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論高齡化社會的教育政策與社會安全制度

Education Systems and Social Security in an Aging Economy

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謝詞

如果說現在是因爲過去的種種所導致的, 在台大6年的學生生活, 會因爲這篇論文的完成而結束, 那我想我應該要感謝這篇論文, 爲了要走到今天這一步, 過程中我認識了許多好朋友。

首先是「虹門」的大家長, 幫主兼指導教授陳虹如老師, 我必須說, 這一年幫主辛苦了! 您犧牲自己的空閒時間, 收留並教養咱們孤苦無依的4人, 希望您回首過去一年的辛苦, 能與我們一樣覺得是值得回憶的, 也希望幫主與我們在人生的道路上一同繼續奮鬥努力。此外, 藉由這次論文, 我也多認識經濟系其他認眞努力的好老師, 我的口試委員毛慶生教授與王泓仁教授, 你們的認眞閱讀讓我感動萬分。

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二年前, 我忘記報名預官考試, 因爲如此, 我沒先去當兵, 當時很懊悔, 但也因爲這樣, 我成了 95 級經研所的一員, 認識了那麼多 tone 很合的朋友, 相處的很 easy 很舒服, 跟你們出去玩都不會擔心無聊, 進研究所前, 有朋友跟我說, 研究所的生活是各過各的, 我想我們班是個反例。

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論文摘要

本文旨在探討高齡化社會下教育政策與養老金制度的關係。我們利用一個內生化生育決策的三期疊代模型,分析在不同的預期壽命與養老金所得替代率下, 實行公立教育體制、私立教育體制與補貼私立教育政策對經濟成長的影響。結果發現若政府沒有實施「隨收隨付制」的養老金制度, 由於公立教育體制會使得家計單位錯估教養小孩的成本,導致家計單位撫養過多小孩, 造成教育資源的稀釋, 故在此一情況下, 採行私立教育體制能得到較高的經濟成長。但若政府實施「隨收隨付制」的養老金制度,現在撫養較多小孩意味著未來能擁有較多勞動力去分擔養老金的稅負,則何種教育體制能夠誘發較高的經濟成長,取決於這個經濟社會的平均預期壽命與所採取的養老金政策,我們發現由於公立教育體制或補助私立教育政策能鼓勵家計單位撫養較多小孩,進而減低養老金稅率對經濟成長的負面影響。故模擬結果發現當一個經濟社會擁有足夠高的平均預期壽命或養老金所得替代率,公立教育體制或補助私立教育政策能導致較高的經濟成長。因此, 我們建議政府當局若是要促進長期經濟成長, 必須同時考慮教育與養老金政策。

關鍵字: 人口老化; 教育體制; 養老金制度
JEL 分類: J14, J13, J18。

Abstract

This paper studies the interaction between two main public policies, education and social security, in an aging economy. We compare the balanced growth rate between different education systems (a private education system, a public education system and a voucher program) at steady state with various life expectancies and pension replacement rates. The results suggest that if govenment does not implement pay-as-you-go (PAYG) social security program, a private education system can induce higher growth at any degree of longevity. In contrast, as government implements PAYG social security program, which education systems can enhance economic growth depends on the life expectancy of an economy and the policy of pension benefit. Our calibrated results reveal that a public education system or a voucher program can yield higher growth than a private education system by encouraging parents to raise children and then reducing the adverse impact of PAYG social security on capital accumulation and growth when an economy with sufficiently high life expectancy and pension replacement ratio. The implication of our analysis indicates that in order to promote economic growth, policy makers should consider these two public policies jointly.

Key Words: Ageing;Education systems;Social security
JEL Classification: J14, J13, J18。

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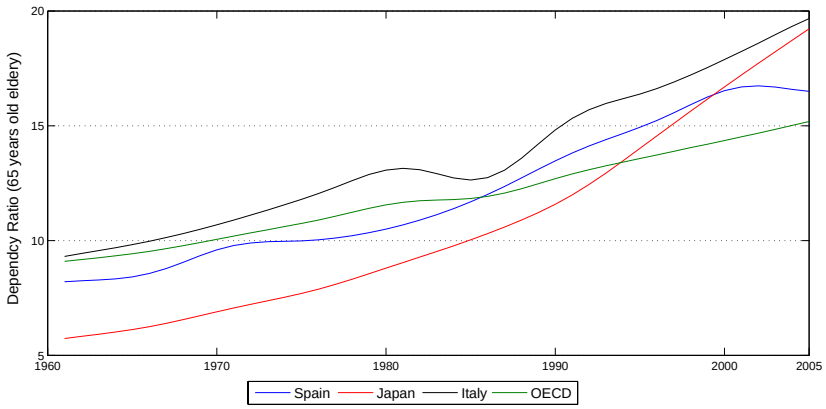
1 Introduction

The phenomenon of population aging has been undergoing in most industrial countries from 1980s. Fig. 1 shows that the ratios of the old (65 year above) to total population in OECD countries in 1960 are below 10% but in 2005 these ratios are nearly 20% because longevity is increasing steadily and birth rate is declining dramatically.

Figure 1: Dependency ratio age 65 above



Sooouce: World Development Indicators

Such big change in population structure leads to so-called ”an aging economy (society)” and brings many challenges and debates for public policies, especially, the design of social security systems and educational policies. For example, the financial sustainability of pay-as-you-go (PAYG) social security systems, which have been adopted by most developed countries virtually (Breyer and Craig 1997), confronts many doubts since population aging leads the more needs (old retiree) for pension benefits but the fewer contributions (young employees) to pension funding (see, such as, Zhang, Zhang and Lee, 2001; Pecchenino and Pollard, 2002).

Another issue rised from this demographic change is whether the increasing olds tend to be against distributing resources on public education for young generation. Several theortical and empirical researches discussed this topic but did not achieve consensus (see, for example, Poterba, 1997; Harris, Evans and Schwab, 2001; Zhang, Zhang and Lee, 2003 ,

Gradstein and Kaganovich, 2004; Grob and Wolter, 2005).

Different from previous studies considering educational policies and social security systems separately, this paper addresses the importance of the coordination between educational policies and social security systems in an aging economy and answer the following questions. First, which education systems is favoring for enhancing economic growth in an aging economy with or without PAYG social security program? Second, can government use educational policies to ease off the heavy tax burden of PAYG social security program accompanied by population aging?

More recently, some studies investigated the link between educational policies and PAYG social security system (see, for example, Kaganovich and Zilcha, 1999; Kemnitz, 2000; Pecchenino and Pollard, 2002; Rojas, 2004; Soares, 2006). On the one hand, most studies in this topic discussed the effect of PAYG social security system on public education policy and assumed fertility is exogenous. Kaganovich and Zilcha (1999) studied the optimal allocation of tax revenue between public education and PAYG social security. Their results suggested that if agents have high levels of altruism toward children and significant concern for their retirement income, then it is optimal to provide PAYG social security program. Because PAYG social security makes agents save less for retiring period and invest more in offspring human capital, which can enhance economic growth and social welfare. Kemnitz (2000) and Soares(2006) also argued that implementing PAYG social security system may generate political support for public education since public education can broaden future PAYG social security benefit of pensioners. Pecchenino and Pollard (2002) considered that if the quality of public education is sufficiently high, lowering PAYG social security tax and increasing the public education funding can uplift economic growth and social welfare.

On the other hand, very few researches think about reverse causality of both policies, that is, how education policies influence PAYG social security system, especially PAYG social security tax burden. To the best of our

knowledge, only Rojas (2004) was from this point of view. Rojas (2004) analyzed the economic effect of subsidizing higher education. He showed that subsidizing higher education not only directly raises the average educational level of the economy but also indirectly leads population aging by reducing average fertility rate, and then resulted in heavier tax suffering of PAYG social security system. Our work also starts from this point of view and we use a three periods overlapping generation model with endogenous fertility to show the possible link between education systems and PAYG social security program in an aging economy.

There has been much recent interest in modeling the impact of education systems on long-run economic growth and income distribution (see, for example, Glomm and Ravikumar 1992; Zhang 1996; Croix and Doepke 2004; Chen 2005). Most of them use the model with exogenous fertility and find public education regime can result in more equal income distribution and bring higher economic growth than private education regime when the initial income inequality is sufficiently high.

However, de la Croix and Doepke (2004) argued that it is worthy to consider fertility behavior and educational policies at the same time. They use the framework of Glomm and Ravikumar (1992) but with endogenous fertility to highlight the fertility differential between the poor (unskilled workers) and the rich (skilled workers) is a key factor when analyzing economic consequence of education systems. They argued that in a public education regime, which provides free education, all parents have the same number and educational investment of their children but in a private education regime, where parents can determine the amount of educational investment for their children, due to quantity-quality tradeoff caused by difference of the opportunity cost of raising children, poor parents have more children and invest less in education per child. Accordingly, if the initial income inequality is sufficiently high, the effect of fertility differential in a private education regime may lead unskilled workers become the majority of labor force, which downsizes average human

capital, and then results in lower economic growth than public education regime.

In this paper, the theoretical model used here is similar to de la Croix and Doepke (2004) but our model adds the setting of the longevity and PAYG social security system, which lets us compare the economic performance between different education systems in an aging economy with or without PAYG social security. At first, we compare the implications of a public and a private education regime for economic growth in an aging economy without PAYG social security program. The closed form solutions reveal that in a private education regime individual will have fewer numbers of children and invest more in education per child. This will generate the higher growth than in a public education regime. Our finding is consistent with the result of homogenous agents case in Glomm and Ravikumar (1992) and low income inequality case in de la Croix and Doepke (2004). However, while considering an economy with PAYG social security system, we find that if longevity or pension replacement rate is sufficiently high, providing public schooling or voucher program (subsidy of private education) may stimulate higher economic growth than adopting private education system by increase fertility rate and then lighten the heavy burden of PAYG social security program.

This paper contributes several new directions to the researches of education systems and social security. From the view of the literatures on education systems, many previous studies pointed out initial income distribution plays an important role in comparing economic growth between a private and public education regime. Our model shows that even if there is no income inquality a public education system still can generate higher growth than a private education system when we consider the factors of longevity and PAYG social security. In other words, our model points out that longevity and PAYG scoial security program matter in comparison of different education systems.

On the other hand, many previous studies try to provide several policy

tools for solving the financial crisis of existing PAYG social security system. For instance, Groezen, Leers and Meijdam (2003) and Oshio (2005) suggested that if government implements PAYG social security system, it has to provide childcare support, such as child allowance, simultaneously for giving incentive to fertility. Because under PAYG social security system children are not only a private good but also a kind of "public good", private optimal number of children is smaller than social optimum (see, for a review, e.g. Cigno, 1992; Groezen and Meijdam, 2008).

Besides providing child allowance to influence fertility behavior, inspired by the wisdom of family economics (Becker, 1973), we know that parents would determine the number of children and the educational investment of children jointly, that is the famous conjecture of quantity-quality tradeoff. Consequently, government can also use educational policies to affect individual fertility behavior. Our results reveal that offering free public education or subsidy of private education can encourage parents to give more births and then alleviate the financial vulnerability of PAYG social security system.

The rest of this paper is organized as follows. Section 2 presents our model economy Section 3 gives the calibration of the model. We compare the economic growth between different educational systems under various longevity and pension replacement ratio. Section 4 shows some empirical supports of our model. Section 5 concludes this paper.

2 The Model Economy

We consider an economy populated by infinite homogeneous agents who live for three periods. Each period is around 30 years, referring to youth, adulthood, and old age. But only p percent adults can survive in their old age[1]. Young agents study in the school, middle-aged agents raise their children and work, and old agents retire to enjoy leisure and survive on

[1]Here, we use p to represent the average life expectancy of an economy. For example, given $p = 0.5$, an economy will have 50% population with life expectancy of 75 years and 50% population with life expectancy of 50 years, which means the average life expectancy in this economy is 62.5 years.

their saving and government transfer. Agents have the same preference and make all their lifetime decisions in adulthood. They care about consumption in middle age c_t^m , consumption in old age c_t^o, how many children they raise n_t, and the human capital of their children h_{t+1}. We can use the following function to represent an individual's utility.

$$\ln(c_t^m) + p\sigma \ln(c_t^o) + \gamma \ln(n_t) + \beta \ln(h_{t+1}) \tag{1}$$

where subscript t represents "generation t" which means agents are in their adulthood at time t. The parameter σ is the discount factor of the utility for the consumption in old age. The parameters γ and β respectively denote the strength of preference over the number of children and the human capital of children. The human capital of the children h_{t+1}depends on parental human capital h_t and school expenditure e_t, which determines the quality of education received from school.

$$h_{t+1} = \lambda (e_t)^{\delta} (h_t)^{1-\delta} \tag{2}$$

where λ is positive constant and $\delta \in (0,1)$. The parameters δ and $1-\delta$ measure the elasticity of school expenditure and parental human capital on human capital per child, repectively. This human capital accumulation technology is constant return to scale in school expenditure and parental human capital. We divide the school expenditure funded privately and publicly, which will be discussed more details about two education regimes in the section 2.3.

2.1 Production

Assume that there are many homogenous firms producing single goods to maximize their profit in a prefectly competitive market. Their production technology is Cobb-Douglas function and satisfies constant return to scale. Hence, aggregate production function in economy become:

$$Y_t = F(K_t, L_t) = AK_t^{\theta} L_t^{1-\theta} \tag{3}$$

where $A > 0$ is the total factor productivity; θ is elasticity of capital stock on output; K_t is aggregate capital stock at time t; $L_t = N_t l_t h_t$ is the aggregate effective labor supply; N_t is total working population (the numbers of adults at time t). We assume that each adult is endowed with 1 unit of time and spend ϕn_t of the time ($\phi \in (0,1)$) to raise their offsprings and the remaining time $l_t = 1 - \phi n_t$ is an individual labor supply. We define per effective labor capital stock (physical-human capital ratio) $m_t = \frac{K_t}{N_t l_t h_t}$ so Eq.(3) can be rewritten in intensive form:

$$y_t = A m_t^{\theta} \tag{4}$$

where $y_t = \frac{Y_t}{N_t l_t h_t}$. Since the market structure in the economy is prefect competitive, firms take the wage rate, w_t and interest (rental) rate, R_t, as given. Firms employ each production fator at the price where is equal to its marginal product:

$$w_t = (1-\theta) A (m_t)^{\theta} \tag{5}$$

$$R_t = \theta A (m_t)^{\theta-1} \tag{6}$$

2.2 PAYG Social security system

Assume that government always maitain a balanced budget to finance the PAYG social security program. Government levies a propotional wage income tax $\tau_t(1-\phi n_t) w_t h_t$ on adults at time t to transfer V_t for elderly at the same time. Social security budget constraint is:

$$N_t \tau_t (1 - \phi n_t) w_t h_t = N_{t-1} p V_t \tag{7}$$

in which $N_{t-1}p$ and N_t[2] can be interpreted as the beneficial and contributed population, respectively. Following Cooley and Soares(1999) and Pecchenino and Pollard(2002), we denote the pension replacement rate B as follows:

[2] We know that $N_t = N_{t-1} n_{t-1}$

$$B = \frac{V_t}{(1 - \phi n_t) w_t h_t} \tag{8}$$

The replacement rate B is the ratio of pension transfer V_t to the wage income of current employees $(1 - \phi n_t) w_t h_t$ and government adjust the social security tax rate τ_t to keep this ratio constant. We can learn the relationship between the demographic structure and the tax rate τ_t from substituting Eq.(8) into Eq.(7) :

$$\tau_t = \frac{N_{t-1} pB}{N_t} = \frac{pB}{n_{t-1}} \tag{9}$$

Eq.(9) indicates that life expectancy increasing or fertility rate declining will lead heavier PAYG social security tax burden.

2.3 Education Systems

2.3.1 A private education system

In a private education regime, an adult needs to choose consumption in the middle age c_{rt}^m, saving for old age s_{rt}, the number of children n_{rt}, and education expenditure per child e_{rt}. Their wage income is taxed at the rate of τ_{rt} for social security. Notice that we denote the variables with subscript r to represent private education regime and u to represent public education regime The budget constraint for an adult is:

$$c_{rt}^m + s_{rt} + e_{rt} n_{rt} = (1 - \tau_{rt})(1 - \phi n_{rt}) w_{rt} h_{rt} \tag{10}$$

The elderly consumption deponds on their middle age saving and social security transfer.In this model, we assume that agents invest their saving in a mutual fund. Only agents surviving to old age can get the return from mutual fund. Thus, the gross rate of return of mutual fund for surviving old is $r_{rt+1} = \frac{R_{rt+1}}{p}$. The budget contraint in an agent's old age become:

$$c_{rt}^o = r_{rt+1} s_{rt} + V_{t+1} \tag{11}$$

According to Eqs.(10) and (11) we can drive the intertemporal budget constraint for an adult:

$$c_{rt}^{m} + e_{rt}n_{rt} + \frac{c_{rt}^{o}}{r_{rt+1}} = (1-\tau_{rt})(1-\phi n_{rt})w_{rt}h_{rt} + \frac{\mathrm{V}_{rt+1}}{r_{rt+1}} \tag{12}$$

Hence, under private education regime an adult at time t solves the following lifetime utility maximization problem:

$$\max_{s_{rt},e_{rt},n_{rt}} U_{rt} = \ln(c_{rt}^{m}) + p\sigma\ln(c_{rt}^{o}) + \gamma\ln(n_{rt}) + \beta\ln(h_{rt+1})$$

$$\text{subject to} \quad c_{rt}^{m} + e_{rt}n_{rt} + \frac{c_{rt}^{o}}{r_{rt+1}} = (1-\tau_{rt})(1-\phi n_{rt})w_{rt}h_{rt} + \frac{\mathrm{V}_{rt+1}}{r_{rt+1}} \tag{13}$$

Definition 1 *(Under a private education regime equilibrium) Given the initial human capital endowments h_0 , preferences ,longevity ,human capital accumulation technology ,production technology, an equilibrium consists of aggregate capital stocks $\{H_{rt}, K_{rt}\}$, sequences of prices $\{w_{rt}, R_{rt}\}$, household decisions $\{c_{rt}^{m}, c_{rt}^{o}\ s_{rt}, n_{rt}, e_{rt}\}$, and policy variables $\{V_{rt}, \tau_{rt}\}$ such that :*

1. given factor prices $\{w_{rt}, R_{rt}\}$ and policy variables $\{V_{rt}, \tau_{rt}\}$, the household decisions $\{c_{rt}^{m}, c_{rt}^{o}\ s_{rt}, n_{rt}, e_{rt}\}$ maximize utility subject to the constraints Eq.(2) and (12);

2. factor prices $\{w_{rt}, R_{rt}\}$ clear markets;

3. labor market clear : $L_{rt} = N_{rt}(1-\phi n_{rt})h_{rt}$

capital market clear : $K_{rt+1} = N_{rt}s_{rt}$[3]

goods market clear : $Y_{rt} = N_{rt}c_{rt}^{m} + pN_{rt-1}c_{rt-1}^{o} + K_{rt+1}$

4. the government's budget constraint Eq. (7) is satisfied;

The first-order conditions for an adult's optimal choices of life-cycle saving s_{rt}, number of children n_{rt}, and educational investment per child e_{rt} under a private education system are:

[3] We assume 100% depreciation for physical capital because one period in our model is 30 years

$$\frac{1}{c_{rt}^{m}} = \frac{\sigma}{c_{rt}^{o}} r_{rt+1} \tag{14}$$

$$\frac{1}{c_{rt}^{m}}(1-\tau_{rt})\phi n_{rt} w_{rt} h_{rt} = \frac{\gamma}{n_{rt}} \tag{15}$$

$$\frac{1}{c_{rt}^{m}} n_{rt} = \frac{\beta}{h_{rt+1}} \lambda \delta (\mathrm{e}_{rt})^{\delta-1} (\mathrm{h}_{rt})^{1-\delta} \tag{16}$$

For maximizing utility, according to Eq.(14), individuals balance the loss in utility from reducing middle age consumption (marginal cost of saving) and the gain in utility from increasing old age consumption (marginal benefit of saving) to determine the quantity of saving for retirement. By Eq.(15) individuals choose the number of children and middle age consumption such that equate the loss in utility from diminishing consumption in adulthood to spend time caring children (marginal cost of raising one child) to the gain in utility from one more child (marginal benefit of raising one child). Under a private education system, adults face the "quantity-quality" tradeoff of their offspring, which is depicted by Eq.(16). Eq.(16) means the loss in utility from cutting the consumption of middle age for financing educational expenditure of children (marginal cost of increasing the level of human capital per child) should be equal to the gain in utility from improvement of offspring human capital (marginal benefit of increasing the level of human capital per child). That is, once adults decide to have more children and then will invest less on offspring human capital.

From the first order conditions of utility maximization Eqs.(14)–(16), budget constraint Eqs.(10)–(11) ,and capital market clear condition $K_{rt+1} = N_{rt} s_{rt}$, we can derive an adult's optimal choices of life-cycle saving s_{rt}, education expenditure per child e_{rt}, and the number of children n_{rt} under a private education system:

$$s_{rt} = \frac{p\sigma\theta(1-\tau_{rt})w_{rt}h_{rt}}{\tau_{rt}(1-\theta)(1+\gamma) + \theta(1+p\sigma+\gamma)} \tag{17}$$

$$e_{rt} = \frac{\beta\delta\phi(1-\tau_{rt})w_{rt}h_{rt}}{\gamma - \beta\delta} \tag{18}$$

$$n_{rt} = \frac{(\gamma - \beta\delta)[\tau_{rt}(1-\theta)+\theta]}{\phi[\tau_{rt}(1-\theta)(1+\gamma)+\theta(1+p\sigma+\gamma)]} \tag{19}$$

2.3.2 A public education system

The only difference between public education system and private education system is that adults do not need to choose school expenditure for their children under public schooling regime. Instead, educational spending is funded through another proportional wage income tax η_{ut} and public school is provided free for all households. Government runs balanced budget to finance public education expenditure.

$$e_{ut}n_{ut} = \eta_{ut}(1-\phi n_{ut})w_{ut}h_{ut} \tag{20}$$

where e_{ut} is public education expenditure per child. Educational tax rate η_{ut} is determined by political process, which will be discussed laterly. The budget constraint for agents in their middle age and old age become :

$$c^m_{ut} + s_{ut} = (1-\tau_{ut}-\eta_{ut})(1-\phi n_{ut})w_{ut}h_{ut} \tag{21}$$

$$c^o_{ut} = r_{ut+1}s_{ut} + V_{ut+1} \tag{22}$$

From Eqs.(21) and (22) we can drive the intertemporal budget constraint for adults under public schooling system:

$$c^m_{ut} + \frac{c^o_{ut}}{r_{ut+1}} = (1-\tau_{ut}-\eta_{ut})(1-\phi n_{ut})w_{ut}h_{ut} + \frac{V_{ut+1}}{r_{ut+1}} \tag{23}$$

Consequently, an adult at time t solves the following lifetime utility maximization problem:

$$\max_{s_{ut},n_{ut}} U_{ut} = \ln(c^m_{ut}) + p\sigma\ln(c^o_{ut}) + \gamma\ln(n_{ut}) + \beta\ln(h_{ut+1})$$

$$\text{subject to} \quad c_{ut}^{m} + \frac{c_{ut}^{o}}{r_{ut+1}} = (1-\tau_{ut}-\eta_{ut})(1-\phi n_{ut})w_{ut}h_{ut} + \frac{V_{ut+1}}{r_{ut+1}} \tag{24}$$

Definition 2 *(Under a public education regime equilibrium) Given the initial human capital endowments* h_o *, preferences ,longevity ,human capital accumulation technology ,production technology, an equilibrium consists of aggregate capital stocks* $\{H_{ut}, K_{ut}\}$*, sequences of prices* $\{w_{ut}, R_{ut}\}$*, household decisions* $\{c_{ut}^{m}, c_{ut}^{o}\ s_{ut}, n_{ut}\}$*, and policy variables* $\{V_{ut}, \tau_{ut}, \eta_{ut}, e_{ut}\}$ *such that :*

1. given factor prices $\{w_{ut}, R_{ut}\}$ *and policy variables* $\{V_{ut}, \tau_{ut}\}$*, the households' decisions* $\{c_{ut}^{m}, c_{ut}^{o}\ s_{ut}, n_{ut}, e_{ut}\}$ *maximize utility subject to the constraints Eq.(2) and (22);*

2. factor prices $\{w_{ut}, R_{ut}\}$ *clear markets;*

3. labor market clear : $L_{ut} = N_{ut}(1-\phi n_{ut})h_{ut}$

capital market clear : $K_{ut+1} = N_{ut}s_{ut}$

goods market clear : $Y_{ut} = N_{ut}c_{ut}^{m} + pN_{ut-1}c_{ut-1}^{o} + K_{ut+1}$

4. the government's budget constraint Eqs.(7) and (20) are satisfied;

5. given households' decisions, the policy variables $\{\eta_{ut}, e_{ut}\}$ *maximize the utility of adult agents;*

The first order conditions for an adult's optimal choices of life-cycle saving s_{rt} and number of children n_{rt} under a public education system are:

$$\frac{1}{c_{ut}^{m}} = \frac{\sigma}{c_{ut}^{o}} r_{ut+1} \tag{25}$$

$$\frac{1}{c_{ut}^{m}}(1-\tau_{ut}-\eta_{ut})\phi n_{ut} w_{ut} h_{ut} = \frac{\gamma}{n_{ut}} \tag{26}$$

The economic intuition of Eqs.(25)–(26) are similar to Eqs.(14)–(15) ,respectively. The individuals equate the marginal rate of substitution between current and old age consumption to the return rate of mutual fund and the marginal rate of substitution between current consumption and a child to the marginal cost of bearing an extra child. The most important

difference between two education systems is that under public education system adults do not need to consider the "quantity-quality" tradeoff of children (Eq.(16)). In other words, since public education is provided free, adults do not think that the gain in utility from an extra child is at expense of the decline of the quality of school of "all households". Therefore, adults in the public education regime have more children than ones in the private education regime because marginal cost of an extra child under public schooling system is cheaper.

From the first order conditions of utility maximization Eqs.(24)–(25), budget constraint Eqs.(21)–(22), and capital market clear condition $K_{ut+1} = N_{ut}s_{ut}$, we can derive an adult's optimal choices of middle age saving s_{ut} and the number of children n_{ut} under a public education system:

$$s_{ut} = \frac{p\sigma\theta(1-\tau_{ut}-\eta_{ut})w_{ut}h_{ut}}{\tau(1-\theta)(1+\gamma)+\theta(1+p\sigma+\gamma)} \tag{27}$$

$$n_{ut} = \frac{\gamma[\tau_{ut}(1-\theta)+\theta]}{\phi[\tau_{ut}(1-\theta)(1+\gamma)+\theta(1+p\sigma+\gamma)]} \tag{28}$$

Next, we describe the political process for educational tax rate. The educational tax rate η_{ut} is determined by majority voting. The adults vote on the η_{ut} to maximize their life-time utilities. The preferred tax rate is chosen by the following indirect utility maximization problem:

$$\max_{\eta_{ut}} U_{ut} = \ln(c^m_{ut}) + p\sigma\ln(c^o_{ut}) + \gamma\ln(n_{ut}) + \beta\ln(h_{ut+1})$$

$$\text{subject to} \quad \text{Eqs.(2), (20) and (27)–(28)} \tag{29}$$

We can obtain the preferred education tax rate as follows:

$$\eta_{ut} = \frac{\beta\delta(1-\tau_{ut})}{1+p\sigma+\beta\delta} \tag{30}$$

where $\eta_{ut} < 1$. Substituting Eq.(30) to the government budget constraint Eq.(20) and the resulting choice for public education expenditure per children is:

$$e_{ut} = \frac{\beta\delta(1-\tau_{ut})w_{ut}h_{ut}(1-\phi n_{ut})}{(1+p\sigma+\beta\delta)n_{ut}} \tag{31}$$

Proposition 1 *For both education regimes, an economy with longer life expectancy has lower fertility rate and higher PAYG social security tax rate.*

Proof. See Appendix ■

Proposition 1 is very intuitive. When a rational individual knows that he/she has longer life span, he/she will work hard and save more for "live-long" retirement. Increasing labor supply is at the cost of bearing fewer children. Consequently, the extension of life expectancy not only results in more living olds directly but also alters an adult's fertility decision, to have fewer offspring, and thereby leads aging population. The implication of population aging for PAYG social security system is that more retiree need for pension benefits but fewer labor force (employees) can contribute pension funding, namely, the tax burden of PAYG social security program will be heavier.

2.4 Growth

In this section, we use the steady-state balanced growth rate to compare the economic performance between private and public education regimes. Along the balanced growth path, the growth rate of physical capital per worker and the growth rate of individual human capital accumulation will be the same as the growth rate of output per capita. The following equations express the above concepts:

$$1+g_k = \frac{\frac{K_{t+1}}{N_{t+1}}}{\frac{K_t}{N_t}} = \frac{A(1-\theta)(m_t)^{\theta-1}S_t}{n_t} \tag{32}$$

$$1+g_h = \frac{h_{t+1}}{h_t} = \lambda\left[A(1-\theta)(m_t)^{\theta}E_t l_t\right]^{\delta} \tag{33}$$

$$1+g = \frac{\frac{Y_{t+1}}{N_{t+1}}}{\frac{Y_t}{N_t}} = 1+g_k = 1+g_h \tag{34}$$

where $S_t = \frac{s_i}{(1-\phi n_t)w_t h_t}$ is the ratio of saving to wage income, $E_t = \frac{e_t}{(1-\phi n_t)w_t h_t}$ is the ratio of education expenditure to wage income, n_t is fertility rate,and $l_t = 1 - \phi n_t$ is an individual labor supply. Eq.(32) devides into Eq.(33) yields the law of motion of m_t.

$$m_{t+1} = \frac{A(1-\theta)s_t}{\lambda[A(1-\theta)e_t l_t]^{\delta} n_t} m_t^{\theta(1-\delta)} \tag{35}$$

At steady state, the fertility rate n_t is a constant and we can neglect the time subscript. From Eqs.(9) and (20), we know that the tax rates τ_t and η_t depends on n. Therefore, the tax rates τ_t and η_t are time-invariant. In addition, the physical-human capital ratio is also constant over time, $m_{t+1} = m_t = m^*$. We substitute m_t in either Eq.(32) or Eq.(33) and obtain the balanced growth rate of output per capita $1 + g$:

$$1 + g_i = \{H(\frac{S_i}{n_i})^{\theta\delta}(E_i)^{\delta(1-\theta)}(l_i)^{\delta(1-\theta)}\}^{\frac{1}{1-\theta(1-\delta)}}, \ i = r, u \tag{36}$$

where $i = r, u$ indicates the private and public education regimes respectively, $H = \lambda^{1-\theta}[A(1-\theta)]^{\delta}$ is a constant. Eq.(36) shows that S_i, E_i and n_i are three determinants of growth. [4] The ratio of saving S_i and education investment E_i to labor income both cause positive effect on the balanced growth rate obviously. However, fertility rate n_i has both positive and negative impacts on the balanced growth rate. Negative one is so-called "resource-dilution effect", that is, bearing more children dilutes educational resources at present time and output per capita in the future. Positive one is "tax-sharing effect"; a higher number of children also implies that there are more labor force for sharing PAYG social security burden in the future.

Proposition 2 *Without PAYG social security program ($B = 0$), that is, an economy with fully-funded social security system or without offering any social security program, given any life expenctancy p an economy with private education system has higher balanced-growth rate than the one with public education system.*

[4] Since labor supply l_i is a function of fertility n_i we do not treat l_i as another growth determinant.

Proof. See Appendix ■

Proposition 2 indicates that when longevity increases but government does not implement PAYG social security program, private education system can stimulate higher economic growth than public education system does. The reason is that the education expenditure in public schooling regime is financed by tax revenue not households themselves, it gives parents incentive to have more children and free ride educational resource. High fertility rate leads ”resource-dilution effect” and thereby has negative impact on balanced growth rate. This result is consistent with the homogenous agent case in Glomm and Ravikumar (1992) and low income inequality case in de la Croix and Doepke (2004). However, as Zhang and Zhang (2001) points out that an increase in longevity also has indirect effect on growth through the higher burden of PAYG social security system. Therefore, it is necessary and interesting to see whether the result will be changed when considering PAYG social security system in our model economy.

Proposition 3 *When implementing PAYG social security system ($B > 0$), an economy with private education system has higher level of PAYG social security tax than the one with public education system. Moreover, a higher social security tax rate reduces the steady-state capital accumulation and balanced growth rate.*

Proof. See Appendix ■

If government implements PAYG social security program,raising more children will have ”tax-sharing effect” by broadening future tax base of PAYG social security program and give posititve impact on economic growth. On balance, implementing PAYG social security system makes children involve a positive externality (Groezen, Leers and Meijdam, 2003). Hence, government can use several policy tools to ”correct” the externality resulted from public pension policy, such as child allowance, which is

discussed a lot by previous studies (Groezen, Leers and Maijdam, 2003 ; Oshio, 2005). Proposition 3 suggests that providing public education may be another policy instrument to encourage parents to bear children and then mitigate the heavier and heavier PAYG social security burden in an aging economy. Furthermore, it also reveals that high level of PAYG social security tax has a negative impact on capital accumulation and balanced growth rate, for this reason, public education system may have possibility to stimulate higher economic growth than private education system if "tax-sharing effect" dominates "resource-dilution effect". The following section, we present the calibrated version of our model to obtain clearer picture of the above two opposite effects when comparing the economic performance between public and private education systems.

3 Computational Experiments

3.1 Calibration

In order to obtain credible quantitative results of our theory, we calibrate our model to match the growth features of the US or other OECD countries. There are five features that we want to match: life expectancy, annual growth rate of output per capita, total fertility rate per woman (TFR=$2n_t$ [5]), the share of education expenditure on output and the tax rate of PAYG social security. Because public school enrollment rate is higher than private school enrollment rate in most countries (Chen, 2005 ; de la Croix and Doepke, 2007), our baseline model, which is calibrated to fit the real world data, is an economy with public education system.

One preiod (generation) in our model is assumed 30 years and agents can survive safely for two preiods, that is, life expectancy in our model economy is at least 60 years old. We set $p = 0.5$ to match life expectancy in the United States at 2000 (about 76 years old). According to standard real business cycle literatures (Docquier and Paddison,2003), we set discount

[5] Since at least two people (a male and a female) can give a birth in the real world, but our model economy is "asexual reproduction" (an agent can have his/her offspring individually.) Therefore, to match the data of total fertility rate per woman, we need to let n multiply 2.

factor (the weight of old age consumption) $\sigma = (0.99)^{30}$.

The parameter $A = 5$ in production function and $\lambda = 3.5$ in human capital accumulation function, which does not influence qualitative results of our model, is used to match long-run growth rate of per capita output 2.5% (i.e. in the US 2.11% and 2.53% in Germany, Zhang and Zhang, 2003).

To calibrate total fertility rate, we need to adjust ϕ the fraction of time devoted to raise children and γ the weight of offspring quantity in the utility function. The studies of Robert Haveman and Barabra Wolfe(1995) and John Knowles(1999) show that parents spend about 15% of their time raising children.Accordingly, we choose $\phi = 0.15$. The parameter γ is assigned to 0.26 (de la Croix and Doepke, 2003) to achieve average total fertility rate per woman 2.11 in United States during 2000 – 2005.

Next, we use the elasticity δ of future human capital (wage income) with respect to public education expenditure and the weight of offspring quality in the utility function β to determine the ratio of public education expenditure to output. Johnson and Stafford(1973) estimated income elasticity for education expenditure was 0.198, another estimation of this figure provided by Card and Kreuger(1992) is 0.2. Since these estimations are similar, we set $\delta = 0.2$.We choose $\beta = 0.72$ such that public education expenditure as a fraction of output fits the corresponding figure (public education expenditure for all level) in high income OECD countries at 2000, which is 4.8%.

The income replacement ratio of PAYG social security B is set to 0.43, which follows Pecchenino and Pollard(2002), for matching the social security contribution rate 19%; this value is between the rates in France and US (Zhang and Zhang, 2003).

The remaining parameter θ is the share of income that goes to physical capital, following the previous literatures (see, for example, Boldrin,2005), we set $\theta = 0.3$ as the calibrated value.The parameters of baseline model is summerized in Table 1.

Table 1: Calibrated values of baseline model

$p = 0.5$	$\lambda = 3.5$	$A = 5$	$B = 0.43$	$\theta = 0.3$
$\delta = 0.2$	$\phi = 0.15$	$\gamma = 0.26$	$\beta = 0.72$	$\sigma = 0.8$

3.2 Comparing private and public education systems

In this section, we compare economic performance between public and private education regimes on the balanced growth path. From propsition 2, we know that a private education regime at steady state has higher growth rate than a public education regime for any degree of longevity in an economy without PAYG social security program.

However, many developed countries execute PAYG social security system, whose tax rate is positive related to life expectancy and pension replacement rate nowadays but negative related to labor force at present (the number of children in last generation). Proposition 3 indicates that an economy with a private education system has to suffer more PAYG social security tax burden than one with a public education system. When the tax burden expands, it will bring about larger distortion of economic activity and slow down the growth rate of GDP per capita. This opens the possibility for a public education system boosting higher economic growth even if there is no income inequality in our simple model.

Next, we want to show that the institution of PAYG social security matters when analyzing the economic effects of two educational systems. The way we use here is by changing two key parameters of PAYG social security tax rate, life expectancy and pension replacement ratio, to emphasize the importance of joint consideration of these two policies.

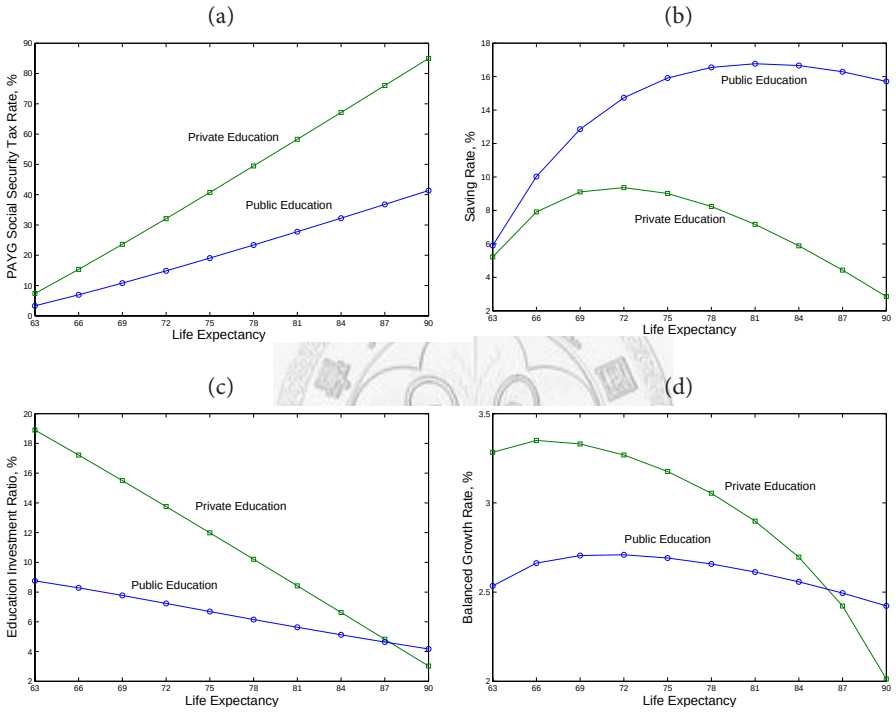
3.2.1 The effect of longevity

To investigate the effect of longevity under different education regimes, we take balanced growth rate comparison by varying the life expectancy over the interval from 63 years to 90 years ($p = 0.1$ to $p = 1.0$).

Fig. 2*a* shows given the ratio of earning replacement $B = 0.43$, the tax burden of PAYG social security increase with the extension of life ex-

pectancy. Due to low fertility rate, a private education regime (green line) has higher level of the PAYG social security tax rate than a public education regime (blue line) and the gap of tax rate between two regimes enlarges as life expectancy raise. Heavy tax burden of PAYG social security program has a very strong negative impact on investment in human and physical capital accumulation.

Figure 2: Life expectancies and education systems



Hence, we can find that a private schooling regime has less physcial capital investment than a public schooling regime at any extent of longevity (see Fig. 2*b*) and has less human capital investment than a public schooling regime at sufficiently high level of life expectancy (see Fig. 2*c*). Because of slow capital accumulation at the stage of high life expectancy (about 87 years), a private education system results in lower economic growth than a public education system (see Fig. 2*d*).

Table 2 gives two numerical examples to summerize the above findings. As shown in the first row of table 2, parents in a public school-

ing regime bear almost twice more number of children than in a private schooling regime. High fertility rate causes two opposite impacts on economic growth, the "resource-dilution effect" and the "tax-sharing effect". However, the relative size of two effects depends on what degree of longevity an economy stays at. In a "young" economy (life expectancy is 63 years), the "tax-sharing effect" is smaller than the "resource-dilution effect", a public education system reuslts in less educational investment and then lower economic growth than a pirvate education system. On the contrary, in an "old" economy (life expectancy is 87 years), the "tax-sharing effect" dominates the "resource-dilution effect". A public education system leads faster capital accumulation and higher growth rate than a private education system. In sum, which education systems is better for long-run growth should hinge on how "old" an economy is (the life expectancy of an economy).

Table 2: Longevity and educational systems

Variables	low longevity (p=0.1)		high longevity (p=0.9)	
	Private	Public	Private	Public
Fertility(TFR)	1.16	2.59	1.01	2.10
Social Security(τ)	7.38	3.31	76.04	36.78
Saving(S)	5.21	5.89	4.43	16.29
Education Investment(E)	18.89	8.75	4.82	4.64
Balanced Growth(g)	3.28	2.53	2.42	2.49

1 Except longevity, all parameters are the same as the setting in baseline model.

3.2.2 The effect of replacement ratio

In this section, we allow government can change her pension policy through varying pension replacement ratio from 10% of average earnings to 100% of average earnings (B = 0.1 to B = 1.0).

If government raises pension replacement ratio (pension benefit for the aged), the level of PAYG social security tax will become higher and then have adverse impacts on physical capital accumulation (see Fig. 3*b*),

human capital accumulation (see Fig. 3*c*) and balanced growth rate (see Fig. 3*d*). Comparing two education systems, we find that a private education system is more sensititve to the change of pension replacement ratio than a public education system, and furthermore a public education system can boost higher economic growth than a private education when government decides to provide sufficiently "rich" pension benefit to the old.

Figure 3: Replacement ratios and education systems

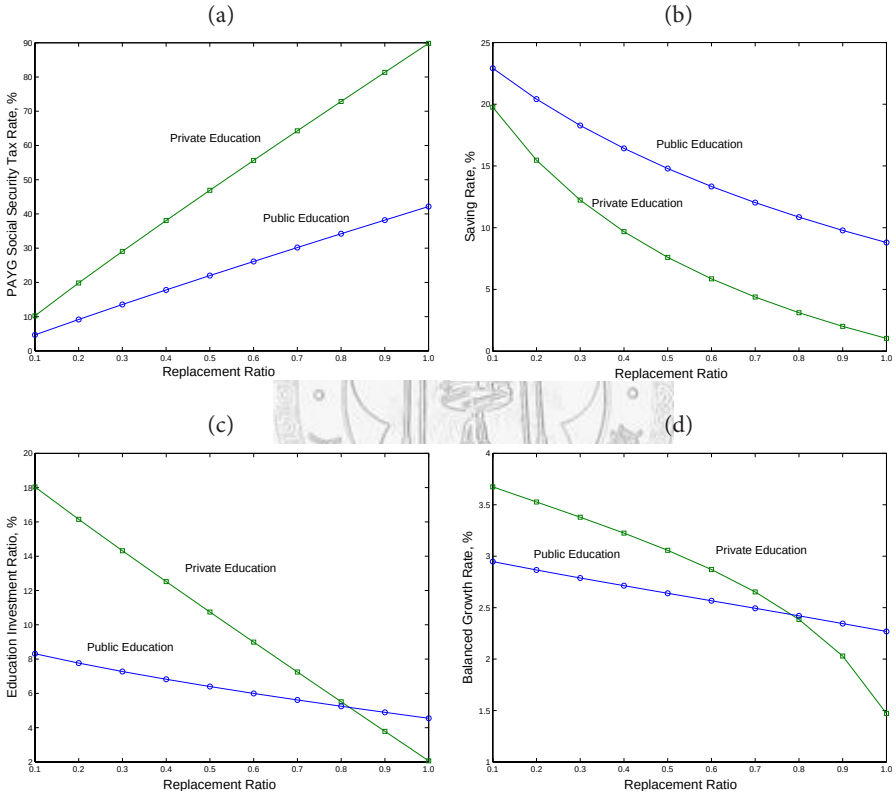


Table 3 gives two specific cases to illustrate that for maximizing economic growth what is the favored choice of education systems under different policies of pension replacement rate. If govenment chooses the policy of low pension replacement ratio (B=0.2), growth rate in a private schooling regime is 3.52% higher than 2.86% in a public schooling regime. Contrariously, if govenment carries out the policy of high pen-

sion replacement rate (B=0.9), it is preferred to adopt a public education system (2.34%) rather than a private education system (2.02%). To sum up, in order to promote economic growth, government should coordinate educational policy and the benefit scheme of PAYG social security.

Table 3: Replacement ratio and educational systems

Variables	low replacement (B=0.2)		high replacement (B=0.9)	
	Private	Public	Private	Public
Fertility(TFR)	1.01	2.18	1.11	2.35
Social Security(τ)	19.82	9.17	81.35	38.20
Saving(S)	15.46	20.41	2.01	9.78
Education Investment(E)	16.15	7.76	3.78	4.89
Balanced Growth(g)	3.52	2.86	2.02	2.34

1 Except replacement ratio, all parameters are the same as the setting in baseline model.

3.3 Policy Implication: Subsidizing Private education

In previous section, we find that, because of low birth rate, a private education system is more sensitive to the variation in life expectancy and pension replacement ratio than a public education system and then leads lower balanced growth rate when an economy with sufficiently high life expectancy and pension replacement rate. For this reason, it seems interesting to see whether growth can be promoted by implementing some policies, such as voucher program (subsidy of private education), which eliminates the educational expenditure per child and then encourage parents give more birth for sharing pension burden in an economy with "great" PAYG social security program.

The intertemporal budget contraint for the households in a private schooling regime with voucher program can be revised as follows:

$$c_{rt}^{m} + (1 - v_{rt}) e_{rt} n_{rt} + \frac{c_{rt}^{o}}{r_{t+1}} = (1 - \tau_{rt} - q_{rt})(1 - \phi n_{rt}) w_{rt} h_{rt} + \frac{V_{rt+1}}{r_{t+1}} \quad (37)$$

where v_{rt} is subsidy rate of private education expenditure and q_{rt} is a propotional tax for financing voucher program. Government also runs balanced budget to subsidize private education and the budget constraint of voucher program is:

$$v_{rt}e_{rt}n_{rt} = q_{rt}(1 - \phi n_{rt})w_{rt}h_{rt} \tag{38}$$

where assume that the scale of voucher program is determined exogenously by government not by voting process.

Table 4 and 5 indicates that compared to private schooling regime (no subsidy, v=0), subsidizing educational fee per child can raise about 0.05–0.07 (50% of subsidy, v=0.5) and 0.09–0.12 (90% of subsidy, v=0.9) total fertility rate and then reduce the tax burden of PAYG social security. The results also reveals that the relationship between the level of subsidy for private education and economic growth depends on the life expectancy of an economy and the policy of pension benefit.

In the case of low longevity or small pension replacement ratio, where "resource-dilution" effect dominates "tax-sharing" effect, more subsidy of educational investment results in lower economic growth. On the contrary, when an economy has high life expectancy or implements the policy of providing large pension benefit for old, "tax-sharing" effect is stronger than "resource-dilution" effect, government should provide more subsidy of educational investment to raise growth rate.

Table 4: Longevity and subsidy of education

Variables	low longevity (p=0.1)			high longevity (p=0.9)		
	v=0	v=0.5	v=0.9	v=0	v=0.5	v=0.9
Fertility(TFR)	1.16	1.23	1.28	1.01	1.06	1.10
Social Security(τ)	7.38	6.98	6.67	76.04	72.80	70.22
Saving(S)	5.21	5.27	5.32	4.43	5.14	5.73
Education Investment(E)	18.89	17.94	17.18	4.82	5.22	5.49
Balanced Growth(g)	3.28	3.23	3.19	2.42	2.49	2.54

1 Except longevity, all parameters are the same as the setting in baseline model.

Table 5: Replacement ratio and subsidy of education

Variables	low replacement (B=0.2)			high replacement (B=0.9)		
	v=0	v=0.5	v=0.9	v=0	v=0.5	v=0.9
Fertility(TFR)	1.01	1.06	1.10	1.11	1.16	1.21
Social Security(τ)	19.82	18.94	18.24	81.35	77.42	74.29
Saving(S)	15.46	15.81	16.10	2.01	2.50	2.91
Education Investment(E)	16.15	15.55	15.08	3.78	4.34	4.73
Balanced Growth(g)	3.52	3.49	3.46	2.02	2.15	2.24

1 Except replacement ratio, all parameters are the same as the setting in baseline model.

4 Empirical Implications

The prediction of our model implies that comparing to a private schooling system, a public schooling system can encourage parents to have more births and then leads to a lower tax rate of PAYG social security in the future. Due to the lack of data, there are few empirical studies examining the impact of educational systems and policies on macroeconomic variables or demographic structure across countries. In this section, we use the internationally comparable data provided by OECD and WDI (World Development Indicators) to investigate preliminary relationships between educational systems, birth rate and social security burden.

Table 6 lists the whole 17 countires in our sample. We especially choose these high income OECD countries for two reasons. First, the life expectancy at birth in these countries are sufficiently high and similar to each other, which matches the demographic feature of our model and also controls the effect of longevity on fertility rate and social security tax rate. Second, some countries, for example Italy and Germany, also satify our standard but their data is not reliable [6].

[6] we also use secondary private school enrollment rate in 1985 from UNSCO to check the reliability of our classification for education systems

Table 6: Education systems, fertility rate and social security

Country	Percentage Share of Private Funding,%	Total Fertility Rate	Chnage in Social security tax rate 1990-2003, %
Australia	15.93	1.87	1.62
Belgium	52.38	1.61	1.38
France	13.11	1.65	2.36
Netherlands	66.72	1.57	0.01
Spain	18.95	1.27	0.70
UK	31.04	1.82	2.64
US	18.44	2.02	2.27
Japan	24.92	1.46	5.51
Denmark	5.89	1.75	0.71
Canada	3.28	1.7	0.19
Luxembourg	4.37	1.69	-3.74
Norway	6.86	1.86	1.97
Finland	5.42	1.81	-1.90
Iceland	1.95	2.22	1.92
Swedn	1.31	2.00	1.13
New Zeland	1.01	2.05	-2.25
Switzerland	6.56	1.51	3.39

To classify education systems in our sample, we follow de la Croix and Doepke (2007) and choose 90% of the public share in all level educaton as a criterion. If an economy has "more" than 90% of public funding for education in 1993, we assort this country to a group of public educaton system. If an economy has "less" than 90% of public funding for education in 1993, we assort this country to a group of private educaton system.

Table 7: Fertility rate and private educational funding

Size of Private Edu.	N.obs	Average Share of Private Funding,%	Total Fertiltiy Rate
Large($\geq$ 10%)	8	30.19	1.66
Samll($\leq$ 10%)	9	4.07	1.84
Mean difference test			-0.18 (t-stat=-1.67)

Fig. 4 and table 6 reveal that the countries with larger share of private funding for education "seem" to have lower birth rates than those with larger share of public funding for education. Computing the correlation between the propotion of private educational spending in 1993 and total fertility rate in 1993, we find that the correlation coeffcient is -0.4070, which is moderately negative. Table 7 provides the mean difference test and shows the difference of total fertility rate between the countries with larger private sector and those with larger public sector is -0.1845, whose

t-statistic is −1.67 and p-value is close to 10% significance.

Figure 4: Fertility rate and private funding on education

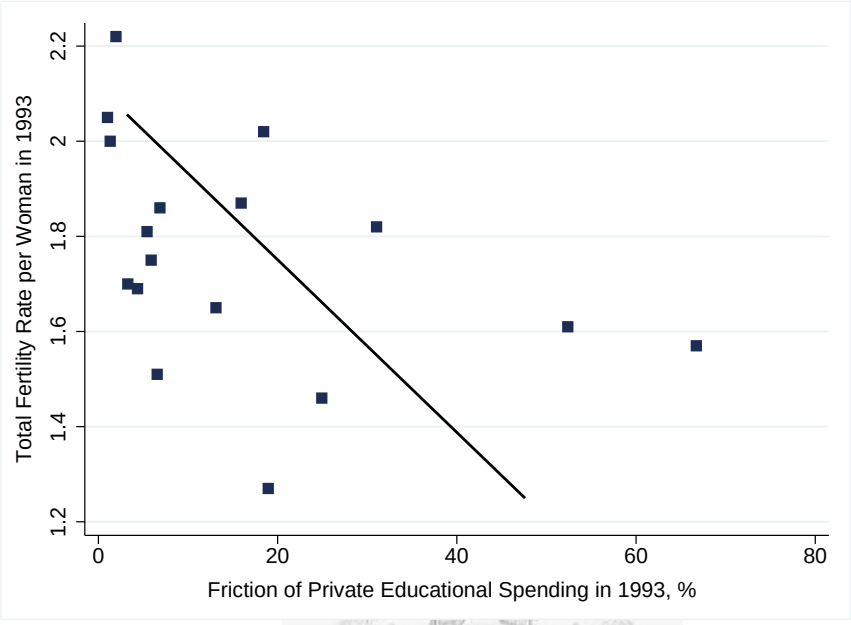
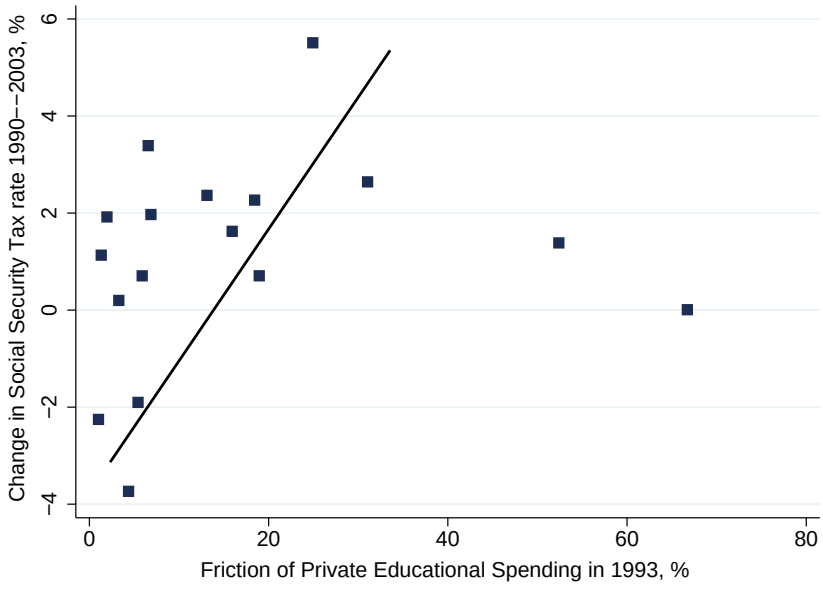


Figure 5: The increase in social security program and private funding on education



The predicted difference of fertility rate is roughly resemble the above empirical evidences. The another implication of our theory needed to

be examined is that countries with public education regime tend to have lower social security tax burden in the future years. Fig. 5 and table 6 show that the countries with larger percentage of private funding on education in 1993 have larger growth in a tax rate of social security during 1990 to 2003 than those with larger percentage of public funding on education in 1993. Some countries with smaller private education sectors, such as Luxembourg, Finland and New Zeland, even have the negative growth in the size of social security program.

Table 8: Growth of social security program and private educational funding

Size of Private Edu.	N.obs	Average Share of Private Funding,%	Increase in Size of Social Security
Large($\geq$ 10%)	8	30.19	2.06
Samll($\leq$ 10%)	9	4.07	0.16
Mean difference test			1.90 (t-stat=-1.92)

Table 8 indicates the difference of growth in social security system between two group is significant (t-stat=−1.92). The tax rate expends 2.06% in the countries with a larger scale of private education but increase only 0.15% in the countries with a smaller scale of private education. According to Ehrlich and Kim (2005), their estimation shows that 1% increase in social secruity tax rate will reduce 0.028% in growth rate per capita. The gap between two groups is almost 2%, that is, the long-run growth rate decreases by 0.056% for the countries with larger size of private education.

5 Conclusion

The design of educational policies and social security program are important issues to modern policy makers, especially to those in developed countries. However, not many previous studies considered these two policies jointly. This paper proposes a three periods overlapping generation model with endogenous fertility to study the interaction between educational systems and PAYG social security program.

We first conclude that If government does not implement PAYG social security program, a private education system can yield higher long-run economic growth than a public education system. This is because free public schooling distorts fertility choice of parents and leads high fertiltiy rate. More children bring a negative ”resource-dilution effect” to an economy and results in less educational investment and slower economic growth.

However, on the other hand, if govenment implements PAYG social security program, a public education system may stimulate higher growth rate than a private education system when an economy has sufficiently old life expectancy or suffciently high pension replacement rate. The reason for this result is that the practice of PAYG social security system makes children have ”tax-sharing effect”, which reduces tax burden of PAYG social security and benefits long-run growth, and furthermore the ”tax-sharing effect” dominates the ”resource-dilution effect” in an economy with high longevity or the policy of high pension benefit.

Thirdly, we also find that government can militgate the financial pressure of PAYG social security program by providing public schooling or voucher program and get some supports from our empirical work.

Our analysis highlights the importance of interaction between educational systems and social security programs in an aging economy. We suggest that to improve economic growth it is necessary for policy makers to think these two policies together.

6 Technical Appendix

Proof of Proposition 1

Proof. Differentiating n_i in Eq.(19) and Eq.(28) with respect to p, it is very straightforward to find the following relation:

$$\frac{\partial n_i}{\partial p} < 0 \quad i = r, u \tag{39}$$

An increase in life expectancy leads agent have fewer children and results in higher PAYG social security tax rate. ■

Proof of Proposition 2

Proof. In order to compare the balanced-growth rate between private education regime and public education regime, it is usful to know how three growth determinants affect growth rate first.

Differentiating $1 + g_i$ in Eq.(36) with respect to n_i, S_i and E_i respectively. We can find:

$$\frac{\partial g_i}{\partial n_i} < 0, \frac{\partial g_i}{\partial S_i} > 0, \frac{\partial g_i}{\partial E_i} > 0 \quad i = r, u \tag{40}$$

Lower fertilty rate, higher saving rate and higher education expenditure can lead higher growth rate.Given any p and $\tau = 0$ in n_i, S_i and E_i respectively we know the following relation:

$$n_{rt} = \frac{(\gamma - \beta\delta)p\sigma}{\phi(1 + p\sigma + \gamma)} < \frac{\gamma p\sigma}{\phi(1 + p\sigma + \gamma)} = n_{ut}$$

$$S_{rt} = \frac{p\sigma}{(1 + p\sigma + \beta\delta)} = \frac{(1 + p\sigma)p\sigma\theta}{(1 + p\sigma + \beta\delta)} = S_{ut}$$

$$E_{rt} = \frac{\beta\delta\phi(1 + p\sigma + \gamma)}{(\gamma - \beta\delta)(1 + p\sigma + \beta\delta)} > \frac{\beta\delta\phi(1 + p\sigma + \gamma)}{\gamma(1 + p\sigma + \beta\delta)} = E_{ut}$$

An economy under private education regime has lower fertility rate, the same saving rate and higher education expenditure than one under

public education regime, which results that balanced-growth rate in private education regime is higher than in public education regime. ■

Proof of Proposition 3

Proof. From Eq.(9) we know lower fertility rate or higher life expectancy will lead higher pay-as-you-go social securtiy tax rate.

$$\frac{\partial \tau_i}{\partial n_i} < 0, \frac{\partial \tau_i}{\partial p} > 0 \quad i = r, u \tag{41}$$

Also form Proposition 2 we find an economy under private education regime has lower fertility rate than one under public education($n_r < n_u$).In our model life expectancy p is exogenous so given any p we know $\tau_r > \tau_u$.

To examine the effect of τ_i on physical/human capital accumulation and balanced growth rate, First, we can differentiate n_i, S_i and E_i with respect to τ_i respectively and find:

$$\frac{\partial n_i}{\partial \tau_i} > 0, \frac{\partial S_i}{\partial \tau_i} < 0, \frac{\partial E_i}{\partial \tau_i} < 0 \quad i = r, u \tag{42}$$

Higher PAYG social security tax rate increase fertility rate (because the substitute effect of tax rate is larger than its income effect), reduce physical and human capital investment and thereby leads lower balanced growth rate. ■

7 Reference

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