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超常材料在不同組成率下之負折射和後退波之研究

Investigation of negative refraction and backward wave in meta-material based on different constitutive relations

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中文摘要

人工超常材料擁有自然界天然材料所沒有之電磁行為。這些特殊現象導致了新的物理機制與工程應用。以物理角度而言，我們可用一等效材料參數來表示。一旦等效參數確立，即可代入組成率方程式並研究其波傳行為。

本論文即已組成率為基礎下探討不一樣之波傳行為。由馬克斯威方程式，我們解析推導出色散關係、共振模態、阻抗與波因廷向量。之後考慮單一介面之波傳問題並利用模態解出反射與穿透係數。其中，針對其中一種非等效性材料:假對掌性材料做波傳研究。本研究發現到在假對掌性材料中波傳模態為兩個橢圓偏振且這兩個不同橢圓模態可分別導致負折射與後退波。最後並用數值高斯光束模擬來驗證此現象。

關鍵字: 超常材料、負折射、後退波、假對掌性材料、非等向性材料

Abstract

Meta-materials are man-made structures which exhibit unusual electromagnetic response. Such extraordinary responses give rise to new physical insights and engineered applications. The most common way to describe this kind of material is by defining effective material parameters connected to constitutive relations. Once constitutive relations are obtained, the wave propagating properties could be explored through basic electromagnetic theory.

In this thesis we investigate wave propagation based on different sets of constitutive relations. Dispersion relation, eigenwaves, impedance together with Poynting vector are derived. Also we derive reflection and transmission coefficients through a single planer interface. In particular, we study plane wave propagation in a special kind of bi-anisotropic medium: pseudochiral medium. It is found that two elliptic eigenwaves appear in pseudochiral material and allow us to realize negative refraction or backward wave. Finally, Gaussian beam propagation is conducted to verify our results.

Keywords: Meta-material, Negative refraction, Backward wave, Pseudo-chiral medium, Anisotropic complex medium.

Contents

Chapter 1 Introduction

Artificial meta-material having simultaneously negative parameters has drawn enormous attention since the first experimental fabrication by Smith [1] in 2000. The theoretical study of such medium was first presented by Veselago in 1967 and the term left-handed material was used to describe this kind of material. Significant physical phenomena and applications include negative refraction [2], backward wave [3], negative Goos-Hanchen shift [4], and the realization of perfect lens [5]. In 2004, Pendry shows that negative refraction could also be realized by isotropic chiral material [6] and its potential applications are then investigated by many researchers [7-9]. More recently, many more complex bi-anisotropic materials are widely theoretical studied to verify the possibility of negative refraction and backward wave [10-14].

Plane wave propagation through inhomogeneous media have long been an important topics in electromagnetic. Bassiri [15] dealt with reflection and transmission coefficients (R,T) of chiral medium in 1988. Later Tretyakov [16] studied R,T for general bi-anisotropic medium using transmission line theory. It is seen that impedance or admittance play crucial role in R T formulations.

In the present thesis, we study harmonic plane wave propagation in various complex media by scalar wave function with time convention $\exp(-i\omega t)$. Dispersion relations, eigenwave, impedance as well as Poynting vector are clearly derived for each case. Reflection and transmission coefficients through inhomogeneous media are also derived analytically and relations to negative refraction or backward waves are discussed. We present a general formula with obvious physical insight and compare with the existing works. Furthermore, we investigate the wave propagation properties in pseudochiral material, in which wave characteristics may differ from those we have known in other bi-anisotropic media. Finally, Gaussian beam propagation simulations are also demonstrated to easily clarify the phenomenon of negative refraction.

Chapter 2 Constitutive Relations

2.1 Constitutive equations

Effects of wave propagation through a linear, isotropic and homogenous medium have been widely studied based on Maxwell equation. Basic remarks include the wave vector, electric field and magnetic field are perpendicular to each other and form a right-handed system; dispersion relation relates the material's dielectric or magnetic susceptibility so that the index of refraction could be defined to estimate the wave velocity. Many materials, however, are anisotropic in nature. Anisotropy is taken into consideration by assuming different behaviors in each direction, in which the physical quantity produced is no longer constant but depends upon the direction. Another characteristic electromagnetic property is the cross effect or chirality, in which the electric and magnetic flux are influenced by magnetic and electric field. These two significant phenomena destroy the symmetry of constitutive relations and in general, could be written as:

$$\begin{aligned}\mathbf{D} &= \overline{\overline{\varepsilon}}\mathbf{E} + \overline{\overline{\xi}}\mathbf{H}\\ \mathbf{B} &= \overline{\overline{\mu}}\mathbf{H} + \overline{\overline{\zeta}}\mathbf{E}\end{aligned} \tag{2.1}$$

where $\overline{\overline{\varepsilon}}$ $\overline{\overline{\mu}}$ are dielectric and magnetic tensors and $\overline{\overline{\xi}}, \overline{\overline{\zeta}}$ are responsible for cross coupling. The term bi-anisotropy is used to describe materials with chiral property.

Note that constitutive relations could be expressed by other forms. Eq. (2.1) is called Condon-Tellegen relation. However, in different problems, the same name is given to parameters which are not exactly the same. Another form of constitutive relations are given by Post, Jaggard and Mickelson [17], which reads

$$\begin{aligned}\mathbf{D} &= \overline{\overline{\varepsilon}}\mathbf{E} + \overline{\overline{\xi}}\mathbf{B}\\ \mathbf{H} &= \overline{\overline{\mu}}^{-1}\mathbf{B} + \overline{\overline{\zeta}}\mathbf{E}\end{aligned} \tag{2.2}$$

Another relations focusing on the nonlocal behavior is given by Drude, which are called Drude-Born-Fedorov relations by Lakhtakia.

$$\begin{aligned}\mathbf{D} &= \overline{\overline{\varepsilon}}\left(\mathbf{E} + \beta\nabla\times\mathbf{E}\right) + \overline{\overline{\xi}}\mathbf{H}\\ \mathbf{B} &= \overline{\overline{\mu}}\left(\mathbf{H} + \beta\nabla\times\mathbf{H}\right) + \overline{\overline{\zeta}}\mathbf{E}\end{aligned} \tag{2.3}$$

One of the relations is adopted depending on which electric and magnetic quantities are used. However, different sets of relations could be related to each other and the results are given by Sihvola and Lindell [18]. Throughout this thesis, we use Condon-Tellegen relation Eq.(2.1) to analyze our problem.

2.2 Lossless, reciprocal medium

Restricting ourselves to lossless media, from conservation of energy we see that dielectric and magnetic tensors must be symmetric. Lossless implies there is no absorption during wave propagation and that every tensor element is real. According to coordinate transformation, every real symmetric matrix could be diagonalized as the form

$$\bar{\bar{\varepsilon}} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}$$
$$\bar{\bar{\mu}} = \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{pmatrix} \quad (2.4)$$

The above coordinate axis is referred to principal coordinate axis, which means that all off-diagonal elements are zero. Let us consider chiral medium. Chirality structure is often encountered in biochemistry. Assuming lossless and reciprocity still hold, from conservation of energy we must have

$$\bar{\bar{\xi}}^{T} = -\bar{\bar{\zeta}} \quad (2.5)$$

$$\bar{\bar{\xi}}^{*} = \bar{\bar{\zeta}} \quad (2.6)$$

which means that $\bar{\bar{\xi}} = -\bar{\bar{\zeta}}$ must be an imaginary number.

Chapter 3 Anisotropy medium

3.1 Dispersion relation

Confine ourselves to anisotropy medium but ignore the cross coupling effect due to chirality. Most materials possess certain degree of anisotropy and could be broadly categorized in terms of: cubic, uniaxial and biaxial as listed in Table 1

	Cubic	Uniaxial	Biaxial
Principal axis	$\begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}$	$\begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_x & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}$	$\begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}$
Material	Diamond	Quartz	Mica

Table 1

In general, from Eqs. (2.1)&(2.4), constitutive equations have the form:

$$\mathbf{D} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} \mathbf{E} \tag{3.1}$$

$$\mathbf{B} = \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{pmatrix} \mathbf{H} \tag{3.2}$$

Insert (5) (6) into Maxwell equations. The following eigenvalue equation is obtained:

$$[(\mathbf{p} \times \mathbf{I}) \bullet \overset{=}{\mu}^{-1} \bullet (\mathbf{p} \times \mathbf{I}) + \overset{=}{\varepsilon}] \cdot \mathbf{E} = 0 \tag{3.3}$$

$$\mathbf{p} = \mathbf{k} / \omega = 1/\omega (k \sin\theta \mathbf{e}_x + k \cos\theta \mathbf{e}_z)$$

where θ denotes angle of wave vector to z axis.

For nontrivial **E,** the determinant of the bracket must be zero, which gives two dispersion relations.

$$\frac{k_x^{\ 2}}{\varepsilon_z} + \frac{k_z^{\ 2}}{\varepsilon_x} = \omega^2 \mu_y \tag{3.4}$$

$$k = \omega \sqrt{\frac{\varepsilon_x \varepsilon_z \mu_y}{\varepsilon_x \sin^2\theta + \varepsilon_z \cos^2\theta}} = k_0 \sqrt{\frac{\varepsilon_x \varepsilon_z \mu_y}{\varepsilon_x \sin^2\theta + \varepsilon_z \cos^2\theta}} \tag{3.5}$$

where $$k_0^{\,2} = \omega^2 \mu_0 \varepsilon_0$$

and

$$\frac{k_x^{\,2}}{\mu_z} + \frac{k_z^{\,2}}{\mu_x} = \omega^2 \varepsilon_y \tag{3.6}$$

$$k = \omega \sqrt{\frac{\mu_x \mu_z \varepsilon_y}{\mu_x \sin^2\theta + \mu_z \cos^2\theta}} = k_0 \sqrt{\frac{\mu_x \mu_z \varepsilon_y}{\mu_x \sin^2\theta + \mu_z \cos^2\theta}} \tag{3.7}$$

From above derivation two wave numbers could be obtained and both of them depend not only on the medium parameters but on propagation direction. To examine our results, simply set $\varepsilon_x = \varepsilon_y = \varepsilon_z$ and $\mu_x = \mu_y = \mu_z$ such that two wave numbers are identical and is not a function of propagation direction, which is an isotropic medium and confirm with elementary electromagnetic theory.

3.2 Eigenmodes solution

Via dispersion relation two wave numbers have been found in term of θ. However, further derivations show that it is easier to express wave number in way of k_x because it is assumed that k_x is a known quantity in later problems. Let us rewrite wave numbers and solve eigenvalue problem for Eq.(3.3). Physically, the corresponding eigenvectors indicate the modes.

From (3.4)

$$p_z = \sqrt{\frac{\varepsilon_x \varepsilon_z \mu_y - p_x^{\,2} \varepsilon_x}{\varepsilon_z}} \tag{3.8}$$

$$\mathbf{E} = E\left(1,\ 0,\ -\frac{\varepsilon_x p_x}{\varepsilon_z p_z}\right) \tag{3.9}$$

From (3.6)

$$p_z = \sqrt{\frac{\mu_x \mu_z \varepsilon_y - p_x^{\,2} \mu_x}{\mu_z}} \tag{3.10}$$

$$\mathbf{E} = E(0,1,0) \tag{3.11}$$

3.3 Impedance analogy

Once electric polarized mode is known, Maxwell equations give the relation between magnetic field and electric field. Transverse impedance is defined as the ratio of transverse electric field to magnetic field.

$$\mathbf{H} = \overline{\overline{\mu}}^{-1}\mathbf{B} = \overline{\overline{\mu}}^{-1}\frac{1}{\omega}\mathbf{k}\times\mathbf{E} = \frac{1}{\omega}(\overline{\overline{\mu}}^{-1}\cdot\mathbf{k}\times\mathbf{I})\mathbf{E} \qquad (3.12)$$

For TM mode $\mathbf{E} = E(1,0,-\frac{\varepsilon_z p_z}{\varepsilon_x p_x})$

$$H_y = \frac{E_x k_z - E_z k_x}{\omega\mu_y} = \frac{p_z^{\,2} + \frac{\varepsilon_x}{\varepsilon_z}p_x^{\,2}}{p_z\mu_y}E_o \qquad (3.13)$$

$$\eta^{TM} = \frac{E_x}{H_y} = \frac{\sqrt{\varepsilon_x\mu_y - p_x^{\,2}\varepsilon_x/\varepsilon_z}}{\varepsilon_x} = \frac{p_z}{\varepsilon_x} \qquad (3.14)$$

For TE mode $\mathbf{E} = E(0,1,0)$

$$H_x = \frac{\sqrt{\mu_x\varepsilon_y - p_x^{\,2}\mu_x/\mu_z}}{\mu_x} = \frac{-p_z}{\mu_x} \qquad (3.15)$$

$$H_z = \frac{p_x}{\mu_z}$$

$$\eta^{TE} = \frac{E_y}{H_x} = \frac{-\mu_x}{p_z} \qquad (3.16)$$

3.4 Poynting vector

Time average Poynting vector indicate the direction of energy flow and is significant when considering plane wave refraction in inhomogeneous media.

TM mode from Eq.(3.9)

$$\mathbf{S} = \begin{pmatrix} S_x \\ S_z \end{pmatrix} = \begin{pmatrix} \frac{k_x}{\omega\varepsilon_z} \\ \frac{k_z}{\omega\varepsilon_x} \end{pmatrix} H_y^{\,2} \qquad (3.17)$$

TE mode from Eq.(3.11)

$$\mathbf{S} = \begin{pmatrix} S_x \\ S_z \end{pmatrix} = \begin{pmatrix} \frac{k_x}{\omega\mu_z} \\ \frac{k_z}{\omega\mu_x} \end{pmatrix} E^2 \qquad (3.18)$$

3.5 Reflection and transmission in inhomogeneous media

Wave vector, dispersion relations, impedance and poynting vectors are derived to solve reflection and refraction problem with inhomogeneous media. Consider a plane wave from a dielectric material entering an anisotropic medium as follow in Fig 1:

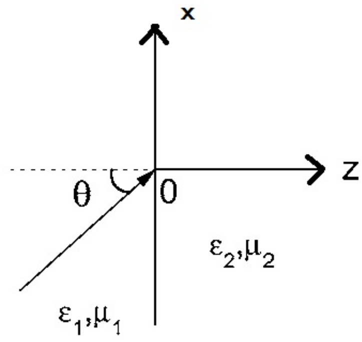


Fig 1

The incident electric and magnetic field could be written as

$$\mathbf{E_i} = E_{im}\left(\cos\theta\mathbf{e}_x - \sin\theta\mathbf{e}_z\right) + E_{ie}\mathbf{e}_y \tag{3.19}$$

$$\mathbf{H_i} = E_{im}/\eta_0 \cdot \mathbf{e}_y + E_{ie}/\eta_0(-\cos\theta\cdot\mathbf{e}_x + \sin\theta\cdot\mathbf{e}_z) \tag{3.20}$$

where E_{im} and E_{ie} refer to the magnitude of TM and TE incidence respectively. Impedance η_0 is defined with respect to incident medium. Similarly, the reflected electric and magnetic field could be written as

$$\mathbf{E_r} = E_{rm}(\cos\theta\cdot\mathbf{e}_x + \sin\theta\cdot\mathbf{e}_z) + E_{re}\mathbf{e}_y \tag{3.21}$$

$$\mathbf{H_r} = -E_{rm}/\eta_0 \cdot \mathbf{e}_y + E_{re}/\eta_0(\cos\theta\cdot\mathbf{e}_x + \sin\theta\cdot\mathbf{e}_z) \tag{3.22}$$

To find the reflection and transmission coefficients E_{rm} and E_{re} it is assumed that the refracted modes are unity in magnitude so that R and T could be regarded directly as the amplitude ratio with respect to incidence field. Therefore, transmitted electric and magnetic fields are expressed as

$$\mathbf{E_t} = E_{te}\cdot\mathbf{e}_y + E_{tm}(\mathbf{e}_x - \frac{\varepsilon_x p_x}{\varepsilon_z p_z}\mathbf{e}_z) \tag{3.23}$$

$$\mathbf{H_t} = \frac{E_{te}}{\eta^{TE}}(\mathbf{e}_x + A\mathbf{e}_z) + \frac{E_{tm}}{\eta^{TM}}\cdot\mathbf{e}_y \tag{3.24}$$

where $\eta^{TM} = \frac{\sqrt{\varepsilon_x\mu_y - p_x^{\,2}\varepsilon_x/\varepsilon_z}}{\varepsilon_x}$ and $\eta^{TE} = \frac{\mu_x}{\sqrt{\mu_x\varepsilon_y - p_x^{\,2}\mu_x/\mu_z}}$ denote TE and TM modes impedance respectively.

Now apply Maxwell boundary condition to obtain reflection and refraction coefficients.

$$
\begin{aligned}
E_{rm} &= \frac{\eta^{TM} - \eta_0 \cos\theta}{\eta^{TM} + \eta_0 \cos\theta} E_{im} \\
E_{tm} &= \frac{2\eta^{TM} \cos\theta}{\eta^{TM} + \eta_0 \cos\theta} E_{im}
\end{aligned}
\tag{3.25}
$$

$$
\begin{aligned}
E_{re} &= \frac{\eta^{TE} \cos\theta - \eta_0}{\eta_0 + \eta^{TE} \cos\theta} E_{ie} \\
E_{te} &= \frac{2\eta^{TE} \cos\theta}{\eta_0 + \eta^{TE} \cos\theta} E_{ie}
\end{aligned}
\tag{3.26}
$$

Follow through Eqs.(3.25)&(3.26), the reflection and transmission coefficients with respect to incident angle could be obtained. Certain examples are shown for $\varepsilon_x = \varepsilon_z = 2.25$ $\varepsilon_x = -\varepsilon_z = 2.25$ in Fig 2&Fig 3.

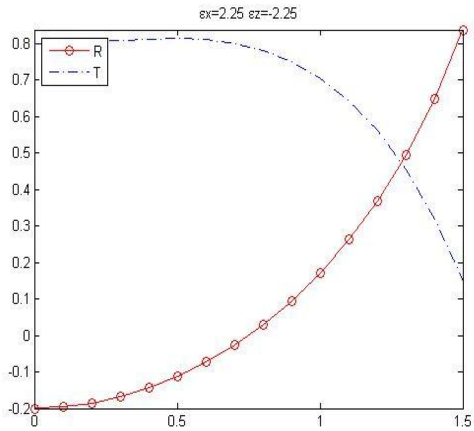


Fig 2

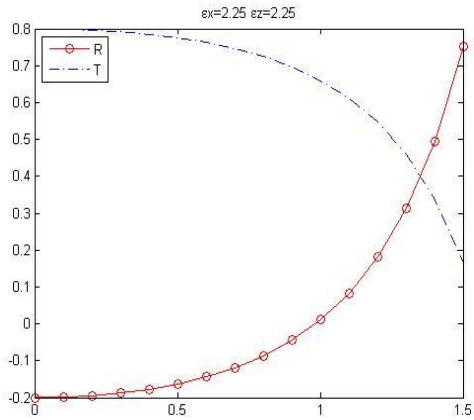


Fig 3

3.6 Negative refraction and backward wave

Many interesting phenomenon emerge when electromagnetic waves incident from a dielectric material to an anisotropic medium. Two of which are negative refraction and backward wave. The definitions of negative refraction and backward wave have been given elsewhere [3]. In brief, negative refraction could be defined when the wave vector in the transmitted medium has an opposite direction with respect to the wave vector along interfacial direction and a plane wave is said to be backward wave if the wave vector has a negative projection onto the Poynting vector. Now suppose permittivity and permeability tensor are along principal axis as defined by Eq.(2.4) and could be negative. Reflection and transmission coefficients, however, could not totally obey Eqs.(3.25)& (3.26) due to the choice of sign of wave vector k_z in Eq.(3.4) or (3.6) and need to be derived in terms of incidence wave vector k_i. Detailed derivations and physical insight have been studied by I.V.Lindell, S.A.Tretyakov et,al [3] and P.A.Belov [14]. The following shows result for case with $\varepsilon_x = 2.25$ $\varepsilon_z = -2.25$ in Fig 4.

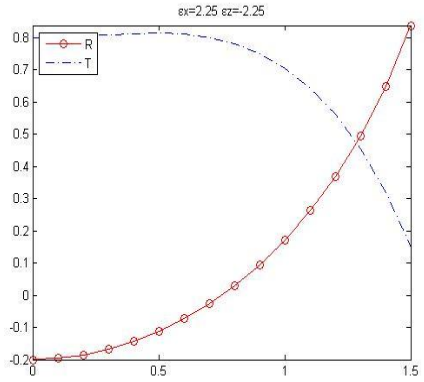


Fig 4

3.7 Slab problem

3.7.1 Layered media

Layered media with multiple interfaces have been an important issue in electromagnetic due to its wide range of application. One of which is to determine the reflection and refraction coefficients and then retrieve material properties by inverse scattering technique. Consider a plane wave normally incident upon a layered structure ABC. Following the method used in[19],the expressions for the fields could be written as

In medium A: (z<-d)

$$
\begin{aligned}
E_i &= E_i e^{ik_1(z+d)} x \\
H_i &= \frac{1}{\eta_A} E_i e^{ik_1(z+d)} y \\
E_r &= E_r e^{-ik_1(z+d)} x \\
H_r &= \frac{-1}{\eta_A} E_r e^{-ik_1(z+d)} y
\end{aligned}
\qquad (3.27)
$$

In medium B: (-d<z<0)

$$
\begin{aligned}
E_B &= E_{Bt} e^{ik_2 z} + E_{Br} e^{-ik_2 z} x \\
H_B &= \frac{1}{\eta_B}\left[E_{Bt} e^{ik_2 z} - E_{Br} e^{-ik_2 z}\right] y
\end{aligned}
\qquad (3.28)
$$

In medium C: (z>0)

$$E_C = E_t e^{ik_3 d} x$$
$$H_C = \frac{1}{\eta_C} E_t e^{ik_3 d} y \tag{3.29}$$

where the wave component E_{Bt} and E_{Br} in medium B are forward and backward wave respectively. Using Maxwell's boundary conditions requiring tangential components of electric and magnetic fields be continuous at two interfaces, we have

At z=-d

$$(E_i + E_r) = E_{Bt} e^{-ik_2 d} + E_{Br} e^{ik_2 d}$$
$$\frac{1}{\eta_A}\left(E_i - E_r\right) = \frac{1}{\eta_B}\left(E_{Bt} e^{-ik_2 d} - E_{Br} e^{ik_2 d}\right) \tag{3.30}$$

At z=0

$$E_{Bt} + E_{Br} = E_t$$
$$\frac{1}{\eta_B}\left[E_{Bt} - E_{Br}\right] = \frac{1}{\eta_C} E_t \tag{3.31}$$

Solve four unknowns based on four equations, we get

$$E_r = \frac{(\eta_2 - \eta_1)(\eta_3 + \eta_2) + (\eta_2 + \eta_1)(\eta_3 - \eta_2) e^{2ik_2 d}}{(\eta_2 + \eta_1)(\eta_3 + \eta_2) + (\eta_2 - \eta_1)(\eta_3 - \eta_2) e^{2ik_2 d}}$$
$$E_t = \frac{4\eta_2 \eta_3 e^{ik_2 d}}{(\eta_2 + \eta_1)(\eta_3 + \eta_2) + (\eta_2 - \eta_1)(\eta_3 - \eta_2) e^{2ik_2 d}}$$
$$E_{Br} = \frac{-i\eta_2(\eta_2 - \eta_3)}{i\eta_2(\eta_1 + \eta_3)\cos(k_2 d) + \eta_2^{\ 2} + (\eta_1 \eta_3)\sin(k_2 d)} \tag{3.32}$$
$$E_{Bt} = \frac{i\eta_2(\eta_2 + \eta_3)}{i\eta_2(\eta_1 + \eta_3)\cos(k_2 d) + \eta_2^{\ 2} + (\eta_1 \eta_3)\sin(k_2 d)}$$

Although the above formulas are based on dielectric material, lossy medium is also applicable if impedance is in term of complex value. Our result can still be verified by transmission line theory.

3.7.2 Effective medium for periodic grating structure

Consider a plane wave normally incident to a periodic grating structure with $\theta_i = 0$ as below in Fig 5

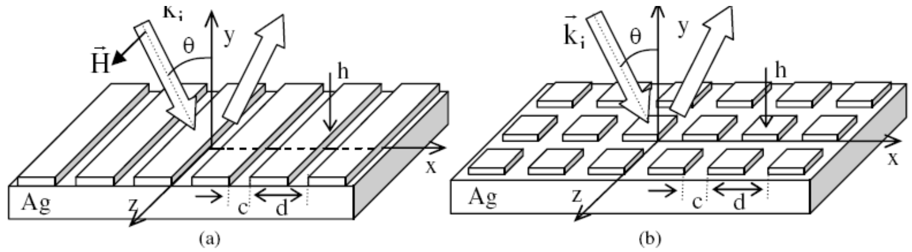


Fig 5 Periodic grating structure in one (a) and two (b) dimensions. Substrate is silver and the coating film is silver-air array with height h and periodicity d.

Suppose the dimension of geometry structure is much smaller than the excitation wavelength. Such material is termed metamaterial and for one dimension periodicity, could be characterized as uniaxial anisotropic medium with effective longitudinal and transverse permeability.

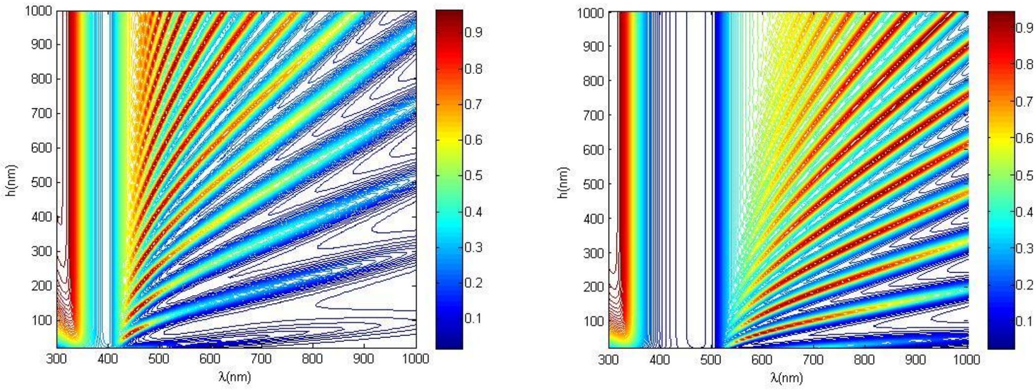
$$\varepsilon_x = \frac{d}{(d-c)/\varepsilon_m + c/\varepsilon_a}$$
$$\varepsilon_y = \varepsilon_z = \frac{d-c}{d}\varepsilon_m + \frac{c}{d}\varepsilon_a \tag{3.33}$$

where

ε_m : permeability of silver

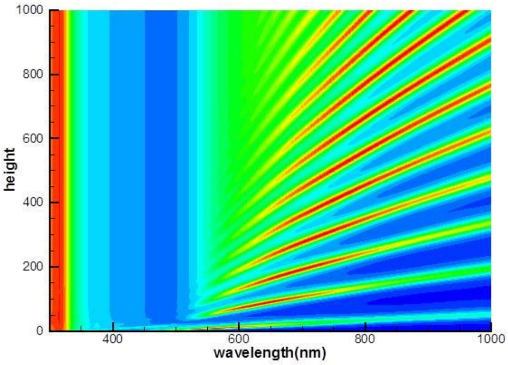
ε_a : permeability of air.

We investigate the metamaterial absorption behavior with regard to wavelength of incident wave as well as height of the structure and check whether effective theory is valid. Let incident wavelength range from 300-1000nm and height from 0-1000nm. We scan for absorption spectrum. Using Eq.(3.33) as layer B impedance, we demonstrate that the smaller periodicity d compared to wavelength, the better the effective medium in Fig 6 resemble real structure Fig 7.

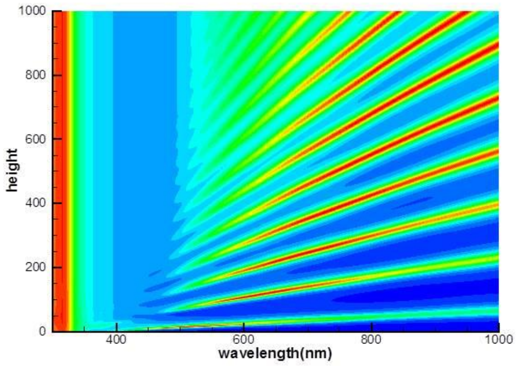


(a) (b)

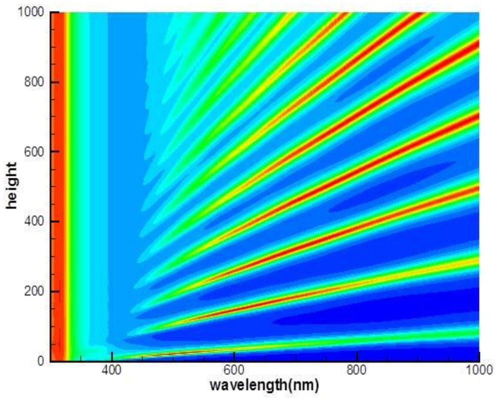
Fig 6 Absorption spectrum of effective medium using c=0.1d (a) and c=0.2d (b). Note that effective material parameter only depend on volume fraction of air and silver, regardless of absolute value of d.



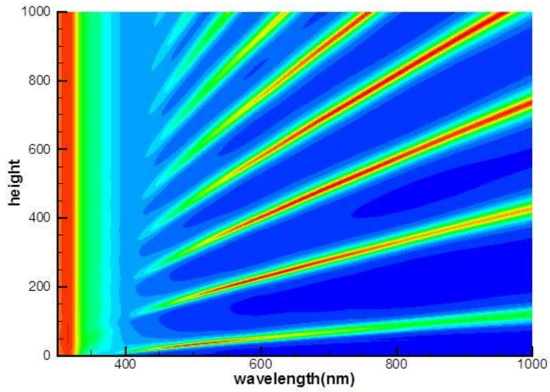
(a)



(b)



(c)



(d)

Fig 7 Absorption spectrum of real structure with c=0.1d while d=10nm (a), d=50nm (b), d=100nm (c) and d=300nm (d).

Chapter 4 Chiral medium

4.1 Dispersion relation

Consider bi-isotropic chiral medium with cross coupling but ignore the effect of anisotropy. Isotropic implies the physical property doesn’t depend on direction. Therefore, the material parameters in constitutive relations could be regarded as scalar quantities, which have the following forms

$$\mathbf{D} = \begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix} \mathbf{E} + \begin{pmatrix} i\gamma & 0 & 0 \\ 0 & i\gamma & 0 \\ 0 & 0 & i\gamma \end{pmatrix} \mathbf{H} \tag{4.1}$$

$$\mathbf{B} = \begin{pmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{pmatrix} \mathbf{H} - \begin{pmatrix} i\gamma & 0 & 0 \\ 0 & i\gamma & 0 \\ 0 & 0 & i\gamma \end{pmatrix} \mathbf{E} \tag{4.1}$$

Follow the same procedure as in anisotropic case and assume wave vector lie on x-z plane. Using Eqs. (4.1) and (4.1) into Eq.(3.3), two wave vectors are given by

$$k_1 = \omega\left(\gamma + \sqrt{\mu\varepsilon}\right) \tag{4.2}$$

$$k_2 = \omega\left(-\gamma + \sqrt{\mu\varepsilon}\right) \tag{4.3}$$

Note that the wave vectors in the isotropic chiral medium are not function of direction angle, which is quite different from anisotropy cases considered in the previous chapter. For nonchiral materials ($\gamma = 0$), the two propagation vectors coincide and become $\omega\sqrt{\mu\varepsilon}$, which is well known in the elementary electromagnetic theory.

4.2 Eigenmodes solution

Once the eigenvalue is obtained, we could go for the eigenvector and determine the wave propagation modes in the material. Based on the results given by Eqs.(4.2) &(4.3), two eigenvectors could be obtained.

For $k_1 = \omega\left(\gamma + \sqrt{\mu\varepsilon}\right)$

$$\mathbf{E}_1 = E(\cos\theta, i, -\sin\theta) \tag{4.4}$$

For $k_2 = \omega\left(-\gamma + \sqrt{\mu\varepsilon}\right)$

$$\mathbf{E}_2 = E(\cos\theta, -i, -\sin\theta) \tag{4.5}$$

From above derivation it can be easily seen that two circularly polarized wave serve as modes for corresponding two wave vectors. The speeds of right circular polarized mode (RHCP) and left circular polarized mode (LHCP) are different due to parameter γ. Note that TE and TM modes cannot be taken as eigenvectors in chiral medium, which is different from dielectric material.

4.3 Impedance analogy

Impedance could be obtained similar to Eq.(3.12). For bi-isotropic material, the results are even simpler.

$$\mathbf{H} = \bar{\bar{\mu}}^{-1}\left(\frac{\bar{k}\times\bar{\bar{I}}}{\omega} - \bar{\bar{\xi}}\right)\mathbf{E} = \bar{\bar{\mathbf{A}}}\mathbf{E} = \begin{pmatrix} \frac{-i\gamma}{\mu} & \frac{-k_z}{\omega\mu} & 0 \\ \frac{k_z}{\omega\mu} & \frac{-i\gamma}{\mu} & \frac{-k_x}{\omega\mu} \\ 0 & \frac{k_x}{\omega\mu} & \frac{-i\gamma}{\mu} \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \tag{4.6}$$

Inserting Eqs.(4.4)&(4.5) into Eq.(4.6), impedance could be obtained.

For $k_1 = \omega\left(\gamma + \sqrt{\mu\varepsilon}\right)$ (RHCP)

$$\mathbf{H}_1 = -i\sqrt{\frac{\varepsilon}{\mu}}\left(\cos\theta, i, -\sin\theta\right) = -i\frac{\mathbf{E}_1}{\eta_1}$$

$$\eta_1 = \sqrt{\frac{\mu}{\varepsilon}} \tag{4.7}$$

For $k_1 = \omega\left(\sqrt{\mu\varepsilon} - \gamma\right)$ (LHCP)

$$\mathbf{H}_2 = i\sqrt{\frac{\varepsilon}{\mu}}\left(\cos\theta, -i, -\sin\theta\right) = i\frac{\mathbf{E}_2}{\eta_2}$$

$$\eta_2 = \sqrt{\frac{\mu}{\varepsilon}} \tag{4.8}$$

Note that $k_x = k\sin\theta, k_z = k\cos\theta$ in above derivation and two eigenmodes share the same impedance. This could be seen as the result of isotropic medium.

4.4 Poynting vector

Time-averaged Poynting vector is defined as $\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}\times\mathbf{H}^*\right]$

For $k_1 = \omega\left(\gamma + \sqrt{\mu\varepsilon}\right)$ (RHCP)

$$\mathbf{S} = \begin{pmatrix} S_x \\ S_z \end{pmatrix} = \begin{pmatrix} \dfrac{\sqrt{\mu\varepsilon}+\gamma}{p_x} \\ \dfrac{p_z(\sqrt{\mu\varepsilon}+\gamma)}{p_x} \end{pmatrix} E^2 = \begin{pmatrix} \sqrt{\dfrac{\mu}{\varepsilon}}\sin\theta \\ \sqrt{\dfrac{\mu}{\varepsilon}}\cos\theta \end{pmatrix} E^2 \tag{4.9}$$

For $k_2 = \omega\left(\sqrt{\mu\varepsilon} - \gamma\right)$ (LHCP)

$$\mathbf{S} = \begin{pmatrix} S_x \\ S_z \end{pmatrix} = \begin{pmatrix} \sqrt{\dfrac{\mu}{\varepsilon}}\sin\theta \\ \sqrt{\dfrac{\mu}{\varepsilon}}\cos\theta \end{pmatrix} E^2 \tag{4.10}$$

Isotropic material shows that the direction of energy flow is the same as direction of wave vector, which means the energy and momentum flows are parallel.

4.5 Reflection and transmission in inhomogeneous media

Consider plane wave problem in inhomogeneous layered medium. An incident plane wave from isotropic medium causes reflection and transmission phenomenon when passing through the interface. Suppose an incident wave can be decomposed into TE and TM modes, which can be written as

$$\mathbf{E_i} = E_{im}\left(\cos\theta_i \mathbf{e}_x - \sin\theta_i \mathbf{e}_z\right) + E_{ie}\mathbf{e}_y \tag{4.11}$$

$$\mathbf{H_i} = \frac{E_{im}}{\eta_0}\mathbf{e}_y + \frac{E_{ie}}{\eta_0}(-\cos\theta_i \mathbf{e}_x + \sin\theta_i \mathbf{e}_z) \tag{4.12}$$

where subscripts m and e refer to TM and TE incidence. Reflected wave, however, are not TE or TM wave as in the nonchiral case. Instead, the reflected wave need to be expressed as the elliptical wave form:

$$\begin{aligned} \mathbf{E}_r &= E_{r\perp}\mathbf{e}_y + E_{r\parallel}(\cos\theta_i \mathbf{e}_x + \sin\theta_i \mathbf{e}_z) \\ \mathbf{H}_r &= \frac{1}{\eta_0}(-E_{r\parallel}\mathbf{e}_y + E_{r\perp}(\cos\theta_i \mathbf{e}_x + \sin\theta_i \mathbf{e}_z)) \end{aligned} \tag{4.13}$$

Transmitted waves are the linear combination of the two circularly polarized waves, which has the form:

$$\mathbf{E}_t = \mathbf{E}_{tR} e^{ik_1(z\cos\theta_1 + x\sin\theta_1)} + \mathbf{E}_{tL} e^{ik_2(z\cos\theta_2 + x\sin\theta_2)} \tag{4.14}$$

where

$$\begin{aligned} \mathbf{E}_{t1} &= E_{tR}(\cos\theta_1 \mathbf{e}_x - \sin\theta_1 \mathbf{e}_z + i\mathbf{e}_y) \\ \mathbf{E}_{t2} &= E_{tL}(\cos\theta_2 \mathbf{e}_x - \sin\theta_2 \mathbf{e}_z - i\mathbf{e}_y) \end{aligned} \tag{4.15}$$

$\mathbf{E}_{t1}$ and $\mathbf{E}_{t2}$ correspond to right and left circular wave respectively. Note that when oblique incidence is taken into account, two polarized wave have different refracted angles due to different values of wave number. Here θ_1 and θ_2 denote refracted angles with respect to RHCP and LHCP. With impedance, magnetic field has the form

$$\mathbf{H}_t = \frac{-i}{Z} \mathbf{E}_{t1} e^{ik_1(z\cos\theta_1 + x\sin\theta_1)} + \frac{i}{Z} \mathbf{E}_{t2} e^{ik_2(z\cos\theta_2 + x\sin\theta_2)} \tag{4.16}$$

where impedance $Z = \sqrt{\dfrac{\mu}{\varepsilon}}$

With Maxwell boundary condition, reflection and refracted coefficients could be obtained.

Reflection coefficients:

$$\begin{pmatrix} E_{r\perp} \\ E_{r\parallel} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} E_{i\perp} \\ E_{i\parallel} \end{pmatrix} \tag{4.17}$$

where

$$\begin{aligned} R_{11} &= \frac{(Z^{Tm2}+\eta^{Tm2})(Z^{Te1}-\eta^{Te1}) + (Z^{Tm1}+\eta^{Tm1})(Z^{Te2}-\eta^{Te2})}{(Z^{Tm2}+\eta^{Tm2})(Z^{Te1}+\eta^{Te1}) + (Z^{Tm1}+\eta^{Tm1})(Z^{Te2}+\eta^{Te2})} \\ R_{12} &= \frac{-2i\eta^{Tm}(Z^{Tm1}-Z^{Tm2})}{(Z^{Tm2}+\eta^{Tm2})(Z^{Te1}+\eta^{Te1}) + (Z^{Tm1}+\eta^{Tm1})(Z^{Te2}+\eta^{Te2})} \\ R_{21} &= \frac{-2i\eta^{Tm}(Z^{Tm1}-Z^{Tm2})}{(Z^{Tm2}+\eta^{Tm2})(Z^{Te1}+\eta^{Te1}) + (Z^{Tm1}+\eta^{Tm1})(Z^{Te2}+\eta^{Te2})} \\ R_{22} &= \frac{(Z^{Tm2}-\eta^{Tm2})(Z^{Te1}+\eta^{Te1}) + (Z^{Tm1}-\eta^{Tm1})(Z^{Te2}+\eta^{Te2})}{(Z^{Tm2}+\eta^{Tm2})(Z^{Te1}+\eta^{Te1}) + (Z^{Tm1}+\eta^{Tm1})(Z^{Te2}+\eta^{Te2})} \end{aligned} \tag{4.18}$$

where

$$Z^{Tm1} = Z\cos\theta_1, Z^{Tm2} = Z\cos\theta_2, Z^{Te1} = Z^{Te2} = Z\cos\theta_i$$

$$\eta^{Tm1} = \eta^{Tm2} = \eta_0\cos\theta_i, \eta^{Te1} = \eta_0\cos\theta_1, \eta^{Te2} = \eta_0\cos\theta_2$$

Transmitted coefficients:

$$\begin{pmatrix} E_{t1} \\ E_{t2} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} E_{i\perp} \\ E_{i\parallel} \end{pmatrix} \tag{4.19}$$

$$T_{11} = \frac{-2iZ^{Te}(\eta^{Tm2} + Z^{Tm2})}{(Z^{Tm2} + \eta^{Tm2})(Z^{Te1} + \eta^{Te1}) + (Z^{Tm1} + \eta^{Tm1})(Z^{Te2} + \eta^{Te2})}$$
$$T_{12} = \frac{2Z^{Te}(Z^{Te2} + \eta^{Te2})}{(Z^{Tm2} + \eta^{Tm2})(Z^{Te1} + \eta^{Te1}) + (Z^{Tm1} + \eta^{Tm1})(Z^{Te2} + \eta^{Te2})}$$
$$T_{21} = \frac{2iZ^{Te}(\eta^{Tm1} + Z^{Tm1})}{(Z^{Tm2} + \eta^{Tm2})(Z^{Te1} + \eta^{Te1}) + (Z^{Tm1} + \eta^{Tm1})(Z^{Te2} + \eta^{Te2})} \quad (4.20)$$
$$T_{22} = \frac{2Z^{Te}(Z^{Te1} + \eta^{Te1})}{(Z^{Tm2} + \eta^{Tm2})(Z^{Te1} + \eta^{Te1}) + (Z^{Tm1} + \eta^{Tm1})(Z^{Te2} + \eta^{Te2})}$$

where

$$Z^{Tm1} = Z\cos\theta_1, Z^{Tm2} = Z\cos\theta_2, Z^{Te1} = Z^{Te2} = Z\cos\theta_i$$
$$\eta^{Tm1} = \eta^{Tm2} = \eta_0\cos\theta_i, \eta^{Te1} = \eta_0\cos\theta_1, \eta^{Te2} = \eta_0\cos\theta_2$$

Our results are identical to that given by Bassiri [15] using Post-Jaggard relations. Here $R_{12}, R_{21}, T_{12}, T_{21}$ are cross polarizations due to the nature of the circular wave and denominator in each coefficient could be viewed as TM TE coupling. In other words, because both circular waves could be seen as a linear combination of TE and TM wave with phase difference 90 degree, two circular eigenmodes appear and linear polarizations are no longer independent basis. However, physical meaning is obvious if we regard chiral materials as a generalization of isotropic material. Suppose $\gamma = 0$, two wave vectors degenerate and only repeated mode exists. From Eqns.(4.18) and (4.20) we reproduce reflection and transmission formula, given by

$$R_{11} = \frac{(Z^{Te} - \eta^{Te})}{(Z^{Te} + \eta^{Te})}$$
$$R_{12} = R_{21} = 0 \quad (4.21)$$
$$R_{22} = \frac{(Z^{Tm} - \eta^{Tm})}{(Z^{Tm} + \eta^{Tm})}$$

$$T_{11} = \frac{-iZ^{Te}}{(Z^{Te} + \eta^{Te})}$$
$$T_{12} = \frac{Z^{Te}}{(Z^{Tm} + \eta^{Tm})}$$
$$T_{21} = \frac{iZ^{Te}}{(Z^{Te} + \eta^{Te})} \quad (4.22)$$
$$T_{22} = \frac{Z^{Te}}{(Z^{Tm} + \eta^{Tm})}$$

where $Z^{Tm} = Z\cos\theta_t, Z^{Te} = Z\cos\theta_i, \eta^{Tm} = \eta_0\cos\theta_i, \eta^{Te} = \eta_0\cos\theta_t$. Cross reflection coefficients vanish and transmitted coefficients are derived for circular modes. Note that in an isotropic medium, both linear and circular polarized modes could serve as basis.

Reflection and transmission coefficients as function of incidence angle are plotted below in Fig 8 through (d) for case of free space entering chiral with parameters $\varepsilon = 4, \mu = 2, \gamma = 0.5$

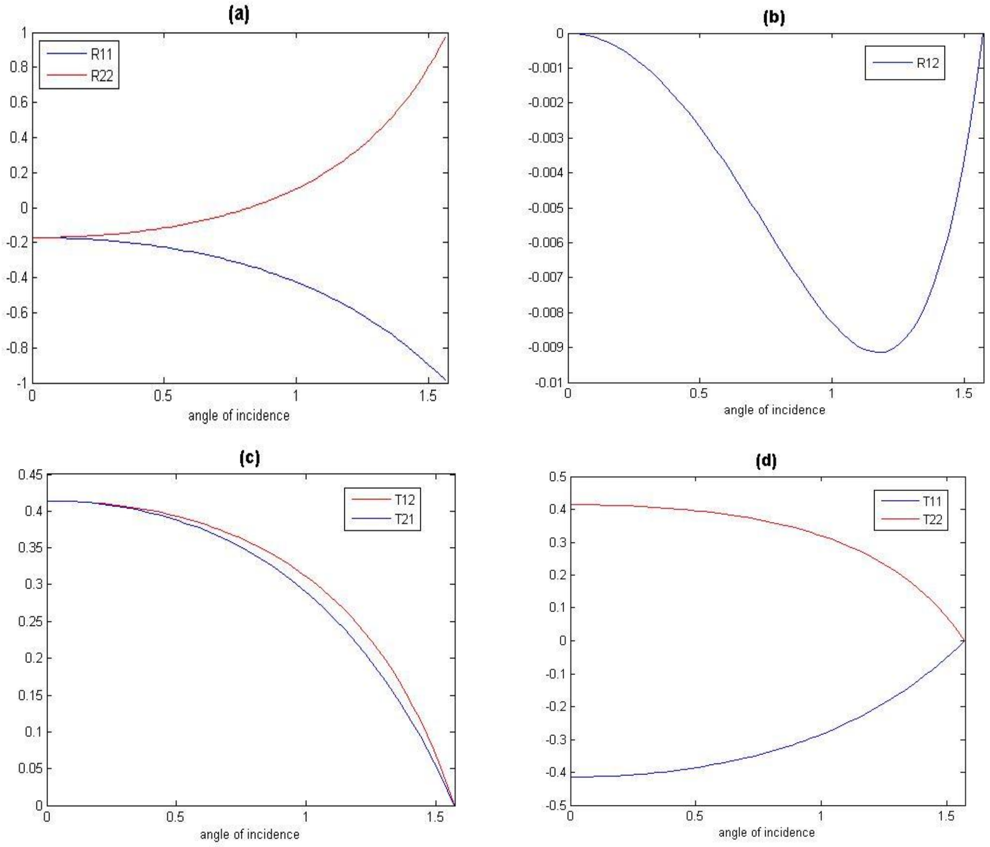


Fig 8 Reflection and transmission coefficients with respect to incident angle of a plane wave incident from free space to a chiral medium with $\varepsilon = 4, \mu = 2, \gamma = 0.5$

We see that reflection cross coefficient $R_{12} = R_{21}$ so that reciprocity theorem is satisfied while transmitted cross coefficient $T_{12} \neq T_{21}$.

Two circular waves determine the field pattern in transmitted medium. Fig 9 shows the schematic diagram of wave propagating from free space to a chiral medium and Fig 10 is the real TE field component (perpendicular to plane of incidence) for an TE oblique incident plane wave with incident angle $\theta_i = 40^0$ coming from free space to a chiral material with $\mu = \varepsilon = 2, \gamma = 0.6$.

Fig 9 (a) shows that the left hand space is the total field of incident and reflected wave while the two circular waves combine for the right hand space. For nonchiral case, $\varepsilon = \mu = 2, \gamma = 0$, two eigenwaves degenerate and only one transmitted wave is observed as expected from basic electromagnetic in Fig 9(b).

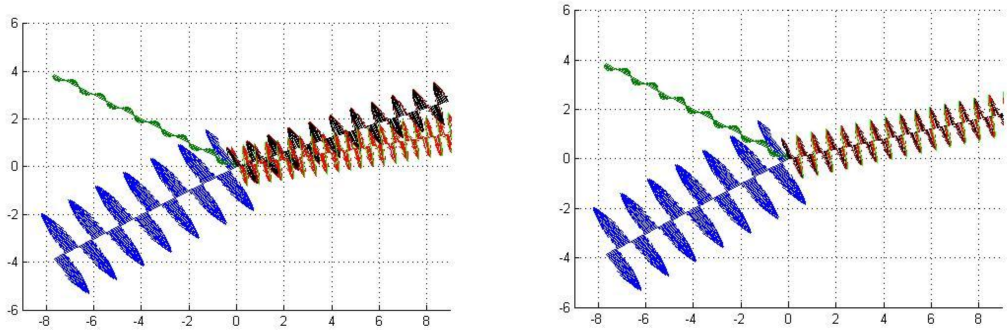


Fig 9 Schematic plot of wave propagation to a chiral (a) and nonchiral (b) medium. Incident wave (blue) split into two circular waves (black and red) while in nonchiral case only one transmitted wave observed.

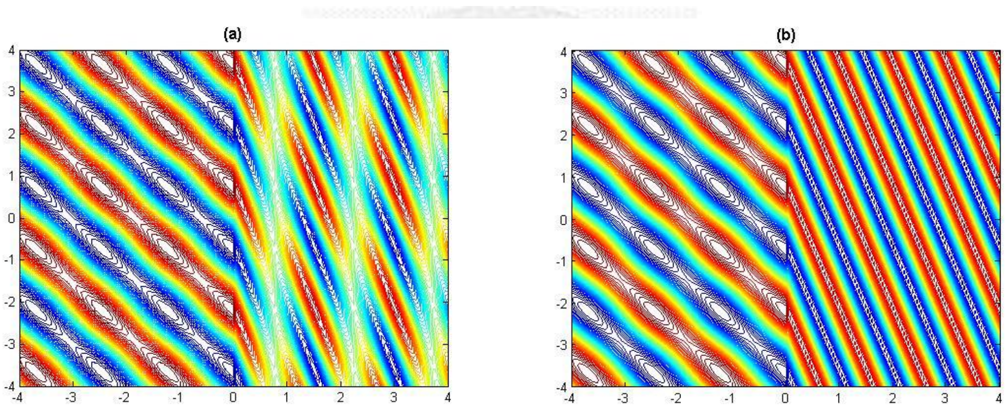


Fig 10 Electric field pattern of a plane wave incident from free space to an inhomogeneous media. (a) Chiral medium results in two circular waves interaction.($\theta_i = 40^\circ, \theta_1 = 14.3^\circ, \theta_2 = 27.3^\circ$) (b) Dielectric medium ($\varepsilon = \mu = 2, \gamma = 0$) with single mode.

4.6 Negative refraction

Special attentions should be given to the left circularly polarized wave in the light of $k_2 = \omega\left(-\gamma + \sqrt{\mu\varepsilon}\right)$, which shows that a negative wave number may emerge when the chiral effect is stronger and negative refraction is possible [9, 20]. Because only the left circularly polarized wave accounts for the negative refraction, the transmitted coefficients correspond to E_{t1} must be small compared with E_{t2}. After some efforts of observation it is concluded that if the incidence wave is a left circular wave, the transmitted coefficients may vanish for the right circularly polarized wave in some special cases. We derive criteria as follow:

Suppose LHCP incidence wave:

$$\mathbf{E}_i = E_{iL}\left(\cos\theta_i\mathbf{e}_x - i\mathbf{e}_y - \sin\theta_i\mathbf{e}_z\right) \tag{4.23}$$

$$\mathbf{H}_i = \frac{E_{iL}}{\eta_0} i(\cos\theta_i\mathbf{e}_x - i\mathbf{e}_y - \sin\theta_i\mathbf{e}_z) \tag{4.24}$$

Refracted wave could be decomposed to RHCP and LHCP components:

$$\mathbf{E}_r = E_{rR}\left(\cos\theta_i\mathbf{e}_x + i\mathbf{e}_y - \sin\theta_i\mathbf{e}_z\right) + E_{rL}\left(\cos\theta_i\mathbf{e}_x - i\mathbf{e}_y - \sin\theta_i\mathbf{e}_z\right) \tag{4.25}$$

$$\mathbf{H}_r = \frac{-i}{\eta_0} E_{rR}\left(\cos\theta_i\mathbf{e}_x + i\mathbf{e}_y - \sin\theta_i\mathbf{e}_z\right) + \frac{i}{\eta_0} E_{rL}\left(\cos\theta_i\mathbf{e}_x - i\mathbf{e}_y - \sin\theta_i\mathbf{e}_z\right) \tag{4.26}$$

Transmitted wave in chiral medium has the same form as (4.14) and(4.15). By Maxwell boundary conditions reflection and transmission problem could be given based on LHCP incidence mode.

$$\begin{aligned}
E_{rR} &= \frac{(Z-\eta_0)(Z+\eta_0)cos\theta_i(cos\theta_1 + cos\theta_2)}{2Z\eta_0 cos\theta_i^2 + 2Z\eta_0 cos\theta_1 cos\theta_2 + (Z^2+\eta_0^2)cos\theta(cos\theta_1 + cos\theta_2)} \\
E_{rL} &= -\frac{2Z\eta_0(cos\theta_i + cos\theta_1)(cos\theta_i - cos\theta_2)}{2Z\eta_0 cos^2\theta_i + 2Z\eta_0 cos\theta_1 cos\theta_2 + (Z^2+\eta_0^2)cos\theta_i(cos\theta_1 + cos\theta_2)} \\
E_{tR} &= \frac{2Z(Z-\eta_0)cos\theta_i(cos\theta_1 - cos\theta_2)}{2Z\eta_0 cos\theta_i^2 + 2Z\eta_0 cos\theta_1 cos\theta_2 + (Z^2+\eta_0^2)cos\theta_i(cos\theta_1 + cos\theta_2)} \\
E_{tL} &= \frac{2Z(Z+\eta_0)cos\theta_i(cos\theta + cos\theta_1)}{2Z\eta_0 cos\theta_i^2 + 2Z\eta_0 cos\theta_1 cos\theta_2 + (Z^2+\eta_0^2)cos\theta_i(cos\theta_1 + cos\theta_2)}
\end{aligned} \tag{4.27}$$

Special case is conducted for $\mu_r = \varepsilon_r = 1$ and $\gamma = 2$, it is seen that $E_{tR} = 0$ and $E_{tL} = 1$ while both reflected waves vanish. In this case wave number $k_2 = -k_0$ so that angle of incidence is the same as transmitted but twists in negative x direction. Schematic diagram is plotted in Fig 11 and the electric field is shown below in Fig 12

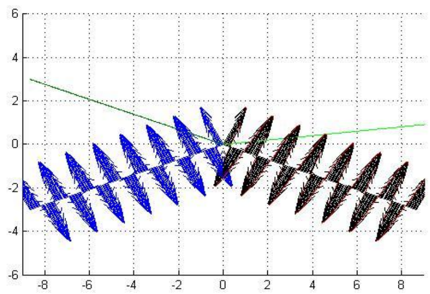


Fig 11 Schematic plot of plane wave with negative refraction. The incident wave is left circular wave(blue) and only left transmitted circular emerge bur twist downward.

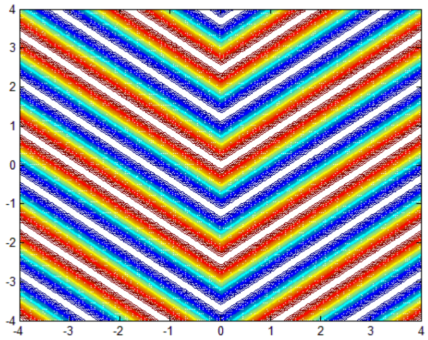


Fig 12 Real field pattern while negative refraction occur.

Although the wave vector (4.3) shows direct implication for negative refraction, however, it is only valid when material is isotropic. Another perspective toward negative refraction is to determine the sign of k_z based on direction of energy flow as in anisotropic case. In reflection and refraction problem energy must flow toward positive z direction such that refraction occurs, which is indicated by z component of Poynting vector. The sign and magnitude of k_x is already determined through phase matching condition so it is convenient to re-derive eigenmode and Poynting vector in terms of k_x. Following the same procedure as above, we could get:

Dispersion relation:

$$p_z = \pm\sqrt{{p_x}^2 + \left(\pm\gamma + \sqrt{\mu\varepsilon}\right)^2} \tag{4.28}$$

Eigenmodes:

For $p_z = \pm\sqrt{{p_x}^2 + \left(\sqrt{\mu\varepsilon} - \gamma\right)^2}$ (LHCP)

$$E = \begin{pmatrix} -p_z / p_x \\ \dfrac{i\left(\sqrt{\mu\varepsilon} - \gamma\right)}{p_x} \\ 1 \end{pmatrix} \tag{4.29}$$

Impedance:

$$Z = i\sqrt{\frac{\mu}{\varepsilon}} \tag{4.30}$$

Magnetic field:

$$\mathbf{H} = i\sqrt{\frac{\mu}{\varepsilon}}\begin{pmatrix} -p_z / p_x \\ \dfrac{-i\left(\gamma - \sqrt{\mu\varepsilon}\right)}{p_x} \\ 1 \end{pmatrix} \tag{4.31}$$

Poynting vector: $\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}\left[\mathbf{E} \times \mathbf{H}^*\right]$

$$\mathbf{S} = \begin{pmatrix} S_x \\ S_z \end{pmatrix} = \begin{pmatrix} \dfrac{\sqrt{\mu\varepsilon} - \gamma}{p_x} \\ \dfrac{p_z(\sqrt{\mu\varepsilon} - \gamma)}{p_x} \end{pmatrix} E^2 \tag{4.32}$$

From above derivation it is easily seen that if $\gamma > \sqrt{\mu\varepsilon}$, k_z must be negative so z component of Poynting vector is positive while x component is negative. This is the phenomenon of negative refraction. In this approach Poynting vector is first taken into account and the sign of k_z is determined by (4.32) instead of(4.10). Eqn.(4.32) demonstrates how Poynting vector relates to wave vector so in the following sections when negative refraction is being considered, we write Poynting vector in term of k_z rather than involve the transmitted angle.

Chapter 5 Bi-anisotropic medium

Although anisotropic and bi-isotropic media have been studied in the previous sections, we now go further to more general linear media, namely, bi-anisotropic media. Bi-anisotropic is introduced by giving more freedom to material parameters in constitutive tensors such that electromagnetic wave characteristic is quite different from conventional material.

5.1 Anisotropic dielectric medium

Let us consider dielectric tensors along principal axis while keep chirality isotropic. This can be done by putting the same toroidal helix in uniaxial or biaxial crystal. Constitutive equations are in the form of Eq.(5.1).

$$\begin{aligned}\mathbf{D} &= \begin{pmatrix}\varepsilon_x & 0 & 0\\ 0 & \varepsilon_y & 0\\ 0 & 0 & \varepsilon_z\end{pmatrix}\mathbf{E} + i\begin{pmatrix}\gamma & 0 & 0\\ 0 & \gamma & 0\\ 0 & 0 & \gamma\end{pmatrix}\mathbf{H}\\ \mathbf{B} &= \begin{pmatrix}\mu_x & 0 & 0\\ 0 & \mu_y & 0\\ 0 & 0 & \mu_z\end{pmatrix}\mathbf{H} + i\begin{pmatrix}-\gamma & 0 & 0\\ 0 & -\gamma & 0\\ 0 & 0 & -\gamma\end{pmatrix}\mathbf{E}\end{aligned} \tag{5.1}$$

Simply suppose $\varepsilon_x = \varepsilon_z, \mu_x = \mu_z$ so that it could be considered as uniaxial case in y direction. Although chirality is still independent of direction , the wave propagation may vary due to non-scalar tensors. More general case has been derived by Semchenko [21], but the results are too complicated in term of transmitted angles. In the present section we demonstrate how it differs from bi-isotropic chiral medium by using wave vector k_x, which is different from [21].

5.1.1 Dispersion relations

Follow the same steps as before, two relations could be given:

$$p_x{}^2 + p_{z+}{}^2 = \gamma^2 + \frac{1}{2}\left(\varepsilon_z\mu_y + \varepsilon_y\mu_z\right) + \rho \tag{5.2}$$

$$p_x{}^2 + p_{z-}{}^2 = \gamma^2 + \frac{1}{2}\left(\varepsilon_z\mu_y + \varepsilon_y\mu_z\right) - \rho \tag{5.3}$$

where

$$\rho = \sqrt{4\gamma^2\left(\varepsilon_y + \varepsilon_z\right)\left(\mu_y + \mu_z\right) + \left(\varepsilon_z\mu_y - \varepsilon_y\mu_z\right)} \tag{5.4}$$

5.1.2 Eigenmode solutions

For mode Eq.(5.2)

$$\mathbf{E}_{+} = \left(1, \chi_{+} i, -\frac{p_x}{p_{z+}}\right) \tag{5.5}$$

For mode Eq.(5.3)

$$\mathbf{E}_{-} = \left(1, -\chi_{-} i, \frac{-p_x}{p_{z-}}\right) \tag{5.6}$$

where

$$\chi_{\pm} = \frac{\rho \pm \left[2\gamma^2\left(\mu_y + \mu_z\right) + \mu_z\left(\mu_z\varepsilon_y - \varepsilon_z\mu_y\right)\right]}{2 p_{z\pm}\gamma\left(\mu_y + \mu_z\right)} \tag{5.7}$$

Here χ denote transverse ratio of electric fields. Note that the eigen-waves are elliptic waves rather than circular waves and this is the consequence of anisotropy effect.

5.1.3 Admittance analogy

$$\mathbf{H} = \overline{\overline{\mu}}^{-1}\left(\frac{\bar{k}\times\overline{\overline{I}}}{\omega} - \overline{\overline{\xi}}\right)\mathbf{E} = \overline{\overline{A}}\mathbf{E} = \begin{pmatrix} \frac{i\gamma}{\mu_x} & \frac{-p_z}{\mu_x} & 0 \\ \frac{p_z}{\mu_y} & \frac{i\gamma}{\mu_y} & \frac{-p_x}{\mu_y} \\ 0 & \frac{p_x}{\mu_z} & \frac{i\gamma}{\mu_z} \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \tag{5.8}$$

$$\mathbf{H}_{\pm} = \left(\mp Y_{\pm}^{TE}, Y_{\pm}^{TM}, A_{\pm}\right) \tag{5.9}$$

where

$$Y_{\pm}^{TE} = \frac{\rho \pm (\varepsilon_y\mu_z - \varepsilon_z\mu_y)}{2\gamma\left(\mu_y + \mu_z\right)} \tag{5.10}$$

$$Y_{\pm}^{TM} = \frac{\varepsilon_y\mu_z + \varepsilon_z\left(\mu_y + 2\mu_z\right) \pm \rho}{2\left(\mu_y + \mu_z\right) p_{z\pm}} \tag{5.11}$$

The upper index TE and TM refer to ratio of transverse x and y component of magnetic fields to x component of electric field and lower index indicates two separate eigenmodes. Note that the sign of Y^{TE} is specially arranged to solve later problem.

5.1.4 Reflection and transmission in layered media

Incident field:

$$\mathbf{E_i} = E_{i\|}\left(\cos\theta_i \mathbf{e}_x - \sin\theta_i \mathbf{e}_z\right) + E_{i\perp}\mathbf{e}_y$$

$$\mathbf{H_i} = \frac{E_{i\|}}{\eta_0}\mathbf{e}_y + \frac{E_{i\perp}}{\eta_0}(-\cos\theta_i \mathbf{e}_x + \sin\theta_i \mathbf{e}_z)$$

Reflected field:

$$\mathbf{E}_r = E_{r\perp}\mathbf{e}_y + E_{r\|}(\cos\theta_i \mathbf{e}_x + \sin\theta_i \mathbf{e}_z)$$

$$\mathbf{H}_r = \frac{1}{\eta_0}(-E_{r\|}\mathbf{e}_y + E_{r\perp}(\cos\theta_i \mathbf{e}_x + \sin\theta_i \mathbf{e}_z))$$

Transmitted field:

$$\mathbf{E}_t = E_{t+}\left(\mathbf{e}_x + \chi_+ i\mathbf{e}_y - \frac{p_x}{p_{z+}\rho_+}\mathbf{e}_z\right) + E_{t-}\left(\mathbf{e}_x - \chi_- i\mathbf{e}_y - \frac{p_x}{p_{z-}\rho_-}\mathbf{e}_z\right) \tag{5.12}$$

$$\mathbf{H}_t = E_{t+}\left(-Y_+^{TE} i\mathbf{e}_x + Y_+^{TM}\mathbf{e}_y\right) + E_{t-}\left(Y_-^{TE} i\mathbf{e}_x + Y_-^{TM}\mathbf{e}_y\right) \tag{5.13}$$

By Maxwell's boundary condition, reflection and refracted coefficients are obtained:

$$\begin{pmatrix} E_{r\perp} \\ E_{r\|} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} E_{i\perp} \\ E_{i\|} \end{pmatrix}$$

$$\begin{aligned}
R_{11} &= \frac{\left(Y + Y_+^{TM}\cos\theta_i\right)\left(\chi_- Y\cos\theta_i - Y_-^{TE}\right) + \left(Y + Y_-^{TM}\cos\theta_i\right)\left(\chi_+ Y\cos\theta_i - Y_+^{TE}\right)}{\left(Y + Y_+^{TM}\cos\theta_i\right)\left(\chi_- Y\cos\theta_i + Y_-^{TE}\right) + \left(Y + Y_-^{TM}\cos\theta_i\right)\left(\chi_+ Y\cos\theta_i + Y_+^{TE}\right)} \\
R_{12} &= \frac{2i\left(\chi_+ Y_-^{TE} - \chi_- Y_+^{TE}\right)Y\cos\theta_i}{\left(Y + Y_+^{TM}\cos\theta_i\right)\left(\chi_- Y\cos\theta_i + Y_-^{TE}\right) + \left(Y + Y_-^{TM}\cos\theta_i\right)\left(\chi_+ Y\cos\theta_i + Y_+^{TE}\right)} \\
R_{21} &= \frac{2i\left(Y_+^{TM} - Y_-^{TM}\right)Y\cos\theta_i}{\left(Y + Y_+^{TM}\cos\theta_i\right)\left(\chi_- Y\cos\theta_i + Y_-^{TE}\right) + \left(Y + Y_-^{TM}\cos\theta_i\right)\left(\chi_+ Y\cos\theta_i + Y_+^{TE}\right)} \\
R_{22} &= \frac{\left(Y - Y_+^{TM}\cos\theta_i\right)\left(\chi_- Y\cos\theta_i + Y_-^{TE}\right) + \left(Y - Y_-^{TM}\cos\theta_i\right)\left(\chi_+ Y\cos\theta_i + Y_+^{TE}\right)}{\left(Y + Y_+^{TM}\cos\theta_i\right)\left(\chi_- Y\cos\theta_i + Y_-^{TE}\right) + \left(Y + Y_-^{TM}\cos\theta_i\right)\left(\chi_+ Y\cos\theta_i + Y_+^{TE}\right)}
\end{aligned} \tag{5.14}$$

$$\begin{pmatrix} E_{t+} \\ E_{t-} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} E_{i\perp} \\ E_{i\|} \end{pmatrix}$$

$$T_{11}=\frac{-2iY\cos\theta_i\left(Y+Y_-^{TM}\cos\theta_i\right)}{\left(Y+Y_+^{TM}\cos\theta_i\right)\left(\chi_-Y\cos\theta_i+Y_-^{TE}\right)+\left(Y+Y_-^{TM}\cos\theta_i\right)\left(\chi_+Y\cos\theta_i+Y_+^{TE}\right)}$$

$$T_{12}=\frac{2Y\cos\theta_i\left(Y_-^{TE}+\chi_-Y\cos\theta_i\right)}{\left(Y+Y_+^{TM}\cos\theta_i\right)\left(\chi_-Y\cos\theta_i+Y_-^{TE}\right)+\left(Y+Y_-^{TM}\cos\theta_i\right)\left(\chi_+Y\cos\theta_i+Y_+^{TE}\right)}$$

$$T_{21}=\frac{2iY\cos\theta_i\left(Y+Y_+^{TM}\cos\theta_i\right)}{\left(Y+Y_+^{TM}\cos\theta_i\right)\left(\chi_-Y\cos\theta_i+Y_-^{TE}\right)+\left(Y+Y_-^{TM}\cos\theta_i\right)\left(\chi_+Y\cos\theta_i+Y_+^{TE}\right)}$$

$$T_{22}=\frac{2Y\cos\theta_i\left(Y_+^{TE}+\chi_+Y\cos\theta_i\right)}{\left(Y+Y_+^{TM}\cos\theta_i\right)\left(\chi_-Y\cos\theta_i+Y_-^{TE}\right)+\left(Y+Y_-^{TM}\cos\theta_i\right)\left(\chi_+Y\cos\theta_i+Y_+^{TE}\right)} \quad (5.15)$$

Here we have derived analytical form of R T in terms of admittance and angle transverse ratio χ for the most general elliptic polarization. The formula (5.14) &(5.15) could be simplified for less complicated case.

5.2 Uniaxially omega medium

Considering a general uniaxially bi-anisotropic media which is composed of symmetric chiral and anti-symmetric omega dyadic and choosing z axis as longitudinal direction, constitutive relations are written in the forms [16].

$$\mathbf{D}=\begin{pmatrix}\varepsilon_t & 0 & 0\\ 0 & \varepsilon_t & 0\\ 0 & 0 & \varepsilon_z\end{pmatrix}\mathbf{E}+i\begin{pmatrix}\gamma_t & \kappa & 0\\ -\kappa & \gamma_t & 0\\ 0 & 0 & \gamma_z\end{pmatrix}\mathbf{H}$$
$$\mathbf{B}=\begin{pmatrix}\mu_t & 0 & 0\\ 0 & \mu_t & 0\\ 0 & 0 & \mu_z\end{pmatrix}\mathbf{H}+i\begin{pmatrix}-\gamma_t & \kappa & 0\\ -\kappa & -\gamma_t & 0\\ 0 & 0 & -\gamma_z\end{pmatrix}\mathbf{E} \quad (5.16)$$

where $\varepsilon_t,\mu_t,\gamma_t$ denote transverse parameters in x-y plane and κ represents omega electromagnetic coupling.

The dispersion relation together with eigenwave solutions have been solved by various authors [22-24] . However, the results are rather complicated due to emerging elliptic polarized waves so that analytical simplification is only valid in special cases. Reflection and transmission in layered media is also considered in [16], but the analytical solution is also limited to certain cases. In the present study we analyze from the simplest case and discuss some optical behavior instead of solving Eqn.(5.16) directly.

Consider an omega media whose magnetoelectric dyadic is anti-symmetric as Eqn.(5.17). The medium could be realized by adding omega shape inclusions in x-z direction [16] and have been studied theoretically by in [25]; potential applications are also studied in [26] . However, the clear reflection and transmission coefficients in layered media are still not presented. In this section we start from the dispersion relation and give exact formula as well as numerical results.

$$\mathbf{D}=\begin{pmatrix}\varepsilon_x & 0 & 0\\ 0 & \varepsilon_y & 0\\ 0 & 0 & \varepsilon_z\end{pmatrix}\mathbf{E}+i\begin{pmatrix}0 & \gamma & 0\\ -\gamma & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\mathbf{H}$$
$$\mathbf{B}=\begin{pmatrix}\mu_x & 0 & 0\\ 0 & \mu_y & 0\\ 0 & 0 & \mu_z\end{pmatrix}\mathbf{H}+i\begin{pmatrix}0 & \gamma & 0\\ -\gamma & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\mathbf{E} \tag{5.17}$$

Eq.(5.17) Constitutive relations correspond to omega medium. Note that $\overline{\overline{\varepsilon}}$ and $\overline{\overline{\mu}}$ may be not uniaxial

5.2.1 Dispersion relation

Following the same procedure as anisotropic cases, dispersion relations are given below

$$\varepsilon_x k_x{}^2+\varepsilon_z k_z{}^2=\omega^2\varepsilon_z\left(\varepsilon_x\mu_y-\gamma^2\right) \tag{5.18}$$

$$\mu_z k_z{}^2+\mu_x k_x{}^2=\omega^2\mu_z\left(\varepsilon_y\mu_x-\gamma^2\right) \tag{5.19}$$

Note that the above results satisfy the duality transformation $\varepsilon\to-\mu,\mu\to-\varepsilon,\xi\to\xi$.

5.2.2 Eigenmode solutions

$\varepsilon_x k_x{}^2+\varepsilon_z k_z{}^2=\omega^2\varepsilon_z\left(\varepsilon_x\mu_y-\gamma^2\right)$:

$$\mathbf{E}=\left(1,0,\frac{-\varepsilon_x p_x}{\varepsilon_z(p_z+i\gamma)}\right) \tag{5.20}$$

$\mu_z k_z{}^2+\mu_x k_x{}^2=\omega^2\mu_z\left(\varepsilon_y\mu_x-\gamma^2\right)$:

$$\mathbf{E}=\left(0,1,0\right) \tag{5.21}$$

which are TM and TE modes respectively. It could be seen that when γ =0, Eqn.(5.20) corresponds to anisotropic case while when $\varepsilon_x=\varepsilon_z$, it is reduced to isotropic case.

5.2.3 Impedance analogy

$$\mathbf{H} = \overline{\overline{\mu}}^{-1}\left(\frac{\overline{k}\times\overline{\overline{I}}}{\omega} - \overline{\overline{\xi}}\right)\mathbf{E} = \overline{\overline{\mathbf{A}}}\mathbf{E} = \begin{pmatrix} 0 & \frac{-p_z + i\gamma}{\mu_x} & 0 \\ \frac{p_z - i\gamma}{\mu_y} & 0 & \frac{-p_x}{\mu_y} \\ 0 & \frac{p_x}{\mu_z} & 0 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \tag{5.22}$$

For TM mode $p_z = \dfrac{\sqrt{-p_x^2\varepsilon_x - \gamma^2\varepsilon_z + \varepsilon_x\varepsilon_z\mu_y}}{\sqrt{\varepsilon_z}}$, $\mathbf{E} = \left(1, 0, \dfrac{-\varepsilon_x p_x}{\varepsilon_z(p_z + i\gamma)}\right)$

$$H_y = Y_{TM}E_x = \frac{1}{Z_{TM}}E_x \tag{5.23}$$

where

$$Z_{TM} = \sqrt{\frac{\mu_y}{\varepsilon_x}}\left[\sqrt{1 - \frac{p_x^2}{\mu_y\varepsilon_z} - \frac{\gamma^2}{\varepsilon_x\mu_y}} + \frac{i\gamma}{\sqrt{\varepsilon_x\mu_y}}\right] \tag{5.24}$$

For TE mode $p_z = \dfrac{\sqrt{-p_x^2\mu_x - \gamma^2\mu_z + \mu_x\mu_z\varepsilon_y}}{\sqrt{\mu_z}}$, $\mathbf{E} = (0, 1, 0)$

$$H_x = \frac{1}{Z_{TE}}E_y \tag{5.25}$$

where

$$Z_{TE} = \sqrt{\frac{\mu_x}{\varepsilon_y}}\frac{1}{1 - \frac{p_x^2}{\mu_z\varepsilon_y}}\left[\sqrt{1 - \frac{p_x^2}{\varepsilon_y\mu_z} - \frac{\gamma^2}{\varepsilon_y\mu_x}} + \frac{i\gamma}{\sqrt{\varepsilon_y\mu_x}}\right] \tag{5.26}$$

Z_{TM} Eq.(5.24) and Z_{TE} Eq(5.26) are the impedances relating the ratio of transverse electric and magnetic field for TM and TE modes respectively. Our results are identical to that of Tretyakov [25] derived by another method.

5.2.4 Poynting vector

Poynting vector is defined as $\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}\times\mathbf{H}^*\right]$

TM mode:

$$\mathbf{E} = \left(-\frac{\varepsilon_z(i\gamma + p_z)}{\varepsilon_x p_x}, 0, 1\right) \tag{5.27}$$

$$\mathbf{H} = \left(0, \frac{-\varepsilon_z}{p_x}, 0\right) \tag{5.28}$$

$$\mathbf{S} = \begin{pmatrix} S_x \\ S_z \end{pmatrix} = \begin{pmatrix} \dfrac{\varepsilon_z}{p_x} \\ \dfrac{\varepsilon_z{}^2 p_z}{\varepsilon_x p_x{}^2} \end{pmatrix} \tag{5.29}$$

TE mode

$$\mathbf{E} = (0,1,0) \tag{5.30}$$

$$\mathbf{H} = \left(\frac{i\gamma - p_z}{\mu_x}, 0, \frac{p_x}{\mu_z}\right) \tag{5.31}$$

$$\mathbf{S} = \begin{pmatrix} S_x \\ S_z \end{pmatrix} = \begin{pmatrix} \dfrac{p_x}{\mu_z} \\ \dfrac{p_z}{\mu_x} \end{pmatrix} \tag{5.32}$$

5.2.5 Reflection and transmission in layered media

Consider plane wave problem in layered structure with host medium being bi-anisotropic omega medium. Since eigenwaves are TE and TM fields, TE and TM can be decoupled separately. Incident wave is expressed as:

$$\mathbf{E_i} = E_{im}\left(\cos\theta_i \mathbf{e}_x - \sin\theta_i \mathbf{e}_z\right) + E_{ie}\mathbf{e}_y$$

$$\mathbf{H_i} = \frac{E_{im}}{\eta_0}\mathbf{e}_y + \frac{E_{ie}}{\eta_0}(-\cos\theta_i \mathbf{e}_x + \sin\theta_i \mathbf{e}_z)$$

Reflected wave:

$$\mathbf{E}_r = E_{r\perp}\mathbf{e}_y + E_{r\|}(\cos\theta_i \mathbf{e}_x + \sin\theta_i \mathbf{e}_z)$$

$$\mathbf{H}_r = \frac{1}{\eta_0}(-E_{r\|}\mathbf{e}_y + E_{r\perp}(\cos\theta_i \mathbf{e}_x + \sin\theta_i \mathbf{e}_z))$$

Transmitted wave:

$$\mathbf{E}_t = E_{tm}\left(\mathbf{e}_x - \frac{\varepsilon_x p_x}{\varepsilon_z\left(i\gamma + p_z\right)}\mathbf{e}_z\right) + E_{te}\mathbf{e}_y \tag{5.33}$$

$$\mathbf{H}_t = \frac{E_{tm}}{\eta^{TM}}\mathbf{e}_y + \frac{-E_{te}}{\eta^{TE}}\mathbf{e}_x + \frac{E_{te}\mu_z}{p_x}\mathbf{e}_z \tag{5.34}$$

where η^{TM} and η^{TE} are given by Eqns.(5.24) and(5.26).

Reflection and transmission coefficients could be obtained by Maxwell's boundary conditions requiring tangential components of fields be continuous.

$$E_{rm} = \frac{\eta^{TM} - \eta_0 \cos\theta}{\eta^{TM} + \eta_0 \cos\theta} E_{im}$$
$$E_{tm} = \frac{2\eta^{TM} \cos\theta}{\eta^{TM} + \eta_0 \cos\theta} E_{im} \tag{5.35}$$

$$E_{re} = \frac{\eta^{TE} \cos\theta - \eta_0}{\eta_0 + \eta^{TE} \cos\theta} E_{ie}$$
$$E_{te} = \frac{2\eta^{TE} \cos\theta}{\eta_0 + \eta^{TE} \cos\theta} E_{ie} \tag{5.36}$$

It is shown that the expressions are the same as anisotropic medium Eqs.(3.25) &(3.26). Although chirality is considered, we have decoupled R T for TM and TE incident due to linearly polarization. We conclude that omega medium with anti-symmetric tensor is somewhat like anisotropic material and the influence of chirality only change the direction of TE and TM modes. To confirm further, if we set $\gamma = 0$, the formula are totally the same as Section Chapter 3

5.3 Uniaxial chiral medium

Let us assume longitudinal chiral material without omega coupling. Eqn.(5.16) is simplified as:

$$\mathbf{D} = \begin{pmatrix} \varepsilon_t & 0 & 0 \\ 0 & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} \mathbf{E} + i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \gamma_z \end{pmatrix} \mathbf{H}$$
$$\mathbf{B} = \begin{pmatrix} \mu_t & 0 & 0 \\ 0 & \mu_t & 0 \\ 0 & 0 & \mu_z \end{pmatrix} \mathbf{H} + i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\gamma_z \end{pmatrix} \mathbf{E} \tag{5.37}$$

Dispersion relation, eigenmodes solutions as well as Poynting vector have been obtained in [10, 27, 28]. Further discussion of mode and energy propagation is also discussed [10]. However, the author doesn't give formula for reflection and refraction coefficients when inhomogeneous media is encountered. Here we give formula and numerical results.

5.3.1 Dispersion relations

$$k_z^{\ 2} + \frac{k_x^{\ 2}}{\rho_\pm} = \omega^2 \mu_t \varepsilon_t \tag{5.38}$$

where

$$\rho_{\pm} = \frac{1}{2}\left[\frac{\varepsilon_z}{\varepsilon_t} + \frac{\mu_z}{\mu_t} \pm \sqrt{\left(\frac{\varepsilon_z}{\varepsilon_t} - \frac{\mu_z}{\mu_t}\right)^2 + \frac{4\gamma^2}{\varepsilon_t \mu_t}} \right] \tag{5.39}$$

5.3.2 Eigenmode solutions

For $p_{z+} = \mu_t \varepsilon_t - \frac{{p_x}^2}{\rho_+}$

$$\mathbf{E}_+ = \left(1, \frac{i\gamma\mu_t}{p_{z+}(\rho_+\mu_t - \mu_z)}, \frac{-p_x}{p_{z+}\rho_+} \right) \tag{5.40}$$

For $p_{z-} = \mu_t \varepsilon_t - \frac{{p_x}^2}{\rho_-}$

$$\mathbf{E}_- = \left(1, \frac{i\gamma\mu_t}{p_{z-}(\rho_-\mu_t - \mu_z)}, \frac{-p_x}{p_{z-}\rho_-} \right) \tag{5.41}$$

Unlike omega chiral material, the eigenwaves are two elliptic waves rather than linearly polarized waves. In other words, TE and TM modes cannot be decoupled.

5.3.3 Admittance analogy

$$\mathbf{H} = \overline{\overline{\mu}}^{-1}\left(\frac{\bar{k}\times\overline{\overline{I}}}{\omega} - \overline{\overline{\xi}} \right)\mathbf{E} = \overline{\overline{\mathbf{A}}}\mathbf{E} = \begin{pmatrix} 0 & \frac{-p_z}{\mu_x} & 0 \\ \frac{p_z}{\mu_y} & 0 & \frac{-p_x}{\mu_y} \\ 0 & \frac{p_x}{\mu_z} & \frac{i\gamma}{\mu_z} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \tag{5.42}$$

$$\mathbf{H}_{\pm} = \left(\frac{-i\gamma}{\left(\rho_{\pm}\mu_t - \mu_z\right)}, \frac{\varepsilon_t}{p_{z\pm}}, \frac{i\beta_{\pm}\sigma^2}{p_x p_{z\pm}} \right) \tag{5.43}$$

where

$$\beta_{\pm} = \frac{\gamma}{\left(\rho_{\pm}\mu_t - \mu_z\right)}, \sigma^2 = \frac{{p_x}^2}{\rho_{\pm}} \tag{5.44}$$

Now we define transverse admittances representing the ratio of transverse electric fields and magnetic fields. Due to elliptic waves, four quantities are given:

$$\begin{aligned} H_{x\pm} &= \mp {Y_{\pm}}^{TE} i E_{x\pm} \\ H_{y\pm} &= {Y_{\pm}}^{TM} E_{x\pm} \end{aligned} \tag{5.45}$$

where

$$Y_{\pm}^{TE} = \frac{\pm\gamma}{\left(\rho_{\pm}\mu_t - \mu_z\right)} \tag{5.46}$$

$$Y_{\pm}^{TM} = \frac{\varepsilon_t}{p_{z\pm}} \tag{5.47}$$

Note that in Eq.(5.45) the sign of Y^{TE} is specially arranged to solve later problem.

5.3.4 Poynting vector

Poynting vector is defined as $\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}\times\mathbf{H}^*\right]$

$$\mathbf{S} = \begin{pmatrix} S_x \\ S_z \end{pmatrix} = \begin{pmatrix} \dfrac{p_x\left(\mu_t\beta_{\pm}^2 + \varepsilon_t\right)}{\rho_{\pm}p_z^2} \\ \dfrac{\left(\mu_t\beta_{\pm}^2 + \varepsilon_t\right)}{p_z} \end{pmatrix} E_x^2 \tag{5.48}$$

5.3.5 Reflection and transmission in layered media

Consider plane wave problem in layered structure with host medium being uniaxial chiral medium. Since eigenwaves are elliptic waves, TE and TM cannot be decoupled separately. Follow the same procedure as before, incident wave is expressed as:

$$\mathbf{E_i} = E_{i\|}\left(\cos\theta_i\mathbf{e}_x - \sin\theta_i\mathbf{e}_z\right) + E_{i\perp}\mathbf{e}_y$$

$$\mathbf{H_i} = \frac{E_{i\|}}{\eta_0}\mathbf{e}_y + \frac{E_{i\perp}}{\eta_0}(-\cos\theta_i\mathbf{e}_x + \sin\theta_i\mathbf{e}_z)$$

Reflected wave:

$$\mathbf{E}_r = E_{r\perp}\mathbf{e}_y + E_{r\|}(\cos\theta_i\mathbf{e}_x + \sin\theta_i\mathbf{e}_z)$$

$$\mathbf{H}_r = \frac{1}{\eta_0}(-E_{r\|}\mathbf{e}_y + E_{r\perp}(\cos\theta_i\mathbf{e}_x + \sin\theta_i\mathbf{e}_z))$$

Transmitted waves:

$$\mathbf{E}_t = E_{t+}\left(\mathbf{e}_x + \chi_+ i\mathbf{e}_y - \frac{p_x}{p_{z+}\rho_+}\mathbf{e}_z\right) + E_{t-}\left(\mathbf{e}_x - \chi_- i\mathbf{e}_y - \frac{p_x}{p_{z-}\rho_-}\mathbf{e}_z\right) \tag{5.49}$$

where

$$\chi_{\pm} = \frac{\pm\gamma\mu_t}{p_{z\pm}\left(\rho_{\pm}\mu_t - \mu_z\right)} \tag{5.50}$$

where $\chi_{\pm}$ is the ratio of y component to x component of electric fields

$$\mathbf{H}_t = E_{t+}\left(-Y_+^{TE} i\mathbf{e}_x + Y_+^{TM}\mathbf{e}_y\right) + E_{t-}\left(Y_-^{TE} i\mathbf{e}_x + Y_-^{TM}\mathbf{e}_y\right) \tag{5.51}$$

where admittances are given by Eqs.(5.46) and (5.47). note that χ is ratio of transverse electric fields while Y are admittances relating transverse electric and magnetic fields.

Let us apply boundary conditions to determine reflection and transmission coefficients: tangential components must be continuous.

$$\begin{aligned}
&\left(E_{im}+E_{r\|}\right)\cos\theta = E_{t+}+E_{t-}\\
&E_{ie}+E_{r\perp} = \left(E_{t+}\chi_{+}-E_{t-}\chi_{-}\right)i\\
&\left(E_{r\perp}-E_{ie}\right)Y_0\cos\theta = \left(-Y_{+}^{TE}E_{t+}+Y_{-}^{TE}E_{t-}\right)i\\
&\left(E_{im}-E_{r\|}\right)Y_0 = Y_{+}^{TM}E_{t+}+Y_{-}^{TM}E_{t-}
\end{aligned} \tag{5.52}$$

$$\begin{pmatrix} E_{r\perp} \\ E_{r\|} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} E_{i\perp} \\ E_{i\|} \end{pmatrix}$$

$$\begin{aligned}
R_{11} &= \frac{\left(Y+Y_{+}^{TM}\cos\theta\right)\left(\chi_{-}Y\cos\theta-Y_{-}^{TE}\right)+\left(Y+Y_{-}^{TM}\cos\theta\right)\left(\chi_{+}Y\cos\theta-Y_{+}^{TE}\right)}{\left(Y+Y_{+}^{TM}\cos\theta\right)\left(\chi_{-}Y\cos\theta+Y_{-}^{TE}\right)+\left(Y+Y_{-}^{TM}\cos\theta\right)\left(\chi_{+}Y\cos\theta+Y_{+}^{TE}\right)}\\
R_{12} &= \frac{2i\left(\chi_{+}Y_{-}^{TE}-\chi_{-}Y_{+}^{TE}\right)Y\cos\theta}{\left(Y+Y_{+}^{TM}\cos\theta\right)\left(\chi_{-}Y\cos\theta+Y_{-}^{TE}\right)+\left(Y+Y_{-}^{TM}\cos\theta\right)\left(\chi_{+}Y\cos\theta+Y_{+}^{TE}\right)}\\
R_{21} &= \frac{2i\left(Y_{+}^{TM}-Y_{-}^{TM}\right)Y\cos\theta}{\left(Y+Y_{+}^{TM}\cos\theta\right)\left(\chi_{-}Y\cos\theta+Y_{-}^{TE}\right)+\left(Y+Y_{-}^{TM}\cos\theta\right)\left(\chi_{+}Y\cos\theta+Y_{+}^{TE}\right)}\\
R_{22} &= \frac{\left(Y-Y_{+}^{TM}\cos\theta\right)\left(\chi_{-}Y\cos\theta+Y_{-}^{TE}\right)+\left(Y-Y_{-}^{TM}\cos\theta\right)\left(\chi_{+}Y\cos\theta+Y_{+}^{TE}\right)}{\left(Y+Y_{+}^{TM}\cos\theta\right)\left(\chi_{-}Y\cos\theta+Y_{-}^{TE}\right)+\left(Y+Y_{-}^{TM}\cos\theta\right)\left(\chi_{+}Y\cos\theta+Y_{+}^{TE}\right)}
\end{aligned} \tag{5.53}$$

$$\begin{pmatrix} E_{t+} \\ E_{t-} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} E_{i\perp} \\ E_{i\|} \end{pmatrix}$$

$$\begin{aligned}
T_{11} &= \frac{-2iY\cos\theta\left(Y+Y_{-}^{TM}\cos\theta\right)}{\left(Y+Y_{+}^{TM}\cos\theta\right)\left(\chi_{-}Y\cos\theta+Y_{-}^{TE}\right)+\left(Y+Y_{-}^{TM}\cos\theta\right)\left(\chi_{+}Y\cos\theta+Y_{+}^{TE}\right)}\\
T_{12} &= \frac{2Y\cos\theta\left(Y_{-}^{TE}+\chi_{-}Y\cos\theta\right)}{\left(Y+Y_{+}^{TM}\cos\theta\right)\left(\chi_{-}Y\cos\theta+Y_{-}^{TE}\right)+\left(Y+Y_{-}^{TM}\cos\theta\right)\left(\chi_{+}Y\cos\theta+Y_{+}^{TE}\right)}\\
T_{21} &= \frac{2iY\cos\theta\left(Y+Y_{+}^{TM}\cos\theta\right)}{\left(Y+Y_{+}^{TM}\cos\theta\right)\left(\chi_{-}Y\cos\theta+Y_{-}^{TE}\right)+\left(Y+Y_{-}^{TM}\cos\theta\right)\left(\chi_{+}Y\cos\theta+Y_{+}^{TE}\right)}\\
T_{22} &= \frac{2Y\cos\theta\left(Y_{+}^{TE}+\chi_{+}Y\cos\theta\right)}{\left(Y+Y_{+}^{TM}\cos\theta\right)\left(\chi_{-}Y\cos\theta+Y_{-}^{TE}\right)+\left(Y+Y_{-}^{TM}\cos\theta\right)\left(\chi_{+}Y\cos\theta+Y_{+}^{TE}\right)}
\end{aligned} \tag{5.54}$$

5.4 Pseudochiral material

Pseudochiral material has drawn highly attention recently. Unlike chiral and omega chiral materials, psedochiral has symmetric cross coupling tensor. The symmetry of cross polarization cause the phenomenon of plane wave propagation even complicated. In general, the constitutive relations have the forms

$$\begin{aligned}\mathbf{D} &= \overline{\overline{\varepsilon}}\mathbf{E} + \overline{\overline{\xi}}\mathbf{H} \\ \mathbf{B} &= \overline{\overline{\mu}}\mathbf{H} + \overline{\overline{\zeta}}\mathbf{E}\end{aligned} \tag{5.55}$$

$$\overline{\overline{\xi}} = \begin{pmatrix} 0 & \xi_{12} & \xi_{13} \\ \xi_{21} & 0 & \xi_{23} \\ \xi_{31} & \xi_{32} & 0 \end{pmatrix} \tag{5.56}$$

$$\overline{\overline{\zeta}} = -\overline{\overline{\xi}} = -\begin{pmatrix} 0 & \xi_{12} & \xi_{13} \\ \xi_{21} & 0 & \xi_{23} \\ \xi_{31} & \xi_{32} & 0 \end{pmatrix} \tag{5.57}$$

where $\overline{\overline{\xi}} = -\overline{\overline{\zeta}}$ satisfies reciprocity condition of Eqns.(2.5) and (2.6).

5.4.1 Dispersion relation

Confine ourselves to a special case of pseudochiral material which has only cross effect along principle z direction. Constitutive relations could be written as:

$$\mathbf{D} = \begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}\mathbf{E} + \begin{pmatrix} 0 & 0 & -i\gamma \\ 0 & 0 & 0 \\ -i\gamma & 0 & 0 \end{pmatrix}\mathbf{H} \tag{5.58}$$

$$\mathbf{B} = \begin{pmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{pmatrix}\mathbf{H} - \begin{pmatrix} 0 & 0 & i\gamma \\ 0 & 0 & 0 \\ i\gamma & 0 & 0 \end{pmatrix}\mathbf{E} \tag{5.59}$$

Follow through the same steps as before, two dispersion relations are obtained.

$$p_x^{\,2} + p_z^{\,2} = \left(\mu\varepsilon - \gamma^2\right) + \frac{2}{\mu\varepsilon}\rho_+ \tag{5.60}$$

$$p_x^{\,2} + p_z^{\,2} = \left(\mu\varepsilon - \gamma^2\right) + \frac{2}{\mu\varepsilon}\rho_- \tag{5.61}$$

where

$$\rho_{\pm} = p_x^{\ 2}\gamma^2 \pm \sqrt{p_x^{\ 2}\gamma^2\left(p_x^{\ 2} - \mu\varepsilon\right)\left(\gamma^2 - \mu\varepsilon\right)} \tag{5.62}$$

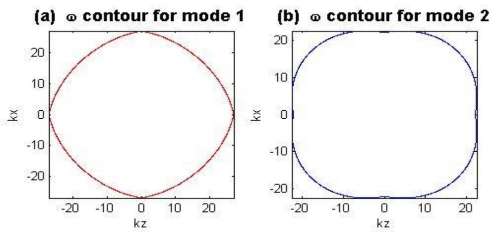
In terms of ω, dispersion relation is given by

$$\omega^2 = \frac{\left(k^2 \pm 2\sqrt{k_x^{\ 2}k_z^{\ 2}/(\varepsilon\mu)}\gamma\right)}{\mu\varepsilon - \gamma^2} \tag{5.63}$$

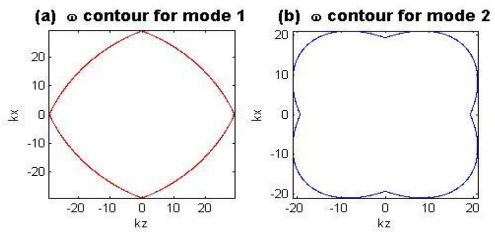
$$k_z^{\ \pm} = \sqrt{1-\left(\frac{\gamma}{\mu\varepsilon}\right)^2}\sqrt{\omega^2\mu\varepsilon - k_x^{\ 2}} \pm |k_x|\frac{\gamma}{\sqrt{\mu\varepsilon}} \tag{5.64}$$

Here $\rho_{\pm}$ are functions of k_x, which implies that there are two terms of k_x in dispersion relations. A small change of materials' parameters may cause significant change of wave vector. Three cases of equifrequency surface with respect to wave vectors are plotted in Fig 13 (1)-(4)

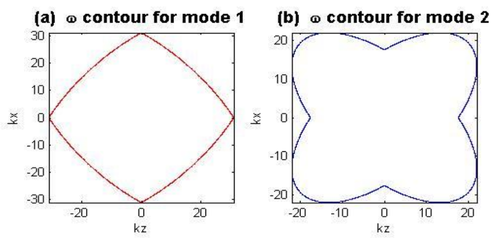
(1)



(2)



(3)



(4)

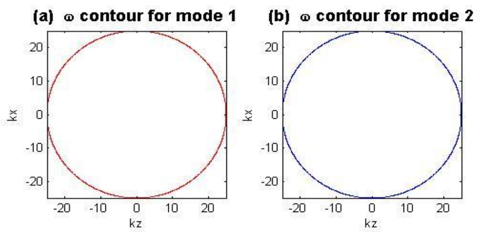


Fig 13 Equiphase diagrams for two eigenmodes (mode1 for ρ_- and mode2 for ρ_+) with material parameters $\varepsilon = \mu = 2$ and $\gamma = 0.4$, 0.8, 1.2 for (1),(2),and (3) while $\gamma = 0$ for (4). Note that eigenwaves degenerate in (4).

The dispersion relations curve show physical significance that the present of γ

influence the wave vector strongly. For isotropic case both wave vectors are circular shapes while when γ gets larger, the curves are shaped to trapezium for mode1 and petal-like for mode2. Furthermore, the petal-shape curve indicates negative wave propagation because the gradient of the frequency or energy may twist in opposite direction compared to wave vector. Unlike hyperbolic curve given from anisotropy medium, we get a petal-like curve that gives rise to negative refraction.

5.4.2 Eigenmode solutions

For mode1 $p_x^2 + p_z^2 = \left(\mu\varepsilon - \gamma^2\right) + \frac{2}{\mu\varepsilon}\rho_+$

$$\mathbf{E}_+ = \left(1, i\chi_+, \frac{-\sigma_+}{p_x p_{z+}\rho_{r+}}\right) \tag{5.65}$$

For mode2 $p_x^2 + p_z^2 = \left(\mu\varepsilon - \gamma^2\right) + \frac{2}{\mu\varepsilon}\rho_-$

$$\mathbf{E}_- = \left(1, -i\chi_-, \frac{-\sigma_-}{p_x p_{z-}\rho_{r-}}\right) \tag{5.66}$$

where

$$\begin{aligned} \chi_\pm &= \frac{i\gamma\rho_{k\pm}}{p_x\rho_{r\pm}} = \frac{\left(\varepsilon\mu - \gamma^2\right)}{\sqrt{\left(\gamma^2 - \mu\varepsilon\right)\left(p_x^{\ 2} - \mu\varepsilon\right)}} = \chi \\ \rho_{\gamma\pm} &= \rho_\pm - \mu\varepsilon\gamma^2 \\ \rho_{k\pm} &= \mp(\rho_\pm - \mu\varepsilon p_x^{\ 2}) \\ \sigma_\pm &= (\gamma^2 + p_x^{\ 2})\rho_\pm - 2\gamma^2 p_x^{\ 2} p_{z\pm}^{\ \ 2} \end{aligned} \tag{5.67}$$

5.4.3 Admittance analogy

$$\mathbf{H} = \overline{\overline{\mu}}^{-1}\left(\frac{\overline{k}\times\overline{\overline{I}}}{\omega} - \overline{\overline{\xi}}\right)\mathbf{E} = \overline{\overline{\mathbf{A}}}\mathbf{E} = \begin{pmatrix} 0 & \frac{-k_z}{\omega\mu} & \frac{-i\gamma}{\mu} \\ \frac{k_z}{\omega\mu} & 0 & \frac{-k_x}{\omega\mu} \\ \frac{-i\gamma}{\mu} & \frac{k_x}{\omega\mu} & 0 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \tag{5.68}$$

Inserting Eqs.(5.65)&(5.66) into Eq.(5.68) and comparing each component, we get

$$\mathbf{H}_\pm = \mp i\sqrt{\frac{\varepsilon}{\mu}}\left(1, \pm i\chi, \frac{\sigma_\pm}{p_x p_{z\pm}\rho_{r\pm}}\right) \tag{5.69}$$

where admittance is defined:

$$Y_{\pm} = \sqrt{\frac{\varepsilon}{\mu}} \tag{5.70}$$

Impedance is defined:

$$Z_{\pm} = \sqrt{\frac{\mu}{\varepsilon}} \tag{5.71}$$

5.4.4 Poynting vector

Poynting vector is defined as $\langle \mathbf{S} \rangle = \frac{1}{2} \mathrm{Re}\left[\mathbf{E} \times \mathbf{H}^* \right]$

$$\mathbf{S}_{\pm} = \begin{pmatrix} S_x \\ S_z \end{pmatrix} = \begin{pmatrix} \dfrac{\chi \sigma_+}{p_x p_z \rho_{r+}} \\ \chi \end{pmatrix} \tag{5.72}$$

5.4.5 Reflection and transmission in layered media

Consider plane wave problem in layered structure with host medium being uniaxial chiral medium. Since eigenwaves are elliptic waves, TE and TM cannot be decoupled separately. Follow the same procedure as before, incident wave is expressed as:

$$\mathbf{E_i} = E_{i\|} \left(\cos\theta_i \mathbf{e}_x - \sin\theta_i \mathbf{e}_z \right) + E_{i\perp} \mathbf{e}_y$$

$$\mathbf{H_i} = \frac{E_{i\|}}{\eta_0} \mathbf{e}_y + \frac{E_{i\perp}}{\eta_0} (-\cos\theta_i \mathbf{e}_x + \sin\theta_i \mathbf{e}_z)$$

Reflected wave:

$$\mathbf{E}_r = E_{r\perp} \mathbf{e}_y + E_{r\|} (\cos\theta_i \mathbf{e}_x + \sin\theta_i \mathbf{e}_z)$$

$$\mathbf{H}_r = \frac{1}{\eta_0} (-E_{r\|} \mathbf{e}_y + E_{r\perp} (\cos\theta_i \mathbf{e}_x + \sin\theta_i \mathbf{e}_z))$$

Transmitted waves:

$$\mathbf{E}_t = E_{t+} \left(\mathbf{e}_x + \chi_+ i \mathbf{e}_y - \frac{\sigma_+}{p_x p_{z+} \rho_{r+}} \mathbf{e}_z \right) + E_{t-} \left(\mathbf{e}_x - \chi_- i \mathbf{e}_y - \frac{\sigma_-}{p_x p_{z-} \rho_{r-}} \mathbf{e}_z \right) \tag{5.73}$$

$$\mathbf{H}_t = -i \frac{1}{\eta} E_{t+} \left(\mathbf{e}_x + \chi_+ i \mathbf{e}_y - \frac{\sigma_+}{p_x p_{z+} \rho_{r+}} \mathbf{e}_z \right) + i \frac{1}{\eta} E_{t-} \left(\mathbf{e}_x - \chi_- i \mathbf{e}_y - \frac{\sigma_-}{p_x p_{z-} \rho_{r-}} \mathbf{e}_z \right) \tag{5.74}$$

Again let us apply boundary conditions to determine reflection and transmission coefficients: tangential components must be continuous.

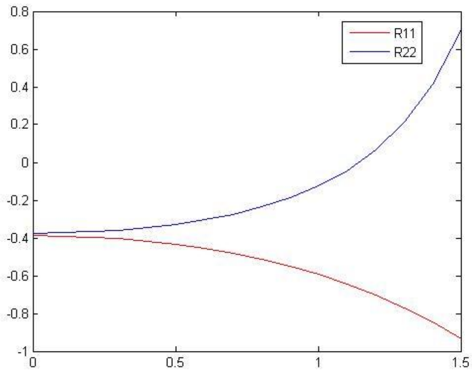
$$
\begin{aligned}
&\left(E_{i\parallel} + E_{r\parallel}\right)\cos\theta = E_{t+} + E_{t-} \\
&E_{i\perp} + E_{r\perp} = \left(E_{t+}\chi_{+} - E_{t-}\chi_{-}\right) i \\
&Y_0\left(E_{r\perp} - E_{i\perp}\right)\cos\theta = Y\left(-E_{t+} + E_{t-}\right) i \\
&Y_0\left(E_{i\parallel} - E_{r\parallel}\right) = Y\left(E_{t+}\chi_{+} + E_{t-}\chi_{-}\right)
\end{aligned}
\tag{5.75}
$$

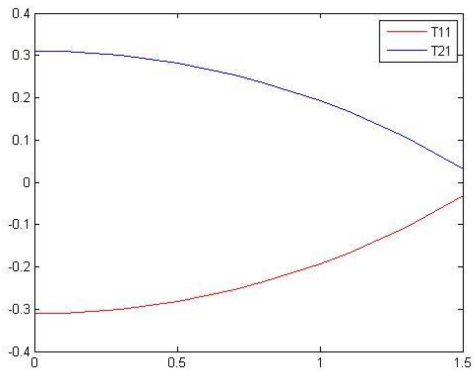
$$
\begin{pmatrix} E_{r\perp} \\ E_{r\parallel} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} E_{i\perp} \\ E_{i\parallel} \end{pmatrix}
$$

$$
\begin{aligned}
R_{11} &= \frac{\left(\chi Y_0 \cos\theta - Y\right)}{\left(\chi Y_0 \cos\theta + Y\right)} \\
R_{12} &= 0 \\
R_{21} &= 0 \\
R_{22} &= \frac{\left(Y_0 - \chi Y \cos\theta\right)}{\left(Y_0 + \chi Y \cos\theta\right)}
\end{aligned}
\tag{5.76}
$$

$$
\begin{pmatrix} E_{t+} \\ E_{t-} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} E_{i\perp} \\ E_{i\parallel} \end{pmatrix}
$$

$$
\begin{aligned}
T_{11} &= \frac{-iY_0 \cos\theta}{\left(\chi Y_0 \cos\theta + Y\right)} \\
T_{12} &= \frac{Y_0 \cos\theta}{\left(Y_0 + \chi Y \cos\theta\right)} \\
T_{21} &= \frac{iY_0 \cos\theta}{\left(\chi Y_0 \cos\theta + Y\right)} \\
T_{22} &= \frac{Y_0 \cos\theta}{\left(Y_0 + \chi Y \cos\theta\right)}
\end{aligned}
\tag{5.77}
$$





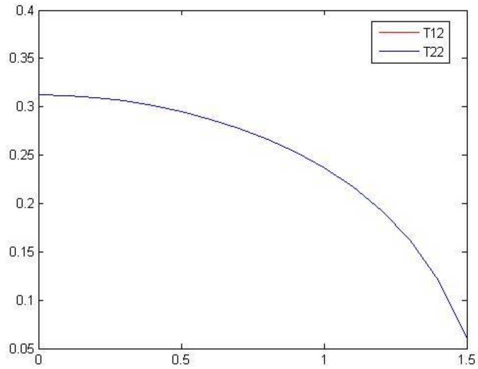


Fig 14. R,T curve with respect to incident angle with host medium $\varepsilon = 5, \mu = 1, \gamma = 0.4$. Note that T11 and T21 is imaginary.

Note that Eqs.(5.76)&(5.77) can be deduced from Eqs.(5.53)&(5.54) by letting $Y^{TM} = \chi Y, Y^{TE} = Y$ or from Eqs.(4.18)&(4.20) by setting $\chi = i / \cos\theta$. This implies the present case is an elliptic polarization which simultaneously posses certain degree of symmetry because transverse admittance is identical. Chiral medium can be seen as isotropic depending on its symmetric tensors whereas uniaxial medium is seen as anisotropic. We conclude that pseduchiral medium is anisotropic by its dispersion relations, however, due to simplified ε, μ, γ, it shows symmetry in admittance analogy. In our analysis, we find that the reflected wave is always linear polarized, which is the consequence of χ is the same for both transmitted waves. In isotropic-chiral material, reflected wave is only linear polarized when Brewster's angle is considered. Reflection of anisotropic medium is linear polarized, however, the transmitted wave is decoupled linear polarized wave. Pseudochiral material, could be regarded as in between, whose reflected wave is always linear while refracted waves are elliptic polarized. Observing RT solution given by Eqs.(5.76)&(5.77), the results are quite similar to that of isotropic host medium but T shows cross coupling. This results are identical to the phenomenon discussed. In general, we conclude that constitutive relations alone determine the characteristic of the waves, which is shown in Table 2

Host medium	Refracted polarization	Reflected polarization	Wave number

anisotropic	Linear	Linear	$\frac{k_x^{\ 2}}{\varepsilon_x}+\frac{k_z^{\ 2}}{\varepsilon_z}=\omega^2\mu_y$ $\frac{k_x^{\ 2}}{\mu_x}+\frac{k_z^{\ 2}}{\mu_z}=\omega^2\varepsilon_y$
Isotropic-chiral	Circular	Elliptic	$k=\omega\left(\sqrt{\mu\varepsilon}\pm\gamma\right)$
pseduochiral	Elliptic	Linear	$k^2=\omega\left(\mu\varepsilon-\gamma^2+\frac{2\rho_\pm}{\mu\varepsilon}\right)$
Bi-anisotropic	Elliptic	Elliptic	

Table 2

5.4.6 Negative refraction

We have investigated many electromagnetic characteristic of pseudo-chiral material. One remarkable phenomenon includes negative refraction occurs for one eigenmode even if all material parameters are positive and at the same time the reflected wave is always linear polarized, which satisfies Brewster angle in Bassiri's paper [15]. Here we demonstrate how this happen.

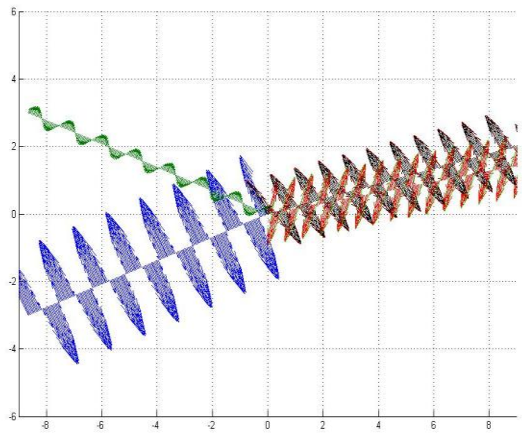
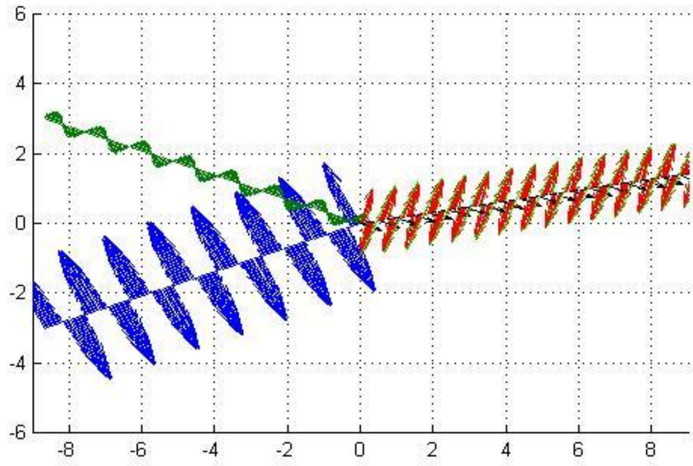
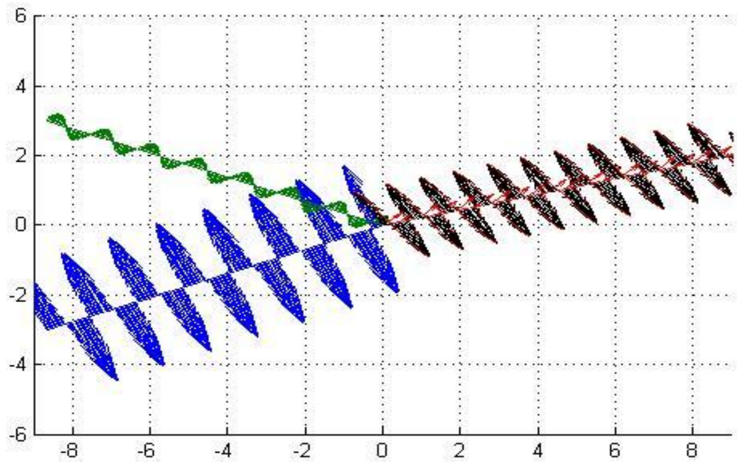


Fig 15.Shematic diagram of a plane wave from free space which is incident on an angle of 30 degree to a pseudo-chiral material. Two refracted waves are shown by mode1 (red line) and mode2 (black line). Note that material parameters are $\varepsilon=\mu=2,\gamma=1.2$ with $\theta_i=30^\circ,\theta_1=14.8^\circ,\theta_2=20.43^\circ$



(a) mode1



(b) mode2

Fig 16. Each eigenmode and corresponding Poynting vector. From (a) it is shown that Poynting vector (black line) is pointed downward, which is negative refraction, while in (b) Poynting vector (red line) is pointed upward showing positive refraction. Note that reflected wave is linearly polarized.

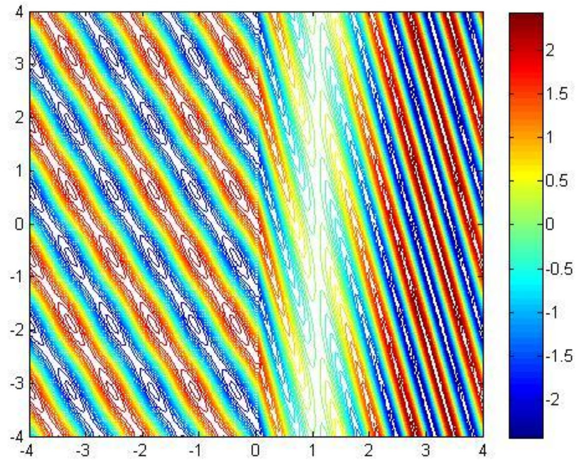
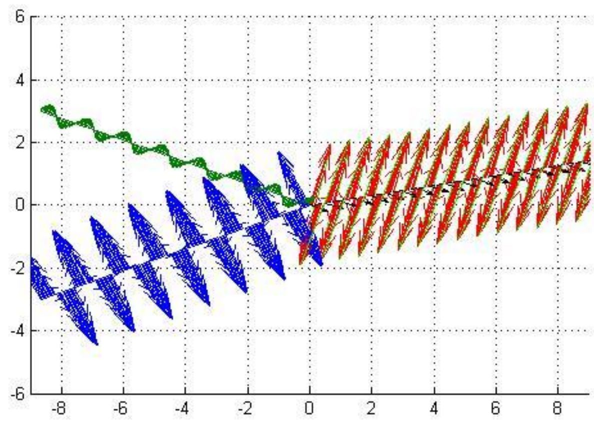
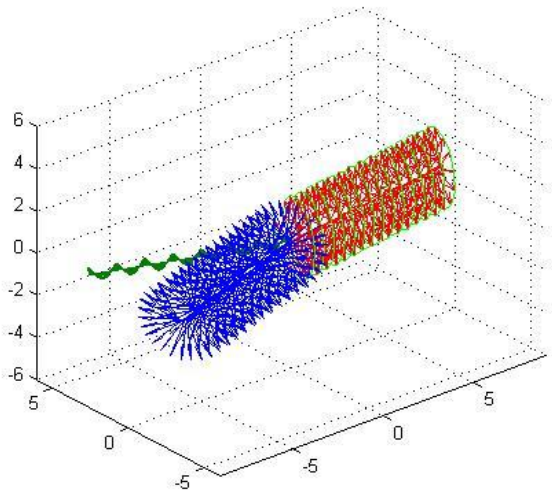


Fig 17. Electric field pattern for Ey component. In the right half plane two refracted elliptic waves combine for the electric field while in the left half plane the result is the combination of reflected and incident waves

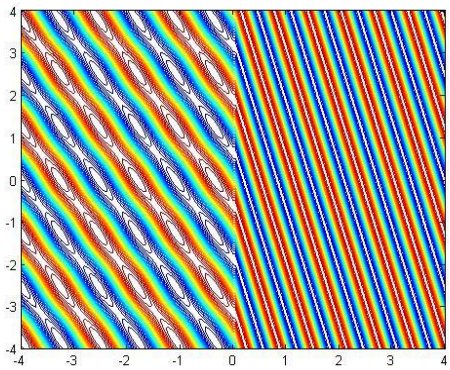
Now we want to design an impedance matching condition such that one of the transmitted waves is vanished. The same method has been performed in dealing with negative refraction in isotropic media in the previous section. Note that a circular incident wave may cause reflected wave even if impedance matching condition is satisfied. Using the same parameter $\varepsilon = \mu = 2, \gamma = 1.2$ and $\theta_i = 30^\circ, \theta_1 = 14.8^\circ, \theta_2 = 20.43^\circ$, however, a right circular wave (RHCP) is given as incident wave. Mode2 is vanished according to Eq.(5.77). The schematic and field pattern are shown in Fig 18.



(a)



(b)



(c)

Fig 18 Schematic diagram for a circular wave (blue) under impedance match condition $\mu / \varepsilon = \mu_0 / \varepsilon_0$. Plot (a) shows the direction of Poynting vector with its corresponding mode and (b) shows it is an elliptic transmitted wave (red).(c) is the electric field pattern. Note that mode2 is not produced.

If a left circular wave (LHCP) is incident under the same condition, mode1 is not produced. This time a positive refraction wave is obtained.

5.4.7 Conditions of negative refraction and backward wave

In this section we discuss the condition of negative refraction and backward wave in detail by examining two eigenwaves derived in Eq.(5.64). To investigate the possible phenomenon of negative refraction, isofrequency contour is plotted with respect to $k_z^{\pm}$ below in Fig 19

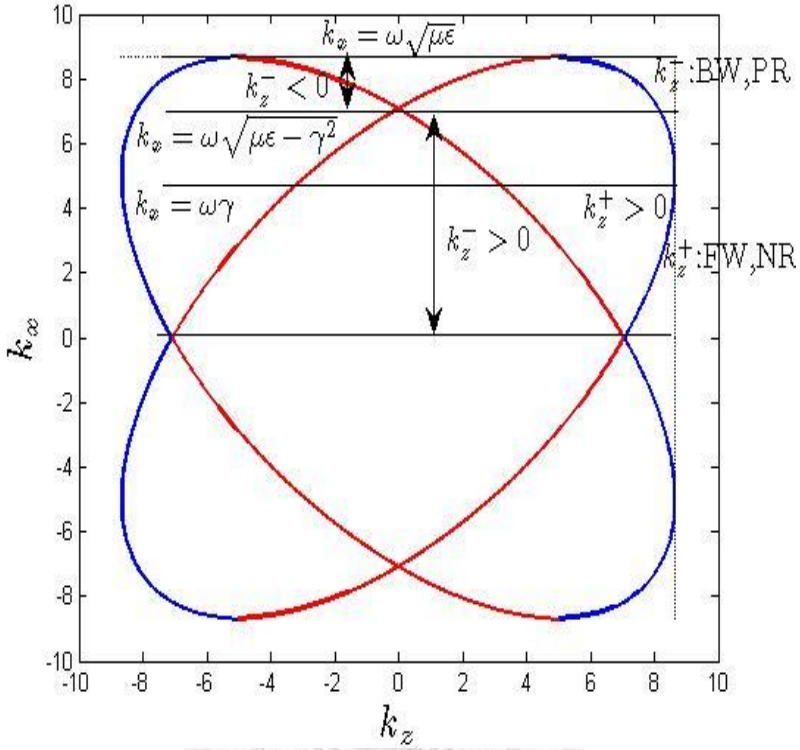


Fig 19 Isofrequency contour with respect to two eigenwaves in pseudochiral medium with parameters $\varepsilon = 3, \mu = \gamma = 1$. Note that red and blue curves correspond to k_z^- and k_z^+ waves respectively.

Without loss of generality, let us focus on the upper half space where $k_x > 0$. Point A, at which group velocity lies directly along the z axis, gives the condition of negative refraction for k_z wave. This point could be evaluated by letting $S_x = 0$, which corresponds to $E_z = H_z = 0$ in Eqs.(5.65)&(5.66).

$$k_x = \omega\gamma \tag{5.78}$$

While frequency is below this point, k_z^+ wave gives rise to negative refraction, which could be easily seen by normal vector of the isofrequency. k_z^- wave, however, corresponds to positive refraction at this region. For point B, at which $k_z^- = 0$, gives us the condition for backward wave. Above this point, in order to keep Poynting vector pointing to positive z direction, k_z^- must be selected for negative sign. Letting $k_z^- = 0$ in Eq.(5.64), we get

$$k_x = \omega\sqrt{\mu\varepsilon - \gamma^2} \tag{5.79}$$

Point C defines the maximum value of k_x. If k_x exceeds this point, then both waves become enavenscent wave. In fact, point C could be obtained by setting $k_z^+ = k_z^-$, which yields

$$k_x = \omega\sqrt{\mu\varepsilon} \tag{5.80}$$

Therefore, it is concluded that points A,B,and C provide us with conditions of negative refraction or backward wave. If $k_x > \omega\gamma$, then negative refraction can be found in k_z^+ wave and positive refraction in k_z^- wave. When $\omega\gamma < k_x < \omega\sqrt{\mu\varepsilon - \gamma^2}$, both eigenwaves support positve refraction. If k_x is further larger such that $\omega\sqrt{\mu\varepsilon - \gamma^2} < k_x < \omega\gamma$, we still have two positive refraction eigenwaves but backward wave occurs for k_z^- mode. Three different regions are indicated in Fig 19.

Chapter 6 Gaussian Beam wave

6.1 Beam propagation through dielectric interface

So far we have focused on plane wave propagation in various complex media, however, plane wave means the wave is on the whole plane and two transmitted waves are not easy to observe because what we see is the mixed profile of the field such as Fig 10&Fig 17. If we want to see two split beams as predicted in chiral medium, a bounded incident beam having Gaussian variation in its cross section is used to simulate laser beam. Following the method used in [29], suppose a Gaussian profile of electric field is incident at z=-h with angle θ, which takes the form:

$$E_i\left(x,-h\right)=\exp\left[-\left(x\cos\theta/w\right)^2+ikx\sin\theta\right] \tag{6.1}$$

The above distribution produces a radiant field which can be represented by Fourier representation.

$$\begin{aligned}\Psi\left(k_x,z=-h\right)&=FT\left(E_i\right)\\&=\int\exp\left(-\left(x\cos\theta/w\right)^2+ikx\sin\theta\right)\exp\left(-jk_xx\right)dx\\&=\frac{\exp\left(-\left(k_x-k\sin\theta\right)^2w^2/4\cos^2\theta\right)}{\cos\theta}\end{aligned} \tag{6.2}$$

$$\begin{aligned}E_i\left(x,z\right)&=IFT\left(\Psi(k_x,z=-h)\exp\left(jk_{0z}z\right)\right)\\&=\frac{1}{2\pi}\int\Psi(k_x,z=-h)\exp\left(jk_{0z}z\right)\exp\left(jk_xx\right)dk_x\end{aligned} \tag{6.3}$$

where $\sqrt{k_0{}^2-k_x{}^2}=k_{0z},\sqrt{k_1{}^2-k_x{}^2}=k_{1z}$. When incident upon a dielectric interface, reflected and transmitted fields are produced. Reflected field could be represented by the Fourier superposition spectrum of reflected wave propagating in minus z direction and the amplitude is that of incident wave multiply $R\left(k_x\right)\exp\left(ik_{0z}h\right)$. $R\left(k_x\right)$ is given by the result in plane wave analysis and $\exp\left(ik_{0z}h\right)$ is needed to satisfy phase matching condition.

$$E_r=IFT\left(R\left(k_x\right)\Psi(k_x,z=-h)\exp\left(-j\sqrt{k_0{}^2-k_x{}^2}\left(z-h\right)\right)\right) \tag{6.4}$$

Similarly, the transmitted field could be represented by Fourier superposition of the refracted wave propagating in positive z direction with amplitude is that of incident wave multiply $T\left(k_x\right)\exp\left(ik_{0z}h\right)$

$$E_t=IFT\left(T\left(k_x\right)\Psi(k_x,z=-h)\exp\left(j\left(k_{1z}z+k_{0z}h\right)\right)\right) \tag{6.5}$$

For dielectric interface, reflection and refraction coefficient is easily obtained for TE wave.

$$R = \frac{\eta_1 \cos\theta_0 - \eta_0 \cos\theta_1}{\eta_0 \cos\theta_1 + \eta_1 \cos\theta_0}$$
$$T = \frac{2\eta_1 \cos\theta_0}{\eta_0 \cos\theta_1 + \eta_1 \cos\theta_0} \tag{6.6}$$

In Fourier spectrum analysis, $\cos\theta$ must be expressed in term of k_x to include all value of k_x in the integral representation. Substitute $\cos\theta_i = \sqrt{1 - k_x{}^2 / k_i^2}$ into Eq.(6.6) , we have

$$R(k_x) = \frac{\mu_1 k_{0z} - \mu_0 k_{1z}}{\mu_1 k_{0z} + \mu_0 k_{1z}} = \frac{\mu_1\sqrt{k_0{}^2 - k_x{}^2} - \mu_0\sqrt{k_1{}^2 - k_x{}^2}}{\mu_1\sqrt{k_0{}^2 - k_x{}^2} + \mu_0\sqrt{k_1{}^2 - k_x{}^2}}$$
$$T(k_x) = \frac{2\mu_1 k_{0z}}{\mu_1 k_{0z} + \mu_0 k_{1z}} = \frac{2\mu_1\sqrt{k_0{}^2 - k_x{}^2}}{\mu_1\sqrt{k_0{}^2 - k_x{}^2} + \mu_0\sqrt{k_1{}^2 - k_x{}^2}} \tag{6.7}$$

The magnetic field could also be expressed as the Fourier integral representation shown by [30].

$$H_{ix} = IFT\left(\Psi(k_x, z=-h)\frac{-k_{0z}}{\omega\mu_0}\exp\left(j\sqrt{k^2 - k_x{}^2}z\right)\right)$$
$$H_{iz} = IFT\left(\Psi(k_x, z=-h)\frac{k_x}{\omega\mu_0}\exp\left(j\sqrt{k^2 - k_x{}^2}z\right)\right) \tag{6.8}$$

where $\frac{-k_{0z}}{\omega\mu_0}, \frac{k_x}{\omega\mu_0}$ represent $-\sin\theta_0, \cos\theta_0$ respectively.

$$H_{rx} = IFT\left(\Psi(k_x, z=-h)R(k_x)\frac{k_{0z}}{\omega\mu_0}\exp\left(-j\sqrt{k_0{}^2 - k_x{}^2}z\right)\right)$$
$$H_{rz} = IFT\left(\Psi(k_x, z=-h)R(k_x)\frac{k_x}{\omega\mu_0}\exp\left(-j\sqrt{k_0{}^2 - k_x{}^2}z\right)\right) \tag{6.9}$$

$$H_{tx} = IFT\left(T(k_x)\Psi(k_x, z=-h)\frac{-k_{1z}}{\omega\mu_1}\exp\left(j\left(k_{1z}z + k_{0z}h\right)\right)\right)$$
$$H_{tz} = IFT\left(T(k_x)\Psi(k_x, z=-h)\frac{k_x}{\omega\mu_1}\exp\left(j\left(k_{1z}z + k_{0z}h\right)\right)\right) \tag{6.10}$$

Time averaged energy flow or Poynting vector, could be expressed as:

$$\langle S\rangle = \sqrt{S_x{}^2 + S_y{}^2 + S_z{}^2} \tag{6.11}$$

where

$$S_x = \frac{1}{2}\mathrm{Re}\left[E_y H_z^{\ *} - E_z H_y^{\ *}\right]$$

$$S_y = \frac{1}{2}\mathrm{Re}\left[E_z H_x^{\ *} - E_x H_z^{\ *}\right]$$

$$S_z = \frac{1}{2}\mathrm{Re}\left[E_x H_y^{\ *} - E_y H_x^{\ *}\right]$$

Matlab simulation has been performed to simulate Gaussian beam propagating through dielectric interface as shown in Fig 20~Fig 23.

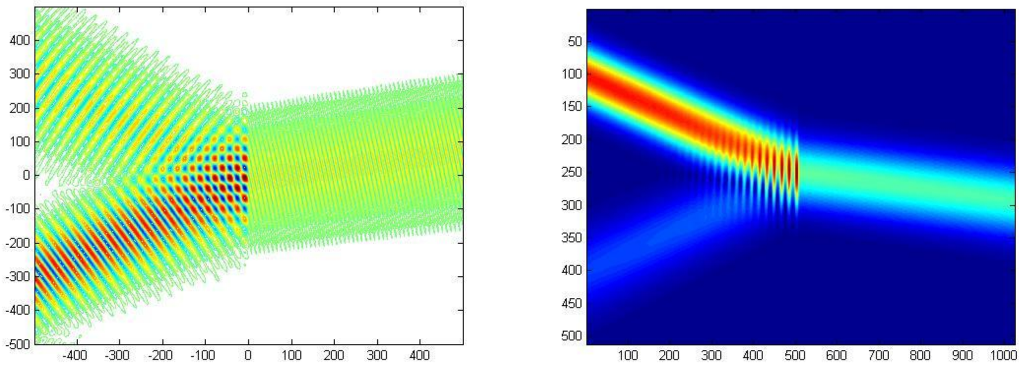


Fig 20 Electric field profile (left) and time averaged energy density (right) of an incidence Gaussian beam incident at angle $\theta_0 = 30^\circ$ with material parameters $\varepsilon_0 = \mu_0 = \mu_1 = 1, \varepsilon_1 = 9$.

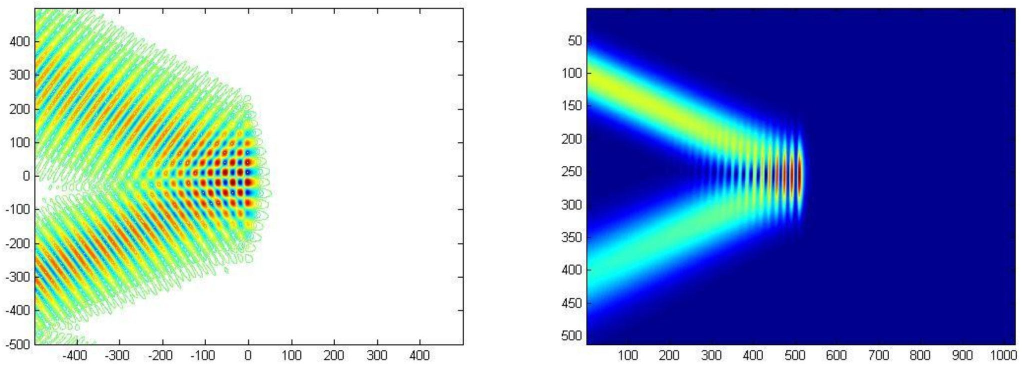


Fig 21 Total reflection of an incident Gaussian beam at angle $\theta_0 = 30^\circ$ with parameters $\varepsilon_0 = 10, \mu_0 = \mu_1 = \varepsilon_1 = 1$.

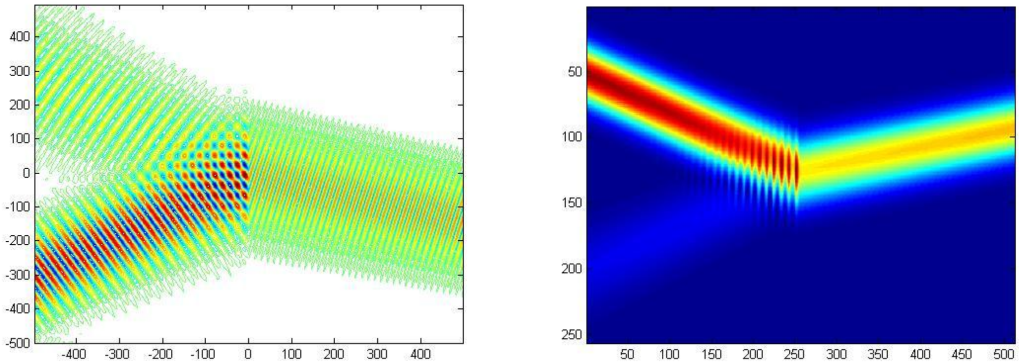


Fig 22 Negative refraction and backward wave resulted by Gaussian beam incident from air to a host medium having simultaneously negative parameters $\varepsilon_1 = -4, \mu_1 = -1$. It is seen that the beam is twisted another side.

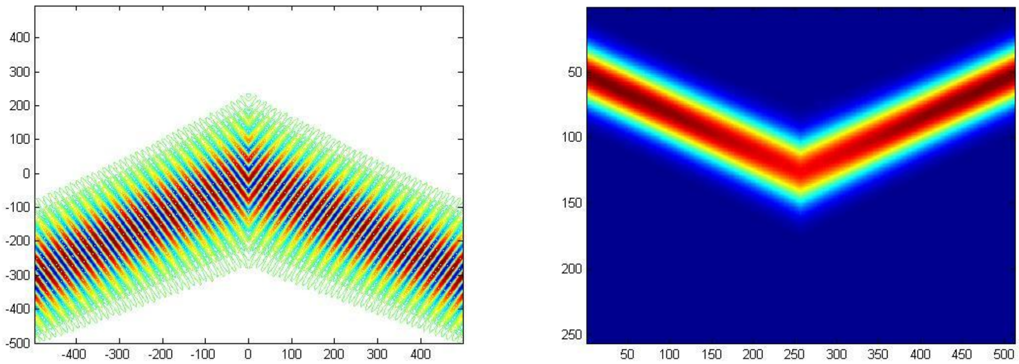


Fig 23 Negative refraction occurs with host medium parameters $\varepsilon_1 = \mu_1 = -1$. In this case, refraction angle is the same as incidence angle. A slab of such material could be used to make perfect lens [5].

Note that in case of negative refraction, $k_{iz} = -\sqrt{k^2 - k_x^{\,2}}$ or $\cos\theta_i = -\sqrt{1 - k_x^{\,2} / k_i^{\,2}}$ in Eqs.(6.6).

6.2 Beam propagation through chiral interface

If dielectric material is replaced by chiral material, two beam waves emerge. The reflection and transmission is given by Eqs.(4.21)&(4.22) to calculate the beam wave fields. Substitute $\cos\theta_i = \sqrt{k_1^{\,2} - k_x^{\,2}}$, $\cos\theta_1 = \sqrt{k_1^{\,2} - k_x^{\,2}}$, $\cos\theta_2 = \sqrt{k_2^{\,2} - k_x^{\,2}}$ in Eqs(6.12) &(6.13), where $k_1 = \omega\left(\sqrt{\mu_2\varepsilon_2} + \gamma\right), k_2 = \omega\left(\sqrt{\mu_2\varepsilon_2} - \gamma\right)$ correspond to wave number of right and left circular waves respectively.

$$
\begin{aligned}
R_{11} &= \frac{(Z\cos\theta_2+\eta_0\cos\theta_i)(Z\cos\theta_i-\eta_0\cos\theta_1)+(Z\cos\theta_1+\eta_0\cos\theta_i)(Z\cos\theta_i-\eta_0\cos\theta_2)}{(Z\cos\theta_2+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_1)+(Z\cos\theta_1+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_2)}\\
R_{12} &= \frac{-2iZ\eta_0\cos\theta_i(\cos\theta_1-\cos\theta_2)}{(Z\cos\theta_2+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_1)+(Z\cos\theta_1+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_2)}\\
R_{21} &= \frac{-2iZ\eta_0\cos\theta_i(\cos\theta_1-\cos\theta_2)}{(Z\cos\theta_2+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_1)+(Z\cos\theta_1+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_2)}\\
R_{22} &= \frac{(Z\cos\theta_2-\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_1)+(Z\cos\theta_1-\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_2)}{(Z\cos\theta_2+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_1)+(Z\cos\theta_1+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_2)}
\end{aligned}
\tag{6.12}
$$

$$
\begin{aligned}
T_{11} &= \frac{-2iZ\cos\theta_i(\eta_0\cos\theta_i+Z\cos\theta_2)}{(Z\cos\theta_2+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_1)+(Z\cos\theta_1+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_2)}\\
T_{12} &= \frac{2Z\cos\theta_i(Z\cos\theta_i+\eta_0\cos\theta_2)}{(Z\cos\theta_2+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_1)+(Z\cos\theta_1+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_2)}\\
T_{21} &= \frac{2iZ\cos\theta_i(\eta_0\cos\theta_i+Z\cos\theta_1)}{(Z\cos\theta_2+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_1)+(Z\cos\theta_1+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_2)}\\
T_{22} &= \frac{2Z\cos\theta_i(Z\cos\theta_i+\eta_0\cos\theta_1)}{(Z\cos\theta_2+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_1)+(Z\cos\theta_1+\eta_0\cos\theta_i)(Z\cos\theta_i+\eta_0\cos\theta_2)}
\end{aligned}
\tag{6.13}
$$

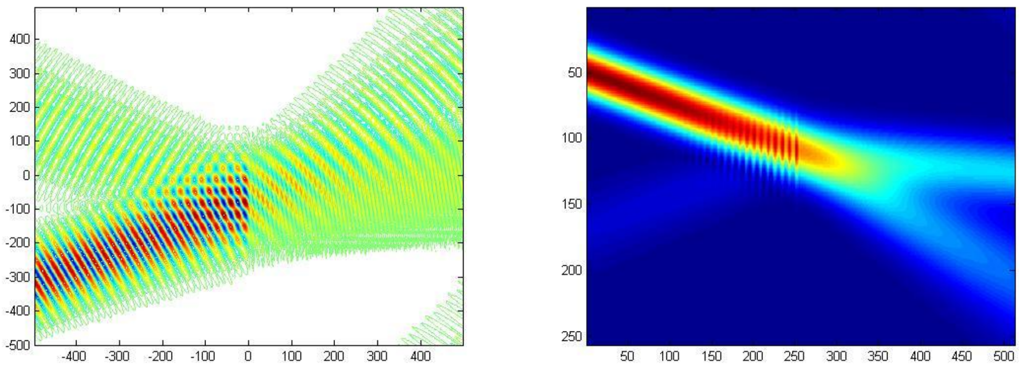


Fig 24 A incident TE beam wave at angle $\theta_i = 25^\circ$ propagate through a chiral medium with material parameters $\mu_1 = 1, \varepsilon_1 = 4, \gamma = 1.3$. Two transmitted beams correspond to RHCP and LHCP respectively.

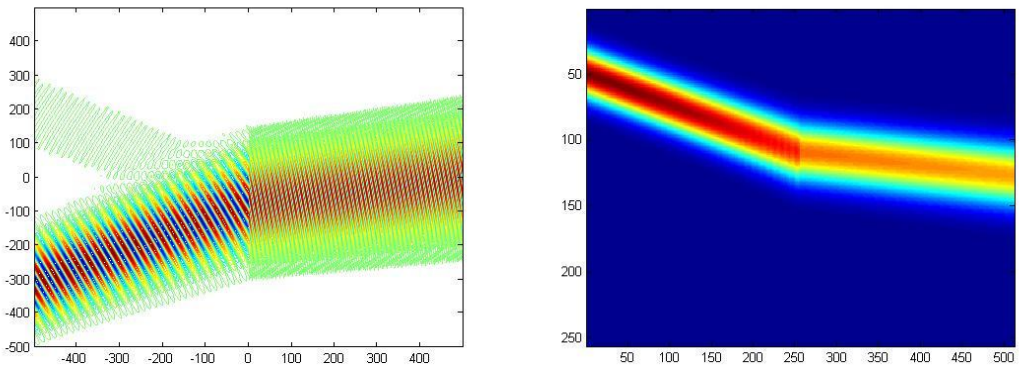
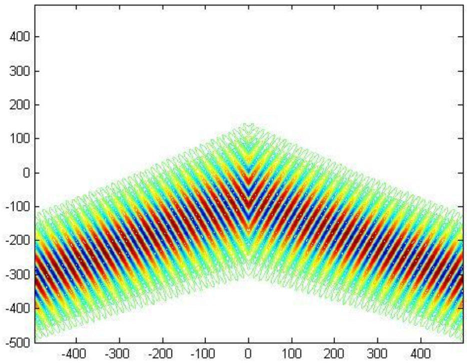


Fig 25 A right circular beam wave incident at angle $\theta_i = 25^\circ$ under impedance match condition with parameters $\mu_1 = \varepsilon_1 = 2, \gamma = 1$. Note that reflection and transmitted left circular waves vanish. The phenomenon is the same as discussed in plane wave.



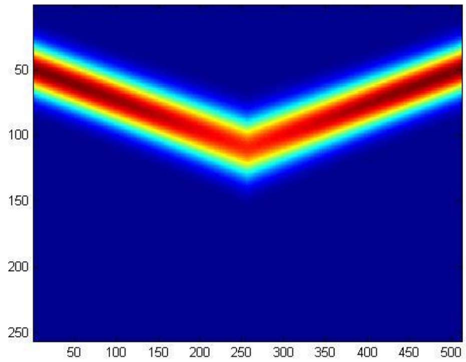
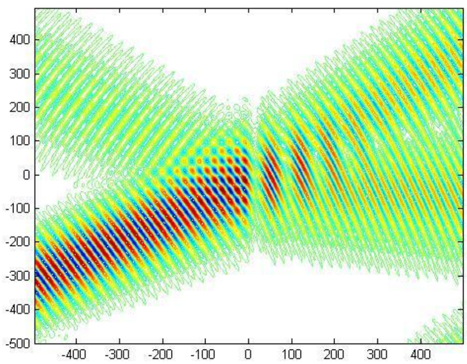


Fig 26 A left circular beam wave incident at angle $\theta_i = 25^\circ$ under impedance match condition $\mu_1 = \varepsilon_1 = 1, \gamma = 2$. Note that reflection and transmitted right circular vanish. The remaining transmitted left circular accounts for negative refraction and a slab of such material could be used to make perfect lens proposed by Pendry.

6.3 Beam propagation through pseudochiral interface

Let us examine beam wave propagation through pseudochiral interface. Like chiral material, two eigenwaves emerge in host medium. The reflection and transmission coefficients for calculating electric field have been derived in Eqs.(5.76)&(5.77). Parameter $\chi = \left(\mu\varepsilon - \gamma^2\right) / \sqrt{\left(\mu\varepsilon - \gamma^2\right)\left(\omega^2 \mu\varepsilon - k_x^{\ 2}\right)}$ is necessary for evaluating each coefficient. Matlab simulation is shown below:



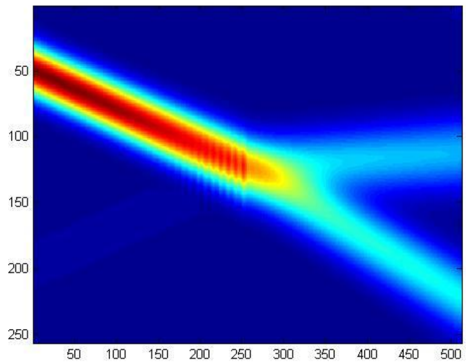


Fig 27. Beam propagation through pseudochiral medium with material parameters $\varepsilon = 2, \mu = 1, \gamma = 0.6$ at incidence angle $\theta = 30^\circ$. Right handed elliptic wave produces negative refraction while left handed elliptic wave produces positive refraction.

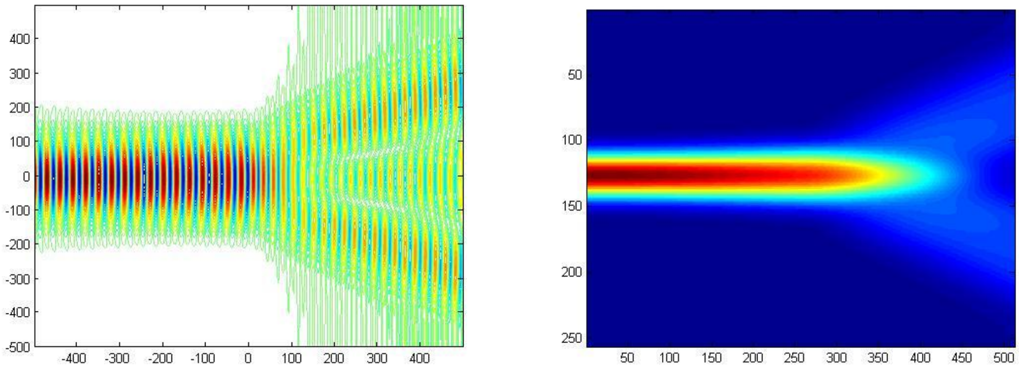


Fig 28. Normal incidence through pseudochiral medium with material parameters $\varepsilon = 2, \mu = 1, \gamma = 0.6$. Two splitting transmitted waves occur.

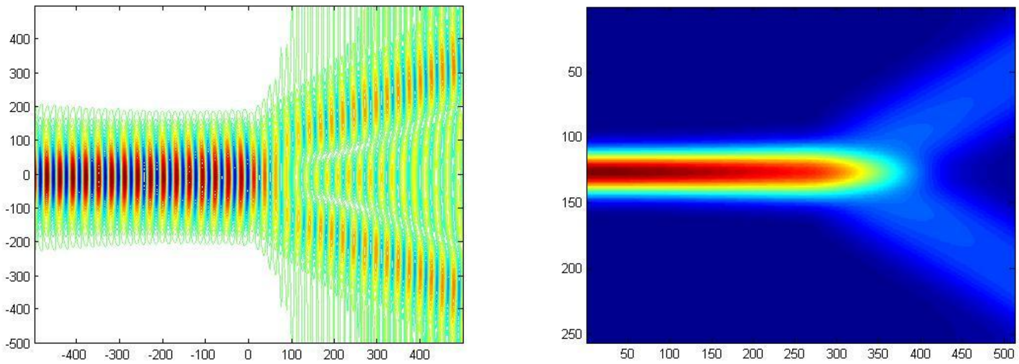


Fig 29. Normal incidence through pseudochiral medium with material parameters $\varepsilon = 2, \mu = 1, \gamma = 0.8$. Two splitting transmitted waves occur, however, transmitted angle is larger as γ gets larger.

It is clearly noted that two splitting beams occur under normal incidence. Such unusual characteristic can be explained by the singular point of isofrequency contour plot under normal incidence. The similar intersecting point could also be found in some modulated photonic crystal structure[31], which leads to undetermined group velocity. According to Notomi, at this point the propagation of light is very sensitive to direction of the wave vector so a slight change of the incident direction may cause opposite direction of group velocity.

To further investigate such splitting behavior, we put dispersion relation Eq.(5.64) into Eq.(6.3). The following analytical field expression could be obtained:

$$E_i = \frac{w}{2q_z}\exp\left(-\frac{x^2+\tilde{\gamma}z^2}{q_z^{\ 2}}\right)\exp\left(jh_0\sqrt{1-\tilde{\gamma}^2}\,z\right)\times$$
$$\left\{2\cosh\left(\frac{2\tilde{\gamma}xz}{q_z^{\ 2}}\right)+j\left[\exp\left(\frac{2\tilde{\gamma}xz}{q_z^{\ 2}}\right)erfi\left(\frac{x-\tilde{\gamma}z}{q_z}\right)-\exp\left(\frac{-2\tilde{\gamma}xz}{q_z^{\ 2}}\right)erfi\left(\frac{x+\tilde{\gamma}z}{q_z}\right)\right]\right\}$$

(6.14)

where

$$q_z = \sqrt{w^2+\frac{2j\beta z}{h_0}},\beta=\sqrt{1-\tilde{\gamma}^2},h_0=\omega\sqrt{\mu\varepsilon} \qquad (6.15)$$

and erfi is imaginary error function. The above equation gives us analytical result of beam propagation under normal incidence. It is seen that the two error functions produce beam steering. If cross polarized factor $\gamma = 0$, then Eq.(6.14) become formula of beam propagation in ordinary dielectric medium.

Chapter 7 Conclusion

In conclusion, in this work we derive dispersion relations, eigenwaves together with Poynting vector of wave propagation in bi-anisotropic medium in analytical form. Reflection and transmission through a planer interface is also formulated. In particular, we explore the wave propagation in pseudochiral material and the condition of negative refraction and backward waves are discussed. Due to symmetry embedded in chirality parameter in pseudochiral structure, reflected wave is always linearly polarized either in TE or TM incidence. Furthermore, it is shown that negative refraction and backward wave could be found in two separate eigenwaves. Such material could be made by simply adding omega shape inclusions in host medium and provides another route to realization of negative refraction.

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