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D-brane in R-R field background

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國立臺灣大學博士學位論文
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本論文係葉啟賢君（D95222008）在國立臺灣大學物理學系、所完成之博士學位論文，於民國 100 年 12 月 30 日承下列考試委員審查通過及口試及格，特此證明

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論文摘要:

本篇論文主要研究在 Ramond-Ramond(R-R)背景場下 D 膜(D-brane)的有效場論。由於 R-R 背景場的存在,這樣的理論在背景場的方向會有體積保持不變的對稱性(volume-preserving diffeomorphism),這是這理論的主要特徵之一。之所以會研究這樣的理論,起源於最近有關 M5 膜在大 C 背景場的有效場論的研究。高一維度的理論可以透過丟掉場在這一維度的自由度,來得到低一維度在低能量極限的有效場論。因此這樣的分析方法常常會有一些多餘的場殘留在低一維度的理論中。要如何分辨哪些場是理論所必須的,而哪些場又是可以被積掉的,是這研究的核心部分。在這篇論文中,我們發現原先所預期出現的規範場被隱藏在某些場內,我們使用了對偶變換的方法來使這樣的規範場在理論中變的明顯。接著我們討論了在這樣的變換下,要如何求出規範場的規範對稱變換以及超對稱變換。我們研究了在規範對稱性以及體積保持不變性之下的協變量(covariant variables)應是什麼樣子的,並利用它們來使理論易於推廣到不同的情形。最後我們利用這理論所具有的超對稱去討論這理論的拓樸性質,即理論所允許的孤立子解。

Abstract

In this paper, we try to understand the low energy effective theory of Dp-brane in large R-R (p-1)-form field background. To construct the effective theory, we start with the M5-brane theory in large C-field background [1, 2]. The C-field background defines the 3-dimensional volume form in M5-brane theory. Hence, the M5-brane theory can be described as a Nambu-Poisson-bracket gauge theory with volume-preserving diffeomorphism symmetry (VPD). After doing double dimensional reduction, we obtain the effective theory of D4-brane in large C-field background [3]. This theory has both the usual U(1) gauge symmetry and the new symmetry VPD. The VPD two-form gauge potential can be understood as the electric-magnetic dual of the one-form gauge field in the D4-brane theory. This theory is described by the one-form gauge field and the dual two-form gauge field at the same time. These results can be generalized to Dp-branes cases. In the last part of thesis, we study the supersymmetry (SUSY) algebra in this theory. We can calculate the central charges from the SUSY algebra in this theory, then we can know the possible topological quantities in this system. This interesting system may help us to understand M-theory, the models with volume-preserving diffeomorphism, the suitable low energy description of Dp-branes in different field backgrounds, some new soliton solutions, and so on.

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Chapter 1

Introduction

In this chapter, we will review several relevant elements of effective theory in a certain background. First of all, we will talk about the effective action of D-brane theory. To understand the string theory, we can start with the calculation of perturbative string scattering amplitudes. On the other hand, the nonperturbative effect of string theory is described by soliton solutions in ten-dimensional supergravity theory. These solitons are the Dp-branes, which are the extended object with p-spatial dimensions. The open string ends on these Dp-branes, hence the low energy effective field theory of Dp-brane can be obtained from the calculation of the open string scattering amplitudes. The Dp-branes theories have two main descriptions. One is the Dirac-Born-Infeld action [4]. Another is the Yang-Mills gauge theory [5]. They share part of the original brane theory in different limits, which are the slowly varying limit or zero slope limit of string theory. There are several good reviews of D-brane theory. For example, the review articles [6–8] are useful.

On the other hand, we want to introduce the well-known case of Dp-branes in constant NS-NS field background [9–12]. We will show noncommutative Yang-Mills theory as the effective field theory of the Dp-branes theory in the low energy limit. The first-order expansion of noncommutative algebra is described by Poisson bracket, which is the generator of Area-Preserving Diffeomorphism (APD). The noncommutative effect depends on the inverse of NS-NS B-field. Hence, the field theory of Poisson-bracket is relevant to Dp-branes in large NS-NS B-field background.

When we want to study the effective field theory in large n-form field background, we can focus on the symmetry in this theory. While the n-form field defines the n-dimensional volume form in the theory, we expect that the effective theory may have n-dimensional volume-preserving diffeomorphism (VPD) symmetry. We will give more detail description of the VPD symmetry, where the symmetry generator is Nambu-Poisson bracket.

In the last part of this chapter, we will review M theory, where similar phenomena can be found. The M5-brane worldvolume theory has their own action with C field, which is called PST action [25]. Recently, people [1, 2] found another action for M5 in large C field background. It is similar to the story of Dp-branes in the NS-NS B-field background. We will give more details of this theory in next chapter.

1.1 Dp-Branes with Different Field Backgrounds

The low energy effective theories of Dp-branes are called Dirac-Born-Infeld (DBI) action. People also study the modification of DBI action in NS-NS and R-R field backgrounds. For the theory to be gauge invariant and anomaly free, we need to replace U(1) field strength F by $B + F$ and add Wess-Zumino terms into the original DBI action. In fact, the NS-NS and R-R fields are the massless mode of the close string spectrum. They are the background fields of open string scattering amplitudes just like the gravitational background. When we use the open string scattering amplitudes to study the effective theories of D-brane in NS-NS and R-R field backgrounds, we may have different interpretations for these background fields. For example, the NS-NS background fields can be absorbed into the field strength of open-string oscillation mode or the open-string metric, then we get different effective theories of D-brane. For these effective theories, we call all of them to be D-brane in field backgrounds. However, this terminology is confusing in this thesis. The effective theory, which we want to talk in this thesis, is the effective theory without manifest background fields. In this case, the effects of background fields hide in the geometry and the symmetry algebra of effective theory. We try to distinguish them in next subsection.

1.1.1 Terminology Explanation

When we want to talk about a theory in some field backgrounds, we need to know what it really means.

Firstly, the meaning of background field is that we neglect the dynamic behavior of this background field. The simple case is that we study the matter field in electric-magnetic fields background, in this case, we neglect the dynamic contribution of EM gauge fields. So the background fields what we means are constant fields. The terminology "theory in constant field background" is the same as the terminology "theory in field background".

Secondly, when we talk about the effective field theory in field background, the effective theory usually does not include the manifest background fields dependence. The effect of background field can appear in effective mass, effective coupling or new geometry. Hence we will have a new field theory, then we can study the equivalent phenomena in the two theories. We do not have a simple example in field theory. However, the phenomena appear frequently in string theory. For example, the effective description of Dp-brane is not unique, we have more than one effective description. The first example is the commutative and noncommutative gauge theory for Dp-brane in NS-NS B field background. People [12] understand this phenomena as the result of different regularization method of open string scattering amplitude analysis. The effective field theory will be different in the different regularization method, they can be related by changing variables. In this case, this change of variables is called the Seiberg-Witten map [12]. However, the two different effective field theories are not really the same after Seiberg-Witten map, they are different by higher derivative terms and total derivative terms. Hence, they stand for the different parts of the full D-brane theory, while they can have overlap in the scaling limit (Appendix C). Hence, in order to distinguish these two situations from other cases, we use the terminology of Dp-brane "with" NS-NS and R-R fields for originally well known DBI action. We use the terminology of Dp-brane "in" NS-NS and R-R fields "background" for the case what we want to talk in this thesis. The effective field theory "in" fields background does not have manifest background fields dependence.

Finally, we study the theory in large field background in the most part of this thesis. In this limit, the effective field theory becomes simpler and easier to analyze.

1.1.2 Dirac-Born-Infeld Action and Yang-Mills Gauge Theory

In this subsection, we want to write down the explicit action form of effective field theory of D-brane. It is called the Dirac-Born-Infeld(DBI) action [4]. Roughly speaking, the DBI action comes from the calculation of open string scattering amplitude. When we calculate the β-function of open string scattering amplitude, because the theory has conformal invariance, the β-function must vanish. From these constraints, we can find the constraints of fields. These fields are the oscillation mode of open string. These constraints of fields can be understood as the equations of motion which are derived from corresponding effective field theory action. The effective action (DBI action) is described by p+1 coordinates $\xi^a, a = 0, 1, \ldots, p$. The DBI action is written as[1] [4]:

$$S_{DBI} = T_p \int d^{p+1}\xi \sqrt{\det(G_{ab} + 2\pi\alpha' F_{ab})}, \tag{1.1}$$

[1] In this chapter, we use the review paper of Dp-brane [6]

here T_p is defined by $\frac{1}{(2\pi)^p g_s \ell_s^{p+1}}$, which is the tension of Dp-brane. It is the generalization of the string tension $T_{F1} = \frac{1}{2\pi\alpha'}$. The p labels the number of spatial dimensions for Dp-brane. The g_s is string coupling and $\ell_s = \sqrt{\alpha'}$ is identified as string length. The G_{ab} is the induced metric in Dp-brane, it is usually complex in the fermionic part. Here, we give the bosonic part of the induce metric:

$$G_{ab} = \eta_{MN}\partial_a X^M \partial_b X^N, \tag{1.2}$$

where M is from 0 to p. We can choose gauge to let $X^a = \xi^a$. So, the remaining scalars in DBI action are the transverse coordinates in target spacetime, and we label them with $2\pi\alpha' X^I \quad I = p+1, \ldots, 9$. Here, we use the factor $2\pi\alpha'$ to make the mass dimension of X^I equal to one. Hence, we can rewrite action as:

$$S_{DBI} = T_p \int d^{p+1}\xi \sqrt{\det(\eta_{ab} + 2\pi\alpha'\partial_a X^I \partial^a X^I + 2\pi\alpha' F_{ab})}. \tag{1.3}$$

The F is the field strength of one form gauge potential A, that is $F = dA$ in Maxwell theory. We can regard the DBI action as the high energy version of Maxwell action. To take the low energy limit $\alpha' \to 0$ and omit the scalar terms, we can get:

$$S_{DBI} = T_p \int d^{p+1}\xi \sqrt{\det \eta_{ab}}(1 - \frac{1}{4}F^{ab}F_{ab} + O(\alpha')). \tag{1.4}$$

The low energy limit makes the D-brane theory to become simpler.

1.1.3 Dp-Branes with NS-NS and R-R Fields

The dynamics of Dp-Brane will be affected by background fields, which come from the closed string NS-NS and R-R sector. In NS-NS sector, we have graviton g_{MN} which is symmetry rank-2 field, and NS-NS B-field $2\pi\alpha' B_{MN}$ which is antisymmetry two-form field. We also have dilaton field Φ, which is a scalar. All of them will modify the form of DBI action. For simplicity, here we only consider the effect of NS-NS B-field. The action of Dp-brane in NS-NS B field background can be written as:

$$S_{DBI} = T_p \int d^{p+1}\xi \sqrt{\det(\eta_{ab} + 2\pi\alpha'\partial_a X^I \partial^a X^I + 2\pi\alpha'(F_{ab} + B_{ab}))}, \tag{1.5}$$

which can be realized by modification of G_{ab}, the induce metric, in following way:

$$G_{ab} = (\eta_{MN} + 2\pi\alpha' B_{MN})\partial_a X^M \partial_b X^N, \tag{1.6}$$

the mixed terms of B and X will vanish for the antisymmetry of B field. The action form can have the gauge symmetry of two form field B with additional shift of one form field A:

$$B \to B + d\Lambda, \qquad A \to A - \Lambda, \tag{1.7}$$

such that $B + F$ term do not transform.

The R-R sectors of close string are some higher ranks form. For example, the Dp-brane can have R-R (p+1)-form,(p-1)-form,...,1-form (or 0-form for odd p), we label them by $C_{p+1}, C_{p-1}, \ldots, C_1$ (or C_0 for odd p).

The action of Dp-brane in R-R field background can be written as [13, 14]:

$$S_{DBI} = T_p \int d^{p+1}\xi \sqrt{\det(G_{ab} + 2\pi\alpha' F_{ab})} + S_{WZ}, \tag{1.8}$$

here the new term is written by:

$$S_{WZ} = \mu_p \int (Ce^{2\pi\alpha' F})_{p+1}, \qquad C \equiv \sum_{n=0}^{8} C_n. \tag{1.9}$$

The notation $(\cdots)_{p+1}$ is to keep the p+1 form inside the parentheses. The μ_p is the electric charge of Dp-brane. In fact, the calculation of open-string scattering amplitude in R-R background is very difficult. People do not know how to quantize the nonlinear sigma model in curved spacetime. However, we can know the field contents, the gauge symmetry, and the supersymmetry from the flat space calculation. Hence, we can use these informations to analyze the effective worldvolume theory of Dp-brane with R-R fields. For example, the Wess-Zumino term (S_{WZ}) is introduced to cancel the gauge anomaly in superstring theory.

While the DBI-like action of multiple Dp-branes is incomplete and unclear (the relevant papers [15, 16]), we can still use non-abelian Yang-Mills action to describe them. Yang-Mills action is the leading term of multiple Dp-branes action after taking zero slope limit ($\alpha' \to 0$).

1.2 Large Field Background Effects

From effective theory viewpoint, the high derivative terms can be omitted in the low energy limit. However, when the system is embedded in large field background, this approximation is not true. The large field background can couple to these high derivative terms, which are still leading in low energy limit. Large field background will be have more differently from original case. Those new effects worth further investigation.

The well-known example is the Dp-branes in constant NS-NS B field background. In this case, the effective field theory is not conventional Yang-Mills field theory, we should use the noncommutative Yang-Mills field theory to suitably describe the effective field theory of Dp-branes in constant NS-NS B field background [9–12]. The noncommutative field theory is a better description of D-brane in NS-NS B field background than original DBI action or Yang-Mills field theory. The reason is the noncommutative theory includes nonlocal behavior, which encodes the information of the higher derivative terms in original theory. As what we mentioned before, the large NS-NS B field coupled to higher derivative terms will remain after taking low energy limit, and noncommutativity emerges.

1.2.1 Dp-Branes in constant NS-NS B-field Background

When we calculate the scattering amplitudes of open string in constant NS-NS B field background, we use another regularization processes called point splitting regularization. The different regularization methods will modify the forms of β-function. After imposing the vanishing β-function, we get the effective field theory of Dp-branes in constant NS-NS B field background. After taking scaling limit, we get the similar Yang-Mills type effective action. For example, the leading terms of three point open string scattering amplitude can be effectively obtained from the action [12]:

$$\frac{(\alpha')^{\frac{3-p}{2}}}{4(2\pi)^{p-2}G_s}\int\sqrt{detG}G^{ab}G^{cd}\hat{F}_{ac}*\hat{F}_{bd}. \tag{1.10}$$

It is called noncommutative Yang-Mills field theory. The noncommutativity is defined by the Moyal product "$*$", such that

$$f(x)*g(x) \;=\; e^{\frac{i}{2}\theta^{ab}\frac{\partial}{\partial\xi^a}\frac{\partial}{\partial\zeta^b}}f(x+\xi)g(x+\zeta)\Big|_{\xi=\zeta=0}. \tag{1.11}$$

The field strength $\hat{F}$ is defined by gauge potential $\hat{a}$ and Moyal product:

$$\hat{F}_{ab}=\partial_a\hat{a}_b-\partial_b\hat{a}_a-i\hat{a}_a*\hat{a}_b+i\hat{a}_b*\hat{a}_a, \tag{1.12}$$

while the gauge symmetry is :

$$\delta_\lambda\hat{a}_a=\partial_a\lambda+i\lambda*\hat{a}_a-ia_a*\lambda. \tag{1.13}$$

When the background B field is large, the noncommutative factor θ becomes small. We can expand the Moyal product to the first order. Hence we will get (in U(1) case) [12]:

$$
\begin{aligned}
f(x) * g(x) &= fg + \frac{i}{2}\theta^{ab}\partial_a f \partial_b g + O(\theta^2), & (1.14)\\
\hat{F}_{ab} &= \partial_a \hat{a}_b - \partial_b \hat{a}_a + \theta^{cd}\partial_c \hat{a}_a \partial_d \hat{a}_b + O(\theta^2), & (1.15)\\
\delta_\lambda \hat{a}_a &= \partial_a \lambda - \theta^{cd}\partial_c \lambda \partial_d \hat{a}_a + O(\theta^2). & (1.16)
\end{aligned}
$$

In this case, the main characteristic in large B field is appearance of the Poisson bracket structure:

$$
\{f, g\}_{pb} = \epsilon^{ab}\partial_a f \partial_b g. \qquad (1.17)
$$

We will discuss it more in next subsection.

1.2.2 Volume-Preserving Diffeomorphism and Nambu-Poisson Bracket

From previous subsection, we can understand the noncommutative gauge fields theory can be described by Moyal product. If we focus on the large NS-NS B field case, theory is handled by Poisson bracket, which is the generator of Area-Preserving Diffeomorphism (APD). In general. for higher ranks field background, they need a general Poisson bracket, which is called Nambu-Poisson bracket [17–21], to be the generator of Volume-Preserving Diffeomorphism (VPD).

To understand the reason why VPD emerges, we can think in following way. The original worldvolume theory has diffeomorphism symmetry:

$$
x^a \to \acute{x}^a = \acute{x}^a(x^a). \qquad (1.18)
$$

When we consider the theory in large field background, the original diffeomorphism symmetry will be broken by background field, the remaining symmetry is volume-preserving diffeomorphism. The n-dimensional volume-preserving diffeomorphism is the reduced symmetry of n-dimensional general coordinate diffeomorphism, which is described by (infinitesimal transformation):

$$
x^a \to \acute{x}^a = \acute{x}^a(x^a) = x^a + \kappa^a, \qquad \partial_a \kappa^a = 0. \qquad (1.19)
$$

where $a = 0, 1, ..., n-1$. The n-dimensional volume-preserving diffeomorphism can be understood as that this transformation parameter κ^a has additional constraint as shown in (1.19). To see how this constraint gives rise to the volume-preserving, we can investigate the Jacobian of coordinate transformation. For example, we can find the Jacobian of coordinate transformation for n=2 reads

$$\epsilon^{ab}\partial_a \acute{x}^0 \partial_b \acute{x}^1 = \{\acute{x}^0, \acute{x}^1\}. \tag{1.20}$$

We consider the coordinate transformation in (1.19), after simple calculation, we can find:

$$\{\acute{x}^0, \acute{x}^1\} = 1 + \partial_a \kappa^a + O(\kappa^2). \tag{1.21}$$

Therefore we can see the constraint $\partial_a \kappa^a = 0$ makes area-preserving. Moreover, higher ranked volume-preserving transformation can be generated by generalize Nambu-Poisson bracket, defined by

$$\{f_1, f_2, \cdots, f_n\} \equiv \epsilon^{a_1 a_2 \cdots a_n} \partial_{a_1} f_1 \partial_{a_2} f_2 \cdots \partial_{a_n} f_n. \tag{1.22}$$

Hence, we can define the VPD transformation as follows

$$\delta_{\Lambda_1,\ldots,\Lambda_{n-1}} x^a = \{\Lambda_1, \ldots, \Lambda_{n-1}, x^a\} = \epsilon^{a_1 \cdots a_{n-1} a} \partial_{a_1} \Lambda_1 \cdots \partial_{a_{n-1}} \Lambda_{n-1} \equiv \kappa^a. \tag{1.23}$$

In the special case of APD (n=2):

$$\delta_\Lambda x^a = \{\Lambda, x^a\}_{pb} = \kappa^a. \tag{1.24}$$

This is the simplest case in this kind of symmetry transformation. We can see the Nambu-Poisson bracket is the generator of VPD.

Now we can ask the next question; what is the field theory with VPD? In fact, we already saw the example of field theory with APD in previous subsection. We can find the symmetry transformation is generated by Poisson bracket. We will see more examples in next three chapters.

1.3 A Review of M Theory

In order to understand more nonperturbative effect of superstring theory, people start to study the M theory. M theory is the complete picture of string theory. The five different perturbative string theories and eleven dimensional supergravity theory can be understood as the different descriptions of M theory. For example, M theory can be understood as strong coupling limit of type IIA superstring theory in one higher dimension. Hence, the low energy effective theory of M theory is eleven dimensional supergravity theory.

From the eleven-dimensional superalgebra analysis, there are two kind high-dimensional central charges [22]. They are carried by M2-brane and M5-brane, which are the extended objects 2-brane and 5-brane in eleven dimensions. Following the analysis of 2-brane and 5-brane soliton solutions in eleven-dimensional supergravity theory, we can know the field contents of the effective worldvolume theory of M2-brane and M5-brane. The action of effective field theory for single M2-brane and M5-brane is well known. The M2-brane effective action is the generalized Nambu-Goto action. The effective action of M5-brane is more difficult because it involves the self-dual two-form gauge potential [23–28]. Recently, there are several interesting papers about self-dual gauge theory [29, 30].

The theory also have a background form fields as string does, and it is the three form field background. The M2-brane couple electrically to the 3-form field. As the research of D-brane in NS-NS B field background, people try to generalize the research into M theory. For example, people [31] tried to study the quantization of open membrane in large C-field background, and they found similar noncommutative behavior as in the open string case because quantization processes naturally adopts Poisson structure. Moreover, people [32] calculated the scattering amplitudes of open membrane in large C-field background which can be described by Nambu-Poisson algebra (or VPD gauge symmetry). It gives the candidate of generalization of Poisson bracket and Moyal product. To study the M theory in large C-filed background helps us understand the way to generalize Moyal product, which gives the way to quantize string. Hence, people try to apply these researches to understand how to quantize membrane. To study M5 in large C-field background has more interesting physic phenomena. This kind theory includes the self-dual two form, the non-abelian gauge algebra and new action form which is different from PST M5 action [25]. We will discuss this topic in next chapter.

Chapter 2

M5 in Large C-Field Background

Recently, Bagger, Lambert and Gustavsson imposed the Lie-3 algebra into Basu-Harvey BPS system to construct the theory of multiple membranes [33–36]. It is called BLG model, which describes the multiple M2-branes system. In this articles, we will not give any more detail of BLG theory. Latter, people [1, 2] started to impose the Nambu-Poisson structure into the three internal dimensions of BLG model, then they found the new description of single M5-brane theory. We denote it as Nambu-Poisson (NP) M5 theory.

2.1 Nambu-Poisson M5 Theory

The Nambu-Poisson algebra is an infinite dimensional Lie-3 algebra, which is used to describe the algebra in BLG model. People [1,2] consider the additional three internal space dimensions ($\mathcal{N}$) with 3-dimensional volume-preserving diffeomorphism, which define the space of Nambu-Poisson bracket. Moreover, the worldvolume of multiple M2-branes ($\mathcal{M}$) and the 3 internal dimensions $\mathcal{N}$ together can be identified as the worldvolume of M5 theory ($\mathcal{M} \times \mathcal{N}$).

These processes will divide the worldvolumes dimensions of M5 into two parts:

$$\{x^{\mu}; y^{\dot{\mu}}\} = \{x^0, x^1, x^2; y^{\dot{1}}, y^{\dot{2}}, y^{\dot{3}}\}. \tag{2.1}$$

Here, the coordinate x^{μ} label the direction on $\mathcal{M}$, which are the longitudinal directions of M2-branes. Another coordinates $y^{\dot{\mu}}$ label the internal directions on $\mathcal{N}$, which are the space of the volume-preserving diffeomorphism. Roughly speaking, this effective M5 description have 3-dimensional VPD, which is the main characteristic of theory in large C field background. Hence, the Nambu-Poisson structure in M5 theory is the first evidence of M5 in large C field background.

The fields contents of NP M5 theory are self-dual two form $(b_{\mu\dot{\mu}}, b_{\dot{\mu}\dot{\nu}})$, five scalar fields X^I, and chiral Majorana fermion Ψ. The two form gauge fields are self-dual, so the degree of freedom (DOF) of two form fields are $\frac{6}{2} = 3$ and we do not need $b_{\mu\nu}$ in this theory. The five scalar fields are the DOF of M5-brane on the transverse directions. The Majorana fermion is reduction from 11 dimension which satisfies the 6-dimension chirality condition $\Gamma^7\Psi = \Psi$. Hence the DOF of fermion are $\frac{1}{2}2^{[\frac{11}{2}]} = 16$. The one half of fermion DOF[1] $(\frac{16}{2})$ are equal to the bosonic DOF $(3+5)$ in NP M5 theory, which is a result of supersymmetry.

In next section, we will give the full action to describe the dynamics of these fields. We will not give all the details about how to derive the action from the BLG theory. The main process of action calculation is to replace the Lie-3 bracket by Nambu-Poisson bracket: $[\bullet, \bullet, \bullet] \to g^2\{\bullet, \bullet, \bullet\}$. The other details can be found in the papers [1, 2].

2.2 Action of Nambu-Poisson M5 Theory

In this section, we want to summarize the main result of action of NP M5 theory. The NP M5 action is the effective description of M5 in large C field background, so the description is well-defined in some suitable scaling limit. We put the discussion of suitable scaling limit in Appendix.

Following the result of the papers [1, 2], the action of NP M5 theory is written as:

$$S = \frac{T_{M5}}{g^2}\left(S_X + S_\Psi + S_{gauge},\right), \qquad S_{gauge} = S_{\mathcal{H}^2} + S_{CS}, \tag{2.2}$$

[1]Only one half of fermionic DOF is really equivalent to bosonic DOF, because the EOM of fermion involves first derivative.

where [2]

$$
\begin{aligned}
S_X &= \int d^3x d^3y \left[-\frac{1}{2}(\mathcal{D}_\mu X^I)^2 - \frac{1}{2}(\mathcal{D}_{\dot\lambda} X^I)^2 \right. \\
&\quad \left. -\frac{1}{2g^2} - \frac{g^4}{4}\{X^{\dot\mu}, X^I, X^J\}^2 - \frac{g^4}{12}\{X^I, X^J, X^K\}^2 \right], \qquad (2.3) \\
S_\Psi &= \int d^3x d^3y \left[\frac{i}{2}\overline{\Psi}\Gamma^\mu \mathcal{D}_\mu \Psi + \frac{i}{2}\overline{\Psi}\Gamma^{\dot\rho}\mathcal{D}_{\dot\rho}\Psi \right. \\
&\quad \left. + \frac{ig^2}{2}\overline{\Psi}\Gamma_{\dot\mu}\Gamma^I\{X^{\dot\mu}, X^I, \Psi\} - \frac{ig^2}{4}\overline{\Psi}\Gamma^{IJ}\Gamma_{\dot 1\dot 2\dot 3}\{X^I, X^J, \Psi\} \right], \qquad (2.4) \\
S_{\mathcal{H}^2} &= \int d^3x d^3y \left[-\frac{1}{12}\mathcal{H}^2_{\dot\mu\dot\nu\dot\rho} - \frac{1}{4}\mathcal{H}^2_{\lambda\dot\mu\dot\nu} \right], \qquad (2.5) \\
S_{CS} &= \int d^3x d^3y\, \epsilon^{\mu\nu\lambda}\epsilon^{\dot\mu\dot\nu\dot\lambda} \left[-\frac{1}{2}\partial_{\dot\mu} b_{\mu\dot\nu}\partial_\nu b_{\lambda\dot\lambda} + \frac{g}{6}\partial_{\dot\mu} b_{\nu\dot\nu}\epsilon^{\dot\rho\dot\sigma\dot\tau}\partial_{\dot\sigma} b_{\lambda\dot\rho}(\partial_{\dot\lambda} b_{\mu\dot\tau} - \partial_{\dot\tau} b_{\mu\dot\lambda}) \right]. \qquad (2.6)
\end{aligned}
$$

In the above we use the notation

$$
\begin{aligned}
X^{\dot\mu}(y) &\equiv \frac{y^{\dot\mu}}{g} + \frac{1}{2}\epsilon^{\dot\mu\dot\kappa\dot\lambda} b_{\dot\kappa\dot\lambda}(y) \equiv \frac{y^{\dot\mu}}{g} + b^{\dot\mu}(y), \qquad (2.7) \\
\{A, B, C\} &\equiv \epsilon^{\dot\mu\dot\nu\dot\rho}\partial_{\dot\mu} A \partial_{\dot\nu} B \partial_{\dot\rho} C. \qquad (2.8)
\end{aligned}
$$

Here, we can find the effective field theory on worldvolume of NP M5 theory are described by Nambu-Poisson bracket.

The covariant derivative is defined by($\Phi = X^I$ or Ψ):

$$
\begin{aligned}
\mathcal{D}_\mu \Phi &\equiv \partial_\mu \Phi - g\{b_{\mu\dot\nu}, y^{\dot\nu}, \Phi\}, \qquad (2.9) \\
\mathcal{D}_{\dot\mu}\Phi &\equiv \frac{g^2}{2}\epsilon_{\dot\mu\dot\nu\dot\rho}\{X^{\dot\nu}, X^{\dot\rho}, \Phi\}. \qquad (2.10)
\end{aligned}
$$

We can find the covariant derivative is defined by two gauge fields: $b_{\mu\dot\mu}$ and $b_{\dot\mu\dot\nu}$. The definition of the 3-form field strength reads

$$
\begin{aligned}
H_{\lambda\dot\mu\dot\nu} &= \partial_\lambda b_{\dot\mu\dot\nu} - \partial_{\dot\mu} b_{\lambda\dot\nu} + \partial_{\dot\nu} b_{\lambda\dot\mu}, \qquad (2.11) \\
H_{\dot\lambda\dot\mu\dot\nu} &= \partial_{\dot\lambda} b_{\dot\mu\dot\nu} + \partial_{\dot\mu} b_{\dot\nu\dot\lambda} + \partial_{\dot\nu} b_{\dot\lambda\dot\mu}, \qquad (2.12)
\end{aligned}
$$

which is no longer covariant under the non-Abelian gauge transformations. The covariant

[2] In original paper [2], they meet the unusually kinetic term of fermions, which has added $\Gamma_{\dot 1\dot 2\dot 3}$ factor. In order to solve the problem, they used the similar unitary transformation: $\Psi = \frac{1}{\sqrt{2}}(1 - \Gamma_{\dot 1\dot 2\dot 3})\Psi'$. Here, we use the symbol Ψ, which was denoted by Ψ' in [2].

3-form field strengths $\mathcal{H}$ should be defined as

$$\begin{aligned}
\mathcal{H}_{\lambda\dot{\mu}\dot{\nu}} &= \epsilon_{\dot{\mu}\dot{\nu}\dot{\lambda}}\mathcal{D}_{\lambda}X^{\dot{\lambda}} \\
&= H_{\lambda\dot{\mu}\dot{\nu}} - g\epsilon^{\dot{\sigma}\dot{\tau}\dot{\rho}}(\partial_{\dot{\sigma}}b_{\lambda\dot{\tau}})\partial_{\dot{\rho}}b_{\dot{\mu}\dot{\nu}}, \qquad (2.13)\\
\mathcal{H}_{\dot{1}\dot{2}\dot{3}} &= g^2\{X^{\dot{1}}, X^{\dot{2}}, X^{\dot{3}}\} - \frac{1}{g} \\
&= H_{\dot{1}\dot{2}\dot{3}} + \frac{g}{2}(\partial_{\dot{\mu}}b^{\dot{\mu}}\partial_{\dot{\nu}}b^{\dot{\nu}} - \partial_{\dot{\mu}}b^{\dot{\nu}}\partial_{\dot{\nu}}b^{\dot{\mu}}) + g^2\{b^{\dot{1}}, b^{\dot{2}}, b^{\dot{3}}\}. \qquad (2.14)
\end{aligned}$$

In fact, the deformations of field strengths come from the VPD symmetry. It is similar to the theory with APD symmetry. We will give the more details of VPD symmetry transformations of fields in next section.

2.3 Symmetry of Nambu-Poisson M5 Theory

In this section, we will show the symmetry in the NP M5 theory. The gauge symmetry of NP M5 theory is the volume-preserving diffeomorphism. On the other hand, the theory has also supersymmetry, which can be used to calculate the BPS states and central charges.

2.3.1 Gauge Symmetry and VPD

The fundamental fields transform under the gauge transformation as

$$\begin{aligned}
\delta_\Lambda\Phi &= g\kappa^{\dot{\rho}}\partial_{\dot{\rho}}\Phi \qquad (\Phi = X^I, \Psi), \qquad (2.15)\\
\delta_\Lambda b_{\dot{\kappa}\dot{\lambda}} &= \partial_{\dot{\kappa}}\Lambda_{\dot{\lambda}} - \partial_{\dot{\lambda}}\Lambda_{\dot{\kappa}} + g\kappa^{\dot{\rho}}\partial_{\dot{\rho}}b_{\dot{\kappa}\dot{\lambda}}, \qquad (2.16)\\
\delta_\Lambda b_{\lambda\dot{\sigma}} &= \partial_{\lambda}\Lambda_{\dot{\sigma}} - \partial_{\dot{\sigma}}\Lambda_{\lambda} + g\kappa^{\dot{\tau}}\partial_{\dot{\tau}}b_{\lambda\dot{\sigma}} + g(\partial_{\dot{\sigma}}\kappa^{\dot{\tau}})b_{\lambda\dot{\tau}}, \qquad (2.17)
\end{aligned}$$

where

$$\kappa^{\dot{\lambda}} \equiv \epsilon^{\dot{\lambda}\dot{\mu}\dot{\nu}}\partial_{\dot{\mu}}\Lambda_{\dot{\nu}}(x, y). \qquad (2.18)$$

The field strengths $\mathcal{H}$ transform like Φ.

The gauge transformations can be more concisely expressed in terms of the new variables $b^{\dot{\mu}}, B_\mu{}^{\dot{\mu}}$

$$\begin{aligned}
b^{\dot{\mu}} &\equiv \frac{1}{2}\epsilon^{\dot{\mu}\dot{\nu}\dot{\lambda}}b_{\dot{\nu}\dot{\lambda}}, \qquad (2.19)\\
B_\mu{}^{\dot{\mu}} &\equiv \epsilon^{\dot{\mu}\dot{\nu}\dot{\lambda}}\partial_{\dot{\nu}}b_{\mu\dot{\lambda}} \qquad (2.20)
\end{aligned}$$

for the gauge fields as

$$\delta_\Lambda b^{\dot\mu} = \kappa^{\dot\mu} + g\kappa^{\dot\nu}\partial_{\dot\nu} b^{\dot\mu}, \tag{2.21}$$

$$\delta_\Lambda B_\mu{}^{\dot\mu} = \partial_\mu \kappa^{\dot\mu} + g\kappa^{\dot\nu}\partial_{\dot\nu} B_\mu{}^{\dot\mu} - g(\partial_{\dot\nu}\kappa^{\dot\mu})B_\mu{}^{\dot\nu}. \tag{2.22}$$

In terms of $B_\mu{}^{\dot\nu}$, the covariant derivative $\mathcal{D}_\mu$ acts as

$$\mathcal{D}_\mu \Phi = \partial_\mu \Phi - gB_\mu{}^{\dot\mu}\partial_{\dot\mu}\Phi. \tag{2.23}$$

Another feature of the gauge transformations is that, in terms of $X^I, \Psi, b^{\dot\mu}$ and $B_\mu{}^{\dot\mu}$, all gauge transformations can be expressed solely in terms of $\kappa^{\dot\mu}$, without referring to $\Lambda_{\dot\mu}$, as long as one keeps in mind the constraint

$$\partial_{\dot\mu}\kappa^{\dot\mu} = 0. \tag{2.24}$$

This gauge transformation can be naturally interpreted as volume-preserving diffeomorphism (VPD)

$$\delta y^{\dot\mu} = g\kappa^{\dot\mu}, \quad \text{with} \quad \partial_{\dot\mu}\kappa^{\dot\mu} = 0. \tag{2.25}$$

The field $b^{\dot\mu}$ is then interpreted as the gauge potential for the VPD in the 3-dimensional space picked by the C-field background.

2.3.2 Supersymmetry

The M5-brane theory is also invariant under the supersymmetry transformations δ_χ and δ_ϵ. We have

$$\delta_\chi \Psi = \chi, \quad \delta_\chi X^I = \delta_\chi b_{\dot\mu\dot\nu} = \delta_\chi b_{\mu\dot\nu} = 0, \tag{2.26}$$

and [3]

$$\delta_\epsilon X^I = i\bar\epsilon\Gamma^I\Psi, \tag{2.27}$$

$$\begin{aligned}\delta_\epsilon \Psi = {} & \mathcal{D}_\mu X^I \Gamma^\mu\Gamma^I\epsilon + \mathcal{D}_{\dot\mu} X^I\Gamma^{\dot\mu}\Gamma^I\epsilon \\ & -\frac{1}{2}\mathcal{H}_{\mu\dot\nu\dot\rho}\Gamma^\mu\Gamma^{\dot\nu\dot\rho}\epsilon - \frac{1}{g}\left(1 + g\mathcal{H}_{\dot1\dot2\dot3}\right)\Gamma_{\dot1\dot2\dot3}\epsilon \\ & -\frac{g^2}{2}\{X^{\dot\mu}, X^I, X^J\}\Gamma^{\dot\mu}\Gamma^{IJ}\epsilon + \frac{g^2}{6}\{X^I, X^J, X^K\}\Gamma^{IJK}\Gamma^{\dot1\dot2\dot3}\epsilon,\end{aligned} \tag{2.28}$$

$$\delta_\epsilon b_{\dot\mu\dot\nu} = -i(\bar\epsilon\Gamma_{\dot\mu\dot\nu}\Psi), \tag{2.29}$$

$$\delta_\epsilon b_{\mu\dot\nu} = -i\left(1 + g\mathcal{H}_{\dot1\dot2\dot3}\right)\bar\epsilon\Gamma_\mu\Gamma_{\dot\nu}\Psi + ig(\bar\epsilon\Gamma_\mu\Gamma^I\Gamma_{\dot1\dot2\dot3}\Psi)\partial_{\dot\nu}X^I. \tag{2.30}$$

[3] ϵ here was denoted by ϵ' in [2].

The SUSY transformation parameters χ, ϵ can be conveniently denoted as an 11D Majorana spinor satisfying the 6D chirality condition

$$\Gamma^7\chi = -\chi, \qquad \Gamma^7\epsilon = -\epsilon. \tag{2.31}$$

They are both nonlinear SUSY transformations, but a superposition of the two,

$$\delta_\chi + g\delta_\epsilon \qquad \text{and} \qquad \chi = \Gamma^{\dot{1}\dot{2}\dot{3}}\epsilon, \tag{2.32}$$

defines a linear SUSY transformation.

2.4 Double Dimensional Reduction

In order to understand that the NP M5 theory describes the M5-brane in large C field background. One can study the relative superstring theory. This relation between M theory and superstring theory can be done by dimensional reduction. There are several ways of dimensional reductions. One way is just to compactify one target space dimension on circle, then M2-brane and M5-brane in eleven dimensions will relate to D2-brane and NS5-brane in ten dimensions. Another way is to compactify one target space dimension and one worldvolume space dimension on a circle at the same time. It is called Double Dimensional Reduction (DDR). After DDR, the M2-brane and M5-brane in eleven dimensions will relate to F1 string and D4-brane in ten dimensions. These objects (F1, D2, D4, and NS5) are the main elements in ⅡA superstring theory in ten dimensions. Similarly, if we compactify one target space dimension on S^1/Z_2, it will relate to the $E_8 \times E_8$ heterotic superstring theory. In this section, we will focus on the DDR method, then we can study the relative D4-brane action of NP M5 theory.

2.4.1 Poisson D4 Description From Nambu-Poisson M5 Theory

In this subsection, we will re-derive the D4 in large NS-NS B field background from the NP M5 theory. Firstly, we know theory D4-brane theory can be obtained from M5-brane theory after double dimensional reduction on a circle. The double dimensional reduction (DDR) means that we do the dimensional reduction on worldvolume and target space at the same time. In original, people [1, 2] want to show the evidence of the NP M5 theory is the effective description of M5-brane in large C field background. Hence, they expect to get the D4 in large NS-NS B field background after compactification the circle, which live in the direction $y^{\dot{3}}$ and has radius R. There are several reasons for this choice. The first thing is the C field background field $C_{\dot{1}\dot{2}\dot{3}}$ will be explained as $B_{\dot{1}\dot{2}}$ after DDR on $y^{\dot{3}}$. The relation between $C_{\dot{1}\dot{2}\dot{3}}$ and $B_{\dot{1}\dot{2}}$ is written by:

$$\int_{y^{\dot{3}}=0}^{y^{\dot{3}}=2\pi R} C_{\dot{1}\dot{2}\dot{3}} dy^{\dot{1}} dy^{\dot{2}} dy^{\dot{3}} \equiv B_{\dot{1}\dot{2}} dy^{\dot{1}} dy^{\dot{2}}. \tag{2.33}$$

The second thing is the Nambu-Poisson bracket will relate to Poisson bracket by this way: $\{f, g, y^{\dot{3}}\} = \epsilon^{\dot{\alpha}\dot{\beta}\dot{3}} \partial_{\dot{\mu}} f \partial_{\dot{\nu}} g \equiv \{f, g\}_{p.b.}$. Here the indices $\dot{\alpha}$ are $\{\dot{1}, \dot{2}\}$.

After integrating out the auxiliary field ($b_{\mu\dot{\alpha}}$) and renaming some fields, we get[4]:

$$\begin{aligned} S_{D4inB} &= \int d^3x d^2y \Big[-\frac{1}{2}(\hat{D}_a X^I)^2 - \frac{1}{4}(\hat{F}_{ab})^2 - \frac{g^2}{4}\{X^I, X^J\}^2 - \frac{1}{2g^2} \\ &\quad + \frac{i}{2}\Big(\overline{\Psi}'\Gamma^a \hat{D}_a \Psi' + g\overline{\Psi}'\Gamma^I\{X^I, \Psi'\}\Big)\Big], \end{aligned} \tag{2.34}$$

where we use the unitary transformation of fermion $\Psi = \frac{1}{\sqrt{2}}(\Gamma_{\dot{3}} + \Gamma^7)\Psi'$ to keep the chirality condition of gaugino (Ψ') on D4-brane: $\Gamma_{\dot{3}}\Psi' = \Psi'$. The gauge field $b_{\mu\dot{3}}$ and $b_{\dot{\alpha}\dot{3}}$ are understood as the one form gauge field in D4-brane theory after DDR. The gauge field $\hat{a}_a := b_{a\dot{3}}$ can be used to define the covariant derivative and field strength:

$$\delta_\Lambda \hat{a}_a = \partial_a \Lambda - g\{\Lambda, \hat{a}_a\}_{p.b.}, \quad \Lambda \equiv \Lambda_{\dot{3}}, \tag{2.35}$$

$$\hat{F}_{ab} = \partial_a \hat{a}_b - \partial_b \hat{a}_a + g\{\hat{a}_a, \hat{a}_b\}_{p.b.}, \tag{2.36}$$

$$\hat{D}_a \Phi = \partial_a \Phi + g\{\hat{a}_a, \Phi\}_{p.b.}. \tag{2.37}$$

This theory describes the D4-brane in large NS-NS B field background($B_{\dot{1}\dot{2}}$).

In this chapter, we show the main characters of NP M5 theory. We give several evidences of the M5-brane in large C field background. For example, the constant term exists in action, the supersymmetry law is nonlinear, the two form gauge field has non-abelian structure, and it reproduces D4-brane in NS-NS B field background, etc. However, we find another possible D4-brane formalism, which can describe the D4-brane in large C field background. It can be achieved by DDR on another circle x^2. We will deal with it in next chapter.

[4]Here the indices 'a' are $\{\mu; \dot{\alpha}\}$.

Chapter 3

D4 in R-R Three Form Background

In this chapter, we will start to consider the effective action of D4-brane in the large three-form background. This is motivated from NP M5 theory, which describes the single M5-brane in large C field background. If we do double dimensional reduction along the codimension of C field. We will get the effective description of D4-brane in large C field background.

3.1 D4-Brane in C Field Background via DDR

To carry out the double dimensional reduction (DDR) for the M5-brane along the x^2-direction, we set

$$x^2 \sim x^2 + 2\pi R, \tag{3.1}$$

and let all other fields to be independent of x^2. As a result we can set ∂_2 to zero when it acts on any field. Here R is the radius of the circle of compactification and we should take $R \ll 1$ such that the 6 dimensional field theory on M5 reduces to a 5 dimensional field theory for D4. To keep zero mode of fields in x^2 direction, we need to explain the meaning of field with component 2. For example, the $b_{\mu\dot{\mu}} \to \{b_{2\dot{\mu}}, b_{\alpha\dot{\mu}}\}$, where $\alpha = 0, 1$ and the field $b_{2\dot{\mu}}$ is understood by one form field on D4-brane theory. Hence, we define

$$b_{\dot{\mu}2} \equiv a_{\dot{\mu}}. \tag{3.2}$$

On the other hand, the Gamma matrix Γ^2 is understood by ten dimensional chirality matrix. It is used to define the chirality condition of fermion (gaugino) in D4-brane theory.

3.1.1 Gauge Transformation of Fields

As what we have mentioned, the fields after DDR are $X^I, \Psi, b_{\alpha\dot{\mu}}, b_{2\dot{\mu}}$, and $b^{\dot{\mu}}$. After DDR, the gauge transformation of fields are:

$$\begin{aligned}
\delta_\Lambda \Phi &= g\kappa^{\dot{\rho}}\partial_{\dot{\rho}}\Phi \qquad (\Phi = X^I, \Psi), &(3.3)\\
\delta_\Lambda b_{\alpha\dot{\sigma}} &= \partial_\alpha \Lambda_{\dot{\sigma}} - \partial_{\dot{\sigma}}\Lambda_\alpha + g\kappa^{\dot{\tau}}\partial_{\dot{\tau}} b_{\alpha\dot{\sigma}} + g(\partial_{\dot{\sigma}}\kappa^{\dot{\tau}})b_{\alpha\dot{\tau}}, &(3.4)\\
\delta_\Lambda b_{2\dot{\sigma}} &= -\partial_{\dot{\sigma}}\Lambda_2 + g\kappa^{\dot{\tau}}\partial_{\dot{\tau}} b_{2\dot{\sigma}} + g(\partial_{\dot{\sigma}}\kappa^{\dot{\tau}})b_{2\dot{\tau}}, &(3.5)\\
\delta_\Lambda b^{\dot{\mu}} &= \kappa^{\dot{\mu}} + g\kappa^{\dot{\nu}}\partial_{\dot{\nu}} b^{\dot{\mu}}. &(3.6)
\end{aligned}$$

We expect that the $U(1)$ gauge symmetry on the D4-brane has its origin in the gauge transformations (2.16), (2.17) on the M5-brane. The gauge transformation parameter Λ_2 shall be identified with the $U(1)$ gauge transformation parameter. This is consistent with the identification of $a_{\dot{\mu}}$ with $b_{\dot{\mu}2}$. The gauge symmetry parametrized by $\Lambda_{\dot{\mu}}$, i.e., the VPD, is also still present on the D4-brane. Hence, we can have the gauge transformation of $a_{\dot{\mu}}$:

$$\delta_\Lambda a_{\dot{\mu}} = \partial_{\dot{\mu}}\lambda + g(\kappa^{\dot{\nu}}\partial_{\dot{\nu}} a_{\dot{\mu}} + a_{\dot{\nu}}\partial_{\dot{\mu}}\kappa^{\dot{\nu}}). \tag{3.7}$$

The gauge symmetry combines U(1) gauge symmetry and volume-preserving diffeomorphism symmetry. This is the first new character of the new D4 theory. The 3-dimensional volume-preserving diffeomorphism is the evidence of D4 in large C-field background. We want to ask how to find the other DOF of one form fields (a_α), and we also want to know how to find the gauge transformation law of a_α. We will deal with it in next section.

3.1.2 Action

After keeping the zero mode of fields in x^2 direction, we get the effective description of five dimensions worldvolume theory. The action is what we expect for the new D4-brane action, which describe the effective action of D4-brane in large C-field background. The complete action form can be represented in different parts. The result of DDR on S_{gauge} is

$$\begin{aligned}
S^{(1)}_{gauge} = &\int d^2x d^3y \Big\{ -\frac{1}{2}\mathcal{H}^2_{\dot{1}\dot{2}\dot{3}} - \frac{1}{4}\mathcal{H}^2_{2\dot{\mu}\dot{\nu}} - \frac{1}{4}\mathcal{H}^2_{\alpha\dot{\mu}\dot{\nu}} \\
&+ \epsilon^{\alpha\beta}\epsilon^{\dot{\mu}\dot{\nu}\dot{\rho}}\partial_\beta a_{\dot{\rho}}\partial_{\dot{\mu}} b_{\alpha\dot{\nu}} + \frac{g}{2}\epsilon^{\alpha\beta}\epsilon_{\dot{\mu}\dot{\nu}\dot{\rho}}\epsilon^{\dot{\mu}\dot{\delta}\dot{\tau}}\epsilon^{\dot{\nu}\dot{\sigma}\dot{\lambda}}\epsilon^{\dot{\rho}\dot{\eta}\dot{\xi}}\partial_{\dot{\delta}} b_{\alpha\dot{\tau}}\partial_{\dot{\sigma}} b_{\beta\dot{\lambda}}\partial_{\dot{\eta}} a_{\dot{\xi}} \Big\},
\end{aligned} \tag{3.8}$$

where we use the definition of $\epsilon^{\alpha\beta 2} \equiv \epsilon^{\alpha\beta}$. The result of DDR on S_X is

$$
\begin{aligned}
S_X^{(1)} \;=\; \int d^2x d^3y \Big\{ & -\frac{1}{2}\mathcal{D}_{\dot{\mu}} X^I \mathcal{D}^{\dot{\mu}} X^I - \frac{1}{2}\partial_\alpha X^I \partial^\alpha X^I + g B_\alpha{}^{\dot{\mu}} \partial_{\dot{\mu}} X^I \partial^\alpha X^I \\
& -\frac{g^2}{2} B_\alpha{}^{\dot{\mu}} B^\alpha{}_{\dot{\nu}} \partial_{\dot{\mu}} X^I \partial^{\dot{\nu}} X^I - \frac{g^2}{8}\epsilon^{\dot{\mu}\dot{\rho}\dot{\tau}}\epsilon_{\dot{\nu}\dot{\sigma}\dot{\delta}} F_{\dot{\rho}\dot{\tau}} F^{\dot{\sigma}\dot{\delta}} \partial_{\dot{\mu}} X^I \partial^{\dot{\nu}} X^I \\
& -\frac{1}{2g^2} - \frac{g^4}{4}\{X^{\dot{\mu}}, X^I, X^J\}^2 - \frac{g^4}{12}\{X^I, X^J, X^K\}^2 \Big\}.
\end{aligned} \tag{3.9}
$$

The result of DDR on S_Ψ is

$$
\begin{aligned}
S_\Psi^{(1)} \;=\; \int d^2x d^3y \Big\{ & \frac{i}{2}\bar{\Psi}\Gamma^\alpha \partial_\alpha \Psi + \frac{i}{2}\bar{\Psi}\Gamma^{\dot{\rho}} \mathcal{D}_{\dot{\rho}} \Psi + g\frac{i}{4}\bar{\Psi}\Gamma^2 \epsilon^{\dot{\mu}\dot{\nu}\dot{\rho}} F_{\dot{\nu}\dot{\rho}} \partial_{\dot{\mu}} \Psi - g\frac{i}{2}\bar{\Psi}\Gamma^\alpha B_\alpha{}^{\dot{\mu}} \partial_{\dot{\mu}} \Psi \\
& + g^2 \frac{i}{2}\bar{\Psi}\Gamma_{\dot{\mu}}\Gamma^I \{X^{\dot{\mu}}, X^I, \Psi\} - g^2 \frac{i}{4}\bar{\Psi}\Gamma^{IJ}\Gamma_{\dot{1}\dot{2}\dot{3}}\{X^I, X^J, \Psi\} \Big\}.
\end{aligned} \tag{3.10}
$$

In this chapter, we will focus on the gauge field part. To understand if the gauge part has a well description of D4 in large C-field background will teach us how to deal with matter fields part. After turning off the mater fields, we only need to consider the equation (3.8). Focus on the action of gauge fields after DDR, we identify $a_{\dot{\mu}}$ as components of the one-form potential on the D4-brane. In terms of the field strength

$$
F_{\dot{\mu}\dot{\nu}} \equiv \partial_{\dot{\mu}} a_{\dot{\nu}} - \partial_{\dot{\nu}} a_{\dot{\mu}}, \tag{3.11}
$$

we can rewrite $\mathcal{H}_{2\dot{\mu}\dot{\nu}}$ as

$$
\mathcal{H}_{2\dot{\mu}\dot{\nu}} = F_{\dot{\mu}\dot{\nu}} + \frac{g}{2}\epsilon_{\dot{\mu}\dot{\nu}\dot{\lambda}}\epsilon^{\dot{\sigma}\dot{\rho}\dot{\tau}}\partial_{\dot{\sigma}} b^{\dot{\lambda}} F_{\dot{\rho}\dot{\tau}}. \tag{3.12}
$$

In the above we see that part of the two-form potential b on the M5-brane transforms into part of the one-form potential a on D4. However, in order to interpret this action as a D4-brane action, we still need to identify the rest of the components a_α of the one-form gauge potential, and to re-interpret $b_{\alpha\dot{\mu}}$ and $b_{\dot{\mu}\dot{\nu}}$ from the D4-brane viewpoint. We also need to find all components of field strength or find all covariant variables in this theory. On the other hand, we also need to understand the new D4 action in usually D4 viewpoint. We will deal with these problems in different sections.

3.2 Dual Transformation

In this section, we use the method which is called dual transformation to find the other components of one form fields (a_α). This one form component is not suddenly adding into the theory. In fact, this method relates the degree of freedom of $b_{\alpha\dot{\mu}}$ to this one form field a_α. It also can be understood as the electric-magnetic duality in 3-dimensional spaces ($y^{\dot{\mu}}$). This is the dual description between the one form $(b_\alpha)_{\dot{\mu}}$ and the zero form (a_α). We will see this fact in this section.

3.2.1 Equivalent Dual Action and Dual One Form Field

In order to understand the physical meaning of the action (3.8), we try to simplify the action by integrating out the remaining components of the 2-form gauge field b as much as possible, since there is no 2-form gauge potential in the usual description of a D4-brane.

First we note that the action (3.8) depends on $b_{\alpha\dot{\mu}}$ only through the variable $B_\alpha{}^{\dot{\mu}}$ (2.20). In terms of $B_\alpha{}^{\dot{\mu}}$, we have

$$\mathcal{H}_{\alpha\dot{\mu}\dot{\nu}} = \epsilon_{\dot{\mu}\dot{\nu}\dot{\lambda}}(\partial_\alpha b^{\dot{\lambda}} - V_{\dot{\sigma}}{}^{\dot{\lambda}} B_\alpha{}^{\dot{\sigma}}), \tag{3.13}$$

where

$$V_{\dot{\nu}}{}^{\dot{\mu}} \equiv \delta_{\dot{\nu}}{}^{\dot{\mu}} + g\partial_{\dot{\nu}} b^{\dot{\mu}}. \tag{3.14}$$

Hence we can rewrite the action (3.8) as

$$\begin{aligned} S^{(2)}[b^{\dot{\mu}}, a_{\dot{\mu}}, B_\alpha{}^{\dot{\mu}}] &= \int d^2x d^3y \Big\{ -\frac{1}{2}\mathcal{H}^2_{\dot{1}\dot{2}\dot{3}} - \frac{1}{4}\mathcal{H}^2_{2\dot{\mu}\dot{\nu}} \\ &\quad -\frac{1}{2}(\partial_\alpha b^{\dot{\mu}} - V_{\dot{\sigma}}{}^{\dot{\mu}} B_\alpha{}^{\dot{\sigma}})^2 + \epsilon^{\alpha\beta}\partial_\beta a_{\dot{\mu}} B_\alpha{}^{\dot{\mu}} + \frac{g}{2}\epsilon^{\alpha\beta} F_{\dot{\mu}\dot{\nu}} B_\alpha{}^{\dot{\mu}} B_\beta{}^{\dot{\nu}} \Big\}. \end{aligned} \tag{3.15}$$

It turns out that it is possible to extract the components a_α on the D4-brane by dualizing the field $B_\alpha{}^{\dot{\mu}}$. We can introduce the Lagrange multiplier $f_{\alpha\dot{\mu}}$ to rewrite the action (3.15) as

$$\begin{aligned} S^{(3)}[b^{\dot{\mu}}, a_{\dot{\mu}}, b_{\alpha\dot{\mu}}, \breve{B}_\alpha{}^{\dot{\mu}}, f_{\beta\dot{\mu}}] &= \int d^2x d^3y \Big\{ -\frac{1}{2}\mathcal{H}^2_{\dot{1}\dot{2}\dot{3}} - \frac{1}{4}\mathcal{H}^2_{2\dot{\mu}\dot{\nu}} - \frac{1}{2}(\partial_\alpha b^{\dot{\mu}} - V_{\dot{\sigma}}{}^{\dot{\mu}} \breve{B}_\alpha{}^{\dot{\sigma}})^2 \\ &\quad + \epsilon^{\alpha\beta}\partial_\beta a_{\dot{\mu}} \breve{B}_\alpha{}^{\dot{\mu}} + \frac{g}{2}\epsilon^{\alpha\beta} F_{\dot{\mu}\dot{\nu}} \breve{B}_\alpha{}^{\dot{\mu}} \breve{B}_\beta{}^{\dot{\nu}} \\ &\quad - \epsilon^{\alpha\beta} f_{\beta\dot{\mu}}[\breve{B}_\alpha{}^{\dot{\mu}} - \epsilon^{\dot{\mu}\dot{\nu}\dot{\rho}}\partial_{\dot{\nu}} b_{\alpha\dot{\rho}}] \Big\}, \end{aligned} \tag{3.16}$$

where we used the notation $\breve{B}$ for a new variable independent of $b_{\alpha\dot{\mu}}$. If we integrate out the Lagrange multiplier $f_{\beta\dot{\mu}}$, we will get $\breve{B}_\alpha{}^{\dot{\mu}} = B_\alpha{}^{\dot{\mu}}$, and the action above reduces back to (3.15).

Instead, we can integrate out $\breve{B}_\alpha{}^{\dot{\mu}}$ and $b_{\alpha\dot{\mu}}$ to dualize the field $B_\alpha{}^{\dot{\mu}}$. First we integrate out $b_{\alpha\dot{\mu}}$, and find the constraint on $f_{\alpha\dot{\mu}}$

$$\epsilon^{\dot{\mu}\dot{\nu}\dot{\lambda}}\partial_{\dot{\mu}} f_{\alpha\dot{\nu}} = 0. \tag{3.17}$$

It implies that, locally

$$f_{\alpha\dot{\mu}} = \partial_{\dot{\mu}} a_\alpha \tag{3.18}$$

for some potential a_α. Hence, after integrating out $b_{\alpha\dot{\mu}}$, we get

$$\begin{aligned} S^{(4)}[b^{\dot{\mu}}, a_{\dot{\mu}}, a_\alpha, \breve{B}_\alpha{}^{\dot{\mu}}] \;&=\; \int d^2x d^3y \Big\{ -\frac{1}{2}\mathcal{H}^2_{\dot{1}\dot{2}\dot{3}} - \frac{1}{4}\mathcal{H}^2_{2\dot{\mu}\dot{\nu}} - \frac{1}{2}(\partial_\alpha b^{\dot{\mu}} - V_{\dot{\sigma}}{}^{\dot{\mu}}\breve{B}_\alpha{}^{\dot{\sigma}})^2 \\ &+\epsilon^{\alpha\beta}\partial_\beta a_{\dot{\mu}}\breve{B}_\alpha{}^{\dot{\mu}} + \frac{g}{2}\epsilon^{\alpha\beta}F_{\dot{\mu}\dot{\nu}}\breve{B}_\alpha{}^{\dot{\mu}}\breve{B}_\beta{}^{\dot{\nu}} - \epsilon^{\alpha\beta}\partial_{\dot{\mu}} a_\beta \breve{B}_\alpha{}^{\dot{\mu}} \Big\}. \end{aligned} \tag{3.19}$$

In order to find the final form of dual action, we should also need to integrate out the $\breve{B}$. We will get the complete form in next subsection.

3.2.2 Action after Dual Transformation

Since the action is at most quadratic in $\breve{B}_\alpha{}^{\dot{\mu}}$, the result of integrating out $\breve{B}_\alpha{}^{\dot{\mu}}$ is the same as replacing $\breve{B}_\alpha{}^{\dot{\mu}}$ by the solution to its equation of motion, which is a constraint

$$V_{\dot{\mu}}{}^{\dot{\nu}}(\partial^\alpha b_{\dot{\nu}} - V^{\dot{\rho}}_{\dot{\nu}}\breve{B}^\alpha{}_{\dot{\rho}}) + \epsilon^{\alpha\beta}F_{\beta\dot{\mu}} + g\epsilon^{\alpha\beta}F_{\dot{\mu}\dot{\nu}}\breve{B}_\beta{}^{\dot{\nu}} = 0. \tag{3.20}$$

The solution of $\breve{B}_\alpha{}^{\dot{\mu}}$, denoted as $\hat{B}_\alpha{}^{\dot{\mu}}$, is given by

$$\hat{B}_\alpha{}^{\dot{\mu}} \equiv (M^{-1})_{\alpha\beta}{}^{\dot{\mu}\dot{\nu}}(V_{\dot{\nu}}{}^{\dot{\sigma}}\partial^\beta b_{\dot{\sigma}} + \epsilon^{\beta\gamma}F_{\gamma\dot{\nu}}), \tag{3.21}$$

where

$$M_{\dot{\mu}\dot{\nu}}{}^{\alpha\beta} \equiv V_{\dot{\mu}\dot{\rho}}V_{\dot{\nu}}{}^{\dot{\rho}}\delta^{\alpha\beta} - g\epsilon^{\alpha\beta}F_{\dot{\mu}\dot{\nu}}, \tag{3.22}$$

and M^{-1} is defined by

$$(M^{-1})_{\gamma\alpha}{}^{\dot{\lambda}\dot{\mu}}M_{\dot{\mu}\dot{\nu}}{}^{\alpha\beta} = \delta^{\dot{\lambda}}{}_{\dot{\nu}}\delta_\gamma{}^\beta. \tag{3.23}$$

After integrating out $\breve{B}_\alpha{}^{\dot{\mu}}$, we get

$$\begin{aligned} S^{(5)}[b^{\dot{\mu}}, a_{\dot{\mu}}, a_\alpha] \;&=\; \int d^2x d^3y \Big\{ -\frac{1}{2}\mathcal{H}^2_{\dot{1}\dot{2}\dot{3}} - \frac{1}{4}(F_{\dot{\nu}\dot{\rho}} + \frac{g}{2}\epsilon_{\dot{\mu}\dot{\nu}\dot{\rho}}\epsilon^{\dot{\sigma}\dot{\delta}\dot{\tau}}\partial_{\dot{\sigma}}b^{\dot{\mu}}F_{\dot{\delta}\dot{\tau}})^2 - \frac{1}{2}\partial_\alpha b^{\dot{\mu}}\partial^\alpha b_{\dot{\mu}} \\ &+\frac{1}{2}(\epsilon^{\alpha\gamma}F_{\gamma\dot{\mu}} + V_{\dot{\mu}}{}^{\dot{\sigma}}\partial^\alpha b_{\dot{\sigma}})(M^{-1})_{\alpha\beta}{}^{\dot{\mu}\dot{\nu}}(\epsilon^{\beta\delta}F_{\delta\dot{\nu}} + V_{\dot{\nu}}{}^{\dot{\lambda}}\partial^\beta b_{\dot{\lambda}}) \Big\}. \end{aligned} \tag{3.24}$$

At the quantum level, there is a one-loop contribution to the action when we integrate out $\breve{B}_\alpha{}^{\dot{\mu}}$. It is

$$\Delta S_{1-loop} = -\frac{\hbar}{2}Tr(Log(M_{\dot{\mu}\dot{\nu}}{}^{\alpha\beta})). \tag{3.25}$$

The action (3.24) is only remotely resembling the familiar Maxwell action for a $U(1)$ gauge theory we expect on the D4-brane. We can find terms resembling $F^2_{\dot{\mu}\dot{\nu}}$ and $F^2_{\alpha\dot{\mu}}$, but the coefficients do not match. The term $F_{\alpha\beta}^2$ is missing. We still have the field $b^{\dot{\mu}}$ which can not be easily integrated out because it has 2nd derivative terms in the action. It appears that we need to keep the field $b^{\dot{\mu}}$, which continues to play the role of the gauge potential for the gauge transformation parametrized by $\Lambda_{\dot{\mu}}$, but we need to identify its physical degrees of freedom in the D4-brane theory.

Having decided to keep the gauge transformations parametrized by $\Lambda_{\dot{\mu}}$ as a new gauge symmetry in the D4-brane theory, we need to define covariant field strengths suitable for the gauge transformations.

3.3 Covariant Variables

In this section, we want to search the covariant field strengths in this new D4 theory. First of all, we need to understand the gauge transformation of all gauge fields, then we can find out what kinds of variables transform covariantly.

3.3.1 Gauge Symmetry after Dual Transformation

The field a_α was introduced by hand and so its gauge transformation rule has to be solved from the requirement that the action $S^{(4)}$ (3.19) to be invariant. For a quick derivation one needs to realize that the Chern-Simons term must be gauge invariant by itself. Plugging in the gauge transformation of $\breve{B}_\alpha{}^{\dot{\mu}}$ [1] and $b^{\dot{\mu}}$, the gauge transformation of the CS term (after integration by part) is

$$
\begin{aligned}
&\delta_\Lambda(\epsilon^{\alpha\beta}\partial_\beta a_{\dot{\mu}}\breve{B}_\alpha{}^{\dot{\mu}} + \frac{g}{2}\epsilon^{\alpha\beta}F_{\dot{\mu}\dot{\nu}}\breve{B}_\alpha{}^{\dot{\mu}}\breve{B}_\beta{}^{\dot{\nu}} - \epsilon^{\alpha\beta}\partial_{\dot{\mu}}a_\beta\breve{B}_\alpha{}^{\dot{\mu}}) \\
= \;& \partial_{\dot{\mu}}\breve{B}_\alpha{}^{\dot{\mu}}\epsilon^{\alpha\beta}[-\partial_\beta\lambda - g(\kappa^{\dot{\sigma}}\partial_{\dot{\sigma}}a_\beta + a_{\dot{\sigma}}\partial_\beta\kappa^{\dot{\sigma}}) + \delta a_\beta]. \qquad (3.26)
\end{aligned}
$$

Hence we get

$$\delta_\Lambda a_\beta = \partial_\beta\lambda + g(\kappa^{\dot{\sigma}}\partial_{\dot{\sigma}}a_\beta + a_{\dot{\sigma}}\partial_\beta\kappa^{\dot{\sigma}}). \qquad (3.27)$$

In our formulation of the self dual gauge field b, the components $b_{\mu\nu}$ do not explicitly show up in the action. Rather they appear when we solve the equations of motion for the rest of the components $b_{\dot{\mu}\dot{\nu}}$ and $b_{\mu\dot{\mu}}$. In [44,45], the components $b_{\mu\nu}$ are used to explicitly exhibit the self duality of the gauge field, and their gauge transformation laws are given by

$$\delta_\Lambda b_{\mu\nu} = \partial_\mu\Lambda_\nu - \partial_\nu\Lambda_\mu + g[\kappa^{\dot{\rho}}(\partial_{\dot{\rho}}b_{\mu\nu}) + (\partial_\nu\kappa^{\dot{\rho}})b_{\mu\dot{\rho}} - (\partial_\mu\kappa^{\dot{\rho}})b_{\nu\dot{\rho}}]. \qquad (3.28)$$

[1] The gauge transformation of $\breve{B}_\alpha{}^{\dot{\mu}}$ should be the same as that of $B_\alpha{}^{\dot{\mu}}$.

Identifying $b_{\beta 2}$ with a_β and setting $\partial_2 = 0$ for DDR, we get exactly the same gauge transformation rule as (3.27) with $\Lambda_2 = \lambda$.

We find that the gauge transformation of $a_{\dot\mu}$ (3.7) and that of a_α (3.27) are of the same form

$$\delta_\Lambda a_A = \partial_A \lambda + g(\kappa^{\dot\nu}\partial_{\dot\nu} a_A + a_{\dot\nu}\partial_A \kappa^{\dot\nu}). \tag{3.29}$$

For the convenience of the reader, let us also give here the gauge transformation of $V_{\dot\nu}{}^{\dot\mu}$, $M_{\dot\mu\dot\nu}{}^{\alpha\beta}$ and $\hat{B}_\alpha{}^{\dot\mu}$:

$$\begin{aligned}
\delta_\Lambda V_{\dot\nu}{}^{\dot\mu} &= g\kappa^{\dot\lambda}\partial_{\dot\lambda} V_{\dot\nu}{}^{\dot\mu} + g(\partial_{\dot\nu}\kappa^{\dot\lambda})V_{\dot\lambda}{}^{\dot\mu}, &(3.30)\\
\delta_\Lambda M_{\dot\mu\dot\nu}{}^{\alpha\beta} &= g[\kappa^{\dot\sigma}\partial_{\dot\sigma} M_{\dot\mu\dot\nu}{}^{\alpha\beta} + (\partial_{\dot\mu}\kappa^{\dot\sigma})M_{\dot\sigma\dot\nu}{}^{\alpha\beta} + (\partial_{\dot\nu}\kappa^{\dot\sigma})M_{\dot\mu\dot\sigma}{}^{\alpha\beta}], &(3.31)\\
\delta_\Lambda \hat{B}_\alpha{}^{\dot\mu} &= \partial_\alpha\kappa^{\dot\mu} + g(\kappa^{\dot\nu}\partial_{\dot\nu}\hat{B}_\alpha{}^{\dot\mu} - \hat{B}_\alpha{}^{\dot\nu}\partial_{\dot\nu}\kappa^{\dot\mu}). &(3.32)
\end{aligned}$$

3.3.2 Covariant Variable with U(1) and VPD Symmetry

In the original NP M5-brane theory, we have the covariant field strengths [2]

$$\begin{aligned}
\mathcal{H}_{\dot1\dot2\dot3} &= \partial_{\dot\mu} b^{\dot\mu} + \frac{1}{2} g(\partial_{\dot\nu} b^{\dot\nu}\partial_{\dot\rho} b^{\dot\rho} - \partial_{\dot\nu} b^{\dot\rho}\partial_{\dot\rho} b^{\dot\nu}) + g^2\{b^{\dot1}, b^{\dot2}, b^{\dot3}\}, &(3.33)\\
\mathcal{F}_{\dot\mu\dot\nu} &\equiv \mathcal{H}_{2\dot\mu\dot\nu} = F_{\dot\mu\dot\nu} + g[\partial_{\dot\sigma} b^{\dot\sigma} F_{\dot\mu\dot\nu} - \partial_{\dot\mu} b^{\dot\sigma} F_{\dot\sigma\dot\nu} - \partial_{\dot\nu} b^{\dot\sigma} F_{\dot\mu\dot\sigma}], &(3.34)
\end{aligned}$$

which survive the DDR. Here we have also rewritten $\mathcal{H}_{2\dot\mu\dot\nu}$, which was given above in (3.12), in a different but equivalent form.

The covariant version of $F_{\alpha\dot\mu}$ can be defined as

$$\mathcal{F}_{\alpha\dot\mu} \equiv \frac{1}{2}\epsilon_{\beta\alpha}\epsilon_{\dot\mu\dot\nu\dot\lambda}\mathcal{H}^{\beta\dot\nu\dot\lambda}. \tag{3.35}$$

This is motivated by the intuition that $\mathcal{F}_{\alpha\dot\mu}$ corresponds to $\mathcal{H}_{\alpha\dot\mu 2}$ in the M5-brane theory, and we used the self duality condition of $\mathcal{H}$ to write down the expression above. Replacing $B_\alpha{}^{\dot\mu}$ by the solution $\hat{B}_\alpha{}^{\dot\mu}$, we can rewrite $\mathcal{H}^{\beta\dot\nu\dot\lambda}$ (3.13) as a function of F_{AB}, $\partial_{\dot\mu} b^{\dot\nu}$ and $\hat{B}_\alpha{}^{\dot\mu}$. (That is, we avoided using $\partial_\alpha b^{\dot\mu}$ directly. The dependence on $\partial_\alpha b^{\dot\mu}$ only appears through $\hat{B}_\alpha{}^{\dot\mu}$.) As a result, we have

$$\mathcal{F}_{\alpha\dot\mu} = V^{-1}{}_{\dot\mu}{}^{\dot\nu}(F_{\alpha\dot\nu} + gF_{\dot\nu\dot\sigma}\hat{B}_\alpha{}^{\dot\sigma}). \tag{3.36}$$

This is also in agreement with the definition of $\mathcal{H}_{\mu\nu\dot\mu}$ defined in [44, 45].

[2] A field $\hat\Phi$ is covariant if its gauge transformation is $\delta_\Lambda\hat\Phi = g\kappa^{\dot\mu}\partial_{\dot\mu}\hat\Phi$.

By inspection, we can guess the covariant form of $F_{\alpha\beta}$. Together with the rest of the covariant field strengths of the $U(1)$ gauge field, we have

$$\begin{aligned}
\mathcal{F}_{\dot{\mu}\dot{\nu}} &= F_{\dot{\mu}\dot{\nu}} + g[\partial_{\dot{\sigma}} b^{\dot{\sigma}} F_{\dot{\mu}\dot{\nu}} - \partial_{\dot{\mu}} b^{\dot{\sigma}} F_{\dot{\sigma}\dot{\nu}} - \partial_{\dot{\nu}} b^{\dot{\sigma}} F_{\dot{\mu}\dot{\sigma}}] \\
&= V_{\dot{\rho}}{}^{\dot{\rho}} F_{\dot{\mu}\dot{\nu}} + V_{\dot{\mu}}{}^{\dot{\rho}} F_{\dot{\nu}\dot{\rho}} + V_{\dot{\nu}}{}^{\dot{\rho}} F_{\dot{\rho}\dot{\mu}}, && (3.37) \\
\mathcal{F}_{\alpha\dot{\mu}} &= V^{-1}{}_{\dot{\mu}}{}^{\dot{\nu}} (F_{\alpha\dot{\nu}} + g F_{\dot{\nu}\dot{\delta}} \hat{B}_{\alpha}{}^{\dot{\delta}}), && (3.38) \\
\mathcal{F}_{\alpha\beta} &= F_{\alpha\beta} + g[-F_{\alpha\dot{\mu}} \hat{B}_{\beta}{}^{\dot{\mu}} - F_{\dot{\mu}\beta} \hat{B}_{\alpha}{}^{\dot{\mu}} + g F_{\dot{\mu}\dot{\nu}} \hat{B}_{\alpha}{}^{\dot{\mu}} \hat{B}_{\beta}{}^{\dot{\nu}}], && (3.39)
\end{aligned}$$

where

$$F_{AB} \equiv \partial_A a_B - \partial_B a_A. \qquad (3.40)$$

Unlike $\mathcal{F}_{\dot{\mu}\dot{\nu}}$ and $\mathcal{F}_{\alpha\dot{\mu}}$, the components $\mathcal{F}_{\alpha\beta}$ can not be directly matched with the field $\mathcal{H}_{\alpha\beta 2}$ in the M5-brane theory, because the latter involves other fields that does not exist in the D4-brane theory.

3.3.3 Action with Covariant Variables

Remarkably, in terms of the covariant field strengths, the action is simply

$$S'_{gauge}[b^{\dot{\mu}}, a_A] = \int d^2x d^3y \left\{ -\frac{1}{2}\mathcal{H}_{\dot{1}\dot{2}\dot{3}}\mathcal{H}^{\dot{1}\dot{2}\dot{3}} - \frac{1}{4}\mathcal{F}_{\dot{\nu}\dot{\rho}}\mathcal{F}^{\dot{\nu}\dot{\rho}} + \frac{1}{2}\mathcal{F}_{\beta\dot{\mu}}\mathcal{F}^{\beta\dot{\mu}} + \frac{1}{2g}\epsilon^{\alpha\beta}\mathcal{F}_{\alpha\beta} \right\}. \quad (3.41)$$

The last term in the Lagrangian resembles the Wess-Zumino term for the C-field.

It appears that we are missing the kinetic term $\mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta}$ in the Lagrangian, and the coefficient of the term $\mathcal{F}_{\alpha\dot{\mu}}\mathcal{F}^{\alpha\dot{\mu}}$ is wrong. However, in the next section we will see that the missing kinetic term arises when we integrate out $b^{\dot{\mu}}$.

3.4 Order Expansion Analysis

In this section, we want to give the detail analysis of the action of D4-brane in large C field background. We will expand the action in different g order. From these calculations, we will see the behavior of the new action, and understand the meaning of $b^{\dot{\mu}}$ in this action. We will find the degree of freedom of $b^{\dot{\mu}}$ is not really independent, it can be understood as the electric-magnetic dual transformation of a_α in worldvolume viewpoint.

3.4.1 Zeroth Order Expansion

In this subsection we show that at the lowest order of g, the D4-brane action (3.41) agrees with the Maxwell action for a $U(1)$ gauge field in the ordinary D4-brane action. First we expand everything to the 1st order

$$
\begin{aligned}
\mathcal{H}_{\dot{1}\dot{2}\dot{3}} &= \partial_{\dot{\mu}} b^{\dot{\mu}} + g\frac{1}{2}(\partial_{\dot{\nu}} b^{\dot{\nu}} \partial_{\dot{\rho}} b^{\dot{\rho}} - \partial_{\dot{\nu}} b^{\dot{\rho}} \partial_{\dot{\rho}} b^{\dot{\nu}}) + \mathcal{O}(g^2), && (3.42)\\
V^{-1}{}_{\dot{\mu}}{}^{\dot{\nu}} &= \delta_{\dot{\mu}}{}^{\dot{\nu}} - g\partial_{\dot{\mu}} b^{\dot{\nu}} + O(g^2), && (3.43)\\
(M^{-1})^{\dot{\mu}\dot{\nu}}{}_{\alpha\beta} &= \delta^{\dot{\mu}\dot{\nu}}\delta_{\alpha\beta} - g[(\partial^{\dot{\mu}} b^{\dot{\nu}} + \partial^{\dot{\nu}} b^{\dot{\mu}})\delta_{\alpha\beta} - \epsilon_{\alpha\beta}F^{\dot{\mu}\dot{\nu}}] + O(g^2), && (3.44)\\
\hat{B}_{\alpha}{}^{\dot{\mu}} &= \partial_\alpha b^{\dot{\mu}} + \epsilon_{\alpha\beta}F^{\beta\dot{\mu}} + g[-\partial^{\dot{\sigma}} b^{\dot{\mu}}\partial_\alpha b_{\dot{\sigma}} - \partial^{\dot{\mu}} b_{\dot{\sigma}}\epsilon_{\alpha\beta}F^{\beta\dot{\sigma}} - \partial_{\dot{\sigma}} b^{\dot{\mu}}\epsilon_{\alpha\beta}F^{\beta\dot{\sigma}} \\
&\quad + \epsilon_{\alpha\beta}\partial^\beta b_{\dot{\sigma}}F^{\dot{\mu}\dot{\sigma}} + F_{\alpha\dot{\sigma}}F^{\dot{\mu}\dot{\sigma}}] + O(g^2). && (3.45)\\
\mathcal{F}_{\beta\dot{\mu}} &= F_{\beta\dot{\mu}} + g(\partial_{\dot{\mu}} b^{\dot{\sigma}}F_{\dot{\sigma}\beta} + \partial_\beta b^{\dot{\sigma}}F_{\dot{\mu}\dot{\sigma}} + \epsilon_{\beta\gamma}F_{\dot{\mu}\dot{\sigma}}F^{\gamma\dot{\sigma}}) + O(g^2) && (3.46)\\
\mathcal{F}_{\alpha\beta} &= F_{\alpha\beta} + g[-F_{\alpha\dot{\mu}}(\partial_\beta b^{\dot{\mu}} + \epsilon_{\beta\gamma}F^{\gamma\dot{\mu}}) - F_{\dot{\mu}\beta}(\partial_\alpha b^{\dot{\mu}} + \epsilon_{\alpha\gamma}F^{\gamma\dot{\mu}})] + O(g^2) && (3.47)
\end{aligned}
$$

To the lowest order of g, the last term in the Lagrangian (3.41) is

$$
\begin{aligned}
\frac{1}{2g}\epsilon^{\alpha\beta}\mathcal{F}_{\alpha\beta} &= \frac{1}{2g}\epsilon^{\alpha\beta}F_{\alpha\beta} + \frac{1}{2}\epsilon^{\alpha\beta}[-F_{\alpha\dot{\mu}}(\partial_\beta b^{\dot{\mu}} + \epsilon_{\beta\gamma}F^{\gamma\dot{\mu}}) - F_{\dot{\mu}\beta}(\partial_\alpha b^{\dot{\mu}} + \epsilon_{\alpha\gamma}F^{\gamma\dot{\mu}})] + \mathcal{O}(g)\\
&\simeq -\epsilon^{\alpha\beta}F_{\alpha\dot{\mu}}\partial_\beta b^{\dot{\mu}} - F_{\alpha\dot{\mu}}F^{\alpha\dot{\mu}} + \mathcal{O}(g)\\
&\simeq \epsilon^{\alpha\beta}\partial_\beta a_\alpha \partial_{\dot{\mu}} b^{\dot{\mu}} - F_{\alpha\dot{\mu}}F^{\alpha\dot{\mu}} + \mathcal{O}(g), && (3.48)
\end{aligned}
$$

up to total derivatives. To the 0-th order of g, the action (3.41) can now be expressed as

$$
\begin{aligned}
S'^{(0)}_{gauge}[b^{\dot{\mu}}, a_A] &= \int d^2x d^3y \left\{ -\frac{1}{2}H^2_{\dot{1}\dot{2}\dot{3}} - \frac{1}{2}\epsilon^{\alpha\beta}F_{\alpha\beta}H_{\dot{1}\dot{2}\dot{3}} - \frac{1}{4}F_{\dot{\mu}\dot{\nu}}F^{\dot{\mu}\dot{\nu}} - \frac{1}{2}F_{\alpha\dot{\mu}}F^{\alpha\dot{\mu}} \right\}\\
&= \int d^2x d^3y \left\{ -\frac{1}{2}(H_{\dot{1}\dot{2}\dot{3}} + F_{01})^2 - \frac{1}{4}F_{AB}F^{AB} \right\}, && (3.49)
\end{aligned}
$$

where $H_{\dot{1}\dot{2}\dot{3}} = \partial_{\dot{\mu}} b^{\dot{\mu}}$ and $A, B = (\dot{\mu}, \alpha)$. Note that $H_{\dot{1}\dot{2}\dot{3}}$ is the only gauge invariant degree of freedom in the gauge potential $b^{\dot{\mu}}$ because there are two independent gauge transformation parameters. [3] Furthermore there is no kinetic term for $b^{\dot{\mu}}$ and so we can integrate it out and then (3.49) becomes exactly the Maxwell action. Integrating out $b^{\dot{\mu}}$ is a duality transformation which imposes the identification

$$
H_{\dot{1}\dot{2}\dot{3}} = -F_{01}. \tag{3.50}
$$

The physical degrees of freedom in $b^{\dot{\mu}}$ is transformed into that of a_α. Although $b^{\dot{\mu}}$ appears as new gauge potentials in the D4-brane theory, they share the same physical degrees of freedom with a_α.

[3] Since the 3 gauge transformation parameters $\kappa^{\dot{\mu}}$ are subject to the condition $\partial_{\dot{\mu}}\kappa^{\dot{\mu}} = 0$, there are only 2 functionally independent degrees of freedom in $\kappa^{\dot{\mu}}$.

3.4.2 First Order Expansion

The first order correction to the action (3.49) is

$$\begin{aligned} S'^{(1)}_{gauge}[b^{\dot{\mu}}, a_A] &= g \int d^2x d^3y \left\{ (-\frac{1}{2} H^2_{\dot{1}\dot{2}\dot{3}} + \frac{1}{2} \partial_{\dot{\mu}} b^{\dot{\nu}} \partial_{\dot{\nu}} b^{\dot{\mu}})(H_{\dot{1}\dot{2}\dot{3}} + F_{01}) \right. \\ &+ H_{\dot{1}\dot{2}\dot{3}} \left(-\frac{1}{2} F_{\dot{\mu}\dot{\nu}} F^{\dot{\mu}\dot{\nu}} + \epsilon^{\alpha\beta} F_{\alpha\dot{\mu}} \partial_\beta b^{\dot{\mu}} \right) - \frac{1}{2} \epsilon_{\alpha\beta} F_{\dot{\mu}\dot{\nu}} F^{\alpha\dot{\mu}} F^{\beta\dot{\nu}} \\ &\left. + F^{\dot{\mu}\dot{\nu}} F_{\dot{\lambda}\dot{\nu}} \partial_{\dot{\mu}} b^{\dot{\lambda}} + F_{\alpha\dot{\mu}} F^{\alpha}{}_{\dot{\nu}} \partial^{\dot{\mu}} b^{\dot{\nu}} - F_{\alpha\dot{\mu}} \partial^\alpha b_{\dot{\nu}} F^{\dot{\mu}\dot{\nu}} \right\}. \end{aligned} \tag{3.51}$$

In order to integrate out $H_{\dot{1}\dot{2}\dot{3}}$, note that we can impose the gauge fixing condition

$$\epsilon^{\dot{\mu}\dot{\nu}\dot{\lambda}} \partial_{\dot{\mu}} b_{\dot{\nu}} = 0, \tag{3.52}$$

so that

$$b_{\dot{\mu}} = \partial_{\dot{\mu}} c \tag{3.53}$$

for some function c. Solving c from

$$H_{\dot{1}\dot{2}\dot{3}} = \partial_{\dot{\mu}} b^{\dot{\mu}}, \tag{3.54}$$

we find

$$b^{\dot{\mu}} = \partial^{\dot{\mu}} \dot{\partial}^{-2} H_{\dot{1}\dot{2}\dot{3}}, \tag{3.55}$$

where $\dot{\partial}^{-2}$ is the inverse operator of the Laplacian $\dot{\partial}^2 \equiv \partial_{\dot{\mu}} \partial^{\dot{\mu}}$. Denoting the Green's function of the Laplacian by G so that

$$\dot{\partial}^2 G(y - y') = \delta^{(3)}(y - y'), \tag{3.56}$$

where y and y' represent the coordinates in the directions $y^{\dot{1}}, y^{\dot{2}}, y^{\dot{3}}$. We have

$$\dot{\partial}^{-2} \phi(y) = \int d^3y' \, G(y - y') \phi(y). \tag{3.57}$$

Plugging (3.55) into the action, we get an action as a functional of $H_{\dot{1}\dot{2}\dot{3}}$ and a_A. To the first order in g, we can integrate out $H_{\dot{1}\dot{2}\dot{3}}$ and the action becomes

$$\begin{aligned} S''_{gauge}[a_A] &= \int d^2x d^3y \left\{ -\frac{1}{4} F_{AB} F^{AB} + g \left[-F_{01} \mathcal{C} - \frac{1}{2} \epsilon_{\alpha\beta} F_{\dot{\mu}\dot{\nu}} F^{\alpha\dot{\mu}} F^{\beta\dot{\nu}} \right. \right. \\ &\left. \left. - F^{\dot{\mu}\dot{\nu}} F_{\dot{\lambda}\dot{\nu}} \partial_{\dot{\mu}} \partial^{\dot{\lambda}} \dot{\partial}^{-2} F_{01} - F_{\alpha\dot{\mu}} F^{\alpha}{}_{\dot{\nu}} \partial^{\dot{\mu}} \partial^{\dot{\nu}} \dot{\partial}^{-2} F_{01} + F_{\alpha\dot{\mu}} F^{\dot{\mu}\dot{\nu}} \partial^\alpha \partial_{\dot{\nu}} \dot{\partial}^{-2} F_{01} \right] \right\}, \end{aligned} \tag{3.58}$$

where

$$\mathcal{C} = -\frac{1}{2} F_{\dot{\mu}\dot{\nu}} F^{\dot{\mu}\dot{\nu}} - \epsilon^{\alpha\beta} F_{\alpha\dot{\mu}} \partial_\beta \partial^{\dot{\mu}} \dot{\partial}^{-2} F_{01}. \tag{3.59}$$

Apparently the action becomes nonlocal at order $\mathcal{O}(g)$.

In principle, using (3.55) to rewrite the action as a functional of a_A and $H_{\dot{1}\dot{2}\dot{3}}$, we can integrate out $H_{\dot{1}\dot{2}\dot{3}}$ to an arbitrary order in g. The resulting action would be a functional of F_{AB} with higher derivatives and $\dot{\partial}^{-2}$.

3.4.3 Electric-Magnetic (EM) duality

In here, we need to emphasize the role of gauge potential of $b^{\dot{\mu}}$. The two form gauge field can be seen as the dual description of one form gauge potential a_α. From the order expansion analyses, we can find the one form field a_α encodes the information of $b_{\alpha\dot{\mu}}$ and $b^{\dot{\mu}}$. The original two form degree in NP M5 theory, after DDR on x^2, the $b_{\dot{\mu}2}$ can be understood by $a_{\dot{\mu}}$. Another two degree of one form field in D4 theory (a_0, a_1) were encoded in other two form fields $b_{\alpha\dot{\mu}}$ and $b^{\dot{\mu}}$. What we do in this chapter will make the result be more manifest. There is an EM duality between $b_{\alpha\dot{\mu}}$ and a_α in the 3-dimensional space $(y^{\dot{\mu}})$. This result can be understood from the calculation of dual transformation. Moreover, when we write down the field strength $\mathcal{F}_{\dot{\mu}\beta}$, we use the solution of equation of motion of $\breve{B}_\alpha{}^{\dot{\mu}}$ $(\hat{B}_\alpha{}^{\dot{\mu}})$. This is the general EM dual relation between $b_{\alpha\dot{\mu}}$ and a_α. The dual transformation always appear in the calculation of M5 after DDR. For example, to connect the D4-brane theory with PST M5 action by DDR, they also need to do dual transformation to get normal D4-brane theory [25]. Finally, after integrating out $\breve{B}_\alpha{}^{\dot{\mu}}$, we still have the two form gauge field $b^{\dot{\mu}}$, this field is also the dual description of one form gauge field a_α in the 5-dimensional worldvolume. This result can be understood from many places. The zeroth order expansion analysis tell us the $\partial_{\dot{\mu}} b^{\dot{\mu}} = -F_{01}$, hence the d.o.f of $b^{\dot{\mu}}$ is relative to d.o.f of a^α. From the relation, we also can know the correct physics degree of freedom of one form gauge field a_A. It can be understood from the E.O.M of a_A and $b^{\dot{\mu}}$. In g^0 order, these equations are simpler:

$$\begin{aligned}
\partial_{\dot{\nu}} F^{\dot{\nu}\dot{\mu}} + \partial_\beta F^{\beta\dot{\mu}} &= 0, &(3.60)\\
\epsilon^{\alpha\beta}\partial_\alpha\partial_{\dot{\mu}} b^{\dot{\mu}} + \partial_{\dot{\mu}} F^{\dot{\mu}\beta} &= 0, &(3.61)\\
\partial_{\dot{\mu}}\partial_{\dot{\nu}} b^{\dot{\nu}} + \frac{1}{2}\epsilon^{\alpha\beta}\partial_{\dot{\mu}} F_{\alpha\beta} &= 0. &(3.62)
\end{aligned}$$

It is easy to simplify them in these two equations:

$$\begin{aligned}
\partial_{\dot{\mu}} b^{\dot{\mu}} &= -\epsilon^{\alpha\beta}\partial_\alpha a_\beta, &(3.63)\\
\partial_A\partial^A a^B &= \partial^B\partial_A a^A. &(3.64)
\end{aligned}$$

There are three independent gauge parameters λ and $\Lambda^{\dot{\mu}}$. We can choose $\Lambda^{\dot{\mu}}$ to make $\epsilon^{\dot{\mu}\dot{\nu}\dot{\rho}}\partial_{\dot{\nu}} b_{\dot{\rho}} = 0$, and we know $\partial_{\dot{\mu}} b^{\dot{\mu}} = -\epsilon^{\alpha\beta}\partial_\alpha a_\beta$, so we fix all degree of freedom of gauge field $b^{\dot{\mu}}$. Now we choose λ to make $\partial_A a^A - \partial_0 a^0 = 0$, then from the second equation, we get $a^0 = 0$. Hence we find we only have 3 independent one form in this g^0 order. For higher g order case, the equation of motion become more complicate, it hard to deal with in this case. However, this result should keep in perturbation theory order by order. Hence, we can fix all d.o.f of $b^{\dot{\mu}}$ field in our action, but this calculation break the VPD gauge symmetry. The complete action of D4 in C field background should be described by one-form gauge field a_A and two-form gauge field $b^{\dot{\mu}}$ at the same time.

Chapter 4

Extension and Application

From the previous chapter, we already show the main part of D4-brane in large R-R 3-form field background. In this chapter, we will do more extension of these results. We focus on several things: the generalization of Dp-brane in large R-R (p-1)-form background, the multiple branes extension, the D4-brane action with matter fields, the supersymmetry transformation law in this D4-brane action, and the possible topological quantities in this theory.

4.1 Dp-Branes in R-R field Background

In fact, the system of D-branes in NS-NS background can be understood from the effective theory, which describes the low energy effect of open string ending on D-branes. The open string is a one-dimensional extended object which can couple to the 2-form NS-NS B field, hence the effective theory is affected by NS-NS B field. On the other hand, the system of D4-brane in R-R 3-form background, which can be understood as the system of D2 ending on D4. The D2-brane can couple to the 3-form R-R C field, hence the effective theory is affected by R-R C field. In general, we can study the Dp-brane in R-R (p-1)-form background. The D(p-2)-brane can end on Dp-brane, which can couple to R-R (p-1)-form field. We expect these theories still with (p-1) dimensions VPD. We also study how to generalize our previous researches to multiple Dp-branes cases. In this thesis, we only consider these extensions in gauge fields, because the gauge parts are easily generalized to multiple Dp-branes from general VPD symmetry.

4.1.1 Generalize VPD in R-R (p-1) Form Field Background

To generalize the story about a single D4-brane to a system of multiple Dp-branes, we notice first that the VPD for a volume $(p-1)$-form is generated by a $(p-1)$-bracket

$$\{f_1, f_2, \cdots, f_{p-1}\} \equiv \epsilon^{\dot{\mu}_1\dot{\mu}_2\cdots\dot{\mu}_{p-1}}\partial_{\dot{\mu}_1}f_1\partial_{\dot{\mu}_2}f_2\cdots\partial_{\dot{\mu}_{p-1}}f_{p-1}. \tag{4.1}$$

We define a $(p-2)$-form gauge potential $b_{\dot{\mu}_1\cdots\dot{\mu}_{p-2}}$ and its dual

$$b^{\dot{\mu}_1} = \frac{1}{(p-2)!}\epsilon^{\dot{\mu}_1\dot{\mu}_2\cdots\dot{\mu}_{p-1}}b_{\dot{\mu}_2\cdots\dot{\mu}_{p-1}}. \tag{4.2}$$

Let

$$X^{\dot{\mu}} = \frac{y^{\dot{\mu}}}{g} + b^{\dot{\mu}} \tag{4.3}$$

and the field strength $\mathcal{H}$ can be defined as

$$\mathcal{H}_{\dot{\mu}_1\dot{\mu}_2\cdots\dot{\mu}_{p-1}} \equiv g^{p-2}\{X^{\dot{\mu}_1}, X^{\dot{\mu}_2}, \cdots, X^{\dot{\mu}_{p-1}}\} - \frac{1}{g} = \partial_{\dot{\mu}}b^{\dot{\mu}} + \mathcal{O}(g). \tag{4.4}$$

In terms of $b^{\dot{\mu}}$ the gauge transformation is exactly of the same form as (3.6), and the parameter $\kappa^{\dot{\mu}}$ is still divergenceless. The only change is that the range of the indices $\dot{\mu}, \dot{\nu}$ becomes $2, 3, \cdots, p$. [1]

4.1.2 Gauge Symmetry and Covariant Variables in Multiple Dp-Branes Theory

While we do not intend to promote the VPD gauge potential $b^{\dot{\mu}}$ to a matrix mostly because we do not know how to modify its gauge transformation law, we shall replace the $U(1)$ potential by a $U(N)$ potential a_A, which is now an $N\times N$ anti-Hermitian matrix of 1-forms. The $U(N)$ gauge transformation of a_A should be defined by

$$\delta a_A = [D_A, \lambda] + g(\kappa^{\dot{\mu}}\partial_{\dot{\mu}}a_A + a_{\dot{\mu}}\partial_A\kappa^{\dot{\mu}}), \tag{4.5}$$

where $D_A \equiv \partial_A + a_A$. It modifies (3.29) only by replacing $\partial_A\lambda$ by $[D_A, \lambda]$. The gauge transformation parameter λ is an $N\times N$ anti-Hermitian matrix but $\kappa^{\dot{\mu}}$ is 1×1. The range of the index A is now $A = 0, 1, 2, \cdots, p$. Decomposing the potential a_A into the $U(1)$ part and the $SU(N)$ part

$$a_A = a_A^{U(1)} + a_A^{SU(N)}, \tag{4.6}$$

[1] The indices $2, 3, \cdots$ would be denoted as $\dot{1}, \dot{2}, \cdots$ in previous chapter.

the gauge transformation of $a_A^{U(1)}$ is exactly the same as before (3.29).

We can define $V_{\dot{\mu}}{}^{\dot{\nu}}$ and $\hat{B}_\alpha{}^{\dot{\mu}}$ using the same expressions (3.14)–(3.21) as before

$$
\begin{aligned}
V_{\dot{\nu}}{}^{\dot{\mu}} &\equiv \delta_{\dot{\nu}}{}^{\dot{\mu}} + g\partial_{\dot{\nu}} b^{\dot{\mu}}, &(4.7)\\
M_{\dot{\mu}\dot{\nu}}{}^{\alpha\beta} &\equiv V_{\dot{\mu}\dot{\rho}} V_{\dot{\nu}}{}^{\dot{\rho}} \delta^{\alpha\beta} - g\epsilon^{\alpha\beta} F^{U(1)}_{\dot{\mu}\dot{\nu}}, &(4.8)\\
\hat{B}_\alpha{}^{\dot{\mu}} &\equiv (M^{-1})^{\dot{\mu}\dot{\nu}}{}_{\alpha\beta}(V_{\dot{\nu}}{}^{\dot{\sigma}}\partial^\beta b_{\dot{\sigma}} + \epsilon^{\beta\gamma} F^{U(1)}_{\gamma\dot{\nu}}), &(4.9)
\end{aligned}
$$

but with the field strength $F^{U(1)}_{\dot{\mu}\dot{\nu}}$ being the $U(1)$ part of the $U(N)$ field strength, so that their gauge transformations remain the same. The range of the indices α, β is still $0, 1$.

The naive definition of field strength $F_{AB} \equiv [D_A, D_B]$ is not covariant. They transform like

$$
\delta F_{AB} = [F_{AB}, \lambda] + g\kappa^{\dot{\mu}}\partial_{\dot{\mu}} F_{AB} + g[(\partial_A \kappa^{\dot{\mu}})F_{\dot{\mu}B} - (\partial_B \kappa^{\dot{\mu}})F_{\dot{\mu}A}]. \tag{4.10}
$$

It turns out that exactly the same expressions as (3.37)–(3.39) give the covariant field strengths. For the convenience of the reader we reproduce them here

$$
\begin{aligned}
\mathcal{F}_{\dot{\mu}\dot{\nu}} &= F_{\dot{\mu}\dot{\nu}} + g[\partial_{\dot{\sigma}} b^{\dot{\sigma}} F_{\dot{\mu}\dot{\nu}} - \partial_{\dot{\mu}} b^{\dot{\sigma}} F_{\dot{\sigma}\dot{\nu}} - \partial_{\dot{\nu}} b^{\dot{\sigma}} F_{\dot{\mu}\dot{\sigma}}]\\
&= V_{\dot{\rho}}{}^{\dot{\rho}} F_{\dot{\mu}\dot{\nu}} + V_{\dot{\mu}}{}^{\dot{\rho}} F_{\dot{\nu}\dot{\rho}} + V_{\dot{\nu}}{}^{\dot{\rho}} F_{\dot{\rho}\dot{\mu}}, &(4.11)\\
\mathcal{F}_{\alpha\dot{\mu}} &= V^{-1}{}_{\dot{\mu}}{}^{\dot{\nu}}(F_{\alpha\dot{\nu}} + gF_{\dot{\nu}\dot{\delta}}\hat{B}_\alpha{}^{\dot{\delta}}), &(4.12)\\
\mathcal{F}_{\alpha\beta} &= F_{\alpha\beta} + g[-F_{\alpha\dot{\mu}}\hat{B}_\beta{}^{\dot{\mu}} - F_{\dot{\mu}\beta}\hat{B}_\alpha{}^{\dot{\mu}} + gF_{\dot{\mu}\dot{\nu}}\hat{B}_\alpha{}^{\dot{\mu}}\hat{B}_\beta{}^{\dot{\nu}}]. &(4.13)
\end{aligned}
$$

They transform like

$$
\delta\mathcal{F}_{AB} = [\mathcal{F}_{AB}, \lambda - g\kappa^{\dot{\mu}}\partial_{\dot{\mu}}]. \tag{4.14}
$$

From this expression it is easy to check that the gauge symmetry algebra is given by

$$
[\delta_1, \delta_2] = \delta_3, \tag{4.15}
$$

where δ_i is the gauge transformation with parameters $\lambda_i, \kappa_i^{\dot{\mu}}$ and

$$
\begin{aligned}
\lambda_3 &= [\lambda_1, \lambda_2] + g(\kappa_2^{\dot{\mu}}\partial_{\dot{\mu}}\lambda_1 - \kappa_1^{\dot{\mu}}\partial_{\dot{\mu}}\lambda_2), &(4.16)\\
\kappa_3^{\dot{\mu}} &= g(\kappa_2^{\dot{\nu}}\partial_{\dot{\nu}}\kappa_1^{\dot{\mu}} - \kappa_1^{\dot{\nu}}\partial_{\dot{\nu}}\kappa_2^{\dot{\mu}}). &(4.17)
\end{aligned}
$$

4.1.3 Ansatz of Action

In view of the D4-brane action (3.41), it is now natural to define the action for the gauge fields on multiple Dp-branes in R-R $(p-1)$-form field background as

$$
\begin{aligned}
S^{Dp}_{gauge}[b^{\dot{\mu}}, a_A] = \int d^2x d^{p-1}y \Big\{ &-\frac{1}{2}\frac{1}{(p-1)!}\mathcal{H}_{\dot{\mu}_1\cdots\dot{\mu}_{p-1}}\mathcal{H}^{\dot{\mu}_1\cdots\dot{\mu}_{p-1}} + \frac{1}{2g}\epsilon^{\alpha\beta}\mathcal{F}^{U(1)}_{\alpha\beta}\\
&-\frac{1}{4}\mathcal{F}^{U(1)}_{\dot{\nu}\dot{\rho}}\mathcal{F}^{\dot{\nu}\dot{\rho}}_{U(1)} + \frac{1}{2}\mathcal{F}^{U(1)}_{\beta\dot{\mu}}\mathcal{F}^{\beta\dot{\mu}}_{U(1)} - \frac{1}{4}\mathrm{tr}\left(\mathcal{F}^{SU(N)}_{AB}\mathcal{F}^{AB}_{SU(N)}\right)\Big\}. \qquad (4.18)
\end{aligned}
$$

If we focus our attention on the $U(1)$ part of the 1-form gauge potential a_A and the VPD gauge potential $b^{\dot{\mu}}$, everything is exactly the same as before. The VPD field strength $\mathcal{H}$ is dual to only the $U(1)$ part of $\mathcal{F}_{01}$. But since the $SU(N)$ part of the field strength $\mathcal{F}_{AB}$ involves the VPD potential $b^{\dot{\mu}}$, the $U(1)$ part of a_A couples to the $SU(N)$ part indirectly through $b^{\dot{\mu}}$. This is different from the usual Yang-Mills theory of $U(N)$ gauge symmetry, for which the $U(1)$ part decouples, but similar to the noncommutative $U(N)$ YM theory.

To the 0-th order in g, the action is

$$S'^{Dp(0)}_{gauge}[b^{\dot{\mu}}, a_A] = \int d^2x d^{p-1}y \left\{ -\frac{1}{2}(H_{23\cdots p} + F_{01}^{U(1)})^2 - \frac{1}{4}F_{AB}^{U(1)} F^{AB}_{U(1)} - \frac{1}{4}\mathrm{tr}\left(F_{AB}^{SU(N)} F^{AB}_{SU(N)} \right) \right\}, \tag{4.19}$$

where $H_{23\cdots p} = \partial_{\dot{\mu}} b^{\dot{\mu}}$. Again, since $H_{23\cdots p}$ is the only component of the field strength for the gauge potential $b^{\dot{\mu}}$, we can integrate it out and the action reduces to that of a Yang-Mills theory in $(p+1)$ dimensions.

In fact, the VPD symmetry allows us to impose the gauge fixing condition

$$\partial^{\dot{\mu}_1} b_{\dot{\mu}_1\dot{\mu}_2\cdots\dot{\mu}_{p-2}} = 0 \qquad \Leftrightarrow \qquad \partial^{\dot{\mu}} b^{\dot{\nu}} - \partial^{\dot{\nu}} b^{\dot{\mu}} = 0. \tag{4.20}$$

This condition allows us to solve $b^{\dot{\mu}}$ in terms of $H_{23\cdots p}$ as

$$b^{\dot{\mu}} = \partial^{\dot{\mu}} \dot{\partial}^{-2} H_{23\cdots p}, \tag{4.21}$$

where $\dot{\partial}^{-2}$ is the inverse of the Laplace operator $\dot{\partial}^2 \equiv \partial_{\dot{\mu}} \partial^{\dot{\mu}}$. Like what we did in Sec. 3.4.2, we can continue to integrate out $H_{23\cdots p}$ at higher orders of g using the relation (4.21) for every term involving $b^{\dot{\mu}}$ in the action, although we would get a nonlocal action in the end. In principle we can write down a nonlocal action of a_A without any trace of $b^{\dot{\mu}}$ or $H_{23\cdots p}$ as an expansion of g to an arbitrary order.

To end this section, we want to emphasize one important thing. The formulation in this section applies to Dp-branes for all $p \geq 2$. For D1-branes, the gauge symmetry introduced by the R-R 0-form (axion) background is the trivial group of diffeomorphism on a single point. There is thus no deformation of the D1-brane theory due to the axion background. Furthermore, as there is no R-R (-1)-form potential, D0-branes are also free from analogous deformations due to R-R backgrounds.

4.2 Couple to Matter fields

In this section, we consider the complete action of NP M5 after DDR on x^2. We use same processes as previous pure gauge fields case. We also can do dual transformation, find covariant variables and solve two form field $\hat{B}_{\alpha}^{\dot{\mu}}$ with matter fields modification. The new thing is the supersymmetry in this case. We should find the way to find the supersymmetry transformation law of the dual field a_α. We will deal with it in next section.

4.2.1 D4 in C Field Background with Matter Fields

In the above we have ignored the matter fields in the NP M5-brane theory. It is straightforward to repeat the derivations above with the matter fields included. To consider the complete action with matter fields: (3.8), (3.9), and (3.10). We do dual transformation in this general case, as the result of action (3.19), then we get

$$\begin{aligned} S^{(4)}[b^{\dot{\mu}}, a_A, \breve{B}_{\alpha}{}^{\dot{\mu}}, X^I, \Psi] \quad = \quad & \int d^2x d^3y \Big\{ -\frac{1}{2}\mathcal{D}_{\dot{\mu}}X^I\mathcal{D}^{\dot{\mu}}X^I - \frac{1}{2}\partial_\alpha X^I\partial^\alpha X^I + g\breve{B}_{\alpha}{}^{\dot{\mu}}\partial_{\dot{\mu}}X^I\partial^\alpha X^I \\ & -\frac{g^2}{2}\breve{B}_{\alpha}{}^{\dot{\mu}}\breve{B}^{\alpha}_{\dot{\nu}}\partial_{\dot{\mu}}X^I\partial^{\dot{\nu}}X^I - \frac{g^2}{8}\epsilon^{\dot{\mu}\dot{\rho}\dot{\tau}}\epsilon_{\dot{\nu}\dot{\sigma}\dot{\delta}}F_{\dot{\rho}\dot{\tau}}F^{\dot{\sigma}\dot{\delta}}\partial_{\dot{\mu}}X^I\partial^{\dot{\nu}}X^I \\ & -\frac{g^4}{4}\{X^{\dot{\mu}}, X^I, X^J\}^2 - \frac{g^4}{12}\{X^I, X^J, X^K\}^2 + \frac{i}{2}\bar{\Psi}\Gamma^{\dot{\rho}}\mathcal{D}_{\dot{\rho}}\Psi \\ & +\frac{i}{2}\bar{\Psi}\Gamma^\alpha\partial_\alpha\Psi + g\frac{i}{4}\bar{\Psi}\Gamma^2\epsilon^{\dot{\mu}\dot{\nu}\dot{\rho}}F_{\dot{\nu}\dot{\rho}}\partial_{\dot{\mu}}\Psi - g\frac{i}{2}\bar{\Psi}\Gamma^\alpha\breve{B}_{\alpha}{}^{\dot{\mu}}\partial_{\dot{\mu}}\Psi \\ & +g^2\frac{i}{2}\bar{\Psi}\Gamma_{\dot{\mu}i}\{X^{\dot{\mu}}, X^I, \Psi\} - g^2\frac{i}{4}\bar{\Psi}\Gamma^{IJ}\Gamma_{\dot{1}\dot{2}\dot{3}}\{X^I, X^J, \Psi\} \\ & -\frac{1}{2g^2} - \frac{1}{2}(\mathcal{H}_{\dot{1}\dot{2}\dot{3}})^2 - \frac{1}{4}\mathcal{F}_{\dot{\nu}\dot{\rho}}\mathcal{F}^{\dot{\nu}\dot{\rho}} - \frac{1}{4}(\epsilon_{\dot{\mu}\dot{\nu}\dot{\rho}}(\partial_\alpha b^{\dot{\mu}} - V_{\dot{\sigma}}{}^{\dot{\mu}}\breve{B}_{\alpha}{}^{\dot{\sigma}}))^2 \\ & +\epsilon^{\alpha\beta}F_{\beta\dot{\mu}}\breve{B}_{\alpha}{}^{\dot{\mu}} + \frac{g}{2}\epsilon^{\alpha\beta}F_{\dot{\mu}\dot{\nu}}\breve{B}_{\alpha}{}^{\dot{\mu}}\breve{B}_{\beta}{}^{\dot{\nu}} \Big\}. \end{aligned} \tag{4.22}$$

With the matter fields included, the action is still no more than quadratic in $\breve{B}_{\alpha}{}^{\dot{\mu}}$ and so we can still integrate it out. This is equivalent to solving the equation of motion for $\breve{B}_{\alpha}{}^{\dot{\mu}}$ and plugging it back into the action. The new equation of motion for $\breve{B}_{\alpha}{}^{\dot{\mu}}$ is

$$V_{\dot{\mu}}{}^{\dot{\nu}}(\partial^\alpha b_{\dot{\nu}} - V^{\dot{\rho}}_{\dot{\nu}}\breve{B}^{\alpha}_{\dot{\rho}}) + \epsilon^{\alpha\beta}F_{\beta\dot{\mu}} + g\epsilon^{\alpha\beta}F_{\dot{\mu}\dot{\nu}}\breve{B}_{\beta}{}^{\dot{\nu}} + g\partial_{\dot{\mu}}X^I\partial^\alpha X^I - g\frac{i}{2}\bar{\Psi}\Gamma^\alpha\partial_{\dot{\mu}}\Psi - g^2\breve{B}^{\alpha}_{\dot{\nu}}\partial_{\dot{\mu}}X^I\partial^{\dot{\nu}}X^I = 0. \tag{4.23}$$

Its solution is

$$\begin{aligned} \hat{B}_{\alpha}{}^{\dot{\mu}} &= (\mathbf{M}^{-1})^{\dot{\mu}\dot{\nu}}{}_{\alpha\beta}(V_{\dot{\nu}}{}^{\dot{\sigma}}\partial^\beta b_{\dot{\sigma}} + \epsilon^{\beta\gamma}F_{\gamma\dot{\nu}} + g\partial_{\dot{\nu}}X^I\partial^\beta X^I - g\frac{i}{2}\bar{\Psi}\Gamma^\beta\partial_{\dot{\nu}}\Psi) \\ &\equiv (\mathbf{M}^{-1})^{\dot{\mu}\dot{\nu}}{}_{\alpha\beta}W^{\beta}_{\dot{\nu}}, \end{aligned} \tag{4.24}$$

where

$$\mathbf{M}_{\dot{\mu}\dot{\nu}}{}^{\alpha\beta} \equiv (V_{\dot{\mu}\dot{\rho}}V_{\dot{\nu}}{}^{\dot{\rho}} + g^2\partial_{\dot{\mu}}X^i\partial_{\dot{\nu}}X^i)\delta^{\alpha\beta} - g\epsilon^{\alpha\beta}F_{\dot{\mu}\dot{\nu}}, \tag{4.25}$$

and $(\mathbf{M}^{-1})^{\dot{\mu}\dot{\nu}}{}_{\alpha\beta}$ is defined by

$$(\mathbf{M}^{-1})^{\dot{\lambda}\dot{\mu}}{}_{\gamma\alpha}\mathbf{M}_{\dot{\mu}\dot{\nu}}{}^{\alpha\beta} = \delta^{\dot{\lambda}}{}_{\dot{\nu}}\delta_{\gamma}{}^{\beta}. \tag{4.26}$$

Finally, we get the action

$$\begin{aligned} S^{(5)}[b^{\dot{\mu}}, a_A, X^I, \Psi] \;=\; & \int d^2x d^3y \Big\{ -\frac{1}{2}\mathcal{D}_{\dot{\mu}}X^I\mathcal{D}^{\dot{\mu}}X^I - \frac{1}{2}\partial_\alpha X^I\partial^\alpha X^I \\ & -\frac{g^2}{8}\epsilon^{\dot{\mu}\dot{\rho}\dot{\tau}}\epsilon_{\dot{\nu}\dot{\sigma}\dot{\delta}}F_{\dot{\rho}\dot{\tau}}F^{\dot{\sigma}\dot{\delta}}\partial_{\dot{\mu}}X^I\partial^{\dot{\nu}}X^I \\ & -\frac{g^4}{4}\{X^{\dot{\mu}}, X^I, X^J\}^2 - \frac{g^4}{12}\{X^I, X^J, X^K\}^2 \\ & +\frac{i}{2}\bar{\Psi}\Gamma^\alpha\partial_\alpha\Psi + \frac{i}{2}\bar{\Psi}\Gamma^{\dot{\rho}}\mathcal{D}_{\dot{\rho}}\Psi + g\frac{i}{4}\bar{\Psi}\Gamma^2\epsilon^{\dot{\mu}\dot{\nu}\dot{\rho}}F_{\dot{\nu}\dot{\rho}}\partial_{\dot{\mu}}\Psi \\ & +g^2\frac{i}{2}\bar{\Psi}\Gamma_{\dot{\mu}}\Gamma^I\{X^{\dot{\mu}}, X^I, \Psi\} - g^2\frac{i}{4}\bar{\Psi}\Gamma^{IJ}\Gamma_{\dot{1}\dot{2}\dot{3}}\{X^I, X^J, \Psi\} \\ & -\frac{1}{2g^2} - \frac{1}{2}(\mathcal{H}_{\dot{1}\dot{2}\dot{3}})^2 - \frac{1}{4}\mathcal{F}_{\dot{\nu}\dot{\rho}}\mathcal{F}^{\dot{\nu}\dot{\rho}} + \frac{1}{2}W^\alpha_{\dot{\mu}}(\mathbf{M}^{-1})^{\dot{\mu}\dot{\nu}}{}_{\alpha\beta}W^\beta_{\dot{\nu}} \Big\}. \end{aligned} \tag{4.27}$$

Here the fields $\mathcal{F}$ are defined by the same expressions as before but with the new definition of $\hat{B}$.

4.2.2 Order Expansion Analysis

In this subsection, we show the result of action with matter fields in g^0 and g^1 order expansion. At the 0-th order of g, the action is just

$$S''^{(0)}[a_A, X^I, \Psi] \simeq \int d^2x d^3y \left\{ -\frac{1}{4}F_{AB}F^{AB} - \frac{1}{2}\partial_A X^I\partial^A X^I + \frac{i}{2}\bar{\Psi}\Gamma^A\partial_A\Psi \right\} \tag{4.28}$$

after we integrate out the VPD gauge fields $b^{\dot{\mu}}$. This defines a Maxwell's theory with neutral bosons X^I and fermions Ψ.In next section, we can find the action is supersymmetry invariant.

For completeness let us also give the expression of the action to the 1st order:

$$
\begin{aligned}
S'^{(1)}[b^{\dot\mu}, a_A, X^I, \Psi] \;\simeq\; & \int d^2x d^3y \Big\{ -\frac{1}{2}\partial_\alpha X^I \partial^\alpha X^I - \frac{1}{2}\partial_{\dot\mu} X^I \partial^{\dot\mu} X^I + \frac{i}{2}\bar\Psi\Gamma^\alpha\partial_\alpha\Psi + \frac{i}{2}\bar\Psi\Gamma^{\dot\mu}\partial_{\dot\mu}\Psi \\
& + g\epsilon^{\alpha\beta}F_{\beta\dot\mu}\partial^{\dot\mu}X^I\partial_\alpha X^I + g\partial^{\dot\mu}X^I\partial_\alpha X^I\partial^\alpha b_{\dot\mu} - g\frac{i}{2}\epsilon^{\alpha\beta}F_{\beta\dot\mu}\bar\Psi\Gamma_\alpha\partial^{\dot\mu}\Psi \\
& - g\frac{i}{2}\bar\Psi\Gamma_\alpha\partial^{\dot\mu}\Psi\partial^\alpha b_{\dot\mu} - g\partial_{\dot\mu}X^I\partial^{\dot\mu}X^I\partial_{\dot\rho}b^{\dot\rho} + g\partial^{\dot\mu}X^I\partial_{\dot\rho}X^I\partial_{\dot\mu}b^{\dot\rho} \\
& + g\frac{i}{2}\bar\Psi\Gamma^{\dot\rho}\partial_{\dot\rho}\Psi\partial_{\dot\nu}b^{\dot\nu} - g\frac{i}{2}\bar\Psi\Gamma^{\dot\rho}\partial_{\dot\nu}\Psi\partial_{\dot\rho}b^{\dot\nu} + g\frac{i}{4}\bar\Psi\Gamma^2\epsilon^{\dot\mu\dot\nu\dot\rho}F_{\dot\nu\dot\rho}\partial_{\dot\mu}\Psi \\
& - \frac{1}{2}\mathcal{H}_{\dot1\dot2\dot3}\mathcal{H}^{\dot1\dot2\dot3} - \frac{1}{4}\mathcal{F}_{\dot\nu\dot\rho}\mathcal{F}^{\dot\nu\dot\rho} - \frac{1}{2}F_{\beta\dot\mu}F^{\beta\dot\mu} - \frac{1}{2}\epsilon^{\alpha\beta}F_{\alpha\beta}\partial_{\dot\mu}b^{\dot\mu} \\
& - g\epsilon^{\alpha\beta}F_{\beta\dot\mu}\partial_\alpha b_{\dot\nu}\partial^{\dot\nu}b^{\dot\mu} + \frac{1}{2}g\epsilon^{\alpha\beta}F_{\dot\mu\dot\nu}\partial_\alpha b^{\dot\mu}\partial_\beta b^{\dot\nu} + gF_{\dot\mu\dot\nu}F^{\alpha\dot\nu}\partial_\alpha b^{\dot\mu} \\
& + gF^{\alpha\dot\nu}F_{\alpha\dot\mu}\partial_{\dot\nu}b^{\dot\mu} + \frac{1}{2}g\epsilon^{\alpha\beta}F_{\beta\dot\mu}F^{\dot\mu\dot\nu}F_{\alpha\dot\nu} + O(g^2) \Big\}. \qquad (4.29)
\end{aligned}
$$

It is more complex, but we can expect the general case with the nonlocal effect after we integrate out the VPD gauge fields $\beta^{\dot\mu}$.

4.2.3 Rewrite Action with Covariant Variables

As same as before, we can check which kind combination of fields will be covariant: $\delta_\Lambda\hat\Phi = g\kappa^{\dot\sigma}\partial_{\dot\sigma}\hat\Phi$, where the $\hat\Phi$ is any variable. We already know these fields are covariant variable as before: $\mathcal{H}_{\dot1\dot2\dot3}$, $\mathcal{F}_{\dot\mu\dot\nu}$, and $\mathcal{D}_{\dot\mu}\Phi$, where Φ are X^I or Ψ. The other covariant variables relate the gauge field $B_2^{\dot\mu}$ and $B_\alpha^{\dot\mu}$, we need to check them.

Firstly, the covariant variable $\mathcal{D}_2\Phi$ become to $(-g\frac{1}{2}\epsilon^{\dot\mu\dot\nu\dot\rho}F_{\dot\nu\dot\rho}\partial_{\dot\mu}\Phi) \equiv \hat D\Phi$ after DDR, we can find the gauge transformation law is:

$$\delta_\Lambda(-g\frac{1}{2}\epsilon^{\dot\mu\dot\nu\dot\rho}F_{\dot\nu\dot\rho}\partial_{\dot\mu}\Phi) \equiv \delta_\Lambda(\hat D\Phi) = g\kappa^{\dot\sigma}\partial_{\dot\sigma}(\hat D\Phi). \qquad (4.30)$$

The each part of $(-g\frac{1}{2}\epsilon^{\dot\mu\dot\nu\dot\rho}F_{\dot\nu\dot\rho}\partial_{\dot\mu}\Phi)$ is not covariant, but the combination of them are covariant. In another case of DDR on $y^{\dot3}$, we can find the covariant derivative vanishing $\mathcal{D}_{\dot3} \to 0$ after we take away zero mode of $y^{\dot3}$. In our new D4 action, we have these addition terms with $\mathcal{D}_2$ in action.

Secondary, we know the original variables with $\breve B_\alpha{}^{\dot\mu}$ are covariant even if we replace it by the solution $\hat B_\alpha{}^{\dot\mu}$. We want to check if the result is same when we include matter fields in theory. The results of gauge transformation of similar fields are given by:

$$\delta_\Lambda \mathbf{M}_{\dot\mu\dot\nu}{}^{\alpha\beta} = g[\kappa^{\dot\sigma}\partial_{\dot\sigma}\mathbf{M}_{\dot\mu\dot\nu}{}^{\alpha\beta} + (\partial_{\dot\mu}\kappa^{\dot\sigma})\mathbf{M}_{\dot\sigma\dot\nu}{}^{\alpha\beta} + (\partial_{\dot\nu}\kappa^{\dot\sigma})\mathbf{M}_{\dot\mu\dot\sigma}{}^{\alpha\beta}], \tag{4.31}$$

$$\delta_\Lambda \mathbf{M}^{-1\dot\mu\dot\nu}{}_{\alpha\beta} = g[\kappa^{\dot\sigma}\partial_{\dot\sigma}\mathbf{M}^{-1\dot\mu\dot\nu}{}_{\alpha\beta} - (\partial_{\dot\sigma}\kappa^{\dot\mu})\mathbf{M}^{-1\dot\sigma\dot\nu}{}_{\alpha\beta} - (\partial_{\dot\sigma}\kappa^{\dot\nu})\mathbf{M}^{-1\dot\mu\dot\sigma}{}_{\alpha\beta}], \tag{4.32}$$

$$\delta_\Lambda W_{\dot\mu}{}^{\alpha} = \partial_\beta\kappa^{\dot\sigma}\mathbf{M}_{\dot\mu\dot\sigma}{}^{\alpha\beta} + g[\kappa^{\dot\sigma}\partial_{\dot\sigma}W_{\dot\mu}{}^{\alpha} + \partial_{\dot\mu}\kappa^{\dot\sigma}W_{\dot\sigma}{}^{\alpha}], \tag{4.33}$$

$$\delta_\Lambda \hat{B}_\alpha^{\ \dot\mu} = \partial_\alpha\kappa^{\dot\mu} + g(\kappa^{\dot\nu}\partial_{\dot\nu}\hat{B}_\alpha^{\ \dot\mu} - \hat{B}_\alpha^{\ \dot\nu}\partial_{\dot\nu}\kappa^{\dot\mu}). \tag{4.34}$$

Hence, we can find these transformations are similar as before. The covariant variables, which are defined by $\hat{B}_\alpha{}^{\dot\mu}$ in previous cases, are also covariant in this general cases.

Thirdly, we want to get more simple action with covariant variables, then we can define the new covariant variable $\hat{\mathcal{F}}_{\alpha\dot\mu}$ by the equation of motion of $\breve{B}_\alpha{}^{\dot\mu}$:

$$\hat{\mathcal{F}}_{\alpha\dot\mu} \equiv V^{-1}{}_{\dot\mu}^{\ \dot\nu}\left\{F_{\alpha\dot\nu} + g[F_{\dot\nu\dot\delta}\hat{B}_\alpha^{\ \dot\delta} + \epsilon_{\alpha\beta}\partial_{\dot\nu}X^I\partial^\beta X^I - \frac{i}{2}\epsilon_{\alpha\beta}\bar\Psi\Gamma^\beta\partial_{\dot\nu}\Psi] - g^2[\epsilon_{\alpha\beta}\hat{B}^\beta_{\dot\rho}\partial_{\dot\nu}X^I\partial^{\dot\rho}X^I]\right\}. \tag{4.35}$$

We also can check the gauge transformation of this covariant variable:

$$\delta_\Lambda\hat{\mathcal{F}}_{\alpha\dot\mu} = g\kappa^{\dot\sigma}\partial_{\dot\sigma}\hat{\mathcal{F}}_{\alpha\dot\mu}. \tag{4.36}$$

After we combine the all covariant variables, we can rewrite the action (4.27) in this way:

$$\begin{aligned}
S'[b^{\dot\mu}, a_A, X^I, \Psi] = \int d^2x d^3y \Big\{ & -\frac{1}{2}\mathcal{H}_{\dot1\dot2\dot3}\mathcal{H}^{\dot1\dot2\dot3} - \frac{1}{4}\mathcal{F}_{\dot\nu\dot\rho}\mathcal{F}^{\dot\nu\dot\rho} + \frac{1}{2}\hat{\mathcal{F}}_{\beta\dot\mu}\hat{\mathcal{F}}^{\beta\dot\mu} + \frac{1}{2g}\epsilon^{\alpha\beta}\mathcal{F}_{\alpha\beta} \\
& -\frac{1}{2g^2} - \frac{1}{2}\mathcal{D}_A X^I\mathcal{D}^A X^I - \frac{1}{2}\hat{D}X^I\hat{D}X^I \\
& -\frac{g^4}{4}\{X^{\dot\mu}, X^I, X^J\}^2 - \frac{g^4}{12}\{X^I, X^J, X^k\}^2 \\
& +\frac{i}{2}\bar\Psi\Gamma^A\mathcal{D}_A\Psi + \frac{i}{2}\bar\Psi\Gamma^2\hat{D}\Psi \\
& +g^2\frac{i}{2}\bar\Psi\Gamma_{\dot\mu}\Gamma^I\{X^{\dot\mu}, X^I, \Psi\} - g^2\frac{i}{4}\bar\Psi\Gamma^{IJ}\Gamma_{\dot1\dot2\dot3}\{X^I, X^J, \Psi\}\Big\}.
\end{aligned} \tag{4.37}$$

In this section, we get the full effective theory of D4-brane in large C field background. However, we do not know how to generalize it to Dp-brane cases. Because the supersymmetry laws of these cases are still unclear. Before the generalization, we want to write down the supersymmetry transformation law of the all fields in the new effective D4-brane theory.

4.3 Supersymmetry Transformation

The full action (4.27) inherits the full supersymmetry from the NP M5-brane theory because DDR preserves global SUSY, and duality transformation is an equivalence relation. Nevertheless it is not totally trivial to derive the explicit SUSY transformation rules for all the variables, in particular those arise as Lagrange multipliers. In this section we want to deal the problem. The supersymmetry transformation of each field after DDR on x^2 are represented by:

$$\begin{aligned}
\delta_\epsilon X^I &= i\bar{\epsilon}\Gamma^I\Psi, && (4.38)\\
\delta_\epsilon \Psi &= \mathcal{D}_\alpha X^I\Gamma^\alpha\Gamma^I\epsilon + \mathcal{D}_{\dot\mu}X^I\Gamma^{\dot\mu}\Gamma^I\epsilon + \frac{1}{2}g\epsilon^{\dot\mu\dot\nu\dot\rho}F_{\dot\nu\dot\rho}\partial_{\dot\mu}X^I\Gamma^2\Gamma^I\epsilon \\
&\quad -\frac{1}{2}\mathcal{F}_{\dot\nu\dot\rho}\Gamma^2\Gamma^{\dot\nu\dot\rho}\epsilon - \frac{1}{2}\mathcal{H}_{\alpha\dot\nu\dot\rho}\Gamma^\alpha\Gamma^{\dot\nu\dot\rho}\epsilon - \left(\frac{1}{g} + \mathcal{H}_{\dot1\dot2\dot3}\right)\Gamma^{\dot1\dot2\dot3}\epsilon \\
&\quad -\frac{g^2}{2}\{X^{\dot\mu}, X^I, X^J\}\Gamma_{\dot\mu}\Gamma^{IJ}\epsilon + \frac{g^2}{6}\{X^I, X^J, X^K\}\Gamma^{IJK}\Gamma^{\dot1\dot2\dot3}\epsilon, && (4.39)\\
\delta_\epsilon b_{\dot\mu\dot\nu} &= -i\bar{\epsilon}\Gamma_{\dot\mu\dot\nu}\Psi, && (4.40)\\
\delta_\epsilon a_{\dot\mu} &= i\bar{\epsilon}\Gamma^2\Gamma_{\dot\mu}\Psi + ig\bar{\epsilon}\Gamma^2\Gamma_{\dot\nu}\Psi\partial_{\dot\mu}b^{\dot\nu} - ig\bar{\epsilon}\Gamma^2\Gamma^I\Gamma^{\dot1\dot2\dot3}\Psi\partial_{\dot\mu}X^I, && (4.41)\\
\delta_\epsilon B_\alpha{}^{\dot\mu} &= -i\bar{\epsilon}\Gamma_\alpha\Gamma_{\dot\lambda}\partial_{\dot\nu}\Psi\epsilon^{\dot\mu\dot\nu\dot\rho}(\delta_{\dot\rho}{}^{\dot\lambda} + g\partial_{\dot\rho}b^{\dot\lambda}) + ig\bar{\epsilon}\Gamma_\alpha\Gamma^I\Gamma^{\dot1\dot2\dot3}\partial_{\dot\nu}\Psi\epsilon^{\dot\mu\dot\nu\dot\rho}\partial_{\dot\rho}X^I. && (4.42)
\end{aligned}$$

We want to know what is the supersymmetry transformation law of new fields after dual transformation. After dual transformation, we get the new fields $\breve{B}_\alpha{}^{\dot\mu}$ and a_α, the new field $\breve{B}_\beta{}^{\dot\nu}$ is not divergenceless ($\partial_{\dot\nu}\breve{B}_\beta{}^{\dot\nu} \neq 0$). How to find the SUSY law of these fields?

4.3.1 Supersymmetry Law of Dual Field

The dual transformation is equivalent to add one term $\epsilon^{\alpha\beta}\partial_{\dot\mu}a_\beta\breve{B}_\alpha{}^{\dot\mu}$ in action and to replace $B_\alpha{}^{\dot\mu}$ by $\breve{B}_\alpha{}^{\dot\mu}$. After doing supersymmetry variation for action (4.22), the new term will have a additional contribution:

$$-\epsilon^{\alpha\beta}\partial_{\dot\mu}\delta_\epsilon a_\beta\breve{B}_\alpha{}^{\dot\mu} - \epsilon^{\alpha\beta}\partial_{\dot\mu}a_\beta\delta_\epsilon\breve{B}_\alpha{}^{\dot\mu}. \qquad (4.43)$$

Because the other terms in action do not include a_α, we should choose $\delta_\epsilon\breve{B}_\alpha{}^{\dot\mu} = \delta_\epsilon B_\alpha{}^{\dot\mu}$ to make the second term vanish, the reason is $\partial_{\dot\mu}\delta_\epsilon\breve{B}_\alpha{}^{\dot\mu} = 0 = \partial_{\dot\mu}\delta_\epsilon B_\alpha{}^{\dot\mu}$. So we get:

$$\delta_\epsilon\breve{B}_\alpha{}^{\dot\mu} = -i\bar{\epsilon}\Gamma_\alpha\Gamma_{\dot\rho}\partial_{\dot\nu}\Psi\epsilon^{\dot\mu\dot\nu\dot\rho} - ig\bar{\epsilon}\Gamma_\alpha\Gamma_{\dot\lambda}\partial_{\dot\nu}\Psi\epsilon^{\dot\mu\dot\nu\dot\rho}\partial_{\dot\rho}b^{\dot\lambda} + ig\bar{\epsilon}\Gamma_\alpha\Gamma^I\Gamma^{\dot1\dot2\dot3}\partial_{\dot\nu}\Psi\epsilon^{\dot\mu\dot\nu\dot\rho}\partial_{\dot\rho}X^I. \qquad (4.44)$$

After calculating the supersymmetry variation of action, we get the terms which depends on $\breve{B}_\alpha{}^{\dot{\mu}}$:

$$\begin{aligned}\delta_\epsilon S &= -\frac{1}{2}ig\delta_\epsilon\overline{\Psi}\Gamma^\alpha\Psi\partial_{\dot{\mu}}\breve{B}_\alpha^{\ \dot{\mu}} - ig\bar{\epsilon}\Gamma^2\Gamma_{\dot{\nu}}\Psi\epsilon^{\alpha\gamma}\partial_\gamma b^{\dot{\nu}}\partial_{\dot{\mu}}\breve{B}_\alpha^{\ \dot{\mu}} \\ &\quad + ig\bar{\epsilon}\Gamma^2\Gamma^I\Gamma^{\dot{1}\dot{2}\dot{3}}\Psi\epsilon^{\alpha\gamma}\partial_\gamma X^I\partial_{\dot{\mu}}\breve{B}_\alpha^{\ \dot{\mu}} \\ &\quad + \epsilon^{\alpha\gamma}\delta_\epsilon a_\gamma\partial_{\dot{\mu}}\breve{B}_\alpha^{\ \dot{\mu}}.\end{aligned} \tag{4.45}$$

Hence, we obtain the SUSY law of dual field:

$$\begin{aligned}\delta_\epsilon a_\beta &= -\frac{1}{2}ig\delta_\epsilon\overline{\Psi}\Gamma^\alpha\Psi\epsilon_{\alpha\beta} + ig\bar{\epsilon}\Gamma^2\Gamma_{\dot{\nu}}\Psi\partial_\beta b^{\dot{\nu}} \\ &\quad - ig\bar{\epsilon}\Gamma^2\Gamma^I\Gamma^{\dot{1}\dot{2}\dot{3}}\Psi\partial_\beta X^I.\end{aligned} \tag{4.46}$$

Here, the transformation law of $\overline{\Psi}$ is:

$$\begin{aligned}\delta_\epsilon\overline{\Psi} &= \bar{\epsilon}\Gamma^I\Gamma^A\mathcal{D}_A X^I + \frac{1}{2}g\bar{\epsilon}\Gamma^I\Gamma^2\epsilon^{\dot{\mu}\dot{\nu}\dot{\rho}}F_{\dot{\nu}\dot{\rho}}\partial_{\dot{\mu}}X^I \\ &\quad - \frac{1}{2}\bar{\epsilon}\Gamma^{\dot{\nu}\dot{\rho}}\Gamma^2\mathcal{F}_{\dot{\nu}\dot{\rho}} - \frac{1}{2}\bar{\epsilon}\Gamma^{\dot{\nu}\dot{\rho}}\Gamma^\alpha\mathcal{H}_{\alpha\dot{\nu}\dot{\rho}} - \bar{\epsilon}\Gamma^{\dot{1}\dot{2}\dot{3}}\left(\frac{1}{g} + \mathcal{H}_{\dot{1}\dot{2}\dot{3}}\right) \\ &\quad - \frac{g^2}{2}\bar{\epsilon}\Gamma^{IJ}\Gamma_{\dot{\mu}}\{X^{\dot{\mu}}, X^I, X^J\} + \frac{g^2}{6}\bar{\epsilon}\Gamma^{\dot{1}\dot{2}\dot{3}}\Gamma^{IJK}\{X^I, X^J, X^K\}.\end{aligned} \tag{4.47}$$

4.3.2 Non-linear Fermion Symmetry of Dual Field

The theory has 16 non-linear fermionic symmetries δ_χ, which shift the fermion by a constant spinor

$$\delta_\chi\Psi = \chi, \quad \delta_\chi X^i = \delta_\chi b^{\dot{\mu}} = \delta_\chi a_{\dot{\mu}} = 0. \tag{4.48}$$

We also can get the SUSY transformation of the new term:

$$\delta_\chi S = \frac{i}{2}g\overline{\chi}\Gamma^\alpha\Psi\partial_{\dot{\mu}}\breve{B}_\alpha^{\ \dot{\mu}} + \epsilon^{\alpha\beta}\delta_\chi a_\beta\partial_{\dot{\mu}}\breve{B}_\alpha^{\ \dot{\mu}}. \tag{4.49}$$

Hence, we obtain the non-linear transformation law of a_α:

$$\delta_\chi a_\alpha = -\frac{i}{2}g\overline{\chi}\Gamma^\beta\Psi\epsilon_{\alpha\beta}. \tag{4.50}$$

4.3.3 Linear Supersymmetry Transformation

The SUSY law of $a_{\dot{\mu}}$ and a_α are not similar, this is the new characteristic in D4 with C-field background. However, we can redefine a new linear supersymmetry law which combine the two SUSY laws in previous subsections, the SUSY law of a_α becomes more similar to the SUSY law of $a_{\dot{\mu}}$. We choose:

$$\delta \equiv \frac{1}{g}\delta_{\chi \to \Gamma_{\dot{1}\dot{2}\dot{3}}\epsilon} + \delta_\epsilon, \tag{4.51}$$

then we can find the SUSY law of a_α becomes:

$$\delta a_\alpha = \delta a_{\dot{\mu}\to\alpha} + \frac{1}{2}\delta\overline{\Psi}\Gamma^\beta\Psi\epsilon_{\alpha\beta}. \tag{4.52}$$

After the linear combination, we can find the lowest order of $\delta\overline{\Psi}$ will start from g^0 order.

4.3.4 Supersymmetry Transformation Law of $\hat{B}_\alpha{}^{\dot{\mu}}$ Field

When we integrate out $\breve{B}_\alpha{}^{\dot{\mu}}$, in classical level, it is same with replacing $\breve{B}_\alpha{}^{\dot{\mu}}$ by $\hat{B}_\alpha{}^{\dot{\mu}}$, which is the solution of E.O.M of $\breve{B}_\alpha{}^{\dot{\mu}}$. Its solution is

$$\begin{aligned}\hat{B}_\alpha{}^{\dot{\mu}} &= (\mathbf{M}^{-1})^{\dot{\mu}\dot{\nu}}{}_{\alpha\beta}(V_{\dot{\nu}}{}^{\dot{\sigma}}\partial^\beta b_{\dot{\sigma}} + \epsilon^{\beta\gamma}F_{\gamma\dot{\nu}} + g\partial_{\dot{\nu}}X^i\partial^\beta X^i - g\frac{i}{2}\bar{\Psi}\Gamma^\beta\partial_{\dot{\nu}}\Psi) \\ &\equiv (\mathbf{M}^{-1})^{\dot{\mu}\dot{\nu}}{}_{\alpha\beta}W^\beta_{\dot{\nu}},\end{aligned} \tag{4.53}$$

where

$$\mathbf{M}_{\dot{\mu}\dot{\nu}}{}^{\alpha\beta} \equiv (V_{\dot{\mu}\dot{\rho}}V_{\dot{\nu}}{}^{\dot{\rho}} + g^2\partial_{\dot{\mu}}X^i\partial_{\dot{\nu}}X^i)\delta^{\alpha\beta} - g\epsilon^{\alpha\beta}F_{\dot{\mu}\dot{\nu}}, \tag{4.54}$$

and $(\mathbf{M}^{-1})^{\dot{\mu}\dot{\nu}}{}_{\alpha\beta}$ is defined by

$$(\mathbf{M}^{-1})^{\dot{\lambda}\dot{\mu}}{}_{\gamma\alpha}\mathbf{M}_{\dot{\mu}\dot{\nu}}{}^{\alpha\beta} = \delta^{\dot{\lambda}}{}_{\dot{\nu}}\delta_\gamma{}^\beta. \tag{4.55}$$

We want to ask if the action is still supersymmetry invariant. Moreover, what is the supersymmetry law of $\hat{B}_\alpha{}^{\dot{\mu}}$. The supersymmetry transformation of the theory is a on-shell formalism. So the $\delta_\epsilon\hat{B}_\alpha{}^{\dot{\mu}}$ and $\delta_\epsilon\breve{B}_\alpha{}^{\dot{\mu}}$ can be different with E.O.M of all fields. In fact, we find the exact results of the difference. The answer is:

$$\delta_\epsilon\hat{B}_\alpha{}^{\dot{\mu}} = \delta_\epsilon\breve{B}_\alpha{}^{\dot{\mu}} - 2(\mathbf{M}^{-1})^{\dot{\mu}\dot{\nu}}{}_{\alpha\beta}(\delta_\epsilon\overline{\Psi}|_{\hat{B}})_{\dot{\nu}}{}^\beta(\text{E.O.M of }\overline{\Psi}). \tag{4.56}$$

Here the notation $(\delta_\epsilon\overline{\Psi}|_{\hat{B}})_{\dot{\nu}}{}^\beta$ means the terms of $\delta_\epsilon\overline{\Psi}$ with $\hat{B}_\alpha{}^{\dot{\mu}}$ field. The explicit form is :

$$(\delta_\epsilon\overline{\Psi}|_{\hat{B}})_{\dot{\nu}}{}^\beta = \frac{1}{2}\bar{\epsilon}\Gamma^{\dot{\rho}\dot{\lambda}}\Gamma^\beta\epsilon_{\dot{\sigma}\dot{\rho}\dot{\lambda}}V_{\dot{\nu}}{}^{\dot{\sigma}} - g\bar{\epsilon}\Gamma^i\Gamma^\beta\partial_{\dot{\nu}}X^i. \tag{4.57}$$

The E.O.M of $\overline{\Psi}$ is:

$$\begin{aligned}(\text{E.O.M of }\overline{\Psi}) &= \frac{i}{2}\Gamma^\alpha\partial_\alpha\Psi + \frac{i}{4}g\Gamma^2\epsilon^{\dot{\mu}\dot{\nu}\dot{\rho}}F_{\dot{\nu}\dot{\rho}}\partial_{\dot{\mu}}\Psi - \frac{i}{2}g\Gamma^\alpha\hat{B}_\alpha{}^{\dot{\mu}}\partial_{\dot{\mu}}\Psi \\ &\quad + \frac{i}{2}\Gamma^{\dot{\rho}}\mathcal{D}_{\dot{\rho}}\Psi + \frac{i}{2}g^2\Gamma_{\dot{\mu}}\Gamma^i\{X^{\dot{\mu}}, X^i, \Psi\} - \frac{i}{4}g^2\Gamma^{ij}\Gamma^{\dot{1}\dot{2}\dot{3}}\{X^i, X^j, \Psi\}.\end{aligned} \tag{4.58}$$

The relation (4.56) reproduces:

$$\delta_\epsilon(\mathbf{M}_{\dot\mu\dot\nu}{}^{\alpha\beta}\hat{B}_\beta{}^{\dot\nu} - W^\alpha_{\dot\mu}) = 0, \tag{4.59}$$

so the result is exact in all order.

Another way to check the relation (4.56) is calculate the zeroth-order expansion of $\hat{B}_\alpha{}^{\dot\mu}$, then we find:

$$\delta_\epsilon\hat{B}_\alpha{}^{\dot\mu}|_{g^0} = \delta_\epsilon\breve{B}_\alpha{}^{\dot\mu}|_{g^0} - i\bar\epsilon\Gamma_\alpha\Gamma^{\dot1\dot2\dot3}\Gamma^{\dot\mu}\Gamma^A\partial_A\Psi. \tag{4.60}$$

The result is also get from the exact answer (4.56).

By the way, the supersymmetry transformation law of fermion is just to replace $\breve{B}_\alpha{}^{\dot\mu}$ with $\hat{B}_\alpha{}^{\dot\mu}$. Now we get all supersymmetry transformation laws of all fields in action (4.27).

4.4 Topological Quantities of D4 in Large C Field Background

In this section, we try to study the topological quantities of D4-brane in C field background. There are several important topological quantities of D-brane researches: the soliton solutions, instanton solutions, monopole solutions, and BPS states. One way to study these topological quantities is to calculate the central charge of superalgebra. Another way is to find the solutions from the equation of motion of fields. However, the researches of topological quantities of D4-brane in C field background are still in progress. Hence, this section does not really finish right now, I just list the main idea of this topic.

4.4.1 Central Charges of Superalgebra

In this subsection, we want to calculate the central charge of D4 in C field background. From the paper [49] or more early papers, the supercurrent of BLG model is given by:

$$J^\mu = \delta_\epsilon\overline{\Psi}\Gamma^\mu\Psi. \tag{4.61}$$

We can derive the $\delta_\epsilon\overline{\Psi}$ from the BLG → NP M5 → D4 in C before dual transform → D4 in C after Dual transform → D4 in C after integrating out $\breve{B}_\alpha{}^{\dot\mu}$. Hence, we can get the central charges in our theory:

$$\bar\epsilon_1\{Q,Q\}\epsilon_2 \approx \int dx^1 d^3y(\delta_{\epsilon_1}\overline{\Psi})\Gamma^0(\delta_{\epsilon_2}\Psi). \tag{4.62}$$

Here the integral range $\int dx^1 d^3y$ is the full space directions in this D4-brane theory and the supersymmetry transformation law of fermion is written down in previous section. These central charges are the possible soliton solutions of D4-brane in C field background. The remaining problem is how to classify these central charges.

4.4.2 Instanton Solutions

We know the noncommutative U(1) gauge theory can have instanton solutions [50], but U(1) gauge theory have no instanton solutions. The reason is the solution space of gauge field is nontrivial in noncommutative U(1) gauge theory. On the other hand, people [51] study D3-brane in graviphoton field strengths (R-R 2-form field) background. They also find some deformation of ADHM constraints by graviphoton background. We want to know if we can find the possible instanton solutions of D4-brane in R-R C field background, because this case is similar to the D4-brane in NS-NS B field background. Moreover, what is the possible instanton solutions of Dp-branes in R-R field background?

Chapter 5

Conclusion and Discussion

5.1 Summary

In this thesis, we give the effective action of Dp-branes in large R-R field background. We motivate the theory from the NP M5 theory after double dimensional reduction on x^2. The first character what we find is that the theory includes volume-preserving diffeomorphism, because of the (p-1)-form field define the (p-1)-dimensional volume form in the effective theory. The trouble what we meet is the theory with auxiliary fields and without manifest one form field a_α. Firstly, we use the dual transformation to make one form field a_α to become manifest. Then we study how to find the gauge symmetry and supersymmetry of one form gauge field a_α. The nontrivial parts are how to find suitable covariant variables in this mixing symmetry (i.e. VPD and U(1) gauge symmetry) system. We solve this problem, and we use these covariant variables to construct the Dp-branes in large R-R (p-1)-form background. After integrating out the auxiliary field $\breve{B}_\alpha{}^{\dot{\mu}}$, we find we still need the two form field $b^{\dot{\mu}}$. This two form field help us to keep system with U(1) gauge symmetry and VPD at same time. We also need the two form field to define these covariant variables. The $b^{\dot{\mu}}$ field is necessary in our theory, we can understand it as the degree of one form field a_α from electric-magnetic dual viewpoints. We give the evidence in order expansion analyses; in zeroth order, we find the relation $\partial_{\dot{\mu}} b^{\dot{\mu}} = -F_{01}$. In general, we should solve the relation in all order, but it is really nontrivial work. Hence, we deal with it by gauge fixing method in first order calculation. Finally, We study the full system which couples with matter fields (X^I,Ψ). We do similar analyses as gauge fields case. The hard problem is to find the supersymmetry transformation law of dual field. We need to calculate the remaining terms of dual action after supersymmetry transformation. Using the result, we can find the way to define the supersymmetry transformation law of dual field. We also study the topological quantities in this system from the central charge of superalgebra. If we can find the new solitons solutions, it will be interesting.

5.2 Discussion

The final purpose in this thesis is to find effective field theory of all extended objects in all possible background fields. However, we are still far from this purpose. One way to extend our recent work is to study the D3-brane in R-R 4-form background from T-duality. Firstly, we do dimensional reduction on x^1, then we view the gauge field a_1 as the transverse direction of D3-brane ($\tilde{X}^1$). Finally we get the effective D3-brane theory in large R-R 4-form background. The R-R 4-form field $D^{(4)}$:

$$D^{(4)} = V^{(1)} \wedge C^{(3)}. \tag{5.1}$$

The $C^{(3)}$ is original 3-form background in D4-brane theory, and the $V^{(1)}$ is the one-form on the transverse direction $\tilde{X}^1$ of D3-brane. Extending this conclusion to Dp-branes, we claim that for a Dp-brane in R-R $(p+1)$-form potential background

$$D^{(p+1)} = V^{(1)} \wedge C^{(p)}, \tag{5.2}$$

where $V^{(1)}$ is transverse and $C^{(p)}$ is parallel to the Dp-brane. The VPD, corresponding to the volume-form $C^{(p)}$, shares the same gauge field degrees of freedom with the component of the momentum p along the direction of $V^{(1)}$.

The other brane systems in large field background are also interesting. For example, we want to know the behavior of NS5-branes in large R-R field background, the effective action of KK monopole in large field background, etc.

There are some interesting papers which describe the possible effective action of Dp-brane in R-R field background [37–39, 51]. However, they only focus on the D3-brane in R-R 2-form background case and the supersymmetry is $\mathcal{N} = \frac{1}{2}$. This topic is called by non-anticommutative field theory, which is motivated from the extension of original anticommutative field theory (supersymmetry theory). How to understand the relation between their work and our new methods is an important problem.

Furthermore, we know the original AdS/CFT correspondence which describe the physics of D-branes in R-R field background. How to apply the correspondence into large R-R field background is another important application.

Appendix A

Conventions and Notations

In this article, we use these indices to label the 6 worldvolume directions:

$$M, N, R = \mu, \nu, \rho, \dot{\mu}, \dot{\nu}, \dot{\rho}\,, \tag{A.1}$$

$$\mu, \nu, \rho = 0, 1, 2\,, \tag{A.2}$$

$$\dot{\mu}, \dot{\nu}, \dot{\rho} = \dot{1}, \dot{2}, \dot{3}\,. \tag{A.3}$$

The metric and Levi-Civita tensor are:

$$g^{MN} = \begin{pmatrix} \eta^{\mu\nu} & 0 \\ 0 & \eta^{\dot{\mu}\dot{\nu}} \end{pmatrix}, \tag{A.4}$$

$$\eta^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{A.5}$$

$$\eta^{\dot{\mu}\dot{\nu}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{A.6}$$

$$\epsilon^{012} = -\epsilon_{012} = 1, \tag{A.7}$$

$$\epsilon^{\dot{1}\dot{2}\dot{3}} = \epsilon_{\dot{1}\dot{2}\dot{3}} = 1. \tag{A.8}$$

The conventions of Gamma matrix are:

$$\begin{aligned}
\{\Gamma^M, \Gamma^N\} &= 2g^{MN}, && \text{(A.9)}\\
(\Gamma^0)^\dagger &= -\Gamma^0, && \text{(A.10)}\\
(\Gamma^{M\neq 0})^\dagger &= \Gamma^{M\neq 0}, && \text{(A.11)}\\
\Gamma^7 &\equiv \Gamma^0\Gamma^1\Gamma^2\Gamma^{\dot{1}}\Gamma^{\dot{2}}\Gamma^{\dot{3}}, && \text{(A.12)}\\
\Gamma^{\mu\nu\rho}\Gamma^{\dot{1}\dot{2}\dot{3}} &= \epsilon^{\mu\nu\rho}\Gamma^7, && \text{(A.13)}\\
\Gamma^{\mu\nu\rho} &= \epsilon^{\mu\nu\rho}\Gamma^{\dot{1}\dot{2}\dot{3}}\Gamma^7, && \text{(A.14)}\\
(\Gamma^7)^2 &= 1. && \text{(A.15)}
\end{aligned}$$

We use these conventions to label the fields and directions in D4-brane theory:

$$\alpha, \beta, \gamma, \delta = 0, 1\ , \quad \text{(A.16)}$$

$$A, B, C = 0, 1, \dot{1}, \dot{2}, \dot{3}\ , \quad \text{(A.17)}$$

$$b_{\dot{\mu}2} \equiv a_{\dot{\mu}}, \quad \text{(A.18)}$$

$$\Lambda_2 \equiv \lambda, \quad \text{(A.19)}$$

$$F_{AB} \equiv \partial_A a_B - \partial_B a_A, \quad \text{(A.20)}$$

$$\epsilon^{\alpha\beta 2} \equiv \epsilon^{\alpha\beta}. \quad \text{(A.21)}$$

On the other hand, we also use the index I to label the transverse directions of brane. In this article, we use $I = 6, 7, 8, 9, 11$ in M5-brane or D4-brane cases.

Appendix B

Some Useful Identities

In order to check supersymmetry, we need to use some identities. Here is the summary.

- Chirarity condition

$$\Gamma^7\Psi = \Psi, \tag{B.1}$$

$$\Gamma^7\epsilon = -\epsilon. \tag{B.2}$$

Hence, we can get:

$$\Gamma^{\mu\nu\rho}\Gamma^{\dot{1}\dot{2}\dot{3}}\psi = \epsilon^{\mu\nu\rho}\Gamma^7\psi = \epsilon^{\mu\nu\rho}\psi, \tag{B.3}$$

$$\Gamma^{\alpha\beta}\Gamma^2\Gamma^{\dot{1}\dot{2}\dot{3}}\psi = \epsilon^{\alpha\beta}\psi, \tag{B.4}$$

$$\Gamma^{\alpha\beta}\Gamma^{\dot{1}\dot{2}\dot{3}}\psi = \epsilon^{\alpha\beta}\Gamma^2\psi, \tag{B.5}$$

$$\Gamma^{\alpha}\Gamma^{\dot{1}\dot{2}\dot{3}}\psi = -\epsilon^{\alpha\beta}\Gamma_\beta\Gamma^2\psi, \tag{B.6}$$

$$\Gamma^{\dot{1}\dot{2}\dot{3}}\psi = -\frac{1}{2}\epsilon^{\alpha\beta}\Gamma_\alpha\Gamma_\beta\Gamma^2\psi, \tag{B.7}$$

$$\Gamma^{\dot{\mu}}\Gamma^{\dot{1}\dot{2}\dot{3}}\psi = \frac{1}{2}\epsilon^{\alpha\beta}\Gamma_\alpha\Gamma_\beta\Gamma^2\Gamma^{\dot{\mu}}\psi. \tag{B.8}$$

- Gamma matrix

There are some useful identities of gamma matrix:

$$\Gamma^A\Gamma^B = \Gamma^{AB} + \eta^{AB}, \tag{B.9}$$

$$\Gamma_{\dot\mu}\Gamma^{\dot\mu} = 3, \tag{B.10}$$

$$\Gamma_{\dot\mu}\Gamma^{\dot\mu\dot\nu} = 2\Gamma^{\dot\nu}, \tag{B.11}$$

$$\Gamma_{\dot\mu}\Gamma^{\dot\mu\dot\nu\dot\rho} = \Gamma^{\dot\nu\dot\rho}, \tag{B.12}$$

$$\epsilon^{\dot\mu\dot\nu\dot\rho} = -\Gamma^{\dot\mu\dot\nu\dot\rho}\Gamma^{\dot1\dot2\dot3}, \tag{B.13}$$

$$\Gamma_{\dot\mu}\epsilon^{\dot\mu\dot\nu\dot\rho} = -\Gamma^{\dot\nu\dot\rho}\Gamma^{\dot1\dot2\dot3}, \tag{B.14}$$

$$\Gamma_{\dot\mu\dot\nu}\epsilon^{\dot\mu\dot\nu\dot\rho} = 2\Gamma^{\dot\rho}\Gamma^{\dot1\dot2\dot3}, \tag{B.15}$$

$$\Gamma^{\dot\mu}\Gamma^{\dot1\dot2\dot3} = \Gamma^{\dot1\dot2\dot3}\Gamma^{\dot\mu}, \tag{B.16}$$

$$\Gamma^\alpha\Gamma_\alpha = 2, \tag{B.17}$$

$$\Gamma^\alpha\Gamma^\beta\Gamma_\alpha = 0. \tag{B.18}$$

- Levi-Civita tensor

$$\epsilon^{\alpha\beta}\epsilon_{\gamma\delta} = -\eta_\gamma{}^\alpha\eta_\delta{}^\beta + \eta_\gamma{}^\beta\eta_\delta{}^\alpha, \tag{B.19}$$

$$\begin{aligned}\epsilon^{\dot\mu\dot\nu\dot\rho}\epsilon_{\dot\sigma\dot\lambda\dot\delta} &= \eta_{\dot\sigma}{}^{\dot\mu}\eta_{\dot\lambda}{}^{\dot\nu}\eta_{\dot\delta}{}^{\dot\rho} + \eta_{\dot\sigma}{}^{\dot\nu}\eta_{\dot\lambda}{}^{\dot\rho}\eta_{\dot\delta}{}^{\dot\mu} + \eta_{\dot\sigma}{}^{\dot\rho}\eta_{\dot\lambda}{}^{\dot\mu}\eta_{\dot\delta}{}^{\dot\nu}\\ &\quad -\eta_{\dot\sigma}{}^{\dot\mu}\eta_{\dot\lambda}{}^{\dot\rho}\eta_{\dot\delta}{}^{\dot\nu} - \eta_{\dot\sigma}{}^{\dot\nu}\eta_{\dot\lambda}{}^{\dot\mu}\eta_{\dot\delta}{}^{\dot\rho} - \eta_{\dot\sigma}{}^{\dot\rho}\eta_{\dot\lambda}{}^{\dot\nu}\eta_{\dot\delta}{}^{\dot\mu},\end{aligned} \tag{B.20}$$

$$\epsilon^{\alpha\beta}\eta^{\gamma\delta} = -\eta^{\alpha\gamma}\epsilon^{\beta\delta} + \eta^{\beta\gamma}\epsilon^{\alpha\delta}, \tag{B.21}$$

$$\epsilon_{\dot\mu\dot\nu\dot\rho}\eta_{\dot\sigma\dot\delta} = \epsilon_{\dot\mu\dot\nu\dot\sigma}\eta_{\dot\rho\dot\delta} + \epsilon_{\dot\nu\dot\rho\dot\sigma}\eta_{\dot\mu\dot\delta} + \epsilon_{\dot\rho\dot\mu\dot\sigma}\eta_{\dot\nu\dot\delta}. \tag{B.22}$$

Appendix C

Suitable Scaling Limit in Different Cases

When we describe the effective field theory of Dp-brane, we need to choose some limit of original exact theory. The effective field theory of open string ending on Dp-brane should be described by some limit. For example, the DBI action of Dp-brane is given by slowly varying limit($\partial F \ll 1$) of original string scattering amplitude analysis. Hence, the DBI action is a effective description of string theory without higher derivative term. Scaling limit(zero slope limit $\alpha' \to 0$) is a low energy limit, which make theory be more easy for analysis. For example, the zero slope limit of DBI action is Yang-Mill action. In this section, we want to describe the suitable low energy limit of theory in different fields background.

C.1 Scaling Limit of Dp-brane in B-field Background

The low energy limit means the theory without string behavior, the first example is Yang-Mill theory. People [12] find the commutative Yang-Mill theory can relate to non-commutative Yang-Mill theory by using Seiberg-Witten map and taking scaling limit. This scaling limit is called Seiberg-Witten limit:

$$\alpha' \sim \sqrt{\epsilon} \to 0, \tag{C.1}$$

$$g_{ij} \sim \epsilon \to 0, \tag{C.2}$$

here i, j is the non-vanish component of B field. This limit is understood the low energy limit of Dp-brane in NS-NS B-field background.

C.2 Scaling Limit of M5 in Large C-field Background

Following the logic in previous section, the NP M5 theory is some special limit of M5 in large C-field background. The reason is the kinetic terms of gauge field which is quadratic ($\mathcal{H}^2$) as Yang-Mill theory case. So, we should ask what is the scale limit of this NP M5 theory. The NP M5 theory can relate to the action of Dp-brane in NS-NS B-field background after DDR, so we can get the clue of scaling limit from this relation. Following the calculations in paper [47], we summary it by below equations:

$$\begin{aligned} \ell_P &\sim \sqrt[3]{\epsilon}, & \text{(C.3)} \\ g_{\mu\nu} &\sim \epsilon^0, & \text{(C.4)} \\ g_{\dot{\mu}\dot{\nu}} &\sim \epsilon, & \text{(C.5)} \\ C_{\dot{\mu}\dot{\nu}\dot{\rho}} &\sim \epsilon^0, & \text{(C.6)} \\ \epsilon &\to 0, & \text{(C.7)} \end{aligned}$$

here the ℓ_P is Plank length and C is background 3-form. The scaling limit will match the scaling limit of D4 in B-field background after DDR on $y^{\dot{3}}$. To understand this result. we introduce the radius of compact direction($y^{\dot{3}}$) $R_{\dot{3}}^{phys}$, the radius can be calculated by this way: $T_{D4} = 2\pi R_{\dot{3}}^{phys} T_{M5} = \frac{2\pi R_{\dot{3}}^{phys}}{(2\pi)^5 \ell_p^6}$. This is the knowledge of M-IIA which D4 is given by M5 after DDR. From the relation $T_{D4} = 2\pi R_{\dot{3}}^{phys} T_{M5}$, it is consistent with the results:

$$R_{\dot{3}}^{phys} = g_s \ell_s, \qquad \ell_P = \sqrt[3]{g_s}\ell_s. \tag{C.8}$$

On the other hand, the C field can relate to B field in D4 by this way:

$$C_{\dot{1}\dot{2}\dot{3}} = \frac{B_{\dot{1}\dot{2}}}{2\pi R_{\dot{3}}^{coord}} = \frac{\sqrt{g_{\dot{3}\dot{3}}} B_{\dot{1}\dot{2}}}{2\pi R_{\dot{3}}^{phys}}. \tag{C.9}$$

From these relation we can find the scaling limit after DDR on $y^{\dot{3}}$:

$$\begin{aligned} \alpha' &\sim \sqrt{\epsilon}, & \text{(C.10)} \\ g_{\dot{1}\dot{2}} &\sim \epsilon, & \text{(C.11)} \\ B_{\dot{1}\dot{2}} &\sim \epsilon^0, & \text{(C.12)} \\ g_s &\sim \sqrt[4]{\epsilon}, & \text{(C.13)} \\ \epsilon &\to 0, & \text{(C.14)} \end{aligned}$$

here the scaling of g_s can get from the constraints of finite Yang-Mills coupling. These relations are same with previous section.

C.3 Scaling Limit of D4 in Large C-field Background

To carry out the double dimensional reduction (DDR) for the M5-brane along the x^2-direction, we set

$$x^2 \sim x^2 + 2\pi R, \tag{C.15}$$

and let all other fields to be independent of x^2. As a result we can set ∂_2 to zero when it acts on any field. Here R is the radius of the circle of compactification and we should take $R \ll 1$ such that the 6 dimensional field theory on M5 reduces to a 5 dimensional field theory for D4. Since the NP M5-brane action is a good low energy effective theory in the limit in previous section, the 5 dimensional field theory is a good low energy effective description of a D4-brane in the limit $\epsilon \to 0$ for

$$\ell_s \sim \epsilon^{1/2}, \qquad g_s \sim \epsilon^{-1/2}, \qquad g_{\alpha\beta} \sim 1, \qquad g_{\dot\mu\dot\nu} \sim \epsilon, \qquad C_{\dot\mu\dot\nu\dot\lambda} \sim 1, \tag{C.16}$$

with

$$g_s\ell_s \ll 1, \tag{C.17}$$

from the perspective of the type II A theory. The indices $\alpha, \beta = 0, 1$ are used to distinguish from the M5-brane indices $\mu, \nu = 0, 1, 2$.

Note that in the scaling limit of NP M5 theory, another three C-field component $C_{012} \sim \epsilon^{-1}$ look like divergence. As a result the B-field component $B_{01} \sim \epsilon^{-1}$ and the noncommutative parameter $\theta^{01} \sim B^{-1} \sim \epsilon$ vanishes in the limit $\epsilon \to 0$. However, the combination $2\pi\alpha' B$ is finite in the limit, and thus the D4-brane is not only in a C-field background but also in the B-field background. Using the nonlinear self-dual relation derived in [23, 24], we can express C_{012} in terms of $C_{\dot1\dot2\dot3}$, and then the B-field background is given by

$$2\pi\alpha' B_{01} = \frac{C_{\dot1\dot2\dot3}}{2\pi}. \tag{C.18}$$

In the convention (normalization of the worldvolume coordinates) of [2], we have

$$C_{\dot1\dot2\dot3} = \frac{1}{g^2} \qquad \Rightarrow \qquad 2\pi\alpha' B_{01} = \frac{1}{2\pi g^2}. \tag{C.19}$$

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