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自發性 CP 破缺相角與 CKM 矩陣相角比較之研究

Spontaneous CP Violating Phase and CKM Matrix Phase

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口試委員會審定書

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本論文係蔡律行君（R95222007）在國立臺灣大學物理學所完成之碩士學位論文，於民國 97 年 7 月 4 日承下列考試委員審查通過及口試及格，特此證明

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陳泉宏

誌謝

很高興能夠完成這篇論文，在此要特別感謝我的指導老師，何小剛老師兩年來的教導，讓我對粒子物理有初步而扎實的認識，也謝謝老師在這段時間不辭辛勞的督促與幫助，論文才得以完成。這裡也要感謝紹龍學長、和進學長，以及中承，常常為我解開疑惑，給我很大的幫助。最後也要感謝父母與兄長們時時刻刻的關心與鼓勵，給我很大的信心與激勵來完成論文。

摘要

在這篇論文我們研究自發性CP破缺相角與Cabibbo-Kobayashi-Maskawa(CKM)矩陣相角的連結。我們先介紹在標準模型中的CP破缺，以及自發性CP破缺，然後提出一類新的模型來連結在CKM混合矩陣中的CP破缺相角和由Higgs位能中自發性CP破缺所產生的CP相角。建立一個多重Higgs伴隨Peccei-Quinn(PQ)對稱的模型來實現這個概念。這個模型有一些有趣的現象學含義。當所有Higgs的質量變大時CP破缺相角並不會消失。一般來說，在樹狀層級時會有中性Higgs傳遞的味變化中性流(FCNC)交互作用。然而，跟一般多重Higgs模型不同的是，FCNC的Yuwaka耦合會被夸克的質量和CKM混合角所確定。我們也詳細研究了關於中性介子的混合與中子的電偶極矩實驗數據所衍生的涵義。

關鍵詞： CKM矩陣相角，自發性CP破缺相角，多重Higgs模型，Yukawa耦合，Peccei-Quinn對稱

ABSTRACT

In this thesis we study the connection between the spontaneous CP violating phase and the Cabibbo-Kobayashi-Maskawa (CKM) matrix phase. At first an introduction to CP violation in the Standard Model is presented, following by the spontaneous CP violation, and then a new class of models is proposed to connect the CP violating phase in the CKM mixing matrix with the CP phases responsible for the spontaneous CP violation in the Higgs potential. A multi-Higgs model with Peccei-Quinn(PQ) symmetry is constructed to realize this idea. This model has some interesting phenomenological implications. The CP violating phase does not vanish when all Higgs masses become large. In general, there are flavor changing neutral current (FCNC) interactions mediated by neutral Higgs bosons at the tree level. Unlike the general multi-Higgs models, however, the FCNC Yukawa couplings are fixed in terms of the quark masses and CKM mixing angles. Implications from experimental data for neutral meson mixing and the neutron electric dipole moment are well-studied.

Keywords: CKM matrix phase, spontaneous CP violating phase, multi-Higgs model, Yukawa couplings, Peccei-Quinn symmetry

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1. INTRODUCTION

1.1 Standard Model

Physicists apply symmetry groups to find the Lagrangians which are associated the fundamental forces. Some symmetries are continuous, such as the standard model gauge group $\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$. Some symmetries are discrete, such as parity P, charge conjugate C, and time reversal T symmetries. Not all symmetries which are important in nature are exact. Broken symmetries are also important. The combination of charge conjugate C and parity P symmetry CP is such a symmetry. In this thesis, we study [1] a possible mechanism for the origin of the broken CP symmetry by making a connection between the CP violating phase in the CKM model and spontaneous CP violating phase in the Higgs potential.

The foundation of standard model is the quark model proposed in 1962 by M. Gell-Mann [2], who suggested the SU(3) as the symmetry to describe mesons and baryons. The concept of quarks is fundamental building block of hadronic matter. Later S. L. Glashow [3], S. Weinberg [4] and A. Salam [5] unified the electromagnetic force and weak force into a gauge symmetry with $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$, and this theory has so far been verified by many experiments. Including the gauge symmetry $\mathrm{SU(3)_C}$ describing color face of quarks, the Standard Model(SM) gauge group is $\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$. Including the leptons and Higgs bosons, the SM particle contents are:

$$\begin{array}{lll} G:(8,1,0) & W:(1,3,0) & B:(1,1,0) \\ L_L:(1,2,-1) & e_R:(1,1,-2) & \\ Q_L:(3,2,1/3) & u_R:(3,1,4/3) & d_R:(3,1,-2/3) \\ H:(1,2,-1) & & \end{array} . \tag{1.1}$$

where G, W, and B are gauge fields corresponding to $\mathrm{SU(3)_C}$, $\mathrm{SU(2)_L}$, and $\mathrm{U(1)_Y}$. The subscript L, R means left handed and right handed particles. The left handed quarks Q_L and leptons L_L are $\mathrm{SU(2)_L}$ doublets, which means $Q_L = (u_L, d_L)$ and $L_L = (\nu_L, e_L)$; The right handed up-type quarks u_R, down-type quarks d_R, and electron e_R are $\mathrm{SU(2)_L}$ singlets. Note that in standard model there are no right handed neutrino ν_R. The Higgs is a $\mathrm{SU(2)_L}$ doublet with $H = (h^0, h^-)$. Here we have not included right handed neutrino ν_R which may be needed. We will treat it when we discuss our model.

We construct the Lagrangian by terms with that the corresponding dimensions are $(1, 1, 0)$. The renormalizable $\mathcal{L}$ is given by

$$\begin{aligned}\mathcal{L} = & - \frac{1}{2}\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}) - \frac{1}{2}\mathrm{Tr}(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ & + \overline{Q}_L i\gamma^\mu D_\mu Q_L + \bar{u}_R i\gamma^\mu D_\mu u_R + \bar{d}_R i\gamma^\mu D_\mu d_R + \overline{L}_L i\gamma^\mu D_\mu L_L + \bar{e}_R i\gamma^\mu D_\mu e_R \\ & + (D_\mu H)^\dagger (D^\mu H) + (\overline{Q}_L H u_R + \overline{Q}_L \widetilde{H} d_R + \overline{L}_L \widetilde{H} e_R + \mathrm{h.c.}) - V(H), \end{aligned} \tag{1.2}$$

where the h.c. means the Hermitian conjugate of the terms in bracket. $\widetilde{H} = -i\sigma_2 H^*$, $V(H)$ is a function of H which is written as

$$V(H) = \mu^2(H^\dagger H) + \lambda(H^\dagger H)^2, \tag{1.3}$$

with μ and λ are coefficients. D_μ is the covariant derivative and has the form

$$D_\mu = \partial_\mu + ig_s\frac{\lambda^a}{2}G^a_\mu + ig\frac{\sigma^b}{2}W^b_\mu + ig'\frac{Y}{2}B_\mu. \ a = 1 \sim 8, \ b = 1 \sim 3; \tag{1.4}$$

where g_s, g, g' are coupling constants of $\mathrm{SU(3)_C}$, $\mathrm{SU(2)_L}$, and $\mathrm{U(1)_Y}$ respectively. Y is the hypercharge of a particle which is just the dimension of representation corresponding to the gauge group $U(1)_Y$. σ^b's are the Pauli matrices and λ^a's are the Gell-Mann matrices.

Note that the term $ig(\sigma^a/2)W^a_\mu$ in D_μ only acts on particles with two-dimensional gauge group $\mathrm{SU(2)_L}$, and $ig_s(\lambda^a/2)G^a_\mu$ acts on quark sectors which have threedimensional SU(3)$_C$. The quantities $G_{\mu\nu}$, $W_{\mu\nu}$ are defined below

$$\begin{aligned}
G_{\mu\nu} &= (\partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu)\frac{\lambda^a}{2} \\
W_{\mu\nu} &= (\partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon^{abc} W^b_\mu W^c_\nu)\frac{\sigma^a}{2} \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu,
\end{aligned} \tag{1.5}$$

where ϵ^{abc} is the totally antisymmetric tensor, and f^{abc} is the structure constant for SU(3).

Now considering the Higgs potential it is given by Eq.(1.3). If one require that the potential does not go to negative infinity and this potential has minima at non-zero H, then the inequalities $\lambda > 0$ and $\mu^2 < 0$ must be satisfied. Assume H has the vacuum expectation value(VEV) $< H >$ as

$$H = \begin{pmatrix} h^0 \\ h^- \end{pmatrix} \rightarrow < H >= \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}, \tag{1.6}$$

Doing differential with v for $V(H)$ at $< H >= v$, we find the minimal condition

$$\left.\frac{\partial V}{\partial h^0}\right|_{h^0=v} = \mu^2 v + \lambda v^3 = 0. \tag{1.7}$$

This condition gives $v = (-\mu^2/\lambda)^{1/2}$, and one can use this expression to replace the variable μ by v.

After spontaneous symmetry breaking, the Higgs doublet becomes

$$H = \begin{pmatrix} \frac{1}{\sqrt{2}}(v + h + ia) \\ h^- \end{pmatrix}, \tag{1.8}$$

where the h is the real part of the neutral Higgs, which is parity even; a is the imaginary part of the neural Higgs, which is pseudoscalar with odd parity. From the mass matrix it is well-known that the three Higgs particles h^+, h^-, and a become massless Goldstone bosons, which are eaten by W^+, W^-, and Z^0, respectively.

Non-zero vacuum expectation value breaks the original electroweak gauge symmetry $\mathrm{SU_L(2)} \times \mathrm{U(1)_Y}$ into $\mathrm{U(1)_{EM}}$, which is the gauge symmetry in the electrodynamics.

The mass of Higgs can be obtained from the Higgs potential after the spontaneous breaking of symmetry, and then the related Lagrangian is written as

$$\mathcal{L} = -\lambda v^2 h^2 - \lambda v h^3 - \frac{\lambda}{4} h^4. \tag{1.9}$$

The first term implies that $m_h^2 = 2\lambda v^2$. The second and third terms are related to three Higgs interaction and four Higgs interactions respectively.

The term $(D_\mu H)^\dagger (D^\mu H)$ produces the masses of gauge fields. H is doublet under the $\mathrm{SU(2)_L}$ gauge transform and thus the covariant derivative is $D_\mu = \partial_\mu + ig(\sigma^b/2)W_\mu^b + ig'(-1/2)B_\mu$. After pulling out the mass matrix of gauge boson $\mathbf{W}$ and B, we find that there are mixing terms between the states W^3 and B. making the following rotation transformation from W^3 and B to new field Z and A,

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \quad \tan\theta_W = \frac{g'}{g}, \tag{1.10}$$

where θ_W is the Weinberg angle, the mixing angle between W^3 and B. $\sin\theta_W$ is an important physical quantity related to the validity of electroweak theory.

The gauge field Z is the well-known Z, which has mass $M_Z = \frac{1}{2}\sqrt{g+g'}v$ which boson. The massless field A is the photon. The mass of Z boson can be seen as from the neutral Higgs imaginary part, i.e. the a field. For the W^1 and W^2 combine to form $W^\pm$, $W^\pm$ has the same mass

$$W_\mu^+ = \frac{W_\mu^1 - iW_\mu^2}{\sqrt{2}}; \; W_\mu^- = \frac{W_\mu^1 + iW_\mu^2}{\sqrt{2}}; \;\; M_{W^\pm} = \frac{1}{2} g v; \tag{1.11}$$

$W^\pm$ field are the our known W boson whose mass is nonzero by absorbing the mass of charge Higgs. From the Z boson mass and Eq.(1.11) there is an important relation between masses of W and Z bosons at the tree level, which is

$$\frac{M_W}{M_Z} = \cos\theta_W. \tag{1.12}$$

The Lagrangian of fermionic kinetic energy terms gives the fermion-gauge couplings because of the covariant derivative D_μ. The Lagrangian consists of photon, W and Z, and the gluon interaction with quarks and leptons, and the form is

$$\begin{aligned}\mathcal{L} = & - g_s(\bar{u}\gamma^{\mu}\frac{\lambda^a}{2}u + \bar{d}\gamma^{\mu}\frac{\lambda^a}{2}d)G^a_{\mu} - e\sum_i Q_i\bar{f}_i\gamma^{\mu}f_iA_{\mu} \\ & - \frac{g}{\sqrt{2}}(\bar{u}\gamma^{\mu}dW^+_{\mu} + \bar{d}\gamma^{\mu}uW^-_{\mu}) - \frac{g}{2\cos\theta_W}\sum_i \bar{f}_i\gamma^{\mu}(c^i_V - c^i_A\gamma_5)f_iZ_{\mu}, \quad (1.13)\end{aligned}$$

where f_i indicates ν, e, u and d, and Q_i are their corresponding charges. c_V and c_A are the coupling constants corresponding to the vector $\bar{f}\gamma^{\mu}f$ and axial vector $\bar{f}\gamma^{\mu}\gamma_5 f$ interaction terms respectively, and this interaction form for Z boson with fermions is the well-known V-A interaction. Sometimes those interactions are expressed in terms of left and right handed interaction, which are $\bar{f}\gamma^{\mu}(1-\gamma_5)/2\ f$ and $\bar{f}\gamma^{\mu}(1+\gamma_5)/2\ f$ with the coupling constants c_R and c_L ,respectively. The general formulas for c_V ,c_A, c_L, and c_R are

$$\begin{aligned} c_V &= I_3 - 2Q_i\sin^2\theta_W; \ \ c_A = I_3; \\ c_L &= 2I_3 - 2Q_i\sin^2\theta_W; \ \ c_R = -2Q_i\sin^2\theta_W, \end{aligned} \quad (1.14)$$

where I_3 is the third component of isospin of particle, which is 1/2 for v_e and u, and $-1/2$ for e and d. c_V, c_A, c_R, c_L for quarks and leptons are shown as follows

	Q	I_3	c_V	c_A	c_L	c_R
ν_e	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
e	-1	-$\frac{1}{2}$	-$\frac{1}{2} + 2\sin^2\theta_W$	-$\frac{1}{2}$	-1 $+ 2\sin^2\theta_W$	$2\sin^2\theta_W$
u	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2} - \frac{4}{3}\sin^2\theta_W$	$\frac{1}{2}$	$1 - \frac{4}{3}\sin^2\theta_W$	-$\frac{4}{3}\sin^2\theta_W$
d	-$\frac{1}{3}$	-$\frac{1}{2}$	-$\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$	-$\frac{1}{2}$	-1 $+ \frac{2}{3}\sin^2\theta_W$	$\frac{2}{3}\sin^2\theta_W$

(1.15)

1.2 CP violation

Parity and charge conjugation are important symmetries in particle physics. Parity was thought as a good symmetry to describe our world until T. D. Lee and C. N. Yang [6] proposed that the parity might be violated in weak interaction in 1956. But it was considered that the combination of parity and charge conjugation was still a good symmetry in all interactions before 1964, when the first evidence for CP violation was observed in the K_0 decay to $\pi\pi$ by J. H. Christenson, J. W. Cronin, V. L. Fitch and R. Turlay [7]. Recently, BaBar [8] and Belle [9] also found CP violation in B meson decays.

1.2.1 P, C, T transformation

Before treating the CP transform of Lagrangian, we introduce the parity, charge conjugation, and time reversal transform for the every kinds of quantum fields in the SM. In the following formulae the superscript p, c, t indicate the field after parity, charge conjugate and time reversal transformation is performed respectively.

Parity transform

The parity transformation in classical physics is to change position from $\mathbf{x}$ into $-\mathbf{x}$. In quantum fields theory, The parity transformed ϕ^p, ψ^p, and A^p_μ of field operators spin-0 ϕ, spin-1/2 ψ and spin-1 A_μ are given below

$$\phi^p(t,\mathbf{x}) = \phi(t,-\mathbf{x});\;\; \psi^p(t,\mathbf{x}) = \gamma_0\psi(t,-\mathbf{x});\;\; A^p_\mu(t,\mathbf{x}) = A^\mu(t,-\mathbf{x}), \tag{1.16}$$

Charge conjugation

The charge conjugation transforms a particle into its anti-particle. The c transformed fields ϕ^c, ψ^c, and A^c_μ are

$$\phi^c(t,\mathbf{x}) = \phi^\dagger(t,\mathbf{x});\;\; \psi^c(t,\mathbf{x}) = i\gamma_2\gamma_0\bar{\psi}^T(t,\mathbf{x});\;\; A^c_\mu(t,\mathbf{x}) = -A_\mu(t,\mathbf{x}), \tag{1.17}$$

where the T above means the transpose of the matrix.

Time reversal

The time reversal is to reversed the time parameter by $t \to -t$ in classical physics. When considering the field operators, we have t transformed fields ϕ^t, ψ^t, and A^t_μ as

$$\phi^t(t,\mathbf{x}) = \phi(-t,\mathbf{x});\;\; \psi^t(t,\mathbf{x}) = i\gamma_1\gamma_3\psi(-t,\mathbf{x});\;\; A^t_\mu(t,\mathbf{x}) = A^\mu(-t,\mathbf{x}). \tag{1.18}$$

When we treat the C,P ,T transformation of Lagrangian or other physical quantities which are made of the field operators, the transformation rules in Eq.(1.16, 1.17, 1.18) are useful.

1.2.2 The Cabibbo-Kobayashi-Maskawa Model

CP violation in the SM was first considered by Kobayashi and Maskawa in 1973 [10]. CP violation came from charged current interaction. Let us discuss this in more detail in the following.

In general, the coupling matrices λ^U and λ^D in the Yukawa interaction of the followings are not diagonal.

$$\overline{Q}_L \lambda^U H U_R + \overline{Q}_L \lambda^D \widetilde{H} D_R + h.c. \,, \tag{1.19}$$

λ^U, λ^D are arbitrary $n \times n$ real matrices for n generations of quarks. The quark mass matrices are given by: $M^U = -\lambda^U v/\sqrt{2}$ and $M^D = -\lambda^D v/\sqrt{2}$. In order to get the quarks mass eigenstates we need to diagonalize these matrices with

$$\begin{aligned} U_L = V_L^u U_L^m; \;\; U_R = V_R^u U_R^m; \\ D_L = V_L^d D_L^m; \;\; D_R = V_R^d D_R^m. \end{aligned} \tag{1.20}$$

where V_L^u ,V_R^u ,V_L^d ,V_R^d are the unitary matrices that diagonalize the coupling matrices as

$$\hat{M}^U = V_L^{U\dagger} M^U V_R^U; \;\; \hat{M}^D = V_L^{D\dagger} M^D V_R^D \tag{1.21}$$

CP violation in the SM resides in charged current interaction of quarks with W boson. The Lagrangian for the $W^\pm$ gauge interaction in the weak interaction basis is given as

$$\mathcal{L} = -\frac{g}{\sqrt{2}}(\overline{U}_L \gamma_\mu D_L W^{\mu+} + \overline{D}_L \gamma_\mu U_L W^{\mu-}). \tag{1.22}$$

When using the quark mass eigenstates, the W-boson gauge interaction becomes

$$\mathcal{L} = -\frac{g}{\sqrt{2}}(\overline{U}_L^m \gamma_\mu V_{\rm CKM} D_L^m W^{\mu+} + \overline{D}_L^m \gamma_\mu V_{\rm CKM}^\dagger U_L^m W^{\mu-}). \tag{1.23}$$

where the $V_{\rm CKM} = V_L^{u\dagger} V_L^d$ is the so-called Kobayashi-Maskawa matrix [10]. The mixing of quarks was first proposed by Cabibbo in 1963 [11], and thus this quark mixing model is also called Cabibbo-Kobayashi-Maskawa(CKM) model.

Using the formula of parity and charge conjugation transformations for field operators, the CP transformation for weak interaction can be found out. Substituting parity transformation Eq. (1.16) into every field operator in weak interaction Eq. (1.23) and then the Lagrangian becomes

$$\begin{aligned}\mathcal{L}^p(t,\mathbf{x}) = & -\frac{g}{\sqrt{2}}(\overline{U}_R^m(t,-\mathbf{x})\gamma_\mu V_{CKM} D_R(t,-\mathbf{x})^m W^{\mu+} \\ & + \overline{D}_R^m(t,-\mathbf{x})\gamma_\mu V_{CKM}^\dagger U_R(t,-\mathbf{x})^m W^{\mu-}),\end{aligned} \tag{1.24}$$

where the left handed couplings change into right handed ones, so this Lagrangian violates under P transformation.

After the parity transformation, we add the charge conjugation into the Lagrangian above. By using the formula Eq.(1.17) as well as the anti-commutating property of spin-1/2 field operators, the CP transformed Lagrangian is given by

$$\begin{aligned}\mathcal{L}^{cp}(t,\mathbf{x}) = & -\frac{g}{\sqrt{2}}(\overline{U}_L^m(t,-\mathbf{x})\gamma_\mu V_{CKM}^* D_L^m(t,-\mathbf{x}) W^{\mu+} \\ & + \overline{D}_L^m(t,-\mathbf{x})\gamma_\mu V_{CKM}^{\dagger *} U_L^m(t,-\mathbf{x}) W^{\mu-}).\end{aligned} \tag{1.25}$$

Eq.(1.25) shows that if the CKM matrix is real, then the form of Lagrangian under CP transformation will be the same as original one except for the $-\mathbf{x}$ parameters. When we consider the action $S = \int \mathcal{L} d^4x$ with real CKM matrix, the difference between $\mathbf{x}$ and $-\mathbf{x}$ will vanish. This means that the CP will be invariant for the weak interaction. If CKM matrix is not real, then the CP will be violated in Eq.(1.23).

For N generations of quarks, V_{CKM} is an $N \times N$ unitary matrix. At first glance it has N^2 independent real parameter. By using the orthogonal matrix property, there are $N(N-1)/2$ angles in the V_{CKM}. So the number of the remaining independent parameters is $N^2 - N(N-1)/2 = N(N+1)/2$. However, we can choose the phase of quarks to eliminate the phases in CKM matrix as

$$\begin{pmatrix} e^{i\alpha_1} & 0 & ... & 0 \\ 0 & e^{i\alpha_2} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & e^{i\alpha_N} \end{pmatrix} \begin{pmatrix} V_{11} & V_{12} & ... & V_{1N} \\ V_{21} & V_{22} & ... & ... \\ ... & ... & ... & ... \\ V_{N1} & ... & ... & V_{NN} \end{pmatrix} \begin{pmatrix} e^{i\beta_1} & 0 & ... & 0 \\ 0 & e^{i\beta_2} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & e^{i\beta_N} \end{pmatrix} \tag{1.26}$$

For N generation there are $2N$ phases which can be eliminated, but we can absorb for example one phase in up quarks into every down quark phases. So total number the phases one can absorb is $2N-1$. In the end, the total independent phases in N generation V_{CKM} is

$$\frac{N(N+1)}{2}-(2N-1)=\frac{(N-1)(N-2)}{2}. \tag{1.27}$$

From Eq.(1.27) it is clear that in order to have irremovable complex phases in the matrix at least three generations of quarks are required, which was first pointed out by Kobayashi and Maskawa in 1973 [10].

With three generations of quarks, It is usually with the expression,

$$V_{\mathrm{KM}}=\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \tag{1.28}$$

where the subscripts indicate which quarks have interaction with W boson.

Because V_{CKM} is complex, the weak interaction of quarks with W boson produces CP violation. It is a convention to parametrize the CKM matrix by three angles and one phases. The original Kobayashi-Maskawa parametrization is given in the following form [10]

$$V_{\mathrm{KM}}=\begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta_{\mathrm{KM}}} & c_1c_2s_3+s_2c_3e^{i\delta_{\mathrm{KM}}} \\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta_{\mathrm{KM}}} & c_1s_2s_3-c_2c_3e^{i\delta_{\mathrm{KM}}} \end{pmatrix} \tag{1.29}$$

where $s_i=\sin\theta_i;\ c_i=\cos\theta_i$, δ_{KM} is the phase which makes the matrix be complex.

Another popular parametrization is from the the Particle Data Group(PDG)[12],

$$V_{\mathrm{CKM}}=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}, \tag{1.30}$$

where $s_{ij}=\sin\theta_{ij};\ c_{ij}=\cos\theta_{ij}$, and the δ_{13} is the complex phase in this parametrization.

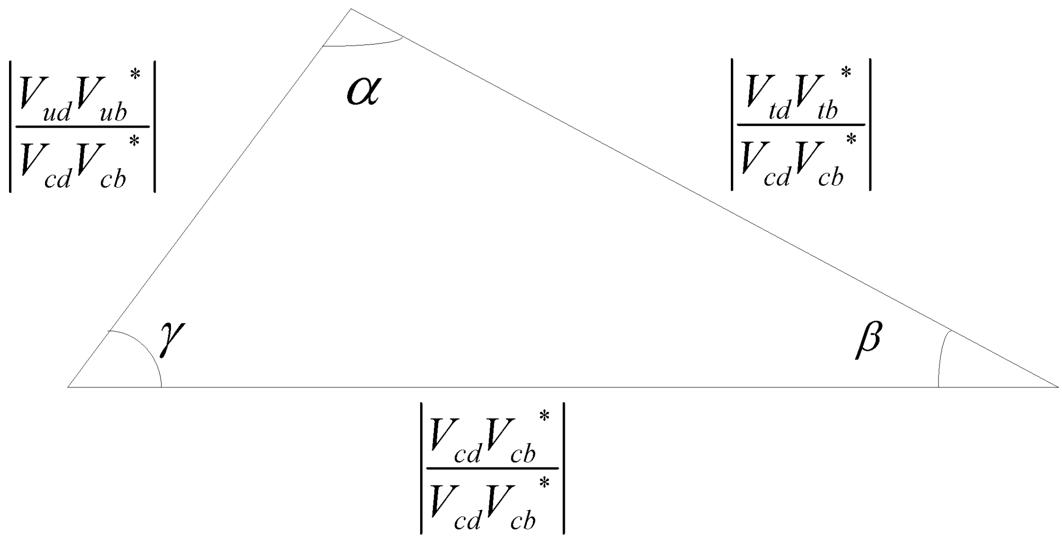

Fig. 1.1: The triangle of one of six CKM matrix unitarity conditions.

Comparing the PDG parametrization and KM parametrization, V_{cb} in expression of KM parametrization is more complicated than that of PDG parametrization. We can determine the PDG parametrization angles more precisely than KM ones, and this is why PDG parametrization is more popular than that of KM.

Wolfenstein [13] proposed a useful parametrization for CKM matrix with the four parameters, λ, A, ρ, and η which has a clear indication of the hierarchy of the individual elements. The V_{CKM} can be expressed in these parameters with order λ^3

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda_3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{1.31}$$

This parametrization makes $-(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*) = \bar{\rho}+i\bar{\eta}$, where $\bar{\rho} = \rho(1-\lambda^2/2)$ and $\bar{\eta} = \eta(1-\lambda^2/2)$ at this order.

Due to the fact that the CKM matrix is unitary, there are orthogonal conditions between different rows or columns. One of them is

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{1.32}$$

This relation can form an triangle on complex plane, as shown in Fig 1.1. There are six different triangles of V_{CKM} matrix, but their areas are the same as $J/2$, where

J is called Jarlskog invariant, which was found in 1985 first by C. Jarlskog [14]

$$\mathrm{Im}[V_{ij}V_{kl}V_{il}^{*}V_{kj}^{*}] = J\sum_{m,n}\epsilon_{ikm}\epsilon_{jln}. \tag{1.33}$$

Write down J in terms of the PDG parametrization and KM parametrization

$$\begin{aligned} J &= c_{12}c_{23}c_{13}^{2}s_{12}s_{23}s_{13}\sin\delta_{13}, \quad \text{in PDG parametrization} \\ J &= s_1^2 s_2 s_3 c_1 c_2 c_3 \sin\delta, \quad \text{in KM parametrization} \end{aligned} \tag{1.34}$$

1.2.3 The determination of V_{CKM} parameters

The test of unitarity property of CKM matrix is very important to examine the validity of the three generations quark mixing mechanism. From lots of the meson decays or semileptonic decays and other experiments, one derive the magnitudes of all nine elements of CKM matrix. Some of the determination will be discussed as follows.

$|\mathbf{V_{ud}}|$

The $|V_{ud}|$ is usually obtained from nuclear beta decay with conserved spin-parity $0^+ \to 0^+$, or the beta decay of neutron and pion. The experimental average of nuclear beta decay is more precise than the others, and it is given as [15]

$$|V_{ud}| = 0.97378 \pm 0.00027(\text{nuclear}). \tag{1.35}$$

$|\mathbf{V_{us}}|$

Determination of $|V_{us}|$ had been performed from different aspect for kaon, like the semileptonic decays, leptonic decay, and also from the ratio of $K \to e\nu$ to $\pi \to e\nu$. The important parameter for kaon semileptonic decay is the form factor f_+ which can gives the determination from $|V_{us}|f_+$. The KLOE collaboration gives [16]

$$|V_{us}| = 0.2253 \pm 0.0007. \tag{1.36}$$

$|\mathbf{V_{cd}}|$

It is precise to determine $|V_{cd}|$ by detecting the process of d or s quarks in hadron interacting with neutrino or antineutrino, which produces a muon and hadron with

c quarks, then the c quark proceeds with semileptonic decay and emit another muon with opposite sign. This dimuon process implies the quantity $\mathcal{B}_\mu |V_{cd}|^2$, where $\mathcal{B}_\mu$ is the average semileptonic branching ratio of charm hadrons. The Particle Data Group[12] used $\mathcal{B}_\mu |V_{cd}|^2 = (0.463 \pm 0.034) \times 10^{-2}$ [20] and the average value of G. D. Lellis [21] and CHORUS [22] which is $\mathcal{B}_\mu = 0.0873 \pm 0.0052$ to get

$$|V_{cd}| = 0.230 \pm 0.011. \tag{1.37}$$

$|\mathbf{V_{cs}}|$

The $|V_{cs}|$ can be determined from the semileptonic decay of D and leptonic decay of D_s. One could choose the semileptonic decay $D \to \pi \ell \nu$. By using the form factor $f_+^{D\to\pi}$ and $f_+^{D\to K}$ calculated from Fermilab Lattice Collaboration [17], with the isospin averaged for semileptonic decay branching ratio from CLEO Collaboration [18], the result is obtained by Artuso [19] with

$$|V_{cs}| = 0.957 \pm 0.017(\text{exp}) \pm 0.093(\text{theory}). \tag{1.38}$$

$|\mathbf{V_{ub}}|$

For the determination of $|V_{ub}|$, the measurement of inclusive semileptonic decay $B \to X_u \ell \nu$ is diffcult to be extracted from the large amount background $B \to X_c \ell \nu$. There are several analysis to determine $|V_{ub}|$. One theoretical extraction is the analysis by Golubev *et al.* [23] from BABAR data [24] for the leptonic momentum spectrum,

$$|V_{ub}| = 4.28 \pm 0.29 \pm 0.29 \pm 0.26 \pm 0.28. \tag{1.39}$$

$|\mathbf{V_{cb}}|$

$|V_{cb}|$ can be determined by semileptonic decays of B to D or D^*. BABAR measured the quantity $\mathcal{F}(1)|V_{cb}| = (34.4 \pm 0.3 \pm 1.1) \times 10^{-3}$ [25] from semileptonic decay $B^0 \to D^{*-} \ell^+ \nu_\ell$, where $\mathcal{F}(1)$ is the axial form factor, which is calculated by unquenched lattice QCD. BABAR use the input $\mathcal{F}(1) = 0.919^{+0.030}_{-0.035}$ [26] to get

$$|V_{cb}| = (37.4 \pm 0.3 \pm 1.2^{+1.2}_{-1.4}) \times 10^{-3}. \tag{1.40}$$

$|\mathbf{V_{td}}|$

The determination of $|V_{td}|$ is from the box diagram for $B_s - \overline{B_s}$ mixing. By using the estimation from results of HPQCD[28] and JLQCD [29], the bag parameter is $f_{B_d}\sqrt{\hat{B}_{B_d}} = 244(26)$MeV [27] and which leads to

$$|V_{td}| = 7.40(79) \times 10^{-3}. \tag{1.41}$$

$|\mathbf{V_{ts}}|$

Using the B meson inclusive rare decay $B \to X_s\gamma$, $|V_{ts}|$ can be determined. Particle Data Group averages those results [30, 31] and gets [12]

$$|V_{ts}| = (40.6 \pm 2.7) \times 10^{-3}. \tag{1.42}$$

$|\mathbf{V_{tb}}|$

The measurement of $|V_{tb}|$ can be extracted from the ratio of t quark decays $R = \mathcal{B}(t \to Wb)/\mathcal{B}(t \to Wq)$. DØ measurement gives $R = 1.03^{+0.19}_{-0.17}$[32] which leads to the lower bound for $|V_{tb}|$

$$|V_{tb}| > 0.78, \tag{1.43}$$

which at 95% confident level. Another method is to using the $p - \bar{p}$ scattering. The parton model for $p - \bar{p}$ includes two mainly channels, which are $q' + \bar{q} \to W^* \to t\bar{b}$ and $q' + g \to qt\bar{b}$. The DØ measurement provides [33]

$$0.68 < |V_{tb}| \leq 1. \tag{1.44}$$

1.2.4 CP violating experimental data and CKM model

CP violation was first discovered in Kaon mixing [7] in 1964. The CP eigenstates for the K_0 and $\overline{K_0}$ system are

$$K_1 = \frac{1}{\sqrt{2}}(K^0 - \overline{K^0});$$
$$K_2 = \frac{1}{\sqrt{2}}(K^0 + \overline{K^0}). \tag{1.45}$$

K_0 is the pesudoscalar particle with odd parity and in charge conjugatation transform $CK_0 = \overline{K}_0$; $C\overline{K}_0 = K_0$. It is obvious that K_1 is CP even eigenstate and K_2 is

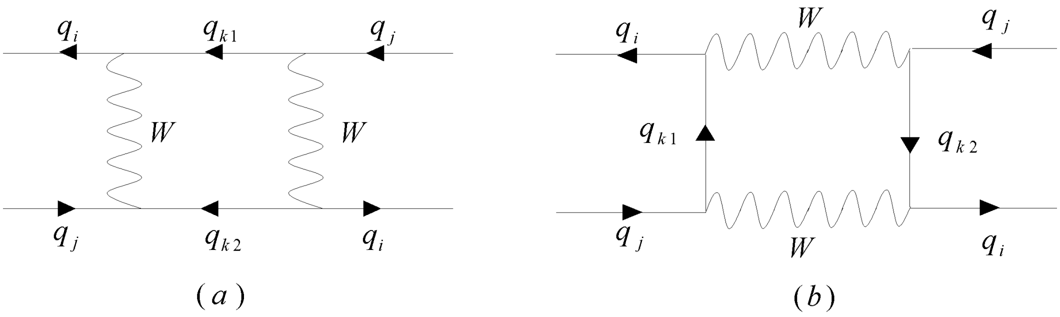

Fig. 1.2: Box diagrams for neutral K-meson mixing

the eigenstate of CP odd. If the CP eigenstates are also the Hamiltonian eigenstates, it means that the CP is conserved under the system.

In general

$$H = M - \frac{i}{2}\Gamma = \begin{pmatrix} M_{11} & M_{12} \\ M_{21}^* & M_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21}^* & \Gamma_{22} \end{pmatrix}, \tag{1.46}$$

where the M and Γ are 2×2 Hermitian matrices, so H is not Hermitian obviously. This non-Hermitian Hamiltonian make the two state system decay during time evolution. The mass eigenstates are

$$\begin{aligned} K_S &= \frac{1}{\sqrt{1+|\tilde{\epsilon}|^2}}(K_1 - \tilde{\epsilon}K_2), \\ K_L &= \frac{1}{\sqrt{1+|\tilde{\epsilon}|^2}}(K_2 + \tilde{\epsilon}K_1). \end{aligned} \tag{1.47}$$

Where $|\tilde{\epsilon}| = (2.44 \pm 0.04) \times 10^{-3}$ [34] is the small value related to the mixing of two CP eigenstates. This formula indicates that the mass eigenstates(energy eigenstates) are not exactly identical to the CP eigenstates. This experimental data is explained by so-called box diagram shown in Fig.1.2 in the SM.

Direct CP violation in Kaon decay into $\pi\pi$ has also been discovered[35]. When treating K meson decay, we usually take the mass eigenstates K_S and K_L as the CP eigenstates instead of K_1 and K_2, because here we discuss only the CP violation from decay. It is convenient to define the quantities related to the decay amplitude

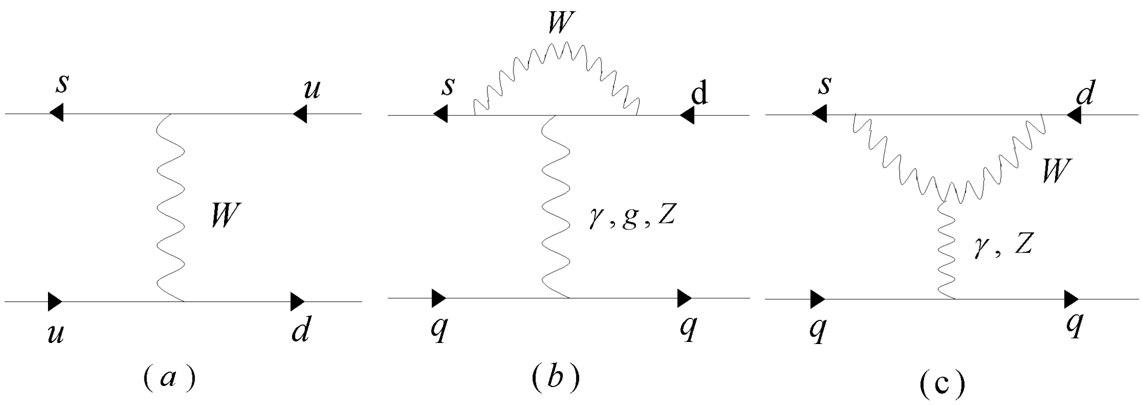

Fig. 1.3: Tree and penguin diagrams for $K \rightarrow \pi\pi$ decays, where q can be u or d

for K meson to study direct CP violation,

$$\begin{aligned}\eta_{00} &= \frac{<\pi^0\pi^0|\mathcal{H}|K_L>}{<\pi^0\pi^0|\mathcal{H}|K_S>};\\ \eta_{+-} &= \frac{<\pi^+\pi^-|\mathcal{H}|K_L>}{<\pi^+\pi^-|\mathcal{H}|K_S>}.\end{aligned} \tag{1.48}$$

CP violation in $K \rightarrow \pi\pi$ is measured by ϵ', which is defined as

$$\begin{aligned}\eta_{00} &= \tilde{\epsilon} - 2\epsilon';\\ \eta_{+-} &= \tilde{\epsilon} + \epsilon'.\end{aligned} \tag{1.49}$$

The Particle Data Group [12] gives the fitting for the value

$$\mathrm{Re}(\epsilon'/\tilde{\epsilon}) = 1.65 \pm 0.26. \tag{1.50}$$

ϵ' is explained in the SM by the tree and penguin diagrams in Fig.1.3.

B decays can provide many tests for CKM model by measuring α, β, and γ in the unitary triangle in Fig 1.1.

β is the relative angle between $V_{td}V_{tb}^*$ and $-V_{cd}V_{cb}^*$ on the complex plane. There are several ways people often take to determine this angle. The most popular process is the $b \rightarrow c\bar{c}s$ process. This is because the amplitude of the tree level and loop diagram has approximately the same phase. One of the process often been used is $B \rightarrow J/\psi K_s$. The $\sin 2\beta$ is extracted from the relation [12]

$$S_f = -\eta_f \sin 2\beta, \tag{1.51}$$

where S_f is the quantity related to time-dependent CP asymmetry in B decays [36, 37], and η_f is the CP eigenvalue of f. The experimental result from average of the related decay by BaBar has the value [38]

$$\sin 2\beta = 0.686 \pm 0.039 \pm 0.015. \tag{1.52}$$

From Fig 1.1 definition the α is the angle between $V_{ud}V_{ub}^*$ and $-V_{td}V_{tb}^*$. It can be extract from $B \to \pi\pi$ process, via the measurement of $S_{\pi^+\pi^-}$ and $C_{\pi^+\pi^-}$. The measurement of BABAR gives that [40]

$$\alpha = 96^{\circ}{}^{+10^\circ}_{-6^\circ}. \tag{1.53}$$

The decay $B \to \rho_0\pi_0$ are also applied to determined α, and the experimental result from Belle gives $68^\circ < \alpha < 95^\circ$ at $CL = 68\%$[41].

γ is the angle between $V_{cd}V_{cb}^*$ and $-V_{ud}V_{ub}^*$. The measurement of γ determination uses the B decay process $B \to DK$. BELLE [42] measured $B^- \to DK^-$, $B^- \to D^*K^-$ and $B^- \to DK^{*-}$ to obtain

$$(\gamma = 53^{+15}_{-18} \pm 3 \pm 9)^\circ \tag{1.54}$$

The process $B \to K^+\pi^-$ are usually applied to test the CP asymmetry, too. The experimental average is $\mathcal{A}_{CP}(B^0 \to K^+\pi^-) = -0.097 \pm 0.012$ by HFAG [43]. This asymmetry can be explained by SM [44, 45, 46].

The global fit for the unitary triangle is summarized in Fig.1.4 which is from CKMfitter [47]. The PDG review[12] provides the fitting values for Wolfenstein parameters, which are

$$\begin{aligned} \lambda &= 0.2272 \pm 0.0010; \quad A = 0.818^{+0.007}_{-0.017}; \\ \overline{\rho} &= 0.221^{+0.064}_{-0.028}; \qquad \overline{\eta} = 0.340^{+0.017}_{-0.045}. \end{aligned} \tag{1.55}$$

Compare these values with the PDG parameters, we can derive $s_{12} = 0.227 \pm 0.001$, $s_{23} = 0.0422 \pm 0.0004$, $s_{13} = 0.00399 \pm 0.00007$, and the phase $\sin\delta_{13} = 0.839 \pm 0.006$. From these values, one obtain[12]

$$J = (3.08^{+0.16}_{-0.18}) \times 10^{-5}. \tag{1.56}$$

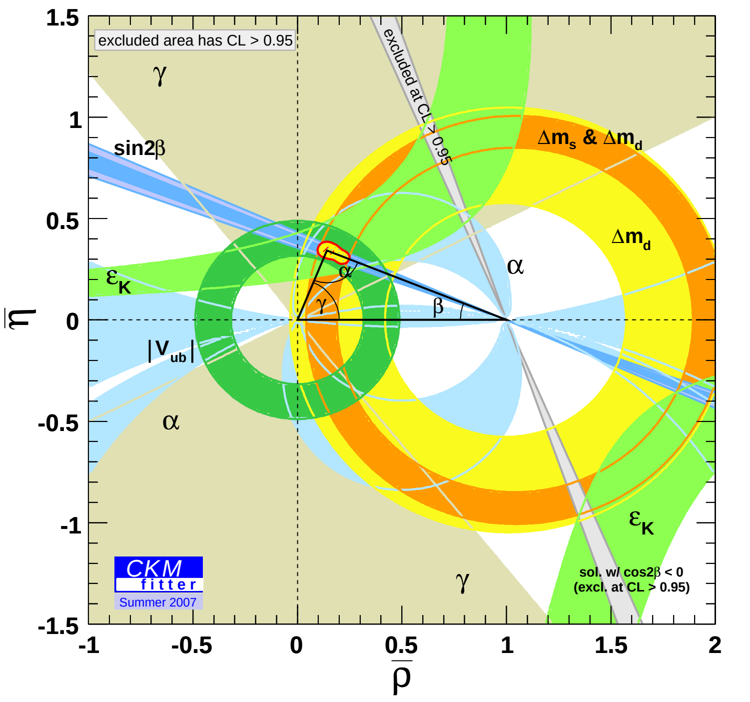

Fig. 1.4: The experimental fit for the $\bar{\rho}$ and $\bar{\eta}$ from CKMfitter Group [47]

From the above discussion for CKM matrix, we can see that the CKM matrix works very well in describing the meson decay, leptonic and semileptonic decay. The phase in CKM matrix generates CP violation, and which is consistent with the experimental result for the CP violation phenomena like K meson mixing, K and B meson decay. However there are still some problems. One of them is the baryongenesis. That is, the amount of particles is more than that of antiparticles in our world, and one necessary condition for this phenomenon is the existence of CP violation. The quantity to estimate the asymmetry of universe is n_B/n_γ, with n_B, n_γ denoting the baryon number density, and photon number density respectively. In high temperature the CKM model can produce about $n_B/n_\gamma \approx \mathcal{O}(10^{-20})$[48], which is too small compared with observation $n_B/n_\gamma \approx 10^{-8}$[49]. There should be another source of CP violation beyond the CKM matrix in our world. Also, the CKM model does not provide the answer where CP violation is originated, but just put in by hand. It is desirable for some understanding of the origin of CP violation. In this thesis we try to study how to connect the CKM matrix phase with the spontaneous CP violating phase for the explanation of the source of CP violation from CKM mechanism.

In chapter 2 we are going to discuss what the spontaneous CP violation is and treat some of the multi-Higgs models. In chapter 3 we will build a new model with the connection between spontaneous CP violating phase and CKM matrix phase. In chapter 4 we will use this model to discuss some phenomenology. In the last chapter we will summarize what we do in this thesis.

2. MULTI-HIGGS MODELS AND SPONTANEOUS CP VIOLATION

Although the CKM matrix and its complex relation explain the CP violation of observation very well, it is still possible that CP is violated from other place. The multi-Higgs model is a popular topic in this area. Such models may also answer that CP violation comes from the so-called spontaneous CP violation(SCPV), a mechanics first proposed by T.D. Lee [50, 51].

When there are more than one Higgs, their vacuum expectation values might have the relative phases difference. If these phases are non-vanishing after symmetry breaking and irreducible in Higgs self-interaction or Yukawa terms with fermions, then they also produce CP violation. Because this kind of CP violation comes from the spontaneous symmetry breaking of Higgs, it is called spontaneous CP violation.

2.1 Two Higgs Doublet Model

In 1973, T. D. Lee proposed a model with two Higgs doublets [50, 51]. The most important property of this model is that if there is a phase difference between VEVs of two Higgs doublets, this phase can give the contribution to the CP violation. The two Higgs doublets are written in the following form [50]

$$
\begin{aligned}
\phi_1 &= e^{i\theta_1} H_1 = e^{i\theta_1} \begin{pmatrix} \frac{1}{\sqrt{2}}(\rho_1 + R_1 + iA_1) \\ h_1^- \end{pmatrix}; \\
\phi_2 &= e^{i\theta_2} H_2 = e^{i\theta_2} \begin{pmatrix} \frac{1}{\sqrt{2}}(\rho_2 + R_2 + iA_2) \\ h_2^- \end{pmatrix}.
\end{aligned} \tag{2.1}
$$

H_1 and H_2 are Higgs with real vacuum expectation values, and R_1, R_2, A_1, and A_2 are real parts and imaginary parts of them. The phase difference between two

Higgs doublets $\delta = \theta_2 - \theta_1$ is the spontaneous CP violating phase. If this phase is non-zero and can't be eliminated by fermion rotation, then this could produce the spontaneous CP violation. The Higgs potential can be built in the form [50]

$$\begin{aligned} V = & - \lambda_1 \phi_1^\dagger \phi_1 - \lambda_2 \phi_2^\dagger \phi_2 + A(\phi_1^\dagger \phi_1)^2 + B(\phi_2^\dagger \phi_2)^2 \\ & + C(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \overline{C}(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \\ & + \frac{1}{2}[(\phi_1^\dagger \phi_2)(D\phi_1^\dagger \phi_2 + E\phi_1^\dagger \phi_1 + F\phi_2^\dagger \phi_2) + \text{h.c.}], \end{aligned} \tag{2.2}$$

where λ_1, λ_2, $A - F$, and $\overline{C}$ are all real numbers. The minimal condition by differentiating with respect to δ can give

$$\cos\delta = -(4D\rho_1\rho_2)^{-1}[E\rho_1^2 + F\rho_2^2]. \tag{2.3}$$

If the right handed side is not required to be 1 or -1, CP is violated spontaneously.

There are eight real scalar fields in two Higgs doublets, and three of them are eaten by $W^\pm$ and Z^0. So there are five physical states in two Higgs doublet model after spontaneous symmetry breaking. The two Higgs model is different from SM because it has charged Higgs bosons. The interaction of charged Higgs and fermions is similar to charged weak interaction. So this model also have more contribution to flavor change process than SM.

Usually the two Higgs doublet models are classified into three types by different Yukawa interactions.

Type I Type I is that one Higgs couples with each fermions, like the Higgs in standard model, and another Higgs does not couple with fermions as below

$$\overline{Q}_L \Gamma_u \phi_1 U_R + \overline{Q}_L \Gamma_d \widetilde{\phi}_1 D_R + \overline{L}_L \Gamma_e \widetilde{\phi}_1 E_R + \text{h.c.}, \tag{2.4}$$

where Γ_u, Γ_d, and Γ_e are real coupling matrices. We construct thise interactions of type I by introducing the discrete symmetry $\phi_2 \to -\phi_2$ and other fields are unchanged, so that ϕ_2 only exists in the Higgs potential with even powers.

Type II Type II is that one Higgs doublet couples with up-type quarks and another one couples with down-type quarks. For example

$$\overline{Q}_L \Gamma_u \phi_1 U_R + \overline{Q}_L \Gamma_d \widetilde{\phi}_2 D_R. + \overline{L}_L \Gamma_e \widetilde{\phi}_2 E_R + \text{h.c.}. \quad (2.5)$$

We can construct type II model by introducing this discrete symmetry:

$$\begin{aligned} \phi_1 &\to \phi_1; \;\; \phi_2 \to -\phi_2; U_R \to U_R; \;\; D_R \to -D_R; \;\; E_R \to -E_R; \\ Q_L &\to Q_L; \;\; L_L \to L_L. \end{aligned} \quad (2.6)$$

Type III The last model type III is the most general Yukawa interactions in which there are two Higgs coupling with each fermion,

$$\begin{aligned} \overline{Q}_L(\Gamma_{u1}\phi_1 + \Gamma_{u2}\phi_2)U_R &+ \overline{Q}_L(\Gamma_{d1}\widetilde{\phi}_1 + \Gamma_{d2}\widetilde{\phi}_2)D_R \\ &+ \overline{L}_L(\Gamma_{e1}\widetilde{\phi}_1 + \Gamma_{e2}\widetilde{\phi}_2)E_R + \text{h.c.} \end{aligned} \quad (2.7)$$

so there could be the FCNC process because one can not diagonalize mass matrix $v_1\Gamma_1 + v_2\Gamma_2$ and coupling matrices $\Gamma_{1,2}$ simultaneously.

Type I and II can not have spontaneous CP violation, because $(\phi_1^\dagger\phi_2)(\phi_1^\dagger\phi_1)$, and $(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_2)$ are not allowed, and this results in $\sin\delta = 0$. The spontaneous CP violating phase δ vanishes. So type I and type II with discrete symmetry have no spontaneous CP violation.

If we hope that the spontaneous CP violation exists, then only type III is allowed. However, type III has the tree level FCNC contribution which is severely constrained by experimental data. Also, there are also too many unknown parameters in the model.

2.2 Weinberg Model

In 1976, Weinberg [52] proposed that by using some discrete symmetry, the model with three or more Higgs doublets gives the spontaneous CP violation without tree level flavor change neutral current. The three Higgs doublet model is called Weinberg model. Three Higgs ϕ_1, ϕ_2, and ϕ_3 are written as follows

$$\phi_k = e^{i\theta_k} H_k = e^{i\theta_k} \begin{pmatrix} \frac{1}{\sqrt{2}}(v_k + R_k + iA_k) \\ H_k^- \end{pmatrix}, \quad k = 1 \sim 3, \tag{2.8}$$

where $\frac{1}{\sqrt{2}} v_k e^{i\theta_k}$ is the vacuum expectation value of neutral part in ϕ_k, and we let $H_k^0 = v_k + R_k + iA_k$. R_k and A_k are the corresponding real part and imaginary part in H_k. Branco extended this idea to arbitrary number of generations with two sets of discrete symmetry [53]

$$\begin{aligned} D_1 &: \quad \phi_1 \to \phi_1;\ \phi_2 \to -\phi_2;\ \phi_3 \to \phi_3;\ Q_L \to Q_L;\ d_R \to d_R;\ u_R \to -u_R \\ D_2 &: \quad \phi_1 \to \phi_1;\ \phi_2 \to \phi_2;\ \phi_3 \to -\phi_3;\ Q_L \to Q_L;\ d_R \to d_R;\ u_R \to u_R. \end{aligned} \tag{2.9}$$

The D_1 implies the constraint that ϕ_1 couples to the up-type quarks singlet u_R and ϕ_2 couples to the down-type ones d_R, which has the same propose as the discrete symmetry for type II of two Higgs doublet model. The D_2 can suppress ϕ_3 not to couple with quarks, but it can couple to leptons. Applying those discrete symmetry can inhibit the tree level FCNC process of Higgs exchange, and the CP violation can arise from the Higgs interaction themselves. The Yukawa terms of Weinberg model are written as

$$\overline{Q}_L \Gamma_u \phi_1 U_R + \overline{Q}_L \Gamma_d \widetilde{\phi}_2 D_R + \overline{L}_L \Gamma_e \widetilde{\phi}_3 E_R + \text{h.c.}. \tag{2.10}$$

These interactions are similar to type II of two Higgs doublet model, and the spontaneous CP violation will be produced in the Higgs potential.

After spontaneous symmetry breaking the Lagrangian is expanded as follows

$$\begin{aligned} L &= -\frac{1}{v_1}\overline{U}_L \hat{M}_u U_R H_1^0 - \frac{1}{v_2}\overline{D}_L \hat{M}_d D_R H_2^0 - \frac{1}{v_2}\overline{L}_L \hat{M}_e E_R H_3^0 \\ &- \frac{\sqrt{2}}{v_1}\overline{D}_L V_{\text{CKM}}^\dagger \hat{M}_u U_R H_1^- + \frac{\sqrt{2}}{v_2}\overline{U}_L V_{\text{CKM}} \hat{M}_d D_R H_2^+ \\ &+ \frac{\sqrt{2}}{v_3}\overline{L}_L V_{\text{CKM}} \hat{M}_e E_R H_3^+ + \text{h.c.}, \end{aligned} \tag{2.11}$$

where $M_u = -\frac{1}{\sqrt{2}}\Gamma_u v_1$, $M_d = -\frac{1}{\sqrt{2}}\Gamma_d v_2$, and $M_e = -\frac{1}{\sqrt{2}}\Gamma_e v_3$. V_{CKM} is assumed to be real matrix here. That is, CP violation does not come from the CKM matrix.

The CP should be arisen from the Higgs self-interaction. The parametrization for three Higgs doublet potential in discussion [54] is

$$\begin{aligned} V = & - \mu_1^2\phi_1^\dagger\phi_1 - \mu_2^2\phi_2^\dagger\phi_2 - \mu_3^2\phi_3^\dagger\phi_3 + h_1(\phi_1^\dagger\phi_1)^2 + h_2(\phi_2^\dagger\phi_2)^2 + h_3(\phi_3^\dagger\phi_3)^2 \\ & + f_{12}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + f_{23}(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) + f_{31}(\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1) \\ & + g_{12}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + g_{23}(\phi_2^\dagger\phi_3)(D\phi_3^\dagger\phi_2) + g_{31}(\phi_3^\dagger\phi_1)(\phi_1^\dagger\phi_3) \\ & + (k_{12}(\phi_1^\dagger\phi_2)^2 + k_{23}(\phi_2^\dagger\phi_3)^2 + k_{31}(\phi_3^\dagger\phi_1)^2 + \text{h.c.}). \end{aligned} \tag{2.12}$$

It can be assumed that the coefficients in above formula are all real. In the potential only two phases $\delta_{12} = \theta_2 - \theta_1$ and $\delta_{23} = \theta_3 - \theta_2$ exist. Differentiating with respect to the two phases, we get a condition below

$$k_{12}\frac{v_2^2}{v_3^2}\sin 2\delta_{12} = k_{23}\frac{v_2^2}{v_1^2}\sin 2\delta_{23} = -k_{13}\sin 2(\delta_{12} + \delta_{23}). \tag{2.13}$$

Eq.(2.13) reflects the fact that the phases δ_{12} and δ_{23} can be nonzero, and CP is violated here.

In this model, there are four charged and five neutral physical Higgs bosons. The mass matrix of Higgs can give CP properties of this model [54]. The resulting mass matrix of charged Higgs has the off diagonal complex elements. It means that the CP violation will arise from the exchanging of charge Higgs. Also, the neutral mass matrix has the mixing terms between scalar and pseudoscalar Higgs which lead to the CP violation under neutral Higgs exchange.

Although Weinberg model can provide the spontaneous CP violation without the tree level FCNC which is inhibited in Lee's two Higgs doublet model, it still has some contradiction which had been provided by many authors[55, 56, 57]. The Weinberg model is decisively ruled out by data on $\sin 2\beta$ measurement in $B \to K_s J/\psi$. In Weinberg model, the upper bound for magnitude of $\sin 2\beta$ is $|\sin 2\beta| < 0.05$[56, 57]. The present experimental data is shown in Eq.(1.52) that $\sin 2\beta = 0.686 \pm 0.039 \pm 0.015$, which means that the Weinberg model has been ruled out. The neutron EDM calculation also rules out the Weinberg model, from which the estimation for neutron EDM has order 10^{-23}e cm [58], whereas the experimental upper bound is $|d_n| < 0.29 \times 10^{-25}$e cm [59].

2.3 The strong CP problem and Peccei-Quinn symmetry

For the Lagrangian in QCD, the term $(\theta g_s^2/32\pi^2)G^{\mu\nu}\widetilde{G}_{\mu\nu}$ is allowed, where $\widetilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$. This term also violates P and CP. This is a possible CP violation in strong interaction.

Because P and CP violation will cause the electric dipole moment(EDM) of particles, the measurement of EDM of particles is important to test the CP violation in standard model. The neutron EDM test is especially important, and in present the experimental upper bound is given [59]

$$|d_n| < 0.29 \times 10^{-25}\text{e cm}. \tag{2.14}$$

SM theoretical calculation of CKM matrix CP violation gives the small value contribution about order less than 10^{-31}e cm [61, 62, 63] without considering strong CP violation. With non-zero θ, d_n can be much larger. Experimental bound for neutron EDM constrains the θ very strongly for $|\theta| \lesssim 10^{-10}$ [12]. This is considered to be unnatural since other couplings with strong interaction are much larger. This is the problem.

For the multi-Higgs model with spontaneous CP violating phase, the strong phase θ would be large [60] at loop level. We need a mechanism to make this phase small.

In 1977, Peccei and Quinn proposed a mechanism to solve this problem [64, 65]. They introduced another global symmetry $\mathrm{U}(1)_{\mathrm{PQ}}$ in the standard model. This symmetry is generated by the chiral transformation defined as follows.

$$\begin{aligned} u &\to e^{i\alpha_u\gamma_5}u; \;\; d \to e^{i\alpha_d\gamma_5}d; \\ \phi_1 &\to e^{i(\alpha_u+\alpha_d)}\phi_1; \;\; \phi_i \to e^{-i(\alpha_u+\alpha_d)}\phi_i; \;\; i \neq 1, \end{aligned} \tag{2.15}$$

where α_u and α_d are the chiral rotational phases for up-type quarks and down-type quarks respectively. The ϕ_1 and other ϕ_i are the multi-Higgs doublets. After the chiral rotation, the strong phase becomes

$$\theta \to \theta - 4\alpha_u - 4\alpha_d. \tag{2.16}$$

Since $\alpha_{u,d}$ are arbitrary phases, one can choose these phase as $\theta = 4(\alpha_u + \alpha_d)$, therefore there is no strong CP phase and also without large contribution to neutron EDM.

Models with PQ symmetry have an axion resulting from spontaneous breaking down of PQ symmetry. No axion has been detected in experiments. One has to make the axion invisible, by extending the original PQ model[66, 67].

3. NEW MODEL BUILDING

The multi-Higgs model can have the spontaneous CP violation(SCPV). This is a nice feature which provides a understanding of the origin of CP violation. But the two Higgs doublet model has tree level FCNC, leading to too many unknown parameters. To improve the situation, Weinberg proposed a three Higgs doublet model which has no tree level FCNC. However, the prediction of Weinberg model for $\sin 2\beta$ is not consistent with experimental data as mentioned before. Here we take the idea [1] that the CP violation is arisen from spontaneous symmetry breaking, but further make the spontaneous CP violating phase be identical to the CP violation in CKM matrix. In the following we build specific models to realize this.

3.1 Making SCPV phase identical to CKM phase

Model(a)

In our new model, we try to build a model with the spontaneous CP violating phase from Higgs identical to the the phase in CKM matrix. We couple two independent Higgs doublets to the up-type quarks and one Higgs doublet to the down type quarks as below

$$\mathcal{L} = \overline{Q}_L(\Gamma_{u1}\phi_1 + \Gamma_{u2}\phi_2)U_R + \overline{Q}_L\Gamma_d\widetilde{\phi}_d d_R + \text{h.c.}, \qquad (3.1)$$

where Γ_{u1} and Γ_{u2} are real 3×3 coupling matrices, and the tilde sign on Higgs means $\widetilde{\phi}_k = -i\sigma_2\phi_k^*$. The ϕ_1, ϕ_2 and ϕ_d are Higgs doublets, which are written as

$$\phi_k = e^{i\theta_k}\begin{pmatrix} \frac{1}{\sqrt{2}}(v_k + R_k + iA_k) \\ h_k^- \end{pmatrix}, \qquad (3.2)$$

where k can be 1, 2, and d. It is convenient to redefine these Higgs doublets so that they have real vacuum expectation values. That is, $\phi_k = e^{i\theta_k}H_k$. Here we call

Eq.(3.1) as model(a).

After spontaneous symmetry breaking, the mass terms of model(a) Lagrangian appear as

$$\mathcal{L}_m = -\overline{U}_L(M_{u1}e^{i\theta_1} + M_{u2}e^{i\theta_2})U_R - \overline{D}_L(M_d e^{-i\theta_d})D_R + \text{h.c.}, \tag{3.3}$$

where $M_{u1,u2} = -\Gamma_{u1,u2}v_{1,2}/\sqrt{2}$ and $M_d = -\Gamma_d v_d/\sqrt{2}$. The phase θ_1 and $-\theta_d$ can be absorbed into the U_R and D_R respectively.

From previous section, Eq.(1.20) shows the relations between flavor eigenstates and mass eigenstates of quarks

$$U_L = V_L^u U_L^m; \;\; U_R = V_R^u U_R^m;$$
$$D_L = V_L^d D_L^m; \;\; D_R = V_R^d D_R^m.$$

We make the M_d to be diagonal without loss of generality. That is, D_L and D_R are already the mass eigenstates D_L^m and D_R^m, and the mixing matrix V_L^d and V_R^d are unit matrices. The mass terms become

$$\mathcal{L}_m = -\overline{U}_L(M_{u1} + M_{u2}e^{i\delta})U_R - \overline{D}_L\hat{M}_d D_R + \text{h.c.}, \tag{3.4}$$

where the relative phase $\delta = \theta_2 - \theta_1$ is the spontaneous CP violating phase in the Yukawa terms, and $\hat{M}_d$ is the diagonal mass matrix. If δ is non-zero, it could cause the spontaneous CP violation in the model. The total mass matrix of up-type quarks can be diagonalized by left and right handed matrices V_R^u and V_L^u. That is

$$\hat{M}_u = V_L^{u\dagger} M_u V_R^u, \tag{3.5}$$

where $\hat{M}_u$ is the diagonal up-quark mass matrix and $M_u = M_{u1} + e^{i\delta}M_{u2}$. To simplify the discussion we assume that V_R^u is a unit matrix. This simplification can help us to make the identity relation between δ and the phase in CKM matrix. Because V_L^d is a unit matrix, from $V_{\text{CKM}} = V_L^{u\dagger}V_L^d$ it is obvious that $V_L^{u\dagger}$ is equal to V_{CKM}. Eq(3.5) becomes $\hat{M}_u = V_{\text{CKM}}(M_{u1} + e^{i\delta}M_{u2})$, and we obtain the relation

$$V_{\text{CKM}}^\dagger = (M_{u1} + e^{i\delta}M_{u2})\hat{M}_u^{-1}. \tag{3.6}$$

At this step we need the explicit CKM parametrization with a phase. First we use the Particle Data Group parametrization which is shown in Eq.(1.30)

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix},$$

This parametrization makes more than one phases in the $V_{\rm CKM}$ elements. That is, one phase δ_{13} in V_{21}, V_{22}, V_{31}, V_{32} and another phase $-\delta_{13}$ in V_{13}. We solve the problem by pulling the phase $-\delta_{13}$ out and then decomposing the $V_{\rm CKM}$ into two matrices as below,

$$\begin{aligned} V_{\rm CKM} &= \begin{pmatrix} e^{-i\delta_{13}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &\times \begin{pmatrix} c_{12}c_{13}e^{i\delta_{13}} & s_{12}c_{13}e^{i\delta_{13}} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}, \end{aligned} \tag{3.7}$$

Absorbing the left handed diagonal matrix by redefining the quark sector U_L. The remaining matrix has the uniform phase in each element. By comparing two side of Eq.(3.6), we introduce the identical relation

$$\delta = -\delta_{13}. \tag{3.8}$$

This relation implies that the CKM phase comes from the spontaneous CP violating phase. Also, this relation is related to the phase δ_{13} which has been measured in experiments, so if the spontaneous CP violating phase δ can be nonzero after solving the minimal condition in Higgs potential, it is independent of the masses of Higgs, and the CP phenomena always exists.

By substituting Eq.(3.8) into Eq.(3.6) we determine the coupling matrices M_{u1}

and M_{u2}

$$
\begin{aligned}
M_{u1} &= \begin{pmatrix} 0 & -s_{12}c_{23} & s_{12}s_{23} \\ 0 & c_{12}c_{23} & -c_{12}s_{23} \\ s_{13} & s_{23}c_{13} & c_{23}c_{13} \end{pmatrix} \hat{M}_u; \\
M_{u2} &= \begin{pmatrix} c_{12}c_{13} & -c_{12}s_{23}s_{13} & -c_{12}c_{23}s_{13} \\ s_{12}c_{13} & -s_{12}s_{23}s_{13} & -s_{12}c_{23}s_{13} \\ 0 & 0 & 0 \end{pmatrix} \hat{M}_u. \qquad (3.9)
\end{aligned}
$$

Note that these matrices M_{u1} and M_{u2} depend on quark masses and the angles in CKM parametrization. This is not true for other multi-Higgs models, with which the spontaneous CP violating phase and CKM phase are concerned.

We apply the same idea to another CKM parametrization, the original Kobayashi Maskawa parametrization in Eq.(1.29)

$$
V_{\mathrm{KM}} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta_{KM}} & c_1c_2s_3 + s_2c_3e^{i\delta_{KM}} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta_{KM}} & c_1s_2s_3 - c_2c_3e^{i\delta_{KM}} \end{pmatrix}.
$$

Then the step of previous discussion for PDG parametrization makes the same relation as Eq.(3.8),

$$
\delta = -\delta_{\mathrm{KM}}. \qquad (3.10)
$$

This relation gives the expression for the mass matrix with respect to KM parametrization,

$$
\begin{aligned}
M_{u1} &= \begin{pmatrix} c_1 & s_1c_2 & s_1s_2 \\ -s_1c_3 & c_1c_2c_3 & c_1s_2c_3 \\ -s_1s_3 & c_1c_2s_3 & c_1s_2s_3 \end{pmatrix} \hat{M}_u; \\
M_{u2} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & -s_2s_3 & c_2s_3 \\ 0 & s_2c_3 & -c_2c_3 \end{pmatrix} \hat{M}_u. \qquad (3.11)
\end{aligned}
$$

In general we can express the two coupling matrices M_{u1} and M_{u2} in terms of the CKM matrix, quark mass matrices, and the spontaneous CP violating phase as follows,

$$\begin{aligned} M_{u1} &= V_{CKM}^{\dagger}\hat{M}_u - \frac{e^{i\delta}}{\sin\delta}\mathrm{Im}(V_{CKM}^{\dagger})\hat{M}_u; \\ M_{u2} &= \frac{1}{\sin\delta}\mathrm{Im}(V_{CKM}^{\dagger})\hat{M}_u. \end{aligned} \tag{3.12}$$

This relation is useful when we treat the Yukawa couplings of Higgs, and it is independent of the parametrization of V_{CKM}. Choosing a specific CKM representation implies a choice of a model.

Model(b)

Now we treat another kind of Yukawa interactions which is called model(b),

$$\mathcal{L} = \bar{Q_L}\Gamma_u\phi_u U_R + \bar{Q_L}(\Gamma_1\widetilde{\phi}_1 + \Gamma_2\widetilde{\phi}_2)d_R. + \mathrm{h.c.}. \tag{3.13}$$

In this Lagrangian two Higgs doublets ϕ_1 and ϕ_2 couple with down-type quarks and one Higgs-doublet ϕ_u is coupled with up-type quarks. $\Gamma_{d(1,2)}$ and Γ_u are 3×3 real matrices. Here the ϕ_1 and ϕ_2 are defined as those in model(a), and ϕ_u is with the same definition as that in Eq.(3.2) and in which k is replaced by u. After symmetry breaking the mass terms is written below

$$\mathcal{L}_m = -\overline{U}_L(M_u e^{i\theta_u})U_R - \overline{D}_L(M_{d1}e^{-i\theta_1} + M_{d2}e^{-i\theta_2})D_R + \mathrm{h.c.}, \tag{3.14}$$

where $M_{d(1,2)} = -\Gamma_{d(1,2)}v_{1,2}/\sqrt{2}$ and $M_u = -\Gamma_u v_u/\sqrt{2}$. Following the same treatment for model(a), we absorb the phases $-\theta_1$ and θ_u into D_R and U_R respectively, and we also treat the M_u as diagonal mass matrix. So $V_L^u = V_R^u = 1$ and the mass terms become

$$\mathcal{L}_m = -\overline{U}_L\hat{M}_u U_R - \overline{D}_L(M_{d1} + M_{d2}e^{-i\delta})D_R + \mathrm{h.c.}. \tag{3.15}$$

The $\hat{M}_u$ indicates that it is diagonal, and δ is the spontaneous CP violating phase with $\delta = \theta_2 - \theta_1$. The diagonal down-type mass matrix $\hat{M}_d$ has the relation to $M_d = M_{d1} + e^{-i\delta}M_{d2}$,

$$\hat{M}_d = V_L^{\dagger d} M_d V_R^d, \tag{3.16}$$

We make $V_R^d = 1$ and the $V_L^{d\dagger}$ is equal to $V_{\mathrm{CKM}}^{\dagger}$. That makes us to express the mass matrices M_{d1} and M_{d2} in terms of V_{CKM} as

$$V_{\mathrm{CKM}} = (M_{d1} + e^{-i\delta}M_{d2})\hat{M}_d^{-1}. \tag{3.17}$$

Using the PDG parametrization with the same argument from Eq.(3.7), and comparing phases in two sides of Eq.(3.17), we can write down the phase relation as follows

$$\delta = -\delta_{13}. \tag{3.18}$$

Note that this relation is the same as Eq.(3.8) discussed in model(a). Also, this relation makes coupling matrices M_{d1} and M_{d2} as

$$M_{d1} = \begin{pmatrix} 0 & 0 & s_{13} \\ -s_{12}c_{23} & c_{12}c_{23} & s_{23}c_{13} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23}c_{13} \end{pmatrix} \hat{M}_d;$$
$$M_{d2} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & 0 \\ -c_{12}s_{23}s_{13} & -s_{12}s_{23}s_{13} & 0 \\ -c_{12}c_{23}s_{13} & -s_{12}c_{23}s_{13} & 0 \end{pmatrix} \hat{M}_d. \tag{3.19}$$

Like the model(a), the mass matrices are determined by down-type quark masses and three angles of CKM matrix parametrization.

Using the KM parametrization we assume the same relation as Eq.(3.10) in model(a), which leads to determine the coupling matrices M_{d1} and M_{d2} as

$$M_{d1} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 & c_1c_2s_3 \\ s_1s_2 & c_1s_2c_3 & c_1s_2s_3 \end{pmatrix} \hat{M}_d;$$
$$M_{d2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -s_2s_3 & s_2c_3 \\ 0 & c_2s_3 & -c_2c_3 \end{pmatrix} \hat{M}_d. \tag{3.20}$$

The general formulas for M_{d1} and M_{d2} in terms of V_{CKM}, $\hat{M}_d$, and the spontaneous CP violating phase δ are

$$M_{d1} = V_{\mathrm{CKM}}\hat{M}_d + \frac{e^{-i\delta}}{\sin\delta}\mathrm{Im}(V_{\mathrm{CKM}})\hat{M}_d;$$
$$M_{d2} = -\frac{1}{\sin\delta}\mathrm{Im}(V_{\mathrm{CKM}})\hat{M}_d. \tag{3.21}$$

In this section we have discussed how to make the spontaneous CP violating phase identical to the CKM matrix phase, this identical relation leads to the determination of couplings matrices, which depends on three angles in CKM parametrization and the quark masses. For different CKM parametrization, the related coupling matrix are also different. In next section we will study a particular multi-Higgs model, and then apply it to our Yukawa coupling models built in this section.

3.2 Multi-Higgs model building

To build a model realizing the idea in the previous section, we also need to consider appropriate Higgs sectors. It has been shown that in order to have the spontaneous CP violation and the PQ symmetry more than two Higgs doublets are required [68, 69]. For SM there is only one Higgs doublet, but at this moment we need a multi-Higgs doublets as described before. We also need the small enough neutron dipole moment with no strong CP problem, so the Peccei-Quinn $\mathrm{U_{PQ}}(1)$ symmetry is introduced. The another scalar field is required to generate invisible axion, and therefore the minimal model is a model with three Higgs doubles and one Higgs singlet.

3.2.1 Higgs potential and CP violating phase

The three Higgs doublets ϕ_1,ϕ_2,ϕ_3,and one Higgs singlet $\widetilde{S}$ are denoted as

$$\begin{aligned}\phi_k &= e^{i\theta_k} H_k = e^{i\theta_k}\begin{pmatrix}\frac{1}{\sqrt{2}}(v_k + R_k + iA_k)\\ h_k^-\end{pmatrix};\\ \widetilde{S} &= e^{i\theta_s} S = e^{i\theta_s}\frac{1}{\sqrt{2}}(v_s + R_s + iA_s).\end{aligned} \tag{3.22}$$

Note that if we hope the axion be invisible, the vacuum expectation value v_s will be large because the interaction of axion is suppressed by $1/v_s$ [66, 67, 70, 71]. We introduce the PQ charge of fermions as the constraints to limit some of the terms in Higgs potential. The PQ charge of Higgs can be chosen as follows,

$$\phi_1 : +1,\ \phi_2 : +1,\ \phi_3 : -1,\ \widetilde{S} : +2. \tag{3.23}$$

We choose ϕ_3 to be ϕ_d in model(a) and ϕ_u in model(b). Using the Eq(3.23) and comparing the form of Yukawa couplings we also write down the PQ charge of fermions for model(a) and (b),

$$\text{Model(a)} \;\; Q_L : 0,\; U_R : -1,\; D_R : -1 \tag{3.24}$$

$$\text{Model(b)} \;\; Q_L : 0,\; U_R : +1,\; D_R : +1 \tag{3.25}$$

With Eq.(3.23) we write down the Higgs potential for three Higgs doublets and one Higgs singlet as follows in terms of H_1, H_2, H_3, and S

$$\begin{aligned}
V = & - m_1^2 H_1^\dagger H_1 - m_2^2 H_2^\dagger H_2 - m_3^2 H_3^\dagger H_3 - m_{12}^2 (H_1^\dagger H_2 e^{i\delta} + \text{h.c.}) \\
& - m_s^2 S^\dagger S + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_t (H_3^\dagger H_3)^2 + \lambda_s (S^\dagger S)^2 \\
& + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_3' (H_1^\dagger H_1)(H_3^\dagger H_3) + \lambda_3'' (H_2^\dagger H_2)(H_3^\dagger H_3) \\
& + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1) + \lambda_4' (H_1^\dagger H_3)(H_3^\dagger H_1) + \lambda_4'' (H_2^\dagger H_3)(H_3^\dagger H_2) \\
& + \frac{1}{2}\lambda_5 ((H_1^\dagger H_2)^2 e^{i2\delta} + \text{h.c.}) + \lambda_6 (H_1^\dagger H_1)(H_1^\dagger H_2 e^{i\delta} + \text{h.c.}) \\
& + \lambda_7 (H_2^\dagger H_2)(H_1^\dagger H_2 e^{i\delta} + \text{h.c.}) + \lambda_8 (H_3^\dagger H_3)(H_1^\dagger H_2 e^{i\delta} + \text{h.c.}) \\
& + d_{12} (H_1^\dagger H_2 e^{i\delta} + \text{h.c.}) S^\dagger S + g_{12} ((H_1^\dagger H_3)(H_3^\dagger H_2) e^{i\delta} + \text{h.c.}) \\
& + f_1 (H_1^\dagger H_1) S^\dagger S + f_2 (H_2^\dagger H_2) S^\dagger S + f_3 (H_3^\dagger H_3) S^\dagger S \\
& + f_{13} (H_1^\dagger H_3 S e^{i(\delta_s + \delta)} + \text{h.c.}) + f_{23} (H_1^\dagger H_3 S e^{i\delta_s} + \text{h.c.}),
\end{aligned} \tag{3.26}$$

where $\delta_s = \theta_3 + \theta_s - \theta_2$. The m's, λ's, f's, and g_{12} are the coefficients in Higgs potential, and all of them are real because in this model the CP violating phenomena is assumed to come from the spontaneous symmetry breaking.

By doing differentiation with respect to δ_s, we can extract one of the minimal condition,

$$f_{13} v_1 v_3 v_s \sin(\delta_s + \delta) + f_{23} v_2 v_3 v_s \sin\delta_s = 0, \tag{3.27}$$

which leads to the relation between δ_s and δ,

$$\tan\delta_s = -\frac{f_{13} v_1 \sin\delta}{f_{23} v_2 + f_{13} v_1 \cos\delta}. \tag{3.28}$$

From the formula above, it is obvious that the phase δ_s depends on the phase δ. If δ is zero, then δ_s vanish. That is, we can regard δ as the only source of CP violation in this model.

In the end of this section, we briefly discuss our model when it is concerned with leptons, also with the right handed neutrino. Using model(a) as the example the Lagrangian is written down as

$$\begin{aligned} L &= \overline{L}_L(Y_1 H_1 + Y_2 H_2 e^{i\delta})\nu_R + \overline{L}_L Y_3 \widetilde{H_3} e_R \\ &+ \overline{\nu}_R^C Y_s S e^{i(\theta_s - 2\theta_1)} \nu_R + \text{h.c.}. \end{aligned} \tag{3.29}$$

In above formula Higgs singlet S is coupled to right handed neutrino couplings, and the PQ charges for leptons are $L_L(0)$, $e_R(-1)$, and $\nu_R(-1)$. The phase $\theta_s - 2\theta_1$comes from VEV phase θ_s of S and phase θ_1 which is absorbed into ν_R.

The mass terms of this Lagrangian is

$$\begin{aligned} L_m &= -\overline{e}_L M_e e_R - \overline{\nu}_L M_D \nu_R \\ &- \frac{1}{2}\overline{\nu}_R^C M_R \nu_R + \text{h.c.}, \end{aligned} \tag{3.30}$$

where the M_e, M_ν, and M_R are

$$\begin{aligned} M_e &= -\frac{1}{\sqrt{2}} Y_3 v_3; \; M_D = -\frac{1}{\sqrt{2}}(Y_1 v_1 + Y_2 v_2 e^{i\delta}) \\ M_R &= -\sqrt{2} Y_s v_s e^{i(\theta_s - 2\theta_1)}. \end{aligned} \tag{3.31}$$

Here the mixing matrix corresponding to CKM matrix is the so-call Pontecove-Maki-Nakagawa-Sakata(PMNS) matrix [72, 73]. It has the relation $V_{\text{PMNS}} = V_L^e V_L^{\nu\dagger}$, with V_L^e and V_L^ν are the mixing matrix of e_L and ν_L respectively. We find that our model corresponding to leptons is more complicated than to quarks, because there is another Majorana mass matrix M_R which does not exist in quark couplings.

3.2.2 Mass matrices of Higgs

For phenomenological studies, the next step is to find the basis of states in which there are two Goldstone bosons in neutral mass matrix. One of which has its mass be eaten by Z boson, and the other one is the axion, and they can be easily removed. For the charge boson the basis we need is the charge Goldstone boson with its mass eaten by W boson. The Goldstone boson eaten by W and Z are not the physical states, and so we will erase them in the Lagrangian. Those Goldstone bosons can be related to the Higgs field as

$$\begin{aligned} h_w^- &= \frac{1}{v}(v_1 h_1^- + v_2 h_2^- + v_3 h_3^-) \\ h_z &= \frac{1}{v}(v_1 A_1 + v_2 A_2 + v_3 A_3) \\ a &= (-v_1 v_3^2 A_1 - v_2 v_3^2 A_2 + v_{12}^2 v_3 A_3 - v^2 v_s A_s)/N_a \end{aligned} \tag{3.32}$$

where the h_w^- and h_z are the Goldstone boson corresponding to the $W^\pm$ and Z^0, and the last one is the axion. The N_a in formula related to axion is $N_a = v\sqrt{v_{12}^2 v_3^2 + v^2 v_s^2}$.

Using Eq.(3.32) and to simplify the formula, we construct the rotation matrices to find the zero mass states,

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_s \end{pmatrix} = \begin{pmatrix} v_2/v_{12} & -v_1 v_3 v_s/N_A & v_1/v & -v_1 v_3^2/N_a \\ -v_1/v_{12} & -v_2 v_3 v_s/N_A & v_2/v & -v_2 v_3^2/N_a \\ 0 & v_{12}^2 v_s/N_A & v_3/v & v_{12}^2 v_3/N_a \\ 0 & v_{12}^2 v_3/N_A & 0 & -v^2 v_s/N_a \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ h_z \\ a \end{pmatrix},$$

$$\begin{pmatrix} h_1^- \\ h_2^- \\ h_3^- \end{pmatrix} = \begin{pmatrix} v_2/v_{12} & v_1 v_3/v v_{12} & v_1/v \\ -v_1/v_{12} & v_2 v_3/v v_{12} & v_2/v \\ 0 & -v_{12}/v & v_3/v \end{pmatrix} \begin{pmatrix} H_1^- \\ H_2^- \\ h_w^- \end{pmatrix}. \tag{3.33}$$

where $N_A = \sqrt{v_{12}^2(v_{12}^2 v_3^2 + v_s^2 v^2)}$.

For the real part neutral Higgs, there is no corresponding Goldstone boson. We rotate the real part Higgs the same way as that for imaginary part Higgs,

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_s \end{pmatrix} = \begin{pmatrix} v_2/v_{12} & -v_1 v_3 v_s/N_A & v_1/v & -v_1 v_3^2/N_a \\ -v_1/v_{12} & -v_2 v_3 v_s/N_A & v_2/v & -v_2 v_3^2/N_a \\ 0 & v_{12}^2 v_s/N_A & v_3/v & v_{12}^2 v_3/N_a \\ 0 & v_{12}^2 v_3/N_A & 0 & -v^2 v_s/N_a \end{pmatrix} \begin{pmatrix} H_1^0 \\ H_2^0 \\ H_3^0 \\ H_4^0 \end{pmatrix}. \tag{3.34}$$

The states after rotation are still not the physical states because the mass matrices are not diagonalized. Here we rotate them in order to find such the states h_w, h_z

and a. The rotational matrices we choose are for convenience, and the states after rotation are still the parity eigenstates.

After rotation we find that the charged mass matrix can be written in the basis (H_1^-, H_2^-, h_w^-) as follows

$$\begin{pmatrix} m^2_{H_1^+H_1^-} & m^2_{H_1^+H_2^-} & 0 \\ m^2_{H_2^+H_1^-} & m^2_{H_2^+H_2^-} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{3.35}$$

Note that $m^2_{H_1^+H_2^-} = (m^2_{H_2^+H_1^-})^*$ and the matrix elements are

$$\begin{aligned} m^2_{H_1^+H_1^-} &= -\frac{\csc\delta}{2v_1v_2^2v_{12}^2}[\sin\delta v_1v_2^2(v_{12}^4(\lambda_4-\lambda_5)+v_3^2(v_2^2\lambda_4'+v_1^2\lambda_4'')) \\ &- g_{12}v_1^2v_2^3v_3^2\sin 2\delta - \frac{f_{13}}{\sqrt{2}}v_3v_s(4v_1^2v_2^2\cos\delta\sin(\delta_s+\delta) \\ &+ 2\csc\delta_s(v_1^4\sin^2(\delta_s+\delta)+v_2^4\sin^2\delta_s))]; \end{aligned} \tag{3.36}$$

$$\begin{aligned} m^2_{H_1^+H_2^-} &= -\frac{v}{2v_2v_{12}^2}[(\lambda_4'-\lambda_4'')v_1v_2^2v_3 - g_{12}v_2v_3(v_1^2e^{-i\delta}-v_2^2e^{i\delta}) \\ &+ \sqrt{2}f_{13}v_s(v_1^2e^{i\delta_s}\frac{\sin(\delta_s+\delta)}{\sin\delta_s}+v_2^2e^{i(\delta_s+\delta)})]; \end{aligned} \tag{3.37}$$

$$\begin{aligned} m^2_{H_2^+H_2^-} &= -\frac{v^2}{2v_{12}^2v_3}[2g_{12}v_1v_2v_3\cos\delta \\ &+ (\lambda_4'v_3v_1^2-\sqrt{2}f_{13}v_1v_s\csc\delta_s\sin\delta+\lambda_4''v_2^2v_3)], \end{aligned} \tag{3.38}$$

where the unphysical state h_w^- is massless, and it also has no mixing with other two states.

Using the basis $(H_1^0, H_2^0, H_3^0, H_4^0, a_1, a_2, a_3, a)$ the neutral mass matrix can be written in the form

$$\begin{pmatrix} m^2_{H_1^0H_1^0} & m^2_{H_1^0H_2^0} & m^2_{H_1^0H_3^0} & m^2_{H_1^0H_4^0} & m^2_{H_1^0a_1} & m^2_{H_1^0a_2} & 0 & 0 \\ m^2_{H_2^0H_1^0} & m^2_{H_2^0H_2^0} & m^2_{H_2^0H_3^0} & m^2_{H_2^0H_4^0} & m^2_{H_2^0a_1} & 0 & 0 & 0 \\ m^2_{H_3^0H_1^0} & m^2_{H_3^0H_2^0} & m^2_{H_3^0H_3^0} & m^2_{H_3^0H_4^0} & m^2_{H_3^0a_1} & 0 & 0 & 0 \\ m^2_{H_4^0H_1^0} & m^2_{H_4^0H_2^0} & m^2_{H_4^0H_3^0} & m^2_{H_4^0H_4^0} & m^2_{H_4^0a_1} & 0 & 0 & 0 \\ m^2_{a_1H_1^0} & m^2_{a_1H_2^0} & m^2_{a_1H_3^0} & m^2_{a_1H_4^0} & m^2_{a_1a_1} & m^2_{a_1a_2} & 0 & 0 \\ m^2_{a_2H_1^0} & 0 & 0 & 0 & m^2_{a_2a_1} & m^2_{a_2a_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3.39}$$

This mass matrix is a real symmetry mass matrix and with off diagonal terms. It means that $m^2_{ij} = m^2_{ji}$. The non-zero matrix elements are listed as follows

$$
\begin{aligned}
m^2_{H_1^0 H_1^0} &= \frac{1}{2v_1 v_2^2 v_{12}^2}[4(\lambda_1 + \lambda_2 - \lambda_3 - \lambda_4) v_1^3 v_2^4 \\
&- 4(\lambda_6 - \lambda_7) v_1^2 v_2^3 (v_1^2 - v_2^2)\cos\delta \\
&+ \lambda_5 v_1 v_2^2 (v_{12}^4 + (v_1^2 - v_2^2)^2 \cos 2\delta) \\
&+ \sqrt{2} f_{13} v_3 v_s (v_1^4 \frac{\sin\delta}{\sin\delta_s} + v_{12}^2 \frac{1}{\sin\delta}(v_1^2 \sin(\delta_s + 2\delta) + v_2^2 \sin\delta_s))]; \quad (3.40) \\
m^2_{H_1^0 H_2^0} &= \frac{1}{2v_{12} N_A}[2v_1 v_2 v_3 v_s(-2\lambda_1 v_1^2 + 2\lambda_2 v_2^2 \\
&+ v_{12}^2(\lambda_3' - \lambda_3'' + \lambda_4' - \lambda_4'' + f_1 - f_2) + (\lambda_3 + \lambda_4)(v_1^2 - v_2^2)) \\
&+ 2\lambda_5 v_1 v_2 (v_1^2 - v_2^2) v_3 v_s \cos 2\delta \\
&+ 2 v_3 v_s \cos\delta(\lambda_6(v_1^4 - 3v_1^2 v_2^2) - \lambda_7(v_2^4 - 3v_1^2 v_2^2) \\
&- (\lambda_8 + d_{12} + g_{12})(v_1^4 - v_2^4)) \\
&+ \frac{f_{13} N_A^2}{\sqrt{2} v_2 v_{12}^2} \csc\delta_s((v_1^2 - v_2^2)\sin\delta + v_{12}^2 \sin(2\delta_s + \delta))]; \quad (3.41) \\
m^2_{H_1^0 H_3^0} &= \frac{1}{v_{12} v}[v_1 v_2 (2\lambda_1 v_1^2 - 2\lambda_2 v_2^2 - (\lambda_3 + \lambda_4)(v_1^2 - v_2^2) \\
&+ (\lambda_3' - \lambda_3'' + \lambda_4' - \lambda_4'') v_3^2 + \lambda_5(-v_1^2 + v_2^2)\cos 2\delta) \\
&+ (-\lambda_6 v_1^2 (v_1^2 - 3v_2^2) + \lambda_7 v_2^2(-3v_1^2 + v_2^2) \\
&- (\lambda_8 + g_{12})(v_1^2 - v_2^2) v_3^2)\cos\delta]; \quad (3.42)
\end{aligned}
$$

$$
\begin{aligned}
m^2_{H^0_1H^0_4} &= \frac{1}{vN_A}[v_1v_2((-2\lambda_1v_1^2+2\lambda_2v_2^2)v_3^2 \\
&+ (\lambda_3+\lambda_4)(v_1^2-v_2^2)v_3^2+(\lambda_3'-\lambda_3''+\lambda_4'-\lambda_4'')v_{12}^2v_3^2 \\
&- (f_1-f_2)v^2v_s^2+\lambda_5(v_1^2-v_2^2)v_3^2\cos 2\delta) \\
&+ (\lambda_6(v_1^4-3v_1^2v_2^2)-\lambda_7(v_2^4-3v_1^2v_2^2) \\
&- (\lambda_8+g_{12})(v_1^4-v_2^4))v_3^2\cos\delta+d_{12}v^2v_s^2\cos\delta]; \quad (3.43) \\
m^2_{H^0_1a_1} &= [(\lambda_6-\lambda_7)v_1v_2-\lambda_5(v_1^2-v_2^2)\cos\delta]\sin\delta; \quad (3.44) \\
m^2_{H^0_1a_2} &= -\frac{f_{13}N_A\sin(\delta+\delta_s)}{\sqrt{2}v_2v_{12}}; \quad (3.45) \\
m^2_{H^0_2H^0_2} &= \frac{1}{2v_3v_sN_A^2}[4v_3^3v_s^3(\lambda_1v_1^4+\lambda_2v_2^4+(\lambda_s+\lambda_t)v_{12}^4+(\lambda_3+\lambda_4)v_1^2v_2^2 \\
&- (\lambda_3'+\lambda_4')v_1^2v_{12}^2-(\lambda_3''+\lambda_4'')v_2^2v_{12}^2-(f_1v_1^2+f_2v_2^2-f_3v_{12}^2)v_{12}^2) \\
&+ 8v_1v_2(\lambda_6v_1^2+\lambda_7v_2^2-(\lambda_8+g_{12}+d_{12})v_{12}^2)v_3^3v_s^3\cos\delta \\
&+ 4\lambda_5v_1^2v_2^2v_3^3v_s^3\cos 2\delta+\sqrt{2}\frac{\sin\delta}{\sin\delta_s}f_{13}v_1(\frac{N_A^4}{v_{12}^4}+4v_{12}^2v_3^4v_s^2)]; \quad (3.46) \\
m^2_{H^0_2H^0_3} &= \frac{v_3v_s}{vN_A}[-2\lambda_1v_1^4-2\lambda_2v_2^4+2\lambda_tv_{12}^2v_3^2-2v_1^2v_2^2(\lambda_3+\lambda_4) \\
&+ (\lambda_3'+\lambda_4')v_1^2(v_{12}^2-v_3^2)+(\lambda_3''+\lambda_4'')v_2^2(v_{12}^2-v_3^2) \\
&- 2\lambda_5v_1^2v_2^2\cos 2\delta-2v_1v_2(2\lambda_6v_1^2+2\lambda_7v_2^2-\lambda_8(v_{12}^2-v_3^2))\cos\delta \\
&+ v_{12}^2(f_1v_1^2+f_2v_2^2+f_3v_3^2)-\sqrt{2}\frac{f_{13}}{v_s}v_1v_3v_{12}^2\frac{\sin\delta}{\sin\delta_s} \\
&+ 2v_1v_2(d_{12}v_{12}^2+g_{12}(v_{12}^2-v_3^2))\cos\delta]; \quad (3.47) \\
m^2_{H^0_2H^0_4} &= \frac{v_{12}^2v_3v_s}{vN_A^2}[2\lambda_1v_1^4v_3^2+2\lambda_2v_2^4v_3^2+2\lambda_tv_3^2v_{12}^4-2\lambda_sv_s^2v_{12}^2v^2 \\
&+ 2v_1^2v_2^2v_3^2(\lambda_3+\lambda_4)-2(\lambda_3'+\lambda_4')v_1^2v_{12}^2v_3^2-2(\lambda_3''+\lambda_4'')v_2^2v_3^2v_{12}^2 \\
&+ 2v_1^2v_2^2v_3^2\lambda_5\cos 2\delta+4v_1v_2v_3^2(\lambda_6v_1^2+\lambda_7v_2^2-\lambda_8v_{12}^2)\cos\delta \\
&+ (v_s^2v^2-v_3^2v_{12}^2)(f_1v_1^2+f_2v_2^2-f_3v_{12}^2) \\
&- \sqrt{2}\frac{f_{13}v_1}{v_3v_s}v_{12}^2(v_{12}^2v_3^2-v_s^2v^2)\frac{\sin\delta}{\sin(\delta_s+\delta)} \\
&+ 2v_1v_2(-2g_{12}v_{12}^2v_3^2+d_{12}(v_s^2v^2-v_{12}v_3^2))\cos\delta]; \quad (3.48) \\
m^2_{H_2a^0_1} &= \frac{v_{12}}{2N_A}[-2\lambda_5v_1v_2v_3v_s\sin 2\delta+2(-\lambda_6v_1^2-\lambda_7v_2^2 \\
&+ (\lambda_8+d_{12}+g_{12})v_{12}^2)v_3v_s\sin\delta+\sqrt{2}\frac{f_{13}}{v_2}\frac{N_A^2}{v_{12}^2}\sin(\delta+\delta_s)]; \quad (3.49)
\end{aligned}
$$

$$
\begin{aligned}
m^2_{H_3^0H_3^0} &= \frac{2}{v^2}[\lambda_1 v_1^4+\lambda_2 v_2^4+\lambda_t v_3^4 \\
&+ (\lambda_3+\lambda_4)v_1^2v_2^2+(\lambda_3'+\lambda_4')v_1^2v_3^2 \\
&+ (\lambda_3''+\lambda_4'')v_2^2v_3^2+\lambda_5 v_1^2v_2^2\cos 2\delta \\
&+ 2v_1v_2\cos\delta(\lambda_6 v_1^2+\lambda_7 v_2^2+\lambda_8 v_3^2+g_{12}v_3^2)]; \qquad (3.50)\\
m^2_{H_3^0H_4^0} &= \frac{v_{12}}{2v^2N_A}[-4v_3^2(\lambda_1 v_1^4+\lambda_2 v_2^4-\lambda_t v_{12}^2v_3^2)-4v_1^2v_2^2v_3^2(\lambda_3+\lambda_4) \\
&+ 2v_1^2v_3^2(v_{12}^2-v_3^2)(\lambda_3'+\lambda_4')+2v_2^2v_3^2(v_{12}^2-v_3^2)(\lambda_3''+\lambda_4'') \\
&- 4v_1v_2v_3^2(2\lambda_6 v_1^2+2\lambda_7 v_2^2-(\lambda_8+g_{12})(v_{12}^2-v_3^2)+d_{12}v_s^2)\cos\delta \\
&- 2v_s^2v^2(f_1v_1^2+f_2v_2^2+f_3v_3^2)+2\sqrt{2}v_1v_3v_sv^2\frac{\sin\delta}{\sin\delta_s}f_{13} \\
&- 4\lambda_5 v_1^2v_2^2v_3^2\cos 2\delta]; \qquad (3.51)\\
m^2_{H_3^0a_1} &= \frac{v_{12}}{v}[2\lambda_5 v_1v_2\cos\delta+\lambda_6 v_1^2+\lambda_7 v_2^2+\lambda_8 v_3^2+g_{12}v_3^2]\sin\delta; \qquad (3.52)
\end{aligned}
$$

$$
\begin{aligned}
m^2_{H_4^0H_4^0} &= \frac{v_{12}^2}{2v^2N_A^2}[4v_3^4(\lambda_1 v_1^4+\lambda_2 v_2^4)+4\lambda_t v_3^4v_{12}^4+4\lambda_s v^4v_s^4 \\
&+ 4v_1^2v_2^2v_3^4(\lambda_3+\lambda_4)-4v_1^2v_3^4v_{12}^2(\lambda_3'+\lambda_4')-4v_2^2v_3^4v_{12}^2(\lambda_3''+\lambda_4'') \\
&+ 4v_1^2v_2^2v_3^4\lambda_5\cos 2\delta+8v_1v_2v_3^4(\lambda_6 v_1^2+\lambda_7 v_2^2-\lambda_8 v_{12}^2)\cos\delta \\
&+ 4v_3^2v_s^2v^2(f_1v_1^2+f_2v_2^2-v_{12}^2f_3) \\
&+ 4\sqrt{2}v_1v_{12}^2v_3v^2v_s\frac{\sin\delta}{\sin\delta_s}f_{13}-8v_1v_2v_3^2(g_{12}v_{12}^2v_3^2-d_{12}v^2v_s^2)\cos\delta]; \\
m^2_{H_4^0a_1} &= \frac{-v_{12}^2}{vN_A}[(\lambda_6 v_1^2+\lambda_7 v_2^2-\lambda_8 v_{12}^2)v_3^2\sin\delta \\
&- (g_{12}v_{12}^2v_3^2-d_{12}v^2v_s^2)\sin\delta+\lambda_5 v_1v_2v_3^2\sin 2\delta]; \qquad (3.53)\\
m^2_{a_1a_1} &= \frac{1}{2v_1v_2^2v_{12}^2}[2\lambda_5 v_1v_2^2v_{12}^4\sin^2\delta \\
&+ \sqrt{2}v_3v_sf_{13}(v_1^2(v_1^2+2v_2^2)\cot\delta\sin(\delta_s+\delta) \\
&+ v_1^4\cot\delta_s\sin(\delta+\delta_s)+v_2^4\frac{\sin\delta_s}{\sin\delta})]; \\
m^2_{a_1a_2} &= \frac{f_{13}N_A}{2\sqrt{2}v_2v_{12}^3}[(v_1^2-v_2^2)\frac{\sin\delta}{\sin\delta_s}+v_{12}^2\frac{\sin(2\delta_s+\delta)}{\sin\delta_s}]; \\
m^2_{a_2a_2} &= \frac{f_{13}v_1N_A^2}{\sqrt{2}v_{12}^4v_3v_s}\frac{\sin\delta}{\sin\delta_s}. \qquad (3.54)
\end{aligned}
$$

The non-zero elements $m^2_{H_ia_j}$ mix real and imaginary part of neutral Higgs field, and they violate CP.

3.3 The Yukawa couplings

Before discussion the phenomenology, we show the Lagrangian for the quarks couplings with Higgs as

$$\begin{aligned}
\mathcal{L}_Y^{(a)} &= \overline{U}_L[\hat{M}_u\frac{v_1}{v_{12}v_2} - (\hat{M}_u - V_{\mathrm{CKM}}\mathrm{Im}(V_{\mathrm{CKM}}^\dagger)\hat{M}_u\frac{e^{i\delta}}{\sin\delta})\frac{v_{12}}{v_1v_2}]U_R(H_1^0 + ia_1) \\
&+ \overline{U}_L\hat{M}_uU_R[\frac{v_3}{v_{12}v}(H_2^0 + ia_2) - \frac{1}{v}H_3^0 + \frac{v_3^2}{v^2v_s}(H_4^0 + ia)] \\
&- \overline{D}_L\hat{M}_uD_R[\frac{v_{12}}{v_3v}(H_2^0 - ia_2) + \frac{1}{v}H_3^0 + \frac{v_{12}^2}{v^2v_s}(H_4^0 - ia)] \\
&+ \sqrt{2}\overline{D}_L[V_{\mathrm{CKM}}^\dagger\hat{M}_u\frac{v_1}{v_2v_{12}} - (V_{\mathrm{CKM}}^\dagger\hat{M}_u - \mathrm{Im}(V_{\mathrm{CKM}}^\dagger)\hat{M}_u\frac{e^{i\delta}}{\sin\delta})\frac{v_{12}}{v_1v_2}]U_RH_1^- \\
&- \sqrt{2}\frac{v_3}{v_{12}v}\overline{D}_LV_{\mathrm{CKM}}^\dagger\hat{M}_uU_RH_2^- - \sqrt{2}\frac{v_{12}}{vv_3}\overline{U}_LV_{\mathrm{CKM}}\hat{M}_dD_RH_2^+ + \mathrm{h.c.}; \\
\mathcal{L}_Y^{(b)} &= \overline{D}_L[\hat{M}_d\frac{v_1}{v_{12}v_2} - (\hat{M}_d + V_{\mathrm{CKM}}^\dagger\mathrm{Im}(V_{\mathrm{CKM}})\hat{M}_d\frac{e^{-i\delta}}{\sin\delta})\frac{v_{12}}{v_1v_2}]D_R(H_1^0 - ia_1) \\
&+ \overline{D}_L\hat{M}_dD_R[\frac{v_3}{v_{12}v}(H_2^0 - ia_2) - \frac{1}{v}H_3^0 + \frac{v_3^2}{v^2v_s}(H_4^0 - ia)] \\
&- \overline{U}_L\hat{M}_uU_R[\frac{v_{12}}{v_3v}(H_2^0 + ia_2) + \frac{1}{v}H_3^0 + \frac{v_{12}^2}{v^2v_s}(H_4^0 + ia)] \\
&- \sqrt{2}\overline{U}_L[V_{\mathrm{CKM}}\hat{M}_d\frac{v_1}{v_2v_{12}} - (V_{\mathrm{CKM}}\hat{M}_d + \mathrm{Im}(V_{\mathrm{CKM}})\hat{M}_d\frac{e^{-i\delta}}{\sin\delta})\frac{v_{12}}{v_1v_2}]D_RH_1^+ \\
&+ \sqrt{2}\frac{v_3}{v_{12}v}\bar{U}_LV_{\mathrm{CKM}}\hat{M}_dD_RH_2^+ + \sqrt{2}\frac{v_{12}}{vv_3}\overline{D}_LV_{\mathrm{CKM}}^\dagger\hat{M}_uU_RH_2^- + \mathrm{h.c.}. \qquad (3.55)
\end{aligned}$$

The above formula shows that the FCNC process is produced from the H_1 and a_1 exchange, because for H_1,a_1 couplings there is a non-diagonal coupling matrix proportional to $V_{\mathrm{CKM}}\mathrm{Im}(V_{\mathrm{CKM}}^\dagger)\hat{M}_ue^{i\delta}/(\sin\delta)$ for up-type quarks in model(a). In model(b) has the same situation with $-V_{\mathrm{CKM}}^\dagger\mathrm{Im}(V_{\mathrm{CKM}})\hat{M}_de^{-i\delta}/(\sin\delta)$ for down-type quarks. Also note that the flavor conserving interaction with H_4^0 and a can be neglected because of the small factor $1/v_s$.

The FCNC coupling matrices for PDG parametrization are expressed as follows

$$\text{For model(a):}\quad V_{\mathrm{CKM}}\mathrm{Im}(V^{\dagger}_{\mathrm{CKM}})\hat{M}_u\frac{e^{i\delta}}{\sin\delta} = \begin{pmatrix} c_{13}^2 & -s_{23}s_{13}c_{13} & -c_{23}^2 s_{13}c_{13} \\ -s_{23}s_{13}c_{13} & s_{23}^2 s_{13}^2 & s_{23}c_{23}s_{13}^2 \\ -c_{23}s_{13}c_{13} & s_{23}c_{23}s_{13}^2 & c_{23}^2 s_{13}^2 \end{pmatrix}\hat{M}_u \tag{3.56}$$

$$\text{For model(b):}\quad -V^{\dagger}_{\mathrm{CKM}}\mathrm{Im}(V_{\mathrm{CKM}})\hat{M}_d\frac{e^{-i\delta}}{\sin\delta} = \begin{pmatrix} c_{12}^2 & s_{12}c_{12} & 0 \\ s_{12}c_{12} & s_{12}^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}\hat{M}_d \tag{3.57}$$

where Eq.(3.56) is FCNC related coupling matrix for up-type quarks in model(a) and Eq.(3.57) is for down-type quarks in model(b). From Eq. (3.56) we find that there exist all the mixing contributions between $u-c$, $u-t$, $c-t$. However, when considering the meson mixing, there are no meson which is constructed by t quark, we only apply the $u-c$ couplings to neutral meson mixing later. The model(b) FCNC coupling matrices shown in Eq.(3.57) produce only the $d-s$ couplings.

The FCNC coupling matrices for KM parametrization are

$$V_{\mathrm{CKM}}\mathrm{Im}(V^{\dagger}_{\mathrm{CKM}})\hat{M}_u\frac{e^{i\delta}}{\sin\delta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & s_2^2 & -s_2c_2 \\ 0 & -s_2c_2 & c_2^2 \end{pmatrix}\hat{M}_u; \tag{3.58}$$

$$-V^{\dagger}_{\mathrm{CKM}}\mathrm{Im}(V_{\mathrm{CKM}})\hat{M}_d\frac{e^{-i\delta}}{\sin\delta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & s_3^2 & -s_3c_3 \\ 0 & -s_3c_3 & c_3^2 \end{pmatrix}\hat{M}_d. \tag{3.59}$$

These formulae show that there are only $s-b$ and $c-t$ couplings in KM parametrization.

4. SOME IMPLICATIONS

After building this model, we try to connect it to some experimental result and find whether this model is consistent with experimental data. We concentrate on effects for neutral meson mixing and neutron electric dipole moment. The multi-Higgs Yukawa coupling model we built leads to the FCNC phenomena, which makes contribution to the neutral meson mixing.

4.1 Neutral meson mixing

The FCNC from Higgs provides the tree level contribution for neutral meson oscillation. It occurs by the exchange of scalar Higgs, pseudoscalar Higgs, or both scalar and pseudoscalar Higgs with the cross terms between them. We write down the Yukawa interaction and the quadratic Higgs interaction for H_1^0 and a_1 as following form

$$\mathcal{L} = \bar{q}_i(a_{ij} + b_{ij}\gamma_5)q_j H_1^0 + i\bar{q}_i(b_{ij} + a_{ij}\gamma_5)q_j a_1 + \lambda_{H_1^0 a_1} H_1^0 a_1, \tag{4.1}$$

where $a_{ij},\ b_{ij}$ are coupling constants of $H_1^0,\ a_1$, and $i,\ j$ quarks, and $\lambda_{H_1^0 a_1}$ is the mixing term between H_1^0 and a_1, with $\lambda_{H_1^0 a_1} = 2m^2_{H_1^0 a_1}$.

The total amplitude for mixing from the sum of three contribution which are shown in Fig 4.1 can be written as follows

$$\begin{aligned}\mathcal{M} = & - \frac{1}{m^2_{H_1^0}}\,\bar{q}_i(a_{ij} + b_{ij}\gamma_5)q_j\bar{q}_i(a_{ij} + b_{ij}\gamma_5)q_j \\ & + \frac{1}{m^2_{a_1}}\,\bar{q}_i(b_{ij} + a_{ij}\gamma_5)q_j\bar{q}_i(b_{ij} + a_{ij}\gamma_5)q_j \\ & - i\frac{\lambda_{H_1^0 a_1}}{m^2_{a_1}m^2_{H_1}}\,\bar{q}_i(a_{ij} + b_{ij}\gamma_5)q_j\bar{q}_i(b_{ij} + a_{ij}\gamma_5)q_j.\end{aligned} \tag{4.2}$$

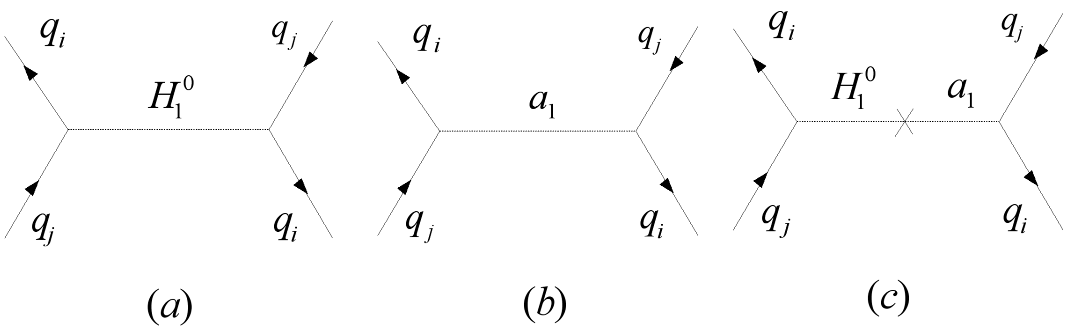

Fig. 4.1: Mixing due to the exchange by (a) H_1^0, (b) a_1, and (c) both H_1^0 and a_1

This amplitude is the sum of the contribution of the s-channel diagrams. The t-channel contribution also needs to be considered. So the total amplitude is the s-channel contribution in Eq.(4.2) with that of Fierz transformation for fermionic fields together. The fermionic Fierz transformation for scalar interaction is $S \to -\frac{1}{4}(S + V + T - A + P)$, with S, V, T, A, P which are the interaction of scalar, vector, tensor, axial vector, and pseudoscalar, respectively. In other words, the $\bar{q}_i(a_{ij} + b_{ij}\gamma_5)q_j\bar{q}_i(a_{ij} + b_{ij}\gamma_5)q_j$ is seen as scalar interaction and can be transformed into

$$\begin{aligned}\frac{1}{3} &\times (-\frac{1}{4})[(\bar{q}_i(a_{ij} + b_{ij}\gamma_5)q_j\bar{q}_i(a_{ij} + b_{ij}\gamma_5)q_j) \\ &+ (\bar{q}_i\gamma_\mu(a_{ij} + b_{ij}\gamma_5)q_j\bar{q}_i\gamma^\mu(a_{ij} + b_{ij}\gamma_5)q_j) \\ &- (\bar{q}_i\gamma_\mu\gamma_5(a_{ij} + b_{ij}\gamma_5)q_j\bar{q}_i\gamma^\mu\gamma_5(a_{ij} + b_{ij}\gamma_5)q_j) \\ &+ (\bar{q}_i\gamma_5(a_{ij} + b_{ij}\gamma_5)q_j\bar{q}_i\gamma_5(a_{ij} + b_{ij}\gamma_5)q_j)], \qquad (4.3)\end{aligned}$$

where the factor 1/3 comes from the constraint of the colorless meson, and note that there are no tensor contribution in this formula. Then we put the amplitude in Eq.(4.2) with the Fierz transformation in above discussion into the scattering amplitude matrix element $\mathcal{M}_{12} = \langle\bar{P}|\mathcal{M}|P\rangle$. We get the matrix element for neutral meson mixing terms from our tree level contribution as follows,

$$\begin{aligned} M_{12} &= \frac{1}{m^2_{H^0_1}}[(b^2_{ij}-\frac{1}{12}(a^2_{ij}+b^2_{ij}))\frac{f_P^2 m_P^3}{(m_i+m_j)^2}+\frac{1}{12}(b^2_{ij}-a^2_{ij})f_P^2 m_P]-\frac{1}{m^2_{a_1}}[(a^2_{ij} \\ &- \frac{1}{12}(a^2_{ij}+b^2_{ij}))\frac{f_P^2 m_P^3}{(m_i+m_j)^2}+\frac{1}{12}(a^2_{ij}-b^2_{ij})f_P^2 m_P] \\ &+ \frac{i2m^2_{H^0_1 a_1}}{m^2_{H^0_1}m^2_{a_1}}\frac{5a_{ij}b_{ij}}{6}\frac{f_P^2 m_P^3}{(m_i+m_j)^2}, \end{aligned} \tag{4.4}$$

where m_P and f_P are the mass and decay constant of meson. Note that the term with $m^2_{H^0_1 a_1}$ is the imaginary part in $\mathcal{M}_{12}$. So it has no contribution to Δm, and also it will cause CP violation in meson mixing. We will mention this later in $K^0-\overline{K^0}$. The quantity $x=\Delta m/\Gamma=2M_{12}/\Gamma$ is useful when we discuss the meson mixing, where Δm is the mass difference in neutral meson, and Γ is the decay width of the meson.

Without considering t quark interaction, the non-zero off diagonal matrix elements of a_{ij} and b_{ij} are shown as follows

PDG model(a)

$$a_{12} = \frac{v_{12}}{2v_1v_2}[-s_{23}s_{13}c_{13}m_c];\quad b_{12}=\frac{v_{12}}{2v_1v_2}[-s_{23}s_{13}c_{13}m_c];$$

PDG model(b)

$$a_{12} = \frac{v_{12}}{2v_1v_2}[s_{12}c_{12}m_s];\quad b_{12}=\frac{v_{12}}{2v_1v_2}[s_{12}c_{12}m_s];$$

KM model(b)

$$a_{23} = \frac{v_{12}}{2v_1v_2}[-s_3c_3m_b];\quad b_{23}=\frac{v_{12}}{2v_1v_2}[-s_3c_3m_b]. \tag{4.5}$$

Note that in above formulae we use the relation $m_u \ll m_c \ll m_t$; $m_d \ll m_s \ll m_b$, and $a_{ij}=a_{ji}$; $b_{ij}=-b_{ji}$.

In numerical analysis the quark masses we are using [74] $m_u(1\text{GeV})=5\text{MeV}$, $m_d(1\text{GeV})=10\text{MeV}$, $m_s(1\text{GeV})=187\text{MeV}$, $m_c(m_c)=1.30\text{GeV}$, $m_b(m_b)=4.34\text{GeV}$ and $m_t=174\text{GeV}$. The meson decay constants which we take are [27] $f_K=156\text{MeV}$, $f_D=201\text{MeV}$, $f_{B_s}=260\text{MeV}$.

$D^0-\overline{D^0}$ mixing

Using the PDG parametrization, for model (a) we only discuss $D^0-\overline{D^0}$ mixing because the mesons with t quark have not been found yet. Here we define

$\tan\beta = v_1/v_2$. BABAR [75] and BELLE [76, 77] experimental results give $x = (5.5 \pm 2.2) \times 10^{-3}$ [78]. Theoretically, we have

$$x \approx 7.5 \times 10^{-5} \frac{1}{\sin^2 2\beta v_{12}^2} \left(\frac{1}{m_{H_1^0}^2} - \frac{1}{m_{a_1}^2}\right)(100\mathrm{GeV})^4 \tag{4.6}$$

The effective Higgs mass, which has the relation with the scalar and pseudoscalar Higgs $1/m_{H_{\text{eff}}}^2 = 1/m_{H_1^0}^2 - 1/m_{a_1}^2$, can be with order 100GeV if one choose $\tan\beta = 40$.

$\underline{K^0 - \overline{K}^0 \text{ mixing}}$

In model(a) there is no contribution to this meson mixing, so we consider the model(b). The only nonzero off-diagonal element is a_{21} and b_{21} which is related to $K^0 - \overline{K}^0$ mixing. The contribution to the this mixing is

$$\frac{\Delta m_K}{m_K} = 4.4 \times 10^{-12} \frac{1}{\sin^2 2\beta v_{12}^2} \left(\frac{1}{m_{H_1^0}^2} - \frac{1}{m_{a_1}^2}\right)(100\mathrm{GeV})^4. \tag{4.7}$$

Using the PDG fit [12] for Δm_K we get $\Delta m_K/m_K = 7.0 \times 10^{-15}$. We find that the effective Higgs mass should be at the scale of order TeV.

From Eq.(4.4), the ratio of $\mathrm{Im}\mathcal{M}_{12}$ to $\mathrm{Re}\mathcal{M}_{12}$ is written as

$$\left|\frac{\mathrm{Im}\mathcal{M}_{12}}{\mathrm{Re}\mathcal{M}_{12}}\right| = \left|\frac{2m_{H_1^0 a_1}^2}{m_{H_1^0}^2 - m_{a_1}^2}\right|. \tag{4.8}$$

Using the experimental value for ϵ in neutral K meson mixing [79] we derive the bound

$$\left|\frac{2m_{H_1^0 a_1}^2}{m_{H_1^0}^2 - m_{a_1}^2}\right| < 6 \times 10^{-3}; \tag{4.9}$$

This bounds will constrain the neutron electric dipole moment from exchange of H_1^0 and a_1^0. We will discuss in next section.

$\underline{B_s^0 - \overline{B}_s^0 \text{ mixing}}$

Here we discuss the KM parametrization with the model(b) because there is no down-type quark mixing by neutral Higgs in other models. The non-zero elements are s and b quark mixing, which is corresponding to the $B_s^0 - \overline{B}_s^0$ mixing. The couplings a_{32} and b_{32} are from Eq.(4.5). Then we have

$$\frac{\Delta m_{B_s}}{m_{B_s}} = 9.5 \times 10^{-12} \frac{1}{\sin^2 2\beta v_{12}^2} \left(\frac{1}{m_{H_1^0}^2} - \frac{1}{m_{a_1}^2}\right)(100\mathrm{GeV})^4 \tag{4.10}$$

If we choose $v_1 = v_2 = v_3$ and using the experimental value for $\Delta m_{B_s} = 17.77\text{ps}^{-1}$ [12], with the estimation that the new physics is allowed to give contribution about 10% of them [80, 81, 82]. Then the mass of Higgs can be about 300GeV.

The above discussion for neutral meson anti-meson mixing provides the bounds for neutral Higgs mass. For model(a) with PDG parametrization the Higgs mass with hundred GeV is allowed in $D^0 - \overline{D^0}$, and for model(b) the Higgs mass can be with the order TeV from $K^0 - \overline{K^0}$ mixing. In KM parametrization, experimental data for Δm_{B_s} makes the Higgs mass with lower bound of 300 GeV.

4.2 Electric dipole moment of neutron

The experimental upper bound for neutron EDM we mentioned in previous is $0.29 \times 10^{-25}\text{e cm}$ ($CL = 90\%$), which is larger for comparing with the standard model prediction. In our model we will use the parameters like VEVs and Higgs mass in previous neutral meson mixing discussion as input to examine whether the neutral EDM we calculated can be close to the experimental upper bound.

At first we consider the one loop contribution to quark EDM. Note that in our model, the exchange of only one Higgs we obtained can not produce the quark EDM because all the couplings with H's and a's are real and pure imaginary, respectively. So we discuss the contribution shown as Fig.4.2(a), in which the EDM contribution comes from the cross terms between H_0^i and a_k. For example, with the flavor conserving interaction, the u quark EDM is generated by exchange of a u quark in the loop. Using H_1^0 and a_1 as the example, the contribution is shown as

$$\begin{aligned} d_u^{H_1^0 a_1} &= \frac{e(2/3)}{32\pi^2}\frac{m_u m^2_{H_1^0 a_1}}{m^2_{H_1^0} - m^2_{a_1}}(-2(\frac{v_1 m_u}{v_{12} v_2} \\ &- s_{13}^2 \frac{v_{12} m_u}{v_1 v_2})^2)[f(m^2_{H_1}, m_u^2) - f(m^2_{a_1}, m_u^2)], \qquad (4.11) \\ \text{where } f(x,y) &= 2\int_0^1 dz \frac{z^2}{x(1-z) + z^2 y}. \end{aligned}$$

This formula shows that the one loop contribution is small because it is proportional to m_u^3 which is small.

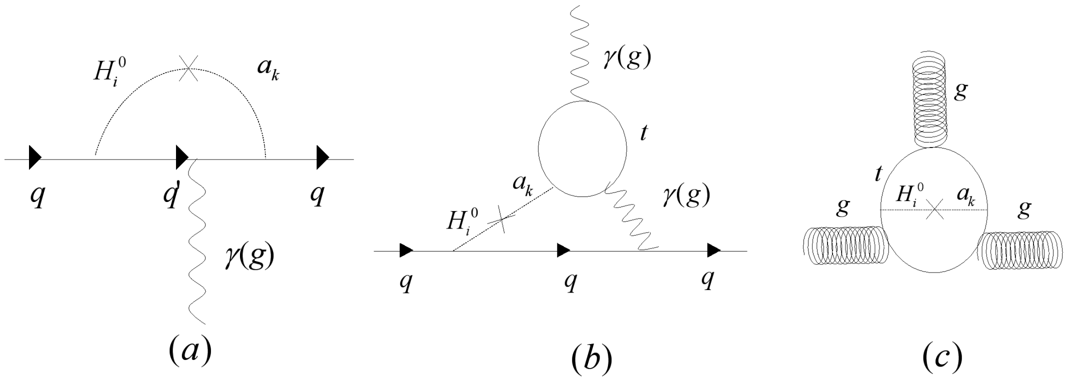

Fig. 4.2: The neutron EDM contribution from (a) quark EDM for q with one loop diagram. The cross sign means the interaction between H_i^0 and a_k (b) quark EDM at two loop diagram, and (c) gluon color EDM

We therefore consider the three dominant two-loop contribution as shown in Fig.4.2(b), the electromagnetic operator $\mathcal{O}^\gamma$ [83, 84], the color EDM $\mathcal{O}^C$ [83, 84], and the gluon color EDM operator $\mathcal{O}^g$ in Fig.4.2(c) proposed by Weinberg [85, 86], which is often called Weinberg operator. These operators are written as

$$\begin{aligned} O^\gamma &= -\frac{d_q}{2} i\bar{q}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu} q, \; O^C = -\frac{f_q}{2} i g_s \bar{q}\sigma_{\mu\nu}\gamma_5 G^{\mu\nu} q, \\ O^g &= -\frac{1}{6} C f_{abc} G^a_{\mu\nu} G^b_{\mu\alpha} \widetilde{G}^c_{\nu\alpha} \end{aligned} \tag{4.12}$$

The corresponding electric dipole moment contributions from Eq.(4.12) are written in the form [61, 62, 63]

$$d_n^\gamma = \eta_d [\frac{4}{3} d_d - \frac{1}{3} d_u]_\Lambda \quad ; \quad d_n^C = e\eta_f [\frac{4}{9} f_d + \frac{2}{9} f_u]_\Lambda; \tag{4.13}$$

$$d_n^g \approx \frac{eM}{4\pi} \xi C, \tag{4.14}$$

where d_n^γ is the radiative contribution from O^γ; d_n^C is the gluon emitted contribution from $\mathcal{O}^C$. d_q, f_q are the contribution to neutron EDM from photon and gluon radiative contribution to quark q respectively, and the subscript Λ indicates that the hadronic energy scale. Eq.(4.14) is the approximation contribution for the color EDM of gluon operator $\mathcal{O}_g^C$, and $M = 1.190$GeV indicates the scale related to the chiral symmetry breaking. The factor C will be defined later. The η_d and η_f [87, 88]

are

$$\begin{aligned}\eta_d &= (\frac{\alpha_s(M_Z)}{\alpha_s(m_b)})^{16/23}(\frac{\alpha_s(m_b)}{\alpha_s(m_c)})^{16/25}(\frac{\alpha_s(m_c)}{\alpha_s(\Lambda)})^{16/27} \approx 0.166,\\ \eta_f &= (\frac{\alpha_s(M_Z)}{\alpha_s(m_b)})^{14/23}(\frac{\alpha_s(m_b)}{\alpha_s(m_c)})^{14/25}(\frac{\alpha_s(m_c)}{\alpha_s(\Lambda)})^{14/27} \approx 0.0117, \qquad (4.15)\end{aligned}$$

which are related to the strong running couplings on scale $m_c,\ m_b,\ m_t,\ M_Z$, and Λ. Also ξ is [89, 90]

$$\begin{aligned}\xi &= (\frac{g(\Lambda)}{4\pi})^3(\frac{\alpha_s(m_b)}{\alpha_s(m_t)})^{-54/23}(\frac{\alpha_s(m_c)}{\alpha_s(m_b)})^{-54/25}(\frac{\alpha_s(\Lambda)}{\alpha_s(m_c)})^{-54/27}\\ &\approx 1.2\times 10^{-4}, \qquad (4.16)\end{aligned}$$

where $g(\Lambda) = 4\pi/6$ [85] is the strong coupling constant at hadronic scale.

The quark EDM q_i, quark color EDM f_i and the factor C in gluon color EDM formula Eq.(4.14) are written as follows

$$d_q = \frac{e\alpha_{em}Q_q}{24\pi^3}m_q G_q;\ \ f_q = \frac{\alpha_s}{64\pi^3}m_q G_q;\ \ C = \frac{1}{8\pi}H_g, \qquad (4.17)$$

where m_q is the mass of quark and Q_q is the charge of quark, and $\alpha_{em},\ \alpha_s$ are electromagnetic coupling constant and strong coupling constant respectively. The factor G_q and H_g are defined as

$$\begin{aligned}G_q &= [(f\left(\frac{m_t^2}{m_{H_l^0}^2}\right) - f\left(\frac{m_t^2}{m_{a_k}^2}\right))\mathrm{Im}Z_{tq}^{lk}\\ &+ (g\left(\frac{m_t^2}{m_{H_l^0}^2}\right) - g\left(\frac{m_t^2}{m_{a_k}^2}\right))\mathrm{Im}Z_{qt}^{lk}\ ];\\ H_g &= (h\left(\frac{m_t^2}{m_{H_l^0}^2}\right) - h\left(\frac{m_t^2}{m_{a_k}^2}\right))\mathrm{Im}Z_{tt}^{lk}, \qquad (4.18)\end{aligned}$$

where $\mathrm{Im}Z_{ij}^{lk} = Y_{ij}^{lk}\lambda_{lk}$, with $Y_{ij}^{lk} = 2a_{ii}^l d_{jj}^k/(m_i m_j)$ and $\lambda_{lk} = m^2_{H_l^0 a_k}/(m^2_{H_l^0} - m^2_{a_k})$. The functions $f,\ g$,and h are

$$\begin{aligned}f(z) &= \frac{z}{2}\int_0^1 dx\frac{1-2x(1-x)}{x(1-x)-z}\ln\frac{x(1-x)}{z};\\ g(z) &= \frac{z}{2}\int_0^1 dx\frac{1}{x(1-x)-z}\ln\frac{x(1-x)}{z};\\ h(z) &= \frac{z^2}{2}\int_0^1 dx\int_0^1 du\frac{u^3x^3(1-x)}{[zx(1-ux)+(1-u)(1-x)]^2}. \qquad (4.19)\end{aligned}$$

Summation of Eq.(4.13) and Eq.(4.14) is the totally contribution to the neutron EDM. That is

$$d_n = d_n^\gamma + d_n^C + d_n^g. \tag{4.20}$$

and we only consider the flavor conserving interaction because the flavor violating contribution is suppressed by s_{12},s_{23},s_{13} for PDG parametrization, or s_1,s_2,s_3 for KM parametrization.

From Eq.(3.40) to Eq.(3.54), we find that for mass mixing terms of scalar and pseudoscalar, there are $m^2_{H_1^0 a_1}$, $m^2_{H_1^0 a_2}$, $m^2_{H_2^0 a_1}$, $m^2_{H_3^0 a_1}$, and $m^2_{H_4^0 a_1}$ which are nonzero. We will not consider the $H_4^0 - a_1$ contribution because the the factor $1/v_s$ in Yukawa couplings suppresses the H_4^0 and a contribution. For model(a) with PDG parametrization, we use H_3^0, a_1 as an example. Writing down all Y_{ij}^{31} in following with i, j indicating quarks,

$$\begin{aligned} Y_{tu}^{31} &= 2(-\frac{1}{v})(\frac{v_1}{v_{12}v_2}); \; Y_{ut}^{31} = 2(-\frac{1}{v})(\frac{v_1}{v_{12}v_2} - \frac{v_{12}}{v_1 v_2}); \\ Y_{tt}^{31} &= 2(-\frac{1}{v})(\frac{v_1}{v_{12}v_2} - \frac{v_{12}}{v_1 v_2}); \\ Y_{td}^{31} &= 0; \; Y_{dt}^{31} = 2(-\frac{1}{v})(\frac{v_1}{v_{12}v_2}). \end{aligned} \tag{4.21}$$

Using the input from previous $D^0 - \overline{D^0}$ discussion for this model with $\tan\beta = 40$, $v_{12} = 240$GeV, and $v_3 = 10$GeV. When the neutral Higgs mass is about order 100GeV, we substitute the functions $f,\ g,\ h$ difference between input by $m_t^2/m_{H_l}^2$ and $m_t^2/m_{a_k}^2$ by $(\Delta f, \Delta g, \Delta h) = (1, 2, 0.1)$.

Substituting Eq.(4.21) into Eq.(4.17, 4.18, 4.13, 4.14), we obtain the relation

$$d_n(H_3^0 - a_1) \approx -3 \times 10^{-25} \frac{m^2_{H_3^0 a_1}}{m^2_{H_3^0} - m^2_{a_1}} \text{ e cm}. \tag{4.22}$$

For the other three kinds of Higgs pair exchange

$$\begin{aligned} d_n(H_2^0 - a_1) &\approx -2 \times 10^{-26} \frac{m^2_{H_2^0 a_1}}{m^2_{H_2^0} - m^2_{a_1}} \text{ e cm}; \\ d_n(H_1^0 - a_1) &\approx -2 \times 10^{-26} \frac{m^2_{H_1^0 a_1}}{m^2_{H_1^0} - m^2_{a_1}} \text{ e cm}; \\ d_n(H_1^0 - a_2) &\approx 8 \times 10^{-27} \frac{m^2_{H_1^0 a_2}}{m^2_{H_1^0} - m^2_{a_2}} \text{ e cm}. \end{aligned} \tag{4.23}$$

So neutron electric dipole moment is dominated by the contribution of $H_3^0 - a_1$ exchange. At this moment $\lambda_{31} = m^2_{H_3^0 a_1}/(m^2_{H_3^0} - m^2_{a_1}) \lesssim 0.1$ is required.

In model(b) with PDG parametrization, we note that there is no up-type quarks coupling with H_1^0 and a_1. If we take the neutral Higgs mass to be with order TeV, we choose $(\Delta f, \Delta g, \Delta h) = (0.2, 0.2, 0.03)$, and then treat the $H_2^0 - a_1$ contribution as

$$d_n \approx -1 \times 10^{-26} \frac{m^2_{H_1^0 a_2}}{m^2_{H_1^0} - m^2_{a_2}} \,\mathrm{e\,cm} \tag{4.24}$$

In KM parametrization of model(a), we consider the $H_1^0 - a_2$ process. If the choice for VEVs is $v_1 = v_2 = v_3$, with the Higgs mass to be 100GeV, then the contribution to neutron EDM for $H_1^0 - a_2$ exchange is

$$d_n \approx 7 \times 10^{-26} \frac{m^2_{H_1^0 a_2}}{m^2_{H_1^0} - m^2_{a_2}} \,\mathrm{e\,cm} \tag{4.25}$$

For small $\lambda_{12} \lesssim 0.4$ this contribution can saturated the upper bound of neutron EDM. Also note that from CP phenomenon in $K^0 - \overline{K^0}$ mixing $H_1^0 - a_1$ contribution is small.

For model(b), $H_1^0 - a_1$ interaction are also not including the interaction with top quarks, so the this interaction will not give the dominate contribution. Taking Higgs mass 100GeV and $v_1 = v_2 = v_3$. The $H_1^0 - a_2$ gives

$$d_n \approx 1 \times 10^{-25} \frac{m^2_{H_1^0 a_2}}{m^2_{H_1^0} - m^2_{a_2}} \,\mathrm{e\,cm}. \tag{4.26}$$

If we choose Higgs mass about 300 GeV, which is the same condition as that for $B_s^0 - \overline{B_s^0}$ mixing discussion. The contribution to neutron EDM will be small

$$d_n \approx 4 \times 10^{-26} \frac{m^2_{H_1^0 a_2}}{m^2_{H_1^0} - m^2_{a_2}} \,\mathrm{e\,cm}. \tag{4.27}$$

Using $\lambda_{12} \lesssim 0.7$ the result can be close to the upper bound.

In above discussion, we treat the two loop contribution to neutron electric dipole moment. Using PDG parametrization in model(a), with effective neutral Higgs mass about 100GeV and $\lambda_{31} \lesssim 0.1$, the result can be close to the experimental bounds. In model(b), the effective Higgs mass we choose is 1TeV, which is the same as that in $K^0 - \overline{K^0}$ mixing. For KM parametrization, we take Higgs mass about 100GeV in model(a) with $\lambda_{12} \lesssim 0.4$ and 300GeV in model(b) with $\lambda_{12} \lesssim 0.7$ to get close results to experimental bound of neutron EDM.

5. CONCLUSION

The CKM matrix can not deal with problems from the baryogenesis, and also it can not deal with the question where CP violation come from. So another source for CP violation is required. With more than one Higgs doublets, these problems may be answered. CP violation can be a result of spontaneous symmetry breaking. That is, spontaneous CP violation. With two Higgs doublets, these are three types model with two Higgs doublets, which is so-called Lee model. Type I and type II introduce the discrete symmetry, and it lead to the vanishing of spontaneous CP violating phase. Type III can make the spontaneous CP violation, and it has tree level FCNC with too many parameters arbitrary. The Weinberg model solves the problem for Lee model. It introduces three Higgs doublets which is the minimal model to have the spontaneous CP violating phase but without tree level FCNC process. However, the Weinberg model has been ruled out by the experimental data for $\sin 2\beta$. This motivation makes us to study new models with the spontaneous CP violation. We summary our work in the following

- We introduce an idea that make the spontaneous CP violating phase be identical to the CKM matrix phase. Two kinds of Yukawa interactions are discussed. One is called model(a) where two Higgs doublet couple to the up-type quarks and one Higgs couples to the down-type quarks. Another one is model(b) with two Higgs doublets couple to the down-type quarks and one Higgs doublets couple to the up-type quarks.

- For model(a) using the PDG parametrization a phase is absorbed into the up-type quarks to make the CKM matrix with uniform phase δ_{13}. We obtain

$$\delta = -\delta_{13},$$

and all coupling matrices are determined. Here δ_{13} is the phase causing spontaneous CP violation.

The same process can be apply to KM parametrization. The phase relation is similar to that of PDG,

$$\delta = -\delta_{\mathrm{KM}}.$$

The model(b) has the same phase relation as that of the model(a).

- We construct a model with three Higgs doublets and one Higgs singlet, with the Pessei-Quinn symmetry to make small enough neutron electric dipole moment. The minimal condition of the Higgs potential makes the spontaneous CP violating phase δ be the only one phase in the Higgs potential, and the spontaneous CP violating phase is the source of CP violation.

- We extract the Goldstone boson eaten by $W^{\pm}$ and Z^0, also the axion by appropriate rotation, and then we derive the corresponding Yukawa couplings. From the couplings we find that the H_4^0 and a interaction are neglected by the factor $1/v_s$. Tree level FCNC only occurs in the interaction by exchanging Higgs H_1^0 and a_1. The coupling matrices are related to the V_{CKM}. When we choose an explicit parametrization for CKM matrix, all couplings can be written in terms of CKM parameters and quark masses.

- Using experimental data on meson and anti-meson mixing, the mass of effective neutral Higgs with the relation $1/m_{\mathrm{eff}}^2 = 1/m_{H_1^0}^2 - 1/m_{a_1}^2$ are constrained.

- We use the result from the previous discussion of meson and anti-meson mixing to discuss the neutron electric dipole moment. It is well-known that the one loop contribution for quarks EDM with exchanging Higgs is small and negligible, so we calculate the two loop contribution from quark electric dipole moment, quark color electric dipole moment, and the gluon color electric dipole moment. The result is shown that the EDM could be close to the present upper bound for neutron electric dipole moment.

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