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利用網路分析金融危機傳染風險
Financial Contagion and Network Analysis

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摘要

金融傳染作為系統風險的決定因素，在過去幾十年間通常是透過網絡模型進行研究。此篇論文中，我們將 Elliott et al. (2014) 所建立的模型一般化，融合了資產重疊（overlapping portfolios）與賤賣資產效應（fire sale）。此論文主要貢獻在於，在透過持股或資產重疊所產生的依賴性上，我們比較了風險分散與傳播渠道的綜合作用，以及賤賣資產如何影響風險分散的有效性。另外，我們指出金融整合度（integration）可分散網絡中的彼此間的依賴性以降低金融傳染的嚴重程度。透過此篇論文的研究，我們強調了制定總體審慎政策時，政策制定者應要考慮不同依賴性來源。

關鍵詞：系統風險、傳染、資產重疊、金融網絡、賤賣資產、風險分散

JEL 分類代號：D85, G01, G11

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Abstract

As one of the determinants of systemic risk, financial contagion has been studied from the perspective of network theory in the past decades. In this thesis, we modify the model designed by Elliott et al. (2014) to generalize the idea of dependency by incorporating overlapping portfolios and fire-sale effect. The main contribution of this thesis is that we compare the risk-sharing effect and the function of transmission channels between share-based and liquidity-based dependency, and how fire-sale affects the effectiveness of diversification. Furthermore, we identify another effect of integration, which could reduce the severity of contagion by averaging the dependency in the networks. Through our study, we stress the importance of considering different sources of dependency when making macroprudential policies.

Keywords: Systemic risk, Contagion, Overlapping portfolio, Financial network, Fire sale, Risk sharing

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Chapter 1

Introduction

The global financial traumas in the past few decades not only harm the global financial markets, but contributes to a prolonged period of depression to real economy all over the world. The growth of international finance intertwines the world so that a disturbance of one market could impact another market miles away. The phenomenon is related to the growth of international finance that intertwines the globe to such an extent that a slight disturbance to one market can have a profound disruption to another country on the other side. The newly-developed domino effect through the interconnected financial markets is coined the terms, systemic risk, to roughly describe a conception that a single or clusters of individual entities can provoke the collapse of an entire system. The concept of systemic risk in finance has been studying extensively in the recent years, both theoretically and empirically.

In a financial system, systemic risk arises mostly from two sources. Firstly, financial institutions have financial exposures to each other, whether directly by equity or debt or indirectly by overlapping portfolios. The financial exposures serve as chains binding institutions to each other so that they're interdependent on each other. A single drop in one institution's value can cause other's to drop. A single collapse of one institution will result in another to fail as well. The domino effect take places when the financial links between institutions act as contagion channels

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to spread distress to one another. Another source is self-fulfilling bank run. As long as there exists multiple equilibria, the change of investor's beliefs will shift the equilibrium to another. This sort of self-fulfilling prophecy largely stems from people's expectation for the health of financial institution. Even the fundamental value doesn't change, as long as some investors believe, or they think some other investors believe that the financial institution is in distress, then the panic will turn their expectation into actual behaviors, and fulfill their expectation. The standard treatment of self-fulfilling bank runs is Diamond and Dybvig (1983) [1]. In this paper, we will only focus on the first source of systemic risk, which I will term it as financial contagion.

A natural way to model the first source of systemic risk is network (graph), which is the typical method to formalize the concept of cascading failures in a variety of academic disciplines such as epidemiology, transportation system or computer science. To answer the question of how a cluster of failures leads to the breakdown of the system, each single entity is mode as a *node*, and the linkages between them are called *edges*, which in different scenarios can represent the acquaintances, wires or similarity etc. For the case of financial contagion, financial exposures as edges are treated as possible contagion channels to spread the failures of nodes to their neighbors. When financial organizations as nodes are linked through financial contracts or any sort of implicit connection, the failures of organizations can spread through to other organizations, and deliver the distress across the financial market. While financial linkages serve as contagion channels to spread the failures, it also provides an effect of risk-sharing when the connections are extensive and strong enough for the organization's values to be distributed among multiple organizations so that the first failure can be avoided, and the distress can be largely shared to the point away from subsequent failures.

In the past, financial contagion is mainly studied in two branches of literature: direct exposure through shareholding or debt contract and indirect exposure through overlapping portfolios. Direct exposure indicates that values of financial

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institutions are interdependent. When one's value drop, the values of its neighbors drop as well. Allen and Gale (2000) [2], Eisenberg and Noe (2001) [3], Elliott, Golub, and Jackson (2014) [4], and Jackson and Pernoud (2019) [5] are some of the pioneers that investigate the financial contagion with direct exposure. Allen and Gale (2000) [2] constructs a stylized financial system, where banks in four different regions are subjected to liquidity shock, and hold interregional claims (exchange parts of their deposits) on other banks to provide a buffer to liquidity shock. They found that the contagion largely depended on the completeness of the structure of interregional claims. In the financial system built by Eisenberg and Noe (2001) [3], financial institutions are connected through debt claims. They developed an algorithm to find a clearing payment equilibrium, and proved the existence of clearing payment vector with certain conditions, with which many succeeding papers built upon. Apart from debt-based dependency, Elliott, Golub, and Jackson (2014) [4] considered an equity-based dependency among organizations, where the contagion starts from the drop of some organization's values, and spreads across the system. They formalized the depth and breadth of financial network, coining the terms integration and diversification, and examine their relationship to contagion. Jackson and Pernoud (2019) extend the model in Elliott, Golub, and Jackson (2014) [4] to include both the dependency from equity and debt.

On the other hand, the indirect exposure is more implicit, which relies on two assumptions: liquidation effect (fire-sale effect) and mark-to-market characteristic of portfolio valuation. A liquidation of assets from failed financial institutions can deteriorate the market, lowering down the asset price (fire-sale or liquidation effect). Other financial organizations holding the same types of assets would have to re-valuate its portfolio accordingly (marked-to-market). As a result, the market values are pushed toward the boundary of failure, and continue the spread of failures. Liquidity effect as a contagion channel has been investigated by many works including Cifuentes et al. (2005) [6], Gai and Kapadia (2010) [7], Caccioli et al. (2014) [8], and Caccioli et al. (2015) [9]. In Cifuentes et al. (2005) [6], they

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studied the roles of liquidaity constraint in a system of interconnected financial organizations. Gai and Kapadia (2010) [7] generalized the model to arbitrary network structure, which showed that financial systems may exhibit a robust-yet-fragile tendency. As for Caccioli et al. (2014) [8], they fully focused on the financial contagion due to overlapping portfolios and liquidity effect, and also demonstrated a robust-yet-fragile financial system.

While differs in methodology and model set-up, most of the previous papers all reached to a conclusion that interconnections among financial organizations display a non-monotonic relationship with the severity of contagion. The reason behind is that the network serves two contradicting purposes in terms of financial contagion. Firstly, the existence of interdependencies allow for risk-sharing, which can help individual institutions be less susceptible to individual liquidity or portfolio shocks. Secondly, the same interdependencies permit more spreading channels to pass on the shock given the first occurrence of failure. Although many papers have studied the contagion with one of the exposures, very little have discussed two kinds of exposures altogether. Therefore, to study whether there exists interaction effect to intensify or mitigate the contagion, I develop a general model consisting of financial network, in which financial institutions connect to each other through equity contract with each making their own investment decisions independently to form an asset-holding bipartite network.

The network models used in this paper are inherited from the one by Elliott et al. (2014) [4], where organizations are interconnected through equity-like financial contracts. Specifically, for each organization, certain amount of shareholdings are distributed among others, through which the values of organizations demonstrate the property of interdependency, forming the so-called financial network. This paper complements, and builds upon it by incorporating the idea of portfolio diversification and fire-sale effect. Through diversifying a portfolio across a multiple types of assets, organizations will inevitably have some overlapping between portfolios that any asset will be held by more than one organization.

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Fire-sale effect represents the price deterioration from asset liquidation. As seen in many financial crises, the accounting principle of mark-to-market makes the values of assets to be recalculated on the basis of market prices. Aside from incurring bankruptcy costs, an insolvency forces the organization to liquidate all the assets on the book for repayment. The liquidation in turn lowers down the market prices by releasing a fair amount of assets in a sudden, and all the organizations that hold the same assets on the book are damaged. As a consequence, overlapping portfolios become another potential channel for defaults to spread when failures lead to liquidation of assets, which in turn deteriorates the prices, and damages the organizations that have the same holdings.

The primary focus of this paper is to understand the mechanisms of financial contagion through direct shareholdings and overlapping portfolios by comparing the effects with random financial and asset-holding network. The main contribution of this paper is the integration of two contagion channels separately studied in two different branches of literature. By studying the interaction between two contagion networks in a wider picture, I show that the damage caused by indirect interconnection through fire-sale effect can overpower the one by direct exposure. More importantly, a new effect of integration is observed.

The paper is organized as follows. Section 2 describes the details of model setup. Section 3 explains the concepts of integration and diversification. Section 4 presents the numerical results from simulation. Section 5 summarizes the results and makes concluding remarks.

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Chapter 2

Model

Organizations and portfolios

Consider an economy consists of m financial institutions (organizations), and n types of investments (assets). Every organization is endowed with a certain amount of wealth w_i, and each constructs their portfolio over n types of assets based on the given asset prices.

Financial network

Organizations are interconnected through a cross-holding network, which can be represented by an adjacency matrix $\mathbf{C}$. The entry C_{ij} is the fraction of organization j held by organization i ($C_{ij} \leq 1$), and C_{ii} is set to be 0. Define the self-ownership matrix as $\hat{\mathbf{C}}$, where the diagonal entries, $\hat{C}_{ii} = 1 - \sum_{j=1}^{m} C_{ji}$, and zero elsewhere. With financial network, the value of organization i is the total amount of its endowment plus the value of its claims on other organizations, and therefore can be calculated as:

$$V_i = w_i + \sum_j C_{ij} V_j \tag{2.1}$$

Write it in matrix form,

$$\mathbf{V} = \mathbf{w} + \mathbf{C}\mathbf{V}$$

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solve it to yield

$$\mathbf{V} = (\mathbf{I}\text{-}\mathbf{C})^{-1}\mathbf{w}$$

As elaborated in Elliott et al.(2014) [4], cross-holding can result in inflated value. To correctly account for the true values of organizations, it's necessary to multiply the cross-holding to self-ownership. Therefore, the non-inflated market value of organization is

$$\mathbf{v} = \hat{\mathbf{C}}(\mathbf{I}\text{-}\mathbf{C})^{-1}\mathbf{w} = \mathbf{A}\mathbf{w} \tag{2.2}$$

, where **A** and can be shown to be a column-stochastic matrix. The entry A_{ij} represents the ultimate claim of j by i, and the diagonal entry A_{ii} can be seen as the claim by external holder (self-ownership), the portion that is not held by other organizations in the network.

Asset-holding network

Aside from the interconnections among organizations represented as financial network, each organization constructs their own portfolio by investing across n types of assets with initial endowment. The portfolio of organization i and the collection of all organization's portfolios can be represented as

$$\mathbf{D}_i = \begin{bmatrix} D_{i1} \\ \vdots \\ D_{ij} \\ \vdots \\ D_{in} \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} \mathbf{D}_1^T \\ \vdots \\ \mathbf{D}_i^T \\ \vdots \\ \mathbf{D}_m^T \end{bmatrix} = \begin{bmatrix} D_{11} & \dots & D_{1n} \\ \vdots & \ddots & \vdots \\ D_{m1} & \dots & D_{mn} \end{bmatrix}$$

, where D_{ij} is the share of asset j held by organization i.

Given initial wealth w and asset prices $\mathbf{p}$, organizations forms their portfolio $\mathbf{D}_i$ so that we will have $\mathbf{w} = \mathbf{D}\mathbf{p}$. As the result, the non-inflated market value of organizations can be rewritten as

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$$\mathbf{v} = \mathbf{A}\mathbf{w} = \mathbf{A}\mathbf{D}\mathbf{p} \tag{2.3}$$

Default

To study the systemic risk originated from a small malfunction of the system, we need to understand what could trigger failures of the system. In the economy, an organizations defaults when the value drops below a certain default threshold θ_i, and the decrease in values come from two sources of shocks: the initial exogenous asset shock, and the deterioration of asset prices resulted from defaults of other organizations. Once an organization defaults, it immediately retires from the networks, and liquidate all shares of its holding assets. As a consequence, the default costs will come in two forms: bankruptcy cost and liquidity cost. When organization defaults, it files bankruptcy, and liquidates all its assets. The liquidation of assets incurs liquidation (fire-sale) effect, which deteriorate the market prices of all liquidated assets. Due to the mark-to-market accounting characteristic, all other organizations holding the liquidated assets should revalue their portfolio based on a newly calculated prices. I rely on the so-called price impact function[1]to estimate the influence of fire sale on market price: $p(q) = pe^{-\alpha q}$, where q $(0 \leq q \leq 1)$ is the cumulative portion of liquidated share of assets, and α is a fire-sale impact parameter, which is set to be 1.0536 later on in the numerical simulation so that the log price reduction is proportional to the amount of liquidation.[2]

Aside from the market price deterioration, bankruptcy cost captures all other types of costs such as legal and accounting fees, loss of human capital etc. Generally speaking, the bankruptcy cost can depend on the market value of each organization: $b_i(v_i) = \beta(v_i)1_{v_i \leq \underline{v_i}}$, where 1 is an indication function equal to 1 if $v_i \leq \underline{v_i}$, and 0 otherwise.

[1]See Bouchaud, J.-P. (2010) [10]

[2]The choice of 1.0536 corresponds to the linear market impact for log-prices (the price drop by 5% when 5% of the asset is liquidated). See Cifuents et al. (2005) [6], Caccolli et al.(2014) [8] for more details

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Default costs imply that market values of organizations are not linearly dependent on the portfolio values, but rather involves discontinuity. As a result, the market value of i can be generalized to:

$$v_i = \sum_{j=1}^{m} A_{ij}(\mathbf{D}_i^T \mathbf{p}(\mathbf{v}) - b_j(v_j))$$

, where

$$\mathbf{p}(\mathbf{v}) = \begin{bmatrix} \vdots \\ p_j \\ \vdots \end{bmatrix},\ p_j = p_j e^{-\alpha q_j(\mathbf{v})},$$

$$q_j(\mathbf{v}) = \sum_{i=1}^{m} \overline{D}_{ij} 1_{v_i \leq \underline{v_i}}\ ,\ \overline{D}_{ij} = \frac{D_{ij}}{\sum_{i=1}^{m} D_{ij}}$$

and

$$b_j(v_j) = \beta(v_j) 1_{v_j < \underline{v_j}}$$

Write it in matrix form

$$\mathbf{v} = \mathbf{A}(\mathbf{D}\mathbf{p}(\mathbf{v}) - \mathbf{b}(\mathbf{v})) \tag{2.4}$$

The above equation demonstrates the relationships between market values of organizations and financial network, asset-holding networks, asset prices and default costs in a nicely closed form.

Multiple Equilibria

To guarantee the existence of solutions to the above equation, I rely on lattice structure[3]and Knaster-Tarski fixed point theorem to show that the equation indeed contains a solution.

[3]Lattice structure is a partially ordered set, where any two elements has a unique supremum and a unique infimum with some pre-defined binary relationship. We say a lattice is complete if, for any of its subset, there exists supremum and infimum.

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Theorem 2.0.1 (Equilibrium existence)**.** *For any financial network, asset-holding network, vector of prices, and vector of bankruptcy costs, there exists at least one equilibrium, and the set of equilibria forms a complete lattice.*

Proof. To show there is always a vector of market value flowing to shareholders, **v** satisfying $\mathbf{v} = \mathbf{A}(\mathbf{Dp}(\mathbf{v}) - \mathbf{b}(\mathbf{v}))$

Let $L = [0, \sum_{j \in N} A_{1j} w_j] \times ... \times [0, \sum_{j \in N} A_{nj} w_j]$, therefore, any possible vector of value is an element in L. Suppose the elements in L is element-wise ordered. That is, for any two vectors $v_1, v_2 \in L$

$$v_1 \leq v_2 \text{ if an only if } v_i^1 \leq v_2^i \text{ for all } i = \{1, ..., n\}$$

As such, $(L, \leq)$ is a partially ordered set. To show that L forms a complete lattice, define subset $S \subset L$ by $S = \{v_p\}_{i=1}^n$. S therefore, has an infimum and supremum given by

$$\text{inf}S = v_1 \wedge ... \wedge v_n, \text{ sup}S = v_1 \vee ... \vee v_n$$

, which are both elements in L. Therefore, L is a complete lattice.

Define the mapping $\Phi(\mathbf{v}) = \mathbf{A}(\mathbf{Dp(v)} - \mathbf{b(v)})$. Because failures are complements:

$$\begin{aligned} v_1 \leq v_2 &\Rightarrow \sum_j D_{1j} p_j \leq \sum_j D_{2j} p_j, \ \beta_i I_{v_1^i \leq \underline{v_i}} \geq \beta_i I_{v_2^i \leq \underline{v_i}} \\ &\Rightarrow Dp(v_1) - b(v_1) \leq Dp(v_2) - b(v_2) \\ &\Rightarrow \Phi(v_1) \leq \Phi(v_2) \end{aligned}$$

Thus, $\Phi(v)$ is an order-preserving function. Because L is a complete lattice, and $\Phi : L \to L$ is an ordering-preserving function, by the Knaster-Tarski fixed point theorem, Φ always has a fixed point in L, and the set of fixed points is a complete lattice. □

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The proof above shows that there is at least one solution to the equation. That is, it's guaranteed that given any structure of cross-holdings, vector of asset prices, portfolios, and default costs, there is at least one combination of market values of organizations, in which the financial contagion stops to continue, and reach to an equilibrium. Nevertheless, the theorem above only guarantees the existence of solution, the discontinuity structure of the value equation (4) allows multiple values of v to solve the equation.

As specified in Elliott et al. (2014) [4], there are two sources of multiple equilibria: self-fulfilling bank run, and interdependence of the values of the organizations. Following Elliott et al. this paper will discuss the second source of multiplicity, and focus on the *best-case equilibrium*[4], which allows us to identify the necessary failures given an exogenous shock.

Next, I demonstrate a standard algorithm to find the best-case equilibrium, which can characterize the hierarchy of cascading failures during the contagion process.

Contagion process

Before the realization of exogenous asset shock, all organizations are assumed to be solvent ($v_i \geq \underline{v}_i$ for all i). An exogenous asset shock triggers the first wave of organization's failures, the defaults result in asset fire-sale, which deteriorates asset prices, and also bring down the values of defaulted organization's neighbors. Some organizations affected by the first wave of failures default, and start the second wave of failures, and so on until no other organizations continue to fail.

To identify the solutions and *hierarchy* of contagion process. A standard algorithm[5](, where we will later use it for numerical simulation) is adopted as follows:

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[4]Within multiple equilibria for the contagion model, best-case equilibrium corresponds to the case, in which as few as organizations fail. As detailed in Elliott et al. (2014) [4], we can identify worst-case equilibrium as well, but with *suitable regularity conditions* , best-case equilibrium can provide a well enough analogue.

[5]The algorithm is standard, and being used in literature of contagion to find extreme points of

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Let Z_t be the set of failed organizations at step $t = 0, 1, ...$ Initialize $Z_0 = \emptyset$. At step $t \geq 1$:

1. Let $\mathbf{b}_{t-1}$ be the vector with element $b_i = \beta_i$ if $i \in Z_{t-1}$ and 0 otherwise.

2. Let $\mathbf{D}_{i,t-1}$ be the organization i's original portfolio at step $t-1$ if $i \notin Z_{t-1}$ or zero vector otherwise

3. Let $\mathbf{q}_{t-1}^T$ to be

$$\mathbf{q}_{t-1}^T = \mathbf{1}(\mathbf{D}_0 - \mathbf{D}_{t-1}) \text{ diag}(\mathbf{1D}_0)^{-1} = \begin{bmatrix} \ldots q_{j,t-1} \ldots \end{bmatrix}$$

, where $q_{j,t-1}$ is the cumulative liquidation portion of asset j at step $t-1$

4. Let $\mathbf{p}_{t-1}$ be the price vector with element $p_{j,t-1} = p_{j,0}e^{-\alpha q_{j,t-1}}$

5. Let Z_t be the set of all k such that entry k of the following vector is negative:

$$\mathbf{A}(\mathbf{D}_{t-1}\mathbf{p}_{t-1} - \mathbf{b}_{t-1}) - \underline{\mathbf{v}} < 0, \text{, where } \mathbf{D}_{t-1} = \begin{bmatrix} \mathbf{D}_{1,t-1} \\ \vdots \\ \mathbf{D}_{m,t-1} \end{bmatrix}$$

6. Terminate if $Z_s = Z_{s-1}$. Otherwise, return to step 1.

When the algorithm terminates at time T, the corresponding Z_T is the best-case equilibrium.

2.1 Integration and diversification

According to Elliott et al. (2014) [4], the structure of financial network can be discussed from two aspects: integration and diversification. In general, a more integrated financial network means the self-ownership of organizations are lower than a less integrated financial network, i.e. the values of organizations are

lattice. See Elliott et al. (2014), Eisenberg and Noe (2001) for more details

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more dependent on each other. On the other hand, a more diversified financial network indicated that the cross-holding of an organization involves more other organizations than a less diversified financial network. To formalize the idea of integration and diversification, I present their formal definitions below as a foundation for numerical simulation.

Integration

I follow Elliott et al. (2014) [4] to define the formal definition of integration of financial network:

Cross-holding **C**' is more integrated than **C** if

$$\sum_{j \neq i} C'_{ji} \geq \sum_{j \neq i} C_{ji}$$

for all i, with strict inequality for some i[6]

As for diversification, there are two types of diversification: organization diversification and portfolio diversification.

Organization diversification

Cross-holdings $\mathbf{C}'$ are more diversified than cross-holdings **C** if

1. $C'_{ij} \leq C_{ij}$ for all i, j such that $C_{ij} > 0$, with strict inequality for some ordered pair (i, j), and
2. $C'_{ij} > C_{ij} = 0$ for some i, j

Portfolio diversification

Aside from the original definitions in Elliott et al. (2014) [4], I incorporate the concept of portfolio diversification into the model. I use Herfindahl–Hirschmann index, which is a typical measure of market concentration to quantify the level of portfolio diversification across different asset-holding network. The index is defined as:

$$H_i = \left[\sum_{j=1}^{n} \overline{w}_{ij}^2 \right]^{-1}$$

[6]It's equivalent to the condition $\hat{C}'_{ii} \leq \hat{C}_{ii}$ for all i, with strict inequality for some i.

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, where $\overline{w}_{ij}$ is the portion of i's wealth invested in asset j. The index equals to 1 if an organization invests all of its wealth into one single asset; conversely, if the organization uniformly invests across k types of assets ($k \leq n$), the index will equal to the degree k[7]

Integration quantifies the extent of organization involvement in the financial network. A larger integration means organizations are more dependent on each other. Organization diversification captures the degree of diversification of dependency. That is, a higher organization diversification means a more diversified cross-holding of organizations. On the other hand, portfolio diversification captures probability of portfolios overlapping and the impact of liquidation effect. A more diversified portfolio indicates a higher chance of portfolio overlapping with others and lower impact on market price when it's liquidated.

The formal definitions above pave the way for the numerical simulation to systematically analyze how the structure of two networks affect the extent and severity of contagion.

2.2 Numerical results

To understand how cascading failure is affected by the interaction between fire-sale effect, integration and diversification. I perform a simulation with three stylized random models with one being the benchmark model similar to the one used in Elliott et al. (2014), and the other two being generalized models with and without fire-sale effect.

I assume $n = m$, and the initial wealth for each organization to be 100. All organizations have a common default threshold $\underline{v}_i = \underline{v} = \theta v$, where $\theta \in (0, 1)$. With the same initial endowment, organizations invest across n types of assets, where all asset prices are initialized to be 1.

[7]If organizations uniformly invest in k types of assets, then $H_i = \sum_{j=1}^{k} \left(\frac{1}{k}\right)^2 = k$. For simplicity, I'll assume that organizations follow a uniform investment strategy, and only adjust k to represent different levels of portfolio diversification for later numerical simulation.

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As for the construction of stylized random networks, I rely on three parameters to represent integration, organization diversification, and portfolio diversification: c, d and e, respectively. As follows, I describe the steps in more detail to construct a random asset-holding network and financial network.

2.2.1 Asset-holding network construction

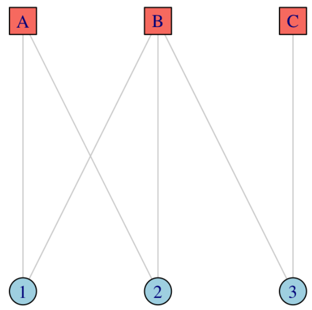

Figure 2.1: A simple bipartite graph with organizations: A, B, C, and assets: 1, 2, 3. Each edge represents the investment. For instance, organization B investments across all three assets.

The asset-holding network can be modeled as a bipartite graph **G**, where there are two types of vertices: organizations and assets. The entry, G_{ik} is the edge between organization i and asset k, which means investment of i in k. Edges can be interpreted as organization's investments, so all the edges coming from an organization is its portfolio. To demonstrate how portfolio diversification affects probability of failure, I follow the steps below to construct a random asset-holding network. With the simple random asset-holding construction, the overall portfolio diversification across all organizations is e, which is exactly the Herfindahl–Hirschmann index.

1. For each organization i, it randomly links to each asset with probability $e/n,\ 0 \leq e \leq n$, where e represents portfolio diversification so that the

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expected degree is e. Specifically saying, G_{ik} is a Bernoulli random variable each taking value 1 with probability e/n and 0 otherwise.

2. Assume each invests uniformly across their connected assets. For example, if organization i is linked to five assets, then it invests $1/5$ of wealth on each. As such, the asset-holding network **D** , where the entry is $D_{ik} = \frac{w_i G_{ik}}{\sum_j G_{ij}}$.

2.2.2 Financial network construction

In addition to the asset-holding bipartite network, organization are connected in a financial network, where I follow the method proposed in Elliott et al. (2014) [4] to construct a random financial network.

1. Generate a directed random network **C** with parameter c, and d representing integration and diversification respectively.

2. Calculate the financial network **A** from **C** as described in the previous section.

With the constructed random networks, the algorithm described previously is used to find the hierarchy of cascading failure, in which organizations fail as few as possible (the best-case equilibrium).

2.2.3 Contagion algorithm

At t of the algorithm, let Z_t be the set of failed organizations. Initialize $Z_0 =$. At time $t \geq 1$

1. Initially shock a random asset j, reducing the market value from 1 to 0, assuming other asset values remain the same. When market deteriorates, each organization i holding the asset j decreases in value by D_{ij}

2. Liquidate the portfolio of insolvent organizations, and get a new asset-holding network $\mathbf{D}'$.

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3. Calculate the new asset prices with the accumulative portion of liquidated asset q', and price impact function $\mathbf{p}'(q) = pe^{-\alpha q'}$

4. Let Z_t be the set of all k such that entry k of the following vector is negative:

   $\mathbf{A}(\mathbf{D}'\mathbf{p}'(\mathbf{v}) - \mathbf{b}(\mathbf{v})) - \underline{\mathbf{v}} < 0$

5. Terminate if $Z_t = Z_{t-1}$. Otherwise, return to step 2.

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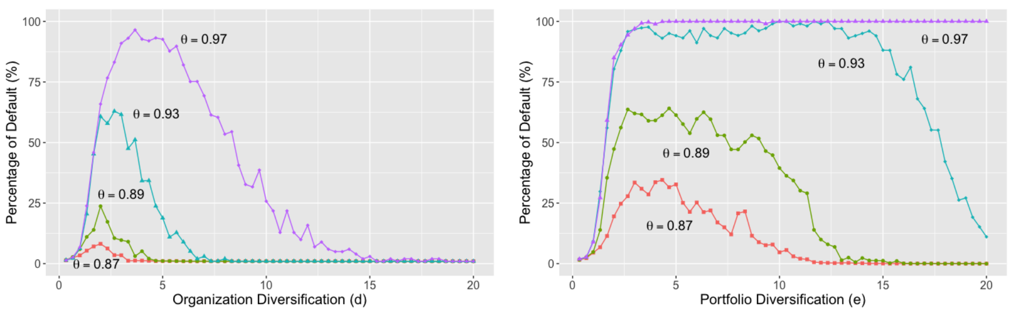
Chapter 3

Numerical Results

I consider n=m=100 nodes for assets and organizations, and simulate on organization diversification d, portfolio diversification e between 1 and 20, and integration c from 0.1 to 0.9. The threshold parameter, θ is ranged from 0.8 to 0.99. I first make a comparison between two benchmark models to investigate the relationship between diversification and contagion for each network. Next, I set up a multi-asset model, where the fire-sale is incorporated to explore the efficiency of diversification. Lastly, I review the effect of integration on reducing the number of failures.

The main outcomes are summarized as follows:



(a) Effect of organization diversification on expected percentage of failure (c=0.5, **D=I**)

(b) Effect of portfolio diversification on expected percentage of failure (**A=I**, $\alpha = 1.0536$)

Figure 3.1: How is percentage of default affected by two types of diversification, $1 \leq d \leq 20$ with several thresholds ($\theta = 0.87$, $\theta = 0.89$, $\theta = 0.93$, $\theta = 0.97$) (Averaged over 100 simulations)

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Which network contributes more to the contagion?

The first result is the comparison between portfolio diversification and organization diversification. Holding other variables fixed, I set one of the network to be identity matrix, and varies one of the two diversification parameter to see how the number of failures change accordingly.

For panel A of Figure 3.1, every organization holds a unique proprietary asset (therefore asset liquidation doesn't affect other organizations) with organization diversification varying while holding integration c fixed. Similar to the case in Elliott et al.(2014), the percentage of organizations failing displays a non-monotone relationship with organization diversification *d*, and the number increases with the default threshold θ.

For panel B of Figure 3.1, rather than diversifying shareholding, organizations diversify the risk of contagion by investing across multiple assets. When $\alpha = 1.0536$, fire-sale effect makes diversification in portfolio less effective in reducing contagion in comparison to the diversification in organization. At the same level of default thresholds, the percentage of failures is higher for portfolio diversification than organization diversification. Additionally, the level of diversification where percentage of failures starts to decrease is larger in the case of asset-holing network, hovering at high percentage longer before going down.

The non-monotonicity of percentage of failures demonstrates the trade-off introduced by both diversification: risk sharing and contagion channel. At low level of diversification, the contagion fails to spread across organizations due to lack of share-based connections or overlapping portfolios. On the other hand, at high level of diversification, both exogenous shock and default cost are shared among larger amount of organizations so that the loss is too small to trigger a further wave of contagion. The most dangerous section lies in where diversification lingers in the middle range, where the shock is not properly mitigated, and the contagion channels are enough for the default to spread.

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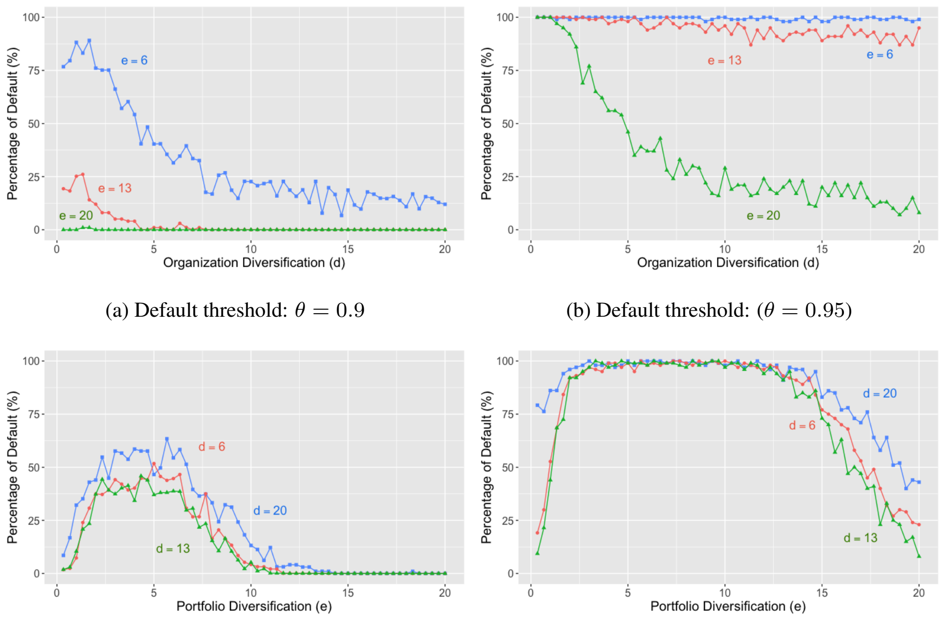
Which diversification is more effective?

Next, I develop a stylized multi-asset model, where organizations are connected through shareholding in a financial network, and they independently invest across a spectrum of assets, forming an asset-holding network. Two diversification parameters are varied to investigate the interaction effect on the extent of contagion.



(a) Default threshold: $\theta = 0.9$ (b) Default threshold: $(\theta = 0.95)$

(c) Default threshold: $(\theta = 0.9)$ (d) Default threshold: $(\theta = 0.95)$

Figure 3.2: How do two types of diversification interact to affect the percentage of default under two thresholds ($\theta = 0.9$, $\theta = 0.95$) ($c = 0.5$, Averaged over 100 simulations)

Panel A and B of Figure 3.2 show the percentage of failures across organization diversification at three different levels of portfolio diversification ($e = 3$, $e = 13$, $e = 20$) with default threshold $\theta = 0.9$ and $\theta = 0.95$.

Apparently, the number of failure declines with default threshold. The non-monotonicity in organization diversification is almost eliminated with a slight increase in portfolio diversification, i.e. given any level of portfolio diversification, the disadvantage of organization's dependency disappeared. However, at low levels of organization diversification, the percentage of failures is brought up by an increase in portfolio diversification. The introduction of portfolio diversification

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only worsens the circumstances if the risk-sharing effect is not enough to wipe out the fire-sale effect.

On the other hand, Panel C and D of Figure 3.2 show the percentage of failures across portfolio diversification at three different levels of organization diversification with the same levels of default threshold being $\theta = 0.9$ and $\theta = 0.95$ (Panel C and D, respectively). As shown in the graphs, the property of non-monotonicity is retained, and In comparison to the previous case, where the number of failures decrease in both types of diversification, higher levels of organization diversification makes no significant difference in reducing average failures for both default thresholds.

The results above show that when organizations are interconnected through a financial and asset-holding network, portfolio diversification becomes the main primary factor that determine the extent of contagion. In contrast to portfolio diversification, organization diversification made little effort in lessening fire-sale effect introduced by portfolio overlapping.

Price impact and diversification

To investigate how the percentage of default is affected by price impact parameter, I perform simulations on five various values of price impact parameters α, which represents different sensitivity of asset price to the cumulative portion of liquidation.[8]

Panel A, B and C in Figure 3.3 display the results with three levels of portfolio diversification. As seen from the graphs, the effect of larger price impact parameter make a visible difference only when portfolio diversification is insufficient. As shown in Panel A, higher price impact parameters corresponds to higher percentage of defaults, such that the risk-sharing effect of organization diversification become insignificant at higher price impact parameters. On the other hand, as portfolio

[8] $\alpha = 0.5129$, 0.7796, 1.0536, 1.3353, 1.625 represent 5%, 7.5%, 10%, 12.5%, 15% price drop, respectively when 10% asset is liquidated.

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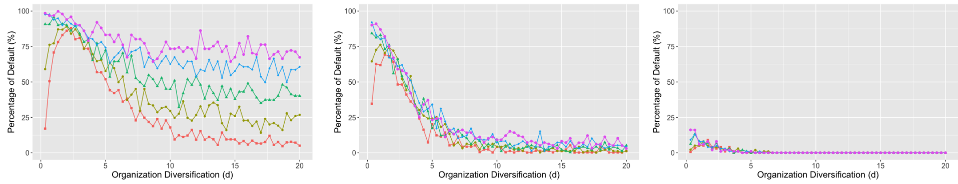
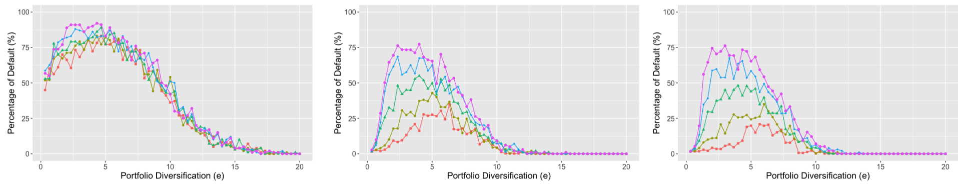


(a) Low portfolio diversification (e=3) (b) Medium portfolio diversification (e=10) (c) High portfolio diversification (e=17)

Figure 3.3: How percentage of Failures across organization diversification affected by different levels of α ($c = 0.5$, $\theta = 0.9$)



(a) Low organization diversification (d=3) (b) Medium organization diversification (d=10) (c) High organization diversification (d=17)

Figure 3.4: How percentage of default across portfolio diversification affected by different levels of α ($c = 0.5$, $\theta = 0.9$)

diversification moves upwards, the marginal effect of price impact parameters diminish to the extent that higher sensitivity to liquidation makes no significant effect on the contagion result.

The main reason behind distinct results between low and higher portfolio diversification is that fire-sale has larger impact on asset prices when the liquidated assets are held by fewer organizations. When portfolio diversification is low, an asset is expected to be held by only a few organizations. As such, the corresponding larger price impact parameter becomes the main contributor to price deterioration, and will then speed up the spreading during the contagion. However, as portfolio diversification move upwards, the portfolios begin to overlap considerably. As a result, the cumulative portion of liquidation becomes the main determining factor of price impact.

On the other hand, Panel A, B and C of Figure 3.4 illustrate the results with three levels of organization diversification. Contrary to the previous case, the impact

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of price deterioration only takes effect at medium or above level of organization diversification. As shown in Panel A, at low level of organization diversification, the average defaults follow the same patterns across five different levels of price impact. Nevertheless, as organization diversification increases to medium level or above, the differences become visible, where higher value of price impact parameter results in higher percentage of failures. Additionally, as shown in Panel B and C, further increase of organization diversification from medium to high level doesn't make significant difference.

When organization diversification is low, shares are distributed to only a few hands. As such, organizations are more sensitive to the direct decline in holding from liquidation than the drop in asset prices. As organization diversification rises, shareholding become more diversified across organizations, and deteriorated price then become the primary factor that determines the magnitude of contagion at each wave. Controlling the sensitivity to accumulative portion of liquidation, price impact parameter therefore dictates the overall size of propagating shock, causing a visible variation to average failure.

When another contagion channel is added, the severity of contagion then is determined jointly with share-based and liquidation-based dependency. Through simulating on different levels of price sensitivity to liquidation, we can determine the relationship between diversification and price impact. As shown from the previous graphs, the severity of contagion is controlled by the relative strength among two aspects of diversification and price impact. When portfolio diversification is above a certain level, sensitivity to liquidation becomes insignificant, and the number of failures is primarily determined by share-based contagion. On the other hand, When organization diversification is high enough, average failures are primarily dominated by liquidation-based contagion, and higher sensitivity to liquidation can result in higher number of defaults.

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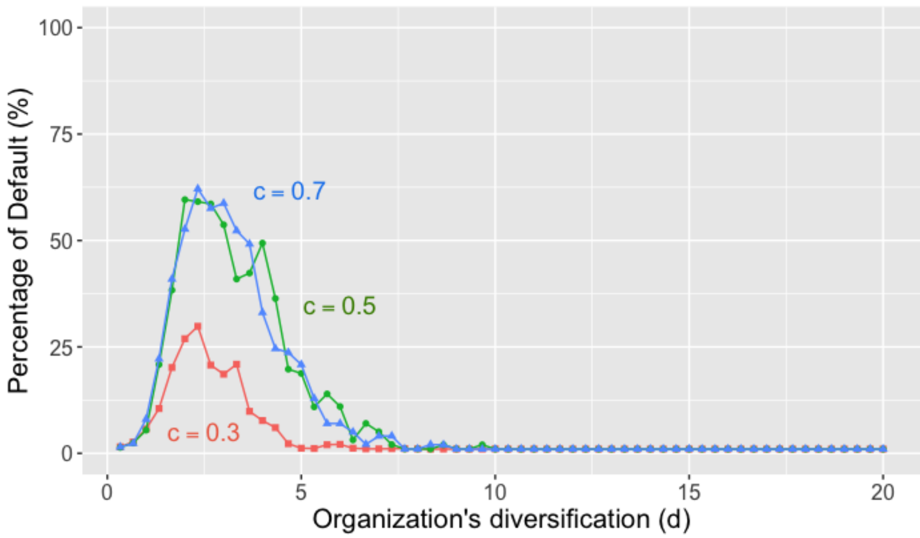

Figure 3.5: **Benchmark model**: Effect of integration on percentage of failures with default threshold $\theta = 0.93$

Integration helps diversification

Aside from the diversification, the integration of financial network is another key factor that determines the severity of contagion. Integration controls the extent of interdependency, which is instrumental in the probability of first failure and the condition for further spread. As explained in Elliott et al. (2014), high levels of integration can prevent first failure from a single exogenous shock; nevertheless, the larger dependency, the more susceptible organizations become to the default of their neighbors. To see if the trade-off brought by integration still remains when asset-holding network is generalized, and fire-sale effect are incorporated, I compare three stylized models, with one of which being the benchmark result in Elliott et al. (2014), and the other two are multi-asset models with and without fire-sale effect. For each of them, the level of integration is varied to see what the role integration plays in the spread of contagion.

For Figure 3.5, the benchmark model illustrates that the increase of integration exacerbates the severity of contagion. The average number of default rises across organization diversification, which implies that although integration can prevent

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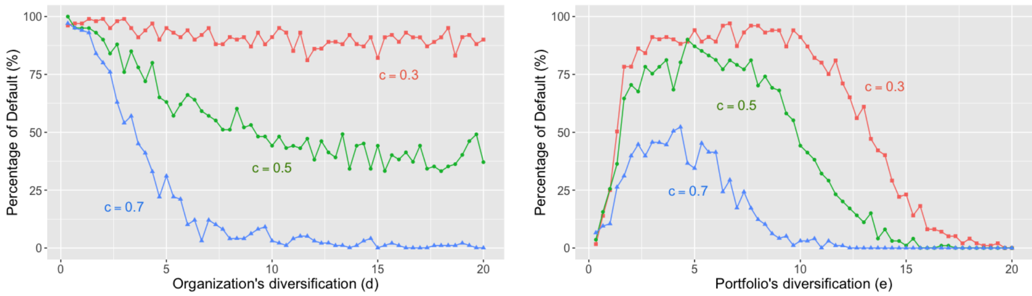
partial first failure, but in general the dependency brought by integration outweighs the benefit of avoidance of first failure. Interestingly, the effect of integration seems to ”hits the bottleneck” when it rises above 0.5 that there is no substantial difference between $c = 0.5$ and $c = 0.7$ in terms of the number of default.



(a) x-axis: organization diversification d ($e = 10$) (b) x-axis: portfolio diversification e ($d = 10$)

Figure 3.6: How percentage of failures across organization and portfolio diversification is affected by integration ($\theta = 0.93$)

On the other hand, the effect of integration is reversed in the multi-asset model, as demonstrated in Figure 3.6. On the contrary, integration now improves the situation, in which the increase of integration significantly lowers the number of default for both cases. To explain the reversing patterns brought by integration and fire-sale effect in multi-asset model, I decompose the percentage of failures through the equation below to understand the trade-off brought by integration.

$$E(X) = E(X|C, A)P(C \cap A) + E(X|C, A')P(C \cap A') \\ + E(X|C', A)P(C' \cap A) + E(X|C', A')P(C' \cap A')$$

, where C is the event of occurrence of contagion, A is the event of occurrence of first failure, and X is the number of defaults.

By construction, if the first wave of failures doesn’t appear, contagion will not occur, and the expected number of conditional failures is zero as well. With $P(C \cap A') = E(X|C', A') = 0$, the equation can be rewritten as:

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$$
\begin{aligned}
E(X) &= E(X|C,A)P(C|A)P(A) + E(X|C',A)P(C'|A)P(A) \\
&\approx E(X|C,A)P(C|A)P(A)
\end{aligned}
$$

By definition, $P(A)$ and $P(C'|A)$ are between zero and one, and $E(X|C',A)$ can only take a limited value so that the primary contributors to the expected number of defaults are $E(X|C,A)$, $P(C|A)$, and $P(A)$, which represents conditional failures, probability of contagion, and probability of first failure, respectively. Next, I will discuss how these three terms together determine the expected number of defaults.

Single-asset model

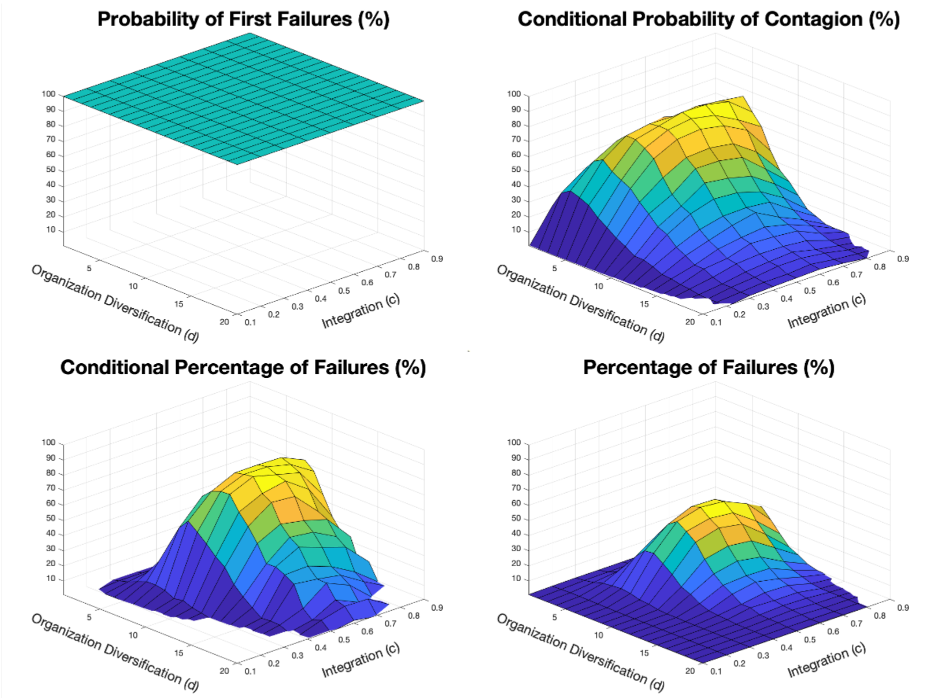

Figure 3.7: Decomposition: $\theta = 0.9$, right axis: integration, left axis: organization diversification (averaged over 1000 simulation)

As shown in Figure 3.7, the decomposition of percentage of failures demonstrates that the conditional probability of contagion and conditional percentage of failures both demonstrates a hump-shaped dependency when the organization

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diversification d increases above 10. Corresponding to the results in Elliott et al. (2014) [4], the percentage of failures displays the property of non-monotonicity.

Moreover, the property of non-monotonicity also appears across integration when organization diversification is above medium level. The hump-shaped occurs because there exists an interaction effect between integration and organization diversification. Given a cross-holding network, the financial network A can be seen as the real dependency network, which incorporates cyclic shareholdings of multiple organization (i.e. i holds j, j holds k, and k holds i). As a result, the percentage of failures is lowered down above medium level of integration.

The notion of dependency can be seen from two aspects: vulnerability and default impact. The row i of dependency network A is the portion of all organizations held by organization i, and the column i represents the portion of shares of organization i held by other organizations. In other words, row i can be regarded as the potential impact on i when other organizations default, and column i represents the potential impact on other organizations incurred by the default of the organization i. In this manner, we can examine how the dependency changes among organizations when the integration takes on different values.

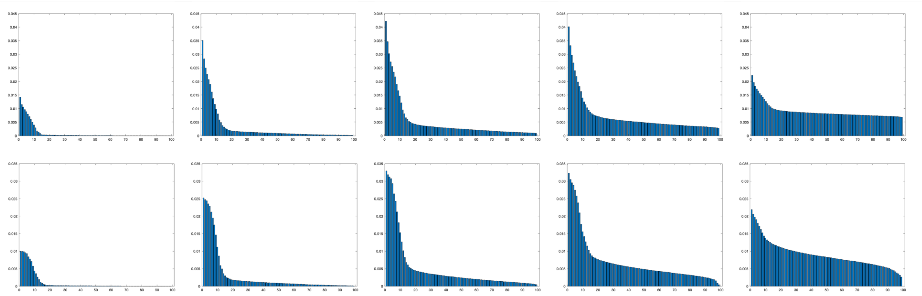

Figure 3.8: Top: *Vulnerability*, Bottom: *Default Impact*

From left to right: *c=0.1, 0.3, 0.5, 0.7, 0.9*

Panel A and B of Figure 3.8 show two cumulative distributions of dependency averaged across all organizations. The five subfigures ordered from left to right represent different levels of integration of 0.1, 0.3, 0.5, 0.7 and 0.9. For each graph, the vertical axes represent normalized dependency, and the horizontal axes

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represent corresponding organizations sorted from the largest dependency to the smallest.

As seen in the figure, at low level of integration, the dependency is mainly distributed among a small group of organizations for both cases of vulnerability and default impact, which take up most of the dependency across all organizations. In other words, the default of an organization doesn't affect most of the organizations much but those who are closely connected to the defaulted organization. With the increase of integration c, the dependency begins to rise, especially for those who have the strongest connection in the very beginning. Nonetheless, the pattern starts to reverse as integration moves across medium level, in which the originally strong dependency among the small group start to decline as the integration continues to increase above medium level. While the organizations with originally strong dependency displays a hump-shaped pattern across integration, the rest of those organizations with originally weak dependency become more dependent all the way from low level of integration to higher level. As a result, integration contributes to a more even distribution of dependency, which consequently reduces the chances of extreme impact from failures, and lowers down the probability of contagion.

As pointed out in Elliott et al. (2014), the effect of integration can reduce the chance of first failure by sharing the impact from exogenous asset shock to multiple organizations. Nonetheless, the cofounding effect between organization diversification and integration and existence of cyclic shareholdings provide an additional layer of risk sharing with a more even distribution of dependency, which avoids the extreme events of default cost.

Multi-asset model without fire-sale effect

Next, I generalize the asset-holding network, allowing every organization to hold different portfolios to see if the generalization will affect the results, and whether or not the dependency-sharing effect of integration will remain,

In Panel A of Figure 3.9, the probability of first failures now varies with organization diversification and integration. Since organizations can now hold

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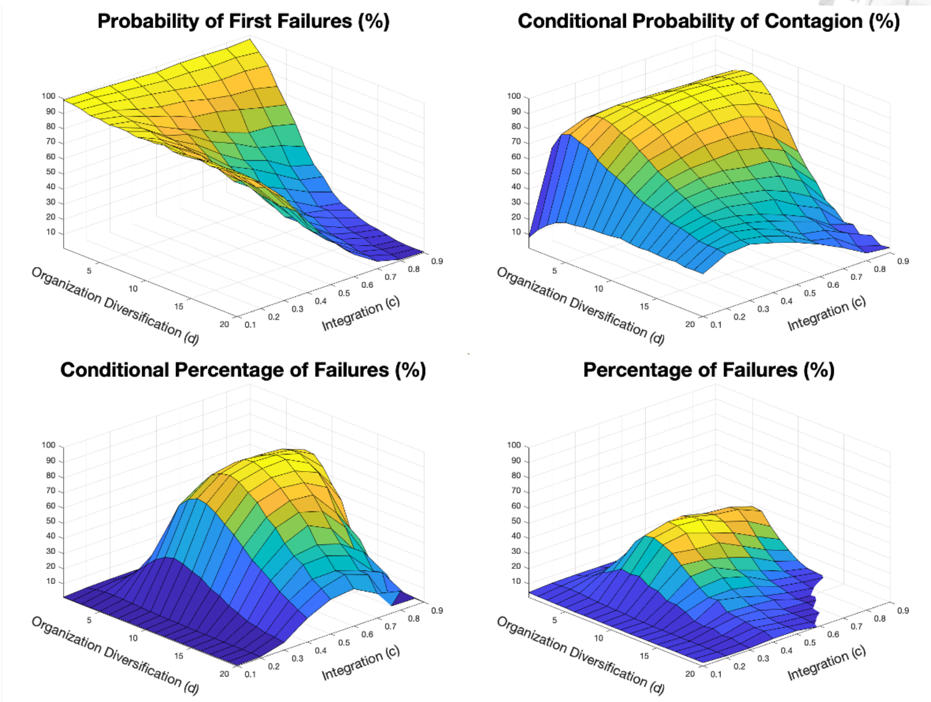

Figure 3.9: Decomposition: $\theta = 0.9$, right axis: integration, left axis: organization diversification (averaged over 1000 simulation)

multiple assets in its portfolio, the initial exogenous asset shock is alleviated so that a sufficiently high levels of organization diversification and integration can now avoid the first failure. As for conditional probability of contagion and conditional percentage of failures in Panel B and C, the numbers both rise especially in the region of low organization diversification and integration. Although having multiple assets in portfolios alleviates the exogenous asset shock, it also allows for the possibility of multiple failures from the initial shock, which serve as multiple origins of contagion. As such, the probability of contagion and percentage of default given multiple contagion origins both increase. With lower probability of first failures, higher conditional probability of contagion and conditional percentage of failures, the overall percentage of failures doesn't display a significant difference, showing a similar pattern to single-asset model.

Although randomly diversified portfolios reduces the chances of failing given an exogenous shock, the effect is offset by multiple failures when the exogenous

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shock hits the right positions. When allowing organizations to have more than one asset in portfolios, the primary consequences are the decrease of probability of occurrence of first failure, and increase of contagion origins if first failure does happen. Under these circumstances, the risk-sharing effects of organization diversification and integration are not influenced much, and higher number of connections with other organizations can still share the default cost, preventing contagion to continue.

Multi-asset model with fire-sale effect

Finally, fire-sale effect is incorporated to see whether an additional contagion channel is related to the confounding effect of organization diversification and integration.

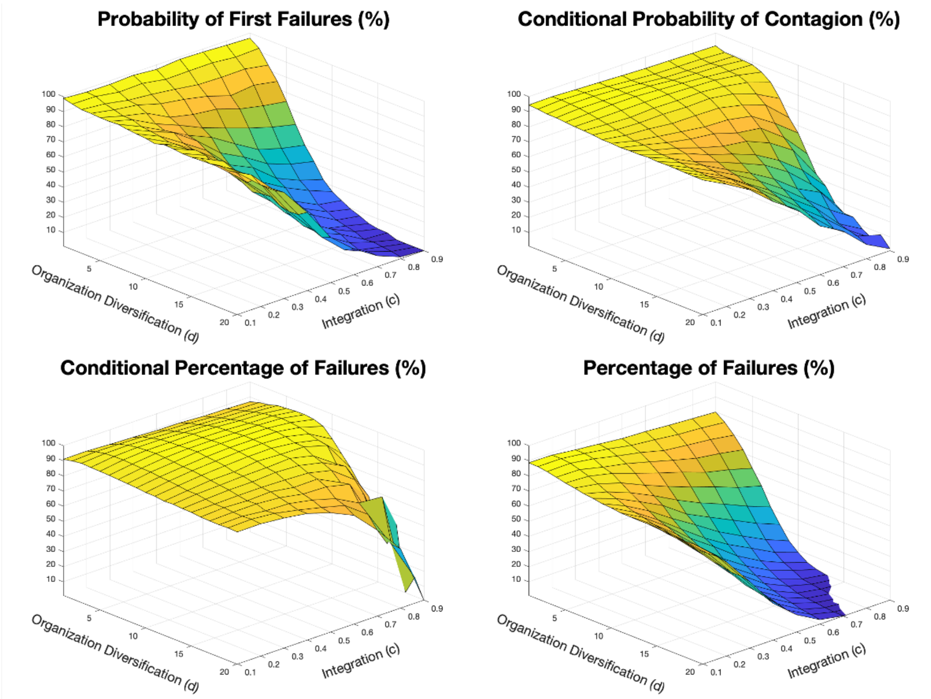

Figure 3.10: Decomposition: $\theta = 0.9$, right axis: integration, left axis: organization diversification (averaged over 1000 simulation)

As shown in Panel A of Figure 3.10, the percentage of first failures is not affected with the inclusion of fire-sale effect in that before the occurrence of first failure, asset liquidation and price deterioration have yet taken any effect. The

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most distinguishable feature brought by fire-sale effect occurs in conditional probability of contagion and conditional percentage of failures, where the numbers in general are worsened, and the hump-shaped distribution disappeared, where the lack of share-based connections between organizations from low level of organization diversification and integration don't necessarily guarantee the avoidance of contagion.

As for Panel B and C of Figure 3.10, at low level of diversification and integration, the numbers of conditional probability of contagion and percentage of failures both approach one hundred percent, and the numbers hover at the same high level for a wide range of organization diversification and integration. The numbers begin to decrease only when the level of organization diversification d is above 15, and integration c is above 0.5.

The introduction of fire-sale effect as an additional contagion channel makes it the dominant force that overpowers the shareholding-based contagion channel, which only accounts for the bankruptcy cost when organizations fail. The price deterioration incurred by liquidation become the primary spreading factor that determine the severity of contagion.

As explained previously, the reason why conditional probability of contagion and percentage of failures are higher in multi-asset model (without fire-sale effect) than those in single-asset model is that the default number in the first wave (due to exogenous asset shock) can be more than one, causing multiple sources to spread the contagion. Now, the inclusion of fire-sale effect makes the failed organizations not only damage shareholders, but those whose portfolios overlap with the default organizations. As a consequence, absence of share-based connections, and large amount of self-owned shares don't necessarily break the contagion, which is now dominated by liquidation-based contagion. When the first failure occurs, the price impact from asset liquidation makes the contagion due to spread, and average failures to rise. As such, the originally low number of default in the region of low organization diversification and integration is raised.

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As shown in Elliott et al. (2014) [4], integration represents the intensity of interdependency among organizations. As with organization diversification, integration generates two opposing forces, creating a trade-off between occurrence of first failure and probability of contagion. The rise in integration strengthens the interdependency among organizations, and therefore reduces the barrier to spreading failures; meanwhile, integration lessens the potential impact from its own assets, making organizations less sensitive to initial asset shock. Nonetheless, the simulation results above point out a third force brought by integration and cyclic shareholding. With a decent amount of organization diversification, the increase in integration can average the overall dependency so that the probability of extreme impact from default is reduced.

Through the decomposition, percentage of failures can be broken down to see which part of the contagion process is affected by integration and diversification. For multi-asset model with fire-sale effect, the cumulative portion of liquidation dominates the probability of contagion, and conditional percentage of failures so that share-based contagion channel controlled by organization diversification and integration become insignificant, and only take influence through probability of first failures.

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Chapter 4

Concluding Remarks

Based on the models modified from Elliott et al. (2014) [4], I investigate the relationship between the original financial network, and the new added asset-holding network to explore how the contagion is affected through the interaction between two types of diversification and integration. In the original model from Elliott et al.(2014) [4], organization diversification displays a pattern of non-monotonicity, in which the strength of risk sharing and contagion channels diverge across diversification on shareholding. I found that while portfolio diversification demonstrates a similar pattern with the absence of financial connection, the outcomes are generally more severe, and the non-monotonic property is magnified that the percentage of default stays at high level longer before going down.

As for multi-asset model, where the two networks coexist, and fire-sale effect is incorporated, the non-monotonic property of organization diversification is eliminated. As such, the price deterioration from asset liquidation provides an additional channel for spreading failures, and only worsen the circumstance when the portfolio diversification is not enough to cancel off the fire-sale effect. On the other hand, the non-monotonicity of portfolio diversification is retained, as opposed to the case for organization diversification. The increase in organization diversification proves ineffective in mitigating the circumstances, no matter the default threshold.

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The most interesting result lies in integration, where it not only reduces the occurrence of first failure, and amplifies the contagious damage as specified in Elliott et al. (2014) [4], but also produces a dependency-sharing effect with organization diversification, through which large damage from a failed organization is distributed to more organizations. As such, the number of failures also displays a hump-shaped tendency across integration.

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