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以感測器網路為基礎的智慧型系統之資料融合決策與控制

Information Fusion, Decision and Control of Sensor Network Based Intelligent Systems

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摘要

從環境中蒐集資料的感測器網路讓許智慧型裝置，例如機器人、智慧型車輛甚致是生物醫療器材的應用與設置成為可行的技術。我們觀察到傳統的方法分開執行感測器網路的訊息融合、決策、與接下來的控制行動，而我們提出了一個創新的智慧型決策架構來做整個這些裝置的系統之模型，而可以更進一步的增進系統效能來超越傳統方法。智慧型決策架構藉由分開事件到觀察的映射，成為兩個映射，分別是從事件到物理量及從物理量到觀測，而改善了傳統估計方法。數學公式化在本篇論文中建構出來而且應用於救火機器人的場景來展示它的有效性。我們還更展示了智慧型決策架構在特定的條件下可以被退化成傳統的決策方法。更重要的，我們可以把這個架構延展而超出傳統機制，到融合多個物理量的觀察然後獲得最佳解條件。對於有限物理量相關性資訊下的決策，我們提出了觀察選擇然後求得其與最佳決策等效之條件。較缺乏嚴謹數學架構的模糊邏輯常被應用於這樣的決策，而我們可以展示具嚴謹定義的決策理論數學架構之觀察選擇可以退化成多觀察模糊邏輯決策。最後，模擬結果顯示我們提出的智慧型決策架構的確改善了決策精準程度然後也增進了系統效能。除了感測器網路，這個架構也可以應用於各種不同的智慧型或感知系統。我們提出了在智慧型決策架構下發展出來的雙向時間分割頻譜偵測來展示除了感測器網路之外的應用。這個方法藉由僅一個點的從獨立感測通道的多重觀察減低了隱藏點問題，而合作頻譜偵測則需要多重點去進行多重觀察。這個方法更進一步的利用了因為地理位置間隔產生的路徑損失之資訊來增進感測效能。分析及模擬結果顯示我們提出的頻譜偵測方法顯著的改善了傳統的頻譜偵測效能。

關鍵字：感測器網路，資訊融合，智慧型決策，資料融合，多重觀察，智慧型系統，機器人，導航，決策理論，感知無線電，頻譜感測，接受器感測，雙向時間分割頻譜

Abstract

Sensor networks to collect various information from environments enable deployment and application of many intelligent devices and systems, such as robots, intelligent vehicles, and even biomedical instruments. Observing traditional approach separately executing information fusion from sensor networks, decision, and later control functions, we propose a novel intelligent decision framework to allow thorough system modeling of such devices, and thus further enhancement beyond traditional approach. Intelligent decision framework improves traditional estimation theory by separating the mapping from event to observation into two mappings, the mapping from observed physical quantity to sensor observation and the mapping from target event to physical quantity. The mathematical formulation is constructed and applied in the firefighting robot navigation scenario to illustrate its effectiveness. We further shows that the intelligent decision framework can be degenerated to traditional decision schemes under special conditions. More importantly, we can extend the framework to fuse observations from multiple kinds of physical quantities and derive the optimal decision, beyond traditional statistical decision mechanisms. For the decision with limited knowledge of the correlations among physical quantities, we propose Observation Selection and derive the equality condition with optimal decision. While fuzzy logic of less strict-sense mathematic structure is commonly employed to resolve this application scenario, we can demonstrate that Observation Selection derived from well-defined decision theory can be degenerated to fuzzy logic of multiple kinds of observations. Finally, simulation results show that the proposed intelligent decision framework indeed improves the accuracy of the decision and enhances system performance. In addition to sensor network, this framework can also be applied in various intelligent system or cognitive systems. We propose a novel cognitive radio spectrum sensing scheme, Dual-way Time-Division Spectrum Sensing, derived under intelligent decision framework to demonstrate the application of this general framework other than sensor network. This scheme mitigates the hidden terminal problem by only one node taking multiple observations from independent sensing channel, while cooperative spectrum sensing needs multiple nodes to perform multiple observation. Moreover, this scheme takes the path-loss due to geographical separation into consideration to improve the sensing performance. Analytical and simulation result shows that the proposed spectrum sensing scheme significantly improves the performance of traditional spectrum sensing.

Keywords: Sensor network, information fusion, intelligent decision, data, fusion, multiple observation, intelligent system, robot, navigation, decision theory, cognitive radio, spectrum sensing, receiver sensing, DTD spectrum sensing

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Chapter 1

Introduction

In this chapter, we provide an overview on the information fusion and its application. Here we explain why information fusion is an important concept nowadays and why application of information fusion in sensor network based intelligent system is a challenging problem. We summarize organization of this thesis in the end of the chapter.

1.1 Information Fusion

Information fusion is a widely applied technique in various areas, including sensor network, GPS navigation systems, image processing and communication systems. The term “information fusion” has been defined as follows [36] : “in the context of its usage in the society, it encompasses the theory, techniques and tools created and applied to exploit the synergy in the information acquired from multiple sources (sensor, databases, information gathered by humans, etc.) in such a way that the resulting decision or action is in some sense better (qualitatively or quantitatively, in terms of accuracy, robustness, etc.) than would be possible if any of these sources were used individually without such synergy exploitation."

The information fusion techniques can be classified in three categories according to the sources: *complementary, Redundant,* and *Cooperative*. The relationship can be illustrated as the following figure [16]:

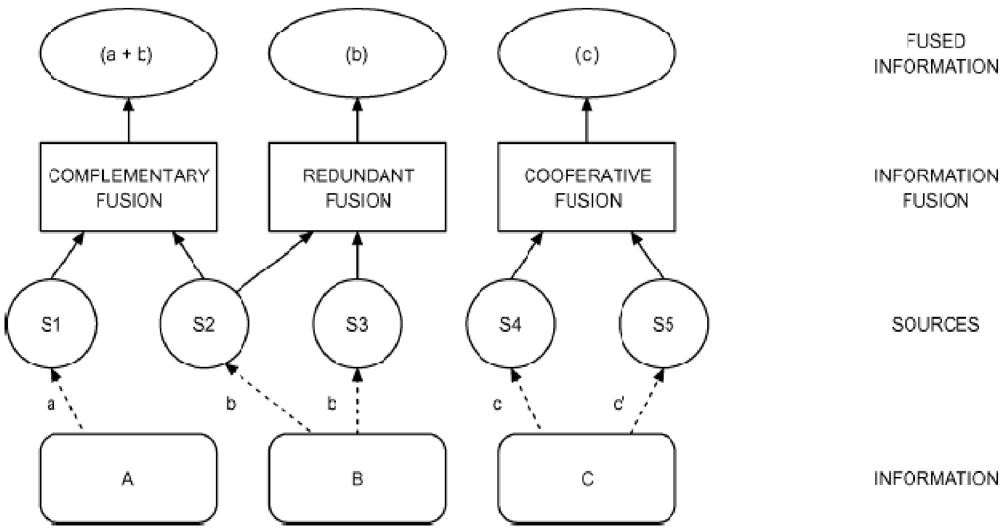


Fig. 1.1 Types of information fusion based on the relationship among the sources.

Then we describe the three types of information fusion [16]:

- *Complementary*. When information provided by the sources represents different portions of a broader scene, information fusion can be applied to obtain a piece of information that is more complete (broader).
- *Redundant*. If two or more independent sources provide the same piece of information, these pieces can be fused to increase the associated confidence.
- *Cooperative*. Two independent sources are cooperative when the information provided by them is fused into new information (usually more complex than the original data) that, from the application perspective, better represents the reality.

In this paper, we focus on the information fusion techniques applied in sensor network cooperating with intelligent systems. We begin with a brief review of information fusion method and algorithms which are able to applied in sensor networks.

1.1.1 Inference

- *Bayesian Inference*:

  In the context of Bayesian inference, the information is represented in terms of conditional probabilities conditioned on the hypothesis we would like to infer and choose. The inference is based on the Bayes' rule:

  $$\Pr(Y|X) = \frac{\Pr(X|Y)\Pr(Y)}{\Pr(X)} \tag{1.1}$$

  The posterior probability $\Pr(Y|X)$ represents the "belief" of hypothesis Y given the information X. With the *a prior* probability $\Pr(Y)$ and conditional probability $\Pr(X|Y)$, we can derive the "belief" of the hypothesis when we have the information X and make inference according to the "belief."

- *Fuzzy Logic*:

  Fuzzy logic is concerned with the formal principles of approximate reasoning, with precise reasoning viewed as a limiting case. Fuzzy logic tries to model the imprecise modes of reasoning that play an essential role in the remarkable human ability to make rational decisions in an environment of uncertainty and imprecision. The following question is an example for the reasoning process that fuzzy logic aims to model.

  "Most of those who live in Belvedere have high incomes. It is probable that Mary lives in Belvedere. What can be said about Mary's income?"

  The question involves many unspecific terms in natural language. Fuzzy logic theorem tries to establish a theoretical framework to deal with this kind of logic and reasoning.

Fuzzy set is the fundamental concept of fuzzy logic. It transforms the traditional set theory to fuzzy set by defining the membership function A,

$$A: X \rightarrow [0,1] \tag{1.2}$$

which maps the members to values in [0,1] to represent its membership in the set. The membership can be some value between 0 and 1 to represent the ambiguity of the concepts or terms involved in the nature language. For example, the description of temperature can be fuzzy set which has the membership function as the following figure:

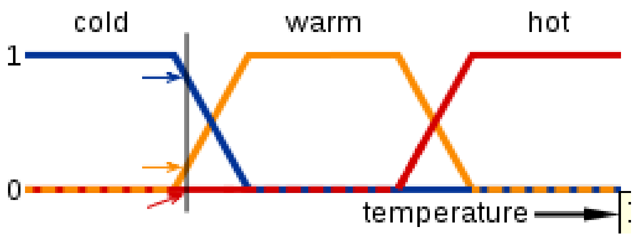


Fig. 1.2 Fuzzy set: description of temperature

The temperature can belong "cold" and "warm" simultaneously in a segment and the membership function of both set in the segment is less than 1 to represent the ambiguity and uncertainty.

Based on the fuzzy set concept, the intersection and union operations, *t-norm* and *t-conorm,* is established. Then with the fuzzy set theory and operations, fuzzy relation and inference can be developed. The fuzzy inference in the form of conditional statement is widely applied in the information fusion problem in sensor network and will be discussed in detail in the succeeding section and chapter 4.

1.1.2 Estimation

- *Maximum Likelihood (ML) and Maximum a posterior (MAP) estimation*:

Estimation methods based on likelihood are suitable when the parameter being estimated is nonrandom. With the likelihood function

$$L(x) = p(\boldsymbol{z}|x) \tag{1.3}$$

where $\boldsymbol{z}$ is the observation vector and x is the parameter we want to estimate. Then the ML estimation is done by maximizing the likelihood function:

$$\hat{x} = \arg\max_{x} p(\boldsymbol{z}|x) \tag{1.4}$$

The MAP estimation aims at estimating a random variable with known probability $p(x)$. Based on Bayesian theory, we can convert the likelihood function $p(\boldsymbol{z}|x)$ to the *a posterior* probability $p(x|\boldsymbol{z})$ with $p(x)$. Then we have the MAP estimation:

$$\hat{x} = \arg\max_{x} p(x|\boldsymbol{z}) \tag{1.5}$$

- *Kalman Filter:*

Kalman Filter is a well-known theory applied in various area including control system, tracking system and sensor network. Kalman filter is first appeared in R.E. Kalman's famous paper in 1960 [44], in which describes a recursive solution to the discrete data linear filtering problem. The Kalman filter addresses the general problem of trying to estimate the state of a discrete-time controlled process that is governed by the linear stochastic difference equation:

$$\boldsymbol{x}(n+1) = \boldsymbol{A}\boldsymbol{x}(n) + \boldsymbol{B}\boldsymbol{u}(n) + \boldsymbol{w}(n) \tag{1.6.1}$$

$$\boldsymbol{y}(n) = \boldsymbol{H}\boldsymbol{x}(n) + \boldsymbol{v}(n) \tag{1.6.2}$$

The random vector $\boldsymbol{w}(n)$ and $\boldsymbol{v}(n)$ and represent the process and measurement noise respectively. In fact, the Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of the squared error [45].

The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. As such, the equations for the Kalman filter fall into two groups: time update equations and measurement update equations.

Prediction Phase
$\hat{\boldsymbol{x}}(n-) = \boldsymbol{A}\hat{\boldsymbol{x}}(n-1) + \boldsymbol{Bu}(n-1)$
$\boldsymbol{P}(n-) = \boldsymbol{AP}(n-1)\boldsymbol{A}^T + \boldsymbol{Q}$
Estimation Phase
$\boldsymbol{K}(n) = \boldsymbol{P}(n-)\boldsymbol{H}^T(\boldsymbol{HP}(n-)\boldsymbol{H}^T + \boldsymbol{R})^{-1}$
$\hat{\boldsymbol{x}}(n) = \hat{\boldsymbol{x}}(n-) + \boldsymbol{K}(n)(\boldsymbol{v}(n) - \boldsymbol{H}\hat{\boldsymbol{x}}(n-))$
$\boldsymbol{P}(n) = (\boldsymbol{I} - \boldsymbol{K}(n)\boldsymbol{H})\boldsymbol{P}(n-)$

Table 1.1 Kalman filter equations.

1.2 Sensor Network Based Intelligent System

Originally, sensor network has been applied in various areas including environment monitoring, forest fire detection, military, drug administration…etc [46-48]. Recent technology advances have lead to the emergence of application of sensor network in various areas including robotic automation system [7,8], intelligent vehicle and transportation system [29,30], and even body area network for biomedical applications [24,49,50]. Most of these systems are operated by deploying the sensor network in the operation environment, such as roadside, buildings, or even inside human body, to monitor the environment and cooperate with intelligent devices, such as a robot or intelligent vehicle, which actively or passively collecting observations from sensor network to perform their tasks. We call these systems Sensor Network Based Intelligent System (SNBIS) in this paper. In SNBISs, the intelligent devices rely on sensor network deployed in the environment to observe the physical world. By collecting the sensor observations, the intelligent device can perceive the environment and perform control actions to execute tasks. Consequently, in order to make correct inference from observations to execute the tasks accurately and efficiently, an intelligent decision framework, which is capable of effectively unified modeling the process from observations of the physical world to the executions of the tasks by the intelligent device, plays an extremely important role in this kind of systems.

There have been many researches on SNBISs already, especially the robot and vehicle being the intelligent devices [7,8,11], while the research on medical applications is emerging [24]. Common application scenarios are obstacle avoidance navigation for the robot [10], localization of robot by cooperative schemes [12], or crash avoidance for intelligent vehicle [30]. The research topics include data collection from network perspective [13], multiple access control [14], robot task allocation [15], and information fusion (or data fusion) to link the observation to the robot's missions, which is the most essential part of the SNBIS. In works regarding this realm, information fusion algorithms [16] such as Kalman filter [20], occupancy grid [21], Baysian inference [19], fuzzy logic [18] are applied to deal with different problems. Many information fusion algorithms can be extended to apply in making decision on multiple kinds of sensor observation. There are also some researches devote in this kind of SNBIS. Most of them focus on heterogeneous sensors with different sensing quality [22,23,28] or fusing observations of different physical quantities by fuzzy logic instead of statistical inference [17].

Although tremendous research effort have devoted in subjects of information fusion or data aggregation in SNBIS, they all focus on specific part of the problem instead of seeking for a general unified theoretical framework from sensor observation to task execution. Application specific works like the algorithms for robot navigation problem [11] is only suitable for small range of applications. The control approaches such as artificial potential field ignore the signal processing while concentrate on control actions corresponding to the environment like obstacle on the road [10,31]. The unified frameworks like Ubiquitous Robotic Space mostly deal with the cooperation among devices and availability of sensor network data to the intelligent devices [25,26]. Problems regarding sensor network monitoring tasks, intruder detection as example, focus on the signal processing but are lack of control action consideration corresponding to the detection event [27]. However, as the development of the intelligent device technology, more and more applications of complex missions in various environments utilizing SNBISs are emerging. Consequently, an unified general theoretical framework from sensor observation to task execution, which is able to serve as a foundation to develop the algorithms and action mechanisms for various application scenarios of SNBIS, is necessary to realize the implementation of SNBIS in various environments. Moreover, this framework must enable the intelligent device to cooperate with various sensors observing different physical phenomenon in the same sensor network to execute the task more efficiently.

1.3 Organization

This paper is organized as follows. We establish the system model of SNBIS intelligent decision framework in Chapter 2. Chapter 3 follows to derive the solution of the application example, firefighting robot navigation problem, by the intelligent decision framework. In Chapter 4, we extend the framework to multiple observation case. Many information fusion schemes including optimal fusion, Observation Selection, Ratio Combining, and Fuzzy Logic. The application example, firefighting robot, follows in Chapter 5. Chapter 6 presents the numerical and simulation results of the application examples of proposed approaches. The application of the intelligent decision framework in Cognitive Radio Spectrum Sensing is presented in Chapter 7. Finally, the conclusion and future work is presented in Chapter 8.

Chapter 2

Intelligent Decision Framework

In this chapter, we give an overview of the intelligent decision framework we proposed to solve the information fusion problem in SNBIS. Then the system model of this framework is established in the succeeding section.

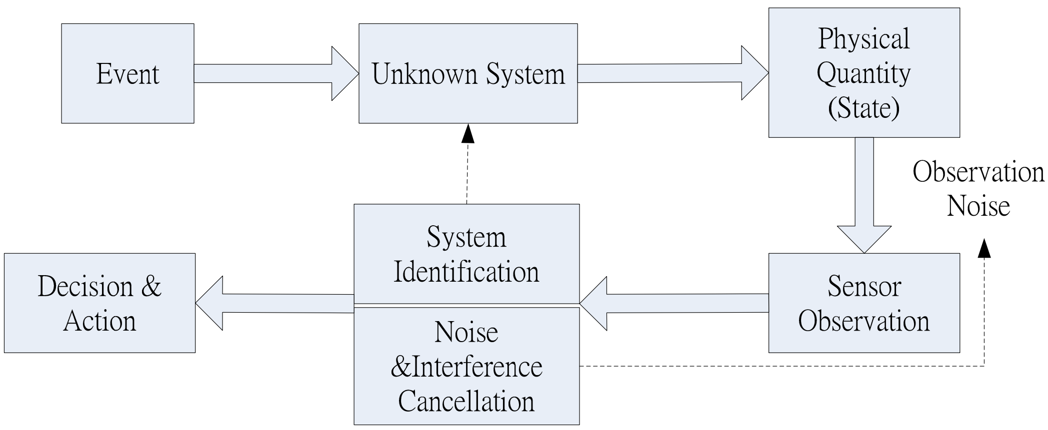


Fig. 2.1 Intelligent decision making mechanism for sensor network based intelligent systems.

2.1 Framework Overview

In this paper, we develop a novel framework called intelligent decision framework.

We derive intelligent decision framework by exploring the relationship between sensor observation and task execution to merge the processes, signal processing, decision, and control action, into a single framework. In SNBISs, the event related to the intelligent devices' tasks induces the physical quantity variations in the environment and sensors are able to transfer the observations of these physical quantities to the intelligent devices to make decision and execute tasks (Fig. 2.1). Thus we can model the sensor observation by two mappings, one from event (event parameter space) to physical quantity (state space) and one from physical quantity (state space) to observation (observation space). Then the intelligent device can make decision to execute tasks (action space) based on the above two mappings according to a cost function, which is the function of event and action. Unlike traditional approaches, these mappings in the framework cover the processes from sensor observation to control action and are applicable in various application scenarios. An example of sensor network navigation for firefighting robot problem (firefighting robot navigation problem in brief) follows to illustrate the application of this framework. The example also shows that the traditional decision scheme is just a degenerated case of our framework under special conditions. Then we extend this framework to intelligent decision framework for observations from multiple kinds of physical quantities. In multiple observation intelligent decision framework, the two mappings, event space to state space and state space to observation space, are extended to account for the variations of different physical quantities induced by the event and observation processes of different sensors. Consequently, we are able to simultaneously model the uncertainty and correlation among different physical quantities and different sensor observation precisions beyond traditional approaches. Optimal decision scheme for multiple observations is developed as well as Observation Selection scheme, which ignores the complex correlation structure among different physical quantities to make decision by limited knowledge of the nature. The condition for equivalence between Observation Selection and optimal decision is zero mutual information between the event and the observations other than selected one. To reduce the computation complexity, Observation Selection by Cramer-Rao bound is developed under specific conditions. In many multiple observation decision applications, fuzzy inference base on fuzzy conditional statement [2], a widely used fuzzy logic control approach, is applied to make decision with less strict-sense mathematic structures. We show that by Observation Selection, we can degenerate the decision scheme to the fuzzy logic controller under the strict mathematical structure of our intelligent decision framework. The example for multiple observation decision also follows up to illustrate the application of the framework and the fuzzy logic formulation.

2.2 System Model

In this section, we construct the system model of intelligent decision for the sensor network based intelligent system. In intelligent decision framework, we formally define and formulate the mathematical relationship between the essential elements involving in the decision process: event parameter, physical quantity related to the event (physical quantity in brief), sensor observation and the control action of the intelligent device. Traditional estimation problem in decision theory directly maps event to sensor observation. However, in order to derive a general framework unifying sensor observation aggregation, decision fusion and control action that is applicable to various environment, we reconstruct two mappings to account the uncertainty involve in the process. The process involves the uncertainty (or incomplete information) of the relationship between event parameter and physical

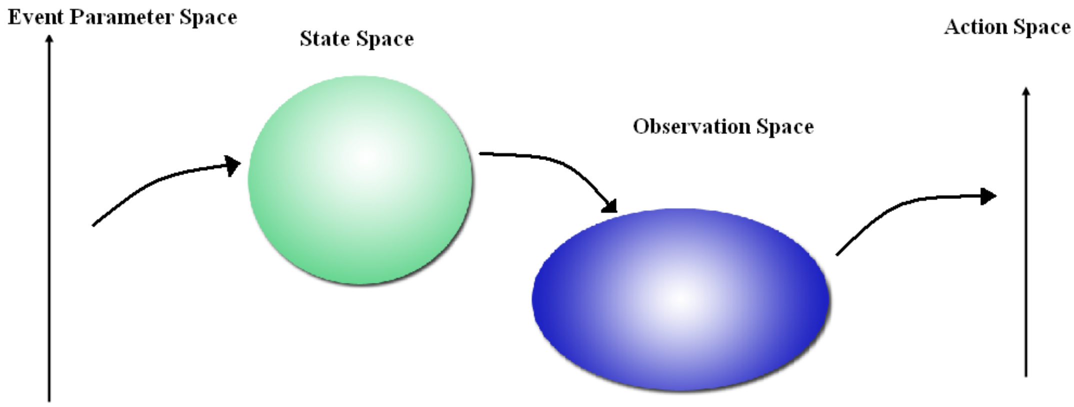


Fig. 2.2 Mathematical structure of intelligent decision making mechanism for sensor network based intelligent systems.

quantities and the uncertainty introduced during observation of the physical quantities (observation noise). According to this concept, we construct the framework of the intelligent decision as follows.

Definition 2-1.1: (*Event Space*) *Event Space* $\boldsymbol{\Theta}$ *is composed of the event parameter, denoted by* $\boldsymbol{\theta}$, *representing the environmental facts or events that are necessary for the intelligent system to make the decision.*

Definition 2-1.2: (*Observation Space*) *Observation Space* $\boldsymbol{O}$ *is composed of the quantity of observations, denoted by* $\boldsymbol{y}$, *from sensors*.

Remark: The observations are the physical quantity plus noise and interference induced during sensor observation.

Definition 2-1.3: (*State Space*) *State space* $\boldsymbol{S}$ *is composed of the observable physical quantity induced by the events. We call them state and denoted by* $\boldsymbol{s}$.

Definition 2-1.4: (*Action space*) *Action space* $\boldsymbol{A}$ *is composed of the decision of actions of the intelligent device, denoted by* $\boldsymbol{a}$.

Definition 2-1.5: (*Utility function*) *The utility function is the reward of the system receiving by making a decision on its action, denoted by* $u = u(\boldsymbol{a}, \boldsymbol{\theta})$

Remark: The utility function must reach its maximum value when the action matches the event parameter, and decrease when the action is more inconsistent with the event parameter. If the system should be panelized by each incorrect decision, we use cost function instead.

Definition 2-2: (*Optimal decision mapping*) *The optimal decision mapping is the mapping* $\Pi: \boldsymbol{O} \rightarrow \boldsymbol{A}$ *that maximize the utility function* $u = u(\boldsymbol{a}, \boldsymbol{\theta})$

We use an example, sensor network navigation for firefighting robot (Fig.2.3), to illustrate the above definitions. The necessary information for firefighting robot's task, reaching the place on fire, is the direction of the place on fire. Hence it is defined to be the event parameter. The fire induces abnormal temperature distribution (or smoke density) in the environment. Consequently, the temperature (smoke) is the physical quantity the sensors should observe. The temperature (smoke density) read on the sensor's thermometer (smoke detector) is the observation aggregated by the firefighting robot. Finally, the control action is the robot's movement direction decided by the sensor observations. Traditional estimation only estimates the exact value of the observed physical quantity considering the observation noise. Hence it can not directly determine the control action. However, our intelligent decision considers the relationship between event and the induced physical quantity and is able to determine the control action according to the utility function under the unified framework. We illustrate the decision mechanism by the mappings between the spaces as follows.

Proposition 2-3: *The optimal decision mapping,* $\Pi: \boldsymbol{O} \rightarrow \boldsymbol{A}$*, is determined from the mapping from event space to state space,* $\Phi: \boldsymbol{S} \rightarrow \boldsymbol{O}$*, and the mapping from state space to observation space,* $\Psi: \boldsymbol{\Theta} \rightarrow \boldsymbol{S}$*, to maximize the utility function* $u = u(\boldsymbol{a}, \boldsymbol{\theta})$

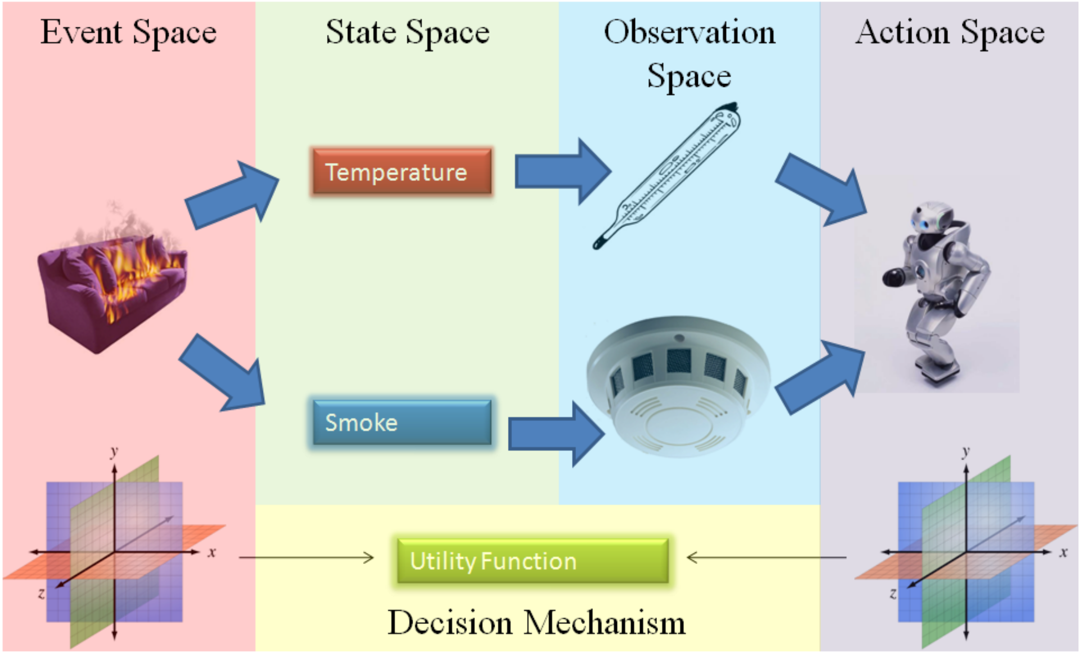


Fig. 2.3 Sensor network navigation for firefighting robot under intelligent decision framework

Remark: From above discussion and definition, we know the state is induced by the event parameter $\boldsymbol{\theta}$ by the mapping $\Psi: \boldsymbol{\Theta} \rightarrow \boldsymbol{S}$. Noise and interference are introduced during sensor observation and establish the mapping $\Phi: \boldsymbol{S} \rightarrow \boldsymbol{O}$. Consequently, we must use $\Psi: \boldsymbol{\Theta} \rightarrow \boldsymbol{S}$ and $\Phi: \boldsymbol{S} \rightarrow \boldsymbol{O}$ to construct the optimal decision mapping $\Pi: \boldsymbol{O} \rightarrow \boldsymbol{A}$ to maximize the utility function $u = u(\boldsymbol{a}, \boldsymbol{\theta})$.

Generally speaking, $\Phi: \boldsymbol{S} \rightarrow \boldsymbol{O}$ involves noise and interference introduced during observation. It can be represented by conditional probability $\mathrm{p}(\boldsymbol{y}|\boldsymbol{s})$ as traditional sensor estimation problem. For the mapping $\Psi: \boldsymbol{\Theta} \rightarrow \boldsymbol{S}$, we have the following proposition:

Proposition 2-4: *The mapping* $\Psi: \boldsymbol{\Theta} \rightarrow \boldsymbol{S}$ *can be represented by the conditional probability* $\mathrm{p}(\boldsymbol{s}|\boldsymbol{\theta})$

Remark: The uncertainty of $\Psi: \boldsymbol{\Theta} \rightarrow \boldsymbol{S}$ comes from the uncertainty or incomplete information of the relationship between the physical quantities we observe and the desired event. We call this "system model uncertainty," or "model uncertainty" in brief. Unlike the mapping $\Phi: \boldsymbol{S} \rightarrow \boldsymbol{O}$ which depends on noise statistics, this mapping depends on the knowledge of the nature and is usually complex.

If this relationship is deterministic and completely known or state and event parameter is the same physical quantity, the mapping degenerates to deterministic or identical mapping. For example, when tracking a fighter, the relationship between the observable physical quantity (radar signal) and the event parameter (fighter's position) is known, the mapping is deterministic. Besides this, with appropriate conditions, we can degenerate Ψ to the deterministic or identical mapping. For example, for firefighting robot navigation problem, the mapping from the direction towards fire (event parameter) to the direction of temperature gradient (state) is an identical mapping if the pattern of the potential field modeling the temperature distribution is radiative. This example will be discussed in detail in next section. For the mapping Ψ to be an identical mapping, we have the following corollary:

Corollary 2-5: Ψ *is an identical mapping if and only if* $p(\boldsymbol{s} = \boldsymbol{\theta} | \boldsymbol{\theta}) = 1$

From above propositions, the mappings, $\Psi: \boldsymbol{\Theta} \rightarrow \boldsymbol{S}$ and $\Phi: \boldsymbol{S} \rightarrow \boldsymbol{O}$, are represented by conditional probabilities. Hence we can interpret the optimal decision in *Definition* II-2 by the following proposition:

Proposition 2-6: (*Optimal decision mapping*) *The optimal decision mapping,* $\Pi: \boldsymbol{O} \rightarrow \boldsymbol{A}$, *following Definition* 2-2*, is the mapping that maximize the a posterior expected utility function* $\mathrm{E}(u(\boldsymbol{a}, \boldsymbol{\theta}) | \boldsymbol{y})$.

Remark: The optimal decision on the action is the action $\hat{\boldsymbol{a}}$ that maximize the *a posterior expected* utility:

$$\hat{\boldsymbol{a}} = \arg\max_{\boldsymbol{a}} \mathrm{E}(u(\boldsymbol{a}, \boldsymbol{\theta})|\boldsymbol{y}) = \arg\max_{\boldsymbol{a}} \int u(\boldsymbol{a}, \boldsymbol{\theta})\mathrm{p}(\boldsymbol{\theta}|\boldsymbol{y})\mathrm{d}\boldsymbol{\theta} \tag{2.1}$$

Baysian Inference

By applying Baysian theory, the *a posterior* probability $\mathrm{p}(\boldsymbol{\theta}|\boldsymbol{y})$ becomes

$$\mathrm{p}(\boldsymbol{\theta}|\boldsymbol{y}) = \frac{\mathrm{p}(\boldsymbol{y}|\boldsymbol{\theta})\mathrm{p}(\boldsymbol{\theta})}{\mathrm{p}(\boldsymbol{y})} \tag{2.2}$$

$\mathrm{p}(\boldsymbol{\theta})$ is the *a prior* distribution of the event. By *Proposition* 2-4 and the mapping $\Phi: \boldsymbol{S} \to \boldsymbol{O}$**,** we can represent $\mathrm{p}(\boldsymbol{y}|\boldsymbol{\theta})$ by the two conditional probabilities

$$\mathrm{p}(\boldsymbol{y}|\boldsymbol{\theta}) = \int \mathrm{p}(\boldsymbol{y}|\boldsymbol{s})\mathrm{p}(\boldsymbol{s}|\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{s} \tag{2.3}$$

And we apply (2.2) and (2.3) to (2.1), we have

$$\hat{\boldsymbol{a}} = \arg\max_{\boldsymbol{a}} \int_{\boldsymbol{\theta}} u(\boldsymbol{a}, \boldsymbol{\theta}) \frac{(\int \mathrm{p}(\boldsymbol{y}|\boldsymbol{s})\mathrm{p}(\boldsymbol{s}|\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{s})\mathrm{p}(\boldsymbol{\theta})}{\mathrm{p}(\boldsymbol{y})}\mathrm{d}\boldsymbol{\theta} \tag{2.4}$$

$$= \arg\max_{\boldsymbol{a}} \iint_{\boldsymbol{s},\boldsymbol{\theta}} u(\boldsymbol{a}, \boldsymbol{\theta})\mathrm{p}(\boldsymbol{y}|\boldsymbol{s})\mathrm{p}(\boldsymbol{s}|\boldsymbol{\theta})\mathrm{p}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}\mathrm{d}\boldsymbol{\theta} \tag{2.5}$$

(2.4) and (2.5) is equal because $\mathrm{p}(\boldsymbol{y})$ is constant for every $\boldsymbol{a}$. The two conditional probabilities, $\mathrm{p}(\boldsymbol{y}|\boldsymbol{s})$ and $\mathrm{p}(\boldsymbol{s}|\boldsymbol{\theta})$, stands for the two mappings, $\Phi: \boldsymbol{S} \to \boldsymbol{O}$ and $\Psi: \boldsymbol{\Theta} \to \boldsymbol{S}$, that involves in the decision mapping $\Pi: \boldsymbol{O} \to \boldsymbol{A}$. Consequently, the decision involves system identification for the modeling uncertainty $\mathrm{p}(\boldsymbol{s}|\boldsymbol{\theta})$ as well as noise and interference cancellation for $\mathrm{p}(\boldsymbol{y}|\boldsymbol{s})$, as depicted in Fig. 2.1. We formulate insightful example of the firefighting robot navigation problem under the intelligent decision framework to demonstrate its application in next chapter.

Chapter 3

Sensor Network Navigation System for Firefighting Robot

In this chapter, we present the sensor network navigation system for firefighting robot problem, an application example of SNBIS, to show the effectiveness of the intelligent decision framework. This example has been investigated in [5]. But we reinvestigate it with the new framework proposed in this paper.

There is a firefighting robot making its way to the place on fire guided by the

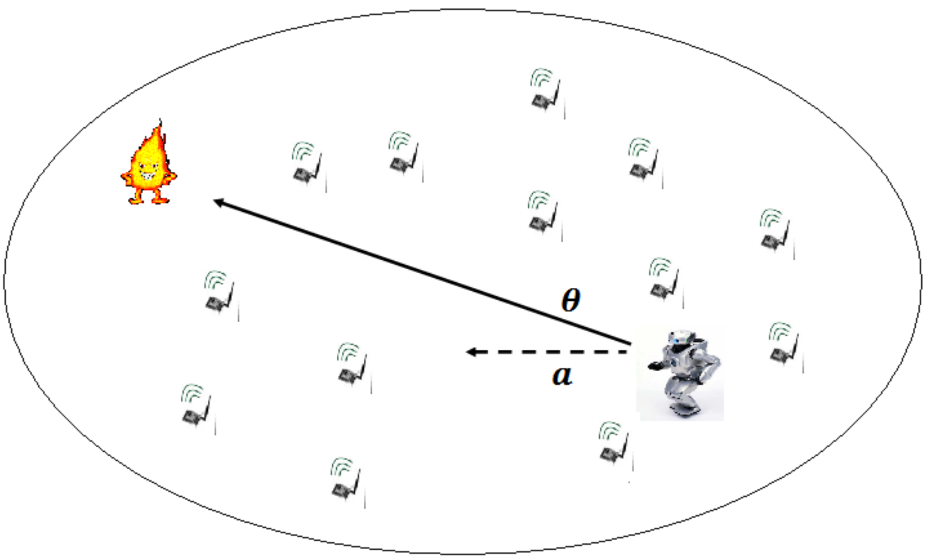


Fig. 3.1 Scenario of Sensor Network Navigation System for Firefighting Robot

sensor network deployed in the environment. The sensors are the thermometers measuring the temperature. The robot collects sensor information (temperature) in this way: it collects the observations from nearby sensor around it and makes decision on movement direction by the observation from sensors and itself. Then the robot walks along the direction until it encounters another group of sensors around it and repeats this action until it reaches the place on fire. The firefighting robot is not equipped with the positioning system. Hence it does not know the precise information of location and direction of reference coordinate system.

3.1 Intelligent Decision Framework for Firefighting Robot

We formulate the decision problem of navigation system for firefighting robot under the intelligent decision framework. We model the temperature distribution induced by the fire as a potential field $f(\boldsymbol{x}, t)$. To simplify the problem, we consider the static potential field $f(\boldsymbol{x})$, that is, ignoring the time variance. In such kind of problems, the intelligent devices are usually lack the information about the nature system $f(\boldsymbol{x})$.

Following *Definition* 2-1.1~1.5 and the remarks, we define the event parameter as the direction of the fire, the observations as the temperatures observed by sensors and the action as the robot's movement direction. The physical quantities (states) are defined to be the direction of highest temperature, namely, the gradient of the potential field, to represent the temperature distribution. The correspondence of spaces in the intelligent decision framework and sensor network navigation for firefighting robot is summarized in Table.3.1. Note that the robot does not know the direction of reference coordinate system. Consequently, the above definitions are using the robot's own coordinate system. Because the robot's goal is to reach the place on fire, we define the utility function to represent how the robot is approaching the place on fire by each decision. Then the utility function is:

$$\mathrm{u}(\boldsymbol{a}, \boldsymbol{\theta}) = \cos(\mathrm{Arg}(\boldsymbol{a}) - \mathrm{Arg}(\boldsymbol{\theta})) \tag{3.1}$$

$\mathrm{Arg}(\boldsymbol{a})$ is the angle with x axis of vector $\boldsymbol{a}$, $\boldsymbol{a}$ is the robot's movement direction (action), and $\boldsymbol{\theta}$ is the direction of the place on fire (event parameter). We assume the length of robot movement between two successive sensor observation collections is unit length. Hence the utility function is proportional to the variance of the distance between the robot and the fire. Putting (3.1) into (2.5), the optimal decision of action $\hat{\boldsymbol{a}}$ becomes:

$$\hat{\boldsymbol{a}} = \arg\max_{\boldsymbol{a}} \int\int \cos(\mathrm{Arg}(\boldsymbol{a}) - \mathrm{Arg}(\boldsymbol{\theta}))\, \mathrm{p}(\boldsymbol{y}|\boldsymbol{s})\mathrm{p}(\boldsymbol{s}|\boldsymbol{\theta})\mathrm{p}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}\, \mathrm{d}\boldsymbol{\theta} \tag{3.2}$$

Intelligent Decision Framework	Robot Navigation
Event parameter	Direction of the fire
Physical quantity	Gradient of temperature
Observation 2	Temperature measurement
Action	Robot's movement direction

Table 3.1 Correspondence of Intelligent decision framework and robot navigation problem

3.2 Sensor Observation Model

We next establish the mapping $\Phi: \boldsymbol{S} \to \boldsymbol{O}$ by constructing the observation model. We begin by defining the observation more precisely. The observation is defined to be the difference between the observed temperature from the sensors around the robot and from the robot itself. The sensor is able to observe $f(\boldsymbol{x}_s)$ plus noise, $\boldsymbol{x}_s$ is the location of the sensor, and robot itself can observe $f(\boldsymbol{x}_{\mathrm{ro}})$ plus noise in its location $\boldsymbol{x}_{\mathrm{ro}}$, thus the potential difference, $f(\boldsymbol{x}_{\mathrm{s}})-f(\boldsymbol{x}_{\mathrm{ro}})$, plus noise is also observable. Hence the relationship between the observations and states can be established approximately by the aid of Taylor Expansion. The Taylor Expansion of the potential field $f(\boldsymbol{x}_{\mathrm{s}})$ of a sensor in $\boldsymbol{x}_{\mathrm{s}}$ is:

$$f(\boldsymbol{x}_{\mathrm{s}})=f(\boldsymbol{a})+\sum_{n_1+n_2+\cdots+n_d=1}^{\infty}\frac{\partial^{n_1}}{\partial x_1^{n_1}}\cdots\frac{\partial^{n_{\mathrm{d}}}}{\partial x_{\mathrm{d}}^{n_{\mathrm{d}}}}\frac{f(x_{\mathrm{ro1}},..,x_{\mathrm{rod}})}{n_1!\ldots n_{\mathrm{d}}!}(x_{\mathrm{s1}}-x_{\mathrm{ro1}})^{n_1}\ldots(x_{\mathrm{sd}}-x_{\mathrm{rod}})^{n_d} \tag{3.3}$$

where

$\boldsymbol{x}_{\mathrm{s}}=[x_{\mathrm{s1}}\ x_{\mathrm{s2}}\ldots x_{\mathrm{sd}}]$, $\boldsymbol{x}_{\mathrm{ro}}=[x_{\mathrm{ro1}}\ x_{\mathrm{ro2}}\ldots x_{\mathrm{rod}}]$

and the potential difference between the robot and the sensor is

$$f(\boldsymbol{x}_{\mathrm{s}})-f(\boldsymbol{x}_{\mathrm{ro}})=\sum_{n_1+n_2+\cdots+n_d=1}^{\infty}\frac{\partial^{n_1}}{\partial x_1^{n_1}}\cdots\frac{\partial^{n_{\mathrm{d}}}}{\partial x_{\mathrm{d}}^{n_{\mathrm{d}}}}\frac{f(x_{\mathrm{ro1}},\cdots,x_{\mathrm{rod}})}{n_1!\ldots n_{\mathrm{d}}!}(x_{\mathrm{s1}}-x_{\mathrm{ro1}})^{n_1}\ldots(x_{\mathrm{sd}}-x_{\mathrm{rod}})^{n_d} \tag{3.4}$$

Assume the dimension d=2. If we approximate the potential difference by the linear term, and note that the relationship between the gradient and the differentiation of the field is

$$f'(\boldsymbol{x})|_{\mathbf{z}}=\frac{\partial f}{\partial x_1}\mathrm{d}x_1+\frac{\partial f}{\partial x_2}\mathrm{d}x_2=\mathbf{z}^{\mathrm{T}}\cdot\vec{\nabla}f(\boldsymbol{x}) \tag{3.5}$$

where

$$\mathbf{z}=[\mathrm{d}x_1\ \mathrm{d}x_2]$$

With Taylor expansion (3.4), when the difference between two location vectors, $\boldsymbol{x}_{\mathrm{s}}$ and $\boldsymbol{x}_{\mathrm{ro}}$, is sufficiently small, we are able to approximate the potential field difference

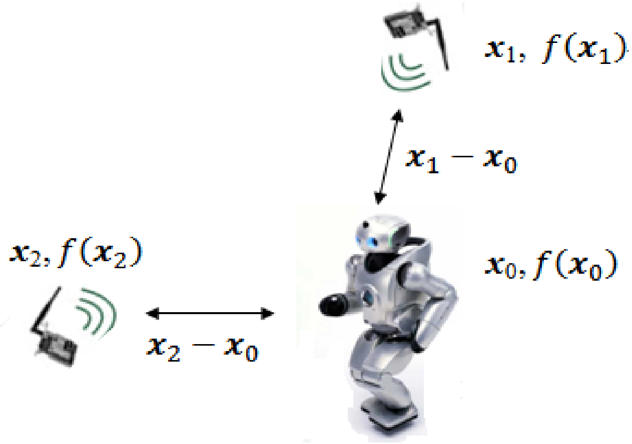


Fig. 3.2 Sensor observation model

by the first order term:

$$f(\boldsymbol{x}_{\mathrm{s}}) - f(\boldsymbol{x}_{\mathrm{ro}}) = \frac{\partial f}{\partial x_1}(x_{\mathrm{s}1} - x_{\mathrm{ro}1}) + \frac{\partial f}{\partial x_2}(x_{\mathrm{s}2} - x_{\mathrm{ro}2}) = (\boldsymbol{x}_{\mathrm{s}} - \boldsymbol{x}_{\mathrm{ro}})^{\mathrm{T}} \cdot \vec{\nabla} f(\boldsymbol{x}_{\mathrm{ro}}) \qquad (3.6)$$

Then we can establish the sensor observation model related to the gradient (state):

$$\boldsymbol{y} = \begin{bmatrix} f'(\boldsymbol{x})|_{(x_s^1 - x_{\mathrm{ro}})} \\ \cdots \\ f'(\boldsymbol{x})|_{(x_s^k - x_{\mathrm{ro}})} \end{bmatrix} + \boldsymbol{n} = \begin{bmatrix} (\boldsymbol{x}_s^1 - \boldsymbol{x}_{\mathrm{ro}})^{\mathrm{T}} \\ \cdots \\ (\boldsymbol{x}_s^k - \boldsymbol{x}_{\mathrm{ro}})^{\mathrm{T}} \end{bmatrix} \cdot \vec{\nabla} f(\boldsymbol{x}) + \boldsymbol{n} \qquad (3.7)$$

$\boldsymbol{n}$ is the difference of observation noise of the sensors and the robot. The robot can infer the gradient (state) by this observation model. By this approximation model and the distribution of $\boldsymbol{n}$, we can derive the conditional distribution $\mathrm{p}(\boldsymbol{y}|\boldsymbol{s})$ where $\boldsymbol{s} = \vec{\nabla} f(\boldsymbol{x})$. Then if we can specify the mapping $\Psi: \boldsymbol{\Theta} \rightarrow \boldsymbol{S}$, we can derive the optimal decision mapping $\Pi: \boldsymbol{O} \rightarrow \boldsymbol{A}$. Generally speaking, this mapping is usually hard to derive directly due to the insufficient information of the unknown system. However, if $\Psi: \boldsymbol{\Theta} \rightarrow \boldsymbol{S}$ is an identical mapping, this problem will reduce to traditional estimation problem in the form of state space model. We derive the degenerate form of state space model in the following.

3.3 Degenerate Problem: State space model

Now we investigate the conditions for the intelligent decision problem of the firefighting robot to degenerate to the traditional state space estimation problem. When the mapping $\Psi: \boldsymbol{\Theta} \rightarrow \boldsymbol{S}$ is an identical mapping, we are able to estimate the event parameter by estimating the state through (3.7), which will be proved in the following. And we can formulate the state space model based on observation model (3.7). Consequently, to degenerate this problem to the traditional form, we must find out the condition for Ψ to be an identical mapping.

We start by investigating $f(\boldsymbol{x})$, which generates the mapping Ψ. When we have precise knowledge of $f(\boldsymbol{x})$ and robot's location $\boldsymbol{x}$, the relationship between state $\boldsymbol{s}$ (gradient of the potential field) and event parameter $\boldsymbol{\theta}$ should be a deterministic function:

$$\boldsymbol{\theta} = h(\boldsymbol{s}, \boldsymbol{x}) \tag{3.8}$$

$h(\boldsymbol{s}, \boldsymbol{x})$ can be determined by $f(\boldsymbol{x})$. However, we assume $f(\boldsymbol{x})$ unknown and no available location information. Consequently, we model their relationship by conditional probability $\mathrm{p}(\boldsymbol{s}|\boldsymbol{\theta})$ according to *Proposition* 2-4. However, the mapping reduces to an identical mapping under appropriate conditions. If we know $f(\boldsymbol{x})$ satisfies the following condition,

$$\vec{\nabla} f(\boldsymbol{x}) = \mathrm{c}(\boldsymbol{x}_d - \boldsymbol{x}),$$

$$\text{for all } \boldsymbol{x}, \text{and } c \text{ is an arbitrary scalar function of } \boldsymbol{x} \tag{3.9}$$

we have

$$\mathrm{p}(\mathrm{Arg}(\boldsymbol{s}) = \mathrm{Arg}(\boldsymbol{\theta}) | \boldsymbol{\theta}) = 1 \tag{3.10}$$

(3-10) is equivalent to $\mathrm{p}(\boldsymbol{s} = \boldsymbol{\theta}|\boldsymbol{\theta}) = 1$ when we consider the expected *a posterior* utility $\mathrm{E}(\cos(\mathrm{Arg}(\boldsymbol{a}) - \mathrm{Arg}(\boldsymbol{\theta})) \mid \boldsymbol{y})$. There are many kinds of potential field satisfying this condition, including electrostatic field and diffusion in free space [6]. As long as the temperature distribution could be modeled by these kinds of potential fields, this condition holds. Then following the concept of *Corollary* 2-5, we can formally prove that the optimal decision mapping can be constructed by estimating state under condition (3.9).

Corollary 3-1: *Estimating state (gradient of potential field) is equivalent to estimating event parameter in the sense of utility function* (3.1) *if* (3.9) *(or equivalent,* (3.10)*) holds.*

Proof:

We need to prove that under the condition (3.10), we have

$$\arg\max_{\boldsymbol{s}'} \mathrm{E}(\cos(\mathrm{Arg}(\boldsymbol{s}') - \mathrm{Arg}(\boldsymbol{s}))\,|\boldsymbol{y}) = \arg\max_{\boldsymbol{a}} \mathrm{E}(\cos(\mathrm{Arg}(\boldsymbol{a}) - \mathrm{Arg}(\boldsymbol{\theta}))\,|\boldsymbol{y})$$

As mentioned above, $\mathrm{p}(\boldsymbol{s} = \boldsymbol{\theta}|\boldsymbol{\theta}) = 1$ is equivalent to (3.10) when calculating $\mathrm{E}(\cos(\mathrm{Arg}(\boldsymbol{a}) - \mathrm{Arg}(\boldsymbol{\theta}))\,|\,\boldsymbol{y})$. Hence we use $\mathrm{p}(\boldsymbol{s} = \boldsymbol{\theta}|\boldsymbol{\theta}) = 1$ instead.

The estimation of gradient is

$$\hat{\boldsymbol{s}} = \arg\max_{\boldsymbol{s}'} \mathrm{E}(\cos(\mathrm{Arg}(\boldsymbol{s}') - \mathrm{Arg}(\boldsymbol{s}))\,|\boldsymbol{y})$$

$$= \arg\max_{\boldsymbol{s}'} \int \cos(\mathrm{Arg}(\boldsymbol{s}') - \mathrm{Arg}(\boldsymbol{s}))\,\mathrm{p}(\boldsymbol{y}|\boldsymbol{s})\mathrm{p}(\boldsymbol{s})\mathrm{d}\boldsymbol{s} \tag{3.11}$$

The maximum expectation of utility function is

$$\max_{\boldsymbol{a}} \mathrm{E}\big(\mathrm{u}(\boldsymbol{a},\boldsymbol{\theta})\big)$$

$$= \max_{\boldsymbol{a}} \mathrm{E}(\cos(\mathrm{Arg}(\boldsymbol{a}) - \mathrm{Arg}(\boldsymbol{\theta}))\,|\boldsymbol{y}) \tag{3.12}$$

$$= \max_{\boldsymbol{a}} \int\int \cos(\mathrm{Arg}(\boldsymbol{a}) - \mathrm{Arg}(\boldsymbol{\theta}))\,\mathrm{p}(\boldsymbol{y}|\boldsymbol{s})\mathrm{p}(\boldsymbol{s}|\boldsymbol{\theta})\mathrm{p}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}\,\mathrm{d}\boldsymbol{\theta} \tag{3.13}$$

$$= \max_{\boldsymbol{a}} \int \cos(\mathrm{Arg}(\boldsymbol{a}) - \mathrm{Arg}(\boldsymbol{s}))\,\mathrm{p}(\boldsymbol{y}|\boldsymbol{s})\mathrm{p}(\boldsymbol{s})\mathrm{d}\boldsymbol{s} \tag{3.14}$$

The equality of (3.13) and (3.14) holds because $\mathrm{p}(\boldsymbol{s} = \boldsymbol{\theta}|\boldsymbol{\theta}) = 1$ and thus

$p(\boldsymbol{s} \neq \boldsymbol{\theta}|\boldsymbol{\theta}) = 0$

Hence

$$\hat{\boldsymbol{a}} = \arg\max_{\boldsymbol{a}} \int \cos(\mathrm{Arg}(\boldsymbol{a}) - \mathrm{Arg}(\boldsymbol{\theta}))\, p(\boldsymbol{y}|\boldsymbol{s})p(\boldsymbol{s})d\boldsymbol{\theta} = \hat{\boldsymbol{s}} \tag{3.15}$$

according to (3.11). Then the Corollary is proved. Q.E.D

Corollary 3.1 shows that under the condition (3.9) on the potential field, the observed physical quantity is identical to the event parameter for the utility function (3.1). Consequently, we are able to use the linear approximation observation model (3.7) to estimate the physical quantity (state) to make action decision without considering event parameter.

In addition to the observation model, we proceed to derive the state transition and formulate the problem into the state space estimation problem. We observe that the gradient is always point to $\boldsymbol{x}_d$ wherever the robot stands on. Hence it is always the same when the robot makes right direction decisions, and would not change significantly even when it makes wrong direction decisions due to the small displacement between two observation collections assumed in the observation model. Recall that the state is defined to be the direction of the destination in robot's own coordinate system and this relative coordinate would rotate when the robot turns its direction, or say makes wrong decision. Hence change of the gradient's direction comes mostly from the relative coordinate rotation instead of the gradient's direction rotation in absolute coordination system when the robot makes wrong decision. We can conclude from above observations that the direction of gradient in the robot's own coordinate system is the difference between in the direction of estimated gradient and the true gradient. Then the state transition is:

$$\boldsymbol{s}_n = \frac{\vec{\nabla} f}{|\vec{\nabla} f|}(\boldsymbol{x}_{n-1}) - \frac{\widehat{\vec{\nabla} f}}{|\vec{\nabla} f|}(\boldsymbol{x}_{n-1}) + \boldsymbol{u} = \boldsymbol{s}_{n-1} - \hat{\boldsymbol{s}}_{n-1} + \boldsymbol{u}_n \tag{3.16}$$

$\boldsymbol{u}$ is a random variable accounts for the direction deviation of the robot's movement due to obstacles, mechanical errors or other non-ideal effects. Here we use normalized gradient to represent its direction. $\boldsymbol{x}_\mathrm{n}$ is the position vector of n'th observation.

The similar navigation problem has been investigated in [5] in a simplified version. We generalized the discrete direction selection solved by Maximum *a posterior* (MAP) hypothesis testing to the continuous vector form (3.17) and relieve the unrealistic assumptions on the observation model to derive the linear approximate observation model (3.7). Then the firefighting robot navigation problem can be formulated by modified state-space model with the estimation of previous state in state transition:

$$\boldsymbol{s}_n = \boldsymbol{s}_{n-1} - \hat{\boldsymbol{s}}_{n-1} + \boldsymbol{u}_n \tag{3.17.a}$$

$$\boldsymbol{y}_n = \boldsymbol{H}_n \boldsymbol{s}_n + \boldsymbol{n}_n \tag{3.17.b}$$

where $\boldsymbol{H}_n = \left|\vec{\nabla} f(\boldsymbol{x}_\mathrm{n})\right| \begin{bmatrix} \mathbf{a}_1{}^\mathrm{T} \\ \dots \\ \mathbf{a}_k{}^\mathrm{T} \end{bmatrix}$, $\mathbf{a}_i$ is also measured in the robot's own coordinate system.

Then we can adapt the widely used MMSE method, Kalman filter, to solve it. In order to apply Kalman filter, we should make a further assumption to limit the error is in a small range in which the approximation

$$\cos(\arg(\boldsymbol{a}) - \arg(\boldsymbol{\theta})) \sim 1 - |\boldsymbol{a} - \boldsymbol{\theta}|^2 \tag{3.18}$$

holds. The assumption assures Kalman filter to fit the optimal decision mapping to maximize the utility function because it makes MMSE decision. However, (3.17.a) and (3.17.b) is different from traditional state-space model of Kalman filter problem due to the additional term, estimation of previous state, in (3.17.a). In fact, in many papers, for example [32], the estimation of previous state has been applied in the prediction problem. In [5], we solve this problem by directly applying MAP hypothesis testing due to the discrete direction form. But when we generalize it to continuous direction, we should solve it by modifying solution of Kalman filter. However, it turns out that the estimation of previous state can be regard as outside input in solving Kalman filter. We prove this in the *Lemma* A1 in Appendix 3. Then with this lemma, we can solve the state space model by Kalman filter approach. The solution is:

Prediction Phase	
$\hat{\boldsymbol{x}}_n^- = \hat{\boldsymbol{x}}_{n-1} - \hat{\boldsymbol{x}}_{n-1} = \boldsymbol{0}$	(3.19)
$\boldsymbol{P}_n^- = \boldsymbol{P}_{n-1}^+ + \boldsymbol{Q}_{n-1}$	(3.20)
Estimation Phase	
$\boldsymbol{K}_n = \boldsymbol{P}_n^- \boldsymbol{H}_n^{\mathrm{T}} (\boldsymbol{H}_n \boldsymbol{P}_n^- \boldsymbol{H}_n^{\mathrm{T}} + \boldsymbol{R}_n)^{-1} = \boldsymbol{P}_n^+ \boldsymbol{H}_n^{\mathrm{T}} \boldsymbol{R}_n^{-1}$	(3.21)
$\hat{\boldsymbol{x}}_n = \boldsymbol{K}_n \boldsymbol{y}_n$	(3.22)
$\boldsymbol{P}_n^+ = (\boldsymbol{I} - \boldsymbol{K}_n \boldsymbol{H}_n) \boldsymbol{P}_n^-$	(3.23)

Table. 3.2 Kalman filter

where

$\boldsymbol{Q}_n = \mathrm{E}(\boldsymbol{u}_n \boldsymbol{u}_n^{\mathrm{T}}),\ \ \boldsymbol{R}_n = \mathrm{E}(\boldsymbol{n}_n \boldsymbol{n}_n^{\mathrm{T}})$

We can use above equations to recursively solve our state space model estimation problem.

Appendix 3.A State-space Model with Estimation of Previous State

In Section 3.3, we derive the state space model of the firefighting robot navigation problem. The state space model (3.17.a), (3.17.b) is different from state space model of Kalman filter due to that it includes the estimation of previous state in state transition equation. However, in this section we show that the estimation of previous state in state transition equation can be regarded as outside input in Kalman filter when deriving the LMMSE state estimation.

Lemma 3.A1: *The estimation of previous state in state transition equation of state-space model is the same as outside input in the state transition equation of Kalman filter when solving the LMMSE state estimation problem.*

Proof:

The state-space model including the estimation of previous state can be formulated as:

$$\mathbf{x}(n) = \mathbf{\Phi}(n, n-1)\mathbf{x}(n-1) + \mathbf{\Psi}(n, n-1)\hat{\mathbf{x}}(n-1) + \mathbf{u}(n) \quad (3.24)$$

$$\mathbf{y}(n) = \mathbf{H}(n)\mathbf{x}(n) + \mathbf{w}(n) \quad (3.25)$$

$$E(\mathbf{u}(n)\mathbf{u}(s)') = \mathbf{Q}(n)\delta_{ns} \quad (3.26)$$

$$E(\mathbf{w}(n)\mathbf{w}(s)') = \mathbf{R}(n)\delta_{ns} \quad (3.27)$$

where $\hat{\mathbf{x}}(n-1)$ is the estimation of previous state $\mathbf{x}(n-1)$.

Based on the innovation process concept in [1], we can derive the LMMSE of $\mathbf{x}(n)$'s as follows:

Define the innovation process:

$$\mathbf{v}(n) \equiv \mathbf{y}(n) - \mathbf{H}(n)\hat{\mathbf{x}}(n|n-1) \quad (3.28)$$

And the covariance of estimation of $\mathbf{H}(n)\mathbf{x}(n)$

$$\mathbf{P}_{\mathbf{z}}(n) = E_s([\mathbf{z}(n) - \hat{\mathbf{z}}(n|n-1)][(\mathbf{z}(n) - \hat{\mathbf{z}}(n|n-1)]') \quad (3.29)$$

$$\mathbf{z}(n) = \mathbf{H}(n)\mathbf{x}(n) \quad (3.30)$$

We use $E_s(\mathbf{X})$ to denote the ensemble average. And we require

$$\mathbf{v}(s) \perp \mathbf{x}(n) - \hat{\mathbf{x}}(n) \quad \text{for } s \le n \quad (3.31)$$

By projection theory

$$\hat{\mathbf{x}}(n) = \sum_{k=1}^{n} E_s(\mathbf{x}(n)\mathbf{v}'(k))\left(\mathbf{P}_{\mathbf{z}}(k) + \mathbf{R}(k)\right)^{-1}\mathbf{v}(k) \quad (3.32)$$

$$= \sum_{k=1}^{n-1} E_s(\mathbf{x}(n)\boldsymbol{\upsilon}'(k))\left(\mathbf{P_z}(k) + \mathbf{R}(k)\right)^{-1}\boldsymbol{\upsilon}(k)$$

$$+ E\left(\mathbf{x}(n)\boldsymbol{\upsilon}'(n)\right)\left(\mathbf{P_z}(n) + \mathbf{R}(n)\right)^{-1}\boldsymbol{\upsilon}(n) \tag{3.33}$$

$$= \mathbf{K}(n)\boldsymbol{\upsilon}(n) + \sum_{k=1}^{n-1} E_s([\boldsymbol{\Phi}(n, n-1)\mathbf{x}(n-1) + \boldsymbol{\Psi}(n, n-1)\hat{\mathbf{x}}(n-1)$$

$$+ \mathbf{u}(n)]\boldsymbol{\upsilon}'(k))\left(\mathbf{P_z}(k) + \mathbf{R}(k)\right)^{-1}\boldsymbol{\upsilon}(k) \tag{3.34}$$

$$= \mathbf{K}(n)\boldsymbol{\upsilon}(n) + \boldsymbol{\Phi}(n, n-1)\hat{\mathbf{x}}(n-1) +$$

$$\boldsymbol{\Psi}(n, n-1)\sum_{k=1}^{n-1} E_s(\hat{\mathbf{x}}(n-1)\boldsymbol{\upsilon}'(k))\left(\mathbf{P_z}(k) + \mathbf{R}(k)\right)^{-1}\boldsymbol{\upsilon}(k) \tag{3.35}$$

In (3.34), we define

$$\mathbf{K}(n) = E(\mathbf{x}(n)\boldsymbol{\upsilon}'(n))\left(\mathbf{P_z}(n) + \mathbf{R}(n)\right)^{-1}$$

and in (3.35), we use that fact that

$$\boldsymbol{\Phi}(n, n-1)\hat{\mathbf{x}}(n-1) = \boldsymbol{\Phi}(n, n-1)\sum_{k=1}^{n-1} E_s(\mathbf{x}(n-1)\boldsymbol{\upsilon}'(k))\left(\mathbf{P_z}(k) + \mathbf{R}(k)\right)^{-1}\boldsymbol{\upsilon}(k) \tag{3.36}$$

The first two terms in RHS is the same as derivation in [1]. Then we deal with the last term:

$$\sum_{k=1}^{n-1} E_s(\hat{\mathbf{x}}(n-1)\boldsymbol{\upsilon}'(k))\left(\mathbf{P_z}(k) + \mathbf{R}(k)\right)^{-1}\boldsymbol{\upsilon}(k) \tag{3.37}$$

$$= \sum_{k=1}^{n-1} E_s([\mathbf{x}(n-1) - (\mathbf{x}(n-1) - \hat{\mathbf{x}}(n-1))]\boldsymbol{\upsilon}'(k))\left(\mathbf{P_z}(k) + \mathbf{R}(k)\right)^{-1}\boldsymbol{\upsilon}(k) \tag{3.38}$$

Because $\mathbf{x}(n-1) - \hat{\mathbf{x}}(n-1)$ is orthogonal to $\boldsymbol{\upsilon}'(k)$ according to (3.31), we have

$$\sum_{k=1}^{n-1} E_s(\hat{\mathbf{x}}(n-1)\boldsymbol{\upsilon}'(k))\left(\mathbf{P_z}(k) + \mathbf{R}(k)\right)^{-1}\boldsymbol{\upsilon}(k) \tag{3.39}$$

$$= \sum_{k=1}^{n-1} E_s(\hat{\mathbf{x}}(n-1)\boldsymbol{\upsilon}'(k))\big(\mathbf{P_z}(k) + \mathbf{R}(k)\big)^{-1}\boldsymbol{\upsilon}(k) \tag{3.40}$$

$$= \hat{\mathbf{x}}(n-1) \tag{3.41}$$

Put (3.41) into (3.35), we have

$$\hat{\mathbf{x}}(n) = \sum_{k=1}^{n} E_s(\mathbf{x}(k)\boldsymbol{\upsilon}'(k))\big(\mathbf{P_z}(k) + \mathbf{R}(k)\big)^{-1}\boldsymbol{\upsilon}(k)$$

$$= \mathbf{K}(n)\boldsymbol{\upsilon}(n) + \boldsymbol{\Phi}(n, n-1)\hat{\mathbf{x}}(n-1) + \boldsymbol{\Psi}(n, n-1)\hat{\mathbf{x}}(n-1) \tag{3.42}$$

The original solution of Kalman filter based on innovation process is

$$\hat{\mathbf{x}}(n) = \mathbf{K}(n)\boldsymbol{\upsilon}(n) + \boldsymbol{\Phi}(n, n-1)\hat{\mathbf{x}}(n|n-1) \tag{3.43}$$

From (3.42), we can observe that $\boldsymbol{\Psi}(n, n-1)\hat{\mathbf{x}}(n-1)$ can be directly added, without alternating the terms in the original form (3.43), to the estimation like an outside deterministic input to the state transition system. Consequently, we know that the estimation of previous state can be regarded as outside input to the system and ordinary algorithm solving state-space model of Kalman filter is able to solve the state space model with estimation of previous state without any modification.

Q.E.D

Chapter 4

Intelligent Decision Framework-Multiple Observation

In previous sections, the intelligent decision framework fuses observations of single kind of physical quantity. However, in many application scenarios of sensor network based intelligent systems, multiple kinds of physical quantities may change in response to the occurrence of an event. For example, fire can induce high temperature and heavy smoke intensity, or even the number of broken sensors can be taken as observations. Intuitively, efficiently taking the observations of more kinds of

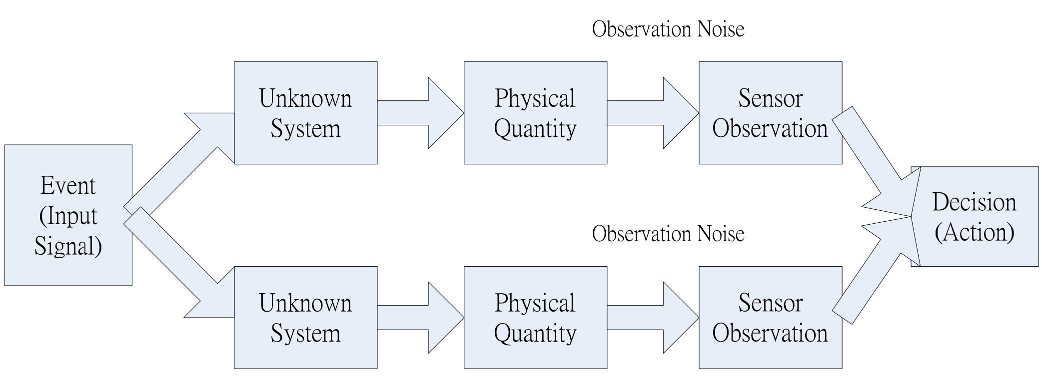
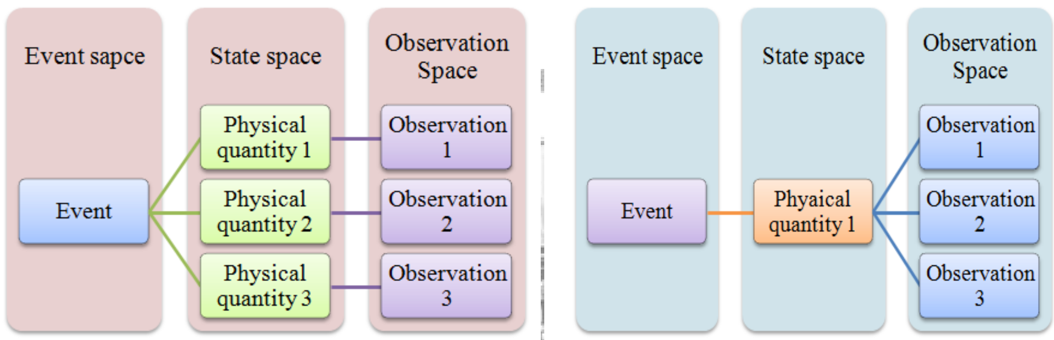


Fig. 4.1 Intelligent decision making mechanism with multi-observation for sensor network based intelligent systems (detail of decision block omitted).

physical quantities into consideration in decision process may improve the system performance. Traditional estimation schemes, which directly map event to observation, are only able to solve the fusion problem of observations of the same physical quantity from sensors with different precisions [23,28]. However, our intelligent decision framework separates the mappings and are able to simultaneously model the uncertain and correlation of different physical quantities and uncertain introduced by different sensor precisions. We formulate the multi-observation intelligent decision by extending the framework in Chapter 2 in the following.



(a) (b)

Fig. 4.2 Comparison of (a) multiple observation intelligent decision framework and (b) traditional multiple observation model

4.1 Optimal Multi-Observation Decision System Model

Extending the framework established in section II, we have the following definitions:

Definition 4-1.1: (*Observation space*) *Observation space is defined to be the Cartesian product of the observation spaces of each kind of observations.*

Remark: Different sensors collect different kinds of observations simultaneously.

Consequently, the observation space is the Cartesian product of the observation spaces corresponding to each kind of sensors. According to the definition, the observation space of multi-observation decision problem becomes $\boldsymbol{O}^K = \boldsymbol{O}_1 \times \boldsymbol{O}_2 \times \ldots . . \times \boldsymbol{O}_K$, K is the number of kinds of observations. We call $\boldsymbol{O}^K$ the observation space and $\boldsymbol{O}_i$ the sub-observation spaces.

Definition 4-1.2: (*State space*) *State space is defined to be the Cartesian product of the state spaces corresponding to each kind of physical quantities.*

Remark: Different kinds of observations are raised from different physical quantities. Consequently, $\boldsymbol{S}^K = \boldsymbol{S}_1 \times \boldsymbol{S}_2 \times \ldots . . \times \boldsymbol{S}_K$.

The definition of event space and action space are the same as single observation decision, and also denoted by $\boldsymbol{\Theta}$ and $\boldsymbol{A}$. Then we can define the optimal multi-observation decision.

We again use the firefighting robot navigation scenario of Chapter 3 with multiple kinds of observations to elucidate the above definitions. The fire can induce

Fig. 4.3 Intelligent Decision framework : multiple observation heavy smoke and high temperature in the environment. If there are two kinds of sensors, thermometer and the smoke detector, in the environment, the firefighting robot is able to collect two kinds of observations, temperature and smoke density. Then observation space $\boldsymbol{O}_1$ is consist of temperature observations, observation space $\boldsymbol{O}_2$ is consist of smoke density observations, and the observation space for decision is $\boldsymbol{O}^2 = \boldsymbol{O}_1 \times \boldsymbol{O}_2$. Similarly, $\boldsymbol{S}_1$ is temperature and $\boldsymbol{S}_2$ is smoke density. The event parameter and action is the same as single observation case. Based on above definitions, we have the definition of optimal multi-observation decision:

Definition 4-2: (*Optimal multi-observation decision*) *The optimal multi-observation decision mapping is the mapping* $\Pi: \boldsymbol{O}^K \rightarrow \boldsymbol{A}$ *that maximize the a posterior expected utility function* $\mathrm{E}(u(\boldsymbol{a},\boldsymbol{\theta})|\, \boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_K)$

We expand the *a posterior* expected utility function

$$\mathrm{E}(u(\boldsymbol{a},\boldsymbol{\theta})|\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_K)$$
$$= \int u(\boldsymbol{a},\boldsymbol{\theta})\mathrm{p}(\boldsymbol{\theta}|\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_K)\mathrm{d}\boldsymbol{\theta}$$
$$= \int_{\boldsymbol{\theta}} u(\boldsymbol{a},\boldsymbol{\theta})\left(\int_{\boldsymbol{s}_1,\boldsymbol{s}_2,\ldots,\boldsymbol{s}_K} \ldots \int \mathrm{p}(\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_K|\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K)\mathrm{p}(\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K|\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}_1 \ldots \mathrm{d}\boldsymbol{s}_K\right)\mathrm{p}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta} \tag{4.1}$$

And the decision mapping is decided by maximizing (4.1)

$$\max_{\boldsymbol{a}} \mathrm{E}(u(\boldsymbol{a},\boldsymbol{\theta})|\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_K)$$
$$= \max_{\boldsymbol{a}} \int_{\boldsymbol{\theta}} u(\boldsymbol{a},\boldsymbol{\theta})\left(\int_{\boldsymbol{s}_1,\boldsymbol{s}_2,\ldots,\boldsymbol{s}_K} \ldots \int \mathrm{p}(\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_K|\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K)\mathrm{p}(\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K|\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}_1 \ldots \mathrm{d}\boldsymbol{s}_K\right)\mathrm{p}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta}$$
$$= \max_{\boldsymbol{a}} \int_{\boldsymbol{\theta}} u(\boldsymbol{a},\boldsymbol{\theta})\left(\int_{\boldsymbol{s}_1,\boldsymbol{s}_2,\ldots,\boldsymbol{s}_K} \ldots \int \prod_{i=1}^{K} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\, \mathrm{p}(\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K|\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}_1 \ldots \mathrm{d}\boldsymbol{s}_K\right)\mathrm{p}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta} \tag{4.2}$$

In (4.2), we assume

$$\mathrm{p}(\boldsymbol{y}_1, \boldsymbol{y}_2, \dots, \boldsymbol{y}_K | \boldsymbol{s}_1, \boldsymbol{s}_2, \dots, \boldsymbol{s}_K)$$

$$= \prod_{i=1}^{K} \mathrm{p}(\boldsymbol{y}_i | \boldsymbol{s}_1, \boldsymbol{s}_2, \dots, \boldsymbol{s}_K) = \prod_{i=1}^{K} \mathrm{p}(\boldsymbol{y}_i | \boldsymbol{s}_i) \quad (4.3)$$

which means that given the states, the observation of each state is independent, and the observations are independent with other states excepted the corresponding state. Hence the mapping $\Phi: \boldsymbol{S}^K \rightarrow \boldsymbol{O}^K$ can be separated in to independent mappings $\Phi_i: \boldsymbol{S}_i \rightarrow \boldsymbol{O}_i$. This is quite reasonable because each sensor observes each physical quantity (state) independently and would not be affected by other states. However, although it is reasonable to assume the independence of the conditional probability $\mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)$'s, the independence among physical quantities given the event parameter does not hold in general. For example, in the scene of fire, the place with high temperature will have high smoke density with high probability. Generally speaking, the physical quantities changed by the same event are highly correlated and the correlation is too complex to derive directly. Hence, we develop the "Observation Selection", a decision mapping to optimally select one observation to make decision without considering the correlations among the physical quantities.

4.2 Observation Selection

In order to avoid dealing with the correlations among the different physical quantities, Observation Selection scheme selects the best observation according to the utility function and makes decision by this selected observation. In Observation Selection, we reduce the general optimal decision of multi-observation problem into selection of a best observation among all kinds of observations to make decision.

(Fig.4.4) We first define the sub-decision mapping:

Definition 4-3: (*Sub-decision mapping*) *The sub-decision mappings are defined to be the mapping from each sub-observation space* $\boldsymbol{O}_i$ *to action space* $\boldsymbol{A}$ *that maximize the a posterior expected utility function* $\mathrm{E}_i(u(\boldsymbol{a},\boldsymbol{\theta})|\boldsymbol{y}_i)$.

We denote the sub-decision mapping by

$\Pi_i: \boldsymbol{O}_i \rightarrow \boldsymbol{A}$

Then the expected *a posterior* utility E_i for sub-decision mapping Π_i is

$$\mathrm{E}_i(u(\boldsymbol{a},\boldsymbol{\theta})|\boldsymbol{y}_i) = \int\int u(\boldsymbol{a},\boldsymbol{\theta})\mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\mathrm{p}(\boldsymbol{s}_i|\boldsymbol{\theta})\mathrm{p}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}_i\,\mathrm{d}\boldsymbol{\theta} \tag{4.4}$$

And the mapping Π_i is decided by

$$\widehat{\boldsymbol{a}} = \max_{\boldsymbol{a}} \mathrm{E}_i(u(\boldsymbol{a},\boldsymbol{\theta})|\boldsymbol{y}_i) \tag{4.5}$$

(4.4) is the same as the single observation expected *a posterior* utility except the observation indices. The mapping considered here is $\boldsymbol{S}_i \rightarrow \boldsymbol{O}_i$ and $\boldsymbol{\Theta} \rightarrow \boldsymbol{S}_i$ instead of $\boldsymbol{S}^K \rightarrow \boldsymbol{O}^K$ and $\boldsymbol{\Theta} \rightarrow \boldsymbol{S}^K$.

Sub-decision mapping divides the observation space into individual observations and makes decision by those sub-observation spaces separately. Observation Selection is the decision scheme to select the best sub-decision mappings. Following the definition of sub-decision mapping, we formally define Observation Selection as follows:

Definition 4-4: (*Observation Selection*) *Observation Selection is the decision mapping that has the largest a posterior expected utility function among all* $\Pi_i, i = 1, \dots, K$

Then the optimal decision mapping Π of Observation Selection is:

$$i^* = \arg\max_i \left(\max_{\boldsymbol{a}} \mathrm{E}_i(u(\boldsymbol{a},\boldsymbol{\theta})|\boldsymbol{y}_i)\right)$$

$$\Pi = \Pi_{i^*} \tag{4.6}$$

Expanding the maximum expected *a posterior* utility, we have

$$\max_i \max_{\boldsymbol{a}} \mathrm{E}_i(u(\boldsymbol{a}, \boldsymbol{\theta})|\boldsymbol{y}_i)$$

$$= \max_i \max_{\boldsymbol{a}} \int\int u(\boldsymbol{a}, \boldsymbol{\theta}) \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i) \mathrm{p}(\boldsymbol{s}_i|\boldsymbol{\theta}) \mathrm{p}(\boldsymbol{\theta}) \mathrm{d}\boldsymbol{s}_i \, \mathrm{d}\boldsymbol{\theta} \quad (4.7)$$

When applying Observation Selection, instead of combing all observations and fusing them to make decision, we select the "best" observation to make decision due to lack of the information regarding correlations among states. For example, the firefighting robot using Observation Selection first chooses among the observations from thermometer and smoke detector then makes decision by the selected observations.

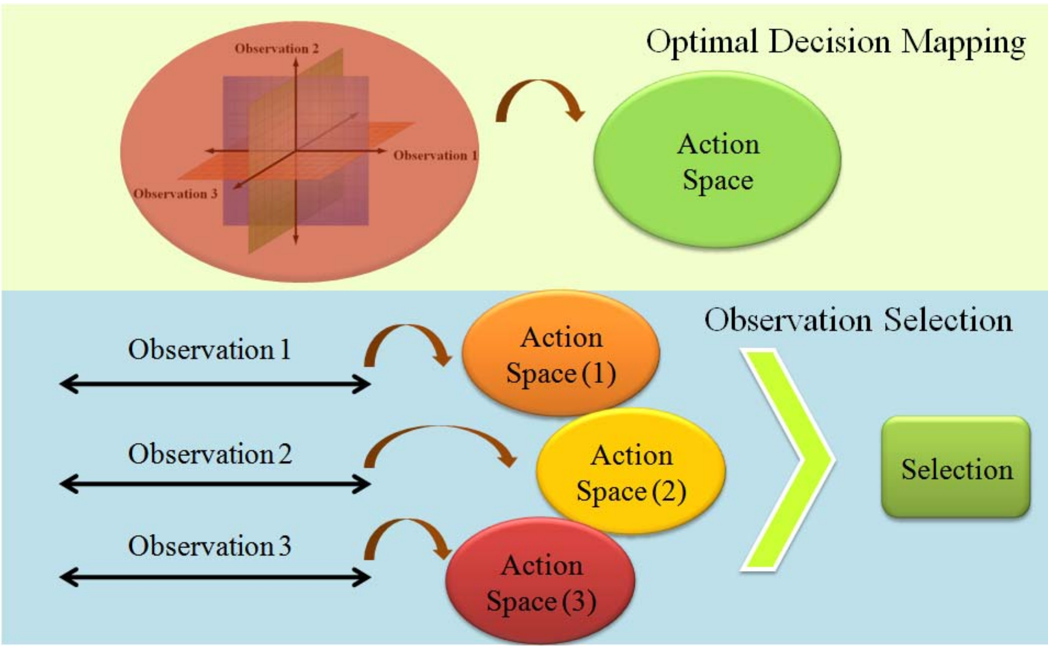


Fig. 4.4 Comparison of optimal decision and Observation Selection

We next investigate under what conditions the Observation Selection being equivalent to the optimal decision mapping. Denote the index of the sub-decision mapping that has the largest maximum expected *a posterior* utility by i^*. Intuitively, if the observations other than the selected one do not provide information when making the decision, the Observation Selection is optimal. In other words, the conditional mutual information of event parameter and the observations other than the selected one given the state of the selected observation, $\mathrm{I}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K;\boldsymbol{\theta}|\boldsymbol{s}_{i^*})$, is zero. In order to prove that conditional mutual information being zero is the sufficient and necessary condition for optimality of Observation Selection, we prove the following lemma first:

Lemma 4-5. $\mathrm{I}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K;\boldsymbol{\theta}|\boldsymbol{s}_{i^*})=0$ *is equivalent to* $\mathrm{p}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K|\boldsymbol{\theta},\boldsymbol{s}_{i^*})$ *being const with respect to* $\boldsymbol{\theta}$ *and* $\boldsymbol{s}_{i^*}$*.*

Proof:

$$\mathrm{I}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K;\boldsymbol{\theta}|\boldsymbol{s}_{i^*})$$

$$=\sum_{\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K,\boldsymbol{\theta},\boldsymbol{s}_{i^*}}\mathrm{p}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K,\boldsymbol{\theta},\boldsymbol{s}_{i^*})\log\left(\frac{\mathrm{p}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K,\boldsymbol{\theta}|\boldsymbol{s}_{i^*})}{\mathrm{p}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K|\boldsymbol{s}_{i^*})\mathrm{p}(\boldsymbol{\theta}|\boldsymbol{s}_{i^*})}\right)$$

$$=0 \tag{4.8}$$

Hence

$$\frac{\mathrm{p}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K,\boldsymbol{\theta}|\boldsymbol{s}_{i^*})}{\mathrm{p}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K|\boldsymbol{s}_{i^*})\mathrm{p}(\boldsymbol{\theta}|\boldsymbol{s}_{i^*})}$$

$$=\frac{\mathrm{p}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K|\boldsymbol{\theta},\boldsymbol{s}_{i^*})}{\mathrm{p}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K|\boldsymbol{s}_{i^*})}=1 \tag{4.9}$$

Then

$$\mathrm{p}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K|\boldsymbol{\theta},\boldsymbol{s}_{i^*})$$

$$=\mathrm{p}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K|\boldsymbol{s}_{i^*}) \tag{4.10}$$

$$=\mathrm{p}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K) \tag{4.11}$$

(4.10) is due to (4.9). (4.11) holds because the observations depend only on its corresponding state and is independent of other states. $\mathrm{p}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K)$ is a constant with respect to $\boldsymbol{\theta}$ and $\boldsymbol{s}_{i^*}$.

Q.E.D

We proceed to prove that $\mathrm{I}(\boldsymbol{y}_1,\dots,\boldsymbol{y}_{i^*-1},\boldsymbol{y}_{i^*+1}\dots,\boldsymbol{y}_K;\boldsymbol{\theta}|\boldsymbol{s}_{i^*})=0$ is the sufficient and necessary condition for optimality of Observation Selection in the following theorem:

Theorem 4-6 *Observation Selection is equivalent to optimal decision mapping if and* *only if* $\mathrm{I}(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{i^*-1}, \boldsymbol{y}_{i^*+1} \ldots, \boldsymbol{y}_K; \boldsymbol{\theta}|\boldsymbol{s}_{i^*}) = 0$

Proof:

From *Lemma* 4-5, we know that if $\mathrm{I}(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{i^*-1}, \boldsymbol{y}_{i^*+1} \ldots, \boldsymbol{y}_K; \boldsymbol{\theta}|\boldsymbol{s}_{i^*}) = 0$, then

$\mathrm{p}(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{i^*-1}, \boldsymbol{y}_{i^*+1} \ldots, \boldsymbol{y}_K|\boldsymbol{\theta}, \boldsymbol{s}_{i^*}) =$

$$\int \ldots \int \prod_{i=1, i\neq i^*}^{K} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i) \cdot \mathrm{p}(\boldsymbol{s}_1, \ldots, \boldsymbol{s}_{i^*-1}, \boldsymbol{s}_{i^*+1} \ldots, \boldsymbol{s}_K|\boldsymbol{\theta}, \boldsymbol{s}_{i^*}) \mathrm{d}\boldsymbol{s}_1 \ldots \mathrm{d}\boldsymbol{s}_{i^*-1} \mathrm{d}\boldsymbol{s}_{i^*+1} \ldots \mathrm{d}\boldsymbol{s}_K \tag{4.12}$$

is a constant with respect to $\boldsymbol{\theta}$ and $\boldsymbol{s}_{i^*}$. We first prove the "if" part and then the "only if" part.

1. "if" part

The *a posterior* utility of optimal decision mapping is

$$\max_{\boldsymbol{a}} \mathrm{E}(u(\boldsymbol{a}, \boldsymbol{\theta})|\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_K)$$

$$= \max_{\boldsymbol{a}} \int_{\boldsymbol{\theta}} u(\boldsymbol{a}, \boldsymbol{\theta}) \left(\int_{\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K} \ldots \int \prod_{i=1}^{K} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\, \mathrm{p}(\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K|\boldsymbol{\theta}) \mathrm{d}\boldsymbol{s}_1 \ldots \mathrm{d}\boldsymbol{s}_K \right) \mathrm{p}(\boldsymbol{\theta}) \mathrm{d}\boldsymbol{\theta} \tag{4.13}$$

$$\int_{\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K} \ldots \int \prod_{i=1}^{K} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\, \mathrm{p}(\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K|\boldsymbol{\theta}) \mathrm{d}\boldsymbol{s}_1 \ldots \mathrm{d}\boldsymbol{s}_K$$
$$= \int_{\boldsymbol{s}_{i^*}} \mathrm{p}(\boldsymbol{y}_{i^*}|\boldsymbol{s}_{i^*}) \mathrm{p}(\boldsymbol{s}_{i^*}|\boldsymbol{\theta}) \left(\int \ldots \int \prod_{i=1, i\neq i^*}^{K} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i) \frac{\mathrm{p}(\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K|\boldsymbol{\theta})}{\mathrm{p}(\boldsymbol{s}_{i^*}|\boldsymbol{\theta})} \mathrm{d}\boldsymbol{s}_1 \ldots \mathrm{d}\boldsymbol{s}_{i^*-1} \mathrm{d}\boldsymbol{s}_{i^*+1} \ldots \mathrm{d}\boldsymbol{s}_K \right) \mathrm{d}\boldsymbol{s}_{i^*} \tag{4.14}$$

$$\frac{\mathrm{p}(\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K|\boldsymbol{\theta})}{\mathrm{p}(\boldsymbol{s}_{i^*}|\boldsymbol{\theta})} = \mathrm{p}(\boldsymbol{s}_1, \ldots, \boldsymbol{s}_{i^*-1}, \boldsymbol{s}_{i^*+1} \ldots, \boldsymbol{s}_K|\boldsymbol{\theta}, \boldsymbol{s}_{i^*}) \tag{4.15}$$

Hence if

$$\int \ldots \int \prod_{i=1, i\neq i^*}^{K} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i) \cdot \mathrm{p}(\boldsymbol{s}_1, \ldots, \boldsymbol{s}_{i^*-1}, \boldsymbol{s}_{i^*+1} \ldots, \boldsymbol{s}_K|\boldsymbol{\theta}, \boldsymbol{s}_{i^*}) \mathrm{d}\boldsymbol{s}_1 \ldots \mathrm{d}\boldsymbol{s}_{i^*-1} \mathrm{d}\boldsymbol{s}_{i^*+1} \ldots \mathrm{d}\boldsymbol{s}_K$$

is a constant, denoted by *c*, with respect to $\boldsymbol{\theta}$ and $\boldsymbol{s}_{i^*}$, then put (4.15) into (4.14), we have

$$\int_{\boldsymbol{s}_1,\boldsymbol{s}_2,\dots,\boldsymbol{s}_K} \dots \int \prod_{i=1}^{K} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\, \mathrm{p}(\boldsymbol{s}_1,\boldsymbol{s}_2,\dots,\boldsymbol{s}_K|\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}_1 \dots \mathrm{d}\boldsymbol{s}_K$$

$$= \int_{\boldsymbol{s}_{i^*}} \mathrm{p}(\boldsymbol{y}_{i^*}|\boldsymbol{s}_{i^*})\mathrm{p}(\boldsymbol{s}_{i^*}|\boldsymbol{\theta})c \cdot \mathrm{d}\boldsymbol{s}_i \qquad (4.16)$$

and by put (4.16) into (4.13), the *a posterior* utility of optimal decision mapping becomes

$$\max_{\boldsymbol{a}} \int u(\boldsymbol{a},\boldsymbol{\theta}) \left(\int \mathrm{p}(\boldsymbol{y}_{i^*}|\boldsymbol{s}_{i^*})\mathrm{p}(\boldsymbol{s}_{i^*}|\boldsymbol{\theta})c \cdot \mathrm{d}\boldsymbol{s}_i \right) \mathrm{p}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta}$$

$$= \max_{\boldsymbol{a}} c \left(\iint_{\boldsymbol{s}_{i^*},\boldsymbol{\theta}} u(\boldsymbol{a},\boldsymbol{\theta})\mathrm{p}(\boldsymbol{y}_{i^*}|\boldsymbol{s}_{i^*})\mathrm{p}(\boldsymbol{s}_{i^*}|\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}_i\mathrm{p}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta} \right) \qquad (4.17)$$

$$= \max_{\boldsymbol{a}} \iint_{\boldsymbol{s}_{i^*}\boldsymbol{\theta}} u(\boldsymbol{a},\boldsymbol{\theta})\mathrm{p}(\boldsymbol{y}_{i^*}|\boldsymbol{s}_{i^*})\mathrm{p}(\boldsymbol{s}_{i^*}|\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}_i\mathrm{p}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta} \qquad (4.18)$$

(4.18) is identical to the maximum *a posterior* probability of Observation Selection in (4.7). Hence we have proved that the two decision mappings are equal.

2. "only if" part

$\mathrm{E}(u(\boldsymbol{a},\boldsymbol{\theta})|\boldsymbol{y}_1,\boldsymbol{y}_2,\dots,\boldsymbol{y}_K)$

$$= \max_{\boldsymbol{a}} \int_{\boldsymbol{\theta}} u(\boldsymbol{a},\boldsymbol{\theta}) \left(\int_{\boldsymbol{s}_1,\boldsymbol{s}_2,\dots,\boldsymbol{s}_K} \dots \int \prod_{i=1}^{K} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\, \mathrm{p}(\boldsymbol{s}_1,\boldsymbol{s}_2,\dots,\boldsymbol{s}_K|\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}_1 \dots \mathrm{d}\boldsymbol{s}_K \right) \mathrm{p}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta}$$

$$= \max_{\boldsymbol{a}} \iint_{\boldsymbol{s}_{i^*},\boldsymbol{\theta}} u(\boldsymbol{a},\boldsymbol{\theta})\mathrm{p}(\boldsymbol{y}_{i^*}|\boldsymbol{s}_{i^*})\mathrm{p}(\boldsymbol{s}_{i^*}|\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}_i\mathrm{p}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta} \qquad (4.19)$$

Because the equation is true for any utility function $u(\boldsymbol{a},\boldsymbol{\theta})$

$$\int_{\boldsymbol{s}_1,\boldsymbol{s}_2,\dots,\boldsymbol{s}_K} \dots \int \prod_{i=1}^{K} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\, \mathrm{p}(\boldsymbol{s}_1,\boldsymbol{s}_2,\dots,\boldsymbol{s}_K|\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}_1 \dots \mathrm{d}\boldsymbol{s}_K$$

$$= c \int_{\boldsymbol{s}_{i^*}} \mathrm{p}(\boldsymbol{y}_{i^*}|\boldsymbol{s}_{i^*})\mathrm{p}(\boldsymbol{s}_{i^*}|\boldsymbol{\theta}) \cdot \mathrm{d}\boldsymbol{s}_i \qquad (4.20)$$

Differentiate with respect to $\boldsymbol{s}_{i^*}$, we have

$c \cdot \mathrm{p}(\boldsymbol{y}_{i^*}|\boldsymbol{s}_{i^*})\mathrm{p}(\boldsymbol{s}_{i^*}|\boldsymbol{\theta}) =$

$$\mathrm{p}(\boldsymbol{y}_{i^*}|\boldsymbol{s}_{i^*})\mathrm{p}(\boldsymbol{s}_{i^*}|\boldsymbol{\theta})\int\dots\int\prod_{i=1,i\neq i^*}^{K}\mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)$$

$$\cdot\ \mathrm{p}(\boldsymbol{s}_1,\dots,\boldsymbol{s}_{i^*-1},\boldsymbol{s}_{i^*+1}\dots,\boldsymbol{s}_K|\boldsymbol{\theta},\boldsymbol{s}_{i^*})\mathrm{d}\boldsymbol{s}_1\dots\mathrm{d}\boldsymbol{s}_{i^*-1}\mathrm{d}\boldsymbol{s}_{i^*+1}\dots\mathrm{d}\boldsymbol{s}_K \tag{4.21}$$

c is constant with respect to $\boldsymbol{s}_{i^*}$ because the terms in (4.20) except *c* are function of $\boldsymbol{y}_i$'s. Consequently, *c* is not a function of $\boldsymbol{s}_{i^*}$.

Hence

$$\int\dots\int\prod_{i=1,i\neq i^*}^{K}\mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\cdot\ \mathrm{p}(\boldsymbol{s}_1,\dots,\boldsymbol{s}_{i^*-1},\boldsymbol{s}_{i^*+1}\dots,\boldsymbol{s}_K|\boldsymbol{\theta},\boldsymbol{s}_{i^*})\mathrm{d}\boldsymbol{s}_1\dots\mathrm{d}\boldsymbol{s}_{i^*-1}\mathrm{d}\boldsymbol{s}_{i^*+1}\dots\mathrm{d}\boldsymbol{s}_K$$

is constant. Q.E.D

Theorem 4-6 is an intuitive result of the concept of mutual information. Mutual information I(*X*;*Y*) is the reduction in the uncertainty of *X* due to the knowledge of *Y* (or uncertainty reduction of *Y* due to knowledge of *X*). Hence Observation Selection is optimal when the knowledge of the observations except the selected one are not able to reduce the uncertainty of the event parameter given the state of the selected observation. Zero mutual information also implies the independence of the two random variables. Hence we also know the optimality condition of Observation Selection can also be stated as the event parameter conditioned on the selected state is independent of the other observations.

4.3 Cramer-Rao bound

Although we can avoid dealing with the complex correlation structure between the states by Observation Selection scheme, the selection rule (4.6) is still tedious. We know that Cramer-Rao bound is the performance bound of a estimation problem in the sense of MMSE. Hence we can select observation by comparing the Cramer-Rao bound of each observation instead of directly calculating the expected *a posterior* utility. The most important advantage gained from calculating Cramer-Rao bound is that the calculation complexity is significantly reduced.

We denote the index of selected observation i^*, the same as (4.6). We first investigate scalar case. Deciding i^* from (4.6) is equivalent to select the observation with the lowest Cramer-Rao bound if the following conditions hold:

(1) Utility function $u(a,\theta)$ is second order. That is,

$$u(a,\theta) = k - c(a-\theta)^2 \tag{4.22}$$

where k is a constant. This condition is necessary because Cramer-Rao bound is the bound for the square error.

(2) The estimator is unbiased. This is the necessary condition to apply Cramer-Rao bound.

(3) The *efficient estimation* exists for the observation with lowest Cramer-Rao bound.

Observation Selection by Cramer-Rao bound can be formulated as follows:

$$i^* = \arg\min_i \left(\mathrm{E}_\theta \left\{ \frac{\partial \ln \mathrm{p}_{\boldsymbol{y}_i,\theta}(\boldsymbol{y}_i,\theta)}{\partial \theta} \right\}^2 \right)^{-1} = \arg\min_i \left(\mathrm{E}_\theta \left\{ \frac{\partial^2 \ln \mathrm{p}_{\boldsymbol{y}_i,\theta}(\boldsymbol{y}_i,\theta)}{\partial \theta^2} \right\} \right)^{-1}$$

$$\Pi = \Pi_{i^*} \tag{4.23}$$

We can calculate $\mathrm{p}_{\boldsymbol{y}_i,\theta}(\boldsymbol{y}_i,\theta)$ by (2.3), that is

$$\mathrm{p}_{\boldsymbol{y}_i,\theta}(\boldsymbol{y}_i,\theta) = \mathrm{p}(\boldsymbol{y}_i|\theta)\mathrm{p}(\theta) = \int \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\mathrm{p}(\boldsymbol{s}_i|\theta)\mathrm{p}(\theta)\mathrm{d}\boldsymbol{s}_i \tag{4.24}$$

Instead of directly calculating the *a posterior* expectation of utility function and taking its maximum, Observation Selection by Cramer-Rao bound calculates the expectation of the logarithm of the probability, which may simplify the calculation procedure.

Then we generalize it to vector case. Cramer-Rao bound for vector parameter is

$$\mathbf{\Sigma}_i \geq J_i(\boldsymbol{\theta})^{-1} = \left(\mathrm{E}_{\boldsymbol{\theta}}\left\{\frac{\partial \ln \mathrm{p}_{\boldsymbol{y}_i,\boldsymbol{\theta}}(\boldsymbol{y}_i,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right\}^2\right)^{-1}$$

$$= \left(\mathrm{E}_{\boldsymbol{\theta}}\left\{\frac{\partial^2 \ln \mathrm{p}_{\boldsymbol{y}_i,\boldsymbol{\theta}}(\boldsymbol{y}_i,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^2}\right\}\right)^{-1} \quad (4.25)$$

where $\mathbf{\Sigma}_i$ is the covariance matrix of the estimators for $\boldsymbol{\theta}$ by observation $\boldsymbol{y}_i$, $J_i(\boldsymbol{\theta})$ is the *Fisher information matrix* [4]. And the Observation Selection by Cramer-Rao bound becomes

$$i^* = \arg\min_i J_i(\boldsymbol{\theta})^{-1}$$

$$\Pi = \Pi_{i^*} \quad (4.26)$$

In order to apply Fisher information matrix, we need to generalize above conditions for Cramer-Rao bound to vector parameter case. Hence we formally formulate the conditions for generalized case into following lemma:

Lemma 4-7: Observation Selection done by Fisher information matrix (Cramer-Rao bound), (4.27) is equivalent to Observation Selection if the following conditions hold:

(1) $u(\boldsymbol{a},\boldsymbol{\theta}) = k - \boldsymbol{x}^{\mathrm{T}}\mathbf{\Gamma}\boldsymbol{x}^{\mathrm{T}}$, $\boldsymbol{x}$ *is arbitrary vector, k is arbitrary const*

$$\mathbf{\Gamma} = (\boldsymbol{a}-\boldsymbol{\theta})^{\mathrm{T}}(\boldsymbol{a}-\boldsymbol{\theta}) \quad (4.27)$$

(2) The estimator is unbiased.

(3) The efficient estimation exists for the observation with lowest Cramer-Rao bound.

Proof:

We state (2) without proof because it is the conditions to apply Cramer-Rao bound and achieve it. We proof (1) here. Denote the expected utility function for sub-decision mapping Π_i in *Definition 4-3* by $\mathrm{E}_i u$.

The inequality in (4.25) means that the difference of the two matrix is positive semi-definite. Consequently, for arbitrary vector $\boldsymbol{x}$, if (3) is satisfied, we have

$$\mathrm{E}_{i^*}u = \mathrm{E}_i(k - \boldsymbol{x}^{\mathrm{T}}\mathbf{\Gamma}\boldsymbol{x}) = k - \boldsymbol{x}^{\mathrm{T}}\mathbf{\Sigma}_{i^*}\boldsymbol{x}^{\mathrm{T}} = k - \boldsymbol{x}^{\mathrm{T}}J_{i^*}(\boldsymbol{\theta})^{-1}\boldsymbol{x} \quad (4.28)$$

Then for other i,

$$\mathrm{E}_i u = k - \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\Sigma}_i\boldsymbol{x}^{\mathrm{T}} \leq k - \boldsymbol{x}^{\mathrm{T}}J_i(\boldsymbol{\theta})^{-1}\boldsymbol{x}^{\mathrm{T}}$$

$$\leq k - \boldsymbol{x}^{\mathrm{T}}J_{i^*}(\boldsymbol{\theta})^{-1}\boldsymbol{x} = \mathrm{E}_{i^*}u \quad (4.29)$$

This condition is equivalent to Observation Selection (4.6).

Q.E.D

By computing the Cramer-Rao bound to implement Observation Selection, the intelligent systems are able to make decision on multiple observations more efficiently. Although multiple observations are able to improve decision performance by providing more information, to optimally fuse those observations, the complexity and necessary knowledge for decision process may be significantly increased. Comparing to optimal decision and ordinary Observation Selection, Observation Selection by Cramer-Rao bound significantly reduce the complexity of decision process. However, this scheme reduces the computational complexity at the cost of more restrictive application conditions as mentioned above.

4.4 Optimal Ratio Combining

Observation Selection utilizes multiple observations to improve performance from single observation by selecting the best observation to make decision on this observation independently of other observations. In fact, data fusion schemes to combine observations from different sensors to enhance the estimation performance are investigated in many works on distributed estimation [35]. Diversity schemes applied in communication systems to resist fading effect by receiving multiple signals to detect the transmitted signal [37]. In most of those works, the different observations are independent and restricted to specific forms, such as scaling the signal and adding noise, to derive the optimal combining coefficient. However, in the scenario considered in this paper, observations are different physical quantities. It is not reasonable to directly combining the observations. Consequently, under the intelligent decision framework, we propose the Ratio Combining scheme to combine the decisions in each sub-decision mapping (4.5) instead of directly combing the observations like traditional weighted combining (Fig.4.5). Unlike Observation Selection selects the best decision among all sub-decision mappings, the decision of Ratio Combing is the linear combination of the decision of each sub-decision mapping. Ratio Combining can be formulated as follows:

$$\boldsymbol{a} = \sum_{i=1}^{K} \beta_i \arg\max_{\boldsymbol{a}_i} \mathrm{E}_i(u(\boldsymbol{a}_i, \boldsymbol{\theta})|\boldsymbol{y}_i) \tag{4.30}$$

Observation Selection is a special case of Ratio Combining achieved by setting $\beta_{i^*} = 1$ for the i^* satisfying (4.6) and other β_i's zero.

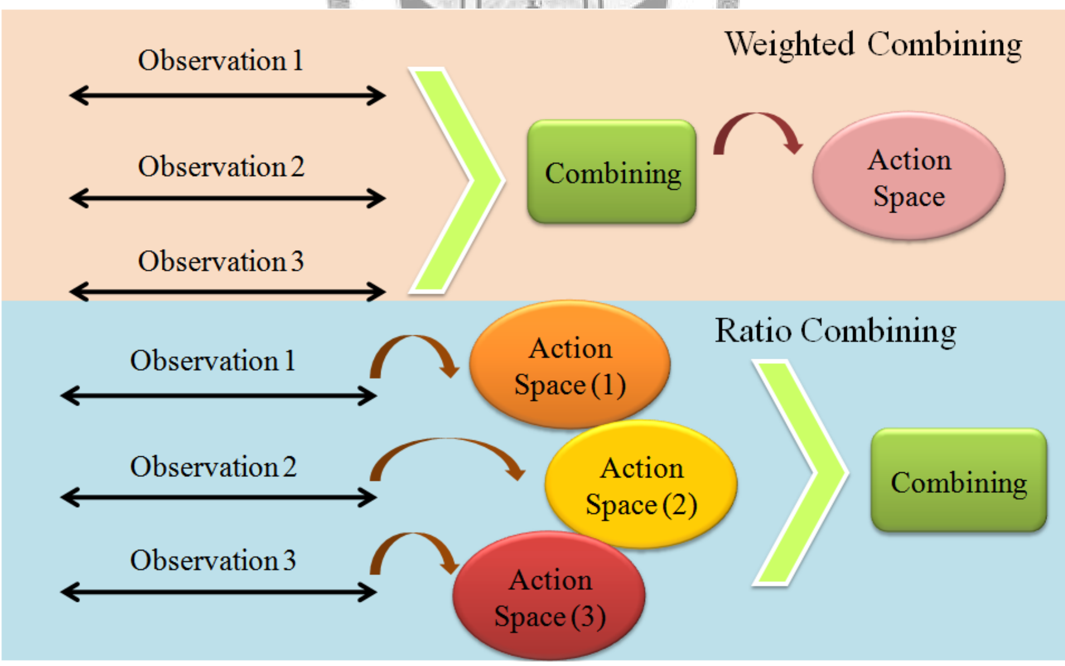


Fig. 4.5 Comparison of Ratio Combining and Observation Selection

The Ratio Combining is optimal when the action $\boldsymbol{a}$ decided by the set of β_i's is equal to the action $\boldsymbol{a}$ decided by the optimal decision mapping. To simplify the formula, we assume the *prior* probability is uniform and the utility function is delta function:

$$u(\boldsymbol{a},\boldsymbol{\theta}) = \delta(\boldsymbol{a}-\boldsymbol{\theta}) \tag{4.31}$$

Then the coefficients β_i of Optimal Ratio Combining can be derived by:

$$\widehat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}}\left(\int_{\boldsymbol{s}_1,\boldsymbol{s}_2,\dots,\boldsymbol{s}_K} \dots \int \prod_{i=1}^{K} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\,\mathrm{p}(\boldsymbol{s}_1,\boldsymbol{s}_2,\dots,\boldsymbol{s}_K|\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}_1 \dots \mathrm{d}\boldsymbol{s}_K\right)$$

$$= \sum_{i=1}^{K} \beta_i \cdot \widehat{\boldsymbol{\theta}}_i = \sum_{i=1}^{K} \beta_i \cdot \arg\max_{\boldsymbol{\theta}_i} \int_{\boldsymbol{s}_i} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\mathrm{p}(\boldsymbol{s}_i|\boldsymbol{\theta}_i)\mathrm{d}\boldsymbol{s}_i \tag{4.32}$$

If we further assume the distribution

$\int_{\boldsymbol{s}_1,\boldsymbol{s}_2,\dots,\boldsymbol{s}_K} \dots \int \prod_{i=1}^{K} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\,\mathrm{p}(\boldsymbol{s}_1,\boldsymbol{s}_2,\dots,\boldsymbol{s}_K|\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}_1 \dots \mathrm{d}\boldsymbol{s}_K$ and $\int_{\boldsymbol{s}_i} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\mathrm{p}(\boldsymbol{s}_i|\boldsymbol{\theta}_i)\mathrm{d}\boldsymbol{s}_i$'s are convex, like multi-variant normal distribution, we can derived β_i's by solving the following integral equations:

$$\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\theta}_i}\left(\int_{\boldsymbol{s}_i} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\mathrm{p}(\boldsymbol{s}_i|\boldsymbol{\theta}_i)\mathrm{d}\boldsymbol{s}_i\right)|_{\boldsymbol{\theta}_i=\widehat{\boldsymbol{\theta}}_i} = 0 \tag{4.33}$$

$$\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\theta}}\left(\int_{\boldsymbol{s}_1,\boldsymbol{s}_2,\dots,\boldsymbol{s}_K} \dots \int \prod_{i=1}^{K} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\,\mathrm{p}(\boldsymbol{s}_1,\boldsymbol{s}_2,\dots,\boldsymbol{s}_K|\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}_1 \dots \mathrm{d}\boldsymbol{s}_K\right)|_{\boldsymbol{\theta}=\widehat{\boldsymbol{\theta}}} = 0 \tag{4.34}$$

$$\widehat{\boldsymbol{\theta}} = \sum_{i=1}^{K} \beta_i \widehat{\boldsymbol{\theta}}_i \tag{4.35}$$

Theoretically, knowing the distributions, we can solve β_i's by above integral equations. However, since current methods can not derive a general close form solution and dealing with specific distribution is not the goal of this paper, we leave the solution of Optimal Ratio Combining to the future work.

4.5 Fuzzy Logic

Fuzzy logic is widely applied in control problems based on sensor observation [17,18]. Fuzzy controller to decide control action on sensor data can use the fuzzy inference rule call *generalized modus ponens*, which is constructed by fuzzy conditional statement [2], follows by the defuzzification procedure. The fuzzy inference is in the form:

Fuzzy conditional statement	IF (*x* is *A*), THEN (*y* is *B*)
Fact	*x* is *A'*
Conclusion	*y* is *B'*

Table 4.1 Fuzzy conditional statement inference

where *x* and *y* is a variable from set X and Y, and *A,A',B* and *B'* are the fuzzy sets on X and Y, respectively.

To compute the fuzzy sets, we can formulate the fuzzy conditional statement by the fuzzy logic relation [33]:

$$\mathrm{R}(x,y)=\mathrm{I}\big(\mathrm{A}(x),\mathrm{B}(y)\big) \tag{4.36}$$

where A(*x*) and B(*y*) is the member function of fuzzy set *A* and *B*, and I is the fuzzy implication function. And the member function of B'(*y*) is

$$\mathrm{B}'(y)=\sup_{x\in\mathrm{X}}\min\big(\mathrm{A}'(x),\mathrm{R}(x,y)\big) \tag{4.37}$$

Then the defuzzification procedure is performed on fuzzy set *B* to decide the control action of the controller.

The fuzzy logic relations applied in fuzzy logic controllers are often constructed in the experience based argument with less strict-sense mathematical structure in most applications. However, following this structure with some modifications, we can degenerate Observation Selection to a fuzzy logic controller based on concrete mathematical structure. The fuzzy conditional statement for our problem is

IF (x is A), THEN (y is B)

Where x is the variable observation space and y is the variable from action space. A represents the value in observation space, and B represents the value in action space. Note that A is an ordinary singleton set instead of fuzzy set. B is the fuzzy set with one element, and the element is decided by (4.5). Then we define the fuzzy relation to be the function of Cramer-Rao bound:

$$R(x_i, y) = \frac{\dfrac{1}{\mathrm{E}\left\{\dfrac{\partial \ln \mathrm{p}_{x_i,\theta}(x_i,\theta)}{\partial\theta}\right\}^2}}{\sum_{j=1}^{K} \dfrac{1}{\mathrm{E}\left\{\dfrac{\partial \ln \mathrm{p}_{x_j,\theta}(x_j,\theta)}{\partial\theta}\right\}^2}} \tag{4.38}$$

$\sum_{j=1}^{K} \frac{1}{\mathrm{E}\left\{\frac{\partial \ln \mathrm{p}_{x_j,\theta}(x_j,\theta)}{\partial\theta}\right\}^2}$ is the normalized term to restrict the summation of Cramer-Rao bound factor to 1, which is necessary to the derivation of the center of area (COA) deffizification procedure in the following. K is the number of kinds of observations. Note that the dependence of the fuzzy relation on y is eliminated because Cramer-Rao bound does not depends on which action the system decided. By this definition of fuzzy relation, the degeneration is equivalent to Observation Selection when conditions for Cramer-Rao bound stated previously hold. And each observation from a particular sub-observation space is one fact:

x_i is A_i, i=1,…,K

A_i is also a singleton set. Hence we have the member function corresponding to each conclusion

$$\mathrm{B}_i(y) = \sup_{x_i \in \boldsymbol{O}_i} \min \ (\mathrm{A}_i(x_i), \mathrm{R}(x_i, y)) = \mathrm{R}(x_{Ai}, y) \tag{4.39}$$

The last equality is due to the fact that A_i is an ordinary singleton set. x_{Ai} denotes the value of the element in A_i. Then the defuzzification procedure can decide the control action by choosing y with the largest value of member function $\mathrm{B}_i(y)$'s in all fuzzy set B_i's.

To summarize, the fuzzy rule degenerated from Observation Selection operates by establishing fuzzy conditional statement by the sub-decision mapping (4.6) and defining its fuzzy relation by Cramer-Rao bound. Then the defuzzification is done by choosing the element with largest value of member function. Note that in the degenerated case, the conditions stated previous for applying Cramer-Rao bound must be satisfied. Moreover, if the condition (4.8) is also satisfied, the fuzzy logic controller is also equivalent to optimal decision (4.2).

On the other hand, we can construct another commonly used defuzzification procedure, center of area (COA), based on Ratio Combining. Similar defuzzification procedures have discussed in many works [33,34]. The COA method defuzzify a fuzzy set A whose member function is $\mathrm{A}(x)$ by the following formula:

$$\bar{x} = \frac{\sum_{x\in A} x \cdot \mathrm{A}(x)}{\sum_{x\in A} \mathrm{A}(x)} \tag{4.40}$$

Hence we can decides the control action by computing y_a from member functions $\mathrm{B}_i(y)$'s:

$$B' = \bigcup_{i=1}^{K} B_i \text{, hance } \mathrm{B}'(y) = \max_i \mathrm{B}_i(y) \tag{4.41}$$

$$y_\mathrm{a} = \frac{\sum_y y \cdot \mathrm{B}'(y)}{\sum_y \mathrm{B}'(y)} = \frac{\sum_{i=1}^{K} y_i \cdot \mathrm{B}_i(y_i)}{\sum_{i=1}^{K} \mathrm{B}_i(y_i)} \tag{4.42}$$

y_i is the only element with nonzero member function in set B_i. In fact, if we put (4.39) and (4.38) into (4.42), the defuzzification procedure is equivalent to Ratio Combining with the coefficients determined by Cramer-Rao bounds. Hence we have the following proposition:

Proposition 4-7: The fuzzy logic control with COA defuzzification procedure can be derived from Ratio Combining with coefficients being inverse proportional to

Cramer-Rao bound of each observation.

Proof:

The construction of fuzzy logic control follows the above derivation. We focus on the proof of the equivalency to Ratio Combining.

$$y_{\mathrm{a}} = \frac{\sum_{i=1}^{K} y_i \cdot \mathrm{B}_i(y_i)}{\sum_{i=1}^{K} \mathrm{B}_i(y_i)} = \frac{\sum_{i=1}^{K} y_i \cdot \mathrm{R}(x_{Ai}, y_i)}{\sum_{i=1}^{K} \mathrm{R}(x_{Ai}, y_i)} = \sum_{i=1}^{K} y_i \cdot \frac{\mathrm{R}(x_{Ai}, y_i)}{\sum_{j=1}^{K} \mathrm{R}(x_{Aj}, y_i)}$$

$$= \sum_{i=1}^{K} y_i \cdot \mathrm{R}(x_{Ai}, y_i) \tag{4.43}$$

(4.43) has exactly the same form as Ratio Combining (4.30), where notation of the decision of each observation is replaced y_i. By (4.38), the coefficient of Ratio Combining is inverse proportional to Cramer-Rao bound. Consequently, the fuzzy logic control with COA defuzzification procedure can be derived from Ratio Combining.

Q.E.D

To sum up, many fuzzy logic controllers make decision on multiple observation by selection or establishing a series of rules (fuzzy conditional statement) corresponding to possible combinations of observation outcomes. Those ideas in essence are almost the same as the fuzzy logic controller with two different defuzzification procedure presented above. In those fuzzy logic controllers, the mapping between multi-dimensional observation space to action is reduced to finite possible combinations mapping or the selected one dimensional mapping. The selection or combination rules are usually derived by the arguments based on experience or experimental results. However, in the derivation above we have shown that the fuzzy inference based on fuzzy conditional statement is able to be established with concrete mathematical structure on the basis of our intelligent decision framework. Moreover, this also implies that much of the fuzzy controllers based on the fuzzy conditional statement are the special cases degenerated from the general decision in our intelligent framework. For example, the fuzzy logic controller derived above is a degenerated case by the condition of Cramer Rao bound mentioned above.

4.6 Performance Comparison of Observation Selection and Ratio Combining

Now we try to compare the performance of the above two defuzzification procedures, which are indeed the Observation Selection and Ratio Combining with Cramer-Rao bound coefficient (Ratio Combining in brief). In fact, this Ratio Combining is a special case which ignores the correlation among observations while takes the quality indicated by Cramer-Rao bound into consideration. We do not include correlation information into Ratio Combining because we compare its performance with Observation Selection, which is assumed to be operated in the circumstances of no correlation information. To be consist with the Observation Selection and Ratio Combining section, here we use the original notation, observation $\boldsymbol{O}_i$'s and action decision $\boldsymbol{a}$, to substitute the notations, x_{Ai} and y_a. To enable us to apply Cramer-Rao bound, we assume the conditions in *Lemma 4-6* are satisfied. Then the condition (4.27) implies that the conditional mean minimizes the expected utility function [9]. Consequently, the decision of the Observation Selection becomes:

$$\boldsymbol{a}_{\mathrm{os}} = \mathrm{E}(\boldsymbol{\theta}|\boldsymbol{O}_{i^*}) = \int \boldsymbol{\theta}\mathrm{p}(\boldsymbol{\theta}|\boldsymbol{O}_{i^*})\,\mathrm{d}\boldsymbol{\theta} \tag{4.44}$$

And the decision of the Ratio Combining becomes

$$\boldsymbol{a}_{\mathrm{rc}} = \sum_{i=1}^{K} \frac{\mathrm{E}(\boldsymbol{\theta}|\boldsymbol{O}_i) \times \dfrac{1}{\mathrm{E}\left\{\dfrac{\partial \ln \mathrm{p}_{\boldsymbol{O}_i,\theta}(\boldsymbol{O}_i,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right\}^2}}{\sum_{j=1}^{K} \dfrac{1}{\mathrm{E}\left\{\dfrac{\partial \ln \mathrm{p}_{\boldsymbol{O}_j,\theta}(\boldsymbol{O}_j,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right\}^2}} = \sum_{i=1}^{K} \frac{\mathrm{E}(\boldsymbol{\theta}|\boldsymbol{O}_i) \times \dfrac{1}{\mathrm{Var}_i(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta})}}{\sum_{j=1}^{K} \dfrac{1}{\mathrm{Var}_j(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta})}}$$

$$= \sum_{i=1}^{K} \frac{\mathrm{E}(\boldsymbol{\theta}|\boldsymbol{O}_i)}{1 + \sum_{j=1, j\neq i}^{K} \dfrac{\mathrm{Var}_i(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta})}{\mathrm{Var}_j(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta})}} \qquad (4.45)$$

To simplify the problem, we make the assumption that the efficient estimator exists for all observations and derive the second equality in (4.45). The optimal decision is

$$\boldsymbol{a}_{\mathrm{opt}} = \mathrm{E}(\boldsymbol{\theta}|\boldsymbol{O}_1, \boldsymbol{O}_2, \ldots, \boldsymbol{O}_K) = \int \boldsymbol{\theta}\, \mathrm{p}(\boldsymbol{\theta}|\boldsymbol{O}_1, \boldsymbol{O}_2, \ldots, \boldsymbol{O}_K) \mathrm{d}\boldsymbol{\theta} \qquad (4.46)$$

We compare the difference of mean square error (MSE) to optimal decision, $\mathrm{E}(\boldsymbol{a}_{\mathrm{os}} - \boldsymbol{a}_{\mathrm{opt}})^2$ and $\mathrm{E}(\boldsymbol{a}_{\mathrm{rc}} - \boldsymbol{a}_{\mathrm{opt}})^2$, to infer which of Observation Selection and Ratio Combining makes better decision gain more utility. In fact, the difference of mean square error is proportional to expected utility function which contains only square terms and constant. Hence we apply it to performance comparison. We investigate the comparison by a simple example of two observations with Gaussian distribution. Assume the distributions of observations conditioned on parameter are normal distributions and bivariate normal distribution:

$$\mathrm{P}(O_1|\Theta) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(o_1 - \theta)^2}{2\sigma_1^2}\right) \qquad (4.47)$$

$$\mathrm{P}(O_2|\Theta) = \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(o_2 - \theta)^2}{2\sigma_2^2}\right) \qquad (4.48)$$

$$\mathrm{P}(O_1, O_2|\Theta) = \frac{1}{2\pi|\boldsymbol{\Lambda}|^{\frac{1}{2}}} \exp\left(-\frac{(\boldsymbol{o} - \theta)^T \boldsymbol{\Lambda}^{-1} (\boldsymbol{o} - \theta)}{2}\right) \qquad (4.49)$$

Where $\boldsymbol{o}=[o_1 \;\; o_2]^{\mathrm{T}}$, $\boldsymbol{\Lambda} = \begin{bmatrix} \sigma_1 & \sqrt{\sigma_1\sigma_2}\rho \\ \sqrt{\sigma_1\sigma_2}\rho & \sigma_2 \end{bmatrix}$, $|\boldsymbol{\Lambda}|$ is the determinant of $\boldsymbol{\Lambda}$. And assume the prior distribution of θ is

$$\mathrm{p}(\Theta) = \frac{1}{\sqrt{2\pi}\sigma_\theta}\exp\left(-\frac{\theta^2}{2\sigma_\theta^2}\right) \tag{4.50}$$

Then we have the *a posterior* distributions

$$\mathrm{P}(\Theta|O_1) = \frac{\frac{1}{\sqrt{2\pi}\sigma_1}\cdot\frac{1}{\sqrt{2\pi}\sigma_\theta}}{\mathrm{p}(O_1)}\exp\left(-\frac{(o_1-\theta)^2}{2\sigma_1^2}-\frac{\theta^2}{2\sigma_\theta^2}\right)$$

$$= k(o_1)\exp\left\{-\frac{1}{2\sigma_{\mathrm{p1}}^2}\left[\theta-\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_1^2}\cdot o_1\right]^2\right\} \tag{4.51}$$

$$\mathrm{P}(\Theta|O_2) = \frac{\frac{1}{\sqrt{2\pi}\sigma_2}\cdot\frac{1}{\sqrt{2\pi}\sigma_\theta}}{\mathrm{p}(O_2)}\exp\left(-\frac{(o_2-\theta)^2}{2\sigma_1^2}-\frac{\theta^2}{2\sigma_\theta^2}\right)$$

$$= k(o_2)\exp\left\{-\frac{1}{2\sigma_{\mathrm{p2}}^2}\left[\theta-\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_2^2}\cdot o_2\right]^2\right\} \tag{4.52}$$

$$\mathrm{P}(\Theta|O_1,O_2) = \frac{\frac{1}{2\pi|\boldsymbol{\Lambda}|^{\frac{1}{2}}}\cdot\frac{1}{\sqrt{2\pi}\sigma_\theta}}{\mathrm{p}(O_1,O_2)}\exp\left(-\frac{1}{2}(\boldsymbol{o}-\theta)^T\boldsymbol{\Lambda}^{-1}(\boldsymbol{o}-\theta)-\frac{\theta^2}{2\sigma_\theta^2}\right)$$

$$= k(o_1,o_2)\exp\left\{-\frac{1}{2(1-\rho^2)\sigma_{\mathrm{p12}}^2}\left[\theta-\left(\frac{o_1}{\sigma_1^2}-\frac{o_1+o_2}{\sigma_1\sigma_2/\rho}+\frac{o_2}{\sigma_2^2}\right)\sigma_{\mathrm{p12}}^2\right]^2\right\} \tag{4.53}$$

Where

$$\sigma_{\mathrm{p1}}^2 = \left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_\theta^2}\right)^{-1}, \sigma_{\mathrm{p2}}^2 = \left(\frac{1}{\sigma_2^2}+\frac{1}{\sigma_\theta^2}\right)^{-1},$$

$$\sigma_{\mathrm{p12}}^2 = \left(\frac{1}{\sigma_1^2}-\frac{2}{\sigma_1\sigma_2/\rho}+\frac{1}{\sigma_2^2}+\frac{1}{\sigma_\theta^2(1-\rho^2)}\right)^{-1}$$

Observing that (4.51)~(4.53) are all in the form of normal distribution, we can derive the conditional mean of each distribution:

$$\mathrm{E}(\Theta|O_1=o_1) = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_1^2}\cdot o_1 \tag{4.54}$$

$$\mathrm{Var}(\Theta|O_1) = \left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_\theta^2}\right)^{-1} \tag{4.55}$$

$$\mathrm{E}(\Theta|O_2=o_2) = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_2^2}\cdot o_2 \tag{4.56}$$

$$\mathrm{Var}(\Theta|O_2) = \left(\frac{1}{\sigma_2^2} + \frac{1}{\sigma_\theta^2}\right)^{-1} \tag{4.57}$$

$$\mathrm{E}(\Theta|O_1, O_2) = \left(\frac{o_1}{\sigma_1^2} - \frac{o_1 + o_2}{\sigma_1\sigma_2/\rho} + \frac{o_2}{\sigma_2^2}\right)\sigma_{\mathrm{p12}}^2 \tag{4.58}$$

$$\mathrm{Var}(\Theta|O_1, O_2) = (1-\rho^2)\sigma_{\mathrm{p}}^2 = (1-\rho^2)\left(\frac{1}{\sigma_1^2} - \frac{2}{\sigma_1\sigma_2/\rho} + \frac{1}{\sigma_2^2} + \frac{1}{\sigma_\theta^2}\right)^{-1} \tag{4.59}$$

Then we can derive the a_{os}, a_{rc}, and a_{opt} (Assume that $\sigma_1^2 < \sigma_2^2$):

$$a_{\mathrm{os}} = \mathrm{E}(\Theta|O_1 = o_1) = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_1^2} \cdot o_1 \tag{4.60}$$

$$a_{\mathrm{rc}} = \sum_{i=1}^{2} \frac{\mathrm{E}(\theta|O_i)}{1 + \sum_{j=1, j\neq i}^{2} \frac{\mathrm{Var}_i(\hat{\theta}-\theta)}{\mathrm{Var}_j(\hat{\theta}-\theta)}} = \frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_1^2}\cdot o_1}{1 + \frac{\mathrm{Var}_1(\hat{\theta}-\theta)}{\mathrm{Var}_2(\hat{\theta}-\theta)}} + \frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_2^2}\cdot o_2}{1 + \frac{\mathrm{Var}_2(\hat{\theta}-\theta)}{\mathrm{Var}_1(\hat{\theta}-\theta)}}$$

$$= \frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_1^2}\cdot o_1}{1 + \frac{\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_\theta^2}\right)^{-1}}{\left(\frac{1}{\sigma_2^2}+\frac{1}{\sigma_\theta^2}\right)^{-1}}} + \frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_2^2}\cdot o_2}{1 + \frac{\left(\frac{1}{\sigma_2^2}+\frac{1}{\sigma_\theta^2}\right)^{-1}}{\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_\theta^2}\right)^{-1}}} = \frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_1^2}\cdot o_1}{1 + \frac{\left(\frac{1}{\sigma_2^2}+\frac{1}{\sigma_\theta^2}\right)}{\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_\theta^2}\right)}} + \frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_2^2}\cdot o_2}{1 + \frac{\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_\theta^2}\right)}{\left(\frac{1}{\sigma_2^2}+\frac{1}{\sigma_\theta^2}\right)}}$$

$$= \frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_1^2}\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_\theta^2}\right)}{\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_\theta^2}\right) + \left(\frac{1}{\sigma_2^2}+\frac{1}{\sigma_\theta^2}\right)}\cdot o_1 + \frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_2^2}\left(\frac{1}{\sigma_2^2}+\frac{1}{\sigma_\theta^2}\right)}{\left(\frac{1}{\sigma_2^2}+\frac{1}{\sigma_\theta^2}\right) + \left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_\theta^2}\right)}\cdot o_2 \tag{4.61}$$

$$a_{\mathrm{opt}} = \left(\frac{o_1}{\sigma_1^2} + \frac{o_1 + o_2}{\sigma_1\sigma_2/\rho} + \frac{o_2}{\sigma_2^2}\right)\sigma_{\mathrm{p12}}^2 \tag{4.62}$$

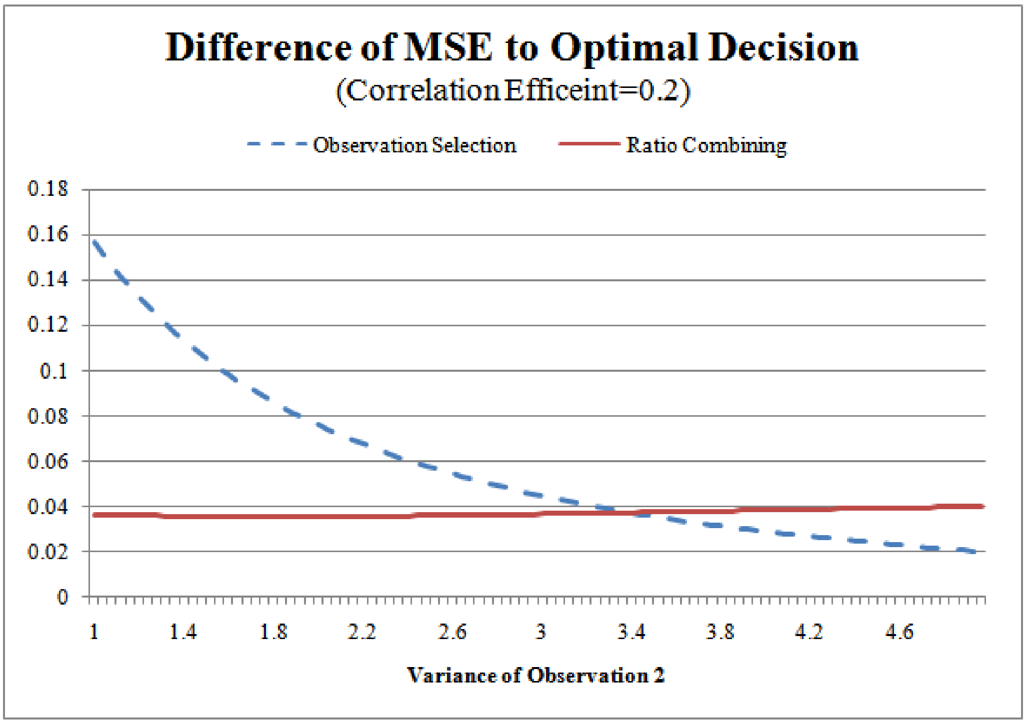
The difference of mean square error, $\mathrm{E}(a_{\mathrm{os}}-\theta)^2 - \mathrm{E}\left(a_{\mathrm{opt}}-\theta\right)^2$ and $\mathrm{E}(a_{\mathrm{rc}}-\theta)^2 - \mathrm{E}\left(a_{\mathrm{opt}}-\theta\right)^2$, with respect to variations of observation variance and correlation are shown in Fig.4.2. In Fig.4.2(a)(b), we set the variance of observation 1, σ_1^2, fixed to 1 and change the variance of observation 2, σ_2^2, while the correlation coefficient ρ is fixed. In 4.2(c)(d), we set the variance of observation 1 fixed to 1 and change the correlation coefficient while the variance of observation 2 is fixed. Note that the smaller distance implies better performance. The figures show that when both observations have similar variances and nearly independent, Ratio Combining notably outperforms selection scheme (Fig.4.2(c) and part of (a)(b)). If the difference between the variances of two observations is not significant, Ratio Combining is still better than Observation Selection when the observations are nearly independent (Fig.4.2(a)). On the other hand, if the observations are highly correlated or the difference between variances is large enough, the performance of Observation Selection can achieve or even exceed the performance of Ratio Combining (Fig.4.2(b)(c)(e)). Then we have the following remark.

Remark 5-1: The numerical result of above performance comparison in low correlation region shows that

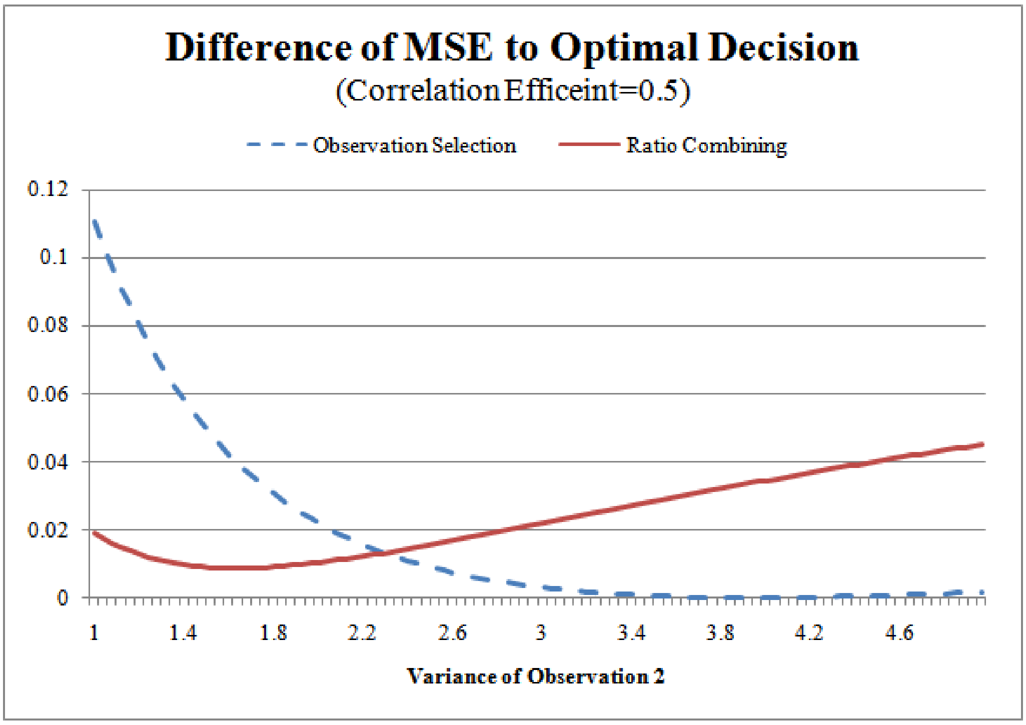
1) *Diversity gain dominates the performance comparison when the correlation among observations is low and the variances of observations are close.*
2) *The inferior observation diminishes the diversity gain when the variances among observations are significantly different.*

These characteristics coincide with the intuitions which are able to establish common experience-based decision rules for multiple observation. However, under intelligent decision framework, we mathematically demonstrate the validity of those intuitive rules and relate the observations from the numerical results to widely-used multiple observation decision schemes such as diversity or selection.

Besides the above discussion of performance comparison in ordinary region, the abnormal behaviors in extreme region also can be well interpreted. In intelligent decision framework, optimal decision takes correlation among observations into consideration while Ratio Combining and Observation Selection ignoring it. This results in two abnormal and opposite behaviors of performance comparison for the extreme cases in which the observations are highly correlated. Ratio Combining and Observation Selection both approach the optimal performance when two observations have the same variance and highly correlated (Fig. IV-2(c)). In fact, when correlation coefficient approaches 1, the two schemes are almost equivalent. However, when the observations are highly correlated and variances are different, the estimation error of optimal decision approaches zero and the mean square differences to optimal decision of both Ratio Combining and Observation Combining jump sharply (Fig. IV-2(d)). This is due to the correlation gain, which utilize the correlation to enhance estimation performance in contrast to the diversity gain by independent observations. In fact, the

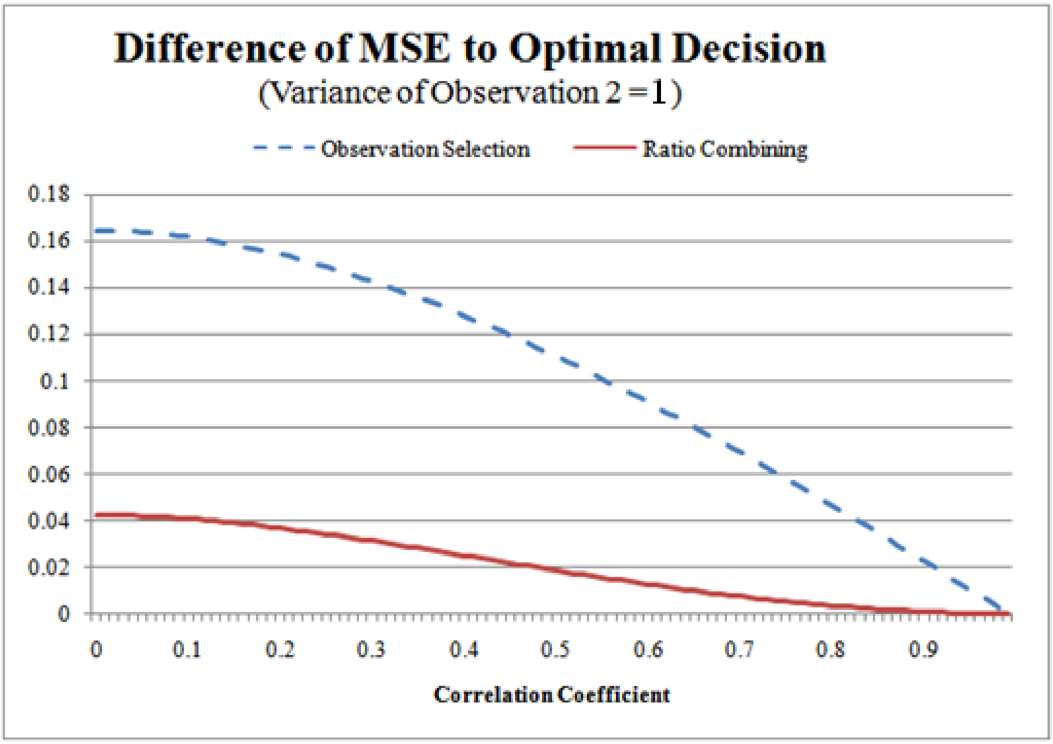


(a)

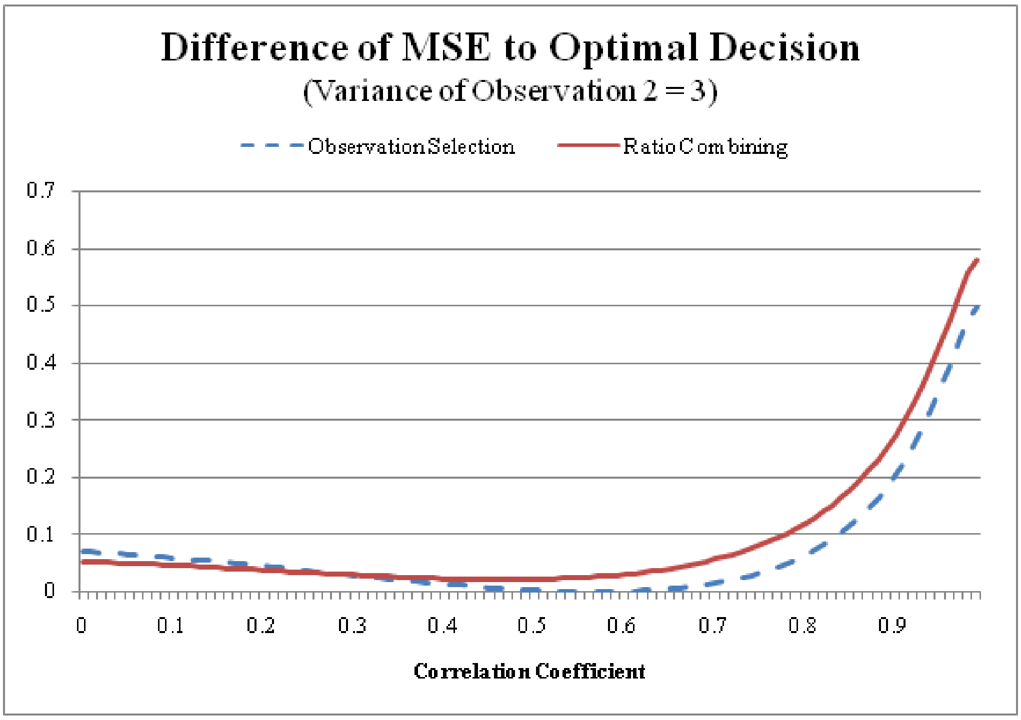


(b)

Fig. 4.2 Difference of MSE to Optimal Decision (a),(b) the correlation coefficient fixed.



(c)



(d)

Fig. 4.2 Difference of MSE to Optimal Decision (c),(d) the variance of observation 2 fixed.

correlation gain enables optimal decision to achieve perfect estimation as the correlation approaches 1. Then we have the following remark:

Remark 2: For the highly-correlated observations

1) *When the variances of observations are the same, the performance of Observation Selection and Ratio Combining both converges to optimal decision as the correlation approaches 1*
2) *When the variances of observation are different, the error of optimal decision converges to zero as the correlation approaches 1 due to correlation gain.*

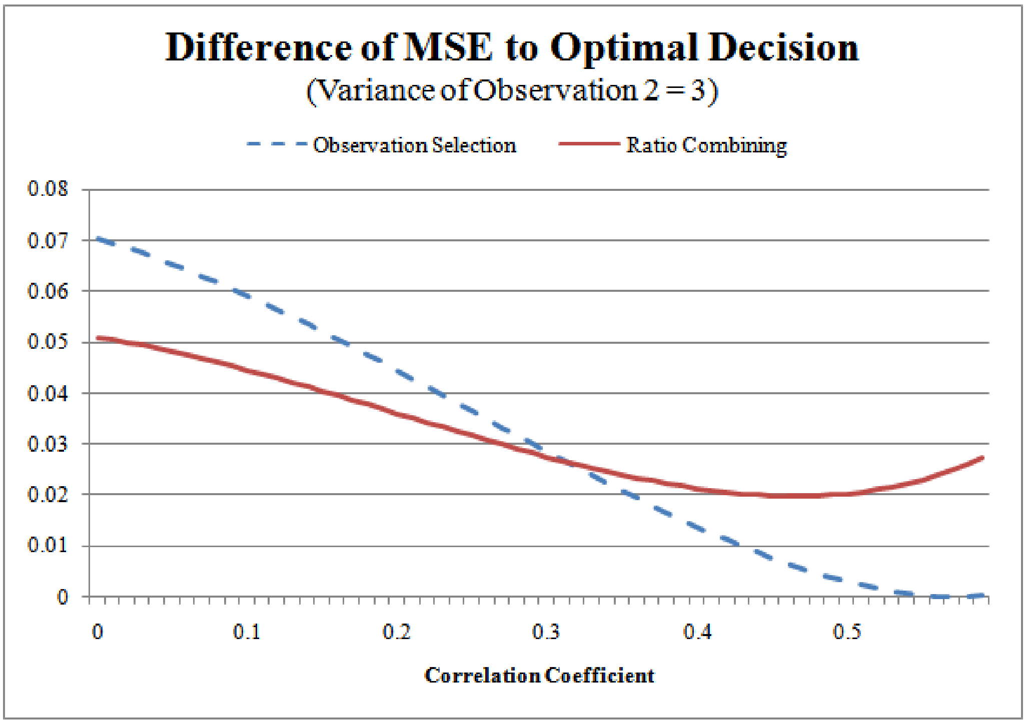


Fig. 4.2 Difference of MSE to Optimal Decision (e) The segment from (d), coefficient range 0~0.6 is highlighted.

We can intuitively explain the performance enhancement by correlation gain stated in *Remark 2-2*. Consider the extreme case, correlation coefficient is 1 and variance of observation 2 is 3. The noise added on observation 2 is exactly three times of noise added on observation 1. Then the difference between two observations is exactly two times of noise added on observation 1 and we can derive that noise and the event parameter, θ. Consequently, the optimal decision taking the correlation coefficient into consideration is able to estimate exact value of the parameter θ while both Ratio Combining and Observation Selection are unable to do so due to ignoring the correlation among observations. This explains the jump in Fig. IV-2(d). To sum up, the performance comparison analysis under intelligent decision framework broaden the scope of multiple observation diversity and correlation gain and explain them more precisely.

Chapter 5

Multi-Observation Sensor Network Navigation System for Firefighting Robot

We extend the application example of firefighting robot navigation problem presented in chapter 3 to multi-observation case by the framework in chapter 4. Now there are more than one kind of sensor observations, such as temperature and smoke, can be collected by the firefighting robot. We formulate the problem as follows.

5.1 Multi-Observation Intelligent Decision System Model

Under similar definitions as chapter 4, we can formulate the optimal decision by (4.2):

$$\hat{\boldsymbol{a}} = \arg\max_{\boldsymbol{a}} \mathrm{E}(\cos(\mathrm{Arg}(\boldsymbol{a}) - \mathrm{Arg}(\boldsymbol{\theta}))\,|\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_K) \quad (5.1)$$

$$= \arg\max_{\boldsymbol{a}} \int_{\boldsymbol{\theta}} \cos(\mathrm{Arg}(\boldsymbol{a})$$

$$- \mathrm{Arg}(\boldsymbol{\theta}))\left(\int_{\boldsymbol{s}_1,\boldsymbol{s}_2,\ldots,\boldsymbol{s}_K} \ldots \int \prod_{i=1}^{K} \mathrm{p}(\boldsymbol{y}_i|\boldsymbol{s}_i)\, \mathrm{p}(\boldsymbol{s}_1,\boldsymbol{s}_2,\ldots,\boldsymbol{s}_K|\boldsymbol{\theta})\mathrm{d}\boldsymbol{s}_1 \ldots \mathrm{d}\boldsymbol{s}_K\right)\mathrm{p}(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta} \tag{5.2}$$

There are K kind of observations, $\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_K$, collected by the decision system and those observations corresponding to different states, $\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K$.

Similarly, Observation Selection becomes

$$i^* = \arg\max_i \left(\max_{\boldsymbol{a}} \mathrm{E}_i(\cos(\mathrm{Arg}(\boldsymbol{a}) - \mathrm{Arg}(\boldsymbol{\theta}))\,|\boldsymbol{y}_i)\right)$$

$$\Pi = \Pi_{i^*} \tag{5.3}$$

For the optimal decision, the firefighting robot fuses all observation, including smoke, temperature, etc, by the joint distribution $\mathrm{p}(\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K|\boldsymbol{\theta})$. On the other hand, Observation Selection is done by selection of the individual best observation for each direction decision.

5.2 Degenerate Problem: Fuzzy Logic Controller.

The multiple observation intelligent decision procedure can be degenerated a fuzzy logic controller under specific conditions as mentioned above. We demonstrate this by the firefighting robot example. For simplicity, we only present the defuzzification procedure utilizing selection while the COA defuzzification procedure is in almost the same formulation. Consider the observations to be temperature and the density of the smoke. Denote observation of temperature to be system 1, and observation of the density of the smoke to be system 2. We assume the condition to apply Cramer-Rao bound to Observation Selection holds. In order to satisfy the conditions, we also make the same assumption as the approximation (3.18) in chapter

3. Follow the procedure derived in last section, the fuzzy logic controller is:

Fuzzy Controller for Firefighting Robot

	Observation 1	Observation 2
Fact	*temperature* is T	*smoke* is S
Conclusion	*action* corresponds to *temperature* is A_T	*action* corresponds to *smoke* is A_S
Defuzzification	Choose the largest of value of member function among A_T and A_S	

Note that the member function of A_T and A_S are determined by (4.37). To put it simply, this fuzzy controller operates by selection of action by its member function derived from the fuzzy inference rule. It is similar to Observation Selection, but the relation is defined by Cramer-Rao bound. Hence it must satisfy the conditions for Cramer-Rao bound to be equivalent to Observation Selection.

Chapter 6

Experiments

In this chapter, we present the numerical result of intelligent decision for sensor network based system. By exploring the sensor observation and decision process, our intelligent decision framework presented in this work is significantly different from previous works on introducing the uncertainty of the relationship between event parameter and physical quantity in addition to the uncertainty of observation process (interference and additive noise). This framework enables us to further develop the optimal decision for multiple observations of different physical quantities. For both single observation and multi-observation cases, we develop the decision schemes and the application example in previous sections. In this section, we simulate the scenarios based on the firefighting robot navigation problem in chapter 3 and 5 with some simplifications, showing that intelligent decision framework really make significant improvement.

6.1 Single Observation

We use the system model adapted from firefighting robot navigation problem in Chapter 3 to demonstrate the performance improvement that the intelligent decision framework may bring. We use the electrostatic field, whose field value is proportional to the inverse of distance to the source, to model our potential field. There are several sources and the firefighting robot aims at finding the target source by intelligently deciding its direction by the observation of the field value (temperature). We model the conditional probability of the two mappings, $\Phi: \boldsymbol{S} \to \boldsymbol{O}$ and $\Psi: \Theta \to \boldsymbol{S}$ as follows:

(1) $\mathrm{p}(\boldsymbol{s}|\boldsymbol{\theta})$ is simplified to $\mathrm{p}(\mathrm{Arg}(\boldsymbol{s})|\mathrm{Arg}(\boldsymbol{\theta}))$ and denoted $\mathrm{p}(\varphi_{\boldsymbol{s}}|\varphi_{\boldsymbol{\theta}})$. $\mathrm{p}(\varphi_{\boldsymbol{s}}|\varphi_{\boldsymbol{\theta}})$ is assumed to be a function of the intervals of the observation of the potential field' value (it is the robot's own observation and is different from observation $\boldsymbol{y}_i$, as mentioned in Chapter 3.) We determine $\mathrm{p}(\varphi_{\boldsymbol{s}}|\varphi_{\boldsymbol{\theta}})$ empirically and fit it to Gaussian distribution. We randomly choose the points in the potential fields belonging to different intervals of the observation of the potential field's value, and derive the mean and variance of the deviation from $\varphi_{\boldsymbol{\theta}}$ to $\varphi_{\boldsymbol{s}}$ to derive $\mathrm{p}(\varphi_{\boldsymbol{s}}|\varphi_{\boldsymbol{\theta}})$ as a Gaussian distribution. Then we apply difference $\mathrm{p}(\varphi_{\boldsymbol{s}}|\varphi_{\boldsymbol{\theta}})$ according to the observation of the potential field's value.

(2) $\mathrm{p}(\boldsymbol{y}|\boldsymbol{s})$ is also simplified to $\mathrm{p}(\mathrm{Arg}(\boldsymbol{y})|\mathrm{Arg}(\boldsymbol{s}))$ and denoted $\mathrm{p}\left(\varphi_{\boldsymbol{y}}|\varphi_{s}\right)$. $\varphi_{\boldsymbol{y}_i}$ is determined by least square error solution of equation (3.7) without noise term. And we approximate $\mathrm{p}\left(\varphi_{\boldsymbol{y}}|\varphi_{s}\right)$ by Gaussian distribution with mean φ_{s}.

To simplify the computation, we assume the deviation of the angle is small and we approximate the utility function by

$$\cos(\varphi_{\boldsymbol{a}} - \varphi_{\boldsymbol{\theta}}) \sim 1 - (\varphi_{\boldsymbol{a}} - \varphi_{\boldsymbol{\theta}})^2$$

The estimator maximizes the utility function, or equivalently, minimizes the term $(\varphi_{\boldsymbol{a}} - \varphi_{\boldsymbol{\theta}})^2$, is the conditional mean estimator [9]. That is,

$$\hat{\varphi}_a = \arg\max_{\varphi_{\boldsymbol{a}}} \iint (1-(\varphi_{\boldsymbol{a}}-\varphi_{\boldsymbol{\theta}})^2)\mathrm{p}(\varphi_{\boldsymbol{y}}|\varphi_{\boldsymbol{s}}).\,\mathrm{p}(\varphi_s|\varphi_{\boldsymbol{\theta}})\mathrm{d}\varphi_s\mathrm{d}\varphi_{\boldsymbol{\theta}}$$

$$= \arg\min_{\varphi_{\boldsymbol{a}}} \iint (\varphi_{\boldsymbol{a}}-\varphi_{\boldsymbol{\theta}})^2\mathrm{p}(\varphi_{\boldsymbol{y}}|\varphi_{\boldsymbol{s}}).\,\mathrm{p}(\varphi_s|\varphi_{\boldsymbol{\theta}})\mathrm{d}\varphi_s\mathrm{d}\varphi_{\boldsymbol{\theta}}$$

$$= \arg\min_{\varphi_{\boldsymbol{a}}} \int (\varphi_{\boldsymbol{a}}-\varphi_{\boldsymbol{\theta}})^2\mathrm{p}(\varphi_{\boldsymbol{y}}|\varphi_{\boldsymbol{\theta}})\mathrm{d}\varphi_{\boldsymbol{\theta}}$$

$$= \mathrm{E}(\varphi_{\boldsymbol{\theta}}|\varphi_{\boldsymbol{y}}) \tag{6.1}$$

where

$$\mathrm{p}(\varphi_{\boldsymbol{y}}|\varphi_{\boldsymbol{\theta}}) = \int \mathrm{p}(\varphi_{\boldsymbol{y}}|\varphi_{\boldsymbol{s}}).\,\mathrm{p}(\varphi_s|\varphi_{\boldsymbol{\theta}})\mathrm{d}\varphi_s \tag{6.2}$$

To simplify the calculation of $\mathrm{p}(\varphi_{\boldsymbol{y}}|\varphi_{\boldsymbol{\theta}})$, we also assume the variances of $\mathrm{p}(\varphi_{\boldsymbol{y}}|\varphi_{\boldsymbol{s}})$ and $\mathrm{p}(\varphi_{\boldsymbol{s}}|\varphi_{\boldsymbol{\theta}})$ are the same. Then $\mathrm{p}(\varphi_{\boldsymbol{y}}|\varphi_{\boldsymbol{\theta}})$ becomes

$$\mathrm{p}(\varphi_{\boldsymbol{y}}|\varphi_{\boldsymbol{\theta}})$$

$$= \int \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(\varphi_{\boldsymbol{y}}-\varphi_{\boldsymbol{s}})^2}{2\sigma^2}\right)\cdot\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(\varphi_{\boldsymbol{s}}-(\varphi_{\boldsymbol{\theta}}+\varphi_k))^2}{2\sigma^2}\right)\mathrm{d}\varphi_s \tag{6.3}$$

$$= \int \frac{1}{2\pi\sigma^2}\exp\left(-\frac{\left(\sqrt{2}\varphi_s-\frac{1}{\sqrt{2}}(\varphi_y+\varphi_{\boldsymbol{\theta}}+\varphi_k)\right)^2+({\varphi_y}^2+(\varphi_{\boldsymbol{\theta}}+\varphi_k)^2)-\frac{(\varphi_y+\varphi_{\boldsymbol{\theta}}+\varphi_k)^2}{2}}{2\sigma^2}\right)\mathrm{d}\varphi_s$$

$$= \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{({\varphi_{\boldsymbol{y}}}^2+(\varphi_{\boldsymbol{\theta}}+\varphi_k)^2)-\frac{(\varphi_{\boldsymbol{y}}+\varphi_{\boldsymbol{\theta}}+\varphi_k)^2}{2}}{2\sigma^2}\right)$$

$$\cdot\frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\left(\sqrt{2}\varphi_{\boldsymbol{s}}-\frac{1}{\sqrt{2}}(\varphi_{\boldsymbol{y}}+\varphi_{\boldsymbol{\theta}}+\varphi_k)\right)^2}{2\sigma^2}\right)\mathrm{d}(\sqrt{2}\varphi_{\boldsymbol{s}}) \tag{6.4}$$

$$= \frac{1}{2\sigma\sqrt{\pi}}\exp\left(-\frac{({\varphi_{\boldsymbol{y}}}^2+(\varphi_{\boldsymbol{\theta}}+\varphi_k)^2)-\frac{(\varphi_{\boldsymbol{y}}+\varphi_{\boldsymbol{\theta}}+\varphi_k)^2}{2}}{2\sigma^2}\right) \tag{6.5}$$

where

$$\int \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left(\sqrt{2}\varphi_{s} - \frac{1}{\sqrt{2}}(\varphi_{y} + \varphi_{\theta} + \varphi_{k})\right)^2}{2\sigma^2}\right) d(\sqrt{2}\varphi_{s}) = 1 \qquad (6.6)$$

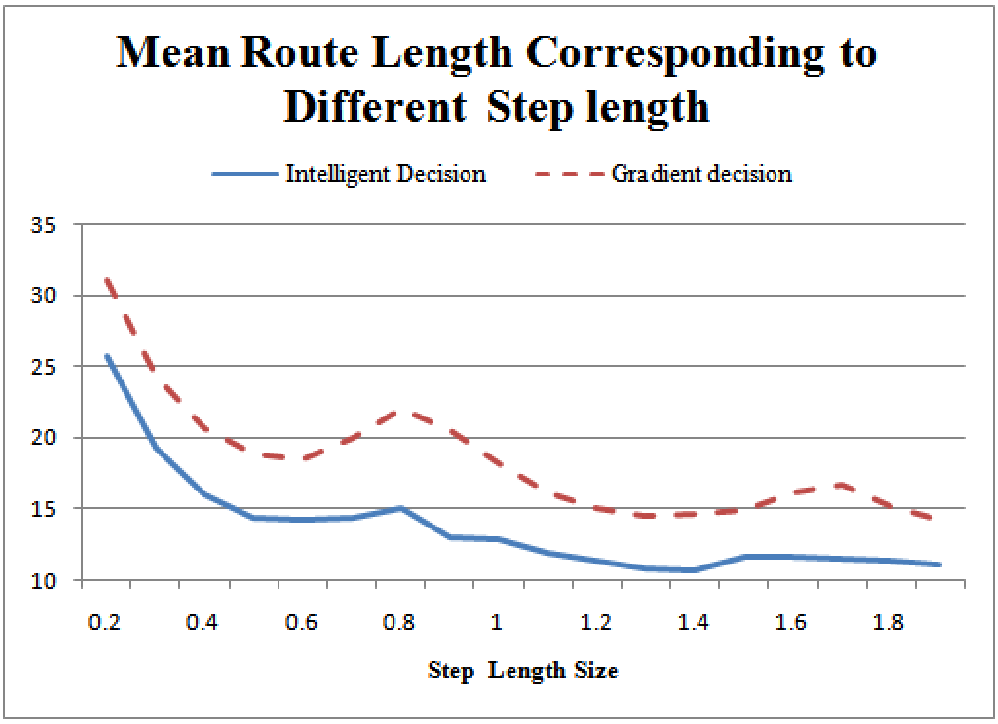
In our simulation, we define the performance to be the route length of the firefighting robot to reach the place on fire. The more accurate the decision is, the shorter route the robot would take. When the distance between fire and the firefighting robot is less than 1 unit length, the robot has reached the place on fire. The firefighting robot's initial position is 8 unit length away from the place on fire. We simulate the firefighting robot navigation scenario with different step length. The step length is the distance the robot traveled between two decisions by observation collection from sensors. Then we compare the performance of our intelligent decision with the decision mechanism which estimates φ_{s}, the direction of gradient, to make decision (gradient decision).

The following figure depicts the results of the experiment. We compare the performance in two aspects:

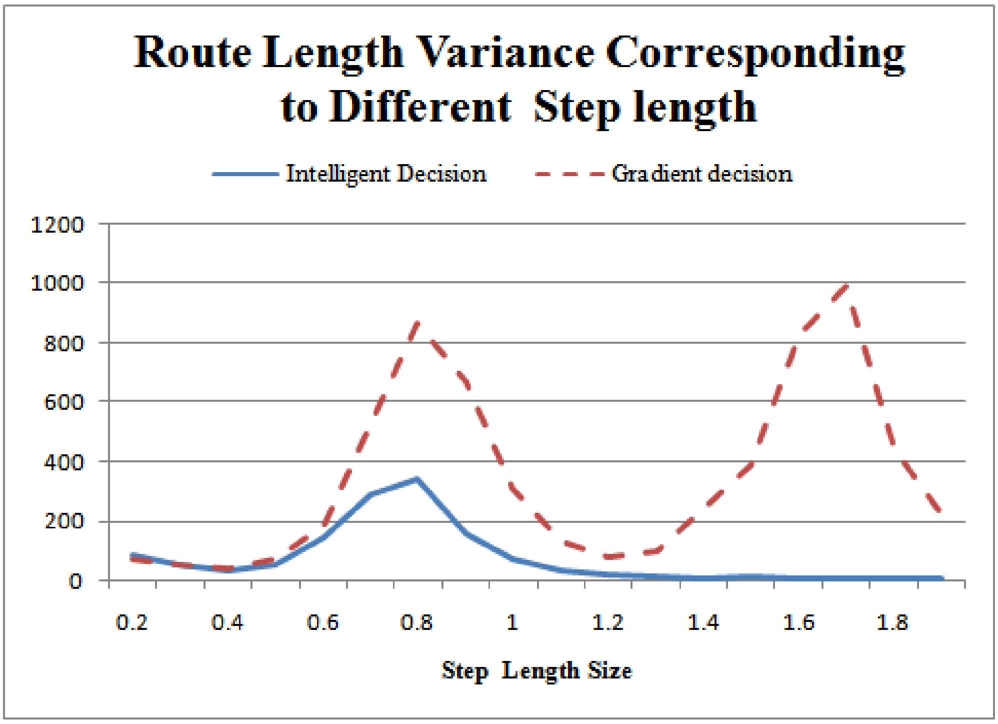
(1) The performance (mean route length) of our intelligent decision scheme always outperforms the traditional scheme (gradient decision) corresponding to various step length. (Fig.6.1(a))

(2) The performance variance of gradient decision is unstable as the system parameter (step length) varies. On the other hand, the intelligent decision is more robust to system parameter variation. (Fig.6.1(b))

Then we investigate how intelligent decision outperforms gradient decision. Note that the event parameter, the direction of the fire, does not always coincide with the physical quantity, the gradient of potential field due to the inclusion of other sources. Hence the intelligent decision performs better and more robust because it takes the



(a)



(b)

Fig. 6.1 The simulation result of single observation decision for the proposed intelligent decision scheme and the traditional scheme (gradient decision).

distribution of the state φ_{s} conditioned on event parameter φ_{θ} into consideration and makes more accurate decision; on the other hand, performance of gradient decision is inferior compared to intelligent decision because it ignores the inconsistence of event parameter and physical quantity. The unstable performance of gradient decision as the step length varies is due to that for some ranges of step length, the robot has higher chance to cross the area affected by the potential field sources other than the target one while for some range of step length, the robot's direction decision is misguided by those sources. The peaks in Fig.6.1(b) corresponds to the ranges of step length in which the robot is misguided in our experiment. We can infer that environment or system parameter variations may degrade the performance of gradient decision significantly, while our intelligent decision under the intelligent decision framework is more robust to environment or system parameter variations because the framework precisely models the environment and take more relevant information into consideration.

6.2 Multiple Observation

We investigate the scenario which has two kinds of observations (*K*=2). We model the two mappings belonging to two observations by Gaussian distribution, the same as the single observation case. However, to demonstrate the performance improvement by Observation Selection corresponding to various situations, the variance of Gaussian distribution is randomly generated from a specific range. Larger variance of $\mathrm{p}(\varphi_{y_i}|\varphi_{s_i})$ stands for less precision of sensor observation and larger variance of $\mathrm{p}(\varphi_{s_i}|\varphi_{\theta})$ stands for weaker correlation between event and physical quantity or less available information for nature. In this simulation scenario, we assume the precision of all kinds of sensors is fixed to focus on the uncertainty of the uncertainty of relationship between physical quantities and event. Hence we release the assumption of the same variance of the two conditional probabilities, $\mathrm{p}(\varphi_{y_i}|\varphi_{s_i})$ and $\mathrm{p}(\varphi_{s_i}|\varphi_{\boldsymbol{\theta}})$ and assume $\mathrm{p}(\varphi_{y_i}|\varphi_{s_i})$ is the same for all observations but $\mathrm{p}(\varphi_{s_i}|\varphi_{\boldsymbol{\theta}})$ is different for different φ_{s_i}'s. To derive the decision in this case, we can follow the way we derive the decision of single observation case: calculate the conditional mean. We first derive $\mathrm{p}(\varphi_{y_i}|\varphi_{\boldsymbol{\theta}})$. To simplify the calculation, we assume the mean of observation is the physical quantity and the mean of physical quantity is the event parameter

$$\mathrm{p}(\varphi_{y_i}|\varphi_{\boldsymbol{\theta}}) = \int \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(\varphi_{y_i} - \varphi_{s_i})^2}{2\sigma_1^2}\right) \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(\varphi_{s_i} - \varphi_{\boldsymbol{\theta}})^2}{2\sigma_2^2}\right) \mathrm{d}\varphi_{s_i}$$

$$= \int \frac{1}{2\pi\sigma_1\sigma_2} \exp\left(-\frac{1}{2}\left(\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)\varphi_{s_i}{}^2 - 2\left(\frac{\varphi_{y_i}}{\sigma_1^2} + \frac{\varphi_{\boldsymbol{\theta}}}{\sigma_2^2}\right)\varphi_{s_i} + \left(\frac{\varphi_{y_i}{}^2}{\sigma_1^2} + \frac{\varphi_{\boldsymbol{\theta}}{}^2}{\sigma_2^2}\right)\right)\right) \mathrm{d}\varphi_{s_i}$$

$$= \int \frac{1}{2\pi\sigma_1\sigma_2} \exp\left(-\frac{1}{2}\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)\left(\varphi_{s_i} - \frac{\left(\frac{\varphi_{y_i}}{\sigma_1^2} + \frac{\varphi_{\boldsymbol{\theta}}}{\sigma_2^2}\right)}{\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)}\right)^2 - \frac{1}{2}\left(\left(\frac{\varphi_{y_i}{}^2}{\sigma_1^2} + \frac{\varphi_{\boldsymbol{\theta}}{}^2}{\sigma_2^2}\right) - \frac{\left(\frac{\varphi_{y_i}}{\sigma_1^2} + \frac{\varphi_{\boldsymbol{\theta}}}{\sigma_2^2}\right)^2}{\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)}\right)\right) \mathrm{d}\varphi_{s_i}$$

$$= \frac{1}{\sqrt{2\pi}\sigma_1\sigma_2\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}\right)^{\frac{1}{2}}}\exp\left(-\frac{1}{2}\left(\frac{{\varphi_{y_i}}^2}{\sigma_1^2}+\frac{{\varphi_{\theta}}^2}{\sigma_2^2}\right)+\frac{1}{2}\frac{\left(\frac{\varphi_{y_i}}{\sigma_1^2}+\frac{\varphi_{\theta}}{\sigma_2^2}\right)^2}{\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}\right)}\right)$$

$$\cdot\int\frac{1}{\sqrt{2\pi}\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}\right)^{-\frac{1}{2}}}\exp\left(-\frac{\left(\varphi_{s_i}-\left(\frac{\varphi_{y_i}}{\sigma_1^2}+\frac{\varphi_{\theta}}{\sigma_2^2}\right)\right)^2}{2\left(\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}\right)^{-\frac{1}{2}}\right)^2}\right)d\varphi_{s_i} \tag{6.7}$$

$$= \frac{1}{\sqrt{2\pi}(\sigma_1^2+\sigma_2^2)^{\frac{1}{2}}}\exp\left(-\frac{1}{2}\left(\frac{{\varphi_{y_i}}^2}{\sigma_1^2}+\frac{{\varphi_{\theta}}^2}{\sigma_2^2}\right)+\frac{\left(\frac{\varphi_{y_i}}{\sigma_1^2}+\frac{\varphi_{\theta}}{\sigma_2^2}\right)^2}{2\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}\right)}\right) \tag{6.8}$$

we have (6.8) because the integration of Gaussian distribution is 1:

$$\int\frac{1}{\sqrt{2\pi}\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}\right)^{-\frac{1}{2}}}\exp\left(-\frac{\left(\varphi_{s_i}-\left(\frac{\varphi_{y_i}}{\sigma_1^2}+\frac{\varphi_{\theta}}{\sigma_2^2}\right)\right)^2}{2\left(\left(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}\right)^{-\frac{1}{2}}\right)^2}\right)d\varphi_{s_i} = 1 \tag{6.9}$$

Assume the *a prior* distribution of event parameter is uniform in 0 to 2π. Then we can use (6.8) to calculate the conditional mean estimator as single observation case.

In our simulation scenario, the variance of each $p(\varphi_{s_i}|\varphi_{\theta})$ of each observation collection is a uniform distributed random number in a specific range while $p(\varphi_{y_i}|\varphi_{s_i})$ of each observation collection is the same for all *i*. We compare the performance of Observation Selection, observation1 only, observation 2 only and a scheme randomly choose the observation to make decision corresponding to different variance range. The performance matrix is the same as single observation case.

Simulation results show that the Observation Selection scheme outperforms the other schemes, especially when the variance increases. As the variance range increasing, the average performances of the decisions on each of the two observations only are decreasing, and the routes length are increasing. Random selection also suffers similar degradation. However, Observation Selection seems more robust to the increase of variance range. We know that the Observation Selection scheme will select the observation with lower randomly generated variance due to the application of Cramer-Rao bound. Consequently, although the range of randomly generated variance is increasing, performance of Observation Selection will significantly decline only when both randomly generated variances are large. On the other hand, other schemes do not aware of the quality of the observation other than their selection and will suffer from performance degradation once the randomly generated variance of the selected observation is large. From the simulation result, we know that Observation Selection can significantly improve the decision performance when the quality of the observations changes a lot and less correlated with each other.

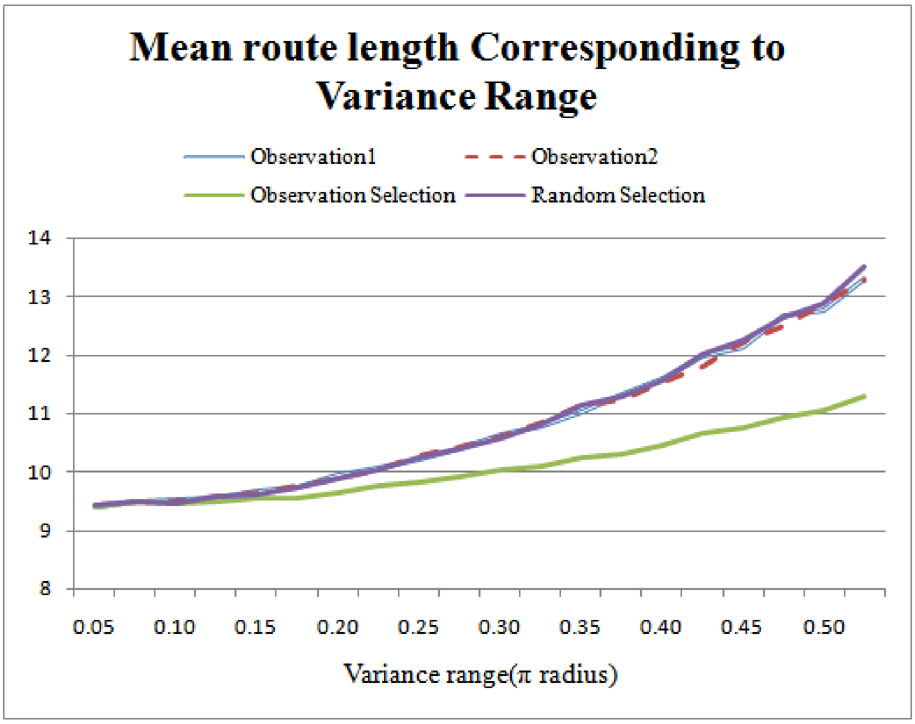


Fig. 6.2 (a) The simulation results, mean route length, of multi observation decision for Observation Selection, observation 1 only, observation 2 only, and the random selection scheme

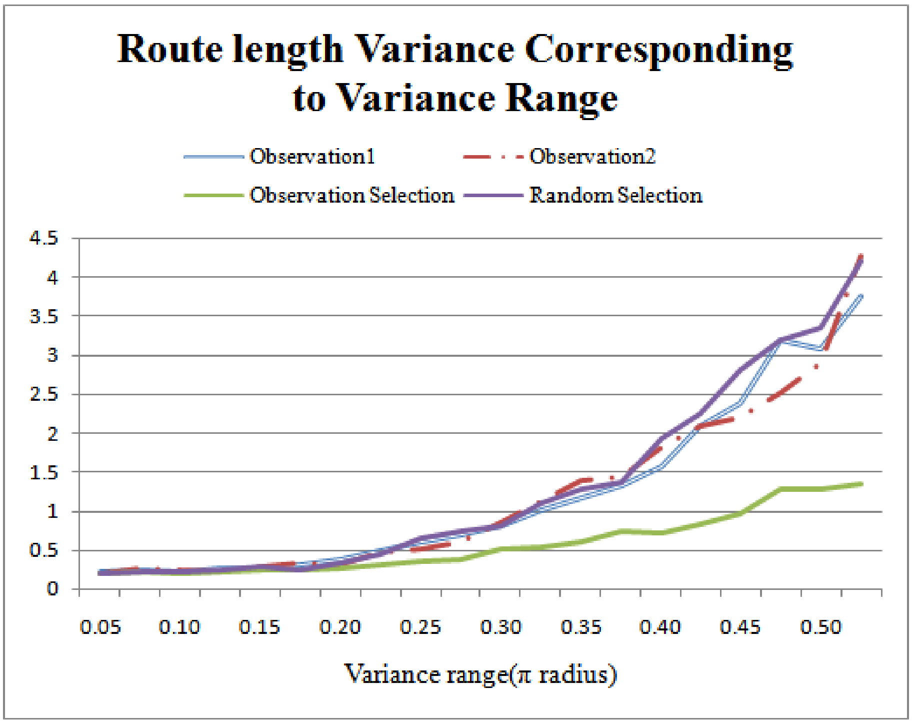


Fig. 6.2 (b) The simulation results, route length variance, of multi observation decision for Observation Selection, observation 1 only, observation 2 only, and the random selection scheme

Chapter 7

Cognitive Radio Spectrum Sensing under Intelligent Decision

Although the application examples in previous chapters are all in the realm of sensor network, the intelligent decision framework can also be applied in various areas. In this chapter, we apply the intelligent decision framework to cognitive radio spectrum sensing to solve the hidden terminal problem. By the application example we present in this chapter, we are able to state that the intelligent decision framework is a general information fusion framework which can be applied in various intelligent or cognitive systems.

7.1 Cognitive Radio Spectrum Sensing

Cognitive radio (CR) terminal based on software define radio (SDR) technology [38] has drawn widely attention as a key technology for future wireless communications. CR terminal is a device which can explore the available spectrum to transmit on and can “adapt” communication to connect to various systems. In order to explore the available spectrum to transmit, CR relies on spectrum sensing to perceive the radio environment and seek for spectrum opportunity to transmit its data. Like the traditional sensing mechanism Collision Sense Multiple Access (CSMA), CR spectrum sensing also suffers from the hidden terminal problem. In this chapter we will investigate the hidden terminal problem in CR spectrum sensing and its solutions including traditional scheme and the proposed scheme. In the following sections, we call primary system transmitter PS-Tx, primary system receiver PS-Rx, CR transmitter CR-Tx and CR receiver CR-Rx.

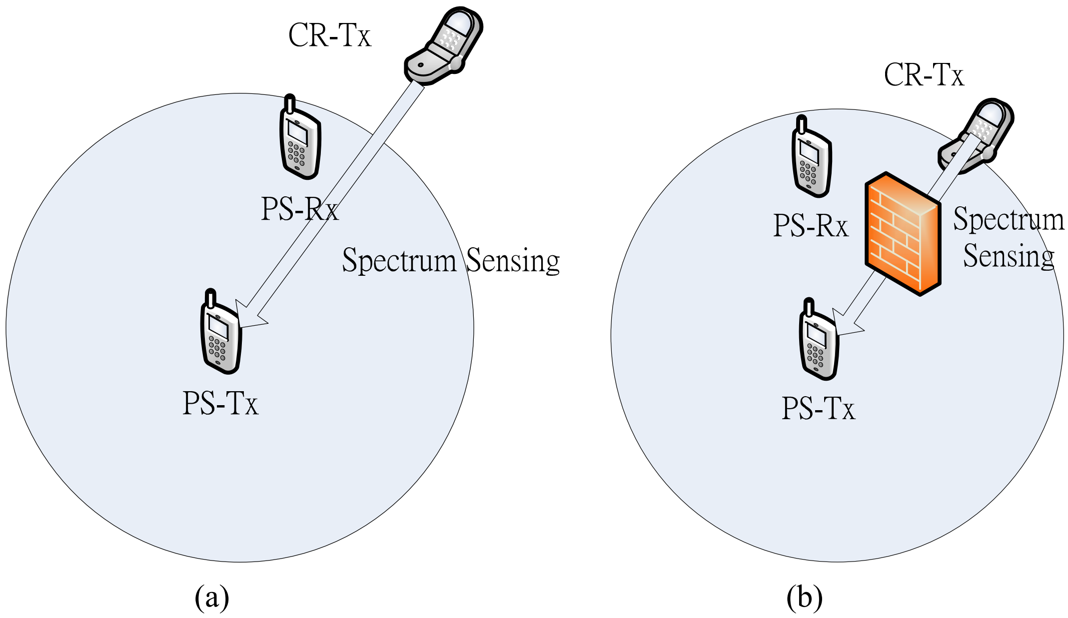


Fig. 7.1 Hidden terminal problem (a) CR is out of transmission range of PS-Tx (b) CR spectrum sensing is blocked by obstacles.

7.1.1 Hidden Terminal Problem and Cooperative Spectrum Sensing

The hidden terminal problem in CR spectrum sensing can be described as follows. If the signal on sensing channel is weak due to the geographical separation but the signal on interference channel happens to be strong, secondary transmission will interfere with primary transmission (Fig.7.1(a)). Moreover, the sensing channel may experience deep fading or blocked by obstacles (Fig.7.1(b)) and result in incorrect sensing and interference to primary transmission.

Traditionally, the hidden terminal problems in spectrum sensing are solved by cooperative spectrum sensing among geographically separated CR nodes (Fig.7.2). The cooperative spectrum sensing creates geographically independent sensing channels and has higher chance to avoid hidden terminal problem. These channels also experience independent fading. However, the cooperative spectrum sensing needs

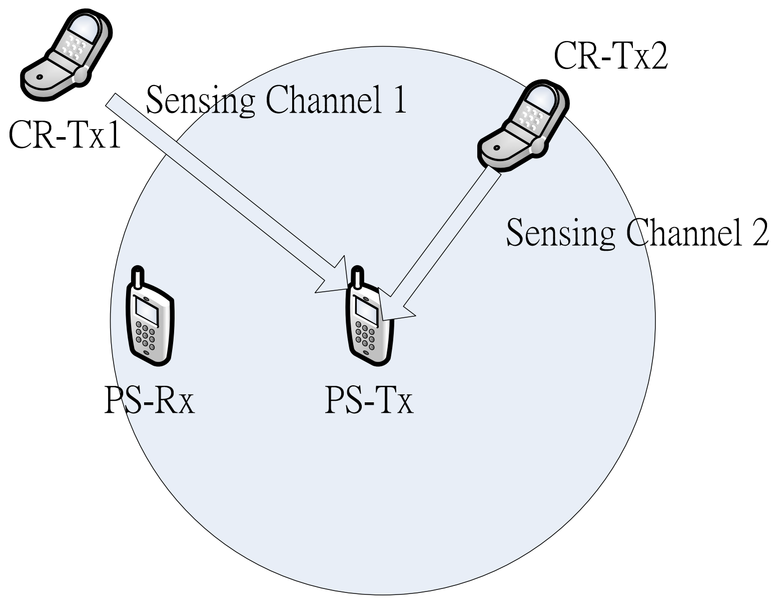


Fig. 7.2 Cooperative Spectrum Sensing

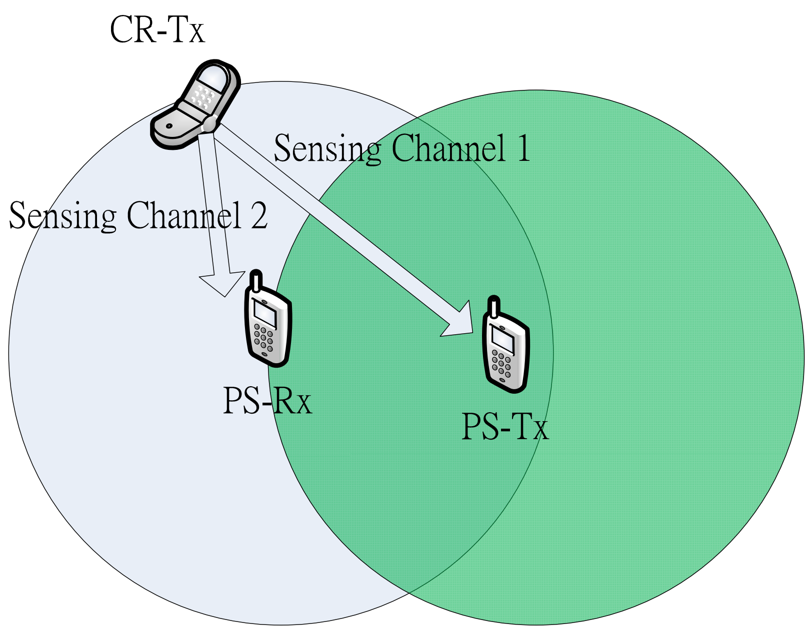


Fig. 7.3 DTD spectrum sensing scheme

control channel between CR nodes and results in lots of communication overhead.

7.1.2 Dual-way Time Division Spectrum Sensing

We develop a novel spectrum sensing scheme, Dual-way Time Division (DTD) Spectrum Sensing, to take the “complete” observation and perform information fusion to make the decision. We create a sensing channel between CR-Tx and PS-Rx by sensing the acknowledgement message (ACK) from PS-Rx to PS-Tx (Fig.7.3). Unlike the cooperative spectrum sensing which observes the same physical quantity (signal from PS-Tx) from different nodes, DTD spectrum sensing observes different physical quantity (signal from PS-Tx and ACK from PS-Rx) by the same node through different sensing channel (Fig.7.4). Hence DTD spectrum sensing extend the sensing dimension beyond cooperative sensing by not only create independent fading channel but also taking observations from different geographical position.

We also can state that this is the “complete” observation of the PS transmission pair because it senses the existence of the signal both from PS-Tx and PS-Rx. The complete observation also brings another performance gain. Consider the situation in Fig.7.5. The cooperative CR-Tx’s are geographically separated nodes. So the presence of the PS transmission for one CR-Tx does not imply the presence of PS transmission for another CR-Tx because they are in the different geographically positions. The underlying reason for the drawback of cooperative sensing is that it tries to use the observation from other nodes to compensate the incomplete observation while the observation may not directly related to the desired event or parameter. For example the observation from other CR-Tx may not necessary imply the existence of the PS transmission in this CR-Tx’s transmission range like in Fig. 7.5. Our spectrum sensing scheme is free from this problem because the CR-Tx take the “complete” observation by its own.

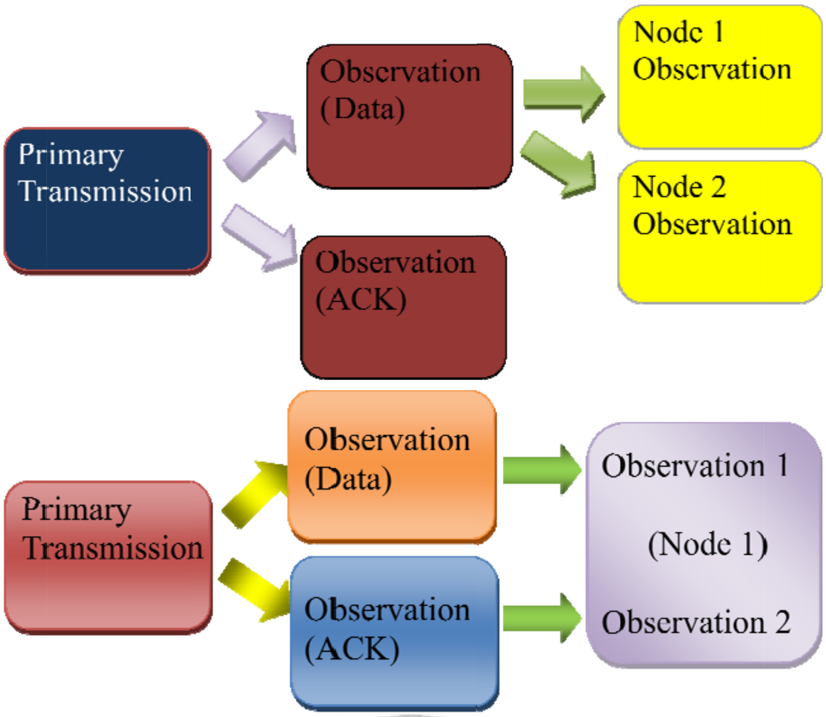


Fig 7.4 Comparison of (a) cooperative spectrum sensing and (b) DTD spectrum sensing

Fig. 7.5 Cooperative Spectrum Sensing- Drawbacks

7.2 Spectrum Sensing Model

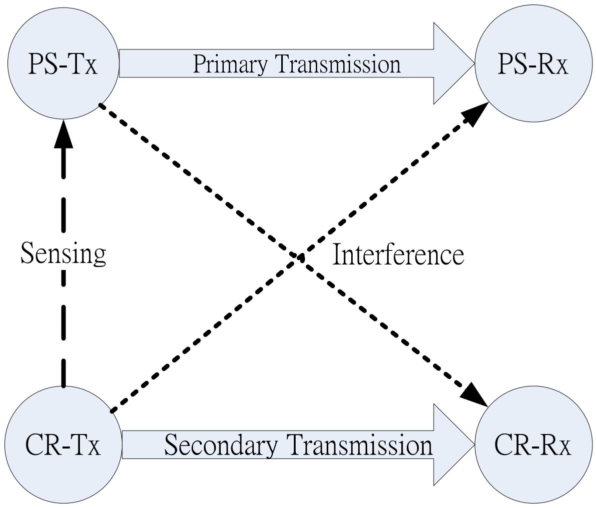


Fig. 7.6 Spectrum sensing system architecture

In this paper, we consider the simplest system with one transmission pair of CR and one of PS (Fig.7.6). The four nodes, CR-Tx, CR-Rx, PS-Tx, PS-Rx, are geographically separated. The received signal strength decreases as the geographical separation increases. The interference between CR and PS occurs when the primary transmission and secondary transmission exist simultaneously. In order to avoid the situation, CR-Tx tries to sense the primary transmission via sensing channel. If the spectrum sensing does not detect the primary transmission, the CR-Tx can transmit. However, traditional spectrum sensing only senses the presence of signal from PS-Tx, which is an "incomplete" observation of transmission between a pair of PS nodes, PS-Tx and PS-Rx. The observation is incomplete because it is the information comes only from PS-Tx but not PS-Rx. The incompleteness of the observation results in the hidden terminal problem. Due to the geographical separation, the absence of signal on sensing channel does not necessary imply the absence of signal on interference channel. Hidden terminal problem occurs when CR-Tx fails to sense the signal on the sensing channel and CR transmission cause interference to PS transmission through the interference channel. We propose DTD spectrum sensing scheme to relieve the hidden terminal problem by information fusion of the complete observation from both

PS-Tx and PS-Rx.

We first establish the system model of PS transmission and CR spectrum sensing.

7.2.1 Primary System (PS) transmission model

In order to sense the primary transmission from PS-Tx and PS-Rx, we assume that the PS transmission follows the following transmission protocol. To simplify the procedures, we assume ACK can be received and decoded by PS-Tx if PS-Rx sends it and the propagation delay is very small thus we neglect it.

Transmission protocol:

1. PS-Tx sends data to PS-Rx.
2. When PS-Rx successfully decodes the data packet, it replies ACK to PS-Tx for each received packet. If the decode is success, go step 3. Otherwise, go step 4.
3. After PS-Tx receives ACK, repeat step 1 for next transmission.
4. If the decode fails, PS-Rx remains silent. Then PS-Tx does not receive the ACK. Now repeat step 1 for the same data transmission.

Note that the PS-Tx wait for one ACK packet duration and proceed to transmit (the same packet or next packet). Under this transmission protocol, the signal in the time domain will periodically alternate between data packet and ACK packet as shown in Fig.7.7 (ACK packet is absent if the decode failure happens).

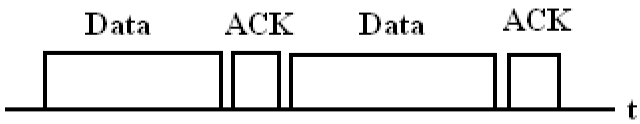


Fig. 7.7 Primary system transmission model

The duration of the data packet is t_{data} and the duration of ACK packet is t_{ACK}.

7.2.2 CR sensing channel model

The received signal in CR-Tx becomes:

$$y(t) = h \cdot s(t) + v(t) \tag{7.1}$$

h is a Rayleigh distributed random variable to account for the multipath fading. Assume the coherence time approximately the same as $t_{\mathrm{data}} + t_{\mathrm{ACK}}$. Hence h is the same in the same data packet duration and ACK packet duration. Because h is Rayleigh distributed, the power of the received signal is exponential distributed with mean denoted by ${\sigma_{hp}}^2$. ${\sigma_{hp}}^2$ is determined by the path loss model and the distance between the signal source and receiver, CR-Tx. To simplify the problem, we use the simplified path-loss model here [39]:

$${\sigma_{hp}}^2 = K_{\mathrm{p}} \left(\frac{d_0}{d}\right)^{\gamma} \tag{7.2}$$

K_{p} is the path-loss constant depending on the antenna characteristics and the average channel attenuation, d_0 is a reference distance for the antenna far field, and γ is the path-loss exponent. Due to the geographical separation of PS-Tx and PS-Rx, the ${\sigma_{hp}}^2$'s are different for sensing signal from the two nodes. $v(t)$ is AWGN and $v(t)$ in different sensing channel is independent.

7.3 Spectrum Sensing Procedure and Algorithm

If we are able to detection the alternating point of the ACK and data packet, we can separately take the two independent observations from different position, both in the dimension of geometry and time. The sensing also performs in different sensing channel. The difference in geographical dimension results in different path loss factor and the sensing channel difference results in independent fading. In fact, we can formulate the spectrum sensing problem under the intelligent decision framework. The following table illustrates the formulation:

Intelligent Decision Framework	Spectrum Sensing
Event	Primary Transmission
Physical quantity 1	PS-Tx transmission data
Physical quantity 2	PS-Rx ACK
Observation 1	Sensing data packet
Observation 2	Sensing ACK

Table 7.1. Formulation under Intelligent decision framework

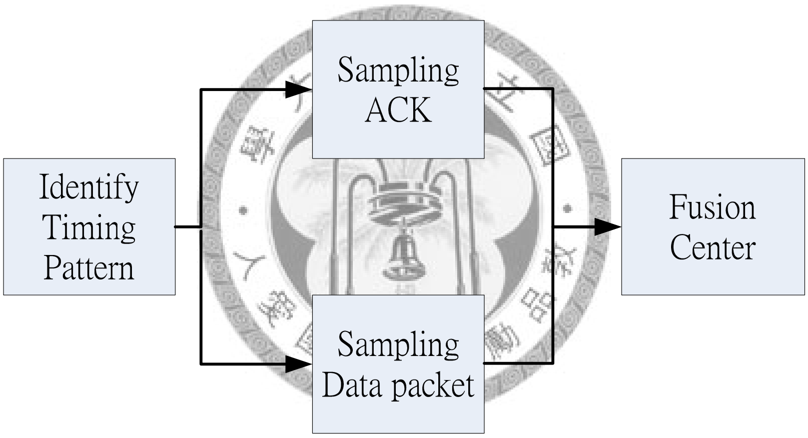


Fig. 7.8 Spectrum Sensing Procedure

The spectrum sensing scheme follows the following procedures (Fig.7.8):

1. Identifying the timing pattern

We assume CR has the knowledge of the duration of data packet and ACK. We also know that because the signals from PS-Tx and PS-Rx experience different path-loss, their power level is different when they arrive in CR-Tx. In other words, the mean power of receiving signal process is different. The Change Detection algorithms [40,41] applied in signal segmentation or remote can be applied here to detect the change in the process mean at the alternating point of data packet and ACK.

With the above knowledge, the following procedures are sufficient to identify the timing pattern of the intertwined ACK and data packet:

(1) Start monitoring at $t = t_0$

(2) Keep monitoring until a change in mean happened or until $t = t_0 + t_{\text{data}}$. Denote the time of the change point t_{ch}. If the change does not happen until t_{data}, $t_{\text{ch}} = t_{\text{data}}$.

(3) If $t_0 - t_{\text{ch}} > t_{\text{ACK}}$, stop the monitoring. If $t_0 - t_{\text{ch}} < t_{\text{ACK}}$, keep monitoring until $t = t_{\text{ch}} + t_{\text{ACK}}$.

Then we can construct the timing pattern by the following inference:

(1) If $t_0 - t_{\text{ch}} > t_{\text{ACK}}$, at the change point t_{ch} the transmission changes from data packet transmission to ACK reply.

(2) If $t_0 - t_{\text{ch}} < t_{\text{ACK}}$ and a change point is observed at $t = t_{\text{ch}} + t_{\text{ACK}}$, at the change point t_{ch} the transmission changes from data packet transmission to ACK reply. Otherwise, at the change point t_{ch} the transmission changes from ACK reply to data packet transmission.

The performance of Change Detection algorithm is better when the process changes significantly. When the process only changes a little, the performance degrades. However, our spectrum sensing scheme has large performance gain when the signal power of ACK and data packet is significantly different, which will be elaborated in the following sections. If the signal power is almost the same, our scheme acts almost like ordinary energy detector. Hence the performance of our spectrum sensing scheme would not be sensitive to the performance of Change Detection because in the region which the performance of Change Detection may degrade, the correct timing of PS transmission would not significantly affect the performance of our spectrum sensing scheme.

2. Derive the test statistics of ACK & data packet

The ACK and data packet detection problem falls back to detect an unknown signal in the fading channel which is a well-studied subject [42]. As mentioned previously, the signal takes the form:

$$y(t) = h \cdot s(t) + v(t) \tag{7.3}$$

The only difference in signal model between ACK and data packet is the path-loss factor which determines the mean of channel fading factor h.

And the test statistic Y is the integration of the square of the received signal:

$$Y = \frac{1}{\sigma_v^2} \int_0^T y(t)^2 dt \tag{7.4}$$

T is the duration of sensing time, σ_v^2 is the standard deviation of the noise. Conditioning on SNR λ, the distribution of Y is:

$$Y \sim \begin{cases} \chi_{2u}^2 & H_0 \\ \chi_{2u}^2(2\lambda) & H_1 \end{cases} \tag{7.5}$$

χ_{2u}^2 is chi-square distribution with $2u$ degrees of freedom. $u = TW$ is the time-bandwidth product. $\chi_{2u}^2(2\lambda)$ is chi-square distribution with $2u$ degrees of freedom and a non-centrality parameter 2λ. Because h follows Rayleigh distribution, λ follows the distribution:

$$f(\lambda) = \frac{1}{\bar{\lambda}} \exp\left(-\frac{\lambda}{\bar{\lambda}}\right) \tag{7.6}$$

where $\bar{\lambda} = \frac{K_{\mathrm{p}}}{\sigma_v^2}\left(\frac{d_0}{d_{\mathrm{Tx}}}\right)^{\gamma}$ for test statistics of data packet and $\bar{\lambda} = \frac{K_{\mathrm{p}}}{\sigma_v^2}\left(\frac{d_0}{d_{\mathrm{Rx}}}\right)^{\gamma}$ for test statistics of ACK.

3. Fusion center

Traditionally, there are several kinds of combining (fusion) scheme to deal with the problem of detecting signal in the fading channel. The most widely used combining schemes are weighted combining and selective combining. Equal gain combining is a special case of weighted combining. In fact, the weighted combining scheme is the Ratio Combining scheme and the selective combining is Observation Selection in the intelligent decision framework. Since primary transmission detection is a detection problem, we slightly modified the Ratio Combining and Observation Selection scheme for estimation problem without modifying the underlying structure.

A. Observation Selection

The fusion center selects the observation to make decision according to the geographical separation. In other words, the fusion center selects the observation of ACK if the distance between PS-Rx and CR-Tx is smaller. Otherwise, it selects data packet observation. The validity of this selection rule is justified by that according to our system model, the expectation of SNR is monotonic decreasing function of distance. Then the selected test statistics is compared with a threshold τ to decide the presence of primary transmission.

B. Ratio Combining

The fusion center combines the test statistics with the weighting coefficient determined by the quality of the observation. The quality of observation can be determined by the detection probability alone because the false alarm probability is the same for the two observations. Hence the weighting coefficient is the ratio of detection probability. Then the test statistic of Ratio Combining, Y_{RC}, is the weighted sum of test statistics of data packet and ACK:

$$Y_{\mathrm{RC}} = \frac{P_{\mathrm{dTx}}}{P_{\mathrm{dTx}} + P_{\mathrm{dRx}}} Y_{\mathrm{Tx}} + \frac{P_{\mathrm{dRx}}}{P_{\mathrm{dTx}} + P_{\mathrm{dRx}}} Y_{\mathrm{Rx}} \tag{7.7}$$

where Y_{Tx} is the test statistics of data packet, Y_{Rx} is the test statistics of ACK packet. Then the weighted sum of test statistics Y_{RC} is compared with a threshold

τ to decide the presence of primary transmission. The detection probability of energy detection in fading channel has been derived in [42],

$$P_{\mathrm{dTx}} = \exp\left(-\frac{\tau}{2}\right)\sum_{n=0}^{u-2}\frac{1}{n!}\left(\frac{\tau}{2}\right)^{n} +$$

$$\left(\frac{1+\bar{\lambda}_{\mathrm{Tx}}}{\bar{\lambda}_{\mathrm{Tx}}}\right)^{u-1}\left(\exp\left(-\frac{\tau}{2(1+\bar{\lambda}_{\mathrm{Tx}})}\right) - \exp\left(-\frac{\tau}{2}\right)\sum_{n=0}^{u-2}\frac{1}{n!}\left(\frac{\tau\bar{\lambda}_{\mathrm{Tx}}}{2(1+\bar{\lambda}_{\mathrm{Tx}})}\right)^{n}\right) \quad (7.8)$$

$$P_{\mathrm{dRx}} = \exp\left(-\frac{\tau}{2}\right)\sum_{n=0}^{u-2}\frac{1}{n!}\left(\frac{\tau}{2}\right)^{n} +$$

$$\left(\frac{1+\bar{\lambda}_{\mathrm{Rx}}}{\bar{\lambda}_{\mathrm{Rx}}}\right)^{u-1}\left(\exp\left(-\frac{\tau}{2(1+\bar{\lambda}_{\mathrm{Rx}})}\right) - \exp\left(-\frac{\tau}{2}\right)\sum_{n=0}^{u-2}\frac{1}{n!}\left(\frac{\tau\bar{\lambda}_{\mathrm{Rx}}}{2(1+\bar{\lambda}_{\mathrm{Rx}})}\right)^{n}\right) \quad (7.9)$$

Note that the τ in $\mathrm{P_{dTx}}$ and $\mathrm{P_{dRx}}$ is the same as the threshold τ. The detection probability is a monotonic increasing function of the mean of SNR distribution, $\bar{\lambda}_{\mathrm{Tx}}$ and $\bar{\lambda}_{\mathrm{Rx}}$. And the mean of SNR distribution is a monotonic decreasing function of the distance between the sensing node and the signal source. Hence the weighting coefficient is the decreasing function of distance, which is intuitively true.

7.4 Performance Analysis and Comparison

In this section, we analyze the performance of Observation Selection and Ratio Combing then compare their performance with other schemes. We analyze the detection probability only because the false alarm probability is the same for the test statistic of data packet and ACK and fusion scheme would not affect the false alarm probability. Hence all the distribution we analyze in this section is under H_1 unless specified.

7.4.1 Observation Selection

For the Observation Selection scheme, the distribution of test statistic conditioned on SNR λ becomes

$$Y_{\mathrm{OS}} \sim \begin{cases} \chi_{2u}^2 & H_0 \\ \chi_{2u}^2(2\lambda) & H_1 \end{cases}$$

λ has the distribution

$$f(\lambda) = \frac{1}{\bar{\lambda}} \exp\left(-\frac{\lambda}{\bar{\lambda}}\right) = \frac{1}{\max(\bar{\lambda}_{\mathrm{Tx}}, \bar{\lambda}_{\mathrm{Rx}})} \exp\left(-\frac{\lambda}{max\left(\bar{\lambda}_{Tx}, \bar{\lambda}_{Rx}\right)}\right) \tag{7.10}$$

$$\bar{\lambda}_{\mathrm{Tx}} = \frac{{\sigma_{h\mathrm{Tx}}}^2}{\sigma_v^2} = \frac{K_{\mathrm{p}}}{\sigma_v^2}\left(\frac{d_0}{d_{\mathrm{Tx}}}\right)^{\gamma}, \bar{\lambda}_{\mathrm{Rx}} = \frac{{\sigma_{h\mathrm{Rx}}}^2}{\sigma_v^2} = \frac{K_{\mathrm{p}}}{\sigma_v^2}\left(\frac{d_0}{d_{\mathrm{Rx}}}\right)^{\gamma} \tag{7.11}$$

Then the detection probability becomes:

$$\mathrm{P}_{\mathrm{dos}} = \exp\left(-\frac{\tau}{2}\right) \sum_{n=0}^{u-2} \frac{1}{n!} \left(\frac{\tau}{2}\right)^n$$

$$+ \left(\frac{1+\bar{\lambda}}{\bar{\lambda}}\right)^{u-1} \left(\exp\left(-\frac{\tau}{2(1+\bar{\lambda})}\right) - \exp\left(-\frac{\tau}{2}\right) \sum_{n=0}^{u-2} \frac{1}{n!} \left(\frac{\tau\bar{\lambda}}{2(1+\bar{\lambda})}\right)^n\right) \tag{7.12}$$

Where

$$\bar{\lambda} = \max(\bar{\lambda}_{\mathrm{Tx}}, \bar{\lambda}_{\mathrm{Rx}})$$

As a comparison, we also analyze the performance of the Selection Combining, which performs selection based on full channel gain information and selects the observation with larger channel gain. By modifying the derivation of the SNR distribution of Selection Combining in i.i.d. Rayleigh fading channel in [43], we can derive the distribution of SNR in Selection Combining in Rayleigh fading channel with different mean.

$$f_{\Lambda_{\mathrm{SC}}}(\lambda) = f_{\Lambda_1,\Lambda_2}(\lambda) = f_{\Lambda_1}(\lambda)\mathrm{P}(\Lambda_2 \le \lambda) + f_{\Lambda_2}(\lambda)\mathrm{P}(\Lambda_1 \le \lambda)$$

$$= \frac{1}{\bar{\lambda}_{\mathrm{Tx}}} \exp\left(-\frac{\lambda}{\bar{\lambda}_{\mathrm{Tx}}}\right)\left(1 - \exp\left(-\frac{\lambda}{\bar{\lambda}_{\mathrm{Rx}}}\right)\right) + \frac{1}{\bar{\lambda}_{\mathrm{Rx}}} \exp\left(-\frac{\lambda}{\bar{\lambda}_{\mathrm{Rx}}}\right)\left(1 - \exp\left(-\frac{\lambda}{\bar{\lambda}_{\mathrm{Tx}}}\right)\right)$$

$$= \frac{1}{\bar{\lambda}_{\mathrm{Tx}}} \exp\left(-\frac{\lambda}{\bar{\lambda}_{\mathrm{Tx}}}\right) + \frac{1}{\bar{\lambda}_{\mathrm{Rx}}} \exp\left(-\frac{\lambda}{\bar{\lambda}_{\mathrm{Rx}}}\right) - \left(\frac{1}{\bar{\lambda}_{\mathrm{Tx}}} + \frac{1}{\bar{\lambda}_{\mathrm{Rx}}}\right) \exp\left(-\lambda\left(\frac{1}{\bar{\lambda}_{\mathrm{Tx}}} + \frac{1}{\bar{\lambda}_{\mathrm{Rx}}}\right)\right)$$

(7.13)

Then the detection probability becomes:

$$\mathrm{P_{dSC}} = \int_{\lambda} \left(\int_{\tau}^{\infty} f_{Y_{\mathrm{SC}}|\lambda,H_1}(y)\mathrm{d}y \right) f_{\Lambda_{\mathrm{SC}}}(\lambda)\mathrm{d}\lambda = \int_{\lambda} \mathrm{Q}_u(\sqrt{2\lambda},\sqrt{\tau}) f_{\Lambda_{\mathrm{SC}}}(\lambda)\mathrm{d}\lambda \qquad (7.14)$$

where $\mathrm{Q}_u(a,b)$ is the generalized Marcum Q-function.

7.4.2 Ratio Combining

In ratio combining, the test statistics becomes weighted sum of the test statistics of data packet and ACK. Their distributions under H_1 (signal presence) are $Y_{\mathrm{Tx}} \sim \chi_{2u}^2(2\lambda_{\mathrm{Tx}})$ and $Y_{\mathrm{Rx}} \sim \chi_{2u}^2(2\lambda_{\mathrm{Rx}})$ respectively. It is common practice in statistics to approximate a weighted sum of non-central chi-square random variables by a single chi-square random variable with different degrees of freedom and an adequate scaling factor [44-46].

$$\sum_i \alpha_i \chi_{2u}^2(\lambda_i) \sim \beta \chi_{\omega}^2 \qquad (7.15)$$

The degree of freedom and scaling factor should be chosen such that both sides have the same first two moments. By the above formula, we can approximate the test statistics of Ratio Combining scheme by the chi-square random variable (conditioning on SNR λ_{Tx} and λ_{Rx}):

$$Y_{\mathrm{RC}} = \left(\frac{P_{\mathrm{dTx}}}{P_{\mathrm{dTx}} + P_{\mathrm{dRx}}} Y_{\mathrm{Tx}} + \frac{P_{\mathrm{dRx}}}{P_{\mathrm{dTx}} + P_{\mathrm{dRx}}} Y_{\mathrm{Rx}} \right) \sim \beta_{\mathrm{RC}} \chi_{\omega_{\mathrm{RC}}}^2 \qquad (7.16)$$

Recall that if $z \sim \chi_{\omega}^2(m)$, then $\mathrm{E}(z) = m + \omega$ and $\mathrm{var}(z) = 2(2m + \omega)$. We can derive β_{RC} and ω_{RC} by the following two equations:

$$\beta_{\mathrm{RC}} \omega_{\mathrm{RC}} = \frac{P_{\mathrm{dTx}}}{P_{\mathrm{dTx}} + P_{\mathrm{dRx}}}(2\lambda_{\mathrm{Tx}} + 2u) + \frac{P_{\mathrm{dRx}}}{P_{\mathrm{dTx}} + P_{\mathrm{dRx}}}(2\lambda_{\mathrm{Rx}} + 2u) \qquad (7.17)$$

$$2{\beta_{\mathrm{RC}}}^2 \omega_{\mathrm{RC}} = 2\left(\frac{P_{\mathrm{dTx}}}{P_{\mathrm{dTx}} + P_{\mathrm{dRx}}} \right)^2 (4\lambda_{\mathrm{Tx}} + 2u) + 2\left(\frac{P_{\mathrm{dRx}}}{P_{\mathrm{dTx}} + P_{\mathrm{dRx}}} \right)^2 (4\lambda_{\mathrm{Rx}} + 2u)$$

(7.18)

Solving these two equations, we can derive:

$$\beta_{\mathrm{RC}} = \frac{\left(\frac{P_{\mathrm{dTx}}}{P_{\mathrm{dTx}} + P_{\mathrm{dRx}}}\right)^2 (2\lambda_{\mathrm{Tx}} + u) + \left(\frac{P_{\mathrm{dRx}}}{P_{\mathrm{dTx}} + P_{\mathrm{dRx}}}\right)^2 (2\lambda_{\mathrm{Rx}} + u)}{\frac{{d_{\mathrm{Rx}}}^{\gamma}}{{d_{\mathrm{Tx}}}^{\gamma} + {d_{\mathrm{Rx}}}^{\gamma}} (\lambda_{\mathrm{Tx}} + u) + \frac{{d_{\mathrm{Tx}}}^{\gamma}}{{d_{\mathrm{Tx}}}^{\gamma} + {d_{\mathrm{Rx}}}^{\gamma}} (\lambda_{\mathrm{Rx}} + u)} \tag{7.19}$$

$$\omega_{\mathrm{RC}} = \frac{2\left(\frac{P_{\mathrm{dTx}}}{P_{\mathrm{dTx}} + P_{\mathrm{dRx}}} (\lambda_{\mathrm{Tx}} + u) + \frac{P_{\mathrm{dRx}}}{P_{\mathrm{dTx}} + P_{\mathrm{dRx}}} (\lambda_{\mathrm{Rx}} + u)\right)^2}{\left(\frac{P_{\mathrm{dTx}}}{P_{\mathrm{dTx}} + P_{\mathrm{dRx}}}\right)^2 (2\lambda_{\mathrm{Tx}} + u) + \left(\frac{P_{\mathrm{dRx}}}{P_{\mathrm{dTx}} + P_{\mathrm{dRx}}}\right)^2 (2\lambda_{\mathrm{Rx}} + u)} \tag{7.20}$$

Then the detection probability becomes:

$$\mathrm{P}_{\mathrm{dRC}} = \int\limits_{\lambda_{\mathrm{Tx}}} \int\limits_{\lambda_{\mathrm{Rx}}} \left(\int_{\tau}^{\infty} f_{Y_{\mathrm{RC}}|\lambda_{\mathrm{Rx}}, \lambda_{\mathrm{Tx}}, \mathrm{H}_1}(y) \mathrm{d}y \right) f_{\Lambda_{\mathrm{Rx}}}(\lambda_{\mathrm{Rx}}) f_{\Lambda_{\mathrm{Tx}}}(\lambda_{\mathrm{Tx}}) d\lambda_{\mathrm{Rx}} \mathrm{d}\lambda_{\mathrm{Tx}} \tag{7.21}$$

where

$$f_{\Lambda_{\mathrm{Rx}}}(\lambda_{\mathrm{Rx}}) = \frac{1}{\bar{\lambda}_{\mathrm{Rx}}} \exp\left(-\frac{\lambda_{\mathrm{Rx}}}{\bar{\lambda}_{\mathrm{Rx}}}\right), \quad f_{\Lambda_{\mathrm{Tx}}}(\lambda_{\mathrm{Tx}}) = \frac{1}{\bar{\lambda}_{\mathrm{Tx}}} \exp\left(-\frac{\lambda_{\mathrm{Tx}}}{\bar{\lambda}_{\mathrm{Tx}}}\right),$$

and distribution of test statistics Y_{RC} conditioning on λ_{Rx} and λ_{Tx} is approximated by chi-square distribution with degrees of freedom ω and scaling factor β. By the Method of Transformations in probability theory, we can derive the distribution of Y_{RC}:

$$f_{Y_{\mathrm{RC}}|\lambda_{\mathrm{Rx}}, \lambda_{\mathrm{Tx}}, \mathrm{H}_1}(y) = f_{\chi^2_{\omega_{\mathrm{RC}}}}\left(\frac{y}{\beta_{\mathrm{RC}}}\right) \left|\frac{1}{\beta_{\mathrm{RC}}}\right|$$

$$= \frac{\left(\frac{1}{2}\right)^{\frac{\omega_{\mathrm{RC}}}{2}}}{\Gamma(\frac{\omega_{\mathrm{RC}}}{2})} \left(\frac{y}{\beta_{\mathrm{RC}}}\right)^{\frac{\omega_{\mathrm{RC}}}{2} - 1} \exp\left(-\frac{y}{2\beta_{\mathrm{RC}}}\right) \frac{1}{\beta_{\mathrm{RC}}} \tag{7.22}$$

Then put the three distribution into the integral equation, we can derive $\mathrm{P}_{\mathrm{dRC}}$.

We compare the Ratio Combining scheme with the traditional Equal Gain Combining (EGC), which is widely applied in cooperative diversity due to its IID Rayleigh fading channel assumption. The test statistics for EGC is:

$$Y_{\mathrm{EGC}} = \frac{1}{2}(Y_{\mathrm{Tx}} + Y_{\mathrm{Rx}})$$

We also can approximate it by the scaled chi-square distribution:

$Y_{\mathrm{EGC}} \sim \beta_{\mathrm{EGC}} \chi^2_{2\omega_{\mathrm{EGC}}}$

Then we have

$$\beta_{\mathrm{EGC}}\omega_{\mathrm{EGC}} = \frac{1}{2}(2\lambda_{\mathrm{Tx}} + 2\lambda_{\mathrm{Rx}} + 4u)$$

$$2{\beta_{\mathrm{EGC}}}^2\omega_{\mathrm{EGC}} = \frac{1}{4} \times 2(4\lambda_{\mathrm{Tx}} + 2u) + \frac{1}{4} \times 2(4\lambda_{\mathrm{Rx}} + 2u)$$

Solving the two equation, we can derive β_{EGC} and ω_{EGC}

$$\beta_{\mathrm{EGC}} = \frac{\lambda_{\mathrm{Tx}} + \lambda_{\mathrm{Rx}} + 2u}{\lambda_{\mathrm{Tx}} + \lambda_{\mathrm{Rx}} + u}$$

$$\omega_{\mathrm{EGC}} = \frac{(\lambda_{\mathrm{Tx}} + \lambda_{\mathrm{Rx}} + 2u)^2}{\lambda_{\mathrm{Tx}} + \lambda_{\mathrm{Rx}} + u}$$

Then following the above procedures, we can derive the detection probability, $\mathrm{P_{dEGC}}$, by putting β_{EGC} and ω_{EGC} in (7.22).

7.5 Numerical Result

In this section, we present the simulation result and numerical result of above analysis. Because we derive the analytical result of Ratio Combining and Equal Gain Combining through approximation, we also present the simulation result to compare with the theoretical approximation. We use the Complement Receiver Operation Curve (CROC) to present the result of performance comparison of the proposed scheme and traditional schemes.

7.5.1 Observation Selection

The Observation Selection scheme is compared with single observation and Selective Combining. Single observation is the traditional spectrum sensing with energy detector which senses the signal from transmitter. Selective Combining scheme selects the Observation with larger SNR, which is an ideal situation but is unrealistic in most cases of spectrum sensing. Here we present the performance of Selective Combining to serves as the performance upper-bound of Observation Selection. We set time-bandwidth product u=2 in the numerical result.

We can observe from the numerical result (Fig.7-9(a)(b)) that as the difference of distance from transmitter and receiver increases, the performance curve of Observation Selection is closer to Selective Combining and away from Single Observation. That is, the performance gain by taking observations from two sensing channels and selecting the observation with larger mean SNR is larger when PS-Rx is closer to CR-Tx. Although the Observation Selection in spectrum sensing cannot achieve full diversity gain due to lake of sensing channel side information (SNR), it indeed improves the performance of the traditional no diversity spectrum sensing scheme.

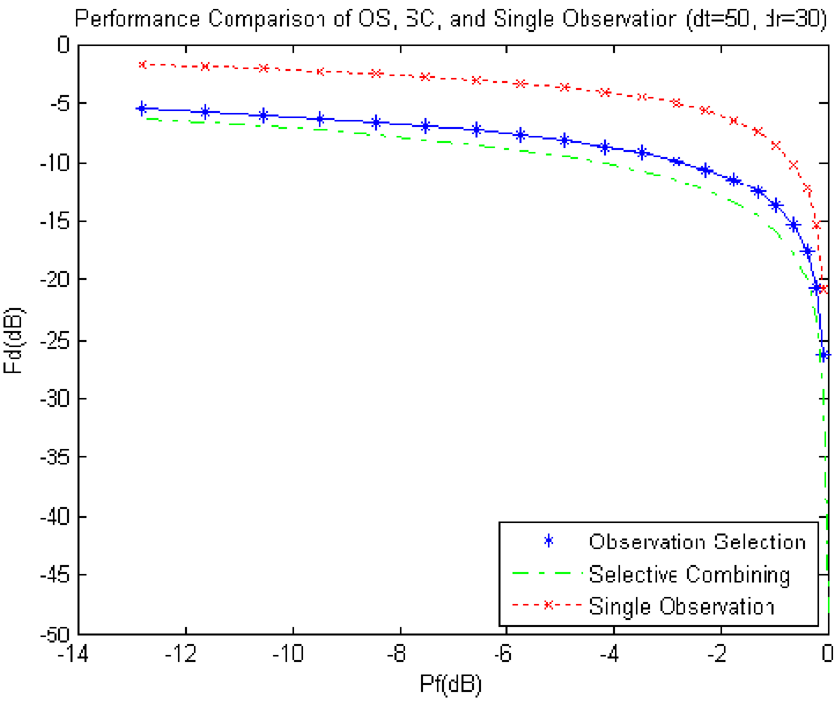


Fig. 7.9 (a) Performance Comparison of Observation, Selective Combining and Single Observation (PS-Tx to CR-Tx =50, PS-Rx to CR-Tx distance = 30)

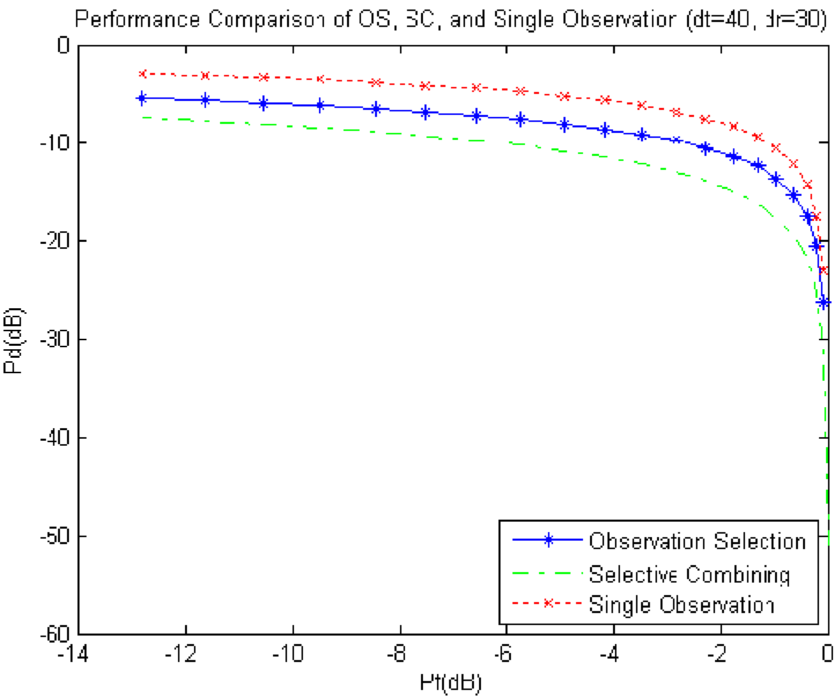


Fig. 7.9 (b) Performance Comparison of Observation, Selective Combining and Single Observation (PS-Tx to CR-Tx =40, PS-Rx to CR-Tx distance = 30)

7.5.2 Ratio Combining

The Ratio Combining scheme is compared with Equal Gain Combining, and Observation Selection. We derive the performance comparison through simulation first, and then compare the approximation derived analytically in last section with the simulation result to see the quality of the approximation. We set time-bandwidth product $u=4$ in the simulation.

Simulation results (Fig.7.10) show that

(1) In the area of extremely low miss detection probability and high false alarm probability, the performance of Ratio Combining is very close to EGC and both schemes are significantly better than Observation Selection.

(2) As miss detection probability becomes higher, the difference between performance of Ratio Combining and EGC becomes larger and the difference between the performance of Ratio Combining and Observation Selection shrinks. We can observe the trend more clear in Fig.7.10(b), which is the segment in low false alarm probability area.

The performance gain of Ratio Combining is due to that it utilizes the geographical position information to infer the average path-loss of the signal and determines the combining coefficient. It becomes larger because as the false alarm probability decreases, the difference between false alarm probabilities of the two observations becomes larger, thus the performance gain of Ratio Combining becomes more significant.

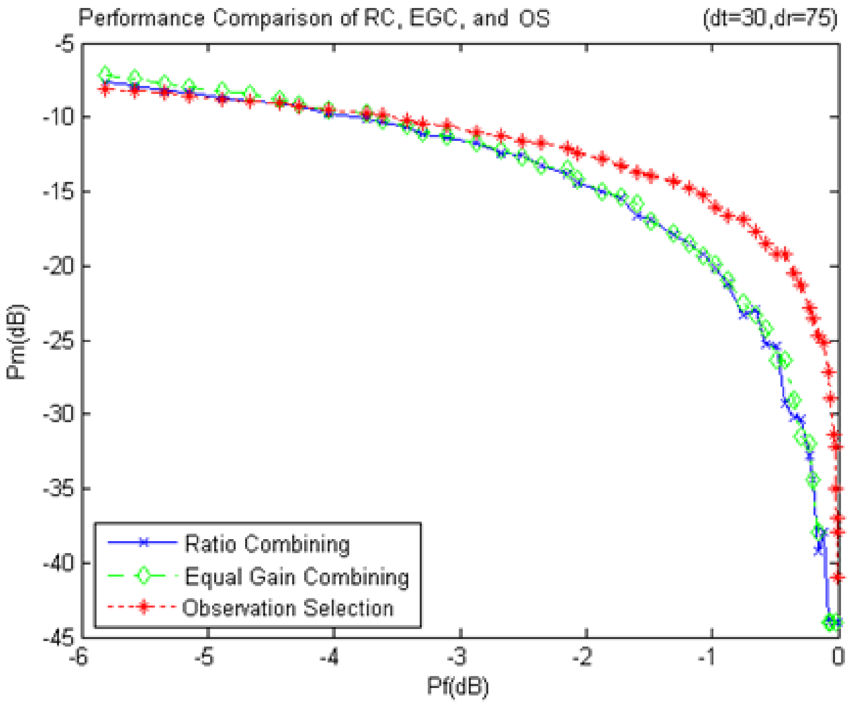


Fig. 7.10 (a) Performance Comparison of Ratio Combining, EGC, and Observation Selection (PS-Tx to CR-Tx =30, PS-Rx to CR-Tx distance = 75)

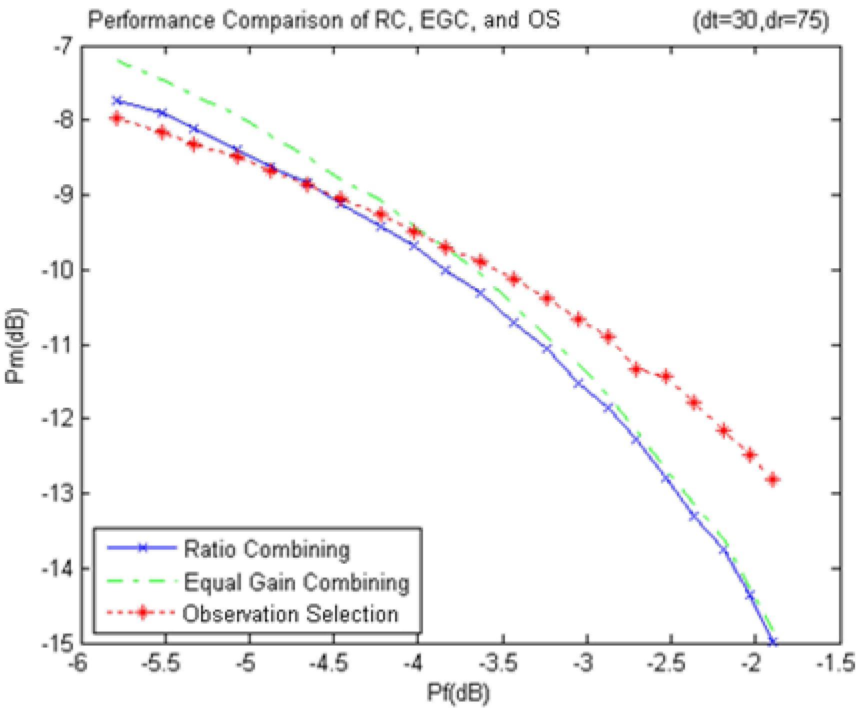


Fig. 7.10 (b) Segment from 7.10(a)

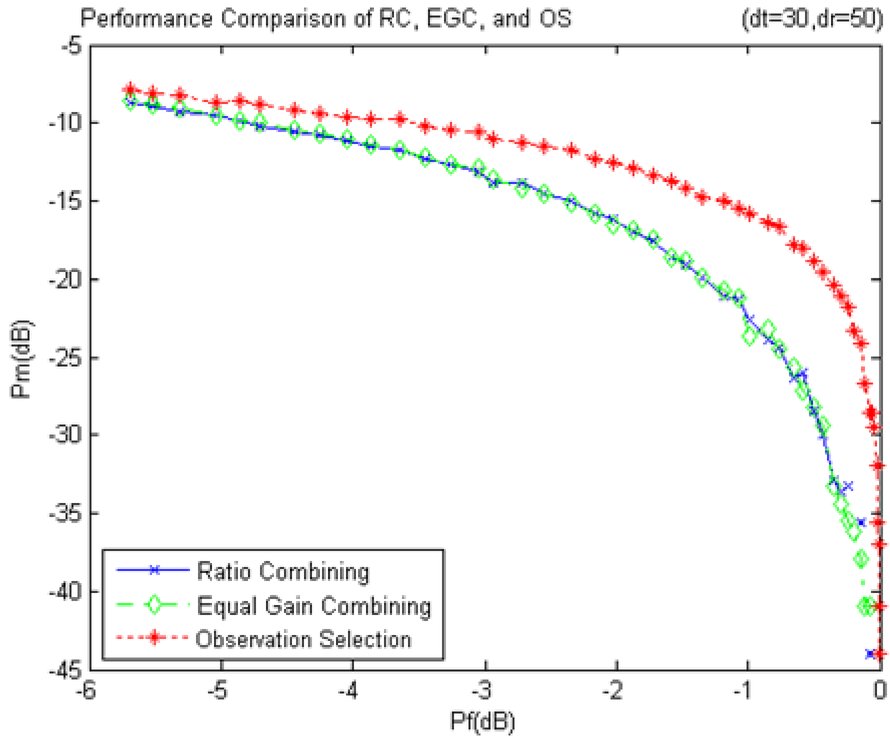


Fig. 7.11 (a) Performance Comparison of Ratio Combining, EGC, and Observation Selection (PS-Tx to CR-Tx =30, PS-Rx to CR-Tx distance = 50)

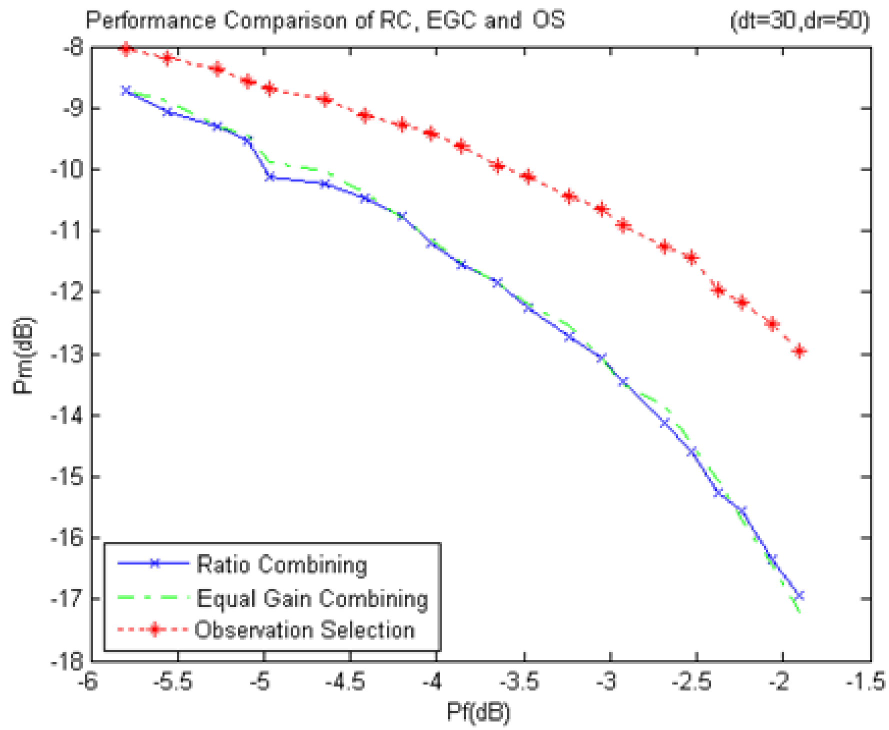


Fig. 7.11 (b) Segment from 7.11(a)

On the other hand, if we run the simulation in the scenario where the difference between the distances from CR-Tx to PS-Tx and to PS-Rx becomes smaller, the difference between the performance of Ratio Combining and Observation Selection becomes larger (Fig.7.11(a)), while the performance of Ratio Combining and EGC becomes closer (Fig.7.11(a)(b)). This can also explained by that when the difference between distances becomes smaller, the diversity gain becomes more significant due to the improvement of the inferior observation's quality. However, the weighting coefficient of the two observations is closer when the inferior observation's quality is improved. Hence the difference between the performance of Ratio Combining and EGC becomes smaller.

By the simulation results, we can infer that to have better sensing performance:

(1) When the two signals experience significantly different path-loss, apply Observation Selection to choose the better one to make decision

(2) When the two signals experience similar path-loss, apply Ratio Combining or EGC to make decision.

Finally we present the comparison of the analytical approximation result and the simulation result of Ratio Combining performance analysis. Fig.7.12 shows that the

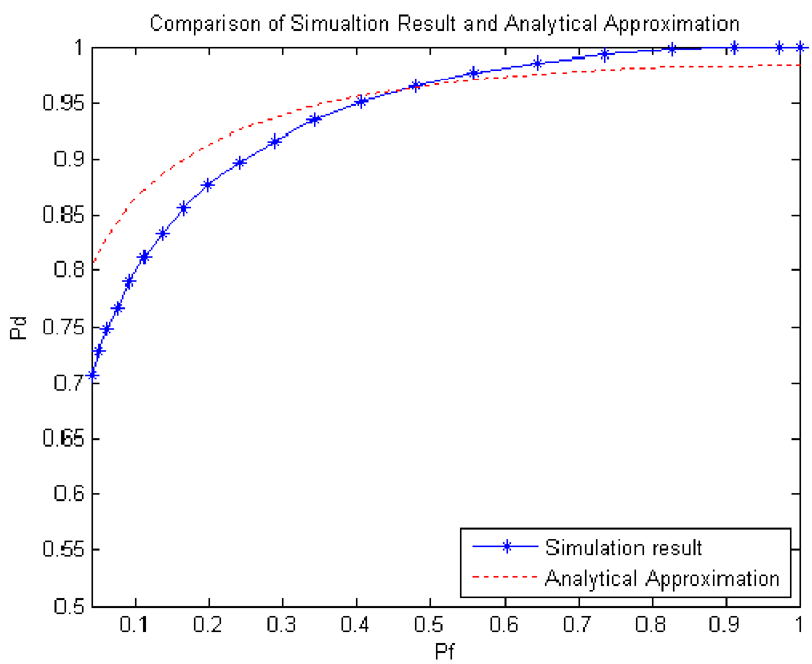


Fig. 7.12 Comparison of simulation result and analytical approximation of ROC for Ratio Combining curve.

approximation is close in the region of high detection probability area, and is relatively poor in low detection probability area.

Chapter 8

Conclusion and Future Work

A general decision framework for SNBIS unifying the information fusion of sensor network, decision and control action is the most crucial theory for implementation of SNBIS in various environments to efficiently execute tasks. The intelligent decision framework proposed in this paper lays the foundation of the mathematical structure and techniques for the unified general decision process of SNBIS. By further exploring the relationship of observation and task execution in SNBIS, this framework separate the traditional event to observation mapping into two mappings, event to physical quantity mapping and physical quantity to observation mapping. Based on the two mappings, we derive the new decision mapping and use the firefighting robot navigation problem to illustrate the application of intelligent decision framework. In this example, we also investigate the special case degenerated to traditional state space estimation problem to show the relationship between our framework and traditional decision techniques. Then we extend the intelligent decision framework to accommodate observations from multiple kinds of physical quantities. Under this framework, optimal decision and Observation Selection for limited knowledge of correlations among physical quantities are formulated and their equivalence condition is derived. Fuzzy logic controller, which is widely used with less strict-sense mathematical structure, can be derived by degenerating Observation Selection scheme. Simulation results show that the intelligent decision outperforms traditional decision schemes and is more robust to environment or system parameter variations in both single observation and multiple observation scenarios.

In addition to SNBIS, we also apply the framework to CR spectrum sensing to mitigate the hidden terminal problem in a novel way. Unlike the traditional cooperative spectrum sensing which relies on cooperation of other nodes to obtain independent sensing data, the proposed spectrum sensing scheme senses the PS-Rx to create another sensing channel and obtain the "complete" observation of the primary transmission. By taking the geographical position information into consideration, we can improve the information fusion of spectrum sensing to achieve the diversity which is traditionally applied in cooperative spectrum sensing.

Based on this framework, research topics, including gathering more information of the conditional distributions by learning, blind estimation without knowing the distribution in this framework, etc, is able to be explored. By sophisticated and accurate decision process, SNBIS is able to be applied in more fascinating future life scenarios like smart home, intelligent health care and medical system to realize them.

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