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平行檢定法於生物製劑學名藥
對等性評估之應用

# On Study of Applications of the Parallel Line Assays to Evaluation of Biosimilar Drug Products 

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## English Abstract

In recent years, biologics market increases rapidly, but the development costs are also very high. Therefore, after the patent of biological products expires, many pharmaceutical companies have invested in the development of biosimilar drug products. But biological products are different from traditional small molecule drug products. Therefore the methods for assessment of biosimilar drug products are also different. Innovators ask for clinical trials to provide the efficacy and safety data to approve biosimialr drug products. However if approval of biosimilar products requires clinical trials, then development cost of biosimilar products will be the same as that of the innovators. As a result, it cannot achieve the goal of cost reduction. Therefore we try to apply the method of the parallel line assay to test whether the approval of the biosimilar should require clinical trials and to evaluate the similarity between the biosimilar products and innovator's biological products. The results of type I error and power from the simulation studies are presented. A numerical example is used to illustrate the application of the proposed method.

Keywords: Biological products, Biosimilar, Parallel line assay, Bioequivalence, Extrapolation

## 中文摘要

近年來生物製劑的市場逐漸增加，但所開發的成本太高。所以在生物製劑的專利期到期之後，許多藥廠紛紛投入生物製劑學名藥的開發。但生物製劑有別於一般傳統的小分子學名藥，所以評估生物製劑學名藥的方法可能不同。生技原廠認為生物製劑學名藥須執行臨床試驗才能被核准。但若生物製劑學名藥須執行大型臨床試騟，則生物製劑學名藥的研發則與發展新的生物製劑相同，而不能達到降低成本造福病患的目的。所以本論文以平行檢定的方法評估生物製劑學名藥是否必須進行大型臨床試驗及提出生物製劑學名藥與原廠生物製劑藥品相似的方法，執行模擬評估所提出方法之第一型錯誤機率和檢定力，並以數值例子來介紹提出方法之應用。

關鍵字：生物製劑，生物製劑學名藥，平行檢定，生物相等性，外插

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## Chapter 1 Introduction

### 1.1 Background and Motivation

Biological drug products are therapeutic moiety manufactured by a living system or organism, such as human, plants, animals or microorganisms, and they are used for medical, prevention and cure of human diseases. It can be applied to cancer, diabetes mellitus, growth impairment, and many other diseases. Many biologic drug products are important life-saving drug products for patients with unmet medical needs. The worldwide sales of biological products in 2008 reached US $\$ 125$ billion dollars which accounts about $20 \%$ of the pharmaceutical industry.

In addition, unlike the chemical drug products, biological drug products are proteins or peptides with much larger molecular weights. In other words, biological drug products are organic compounds made of amino acids, which possess primary, secondary, tertiary, and quaternary structures. Biologic products are sensitive to heat, light and shock, and it can easily be contaminated (Keith O, 2007). Therefore, the manufacture of biological agents has to be noted, such as types of bacteria, the cells to create the protein, and the quality of production equipments. Therefore the biological
drugs are very expensive and can reach up to US $\$ 100,000$ per patient per year. This high cost denies the accessibility of most of patients to the most of life-saving biological products.

Many best-selling biological products in recent years will face patent expiration. Table 1.1 shows that several biological products have reached the end of their patent protection. Therefore, generic versions of the innovator's biological products can be made available. These generic versions of the innovator's biological products are called biosimalar products by the European Union and follow-on products in the United States. In 2008, the US Congressional Budget Office predicted a savings of US $\$ 25$ billion dollars in the next 10 years. It is therefore hoped that due to competition and avoidance of unnecessary drug testing and clinical trials, the price of the biosimilar drug products can be reduced and become affordable for most needed patients.

Biosimilars or follow-on biologics are not like the more common small-molecule chemical drugs. Biologics generally exhibit high molecular complexity and may be quite sensitive to manufacturing process changes. Table 1.2 provides comparisons of the differences between small-molecule chemical generics and biosimilar products

Biosimilar products and small molecule generics are different. Therefore, assessment of biosimilar products may also be different. The innovators of the biological products suggest that biosimilar products should not be approved without
data from large clinical trials. However if approved of biosimilar products requires clinical trials, then development cost of biosimilar products will be the same as that of the innovators. As a result, it cannot achieve the goal of cost reduction to benefit the patients.

Therefore, we try to apply the method of parallel line assay to verify whether the clinical trials are required for approval of the biosimilar products and to assess the equivalence between the innovator and biosimilar products if validated characteristics of the drug products are reliable predictors of clinical responses.

### 1.2 Bioequivalence

For small molecular chemical compounds, the measure of bioavailability include the peak concentration $\left(\mathrm{C}_{\max }\right)$ and the area under the concentration - time curve (AUC). The assessments of traditional chemical generic drugs are based on pharmacokinetics measures and the approval of traditional chemical generic drugs is based on the following Fundamental Bioequivalence Assumption (Chow and Liu, 2008)

When two drug products are equivalent in the rate and extent to which the active drug ingredient or therapeutic moiety is absorbed and becomes available at the site of drug action, it is assumed that they will be therapeutically equivalent and can be used interchangeably.

Next, we briefly review the regulations for approval of the traditional chemical generic
drug products of small molecules.

## Europe

According to European regulations two medicinal products are bioequivalent if they are pharmaceutically equivalent or pharmaceutical alternatives and if their bioavailability after administration in the same molar dose is similar to such a degree that their effects, with respect to both efficacy and safety, will be essentially the same. This can be demonstrated if the $90 \%$ confidence intervals of the means ratios between the two preparations based on $\mathrm{C}_{\text {max }}$ and AUC lie in the range of $(0.80,1.25)$ or $(-0.2231$, 0.2231 ) for the log-transformed data.

## United States

The FDA considers two products are bioequivalent if based on $\mathrm{C}_{\text {max }}, \mathrm{AUC}_{(0-\mathrm{t})}$ and $\operatorname{AUC}_{(0-\infty)}$, the $90 \% \mathrm{CI}$ of the ratio of the means of the test (e.g. generic formulation) to the reference (e.g. innovator brand formulation) is within $80.00 \%$ to $125.00 \%$.

## Taiwan

The Department of Health of Taiwan also has similar regulations for approval of generic products with respect to $\mathrm{C}_{\text {max }}, \mathrm{AUC}_{(0-\mathrm{t})}$ and $\mathrm{AUC}_{(0-\infty)}$ based on the logarithm scale. If the $90 \%$ confidence interval of the mean ratio between generic and innovator of products is within $(0.8,1.25)$, then the generic drug product is claimed to the bioequivalent to the innovator's drug product.

Since approval of the traditional chemical generic drugs is not require conducting expensive and large clinical trials, the costs of chemical generic drugs are quite inexpensive, usually $1 / 5$ to $1 / 2$ of the innovator products. Therefore many chemical generic drugs become affordable to many needed patients. On the other hand, the biological products and traditional generic drug are fundamentally different. Currently, there is no regulation to approve the follow-on products in the United States. The European Union requires the efficacy and safety data on clinical trials to approve biosimilar products. Therefore it cannot reduce the cost of biosimailar products, and cannot achieve the goal of reducing the costs to the needed patients. The main reason is that even if characteristics of the biosimilar products are equivalent to those of innovator products, it does not guarantee that the efficacy and safety of the biosimilar products are similar to those of the innovator biological products.

Table1.1: Patent status of leading biopharmaceuticals

| Product | Active <br> Substance | Company | Patent <br> Expiry <br> (year) | Worldwide sales |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  | (2005 in \$million) |  |
| Cerezyme | imiglucerase | Genzyme | 2001 | 933 |
| Humulin | human insulin | Eli Lily | 2001 | 1005 |
| Novolin | human insulin | Novo Nordisk | 2001 | 1618 |
| Intron-A | interferon | Schering-Plough | 2002 | 287 |
|  | alpha2b |  |  |  |
| Avonex | interferon-beta | Biogen Idec | 2003 | 1543 |
| Humatrope | somatropin | Eli Lily | 2003 | 414 |
| Nutropin | somatropin | Genentech | 2003 | 370 |
| Procrit | erythropoietin | J\&J | 2004 | 3324 |
| Epogen | erythropoietin | Amgen | 2004 | 2455 |
| Neupogen | figrastim | Amgen | 2006 | 1216 |
|  |  |  | Total | $\mathbf{1 3 , 1 6 5}$ |

Source: Generic Pharmaceutical Association and Company Reports(2005)

Table1.2: The differences between small-molecule generics and biosimilar products.

|  | Small molecule generics | Biosimilars |
| :---: | :---: | :---: |
| Product characteristics | - Small molecules <br> - Often very stable <br> - Mostly without a device | - Large, complex molecules <br> - Stability requires special handling <br> - Device is often a key differentiator |
| Production | - Produced by chemical synthesis | - Produced in living organisms <br> - Highly sensitive to manufacturing changes <br> - Often comparatively high costs |
| Regulation | - Abbreviated registration procedures in Europe and US <br> - Usually enjoy "substitutability" status | - Regulatory pathway now defined by EMEA <br> - "Comparability" status <br> - No pathway yet in US under BLA |
| Marketing | - No or limited detailing to physicians <br> - Key role of wholesalers and payors <br> - Market substitution in pharmacies <br> - High price discounts | - Detailing to (specialist) physicians required <br> - Pharmacists may not substitute <br> - Price discounts smaller; price sensitivity is product specific |

http://www.sandoz.com/site/en/product_range/more_about_biosimilars/index.shtml

## Chapter 2 Literature Review

In this chapter, we briefly review the method of assessment of the bioequivalence of traditional chemical generic drugs and the parallel line assay.

### 2.1 Interval Hypotheses

The assessment of average bioequivalence is based on the comparison of bioavailability profiles between formulations. However, in practice, it is recognized that no two formulations will have exactly the same bioavailability profiles. Therefore, if the profiles of the two formulations differ by less than a (clinically) meaningful limit, the profiles of the two formulations may be considered equivalent. Following this concept, Schuirmann (1981) first introduced the use of interval hypotheses for assessing average bioequivalence.

Let $\mu_{T}$ be the average bioavailability of the test ( T ) formulation and $\mu_{R}$ the average bioavailability of the reference ( R ) formulation. The interval hypotheses for average bioequivalence on the log-scale can be formulated as

$$
\left\{\begin{array}{l}
H_{0}: \mu_{T}-\mu_{R} \leq \theta_{L} \text { or } \mu_{T}-\mu_{R} \geq \theta_{U}  \tag{2.1}\\
\text { vs. } \\
H_{a}: \theta_{L}<\mu_{T}-\mu_{R}<\theta_{U}
\end{array}\right.
$$

Where $\theta_{L}=-\theta_{U}=-0.2231$ are some clinically meaningful limits.

The concept of interval hypotheses Eq. (2.1) is to show average bioequivalence by rejecting the null hypothesis of inbioequivalence.

## Schuirmann's Two One-Sided Tests Procedure

The interval hypotheses (2.1) can be decomposed into two sets of one-sided hypotheses

$$
\begin{gather*}
\mathrm{H}_{01}: \mu_{\mathrm{T}}-\mu_{\mathrm{R}} \leq \theta_{\mathrm{L}} \text { vs. } \mathrm{H}_{\mathrm{a} 1}: \mu_{\mathrm{T}}-\mu_{\mathrm{R}}>\theta_{\mathrm{L}} \\
\text { and }  \tag{2.2}\\
\mathrm{H}_{02}: \mu_{\mathrm{T}}-\mu_{\mathrm{R}} \geq \theta_{\mathrm{U}} \text { vs. } \mathrm{H}_{\mathrm{a} 2}: \mu_{\mathrm{T}}-\mu_{\mathrm{R}}>\theta_{\mathrm{U}}
\end{gather*}
$$

The first set of hypotheses is to verify that the average bioavailability of the test formulation is not too low, whereas the second set of hypotheses is to verify that the average bioavailability of the test formulation is not too high. A relatively low (or high) average bioavailability may refer to the concern of efficacy (or safety) of the test formulation. If one concludes that $\theta_{L}<\mu_{T}-\mu_{R}$ (i.e., reject $\mathrm{H}_{01}$ ) and $\mu_{T}-\mu_{R}<\theta_{U}$ (i.e., reject $\mathrm{H}_{02}$ ), then it has been concluded that

$$
\theta_{L}<\mu_{T}-\mu_{R}<\theta_{U}
$$

$\mu_{\mathrm{T}}$ and $\mu_{\mathrm{R}}$, thus, are equivalent. The rejection of $\mathrm{H}_{01}$ and $\mathrm{H}_{02}$, which leads to the conclusion of average bioequivalence, is equivalent to rejecting $\mathrm{H}_{0}$ in Equation 2.1. Schuirmann (1981, 1987) first introduced the two one-sided tests procedure based on Equation 2.2 for assessing average bioequivalence between formulations. The proposed two one-sided tests procedure suggests the conclusion of equivalence of $\mu_{\mathrm{T}}$ and $\mu_{\mathrm{R}}$ at
the a level of significance if, and only if, $\mathrm{H}_{01}$ and $\mathrm{H}_{02}$ in Equation 2.2 are rejected at a predetermined a level of significance. Under the normality assumptions, the two sets of one-sided hypotheses can be tested with ordinary one-sided $t$ tests. We conclude that $\mu_{\mathrm{T}}$ and $\mu_{\mathrm{R}}$ are average equivalent if

$$
\mathrm{T}_{\mathrm{L}}=\frac{\left(\overline{\mathrm{Y}}_{\mathrm{T}}-\overline{\mathrm{Y}}_{\mathrm{R}}\right)-\theta_{\mathrm{L}}}{\hat{\sigma}_{\mathrm{d}} \sqrt{\frac{1}{\mathrm{n}_{\mathrm{R}}}+\frac{1}{\mathrm{n}_{\mathrm{T}}}}}>\mathrm{t}\left(\alpha, \mathrm{n}_{\mathrm{R}}+\mathrm{n}_{\mathrm{T}}-2\right)
$$

and

$$
\mathrm{T}_{\mathrm{U}}=\frac{\left(\overline{\mathrm{Y}}_{\mathrm{T}}-\overline{\mathrm{Y}}_{\mathrm{R}}\right)-\theta_{\mathrm{U}}}{\hat{\sigma}_{\mathrm{d}} \sqrt{\frac{1}{\mathrm{n}_{\mathrm{R}}}+\frac{1}{\mathrm{n}_{\mathrm{T}}}}}<-\mathrm{t}\left(\alpha, \mathrm{n}_{\mathrm{R}}+\mathrm{n}_{\mathrm{T}}-2\right)
$$

where $\bar{Y}_{T}, \bar{Y}_{R}$ and $\hat{\sigma}_{d}$ are the least squares estimator of the test and reference means and square root of error mean square.

The two one-sided $t$ tests procedure is operationally equivalent to the classic (shortest) confidence interval approach; that is, if the classic (1-2 $\alpha$ ) $100 \%$ confidence interval for $\mu_{\mathrm{T}}$ and $\mu_{\mathrm{R}}$ is within $\left(\theta_{\mathrm{L}}, \theta_{\mathrm{U}}\right)$, then both $\mathrm{H}_{01}$ and $\mathrm{H}_{02}$ are also rejected at the $\alpha$ level by the two one-sided t tests procedure (Chow and Liu, 2008).

### 2.2 Two-by-Two Crossover Design

Several useful designs for assessing bioequivalence in variety of situations were discussed in literature (Chow and Liu, 2008). The most commonly used statistical design for comparing average bioavailability between two formulations of a drug
probably is a two-sequence, two-period, crossover design, we shall refer to this design as the standard $2 \times 2$ crossover design. For the standard $2 \times 2$ crossover design, each subject is randomly assigned to either sequence RT or sequence TR at two dosing periods. In other words, subjects within $R T(T R)$ receive formulation $R(T)$ at the first dosing period and formulation $\mathrm{T}(\mathrm{R})$ at the second dosing period. The dosing periods are separated by a washout period of sufficient length for the drug received in the first period to be completely metabolized or excreted from the body. An example of a $2 \times 2$ crossover design is illustrated in Figure 2.1.

Because the biological products and traditional small molecular chemical generic drugs are fundamentally different, therefore the methods for evaluation of bioequivalence based on pharmacokinetics measures are not applicable to assess the biosimilar products. As a result, we propose to employ the parallel line assay to evaluate biosimilar products. In the next two sections, biological assay and parallel line assay will briefly reviewed.

### 2.3 Biological assay

Bioassay is a preparation (such as drugs, hormones, etc.) added to the subject. It is the method to determine the nature, constitution, and potency of the preparation by the reaction of subjects. The preparation added to the subject can have a single dose, or
several different doses. Therefore the subjects produce a response value or multiple response values, we can test the potency of the preparation by these values. The purpose of testing is usually to compare the relative potency of two or more preparations.

So we always take a common nature of the preparation known as standard preparation, while the other is the unknown nature of the newly developed preparation, referred to as test preparation. The two preparations produce the difference between response values in the same dose, i.e., for inference of the potency of the test preparation. Generally we use the indirect assay. Indirect assay consists of quantitative responses and quantal responses. Indirect assay with quantitative responses also involves parallel line assay and slope ratio assay.

For our current research, we propose to use the parallel line assay to evaluation of the similarity of biosimilar products and its innovator's biological products. In next section we briefly reviewed parallel line assay.

### 2.4Parallel Line Assay

Liu and Chow (2010) refer to group means of a well-defined product characteristic can be computed for each dose level for the biosimilar and innovator's product respectively. Using the group means of the well-defined characteristic as the independent variable, a simple linear regression equation can be fitted to primary
efficacy endpoint for the biosimilar and innovator's biological products, respectively. It follows that the concept of relative potency in the parallel-line bioassay can be then employed to investigate extrapolability of equivalence in product characteristic to equivalence in efficacy.

Parallel line assay is to compare potency of two preparations under a linear regression model. The first step is to test for a linear regression model. If the relationship between the product characteristic and clinical responses are linear, then the second step is to test whether regression lines are parallel. In the setting, the innovative biological product is treated as the standard preparation, and the biosimilar product is referred to as the test preparation.

Let the mean characteristics at different doses be the independent variable (x), and clinical efficacy response be the dependent variable (Y). Then the regression model of the reference preparation is given as

$$
\begin{equation*}
E\left(Y_{R}\right)=\alpha+\beta z^{\lambda}=\alpha+\beta x \tag{2.3}
\end{equation*}
$$

Where $\alpha, \beta, \lambda$ are unknown constants, and $x=z^{\lambda}$

When $\lambda=0, x=\log z$,

$$
\begin{equation*}
E\left(Y_{R}\right)=\alpha+\beta \log z=\alpha+\beta x . \tag{2.4}
\end{equation*}
$$

Under the same reaction, the ratio of the two preparation doses is called the relative potency ( $\rho$ )

$$
\rho=\frac{\mathrm{Z}_{\mathrm{S}}}{\mathrm{Z}_{\mathrm{T}}}, \mathrm{Z}_{\mathrm{S}}=\rho \mathrm{Z}_{\mathrm{T}}
$$

It follows that

$$
\begin{align*}
& E\left(Y_{\mathrm{T}}\right)=\alpha+\beta(\rho z)^{\lambda}=\alpha+\beta \rho^{\lambda} \mathrm{z}^{\lambda} \\
& \quad=\alpha+\beta \rho^{\lambda} \mathrm{x} . \tag{2.5}
\end{align*}
$$

When $\lambda=0$

$$
\begin{align*}
& E\left(Y_{\mathrm{T}}\right)=\alpha+\beta(\log \rho+\log z)=\alpha+\beta \log \rho+\beta \log z \\
& \quad=\alpha^{\prime}+\beta \mathrm{x} \tag{2.6}
\end{align*}
$$

It is also a linear function. Therefore the regression line for the response preparation in
(2.4) and that for the test preparation in (2.6) have same slope ( $\beta$ ) but different intercepts.

### 2.4.1 Testing linearity of a regression model

For lack of fit test for a linear regression model, multiple efficacy responses must observed for each mean product characteristic. The data structure for testing linearity is given Table 2.1.

The corresponding ANOVA table for lack of fit test is given in Table 2.2.
where $n=\sum n_{i}, \quad y=\sum \sum y_{i j}, T_{i}=\sum_{j=1}^{n_{i}} y_{i j}, x=\sum n_{i} x_{i}$

$$
\begin{aligned}
& S_{x x}=\sum n_{i} x_{i}^{2}-\frac{x^{2}}{n} \\
& S_{x y}=\sum \sum x_{i} y_{i j}-\frac{x y_{n}}{n} \\
& S_{y y}=\sum \sum y_{i j}^{2}-\frac{y_{.2}^{2}}{n}
\end{aligned}
$$

The F value of lack of fit test is $F_{0}=\frac{M S L}{M S P}$

If $F_{0}<F_{\alpha, k-2, n-k}$, then we fail to reject the null hypothesis of no lack of fit.

### 2.4.2 Testing for Parallel Lines

Next, we want to know whether two linear regression lines are parallel. In other words we need to check whether the slopes are equal between the two regression lines. The linear regression models for the test and reference preparation are given as respectively.

$$
Y_{R}=\alpha_{R}+\beta_{R} x_{R}+\varepsilon_{R},
$$

and

$$
Y_{T}=\alpha_{T}+\beta_{T} x_{T}+\varepsilon_{T} .
$$

The corresponding estimated linear regression equations are given as respectively.

$$
\begin{gathered}
\hat{Y}_{R}=a_{R}+b_{R} x_{R} \text { and } \hat{Y}_{T}=a_{T}+b_{T} x_{T}, \\
\text { where } b_{R}=\frac{{ }_{R} S_{x y}}{{ }_{R} S_{x x}^{*}}, a_{R}=\bar{y}_{R}-b_{R} \bar{x}_{R} . \\
b_{T}=\frac{{ }_{T} S_{x y}}{{ }_{T} S_{x x}}, a_{T}=\bar{y}_{T}-b_{T} \bar{x}_{T} .
\end{gathered}
$$

${ }_{R} S x y$ and ${ }_{T} S x y$ are the connected sums of cross products between mean characteristic sand clinical responses for reference and test preparations, similarly ${ }_{R} S x x$ and ${ }_{T} S x x$ are the sums of squares of the mean characteristics of the reference and test preparations, $\bar{y}_{R}$ and $\bar{y}_{T}$ are the average clinical responses for the reference and test preparations, and $\bar{x}_{R}$ and $\bar{x}_{T}$ are the average of the mean characteristics for the reference and test preparations, respectively.

The hypothesis of equal slopes is given as
$H_{0}: \beta_{R}=\beta_{T}$ v.s. $H_{a}: \beta_{R} \neq \beta_{T}$.

Let $\mathrm{S}_{\mathrm{R}}{ }^{2}$ and $\mathrm{S}_{\mathrm{T}}{ }^{2}$ be the residue mean squares for the reference and test preparations respectively as follows
$S_{R}^{2}=\frac{{ }_{R} S y y-b_{R} \cdot{ }_{R} S x y}{n_{R}-2}$, and
$S_{T}^{2}=\frac{{ }_{T} S y y-b_{T} \cdot{ }_{T} S x y}{n_{T}-2}$.
If $F=\frac{\max \left\{S_{R}^{2}, S_{T}^{2}\right\}}{\min \left\{S_{R}^{2}, S_{T}^{2}\right\}}<F\left(\alpha, n_{R}-2, n_{T}-2\right)$
We fail to reject the null hypothesis of homogeneity of the error variances between the reference and test formulations.

If the residual variances of the reference and test preparation are homogeneous, the test statistic for equality of the two slopes is given as
$t=\frac{b_{R}-b_{T}}{\sqrt{V\left(b_{R}-b_{T}\right)}}$,
where $\hat{V}\left(b_{R}-b_{T}\right)=\hat{V}\left(b_{R}\right)+\hat{V}\left(b_{T}\right)=S_{p}^{2}\left(\frac{1}{{ }_{R} S x x}+\frac{1}{{ }_{T} S x x}\right) \quad$,
and $S_{p}^{2}=\frac{\left(n_{R}-2\right) S_{R}^{2}+\left(n_{T}-2\right) S_{T}^{2}}{n_{R}+n_{T}-4}$

If $\mathrm{t}>\mathrm{t}\left(\alpha / 2, \mathrm{n}_{\mathrm{R}}+\mathrm{n}_{\mathrm{T}}-4\right)$, the null hypothesis of equal slope is rejected at the $\alpha$ significance level.

If the null hypothesis of equal slopes is not rejected at the $\alpha$ significance level, then the common slope is estimated as:

$$
b_{c}=\frac{{ }_{R} S x y+{ }_{T} S x y}{{ }_{R} S x x+{ }_{T} S x x}
$$

It follow that estimated linear regression lines for the reference and test preparation are given as respectively,

$$
\begin{aligned}
& \hat{y}_{R}=a_{R}+b_{c} x_{R} \\
& \hat{y}_{T}=a_{T}+b_{c} x_{T}
\end{aligned}
$$

### 2.4.3 Relative Potency

Assume that the test preparation is more potent than the reference preparation, then the regression line of the test preparation is above that of the clinical reference preparation. When responses of two drugs are the same, let the dose of the test preparation be $\mathrm{x}_{0}$.

Denote $\Delta$ as the horizontal distance between the two regression lines. Then $\mathrm{x}_{0}$ dose of the test preparation effect is the same with the $x_{0}+\Delta$ dose of the reference preparation effect under the $\log _{10}$ scale. Therefore the $10^{\Delta}$ dose of the test preparation effect is the same with the $10^{x_{0}+\Delta}=10^{x_{0}+\log R}$ dose of the reference preparation effect under the original scale. $10^{x_{0}+\log ^{R}}=10^{x_{0}} \cdot 10^{\log _{10} R}=R \cdot 10^{x_{0}}$, where $\Delta=\log _{10} R$. The left hand side of the above equation is the dose of test preparation where the right hand side of the above equation is the dose of reference preparation multiplied by R. In other words, the potency of the test preparation is R times of the reference preparation.

An estimate of $\Delta$ is given as

$$
\begin{gathered}
\hat{\Delta}=\frac{a_{R}-a_{T}}{b_{c}}=\bar{x}_{T}-\bar{x}_{R}+\frac{\bar{y}_{R}-\bar{y}_{T}}{b_{c}} \\
\hat{R}=\text { anti } \log _{10} \hat{\Delta}=10^{\hat{\Delta}}
\end{gathered}
$$

## The confidence interval of Relative Potency

Let $\left(x_{1}, x_{2}\right)$ is bivariate normal distribution．Its expectation is $\left(\mu_{1}, \mu_{2}\right)$ with variance $\left(\sigma^{2} v_{11}, \sigma^{2} v_{22}\right)$ ，and covariance $\sigma^{2} v_{12}$（沈明來，2000）．

If $x_{1}, x_{2}$ are the sample mean，then $v_{11}=\frac{1}{n_{1}}, v_{22}=\frac{1}{n_{2}} 。$
If $x_{1}, x_{2}$ are the regression slopes，then $v_{11}=\frac{1}{\sum\left(x_{1}-\bar{x}_{1}\right)^{2}}, v_{22}=\frac{1}{\sum\left(x_{2}-\bar{x}_{2}\right)^{2}}$
Let $\theta=\frac{\mu_{1}}{\mu_{2}}$ ，its estimator is $\hat{\theta}=\frac{x_{1}}{x_{2}}$ ，and $S^{2}$ is an estimate of $\sigma^{2}$
Let $x_{1}-\hat{\boldsymbol{\theta}} x_{2}$ ．The variance of $x_{1}-\hat{\theta} x_{2}$ is given as
$\operatorname{Var}\left(x_{1}-\hat{\theta} x_{2}\right)=\sigma^{2}\left(v_{11}-2 \theta v_{12}+\theta^{2} v_{22}\right)$

It follows that
$x_{1}-\hat{\theta} x_{2} \sim N\left(0, \sigma^{2}\left(v_{11}-2 \theta v_{12}+\theta^{2} v_{22}\right)\right)$

Therefore $\frac{x_{1}-\hat{\theta} x_{2}}{\sqrt{S^{2}\left(v_{11}-2 \theta v_{12}+\theta^{2} v_{22}\right)}}$ ，follows a central t－distribution with $n_{R}+n_{T}=4$ degrees of freedom．

The probability statement of constructing an $100(1-\alpha) \%$ confidence interval for $\theta$ is given as
$\operatorname{Pr}\left\{\frac{\left(x_{1}-\hat{\theta} x_{2}\right)^{2}}{S^{2}\left(v_{11}-2 \theta v_{12}+\theta^{2} v_{22}\right)} \leq t_{\alpha / 2}^{2}\right\}=1-\alpha$
Where $t_{\alpha / 2}$ is the upper $\alpha / 2$ quantifile of a central distribution with $n_{R}+n_{T}-4$ degrees of freedom.

It turns out that it is a quadratic equation of $\hat{\theta}$ as expressed in the following
$\hat{\theta}^{2}\left(x_{2}^{2}-t_{\alpha / 2}^{2} S^{2} v_{22}\right)-2 \hat{\theta}\left(x_{1} x_{2}-t_{\alpha / 2}^{2} S^{2} v_{12}\right)+\left(x_{1}^{2}-t_{\alpha / 2}^{2} S^{2} v_{11}\right) \leq 0$

The two roots of the above quadratic equation constitute the lower and upper limits of the $100(1-\alpha) \%$ confidence interval for $\theta$ as
$\hat{\theta}=\frac{1}{x_{2}^{2}-t^{2} S^{2} v_{22}}\left\{\left(x_{1} x_{2}-t^{2} S^{2} v_{12}\right) \pm \sqrt{\left(x_{1} x_{2}-t^{2} S^{2} v_{12}\right)^{2}-\left(x_{2}^{2}-t^{2} S^{2} v_{22}\right)\left(x_{1}^{2}-t^{2} S^{2} v_{11}\right)}\right\}$
It can also be rearranged as
$\hat{\theta}=\frac{1}{1-\frac{t^{2} S^{2} v_{22}}{x_{2}^{2}}}\left\{\left(\frac{x_{1}}{x_{2}}-\frac{t^{2} S^{2} v_{12}}{x_{2}^{2}}\right) \pm \sqrt{\left(\frac{x_{1}}{x_{2}}-\frac{t^{2} S^{2} v_{12}}{x_{2}^{2}}\right)^{2}-\left(1-\frac{t^{2} S^{2} v_{22}}{x_{2}^{2}}\right)\left(\frac{x_{1}^{2}}{x_{2}^{2}}-\frac{t^{2} S^{2} v_{11}}{x_{2}^{2}}\right)}\right\}$
Let $g=\frac{t^{2} S^{2} v_{22}}{x_{2}^{2}}, \hat{\theta}=\frac{x_{1}}{x_{2}}$
To get $\frac{1}{1-g}\left\{\hat{\theta}-g \frac{v_{12}}{v_{22}} \pm \frac{t S}{x_{2}} \sqrt{(1-g) v_{11}-2 \hat{\theta} v_{12}+\hat{\theta}^{2} v_{22}+g \frac{v_{12}^{2}}{v_{22}}}\right\}$
If $\mathrm{x}_{1}$ and $\mathrm{x}_{2}$ are independent, $v_{12}=0$
It follows that $\frac{1}{1-g}\left\{\hat{\theta} \pm \frac{t S}{x_{2}} \sqrt{(1-g) v_{11}+\hat{\theta}^{2} v_{22}}\right\}$.

Let $x_{1}=\bar{y}_{T}-\bar{y}_{R}, x_{2}=b_{c}$
$v_{11}=\frac{1}{n_{T}}+\frac{1}{n_{R}}$
$v_{22}=\frac{1}{\sum\left(x_{T}-\bar{x}_{T}\right)^{2}+\sum\left(x_{R}-\bar{x}_{R}\right)^{2}}$
and $\hat{\Delta}-\left(\bar{x}_{R}-\bar{x}_{T}\right)=\frac{\bar{y}_{T}-\bar{y}_{R}}{b_{c}}=\hat{\theta}$
A $100(1-\alpha) \%$ confidence interval of $\Delta$ is given as

$$
\begin{equation*}
\left[\left(\bar{x}_{R}-\bar{x}_{T}\right)+\frac{1}{1-g}\left\{\left(\hat{\Delta}-\bar{x}_{R}+\bar{x}_{T}\right) \pm \frac{t_{a / 2} S}{b_{c}} \sqrt{(1-g)\left(\frac{1}{n_{R}}+\frac{1}{n_{T}}\right)+\frac{\left(\hat{\Delta}-\bar{x}_{R}+\bar{x}_{T}\right)^{2}}{\sum\left(x_{T}-\bar{x}_{T}\right)^{2}+\sum\left(x_{R}-\bar{x}_{R}\right)^{2}}}\right\}\right] \tag{2.8}
\end{equation*}
$$

If $\mathrm{g}<0.1$, then it can be ignored in the equation (2.8), an $100(1-\alpha) \%$ confidence interval of $\Delta$ which is given as
$\Delta_{U}, \Delta_{L}=\left(\bar{x}_{R}-\bar{x}_{T}\right)+\left\{\left(\hat{\Delta}-\bar{x}_{R}+\bar{x}_{T}\right) \pm \frac{t_{\alpha_{2}} S}{\left|b_{c}\right|}\left[\left(\frac{1}{n_{R}}+\frac{1}{n_{T}}\right)+\frac{\left(\hat{\Delta}-\bar{x}_{R}+\bar{x}_{T}\right)^{2}}{\sum\left(x_{R}-\bar{x}_{R}\right)^{2}+\sum\left(x_{T}-\bar{x}_{T}\right)^{2}}\right]^{\frac{1}{2}}\right\}$
If $\bar{x}_{R}=\bar{x}_{T}$ then equation (2.9) can be reduced to
$\Delta_{U}, \Delta_{L}=\hat{\Delta} \pm \frac{t_{\alpha /} S}{\left|b_{c}\right|}\left[\left(\frac{1}{n_{R}}+\frac{1}{n_{T}}\right)+\frac{\hat{\Delta}^{2}}{\sum\left(x_{R}-\bar{x}_{R}\right)^{2}+\sum\left(x_{T}-\bar{x}_{T}\right)^{2}}\right]^{\frac{1}{2}}$

Table 2.1: The data structure for testing linearity.

| Independent <br> variable | $x_{1}$ | $x_{2}$ | $\ldots$ | $x_{k}$ |
| :--- | :---: | :---: | :---: | :---: |
| Dependent <br> variable | $y_{11}$ | $y_{21}$ | $\ldots$ | $y_{k 1}$ |
|  | $y_{12}$ | $y_{22}$ | $\ldots$ | $y_{k 2}$ |
|  | $\cdot$ | $\cdot$ |  | $\cdots$ |
|  | $\cdot$ | $\cdot$ |  | $\cdots$ |
| Sum | $y_{1 n_{1}}$ | $y_{2 n_{2}}$ | $\ldots$ | $y_{k n_{k}}$ |
| Replications | $T_{1}$ | $T_{2}$ | $\ldots$ | $T_{k}$ |

Table2.2: The ANOVA table of lack of fit test

| Source of Variation | df | Sum of Squares | Mean <br> Squares | F-statistics |
| :--- | :--- | :--- | :--- | :--- |
| Regression(R) | 1 | $S S R=S_{x y}^{2} / S_{x x}$ | MSR |  |
| Residual error(E) | $\mathrm{n}-2$ | $S S E=S_{y y}-S S R$ | MSE |  |
| Lack of fit (L) | $\mathrm{k}-2$ | $S S L=S S E-S S P$ | MSL | $F_{0}=M S L / M S P$ |
| Pure error(P) | $\mathrm{n}-\mathrm{k}$ | $S S P=\sum \sum y_{i j}^{2}-\sum T_{i}^{2} / n_{i}$ | MSP |  |
| Total(T) | $\mathrm{n}-1$ | $S S T=S_{y y}$ |  |  |



Figure 2.1: The standard two-by two crossover design

## Chapter 3 Proposed Methods

In the previous chapter we reviewed the statistical methods for parallel line assay. In this chapter we propose the applications of the parallel line assay to evaluation of biosimilar products.

### 3.1 Design

Design (a) in Figure 3.1 proposed by Liu and Chow (2010) consists of two dose-response trials, one for the biosimilar product and one for the innovator's biological product, each with at least three dose levels with a placebo group. Eligible patients are first randomized into biosimilar or innovator groups. Within each group, patients are randomized again to receive one of the doses for the respective products. Well-defined product characteristics and primary efficacy endpoints are evaluated for all patients at their respective doses. Suppose that after a statistically significant relationship as represented by a simple linear regression equation can be established between the well-defined product characteristics and the primary efficacy endpoint through dose levels for the innovator's product, maybe after a suitable transformation. If a similar linear relationship can be also obtained for the biosimilar product and its corresponding linear regression equation is very close to the one for the innovator's
product, then equivalence in efficacy based on the primary efficacy endpoint may be claimed. Because the innovator's product has been approved by the regulatory agencies due to its confirmed efficacy, therefore the objective of Design (a) is not to establish the efficacy of either biological products but to establish the similar patterns of the relationship between the well-defined product characteristics and primary efficacy endpoint for the two products. As a result, the sample size of Design (a) can be reduced significantly.

### 3.1.1. Fundamental Biosimilarity Assumption

Similar to the chemical generic drug products, approval for the biosimilar drug products can be treated as the evaluation of post-approval changes. (ICH Q5E, 2005) and this post-approval change is the change of drug manufacturers. Therefore, as indicated by Liu and Chow (2010) if the biosimilar drug products and the corresponding innovator's biological products appear highly similar, and if in addition, based on the accumulated experience relevant information and data, minute differences observed in the product characteristics are expected to have no clinically meaningful adverse effect of safety and efficacy profiles. Under this circumstance, biosimilar drug products and innovator's products can be considered similar. Therefore, except for the traditional pivotal bioequivalence study, no further data from pivotal phase III trials should be
requested. However, the above statement is based on a crucial assumption that at least one of the product characteristics is a validated and reliable predictor of the safety and efficacy profiles of the biological products. As a result, the Fundamental Biosimilarity Assumption (FBA) is

When a biosimilar product is claimed to be biosimilar to an innovator's product based on some well-defined product characteristics and is therapeutically equivalent provided that the well-defined product characteristics are validated and reliable predictors of safety and efficacy of the products.

For the chemical generic products, the well-defined product characteristics are the exposure measures for early, peak, and total portions of the concentration-time curve. The Fundamental Bioequivalence Assumption is assumed that the equivalence in the exposure measures implies therapeutically equivalent. However, due to the complexity of the biosimilar drug products, one has to verify that some validated product characteristics are indeed reliable predictors of the safety and efficacy. It follows that the design and analysis for evaluation of equivalence between the biosimilar drug product and innovator product are substantially different from those of the chemical generic products.

For design (a) Liu and Chow (2010) suggest that, the standard statistical method for analysis of parallel line assays can be used to construct the (1-2 $\alpha$ ) $100 \%$ confidence
interval for the relative potency in the following steps:

Step 1: Fit a linear regression equation to the primary efficacy endpoint with the group mean of the product characteristics at each dose level as the independent variable separately for the biosimilar and innovator's biological products. This may be done after a suitable transformation.

Step 2: If the estimate of the slope of any one product is not significant at the pre-defined level, then conclude that no simple relationship can be established between the product characteristic and primary endpoint and hence, a full clinical evaluation of the biosimilar product is required. Otherwise, go to Step 3.

Step 3: Test whether the two estimated simple linear regressions are parallel at the predefined significance level. If the two estimated linear regressions are not parallel, then further clinical evaluation of the biosimilar product is warranted. Otherwise, proceed to Step 4.

Step 4: Compute estimated relative potency and its corresponding ( $1-2 \alpha$ ) $100 \%$ confidence interval. If the $(1-2 \alpha) 100 \%$ confidence interval for the relative potency is within the predefined margins $\left(\delta_{\mathrm{L}}, \delta_{\mathrm{U}}\right)$, then equivalence in the product characteristic can be extrapolated to equivalence in the primary efficacy endpoint at the $\alpha$ significance level. Otherwise, further clinical investigation of the biosimilar product is needed.

The objective of application of Design (a) is not to establish the efficacy of
biosimilar product but rather to test whether the relative potency is within some pre-defined limits. We will use this design to assess the biosimilar products. Let Y be clinical responses, and X be the sample means of biomarkers or PK measures under some doses to establish the regression model.

### 3.2 The procedure and methods based on relative potency of product characteristics

Figure 3.2 provides a graphic depiction of the application of the parallel line assay to evaluation of the extrapolability of equivalence in product characteristic to equivalence in efficacy.

The interval hypothesis based on the relative potency is given as
$\left\{\begin{array}{l}H_{0}: \Delta \leq \delta_{L} \text { or } \Delta \geq \delta_{U} \\ \text { v.s. } \\ H_{a}: \delta_{L} \leq \Delta \leq \delta_{U},\end{array}\right.$
where $\delta_{\mathrm{L}}$ and $\delta_{\mathrm{U}}$ are some pre-defined lower and upper margins.

If the $100(1-2 \alpha) \%$ confidence interval for $\Delta$ given in (2.8) is within $\left(\delta_{L}, \delta_{U}\right)$ then null hypothesis (3.1) is rejected and the biosimilar product is similar to the innovator product at the $\alpha$ significance level.

### 3.3 The procedure and methods based on clinical responses

The method for the relative potency mention above is based on the horizontal distance for a fixed clinical response. However, on the other hand, sometimes it is of interest to see whether the clinical responses of the biosimilar and innovator's products are similar at a fixed level of mean characteristic. This can be accomplished using the vertical distance. A graphical representation of the concept is given in Figure 3.3.

Let $\theta$ be the vertical distance between the two regression lines.
$\theta=E\left(Y_{R}\right)-E\left(Y_{T}\right)=\alpha_{R}-\alpha_{T}$.

The interval hypothesis based on $\theta$ is given as:
$\left\{\begin{array}{l}H_{0}: \theta \leq \theta_{L} \text { or } \theta \geq \theta_{U} \\ \text { v.s. } \\ H_{a}: \theta_{L} \leq \theta \leq \theta_{U} .\end{array}\right.$

An estimate of $\theta$ is given as $\hat{\theta}=\hat{Y}_{R}-\hat{Y}_{T}$.

For a fixed mean characteristic $\mathrm{x}_{0}$, the expected value of $\hat{\theta}$ is the difference of the intercepts of the two regression line between the innovator's and biosimilar products.

$$
E\left(\hat{\theta} \mid x_{0}\right)=E\left(\hat{Y}_{R} \mid x_{0}\right)-E\left(\hat{Y}_{T} \mid x_{0}\right)=\left(\alpha_{R}+\beta_{c} x_{0}\right)-\left(\alpha_{T}+\beta_{c} x_{0}\right)=\alpha_{R}-\alpha_{T} .
$$

$\hat{\boldsymbol{\theta}}$ is an unbiased estimation of $\theta$.
Since $\hat{Y}_{T}$ and $\hat{Y}_{R}$ are independent, it follows that the variance of $\hat{\boldsymbol{\theta}}$ is given as

$$
\begin{aligned}
& \operatorname{Var}\left(\hat{\theta} \mid x_{0}\right)=\operatorname{Var}\left(\hat{Y}_{R}-\hat{Y}_{T} \mid x_{0}\right)=\operatorname{Var}\left(\hat{Y}_{R} \mid x_{0}\right)+\operatorname{Var}\left(\hat{Y}_{T} \mid x_{0}\right) \\
& =\operatorname{Var}\left(a_{R}+b_{c} x_{0}\right)+\operatorname{Var}\left(a_{T}+b_{c} x_{0}\right) .
\end{aligned}
$$

On the other hand
$\operatorname{Var}\left(\hat{Y}_{T} \mid x_{0}\right)=\operatorname{Var}\left(a_{T}\right)+2 x_{0} \operatorname{Cov}\left(a_{T}, b_{c}\right)+x_{0}{ }^{2} \operatorname{Var}\left(b_{c}\right)$
$=\left(\frac{1}{n}+\frac{\bar{x}^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}\right) \sigma_{T}{ }^{2}+2 x_{0} \frac{-\bar{x} \sigma_{T}{ }^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}+\frac{x_{0}{ }^{2} \sigma_{T}{ }^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}$
$=\frac{\sigma_{T}{ }^{2}}{n}+\frac{\left(x_{0}-\bar{x}\right)^{2} \sigma_{T}{ }^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}=\sigma_{T}{ }^{2}\left[\frac{1}{n}+\frac{\left(x_{0}-\bar{x}\right)^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}\right]$.
Similarly
$\operatorname{Var}\left(\hat{Y}_{R} \mid x_{0}\right)=\sigma_{R}{ }^{2}\left[\frac{1}{n}+\frac{\left(x_{0}-\bar{x}\right)^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}\right]$.

It follows that
$\operatorname{Var}\left(\hat{\theta} \mid x_{0}\right)=\left(\sigma_{R}{ }^{2}+\sigma_{T}{ }^{2}\right)\left[\frac{1}{n}+\frac{\left(x_{0}-\bar{x}\right)^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}\right]$.
Then an estimate of variance of $\hat{\theta}$ is given as
$\hat{\operatorname{Var}}\left(\hat{\theta} \mid x_{0}\right)=\left(S_{R}{ }^{2}+S_{T}{ }^{2}\right)\left[\frac{1}{n}+\frac{\left(x_{0}-\bar{x}\right)^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}\right]$ and
$\hat{\theta} \sim N\left(\left(\alpha_{R}-\alpha_{T}\right),\left(\frac{1}{n}+\frac{\left(x_{0}-\bar{x}\right)^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}\right)\left(\sigma_{R}^{2}+\sigma_{T}{ }^{2}\right)\right)$

As a result,
$a(1-2 \alpha) 100 \%$ C.I. of $\hat{\theta}$ is given as $\left[\left(a_{R}-a_{T}\right) \pm t_{2(n-2), \alpha} \sqrt{\hat{\operatorname{Var}}(\hat{\boldsymbol{\theta}})}\right]$
Let $S_{R}{ }^{2}=\frac{\sum\left(Y_{i}-a_{R}-b_{c} x_{i}\right)^{2}}{n-2}$, and $S_{T}{ }^{2}=\frac{\sum\left(Y_{i}-a_{T}-b_{c} x_{i}\right)^{2}}{n-2}$ be given in section 2.4.2.
If the $100(1-2 \alpha) \%$ confidence interval for $\hat{\theta}$ is within $\left(\theta_{L}, \theta_{U}\right)$, then the biosimilar product is similar to the innovator product at the $\alpha$ significance level.

The rejection region for the proposed procedure for hypothesis (3.2) is given as

$$
\begin{aligned}
& R\left\{(\theta, s) \mid \hat{\theta}-t s>\theta_{L} \text { and } \hat{\theta}+t s<\theta_{U}\right\} \\
& =R\left\{(\theta, s) \mid \theta_{L}+t s<\hat{\theta}<\theta_{U}-t s\right\}, \text { where } s=\sqrt{\hat{\operatorname{Var}}(\hat{\theta})}
\end{aligned}
$$

It follows that the power function is given as

$$
\Phi(\theta)=P\left(\left|\theta_{L}+t s<\hat{\theta}<\theta_{U}-t s\right| \theta \in \Theta\right)
$$

where $\Theta=\left(\theta_{L}, \theta_{U}\right)$.

For simplicity, we only consider the situation of symmetric equivalence margin where $\theta_{U}=-\theta_{L}=\theta_{0}$.

Next we determine the require sample size for at least $1-\beta$ power, when $\theta=0$ and $n_{T}=n_{R}=n$.

Since $\Phi(0)=P\left[-\theta_{0}+t s<\hat{\theta}<\theta_{0}-t s \mid \theta=0\right] \geq 1-\beta$

It follows that
$\frac{\theta_{0}}{\sqrt{\left(\frac{1}{n}+\frac{\left(x_{0}-\bar{x}\right)^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}\right)\left(S_{R}{ }^{2}+S_{T}{ }^{2}\right)}}-t(\alpha, 2 n-2) \leq t\left(\frac{\beta}{2}, 2 n-2\right)$,
$\Rightarrow \frac{\theta_{0}}{\sqrt{\left(\frac{1}{n}+\frac{\left(x_{0}-\bar{x}\right)^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}\right)\left(S_{R}^{2}+S_{T}^{2}\right)}} \leq t\left(\frac{\beta}{2}, 2 n-2\right)+t(\alpha, 2 n-2)$.

The required sample size per drug at least 1- $\beta$ power is given as

$$
\begin{equation*}
n=\left\{\left\{\left(\frac{\theta_{0}}{t\left(\frac{\beta}{2}, 2 n-2\right)+t(\alpha, 2 n-2)}\right)^{2} \frac{1}{\left(S_{R}{ }^{2}+S_{T}{ }^{2}\right)}\right\}-\frac{\left(x_{0}-\bar{x}\right)^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}\right\}^{-1} . \tag{3.4}
\end{equation*}
$$

When $\theta<\theta_{0} \neq 0$ and $n_{T}=n_{R}=n$, an approximate formula for sample size determination per drug is given as

$$
\begin{equation*}
n=\left\{\left\{\left(\frac{\theta_{0}-\theta}{t\left(\frac{\beta}{2}, 2 n-2\right)+t(\alpha, 2 n-2)}\right)^{2} \frac{1}{\left(S_{R}^{2}+S_{T}^{2}\right)}\right\}-\frac{\left(x_{0}-\bar{x}\right)^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}\right\}^{-1} . \tag{3.5}
\end{equation*}
$$

Then the sample size per drug can be allocated to each dose. For example, with each allocation of k doses, the required sample size per dose is $\mathrm{n} / \mathrm{k}$.


Figure 3.1: Design (a) for evaluation of extrapolation ability


Figure 3.2: Application of the parallel line assay to evaluation of the extrapolability of based on product characteristics


Figure 3.3: Application of the parallel line assay to evaluation of the extrapolability of based on clinical responses.

## Chapter 4 Numerical Example

In this chapter we apply the proposed statistical methods derived in chapter 3 to a numerical data set. We want to test if the AUC in the innovative drug product and biosimilar are close, then clinical response values are also close.

Because we can not get real data, we use the Fortran to generate the data to evaluate the biosimilar drug. The purpose of this section is to illustrate the proposed methods with a numerical data set. The steps for generation of data are fronded below :

Step 1: Determine the relationship between dose and AUC. For each dose generate 20 AUC under normal assumption with mean $=10,20,30,40,50$ and variance $=1$.

Step 2: To calculate the mean of $\operatorname{AUC}\left(\mathrm{x}_{\mathrm{R}}\right.$ or $\left.\mathrm{x}_{\mathrm{T}}\right)$ and use the mean to generate the data set of clinical responses(y) based on the following linear regression models.
$Y_{R}=100-x_{R}+\varepsilon_{R}$
$Y_{T}=100-x_{T}+\varepsilon_{T}$
$\varepsilon_{R}$ and $\varepsilon_{T}$ are generated from normal assumption with mean $=0$ and variance $=144$

Both proposed methods are illustrated with the above data set which is given in Appendix A.

The mean characteristics and mean clinical response with the correspond standard deviation for biosimilar and innovator's products are provided in Table 4.1. Table 4.2
gives the ANOVA tables for biosimilar and innovator's products.

The linear regression line for the biosimilar and innovator's products are given respectively as $\hat{y}_{T}=100-x_{T}, \hat{y}_{R}=100-x_{R}$. First, we should test the linear regression model. The F -values for the lack of fit test for both products, because $\mathrm{F}_{\mathrm{R}, \mathrm{LOF}}=0.174$ and $\mathrm{F}_{\mathrm{T}, \mathrm{LOF}}=0.876$ which are less than $\mathrm{F}(0.05,3,95)=2.7$, therefore we fail to reject the null hypothesis that the two regression models are linear at the 5\% significance level.

Next we verify whether the two linear regression lines are parallel, the corresponding sums of square and cross products are provided in Table 4.3.

The residual mean squares for the innovator's and biosimilar products are given as

$$
\begin{aligned}
& S_{R}^{2}=\frac{{ }_{R} S y y-b_{R} \cdot{ }_{R} S x y}{n_{r}-2}=118.334 \quad \text { and } \\
& S_{T}^{2}=\frac{{ }_{T} S y y-b_{T} \cdot{ }_{T} S x y}{n_{t}-2}=140.067 \\
& F=1.184<F(0.1,98,98)=2.14
\end{aligned}
$$

Therefore the null hypothesis of homogenous variances is not rejected at the $5 \%$ significance level.

The pooled residual mean squares is then estimates as

$$
S_{p}^{2}=\frac{1}{2}(118.334+140.067)=129.2
$$

It follow the estimated variance of $b_{R}-b_{T}$ is

$$
V\left(b_{R}-b_{T}\right)=129.2\left[\frac{1}{19612.75}+\frac{1}{19612.75}\right]=0.0132
$$

Therefore the test statistic for hypothesis of equal slope is

$$
t=\frac{|-0.8437-(-1.002)|}{\sqrt{0.0132}}=1.379<t_{0.05,196}=1.65
$$

As results, null hypothesis of equal slopes is not rejected at 5\% significance level

The estimated common slope is then given as

$$
b_{c}=\frac{-16548.043+(-19652.91)}{19612.75+19612.75}=-0.9229
$$

It follow that the two estimated linear regression lines are given as:

$$
\begin{aligned}
& y_{R}=97.2205-0.9929 x \\
& y_{T}=96.935-0.9929 x
\end{aligned}
$$

An estimated relative potency is given as

$$
\hat{\Delta}=\frac{a_{R}-a_{T}}{b_{c}}=\frac{97.2205-96.935}{-0.9229}=-0.2875
$$

Since

$$
g=\frac{t_{\frac{0,1}{2}, 196}^{2} S^{2}}{b_{c}^{2}\left[\sum\left(x_{r}-\bar{x}_{r}\right)^{2}+\sum\left(x_{t}-\bar{x}_{t}\right)^{2}\right]}=\frac{1.65^{2} \cdot 129.2}{(-0.9229)^{2}(19612.75+19612.75)}=0.011<0.1
$$

We can ignore $g$-value is computation of the $90 \%$ confidence interval for $\Delta$. The lower and upper limits of the $90 \%$ confidence interval for $\Delta$.

$$
\begin{aligned}
& \left(\hat{\Delta}_{U}, \hat{\Delta}_{L}\right)=\hat{\Delta} \pm \frac{t_{\alpha / 2} S}{\left|b_{c}\right|}\left[\left(\frac{1}{n_{R}}+\frac{1}{n_{T}}\right)+\frac{\hat{\Delta}^{2}}{\sum\left(x_{R}-\bar{x}_{R}\right)^{2}+\sum\left(x_{T}-\bar{x}_{T}\right)^{2}}\right]^{\frac{1}{2}} \\
& =-0.2875 \pm \frac{1.65 \cdot \sqrt{129.2}}{|-0.9229|}\left[\left(\frac{1}{100}+\frac{1}{100}\right)+\frac{(-0.2875)^{2}}{19612.75+19612.75}\right]^{\frac{1}{2}} \\
& =-0.2875 \pm 2.874 \\
& =(-3.162,2.5865)
\end{aligned}
$$

The $90 \%$ confidence interval for $\hat{\Delta}$ is ( $-3.162,2.5865$ ). Let $-\delta_{L}=\delta_{U}=10$. Since the $90 \%$ confidence interval $(-3.162,2.5865)$ is contained within $(-10,10)$, then we can
conclude that the biosimilar product is similar to the innovator product at $5 \%$ significance level.

On the other hand, the estimated vertical distance of the two regression lines is
$\hat{\theta}=a_{R}-a_{T}=0.2855$

Its corresponding estimated variance is given as

$$
\hat{\operatorname{Var}}(\hat{\theta})=\left(\frac{1}{n}+\frac{\left(x_{0}-\bar{x}\right)^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}\right)\left(S_{R}^{2}+S_{T}^{2}\right)=\left(\frac{1}{100}+\frac{(30-30.0615)^{2}}{19612.75^{2}}\right)(2 \times 129.2)=2.584
$$

It follows that the $90 \%$ confidence interval for $\theta$ is $(-2.3712,2.9422)$
If the equivalence limits $\theta_{0}$ is set at 10 , since the $90 \%$ confidence interval $(-2.3712$,
2.9422 ) is completely within $(-10,10)$, then the null hypothesis is rejected and biosimilar product is similar to the innovator product at 5\% significance level.

Suppose that $\theta=0$, the required sample size for hypothesis (3.2) for $80 \%$ power at the 5\% significance level is given as

$$
n=\left\{\left\{\left(\frac{\Delta}{t\left(\frac{\beta}{2}, 2 n-2\right)+t(\alpha, 2 n-2)}\right)^{2} \frac{1}{\left(S_{R}{ }^{2}+S_{T}{ }^{2}\right)}\right\}-\frac{\left(x_{0}-\bar{x}\right)^{2}}{\sum\left(x_{i}-\bar{x}\right)^{2}}\right\}^{-1}=28.25
$$

The sample size is 29 per group. However, there are 5 doses, therefore we round 29 up to 30 so that with an equal allocation, the sample size for each dose is 6 per products.

Table 4.1: The Descriptive Statistics of innovator's product and biosimilar product

| Mean characteristic of <br> Innovator's products | Mean | 10.06 | 20.44 | 30.18 | 39.71 | 49.93 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Standard deviation | 1.5 | 0.98 | 0.87 | 1.12 | 1.54 |
| Mean characteristic of <br> Biosimilar products | Mean | 9.65 | 19.93 | 28.9 | 40.15 | 50.18 |
|  | Standard deviation | 0.59 | 1 | 0.84 | 0.93 | 0.86 |
| Innovator's products | Mean | 85.69 | 78.74 | 68.61 | 62.11 | 52.23 |
|  | Standard deviation | 9.2 | 12.03 | 10.99 | 11.55 | 11.12 |
| Biosimilar products | Mean | 87.46 | 80.42 | 68.9 | 62.23 | 46.95 |
|  | Standard deviation | 11.79 | 11.06 | 11.46 | 10.85 | 13.87 |

Table 4.2: The ANOVA table of lack of fit test

| Source of Variation | df | Innovator's product |  |  | Biosimialr product |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Sum of <br> Squares | Mean <br> Squares | F-statistics | Sum of Squares | Mean <br> Squares | F-statistics |
| Regression(R) | 1 | 13962.225 | 13962.225 |  | 19693.152 | 19693.152 |  |
| Residual error(E) | 98 | 11596.045 | 118.327 |  | 13725.538 | 140.057 |  |
| Lack of fit ( $\mathbf{L}$ ) | 3 | 63.448 | 21.149 | 0.174 | 369.398 | 123.133 | 0.876 |
| Pure error(P) | 95 | 11532.600 | 121.396 |  | 13356.140 | 140.591 |  |
| $\operatorname{Total}(\mathbf{T})$ | 99 | 25558.270 |  |  | 33418.690 |  |  |

Table 4.3: The Descriptive Statistics of innovator's product and biosimilar product

| group | $\mathbf{n}$ | Sxx | Sxy | Syy | xbar | ybar | a | b |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Innovator's | 100 | 19612.75 | -16548.04 | 25558.27 | 30.0615 | 69.4769 | 94.841 | -0.8437 |
| Biosimilar | 100 | 19612.75 | -19652.91 | 33418.69 | 30.0615 | 69.1914 | 99.315 | -1.002 |

# Chapter 5 Simulation Study 

In this chapter, Compaq Visual Fortran 6.5 and IMSL's STAT/LIBRARY FORTRAN subroutines RNNOR were used in simulation study to generate the data to do simulation to compare the coverage probability and power with different sample size, variance and dose. A flow chart of the simulation study is given in Figure 5.1.

### 5.1 Simulation Process

First we generate the data of AUC, for each dose generates 20 AUC under normal assumption with specified means and variances. The number of doses is $3,5,7$. In addition, the RNNOR subroutine of Compaq Visual Fortran 6.5/IMSL was employed to generate normal deviates with variance 1 and dose 10 to k 10 by 10 , where k is the number of the dose. The responses were generated using following regression equation, and where $\mathrm{x}_{\mathrm{R}}\left(\mathrm{x}_{\mathrm{T}}\right)$ are the mean AUC at each dose.

$$
\begin{aligned}
& Y_{R}=100-x_{R}+\varepsilon_{R} \\
& Y_{T}=100-x_{T}+\varepsilon_{T} .
\end{aligned}
$$

$\varepsilon$ are i.i.d. with mean 0 and standard deviation from 12 to 24 by 6 .

Then we use the statistical method mentioned in Chapter 3 to determine the $90 \%$ confidence interval. Then we repeated for 10000 times to determine the empirical power, empirical size and coverage probability. The equivalence limits are set to be $\pm 10$ for
both methods based on product characteristic and clinical responses. The empirical size is estimated the proportion of 10000 random samples for which the confidence intervals are totally contained within $(-10,10)$ when $\mu_{T}-\mu_{R}= \pm 10$.

The empirical power is calculated as the proportion of 10000 random samples in which the confidence interval falls entirely within ( $-10,10$ ), where $\mu_{\mathrm{T}}-\mu_{\mathrm{R}}=-5,0$ and 5.

The coverage probability is computed as the proportion of 10000 confidence intervals containing the specified $\mu_{\mathrm{T}}-\mu_{\mathrm{R}}=-10$ to 10 by 5

At the 5\% nominal significance level, a simulation study with 10000 samples that $95 \%$ of the empirical sizes will be between 0.0457 and 0.0542 if the proceed procedure can adequately control the type one error rate at its nominal level. Similarly, $90 \%$ coverage probability will be between 0.8941 and 0.9059 if the proceed procedure can provide adequate coverage probability at the $90 \%$ level.

### 5.2 Simulation Result

The simulation results based on relative potency are given from Table 5.1 to Table 5.3. In general, the empirical size decreases as the variability increases or the number of the doses increase. $36.67 \%$ (33/90) of empirical sizes are within ( $0.0457,0.0542$ ). This indicates that the method based on relative potency can not adequately control the size.

When $\sigma=12$ and the number of the doses is fewer than 5 , the method based on relative potency yields the empirical sizes greater than 0.0542 . Therefore, the method based on relative potency is liberal when the variability is small and the number of doses is fewer. However when $\sigma=24$ and the number of doses is 5 or above, the empirical sizes are below 0.0457 . This reveals that the method based on relative potency becomes much more conservative when the variability is higher and the number of doses is larger.

The simulation results based on clinical responses are provided from Table 5.4 to Table 5.6. When $\sigma=12$, all 30 empirical sizes are within ( $0.0457,0.0542$ ). However, when $\sigma>12$, all empirical size are below 0.0457 . The empirical sizes based on clinical responses are smaller than 0.01 when $\sigma=24$. Therefore, the method based on clinical responses can adequately control the size when variability is small. It becomes extremely conservative when variability is large.

The sample size for the simulation study is to provide a $80 \%$ power when $\mu_{\mathrm{T}}-\mu_{\mathrm{R}}=0$ and $\sigma=12$ using the formula based on the clinical responses. This is confirmed by the fact that empirical power is at least $80 \%$ when $\mu_{\mathrm{T}}-\mu_{\mathrm{R}}=0$ and $\sigma=12$. On the other hand, when $\mu_{\mathrm{T}}-\mu_{\mathrm{R}}=0$ and $\sigma=12$, the empirical power based on the relative potency are around $80 \%$ though they are smaller than those based on clinical response.

In general, the empirical power for both methods decreases dramatically as the
variability increases for the sample size investigated in the simulation study. On the other hand, the empirical power increases slightly as the number of doses increases or the sample size increases. None of the coverage probabilities given from Table 5.1 to Table 5.6 are smaller than 0.8941 . This suggested that the confidence intervals constructed by both methods can provide sufficient probability.

Table 5.1: Simulation result of power and coverage probability based on relative potency under $\sigma=12$

| dose | n |  | -10 | -5 | 0 | 5 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 26 | power | 0.0648 | 0.4130 | 0.7303 | 0.4198 | 0.0703 |
|  |  | coverage probability | 0.9231 | 0.9200 | 0.9220 | 0.9214 | 0.9182 |
| 3 | 27 | power | 0.0706 | 0.4250 | 0.7394 | 0.4223 | 0.0706 |
|  |  | coverage probability | 0.9186 | 0.9174 | 0.9157 | 0.9170 | 0.9161 |
| 3 | 28 | power | 0.0686 | 0.4373 | 0.7684 | 0.4387 | 0.0726 |
|  |  | coverage probability | 0.9188 | 0.9174 | 0.9183 | 0.9162 | 0.9151 |
| 3 | 29 | power | 0.0672 | 0.4475 | 0.7831 | 0.4451 | 0.0672 |
|  |  | coverage probability | 0.9186 | 0.9177 | $0.9198$ | $0.9173$ | $0.9182$ |
| 3 | 30 | power | 0.0665 | 0.4574 | 0.7961 | 0.4622 | 0.0675 |
|  |  | coverage probability | 0.9204 | 0.9206 | 0.9185 | 0.9203 | 0.9193 |
| 5 | 26 | power | 0.0585 | 0.4196 | 0.7776 | 0.4330 | 0.0560 |
|  |  | coverage probability | 0.9078 | 0.9103 | 0.9081 | 0.9070 | 0.9100 |
| 5 | 27 | power | 0.0588 | 0.4346 | 0.7915 | 0.4396 | 0.0590 |
|  |  | coverage probability | 0.9034 | 0.8991 | 0.9021 | $0.9014$ | $0.9017$ |
| 5 | 28 | power | 0.0593 | 0.4457 | 0.8153 | 0.4475 | $0.0601$ |
|  |  | coverage probability | $0.9038$ | 0.9029 | $0.9051$ | 0.9041 | $0.9032$ |
| 5 | 29 | power | 0.0546 | $0.4615$ | $0.8359$ | $0.4606$ | $0.0570$ |
|  |  | coverage probability | 0.9071 | 0.4615 | 0.8359 | 0.9608 | 0.9043 |
| 5 | 30 | power | 0.0545 | 0.4692 | 0.8441 | 0.4794 | 0.0560 |
|  |  | coverage probability | 0.9090 | 0.9069 | 0.9061 | 0.9057 | 0.9075 |
| 7 | 26 | power | 0.0535 | 0.4195 | 0.7882 | 0.4347 | 0.0537 |
|  |  | coverage probability | 0.9059 | 0.9070 | 0.9048 | 0.9042 | 0.9058 |
| 7 | 27 | power | 0.0567 | 0.4380 | 0.8038 | 0.4410 | 0.0558 |
|  |  | coverage probability | 0.8977 | 0.8957 | 0.8986 | 0.8986 | 0.8977 |
| 7 | 28 | power | 0.0550 | 0.4472 | 0.8315 | 0.4499 | 0.0553 |
|  |  | coverage probability | 0.9015 | 0.8999 | 0.9005 | 0.9018 | 0.9005 |
| 7 | 29 | power | 0.0513 | 0.4626 | 0.8510 | 0.4613 | 0.0532 |
|  |  | coverage probability | 0.9054 | 0.9068 | 0.9032 | 0.9025 | 0.9021 |
| 7 | 30 | power | 0.0504 | 0.4744 | 0.8624 | 0.4816 | 0.0536 |
|  |  | coverage probability | 0.9042 | 0.9028 | 0.9032 | 0.9022 | 0.9028 |

Table 5.2: Simulation result of power and coverage probability based on relative potency under $\sigma=18$
$\left.\begin{array}{ccccccc}\hline \text { dose } & \mathrm{n} & & -10 & -5 & 0 & 5 \\ \hline 3 & 26 & \text { power } & 0.0617 & 0.1979 & 0.2967 & 0.2041\end{array}\right) 0.0676$

Table 5.3: Simulation result of power and coverage probability based on relative potency under $\boldsymbol{\sigma}=\mathbf{2 4}$

| dose | n |  | -10 | -5 | 0 | 5 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 26 | power | 0.0420 | 0.0907 | 0.1207 | 0.0967 | 0.0489 |
|  |  | coverage probability | 0.9303 | 0.9516 | 0.9667 | 0.9520 | 0.9253 |
| 3 | 27 | power | 0.0479 | 0.0992 | 0.1245 | 0.1054 | 0.0505 |
|  |  | coverage probability | 0.9245 | 0.9519 | 0.9662 | 0.9550 | 0.9230 |
| 3 | 28 | power | 0.0506 | 0.1060 | 0.1428 | 0.1148 | 0.0526 |
|  |  | coverage probability | 0.9235 | 0.9485 | 0.9663 | 0.9494 | 0.9218 |
| 3 | 29 | power | 0.0521 | 0.1208 | 0.1471 | 0.1087 | 0.0481 |
|  |  | coverage probability | 0.9266 | 0.9515 | 0.9625 | 0.9477 | 0.9273 |
| 3 | 30 | power | 0.0521 | 0.1184 | 0.1576 | 0.1194 | 0.0541 |
|  |  | coverage probability | 0.9267 | 0.9516 | 0.9633 | 0.9513 | 0.9250 |
| 5 | 26 | power | 0.0237 | 0.0504 | 0.0688 | 0.0507 | 0.0239 |
|  |  | coverage probability | 0.9271 | 0.9311 | 0.9327 | 0.9307 | 0.9292 |
| 5 | 27 | power | 0.0236 | 0.0510 | 0.0675 | 0.0545 | 0.0248 |
|  |  | coverage probability | 0.9223 | 0.9212 | 0.9229 | $0.9221$ | $0.9208$ |
| 5 | 28 | power | $0.0256$ | $0.0613$ | 0.0842 | $0.0650$ | $0.0275$ |
|  |  | coverage probability | 0.9235 | 0.9253 | 0.9268 | 0.9262 | 0.9236 |
| 5 | 29 | power | 0.0274 | 0.0701 | 0.0915 | 0.0638 | 0.0275 |
|  |  | coverage probability | 0.9256 | 0.9259 | 0.9254 | 0.9254 | 0.9219 |
| 5 | 30 | power | 0.0292 | 0.0700 | 0.1026 | 0.0734 | 0.0295 |
|  |  | coverage probability | 0.9252 | 0.9259 | 0.9266 | 0.9238 | 0.9245 |
| 7 | 26 | power | 0.0148 | 0.0349 | 0.0441 | 0.0336 | 0.0152 |
|  |  | coverage probability | 0.9185 | 0.9188 | 0.9173 | 0.9159 | 0.9166 |
| 7 | 27 | power | 0.0162 | 0.0330 | 0.0452 | 0.0374 | 0.0157 |
|  |  | coverage probability | 0.9088 | 0.9065 | 0.9073 | 0.9090 | 0.9088 |
| 7 | 28 | power | 0.0173 | 0.0413 | 0.0541 | 0.0451 | 0.0169 |
|  |  | coverage probability | 0.9101 | 0.9091 | 0.9112 | 0.9108 | 0.9108 |
| 7 | 29 | power | 0.0187 | 0.0543 | 0.0671 | 0.0458 | 0.0207 |
|  |  | coverage probability | 0.9150 | 0.9160 | 0.9146 | 0.9129 | 0.9125 |
| 7 | 30 | power | 0.0184 | 0.0510 | 0.0743 | 0.0536 | 0.0207 |
|  |  | coverage probability | 0.9134 | 0.9124 | 0.9111 | 0.9101 | 0.9105 |

Table 5.4: Simulation result of power and coverage probability based on clinical response under $\boldsymbol{\sigma}=\mathbf{1 2}$

| dose | n |  | -10 | -5 | 0 | 5 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 26 | power | 0.0478 | 0.4161 | 0.8013 | 0.4279 | 0.0455 |
|  |  | coverage probability | 0.9067 | 0.9067 | 0.9067 | 0.9067 | 0.9067 |
| 3 | 27 | power | 0.0498 | 0.4355 | 0.8130 | 0.4337 | 0.0502 |
|  |  | coverage probability | 0.9000 | 0.9000 | 0.9000 | 0.9000 | 0.9000 |
| 3 | 28 | power | 0.0488 | 0.4472 | 0.8396 | 0.4435 | 0.0495 |
|  |  | coverage probability | 0.9017 | 0.9017 | 0.9017 | 0.9017 | 0.9017 |
| 3 | 29 | power | 0.0478 | 0.4636 | 0.8592 | 0.4536 | 0.0485 |
|  |  | coverage probability | 0.9037 | 0.9037 | 0.9037 | 0.9037 | 0.9037 |
| 3 | 30 | power | 0.0474 | 0.4707 | 0.8703 | 0.4771 | 0.0489 |
|  |  | coverage probability | 0.9037 | 0.9037 | 0.9037 | 0.9037 | 0.9037 |
| 5 | 26 | power | 0.0475 | 0.4149 | 0.8020 | 0.4279 | 0.0459 |
|  |  | coverage probability | 0.9066 | 0.9066 | 0.9066 | 0.9066 | 0.9066 |
| 5 | 27 | power | 0.0496 | 0.4363 | 0.8125 | 0.4376 | 0.0502 |
|  |  | coverage probability | 0.9002 | 0.9002 | 0.9002 | 0.9002 | 0.9002 |
| 5 | 28 | power | 0.0489 | 0.4464 | 0.8405 | 0.4437 | 0.0496 |
|  |  | coverage probability | 0.9015 | 0.9015 | 0.9015 | 0.9015 | 0.9015 |
| 5 | 29 | power | 0.0479 | 0.4639 | 0.8602 | 0.4541 | 0.0484 |
|  |  | coverage probability | 0.9037 | 0.9037 | 0.9037 | 0.9037 | 0.9037 |
| 5 | 30 | power | 0.0474 | 0.4706 | 0.8707 | 0.4767 | 0.0488 |
|  |  | coverage probability | 0.9038 | 0.9038 | 0.9038 | 0.9038 | 0.9038 |
| 7 | 26 | power | 0.0480 | 0.4153 | 0.8022 | 0.4291 | 0.0461 |
|  |  | coverage probability | 0.9059 | 0.9059 | 0.9059 | 0.9059 | 0.9059 |
| 7 | 27 | power | 0.0498 | 0.4361 | 0.8128 | 0.4370 | 0.0502 |
|  |  | coverage probability | 0.9000 | 0.9000 | 0.9000 | 0.9000 | 0.9000 |
| 7 | 28 | power | 0.0489 | 0.4473 | 0.8399 | 0.4445 | 0.0494 |
|  |  | coverage probability | 0.9017 | 0.9017 | 0.9017 | 0.9017 | 0.9017 |
| 7 | 29 | power | 0.0475 | 0.4634 | 0.8595 | 0.4546 | 0.0486 |
|  |  | coverage probability | 0.9039 | 0.9039 | 0.9039 | 0.9039 | 0.9039 |
| 7 | 30 | power | 0.0477 | 0.4714 | 0.8701 | 0.4768 | 0.0483 |
|  |  | coverage probability | 0.9040 | 0.9040 | 0.9040 | 0.9040 | 0.9040 |

Table 5.5: Simulation result of power and coverage probability based on clinical response under $\sigma=18$

| dose | n |  | -10 | -5 | 0 | 5 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 26 | power | 0.0369 | 0.1464 | 0.2477 | 0.1527 | 0.0354 |
|  |  | coverage probability | 0.9067 | 0.9067 | 0.9067 | 0.9067 | 0.9067 |
| 3 | 27 | power | 0.0395 | 0.1646 | 0.2732 | 0.1667 | 0.0402 |
|  |  | coverage probability | $0.9000$ | $0.9000$ | 0.9000 | $0.9000$ | 0.9000 |
| 3 | 28 | power | 0.0407 | 0.1839 | 0.3006 | 0.1752 | 0.0396 |
|  |  | coverage probability | 0.9017 | 0.9017 | 0.9017 | 0.9017 | 0.9017 |
| 3 | 29 | power | 0.0414 | 0.1972 | 0.3236 | 0.1696 | 0.0427 |
|  |  | coverage probability | $0.9037$ | $0.9037$ | $0.9037$ | $0.9037$ | $0.9037$ |
| 3 | 30 | power | 0.0423 | 0.2046 | 0.3595 | 0.2140 | 0.0449 |
|  |  | coverage probability | 0.9037 | 0.9037 | 0.9037 | 0.9037 | 0.9037 |
| 5 | 26 | power | 0.0367 | 0.1471 | 0.2484 | 0.1532 | 0.0358 |
|  |  | coverage probability | 0.9066 | 0.9066 | 0.9066 | 0.9066 | 0.9066 |
| 5 | 27 | power | 0.0395 | 0.1650 | 0.2738 | 0.1661 | $0.0402$ |
|  |  | coverage probability | 0.9002 | 0.9002 | $0.9002$ | $0.9002$ | $0.9002$ |
| 5 | 28 | power | 0.0408 | 0.1853 | 0.3010 | 0.1761 | $0.0396$ |
|  |  | coverage probability | 0.9015 | $0.9015$ | 0.9015 | 0.9015 | 0.9015 |
| 5 | 29 | power | $0.0415$ | 0.1977 | $0.3244$ | $0.1961$ | $0.0426$ |
|  |  | coverage probability | 0.9037 | 0.9037 | 0.9037 | 0.9037 | $0.9037$ |
| 5 | 30 | power | 0.0423 | 0.2045 | 0.3604 | 0.2144 | 0.0447 |
|  |  | coverage probability | 0.9038 | 0.9038 | 0.9038 | 0.9038 | 0.9038 |
| 7 | 26 | power | 0.0371 | 0.1474 | 0.2477 | 0.1539 | 0.0362 |
|  |  | coverage probability | 0.9059 | 0.9059 | 0.9059 | 0.9059 | 0.9059 |
| 7 | 27 | power | 0.0398 | 0.1643 | 0.2735 | 0.1661 | 0.4010 |
|  |  | coverage probability | 0.9000 | 0.9000 | 0.9000 | 0.9000 | 0.9000 |
| 7 | 28 | power | 0.0408 | 0.1860 | 0.3014 | 0.1747 | 0.0392 |
|  |  | coverage probability | 0.9017 | 0.9017 | 0.9017 | 0.9017 | 0.9017 |
| 7 | 29 | power | 0.0409 | 0.1980 | 0.3243 | 0.1962 | 0.0429 |
|  |  | coverage probability | $0.9039$ | $0.9039$ | 0.9039 | 0.9039 | 0.9039 |
| 7 | 30 | power | 0.0427 | 0.2051 | 0.3603 | 0.2144 | 0.0442 |
|  |  | coverage probability | 0.9040 | 0.9040 | 0.9040 | 0.9040 | 0.9040 |

Table 5.6: Simulation result of power and coverage probability based on clinical response under $\boldsymbol{\sigma}=\mathbf{2 4}$

| dose | n |  | -10 | -5 | 0 | 5 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 26 | power | 0.0026 | 0.0087 | 0.0087 | 0.0065 | 0.0031 |
|  |  | coverage probability | 0.9067 | 0.9067 | 0.9067 | 0.9067 | 0.9067 |
| 3 | 27 | power | 0.0045 | 0.0077 | 0.0117 | 0.0087 | 0.0049 |
|  |  | coverage probability | 0.9000 | 0.9000 | 0.9000 | 0.9000 | $0.9000$ |
| 3 | 28 | power | 0.0042 | 0.0108 | 0.0173 | 0.0126 | 0.0040 |
|  |  | coverage probability | 0.9017 | 0.9017 | 0.9017 | 0.9017 | 0.9017 |
| 3 | 29 | power | 0.0052 | 0.0146 | 0.0220 | 0.0136 | 0.0059 |
|  |  | coverage probability | 0.9037 | 0.9037 | 0.9037 | 0.9037 | $0.9037$ |
| 3 | 30 | power | 0.0071 | 0.0215 | 0.0287 | 0.0190 | 0.0087 |
|  |  | coverage probability | 0.9037 | 0.9037 | 0.9037 | 0.9037 | 0.9037 |
| 5 | 26 | power | 0.0026 | 0.0086 | 0.0085 | 0.0063 | 0.0033 |
|  |  | coverage probability | 0.9066 | 0.9066 | 0.9066 | 0.9066 | 0.9066 |
| 5 | 27 | power | 0.0043 | 0.0078 | 0.0123 | 0.0091 | 0.0046 |
|  |  | coverage probability | 0.9002 | 0.9002 | 0.9002 | $0.9002$ | $0.9002$ |
| 5 | 28 | power | 0.0046 | 0.0111 | 0.0170 | 0.0126 | $0.0037$ |
|  |  | coverage probability | $0.9015$ | 0.9015 | 0.9015 | 0.9015 | 0.9015 |
| 5 | 29 | power | $0.0052$ | $0.0149$ | 0.0219 | $0.0137$ | $0.0059$ |
|  |  | coverage probability | 0.9037 | 0.9037 | 0.9037 | 0.9037 | $0.9037$ |
| 5 | 30 | power | 0.0072 | 0.0210 | 0.0283 | 0.0190 | 0.0088 |
|  |  | coverage probability | 0.9038 | 0.9038 | 0.9038 | 0.9038 | 0.9038 |
| 7 | 26 | power | 0.0027 | 0.0085 | 0.0088 | 0.0058 | 0.0037 |
|  |  | coverage probability | 0.9059 | 0.9059 | 0.9059 | 0.9059 | 0.9059 |
| 7 | 27 | power | 0.0044 | 0.0079 | 0.0124 | 0.0092 | 0.0046 |
|  |  | coverage probability | 0.9000 | 0.9000 | 0.9000 | 0.9000 | 0.9000 |
| 7 | 28 | power | 0.0043 | 0.0114 | 0.0175 | 0.0122 | 0.0039 |
|  |  | coverage probability | 0.9017 | 0.9017 | 0.9017 | 0.9017 | 0.9017 |
| 7 | 29 | power | 0.0052 | 0.0148 | 0.0220 | 0.0138 | 0.0061 |
|  |  | coverage probability | 0.9039 | 0.9039 | 0.9039 | 0.9039 | 0.9039 |
| 7 | 30 | power | 0.0072 | 0.0213 | 0.0289 | 0.0190 | 0.0084 |
|  |  | coverage probability | 0.9040 | 0.9040 | 0.9040 | 0.9040 | 0.9040 |



Figure 5.1: Flow chart of simulation process

## Chapter 6 Discussion and Conclusion

Following the results of simulation, when the variance increases, the error of size decrease, and the empirical power also decreases. And the number of doses also affects the empirical power. Under the same sample size, when the number of doses increases, the empirical power also increases. In summary, when sample size is large enough with an reasonable variance, we can choose the dose appropriately.

From the
$\left[\left(\bar{x}_{R}-\bar{x}_{T}\right)+\frac{1}{1-g}\left\{\left(\Delta-\bar{x}_{R}+\bar{x}_{T}\right) \pm \frac{t_{\alpha} S}{\left|b_{c}\right|} \sqrt{(1-g)\left(\frac{1}{n_{R}}+\frac{1}{n_{T}}\right)+\frac{\left(\Delta-\bar{x}_{R}+\bar{x}_{T}\right)^{2}}{\sum\left(x_{R}-\bar{x}_{R}\right)^{2}+\sum\left(x_{T}-\bar{x}_{T}\right)^{2}}}\right\}\right]$,
we will therefore wish to choose the design so that
i. t-value is small,
ii. the common slope $b_{c}$ is large,
iii. $\left(\frac{1}{n_{R}}+\frac{1}{n_{T}}\right)$ is small,
iv. $\left(\Delta-\bar{x}_{R}+\bar{x}_{T}\right)$ is small, and
v. $\quad \sum S_{x x}$ is large.

Then the relative potency will be small (Finney, 1979).

When the variance increases, the empirical power decreases. So we could increase the sample size to increase empirical power. But when we increase the sample size the cost of the experiment also increases.

In the Chapter 2, we want to know whether two linear regression lines are parallel. In other words we need to check whether the slopes are equal between the two regression lines. So we let the null hypothesis be $\beta_{R}=\beta_{T}$ and the alternative hypothesis be $\beta_{R} \neq \beta_{T}$. When the Eq. (2.7) less than $\mathrm{t}\left(\alpha / 2, \mathrm{n}_{\mathrm{R}}+\mathrm{n}_{\mathrm{T}}-4\right)$, we fail to reject the null hypothesis. But it dose not imply that the slopes are equal between the two regression lines. Therefore, we can apply the concept of equivalence to formulate the null and alternative hypothesis. As a result, when we reject the null hypothesis, it ensures that the slopes are equivalent between the two regression lines.

In the Chapter 3 we proposed the sample size based on clinical endpoints. However, the sample size based on relative potency is not provided. This is the future goal of our research. In the Chapter 5 we generate the data of AUC under normal assumption with mean and standard deviation 1 when we increase the standard deviation to 4 and 8 . The additional simulation results on the coverage probability, empirical power and empirical size are quite consistent.

In this thesis we assume the clinical response is a continuous variable. But many clinical responses are categorical data or censored data. Application of the parallel line assay to assess biosimilar products may be adapted using the logistic regression for binary data and the proportional hazard model for censored data. However further research is required for these applications.

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http：／／en．wikipedia．org／wiki／Biosimilar（2010．04）

## Appendix A

The data of numerical example

Table A.1: The data of AUC

| Innovator's products |  |  |  |  | Biosimilar products |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9.86 | 9.40 | 9.17 | 11.38 | 10.66 | 9.96 | 8.76 | 8.49 | 9.50 | 9.62 |
| 12.68 | 10.41 | 11.76 | 10.03 | 10.97 | 9.83 | 9.98 | 8.91 | 8.22 | 9.50 |
| 9.28 | 10.35 | 9.84 | 8.76 | 10.16 | 9.52 | 9.93 | 10.70 | 10.32 | 8.41 |
| 8.01 | 10.26 | 11.33 | 7.81 | 9.01 | 10.18 | 9.53 | 10.82 | 10.04 | 10.70 |
| 21.34 | 20.53 | 20.22 | 19.42 | 21.55 | 19.36 | 21.95 | 19.81 | 19.55 | 19.72 |
| 20.57 | 20.92 | 20.38 | 19.95 | 20.46 | 19.88 | 19.22 | 19.93 | 18.81 | 20.13 |
| 21.09 | 19.97 | 21.07 | 19.93 | 20.54 | 20.16 | 19.53 | 19.00 | 18.74 | 20.57 |
| 21.28 | 16.97 | 21.24 | 21.00 | 20.30 | 19.23 | 20.03 | 22.00 | 21.97 | 19.02 |
| 30.30 | 28.68 | 31.17 | 29.57 | 30.88 | 30.75 | 29.28 | 28.67 | 28.81 | 29.00 |
| 31.09 | 30.33 | 29.83 | 30.47 | 29.93 | 30.33 | 29.43 | 31.16 | 29.10 | 29.80 |
| 31.75 | 29.77 | 29.18 | 29.84 | 28.20 | 29.28 | 30.66 | 28.77 | 30.75 | 30.23 |
| 30.38 | 29.59 | 30.88 | 29.96 | 31.80 | 31.77 | 29.32 | 30.76 | 30.74 | 29.36 |
| 39.14 | 40.50 | 39.70 | 38.79 | 39.98 | 38.00 | 39.91 | 40.09 | 39.87 | 38.73 |
| 40.65 | 39.63 | 39.69 | 39.29 | 41.00 | 40.89 | 39.08 | 40.13 | 42.43 | 40.73 |
| 39.39 | 42.07 | 37.82 | 39.21 | 38.24 | 40.73 | 39.34 | 40.86 | 39.35 | 41.12 |
| 38.33 | 39.76 | 39.37 | 40.16 | 41.40 | 40.28 | 40.28 | 40.03 | 40.78 | 40.42 |
| 50.45 | 50.59 | 48.94 | 52.19 | 50.73 | 50.79 | 51.51 | 51.61 | 48.87 | 50.73 |
| 50.06 | 48.31 | 49.34 | 51.14 | 49.50 | 50.07 | 50.19 | 50.47 | 49.26 | 50.37 |
| 49.37 | 49.34 | 50.03 | 47.96 | 49.58 | 49.62 | 49.59 | 48.59 | 50.56 | 48.95 |
| 52.67 | 49.02 | 48.31 | 51.21 | 49.86 | 50.27 | 51.05 | 49.72 | 49.49 | 51.87 |

Table A.2: The data of clinical response

| Innovator's products |  |  |  |  | Biosimilar products |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 89.47 | 75.04 | 71.79 | 83.91 | 85.42 | 111.51 | 93.15 | 71.99 | 70.92 | 98.87 |
| 87.86 | 89.66 | 76.82 | 68.58 | 83.89 | 88.17 | 95.34 | 88.47 | 97.56 | 81.15 |
| 84.22 | 89.12 | 98.40 | 93.83 | 70.86 | 100.52 | 89.91 | 87.78 | 63.14 | 80.89 |
| 92.15 | 84.30 | 99.79 | 90.38 | 98.38 | 90.40 | 70.60 | 84.44 | 95.64 | 88.69 |
| 71.87 | 102.98 | 77.28 | 74.22 | 76.19 | 101.77 | 98.56 | 80.20 | 78.89 | 59.28 |
| 78.13 | 70.26 | 78.74 | 65.34 | 81.08 | 82.88 | 80.33 | 86.99 | 88.19 | 78.83 |
| 81.50 | 73.90 | 67.57 | 64.45 | 86.42 | 67.17 | 89.04 | 72.38 | 69.72 | 79.79 |
| 70.35 | 79.94 | 103.59 | 103.25 | 67.78 | 83.11 | 75.70 | 60.67 | 91.30 | 83.61 |
| 78.88 | 61.20 | 53.86 | 55.52 | 57.78 | 54.74 | 82.10 | 68.82 | 76.17 | 63.48 |
| 73.74 | 62.94 | 83.74 | 59.06 | 67.46 | 84.83 | 67.55 | 78.84 | 61.37 | 68.29 |
| 61.19 | 77.69 | 55.05 | 78.82 | 72.62 | 50.09 | 57.12 | 79.71 | 64.08 | 78.60 |
| 91.10 | 61.63 | 78.96 | 78.75 | 62.19 | 61.25 | 93.50 | 54.45 | 69.57 | 63.50 |
| 36.26 | 59.18 | 61.37 | 58.71 | 45.04 | 67.11 | 59.46 | 59.67 | 38.09 | 61.58 |
| 70.99 | 49.27 | 61.82 | 89.42 | 69.01 | 67.19 | 45.64 | 67.16 | 55.15 | 51.97 |
| 69.08 | 52.42 | 70.58 | 52.44 | 73.72 | 70.10 | 67.08 | 55.87 | 55.44 | 75.09 |
| 63.68 | 63.60 | 60.71 | 69.65 | 65.31 | 60.81 | 64.71 | 67.42 | 89.67 | 65.37 |
| 59.52 | 68.23 | 69.36 | 36.53 | 58.88 | 65.73 | 50.96 | 34.17 | 24.38 | 39.24 |
| 50.90 | 52.33 | 55.74 | 41.17 | 54.52 | 66.40 | 69.73 | 58.73 | 41.97 | 44.85 |
| 45.55 | 45.20 | 33.16 | 56.82 | 37.53 | 40.27 | 55.25 | 43.57 | 31.50 | 41.50 |
| 53.32 | 62.62 | 46.71 | 43.96 | 72.51 | 33.48 | 25.41 | 55.35 | 50.24 | 66.22 |

## Appendix B

## Fortran code for simulation program

！sstats．lib smaths．lib sstatd．lib smathd．lib
PROGRAM MAIN
！use IMSL
INTEGER ISEED，ISEED1，NOUT，NR，i，j，k，q
REAL XM，XSTD，t（5），x＿bar，yt＿bar，yr＿bar，Syyr，Syyt， Sxx，Sxy，nm，m，g，tsc，mu，ml，DFD，DFN，F，FIN，P，DF，TV，TIN
REAL R1（6），R2（30），R3（30），yr（30），yt（30），x（30）
EXTERNAL RNNOA，RNSET，UMACH，SSCAL，SADD，FIN ，TIN
CALL UMACH（2，NOUT）
OPEN（8，FILE＝＇M confidence interval．txt＇）
print＊，＂q＝＂
read（＊，＊）q
$!====$ 產生 100 筆亂數＝ニニ＝ニ＝ニ＝＝＝ニ＝ニ＝＝＝＝＝＝
ISEED＝ 25557
！ISEED1＝ 234
CALL RNSET（ISEED）
！CALL RNSET（ISEED1）
do $\mathrm{k}=1,10000$
！＝＝＝＝＝＝＝＝＝＝＝＝＝＝＝＝＝＝＝＝＝＝ $\mathrm{t}, \mathrm{F}$ value＝＝＝＝＝＝＝＝＝＝
NR2 $=30$
$\mathrm{P}=0.9$
DFN＝NR2－2
DFD＝NR2－2
$\mathrm{F}=\mathrm{FIN}(\mathrm{P}, \mathrm{DFN}, \mathrm{DFD})$
！print＊，F
$\mathrm{P}=0.95$
$\mathrm{DF}=2 * \mathrm{NR} 2-4$
$\mathrm{TV}=\mathrm{TIN}(\mathrm{P}, \mathrm{DF})$
！print＊，TV

## 

NR1＝ 6
XSTD $=1$
do $\mathrm{I}=1,5$ ！產生 mean $=10,20,30,40,50$
$\mathrm{XM}=\mathrm{I}^{*} 10$
！CALL RNSET（ISEED）
CALL RNNOA（NR1，R1）
CALL SSCAL（NR1，XSTD，R1，1）
CALL SADD（NR1，XM，R1，1）
！WRITE（NOUT，99999）R1
99999 FORMAT（＇normal random deviates：＇，20F8．4）
$\mathrm{t}(\mathrm{I})=\mathrm{SUM}(\mathrm{R} 1) / \mathrm{NR} 1$
end do
！＝＝＝＝＝＝＝＝＝＝＝＝＝＝＝＝＝將 5 個 array 轉成 100array＝＝＝＝＝＝＝＝＝＝＝＝＝＝
$\mathrm{NR} 2=30$
average＿t＝sum $(\mathrm{t}) / 5$
！WRITE（8，＊）＇Mean of X is＇， X
do $\mathrm{i}=1,5$
do $\mathrm{j}=1, \mathrm{NR} 2$
if $(((\mathrm{i}-1) * 6$ ．lt．j）．and．（j ．le． $\mathrm{i} * 6))$ then
$x(j)=t(i)$
end if
end do
end do
！print＊，＇x array＇，x
x＿bar＝sum（x）／NR2
！print＊，＇x＿bar＇，x＿bar
！＝＝＝＝＝＝＝＝＝＝＝error term $\sim \mathrm{N}(0,12)=============$
CALL UMACH（2，NOUT）
NR2 $=30$
XSTD $=36$
XM＝0
！CALL RNSET（ISEED）
CALL RNNOA（NR2，R2）
CALL SSCAL（NR2，XSTD，R2，1）
CALL SADD（NR2，XM，R2，1）
！WRITE（NOUT，9999）R2
9999 FORMAT（＇error normal random ：＇，100F8．4）

CALL UMACH (2, NOUT)
NR2 $=30$
$\mathrm{XSTD}=36$
XM=0
!CALL RNSET (ISEED1)
CALL RNNOA (NR2, R3)
CALL SSCAL (NR2,XSTD,R3,1)
CALL SADD (NR2,XM,R3,1)
!WRITE (NOUT,999) R3

999 FORMAT ('error normal random : ', 100F8.4)
!===================x_bar, y_bar=========================
$\mathrm{yr}=100+\mathrm{q}-\mathrm{x}+\mathrm{R} 2$
$\mathrm{yt}=100-\mathrm{x}+\mathrm{R} 3$
!PRINT*,' yr is', yr
!PRINT*,' yt is', yt
!print*,'x_bar',x_bar
yr_bar=sum(yr)/NR2
!PRINT*,' yr_bar is',yr_bar
yt_bar=sum(yt)/NR2
!PRINT*,' yt_bar is', yt_bar

Sxx=sum((x-x_bar)**2)
!PRINT*,' Sxx is', Sxx
Syyr=sum((yr-yr_bar)**2)
!PRINT*,' Syyr is', Syyr
Syyt=sum((yt-yt_bar)**2)
!PRINT*,' Syyt is', Syyt
Sxyr=sum(((x-x_bar)*(yr-yr_bar)))
!PRINT*,' Sxyr is', Sxyr
Sxyt=sum(((x-x_bar)*(yt-yt_bar)))
!PRINT*,' Sxyt is', Sxyt
$!========================$
br=Sxyr/Sxx
!print*,'br=',br
ar=yr_bar-br*x_bar

```
!print*',ar=',ar
bt=Sxyt/Sxx
!print*','bt=',bt
at=yt_bar-bt*x_bar
!print*,'at=',at
!============= ==============
Sr_square=(syyr-(br*sxyr))/(NR2-2)
!print*,'Sr^2=',Sr_square
St_square=(syyt-(bt*sxyt))/(NR2-2)
!print*,'St^2=',St_square
F1=Sr_square/St_square
!print*','F=',F
if(F1 .lt. F)then
Sp_square=(Sr_square+St_square)/2
!print*,'Sp^2',Sp_square
end if
V=Sp_square*(2/Sxx)
t1=(abs(br-bt))/(sqrt(v))
if(t1< TV) then
bc=(Sxyr+Sxyt)/(Sxx+Sxx)
!print*,'bc=',bc
end if
ar_new=yr_bar-bc*x_bar
at_new=yt_bar-bc*x_bar
!print*,'ar_new=',ar_new
!print*,'at_new=',at_new
M=(ar_new-at_new)
!print*,'M=',M
sigma_t=sum(((yt-at_new-bc*x)**2)/(NR2-2))
!print*,sigma_t
sigma_r=sum(((yr-ar_new-bc*x)**2)/(NR2-2))
var=((1.0/NR2)+(((30-x_bar)**2)/Sxx))*(sigma_t+sigma_r)
var_root=sqrt(var)
!print*,var_root
```

```
\(\mathrm{mu}=\mathrm{M}+(\mathrm{tv} * \mathrm{sqrt}(\mathrm{var}))\)
\(\mathrm{ml}=\mathrm{M}-(\mathrm{tv} * \mathrm{sqrt}(\mathrm{var}))\)
!print**,k,M,ml,mu
!=====================count power==========
cc1=0
if((ml > -10) .AND. \((\mathrm{mu}<10)) \mathrm{cc} 1=1\)
oc1=oc1+cc1
!====== ======count coverage probability
cc2=0
\(\mathrm{if}((\mathrm{ml}<\mathrm{q})\).AND. \((\mathrm{mu}>\mathrm{q})) \mathrm{cc} 2=1\)
oc2=oc2+cc2
!========sample size======================
sample \(=9.1827^{*}(1 /(\) Sp_square*2))
size=(sample-(((30-x_bar)**2)/Sxx))
sample_size=1/size
!print*,sample_size
!write(8,*) k,ml,mu
!,sample_size
!write(8,*) oc1,oc2
end do
write(*,*)"power=", oc1
write(*,*)"converage probability=", oc2
END program
```

!sstats.lib smaths.lib sstatd.lib smathd.lib

## PROGRAM MAIN

!use IMSL
INTEGER ISEED,ISEED1, NOUT, NR,i,j,k,muk,power,oc1,oc2,q
REAL XM,XSTD,t(5),x_bar,yt_bar,yr_bar,Syyr,Syyt, nm,
Sxx,Sxy,m,g,tsc,mu,ml,t1,t11,t111,t1111,t11111,DFD,DFN,F,FIN,P,DF,TV,TIN
REAL R1(7),R11(5),R111(3),R1111(5),R11111(7),R2(27),R3(27),yr(27),yt(27),x(27)
EXTERNAL RNNOA, RNSET, UMACH, SSCAL, SADD, FIN , TIN
CALL UMACH (2, NOUT)

OPEN（8，FILE＝＇M confidence interval．txt＇）

```
print*,"q="
read(*,*) q
!====產生 100 筆亂數====================
ISEED = 12345
!ISEED1 = 234
CALL RNSET (ISEED)
!CALL RNSET (ISEED1)
do k=1,10000
```

NR2=27
$\mathrm{P}=0.9$
DFN=NR2-2
DFD=NR2-2
F=FIN(P,DFN,DFD)
!print*, F
$\mathrm{P}=0.95$
$\mathrm{DF}=2 * \mathrm{NR} 2-4$
TV=TIN(P,DF)
!print*, TV
NR1 $=7$
XSTD $=1$
XM = 10
!CALL RNSET (ISEED)
CALL RNNOA (NR1, R1)
CALL SSCAL (NR1,XSTD,R1,1)
CALL SADD (NR1,XM,R1,1)
!WRITE (NOUT,99999) R1
99999 FORMAT ('normal random deviates: ', 20F8.4)
t1=SUM(R1)/NR1
!write(*,*)"
t1"
!write(*,*) t1
!====================120 $=20================$
NR11 = 5
XSTD $=1$
$\mathrm{XM}=20$
!CALL RNSET (ISEED)
CALL RNNOA (NR11, R11)
CALL SSCAL (NR11,XSTD,R11,1)
CALL SADD (NR11,XM,R11,1)
!WRITE (NOUT,99998) R11
99998 FORMAT ('normal random deviates: ', 20F8.4)
t11=SUM(R11)/NR11
!write(*,*)"--------------t11"
!write(*,*) t11
!====================2ean $=30==================$
NR111 = 3
XSTD $=1$
XM = 30
!CALL RNSET (ISEED)
CALL RNNOA (NR111, R111)
CALL SSCAL (NR111,XSTD,R111,1)
CALL SADD (NR111,XM,R111,1)
!WRITE (NOUT,99997) R111
99997 FORMAT ('normal random deviates: ', 20F8.4)
t111=SUM(R111)/NR111
!write(*,*)"--------------111"
!write(*,*) t111

NR1111= 5
XSTD $=1$
XM = 40
!CALL RNSET (ISEED)
CALL RNNOA (NR1111, R1111)
CALL SSCAL (NR1111,XSTD,R1111,1)
CALL SADD (NR1111,XM,R1111,1)
!WRITE (NOUT,99996) R1111
99996 FORMAT ('normal random deviates: ', 20F8.4)
t1111=SUM(R1111)/NR1111
!write(*,*)"-------------t1111"
!write(*,*) t1111
$!===================$ mean $=50=====================$
NR11111 = 7

```
XSTD = 1
XM = 50
!CALL RNSET (ISEED)
CALL RNNOA (NR11111, R11111)
CALL SSCAL (NR11111,XSTD,R11111,1)
CALL SADD (NR11111,XM,R11111,1)
!WRITE (NOUT,99995) R11111
99995 FORMAT ('normal random deviates: ', 20F8.4)
t11111=SUM(R11111)/NR11111
!write(*,*)"
```

$\qquad$

``` t11111"
```

```
!write(*,*) t11111
```

!write(*,*) t11111
!===== ========combine array================================
x=(/R1,R11,R111,R1111,R11111/)
x_bar=(t1+t11+t111+t1111+t11111)/5
!write(*,*)"

```
\(\qquad\)
```

!write(*,*) x
!write(*,*) x_bar
!============error term ~ N(0,12)
CALL UMACH (2, NOUT)
NR2 $=27$
XSTD $=12$
XM=0
!CALL RNSET (ISEED)
CALL RNNOA (NR2, R2)
CALL SSCAL (NR2,XSTD,R2,1)
CALL SADD (NR2,XM,R2,1)
!WRITE (NOUT,9999) R2
9999 FORMAT ('error normal random : ', 100F8.4)
CALL UMACH (2, NOUT)
NR2 $=27$
XSTD $=12$
XM=0
!CALL RNSET (ISEED1)

```

CALL RNNOA (NR2, R3)
CALL SSCAL (NR2,XSTD,R3,1)
CALL SADD (NR2,XM,R3,1)
!WRITE (NOUT,999) R3

999 FORMAT ('error normal random : ', 100F8.4)
\(!================x \_b a r, y \_b a r===================\)
\(y r=100+q-x+R 2\)
\(\mathrm{yt}=100-\mathrm{x}+\mathrm{R} 3\)
!PRINT*,' yr is', yr
!PRINT*,' yt is', yt
!print*,'x_bar',x_bar
yr_bar=sum(yr)/NR2
!PRINT*,' yr_bar is',yr_bar
yt_bar=sum(yt)/NR2
!PRINT*,' yt_bar is', yt_bar
\(\operatorname{Sxx}=\operatorname{sum}\left(\left(x-x \_b a r\right) * * 2\right)\)
!PRINT*,' Sxx is', Sxx
Syyr=sum((yr-yr_bar)**2)
!PRINT*,' Syyr is', Syyr
Syyt=sum((yt-yt_bar)**2)
!PRINT*,' Syyt is', Syyt
Sxyr=sum(((x-x_bar)*(yr-yr_bar)))
!PRINT*,' Sxyr is', Sxyr
Sxyt=sum(((x-x_bar)*(yt-yt_bar)))
!PRINT*,' Sxyt is', Sxyt
\(\mathrm{br}=\mathrm{Sxyr} /\) Sxx
!print*,'br=',br
ar=yr_bar-br*x_bar
!print*,'ar=',ar
\(\mathrm{bt}=\) Sxyt/Sxx
!print*,'bt=',bt
at=yt_bar-bt*x_bar
!print*,'at=',at
```

Sr_square=(syyr-(br*sxyr))/(NR2-2)
!print*,'Sr^2=',Sr_square
St_square=(syyt-(bt*sxyt))/(NR2-2)
!print*,'St^2=',St_square
F1=Sr_square/St_square
!print*,'F=',F
if(F1 .lt. F)then
Sp_square=(Sr_square+St_square)/2
!print*,'Sp^2',Sp_square
end if
V=Sp_square*(2/Sxx)
t1=(abs(br-bt))/(sqrt(v))
if(t1< TV) then
bc=(Sxyr+Sxyt)/(Sxx+Sxx)
!print*,'bc=',bc
end if
ar_new=yr_bar-bc*x_bar
at_new=yt_bar-bc*x_bar
!print*,'ar_new=',ar_new
!print*,'at_new=',at_new
M=(at_new-ar_new)/bc
!print*','M=',M
g=((TV**2)*Sp_square)/((bc**2)*(Sxx+Sxx))
!print*,'g=',g
tsc=(TV*sqrt(Sp_square))/abs(bc)
nm=(2.0/NR2)+((M**2)/(Sxx+Sxx))
!print*,nm
mu=M+(tsc*sqrt(nm))
ml=M-(tsc*sqrt(nm))
!print*,k,M,ml,mu
!====================count power=====================
cc1=0
if((ml > -10) .AND. (mu < 10)) cc1 = 1
oc1=oc1+cc1

```
!==== =======count converage probability===============
cc2=0
\(\mathrm{if}((\mathrm{ml}<\mathrm{q})\). AND. \((\mathrm{mu}>\mathrm{q})) \mathrm{cc} 2=1\)
oc2=oc2+cc2
!========sample size=============
sample=9.1827*(1/(Sp_square*2))
size=(sample-(((30-x_bar)**2)/Sxx))
sample_size \(=1 /\) size
!print*,sample_size
write (8,*) k,ml,mu
!,sample_size
!write(8,*) oc 1,oc2
end do
write(*,*)"power=", oc1
write(*,*)"coverage probability=", oc2
END program```

