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## 休閒外部性與最適財政政策

# Leisure Externalities and Optimal Fiscal Policy 

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## 中文摘要

消費與休閒兩者都是構成個人效用的重要因子。在研究「習慣養成（habit formation）」 的文獻中，消費的內部與外部習慣皆被大量地探討，其中後者可以視爲消費的一種外部性。然而卻很少研究涉及休閒的外部性——盡管它是很可能存在的。本篇論文目的在於填補這部分的文獻聞漏。我們從一個高度特化的簡單模型開始，在允許模型經濟存在休閒外部性的情況下探討其政策含義。接著我們擴充上述模型，同時允許代表性個人在休閒的選擇上存在跨期替代或跨期互補的偏好。分析結果顯示休閒的跨期偏好與休閒的外部性兩者之間的互動會影響最適稅率的景氣循環性質。

關鍵字：休閒外部性，習慣養成•最適財政政策，跨期替代，休閒共享


#### Abstract

Consumption and leisure are both important components of individual utility. In the habit formation literature, both internal and external consumption habits have been widely investigated. However, when it comes to leisure externalities, little works have been done despite the fact that it is economically plausible. This paper aims to fill in this gap. A stylized simple model is constructed to allow for leisure externalities, and its policy implications are studied. A richer model with both leisure externalities and intertemporal substitution (or complementarity) is also studied, and the interaction between such intertemporal preference and externalities is found to be crucial in the determinant of the cyclical property of optimal tax.

Keywords: Leisure externalities, habit formation, optimal fiscal policy, intertemporal substitution, leisure coordination.


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## 1 Introduction

Time-separable leisure choice in the utility function is widely adopted in modern macroeconomics despite its lacking of empirical evidence. Such setting implies unreasonable independence between leisure choice today and that of yesterday.

In Kydland and Prescott (1982), a time-nonseparable leisure is introduced into a standard growth model with time-to-build technology in order to provide greater intertemporal substitution of leisure, which in turn gives rise to the vivid aggregate fluctuations of their equilibrium model. Hotz, Kydland, and Sedlacek (1988) then provide empirical evidence that supports time-nonseparable leisure setting and suggests the notion of intertemporal substitution in labor supply. However, the empirical results of Eichenbaum, Hansen, and Singleton (1988), Kennan (1988) and Bover (1991) are against intertemporal substitution in labor supply and in favor of intertemporal complementarity, i.e., the internal habits argument. Though the notion of intertemporal preferences itself appears to be controversial, the null of time-separable utility is frequently rejected in empirical field, both in consumption and in leisure ${ }^{1}$ The former relates to the growing literature in consumption habit formation and has been extensively investigated. However, the latter, timenonseparable leisure, attracts relatively less attention. Several studies do focus on the existence of intertemporal preference, and some of them suggest the importance of such feature. Wen (1998) finds that when intertemporal complementarity of leisure is allowed, RBC model can fit better to the U.S. data in terms of a Watson test. Guo and Janko (2009) find better performance to mimic Canada data after internal habits in leisure is incorporated into a small open economy RBC model with variable capital utilization.

In addition to intertemporal preference on leisure, the existence of leisure ex-

[^0]ternalities is also possible. In the consumption habit formation literature, both internal and external habits have been widely studied. The latter can be viewed as consumption externalities, which points out the possible effect of own consumption level on other people. However, when it comes to leisure externalities, little works have been done in the literature despite the fact that it is economically plausible. Jenkins and Osberg (2005) propose the idea of "leisure coordination," which suggests that an overall increase in economy-wide working hours may have negative effect on individual leisure choice due to the difficulty to share leisure time with others. That is, an increase in overall working hours will decrease individual's marginal utility of leisure. They then provide empirical evidence that supports this view. Weder (2004) also gives a similar reasoning: externalities from leisure could be the outcome of coordination spillovers in communal leisure activities.

Existing studies on leisure externalities mainly come from growth literature. Azariadis et al. (2009) argue that leisure externalities (possibly result from cultural difference) can explain the large differences in hours worked between people in United States and in Europe. Pintea (2006) examines a growth model with positive leisure externalities and the resulting inefficiency. Gómez (2008) studies the effect of both consumption and leisure externalities on the equilibrium growth path of an endogenous growth model. In this paper, nonetheless, the focus is on its policy implication under a simple RBC-like context. One relevant study is Lettau and Uhlig (2000). They show that a RBC model with external leisure habits generates counterfactually over-smoothing of labor supply across time. This paper differs from theirs in at least three dimensions: first, leisure externalities are allowed to be either positive or negative; second, optimal fiscal policy in response to such inefficiency and its implied cyclicality property are of main interest; third, intertemporal preference on leisure is studied in the extended version of the model, given the great amount of empirical evidence already mentioned, so that experi-
ments on the interaction between leisure externalities and cross-time substitution or complementarity are conducted to provide further theoretical insight about non-standard leisure preference. Another relevant study is Ljungqvist and Uhlig (2000), who focus on consumption externalities instead of leisure.

The main findings of this paper are summarized here. When there are negative leisure externalities on utility, it will be beneficial for government to subsidize labor income to encourage people to work more since they work too little compared to the socially optimal level. Such optimal tax is acyclical as long as the externalities enter into utility contemporaneously, and individual does not feature time-nonseparable preference on own leisure. However, if either the externalities enter into utility with lags, or individual choice of leisure is time-dependent, optimal tax will have cyclicality consequences. The dynamics of tax is crucially contingent on both the risk aversion coefficient and the interaction between intertemporal preference and the sign of externalities.

The remaining of this paper is arranged as follows. Section 2 describes the benchmark model economy with leisure externalities; section 3 studies the optimal fiscal policies that aim to restore first-best optimality and the implied cyclicality consequences; section 4 extends the model to simultaneously allow for different intertemporal preferences on leisure; section 5 concludes.

## 2 The benchmark model

### 2.1 Utility function

Consider a period utility of the form:

$$
\begin{equation*}
U(c, h, H)=\frac{c^{1-\gamma}-1}{1-\gamma}-A \frac{(h+\alpha H)^{1+\omega}-1}{1+\omega}, \tag{1}
\end{equation*}
$$

with time endowment standardized to unity:

$$
\begin{equation*}
1=l+h, \tag{2}
\end{equation*}
$$

where $c$ denotes own consumption, $l$ denotes leisure, $h$ denotes individual labor hours and $H$ denotes economy-wide average labor hours; the risk aversion coefficient $\gamma>0$ and the (inverse of) Frisch labor supply elasticity, $\omega>0 \|^{2} H$ enters into the utility as the source of leisure externalities.

The reason for adopting a consumption-leisure-separable form is to distinguish the leisure externalities from that of consumption. If consumption and leisure enter into utility in a nonseparable way, the interpretation becomes somewhat ambiguous since the marginal utility of consumption is also contingent on leisure, which is in turn subject to externalities.

The parameter $\alpha$ determines the sign and magnitude of externalities. A relevant interval restriction $\alpha \in(-1,1)$ ensures that leisure generates positive utility. If $\alpha>0$, we have $\frac{\partial U(\cdot)}{\partial H}<0$ so that economy-wide average labor hours have a negative effect on individual's utility; if $\alpha<0$, we have $\frac{\partial U(\cdot)}{\partial H}>0$, such externalities then change sign. Detailed discussion about effects of externalities on utility is given in the following subsection.

### 2.2 Channels through which leisure externalities affect utility

In the study on consumption externalities in Dupor and Liu (2003), it is identified that such externalities may have two effects. One is its direct effect on utility $\left(\frac{\partial U(\cdot)}{\partial C}\right)$ and the other is its impact on the marginal rate of substitution of leisure for consumption $\left(\frac{\partial M R S}{\partial C}\right)$. (The uppercase $C$ denotes economy-wide average consumption which individual takes as given.) According to them, the former is referred to as "jealousy" or "admiration" and the latter is referred to as "keeping up with or running away from the Joneses." These two features do not nessecarily imply each other. That is, a jealous agent may feature either keeping up with the Joneses (KUJ) or running away from the Joneses (RAJ), and vice versa. This is

[^1]summarized in Table 1.

| Mathematical notation | Interpretation |
| :---: | :--- |
| $\frac{\partial M R S}{\partial C}>0$ | Keeping up with the Joneses (KUJ) |
| $\frac{\partial M R S}{\partial C}<0$ | Running away from the Joneses (RAJ) |
| $\frac{\partial U(\cdot)}{\partial C}>0$ | Admiration |
| $\frac{\partial U(\cdot)}{\partial C}<0$ | Jealousy |

Table 1: Possible effects of consumption externalities on utility

Dupor and Liu (2003) also show that equilibrium under or over-consumption is a direct result from admiration or jealousy, and that the existence of KUJ or RAJ plays a role to amplify or dampen such inefficiency. Though their argument is based on consumption externalities, the same logic can apply to leisure externalities as well. Given the consumption-leisure-separable utility functional form setup in equation (1), there are also two effects that can be mathematically identified: one is its direct impact on utility $\left(\frac{\partial U(\cdot)}{\partial H}\right)$ and the other is its impact on the marginal utility of leisure $\left(\frac{\partial\left(\frac{\partial U(\cdot)}{\partial h}\right)}{\partial H}\right)$. For convenience, in the following we may refer to the first term, the direct externalities effect, as jealousy or admiration and the second term, effect on marginal utility, as KUJ or RAJ. Readers should recognize that this definition is more functional form-specific and thus somewhat different from that of Dupor and Liu (2003).

The direct externalities effect derived from equation (1) is:

$$
\begin{equation*}
\frac{\partial U(c, h, H)}{\partial H}=-\alpha A(h+\alpha H)^{\omega} \tag{3}
\end{equation*}
$$

which is positive for $\alpha<0$ and negative for $\alpha>0$. Since $\frac{\partial U(c, h, H)}{\partial H}=-\frac{\partial U(c, h, H)}{\partial L}(L$ denotes economy-wide leisure), the case of $\alpha>0(<0)$ denotes a positive (negative) leisure externalities effect of economy-wide average leisure on individual's
utility. Instead, the effect of externalities on marginal utility is computed as:

$$
\begin{equation*}
\frac{\partial\left(\frac{\partial U(\cdot)}{\partial h}\right)}{\partial H}\left(=\frac{\partial\left(\frac{\partial U(\cdot)}{\partial l}\right)}{\partial L}\right)=-\alpha \omega A(h+\alpha H)^{\omega-1} \gtreqless 0 . \tag{4}
\end{equation*}
$$

For $\alpha<0$, an increase in economy-wide working hours will increase (decrease) individual's marginal utility (disutility) of labor hours, or put it differently, a decrease in economy-wide leisure has a negative effect on individual's marginal utility of leisure. This is referred to as "leisure coordination" through out this paper. For $\alpha>0$, the effect just reverses: a decrease in economy-wide leisure increases individual's marginal utility of leisure. This can be interpreted as the congestion effect, as pointed out by Gómez (2008). Table 2 summarizes this paragraph.

| Parameter value | Mathematical notation | Interpretation |
| :---: | :---: | :--- |
| $\alpha<0$ | $\frac{\partial\left(\frac{\partial U(\cdot)}{\partial h}\right)}{\partial H}>0$ | KUJ or "leisure coordination" |
| $\alpha>0$ | $\frac{\partial\left(\frac{\partial U(\cdot)}{\partial h}\right)}{\partial H}<0$ | RAJ or "congestion effect" |
| $\alpha>0$ | $\frac{\partial U(\cdot)}{\partial H}<0$ | Admiration |
| $\alpha<0$ | $\frac{\partial U(\cdot)}{\partial H}>0$ | Jealousy |

Table 2: Possible effects of leisure externalities on consumption-leisure-separable utility

Equation (1) is analogous to that of Ljungqvist and Uhlig (2000), where consumption externalities rather than leisure externalities are investigated. It implicitly implies agent will feature jealousy together with KUJ (so that $\alpha<0$ ), or admiration together with RAJ (so that $\alpha>0$ ). The former means that people feel bad when others get better (consume more), and would like to "compete" with others by increase own consumption-marginal utility of consumption is larger for higher economy-wide average consumption. In Azariadis et al. (2009), however, they assume utility of the form:

$$
\begin{equation*}
U=\ln c+\psi \frac{\left(l L^{\gamma}\right)^{1-\sigma}-1}{1-\sigma} \tag{5}
\end{equation*}
$$

where $l$ denotes own leisure and $L$ denotes economy-wide average leisure. Utility of this form is able to disconnect the KUJ and jealousy through the parameter pair $(\gamma, \sigma)]^{3}$ According to the preferred calibration in their paper, KUJ is associated with admiration to represent the leisure culture in Europe and RAJ is associated with jealousy to represent the workaholic labor market in the US.

Empirical evidence mentioned in the previous section tends to support a negative working hours externalities on marginal utility of leisure. But whether there is jealousy or admiration effect on leisure is indeed another question of empirical interest. Though there has been much evidence suggesting jealousy in consumption, for example, see Luttmer (2005) and Dynan and Ravina (2007), little empirical works to our knowledge have said words about jealousy in leisure (or admiration either). This paper examines both of them, under the context of a separable utility.

### 2.3 The model economy

Imagine that there lives a representative agent having utility specified as equation (11), in periods of discrete time over an infinite-horizon, with a linear production technology:

$$
\begin{equation*}
y_{t}=\theta_{t} h_{t} \tag{6}
\end{equation*}
$$

where $h_{t}$ is hours devoted to working; $y_{t}$ is the final real output and $\theta_{t}$ is a random shock to production, with mean $\theta=1$. Capital is assumed fixed in this simple model for simplicity. Government levies tax rate $\tau_{t}$ on labor income and rebates them to household each period. The transfer payment then is:

$$
\begin{equation*}
\nu_{t}=\tau_{t} y_{t} . \tag{7}
\end{equation*}
$$

Household aims to maximize their expected life-time utility beginning at period

[^2]0 :

$$
\begin{equation*}
E_{0} \sum_{t=0}^{\infty} \beta^{t} U\left(c_{t}, h_{t}, H_{t}\right) \tag{8}
\end{equation*}
$$

subject to the budget constraint:

$$
\begin{equation*}
c_{t}=\left(1-\tau_{t}\right) y_{t}+\nu_{t}, \forall t \tag{9}
\end{equation*}
$$

along with the time endowment as equation (2) and the production technology as equation (6). Since individuals do not take into consideration the effect of own leisure choice on the others, taking economy-wide level of hours worked as given when they maximize their utility, the existence of leisure externalities can potentially render suboptimal outcome to the economy as a whole and thus allow room for beneficial government intervention to restore the efficiency.

Ljungqvist and Uhlig (2000) study the business cycle property of such optimal tax in the case of consumption externalities, and find that the optimal tax moves procyclically if externalities enter into utility with a one-period lag (which they refer to as "catching up with the Joneses"). So a traditional Keynesian style demand-management policy appears, though for rather unconventional reason: the economy is optimally cooling downed with higher taxes when it is overheating in booms and is optimally stimulated with lower taxes in recessions to keep consumption up. In the following section, an analogous analysis is first conducted for the case of leisure externalities.

## 3 Optimal taxation

### 3.1 Contemporaneous externalities

Assume leisure externalities enter into utility with no lags, so that individual's period utility depends not only on current own consumption and own leisure times but also on current level of economy-wide average leisure times. Given model structure already described in section 2, the mathematical maximization problem
for individual can be depicted as follows:

$$
\max _{c_{t}, h_{t}} E_{0} \sum_{t=0}^{\infty} \beta^{t}\left[\frac{c_{t}^{1-\gamma}-1}{1-\gamma}-A \frac{\left(h_{t}+\alpha H_{t}\right)^{1+\omega}-1}{1+\omega}\right]
$$

subject to

$$
\begin{equation*}
c_{t}=\left(1-\tau_{t}\right) \theta_{t} h_{t}+\nu_{t}, \forall t, \tag{10}
\end{equation*}
$$

A Lagrangian is formed (with multiplier $\lambda_{t}$ on constraint equation (10)) which gives the following first order conditions:

$$
\begin{align*}
& \text { w.r.t. } c_{t}: c_{t}^{-\gamma}=\lambda_{t} \text {, }  \tag{11}\\
& \text { w.r.t. } h_{t}: A\left(h_{t}+\alpha H_{t}\right)^{\omega}=\left(1-\tau_{t}\right) \lambda_{t} \theta_{t} \text {. } \tag{12}
\end{align*}
$$

Along with market clearing condition requiring that $c_{t}=y_{t}\left(=\theta_{t} h_{t}\right)$ and symmetric equilibrium requiring that $h_{t}=H_{t}$, we can solve for optimal labor supply to be:

$$
\begin{equation*}
h_{t}=\left[\frac{\left(1-\tau_{t}\right) \theta_{t}^{1-\gamma}}{A(1+\alpha)^{\omega}}\right]^{\frac{1}{\omega+\gamma}} \tag{13}
\end{equation*}
$$

Notice that in this simple model we have labor supply that responses to technology shock negatively (or positively) as long as $\gamma>1$ (or $<1$ ):

$$
\begin{equation*}
\frac{\partial h_{t}}{\partial \theta_{t}}=\frac{1-\tau_{t}}{\omega+\gamma}\left[\frac{\left(1-\tau_{t}\right) \theta_{t}^{1-\gamma}}{A(1+\alpha)^{\omega}}\right]^{\frac{1}{\omega+\gamma}-1}(1-\gamma) \theta_{t}^{-\gamma} \tag{14}
\end{equation*}
$$

When $\gamma=1$ the wealth effect exactly offsets the substitution effect and thus labor supply will not response to technology shock at all. We will show later that this amount of labor supply is not socially optimal. We seek for potentially first-best solution by considering a central planner solution, where the government internalizes leisure externalities for the economy. That is, the government take into consideration $H_{t}=h_{t}$ and $\nu_{t}=\tau_{t} \theta_{t} h_{t}$. Its optimization problem then can be depicted as follows:

$$
\begin{equation*}
\max _{c_{t}, h_{t}} E_{0} \sum_{t=0}^{\infty} \beta^{t}\left[\frac{c_{t}^{1-\gamma}-1}{1-\gamma}-A \frac{\left[(1+\alpha) h_{t}\right]^{1+\omega}-1}{1+\omega}\right], \tag{15}
\end{equation*}
$$

subject to

$$
\begin{equation*}
c_{t}=y_{t}\left(=\theta_{t} h_{t}\right), \quad \forall t \tag{16}
\end{equation*}
$$

with relevant first order conditions:

$$
\begin{align*}
& \text { w.r.t. } c_{t}: c_{t}^{-\gamma}=\lambda_{t} \text {, }  \tag{17}\\
& \text { w.r.t. } h_{t}: A(1+\alpha)^{1+\omega} h_{t}^{\omega}=\lambda_{t} \theta_{t} \text {. } \tag{18}
\end{align*}
$$

Using market clearing condition we can again solve for optimal labor supply to be:

$$
\begin{equation*}
h_{t}^{*}=\left[\frac{\theta_{t}^{1-\gamma}}{A(1+\alpha)^{1+\omega}}\right]^{\frac{1}{\omega+\gamma}} . \tag{19}
\end{equation*}
$$

Compared to equation (13), we can find that the tax that supports the first-best optimality must be:

$$
\begin{equation*}
\tau_{t}^{*}=\frac{\alpha}{1+\alpha} . \tag{20}
\end{equation*}
$$

This result suggests that the government should subsidize household for the case of $\alpha<0$ (jealousy together with leisure coordination), and tax household for the case of $\alpha>0$ (admiration together with congestion effect). The intuition goes as follows. When people are jealous, we have $\frac{\partial U(\cdot)}{\partial L}<0\left(\frac{\partial U(\cdot)}{\partial H}>0\right)$, the economywide average leisure has a negative effect on individual's utility. But each agent does not recognize such negative effect of their own choice on other people. As a result, the economy as a whole ends up consuming too much leisure, and thus too little labor hours. In addition, for $\alpha<0$ we also have $\frac{\partial\left(\frac{\partial U(.)}{\partial h}\right)}{\partial H}>0$, lower average labor hours lead to higher marginal utility of leisure since people are easy to find time enjoyed together, such leisure coordination effect further amplifies the inefficient over-consuming of leisure.

Mathematically, when tax is set at zero, the optimal labor hours from equation 13. can be solved to be $h_{t}=\left[\frac{\theta_{t}^{1-\gamma}}{A(1+\alpha)^{\omega}}\right]^{\frac{1}{\omega+\gamma}}$, while the first-best solution is described by equation (19). For $\alpha<0$, It is readily seem that the first-best solution renders higher equilibrium quantity of labor supply (notice that $\frac{1}{1+\alpha}>1$ for $-1<\alpha<0$ ).

As this is the case, it is desirable for government to subsidize labor income so that each individual will face the correct marginal trade-off between consumption and leisure. This is described by the first order condition with respect to labor (under symmetric equilibrium condition):

$$
\begin{align*}
& \underbrace{A\left[(1+\alpha) h_{t}\right]^{\omega}} \quad \underbrace{\lambda_{t}} \cdot \underbrace{\left(1-\tau_{t}\right) \theta_{t}} \text {. } \\
& \text { optimal labor choice marginal utility } \\
& \text { marginal income earned } \\
& \text { (marginal disutility of labor) from consumption from labor supply } \tag{21}
\end{align*}
$$

The "true" marginal disutility of labor is the left hand side of equation (18), which is lower than the "perceived" marginal disutility of labor when $\alpha<0$. People are thus under-working. When individuals' labor incomes are (optimally) subsidized, they choose to work more, in such a way that the externalities can be fully internalized via a negative tax.

Since the government can effectively correct this distortion period by period, the optimal tax does not have any cyclical property at all. Optimal tax $\tau_{t}$ is a constant not subject to changes in technology shock $\theta_{t}$. Such result is in line with Ljungqvist and Uhlig (2000), where they find contemporaneous consumption externalities do not generate any tax-induced cyclical consequence. $\underbrace{4}$

### 3.2 Externalities with a one-period lag

Next we assume the externalities enter into period utility with a one-period lag.5 This is the notion of "catching up with the Joneses" as a kind of consumption externalities first introduced in the equity premium puzzle literature $\sqrt{6}$ In this

[^3]paper the idea is applied to leisure externalities. Under such environment, for $\alpha<0$, if others enjoy more leisure today, individual will experience a higher marginal utility from an additional unit of leisure time in the future-a desire to catch up with the Joneses. The case of $\alpha>0$ can be similarly argued.

Individual's expected life-time utility in this case becomes:

$$
\begin{equation*}
E_{0} \sum_{t=0}^{\infty} \beta^{t} U\left(c_{t}, h_{t}, H_{t-1}\right)=E_{0} \sum_{t=0}^{\infty} \beta^{t}\left[\frac{c_{t}^{1-\gamma}-1}{1-\gamma}-A \frac{\left(h_{t}+\alpha H_{t-1}\right)^{1+\omega}-1}{1+\omega}\right] \tag{22}
\end{equation*}
$$

which is subject to the budget constraint (10). Optimality requires that:

$$
\begin{align*}
c_{t}^{-\gamma} & =\lambda_{t}  \tag{23}\\
A\left(h_{t}+\alpha H_{t-1}\right)^{\omega} & =\left(1-\tau_{t}\right) \lambda_{t} \theta_{t} . \tag{24}
\end{align*}
$$

The central planner's problem is constructed as follows:

$$
\begin{equation*}
\max E_{0} \sum_{t=0}^{\infty} \beta^{t}\left[\frac{c_{t}^{1-\gamma}-1}{1-\gamma}-A \frac{\left(h_{t}+\alpha h_{t-1}\right)^{1+\omega}-1}{1+\omega}\right] \tag{25}
\end{equation*}
$$

subject to

$$
\begin{equation*}
c_{t}=\theta_{t} h_{t}, \quad \forall t, \tag{26}
\end{equation*}
$$

with first order conditions:

$$
\begin{align*}
\text { w.r.t. } & c_{t}
\end{align*}: c_{t}^{-\gamma}=\lambda_{t}, ~ 子\left(h_{t}+\alpha h_{t-1}\right)^{\omega}+A \alpha \beta\left(E_{t} h_{t+1}+\alpha h_{t}\right)^{\omega}=\lambda_{t} \theta_{t} . ~ \$ ~ w . r . t . ~ h_{t}: A\left(\begin{array}{ll} 
& \tag{27}
\end{array}\right.
$$

### 3.2.1 Steady state solution

Define steady state of the system as when technology shock $\theta_{t}$ remains at its mean value $\theta_{t}=\theta(=1)$. Then the steady state optimal labor supply in social planner's solution can be computed as:

$$
\begin{equation*}
h=\left[\frac{\theta^{1-\gamma}}{A(1+\alpha \beta)(1+\alpha)^{\omega}}\right]^{\frac{1}{\omega+\gamma}} . \tag{29}
\end{equation*}
$$

By comparison with the steady state solution of individual's problem, we can easily figure out the optimal steady state tax rate:

$$
\begin{equation*}
\tau=\frac{\alpha \beta}{1+\alpha \beta} \tag{30}
\end{equation*}
$$

This result is again in line with Ljungqvist and Uhlig (2000). Since in this case there is a future effect of today's leisure choice, such effect is discounted by $\beta$ to represent its present value and thus the tax (or subsidy) dealing with that value is also scaled down by the same factor $\beta$.

### 3.2.2 Transitional dynamics

Since equilibrium solution off the steady state does not have closed form due to non-linearity of leisure in utility, the model is solved numerically with moderate calibration $\sqrt[7]{ }$ Process for technology is assumed to follow a AR(1). System of all equations is reproduced below:

$$
\begin{align*}
& \text { (F.o.c. w.r.t. } \left.c_{t}\right): c_{t}^{-\gamma}=\lambda_{t}  \tag{31}\\
& \left(\text { F.o.c. w.r.t. } h_{t}\right): A\left(h_{t}+\alpha h_{t-1}\right)^{\omega}+A \alpha \beta\left(E_{t} h_{t+1}+\alpha h_{t}\right)^{\omega}=\lambda_{t} \theta_{t},  \tag{32}\\
& \quad(\text { Technology }): y_{t}=\theta_{t} h_{t}  \tag{33}\\
& \text { (Market clearing) }: c_{t}=y_{t} \tag{34}
\end{align*}
$$

$$
\begin{equation*}
\left(\text { Process for technology) : } \ln \theta_{t+1}=\rho \ln \theta_{t}+\varepsilon_{t+1}\right. \text {. } \tag{35}
\end{equation*}
$$

Optimal tax can be tracked by:

$$
\begin{equation*}
\tau_{t}=1-\frac{A\left(h_{t}+\alpha h_{t-1}\right)^{\omega}}{\theta_{t}^{1-\gamma} h_{t}^{-\gamma}} \tag{36}
\end{equation*}
$$

which is derived from the combination of equation (23), (24), and (26).

Calibration Calibration strategy is described as follows. One period of model economy corresponds to a quarter. Subjective discount factor $\beta=0.99$ so that the

[^4]implied quarterly real interest rate equals 0.01 ; the inverse of Frisch labor supply elasticity $\omega=1$, following Christiano et al. (2005); coefficient of risk averse $\gamma=2$, following Schmitt-Grohé and Uribe (2007); preference weight on labor hours $A$ is calibrated such that the steady state labor hours $h=0.2$; persistence of technology shocks $\rho=0.8556$ and the variance of innovation $\varepsilon_{t+1}$ is set at $\sigma_{\varepsilon}^{2}=0.0064^{2}$, both are values that Schmitt-Grohé and Uribe (2007) obtain from the estimation of US data.

System of equations (31) to (36) is first-order approximated around its steady state. Two experiments about parameter $\alpha$ are conducted: $\alpha=-0.3$ and $\alpha=0.3$. These numbers are arbitrarily set since there is no empirical work in the literature that suggests any estimate about it. This paper thus emphasizes on the qualitative outcome, given a highly stylized model economy. The numerical choice of $\alpha$ does not qualitatively change the overall results in this paper.

Case I: $\alpha=-0.3$ For this case (catching up with the Joneses), optimal tax is found to be strongly countercyclical with $\operatorname{corr}\left(\tau_{t}, y_{t}\right)=-0.9235$. When the model economy is booming, the government subsidizes more. The responses of variables of interest to a one-standard deviation positive technology shock are plotted in Figure 1. Notice that the government is always subsidizing households since people work too little at the equilibrium, but the magnitude of subsidy differs across time: it is larger initially after shock and gradually regress to its steady state value. That is, the degree to which working hours are considered too little is varying over time. It is considered larger at the beginning of the shock period and is decreasing toward its steady state level over time.

Case II: $\alpha=0.3$ For the case of $\alpha>0$, the result above is reversed. Optimal tax now tends to be procyclical with $\operatorname{corr}\left(\tau_{t}, y_{t}\right)=0.6924$. The responses of variables of interest to a one-standard deviation positive technology shock are plotted in


Figure 1: Impulse response functions for the case of $\alpha=-0.3$


Figure 2: Impulse response functions for the case of $\alpha=0.3$

Figure 2. The government is always taxing households since people work too much at the equilibrium. The magnitude of such tax is again varying over time, which results in cyclical consequences of optimal tax.

### 3.3 Discussion

Two important findings arise, based on the results of previous numerical experiments. First, when leisure externalities enter into utility contemporaneously, the optimal tax is found to be a constant over time, i.e., the optimal tax does not have cyclical property. A constant optimal tax arises because the degree to which people are working too little (or too much) remains the same over time, so that the government is able to correct such distortion period-by-period with the same amount of tax; second, if leisure externalities enter into utility with lags, optimal tax exhibits cyclicality since the degree to which people are working too little (or too much) is changing over time when the model is subject to a random shock. The behavior of optimal tax is found to be countercyclical for $\alpha<0$ and procyclical for $\alpha>0$. This cyclical behavior of optimal tax, however, can be more complex in the sense that it may not only depend on the externalities parameter $\alpha$ but also depend on other deep parameters that affect the model's dynamics of labor hours. We proceed to further examine such possibility below.

### 3.3.1 Results for different risk aversion settings

The above two experiments are conducted under a parameter choice of risk aversion $\gamma=2$, which gives rise to a countercyclical labor supply. Will the cyclical behavior of tax depends on the cyclical behavior of labor supply? Here we do two more experiments with different values of $\gamma$ to deal with such question, and the answer is: Yes, the cyclical behavior of tax does depend on the cyclical behavior of labor supply, which is governed by the relative strength between income effect (people work less for being richer) and substitution effect (people work more for leisure
being relatively costlier). According to equation (14), when $\gamma>1$ we have income effect larger than substitution effect (absolutely) and when $\gamma<1$ we have the opposite.

Figure 3 presents impulse response functions for the case of both a.) $\gamma=1.5$ and b.) $\gamma=0.5$ for comparison, under $\alpha=-0.3$. Dot-markered lines correspond to tax. For the case of $\gamma=1.5$ (the solid lines) we have similar results compared to that of $\gamma=2$, which is already discussed. While for the case of $\gamma=0.5$, the whole story just reverses. In addition to procyclical labor hours, optimal tax also becomes procyclical. Results under $\alpha=0.3$ is similarly affected by $\gamma$ and are not presented here to save space.


Figure 3: Dynamics of labor hours and optimal tax for different risk aversion

Interpretation If $\gamma=0$, labor supply is constant over time and so does optimal tax. If $\gamma \neq 0$, the dynamics of labor hours gives rise to dynamic behavior of the degree to which labor hours deviate from its socially optimal level. For the case
of $\alpha=-0.3$, for example, the government's goal is to encourage working. Now if labor supply responses positively to shocks $(\gamma<1)$, the degree to which labor hours deviate from its socially optimal level is reduced. As a consequence, the government can subsidize less when the economy is booming. When labor supply responses negatively to shocks $(\gamma>1)$, the story reverses. The optimal tax itself indeed can be viewed as a measure of the degree to which labor hours deviate from its socially optimal level.

### 3.3.2 Results for different persistency of technology

Next we consider experiments with different parameter choices in persistency of technology $\rho$. $\gamma$ is set to 2 and $\alpha=-0.3$, other calibration the same as in previous settings. Basically, we find the correlation between tax and real output remain negative, with absolute value decreasing as persistency increases. This is summarized in Table 3. The impulse response of tax under different persistency of technology is summarized in Figure 4. The cyclicality of tax seems not to dramatically change with different settings about shock persistency. The correlation of $\left(\tau_{t}, y_{t}\right)$ drops by half if the shock process approaches unit root, but still implies countercyclicality of tax.

| Process for shock: $\ln \theta_{t+1}=\rho \ln \theta_{t}+\varepsilon_{t+1}$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| persistency | i.i.d. $(\rho=0)$ | $\rho=0.25$ | $\rho=0.50$ | $\rho=0.75$ | $\rho=0.99$ |
| $\operatorname{corr}\left(\tau_{t}, y_{t}\right)$ | -0.9979 | -0.9951 | -0.9878 | -0.9616 | -0.4869 |

Table 3: Correlation between tax and output under different persistency of technology

### 3.4 Short summary

When there is jealousy in leisure, the equilibrium labor hours are too little compared to that of socially optimal level. It is then beneficial for the government to


Figure 4: Impulse response of optimal tax under different persistency of technology
subsidize labor income, encouraging people to work more. Optimal tax that eliminates the efficiency loss due to this kind of leisure externalities is found to have cyclicality when leisure externalities enter into utility with lags, as in Ljungqvist and Uhlig (2000). The pattern, however, is more complex than the case of consumption externalities. Specifically, the cyclicality of optimal tax is crucially contingent on the business cycle property of labor hours. When labor supply is procyclical, optimal tax (in form of subsidy) is also procyclical. When labor supply is countercyclical, optimal tax is also countercyclical. Exactly the same logic applies to the case where there is admiration in leisure.

## 4 Intertemporal preferences and externalities

In this section we will show that even if externalities enter into utility without lags, the corresponding optimal tax can still have cyclical behavior, as long as individual's utility features intertemporal preference on leisure.

Based on the empirical findings mentioned in the introduction, in this section a utility featuring both time-nonseparability in own leisure and contemporaneous leisure externalities is investigated. Individual's instantaneous utility function is now given as:

$$
\begin{equation*}
U\left(c_{t}, h_{t}, h_{t-1}, H_{t}\right)=\frac{c_{t}^{1-\gamma}-1}{1-\gamma}-A \frac{\left(h_{t}+\eta h_{t-1}+\alpha H_{t}\right)^{1+\omega}-1}{1+\omega}, \tag{37}
\end{equation*}
$$

with parameter restriction $\eta+\alpha \in(-1,1)$ imposed to ensure that leisure generates positive utility. Parameter $\eta$ governs the intertemporal preference setting:

$$
\begin{align*}
\frac{\partial^{2} U(\cdot)}{\partial h_{t-1} \partial h_{t}} & =\frac{\partial}{\partial h_{t-1}}\left[-A\left(h_{t}+\eta h_{t-1}+\alpha H_{t}\right)^{\omega}\right] \\
& =-A \omega \eta\left(h_{t}+\eta h_{t-1}+\alpha H_{t}\right)^{\omega-1} \gtrless 0 . \tag{38}
\end{align*}
$$

When $\eta>0$, we have $\frac{\partial^{2} U(\cdot)}{\partial h_{t-1} \partial h_{t}}<0$, the current and past values of hours worked are intertemporal substitutes. This is interpreted as the fatigue effects of labor, as pointed out by Guo and Janko (2009). When $\eta<0$, we have $\frac{\partial^{2} U(\cdot)}{\partial h_{t-1} \partial h_{t}}>0$, the current and past values of hours worked become intertemporal complements. This is the internal habits formation.

The rest of the model economy is the same as what has been depicted in section 2. The individual's optimization problem is to maximize life-time expected utility:

$$
\begin{equation*}
E_{0} \sum_{t=0}^{\infty} \beta^{t} U\left(c_{t}, h_{t}, h_{t-1}, H_{t}\right), \tag{39}
\end{equation*}
$$

subject to budget constraint (10) and production technology (6). First order conditions are:

$$
\begin{align*}
\text { w.r.t. } & c_{t}:  \tag{40}\\
\text { w.r.t. } h_{t}^{-\gamma}=\lambda_{t}, & \left(1-\tau_{t}\right) \lambda_{t} \theta_{t}=A\left(h_{t}+\eta h_{t-1}+\alpha H_{t}\right)^{\omega} \\
& +A \beta \eta E_{t}\left(h_{t+1}+\eta h_{t}+\alpha H_{t+1}\right)^{\omega}, \tag{41}
\end{align*}
$$

$$
\begin{equation*}
\text { w.r.t. } \lambda_{t}: c_{t}=\left(1-\tau_{t}\right) \theta_{t} h_{t}+\nu_{t} \text {. (the budget constraint) } \tag{42}
\end{equation*}
$$

Government's problem is again solved from a social planner's perspective:

$$
\begin{equation*}
\max _{c_{t}, h_{t}} E_{0} \sum_{t=0}^{\infty} \beta^{t}\left[\frac{c_{t}^{1-\gamma}-1}{1-\gamma}-A \frac{\left[(1+\alpha) h_{t}+\eta h_{t-1}\right]^{1+\omega}-1}{1+\omega}\right], \tag{43}
\end{equation*}
$$

subject to market clearing condition (34). First order conditions concerning optimal choice about consumption and leisure are:

$$
\begin{align*}
c_{t}^{-\gamma}= & \lambda_{t}  \tag{44}\\
\lambda_{t} \theta_{t}= & (1+\alpha) A\left(h_{t}+\eta h_{t-1}+\alpha h_{t}\right)^{\omega} \\
& +A \beta \eta E_{t}\left(h_{t+1}+\eta h_{t}+\alpha h_{t+1}\right)^{\omega} . \tag{45}
\end{align*}
$$

### 4.1 Steady state solution

By comparing equation (41) and (45), with market clearing condition $c_{t}=\theta_{t} h_{t}$, symmetric condition $h_{t}=H_{t}$, and equation (44), we can find the steady state labor supply:

$$
\begin{equation*}
h=\left[\frac{(1-\tau) \theta^{1-\gamma}}{A(1+\beta \eta)(1+\alpha+\eta)^{\omega}}\right]^{\frac{1}{\omega+\gamma}}, \tag{46}
\end{equation*}
$$

and the optimal tax:

$$
\begin{equation*}
\tau=\frac{\alpha}{1+\alpha+\beta \eta} \tag{47}
\end{equation*}
$$

### 4.2 Transitional dynamics

To obtain the transitional dynamics, the model is solved numerically with firstorder Taylor approximation around its steady state. Optimal tax can be tracked by:

$$
\begin{equation*}
\tau_{t}=1-\frac{A\left[(1+\alpha) h_{t}+\eta h_{t-1}\right]^{\omega}+A \beta \eta E_{t}\left[(1+\alpha) h_{t+1}+\eta h_{t}\right]^{\omega}}{\theta_{t}^{1-\gamma} h_{t}^{-\gamma}}, \tag{48}
\end{equation*}
$$

which is derived from the combination of equation (40) and (41). Calibration is the same as in section 3, except for a new parameter $\eta$ which appears here to allow for intertemporal preference on own leisure.

Experiments of four different settings about parameter pair $(\eta, \alpha)$ are investigated. They are $(\eta=-0.3, \alpha=-0.3),(\eta=0.3, \alpha=0.3),(\eta=-0.3, \alpha=0.3)$, and $(\eta=0.3, \alpha=-0.3)$, respectively. Mainly, we find countercyclical tax for the former two cases and procyclical tax for the latter two cases. This is summarized in Table 4.

|  | $\alpha=-0.3$ |  | $\alpha=0.3$ |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $\eta=-0.3$ | $\eta=0.3$ | $\eta=-0.3$ | $\eta=0.3$ |
| $\operatorname{corr}\left(\tau_{t}, y_{t}\right)$ | -0.91 | 0.50 | 0.94 | -0.81 |
| steady state of $\tau_{t}$ | -0.74 | -0.30 | 0.30 | 0.19 |

Table 4: Results of experiments for different parameter pair

The model economy is at its suboptimal as long as $\alpha \neq 0$. For $\alpha<0$ (jealousy) agents are under-working and for $\alpha>0$ (admiration) agents are over-working. Optimal tax thus is negative for the former and positive for the latter. The cyclicality of optimal tax, however, appears to be crucially affected by intertemporal preference. This is further discussed in the next subsection.

### 4.3 Discussion

The difference between a positive $\eta$, intertemporal substitution, and a negative $\eta$, intertemporal complementarity, is illustrated by the impulse response of labor hours in Figure 5. Consider first the case of no government intervention (i.e., $\tau_{t}=0$ for all $\left.t\right)$. The solid line is for the case of $\eta=-0.3$ and the dotted line is for $\eta=0.3$ through out the following figures.

For the case of intertemporal complementarity, since agent is reluctant to change leisure time dramatically, the adjustments move in an inverse hump-shaped style; for the case of intertemporal substitution, the initial response of hours worked is more negative, for agent is more willing to exchange future leisure for current


Figure 5: Impulse response of hours worked. under $\tau_{t}=0$ and $\alpha=-0.3$
leisure. The pattern is roughly the same for the case of $\alpha=0.3$, which is illustrated in Figure 6, for completeness.

Notice that the difference mainly takes place at the first two periods after shock. The subsequent behaviors are more similar. This is due to the utility setting which only specifies a one-period intertemporal preference. In a more general setting, for instance, the recursive formulation adopted in Ljungqvist and Uhlig (2000) but to leisure rather than consumption, we shall expect the IRFs to behave more differently for subsequent periods after shock, as well. We proceed to focus on the behavior of optimal tax under different parameter settings.

### 4.3.1 Case of jealousy $(\alpha=-0.3)$

When optimal tax (in the form of labor income subsidy) takes place to restore efficiency, it will subsidize more as the economy is booming due to a positive technology shock, which triggers a decrease in labor hours that is considered too


Figure 6: Impulse response of hours worked. under $\tau_{t}=0$ and $\alpha=0.3$
much. In both cases, the positive and the negative $\eta$, the initial response of optimal tax (subsidy) is a drop (rise) in the rate. However, the subsequent behavior differs greatly for different intertemporal preference settings. This can be clearly seen in the IRFs plotted in Figure 7.

For the case of intertemporal complementarity (the solid line), optimal tax drops and then return to its mean gradually; for the case of intertemporal substitution (the dotted line), however, tax initially drops and then rises above its mean, and gradually returns to the steady state level. The latter kind of dynamics results in a positive unconditional correlation between output and tax.

Interpretation The intuition goes as follows. When there is internal habits of leisure, it means that any increase in labor hours triggers relatively less disutility for individual in the next period (people "get used to it"). So when the goal of optimal tax is to raise additional labor hours, the subsidy can easily continue


Figure 7: Impulse response functions of optimal taxes under different $\eta$ and $\alpha=-0.3$


Figure 8: Impulse response functions of hours worked under different $\eta$, with or without tax and $\alpha=-0.3$
to encourage people doing so in subsequent periods after shock. However, this is not the case when there is instead a strong intertemporal substitution leisure preference, that is, the "fatigue effect." As fatigue effect takes place, higher working hours currently trigger relatively larger disutility in the next period. As a result, the government can not subsidize too much; otherwise people will end up "hating" to work too much. This suggests a smaller decrease in tax initially after a shock, followed by latter increase in tax (while still in the form of negative tax, i.e., subsidy). The IRFs of hours worked under optimal taxation for the first three periods after shock are also plotted in Figure 8. IRFs with optimal taxation are x -markered. The efficient response is less negative for both cases. The quantitative difference is found to be rather small for $\eta=0.3^{8}$

### 4.3.2 Case of admiration $(\alpha=0.3)$

On the contrary, admiration effect renders equilibrium labor supply inefficiently too high. So the government should levy positive tax on labor income to restore efficiency. The dynamics of optimal taxes are plotted in Figure 9, The dynamics of labor hours with optimal taxation are also plotted in Figure 10. Again the quantitative difference for $\eta=0.3$ is rather small. (Even though it is hard to distinguish from the plot, the x -markered IRF does lie slightly below the nogovernment case for initial and also subsequent periods.)

Interpretation Now the government's goal is to lower working hours by positive tax. For the case of internal habits, people easily get used to work less so that the tax can continue to discourage people doing so; however, for the case of fatigue

[^5]

Figure 9: Impulse response functions of optimal taxes under different $\eta$ with $\alpha=0.3$


Figure 10: Impulse response functions of hours worked under different $\eta$, with or without tax and $\alpha=0.3$
effect, the tax response should be smaller to ensure that people do not substitute too much current working hours for future hours.

To sum up, when individual features intertemporal preference on leisure, the dynamics of optimal tax play a role that "fine-tunes" the dynamics of labor hours. This is reflected in the relatively small standard deviation of tax in the case of $\eta>0$ than the case of $\eta<0$ (for both positive and negative $\alpha$ ). For example, the standard deviation of tax for case of internal habits is approximately six-time larger than that of fatigue effect under $\alpha=-0.3$. The rationale is that a strong current response in tax will trigger individual to act in a more inefficient way in the next period: when $\alpha<0$ and the government's goal is to raise hours of working, though decrease in tax can help achieve the goal currently, too much decrease in current tax will also make people more reluctant to work more in the next period, which contradicts the goal. Exactly the same logic applies to the case of $\alpha>0$. The dynamic behavior of externalities-internalizing optimal tax is concluded to be crucially contingent on the interaction between intertemporal leisure preference and the sign of externalities itself.

### 4.3.3 Risk aversion revisited

In section 3.3.1 we have shown that in addition to the sign of leisure externalities $\alpha$, the risk aversion coefficient $\gamma$ also affect the cyclicality of optimal tax. This issue is re-examined here under the context of intertemporal preference given the utility function in equation (37). The results for unconditional correlation between $\left(\tau_{t}, y_{t}\right)$ is summarized in Table 5. The overall results are consistent with the findings in section 3.3.1. When $\gamma<1$, we have procyclical labor hours that reverse all the cyclicality patterns of optimal tax given parameter pair ( $\eta, \alpha$ )

|  | $\alpha=-0.3$ |  | $\alpha=0.3$ |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: |
|  |  | $\eta=-0.3$ | $\eta=0.3$ | $\eta=-0.3$ | $\eta=0.3$ |
| $\gamma=2$ | $\operatorname{corr}\left(\tau_{t}, y_{t}\right)$ | -0.91 | 0.50 | 0.94 | -0.81 |
|  | steady state of $\tau_{t}$ | -0.74 | -0.30 | 0.30 | 0.19 |
| $\gamma=0.5$ | $\operatorname{corr}\left(\tau_{t}, y_{t}\right)$ | 0.97 | -0.67 | -0.98 | 0.92 |
|  | steady state of $\tau_{t}$ | unchanged |  |  |  |

Table 5: Results of experiments for different risk aversion coefficients

## 5 Concluding remarks

Consumption and leisure are both considered to be important components of individual utility. For the most recent decade, consumption externalities have been extensively studied. However, when it comes to leisure externalities, little works have been done despite the fact that it is also economically plausible. This paper aims to fill in this gap.

Similar to Ljungqvist and Uhlig (2000) who study consumption externalities, we also find that when leisure externalities enter into utility without lags, optimal tax is acylical, as long as individual does not feature intertemporal preference on leisure. This is because optimal tax itself serves as a measure of the degree to which labor hours deviate from its socially optimal level. Such "degree of deviation from efficiency" is constant for the case of contemporaneous externalities but has its own dynamics for the case of lag externalities. The degree of deviation from efficiency will be a function of labor hours choice today and that of yesterday if leisure externalities instead enter into utility with lags. As a result, the dynamics of labor hours will give rise to the dynamics of that deviation, or the optimal tax. If labor hours respond negatively to shock, we have the result that optimal tax is countercyclical for the case of negative leisure externalities and is procyclical for
the case of positive externalities. If labor hours response positively to shock, the pattern reverses.

Given the existing empirical evidence suggesting the presence of intertemporal preference on leisure, either as the notion of "fatigue effect" or "internal habits," we further examine an utility having both intertemporal preference and contemporaneous externalities. Optimal tax is found to have cyclical property. The results suggest that the interaction between intertemporal preference and externalities is important in determining the cyclicality of tax. In addition, we also find that the dynamics of labor hours itself plays another crucial role in the determinant of cyclicality of tax. Since such dynamics is strongly governed by the deep parameter, the risk aversion coefficient, in the stylized model economy, we have a clear dichotomical argument stating that any cyclical behavior of tax given a pair of ( $\eta, \alpha$ ) will totally reverse when $\gamma$ is changed from one range $(\gamma>1)$ to the other $(\gamma<1)$.

We conclude by discussing future research directions. First, a model with capital accumulation and with leisure externalities can be investigated. Model with capital and government spending can effectively distinguish the dynamics of consumption and output, which in turn means that tax levied on consumption, capital and on total income may serve as different policy instruments. An immediate question is to ask which of the policy instrument can give rise to the best welfare outcome. Second, the assumption of balanced government budget can be further relaxed to allow for the issuance of public debt, as in Chari, Christiano, and Kehoe (1994), so that the optimal fiscal policy will be more sophisticated. Third, the optimal taxation in response to both leisure and consumption externalities can be studied under such a rich model. When both leisure and consumption are subject to externalities, it seems that we may need two instruments to deal with each other. The resulting optimal taxation can be studied.

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[^0]:    ${ }^{1}$ For example, see Mankiw, Rotemberg, and Summers (1985), Alogoskoufis (1987), Heckman (1993), Altonji (1986) and Ball (1990).

[^1]:    ${ }^{2}$ The Frisch labor elasticity captures the substitution effect of a change in wage rate on labor supply, i.e., the elasticity of hours worked to wage rate given a constant marginal utility of consumption.

[^2]:    ${ }^{3}$ Pintea (2006) adopts a similar approach but assumes consumption and leisure to be nonseparable.

[^3]:    ${ }^{4}$ Given existing empirical evidence, they only examine the case of consumption jealousy together with keeping up with the Joneses.
    ${ }^{5}$ Another way, maybe more general, to model such externalities is to assume a (external) habit stock that is a moving average of all past average leisure times. In this paper, however, we do not adopt this strategy, according to principle of parsimony.
    ${ }^{6}$ See Abel (1990), Campbell and Cochrane (1999), among others.

[^4]:    ${ }^{7}$ Ljungqvist and Uhlig (2000) do have closed-form solution for their model with consumption externalities since their utility is linear in leisure. In this paper, however, the focus on leisure makes it undesirable to assume linear leisure in utility.

[^5]:    ${ }^{8}$ Notice that in Figure 8 for the case of $\eta=0.3$, it is observed that the IRF of hours worked without tax lies above the IRF of hours worked with optimal tax for the subsequent periods. This, however, does not mean that agent is over-working in the subsequent periods. The steady state level of hours worked without tax is itself below the level of first-best solution. Figure 8 plots the deviation from the steady state for each case, with or without tax.

