 when the same constraint was applied. This result demonstrates the effectiveness of the recovery procedure following the correction of the readout window misalignment.

3D track efficiency	w/o the the constraint of p -value < 0.03	with the the constraint of p -value < 0.03
Board0	170/797 $\approx 21.32\%$	32/204 $\approx 15.68\%$
Board1	148/797 $\approx 18.56\%$	42/204 $\approx 20.58\%$
Board2	159/797 $\approx 19.94\%$	50/204 $\approx 24.50\%$
Board3	142/797 $\approx 17.81\%$	32/204 $\approx 15.68\%$
Overall	19845/25155 $\approx 78.89\%$	5481/6982 $\approx 78.50\%$
Overall	13364/15615 $\approx 85.58\%$	4747/5757 $\approx 82.45\%$

Table 4.2: Efficiency of the 3D tracker before correcting the readout window misalignment in run #5609 of experiment 10.

	w/o the the constraint of p -value < 0.03	with the the constraint of p -value < 0.03
2D track efficiency	304/329 $\approx 92.40\%$	106/110 $\approx 96.36\%$
2D event efficiency	193/203 $\approx 95.07\%$	85/87 $\approx 97.70\%$
3D track efficiency	299/329 $\approx 90.88\%$	106/110 $\approx 96.36\%$
3D event efficiency	191/203 $\approx 94.08\%$	85/87 $\approx 97.70\%$

Table 4.3: Efficiency of the 3D tracker after correcting the readout window misalignment in run #1397 of experiment 12.

We also divided our tests into several categories to evaluate the performance of the self-timing system. In this test setup, the persisted CC was used as the coarse timing reference for the 13-bit TDC priority time, and simulated perfect 2D information (with zero resolution) was provided as input.

We applied a series of parameter adjustments labeled as follows:

- A: Adjusted the parameters of the 3D finder:
 - Azimuthal window size changed from (13, 10, 10, 10) to (11, 10, 10, 10)



- Favored TS configuration changed from (3, 3, 2, 3) to (3, 3, 3, 3)
- B: Selected candidate TSs in the 3D finder in the valid drift-time range $[-100 \text{ ns}, 512 \text{ ns}]$.
- C: Removed the same drift-time selection criterion to the 3D fitter.

The results are shown in Fig. 4.14, where a gradual improvement in the z_0 resolution is observed from left to right. Based on these findings, the valid drift-time range was subsequently extended from $[-100 \text{ ns}, 512 \text{ ns}]$ to $[-512 \text{ ns}, 512 \text{ ns}]$ to capture earlier hits that were previously excluded.

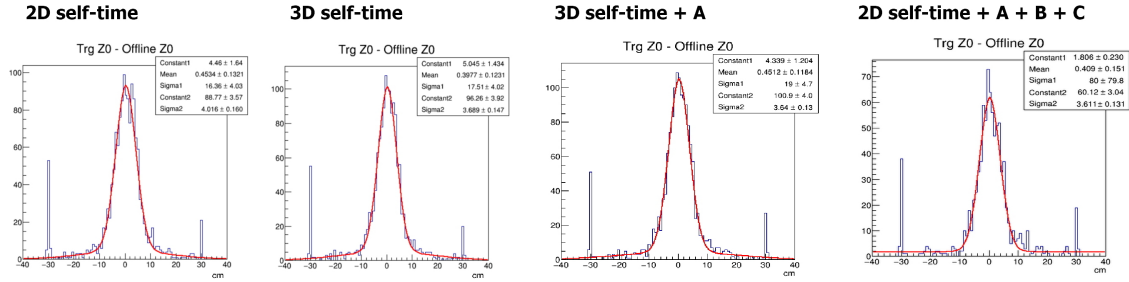


Figure 4.14: Distributions of z_0 (trigger minus offline) for different event categories, used to assess the performance of the self-timing system. Each histogram is fitted with a double-Gaussian function, and the mean and resolution parameters are shown to illustrate the precision and possible biases of the timing alignment.

4.7.4 Missing Track Segment Hits in 3D Finder

The observed issue is that all TSs within the azimuthal window of SL7 are missing. Fig. 4.15 shows the distribution of related TS positions of matched 3D tracks within the azimuthal window for each stereo SL. The absence of expected wire hits in the 3D finder is clearly visible and leads to degraded tracking performance. The rightmost bin in the figure represents the default value assigned when no related TS is found. Fig. 4.16 presents a comparison of signal versus background trade-off curves before and after the correction. The improved curve (solid purple) demonstrates the effectiveness of the recovery.

Distribution of related TS of matched 3D tracks in the ten-wire/eleven wire window

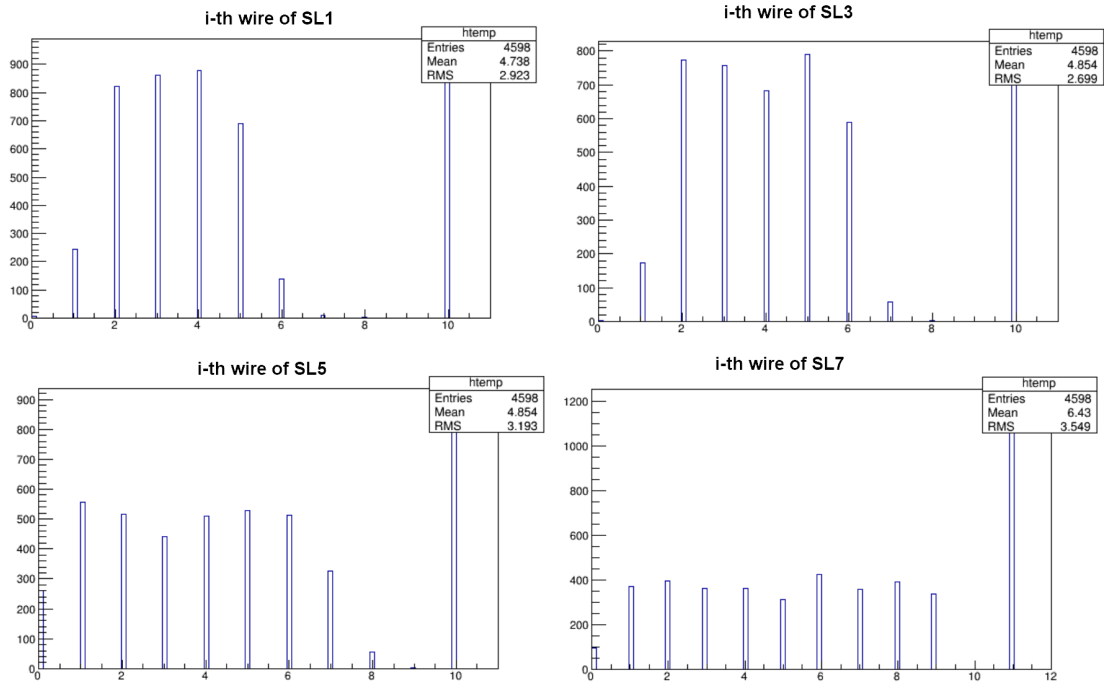


Figure 4.15: Distribution of related TS positions of matched 3D tracks within the azimuthal window for each stereo SL. The rightmost bin represents the default value when no related TS is found.

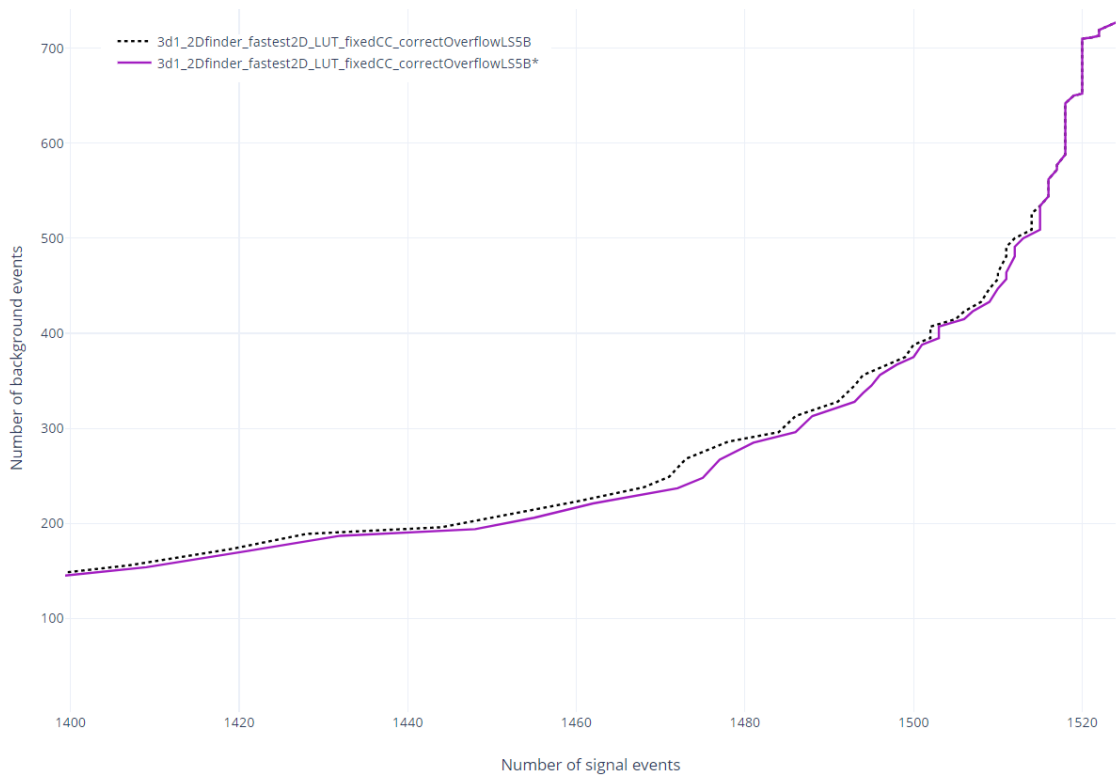


Figure 4.16: Comparison of signal vs. background trade-off curves before (dashed black) and after (solid purple) correction.

4.7.5 Serializer and Deserializer Errors

The observed issue was that the 3D track flag (SYSCLKCC) intermittently disappeared, and the latency of the 3D tracker varied across runs. This behavior was caused by the problem is that the serializer and deserializer timing is driven by a 9-bit system clock counter (SYSCLKCC), which was not synchronized with the delayed 2D inputs. Therefore, when SYSCLKCC started at an arbitrary phase, the slice order (0, 1, 2, 3) could rotate (e.g. (1, 2, 3, 4)), giving run-dependent 3D latency. To resolve this, the SYSCLKCC counter is reset whenever the 2D clock counter repeats the same value for three consecutive system clocks, locking the 3D phase to the 2D input.

4.8 Performance

This section provides a systematic evaluation of the 3D tracking algorithm's performance. It defines the selection criteria for good tracks and signal events and then introduces key performance metrics such as tracking efficiency and event efficiency. The section also outlines the methods used to quantify these metrics using real beam data, establishing the practical effectiveness of the firmware implementation.

4.8.1 Simulation Method

Fig. 4.17 illustrates the schematic of the simulation method. Offline-reconstructed tracks and trigger tracks were matched to identify overlapping events. From this matched sample, only a subset of events was extracted to provide the full input and output information of the trigger tracks for simulation.

Event efficiency was limited to 86% due to bugs present at the time. In addition, only one-fourth of the trigger data (from a single board) was used, and not all events could be simulated. This is because TS hits in the firmware are persisted for an extended period. To avoid the influence of TS hits persisted from before the previous 48 data clock trigger timing window—which are unavailable to the simulation—only

3D track information from the later part of the window was extracted for simulation. This selection ensured that the simulation used the cleanest data, free from prior interference.

During the simulation process, sequential event-by-event data were input into ISE, as illustrated in Fig. 4.18. To prevent contamination of the output from TS hits persisted across events, blank signals were inserted between events as a buffer.

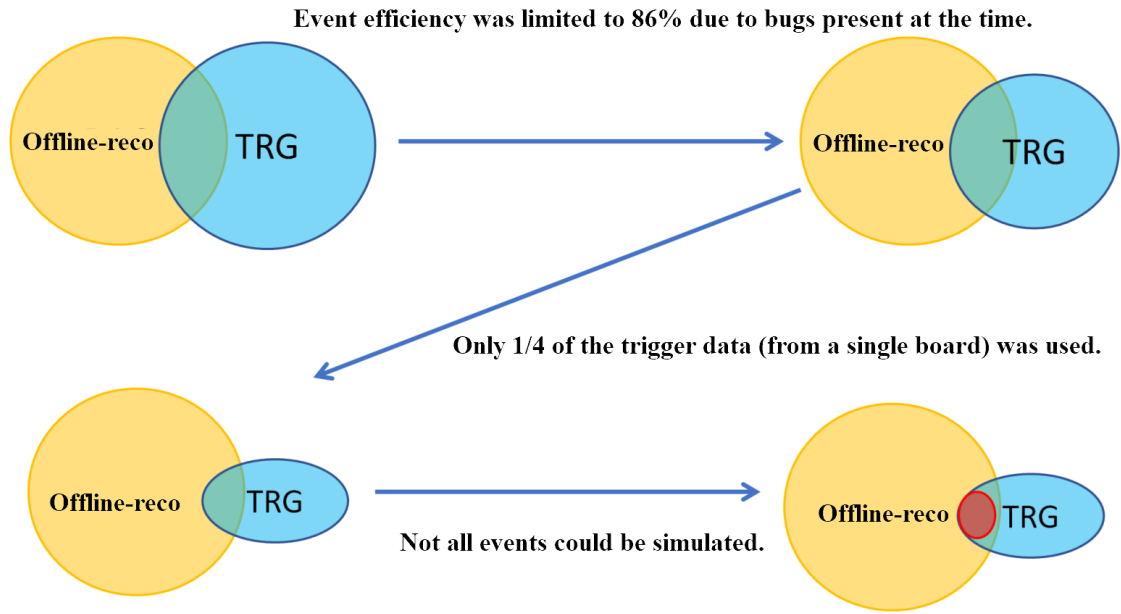


Figure 4.17: Schematic of simulation method.

m_evt	iCik	v_t3dtrk0_evtTime	v_t3dtrk0_evtTimeValid	v_t2d0_charge	v_t2d0_oldfnt	v_t2d0_fnf	v_t2d0_rho_s	v_t2d0_phi	v_t2d0ts0_id	v_t2d0ts2_id
v_evt	v_clk	v_eventTime	v_eventTimeValid							
41472	44	132	1	0	1	0	0	0	0	0
41472	44	132	1	0	1	0	0	0	0	0
41472	44	132	1	0	1	0	0	0	0	0
41472	44	132	1	0	1	0	0	0	0	0
41472	45	132	1	0	1	0	0	0	0	0
41472	45	132	1	0	1	0	0	0	0	0
41472	45	132	1	0	1	0	0	0	0	0
41472	45	132	1	0	1	0	0	0	0	0
41472	46	132	1	0	1	0	0	0	0	0
41472	46	132	1	0	1	0	0	0	0	0
41472	46	132	1	0	1	0	0	0	0	0
41472	46	132	1	0	1	0	0	0	0	0
41472	47	132	1	0	1	0	0	0	0	0
41472	47	132	1	0	1	0	0	0	0	0
41472	47	132	1	0	1	0	0	0	0	0
41472	47	132	1	0	1	0	0	0	0	0

Buffer

76032	0	114	1	0	1	0	0	0	0	0
76032	0	114	1	0	1	0	0	0	0	0
76032	0	114	1	0	1	0	0	0	0	0
76032	0	114	1	0	1	0	0	0	0	0
76032	1	114	1	0	1	0	0	0	0	0
76032	1	114	1	0	1	0	0	0	0	0
76032	1	114	1	0	1	0	0	0	0	0
76032	1	114	1	0	1	0	0	0	0	0
76032	2	114	1	0	1	0	0	0	0	0
76032	2	114	1	0	1	0	0	0	0	0
76032	2	114	1	0	1	0	0	0	0	0
76032	2	114	1	0	1	0	0	0	0	0
76032	3	114	0	0	1	0	0	0	0	0
76032	3	114	0	0	1	0	0	0	0	0
76032	3	114	0	0	1	0	0	0	0	0

Figure 4.18: Schematic of sequential event-by-event data input to ISE for simulation.

4.8.2 Selections

After debugging and validation, we performed systematic performance studies. To this end, we applied some selections to the trigger tracks and offline reconstructed tracks. First, we defined the good track. Based on this definition, we defined a signal event as having the number of good tracks greater than one; otherwise, it was considered a background event, as detailed below:

- Trigger tracks
 - No selections. All trigger tracks (track0/1/2/3) are included.
- Offline reconstructed tracks

- Definition of **signal event**: number of **good tracks** > 1 (otherwise **background event**)
- Definition of **good track**:
 - * $-1 \text{ cm} < z_0 < 1 \text{ cm}$, $-0.5 \text{ cm} < d_0 < 0.5 \text{ cm}$
 - * $P_T > 0.3 \text{ GeV}$
 - * SL0 and SL8 should be hit.
 - * Total number of CDCHit > 20 .



4.8.3 Matching Algorithm

A variable $\Delta Trace = \sqrt{\frac{\sum \Delta \phi_i^2}{N}}$ was defined for matching offline-reco tracks and trigger tracks in two dimension, where ϕ_i is azimuthal angle of the i -th TS for trigger tracks (for offline-reco tracks, the average wire hit angles were used instead); N is the number of matched TS hits, as shown in Fig. 4.19. This variable quantified the angular alignment between offline reconstructed tracks and trigger tracks in the transverse plane. We required $N > 3$ and $\Delta Trace$ was cut on 0.15.

The matching was not restricted to one-to-one correspondence. An offline-reco track was categorized as “matched” if it matched any trigger track within the same event. An event was considered matched if it contained at least one matched offline-reco track. In this study, the tracking efficiency is defined as the number of matched good tracks divided by the total number of good tracks, while the event efficiency is defined as the number of matched signal events divided by the total number of signal events.

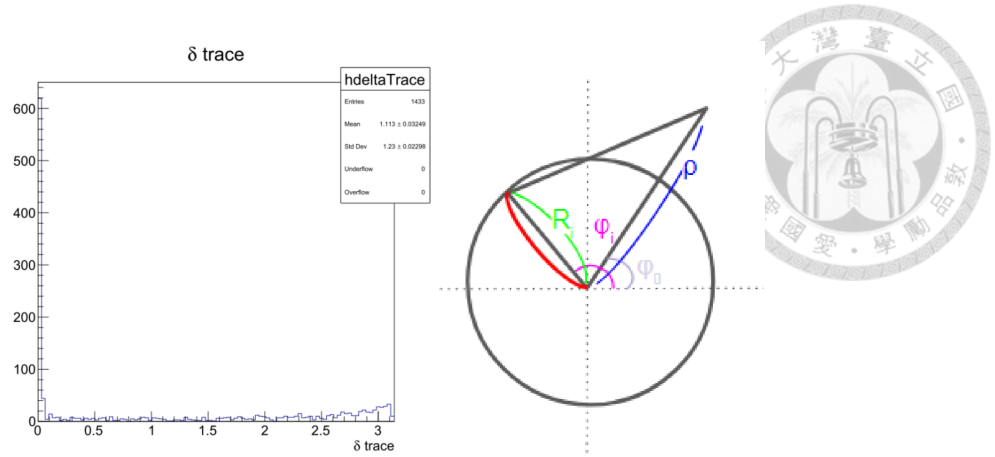


Figure 4.19: Schematic of matching algorithm.

4.8.4 Results

Fig. 4.20 shows the distributions of trigger tracks for geo-finder, hadron skim, while Fig. 4.21 presents the corresponding distributions of offline-reco tracks. Three clear dips are observed in the ϕ_0 distribution. This characteristic shape will serve as a reference for later comparison with the ϕ_0 distribution after removing HLT filtering, in order to highlight regions where signal loss occurs. Fig. 4.22 shows the correlation between trigger and offline-reco tracks for the same dataset. The two are linearly correlated, and their differences follow a Gaussian distribution.

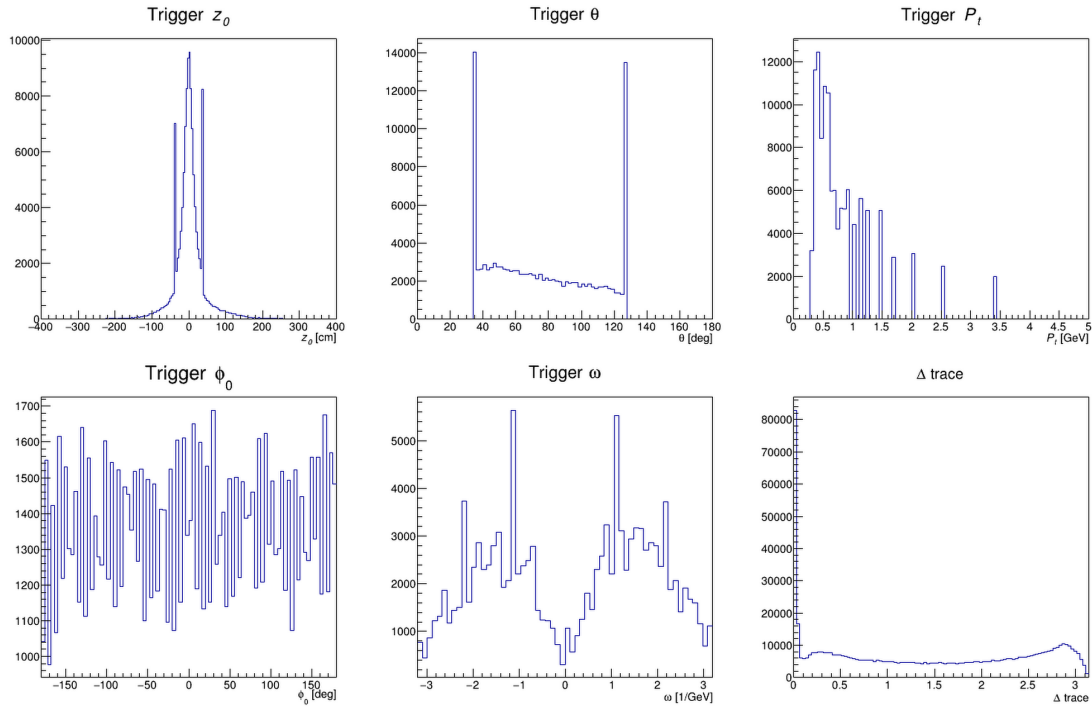


Figure 4.20: Distributions of trigger tracks (w/o selections & matching), run #883 of experiment 18 (geo-finder), hadron skim.

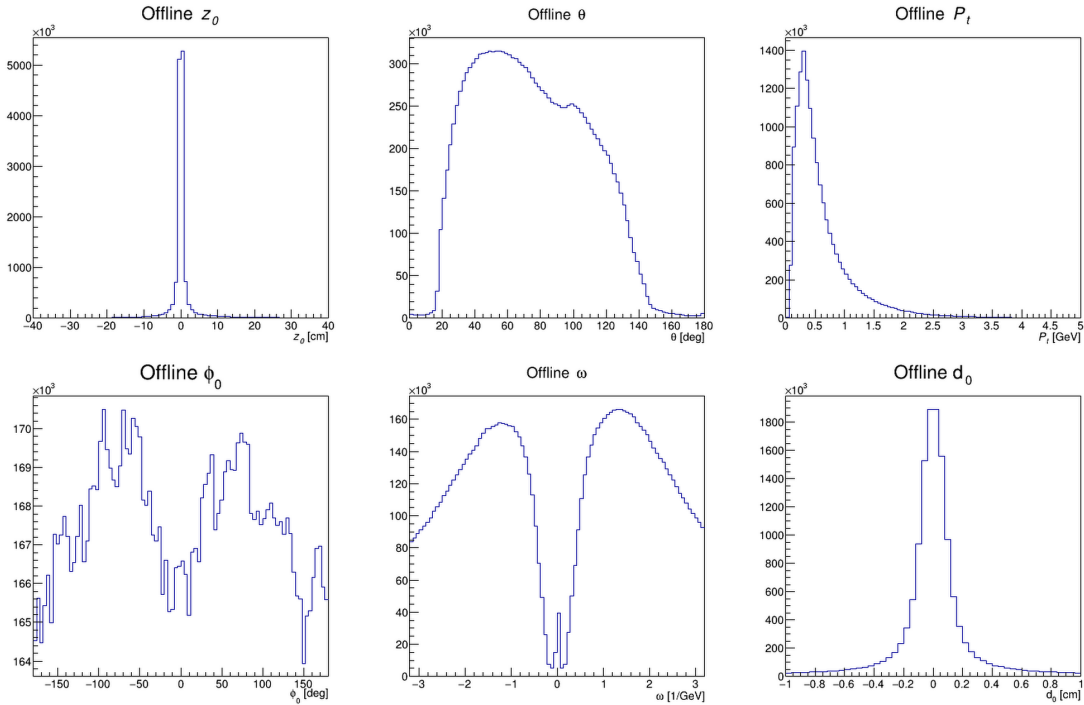


Figure 4.21: Distributions of offline-reco tracks (w/o selections & matching), run #883 of experiment 18 (geo-finder), hadron skim.

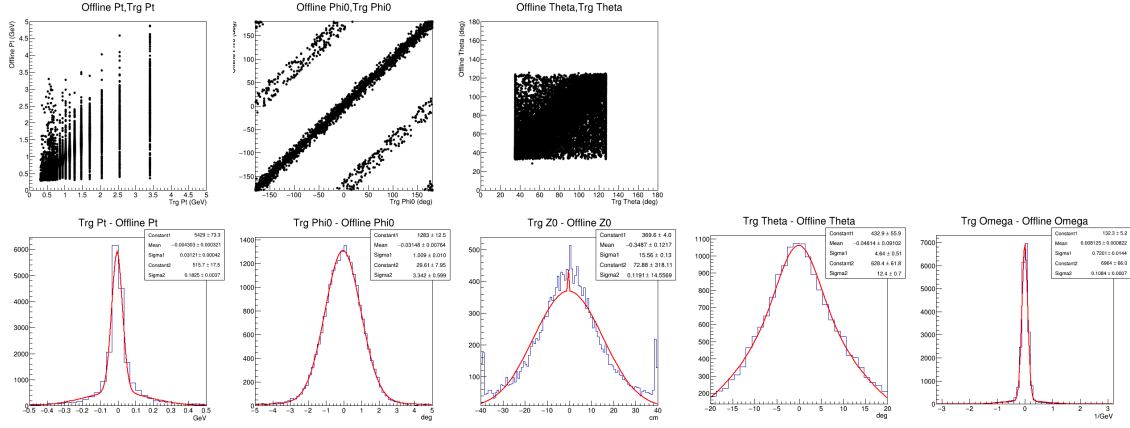


Figure 4.22: Trigger tracks vs. offline-reco tracks (w/ selections & matching), run #883 of experiment 18 (geo-finder), hadron skim.

Fig. 4.23 shows the distributions of trigger tracks without HLT filtering for geo-finder, hadron skim, while Fig. 4.24 presents the corresponding offline-reco track distributions. Compared to the distributions after HLT filtering, the ϕ_0 distribution here appears relatively flat, indicating uniform acceptance across the angular range. This contrast suggests that HLT filtering introduces an angular bias, likely suppressing tracks in specific ϕ_0 regions. Fig. 4.25 shows the correlation between trigger and offline-reco tracks without HLT filtering. As before, the two distributions are linearly correlated, and their differences follow a Gaussian distribution.

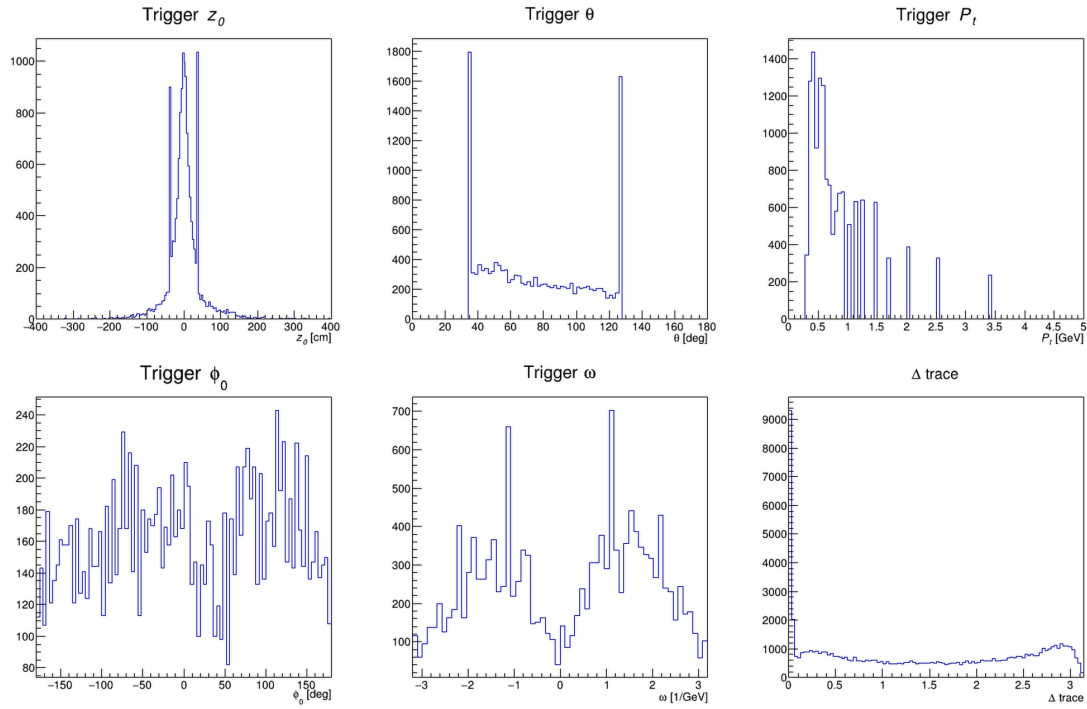


Figure 4.23: Distributions of trigger tracks (w/o selections & matching), run #1434 of experiment 18 (geo-finder, w/o HLT filtering), hadron skim.

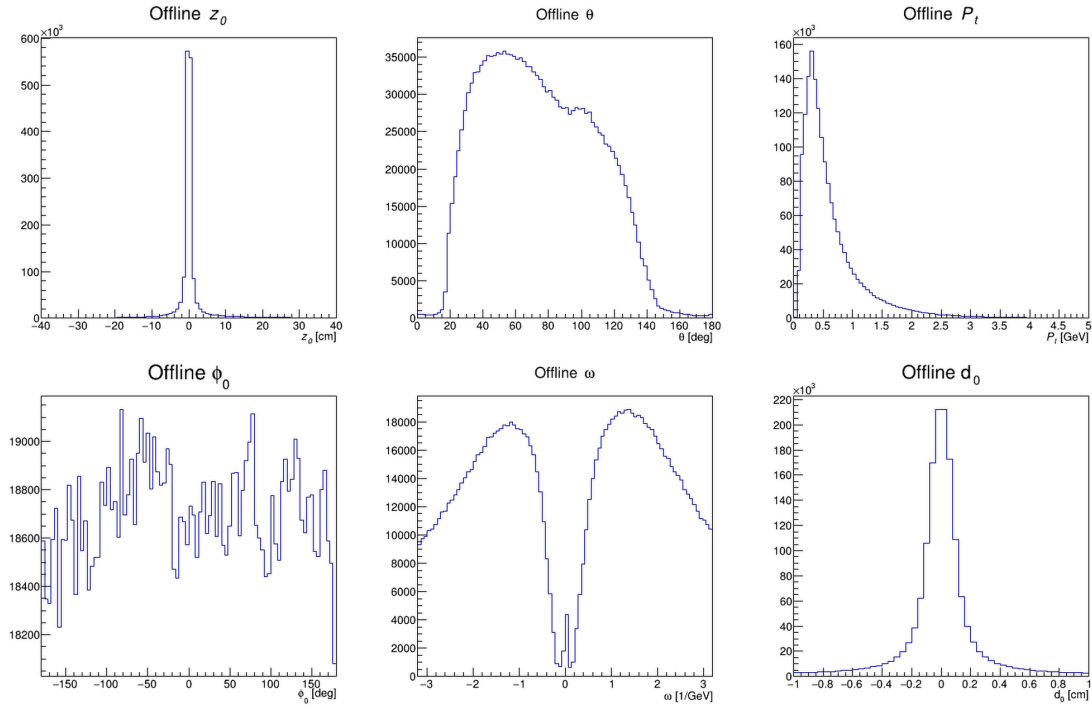


Figure 4.24: Distributions of offline-reco tracks (w/o selections & matching), run #1434 of experiment 18 (geo-finder, w/o HLT filtering), hadron skim.

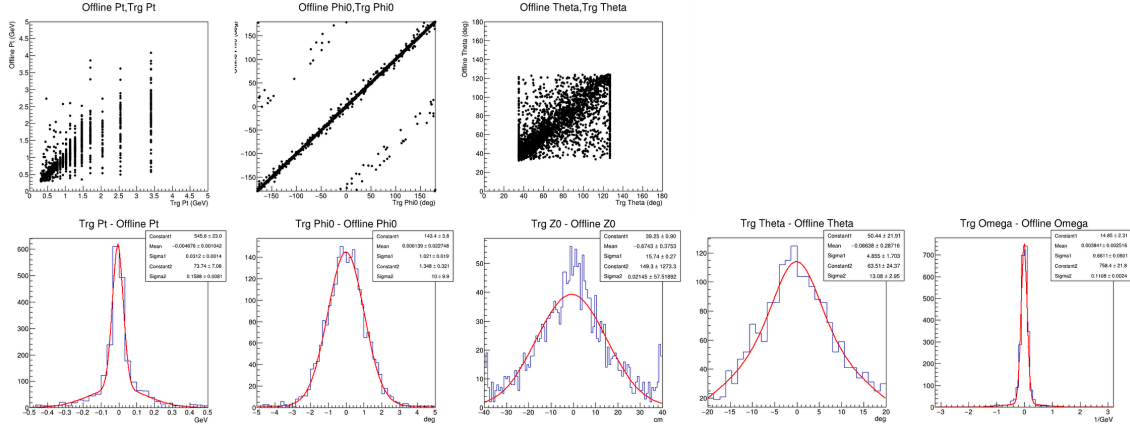


Figure 4.25: Trigger tracks vs. offline-reco tracks (w/ selections & matching), run #1434 of experiment 18 (geo-finder, w/o HLT filtering), hadron skim.

Fig. 4.26 shows the distributions of trigger tracks for LUT-finder, hadron skim, while Fig. 4.27 presents the corresponding offline-reco track distributions. Fig. 4.28 shows the correlation between trigger and offline-reco tracks for the same dataset. As in previous cases, the two distributions are linearly correlated, and their differences follow a Gaussian distribution. Compared to the geo-finder results, the resolution achieved by the LUT-finder is noticeably improved.

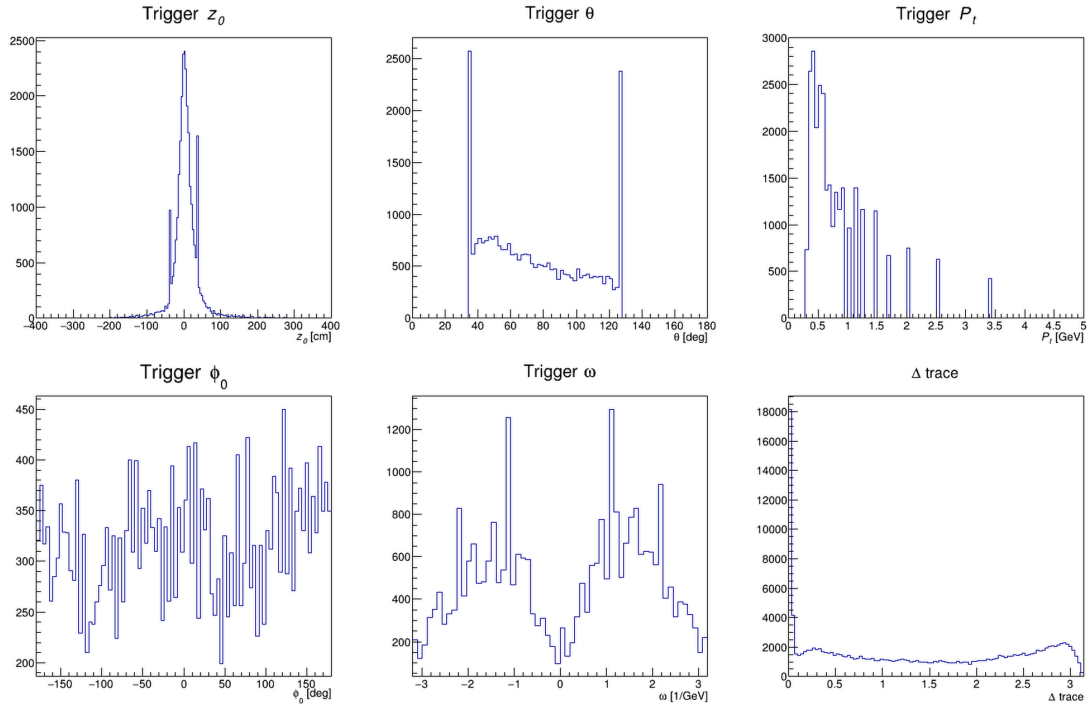


Figure 4.26: Distributions of trigger tracks (w/o selections & matching), run #1811 of experiment 18 (LUT-finder), hadron skim.

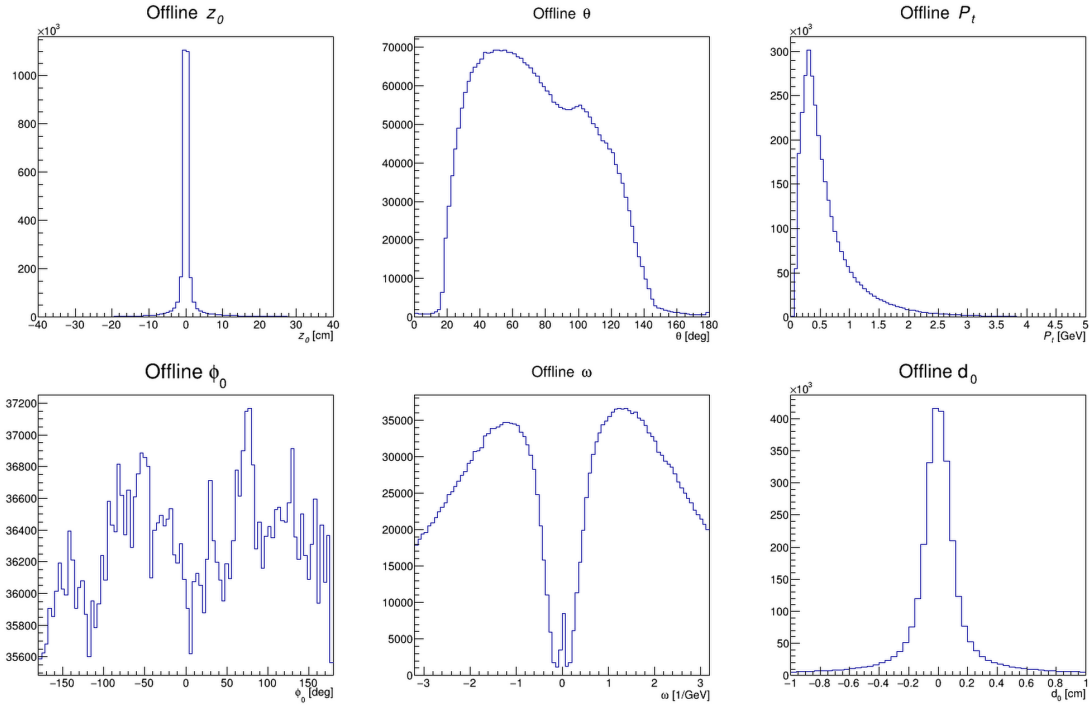


Figure 4.27: Distributions of offline-reco tracks (w/o selections & matching), run #1811 of experiment 18 (LUT-finder), hadron skim.

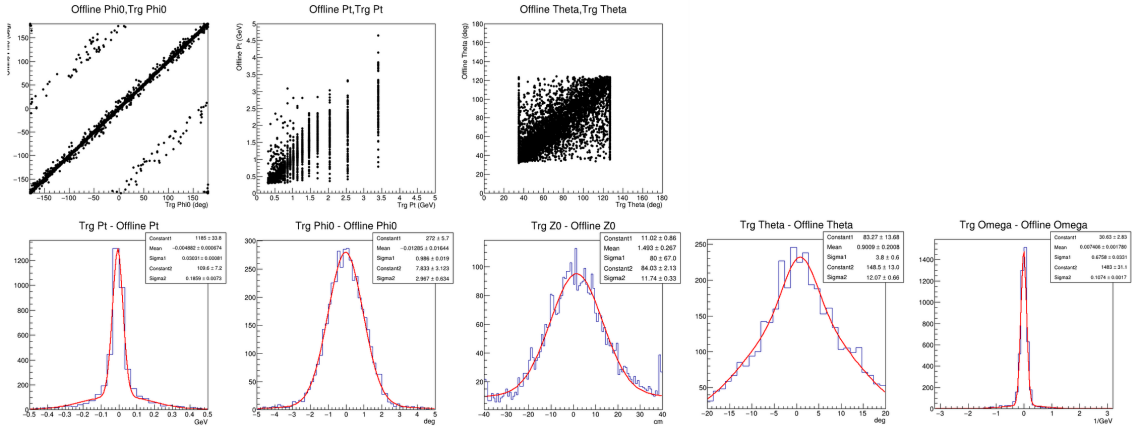


Figure 4.28: Trigger tracks vs. offline-reco tracks (w/ selections & matching), run #1811 of experiment 18 (LUT-finder), hadron skim.

The z_0 resolution is presented as a function of different offline-reco z_0 cuts across various charge and selection conditions in the following plots. Fig. 4.29 presents the result with all charges included, showing a clear degradation in resolution as the offline-reco z_0 value increases. Fig. 4.30 and Fig. 4.31 show the same distributions for positively and negatively charged tracks, respectively. In addition to the general worsening of resolution with larger offline-reco z_0 , a charge-dependent bias becomes

evident. Fig. 4.32 compares the z_0 resolution for all charges between raw data (top) and hadron skim (bottom), further illustrating the effect of selection on the resolution performance.

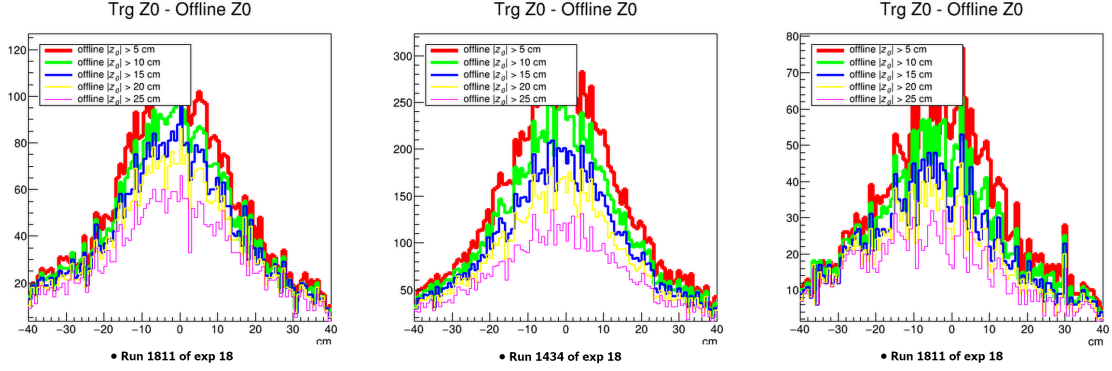


Figure 4.29: z_0 resolution vs. different offline-reco z_0 cut (w/o selections; w/ matching), all charges.

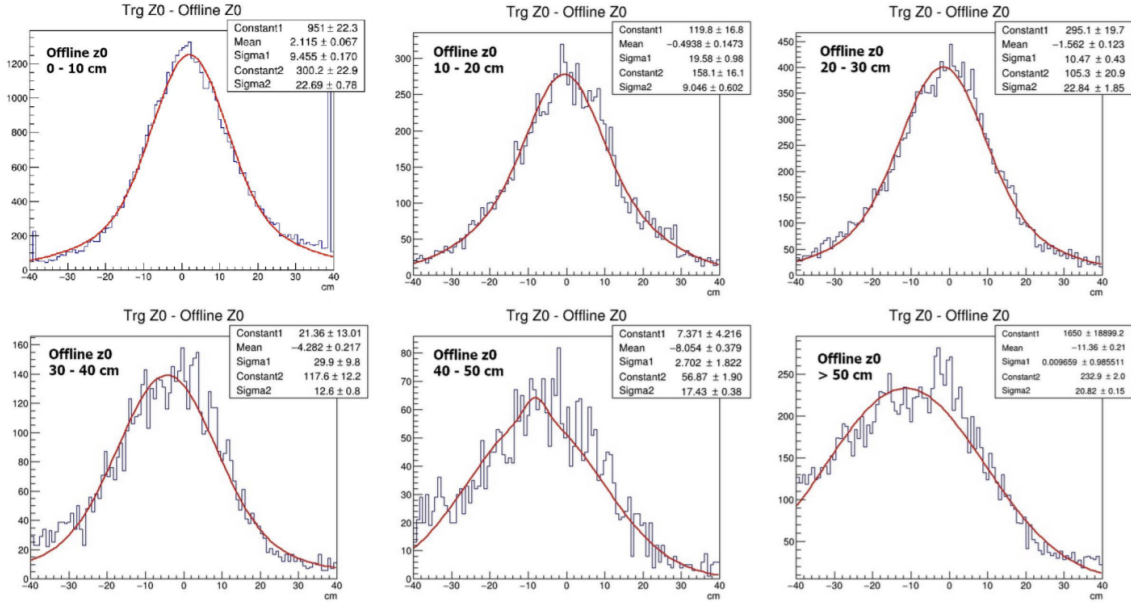


Figure 4.30: z_0 resolution vs. different offline-reco z_0 cut (w/o selections; w/ matching), positive charge.

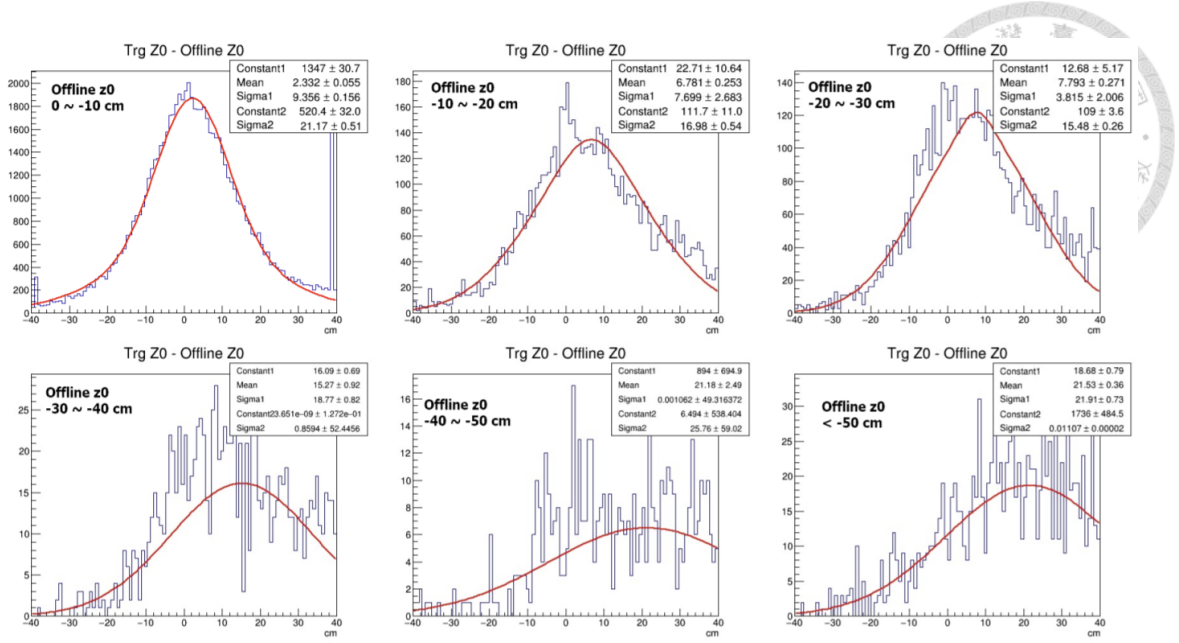


Figure 4.31: z_0 resolution vs. different offline-reco z_0 cut (w/o selections; w/ matching), negative charge.

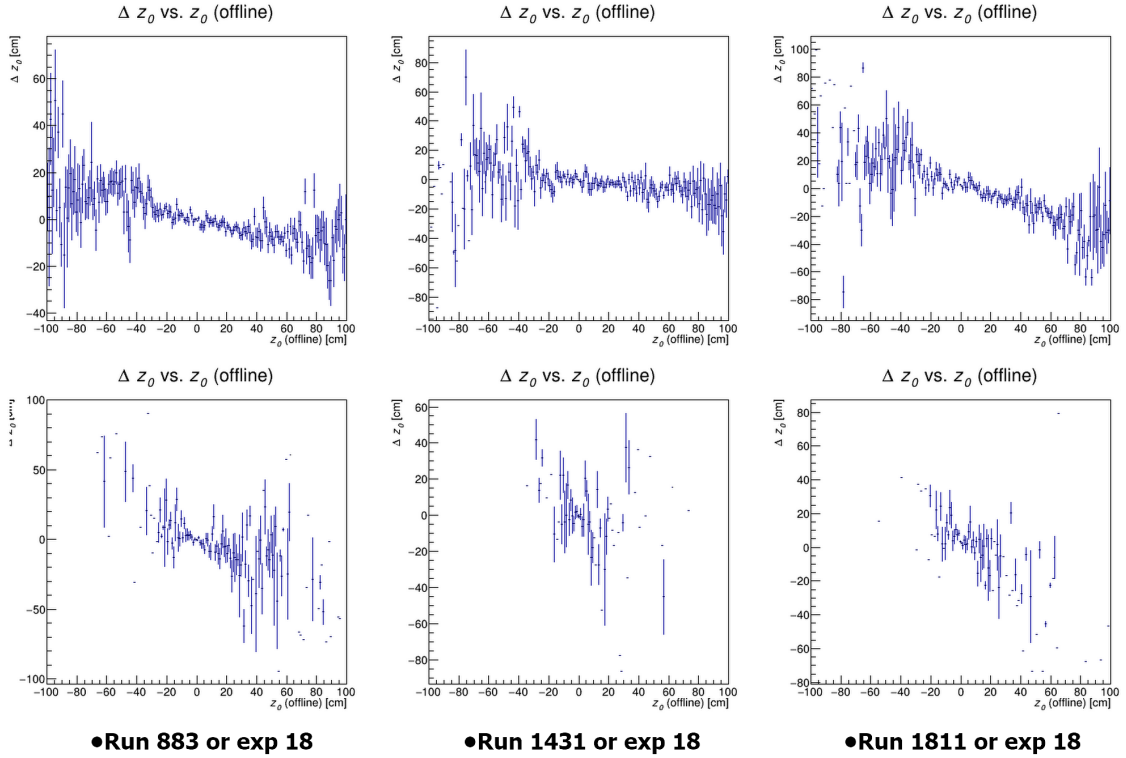


Figure 4.32: Δz_0 vs. offline-reco z_0 (w/o selections; w/ matching).

Table 4.4 along with Tables 4.5 and 4.6 present the tracking efficiency and event efficiency for different runs using the hadron skim dataset. A relatively high background level is observed in these results. However, this is not considered to be

genuine background, as the hadron skim is expected to have already removed such contributions. Instead, the apparent background may arise from mismatches between trigger and offline-reco tracks, or from limitations in the reconstruction performance.

	Tracking efficiency		Event efficiency	
z_0 cut	Efficiency	Background	Efficiency	Background
∞	84.64%	20.85%	99.40%	69.33%
35 cm	75.59%	17.17%	98.72%	64.45%

Table 4.4: Tracking efficiency and event efficiency of run #883, experiment 18 (geo-finder) with different z_0 cuts, hadron skim.

	Tracking efficiency		Event efficiency	
z_0 cut	Efficiency	Background	Efficiency	Background
∞	81.71%	21.74%	99.25%	66.47%
35 cm	73.07%	17.86%	97.99%	62.61%

Table 4.5: Tracking efficiency and event efficiency of run #1434, experiment 18 (geo-finder, w/o HLT filtering) with different z_0 cuts, hadron skim.

	Tracking efficiency		Event efficiency	
z_0 cut	Efficiency	Background	Efficiency	Background
∞	82.95%	22.81%	99.53%	74.70%
35 cm	78.05%	20.57%	99.07%	71.14%

Table 4.6: Tracking efficiency and event efficiency of run #1811, experiment 18 (LUT-finder) with different z_0 cuts, hadron skim.

In contrast, Tables 4.7, 4.8, and 4.9 summarize the results obtained from raw data. These results demonstrate excellent tracking efficiency and reasonably effective background rejection. Furthermore, the LUT-finder consistently outperforms the geo-finder, achieving both higher efficiency and superior background suppression across all runs.

	Tracking efficiency		Event efficiency	
z_0 cut	Efficiency	Background	Efficiency	Background
∞	87.68%	27.60%	99.04%	30.36%
35 cm	81.59%	20.87%	98.22%	25.47%

Table 4.7: Tracking efficiency and event efficiency of run #883, experiment 18 (geo-finder) with different z_0 cuts.

	Tracking efficiency		Event efficiency	
z_0 cut	Efficiency	Background	Efficiency	Background
∞	86.88%	32.15%	98.92%	23.95%
35 cm	80.22%	21.39%	97.58%	17.74%

Table 4.8: Tracking efficiency and event efficiency of run #1434, experiment 18 (geo-finder, w/o HLT filtering) with different z_0 cuts.

	Tracking efficiency		Event efficiency	
z_0 cut	Efficiency	Background	Efficiency	Background
∞	86.22%	28.22%	99.46%	28.67%
35 cm	82.51%	23.25%	98.88%	25.16%

Table 4.9: Tracking efficiency and event efficiency of run #1811, experiment 18 (LUT-finder) with different z_0 cuts.

Fig. 4.33 compares the signal versus background trade-off performance of the LUT-finder and the geo-finder under two CC alignment strategies. The results indicate three clear trends: (1) overflow correction provides improved performance compared to the simple CC-4 shift; (2) 2D self-timing yields better background rejection than 3D self-timing; and (3) the LUT-finder consistently outperforms the geo-finder across all configurations.

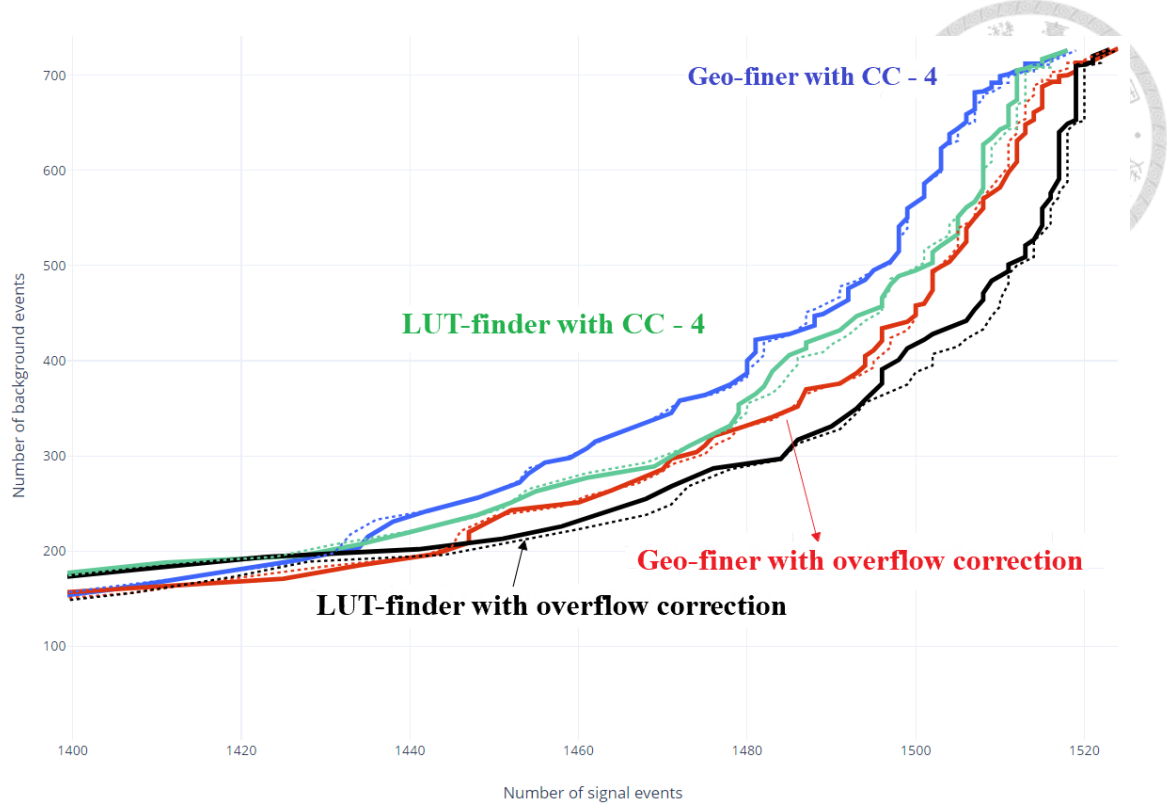


Figure 4.33: Comparison of signal vs. background trade-off curves for LUT-finder and geo-finder using the present CC under different alignment strategies. Green and blue lines represent CC-4 alignment for LUT- and geo-finder, respectively, while black and red lines show results with overflow correction. Solid lines correspond to 3D self-timing; dashed lines to 2D self-timing.

The trigger-bit efficiency logic for the Belle II L1 trigger was systematically studied by Joon-Nyong Chang in 2024 [27], providing quantitative evaluations of bit-wise contributions and optimization strategies for combined trigger logic. We also conducted a bit efficiency study, focusing on the dependence of bit efficiency on offline-reco track parameters: ϕ_0 , p_T , and θ . This study specifically targets z -related bit efficiency using the hadron skim dataset from run #313 of experiment 17 (2021/04/05).

Definitions and Requirements:

- **Bit efficiency** is defined as:

$$\text{Efficiency} = \frac{\text{xxx} \& (\text{c4} | \text{hie})}{\text{c4} | \text{hie}}$$

where **xxx** represents the trigger bit under study.

- Events are required to contain at least one offline-reco track.
- If multiple offline-reco tracks exist in an event, the track with the highest transverse momentum p_T is selected for efficiency evaluation.



Definitions of Trigger Bits:

c4 — ≥ 4 ECL clusters, no Bhabha veto, no injection veto.

hie — High ECL total energy, no Bhabha veto, no injection veto.

fff — At least 2 full CDC tracks, no injection veto.

ffy — At least 2 full CDC tracks and ≥ 1 3D NeuroTrack, no injection veto.

ffz — At least 2 full CDC tracks and ≥ 1 3D CDC track, no injection veto.

ffo — At least 2 full CDC tracks with opening angle $> 90^\circ$, no injection veto.

fyo — At least 1 full CDC track and ≥ 1 3D NeuroTrack with opening angle $> 90^\circ$, no Bhabha veto, no injection veto.

fzo — At least 1 full CDC track and ≥ 1 3D CDC track with opening angle $> 90^\circ$, no Bhabha veto, no injection veto.

ffb — At least 1 full CDC track with back-to-back topology, no Bhabha veto, no injection veto.

fyb — At least 1 full CDC track and ≥ 1 3D NeuroTrack with back-to-back topology, no Bhabha veto, no injection veto.

fzb — At least 1 full CDC track and ≥ 1 3D CDC track with back-to-back topology, no Bhabha veto, no injection veto.

Tables 4.10 summarizes the efficiency of individual trigger bits and their corresponding logical AND and OR combinations. Fig. 4.33 shows the time evolution of the trigger rates for the individual bits and for their logical AND/OR combinations in run #313 of experiment 17. Immediately after deploying the geo-finder firmware, the

ffz (**fzo**) rate was nearly identical to **fff** (**ffo**). Although GDL/GRL reported 3D tracks with a $z_0 < 35$ cm requirement, the overall rate decreased by only $\sim 10\%$ relative to the 2D rate, indicating that the z_0 cut was effectively not applied. The issue was traced to misconfigured parameters in GDL/GRL. After restoring the correct settings, the rates of the z -tagged bits dropped markedly and became comparable to the corresponding y -tagged bits.

Bit	Efficiency	Bit	Efficiency	Bit	Efficiency
fff	0.770	ffo	0.789	ffb	0.626
ffy	0.756	fyo	0.776	fyb	0.617
ffz	0.721	fzo	0.739	fzb	0.585
ffy && ffz	0.711	fyo && fzo	0.730	fyb && fzb	0.579
ffy ffz	0.766	fyo fzo	0.784	fyb fzb	0.623

Table 4.10: Efficiency of different trigger bits and their corresponding logical AND and OR combinations in run #313, experiment 17.



Figure 4.34: Trigger rates (Hz) of individual bits **fff**, **ffy**, **ffz** and **ffo**, **fyo**, **fzo**, together with their logical AND/OR combinations, during exp. 17 run #313. Following the geo-finder firmware update, **ffz**/**fzo** initially matched **fff**/**ffo** because GDL/GRL parameters were misconfigured, rendering the $z_0 < 35$ cm cut ineffective; after correcting the settings, the z -related rates decreased to the expected level, comparable to the y -related bits.

Fig. 4.35 illustrates the dependence of bit efficiency on offline-reco ϕ_0 , p_T , and θ for the geo-finder, using the hadron skim dataset. Fig. 4.36 presents the same dependence without HLT filtering, also for the geo-finder and hadron skim dataset. Fig. 4.37 depicts the dependence of bit efficiency on offline-reco ϕ_0 , p_T , and θ for the LUT-finder, based on the hadron skim dataset. The dips at both ends of the angular range correspond to tracks moving along the beam direction. These regions have limited detector coverage, leading to fewer hits per track. As a result, tracking efficiency drops due to geometric limitations.

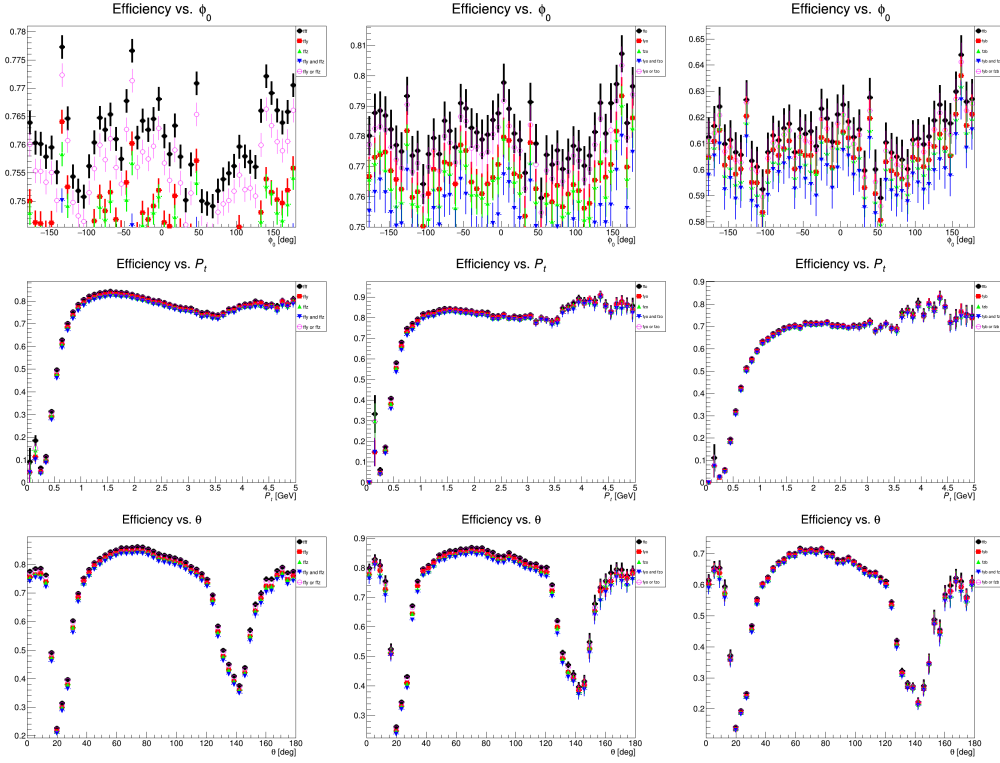


Figure 4.35: Dependence of the bit efficiency on offline-reco ϕ_0 , p_T , and θ (w/o selections & matching), run #883 of experiment 18 (geo-finder), hadron skim.

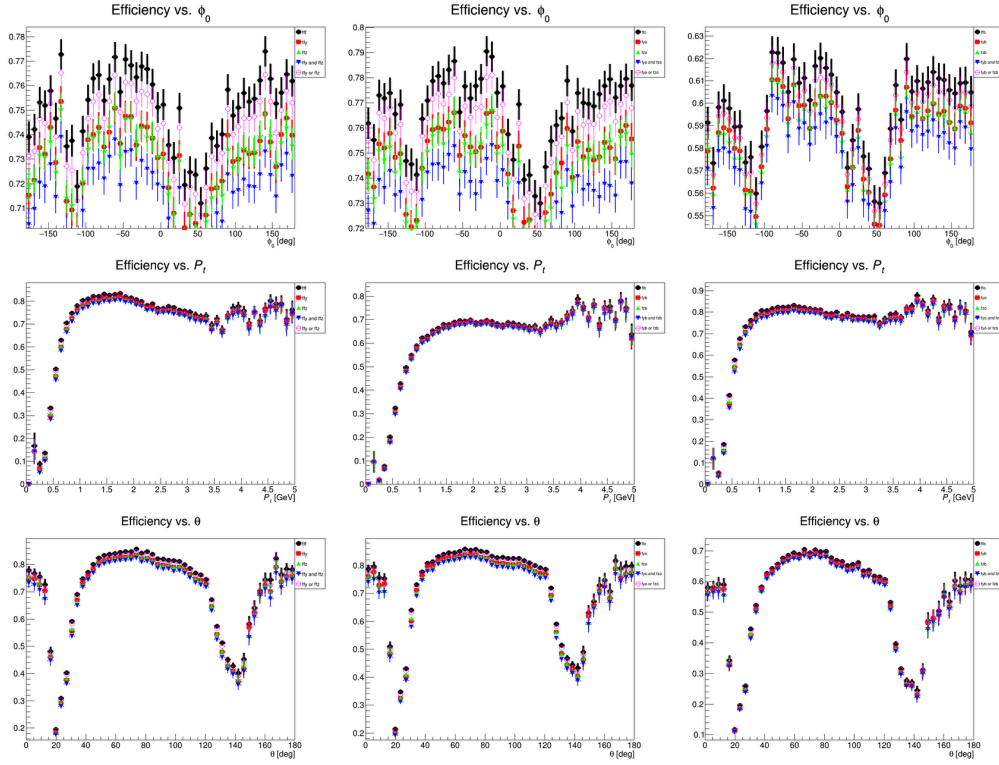


Figure 4.36: Dependence of the bit efficiency on offline-reco ϕ_0 , p_T , and θ (w/o selections & matching), run #1434 of experiment 18 (geo-finder, w/o HLT filtering), hadron skim.

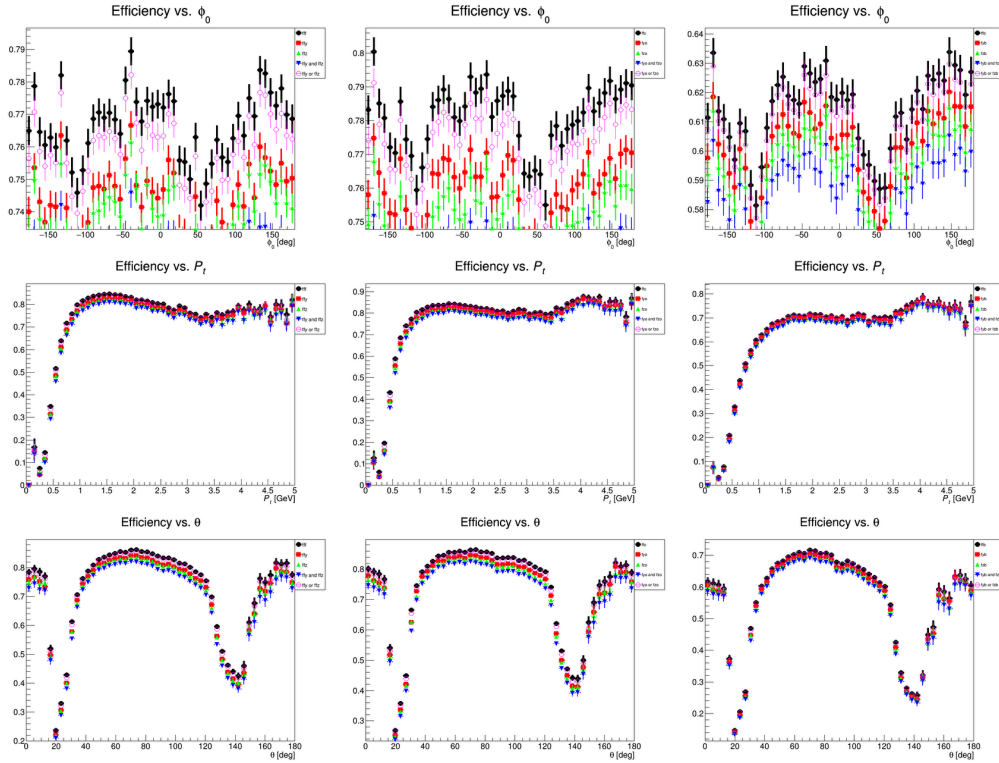


Figure 4.37: Dependence of the bit efficiency on offline-reco ϕ_0 , p_T , and θ (w/o selections & matching), run #1811 of experiment 18 (LUT-finder), hadron skim.

4.9 Discussion

The performance evaluation of the 3D tracker demonstrates its capability to accurately reconstruct track parameters and effectively reject beam-induced background. Using realistic simulation inputs and dedicated matching algorithms, the system achieves per-track efficiencies of approximately 85% and event-level efficiencies exceeding 98%. The LUT-finder algorithm significantly outperforms the earlier geo-finder, offering sharper z_0 resolution and lower background rates across various alignment strategies and timing modes. Notably, imposing a moderate z_0 constraint further reduces false triggers without compromising signal efficiency.

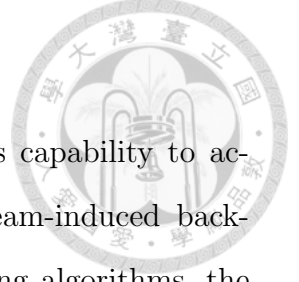
Overall, the 3D tracker meets the real-time requirements of the Belle II L1 trigger. The firmware's robustness is validated under both hadron-skimmed and raw datasets, and the improvements in algorithm design—especially the adoption of LUT-based segment finding and overflow correction—contribute to measurable gains in precision and background suppression. These results confirm the system's readiness for stable operation under high-luminosity conditions.

4.10 Summary

We successfully implemented a 3D tracking method using the LUT-finder algorithms for the Belle II L1 trigger, significantly enhancing track reconstruction accuracy and background rejection efficiency.

The performance of the tracker is thoroughly validated with simulation and experimental data, effectively resolving synchronization and timing issues to ensure reliable performance under real-world conditions.

With the robust 3D tracking framework firmly established, future prospects for optimization include further refinement of the FPGA firmware, enabling full utilization of the LUT-finder's superior capabilities within hardware constraints.





Chapter 5

Monte Carlo Study of

$$\overline{B}^0 \rightarrow J/\psi \Lambda^0 \overline{n}$$

5.1 Motivation

The decay $\overline{B}^0 \rightarrow J/\psi \Lambda^0 \overline{n}$ is a rare, color-suppressed baryonic B decay featuring a neutral anti-baryon and charmonium in the final state. It provides a unique testing ground for studying QCD color dynamics, threshold enhancement, anti-neutron reconstruction, and possible intermediate exotic states, while also contributing to the broader understanding of baryonic hadronization mechanisms in heavy flavor physics. Fig.5.1 shows the the leading quark-level Feynman diagram of $\overline{B}^0 \rightarrow J/\psi \Lambda^0 \overline{n}$.

Related baryonic B decays have been extensively investigated in Belle, such as the measurements of $B^+ \rightarrow p \overline{\Lambda} K^+ K^-$ and $B^+ \rightarrow p \overline{\Lambda} K^+ K^+$ with determinations of branching fractions and the first observation of $\eta_c \rightarrow \overline{p} \Lambda K^- + \text{c.c.}$ by Pei-Cheng Lu in 2020 [28], as well as MC studies of the two-body mode $B^+ \rightarrow p \overline{\Lambda}(1520)$ providing upper-limit estimates to guide experimental searches by Kuan-Po Lin in 2017 [29]. A similar study of $B^0 \rightarrow J/\psi \eta$, involving a charmonium state and a neutral meson, was carried out by Yuan-Hao Hsu in 2021 [30].

Neutral hadron and proton identification in the Belle and Belle II experiments

have been pursued through several complementary approaches. Wei-Che Lin (2025) [31] investigated the potential decay mode $B^0 \rightarrow \phi K_L^0$ using 699.235 fb^{-1} of MC samples and applied a multivariate classifier to separate K_L^0 from background, Jheng-Hao Su (2023) [32] developed a K_L^0 PID method based solely on ECL information with a Fast Boosted Decision Tree (FastBDT) classifier and compared it with KLM-based reconstruction in $B^0 \rightarrow J/\psi K_L^0$ decays, Yuan-Ru Lin (2021) [33] designed an anti-neutron identification algorithm using ECL information with a deep neural network, calibrated with $\bar{p}$ from $\bar{\Lambda} \rightarrow \bar{p}\pi^+$ decays under the isospin symmetry assumption, and validated in baryonic B decays, and Wei-Ling Yeh (2009) [34] established a proton identification method by combining TOF, ACC, and CDC information, validated with $\Lambda \rightarrow p\pi^-$ decays and applied to baryonic B decays.

Building on these studies, we study the baryonic B decay mode $\bar{B}^0 \rightarrow J/\psi (J/\psi \rightarrow e^+e^-, \mu^+\mu^-) \Lambda^0 (\Lambda^0 \rightarrow p^+\pi^-) \bar{n}$ by applying a MVA-based anti-neutron PID method developed in collaboration. This method assigns a score ranging from 0 to 1 to identify $\bar{n}$ candidates from ECL clusters in Belle II data. For reference, the branching fraction of the comparable decay mode $B^- \rightarrow J/\psi (J/\psi \rightarrow \mu^+\mu^-) \Lambda^0 (\Lambda^0 \rightarrow p^+\pi^-) \bar{p}$ is $(1.46 \pm 0.12) \times 10^{-5}$ (PDG2025).

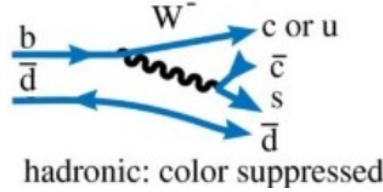


Figure 5.1: Leading quark-level Feynman diagram of the hadronic color-suppressed decay of $\bar{B}^0 \rightarrow J/\psi \Lambda^0 \bar{n}$.

5.2 Data Samples

One hundred thousand $B^0\bar{B}^0$ events are generated using phase space decay model independently as signal MC. The MC sample corresponds to an integrated luminosity of $\int L dt = 200 \text{ fb}^{-1}$, with the target decay mode $\bar{B}^0 \rightarrow J/\psi \Lambda^0 \bar{n}$ simulated assuming a branching fraction of 8.6×10^{-5} . For background study and final fitting,

we use an MC-15 run-independent MC sample, processed with release-08.



5.3 Methods

We reconstruct the decay involving a long-lived neutral hadron using both the IP constraint and the B meson mass constraint. The IP constraint assumes that the direction of the candidate's momentum points from the IP to the ECL cluster, while the B mass constraint uses four-momentum conservation to determine the magnitude of the $\bar{n}$ momentum.

We use P_B^* , the momentum of the B meson in the CM frame, as the signal discriminant variable, which is expected to peak at approximately 0.33 GeV/ c , given that

$$E_B^* \approx 5.29 \text{ GeV}, \quad P_B^* \approx 0.33 \text{ GeV}/c.$$

The reconstruction is based on the conservation of four-momentum, expressed as:

$$(E_x + E_{\bar{n}})^2 = M_B^2 + (\vec{p}_x + \vec{p}_{\bar{n}})^2, \quad (5.1)$$

where $E_{\bar{n}}^2 = M_{\bar{n}}^2 + p_{\bar{n}}^2$.

Solving for $p_{\bar{n}}$, we obtain:

$$p_{\bar{n}} = \frac{\Delta M^2 (\vec{p}_x \cdot \hat{p}_{\bar{n}}) \pm E_x \sqrt{(\Delta M^2)^2 - 4M_{\bar{n}}^2 (E_x^2 - (\vec{p}_x \cdot \hat{p}_{\bar{n}})^2)}}{2(E_x^2 - (\vec{p}_x \cdot \hat{p}_{\bar{n}})^2)}, \quad (5.2)$$

where $\Delta M^2 \equiv M_B^2 - M_x^2 - M_{\bar{n}}^2$.

Finally, the fitting variable is defined as:

$$p_B^* \equiv \sqrt{(E_x^* + E_{\bar{n}}^*)^2 - M_B^2}, \quad (5.3)$$

where

$$E_{\bar{n}}^* = \gamma \left(E_{\bar{n}} - (\vec{\beta} \cdot \hat{p}_{\bar{n}}) p_{\bar{n}} \right) \quad (5.4)$$

is the Lorentz-transformed energy of the anti-neutron from the lab frame to the CM

frame.



5.4 Event Reconstruction and Selection

This section describes the reconstruction of physics objects and the selection criteria applied to isolate signal events of interest. The analysis relies on a series of algorithms and detector-specific calibrations to identify charged and neutral particles, reconstruct decay vertices, and compute relevant kinematic quantities. To enhance the signal-to-background ratio, a combination of baseline quality cuts and topology-based requirements is employed, tailored to the expected features of the decay channel under study.

5.4.1 Bremsstrahlung Correction

The motivation for Bremsstrahlung correction lies in the fact that electrons lose energy via Bremsstrahlung when traversing detector material, causing their reconstructed momentum to be underestimated. This smears mass peaks like $J/\psi \rightarrow e^+e^-$. Muons, being much heavier, emit negligible Bremsstrahlung and thus do not require such correction.

The Bremsstrahlung correction method implemented in Belle II involves an algorithm that searches for ECL clusters near the electron's track direction. If matched and not linked to other tracks, the photon energy is added to the electron's momentum, improving energy resolution through a dedicated Bremsstrahlung recovery module. Fig.5.2 shows the mass distribution of $J/\psi \rightarrow e^+e^-$ from MC true B meson decays before and after Bremsstrahlung correction.

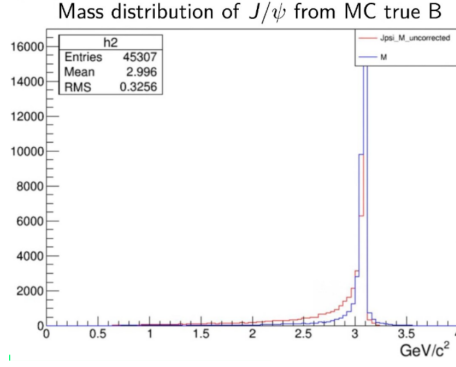


Figure 5.2: Mass distribution of $J/\psi \rightarrow e^+e^-$ from MC true B meson decays before (red) and after (blue) Bremsstrahlung correction.

5.4.2 Selections

With the Bremsstrahlung correction applied to improve the electron momentum resolution, we proceed to the event selection criteria used to isolate the signal.

First, a preselection is applied to ensure basic event quality, requiring well-reconstructed charged tracks that satisfy tight impact parameter cuts and pass the standard `higheff` definition, which corresponds to a PID likelihood greater than 0.002 and good track quality.

For PID, muon and electron candidates used in the reconstruction of J/ψ are required to have `muonID` and `electronID` scores greater than 0.5, respectively. The J/ψ candidates are selected within an invariant mass window of $3.04 < M < 3.13$ GeV/c^2 for the $\mu^+\mu^-$ channel, and $2.95 < M < 3.13$ GeV/c^2 for the e^+e^- channel, reflecting the broader mass resolution of the electron mode.

Λ^0 candidates are reconstructed from a proton and a charged pion. The proton is required to satisfy `protonID` > 0.5 , while the pion must pass `pionID` > 0.1 . The invariant mass of the $p\pi^-$ pair must lie within the window $1.111 < M < 1.121$ GeV/c^2 . In addition, we require that the cosine of the angle between the reconstructed momentum and the vector connecting the IP and the decay vertex be greater than zero, to ensure the consistency of the decay topology.

The standard Λ^0 of Belle II is comprised of Λ^0 from reconstruction and Λ^0 from v^0 particles obtained using online information. The standard Λ^0 sample in Belle II

consists of two sources: one obtained from full offline reconstruction, and the other from v^0 candidates identified using online information. The offline reconstruction uses well-calibrated track fitting and vertexing to reconstruct ($\Lambda^0 \rightarrow p\pi^-$) decays with high precision, while the online v^0 finder provides fast identification of displaced secondary vertices using simplified algorithms during data acquisition or high-level triggering. Although the online method is faster, the offline reconstruction yields higher purity and resolution, and is typically used for final physics analyses.

$\bar{n}$ candidates are required to originate from an ECL cluster (i.e., satisfy `isFromECL`) and pass the MVA-based anti-neutron PID requirement `nbarID` > 0.5.

Finally, $\overline{B^0}$ candidates must satisfy $P_B^* < 1 \text{ GeV}/c$, where P_B^* is the momentum of the B candidate in the CM frame.

The selection criteria are summarized below:

- J/ψ : $3.04 (2.95) < M < 3.13 (3.13) \text{ GeV}^2/c$ for J/ψ reconstructed by $\mu^\pm(e^\pm)$
 - $\mu^\pm(e^\pm)$: `higheff` and `muonID/(electronID)` > 0.5
- Λ^0 : standard Λ^0
 - p^+ : `protonID` > 0.5
 - π^- : `pionID` > 0.1
 - $1.11 < M < 1.12 \text{ GeV}^2/c$
 - `cosAngleBetweenMomentumAndVertexVector` (cosine of the angle between momentum and vertex vector (vector connecting IP and fitted vertex) of this particle) > 0
- $\bar{n}$: `isFromECL` (created from an ECL Cluster) and `nbarID` > 0.5
- $\overline{B^0}$: $P_B^* < 1 \text{ GeV}/c$

`higheff` of standard charged particles := **PID** > 0.002 and **good track**

- **good track**: $dr < 0.5 \text{ cm}$, $|dz| < 2.0 \text{ cm}$, `nCDC` > 20, θ in CDC acceptance)
 - **PID**: $\mathcal{L}_x / (\mathcal{L}_e + \mathcal{L}_\mu + \mathcal{L}_\pi + \mathcal{L}_K + \mathcal{L}_p + \mathcal{L}_d)$, where x is e, μ, π, K, p , or d .

5.4.3 Vertex fitting

Fig. 5.3 illustrates the vertex fitting strategy for the $\overline{B}^0 \rightarrow J/\psi \Lambda^0 \bar{n}$ decay. To improve the resolution of the kinematics, TreeFit is used for J/ψ (with mass and IP constraints) and $\overline{B}^0$, fitting the entire decay chain simultaneously; although the anti-neutron is neutral and excluded from the vertex constraint, its kinematics are included to enforce momentum and energy conservation; and for Λ^0 candidates, KFit is used to refine the vertex by applying kinematic constraints such as mass, vertex, and momentum conservation.

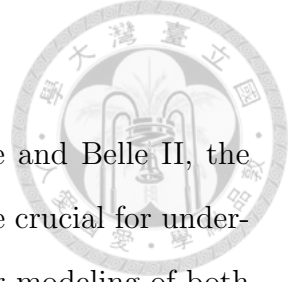
- KFit: KFit is a kinematic fitting tool that refines particle momenta by applying constraints such as: mass constraints; vertex constraints (e.g., requiring all decay products to originate from the same point); Momentum conservation (e.g., ensuring total momentum is conserved in a decay).
- TreeFit: TreeFit is a more advanced kinematic fitter that refines full decay chains in a hierarchical (tree-like) structure. Instead of fitting individual particles separately, TreeFit: Considers the entire decay tree at once; Iteratively adjusts the momenta of all intermediate states while maintaining physics constraints; Is particularly useful for complex multi-particle decay.



Figure 5.3: Vertex fitting strategy for the $\overline{B}^0 \rightarrow J/\psi \Lambda^0 \bar{n}$ decay.

5.4.4 Best Candidate Selection

For these remaining events, the best candidate is selected with the highest χ^2 probability from the Λ^0 vertex fit. If multiple candidates remain, the one with the highest χ^2 probability from the $\overline{B}^0$ vertex fit is chosen. The efficiency after applying all selection criteria is measured to be 9.11%.



5.5 Fitting Strategy

In the context of high-energy physics experiments such as Belle and Belle II, the concepts of **Self-Cross-Feed (SCF)** and **Cross-Feed (CF)** are crucial for understanding signal reconstruction and background modeling. Proper modeling of both SCF and CF is essential in maximum likelihood fits, signal yield extraction, and branching ratio calculations. SCF is usually included as a component of the signal with its own shape (e.g., wider Gaussian or non-parametric shape), while CF is often treated as a peaking background and modeled independently.

Self-Cross-Feed refers to events that originate from the signal decay mode but are *misreconstructed* in such a way that they deviate from the nominal signal distribution in one or more discriminating variables. These events:

- Come from the correct physical decay (i.e., true signal).
- Are misreconstructed due to particle misidentification or incorrect candidate combination.
- Appear as wide tails, shifted peaks, or distorted shapes in the signal region.
- May resemble background events in distributions, though they are signal by origin.

Mathematically, the SCF fraction is often defined as:

$$k = \frac{N_{\text{SCF}}}{N_{\text{signal}}},$$

where N_{SCF} is the number of misreconstructed signal events and N_{signal} is the number of correctly reconstructed signal events.

Cross-Feed is a broader term encompassing any events from *other decay modes* that are misreconstructed and **feed into** the signal region of the studied decay. Cross-Feed includes:

1. **Self-Cross-Feed** as a subset.

2. Events from different decay channels that mimic the target signal due to misreconstruction.
3. Background-like events that produce peaking structures (peaking background).

For example, in a $B^0 \rightarrow D^- \pi^+$ analysis, misidentification of K^+ as π^+ from a $B^0 \rightarrow D^- K^+$ decay can cause this mode to feed into the signal sample, creating a peak close to the true signal. Such cross-feed effects must be carefully modeled in fits.

For the PDFs for P_B^* distributions, the signal shape is primarily modeled using two Gaussian functions with a shared mean, capturing the core distribution. To account for signal tails caused by misreconstructed events—such as inaccuracies in the direction of the neutron—a second-order polynomial component is added. The signal yield is defined as the integral of the two Gaussian components. For the background, an inverted hyperbolic secant function is used to describe its distribution. To extract the parameters of the signal and self-cross-feed (SCF) components, a fit is first performed on signal MC events using the signal probability density function (PDF). Subsequently, the combined background, including both $B\bar{B}$ and continuum $q\bar{q}$ contributions from MC, is fitted with the background PDF to validate the appropriateness of the chosen background model. The fraction and σ of the two Gaussian functions are fixed, and the other parameters are floating in the final fit. Fig.5.4 shows the fits to the P_B^* distribution in MC.

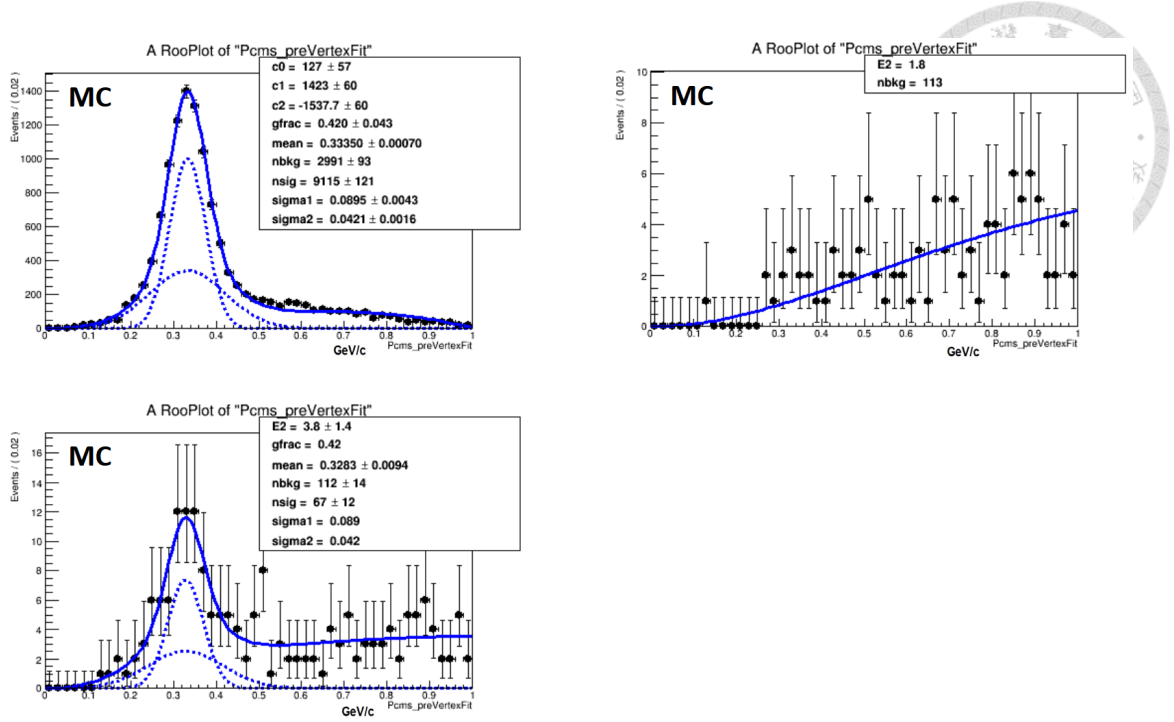
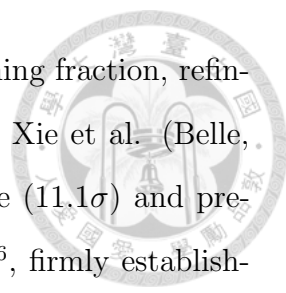


Figure 5.4: Fits to the P_B^* distribution in MC. Left: Signal MC sample fitted using a double-Gaussian model plus a second-order polynomial to model tails from misreconstruction. Top right: Background MC sample fitted with an inverted hyperbolic secant function. Bottom right: Combined signal + background MC fitted using the full PDF model with fixed Gaussian shape parameters and floating background parameters.

5.6 Control Sample

A control sample is used to validate the reconstruction and selection procedures as well as to estimate systematic uncertainties. The sample is chosen to resemble the signal in key kinematic and detector response aspects while being statistically independent from the final selection. By comparing distributions between data and MC simulation, potential discrepancies in modeling or detector performance can be identified and corrected before extracting the final results from the signal region.

Studies of the decay mode $B^- \rightarrow J/\psi \Lambda^0 \bar{p}$ and $B^+ \rightarrow J/\psi \bar{\Lambda}^0 p$ have provided valuable insights into baryonic charmonium production and potential intermediate resonance structures. Aubert et al. (BaBar, 2003) [35] first presented evidence for this decay, measuring a branching fraction of approximately $(12^{+9}_{-6}) \times 10^{-6}$, and setting initial constraints on related decay modes. Subsequently, Zang et al. (Belle,



2004) [36] established an upper limit of 4.1×10^{-5} for the branching fraction, refining the search but without confirming the BaBar signal. Later, Xie et al. (Belle, 2005) [37] conclusively observed the decay with high significance (11.1σ) and precisely measured the branching fraction as $(11.6 \pm 2.8_{-2.3}^{+1.8}) \times 10^{-6}$, firmly establishing the decay's existence. Most recently, the CMS Collaboration (2019) [38] used high-statistics proton-proton collision data to measure the branching fraction ratio relative to the $B^+ \rightarrow J/\psi K^{*+}$ decay, confirming previous results and performing a detailed angular analysis that indicated significant contributions from excited strange resonances decaying into the $\Lambda^0 p$ system.

5.6.1 Motivation and Methods

The decay $B^- \rightarrow J/\psi \Lambda^0 \bar{p}$ is used as a control sample to validate the reconstruction and PID methodology. Specifically, the momentum magnitude of the anti-proton is reconstructed using the same kinematic method as applied to the anti-neutron, rather than relying on CDC tracking information. Moreover, the PID information for the anti-proton is obtained from the matched ECL cluster, following the same procedure used for the anti-neutron.

To evaluate the performance of this reconstruction method, we compare it against a baseline approach that reconstructs the anti-proton momentum using CDC tracking information, without applying anti-neutron PID or kinematic constraints. In the baseline method, the discriminating variables used to extract the signal are the beam-constrained mass $M_{bc} \equiv \sqrt{E_{\text{beam}}^2 - |\vec{p}_B|^2}$, and the energy difference, $\Delta E \equiv E_B - E_{\text{beam}}$, where all quantities are evaluated in the CM frame. Both variables are widely used in B -factory experiments to separate signal and background: signal events tend to peak at $M_{bc} \approx m_B$ and $\Delta E \approx 0$, where m_B is the nominal B -meson mass.

In contrast, the proposed method reconstructs the decay using the $\bar{p}$ momentum inferred from the mother mass and IP constraints, and uses P_B^* , the momentum of the B candidate in the CM frame, as the discriminating variable. By comparing the

extracted signal yields and branching fractions between the two methods, we validate the accuracy and reliability of the proposed reconstruction and PID approach.

Finally, the branching fraction extracted from this control sample is compared to the known world-average value, thereby verifying the validity of the proposed reconstruction and PID approach.

5.6.2 Data Samples and Selections

One hundred thousand $B\bar{B}$ events are generated using a phase space decay model as the signal MC sample. Half of the events decay into e^+e^- , and the other half into $\mu^+\mu^-$. For the data sample, the proc13 and latest prompt datasets processed with release-08 are used in this study. The selection criteria for $B^- \rightarrow J/\psi \Lambda^0 \bar{p}$ follow those of $\bar{B}^0 \rightarrow J/\psi \Lambda^0 \bar{n}$ with an additional requirement of `protonID` > 0.5 applied on the $\bar{p}$.

5.6.3 Modified scheme for the Electromagnetic Calorimeter matching of $\bar{p}$

For $B^- \rightarrow J/\psi \Lambda^0 \bar{p}$, the intrinsic three-body baryonic kinematics result in low $\bar{p}$ momentum, as shown in Fig.5.5. This leads to low ECL cluster matching efficiency, causing significant signal loss. Only 43.88% of signals have matched ECL clusters. After applying `nbarID` > 0.5 , signal retention further drops to 12.98%.

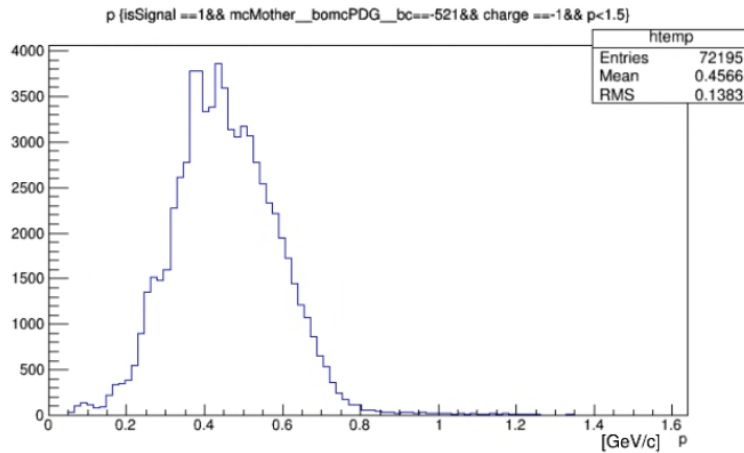


Figure 5.5: Momentum distribution of $\bar{p}$.

The `track` variable returns a count of the number of ECL clusters matched to the track. This is always 0 or 1 with newer versions of ECL reconstruction. For a particle matched to a ECL cluster, two types of hypotheses are attached to the matched ECL cluster. One is neutral hadron hypothesis (`neutralHadron`). Another is n-photon hypothesis (`n-photon`). A particle object chooses the most energetic ECL cluster having n-photon hypothesis as the matched ECL cluster; however the original matching ignores `neutralHadron` clusters that are important for $\bar{p}$ reconstruction.

To address this limitation, we designed the modified matching scheme to prioritize the selection of ECL clusters with `neutralHadron` hypothesis:

- For $\bar{p}$, choose the most energetic cluster with `neutralHadron` hypothesis if available.
- If both hypotheses are present, select `neutralHadron`; otherwise, use the available one.

Fig.5.6 and Fig.5.7 portray the original scheme and the modified scheme for the ECL matching of $\bar{p}$, respectively.

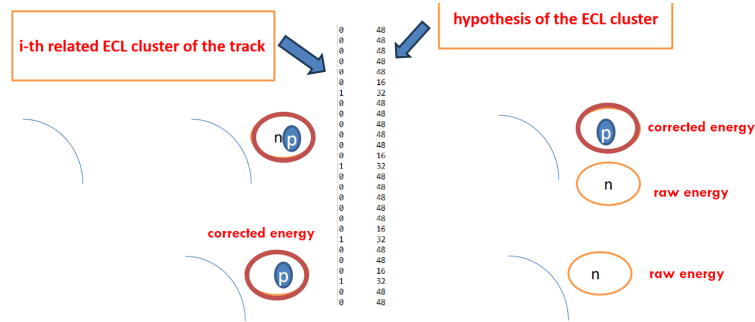


Figure 5.6: Original scheme for the ECL matching of $\bar{p}$. p (n): n-photon (neutralHadron) hypothesis

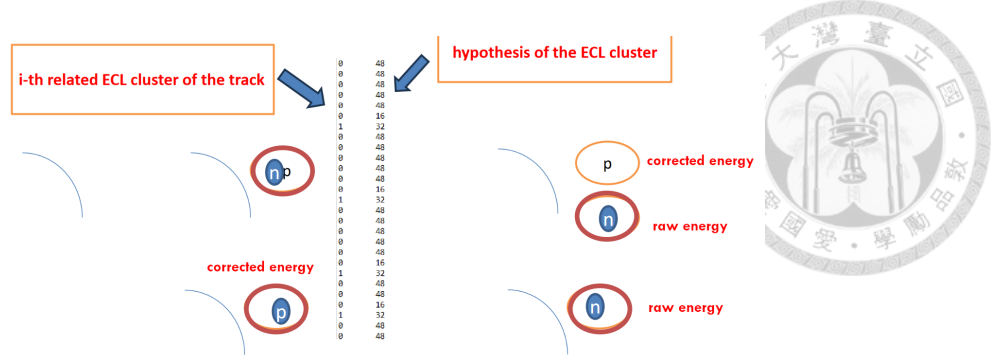


Figure 5.7: Modified scheme for the ECL matching of $\bar{p}$. p (n): n-photon (neutronHadron) hypothesis

The study of $\bar{p}$ detection efficiency under annihilation effects in ECL clusters shows the results are consistent with our previous dedicated study on anti-neutron PID (BELLE2-NOTE-TE-2020-016 3 Version 3.2.0: Particle Identification of Anti-neutron). Fig.5.8 shows distributions and efficiency of $\bar{p}$ and $\bar{n}$ after selections.

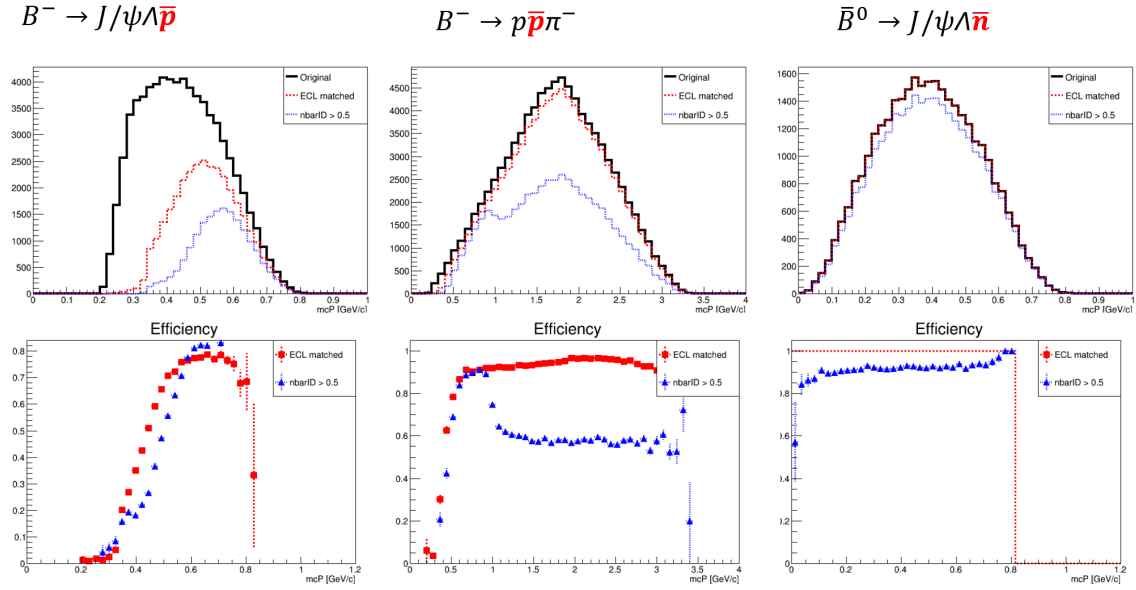


Figure 5.8: Distributions (top) and efficiency (bottom) of $\bar{p}$ and $\bar{n}$ after selections. Top **black**: original distribution; **red**: ECL matched; **blue**: ECL matched with $nbarID > 0.5$. Bottom **red**: ratio of red/black in the top plots; **blue**: ratio of blue/red in the top plots.

Fig.5.9 shows distributions of cluster energy and anti-neutron PID MVA output for $\bar{p}$ before (top) and after (bottom) the application of the modified ECL matching scheme, demonstrating the effectiveness of the recovery. The remaining signal after recover is 24.06% — a 45.18% of the signals have matched ECL clusters, and 53.25%

of those remaining signals survive after $\text{nbarID} > 0.5$ cut. This modified scheme nearly doubles signal efficiency, though systematic challenges in $\overline{B}^0 \rightarrow J/\psi \Lambda^0 \bar{n}$ remain due to low momentum of $\bar{p}$.

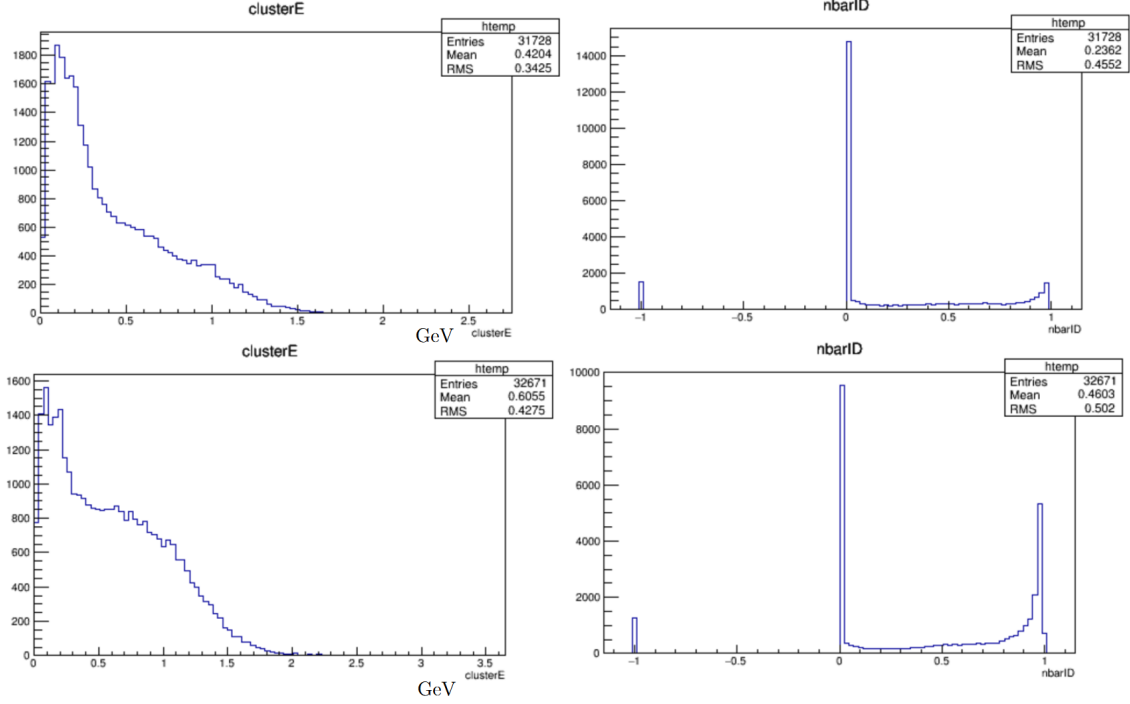


Figure 5.9: Distributions of cluster energy and anti-neutron PID MVA output for $\bar{p}$ before (top) and after (bottom) the application of the modified ECL matching scheme.

5.6.4 Results and Discussion

This modified reconstruction scheme nearly doubles the signal efficiency by including ECL clusters with all available hypotheses in the matching algorithm, which significantly improves the recovery of low-momentum $\bar{p}$ candidates in the decay $B^- \rightarrow J/\psi \Lambda^0 \bar{p}$; however, the evaluation of systematic uncertainties for this decay mode remains challenging due to the inherent limitations associated with low-momentum particle reconstruction.

Due to the reconstruction challenges mentioned above, anti-neutron PID is not applied to $\bar{p}$ in this stage of the evaluation; in particular, the requirement $\text{protonID} > 0.5$ is not enforced. Fig. 5.10 compares two reconstruction methods for the control sample $B^- \rightarrow J/\psi \Lambda^0 \bar{p}$. The branching fraction reported by the PDG for this decay

is 1.46×10^{-5} . We use this value, along with reconstruction efficiencies, to estimate the expected signal yield in real data and compare it with the result obtained from signal fitting.

- When the anti-proton momentum is reconstructed using CDC tracking information, the expected yield is

$$1.46 \times 10^{-5} \times 1.1 \div 2 \times 10^6 \times 0.0596 \times 0.639 \times 318.83 \times 0.130 = 12.68,$$

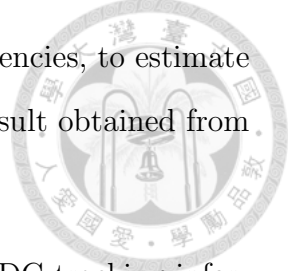
under an integrated luminosity of $\int L dt = 318.83 \text{ fb}^{-1}$.

- When the anti-proton momentum is reconstructed using the same kinematic method as applied to the anti-neutron (based on the mother mass and IP constraints), the expected yield is

$$1.46 \times 10^{-5} \times 1.1 \div 2 \times 10^6 \times 0.0596 \times 0.639 \times 356.63 \times 0.132 = 14.40,$$

under an integrated luminosity of $\int L dt = 356.63 \text{ fb}^{-1}$.

The results from both methods are consistent and agree well with the expected PDG value. This comparison validates the proposed anti-neutron PID-based reconstruction approach and demonstrates its effectiveness.



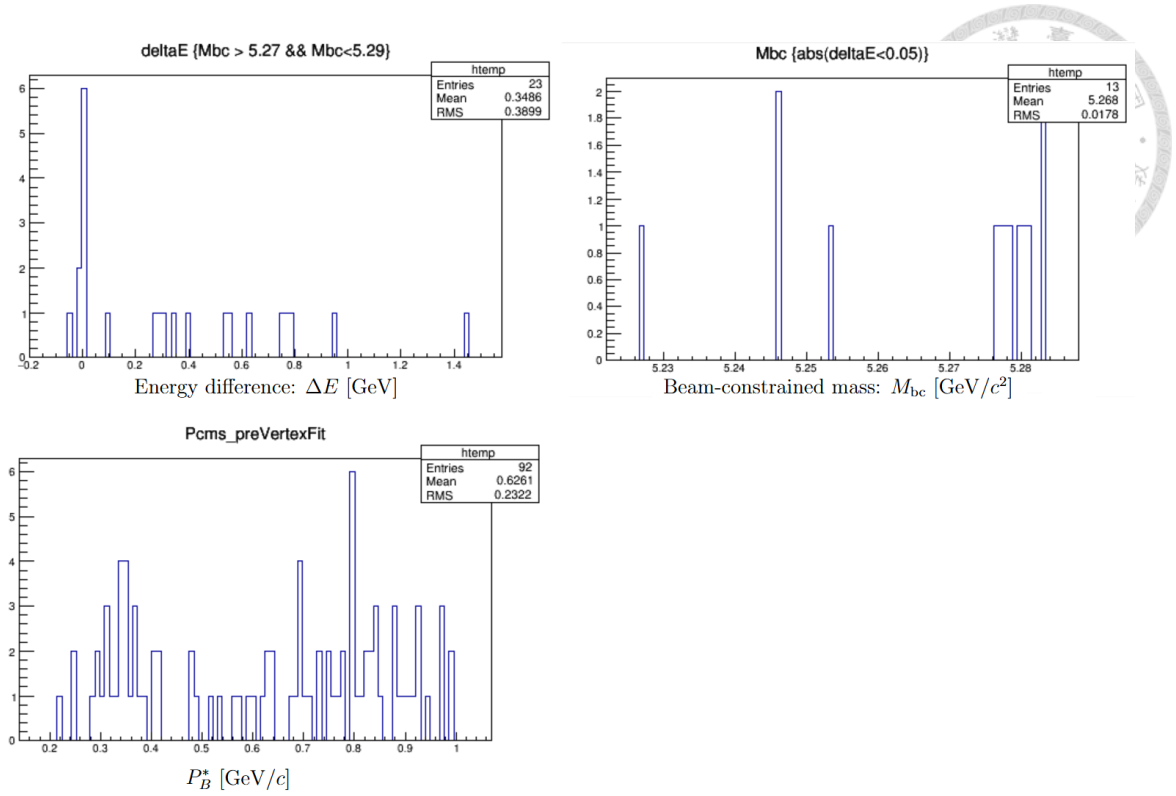


Figure 5.10: Comparison of reconstruction methods for the control sample $B^- \rightarrow J/\psi \Lambda^0 \bar{p}$. The upper plots show the distributions of the energy difference ΔE (left) and the beam-constrained mass M_{bc} (right), where the anti-proton momentum is reconstructed using CDC tracking information. The lower plot shows the distribution of P_B^* , the momentum of the B candidate in the CM frame, where the anti-proton momentum is reconstructed using the same kinematic method as applied to the anti-neutron, based on the mother mass and IP constraints.

5.7 Upper Limit Estimation

We use the profile likelihood method to estimate the upper limit. The Profile Likelihood Method, used by RooFit's **ProfileLikelihoodCalculator**, is based on the following:

- **1. Likelihood Ratio Test:** The likelihood ratio is defined as:

$$\Lambda(\theta_0) = \frac{\mathcal{L}(\theta_0)}{\mathcal{L}(\hat{\theta})}$$

- $\mathcal{L}(\theta_0)$ is the likelihood evaluated at a fixed parameter value θ_0 .
- $\mathcal{L}(\hat{\theta})$ is the maximum likelihood over all parameter values (the global maximum).

– The test statistic is:

$$-2 \ln \Lambda(\theta_0) = -2 \left(\ln \mathcal{L}(\theta_0) - \ln \mathcal{L}(\hat{\theta}) \right)$$



- **2. Wilks' Theorem:** For a sufficiently large sample size, the distribution of $-2 \ln \Lambda(\theta_0)$ follows a chi-squared distribution with k degrees of freedom, where k is the number of parameters of interest:

$$-2 \ln \Lambda(\theta_0) \sim \chi_k^2$$

- **3. Confidence Interval Estimation:** The confidence interval or upper limit is determined by finding the range of θ_0 where $-2 \ln \Lambda(\theta_0)$ falls within the critical values of the chi-squared distribution. For example:

– For a 95% confidence level: $-2 \ln \Lambda(\theta_0) = 3.84$ (for a single parameter).

The advantages are listed below:

- It is a standard frequentist approach.
- Applicable to arbitrary likelihood models.
- RooFit automatically handles marginalization by maximizing the likelihood over nuisance parameters.

The upper limit of the number of signals MC is estimated to be **71.74** using the profile likelihood method.

5.8 Results

Fig. 5.11 shows the P_B^* distributions of the reconstructed meson. The expected yield under $\int L dt = 200 \text{ fb}^{-1}$ luminosity is

$$8.6 \times 10^{-5} \times 1.05 \div 2 \times 10^6 \times 0.119 \times \mathbf{0.0911} \times 0.64 \times 200 = 62.65,$$

assuming a branching fraction of 8.6×10^{-5} , which is consistent with the fitted yield of 67 ± 12 .

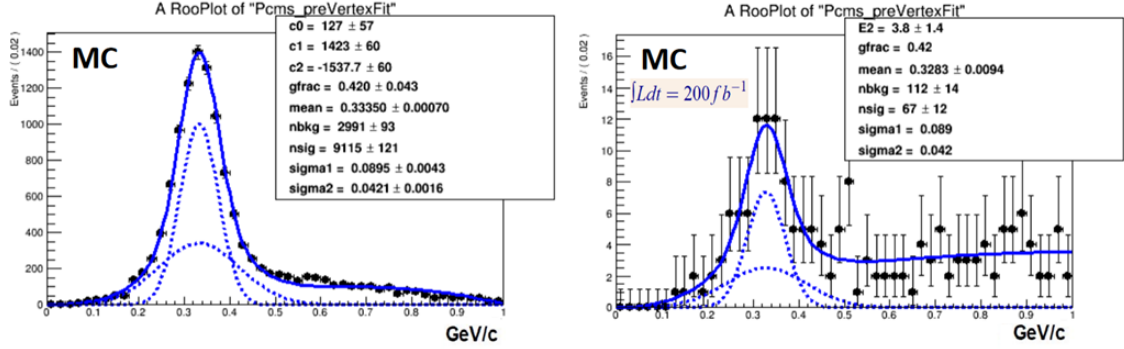


Figure 5.11: P_B^* distributions of the reconstructed meson. Left: Pure signal MC sample; Right: Combined sample of signal MC and generic MC background, scaled to an integrated luminosity of $\int L dt = 200 \text{ fb}^{-1}$.

5.9 Discussion

The observed agreement between expected and fitted yields supports the feasibility of observing this decay mode with Belle II data. The successful reconstruction of $\bar{n}$ using calorimeter-based PID is a promising step toward future studies of baryonic B decays involving neutral final-state particles.

The baryonic decay mode $B^- \rightarrow J/\psi \Lambda^0 \bar{p}$ has a similar Feynman diagram to $\bar{B}^0 \rightarrow J/\psi \Lambda^0 \bar{n}$, and its branching fraction is $(1.46 \pm 0.12) \times 10^{-5}$ according to the PDG. According to the luminosity information for Run 1, Belle II accumulated an integrated luminosity of approximately $\int L dt = 363 \text{ fb}^{-1}$ by the end of 2023. Assuming that the decay $\bar{B}^0 \rightarrow J/\psi \Lambda^0 \bar{n}$ has the same branching fraction, the expected signal yield under the Run 1 dataset is estimated to be approximately 19.20 events. As the integrated luminosity continues to increase in future runs, the expected yield will scale proportionally. This indicates strong potential for future studies of this decay mode using the anti-neutron PID method.

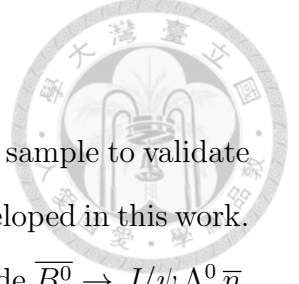
5.10 Summary

We study the baryonic B decay mode $B^- \rightarrow J/\psi \Lambda^0 \bar{p}$ as a control sample to validate the reconstruction and MVA-based anti-neutron PID method developed in this work. This channel shares similar kinematic features with the signal mode $\bar{B}^0 \rightarrow J/\psi \Lambda^0 \bar{n}$, but offers the advantage of a reconstructable charged anti-proton.

Due to the intrinsically low momentum of the $\bar{p}$ in this three-body baryonic decay, the ECL cluster matching efficiency is limited, and applying the anti-neutron PID requirement (`nbarID` > 0.5) drastically reduces the remaining signal to 12.98%. To mitigate this loss, we implement a modified ECL matching scheme that considers clusters with all available hypotheses and prioritizes those with the `neutralHadron` tag. This improvement nearly doubles the signal retention.

Nevertheless, the challenges associated with low-momentum particle reconstruction remain, and systematic uncertainties in the decay $\bar{B}^0 \rightarrow J/\psi \Lambda^0 \bar{n}$ continue to be difficult to evaluate. The validation using the B^- control sample demonstrates the effectiveness of the reconstruction strategy, but also highlights the intrinsic limitations of anti-baryon detection in such kinematic regimes.

We investigate the baryonic decay mode $\bar{B}^0 \rightarrow J/\psi (J/\psi \rightarrow e^+e^-, \mu^+\mu^-) \Lambda^0 (\Lambda^0 \rightarrow p^+\pi^-) \bar{n}$ using a MC simulation to apply an MVA-based anti-neutron PID method developed in our collaboration. The comparison between the expected yield and the fitting yield shows good agreement, demonstrating the effectiveness of the anti-neutron PID method. Based on Run 1 luminosity and assuming a branching fraction similar to $B^- \rightarrow J/\psi \Lambda^0 \bar{p}$, the expected yield is about 19.20 events. This suggests high potential for observing this mode with current and future Belle II data.





Chapter 6

Conclusions

The 2D tracking system, built upon an inherited framework, was successfully validated with both cosmic-ray and beam data. It served as a reliable base for subsequent developments, including the 3D tracker and high-rate upgrades within the L1 trigger system.

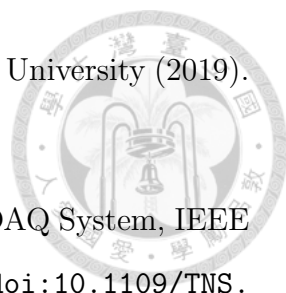
The 3D tracking system was developed and validated using both geo-finder and the improved LUT-finder, achieving better efficiency and background rejection. The system underwent thorough testing with simulation and beam data, providing a solid foundation for further optimization in the Belle II experiment.

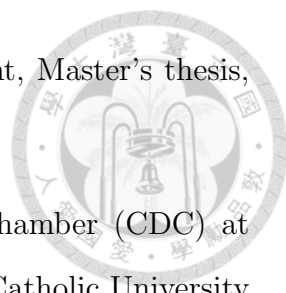
The anti-neutron PID methodology based on MVA was successfully validated and demonstrated to be applicable to the decay $\overline{B}^0 \rightarrow J/\psi \Lambda^0 \overline{n}$ using MC simulation. The good agreement between expected and fitted signal yields confirms both the reliability of the method and its effectiveness in this physics analysis, highlighting the potential of this decay channel for future studies in the Belle II experiment.



Bibliography

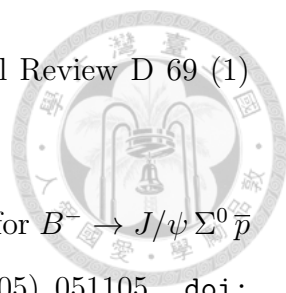
- [1] T. Abe et al., Belle II Technical Design Report (Nov. 2010). [arXiv:1011.0352](#),
[doi:10.48550/arXiv.1011.0352](#).
- [2] M. Yamauchi, Super KEKB, a high luminosity upgrade of KEKB, Nuclear Physics B - Proceedings Supplements 111 (1-3) (2002) 96–105. [doi:10.1016/S0920-5632\(02\)01690-0](#).
- [3] P. Lewis et al., First measurements of beam backgrounds at SuperKEKB, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 914 (2019) 69–144. [doi:10.1016/j.nima.2018.05.071](#).
- [4] N. Taniguchi, Central Drift Chamber for Belle-II, Journal of Instrumentation 12 (06) (2017) C06014–C06014. [doi:10.1088/1748-0221/12/06/C06014](#).
- [5] T. Uchida et al., Readout Electronics for the Central Drift Chamber of the Belle II Detector, in: 2011 IEEE Nuclear Science Symposium Conference Record, IEEE, Valencia, Spain, 2011, pp. 694–698. [doi:10.1109/NSSMIC.2011.6154084](#).
- [6] N. Taniguchi et al., All-in-one readout electronics for the Belle-II Central Drift Chamber, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 732 (2013) 540–542. [doi:10.1016/j.nima.2013.06.096](#).
- [7] Chia-Te Chen, The Energy Correction of Belle II ECL detector by Using Con-

- 
- volutional Neural Network, Master's thesis, National Taiwan University (2019).
doi:10.6342/NTU201900280.
- [8] R. Itoh et al., Data Flow and High Level Trigger of Belle II DAQ System, IEEE Transactions on Nuclear Science 60 (5) (2013) 3720–3724. doi:10.1109/TNS.2013.2273091.
- [9] Y.-T. Lai et al., Development of the Level-1 track trigger with Central Drift Chamber detector in Belle II experiment and its performance in SuperKEKB 2019 Phase 3 operation, Journal of Instrumentation 15 (06) (2020) C06063–C06063. doi:10.1088/1748-0221/15/06/C06063.
- [10] Y.-T. Lai et al., Design of the Global Reconstruction Logic in the Belle II Level-1 Trigger system, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 1078 (2025) 170577. doi:10.1016/j.nima.2025.170577.
- [11] D.-W. Lan, The Stability Study of High Speed Data Transmission on High Energy Experiments Based on FPGA, Master's thesis, National United University (2019).
- [12] Yun-Tsung Lai, Search for $D^0 \rightarrow \nu \bar{\nu}$ at Belle, and Belle II CDCTRG system firmware design., Ph.D. thesis, National Taiwan University (Feb. 2017). doi:10.6342/NTU201601179.
- [13] K.-T. Kim, Development of Track Segment Finder for Central Drift Chamber Based Level-1 Trigger System in the Belle II Experiment, Ph.D. thesis, Korea University (2021).
- [14] Tzu-An Sheng, Implementing the 2D track reconstruction for the Level 1 trigger of the Belle II experiment, Master's thesis, National Taiwan University (2018). doi:10.6342/NTU201802022.
- [15] S. hyung Lee, Development of a real time framework for the Belle II at the SuperKEKB, Master's thesis, Korea University (2010).

- 
- [16] C.-H. Huang, CDC 2-D Track finding at Belle-II experiment, Master's thesis, Fu Jen Catholic University (2011).
 - [17] H.-P. Chiu, 2D Track Reconstruction for Central Drift Chamber (CDC) at Belle-II : VHDL Firmware Design, Master's thesis, Fu Jen Catholic University (2013).
 - [18] Y.-F. Kuo, 2D Track Reconstruction for Central Drift Chamber (CDC) at Belle II: TSIM Design, Master's thesis, Fu Jen Catholic University (2014).
 - [19] K.-Y. Chen, Updated 2D Tracker TSIM Design for Central Drift Chamber at Belle-II, Master's thesis, Fu Jen Catholic University (2015).
 - [20] Z.-X. Chen, Updated 2D Tracker Firmware Design for Central Drift Chamber at Belle-II, Master's thesis, Fu Jen Catholic University (2016).
 - [21] S. Pohl, Track Reconstruction at the First Level Trigger of the Belle II Experiment, Ph.D. thesis, Ludwig-Maximilians-Universität München (Apr. 2018). doi:10.5282/edoc.22085.
 - [22] Ping Ni, Upgrade of Two Dimensional Track Trigger on Central Drift Chamber aimed for Belle II Targeted Luminosity, Master's thesis, The University of Tokyo (Aug. 2022).
 - [23] J. bak Kim, A 3-dimensional fast fitter for central drift chamber based level 1 trigger system in the Belle II experiment, Master's thesis, Korea University (2012).
 - [24] E. Won et al., Three-dimensional fast tracker for the central drift chamber based level-1 trigger system in the Belle II experiment, Journal of the Korean Physical Society 72 (1) (2018) 33–37. doi:10.3938/jkps.72.33.
 - [25] J. Kim et al., A software framework for pipelined arithmetic algorithms in field programmable gate arrays, Nuclear Instruments and Methods in Physics Re-

search Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 883 (2018) 83–89. doi:10.1016/j.nima.2017.11.064.

- [26] Hiroto Sudo, Development of the Three-Dimensional Track Reconstruction Algorithm for the Belle II Level-1 Trigger, Master's thesis (Jan. 2024).
- [27] J.-N. Chang, Trigger Efficiency Logic Study on the Belle II Experiment, Master's thesis, Korea University (2024).
- [28] Pei-Cheng Lu, $B^+ \rightarrow p \bar{\Lambda} K^+ K^-$ and $\bar{p} \Lambda K^+ K^+$, Ph.D. thesis, National Taiwan University (2020). doi:10.6342/NTU202000406.
- [29] Kuan-Po Lin, Monte Carlo Study of the Decay $B^+ \rightarrow p(1520)$ at Belle, Master's thesis, National Taiwan University (2017). doi:10.6342/NTU201700568.
- [30] Y.-H. Hsu, The Study of $B^0 \rightarrow J/\psi \eta$ in the Belle II Experiment at KEK, Master's thesis, Fu Jen Catholic University (2021).
- [31] Wei-Che Lin, MC study for $B^0 \rightarrow \phi K_L^0$ decay at Belle II, Master's thesis, National Taiwan University (2025). doi:10.6342/NTU202500482.
- [32] Jheng-Hao Su, The study of K_L^0 identification with ECL information at the Belle II experiment, Master's thesis, National Taiwan University (2023). doi:10.6342/NTU202301289.
- [33] Yuan-Ru Lin, The Study of Antineutron Identification at Belle Experiment, Master's thesis, National Taiwan University (2021). doi:10.6342/NTU202103402.
- [34] Wei-Ling Yeh, The Study of proton identification At Belle, Master's thesis, National Taiwan University (2009). doi:10.6342/NTU.2009.01730.
- [35] B. Aubert et al., Evidence for $B^+ \rightarrow J/\psi p \bar{\Lambda}$ and search for $B^0 \rightarrow J/\psi p \bar{p}$, Physical Review Letters 90 (23) (2003) 231801. doi:10.1103/PhysRevLett.90.231801.

- 
- [36] S. L. Zang et al., Search for $B^- \rightarrow J/\psi \Lambda \bar{p}$ decay, Physical Review D 69 (1) (2004) 017101. doi:10.1103/PhysRevD.69.017101.
- [37] Q. L. Xie et al., Observation of $B^- \rightarrow J/\psi \Lambda \bar{p}$ and searches for $B^- \rightarrow J/\psi \Sigma^0 \bar{p}$ and $B^0 \rightarrow J/\psi p \bar{p}$ decays, Physical Review D 72 (5) (2005) 051105. doi:10.1103/PhysRevD.72.051105.
- [38] The CMS collaboration et al., Study of the $B^+ \rightarrow J/\psi \bar{\Lambda} p$ decay in proton–proton collisions at $\sqrt{s} = 8$ TeV, Journal of High Energy Physics 2019 (12) (2019) 100. doi:10.1007/JHEP12(2019)100.



Appendix A

Track Trajectory Parameterization

A helix is described by 5 parameters, computed at the POCA of the track to the origin of the coordinate system:

- $d_0 \in [-\infty, +\infty]$: the signed distance of the POCA on the transverse plane. The sign is the same as the z component of the angular momentum with respect to the origin;
- $z_0 \in [-\infty, +\infty]$: the z coordinate of the POCA;
- $\phi_0 \in [-\pi, +\pi]$: the angle defined by the transverse momentum at the POCA and the x axis;
- $\omega \in [-\infty, +\infty]$: the inverse of the curvature radius, signed with the assumed charge of the particle;
- $\tan \lambda \in [-\infty, +\infty]$: the tangent of the angle defined by the momentum at the POCA and the transverse xy -plane.

Fig. A.1 illustrates the geometrical interpretation of the five helix parameters.

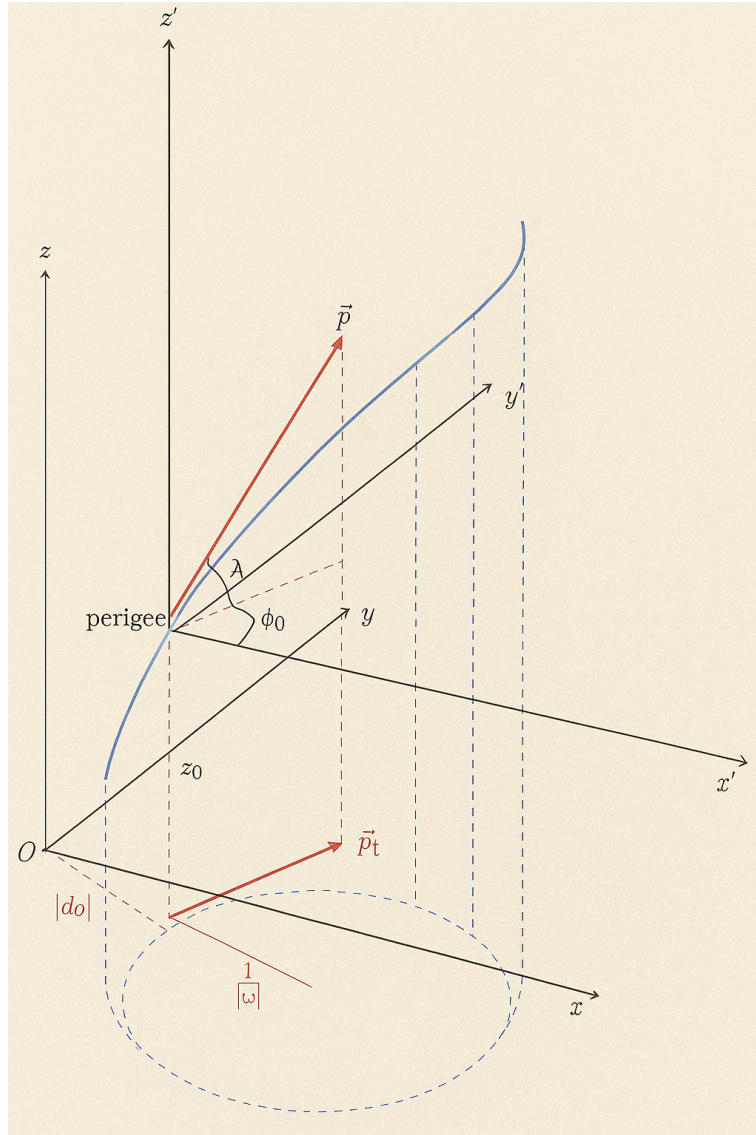


Figure A.1: Geometrical interpretation of the five helix parameters used to describe a charged particle's trajectory in a uniform magnetic field (adapted from Ref. [14]). The helix is defined with respect to the POCA to the origin in the x - y plane. The parameters d_0 , ϕ_0 , ω , z_0 , $\tan \lambda$ uniquely determine the 3D path of the particle.



Appendix B

Performance of 3D

This appendix evaluates the performance of the 3D tracking algorithm developed for the L1 trigger. The study includes efficiency, resolution, and residual distributions obtained from both simulation and data.

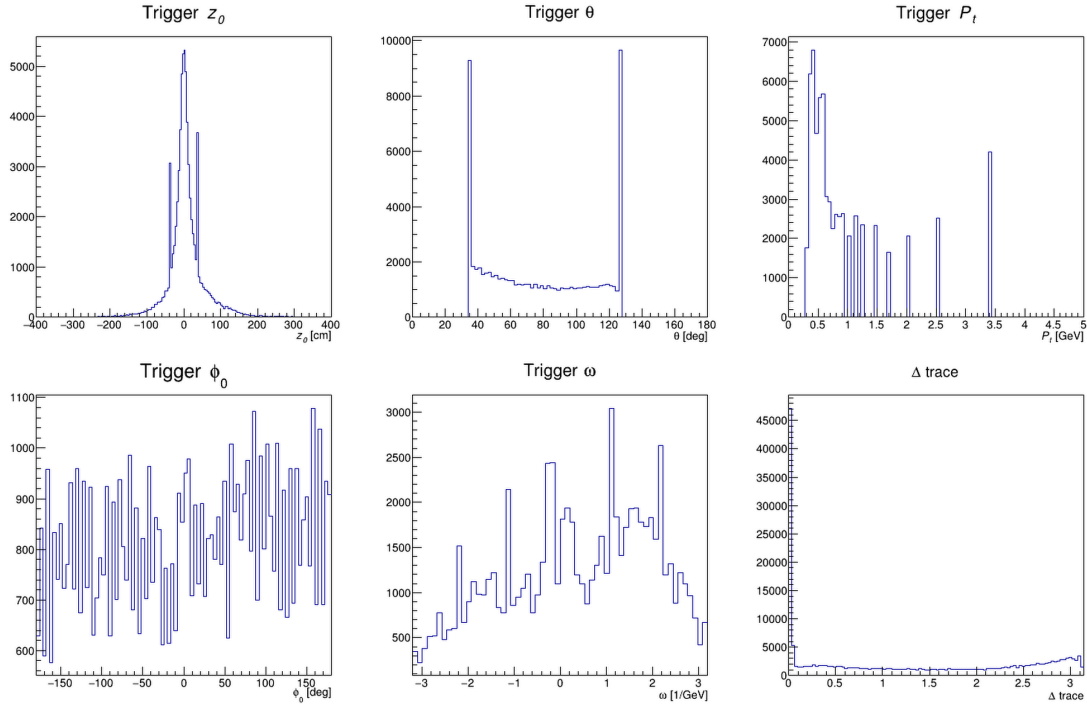


Figure B.1: Distributions of trigger tracks (w/o selections & matching), run #883 of experiment 18 (geo-finder).

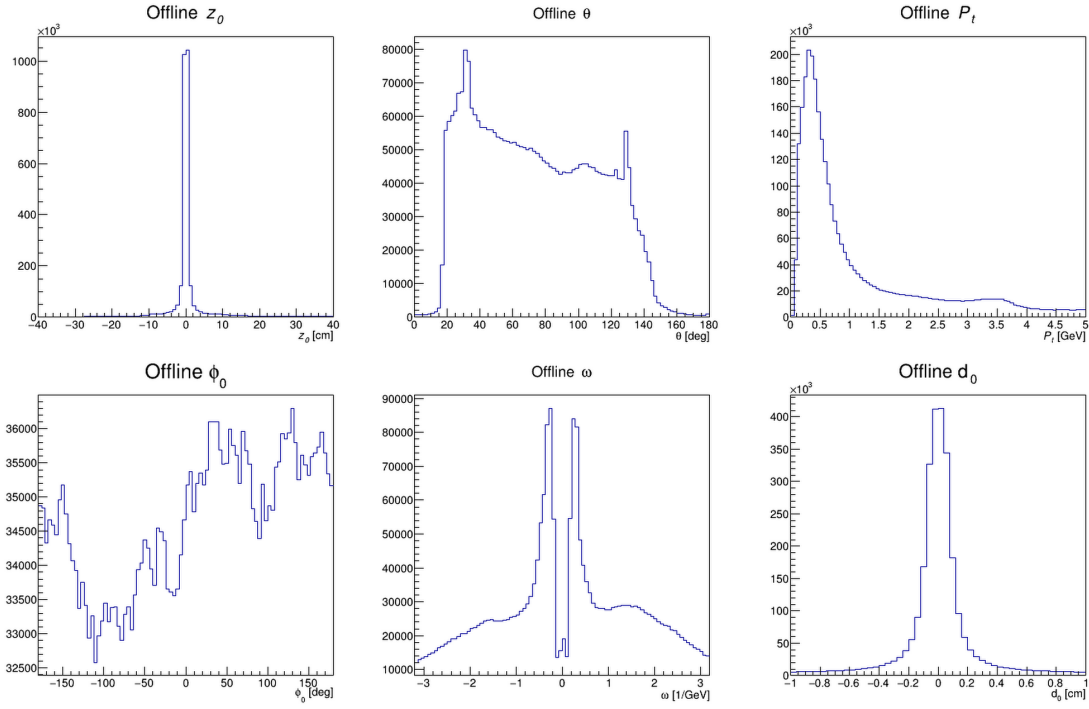


Figure B.2: Distributions of offline-reco tracks (w/o selections & matching), run #883 of experiment 18 (geo-finder).

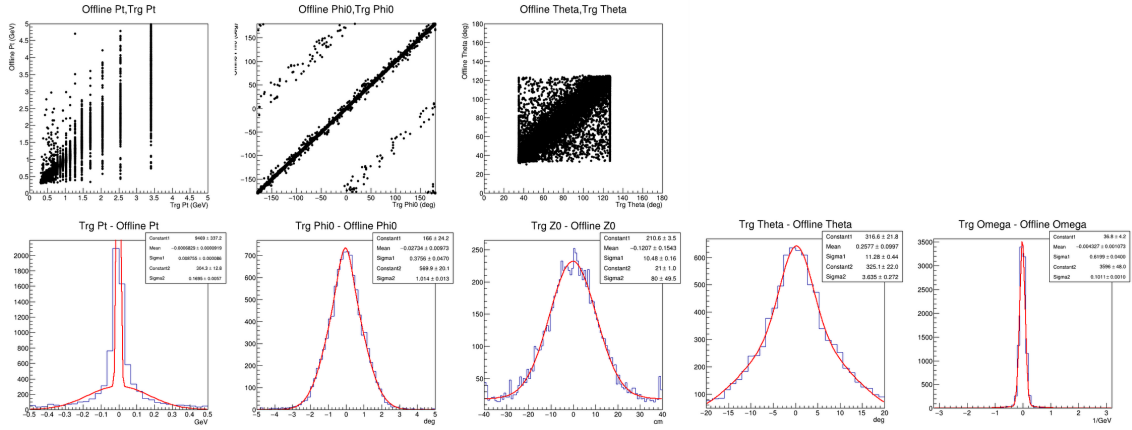


Figure B.3: Trigger tracks vs. offline-reco tracks (w/ selections & matching), run #883 of experiment 18 (geo-finder).

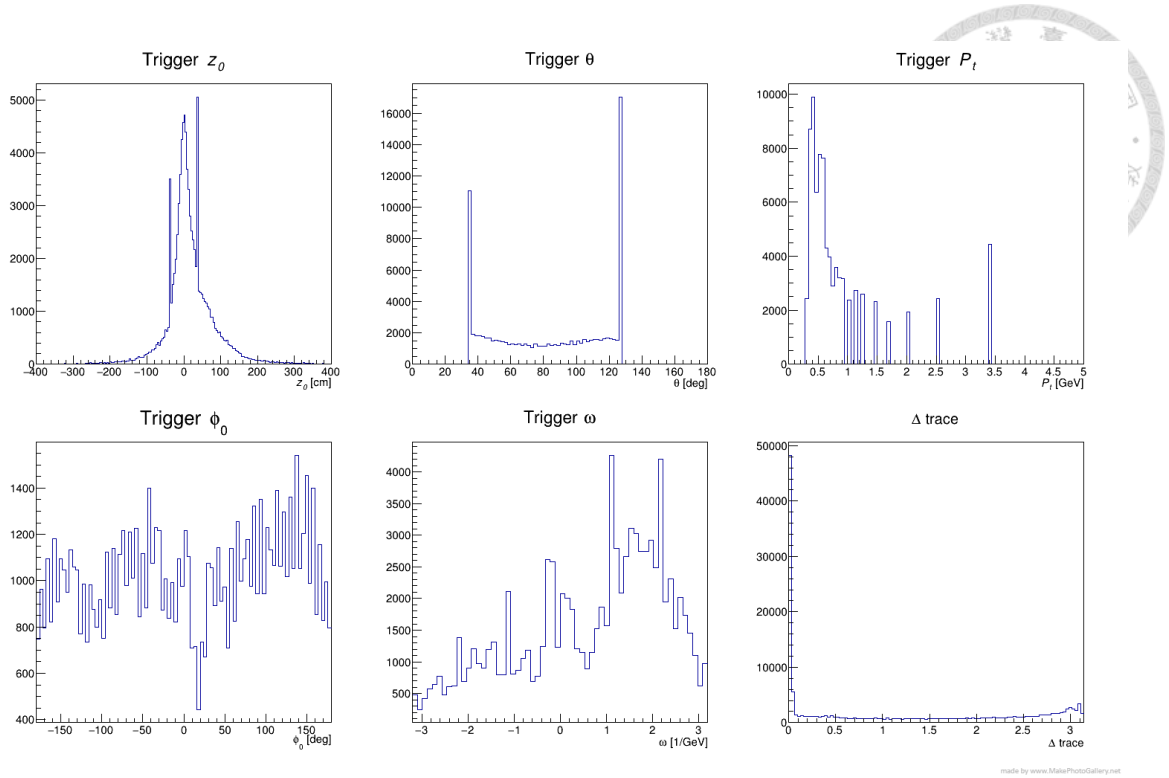


Figure B.4: Distributions of trigger tracks (w/o selections & matching), run #1434 of experiment 18 (geo-finder, w/o HLT filtering).

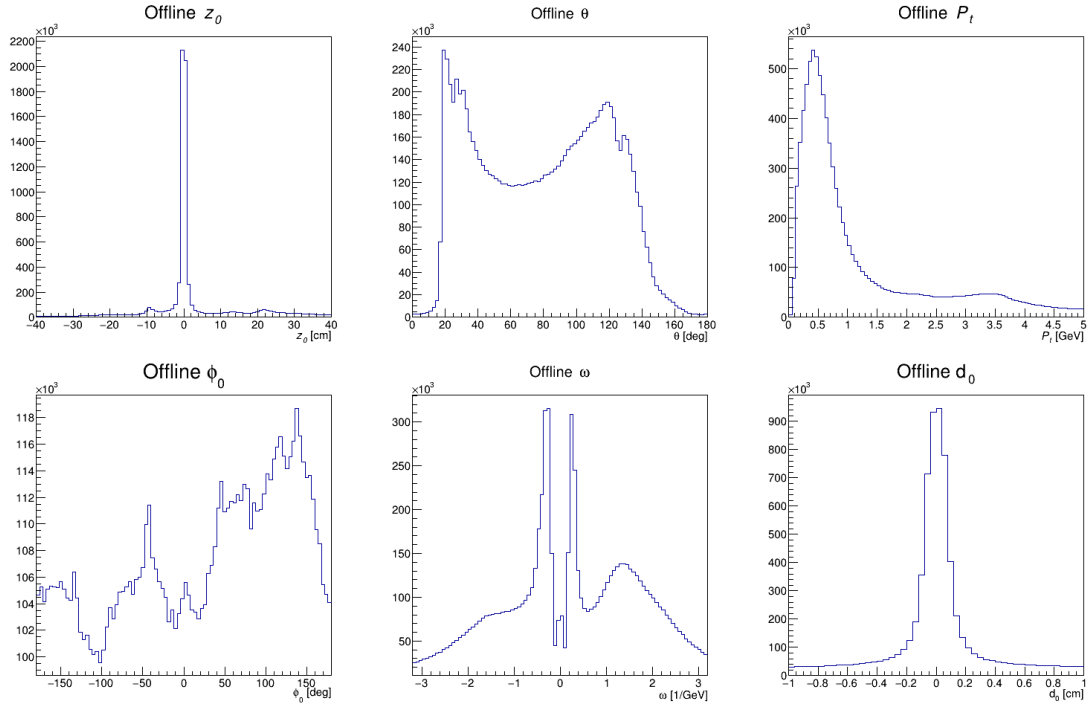


Figure B.5: Distributions of offline-reco tracks (w/o selections & matching), run #1434 of experiment 18 (geo-finder, w/o HLT filtering).

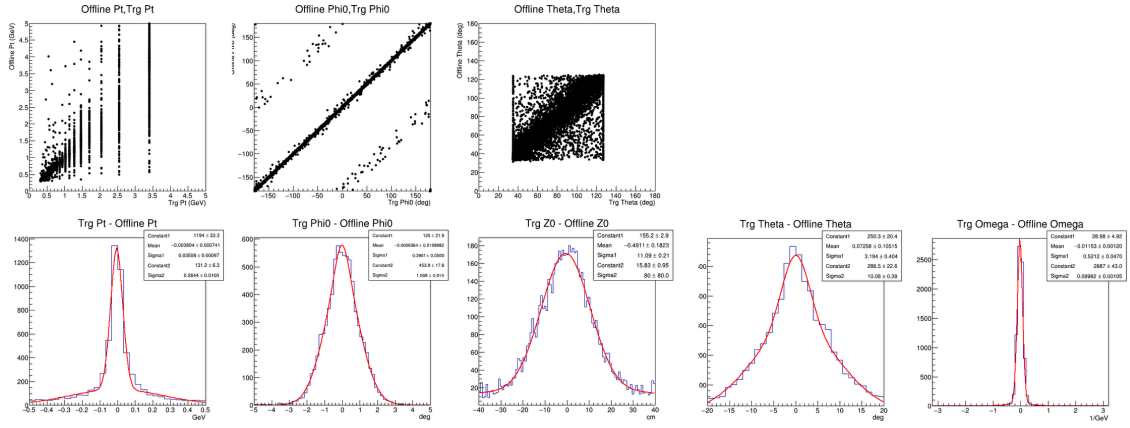


Figure B.6: Trigger tracks vs. offline-reco tracks (w/ selections & matching), run #1434 of experiment 18 (geo-finder, w/o HLT filtering).

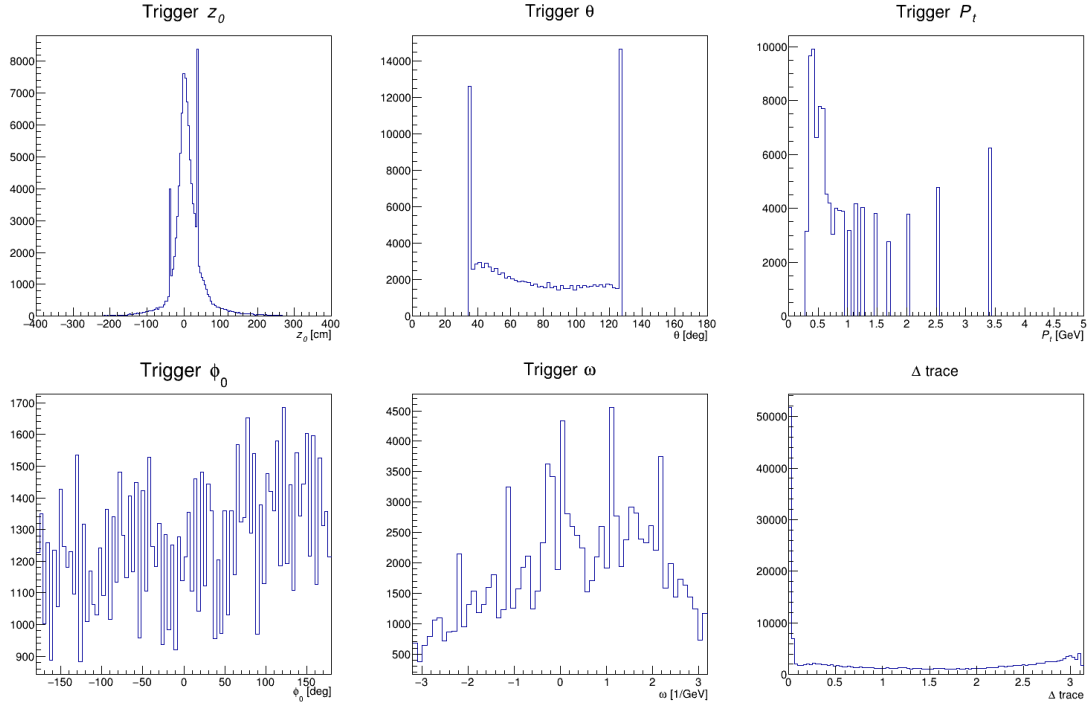


Figure B.7: Distributions of trigger tracks (w/o selections & matching), run #1811 of experiment 18 (LUT-finder).

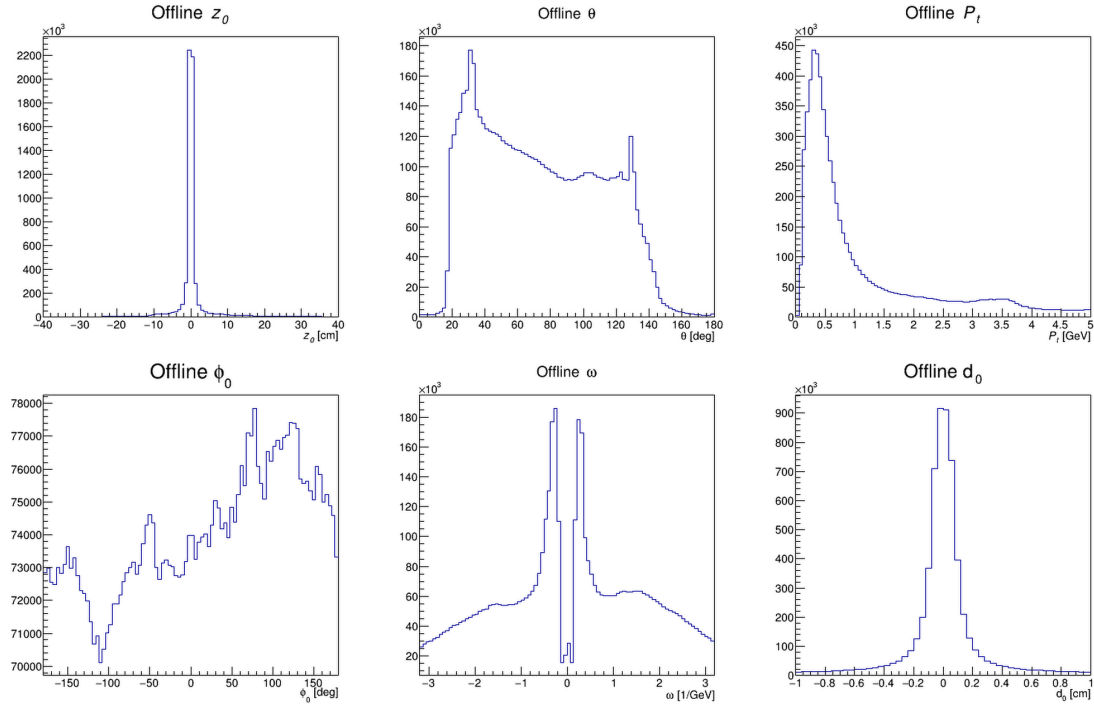


Figure B.8: Distributions of offline-reco tracks (w/o selections & matching), run #1811 of experiment 18 (LUT-finder).

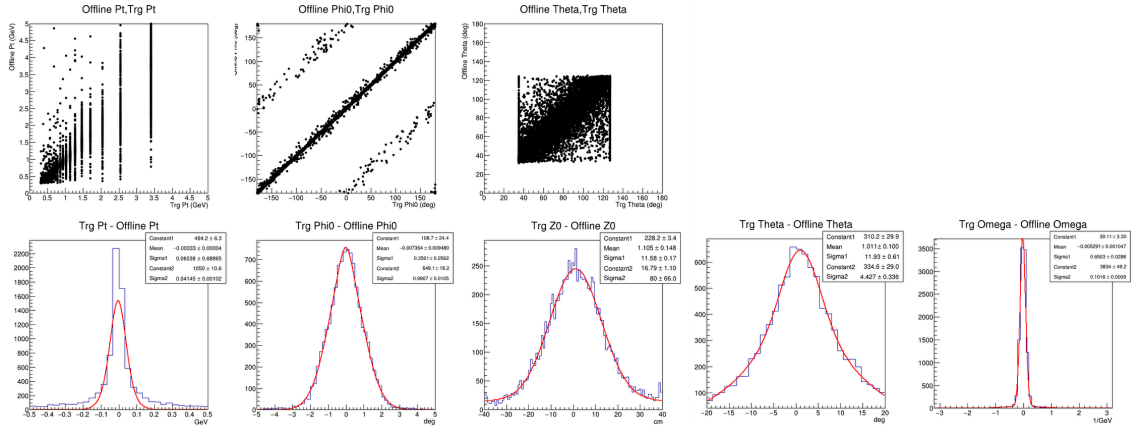


Figure B.9: Trigger tracks vs. offline-reco tracks (w/ selections & matching), run #1811 of experiment 18 (LUT-finder).

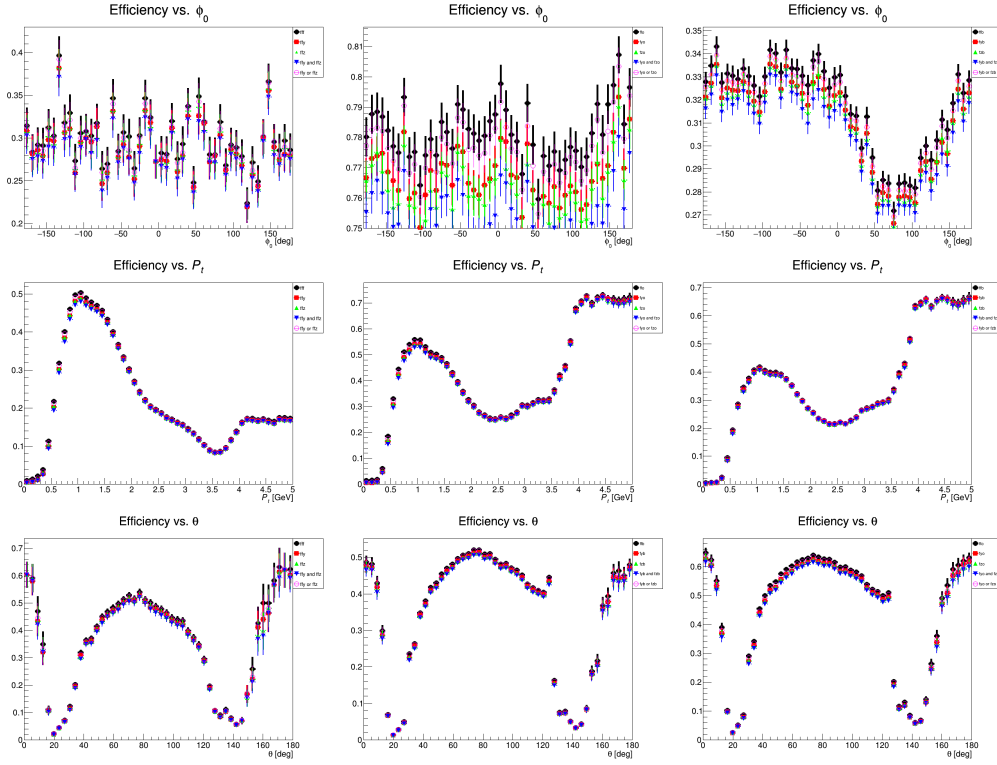


Figure B.10: Dependence of the bit efficiency on offline-reco ϕ_0 , p_T , and θ (w/o selections & matching), run #883 of experiment 18 (geo-finder).

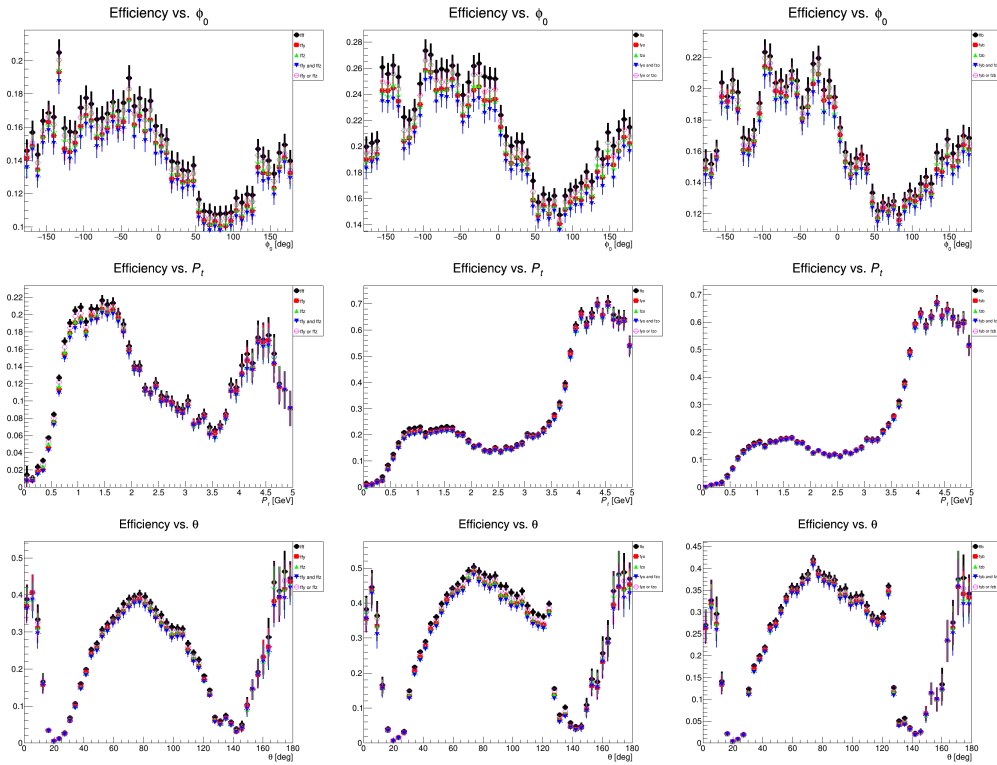


Figure B.11: Dependence of the bit efficiency on offline-reco ϕ_0 , p_T , and θ (w/o selections & matching), run #1434 of experiment 18 (geo-finder, w/o HLT filtering).

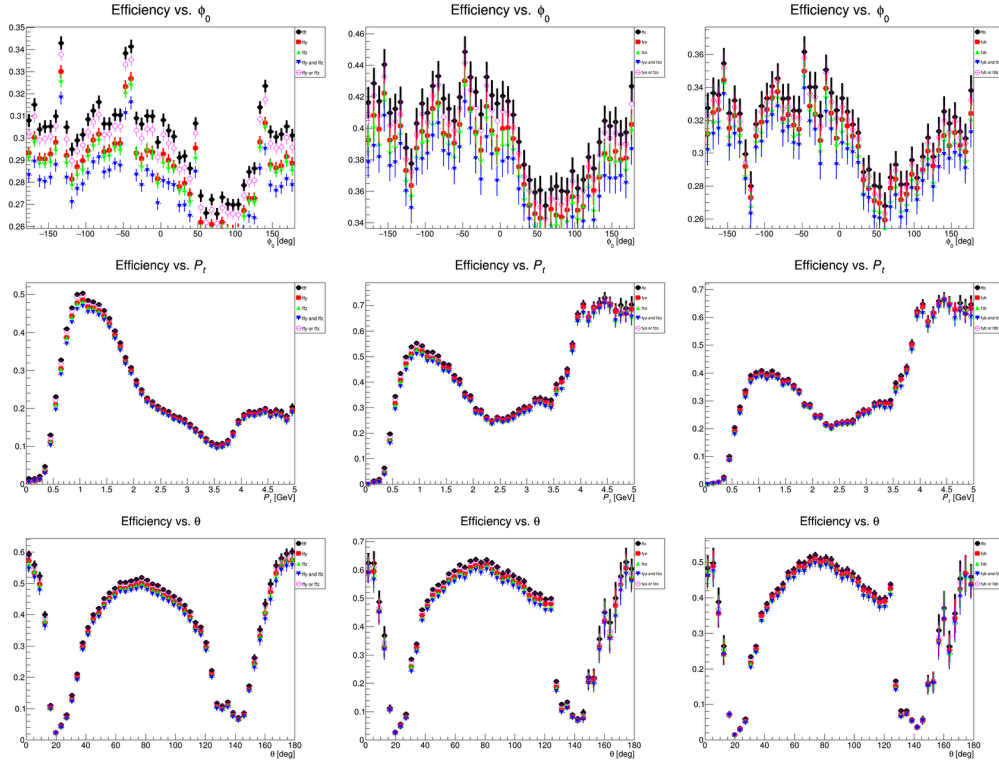


Figure B.12: Dependence of the bit efficiency on offline-reco ϕ_0 , p_T , and θ (w/o selections & matching), run #1811 of experiment 18 (LUT-finder).