國立臺灣大學管理學院財務金融研究所

碩士論文

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Master Thesis

# 隱含變異數彈性與槓桿效應

# Implied Elasticity of CEV model and Leverage Effect

王炤凱

Chao-Kai Wang

指導教授：李存修 博士

Advisor: Tsun-Siou Lee, Ph.D.

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# 國立臺灣大學（碩）博士學位論文
# 口試委員會審定書

## 隱含變異數彈性與槓桿效應
## Implied Elasticity of CEV Model and Leverage Effect

本論文係王炤凱君（R04723063）在國立臺灣大學財務金融學系、所完成之碩（博）士學位論文，於民國 106 年 6 月 21 日承下列考試委員審查通過及口試及格，特此證明

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（指導教授）

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# 摘要

近幾十年來許多不同理論用來解釋股價報酬率與波動度變動率之間存在負向關係的原因。 Black（1976）首先將其歸因於財務槓桿的影響，隨著公司股價下跌，槓桿率將會提高，槓桿率上升導致波動度變大，本文提出了新的方法來驗證財務槓桿效應的存在，根據固定彈性變異數定價模型，從市場的選擇權交易資訊中隱含出變異數彈性。實證結果變異數彈性顯示股票報酬率與波動度變動率之間呈現負相關，且彈性是槓桿率的函數，間接證明了財務槓桿效應，同時研究發現槓桿效應與公司規模之間呈現負向關係。

關鍵字：波動度、固定彈性變異數模型、變異數彈性、槓桿效應、報酬率與波動度變動率關係



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# ABSTRACT

An inverse relationship between the stock return and volatility changes has been interpreted by many different theories during recent decades. First documented by Black (1976) who attributes it to the effects of financial leverage. As firm’s stock price falls, it would become highly leveraged, and this increase in leverage leads to higher equity-return volatility. This thesis provides a new method to support financial leverage effect by implying the elasticity from Constant Elasticity of Variance model. Evidence shows a negative relation between the stock return and volatility changes. Also the elasticity is a function of leverage ratio which implies the effect of financial leverage. Furthermore, result indicates a negative relation between the leverage effect and the firm size.

Keywords: Volatility; Leverage effect; CEV model; Elasticity of variance; Return/Changes of volatility relation.



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# Chapter 1 Introduction

One of the most enduring empirical issues in equity markets is the cause that stock-return volatility declines after stock price rises. First explained by Black (1976), in the seminal paper, he provides a compelling explanation in term of firm's leverage effect: a negative return indicates a drop in the value of the firm's equity. Then it would increase its leverage which causes higher equity-return volatility. Cox (1975) comes up with the Constant Elasticity of Variance option model taking volatility as a deterministic function of stock price. Beckers (1980), Macbeth (1982) estimate the coefficient under CEV model and justify the leverage effect. Christie (1982), Cheung and Ng (1992) show the negative relation between the stock return and volatility changes and discuss that the elasticity is a function of leverage ratio. However, Rubinstein (1983) provides a Displaced Diffusion model that the equity volatility depends on both the distribution of the risky and riskless assets and the debt-to-equity ratio. There may exists a positive relation between the stock return and volatility changes. Duffee (2002), with the dynamic displaced diffusion model, shows the balance sheet effect that there is a positive relation between volatility changes and stock return at firm level. Hasanhodzic and Lo (2011) argue that an inverse relationship between the stock prices and volatility is not driven by the leverage effect.

In this paper, I present a new method to check whether the leverage effect is able to explain the inverse relationship between the stock return and volatility changes by implying the elasticity from the stock options under CEV pricing model. it provides clear evidence that the elasticity of variance is a function of leverage ratio both in aggregate and firm level, consistent with Black (1976), Christie (1982), Cheung and Ng (1992). The firm size is also a factor that influences the leverage effect. Evidence shows a negative relationship between the firm size and leverage effect. The paper is organized as follows.



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Section 2 is the literature review, I provide the literature about financial leverage effect and the characteristics of CEV model; Section 3 is the introduction of the methodology including the implied elasticity methodology as well the regression models to empirically test the leverage effect and elasticity hypotheses; Section 4 is the data along with the empirical evidence. I describe where to get the data and why use the data. There are conclusion and some comments in the final section.



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# Chapter 2 Literature Review

## 2.1 Review of leverage effect and other theories

Black (1976) is viewed as the pioneer of the leverage effect literature. He conducts first empirical test using sample of 30 stocks (mostly Dow Jones Industrials). For each stock, Black constructs monthly estimates of stock return volatility from 1962-1975 by aggregating squared daily returns and he defines the volatility changes as the difference between volatility estimate of the current and previous period divided by the volatility of the previous period. His results suggest a negative relation between the volatility changes and stock return; furthermore, the coefficient is usually less than one. He proposes the leverage effect to explain the results. A drop of the stock prices would cause a negative return and would rise the leverage ratio (i.e. debt/equity ratio) given the fixed level of liabilities. A rise at the leverage ratio leads to the rise of the volatility of the stock.

Different from Black (1976), Rubinstein (1983) conducts the Displaced Diffusion model. He argues that the stock volatility depend on both the distribution on the risky and riskless assets as well as the debt/equity ratio. If value of a firm rise quickly and more than the risk-free return, it would be due to the rise of the value of riskier assets and thus the overall volatility of the firm tend to increase. There would be a positive relationship between the stock return and volatility changes. Other theories for the inverse relation between stock volatility changes and lagged return been proposed. The time-varying risk premia literature, also called volatility feedback effect. An increase in return volatility lead to an increase in expected future return, thus a decline in the current stock price. Wu (2001) argues that the volatility feedback effect dominates the leverage effect empirically.

Beckers (1980) turns the volatility term of the CEV model into a regression model



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for empirical research data from 1972 to 1977 with 47 different stocks. He concludes the stock elasticity to be significantly less than two, mostly are negative and finds that the CEV pricing model would calculate a higher price for at-the-money and in-the-money options comparing with BS model. Christie (1982) constructs quarterly estimates of return volatility for 379 firms over 1962-1978 and finds the cross sectional mean elasticity to be -0.23, consistent with the leverage effect. In addition, he uses the CEV model to test the elasticity hypothesis. He concludes that the elasticity is a function of financial leverage based on leverage quartiles and on sub-periods but without any statistical examination. Cheung and Ng (1992) examine the negative relation between stock price and subsequent volatility under daily returns of 251 NYSE-AMEX stocks from 1962-1989. Using exponential GARCH model, the mean coefficient of the elasticity is negative, consistent with the leverage effect. They apply the Spearman rank correlation test to test the relation between the elasticity and the debt/equity ratio as well as debt/equity ratio and the firm size. They find a negative (positive) correlation between the elasticity and the debt/equity ratio (debt/equity ratio and the firm size), implying that the elasticity is positive link to the firm size, in accordance with Christie (1982). Furthermore, result shows that correlation changes over time.

Duffee (1995) reveals that an inverse relation between the stock return and future volatility changes at the firm level is due to the positive contemporaneous relation between the stock return and volatility changes while the relation in aggregate level still be negative. Using 2500 firms on the AMEX or NYSE from 1971-1991. He finds a positive relation between the elasticity and the debt/equity ratio but the correlation between the lagged elasticity and debt/equity ratio still be negative. He considers the survival bias in the empirical test, concluding that survivorship bias is of great importance and finds positive significant relation between the elasticity and the firm size.



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Duffee (2002) provides a theory of balance-sheet effect to explain a positive contemporaneous relationship between stock return and volatility changes at the firm level. Follow by Duffee (1995), he uses dynamic displaced diffusion model to explain the positive contemporaneous relation. Duffee (2002) conducts the second stage regression about the elasticity coefficient and some accounting variables (i.e. debt/equity, market value, book-to-market ratio) and finds that there is not any statistical significance between the elasticity and debt/equity ratio whether in aggregate or firm level.

Hasanhodzic and Lo (2011) propose that the inverse relation between the stock prices and volatility is not because of the leverage effect. Using the sample of all-equity-financed companies and debt-financed companies from 1972-2008 but they does not take account the operating leverage. Their result shows the former is as negative as the latter. They provide that the inverse relation be due to human cognitive perception of risk.

In my thesis, unlike the Spearman rank test, I run a regression between the implied elasticity and debt/equity ratio. I also discuss the relationship between the leverage effect and firm size.

## 2.2 Review of Constant Elasticity of Variance model

Cox (1975) comes up with the CEV model to describe the random walk of the stock. Take volatility as a function of stock price.

The stock price process is

$$dS = \mu S dt + \delta S^{\beta/2} dz \,. \tag{1}$$

Where $\mu$ is for stock average growth rate, $\beta$ is elasticity of variance and $\delta$ is a constant. dz is a Weiner process.

$$\sigma(S) = \delta S^{(\beta-2)/2}. \tag{2}$$

When $\beta$ equals two, it is simply the lognormal distribution.



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Cox and Ross (1976) solve the stochastic differential equation using modified Bessel function and write down the transition probability function as follow.

$$\mathrm{f}(S_T|S_t, \mathrm{T} > \mathrm{t}) = (2-\beta)k^{\frac{1}{2-\beta}}\left(x\omega^{1-2\beta}\right)^{\frac{1}{4-2\beta}}e^{-x-\omega}I_{\frac{1}{2-\beta}}\left(2\sqrt{x\omega}\right) \quad (3)$$

Where $\mathrm{k} = \frac{2(r-a)}{\delta^2(2-\beta)\left[e^{(r-a)(2-\beta)\tau}-1\right]}$; $\mathrm{x} = \mathrm{k}S_t^{2-\beta}e^{(r-a)(2-\beta)\tau}$; $\omega = \mathrm{k}S_T^{2-\beta}$

$I_k(x) = \sum_{r=0}^{\infty}\frac{(\frac{x}{2})^{2r+k}}{r!\Gamma(r+1+k)}$; r is the risk-free rate; a is the dividend yield; $\tau = T - t$.

Macbeth (1980) chooses six different stocks with option trading and calculate the daily return in order to find the parameter $\delta$ and the constant elasticity of variance. Emanual and Macbeth (1982) extend the research period of Macbeth (1980). They also suggest that the CEV model outperforms the BS model in that CEV model with another variable comparing with BS model, CEV model is more flexible than BS model. Therefore it gets smaller error but only when the period is short. They assert that the CEV model can account for approximately 20% of volatility changes when elasticity is below 2 and CEV model allows for us to predict future volatility. Schroder (1989) shows that the CEV option pricing formula expressed as a function of the non-central chi-square distribution.

$\beta < 2$: The European call price with exercise price E is

$$\mathrm{C} = \mathrm{E}(\max(0, S_T - \mathrm{E})) = e^{-r\tau}\int_E^{\infty}\mathrm{f}(S_T|S_t, \mathrm{T} > \mathrm{t})(S_T - E)dS_T \quad (4)$$

Non-central chi-square probability density function written as

$$p_{x_v'^2(\kappa)}(x) = \frac{1}{2}\left(\frac{x}{\kappa}\right)^{\frac{(v-2)}{4}}e^{-\frac{1}{2}(\kappa+x)}I_{\frac{(v-2)}{2}}\left(\sqrt{\kappa x}\right), x > 0 \quad (5)$$

Where non-central chi-squared with $v$ degrees of freedom, and non-centrality parameter $\kappa$.



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$$\beta < 2, \qquad C = S_t e^{-\alpha\tau} Q\left[2y; 2+\frac{2}{2-\beta}, 2x\right] - E e^{-r\tau}\{1 - Q\left[2x; \frac{2}{2-\beta}, 2y\right]\} \tag{6}$$

$$\beta > 2, \qquad C = S_t e^{-\alpha\tau} Q\left[2y; \frac{2}{\beta-2}, 2x\right] - E e^{-r\tau}\{1 - Q\left[2x; 2+\frac{2}{\beta-2}, 2y\right]\} \tag{7}$$

Lee et al.(2004) compare the performance of CEV and BS model with the RMSE, MAE and MAPE. They show that CEV model in terms of the non-central chi-square distribution performs better than Black-Scholes model in pricing the S&P 500 index call options. Moreover, it is more convenient to utilize the CEV model in term of the non-central chi-square distribution.



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# Chapter 3 Methodology and Hypotheses

## 3.1 Methodology

In order to imply the elasticity from the options, I follow the CEV option pricing formula by Schroder (1989) using formula (6) and (7). The parameters in $CEV(S, K, r, q, t, \sigma, \beta)$: S is the spot price of underlying asset; K is the strike price; r is the risk-free rate of Federal funds; q is the dividend yield; t is the maturity time of the option; $\sigma$ is the historical volatility (year), a year ahead rolling window; $\beta$ is the elasticity I would like to imply. Elasticity represents the relation between the stock return and volatility changes.

*(Pseudo Code)*

$CEV(S, K, r, q, t, \sigma, \beta)$

*Case beta*

$\beta < 2$ *: Formula (6)*

$\beta = 2$ *: BS formula*

$\beta > 2$ *: Formula (7)*

*End*

$$\Pi(\beta) = |P_{real} - P_{model}| = |P_{real} - CEV(S, K, r, q, t, \sigma, \beta)| < \varepsilon \qquad (*)$$

Market data of option price is $P_{real}$ and option price calculated by CEV model is $P_{model}$. I define $\Pi(\beta)$ is a non-linear one variable function, the concept to solve this problem is same as finding the root of a non-linear equation. I use the secant method algorithm to find the root of this equation. The answer would be the implied elasticity. The reason for me to use the secant method is that it is easier to implement than other methods, not like Newton, which has to calculate the first order derivatives. Secant method is the trade-off between the accuracy and calculation. Moreover, I use the parallel



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computing technique up to four CPUs in MATLAB in order to speed up the processing time. On average, it takes less than a second to find each implied elasticity.

*Algorithm - Secant Method (Pseudo Code)*

*Given an equation f(x)*

*Let the initial guesses be* $X_0$ *and* $X_1$

*Do*

$$X_{i+1} = X_i - \frac{f(X_i)(X_i - X_{i-1})}{f(X_i) - f(X_{i-1})},\ i = 1,2,3 \ldots$$

*While* $(f(X_{i+1}) > \varepsilon \text{ or } |X_{i+1} - X_i| > \varepsilon)$

*Repeating the process*

*End (when* $f(X_{i+1}) < \varepsilon \text{ or } |X_{i+1} - X_i| < \varepsilon)$

$X_{i+1}$ *is the implied elasticity of variance.* (*)

## 3.2 Hypotheses

*Hypothesis 1: Implied elasticity is a function of leverage*

Christie (1982) proposes the evidence that a negative relationship between stock return and volatility changes and shows that elasticity is a function of leverage (i.e. debt/equity ratio) by grouping leverage quartiles but without any statistical examination. I test the elasticity hypothesis by running the regression both in aggregate and firm level. Because of the data frequency of debt/equity ratio, I rearrange the implied elasticity of the S&P 500 to monthly period and the implied elasticity of the 30 equities to quarterly period. Then I run two regressions. In firm-level panel data, I test correlated random effects with Hausman test and the result supports the fixed effect to implement in firm-level regression. I also use the interaction effect regression in aggregate level to test the difference between the implied elasticity and the debt/equity ratio through financial crisis.

Debt/equity ratio definition is the value of short-term liabilities plus the long-term



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liabilities divided by the market value of the equity.

Aggregate-level

$$Beta_{market,t} = intercept + \alpha_1 D/E_{\ market,t} + VIX_t + \varepsilon_t. \quad (9)$$

$$Beta_{market,t} = intercept + \alpha_1 D/E_{\ market,t} + \theta_1 Crisis \times D/E_{\ market,t} + VIX_t + \varepsilon_t. \quad (10)$$

Where $Beta_{market,t}$ is the implied elasticity from S&P 500 index options and $D/E_{\ market,t}$ is the total market debt-to-equity ratio. $Crisis$ is the dummy variable discerning before and after the financial crisis at 2008. After year, 2008 is 1, otherwise 0. VIX (Chicago Board Options Exchange Volatility Index) is the control variable. $\varepsilon_t$ is the error term. In the aggregate level, I use the VIX index as the control variable because VIX represents the implied volatility of S&P 500 index. I want to check whether debt/equity ratio has any additional explanation after controlling VIX index.

Firm-level (Panel data with fixed effect)

$$Beta_{i,t} = intercept + \alpha_1 D/E_{\ i,t} + D_i + B/M_{i,t} + \omega_{i,t}\ . \quad (11)$$

Where $Beta_{i,t}$ is the implied elasticity from 30 equity call options over period and $D/E_{\ i,t}$ represents the firm's debt-to-equity ratio. $D_i$ is the dummy variable for the firm. $\omega_{i,t}$ captures everything that is left unexplained about $Beta_{i,t}$. $B/M_{i,t}$ is the book-to market value ratio. Duffee (2002) runs the regression between the elasticity and debt/equity ratio as well as book-to-market ratio. I use B/M as the control variable.

*Hypothesis 2: a negative relationship between leverage effect and firm size*

$$Beta_{i,t} = intercept + \alpha_1 D/E_{\ i,t} + \pi_1 FIRM \times D/E_{\ i,t} + B/M_{i,t} + D_t + \omega_{i,t} \quad (12)$$

Where $Beta_{i,t}$ is the implied elasticity from 30 equities call option over period. $FIRM$ is dummy variable controlling for the firm size. I select the firms with market value more than 30 billion be large firms. $\omega_{i,t}$ is the error term. $D_t$ is the year dummy.



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# Chapter 4 Data and Empirical Evidence

## 4.1 Data

My sample consists of daily stock prices from the University of Chicago's Center for Research in Security Prices, and the option data are from OptionMetrics Ivy DB US. Fundamental data are from the CRSP/Compustat Merged Database (Fundamentals Quarterly) and some come from the Bloomberg. I choose the standardize at-the-money S&P 500 index call options at the OptionMetrics with two months maturity time from 2006 – 2015 daily data and standardized call options of 30 equities from 2007 – 2012 daily data for two reasons. First, from implying the elasticity from out-of-the money to in-the-money, there is a positive skewness phenomenon. I choose at-the-money options represent the average implied elasticity. Second, because Macbeth (1982) shows that CEV model outperforms BS model in short period. I select the S&P 500 index option as the proxy of aggregate level and select the 30 most active trading equity call options as the proxy of firm level.

## 4.2 Empirical evidence

Mean elasticity is -0.67 (monthly period) in S&P 500 index and -0.43 in equities (quarterly period). From the results of implied elasticity both in aggregate and firm level, evidence supports a negative relationship between the stock return and volatility changes. It concurs with Christie (1982), Cheung and Ng (1992), Duffee (1995), Hasanhodzic and Lo (2011). Implied elasticity, comparing the coefficient with Christie (1982), Cheung and Ng (1992), is more volatile in both aggregate and firm level, with the maximum just below 2 (1.758 in aggregate level; 1.995 in firm level) and minimum around -4 (-4.315 in aggregate level; -3.673 in firm level). The result matches Beckers (1980) that most of elasticities are below two under CEV model. Besides, the debt/equity ratio of financial



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service firms is larger than non-financial firms. From Fig. 2.2 and Fig. 2.3, I notice that both aggregate and firm[1]-level implied elasticities change with the same pattern. From histogram, it tells the value density of implied elasticity in aggregate and firm level. It matches with the theories and empirical evidence. In my sample period, 89 out of 120 elasticities are negative in aggregate level while 466 out of 715 elasticities are negative in firm level. Most of the implied elasticities are between zero and negative one, in accordance with the time series figure. Only very few period the implied elasticity vibrates very dramatically.

[1] Firms are：AAPL, ABX, AMZN, BAC, CAT, CHK, CMCSA, CSCO, CVX, CY, DOW, FCX, GE, GS, INTC, JPM, KO, MSFT, MU, MYL, NFLX, NVDA, PFE, PG, SBUX, WDC, X, XOM, IBM, WMT.



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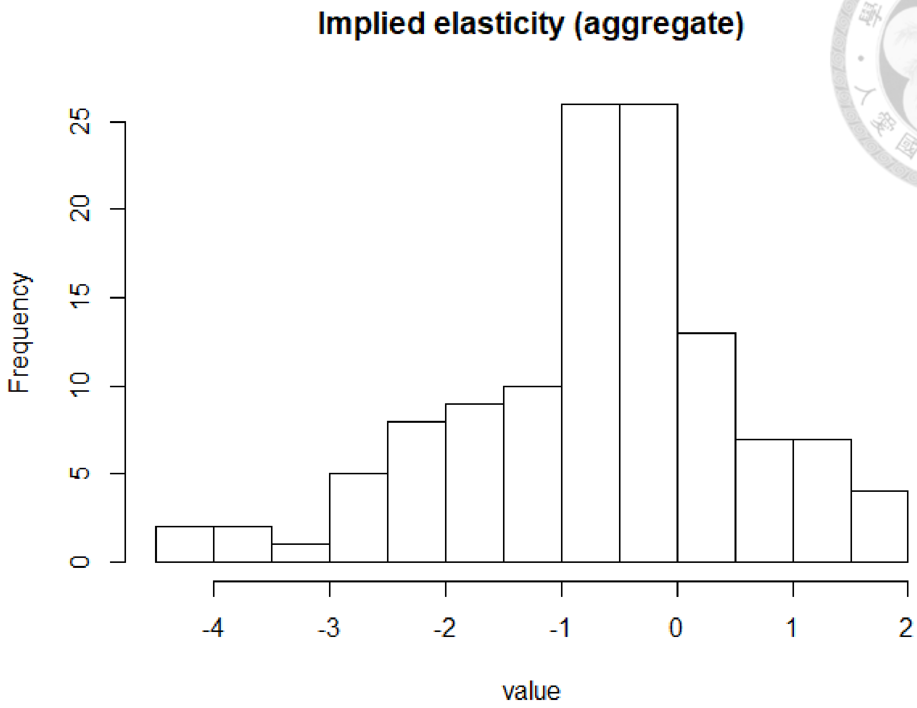


**Fig. 1.1 Histogram of the implied elasticity from S&P 500 index call options.**

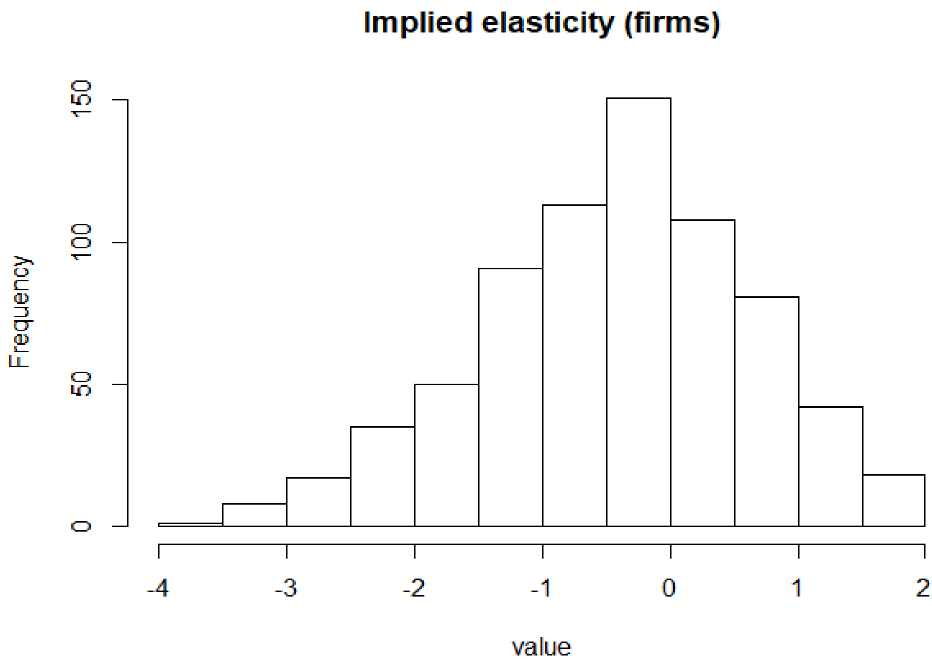


**Fig. 1.2 Histogram of the implied elasticity from 30 equity call options.**



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**Table 1: The summary statistics of implied elasticity and other variables.**

| Statistic (firm-level) | N | Mean | St. Dev. | Min | Max |
|---|---|---|---|---|---|
| Implied elasticity(firms) | 715 | -0.430 | 1.056 | -3.673 | 1.995 |
| Debt/equity ratio (all firms) | 715 | 2.747 | 3.691 | 0.240 | 26.892 |
| (non-financial firms) | 644 | 1.717 | 1.526 | 0.240 | 10.136 |
| Book-to-market ratio | 715 | 0.556 | 0.517 | 0.019 | 5.487 |
| **Statistic (aggregate level)** | **N** | **Mean** | **St. Dev.** | **Min** | **Max** |
| Implied elasticity (aggregate) | 120 | -0.670 | 1.239 | -4.315 | 1.758 |
| Debt/equity ratio (total) | 120 | 0.296 | 0.056 | 0.233 | 0.393 |
| VIX | 120 | 20.604 | 9.092 | 10.420 | 59.890 |

This table provides the simple statistical examination of implied elasticity and debt/equity ratio also the control variables (aggregate and firm level). Aggregate data from January 2006 to December 2015 monthly period and firm data from January 2007 to December 2012 quarterly. Total market debt/equity ratio acquired from Bloomberg; Firm data from Compustat,

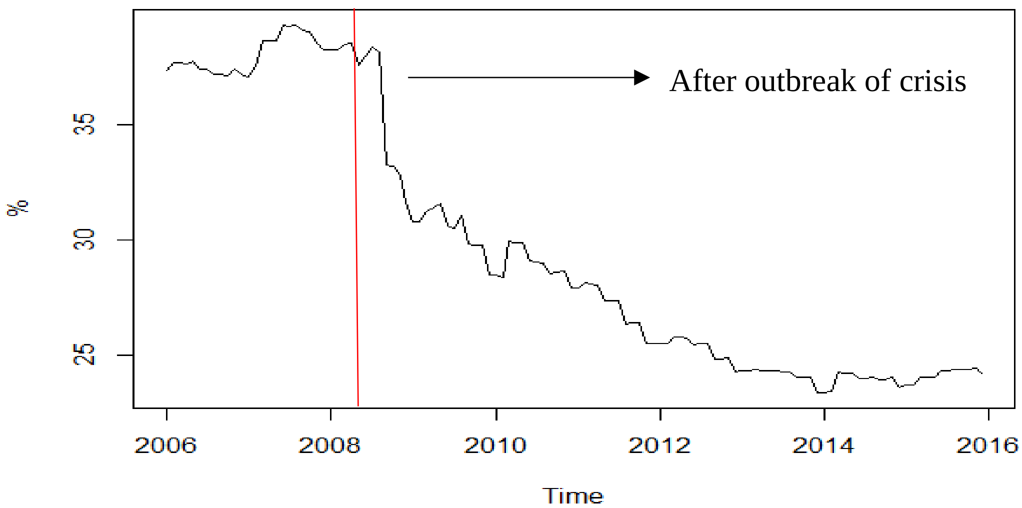


**Fig. 2.1 Total debt/equity ratio**



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Furthermore, I find that there is a negative relation between implied elasticity and the VIX index. I test relation with the Pearson's correlation. It rejects the null hypothesis, a negative correlation 0.32 between the implied elasticity and the VIX index. Focus on the period when the VIX (fear index) is extremely high, the implied elasticity is extremely low which means that the negative relation between the stock return and volatility is stronger in the market downturn period. That is the reason I use VIX index as the control variable. I want to compare the difference between the implied elasticity and the VIX index. From Fig.2.2, blue line represents the VIX price in monthly period and the implied elasticity is the black line. There is a pattern through the implied elasticity of the firm; most of firm's implied elasticities vary with the aggregate elasticity. The range of implied elasticity at the firm level is wider when the aggregate market is in its downturn.

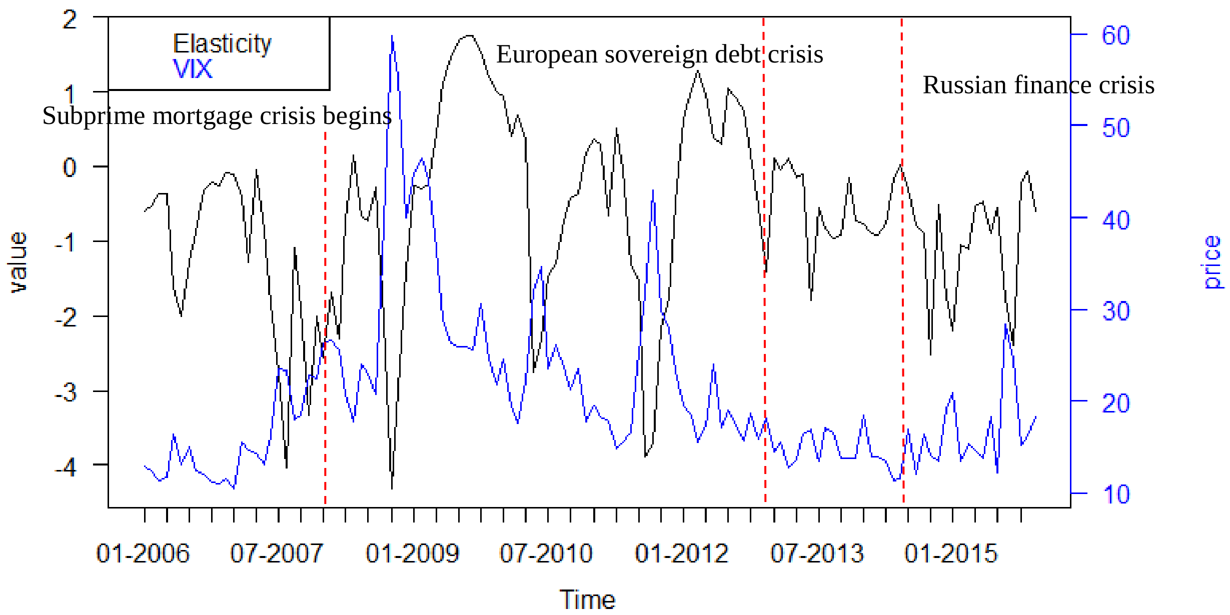


**Fig. 2.2 Time series of the aggregate implied elasticity and the VIX index.**
This graph shows the monthly-implied elasticity and the VIX price from January 2006 to December 2015. The value of implied elasticity is at the left axis and the price of the VIX is at the right axis.



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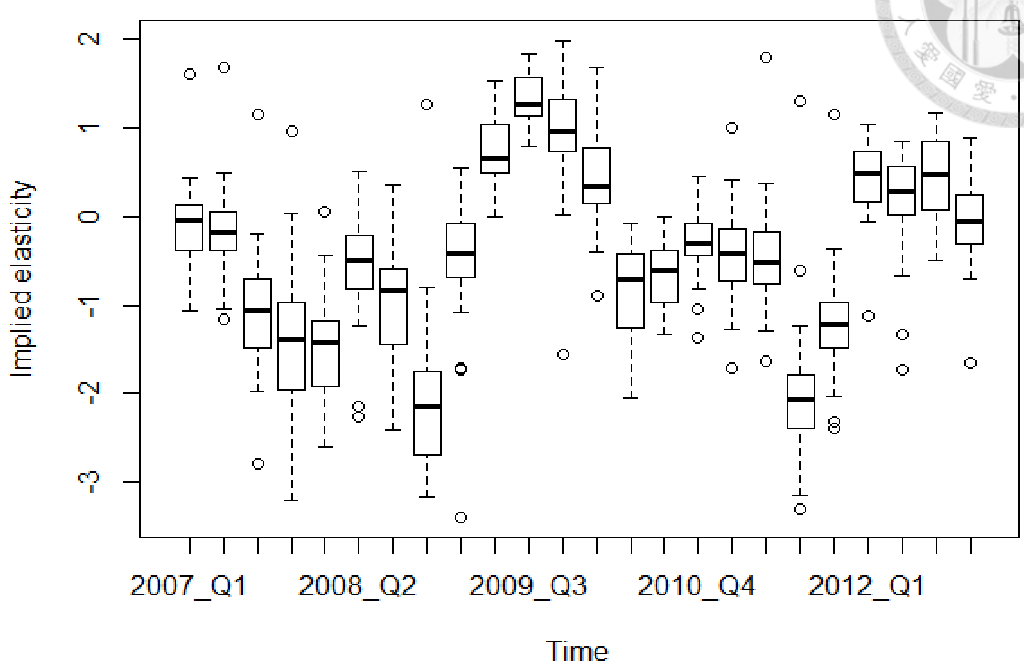


**Fig. 2.3 Time series of the implied elasticity from 30 equities.**
This graph presents Box chart of the 30 firm's elasticities from 2007 Q1 to 2012 Q4. Despite some outliers, most of the firms' implied elasticities follow the same pattern that coincides with the aggregate level through the period.

I test the leverage effect using linear regressions where the dependent variable is the implied elasticity and the independent variable is the debt/equity ratio, slightly different from Black (1976), Christie (1982) and Duffee (1995). In the previous study, they focus on the rank correlation. They would have to do second stage regression in order to examine the relation between the elasticity and other variables. The result shows that implied elasticity is a function of leverage (i.e. debt/equity ratio) and a negative relation between the implied elasticity and the debt/equity ratio with statistical significant level both in aggregate and firm level ( result for excluding the financial service firms is still negative but not significant). It implies the inverse relation between the stock return and volatility changes is due to the leverage effect by Black (1976).



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In aggregate level, the coefficient is -0.037 with significant p-value less than 0.1. Because from Fig.2.1, there is a structural break of total debt/equity ratio. I set the dummy variable that get 1 when time after year, 2008 to examine the interaction effect after financial crisis. The result shows that after crisis, the coefficient is 0.06 while the period prior to crisis is -0.623. The relation between the implied elasticity and the debt/equity ratio is stronger before the crisis.

In firm level, I check the Hausman test and its p-value is 0.03613, rejecting the null hypothesis, which means fixed effect is more appropriate for this regression. Panel data coefficient is -0.077 with significant p-value less than 0.05 (coefficient for excluding financial service firms is -0.03 without significance. I also run regressions for each firms like Christie (1982) and the mean coefficient[2] is -0.293. Empirical evidence reaffirms the existence of leverage effect, consistent with the Christie (1982), Cheung and Ng (1992). Owning to the empirical evidence between the debt/equity ratio and firm size by Cheung and Ng (1992) and Duffee (1995), I further check the relation between the firm size and leverage effect with control variable controlling for the large and small firms. It shows that the smaller the firm size, more negative is the relation between the implied elasticity and the debt/equity ratio. From Table 2 the coefficient even turns positive in large firms while a much smaller magnitude (0.004). The coefficient of the small firm is -0.102.

[2] I run regressions between the implied elasticity and the debt/equity ratio for thirty firms, then average the coefficient.



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**Table 2: Aggregate-level regression results**

1. $Beta_{market,t} = intercept + \alpha_1 D/E_{\ market,t} + VIX_t + \varepsilon_t$.
2. $Beta_{market,t} = intercept + \alpha_1 D/E_{\ market,t} + \theta_1 Crisis\ \times D/E_{\ market,t} + VIX_t + \varepsilon_t$.

| | *Dependent variable:* | |
|---|---|---|
| | Implied elasticity | |
| | (1) | (2) |
| *D/E* | -0.037* | -0.623** |
| | (0.019) | (0.287) |
| *VIX* | -0.041*** | -0.066*** |
| | (0.012) | (0.013) |
| $D/E\ \times\ Crisis$ | | 0.683** |
| | | (0.286) |
| $Intercept$ | 1.287** | 23.508** |
| | (0.601) | (10.890) |
| $intercept{:}\,Crisis$ | | -24.260** |
| | | (10.894) |
| Observations | 120 | 120 |
| $R^2$ | 0.132 | 0.263 |
| Adjusted $R^2$ | 0.117 | 0.237 |
| Residual Std. Error | 1.164 (df = 117) | 1.082 (df = 115) |
| F Statistic | 8.861*** (df = 2; 117) | 10.254*** (df = 4; 115) |
| *Note:* | | *p<0.1;**p<0.05;***p<0.01 |

This table shows two regressions between implied elasticity and the debt/equity ratio in aggregate level; (1) and (2) (including the prior and after financial crisis). Both using OLS simple regressions. The Debt/equity ratio is the main effect. Estimated regression coefficients and their associated standard deviation are in the parentheses. Also the $R^2$ is presented. Dataset from January 2006 to December 2015 monthly data. Database from CRSP, OptionMetric and Bloomberg.



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**Table 3: Firm-level regression results**

1. $Beta_{i,t} = intercept + \alpha_1 D/E_{\ i,t} + D_i + B/M_{i,t} + \omega_{i,t}$.
2. $Beta_{i,t} = intercept + \alpha_1 D/E_{\ i,t} + D_i + B/M_{i,t} + \omega_{i,t}$. (Non-financial firm)
3. $Beta_{i,t} = intercept + \alpha_1 D/E_{\ i,t} + \pi_1 FIRM \times D/E_{\ i,t} + B/M_{i,t} + D_t + \omega_{i,t}$

| | *Dependent variable:* | | |
|---|---|---|---|
| | Implied elasticity | | |
| | (1) | (2) | (3) |
| *D/E* | -0.077** | -0.030 | -0.102*** |
| | (0.031) | (0.061) | (0.038) |
| *B/M* | 0.282*** | 0.080 | -0.096 |
| | (0.104) | (0.091) | (0.062) |
| *D/E* $\times FIRM$ | | | 0.106*** |
| | | | (0.039) |
| $Intercept$ | -1.030*** | -1.020*** | -0.536*** |
| | (0.214) | (0.210) | (0.106) |
| $intercept{:}\,FIRM$ | | | -0.166* |
| | | | (0.095) |
| Observations | 715 | 644 | 715 |
| $R^2$ | 0.072 | 0.064 | 0.413 |
| Adjusted $R^2$ | 0.030 | 0.022 | 0.405 |
| Residual Std. Error | 1.040 (df = 683) | 1.010 (df = 615) | 0.814 (df = 705) |
| F Statistic | 1.702** (df = 31; 683) | 1.509** (df = 28; 615) | 55.057*** (df = 9; 705) |
| *Note:* | | | *p<0.1; **p<0.05; ***p<0.01 |

This table shows the regressions between the implied elasticity and the debt/equity ratio in firm level (panel data firm fixed effect). Control variable is book-to-market ratio. Regression (1) tests the relation for all firms and regression (2) tests the relation excluding the financial-service firms. Regression (3) further adds year dummy and dummy variable for large firms. Estimated regression coefficients and their associated standard deviation are in the parentheses. Also the $R^2$ is presented. Dataset from January 2007 to December 2012 quarterly data. Database from CRSP, OptionMetric and Compustat.



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## 4.3 Robustness test

In the second hypothesis, I prove a negative relation between the firm size and leverage effect. I run the regression with interaction effect to justify the leverage effect in small firms. In order to confirm this result, I equally divide all sample into three subsamples by market capitalization. Then I run regression between the implied elasticity and the debt/equity ratio separately in each group adding the year dummy and book-to-market ratio. Result shows in Table 4. It supports that smaller firms have a stronger leverage effect.

**Table 4: Regression results (subsampling by market capitalization)**

$$Beta_{i,t} = intercept + \alpha_1 D/E_{\ i,t} + B/M_{i,t} + D_t + \omega_{i,t}.$$

| | *Dependent variable:* Implied elasticity | | |
|---|---|---|---|
| | Small firms | Medium firms | Large firms |
| *Debt/equity ratio* | -0.087** | 0.001 | 0.017 |
| | (0.037) | (0.011) | (0.016) |
| *B/M* | -0.133 | -0.300** | -0.028 |
| | (0.178) | (0.148) | (0.073) |
| *Intercept* | -0.297** | -0.427** | -1.066*** |
| | (0.150) | (0.180) | (0.125) |
| Observations | 239 | 238 | 238 |
| $R^2$ | 0.409 | 0.383 | 0.490 |
| Adjusted $R^2$ | 0.391 | 0.364 | 0.474 |
| Residual Std. Error | 0.814 (df = 231) | 0.833 (df = 230) | 0.778 (df = 230) |
| F Statistic | 22.871*** (df = 7; 231) | 20.398*** (df = 7; 230) | 31.561*** (df = 7; 230) |
| *Note:* | | | *p<0.1; **p<0.05; ***p<0.01 |

This table shows the regressions for three subgroups. I equally divide total sample into three subgroups by its market capitalization. B/M is the book-to-market ratio. I also add the year dummy variable $D_t$. Estimated regression coefficients and their associated standard deviation are in the parentheses. Also the $R^2$ is presented. Dataset from January 2007 to December 2012 quarterly data. Database from CRSP, OptionMetric and Compustat.



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# Chapter 5 Conclusion

Plenty of discussions in empirical studies argue that whether financial leverage effect is capable of explaining the negative relationship between stock return and volatility changes. In my thesis, I provide a new way to support the leverage effect by Black (1976). There are two advantages for implying elasticity from CEV model. First, by Macbeth (1982) and Lee.et.al (2004), CEV model is of better performance than BS model because there are an extra parameter in the CEV model that is the elasticity. It would be rational that the implied elasticity reflects the information on the stock market with less measurement errors. Second, the implied elasticity is useful for both practitioners and researchers to analyze or utilize because it acts like a measure relating to the risk. I find that the implied elasticity in equity is more volatile than aggregate market. It is likely that there are more than one source of risks in the equity level. Implied elasticity in equity captures both systematic and idiosyncratic risk. Thus, it would be more volatile. Both aggregate and firm level carry the characteristic of asymmetric volatility oscillating dramatically at the market downturn.

Implied elasticity shows a negative relationship between the stock return and volatility changes. From the histogram above, the results are consistent with the theories and the empirical tests by early research. Notwithstanding some of the implied elasticities are positive, two third of the value are still negative. One reason for some of the implied elasticities are positive is that perhaps there are other factors in certain period dominating the leverage effect. Financial leverage effect only explains a portion of volatility, but there are too many factors that would influence the stock volatility in the market. For example, the behavioral interpretation of the leverage effect, the balance sheet effect by Duffee (2002) and displaced diffusion model by Rubinstein (1983) may attribute to some



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anomalies.

Clearly, the evidence affirms the leverage effect by Black (1976) while Hasanhodzic and Lo (2011) do not consider the operating leverage showing that there is still an inverse relation between the stock prices and volatility without leverage effect. Also this thesis expands the result from Christie (1982), Cheung and Ng (1992), which the earlier research focuses more on rank correlation than on a parameter relation. The implied elasticity method is an easier way to examine that the elasticity is function of leverage ratio (i.e. debt/equity ratio). In aggregate level, the relationship is stronger before the financial crisis in year 2008 when the total market debt/equity ratio is higher. Curiously, total market debt/equity ratio is not resilient to bound back to its former level since the outbreak of financial tsunami nine years ago. One interpretation is that companies have been continually delevering their balance sheets. In other words, they have been cutting back on debt while S&P 500 keeps booming. In firm level, the result is significant for including the financial-service firms. Moreover, I run regressions separately for 30 equities. The mean of 30 coefficients still be negative (-0.2932), consistent with the panel data result. Besides, there is evidence showing the positive relation between the elasticity and the firm size by Cheung and Ng (1992), Duffee (1995). In the last regression, I set the firm size dummy variable and find a negative relation between the firm size and leverage effect. I also run regressions in subsamples to justify the relation. For one reason, the minute relation between the firm size and leverage effect in large firms is that the large firms usually have more stable financial structures and typically low debt/equity ratio. Leverage effect may not able to explain the volatility changes among large firms. Result of small firms is in accordance with Cheung and Ng (1992) that leverage effect has a stronger explanation in small firms.



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