國立臺灣大學管理學院資訊管理學系

碩士論文

Department of Information Management

College of Management

National Taiwan University

Master Thesis

考慮設施種類與顧客自我選擇之公共服務場所選址與設施規劃

A Multi-types Capacitated Facility Location Problem with Customer Preferences

郭芸彤

Yun-Tung Kuo

指導教授：孔令傑 博士

Adviser: Ling-Chieh Kung, Ph.D.

中華民國 111 年 7 月

July, 2022

doi:10.6342/NTU202203368

摘要

設施選址問題長年以來受到了廣泛的討論。在一般的設施選址問題中，決策者需要決定設施的位址、以及分配哪些使用者該前往哪些設施。然而，當我們討論到服務性設施時，我們會發現使用者會對不同設施擁有不同偏好，在這種情況下，使用者的行為便不能被決策者強制決定。除此之外，現實中的設施是具有負載量限制的，這使得使用者無法選擇已經滿載的設施。再者，不同的設施可能會提供不同的服務，成為另一個設施差異性的來源。因此，將不同的服務種類納入考慮能讓我們的問題更貼近現實情況。

在我們的研究中，我們考慮一個具有不同服務類型、附載量有限、且使用者偏好各異的設施選址問題，決策者需要決定設施的位址、規模、及其所提供的服務類型，目標是在給定的預算內最大化設施的總服務人數。為解決此問題，我們建立了一個混和整數規劃模型與一個以貪婪法為底、結合最大流問題的啟發式演算法，透過數值實驗，可以看到我們的演算法能在可接受的時間範圍內得到接近最佳解的結果。

關鍵字： 設施選址，服務性設施選址，有限容量設施選址，啟發式演算法，最大流問題

doi:10.6342/NTU202203368

Abstract

The facility location problems have been widely discussed for decades. In a typical facility location problem, a decision maker decides where to build facilities among some given locations. However, when it comes to service facilities facing end consumers, customers would have different preferences toward them. In this case, whether one customer should visit one specific facility cannot be determined by the decision maker. Besides, facilities have limited capacities, so customers cannot go to the one is fully occupied. Moreover, once a facility is built, it may provide several types of services and make facilities different from each others. Therefore, taking the service types into account may make our problem closer to reality.

In our research, we consider a multi-types capacitated facility location problem with preference. The decision maker plans to choose locations and scale levels to build facilities and decide what services should they provide. The problem aims to maximize total served customers within budget constraint. We formulate a mixed integer programming model and provide a greedy-based heuristic algorithm (GSA) with maximum flow to solve this problem. In numerical study, we find that our algorithm can provide near-optimal solutions in reasonable time.

Keywords: Facility location, Service facility location, Capacitated facility location, Heuristic algorithm, Maximum flow

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Chapter 1

Introduction

1.1 Background and motivation

The facility location problems have been widely discussed for decades. In a typical facility location problem, a decision maker decides where to build facilities among some given locations. Typical objectives of the problem include profit maximization, cost minimization, etc., for the decision maker. Some facility location problems are uncapacitated, i.e., facilities are not subject to capacity constraints. On the contrary, there may be capacitated facility location problem, in which facilities have limited amount of capacity. Traditional applications lie in the fields of supply chain management (Pirkul and Jayaraman, 1998), logistics (Lu and Bostel, 2007), and operations management (Harkness and ReVelle, 2003).

When it comes to service facilities facing end consumers, several extensions for the facility location problem have been studied in recent years. One extension is to consider

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user preference, which is critical when one builds service facilities like retail stores, parks, hospitals, public bike stations, etc. Naturally, these service facilities are heterogeneous to customers, i.e., customers would have different preferences for them. In this case, whether one customer should visit one specific facility cannot be determined by the decision maker, and each of the customer will choose the most preferred facility among the built ones. Applications of facility location problem with user preference can be in many industries. For example, building parks and hospitals (Hiassat, 2017), emergency response facility (Li et al., 2011), retail stores (Hanjoul and Petters, 1987), among others.

A further extension is to incorporate the service types. In many cases, once a facility is built, it may provide several types of services and make facilities different from each others. Different services may have different characterizations, e.g. different costs or capacities, and customers may also have different interests or requirements toward them. There are many researches work on such topic in many fields. For instance, transportation (Xi et al., 2013), health care (Stummer et al., 2004), retail stores (Wang et al., 2014), etc. Therefore, taking the service types into account may make our model closer to reality.

One major motivating applications of our proposed model is to build public sport facilities. With medical technology progressing and economic growth, population aging is a widespread issue recently. To improve the welfare of an aging society, it is suggested for a government to increase the frequency and strength of regular exercise of the elder (Laforge et al., 1999). To make this happen, constructing enough public sport facilities that are appropriate to the elder is crucial. To make a good construction decision, the first step is being able to estimate the benefit of building some facilities, which may be measured by the number of elders using built facilities. An important feature of this problem is that

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the government cannot specify a facility or a service for a citizen; instead, each citizen will make her/his own choice. Whether a construction plan may really benefit citizens cannot be determined if customer preference is ignored. Note that sporting habits vary between citizens (Nagy and Tobak, 2015), and different types of sport centers or infrastructures may affect users' desires (Afthinos et al., 2005). Therefore, if the government neglects elders' service-type preferences, and assumes that all elders will use any available sport service, it will overestimate the effective capacity of a facility. Nevertheless, dealing with different sport services may lead to consideration of costs and places they take. The former affect the budget constraint and the latter have influence on the decisions of scale levels. Together makes this problem more realistic. Formulating a model that includes service-type-dependent customer preference is thus required for this application. To the best of our knowledge, no previous study simultaneously takes aforementioned factors into account. Therefore, the investigation of a multi-types capacitated facility location problem with customers' preferences contributes to the literature.

1.2 Research objectives

In this study, we consider a multi-types capacitated facility location problem with preference. A decision maker first plans to choose the locations and scale levels to build facilities from a set of candidate locations. The scale level determines the total floor area that could be taken up by services. Decision maker then chooses the types of services that each facility to provide. The types of services determine the maximal number of customers could be served in this facility. There are groups of customers with different

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population sizes locating at various places. Customers in the same group have identical preference over each service in each facility. However, they may hold different preferences over different services or different facilities. Among all built facilities and the services they provide, if some services are still available, a customer will choose her/his most preferred facility-service pair to use that service in that facility. We assume that a customer always has the option of staying at home without visiting any facility, which gives her/him a null utility. If one finds that no facility-service pair may give her/him a positive utility, she/he will stay at home. The decision maker acts to maximize the number of served customers subject to a budget constraint.

To solve this problem, we develop two different solution approaches. The first one is through formulating a mixed integer program. When one seeks for an exact solution, she/he may obtain it by solving the program using any existing algorithm (such as branch and bound). Chiang (2017) proposes a single-level model that incorporates preference and capacity based on Hanjoul and Petters' formulation (1987). Nevertheless, this model using sets to express customers' preference which has high time complexity causing the problem harder to be solved in acceptable time. Chuang (2020) reformulates the model in Chiang (2017) with the idea in Camacho-Vallejo et al. (2014a). He also further extends the model with time-dependent preferences. However, these works omit the multi-types issue. Inspired by their contributions, we formulate a single-level mixed integer program for our multi-types capacitated facility location problem with customer preferences, which are allowed to be service-type-dependent.

Given that the problem is NP-hard, in many cases a heuristic algorithm is more realistic. The proof of our problem is NP-hard will be provided in the following chapters.

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To this end, we are going to propose a heuristic algorithm and test its performance.

1.3 Research plan

In the next chapter, we will review some literature about the variants of facility location problems and maximal covering location problems, especially which considering customers' preferences. The mathematical model of the problem is described in Chapter 3, in which we will present a single-level mixed-integer model of multi-types capacitated facility location problem with customer preferences. Expected results are then described in detail in Chapter 4.

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Chapter 2

Literature Review

A facility location problem is a problem where a decision maker decides where to locate facilities and how to assign customers to those built facilities (Francis and White, 1974). The first capacitated facility location models are proposed by Kuehn and Hamburger (1963) and Balinski (1965). They aim to find the optimal locations to minimize the facility construction and customer transportation costs. Problem types vary according to the objective function and constraints. Some important examples include the p-center problem, set covering problem, and maximal covering problems.

2.1 Capacitated facility location problem

Sridharan (1995) solves this problem by proposing an "add and drop" algorithm based on Lagrangian relaxation. Similarly, Tragantalerngsak et al. (2000) propose a solution approach by combining branch and bound and Lagrangian relaxation. Melkote and Daskin (2001) propose a combined facility location design problem, which is a capacitated fa-

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cility location problem with endogenous underlying network topology. Wu et al. (2006) consider a capacitated facility location problem with general setup costs, the fixed location setup cost and the facility setup cost decided by the size of the facility built in the location. However, none of these works consider customers' self-selection.

In fact, they put emphasis on the supply side and assume that the decision maker may assign "customers" (which are almost always retail stores) to specific facilities like firms, plants, machines, etc. Instead, as we mentioned in Chapter 1, in view of the service-side problem, we consider the facility location problem including both customers' preferences and capacity constraints.

2.2 Facility location problem with preference

The facility location problem with user preference is usually formulated as a bi-level program, where the upper-level problem is for the decision maker to decide locations to build facilities and the lower-level one is for customers to choose facilities to visit. Three reformulations of the uncapacitated bi-level problem using sets to express user preference are proposed by Hanjoul and Petters (1987). Two greedy-based heuristic algorithms are also presented. Some other researchers, e.g., Hansen et al. (2004), Cánovas et al. (2007), and Vasil'ev et al. (2009), reformulate the model using similar ideas. However, the reformulation method using sets has been shown to be not so efficient.

Camacho-Vallejo et al. (2014b) present two reformulations. Using the primal-dual relationship and complementary slackness of the lower level, they obtain two linearized single-level facility location models. Camacho-Vallejo et al. (2014a) propose another

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reformulation. Adding the most preferred assignment constraint, they turn the bi-level model into a single-level one. The computational results in Casas-Ramírez and Camacho-Vallejo (2017) shows that the reformulation in Camacho-Vallejo et al. (2014a) requires less time compared to which in Camacho-Vallejo et al. (2014b) to obtain the optimal solutions. Based on the models designed by Camacho-Vallejo et al. (2014b) and Camacho-Vallejo et al. (2014a), Casas-Ramírez et al. (2018) present a valid bound of the preference-considered capacitated facility location problem but without budget constraint. They use a cross entropy method to solve the upper level problem, and then solve the lower level problem using a greedy randomized adaptive procedure. Calvete et al. (2020) further point out that focusing on number of customers instead of total demand, people can avoid the NP-hard issue of lower-level problem in Casas-Ramírez et al. (2018). They therefore propose a single-level reformulation based on duality theory without the inclusion of additional binary variables.

2.3 Maximal covering location problem with preference

Since our objective is to maximize total number of served customers, one related branch of facility location problem is maximal covering location problem. This problem usually tries to minimize number of facilities with all demand points are covered, or maximize the covered demand with a limited number of facilities. Since it was first introduced in Church and ReVelle (1974), many extensions have been proposed. Farahani et al. (2012) and García and Marín (2015) provide excellent reviews of different variants of this topic.

doi:10.6342/NTU202203368

Recently, there are some works incorporating customer preferences with covering problems. For example, Lee and Lee (2012) maximize the weighted covered demand considering customer preferences, but customers' behaviors are controlled by decision maker. They introduce a mixed integer programming model and a heuristic solution based on Lagrangian relaxation. In Casas-Ramírez et al. (2020), they propose a bi-level model to capture customers' self-selection without budget and capacity constraints. They also reformulate it into single level with the method proposed in Camacho-Vallejo et al. (2014b). Mrkela and Stanimirović (2021) further consider an uncapacitated maximal covering location problem with preference and limited budget. In their work, customers' preferences are represented as ordered sets, and a Variable Neighborhood Search is proposed as solution approach.

2.4 Capacitated facility location problem with preference to maximize total served demand

Our goal is to build several finite-capacity facilities providing several services under a budget constraint and maximize the total number of customers with service-type-dependent preferences.

To the best of our knowledge, Kim and Kim (2013) is the first one that explicitly includes both customer preference and facility capacity in a single model. They consider different preferences of customers depending on their income levels. An integer programming formulation and a Lagrangian relaxation-based heuristic algorithm are present.

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Instead of the restricted settings in Kim and Kim (2013), Chiang (2017) introduces a general form of the integer programming model. He proposes a greedy algorithm for solving that NP-hard problem. In each iteration, the benefit evaluation problem is transformed into a maximum flow problem, and the location with the highest benefit-to-cost ratio is selected.

Based on Chiang's work (2017), Chuang (2020) first constructs a time-dependent single-level mixed integer program where using sets to express customers' preferences. He then reformulates the problem with a novel approach, and modifies the algorithm proposed in Chiang (2017) by utilizing properties among neighboring solutions. These two improvements are shown to be much efficient than previous ones.

In our study, we extend the formulation in Chuang (2020) and introduce an algorithm to incorporate the service types.

2.5 The difference between previous research and our study

The main difference between our model and previous literature are summarized below.

First, the objective of out model is to maximize the total served demand within budget constraint. This is different from most facility location problems, which minimize the total cost or maximize the total profit. Mainly motivated by public service infrastructures, our study puts focus on serving as many customers as possible.

Second, in addition to the difference of objective function, the decision maker in our

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study decides how many facilities should be built based on market conditions. Therefore, our study is different from most maximal covering problems, which typically try to maximize the total served demand with exact given number of facilities.

Third, we expand the problem with service types. In general, customers would not use any available service provided by a facility. Moreover, even for the same service, customers would have various preferences if it provided by different facilities. The two issues cause demand overestimated. Therefore, taking service types into account could make the model closer to the real situation.

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Chapter 3

Problem Description and Formulation

3.1 Problem description

In this section, the problem statement and formation of our multi-types capacitated facility location problem with preference are provided.

We consider a decision maker deciding where to build facilities along with the scale levels and different service types. Let $J = \{1, 2, 3, ..., |J|\}$ denote the set of locations where a facility may be built, and $K = \{1, 2, 3, ..., |K|\}$ represent the set of scale levels that for each facility decision maker may choose from. For ease of exposition, we may call the facility built at location j as facility j from time to time. One of the decision maker's decision is to choose locations to build facilities at a certain scale level. To model this, let $y_{jk} \in \{0, 1\}$ be 1 if a facility is built at location j with scale k or 0 otherwise.

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Consider a virtual facility 0, the meaning of which will be explained later. Let $J \cup \{0\}$ be denoted as J_0. According to above definitions, we have the constraints

$$\sum_{k \in K} y_{jk} \leq 1 \quad \forall j \in J_0, \tag{3.1}$$

which ensure that the decision maker can only build at most one facility with one scale level at each location.

Let $W = \{1, 2, 3, ..., |W|\}$ be the set of service types that decision maker may choose for each facility. For ease of exposition, we may call the service with type w as service w from time to time. Another decision maker's decision is to decide what types of services should a facility provides. Consider $z_{jw} \in \{0, 1\}$ as 1 if service w is provided by facility j or 0 otherwise. Assume that H_k is the total floor area of facility with scale k and h_w is the place required for setting up service w. Again, we introduce a virtual service 0 whose meaning will be explained later. Let W_0 to represent $W \cup \{0\}$. We then have

$$\sum_{w \in W_0} h_w z_{jw} \leq \sum_{k \in K} H_k y_{jk} \quad \forall j \in J_0, \tag{3.2}$$

which state that total space taken by all provided services in the facility cannot exceed the floor area of this facility.

Building facilities and providing services are costly. Let the parameters f_{jk} and f_w be the costs of building facility at location j with scale k and providing service w, respectively. Without loss of generality, we assume that $0 \leq f_{j,1} \leq f_{j,2} \leq ... \leq f_{j,|K|}$ for all facility j.[1] If B is the decision maker's budget amount, the following constraint

$$\sum_{j \in J_0} \left(\sum_{k \in K} f_{jk} y_{jk} + \sum_{w \in W_0} f_w z_{jw} \right) \leq B, \tag{3.3}$$

[1]If the number of the scale level candidates of location j is less than $|K|$, set $f_{j,k'}$ to infinite for those $k' \in K$ which cannot be chosen for location j.

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requires that the total construction cost should not exceed the given budget B.

We define $I = \{1, 2, 3, ..., |I|\}$ as the set of customer locations. Similarly, we call customers at location i as customer i. Let $x_{ijw} \in [0, 1]$ to present the proportion of customer i served by service w in facility j. For a built facility, the space which is not taken by any service may still be utilized by some customers. We consider those empty spaces as service $\tilde{w}$. For ease of notation, we denote $W \cup \{\tilde{w}\}$ and $W \cup \{0, \tilde{w}\}$ as $\tilde{W}$ and $\tilde{W}_0$, respectively. Therefore, knowing that the total proportion of customers visiting facilities and services cannot exceed 1, we then may have the following constraints

$$\sum_{j \in J_0} \sum_{w \in \tilde{W}_0} x_{ijw} \leq 1 \quad \forall i \in I. \tag{3.4}$$

Note that x_{ijw} are fractional because facilities and services are capacitated, so it is possible for customers at the same location to make different decisions.

Suppose d_i is the population size at customer location i, and called as the demand of customer i from time to time. In addition, a service w is able to serve at most q_w customers, which considered as the capacity of service w. We have

$$\sum_{i \in I} d_i x_{ijw} \leq q_w z_{jw} \quad \forall j \in J_0, w \in W_0, \tag{3.5}$$

which ensure that the customers can only go to the location where a facility is built and a service is provided. Besides, the number of customers from all locations cannot exceed the capacity of the service.

As mentioned above, we consider those empty spaces as a special service $\tilde{w}$. We assume that service $\tilde{w}$ has cost 0 and can serve l customers per unit area. To model the

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limited capacity of $\tilde{w}$, we consider

$$\sum_{i \in I} d_i x_{i,j,\tilde{w}} \leq l \bigg(\sum_{k \in K} H_k y_{jk} - \sum_{w \in W_0} h_w z_{jw} \bigg) \quad \forall j \in J_0, \tag{3.6}$$

which is similar as constraint (3.5). The only difference is the capacity of service $\tilde{w}$ in a facility is determined as the floor area of that facility subtract the place taken by all services provided by the facility.

To model customers' preferences, assume that customer i has a preference level over service w set up in facility j, represented by p_{ijw}. We have $p_{i,j_1,w_1} > p_{i,j_2,w_2}$ if customer i prefers service w_1 in facility j_1 to service w_2 in facility j_2. For those customers at the same location, we assume that they have identical preference for the same service provided by the same facility. Recall that it is always possible that all built facilities have no attractive services for a customer, and the customer may choose to stay at home without visiting any facility. To model this, we introduce the aforementioned virtual facility, facility 0, and virtual service, service 0. This facility 0 has no construction cost (i.e., $f_{0,k} = 0$ for all $k \in K$), while this service 0 has infinite capacity (i.e, q_0 is infinite), without needing any place or cost (i.e., $h_0 = f_0 = 0$), can only be built in facility 0 (i.e., $z_{j,0} = 0$ for all $j \in J$), and zero preference level for all customers (i.e., $p_{i,0,0} = 0$ for all $i \in I$). Moreover, the special case $y_{0,k}$ for any k is always 1 since customers can decide to stay at home.

We are ready to formulate the decision maker's objective function. Recall that facility 0 and service 0 are virtual. Only J and $\tilde{W}$ include the true number of customers served by the construction plan. Therefore, the total number of customers going to built facilities to use provided services can be described as

$$\sum_{i \in I} \sum_{j \in J} \sum_{w \in \tilde{W}} d_i x_{ijw}. \tag{3.7}$$

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The decision maker's objective is to maximize such a quantity.

After facilities are built and services are provided, each customer either chooses one facility to use one service or stay at home. That decision is made according to her/his preferences. We assume that customers' preferences are exogenous; i.e., the preference over one service will not be affected by other customers' decisions or whether other facilities are built or not.

We now need to add constraints to ensure that a customer cannot go to a facility-service pair if there is another more preferred pair that is still available. To do this, we define three auxiliary variables t_{jw}, a_i, and $\bar{x}_{ijw}$. A binary variable t_{jw} is 1 if service w provided by facility j is still available in equilibrium with respect to the capacity constraints. i.e.,

$$q_w - \sum_{i \in I} d_i x_{ijw} \leq M t_{jw} \quad \forall j \in J_0, w \in W_0, \tag{3.8}$$

and

$$l\left(\sum_{k \in K} H_k y_{jk} - \sum_{w \in W_0} h_w z_{jw}\right) - \sum_{i \in I} d_i x_{i,j,\tilde{w}} \leq M t_{j\tilde{w}} \quad \forall j \in J_0, \tag{3.9}$$

where M is a positive and sufficiently large constant. If $t_{jw} = 0$, constraints (3.5) and (3.8), or constraints (3.6) and (3.9) together ensure that service w provided by facility j is fully occupied. On the contrary, if $t_{jw} = 1$, constraints (3.8) and (3.9) are relaxed, and service w provided by facility j still have residual capacity.

The variable a_i represents customer i's preference of the most preferred facility-service pair among all available ones. Therefore, we have

$$a_i \geq p_{ijw} t_{jw} \quad \forall i \in I, j \in J_0, w \in \tilde{W}_0. \tag{3.10}$$

Furthermore, we define the binary variable $\bar{x}_{ijw}$ is 1 if at least one customer in location

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i use service w provided by facility j. In effect, $\bar{x}_{ijw}$ is 1 implies that either service w provided by facility j is out of capacity in equilibrium or that facility-service pair is the most preferred out of all available ones. We may use the following constraints

$$p_{ijw}\bar{x}_{ijw} + M(1 - \bar{x}_{ijw}) \geq a_i \quad \forall i \in I, j \in J, w \in \tilde{W}_0, \tag{3.11}$$

and

$$\bar{x}_{ijw} \geq x_{ijw} \quad \forall i \in I, j \in J_0, w \in \tilde{W}_0, \tag{3.12}$$

to model customers' preference-based decision making process under capacity constraints. For the available facility-service pair which is most preferred by customer i, we have $p_{ijw} = a_i$ according to (3.10), and thus $\bar{x}_{ijw}$ can be 1. For all the remaining pairs, $\bar{x}_{ijw}$ is forced to be 0 by constraint (3.11). Constraint (3.12) then ensures that the only available facility-service pair that a customer may choose is her/his top-priority choice.

Collectively, we may formulate the decision maker's problem as

$$\begin{aligned}
\max \quad & (3.7) \\
\text{s.t.} \quad & (3.1)\text{–}(3.6) \\
& (3.8)\text{–}(3.12) \\
& x_{ijw} \geq 0 \quad \forall i \in I, j \in J_0, w \in \tilde{W}_0 && (3.13) \\
& y_{jk} \in \{0,1\} \quad \forall j \in J_0, k \in K && (3.14) \\
& z_{jw} \in \{0,1\} \quad \forall j \in J_0, w \in W_0 && (3.15) \\
& t_{jw} \in \{0,1\} \quad \forall j \in J_0, w \in \tilde{W}_0 && (3.16) \\
& a_i \geq 0 \quad \forall i \in I && (3.17) \\
& \bar{x}_{ijw} \in \{0,1\} \quad \forall i \in I, j \in J_0, w \in \tilde{W}_0. && (3.18)
\end{aligned}$$

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It is worth mentioning that, as the objective function is to maximize the number of served customers, it is acceptable for a customer to use any services provided by any facility as long as she/he obtains a positive utility (so that visiting the facility is better than staying at home). In other words, with the current objective function, a decision maker facing this problem does not need to precisely estimate all the preference levels p_{ijw}. Instead of this, the decision maker only needs to estimate whether a given preference level p_{ijw} is positive or not. In this study, we still choose to formulate the problem with p_{ijw} as a real number to retain the generality of our model.

We may show that the problem is NP-hard by reducing the problem studied in Chiang (2017) to our problem. This is trivial by observing that his problem is a special case of ours with $|W| = 0$, i.e., any facility can only provide service $\tilde{w}$. In this case, the capacity of each facility is only determined by its floor area.

Table 3.1 introduces all the decision variables mentioned above, and Table 3.2 introduces all the sets and parameters mentioned above.

3.2 An illustrative example

The following example shows how these constraints express the relationships among service types, limited area, and customer preferences. In this example, there are three locations where facilities can be built at, and two services types can be chosen from for each service. Note that we also take empty space as one service that customers may visit, which denoted as $\tilde{w}$. Suppose that customers all live in the same location, say location 1, the preferences of customers toward different facilities and different services are listed

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	Variables
x_{ijw}	A nonnegative variable that verifies the proportion of customer i using service w provided by facility j.
y_{jk}	A binary variable that verifies whether a facility is built at location j with scale level k, 1 if a facility is built or 0 otherwise.
z_{jw}	A binary variable that verifies whether a service w is provided by facility j, 1 if a service is provided or 0 otherwise.
t_{jw}	A binary variable that verifies whether service w provided by facility j is available in equilibrium, 1 if service is available or 0 otherwise.
a_i	A nonnegative variable that verifies the preference of customer i's most preferred facility-service pair among all available ones.
$\bar{x}_{ijw}$	A binary variable that verifies whether there is at least one customer in location i using service w provided by facility j, 1 if there is or 0 otherwise.

Table 3.1: Notations of variables

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Set and Parameters	
I	The set of customer locations, $I = \{1, 2, ..., \|I\|\}$.
J	The set of locations where a facility may be built, $J = \{1, 2, ..., \|J\|\}$.
K	The set of scale levels that decision maker may choose from for each facility, $K = \{1, 2, ..., \|K\|\}$.
W	The set of service types that decision maker may choose from for each facility, $W = \{1, 2, ..., \|W\|\}$.
d_i	The population size at customer location i.
f_{jk}	The fixed cost of building facility at location j with scale k.
H_k	The floor area of facility with scale k.
f_w	The fixed cost of providing service w.
h_w	The place taken by service w.
q_w	The maximal number of customers could be served by service w.
p_{ijw}	The preference of customer i on service w provided by facility j.
l	The maximal number of customers could be served per unit area which not taken by any service.
B	The total budget.
M	A very large positive number.

Table 3.2: Notations of parameters and sets

doi:10.6342/NTU202203368

in Table 3.3. We assume that the total demand of the customers at location 1 is 1.

(j, w)	$p_{1,j,w}$	(j, w)	$p_{1,j,w}$
$(1, 1)$	0.1	$(3, 1)$	-0.7
$(1, 2)$	0.2	$(3, 2)$	-0.8
$(1, \tilde{w})$	0.3	$(3, \tilde{w})$	-0.9
$(2, 1)$	0.4		
$(2, 2)$	0.5		
$(2, \tilde{w})$	0.6		

Table 3.3: The preference of the example

In this case, no one will go to facility 3 to use any service since the customers hold negative preference over pairs $(3, 1)$, $(3, 2)$, and $(3, \tilde{w})$, which is lower than the zero utility of staying at home. Therefore, facility 3 does not need to be taken into consideration. We then consider two scale levels can be chosen for each facility. If a facility is built with scale 1, its total floor area is 30 unit. On the other hand, if it is with scale 2, the total floor area is 15 unit. Assume that service 1 takes place 15 unit area and can provide capacity 0.6, while services 2 needs 10 unit area and can satisfy at most 0.5 demand. If there are spaces not taken by any facility, it is considered as empty space and provide capacity 0.01 per unit area. Suppose we build facility 1 with scale 1 and facility 2 with scale 2, we may see that the latter cannot provide both service 1 and 2 and the same time due to the area constraint (3.2). We make facility 1 provide service 1 and 2, while facility 2 only have service 1. Therefore, it remains 5 unit empty space for facility 1 and the floor area of facility 2 is fully occupied. We conclude the capacity and required area

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of each provided service in each facility in Table 3.4.

(j, w)	$p_{1,j,w}$	capacity	required area
$(1, 1)$	0.1	0.6	15
$(1, 2)$	0.2	0.5	10
$(1, \tilde{w})$	0.3	0.05	5
$(2, 1)$	0.4	0.6	15

Table 3.4: The capacity of the example

If the customers choose to be served by service 1 in facility 1, empty space in facility 1, and service 1 in facility 2 with proportion 0.35, 0.05, and 0.6 respectively, the status is presented in Table 3.5. Since pair $(1, \tilde{w})$ and $(2, 1)$ are fully occupied (i.e. $t_{1,\tilde{w}} = t_{2,1} = 0$), according to (3.8) and (3.9), the preference of the most preferred available pair among available ones is $p_{1,1,2}$ (i.e. $a_1 = 0.2$). Constraint (3.11) restricts $\bar{x}_{1,1,1}$ to be 0 since it is not the most preferred one. However, this creates a contradiction with the result given by constraint (3.12), where $\bar{x}_{1,1,1} = 1$. The above discussion is summarized in Table 3.5.

(j, w)	$p_{1,j,w}$	$x_{1,j,w}$	residual capacity	t_{jw}	$\bar{x}_{1,j,w}$ in (3.11)	$\bar{x}_{1,j,w}$ in (3.12)
$(1, 1)$	0.1	0.35	0.25	1	0	1
$(1, 2)$	0.2	0	0.5	1	1	1
$(1, \tilde{w})$	0.3	0.05	0	0	1	1
$(2, 1)$	0.4	0.6	0	0	1	1

Table 3.5: The case violating preference constraint

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The above solution cannot be a valid equilibrium outcome is due to the fact that there exists an service having residual capacity that is more preferred than one that is chosen by some customers (i.e, $p_{1,1,2} > p_{1,1,1}$ while $(j, w) = (1, 2)$ still has residual capacity). For this example, the only valid equilibrium that satisfies all preference constraints is listed in Table 3.6. In this case, the customers choose to use service 2 in facility 1, service $\tilde{w}$ in facility 1, and service 1 in facility 2 with proportion 0.35, 0.05, and 0.6, respectively. Similarly, according to constraint (3.10), the value of a_1 is 0.2, and no constraint is violated.

(j, w)	$p_{1,j,w}$	$x_{1,j,w}$	residual capacity	t_{jw}	$\bar{x}_{1,j,w}$ in (3.11)	$\bar{x}_{1,j,w}$ in (3.12)
$(1, 1)$	0.1	0	0.6	1	0	0
$(1, 2)$	0.2	0.35	0.15	1	1	1
$(1, \tilde{w})$	0.3	0.05	0	0	1	1
$(2, 1)$	0.4	0.6	0	0	1	1

Table 3.6: The case satisfying preference constraint

In short, our formulation guarantees that if there is still any service having residual capacity that a customer prefers more, the customer will go to the more preferred one instead of others. Therefore, each customer will act to maximize her/his preference.

doi:10.6342/NTU202203368

Chapter 4

The Algorithm

4.1 Overview

In this chapter, a iterative greedy heuristic algorithm, GSA, for our problem is proposed. In each iteration of our proposed algorithm, we obtain the recommended service types for each unbuilt facility with each scale level with the service selection algorithm, STSMF. After recommended service types are decided, we choose the most recommended scale level for each facility. Among all unbuilt facilities with their recommended scale levels and services, we add the facility with the better performance ratio into the construction plan. The next iteration is proceeded until the terminate conditions met.

In the following sections, we provide detailed description of our proposed algorithm, Greedy selection algorithm (GSA).

doi:10.6342/NTU202203368

4.2 Benefit evaluation

As described in Chapter 3, among the built facilities and the services they provide, customers will choose where to go according to their preference order. Therefore, suppose we have a set of built facilities with determined scale levels, represented by $y = [y_{jk}]_{j\in J,k\in K}$, and sets of services they provide, i.e., $z = [z_{jw}]_{j\in J,w\in W}$. We have to find the number of customers served by this plan, i.e., to solve the objective value $\zeta(y,z)$ of the following program

$$\zeta(y,z) = \max \quad \sum_{i\in I}\sum_{j\in J}\sum_{w\in \tilde{W}} d_i x_{ijw}$$

$$\text{s.t.} \quad (3.4)\text{–}(3.6), (3.8)\text{–}(3.13), (3.16)\text{–}(3.18),$$

which named as benefit evaluation problem. Chiang (2017) proposes a way to transform the facility location problem with given construction plan and customer preferences to a maximum flow problem. We now show how to extend this method to incorporate service types, which is not considered in his work. Given a construction plan, i.e., given y and z, we construct a directed acyclic graph whose structure is similar to that in Figure 4.1. For each customer location i, we add a customer node C_i into the graph. A source node S is created and connected to customer node $\mathrm{C_i}$ with capacity d_i for all i in I. For facility j in J, if it is built at scale level k under the given construction plan (i.e., $y_{jk} = 1$), we add a node $\mathrm{F_{jw}}$ for each service w it provides (i.e., $z_{jw} = 1$). Moreover, if facility j is built and its floor area is not fully occupied (i.e., $H_k y_{jk} - \sum_{w\in W} h_w z_{jw} > 0$), we add another node $\mathrm{F_{j\tilde{w}}}$ representing the empty space. A destination node D is created and linked from node $\mathrm{F_{jw}}$. For each edge connecting node D and node $\mathrm{F_{jw}}$, if w is not $\tilde{w}$, the edge is with capacity q_w; otherwise, the capacity of the edge is determined by the empty space

doi:10.6342/NTU202203368

capacity l and the remaining floor area, i.e., $l(H_k y_{jk} - \sum_{w \in W} h_w z_{jw})$. Finally, a link from node $\mathrm{C_i}$ to node $\mathrm{F_{jw}}$ is added with infinite capacity if $p_{ijw} > 0$.

Figure 4.1 is an example for three customers, and a construction plan that builds two facilities where one provides one service and has empty spaces, while the other provides two services and the floor area is fully occupied. According to the way we connect $\mathrm{C_i}$ and $\mathrm{F_{jw}}$, we may see, e.g., customer 1 is unwilling to get served by service 1 provided by facility 2. Once the graph is constructed, its maximum flow may be solved. For the maximum flow instance made from a construction plan with given y and z, we denote the maximized flow value as $\xi(y, z)$.

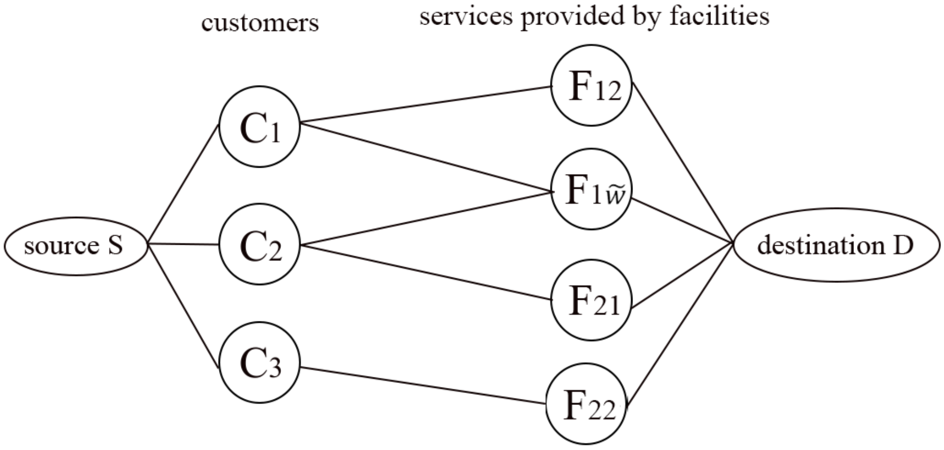

Figure 4.1: An example of the maximum flow graph

Note that the information about preference levels p_{ijw} is largely omitted in the constructed maximum flow problem: It only matters whether $p_{ijw} > 0$ or not. It is thus not surprising that the solutions to the two problems are not always the same. Nevertheless, as for our main purpose is to obtain the objective value, the constructed maximum flow problem is enough. We now show in Theorem 1 that the maximized flow value $\xi(y, z)$ of the maximum flow problem constructed given a construction plan with y and z is always

doi:10.6342/NTU202203368

identical to $\zeta(y,z)$, the objective value of the benefit evaluation problem we mentioned above.

Theorem 1. *For any construction plan with given y and z, we have $\zeta(y,z) = \xi(y,z)$.*

Proof. Given any y and z, each flow on the graph we constructed is bounded by demand of customer and capacity of service. Therefore, the capacity constraints in benefit evaluation problem are obeyed. For every node $\mathrm{C_i}$, if the maximum flow flows out according to the preference levels in equilibrium, the customers' decisions in graph are exactly same as which in problem. It turns out $\zeta(y,z) = \xi(y,z)$. Based on this maximum flow instance, assume that there is at least one flow flowing out some node $\mathrm{C_i}$ changes to the less preferred edge. If this change does not decrease $\xi(y,z)$, i.e., does not make any edge out of capacity, $\zeta(y,z) = \xi(y,z)$. If it does, this new instance is impossible to be an outcome of the maximum flow problem. In conclusion, even we allow the customers to choose the facility which is not her/his most preferred in constructed maximum flow, $\zeta(y,z) = \xi(y,z)$ still holds. □

4.3 Service types selection with maximum flow

In each iteration of GSA, for each unbuilt facility, we try to select its recommended providing services with different scale levels. To do this, given facility j and scale k, we introduce two performance ratios $Ratio_1$ and $Ratio_2$ as our selection rules.

We first define a variable $\bar{y} \in \{0,1\}$ and a function $\mathscr{f}(j,k,w,\bar{y})$. The variable $\bar{y}$ is 1 if we have selected at least one type of services for the facility which we are considering, and 0 otherwise. The function $\mathscr{f}(j,k,w,\bar{y})$ is the marginal construction cost. More precisely,

doi:10.6342/NTU202203368

if $\bar{y}$ is 1, $\not{f}(j,k,w,\bar{y})$ is f_w, the cost of the service w. On the other hand, $\not{f}(j,k,w,\bar{y})$ is $f_{jk}+f_w$, the fixed cost of building facility at location j with scale level k the service w providing service w, when $\bar{y}$ is 0. With above definitions, we now introduce two functions $\mathscr{R}_1(j,k,w,\bar{y}|y,z)$ and $\mathscr{R}_2(j,k,w,\bar{y}|y,z)$ for calculating performance ratios $Ratio_1$ and $Ratio_2$, respectively. The function $\mathscr{R}_1(j,k,w,\bar{y}|y,z)$ is the benefit-to-construction-cost ratio by building facility providing service w at location j with scale k into a construction plan, while $\mathscr{R}_2(j,k,w,\bar{y}|y,z)$ is the benefit-to-required-area ratio. Let $y^o(j_0,k_0,\bar{y}|y)$ be the original construction plan before adding facility j_0 at scale k_0 to form plan y, i.e.,

$$y^o_{jk}(j_0,k_0,\bar{y}|y)=\begin{cases} y_{jk}-1, & \text{if } (j,k)=(j_0,k_0) \text{ and } \bar{y}=0 \\ y_{jk}, & \text{otherwise} \end{cases} \qquad \forall j\in J, k\in K.$$

Similarly, let $z^o(j_0,S|z)$ be the original construction plan before selecting services from set S for facility j_0 to form plan z, i.e.,

$$z^o_{jw}(j_0,S|z)=\begin{cases} z_{jw}-1, & \text{if } j=j_0, \text{ and } w\in S \\ z_{jw}, & \text{otherwise} \end{cases} \qquad \forall j\in J, w\in \tilde{W}.$$

The ratio functions are then defined as

$$\mathscr{R}_1(j,k,w,\bar{y}|y,z)=\frac{\text{marginal objective value}}{\text{marginal construction cost}}=\frac{\zeta(y,z)-\zeta\Big(y^o(j,k,\bar{y}|y),z^o(j,\{w\}|z)\Big)}{\not{f}(j,k,w,\bar{y})},$$

and

$$\mathscr{R}_2(j,k,w,\bar{y}|y,z)=\frac{\text{marginal objective value}}{\text{marginal required area}}=\frac{\zeta(y,z)-\zeta\Big(y^o(j,k,\bar{y}|y),z^o(j,\{w\}|z)\Big)}{h_w}.$$

In each iteration of STSMF, for unbuilt facility j with scale level k, we first calculate $Ratio_1$ and $Ratio_2$ of all unchosen service types. After performance ratios are calculated,

doi:10.6342/NTU202203368

let w_1^* be the service with the highest $Ratio_1$ and w_2^* with the highest $Ratio_2$ among others. Between w_1^* and w_2^*, we choose the one increasing the objective value more into the construction plan. The next iteration is proceeded until all services have been chosen, or we run out of budget, or there is no enough floor area for any unchosen services. For convenience, we define two functions $\mathscr{F}(y,z)$ and $\mathscr{H}(j,z)$. $\mathscr{F}(y,z)$ is the total construction cost given a construction plan with y and z, i.e.,

$$\mathscr{F}(y,z) = \sum_{j\in J}\sum_{k\in K}\sum_{w\in W} f_{jk}y_{jk} + f_w z_{jw}.$$

$\mathscr{H}(j,z)$ is the requires area of services at facility j given a construction plan z, i.e.,

$$\mathscr{H}(j,z) = \sum_{w\in W} h_w z_{jw}.$$

After the recommended service types are chosen, as mentioned in Chapter 3, there is a possible construction plan for the facility that not providing any service, i.e., only provides service $\tilde{w}$. Considering this possibility, we first calculate the performance ratio of $\tilde{w}$ with function $\mathscr{R}_1(j,k,\tilde{w},\bar{y}=0|y,z)$ and compare it with the performance ratio of the recommended services. To be more specific, given a set of recommended services, say S^W, we define a new function $\mathscr{R}(j,k,S^W|y,z)$ as

$$\begin{aligned}\mathscr{R}(j,k,S^W|y,z) &= \frac{\text{marginal objective value}}{\text{marginal construction cost}}\\ &= \frac{\zeta(y,z) - \zeta\Big(y^o(j,k,\bar{y}=0|y), z^o(j,S^W|z)\Big)}{\mathscr{F}(y,z) - \mathscr{F}\Big(y^o(j,k,\bar{y}=0|y), z^o(j,S^W|z)\Big)},\end{aligned}$$

for calculating its performance ratio. Finally, if the performance ratio of $\tilde{w}$ is better, the facility only provides empty spaces; otherwise, it provides services in the set of recommended ones.

The algorithm is summarized in Algorithm 1.

doi:10.6342/NTU202203368

Algorithm 1 Service types selection with maximum flow (STSMF)

Require: y, z, j, k

$S^W \leftarrow \emptyset$, $\bar{y} \leftarrow 0$, $y_{jk} \leftarrow 1$

$aRatio \leftarrow \mathscr{R}_1(j,k,\tilde{w},\bar{y}|y,z)$

repeat

    $Ratio_1 \leftarrow 0$, $Ratio_2 \leftarrow 0$, $w_1^* \leftarrow 0$, $w_2^* \leftarrow 0$, $obj_{w_1^*} \leftarrow 0$, $obj_{w_2^*} \leftarrow 0$

    for $w \in W \setminus S^W$ **do**

        if $\mathscr{F}(y,z) + f_{jw} \leq B$ and $\mathscr{H}(j,z) + h_w \leq H_k$ **then**

            $z_{jw} \leftarrow 1$

            for $n \in \{1,2\}$ **do**

                $aRatio_n \leftarrow \mathscr{R}_n(j,k,w,\bar{y}|y,z)$

                if $aRatio_n > Ratio_n$ **then**

                    $Ratio_n \leftarrow aRatio_n$, $w_n^* \leftarrow w$, $obj_{w_n^*} \leftarrow \zeta(y,z)$

                end if

            end for

            $z_{jw} \leftarrow 0$

        end if

    end for

    $w^* \leftarrow \operatorname{argmax}_{w_n^*} obj_{w_n^*}$

    $z_{jw^*} \leftarrow 1$, $S^W \leftarrow S^W \cup \{w^*\}$, $\bar{y} \leftarrow 1$

until $(w_1^*, w_2^*) = (0,0)$

$Ratio \leftarrow \mathscr{R}(j,k,S^W|y,z)$

if $aRatio > Ratio$ **then**

    $S^W \leftarrow \{\tilde{w}\}$, $Ratio \leftarrow aRatio$

end if

return S^W, $Ratio$

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4.4 The greedy algorithm

In each iteration of our proposed algorithm, GSA, we are given an original construction plan. We decide the recommended services for each unbuilt facility and each candidate scale level with Algorithm STSMF, introduced in Section 4.3. After recommended service types are decided, we choose the scale level with the highest performance ratio for each facility. Among all unbuilt facilities with their recommended scale levels and services, we add the facility with the better performance ratio into the construction plan. The next iteration is proceeded until all facilities are built or there is not enough budget to build any new facility. The pseudocode of our proposed Greedy selection algorithm (GSA) is presented in Algorithm 2.

4.4.1 Time complexity analysis

Let V and E be the number of the nodes and edges in the graph for evaluating the objective function $\zeta(y, z)$. The numbers of V and E satisfy $|V| \leq |I| + |J|(|W| + 1) + 2$ and $|E| \leq |I| + (|I| + 1)|J|(|W| + 1)$. A classic way to solve maximum flow problem is to use the Edmonds-Karp algorithm proposed in Edmonds and Karp (1972), which is an implementation of Ford-Fulkerson method using breadth-first search in finding the augmenting path. The Edmonds-Karp algorithm provide a solution with a $O(|V||E|^2)$ bound. Therefore, in our problem, the time complexity of benefit evaluation in our algorithm is

$$O(|V||E|^2) = O\Big(|I|^2|J|^2|W|^2(|I| + |J||W|)\Big).$$

In each iteration of STSMF, i.e., Algorithm 1, it takes $O\Big(|I|^2|J|^2|W|^2(|I| + |J||W|)\Big)$

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Algorithm 2 Greedy Selection Algorithm (GSA)

$y \leftarrow 0$, $z \leftarrow 0$, $S^J \leftarrow \emptyset$
repeat
  $(j^*, k^*) \leftarrow (0, 0)$
  for $j \in J \setminus S^J$ **do**
    for $k \in K$ **do**
      $S^W_{jk} \leftarrow \emptyset$, $Ratio^W_{jk} \leftarrow 0$
      if $\mathscr{F}(y, z) + f_{jk} \leq B$ **then**
        $S^W_{jk}, Ratio^W_{jk} \leftarrow \mathrm{STSMF}(y, z, j, k)$
      end if
    end for
  end for
  $(j^*, k^*) \leftarrow \mathrm{argmax}_{(j,k)}\, Ratio^W_{jk}$
  if $(j^*, k^*) \neq (0, 0)$ **then**
    $y_{j^*k^*} \leftarrow 1$, $S^J \leftarrow S^J \cup \{j^*\}$
    if $S^W_{j^*k^*} \neq \{\tilde{w}\}$ **then**
      for $w \in S^W_{j^*k^*}$ **do**
        $z_{j^*w} \leftarrow 1$
      end for
    end if
  end if
until $(j^*, k^*) = (0, 0)$
return y, z

doi:10.6342/NTU202203368

to compute the number of the customers served by the construction plan. The algorithm will be executed for $O(|W|^2)$ iterations. Therefore, the time complexity is

$$O\Bigg(|I|^2|J|^2|W|^4(|I|+|J||W|)\Bigg).$$

For GSA, Algorithm 2 we proposed, it finds the recommended services with STSMF in each iteration. Algorithm 1 will be executed for $O(|J|^2|K|)$ times. Therefore, the total complexity is

$$O\Bigg(|I|^2|J|^4|K||W|^4(|I|+|J||W|)\Bigg).$$

4.4.2 Incremental maximum flow

In each iteration of STSMF, we try all services to add one into the construction plan with solving the maximum flow problem, which takes long computational time. However, in our algorithm, there are only slight changes of the graph comparing to which in previous iteration, so most of the flow are still the same. We only need to find new augmenting path and the backward path after adding new service nodes and edges. More precisely, adding a new edge is equivalent to changing one edge's capacity from zero to some positive number on the previous solved graph. The difference between previous maximum flow and the new one will only be determined by those vertices affected by this insertion. Therefore, whenever we add a new service into the construction plan, instead of creating a whole new graph, we add vertices and edges to the previous solved graph then continue solving the maximum flow problem.

Kumar and Gupta (2003) propose an incremental algorithm for maximum flow problem of inserting an edge in the graph. The time complexity of the algorithm is $O(|\Delta V|^2|E|)$,

doi:10.6342/NTU202203368

where $|\Delta V|$ is the number of affected vertices and $|E|$ is the number of edges. When an edge is inserted into the graph, there may exist new augmenting path from source to sink through the new edge. The affected vertices are those that lie on at least one augmenting path. That is, when we add a new service node $\mathrm{F_{jw}}$ into the construction plan, it is equivalent to insert $|I|$ edges, i.e. $(\mathrm{C_i}, \mathrm{F_{wj}})$ for all $i \in I$, to the graph and affects $|\Delta V| = O(|I|)$ nodes. Therefore, the time complexity of adding a new service node to the graph is

$$O(|\Delta V|^2|E|) = O\left(|I|^2\left(|I||J||W|\right)\right) = O(|I|^3|J||W|),$$

and the time complexity of STSMF is

$$O(|I|^3|J||W|^3).$$

For GSA, the total time complexity becomes

$$O(|I|^3|J|^3|K||W|^3).$$

The implementation with incremental maximum flow indeed reduces the time complexity of GSA.

4.5 Service types selection with maximum flow estimation

In GSA, given a construction plan with y and z, we have to calculate the corresponding number of served customers. However, considering large number of customers and facilities, Algorithm 1 requires long computational time for calculating maximum flow. Therefore, we propose a new algorithm STSMFE. It uses the estimation algorithm proposed in Chiang (2017) to evaluate the objective value and extend it with service types.

doi:10.6342/NTU202203368

With the aid of the estimation, we could replace STSMF in Algorithm 2 with STSMFE, and make the algorithm more efficient.

Let I_{jw} be the set of customers that $p_{ijw} > 0$ for service $w \in \tilde{W}$ provided by facility j and JW_i be the set of facility-service pairs (j, w) that $p_{ijw} > 0$ for customer i. For the service $w \in W$ provided by facility j, we denote $q'_{jw} = q_w$. On the other hand, for the empty space, i.e., service $\tilde{w}$, in facility j, we denote its capacity $q'_{j\tilde{w}}$ as $l(H_k y_{jk} - \sum_{w \in W} h_w z_{jw})$ for given construction plan with y and z. Therefore, we can then define two variable α_i and β_{jw} where the former is the potential demand of customer i. We define

$$\alpha_i = \min \left\{ d_i, \sum_{(j,w) \in JW_i} q'_{jw} z_{jw} \right\}.$$

Similarly, β_{jw} is the potential supply of service w provided by facility j as

$$\beta_{jw} = \min \left\{ \sum_{i \in I_{jw}} d_i, q'_{jw} z_{jw} \right\}.$$

The estimated objective value $\zeta'(y, z)$ of the construction plan y and z is the minimum of the potential demand and supply. That is

$$\zeta'(y, z) = \min \left\{ \sum_{i \in I} \alpha_i, \sum_{j \in J} \sum_{w \in \tilde{W}} \beta_{jw} \right\}.$$

In our algorithm with flow estimation reduction, we replace $\zeta(y, z)$ in Algorithm 1 with its estimated value $\zeta'(y, z)$. Therefore, we can also replace $\mathscr{R}_1(j, k, w, \bar{y}|y, z)$, $\mathscr{R}_2(j, k, w, \bar{y}|y, z)$ and $\mathscr{R}(j, k, S^W|y, z)$ with $\mathscr{R}'_1(j, k, w, \bar{y}|y, z)$, $\mathscr{R}'_2(j, k, w, \bar{y}|y, z)$ and $\mathscr{R}'(j, k, S^W|y, z)$, respectively, where

$$\mathscr{R}'_1(j, k, w, \bar{y}|y, z) = \frac{\zeta'(y, z) - \zeta'\Big(y^o(j, k, \bar{y}|y), z^o(j, \{w\}|z)\Big)}{\ell(j, k, w, \bar{y})},$$

doi:10.6342/NTU202203368

$$\mathscr{R}_2'(j,k,w,\bar{y}|y,z) = \frac{\zeta'(y,z) - \zeta'\bigg(y^o(j,k,\bar{y}|y), z^o(j,\{w\}|z)\bigg)}{h_w},$$

and

$$\mathscr{R}'(j,k,S^W|y,z) = \frac{\zeta'(y,z) - \zeta'\bigg(y^o(j,k,\bar{y}=0|y), z^o(j,S^W|z)\bigg)}{\mathscr{F}(y,z) - \mathscr{F}\bigg(y^o(j,k,\bar{y}=0|y), z^o(j,S^W|z)\bigg)}.$$

The pseudocode of the Service types selection with maximum flow estimation is presented in Algorithm 3. The only difference between Algorithms 1 and 3 is using $\zeta(y,z)$ or $\zeta'(y,z)$ for benefit evaluation in each iteration.

4.5.1 Time complexity analysis

Given a construction plan with y and z, the solution of $\zeta'(y,z)$ can be found in $O(|I||J||W|)$, and the time complexity of STSMFE is

$$O(|I||J||W|^3).$$

Therefore, the total complexity of Algorithm 2 with STSMFE is

$$O(|I||J|^3|K||W|^3).$$

which reduce the time complexity of our algorithm.

4.6 An illustrative example

In this section, we demonstrate an example of the STSMFE algorithm to better explain how it works. Given a construction plan already with service 2 provided by facility 1 ($z_{12} = 1$) with scale 1 ($y_{11} = 1$), suppose we are going to run STSMF and STSMFE for

doi:10.6342/NTU202203368

Algorithm 3 Service types selection with maximum flow estimation (STSMFE)

Require: y, z, j, k

$S^W \leftarrow \emptyset$, $\bar{y} \leftarrow 0$, $y_{jk} \leftarrow 1$

$aRatio \leftarrow \mathscr{R}'_1(j, k, \tilde{w}, \bar{y}|y, z)$

repeat

  $Ratio_1 \leftarrow 0$, $Ratio_2 \leftarrow 0$, $w_1^* \leftarrow 0$, $w_2^* \leftarrow 0$, $obj_{w_1^*} \leftarrow 0$, $obj_{w_2^*} \leftarrow 0$

  for $w \in W \setminus S^W$ **do**

    if $\mathscr{F}(y, z) + f_{jw} \leq B$ and $\mathscr{H}(j, z) + h_w \leq H_k$ **then**

      $z_{jw} \leftarrow 1$

      for $n \in \{1, 2\}$ **do**

        $aRatio_n \leftarrow \mathscr{R}'_n(j, k, w, \bar{y}|y, z)$

        if $aRatio_n > Ratio_n$ **then**

          $Ratio_n \leftarrow aRatio_n$, $w_n^* \leftarrow w$, $obj_{w_n^*} \leftarrow \zeta(y, z)$

        end if

      end for

      $z_{jw} \leftarrow 0$

    end if

  end for

  $w^* \leftarrow \operatorname{argmax}_{w_n^*} obj_{w_n^*}$

  $z_{jw^*} \leftarrow 1$, $S^W \leftarrow S^W \cup \{w^*\}$, $\bar{y} \leftarrow 1$

until $(w_1^*, w_2^*) = (0, 0)$

$Ratio \leftarrow \mathscr{R}'(j, k, S^W|y, z)$

if $aRatio > Ratio$ **then**

  $S^W \leftarrow \{\tilde{w}\}$, $Ratio \leftarrow aRatio$

end if

return S^W, $Ratio$

doi:10.6342/NTU202203368

unbuilt facility $j = 2$ with scale level $k = 2$. Moreover, we assume that there are three customers and four service types can be chosen. The budget of the construction plan $B = 60$. The customer demands is listed in Table 4.1, while facility construction costs and total floor areas are in Table 4.2. Service costs, required areas and capacities are listed in Table 4.3. In Table 4.4, we label the preference levels larger than 0 with the plus sign (+) for each customer.

customer i	demand d_i
1	6
2	8
3	10

Table 4.1: Example demands

facility j	scale k	construction cost f_{jk}	total floor area H_k
1	1	10	35
2	2	20	40

Table 4.2: Example construction costs and total floor area of facilities

Let the capacity of empty space $\tilde{w}$ to be 0.6 per area. Since in our construction plan, we have $y_{11} = 1$ and $z_{12} = 1$, there are 10 unit area of empty space for facility 1. Therefore, the capacity of service $\tilde{w}$ provided by facility 1 is $0.6 \times 10 = 6$.

We now start to run STSMF and STSMFE on this example. In iteration 1, the estimation step of first iteration, since no service is recommended by facility 2, we compute

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service w	cost f_w	required area h_w	capacity q_w
1	2	5	2
2	12	25	8
3	12	35	10
4	22	50	12

Table 4.3: Example service costs, required areas, and capacities

customer i	(j, w) : facility j providing service w						
	$(1, 2)$	$(1, \tilde{w})$	$(2, 1)$	$(2, 2)$	$(2, 3)$	$(2, 4)$	$(2, \tilde{w})$
1	+	+			+		
2			+	+	+	+	
3					+	+	+

Table 4.4: Example preferences (positive ones are marked)

the estimated flow $\zeta'(y, z)$ of all services. The estimated flow $\zeta'(y, z)$ and the performance ratios $Ratio'_1$ and $Ratio'_2$ calculated by $\mathscr{R}'_1(j, k, w, \bar{y}|y, z)$ and $\mathscr{R}'_2(j, k, w, \bar{y}|y, z)$ are listed in Table 4.5. We also list the exact objective value $\zeta(y, z)$ and its corresponding performance ratio $Ratio_1$ and $Ratio_2$ of our STSMF algorithm, which solves exact objective value $\zeta(y, z)$ with maximum flow in every iteration, in the table. Note that we do not have to consider service 4 for facility 2 since there is no enough floor area for it.

Take service 1 for example. If we add it into the construction plan, we have empty space $40 - 5 = 35$ unit area, leading to capacity $35 \times 0.6 = 21$ for service $\tilde{w}$ provided

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service w	capacity of service $\tilde{w}$	$\zeta(y,z)$	$\zeta'(y,z)$	$Ratio_1$	$Ratio'_1$	$Ratio_2$	$Ratio'_2$
1	21	18	18	$\frac{9}{11}$	$\frac{9}{11}$	$\frac{18}{5}$	$\frac{18}{5}$
2	9	23	23	$\frac{23}{32}$	$\frac{23}{32}$	$\frac{23}{25}$	$\frac{23}{25}$
3	6	22	24	$\frac{22}{16}$	$\frac{3}{4}$	$\frac{22}{35}$	$\frac{24}{35}$
$\tilde{w}$	24	16	16	$\frac{4}{5}$	$\frac{4}{5}$	-	-

Table 4.5: Estimated flows and ratios in iteration 1 of the example

by facility 2. According to customer preferences, customer 1 is willing to get served by service 2 and $\tilde{w}$ at facility 1, while customer 2 and 3 are interested in service 1 and $\tilde{w}$ provided by facility 2, respectively. The construction plan can be transformed into Figure 4.2 as a maximum flow problem we mentioned in Section 4.2. The numbers on the edges in Figure 4.2 are their capacities.

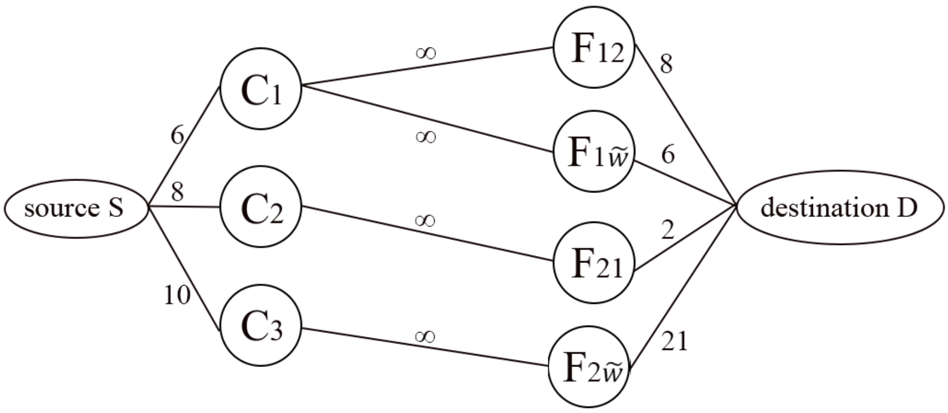

Figure 4.2: The maximum flow graph of the illustrative example

Thus, for STSMF, the maximum flow $\zeta(z,y)$ is 18. For STSMFE, the potential customer demand of each customer is $\alpha_1 = \min\{d_1, q'_{1,2} + q'_{1,\tilde{w}}\} = \min\{6, 14\}$, $\alpha_2 = \min\{d_2, q'_{2,1}\} = \min\{8, 2\}$ and $\alpha_3 = \min\{d_3, q'_{2,\tilde{w}}\} = \min\{10, 21\}$, respectively. The

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potential facility supply of each service nodes are $\beta_{1,2} = \min\{d_1, q'_{1,2}\} = \min\{6, 8\}$, $\beta_{1,\tilde{w}} = \min\{d_1, q'_{1,\tilde{w}}\} = \min\{6, 6\}$, $\beta_{2,1} = \min\{d_2, q'_{2,1}\} = \min\{8, 2\}$, and $\beta_{2,\tilde{w}} = \min\{d_3, q'_{2,\tilde{w}}\} = \min\{10, 21\}$, respectively. Therefore, the estimated flow of adding service 1 at facility 2 is $\zeta'(y, z) = \min\{\sum_{i\in I} \alpha_i, \sum_{j\in J} \sum_{w\in\tilde{W}} \beta_{jw}\} = \min\{6 + 2 + 10, 6 + 6 + 2 + 10\} = 18$. Since we have not chosen any service for facility 2 yet, $\bar{y} = 0$. Therefore, $\not{f}(j = 2, k = 2, w = 1, \bar{y} = 0) = f_{jk} + f_w = f_{22} + f_1 = 22$ and its required area h_1 is 5. We then have $Ratio_1 = Ratio'_1 = \frac{18}{22} = \frac{9}{11}$ and $Ratio_2 = Ratio'_2 = \frac{18}{5}$.

According to Table 4.5, we choose service 1 since both its performance is the highest one no matter with STSMF or STSMFE. After adding it into the construction plan, we end this iteration and proceed to the second iteration with only service 2 and 3 remain as candidates. The result is shown in Table 4.6.

service w	capacity of service $\tilde{w}$	$\zeta(y, z)$	$\zeta'(y, z)$	$Ratio_1$	$Ratio'_1$	$Ratio_2$	$Ratio'_2$
2	6	20	20	$\frac{5}{3}$	$\frac{5}{3}$	$\frac{4}{5}$	$\frac{4}{5}$
3	0	16	22	$\frac{4}{3}$	$\frac{11}{6}$	$\frac{16}{35}$	$\frac{22}{35}$

Table 4.6: Estimated flows and ratios in iteration 2 of the example

At the end of the third iteration, for using STSMF algorithm, we add service 2 into the construction plan since both $Ratio_1$ and $Ratio_2$ of service 2 are the highest. However, we find that if we apply STSMFE algorithm, we will choose service 3 since service 3 has higher $Ratio'_1$ and also has larger estimated flow value $\zeta'(y, z)$.

After the second iteration, there is no enough floor area at facility 2 to provide any other service, so both STSMF and STSMFE end. Therefore, for STSMF algorithm,

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the final recommended service types for facility 2 with scale 2 are service 1 and 2, with the exact objective value $\zeta(y, z) = 20$. On the other hand, our STSMFE algorithm recommends service 1 and 3. Notice that the actual flow value provided by these services is 16. In this example, we find that STSMFE sometimes may overestimate and choose a solution worse than STSMF.

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Chapter 5

Numerical Study

5.1 Experiment setting

To present the experimental results of the algorithm, we adopt five factors to analyze the performance under different circumstances. The first factor is the size of the problem, which considers three sizes, small, medium and large, respectively. We set $|I| = 30$, $|J| = 25$ as the small size, $|I| = 60$, $|J| = 50$ as the medium size and $|I| = 90$, $|J| = 75$ as the large size for the problem. The second factor is the number of scales which each facility has. We consider two scenarios: one is each facility has only one scale and the other is each facility has three scale levels to choose from. The third factor is the numbers of service types which are under three scenarios: problems with three service types, with six service types, and without any service, i.e., only empty spaces are allowed.

The fourth factor is customer's preference. In our experiment, the customer and facility locations are mapped onto a 2-dimensional coordinate. Let d_{ij} be the Euclidean

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distance between customer i and facility j, and we set customer i's location preference to facility j to be $\frac{1}{d_{ij}^2}$. We denote r_i^J, r_i^I, and r_{iw}^W as customer i's location preference, individual preference, and service preference of service w, respectively. Put above factors together, the preference of customer i to service w provided by facility j, i.e., p_{ijw}, is $r_i^J(\frac{1}{d_{ij}^2} + r_i^I) + r_{iw}^W$. There are two scenarios of this factor: low divergence and high divergence. The former randomly distributes r_i^J from 0.75 to 1.25, r_i^I from -0.5 to 0.5, and r_{iw}^W from -2 to 2. In the latter type, r_i^J, r_i^I, and r_{iw}^W are randomly distributed from 0.5 to 1.5, from -2 to 2, and from -10 to 10, respectively.

The last factors is the budget of the construction plan, having two types in the experiment. The tight budget is set to 40 percent of the total construction cost and the loose budget is set to 80 percent.

The above five factors generate $3 \times 2 \times 3 \times 2 \times 2 = 72$ scenarios, and we generate 30 instances for each scenario. The experiments were performed on a PC with a 3.2 GHz Intel(R) Core i7-5820K processor and 16 GB RAM. The heuristic algorithm was implemented in Spyder 4.0 using python 3.8. And the MIP model was solved using 41 Gurobi 8.1 and implemented through Gurobi python.

5.2 Solution performance

In this section, let ζ be the objective value of the solution found by GSA with STSMFE, and ζ^* be which found by mathematical model. We denote the average value of $\frac{\zeta}{\zeta^*}$ as AVG, and the minimum of it as MIN.

In Table 5.1, we find that both average and the worst performance of our algorithm

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are better when the instance size increases. When the problem size increases, the effect of making single non-optimal decision may impact less on the total objective value. Moreover, the number of iterations increase with the problem size. Therefore, even if the algorithms choose a non-optimal solution in an iteration, there are more chances for them to further improve the objective value in the following iterations.

Instance size	AVG	MIN
Small	0.9453	0.6260
Medium	0.9671	0.7744
Large	0.9763	0.8086

Table 5.1: Numerical result of problem size

Table 5.2 shows that the performance of our algorithm does not change so much in two scenarios. Typically, if a facility has multiple scale levels can choose, there are more chances to pick the non-optimal solutions. Therefore, the problem becomes more difficult. However, our algorithm has many chances to fix the non-optimal decisions. If we select the non-optimal scale which larger than the optimal one, it may cost more and sacrifice the budget of building other facilities. Even so, larger scale provides more floor area to provide more services. Similarly, if our non-optimal scale selection is smaller than the optimal one, it may bring a limitation of service types that the facility can choose from. However, smaller scale costs less, leading to more budget to build other facilities and services. As a result, the non-optimal solutions have less influence on the objective value.

In Table 5.3, we can first observe that the performance of the scenario without any service is better. In fact, if there is no service can be provided by any facility, the capacity

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Number of scale levels	AVG	MIN
1	0.9641	0.6260
3	0.9617	0.6673

Table 5.2: Numerical result of number of scale level

of each facility is only determined by their floor area, or more precisely, its scale level. Therefore, our problem is then reduced to the problem in Chiang (2017). In this scenario, since we do not have to decide z in the construction plan, it becomes an easier problem and not surprising that it has a better performance. Moreover, Table 5.3 also shows that the performance of our algorithm only slightly decreases as the number of service types increases. When there are more service types, there are more non-optimal solutions and makes it more difficult to choose the optimal one. However, more service types significantly increases the number of combinations of provided services. The number of customers' options also increases and turns out more chances that customers can still be served by other facilities and services. Therefore, number of service types only has little effect on the objective value.

Number of service types	AVG	MIN
0	0.9728	0.7791
3	0.9593	0.6960
6	0.9565	0.6260

Table 5.3: Numerical result of number of service types

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Table 5.4 indicates that the performance is better when the customers' preferences have low divergence. When customers' preferences are pretty similar, the customer behaviors are more controllable. Therefore, our algorithm could better estimate the problem and easier to get good solutions.

Preference divergence	AVG	MIN
Low	0.9970	0.7778
High	0.9288	0.6260

Table 5.4: Numerical result of customer preference

Finally, Table 5.5 shows that our algorithm performs better when the budget is loose. If we have more budget, we are able to build more facilities and provide more services. Even though the algorithm selects a non-optimal solution in an iteration, we have more chances to make customers be served by other facilities and services in the following processes. Consequently, the result of our algorithm gets closer to the optimal objective value easier when the budget is loose.

Budget	AVG	MIN
Tight	0.9502	0.6260
Loose	0.9755	0.7364

Table 5.5: Numerical result of budget

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5.3 Time performance

In this section, we test the computation time of our algorithm under different problem sizes with three experiments. First, we set the number of facilities $|J| = 50$ and the number of services $|W| = 6$. The number of customer $|I|$ increases from 60 to 240. Next, $|I|$ and $|W|$ are set to be 60 and 6, respectively, while $|J|$ is set from 50 to 200. In the last experiment, $|I|$ is fixed at 60 and $|J|$ is fixed at 50. We test the algorithm under different $|W|$ from 6 to 96. For the other factors, there are two scenarios of scale, one or three scale levels can be chosen, two scenarios of preference, low or high divergence, and two scenarios of budget, tight or loose. We generate 10 instances for each scenario.

The average of computation result of the first experiment is listed in Table 5.6. Meanwhile, Figure 5.1 visualizes the result of Table 5.6. It fits a linear function of $|I|$ with $R^2 = 0.999$. In Section 4.5.1 we claim that GSA with STSMFE has a time complexity $O(|I||J|^3|K||W|^3)$. Figure 5.1 helps us verify this result.

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$\lvert I \rvert$	Time (sec)
60	3.92
90	5.82
120	7.86
150	9.80
180	11.95
210	14.05
240	16.41

Table 5.6: Computation time of different $|I|$

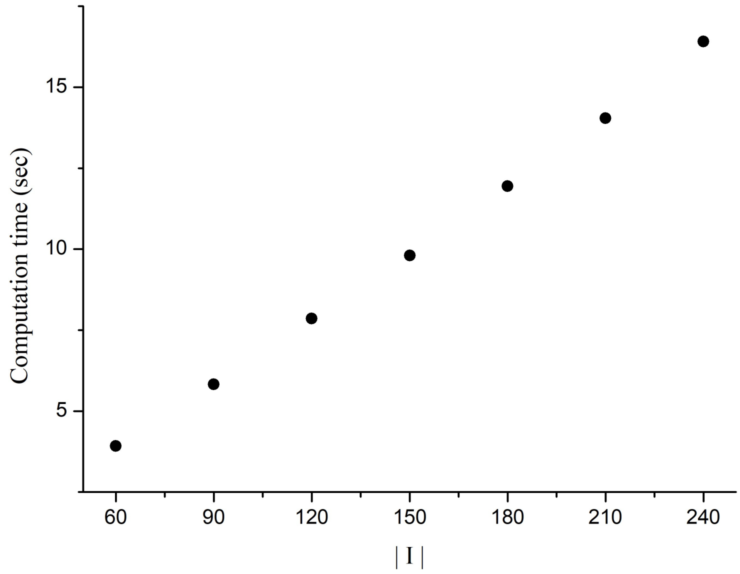

Figure 5.1: Computation time of different $|I|$

Table 5.7 and Figure 5.2 show the computation time of our algorithm under different number of facilities $|J|$, while Table 5.8 and Figure 5.3 present which under different number of services $|W|$. We find that Figures 5.2 fits a quadratic polynomial function of $|J|$ with $R^2 = 0.998$, and Figure 5.3 fits another quadratic polynomial function of $|W|$ with $R^2 = 0.997$. Although the theoretical time complexity of our algorithm is proportional to $|J|^3$ and $|W|^3$, we may see that in Algorithm 2 and 3, we remove the selected facilities and services from the candidate lists. i.e., in every iterations of GSA and STSMFE, the numbers of considered facilities and services are strictly decreasing. Therefore, our algorithm is efficient in practical and this can be supported via the quadratic fitting in Figures 5.2 and 5.3.

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$\lvert J \rvert$	Time (sec)
50	3.92
75	11.74
100	24.97
125	46.37
150	77.28
175	116.98
200	117.31

Table 5.7: Computation time of different $\lvert J \rvert$

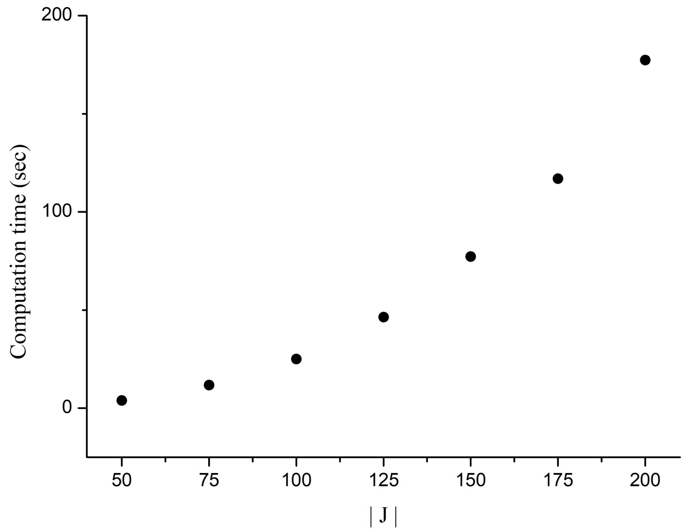

Figure 5.2: Computation time of different $\lvert J \rvert$

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$\lvert W \rvert$	Time (sec)
6	3.92
16	27.76
26	72.10
36	138.40
46	232.51
56	356.58
66	507.48
76	697.44
86	885.97
96	1101.06

Table 5.8: Computation time of different $|W|$

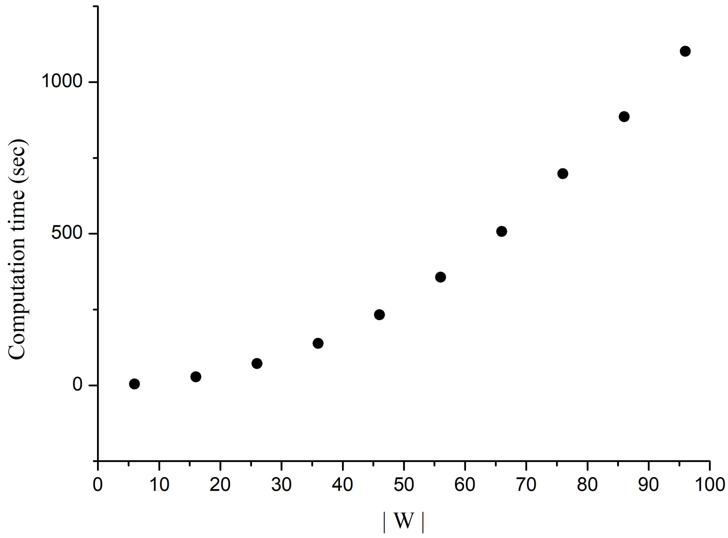

Figure 5.3: Computation time of different $|W|$

doi:10.6342/NTU202203368

Chapter 6

Conclusion

In this study, we consider a multi-types capacitated facility location problem with customer preferences. Inspired by previous literature, we formulate a single-level mixed integer program with objective to maximize the total number of served customers. Since the problem is NP-hard, we develop a greedy-based heuristic algorithms with maximum flow network and flow estimation. Through our numerical study, we find that our algorithm can provide near-optimal solutions in reasonable time.

There are several ways to extend this study. First, in our setting, the decision maker only has to consider a facility should provide a service or not, while the amount is not taken into account. For example, if the decision maker is building sport facilities, she/he determines not only whether a sport center should provide a basketball court, but also how many basketball courts should a sport center have. Extending our model and algorithm to include this feature will make them more applicable in practice. Another research direction is to investigate the proposed algorithm from a more theoretical perspective to see whether there is a worst-case performance guarantee. An investigation on this may

doi:10.6342/NTU202203368

generate theoretical contributions to the literature of discrete optimization.

doi:10.6342/NTU202203368

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