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Kou 的跳躍擴散模型擴展：
納入雙伽瑪分佈的跳躍幅度

An Extension of Kou's Jump-Diffusion Model to
Incorporate Double-Gamma Distributed Jump Sizes

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本論文係胡祖望君(R10723059)在國立臺灣大學財務金融學系暨研究所完成之碩士學位論文，於民國 112 年 11 月 8 日承下列考試委員審查通過及口試及格，特此證明

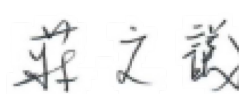
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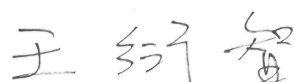
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Abstract

This paper aims to extend Kou's double exponential jump-diffusion model to the double gamma jump-diffusion model. We employ the Fast Fourier Transform method to obtain option prices for both models. By extending the model to the double gamma distribution not only reproduces the results of Kou's model but also provides enhanced flexibility in simulating the market, allowing for fine-tuning of the model. The numerical results show how the double gamma distribution variables affect the moments leading to changes in the call value.

Keywords: Jump-diffusion model, Exponential distribution, Gamma distribution, Fast Fourier Transform, Option value





摘要

本論文主要在研究在將 Kou 的雙指數跳躍擴展成雙伽瑪跳躍擴散模型。我們採用快速傅立葉轉換方法來獲取兩個模型的選擇權價格。通過將模型擴展為雙伽瑪分佈不僅能夠再現 Kou 模型的結果，還能夠提供更大的靈活性來模擬市場，允許對模型進行微調。

關鍵字：跳躍擴散模型、指數分佈、伽馬分佈、快速傅立葉轉換、選擇權定價

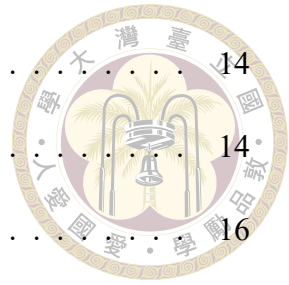




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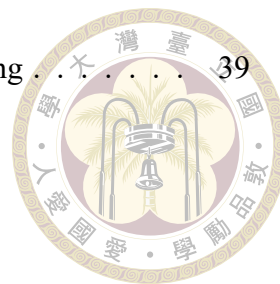




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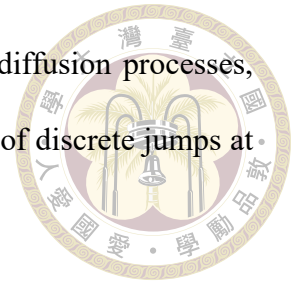


Chapter 1 Introduction

Option pricing is a fundamental topic in financial mathematics that plays a crucial role in valuing derivative securities and managing risk. An option is a financial contract that gives the holder the right, but not the obligation, to buy (in the case of a call option) or sell (in the case of a put option) an underlying asset at a predetermined price (strike price) within a specified period (expiration date). To determine the fair value of an option, various mathematical models and pricing methods have been developed. These models aim to capture the complexities of financial markets and provide a framework for estimating the value of options.

The jump-diffusion model is a powerful mathematical framework used in finance and economics to capture the dynamics of asset prices and market behavior. It provides a more realistic representation of price movements by incorporating sudden and unpredictable jumps or discontinuities in addition to continuous diffusion processes. In traditional continuous diffusion models like the Black-Scholes model, asset price movements are assumed to follow a smooth and continuous path. However, financial markets often experience unexpected events or news releases that can lead to abrupt changes in prices, such as earnings announcements, mergers and acquisitions, or macroeconomic shocks. These discrete jumps are not adequately captured by continuous diffusion models alone. The jump-diffusion model addresses this limitation by introducing a jump component into

the price dynamics. It combines the characteristics of continuous diffusion processes, represented by stochastic differential equations, with the occurrence of discrete jumps at random time intervals.





Chapter 2 Motivation and Literature Review

2.1 Literature Review

The pricing of options in financial markets requires the consideration of various factors, including asset price movements, volatility, and the occurrence of discrete jumps. Jump-diffusion models provide a flexible framework to capture these dynamics, incorporating both continuous diffusion processes and sudden, unpredictable jumps. Pricing options under jump-diffusion models is a challenging task due to the complexity of these models. However, the utilization of the Fast Fourier Transform (FFT) has proven to be an efficient and accurate approach in tackling this challenge.

2.1.1 Option pricing models

In the field of quantitative finance, various mathematical models have been developed to describe and understand the behavior of financial markets and the pricing of derivative securities. Here we will introduce four prominent models in this realm, Black-Scholes model, Merton's jump-diffusion model, Heston's stochastic volatility model, and Kou's jump-diffusion model. Each of these models introduces different elements and as-

sumptions to capture specific aspects of market dynamics.

Kou's model is a double exponential jump-diffusion model [5]. This model represents an important advancement over the traditional Black-Scholes-Merton model by incorporating sudden, significant, and discontinuous price movements known as "jumps" and the sizes of these jumps follow the double exponential distribution. Financial markets are inherently complex and subject to various unpredictable events, including corporate news, economic reports, and unforeseen shocks. Kou's model recognizes these complexities by introducing jumps, or abrupt price changes, into the modeling framework. These jumps are crucial for capturing the infrequent but impactful events that can significantly affect asset prices.

Models contributes to our understanding of financial markets and offers valuable insights into option pricing, risk management, and portfolio optimization. They provide tools to address different aspects of market dynamics, including continuous diffusion processes, jumps, asymmetric price movements, and stochastic volatility.

2.1.2 Numerical methods for option pricing

Option pricing is a fundamental topic in financial mathematics, enabling investors and traders to assess the value of derivative securities. Over the years, various methods have been developed to calculate option prices, each with its own strengths and applicability to different scenarios. Here we will introduce four prominent methods used for option pricing, the analytical method, finite-difference method, Monte Carlo simulation, and the Fast Fourier Transform (FFT).

The analytical method, also known as closed-form solutions, is a mathematical ap-

proach that provides explicit formulas for option prices. The most famous example is the Black-Scholes model, which offers closed-form solutions for European-style options. Analytical methods rely on certain assumptions, such as constant volatility and risk-free interest rates, to simplify the pricing equations. They provide quick and precise results, making them suitable for options with simple payoffs and under standard market conditions.

“Option Valuation Using the Fast Fourier Transform” by Carr and Madan, published in 1999 [1], presents a groundbreaking approach to option valuation based on the FFT. Their research explores the application of FFT in efficiently and accurately pricing options, providing significant contributions to the field of quantitative finance. The advantages of the FFT approach includes its ability to handle complex option payoffs and its fast computational speed compared to traditional numerical methods.

Each method has its advantages and trade-offs. Analytical methods provide closed-form solutions for simple option contracts, while the FFT method’s advantage is its ability to handle complex option payoffs and its fast computational speed compared to traditional numerical methods.

2.2 Motivation

The probability density functions (PDF) of the double exponential distribution and the double gamma distribution are

$$p(x) = p\rho_1 e^{-\rho_1 x} \mathbb{1}_{x>0} + (1-p)\rho_2 e^{\rho_2 x} \mathbb{1}_{x\leq 0} \quad (2.1)$$

and

$$p(x) = p \frac{\rho_1^{\vartheta_1}}{\Gamma(\vartheta_1)} x^{\vartheta_1-1} e^{-\rho_1 x} \mathbf{1}_{x>0} + (1-p) \frac{\rho_2^{\vartheta_2}}{\Gamma(\vartheta_2)} (-x)^{\vartheta_2-1} e^{\rho_2 x} \mathbf{1}_{x\leq 0}, \quad (2.2)$$

respectively. The double exponential distribution's mean and variance are

$$\text{mean} = \mu = p \frac{1}{\rho_1} + (1-p) \frac{1}{\rho_2}$$

and

$$\begin{aligned} \text{variance} &= \int_{-\infty}^{\infty} [x - \mu]^2 p(x) dx \\ &= p \frac{\rho_1 \mu (\rho_1 \mu - 2) + 2}{\rho_1^2} + (1-p) \frac{\rho_2 \mu (\rho_1 \mu + 2) + 2}{\rho_2^2}, \end{aligned}$$

respectively. The double gamma distribution's mean and variance are

$$\text{mean} = \mu = p \frac{\vartheta_1}{\rho_1} + (1-p) \frac{\vartheta_2}{\rho_2} \quad (2.3)$$

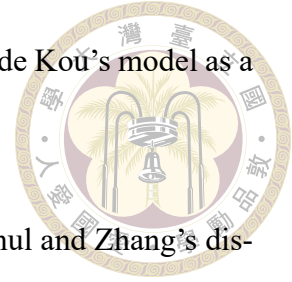
and

$$\begin{aligned} \text{variance} &= \int_{-\infty}^{\infty} [x - \mu]^2 p(x) dx \\ &= p \left(\frac{(\vartheta_1 + 1)\vartheta_1}{\rho_1^2} - \frac{2\mu\vartheta_1}{\rho_1} + \mu^2 \right) + \\ &\quad (1-p) \left(\frac{(\vartheta_2 + 1)\vartheta_2}{\rho_2^2} + \frac{2\mu\vartheta_2}{\rho_2} + \mu^2 \right), \end{aligned} \quad (2.4)$$

respectively [6].

We can see that the double exponential distribution (2.1) has three variables (p , ρ_1 and ρ_2), and the double gamma distribution has five (p , ρ_1 , ρ_2 , ϑ_1 and ϑ_2). The double exponential distribution can be seen to be a special case of the double gamma distribution because by setting the double gamma's ϑ_1 and ϑ_2 to 1, turning equation (2.2) into equation (2.1). For both distributions, the variable p determines the probability of upward jumps. With the extra degrees of freedom that the double gamma distribution has, it can create heavier tails, which captures greater extreme events to better fit the market. So

not only does the double gamma jump-diffusion (DGJD) model include Kou's model as a special case, but it also has the flexibility to better fit the market.



The DGJD model, the focus of this thesis, is a special case of Thul and Zhang's displaced double gamma jump-diffusion model [10]. The displacement is like a barrier between the positive and negative gamma distributions, and the DGJD model is their model with zero displacement. Thul and Zhang use the Esscher transform to obtain the closed-form solution for the option value. But their analytical solution is time-consuming to evaluate. In contrast, we use the Fast Fourier Transform to obtain the option values numerically, which is expressed in terms of a characteristic function (CF). Our method can reduce the computation time to milliseconds while maintaining high accuracy. The savings arise because their closed-form solution involves calculating a large amount of double summations whereas ours only requires one summation.





Chapter 3 Our Proposed Model and Numerical Methods

3.1 The jump-diffusion model

In this section we go through Kou's jump-diffusion model [6] and then extend it to incorporate the double gamma jump-size distribution.

3.1.1 Stochastic differential equation for the jump-diffusion model

In Kou's double exponential jump-diffusion model (KJD), S_t (stock price) is assumed to follow the following stochastic differential equation (SDE) [4, 9]

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t + dJ_t \quad (3.1)$$

where

$$J_t = \sum_{j=1}^{N_t} (Y_j - 1)$$

$$W_t \sim \text{Normal}(0, t)$$

$$N_t \sim \text{Poisson}(\lambda t)$$

Here W_t and N_t are assumed to be independent. In Kou's model, the logarithmic random jump size $\ln Y_j$ is assumed to follow an asymmetric double exponential distribution and is independent of W_t and N_t . So if x follows the double gamma distribution, then $Y_j = e^x$.

Process J_t is a compound Poisson process where Y_j are independent and identically distributed and the number of jumps N_t is a Poisson process with jump intensity λ . Although the jump part is independent of the Brownian motion, it contributes to the drift part of S_t and the size it contributes is

$$E[(Y_j - 1)dN] = E[Y_j - 1]E[dN_t] = k_d \lambda dt$$

Above k_d denotes $E[Y_j - 1]$. The d in k_d refers to the distribution, and can be either exp for the double exponential distribution or gamma for the double gamma distribution.

Now the SDE from equation (3.1) can be written as

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t + \prod_{j=1}^{dN_t} (Y_j - 1)$$

or, equivalently,

$$dS_t = \mu S_t dt + \sigma S_t dW_t + S_t \prod_{j=1}^{dN_t} (Y_j - 1) \quad (3.2)$$

Because when multiple jumps occurs, S_t turns into $S_t Y_1 Y_2 Y_3 \dots$, we can write dS_t as

$$dS_t = S_t Y_1 Y_2 Y_3 \dots - S_t = S_t \prod_{j=1}^{dN_t} Y_j - S_t = S_t \prod_{j=1}^{dN_t} (Y_j - 1)$$

Apply Itô's lemma to equation (3.2) to get

$$\begin{aligned} d\ln S_t &= \frac{1}{S_t} dS_t + \frac{1}{2} \frac{1}{S_t^2} dS_t^2 + \ln\left(\prod_{j=1}^{dN_t} S_t Y_j\right) - \ln(S_t) \\ &= \frac{1}{S_t} dS_t + \frac{1}{2} \frac{1}{S_t^2} dS_t^2 + \sum_{j=1}^{dN_t} \ln(Y_j) \end{aligned} \quad (3.3)$$



And since we know what dS_t/S_t looks like, we can insert it into equation (3.3) to obtain

$$d\ln S_t = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dW_t + \sum_{j=1}^{dN_t} \ln(Y_j) \quad (3.4)$$

Now integrate equation (3.4) on both sides from 0 to time t to obtain

$$\begin{aligned} \ln S_t - \ln S_0 &= \left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma(W_t - W_0) + \sum_{j=1}^{N_t - N_0} \ln(Y_j) \\ &= \left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma W_t + \sum_{j=1}^{N_t} \ln(Y_j) \end{aligned} \quad (3.5)$$

We can rewrite equation (3.5) in the following form

$$S_t = S_0 e^{v_t t + \sigma W_t + X_t}$$

or, equivalently,

$$S_t = S_0 e^{x_t}$$

where

$$x_t = v_t t + \sigma W_t + X_t \quad (3.6)$$

$$v_t = \left(\mu - \frac{1}{2}\sigma^2\right) \quad (3.7)$$

$$X_t = \sum_{j=1}^{N_t} \ln(Y_j)$$

and $\ln(Y_j)$ is the logarithmic jump size. In Kou's model $\ln(Y_j)$ follows the double expo-

nential distribution and in the DGJD model $\ln(Y_j)$ follows the double gamma distribution.



3.1.2 Martingale condition of jump-diffusion model

The return is

$$R_t = \ln \left(\frac{S_t}{S_0} \right) = v_t t + \sigma W_t + X_t \quad (3.8)$$

In a risk-neutral world

$$E^Q[S_t] = E^Q[S_0 e^{R_t}] = S_0 e^{(r-q)t} \quad (3.9)$$

where q is the continuous dividend of the underlying asset. Divide equation (3.9) by S_0 and substitute R_t with equation (3.8) to obtain

$$E^Q[e^{R_t}] = E^Q[e^{v_t t + \sigma W_t + X_t}] = e^{(r-q)t} \quad (3.10)$$

Expand equation (3.10) to yield

$$E^Q[e^{v_t t + \sigma W_t + X_t}] = E^Q[e^{v_t t}] E^Q[e^{\sigma W_t}] E^Q[e^{X_t}] = e^{(r-q)t} \quad (3.11)$$

Substitute v_t from equation (3.7) into equation (3.11) to obtain

$$E^Q[e^{v_t t}] E^Q[e^{\sigma W_t}] E^Q[e^{X_t}] = e^{(\mu - 0.5\sigma^2)t} E^Q[e^{\sigma W_t}] E^Q[e^{X_t}] = e^{(r-q)t} \quad (3.12)$$

It is well-known that

$$E^Q[e^{\sigma W_t}] = e^{\sigma^2 t / 2} \quad (3.13)$$

Substituting $E^Q[e^{\sigma W_t}]$ from equation (3.13) into equation (3.12) and taking the nat-

ural log of and dividing it by t , we have

$$\mu = (r - q) - \frac{\ln(E^Q[e^{X_t}])}{t} \quad (3.14)$$



On the other hand,

$$E^Q[e^{X_t}] = E^Q \left[\exp \left(\sum_{j=1}^{N_t} \ln(Y_j) \right) \right]$$

where $\ln(Y_j)$ is the logarithmic jump size, $\ln(Y_1), \ln(Y_2), \dots$ are independent random variables, and N_t is a non-negative, integer-valued random variable independent of $\ln(Y_j)$.

With Wald's lemma [13],

$$\begin{aligned} E^Q[e^{X_t}] &= E^Q \left[\exp \left(\sum_{j=1}^{N_t} \ln(Y_j) \right) \right] \\ &= \sum_{n \geq 0} E^Q \left[\exp \left(\sum_{j=1}^n \ln(Y_j) \right) \right] P(N_t = n) \\ &= e^{-\lambda t} \sum_{n \geq 0} \frac{\lambda^n}{n!} t^n E^Q \left[\exp \left(\sum_{j=1}^n \ln(Y_j) \right) \right] \\ &= e^{-\lambda t} \sum_{n \geq 0} \frac{\lambda^n}{n!} t^n \prod_{j=1}^n E^Q [e^{\ln(Y_j)}] \\ &= e^{-\lambda t} \sum_{n \geq 0} \frac{\lambda^n}{n!} t^n (E^Q [e^{\ln(Y_j)}])^n \\ &= e^{-\lambda t} \sum_{n \geq 0} \frac{(\lambda t E^Q [e^{\ln(Y_j)}])^n}{n!} \\ &= e^{-\lambda t} e^{\lambda t E^Q [e^{\ln(Y_j)}]} \\ &= \exp(\lambda t (E^Q [e^{\ln(Y_j)}] - 1)) \\ &= \exp(\lambda t (E^Q [Y_j] - 1)) \end{aligned}$$

(see [7]). We know that $k_d = E[Y_j - 1]$, hence

$$E^Q[e^{X_t}] = e^{\lambda t k_d} \quad (3.15)$$



Substituting $E^Q[e^{X_t}]$ from equation (3.15) into equation (3.14) and taking the natural log of and dividing it by t , we have

$$\mu = r - q - \lambda k_d \quad (3.16)$$

3.2 The Fast Fourier Transform (FFT)

3.2.1 Methodology

We will use a numerical approach to pricing options under the jump-diffusion model with the use of the CF. This method was introduced by Carr and Madan [1] and is based on the Fast Fourier Transform (FFT) [2, 8] for two reasons. First, the algorithm offers an advantage in speed. Second, the CF of the log price is known and has a simple form, while the density is usually not known in closed-form. This method assumes that the CF of the log price is provided analytically. The basic idea of the method is to develop analytical expressions of the CF of the jump-diffusion model and to obtain the option price using the inverse Fourier transformation. Let $C_T(k)$ denote the price of a European call option at maturity T , the share price $S_T = \exp(s)$ and strike $K = \exp(k)$:

$$C_T(k) = e^{-rT} E^Q[(S - K)^+] = \int_k^\infty e^{-rT} (e^s - e^k) q_T(s) ds$$

where q_T is the risk-neutral density of $s_T = \ln(S_T)$. Note that $C_T(k)$ tends to S_0 as k tends

to $-\infty$; hence $C_T(k)$ is not square-integrable. This is undesirable because the Fourier transform and its inversion work only for square-integrable functions. To obtain a square-integrable function, we consider the modified call price $c_T(k)$ defined by

$$c_T(k) = \exp(\alpha k) C_T(k)$$

Now $c_T(k)$ is square-integrable for a suitable $\alpha > 0$ as shown by Carr and Madan [1]. The Fourier transform of c_T is

$$\begin{aligned} \psi_T(v) &= \int_{-\infty}^{\infty} e^{ivk} c_T(k) dk \\ &= \int_{-\infty}^{\infty} e^{ivk} \int_k^{\infty} e^{\alpha k} e^{-rT} (e^s - e^k) q_T(s) ds dk \\ &= \int_{-\infty}^{\infty} e^{-rT} q_T(s) ds \int_{-\infty}^s (e^{\alpha k+s} - e^{(\alpha+1)k}) e^{ivk} dk \\ &= \int_{-\infty}^{\infty} e^{-rT} q_T(s) \left(\frac{e^{(\alpha+1+iv)s}}{\alpha+iv} - \frac{e^{(\alpha+1+iv)s}}{\alpha+1+iv} \right) ds \\ &= \frac{e^{-rT} \Phi_x(v - ((\alpha+1)i))}{(\alpha+iv)(\alpha+1+iv)} \end{aligned}$$

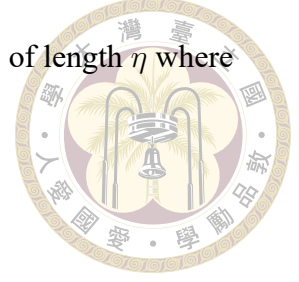
as shown by Carr and Madan [1]. Now, we get the option price in terms of ψ_T using the inverse Fourier transformation:

$$\begin{aligned} C_T(k) &= \exp(-\alpha k) c_T(k) \\ &= \frac{\exp(-\alpha k)}{2\pi} \int_{-\infty}^{\infty} e^{-ivk} \psi_T(v) dv \\ &\approx \frac{\exp(-\alpha k)}{\pi} \int_0^B e^{-ivk} \psi_T(v) dv \end{aligned}$$

where B is some suitably big number [1]. Set N equals sub-intervals of length η where

$$\eta = B/N$$

$$v_j = \eta(j-1), \text{ for } j = 1, \dots, N+1$$



We will approximate the call value as

$$C_T(k) \approx \frac{\exp(-\alpha k)}{\pi} \sum_{j=1}^N \int_{v_j}^{v_{j+1}} e^{-ivk} \psi_T(v) dv \quad (3.17)$$

Use the trapezoidal rule to approximate each sub-integral in equation (3.17) as follows

$$C_T(k) \approx \text{Re} \left[\frac{\exp(-\alpha k)}{\pi} \sum_{j=1}^N e^{-iv_j k} \psi_T(v_j) w_j \right] \quad (3.18)$$

where

$$w_j = \begin{cases} \eta/2 & \text{if } j = 1 \\ \eta & \text{otherwise} \end{cases}$$

This is the call option value $C_T(k)$ with a strike whose logarithm is k .

3.2.2 The CF of the DGJD model

The CF for a function $f(x)$ is

$$\Phi_f(v) = \int_{-\infty}^{\infty} e^{ivx} f(x) dx = E[e^{ivf(x)}]$$

Recall that N_t is Poisson distributed with

$$P[N_t = n] = e^{-\lambda t} \frac{(\lambda t)^n}{n!}$$

If $x_t = \ln(S_t/S_0)$ in equation (3.6) is a Lévy processes, then its CF can be written as $\Phi_x(v) = E[e^{ivx_t}]$ and has the Lévy-Khintchine representation:



$$\begin{aligned}\Phi_x(v) &= E[e^{ivx_t}] \\ &= \exp \left[ivv_t t - \frac{v^2 \sigma^2 t}{2} + \lambda t \int [e^{ivx} - 1] p(x) dx \right] \\ &= \exp \left[ivv_t t - \frac{v^2 \sigma^2 t}{2} + \lambda t (\phi_{\text{Jump}}(v) - 1) \right]\end{aligned}\quad (3.19)$$

where $p(x)$ is the PDF of the logarithmic jump-size distribution in equation (2.2) and $\phi_{\text{Jump}}(v)$ is the CF of $p(x)$ [3, 9]. In Kou's model, $p(x)$ follows the double exponential distribution whereas in the DGJD model, $p(x)$ follows the double gamma distribution.

Recall the DGJD model's logarithmic jump size $\ln(Y_j)$ follows the double gamma distribution. Now we will derive the double gamma distribution's PDF, MGF, k_d , CF and finally the DGJD model's CF [10, 12]. The MGF of the double gamma distribution is

$$E[e^{tv}] = p \left(1 - \frac{t}{\rho_1} \right)^{-\vartheta_1} + (1-p) \left(1 + \frac{t}{\rho_2} \right)^{-\vartheta_2}$$

The k_{gamma} for the double gamma distribution is thus

$$\begin{aligned}k_{\text{gamma}} &= E^Q[e^{\ln Y}] - 1 = E[e^v] - 1 \\ &= p \left(1 - \frac{1}{\rho_1} \right)^{-\vartheta_1} + (1-p) \left(1 + \frac{1}{\rho_2} \right)^{-\vartheta_2} - 1\end{aligned}$$

and the CF for the jump-size distribution is

$$\phi_{\text{Jump}}(v) = p \left(1 - \frac{iv}{\rho_1} \right)^{-\vartheta_1} + (1-p) \left(1 + \frac{iv}{\rho_2} \right)^{-\vartheta_2}\quad (3.20)$$

By inserting equations (3.7), (3.14)–(3.16) and (3.20) into equation (3.19) we can obtain the DGJD model's CF as

$$\Phi_x(v) = \exp \left[iv \left(r - q - \lambda k_{\text{gamma}} - \frac{1}{2} \sigma^2 \right) t - \frac{v^2 \sigma^2 t}{2} \right] \exp \left[\lambda t \left(p \left(1 - \frac{iv}{\rho_1} \right)^{-\vartheta_1} + (1-p) \left(1 + \frac{iv}{\rho_2} \right)^{-\vartheta_2} - 1 \right) \right]$$



The limitation factors for the DGJD model are

$$\rho_1 > 1, \rho_2 > 0$$

$$\vartheta_1 > 0, \vartheta_2 > 0$$

3.3 Moments of the jump-size distribution

We now obtain the moments of the double gamma distribution in terms of $p, \rho_1, \rho_2, \vartheta_1$ and ϑ_2 [10, 12]. Recall the PDF, mean, and variance of the double gamma distribution in equations (2.2), (2.3) and (2.4). The double gamma distribution's skewness equals

$$\begin{aligned} \text{skewness} &= \int_{-\infty}^{\infty} [x - \mu]^3 p(x) dx \\ &= p \left(\frac{(\vartheta_1 + 2)(\vartheta_1 + 1)\vartheta_1}{\rho_1^3} - \frac{3\mu(\vartheta_1 + 1)\vartheta_1}{\rho_1^2} + \frac{3\mu^2\vartheta_1}{\rho_1} - \mu^3 \right) + \\ &\quad (1-p) \left(\frac{-(\vartheta_2 + 2)(\vartheta_2 + 1)\vartheta_2}{\rho_2^3} - \frac{3\mu(\vartheta_2 + 1)\vartheta_2}{\rho_2^2} - \frac{3\mu^2\vartheta_2}{\rho_2} - \mu^3 \right) \end{aligned} \quad (3.21)$$

The double gamma distribution's kurtosis equals

$$\begin{aligned} \text{kurtosis} &= \int_{-\infty}^{\infty} [x - \mu]^4 p(x) dx \\ &= p \left(\frac{(\vartheta_1 + 3)(\vartheta_1 + 2)(\vartheta_1 + 1)\vartheta_1}{\rho_1^4} - \frac{4\mu(\vartheta_1 + 2)(\vartheta_1 + 1)\vartheta_1}{\rho_1^3} + \frac{6\mu^2(\vartheta_1 + 1)\vartheta_1}{\rho_1^2} - \frac{4\mu^3\vartheta_1}{\rho_1} + \mu^4 \right) + \\ &\quad (1-p) \left(\frac{(\vartheta_1 + 3)(\vartheta_1 + 2)(\vartheta_1 + 1)\vartheta_1}{\rho_1^4} + \frac{4\mu(\vartheta_1 + 2)(\vartheta_1 + 1)\vartheta_1}{\rho_1^3} + \frac{6\mu^2(\vartheta_1 + 1)\vartheta_1}{\rho_1^2} + \frac{4\mu^3\vartheta_1}{\rho_1} + \mu^4 \right) \end{aligned} \quad (3.22)$$



Chapter 4 Numerical Analysis Results

4.1 Sanity test

First, we will compare our results with the results in Toivanen [11] to make sure that the FFT method is correct. Toivanen priced the European call options under Kou's model with the following variables and option specifics

$$K = 100, \sigma = 0.15, r = 0.05, q = 0, T = 0.25$$

$$\lambda = 0.1, \rho_1 = 3.0465, \rho_2 = 3.0775, p = 0.3445$$

The variables K , σ , r , q and T will be fixed throughout the experiments. Jump intensity λ will also be fixed throughout the experiments while the other variables from the jump distribution part will be varied. For the DGJD model to reproduce Kou's model we also set

$$\vartheta_1 = 1, \vartheta_2 = 1$$

We apply the trapezoidal scheme to evaluate equation (3.18) with step size $\eta = 0.01$, number of summations $N = 2^{14}$ and $B = \eta N$. Table 4.1 shows our FFT pricing can reproduce Toivanen's results.

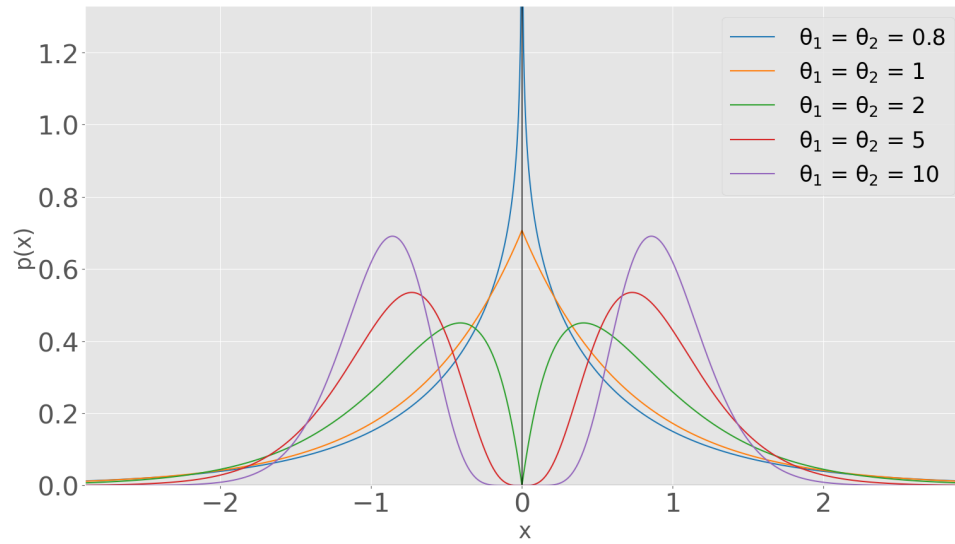
Table 4.1: Comparing our results with Toivanen's results

Stock price	Toivanen's results	Our results	Error
90	0.672677	0.672677331	3.31E - 07
100	3.973479	3.973478849	1.5E - 07
110	11.794583	11.79458299	9.74E - 09

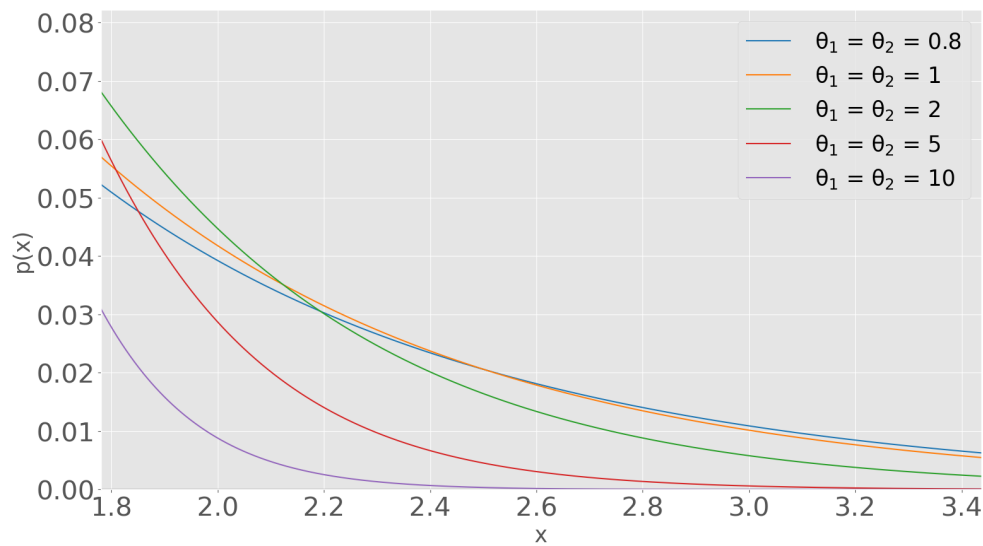
4.2 Influences of the variables on the double gamma distribution

The variables and the option specifics $K = 100$, $\sigma = 0.3$, $r = 0.05$, $q = 0.01$ and $T = 0.25$ are from Schmelzle [8]. The Poisson mean number of events $\lambda = 0.1$ is from Toivanen [11].

In this section, we will demonstrate the influences of the variables (p , ρ_1 , ρ_2 , ϑ_1 and ϑ_2) on the double gamma (DG) distribution. In Figure 4.1 we can see that with a fixed mean and variance, when we increase ϑ_1 and ϑ_2 in lockstep, the distributions will spread out and both the left and right tails will become thinner. The orange curve with $\vartheta_1 = \vartheta_2 = 1$ plots the double exponential (DE) distribution. Figure 4.2 plots the distributions with a fixed mean and variance when ϑ_2 alone is increased. Note that the distributions where $x < 0$ shift to the left and the left tails become thinner. As the tails on the right become slightly heavier and the tails on the left become slightly thinner, more extreme positive events will occur while fewer extreme negative events will occur. Positive events are events that lead stock prices to jump up causing call value to increase and negative events are events that lead stock prices to jump downward causing call value to decrease. So having a heavier tail on the right increases call value and having a heavier tail on the left decreases call value.

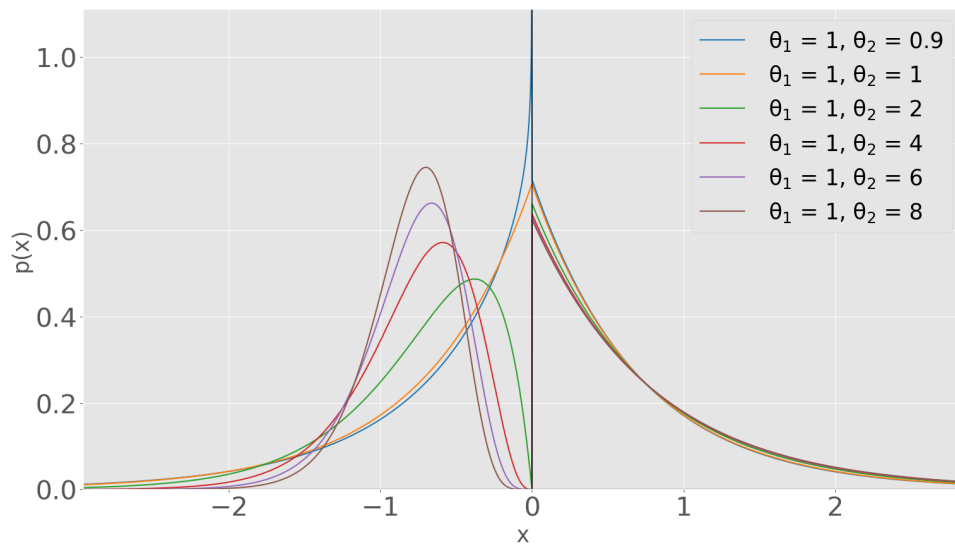


(a) Overall

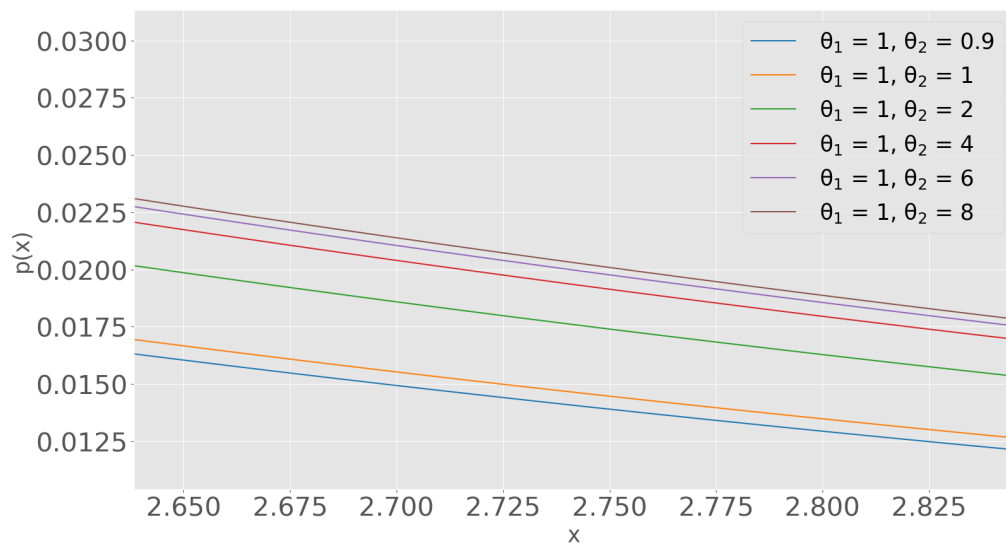


(b) Right tail

Figure 4.1: Influences of the identically set variables ϑ_1 and ϑ_2 on the DG distribution with mean 0 and variance 1



(a) Overall



(b) Right tail

Figure 4.2: Influences of variable ϑ_2 on the DG distribution with mean 0 and variance 1

4.3 Influences of the DG distribution on call prices



This section analyzes the numerical phenomenon of how changes in variables impact the call option values under the DGJD model and compares them with Kou's model.

4.3.1 Results with fixed moments as variables are varied

Table 4.2 shows that by setting the variables $\vartheta_1 = \vartheta_2 = 1$, the FFT method produces option prices agreeing with Kou's model.

The mean, variance, skewness and kurtosis for the DG distribution are given by, respectively, equations (2.3), (2.4), (3.21) and (3.22). The variables $\rho_1, \rho_2, \vartheta_1, \vartheta_2$ and p all have impact on the moments of the DG distribution; therefore, as we change a set of variables, for example ϑ_1 and ϑ_2 , the other variables ρ_1, ρ_2 and p must also change if mean and variance are to remain unchanged.

From Table 4.2, we also see that, when mean and variance are fixed, as p increases, the call value decreases under both models. This may sound counter-intuitive: As p stands for the likelihood of jumps, should not increasing it lead to a higher call value instead? Observe two DG distributions both with mean 0 and variance 1, one distribution with $p = 0.4$ and skewness 0.8660 and the other with $p = 0.6$ and skewness -0.8660 , while both having the same kurtosis of 7. As the only difference in moments is the skewness, it suggests that the difference in the call value is due to the difference in skewness: When mean, variance and kurtosis are fixed, the larger the skewness, the higher the call value. Figure 4.3 shows the distribution of these two scenarios above: $p = 0.4$ and $p = 0.6$. We can see that for $x > 0$, $p = 0.6$ has a larger area under the distribution than $p = 0.4$, but

$p = 0.4$ with mean, variance and kurtosis fixed has a heavier tail on the right. Having a heavier tail on the right means that extreme positive events are more likely to occur, leading to higher call values. On the other hand, $p = 0.6$ has a comparatively heavier tail on the left, meaning that extreme negative events are more likely to occur, leading to lower call values. We can conclude that having a heavier tail on the right leads to a higher call value and having a heavier tail on the left leads to a smaller call value, vice versa.

Table 4.2: Call values with mean 0 and variance 1

variables			moments		call values		
p	ρ_1	ρ_2	skewness	kurtosis	$S_0 = 90$	$S_0 = 100$	$S_0 = 110$
Kou's model; mean = 0, variance = 1							
0.40	1.1547	1.7320	0.8660	7	6.5503	9.8194	15.38
0.45	1.2792	1.5634	0.42640	6.2424	4.87	8.5062	14.5573
0.50	1.4142	1.4142	0	6	4.1806	7.9886	14.2468
0.55	1.5634	1.2792	-0.42640	6.2424	3.786	7.6961	14.0723
0.60	1.7320	1.1547	-0.8660	7	3.5162	7.4959	13.9509
0.65	1.9272	1.0377	-1.3342	8.3736	3.3091	7.3406	13.8538
DGJD model; mean = 0, variance = 1, $\vartheta_1 = \vartheta_2 = 1$							
0.40	1.1547	1.7320	0.8660	7	6.5503	9.8194	15.38
0.45	1.2792	1.5634	0.42640	6.2424	4.87	8.5062	14.5573
0.50	1.4142	1.4142	0	6	4.1806	7.9886	14.2468
0.55	1.5634	1.2792	-0.42640	6.2424	3.786	7.6961	14.0723
0.60	1.7320	1.1547	-0.8660	7	3.5162	7.4959	13.9509
0.65	1.9272	1.0377	-1.3342	8.3736	3.3091	7.3406	13.8538

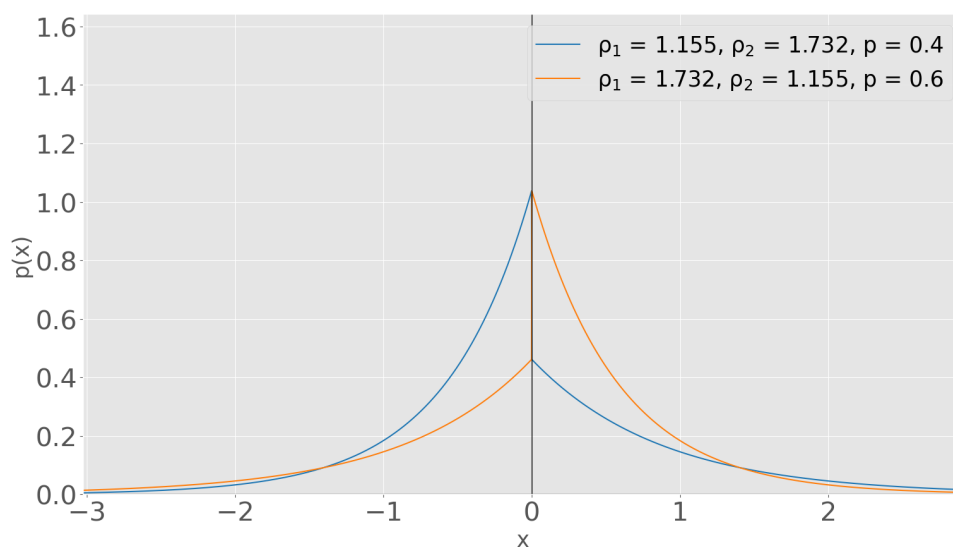


Figure 4.3: The DG distribution with mean 0 and variance 1 but ρ_1 , ρ_2 and p varied

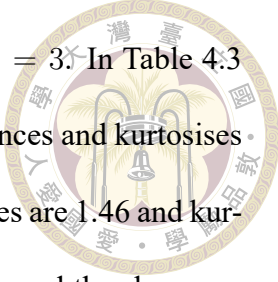


Table 4.3 is divided into two sections $\rho_1 = \rho_2 = 2$ and $\rho_1 = \rho_2 = 3$. In Table 4.3 we look at the first and third rows of all two sections where their variances and kurtosis agree; for example, in the first section, the first and third rows' variances are 1.46 and kurtosis are 7.3752. As p increases from 0.4 to 0.6, the mean increases and the skewness decreases, and the call value increases. But the previous paragraph said when mean, variance and kurtosis are fixed, the larger the skewness the higher the call value. Hence it seems that the mean has a larger impact on call value than skewness. From Figure 4.4 we can see that $p = 0.6$ (orange) results in a larger area under the distribution for $x > 0$ than $p = 0.4$ (blue) and has a heavier tail on the right, leading to increased call value. Now, in Table 4.3 we look at the second rows of both sections where their means and skewness both are 0 and $p = 0.5$. As ρ_1 and ρ_2 increase from 2 to 3, variance and kurtosis decrease. From Figure 4.5 we can see that when mean and skewness are fixed, as variance and kurtosis decrease (blue to orange), the distribution's left and right tails become less heavy by the same amount because of symmetry and the probability mass is more centered around $x = 0$, causing the call value to decrease.

Table 4.3: Call values with different means and variances

variables			moments				call values		
p	ϑ_1	ϑ_2	mean	variance	skewness	kurtosis	$S_0 = 90$	$S_0 = 100$	$S_0 = 110$
DGJD model; $\rho_1 = \rho_2 = 2$									
0.4	2	2	-0.2	1.46	0.284	7.3752	4.2397	8.119	14.4478
0.5	2	2	0.0	1.5	0.0	7.5	4.6802	8.4185	14.5783
0.6	2	2	0.2	1.46	-0.284	7.3752	5.1276	8.7323	14.7234
DGJD model; $\rho_1 = \rho_2 = 3$									
0.4	2	2	-0.1333	0.6488	0.0841	1.4568	3.0692	7.2142	13.8431
0.5	2	2	0.0	0.6666	0.0	1.4814	3.221	7.2994	13.8561
0.6	2	2	0.1333	0.6488	-0.0841	1.4568	3.3744	7.3876	13.8717

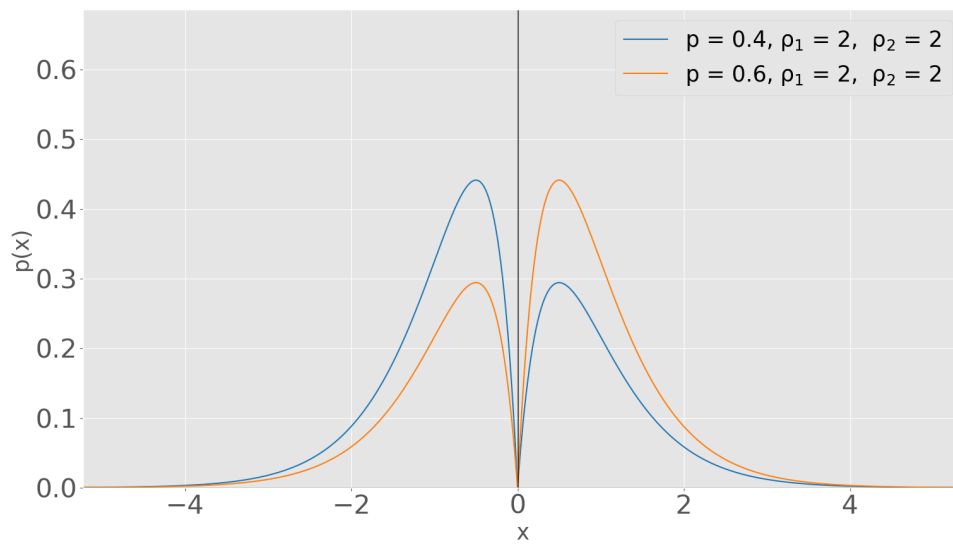


Figure 4.4: The DG distribution with fixed variance and kurtosis but p varied

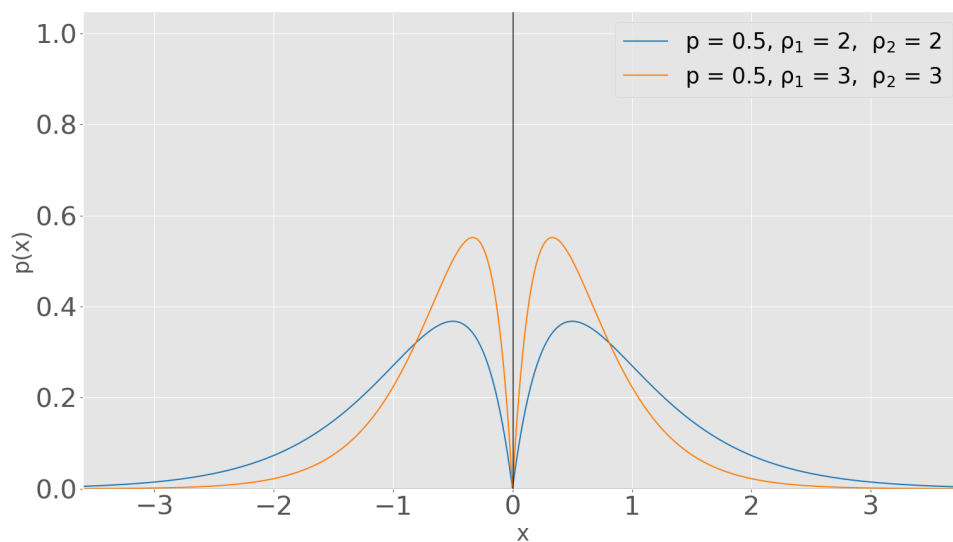


Figure 4.5: The DG distribution with mean 0 and skewness 0 but ρ_1 and ρ_2 varied

Table 4.4 is divided into three sections, $\vartheta_1 = \vartheta_2 = 0.8$, $\vartheta_1 = \vartheta_2 = 1$ and $\vartheta_1 = \vartheta_2 = 2$. In Table 4.4 we look at the same rows of all three sections where their means and variances agree; for example, for the second rows of all three sections, all three means are 0 and variances are 1. As ϑ_1 and ϑ_2 both increase from 0.8 to 2, variables ρ_1 and ρ_2 must also increase to keep mean and variance fixed, leading to increased skewness and decreased kurtosis. From Figures 4.6, 4.7 and 4.8 we can see that when mean and variance are fixed, as skewness increases and kurtosis decreases (blue to orange to green), the distribution's left and right tails become less heavy but the left is a lot less heavy than the right, causing the call value to increase.

Table 4.4: Call values when mean and variance are fixed but $\vartheta_1, \vartheta_2, \rho_1, \rho_2$ are varied

variables		moments				call values		
ρ_1	ρ_2	mean	variance	skewness	kurtosis	$S_0 = 90$	$S_0 = 100$	$S_0 = 110$
DGJD model; $\vartheta_1 = \vartheta_2 = 0.8, p = 0.8$								
3.3941	0.8485	0	0.5	-1.2374	6.0034	2.5681	6.756	13.4529
2.4	0.6	0	1.0	-3.45	24.0138	2.8382	6.9643	13.5895
1.9595	0.4898	0	1.5	-6.4299	54.0312	3.1322	7.1836	13.7244
DGJD model; $\vartheta_1 = \vartheta_2 = 1, p = 0.8$								
4	1	0	0.5	-1.125	4.875	2.5758	6.7691	13.4687
2.8284	0.7071	0	1.0	-3.1819	19.50	2.8413	6.9757	13.6061
2.3094	0.5773	0	1.5	-5.8456	43.875	3.1164	7.1822	13.7347
DGJD model; $\vartheta_1 = \vartheta_2 = 2, p = 0.8$								
6.9282	1.7320	0	0.5	-0.8660	2.7083	2.5998	6.8085	13.5152
4.8989	1.2247	0	1.0	-2.4494	10.8333	2.8655	7.0187	13.6592
4.0	1.0	0	1.5	-4.5	24.375	3.1187	7.2108	13.781

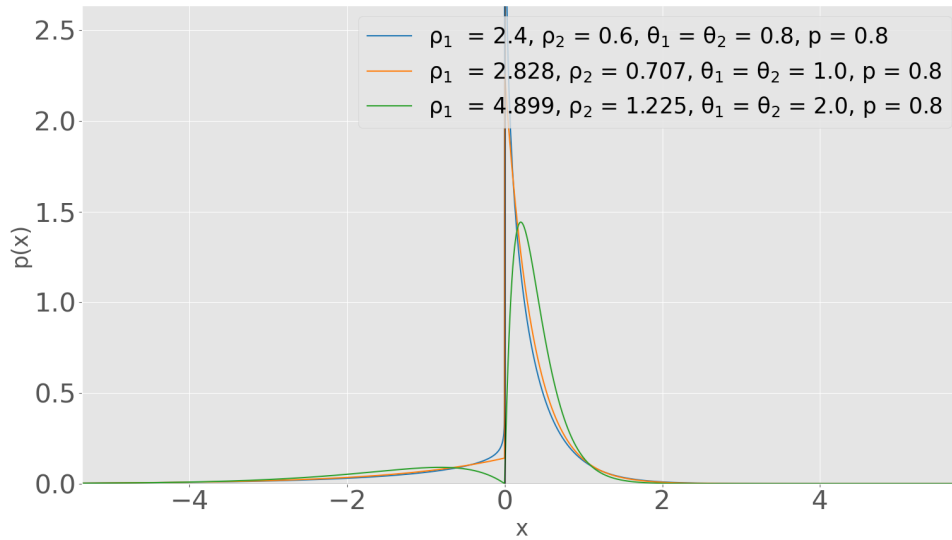


Figure 4.6: The DG distribution with mean 0, variance 1 and $p = 0.8$ but $\vartheta_1, \vartheta_2, \rho_1, \rho_2$ are varied

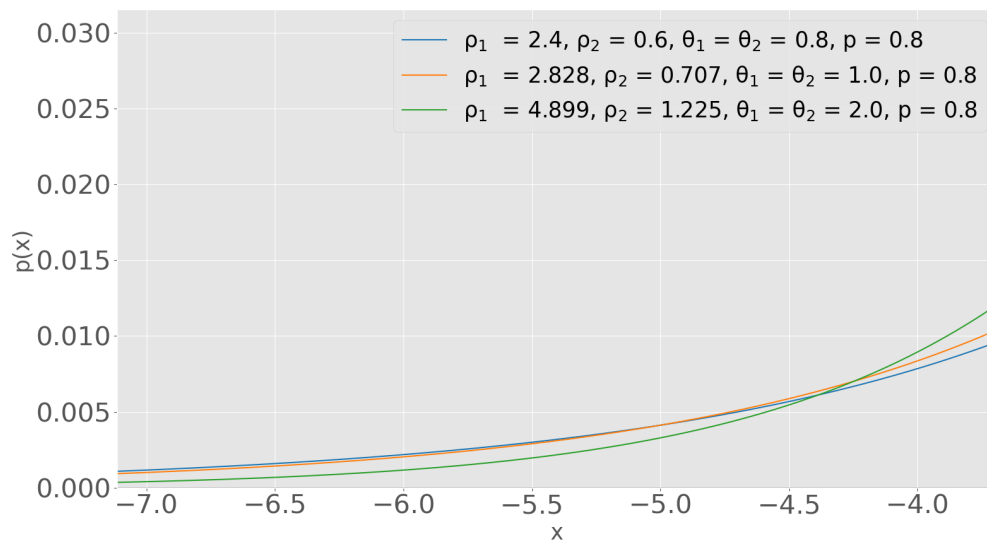


Figure 4.7: The DG distribution's left tail with mean 0, variance 1 and $p = 0.8$ but $\vartheta_1, \vartheta_2, \rho_1, \rho_2$ are varied

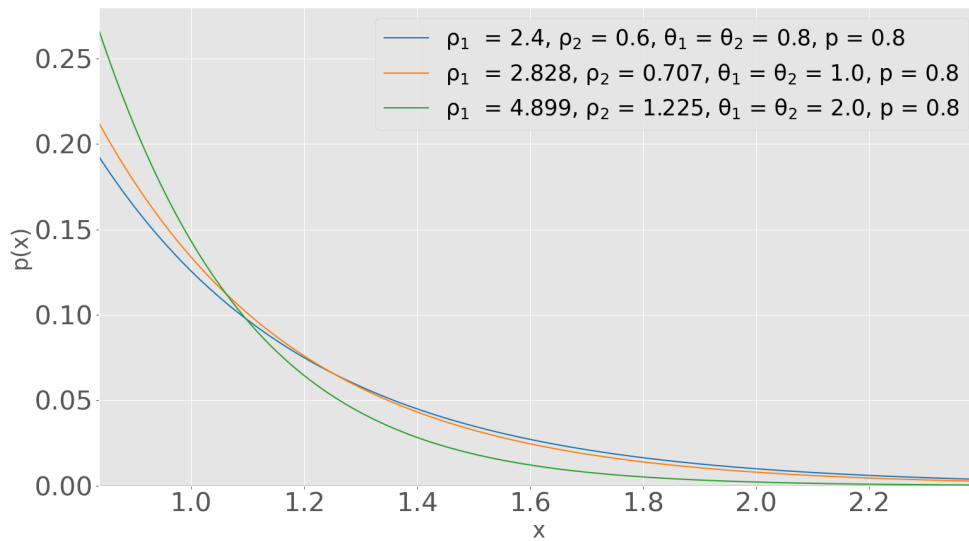


Figure 4.8: The DG distribution's right tail with mean 0, variance 1 and $p = 0.8$ but $\vartheta_1, \vartheta_2, \rho_1, \rho_2$ are varied

Table 4.5 is divided into two sections $\rho_1 = \rho_2 = 3$ and $\rho_1 = \rho_2 = 4$. In each section we can see that when mean is fixed, as variables ϑ_1 and ϑ_2 increase, the distribution's variance, skewness and kurtosis increase. From Figure 4.9 we can see that when mean is fixed, as variance, skewness and kurtosis increase (blue to orange), the distribution's left and right tails become heavier but the right is a lot heavier than the left, causing call value to increase. Next we look at the same rows of both sections where their means and variances agree; for example, for the second rows of both sections, both means are 0.5 and variances are 1. As ρ_1 and ρ_2 both increase from 3 to 4, variables ϑ_1 and ϑ_2 must also increase to keep mean and variance fixed, leading to decreased skewness and kurtosis. From Figure 4.10 we can see that when mean and variance are fixed, as skewness and kurtosis decrease (blue to orange), the distribution's right tail becomes less heavy and the left tail becomes heavier, causing the call value to decrease.

Table 4.5: Call values when mean and variance are fixed but variables are varied (2)

variables		moments				call values		
ϑ_1	ϑ_2	mean	variance	skewness	kurtosis	$S_0 = 90$	$S_0 = 100$	$S_0 = 110$
DGJD model; $\rho_1 = \rho_2 = 3, p = 0.5$								
3.1794	0.1794	0.5	0.5	0.3910	1.0398	4.2577	7.9417	14.0639
4.0413	1.0413	0.5	1.0	0.5346	2.8795	5.5782	9.0111	14.7914
4.7081	1.7081	0.5	1.5	0.6457	5.4720	6.9797	10.1730	15.5982
DGJD model; $\rho_1 = \rho_2 = 4, p = 0.5$								
4.3722	0.3722	0.5	0.5	0.2849	0.7809	4.1665	7.8786	14.0320
5.5311	1.5311	0.5	1.0	0.3935	2.3046	5.3791	8.8632	14.7091
6.4244	2.4244	0.5	1.5	0.4772	4.5185	6.6384	9.9033	15.4331

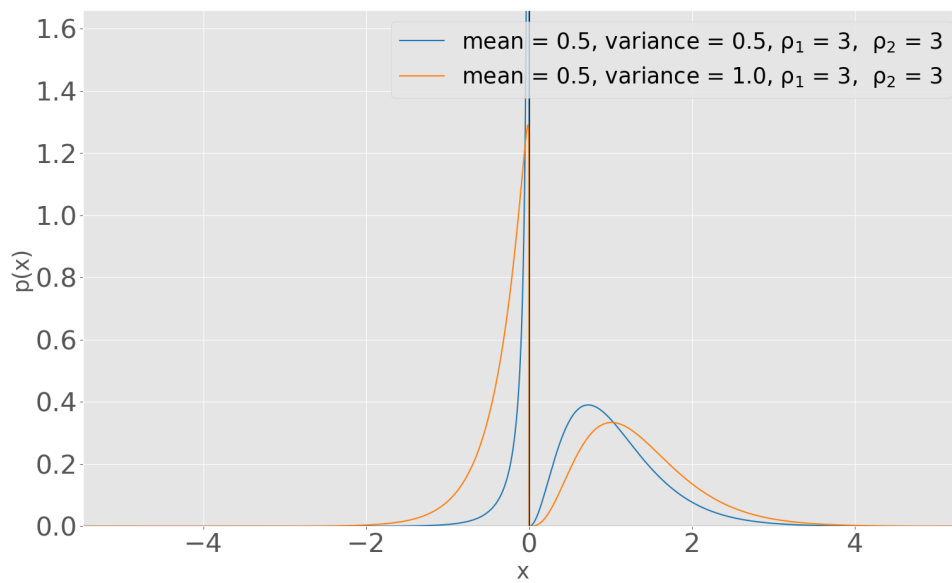


Figure 4.9: The DG distribution with fixed mean, $p = 0.5$ and $\rho_1 = \rho_2 = 3$ but variance is varied

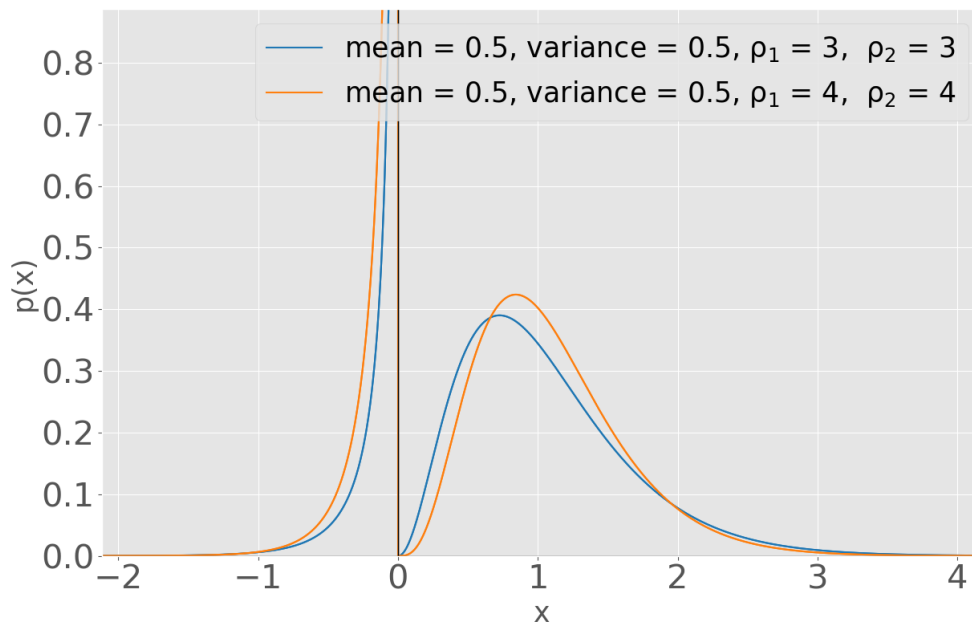


Figure 4.10: The DG distribution with fixed mean, variance and $p = 0.5$ but ρ_1, ρ_2 are varied

Table 4.6 is divided into two sections $p = 0.75$ and $p = 0.8$. In each section we can see that when mean is fixed, as variables ϑ_1 and ϑ_2 increase, the distribution's variance and kurtosis increase while skewness decreases. From Figure 4.11 we can see that when mean is fixed, as both variance and kurtosis increase and skewness decreases, the distribution's right and left tails become heavier but the right is a lot heavier than the left, causing call value to increase. Next we look at the same rows of both sections where their means and variances agree; for example, for the second rows of both sections, both means are 0.5 and variances are 1. As p increases from 0.75 to 0.8, variables ρ_1 and ρ_2 must also decrease to keep mean and variance fixed, leading to decreased skewness and increased kurtosis. From Figure 4.12 we can see that when mean and variance are fixed, as skewness decreases and kurtosis increases (blue to orange), the distribution's right tail becomes less heavy and the left tail becomes heavier, causing call value to decrease.

Table 4.6: Call values when mean and variance are fixed but variables are varied (3)

variables		moments				call values		
ρ_1	ρ_2	mean	variance	skewness	kurtosis	$S_0 = 90$	$S_0 = 100$	$S_0 = 110$
DGJD model; $\vartheta_1 = 2, \vartheta_2 = 2, p = 0.75$								
2.5358	5.4641	0.5	0.5	0.19198	1.0134	4.1723	7.917	14.0884
2.1254	2.4305	0.5	1.0	-0.1683	4.0435	5.2513	8.7807	14.6717
1.8997	1.7266	0.5	1.5	-0.9152	9.8028	6.4028	9.7104	15.2891
DGJD model; $\vartheta_1 = 2, \vartheta_2 = 2, p = 0.8$								
2.6666	4.0000	0.5	0.5	0.0624	1.0546	4.0831	7.85688	14.06057
2.2678	1.9464	0.5	1.0	-0.6298	4.9983	4.95397	8.54935	14.52481
2.0434	1.4135	0.5	1.5	-1.8243	12.8523	5.82257	9.24174	14.97784

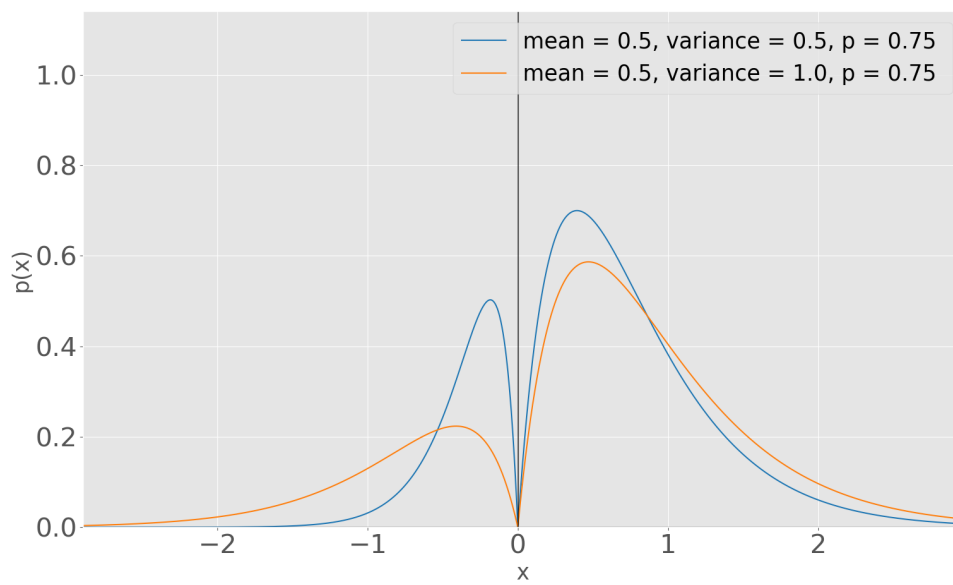


Figure 4.11: The DG distribution with fixed mean and $p = 0.5$ but variance is varied

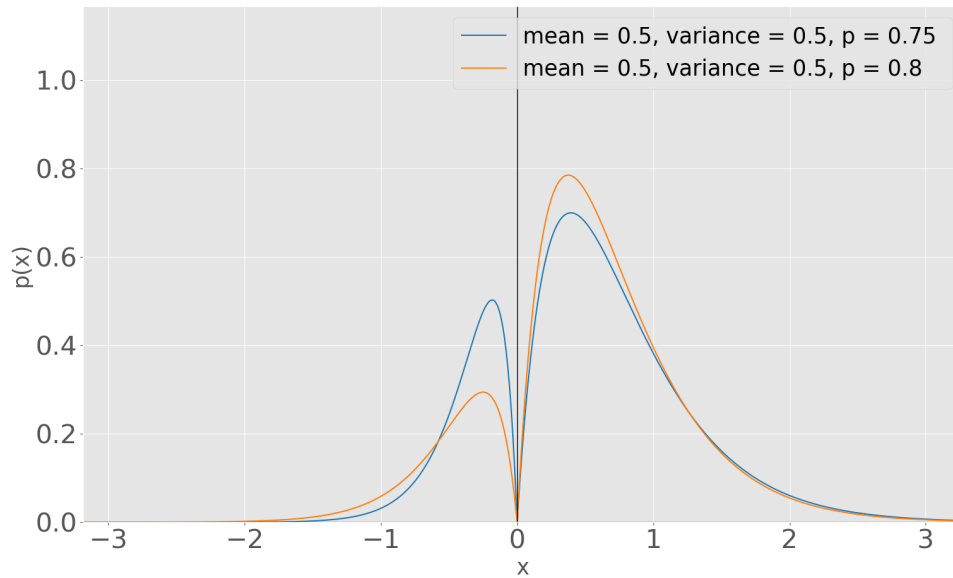


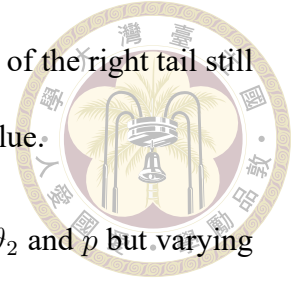
Figure 4.12: The DG distribution with fixed mean and variance but ρ_1 , ρ_2 and p are varied

From the discussions above, we can see that different combinations of variables ρ_1 , ρ_2 , ϑ_1 , ϑ_2 and p yield different DG distribution moments, thus causing the thickness of the distribution's tails and shape of the distribution to change, and these changes cause call values to change. Having a larger area under the distribution where $x > 0$, a heavier tail on the right and a thinner tail on the left makes extreme positive events more likely to occur, and extreme negative events less likely to occur. Recall that positive events are those that lead stock prices to jump up and negative events are events that lead stock prices to jump down. The former benefits the call value; the latter is detrimental.

4.3.2 Results with fixed ϑ_1 , ϑ_2 and p but varying ρ_1 and ρ_2

In Subsection 4.3.1 most of the DG distributions were not symmetric. In Subsections 4.3.2, 4.3.3 and 4.3.4, most of the DG distributions will be symmetric. When this is the case, the changes in the thickness of the left and right tails will be canceled out in the distribution. But the payoff of the call implies the upward movements impact the call

value more than the downward ones. So the increase in the thickness of the right tail still affects call value. The value S_0 relative to K also impacts the call value.



In this subsection, we will look at the call value with fixed ϑ_1, ϑ_2 and p but varying ρ_1 and ρ_2 . Figure 4.13 shows that when variables $\vartheta_1 = \vartheta_2 = 2$ and $p = 0.5$ are fixed, as ρ_1 and ρ_2 both increase from 1.4 to 3 (blue to orange to green), the distribution's probability mass is more centered around $x = 0$ while the right tail becomes less heavy, causing the call value to decrease as shown in Figure 4.14 (blue to orange to green). Figure 4.13 also shows that when variables $\vartheta_1 = \vartheta_2 = 2$ and $\rho_1 = \rho_2 = 3$ are fixed, as p increases from 0.5 to 0.9 (green to red), the distribution's right tail becomes heavier and the left tail becomes less heavy while the probability mass greatly shifts to positive x , causing the call value to increase as shown in Figure 4.14 (green to red).

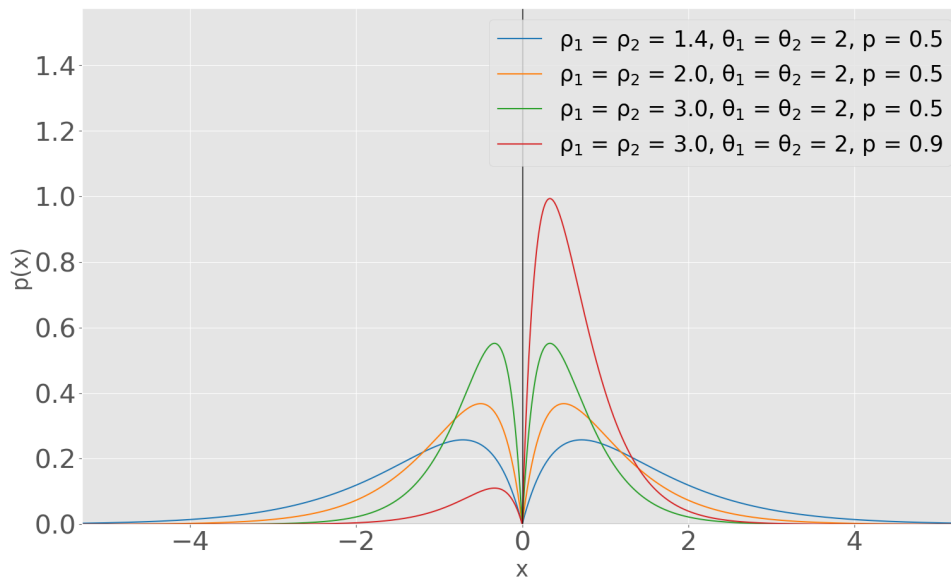
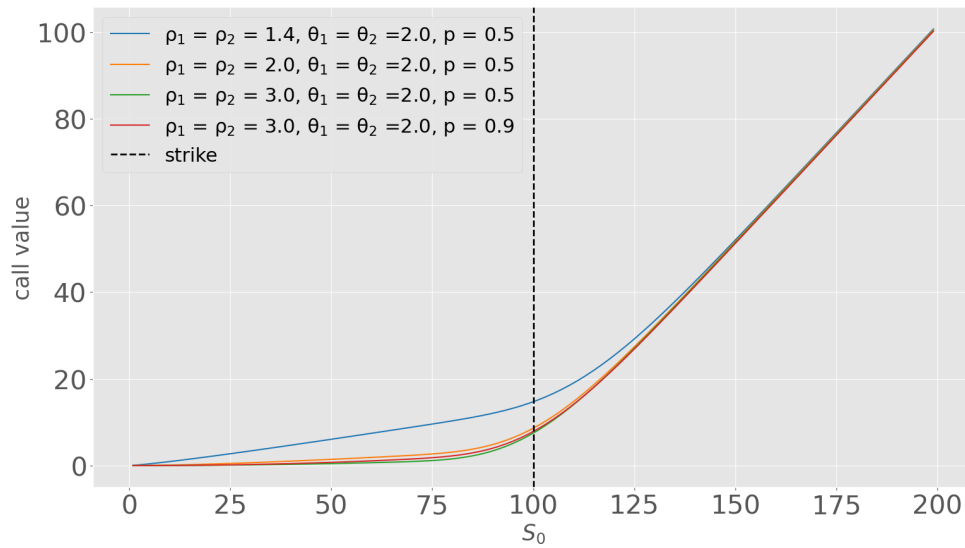
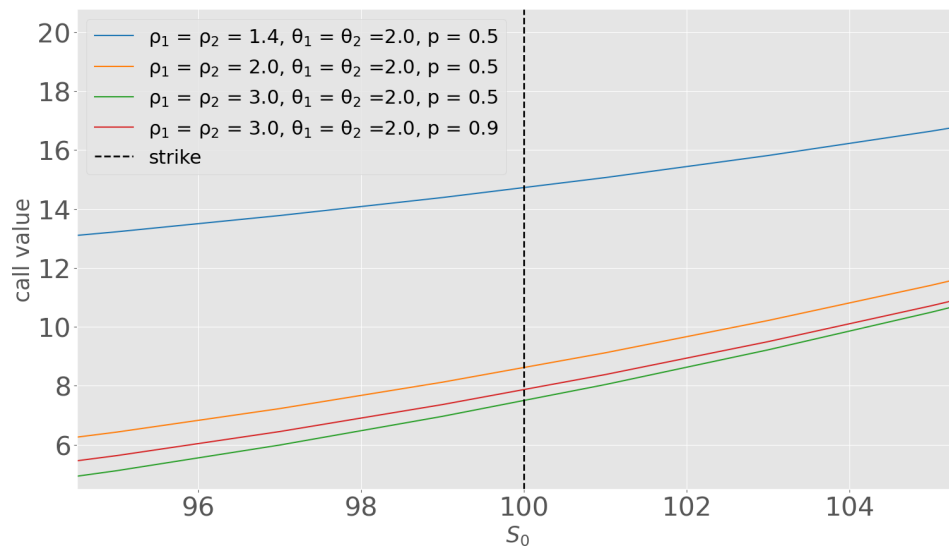


Figure 4.13: The DG distribution with fixed ϑ_1, ϑ_2 and p but ρ_1 and ρ_2 are varying



(a) Overall



(b) Magnified

Figure 4.14: Call values with fixed ϑ_1, ϑ_2 and p but ρ_1 and ρ_2 are varying

4.3.3 Results with fixed ρ_1, ρ_2 and p but varying ϑ_1 and ϑ_2

In this subsection, we will look at the call value with fixed ρ_1, ρ_2 and p but varying ϑ_1 and ϑ_2 . Figure 4.15 shows that when variables $\rho_1 = \rho_2 = 2$ and $p = 0.5$ are fixed, as ϑ_1 and ϑ_2 both increase from 1.4 to 3 (blue to orange to green), the distribution's probability mass is less centered around $x = 0$ while the right tail becomes heavier, causing the call value to increase as shown in Figure 4.16 (blue to orange to green). Figure 4.15 also shows that when variables $\vartheta_1 = \vartheta_2 = 3$ and $p = 0.5$ are fixed, as ρ_1 and ρ_2 both increase from 2 to 3 (green to red), the distribution's probability mass is more centered around $x = 0$ while the right tail becomes less heavy, causing the call value to decrease as shown in Figure 4.16 (green to red).

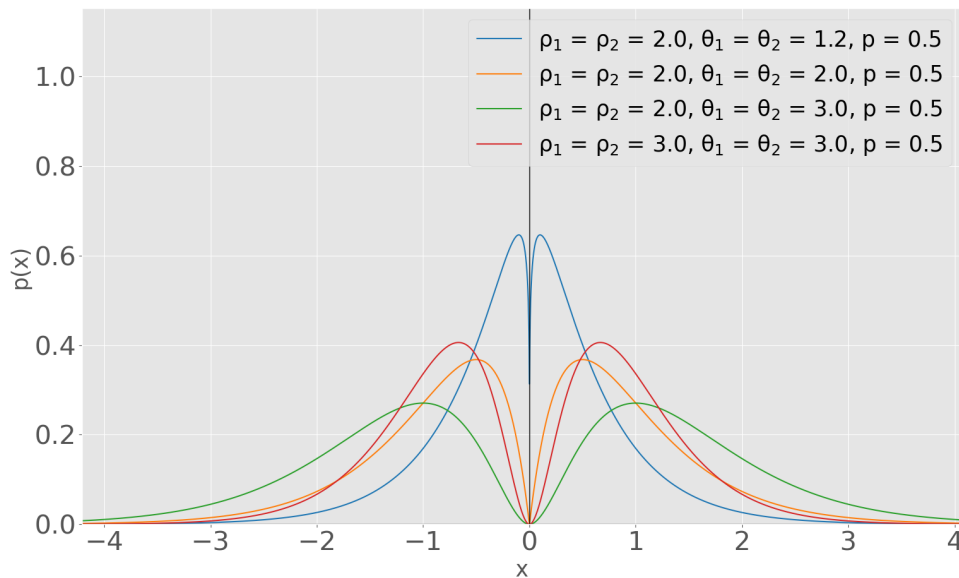
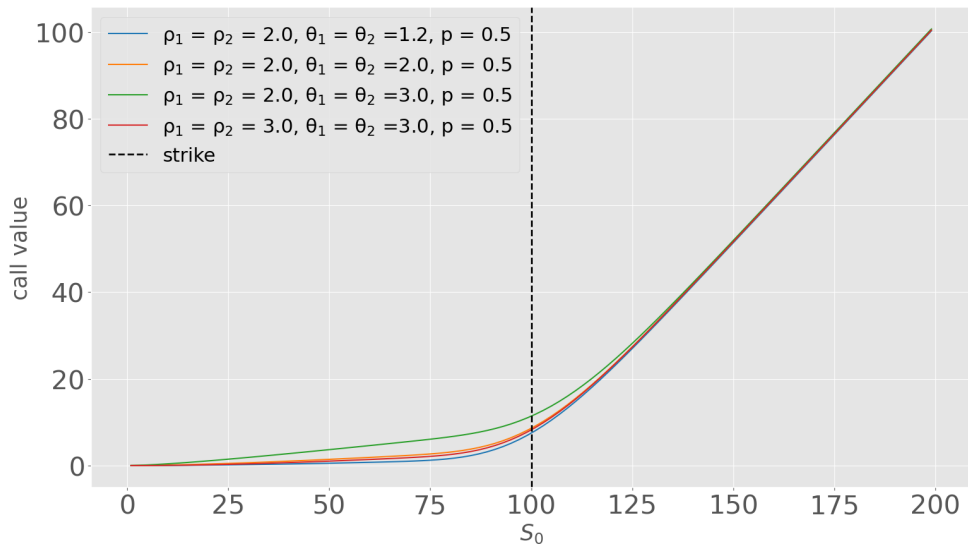
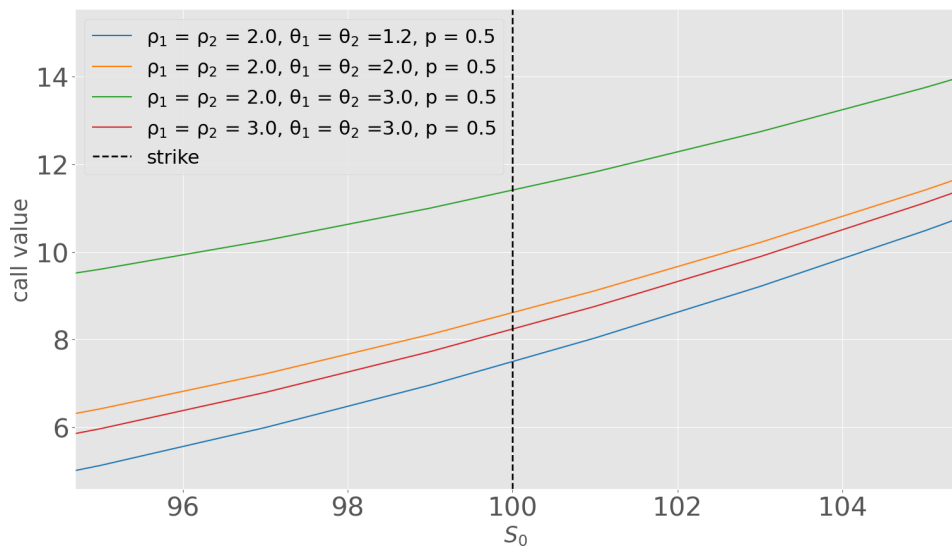


Figure 4.15: The DG distribution with fixed ρ_1, ρ_2 and p but ϑ_1 and ϑ_2 are varying



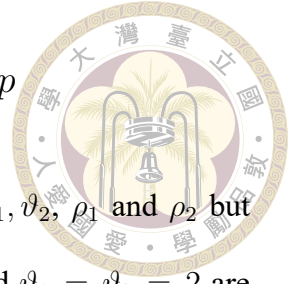
(a) Overall



(b) Magnified

Figure 4.16: Call values with fixed ρ_1, ρ_2 and p but ϑ_1 and ϑ_2 are varying

4.3.4 Results with fixed $\vartheta_1, \vartheta_2, \rho_1$ and ρ_2 but varying p



In this subsection, we will look at the call value with fixed $\vartheta_1, \vartheta_2, \rho_1$ and ρ_2 but varying p . Figure 4.17 shows that when variables $\rho_1 = \rho_2 = 2$ and $\vartheta_1 = \vartheta_2 = 2$ are fixed, as p increases from 0.1 to 0.9 (blue to orange to green), the distribution's probability mass greatly shifts to positive x while the right tail becomes heavier and the left tail less heavy, causing the call value to increase as shown in Figure 4.18 (blue to orange to green). Figure 4.17 also shows that when variables $\rho_1 = \rho_2 = 2$ and $p = 0.9$ are fixed, as ϑ_1 and ϑ_2 both increase from 0.5 to 2 (red to green), the distribution's probability mass greatly shifts to positive x while the left and right tails become heavier (the right tail more so), causing the call value to increase as shown in Figure 4.18 (red to green).

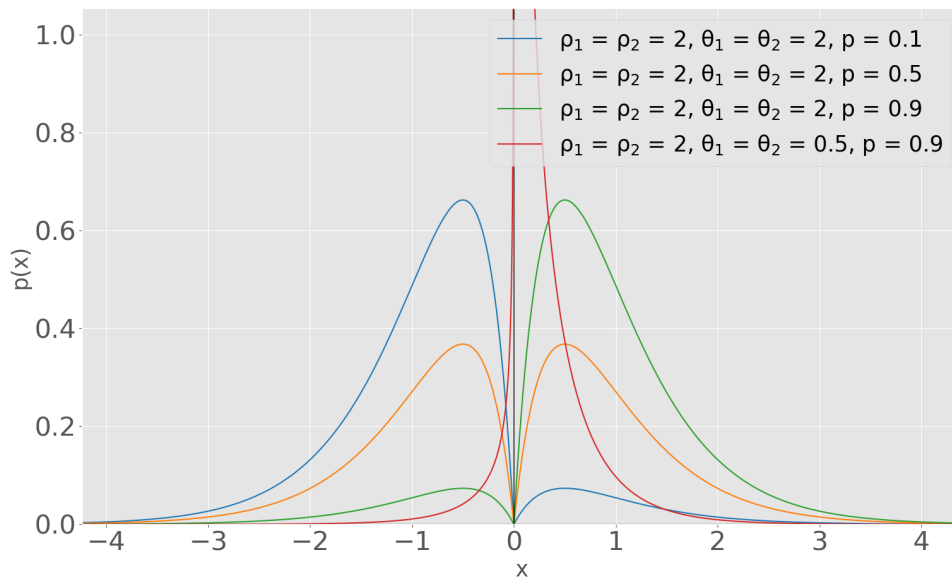
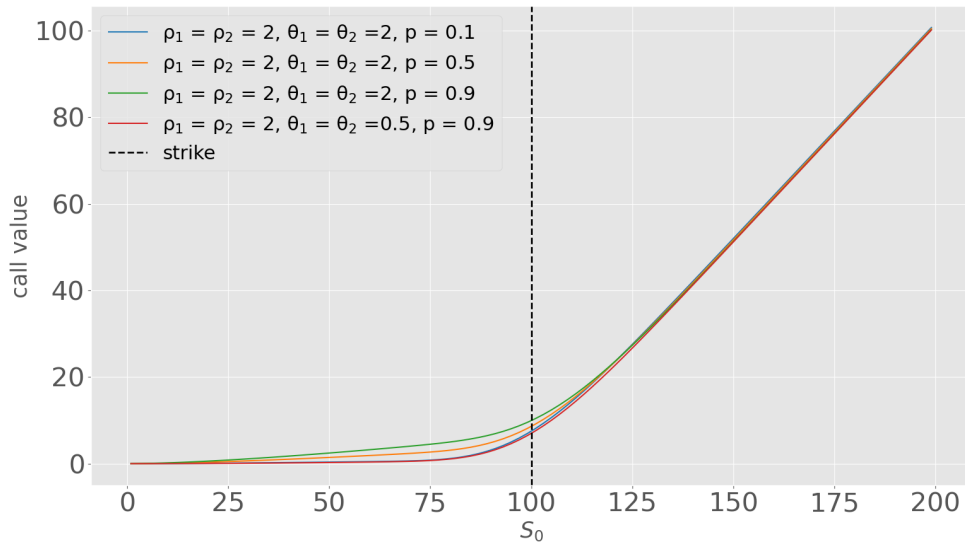
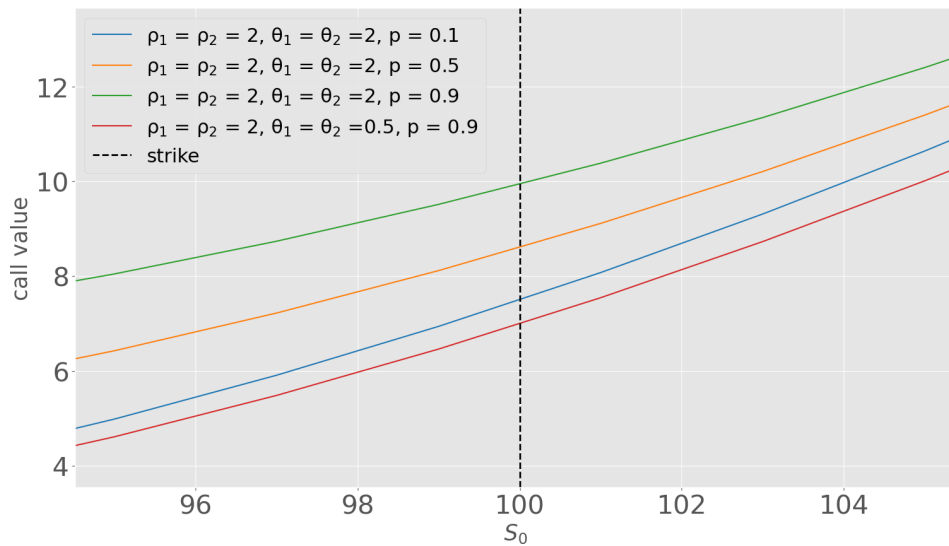


Figure 4.17: DG distribution with fixed $\vartheta_1, \vartheta_2, \rho_1$ and ρ_2 but p is varying



(a) Overall



(b) Magnified

Figure 4.18: Call value results with fixed $\vartheta_1, \vartheta_2, \rho_1$ and ρ_2 but p is varying





Chapter 5 Conclusions

In our research on extending Kou's double exponential jump diffusion model to the double gamma jump-diffusion (DGJD) model, we draw the following conclusions.

First, the DGJD model can reduce to Kou's model by setting variables $\vartheta_1 = \vartheta_2 = 1$. With the extra degrees of freedom, the double gamma distribution can create heavier tails, which captures greater extreme events to better fit the market.

Second, we showed that with different combinations of variables $\rho_1, \rho_2, \vartheta_1, \vartheta_2$ and p , the DG distribution will have different moments. The moments affect the distribution's shape, causing the change in call value. Having a heavier right tail increases the call value and having a heavier left tail decreases the call value. The shape of the probability mass also affects the call value, having a greater positive probability mass also increases the call value.





Chapter 6 Future Works

In the future works, we hope to integrate this model with empirical data and apply it to simulate real-world markets and to assess whether our model fits the market better compared to Kou's model.

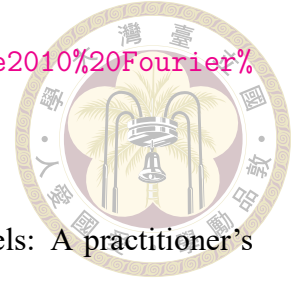




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