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多通道隨機接取之設計與分析：透過隨機幾何之方法

Design and Analysis of Multi-Channel Random Access: A Stochastic Geometry Approach

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本論文係高靖越君（學號 R99942079）在國立臺灣大學電信工程學研究所完成之碩士學位論文，於民國 101 年 6 月 27 日承下列考試委員審查通過及口試及格，特此證明

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中文摘要

在這份論文中，我們研究使用者在多個正交頻率通道環境下之分散式無線隨意網路中的效能，並提出合適的隨機接取策略。在這類問題中，無線網路的空間分布特性通常是被忽略的；而我們透過來自隨機幾何理論的數學工具，對於這樣的問題推導出了其效能指標的解析解。藉由賽局理論的觀點，我們建構了具有利己特性、在網路中共享頻譜資源之使用者的互動模型；並且，我們將這樣情境下的效能結果，與當網路中具有一特定中央控管者或是當使用者具有合作性時之效能解析解進行比較。此外，我們也對利用通道旁消息於接取決策之影響進行探討。其中，我們發現若傳送端擁有通道狀態資訊，通道的多樣性可以被利用，並將有助於使用者效能之提升；但是，當使用者的密度高，並且使用者知道所有通道的可用性時，效能反而會因此降低，也就是產生了類似布雷斯悖論的現象。最後，我們提出幾個能更進一步提升網路效能的機制。藉著利用時域上的資源，我們提出一個接取控制方法以減輕高使用者密度時的干擾問題。另一方面，在無線電裝置具有同時接取多個通道的能力、並使用者密度低時，可透過最大比率合成來利用頻率之多樣性以增進效能。

關鍵詞: 多通道、隨機接取、無線隨意網路、隨機幾何、賽局理論、通道旁消息。

Abstract

In this thesis, we study the performance of and devise appropriate random access strategies for users in a decentralized wireless ad hoc network operating on multiple orthogonal frequency channels. The spatial factor of such problems which is still lack of study is considered with the help of tools from stochastic geometry, from which we derived closed-form expressions for performance metric. The interactions between selfish users sharing the radio resources are modeled with a game-theoretic point of view, and the performance are compared with that in the case when there's a central entity or when the users are cooperative, where we also provide explicit characterizations. The impacts of utilizing channel side information when making access decisions are also explored, where with local channel state information (CSI) available at the transmitter, channel diversity can be exploited and user performance can be improved; but when channel availability information are known by users, a Braess-like paradox, where when more information is provided the performance however degrades, can occur when the user density of the network is high. Finally, mechanisms that may further improve the network performance are introduced. Taking advantage of the time domain resource, an access barring method is proposed to alleviate the interference problem with high user density. On the other hand, frequency diversity can be exploited at low user density with Maximal

Ratio Combining (MRC) to improve the performance when the radio devices are capable of accessing multiple channels simultaneously.

Keywords: Multiple channels, random access, wireless ad hoc networks, stochastic geometry, game theory, channel side information.

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Chapter 1

Introduction

1.1 Preface

In broadband wireless networks where multiple orthogonal frequency channels are available for operation, it becomes a critical issue that radio devices choose suitable communication parameters to best utilize the resources of the network. This is a specifically important problem with spectrum-agile radios. Empowered by software-defined radio technologies such as Cognitive Radios (CR) [3], devices are able to dynamically tune to different frequency channels for channel access in a more intelligent manner. Since different frequency channels may provide different channel qualities, which would lead to different throughput experienced by a user, the choice of frequency channel for data transmission will highly affect the performance of communication. Thus, a user in the network should make channel selection decisions properly based on the knowledge of the spectrum environment.

However, since the wireless channel is a shared medium, the performance of communication a user experience is not only determined by the access decisions of itself, but the influences caused by of sharing spectrum resources with other users in the network should also be taken into consideration. In particular, when there are other users performing data transmission in the same channel, co-channel interference may lead to a substantial degradation of the spectrum performance. Therefore, there is an inevitable concern on the impact of the interactions among the users in the network. Furthermore, in ad hoc networks where centralized coordination is not possible and each user makes decisions autonomously according to their own objectives, the medium access strategy taken by a user needs to be carefully devised.

Due to the distributed nature of wireless ad hoc networks, the spatial distribution of devices/users is an important factor that might affect the network performance. Many studies on medium access has either ignored or simplified the assumptions for these aspects in small or local networks. However, for large scale wireless ad hoc networks, they can no longer be ignored.

In the following we first give a brief review of previous works to demonstrate the motivation of our work.

1.2 Background study

1.2.1 Traditional random access

Since 1970, when the landmark work by Abramson [4] was proposed to enable the distributed sharing of a common communication medium by a large amount of users with a simple protocol – ALOHA. There has been a tremendous amount of study on random access, where the performance and properties of ALOHA and its improved version – slotted ALOHA [5, 6] were of specific interest. It is of special concern on their performance with some given data traffic of users, and the interactions between and the dynamics of traffic intensity and the corresponding throughput. Analysis with simplified assumptions can be found in [7], in which an infinite number of users without buffering are considered, and the stability of slotted ALOHA were studied in more realistic assumptions with queueing theoretic analysis in [8–10]. In spite of the simplicity of the slotted ALOHA protocol, its queueing analysis turns out to be a extremely difficult task due to the complexity of interactions among the queues [11, 12]. Another line of study is the throughput region of random-access, defined to be the set of all achievable long-term average rate that can be obtained by varying access probabilities [13], and was fully characterized in [14].

1.2.2 Random access in wireless ad hoc networks

Spatial reuse is a key feature in spatially distributed network say, wireless ad hoc networks [15]. System throughput may be enhanced by allowing simultaneous data transmissions at different locations of the network. By the distributed nature of wireless ad hoc networks, the probabilistic reception due to wireless channel and the spatial distribution of users cannot be ignored. However, most of the studies of random access mentioned in the previous section did not take these aspects into consideration. Most of the works assumed a simplified collision channel model, in which any simultaneous transmission leads to a failure of reception. Furthermore, they mainly considered or implicitly assumed a star topology, where all users want to access a single base station or access point. These assumptions and scenarios obviously does not fit into the case for wireless ad hoc networks.

The study of slotted ALOHA with multipacket reception (MPR), which includes capture models, was introduced in [12, 16]. By such a model, the effects of probabilistic receptions in wireless channels are captured. However, the models they considered are still based on a star topology and do not take the spatial distribution of users into consideration. The results on throughput region by [14] was extended to a general topology in [13]. With an graph-based model of interfering links, the model in [13] somehow studied the spatial separation of nodes in the sense that only links with an edge between the corresponding nodes in the interference graph has the chance to interact. However, such graph based model is still simplistic since transmission of signals over wireless channels actually make all the transmitting links interact by the interference they imposed. Thus, there is a need for a more precise modeling for the spatial distribution of wireless ad hoc networks.

1.2.3 Spatial analysis of random access with stochastic geometry

Stochastic geometry [17, 18] provides a general model for the study of spatial distribution of nodes in wireless networks. By considering the interference imposed by the active transmitters in the network, the performance of users (outage/throughput, etc.) in the network can be characterized and in some cases with a closed form expression, giving insights to the design and analysis of medium access protocols [19, 20] for networks that are spatially distributed.

In [21], a spatial reuse ALOHA (SR-ALOHA) protocol was introduced, and the study was extended in [20]. The protocol is very simple that a coin is tossed by each user and it accesses the channel if it gets heads. The bias of the coin is optimized so that the best performance of the network could be obtained. Despite its simplicity, the protocol captures how the best spatial reuse can be achieved by considering spatially distributed users and by introducing novel closed form expressions for spatial averages (e.g. outage, throughput).

The studies in [20, 21] are closely related to the concept of transmission capacity in [22], which is defined as the number of successful transmissions taking place in the network per unit area subjecting to a constraint on outage probability. By studying the transmission capacity, some important performance indicators of a MAC protocol such as throughput and area spectral efficiency can be captured. In fact, as pointed out in [22], the transmission capacity is in fact the MAC layer throughput of a wireless network under ALOHA subject to an outage probability constraint.

However, it should be noted that most works studied with stochastic geometry are focused on scenarios with only a single channel [20, 23, 24]. In [25], multiple channels are considered with respect to transmission capacity. Nonetheless, the work of [25] focused on how many sub-bands a certain total system bandwidth should be partitioned into such that the transmission capacity is maximized, but not how a given number of channels should be accessed appropriately.

1.2.4 Multi-channel random access in wireless networks

As mentioned earlier, in modern broadband wireless networks where multiple orthogonal frequency channels are available, it is of utter importance on the proper access of the channels. As identified in [26], two major challenges of multi-channel MAC lies in channel selection and collision avoidance/resolution [27] by distributed users in the network which are discussed in the following.

The problems of channel selection in multi-channel MAC is about choosing suitable channel by the transmitter/receiver for data transmission. Practical issues and algorithms for channel negotiation between transmitters and receivers can be found in [28–30] and the references therein.

Recently, the study of choosing proper frequency channel for channel access has been extended and has attracted tremendous attention in the context of Dynamic Spectrum Access (DSA) [31–36]. In [31], a Partially Observable Markov Decision Process (POMDP) framework was proposed for the sensing and access strategies of device in decentralized ad hoc networks. The POMDP framework exploits the time-domain knowledge of the spectrum environment so that channel selection decisions are made to maximize the long-term performance, but the model did not explicitly take the multiuser environment into consideration, which is every important in a netowrk. In [32], access strategies in the presence of multiple user competition are studied along with tools in classical bandit problems. However, the restriction that users impose equal impact on each other due to a complete contention relationship ignores the important factor of the spatial distribution of network devices. Partial interfering relationship between network devices was treated in [33] with a graph model. But still, the assumption of a simplified collision model of transmission in the works above makes it unrealistic in wireless environment where interference from other users goes through path loss and effects of channel fading which jointly affects the performance of wireless transmissions.

As for collision avoidance/resolution, we should recall that the fundamental problem of MAC is to reduce the impact of collisions over the shared medium. In [27], a generalized tree expansion structure was proposed for a centralized network. The collision resolution tree expansion (CRTE) is for handling collisions, while the collision anticipation tree expansion (CATE) is for splitting the contending users. Some solutions such as the RAP family [37,38] had been proposed for centralized networks with star topologies and the key idea lying behind is how the users are split into groups to avoid/resolve contention.

1.2.5 User interactions in random access with game theory

In wireless networks, the users may be closely related to each other in the decision they make and the performance they can achieve. The actions. Game theory has been applied to study the strategic interaction of users with random access in wireless networks in [39–41]. Selfish behaviors and their resulting performance are studied at the Nash equilibria. However, in these works, the network models considered are with a single channel only, and the spatial distribution of nodes are not well characterized. In [42], the authors considered the spatial distribution of nodes with stochastic geometry and the interaction between users with game theory. Pricing are used in order to let the selfish users achieve the socially optimal performance. However, the issue of how appropriate access strategy should be devised when there are multiple channels was not taken care of.

1.2.6 Channel-aware random access

Another issue in wireless ad hoc network with random access is the impact of some channel side information in the process of channel access decision-making [40, 41, 43, 44]. By exploiting local channel state information (CSI), [44] proposed a decentralized joint physical-MAC layer optimization that exploits multiuser diversity in multi-channel wireless ad hoc networks. The network model considered in [44] is however based on a simplified graphical collision model, where some important factors such as interference in the network are not completely characterized. In [40, 41], the interaction between users with CSI are considered with game theory, and it was found in [41] that the extra information of CSI may however degrades the performance. Nonetheless, the network they consider are with a single channel only and do not take multi-channel into consideration. And still, the aspects of spatial distribution of nodes are lack of precise characterization.

1.3 Motivation and goal of thesis

As can be seen in the previous section, there has been a lot of works in the literature on multi-channel random access, interactions between users with game theory and the spatial aspects of wireless networks with stochastic geometry. However, to the best of our knowledge, there has not yet been a joint consideration of all the aspects listed below:

- Multi-channel random access taking the channel qualities/characteristics into consideration in channel selection.
- Game-theoretic modeling of the interactions of users in the network for contention of resources and the corresponding performance analysis.
- Characterization of the spatially distributed nature of wireless ad hoc network with stochastic geometry and the interaction of users in the sense of the interference they cause imposed on each other.

The main focus of this thesis is to take all the aspects listed above into consideration. We analyze the behaviors of and devise proper channel access strategies for users in multi-channel wireless ad hoc networks. With the help of tools from stochastic geometry, we incorporate important spatial aspects of wireless networks such as path loss and channel fading that governs the performance of a wireless transmission. In addition, a game-theoretic point of view, which provides a good modeling framework for the study of interactions between decentralized competing users of network resources, is proposed to suggest the users of suitable channel selection strategies. The case where there is a central entity controlling the users or when the users are cooperative is also discussed, and the results are compared to those with the game-theoretic solution.

Then, the impact of some channel side information is examined, and the best strategy for channel access with such information is devised along with the corresponding performance comparisons. Finally, we identify and examine mechanisms that may further improve the performance of the network. On one hand, for that with the growing density of network devices, the interference-limited nature of network will highly degrades the channel; we propose a access barring method that exploits the time domain resource by which the throughput of the users can be substantially improved. On the other hand, when the users have the capability of accessing multiple channels simultaneously, frequency diversity of the channels is exploited with Maximal Ratio Combining (MRC) to increase the throughput performance.

Chapter 2

Preliminaries

2.1 Poisson Point Process

2.1.1 Stationary PPP

A stationary Poisson point process (PPP) [19] with density λ is characterised by the following properties

- The number of points in a set $A \subset \mathbb{R}^2$ with area $|A|$ is a Poisson random variable with mean $\lambda|A|$. That is,

$$\mathsf{Pr}\left(\ (A) = k\right) = \frac{(\lambda|A|)^k}{k!} e^{-\lambda|A|} \tag{2.1}$$

  where (A) denotes the number of points in A.

- The number of points in disjoint sets are independent.

Throughout this work, we use $\Psi = \{X_i\}$ to denote the locations of (possibly a subset of) nodes in a network and $\Pi(\lambda)$ to denote a stationary PPP with density λ. With $\Psi = \Pi(\lambda)$, we mean that the nodes specified by Ψ are distributed according to $\Pi(\lambda)$.

2.1.2 Poisson shot noise process

The Poisson shot noise [19] is defined as

$$I = \sum_{X_i \in \Pi(\lambda)} H_i P |X_i|^{-\alpha} \tag{2.2}$$

where H_i is a random variable and P is a constant. The Poisson shot noise process can be used to model the interference received at a node in the network, where P represents the transmission power and H_i represents the channel fading coefficient.

The Laplace transform of I, is defined and could be obtained as

$$\mathcal{L}\{I\} \triangleq \mathsf{E}[e^{-sI}] = \exp\left(-\lambda\pi P^{\delta}\mathsf{E}[H_i^{\delta}]\Gamma(1-\delta)s^{\delta}\right) \tag{2.3}$$

where $\delta \triangleq \frac{2}{\alpha}$ and $\Gamma(z) = \int_0^\infty t^{z-1}d^{-t}dt$ is the Gamma function.

The probability density function (pdf) and cumulative distribution function (cdf) of I has a closed form expression for a specific case where $\alpha = 4$ (i.e. $\delta = \frac{1}{2}$), where I has a Lévy distribution with parameter $\gamma = \frac{1}{2}\lambda^2\pi^3 P\left(\mathsf{E}[H_i^{\frac{1}{2}}]\right)^2$ [22], pdf

$$f_I(x) = \sqrt{\frac{\gamma}{2\pi}}\frac{e^{-\frac{\gamma}{2x}}}{x^{\frac{3}{2}}} \tag{2.4}$$

and cdf

$$F_I(x) = \operatorname{erfc}\left(\sqrt{\frac{\gamma}{2\pi}}\right) \tag{2.5}$$

where $\operatorname{erfc}(s) = \frac{2}{\sqrt{\pi}}\int_s^\infty \exp(-t^2)dt$ is the standard complementary error function. When H_i is exponentially distributed with mean m which corresponds to the important case with Rayleigh fading, we have $\mathsf{E}[H_i^{\frac{1}{2}}] = \frac{\sqrt{m\pi}}{2}$, and I is then Lévy-distributed with $\gamma = \frac{1}{8}\lambda^2\pi^4 mP$. Specifically, the cdf of I is given by

$$F_I(x) = \operatorname{erfc}\left(\frac{\lambda\pi^2 P^{\frac{1}{2}} m^{\frac{1}{2}}}{4x^{\frac{1}{2}}}\right) \tag{2.6}$$

2.1.3 The thinning of a PPP

Consider a set $\{\Psi^{(p_i)}\}_{i=1}^{K}$ of K point processes obtained from assigning each point in $\Pi(\lambda)$ to $\Psi^{(p_i)}$ independently with probability p_i such that $\sum_{i=1}^{K} p_i = 1$ (i.e. thinning [19]). We will have each of the resulting point processes in $\{\Psi^{(p_i)}\}_{i=1}^{K}$ a PPP with density $\mu_i \triangleq p_i\lambda$ independent of other point processes in the set. That is, we have $\Psi^{(p^{(i)})} = \Pi(p_i\lambda)$.

2.2 Performance metrics of MAC with spatial considerations

In this section, we brief some performance metrics that has been proposed in the literature concerning the spatially distributed nature of wireless networks with stochastic geometry.

In [45], the notion of *transmission capacity* is proposed for analysing the performance of MAC protocols in spatially distributed wireless ad hoc networks. Fundamentally, the transmission capacity captures the area spectral efficiency, reliability, and throughput of a random access protocol. We start by giving some assumptions and definitions about transmission capacity, which can be found in detail in [22]:

1. The network is considered with a single snapshot.

2. The network consists of transmitter-receiver pairs. A transmitter and its desired receiver are with a fixed distance r from each other.

3. Each receiver treats interference as noise. The rate of a transmitter-receiver pair is given by the Shannon capacity.

4. The transmitters form a homogeneous PPP in the $\mathbb{R}^2$ plane.

5. Every transmitter decides independently whether to transmit with a common probability p_{tx}.

Given the assumptions above, we give the notion of *outage probability*.

Definition 1 (Outage probability, [22]). *The outage probability of a transmitter in the network is defined as*

$$\nu \triangleq \mathsf{Pr}\left(\log_2\left(1 + SINR\right) < R\right) \tag{2.7}$$

where R is the information rate of the transmitter-receiver pair and SINR is the signal-to-interference-ratio at the receiver.

It can be easily seen that the outage probability is dependent on the density of active transmitters in the network due to interference. Thus, we denote the outage probability with density of active transmitters in the network being λ as $\nu(\lambda)$.

As for the performance of slotted ALOHA in a system perspective, we have the following definition for MAC layer throughput.

Definition 2 (MAC layer throughput, [22]). *The MAC layer throughput of a wireless network with slotted ALOHA, where the active transmitters form a PPP with density λ is*

$$\Lambda(\lambda) \triangleq \lambda(1 - \nu(\lambda)) \tag{2.8}$$

The MAC layer throughput is with units of successful transmission per unit area, and captures the efficiency of a random access protocol in a spatial context as for a system perspective.

The transmission capacity is then defined in the following.

Definition 3 (Transmission capacity, [22]). *The transmission capacity of the network is defined as the maximum spatial density of successful transmissions subject to an outage probability constraint ν^*. That is,*

$$TC(\nu^*) \triangleq \nu^{-1}(\nu^*)(1 - \nu^*) \tag{2.9}$$

Note that $\nu^{-1}(\nu^*)$ is the density of transmitter that can make the outage constraint ν^* satisfied.

There is a close relationship between transmission capacity and the MAC layer throughput. In fact, the transmission capacity is the MAC layer throughput maximization constrained on the outage probability, as was shown in [22]. Specifically, we have $\mathrm{TC}(\nu^*) = \lambda^*$, where λ^* solves the following optimization problem:

$$\begin{aligned} \max_{\lambda} \quad & \lambda(1-\nu(\lambda)) \\ \text{s.t.} \quad & \nu(\lambda) \leq \nu^* \end{aligned}$$

Another line of study of the performance with spatial aspects of wireless network is the *transport capacity* as introduced in the seminal work by Gupta and Kumar [46]. Defined as the maximum distance-weighted sum rate of communication over all pairs of nodes, transport capacity optimizes all scheduling and routing protocols and the focus is on the how the sum rate scales asymptotically in the number of nodes [22]. In spite of its generality, the results provided by scaling laws [47] are less specific about the merit of a MAC protocol. Still another line of study is of an information-theoretic approach, which is well summarized in [48]. However, it is more suitable for the study of small isolated networks [22].

2.3 Game theory and decision making

Game theory is a set of tools that can be used to help us understand the phenomena we might observe in the interaction between multiple decision makers. In the following, we state some simplified concepts and results in traditional game theory that might be used in this work.

A game is a model that describes the interactions between some decision makers, called players. Each player in a game make their own decision, called action. We can represent the set of N players by a set $\mathcal{N} = \{1, 2, \cdots, N\}$, and the set from which a player $i \in \mathcal{N}$ choose its action by A_i. A collection of actions of the players $a = (a_1, a_2, \cdots, a_N)$ is called an action profile, and is also referred to as an outcome. Associated with each player $i \in \mathcal{N}$, a utility function $u_i : \mathcal{A} \to \mathbb{R}$ describes the preferential relation of the outcomes of a game, where $\mathcal{A} = A_1 \times A_2 \times \cdots \times A_N$ is the set of outcomes. A higher utility suggests a higher preference of a user to the outcome, and here we consider rational players that each of them take actions with the purpose of maximizing their own utilities. In summary, we have:

Definition 4. *A game, denoted by the triple $\langle \mathcal{N}, (A_i), (u_i) \rangle$, consists of the following basic elements:*

- *A finite set $\mathcal{N} = \{1, 2, \cdots, N\}$ of N players (decision makers).*
- *A nonempty set of actions (possible decisions) A_i for each player $i \in \mathcal{N}$.*
- *Associated with each player $i \in \mathcal{N}$, a utility function $u_i : \mathcal{A} \to \mathbb{R}$.*

Let a_{-i} denote the action profile of players other than i. The notion of a Nash equilibrium describes a steady state outcome of a game in which players make rational decisions. More specifically, we have the following definitions.

Definition 5. *An action profile $a^* = (a_1^*, a_2^*, \cdots, a_N^*) \in \mathcal{A}$ is a Nash equilibrium if for any player $i \in \mathcal{N}$,*

$$u_i\left(a_i^*, a_{-i}^*\right) \geq u_i\left(a_i, a_{-i}^*\right) \text{ for all } a_i \in A_i \tag{2.10}$$

We now introduce the notion of mixed strategies, where the players play their actions in a probabilistic manner.

Definition 6. *Consider the game $\langle \mathcal{N}, (A_i), (u_i) \rangle$. Suppose $A_i = \{s_1^{(i)}, s_2^{(i)}, \cdots, s_K^{(i)}\}$. Then a mixed strategy for a player i is a probability distribution $p^{(i)} = (p_{s_1}^{(i)}, p_{s_2}^{(i)}, \cdots, p_{s_K}^{(i)})$ over the action set A_i, where $0 \leq p_k^{(i)} \leq 1$ for $k = 1, 2, \cdots, K$ and $\sum_{j=1}^{K} p_j^{(i)} = 1$. A mixed strategy profile $p = (p^{(1)}, \cdots, p^{(K)})$ is a collection of mixed strategies of the players.*

The utility function of player i with players using mixed strategies can then be defined as

$$U_i\left(p^{(i)}, p^{(-i)}\right) = \sum_{a \in \mathcal{A}} \left(\prod_{j \in \mathcal{N}} p_{a_j}^{(j)} \right) u_i\left(a\right) \tag{2.11}$$

where $p^{(-i)}$ denotes the mixed strategies adopted by players other than i.

Now, we can define the Nash equilibrium associated with mixed strategies.

Definition 7. *A mixed strategy profile p^* is a mixed strategy Nash equilibrium if for any user $i \in \mathcal{N}$,*

$$U_i\left(p^{*(i)}, p^{*(-i)}\right) \geq U_i\left(p^{(i)}, p^{*(-i)}\right) \text{ for all } p^{(i)} \in \triangle(A_i) \tag{2.12}$$

where $\triangle(A_i)$ is the set of all probability distributions over A_i.

The following lemma will be useful in this work in solving for a Nash equilibrium of a game.

Lemma 1 ([49], Lemma 33.2)**.** *A mixed strategy profile is a mixed strategy Nash equilibrium of a finite game if and only if for each player it is indifferent between the actions in the support of the equilibrium, where the support is defined as the set of actions which are assigned non-zero probabilities by its mixed strategy in the mixed strategy profile.*

By Lemma 1, we have that p^* is a mixed strategy Nash equilibrium if for any player $i \in \mathcal{N}$

$$U_i\left(a_j, p^{*(-i)}\right) = U_i\left(a_{j'}, p^{*(-i)}\right), \quad \forall a_j, a_{j'} \in A_i, p_{a_j}, p_{a_{j'}} \neq 0 \tag{2.13}$$

where we have defined $U_i\left(a_j, p^{(-i)}\right)$ as the utility obtained by user i when it use the mixed strategy $p_{a_j}^{(i)} = 1$ and $p_{a_{j'}}^{(i)} = 0$ for $a_j \neq a_{j'} \in A_i$.

Chapter 3

System Model

3.1 Network topology and channel model

We consider a wireless ad hoc network in which the transmitters (which we will also call "users") are distributed with locations specified by a homogeneous Poisson Point Process (PPP) with density λ on the 2-D plane. With $\Psi = \{X_i\}$ denoting the locations of the transmitters, we have $\Psi = \Pi(\lambda)$. We assume that each transmitter transmits with power P to the its target receiver of distance r away.

Assume the network operates on K frequency channels with equal bandwidth, denoted by the set $\mathcal{K} = \{1, 2, ..., K\}$. A channel $k \in \mathcal{K}$ can be either available for channel access or not, and a user can utilize a channel for transmission only if the channel is available. Let A_k denote the indicator variable if channel k is available

$$A_k = \begin{cases} 1, & \text{channel } k \text{ is available} \\ 0, & \text{o.w.,} \end{cases} \tag{3.1}$$

and we assume that $A_k \sim \text{Bernoulli}(\theta_k)$, where θ_k is called the channel available probability. If we have $\theta_k = 1 \ \forall k \in \mathcal{K}$, this corresponds to traditional random access. When the channel availability probabilities of different channels are different, it fits the scenario of Cognitive Radio Networks (CRN) [50] where secondary users (SUs) do spectrum sensing[1] to identify the state of a licensed channel and access only if the channel is sensed to be free of primary users where the channel available probability θ_k can be considered as the probability of presence of primary users. We assume that $0 < \theta_k \leq 1$ for that it would be trivial to consider a channel with zero available probability.

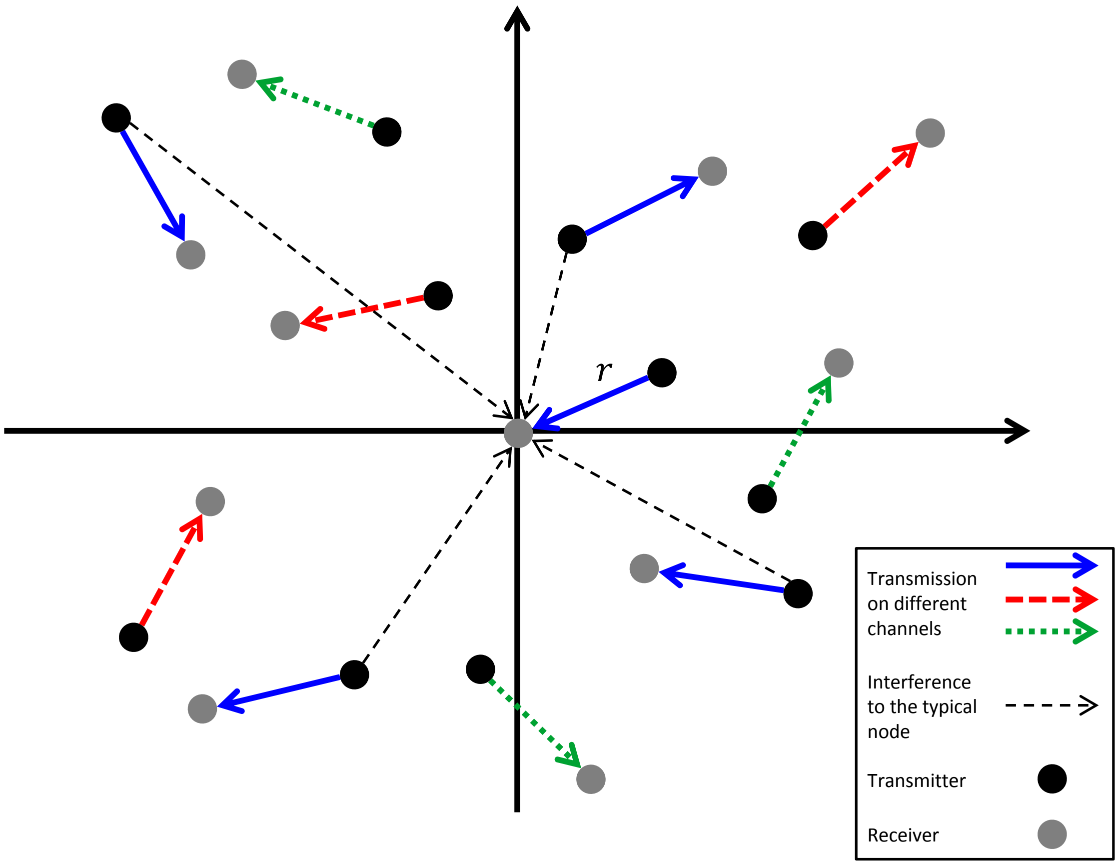


Figure 3.1: The locations of the transmitters in the network are distributed according to a Poisson Point Process, and the intended receiver is of distance r away from a transmitter. The transmitters are represented by the black dots while the receivers are represented by the gray dots, and a typical receiver is placed at the origin. Different colors and line-styles of the bold arrows represent transmissions in different channels. The dashed arrows in black represent interference relationship to the typical receiver. This is the modified multi-channel version of the network topology in [2].

We assume the channel undergoes a general fading with fading coefficients $\{H_{ij,k}\}$, where i and j denotes different users in the network and k is the index of correspond-

[1] In our work, we assume perfect spectrum sensing such that the actual spectrum availabilities can be obtained. The issue of sensing error is out of the scope of this work.

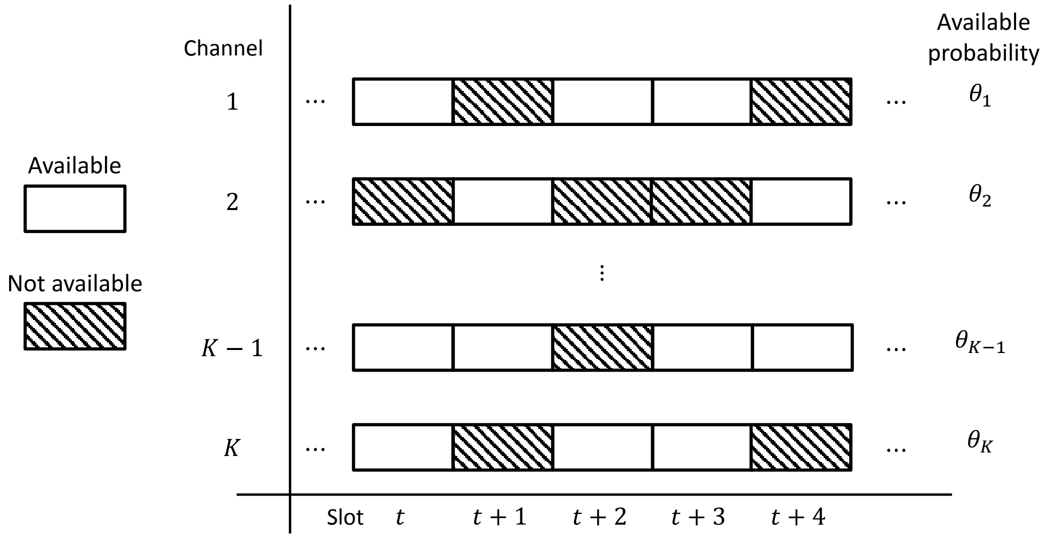


Figure 3.2: The network operates under a synchronized slotted structure with K frequency channels. In each slot, a channel may be available or not, and the probability of a channel k being available is θ_k. Note that the available probabilities of different channels might be different.

ing channel. We assume the fading coefficients between different users are i.i.d., but may be non-i.i.d. over different channels. The fading coefficient $H_{ij,k}$ is characterized by its cumulative distribution function (cdf) $F_{H_{ij,k}}(h)$ and probability density function (pdf) $f_{H_{ij,k}}(h)$. The channel strength is determined jointly by pathloss and fading, i.e. the received signal power at node j due to node i at distance d away in channel k is $H_{ij,k}d^{-\alpha}$, where $\alpha > 2$ is the pathloss exponent. For convenience of notation, we represent the fading coefficient of the link for the desired signal in channel k as $H_{S,k}$ and that for the interference link in channel k as $H_{I,k}$.

3.2 MAC mechanism

We assume the network operates under a time-slotted and synchronized structure. Each user in the network selects one of the K frequency channels in each time slot for channel access according to some access strategy based on its knowledge (e.g. channel statistics) of the system . We assume the users in the network are non-cooperative, and each of them intends to maximize its own performance metric. In particular, we set the performance metric of a user to be the throughput, which is defined in terms of the average number of successful channel access in a time slot given its access strategy. A successful channel access in a time slot can occur when the channel is both available (i.e. $A_k = 1$ for channel k) and the transmission in that channel does not encounter an outage, where an outage happens if the channel cannot support the information rate R of a transmission. That is, an outage occurs in channel k if

$$\log_2\left(1 + \frac{H_{S,k}Pr^{-\alpha}}{N_0 + I_k}\right) < R, \tag{3.2}$$

where I_k is the interference power in channel k and N_0 is the power of the background noise. Thus, the throughput in a channel k (given the user has chosen channel k for access) can be written as

$$\mathcal{T}_k = \theta_k \mathsf{Pr}\left(\log_2\left(1 + \frac{H_{S,k}Pr^{-\alpha}}{N_0 + I_k}\right) \geq R\right) \tag{3.3}$$

When a user adopts a probabilistic channel access strategy, say $\mathbf{p} = [p_1, \ldots, p_K]$, such that channel k is chosen for access with probability p_k, the throughput of the user can be written as

$$\mathcal{T} = \sum_{k\in\mathcal{K}} p_k \mathcal{T}_k = \sum_{k\in\mathcal{K}} p_k \theta_k \mathsf{Pr}\left(\log_2\left(1 + \frac{H_{S,k}Pr^{-\alpha}}{N_0 + I_k}\right) \geq R\right) \tag{3.4}$$

Finally, the performance in a system perspective which we call the "system throughput" is defined by multiplying the user throughput by the network density, i.e. $\lambda\mathcal{T}$, which has units in successful transmission per unit area [42].

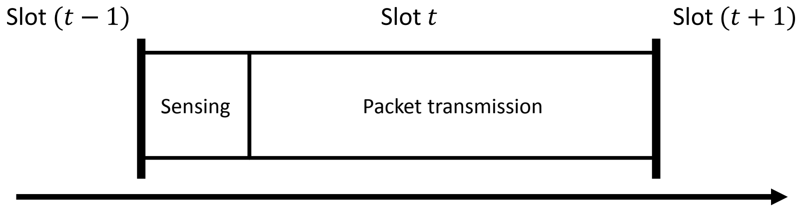


Figure 3.3: A user performs sensing at the beginning of a slot to acquire the availability of a channel, and it can perform channel access and packet transmission on a channel only when it is available.

3.3 General forms of the performance metric with stochastic geometry

Given the definition in (3.3), we can see that the throughput in channel k is determined jointly by the channel available probability θ_k, and the statistics of the desired signal power, $H_{S,k}$, and the statistics of the interference power I_k. By considering a typical user[2] where its receiver is centered at the origin of the network, we can derive the throughput in channel k with tools and techniques from stochastic geometry.

The interference power I_k seen by the typical receiver is the sum of the signal power from the other transmitters in the network attenuated by path loss and channel fading. By letting Ψ_k denote the set of nodes in Ψ taking transmission in channel k, we have $I_k = \sum_{X_i \in \Psi_k} H_{i0,k} P |X_i|^{-\alpha}$, where $H_{i0,k}$ is the fading coefficient of node i to the typical receiver. If the users in Ψ adopts a channel access strategy such that channel k is chosen for access with probability p_k, we then have by the thinning property of PPP that $\Psi_k = \Pi(p_k \lambda)$. The general expression for the throughput in channel k can the be obtained in the following theorem.

Theorem 1. *Given the channel access strategy that a channel k is selected with probability p_k by the users in $\Psi = \Pi(\lambda)$, the throughput in channel k is*

$$\mathcal{T}_k = \theta_k \int_{-\infty}^{\infty} \exp(-\phi N_0 t) \cdot \exp\left(-p_k \lambda \pi r^2 (2^R - 1)^\delta \mathsf{E}[H_{I,k}^\delta] \Gamma(1-\delta) t^\delta\right) \widetilde{h}_{S,k}(t) dt \tag{3.5}$$

where

$$\phi \triangleq (2^R - 1) P^{-1} r^\alpha \tag{3.6}$$

and $\widetilde{h}_{S,k}(t)$ is the inverse Laplace transform of the complementary cumulative distribution function (ccdf) of the of the fading coefficient $H_{S,k}$ for the desired signal in

[2]By Slivnyak's theorem [17], the performance perceived by the typical pair of transmitter and receiver represents that of the node-average performance in the network [2].

channel k.

Proof. With the thinning property of PPP, we have

$$I_k = \sum_{X_i \in \Pi(p_k\lambda)} H_{i0,k} P |X_i|^{-\alpha}$$

By the definition in (3.3), we have

$$\begin{aligned}
\mathcal{T}_k &= \theta_k \mathsf{Pr}\left(\log_2\left(1 + \frac{H_{S,k} P r^{-\alpha}}{N_0 + I_k}\right) \geq R\right) \\
&= \theta_k \mathsf{Pr}\left(H_{S,k} \geq (2^R - 1) P^{-1} r^{\alpha} (N_0 + I_k)\right) \\
&\overset{(a)}{=} \theta_k \mathsf{E}\left[\int_{-\infty}^{\infty} \exp\left(-(2^R - 1) P^{-1} r^{\alpha} (N_0 + I_k) t\right) \widetilde{h}_{S,k}(t) dt\right] \\
&= \theta_k \int_{-\infty}^{\infty} \exp\left(-(2^R - 1) P^{-1} r^{\alpha} N_0 t\right) \mathsf{E}\left[\exp\left(-(2^R - 1) P^{-1} r^{\alpha} I_k t\right)\right] \widetilde{h}_{S,k}(t) dt \\
&\overset{(b)}{=} \theta_k \int_{-\infty}^{\infty} \exp\left(-(2^R - 1) P^{-1} r^{\alpha} N_0 t\right) \\
&\qquad \cdot \exp\left(-p_k \lambda \pi r^2 (2^R - 1)^{\delta} \mathsf{E}[H_{I,k}^{\delta}] \Gamma(1 - \delta) t^{\delta}\right) \widetilde{h}_{S,k}(t) dt
\end{aligned}$$

where (a) follows by using

$$\mathsf{Pr}\left(H_{S,k} \geq s\right) = \int_{-\infty}^{\infty} e^{-st} \widetilde{h}_{S,k}(t) dt$$

where $\widetilde{h}_{S,k}(t) = \mathcal{L}^{-1}\{\overline{F}_{H_{S,k}}(s)\}$ is the inverse Laplace transform of the ccdf $\overline{F}_{H_{S,k}}(s)$ of the fading coefficient $H_{S,k}$ for the desired signal in channel k. (b) follows by using the Laplace transform of the 2D Poisson shot noise process I_k

$$\mathsf{E}[e^{-sI_k}] = \exp\left(-\mu_k \pi P^{\delta} \mathsf{E}[H_{I,k}^{\delta}] \Gamma(1 - \delta) s^{\delta}\right) \tag{3.7}$$

where μ_k is the density of Ψ_k which is $p_k\lambda$ here. The results follows by letting $\phi \triangleq (2^R - 1) P^{-1} r^{\alpha}$ for convenience of notation. □

In the following and throughout the remainder of the thesis, we consider the case when the channels are Rayleigh faded, and in each channel $k \in \mathcal{K}$ with average power $\mathsf{E}[H_{S,k}] = m_{S,k}$ and $\mathsf{E}[H_{I,k}] = m_{I,k}$ for the desired signal and interference respectfully. Thus, we have $\mathsf{E}[H_{I,k}^{\delta}] = m_{I,k}^{\delta}\Gamma(1+\delta)$, and $\widetilde{h}_{S,k}(t) = \delta(t - m_{S,k}^{-1})$, where $\delta(t)$ is the Dirac delta function. Without loss of generality, we let $m_{I,k} = 1$ and $m_{S,k} = m_k$. The results are given in the following corollary.

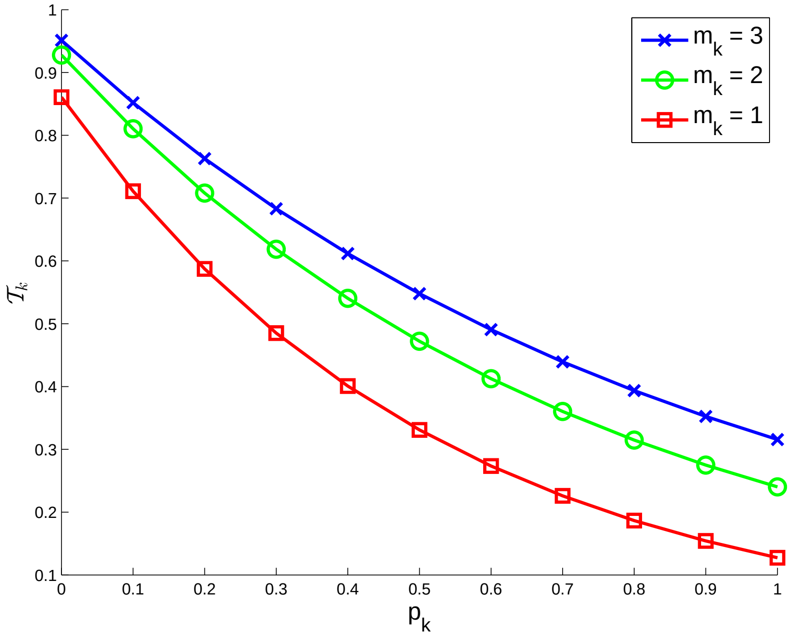
Corollary 1. *Given the assumptions as in Theorem 1, when the channel undergoes Rayleigh fading such that* $\mathsf{E}[H_{S,k}] = m_k$ *and* $\mathsf{E}[H_{I,k}] = 1$, *the throughput in a channel* k *is*

$$\mathcal{T}_k = \theta_k \exp(-\phi N_0 m_k^{-1}) \exp(-p_k \rho m_k^{-\delta}) \tag{3.8}$$

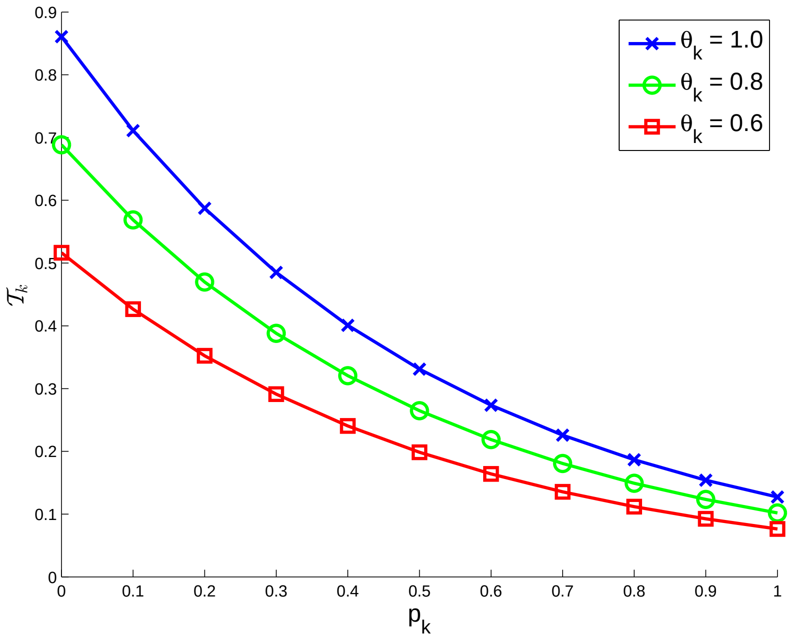
where

$$\rho \triangleq \lambda \pi r^2 (2^R - 1)^{\delta} \Gamma(1+\delta)\Gamma(1-\delta) \tag{3.9}$$

Figure 3.4 gives a plot of the user throughput $\mathcal{T}_k$ in channel k as given by equation (3.8). We can see that the throughput of a user in a channel will be dependent on the density of transmitting users in that channel $p_k\lambda$, and the channel qualities m_k and θ_k. With better channel qualities (higher values of m_k and θ_k), the throughput in channel k will be higher. However, if the channel is accessed more frequently by the users in the network (a higher value of p_k), the throughput in that channel will degrade.



(a) With fixed $\theta_k = 1$ and different m_k



(b) With fixed $m_k = 1$ and different θ_k

Figure 3.4: User throughput $\mathcal{T}_k$ in channel k versus p_k.

3.4 Remarks

3.4.1 Connections to other models

As mentioned earlier, most works on multi-channel random access ignores physical channel characteristics or spatial distribution of users and considers either graph-based collision models that do not take into account the aggregate effect of network or capture models (SINR capture or power capture) ignoring spatial distribution of users in the network. Compared to previous works, we employ stochastic geometry to characterize the aggregate effect (interference) caused by spatially distributed users such that a more precise modeling is provided.

Some connections can be found between our model with those of previous works. By considering the limiting case when $\alpha \to \infty$ with the interference-limited regime ($N_0 = 0$), our model for successful reception reduces to the *protocol model* [46], where transmission by the node X_i to node X_j over channel k is successful if

$$|X_k - X_j| > |X_i - X_j| = r \tag{3.10}$$

for every other node X_k transmitting in channel k. That is, transmission by a typical user in channel k is successful only when there are no other user transmitting in the same channel within the disc of radius r centered at the desired receiver of the typical user. This model is often used in graph-based analysis of multi-channel MAC where corresponding interference graphs are further constructed [33].

Another connection to the traditional non-spatial random access scheme where N users attempts to access a single receiver (base station or access point) assuming collision channel model can also be found. In that case, the N users try to make access through one or multiple, say K, communication channels to the receiver, and if two or more users transmit in the same channel simultaneously, the transmissions in that channel fails. In the following, we define the model for *non-spatial multi-channel random access with collision channels*. With N users attempting to access a common receiver through K channels as previously described and with similar assumptions as our model but in a non-spatial context, the access by a user in channel k is successful if channel k is available and no other users are transmitting in channel k. If each user chooses channel k for access with probability p_k, the expected user throughput in channel k is

$$\widetilde{\mathcal{T}}_k = \theta_k (1 - p_k)^{N-1} \tag{3.11}$$

The model defined above is similar to that in [32] for cognitive medium access, and will be used later for discussion of our model in the non-spatial context.

3.4.2 On the transmitter-receiver distance

Throughout this thesis, the distance between a transmitter and its receiver is assumed to be a fixed constant r as described in Section 3.1. In this section, we show that how this assumption can be dropped so that the distance for each transmitter-receiver pair is now a random variable, denoted by $\hat{r}$, characterized by its pdf $f_{\hat{r}}(r)$.

Consider the interference-limited regime ($N_0 = 0$) with Rayleigh fading for simplicity. With the pdf of $\hat{r}$ being $f_{\hat{r}}(r)$, the equation for throughput of a user in channel k can now be derived as

$$\begin{aligned}
\mathcal{T}_k &= \theta_k \mathsf{Pr}\left(\log_2\left(1 + \frac{H_{S,k} P \hat{r}^{-\alpha}}{I_k}\right) \geq R\right) \\
&= \theta_k \mathsf{Pr}\left(H_{S,k} \geq (2^R - 1) P^{-1} \hat{r}^{\alpha} I_k\right) \\
&= \theta_k \mathsf{E}\left[\int_{-\infty}^{\infty} \exp\left(-(2^R - 1) P^{-1} \hat{r}^{\alpha} I_k t\right) \widetilde{h}_{S,k}(t) dt\right] \\
&= \theta_k \int_{-\infty}^{\infty} \mathsf{E}\left[\exp\left(-(2^R - 1) P^{-1} \hat{r}^{\alpha} I_k t\right)\right] \widetilde{h}_{S,k}(t) dt \\
&= \theta_k \int_{-\infty}^{\infty} \mathsf{E}\left[\exp\left(-p_k \lambda \pi \hat{r}^2 (2^R - 1)^{\delta} \Gamma(1+\delta) \Gamma(1-\delta) t^{\delta}\right)\right] \widetilde{h}_{S,k}(t) dt \\
&= \theta_k \mathsf{E}\left[\exp(-p_k \widetilde{\rho} m_k^{-\delta} \hat{r}^2)\right] \\
&= \theta_k \int_0^{\infty} \exp(-p_k \widetilde{\rho} m_k^{-\delta} r^2) f_{\hat{r}}(r) dr
\end{aligned}$$

where $\widetilde{\rho} \triangleq \lambda \pi (2^R - 1)^{\delta} \Gamma(1+\delta) \Gamma(1-\delta)$.

Now, consider the scenario where the potential receivers of the network form a PPP with density λ_{rx} denoted by Ψ_{rx}, and each transmitter in Ψ chooses the nearest node in Ψ_{rx} as its target receiver. In this case, the cdf of the distance $\hat{r}$ for a transmitter-receiver pair can be found to be

$$F_{\hat{r}}(r) = 1 - e^{-\lambda_{\mathrm{rx}}\pi\bar{r}^2} \tag{3.12}$$

by considering the non-void probability of a circle with radius r with respect to a PPP with density λ_{rx}. Thus, the pdf of $\hat{r}$ is

$$f_{\hat{r}}(r) = 2\pi r\lambda_{\mathrm{rx}} e^{-\lambda_{\mathrm{rx}}\pi r^2} \tag{3.13}$$

and in fact, $\hat{r}$ is Rayleigh distributed with mean $\frac{1}{2\sqrt{\lambda_{\mathrm{rx}}}}$. The throughput in channel k now becomes[3]

$$\begin{aligned}
\mathcal{T}_k &= \theta_k \int_0^{\infty} \exp(-p_k\widetilde{\rho} m_k^{-\delta} r^2) 2\pi r\lambda_{\mathrm{rx}} e^{-\lambda_{\mathrm{rx}}\pi r^2} dr \\
&= 2\pi\lambda_{\mathrm{rx}}\theta_k \int_0^{\infty} r e^{-r^2\left(\lambda_{\mathrm{rx}}\pi + p_k\widetilde{\rho} m_k^{-\delta}\right)} dr \\
&= \frac{\lambda_{\mathrm{rx}}\pi\theta_k}{\lambda_{\mathrm{rx}}\pi + p_k\widetilde{\rho} m_k^{-\delta}} \\
&= \frac{\theta_k}{1 + p_k\frac{\lambda}{\lambda_{\mathrm{rx}}}(2^R - 1)^{\delta}\Gamma(1+\delta)\Gamma(1-\delta) m_k^{-\delta}}
\end{aligned} \tag{3.14}$$

where we have used the integral $\int_0^{\infty} x e^{-ax^2} dx = \frac{1}{2a}$. Analysis of access strategies can thus be made.

[3]Note that we have implicitly assumed that each receiver is capable of receiving more than one number of transmissions at the same time for two or more transmitters may choose the same receiver with this model. More discussions on such assumption can be found in [20] and is not our focus here.

Chapter 4

Multi-channel Random Access Without Channel Side Information

In this chapter, we consider multi-channel random access without channel side information (more precisely, channel state information and channel availability), which will be treated in the next chapter. We devise the optimal access strategy for the uncoordinated users of the network using a game-theoretic point of view. In addition, we characterizes the optimal access strategy and its corresponding performance when there's a central entity available. Finally, we apply our results to some special cases, and discuss about their performance which gives some insights to the designs of different networks.

4.1 Game theoretic design of access strategy

According to the medium access mechanism described in Section 3.2, a user chooses among the K channels for access with a strategy intended to maximize its own performance metric. Intuitively, a user can achieve this goal if it chooses the channel k with the highest available probability θ_k or with the highest mean of channel gain m_k for channel access. However, we should note that the throughput in a channel defined in (3.3) is characterized not only by the channel available probability or channel fading statistics, but also by the received interference power in that channel. If all the users naively choose the same channel with the best available probability or best channel fading statistics for access, the chosen channels might be crowded with users which lead to high interference and thus high outage probability, and ultimately results in low throughput in that channel. On the contrary, choosing a channel with a lower channel available probability or relatively worse channel fading statistics may however result in a better performance if that channel is not so crowded.

The main purpose here is to devise appropriate channel access strategies for the users in the network. From the discussion above, we can see that while the users makes channel selection decisions independently with a goal of maximizing their own performance, each of their performance is dependent on the others' decisions. This leads to the formulation of a game-theoretic problem of the multi-channel random access problem.

Specifically, by taking the users in the network as the players in a game, we have the following *multi-channel random access game*. The channel selection k of a user for channel access is the action of a player in the game, and the throughput $\mathcal{T}_k$ of choosing channel k for access is the utility of the corresponding action. Without loss of generality, we assume that a user use a mixed strategy where a channel is chosen with a specific probability for access.

Due to the homogeneity of PPP and since each user in the network has the same objective that its throughput be maximized, we can restrict our attention to symmetric Nash Equilibria (SNE), in which every user would use the same mixed strategy $\mathbf{p}^*$ for channel access at the equilibrium. By considering SNE, the original game which consists of an infinite number of users can be transformed to a equivalent two-player game, where one player is a typical user and the other player represents all other users in the network. We denote the utility of the typical node with mixed strategy $\mathbf{p}' = [p'_1, \ldots, p'_K]$ as $U(\mathbf{p}', \mathbf{p})$, where $\mathbf{p} = [p_1, \ldots, p_K]$ is the mixed strategy of all the other users. When the channels are Rayleigh faded, the utility function could be written according to Corollary 1 as

$$U(\mathbf{p}', \mathbf{p}) = \sum_{k \in \mathcal{K}} p_k' \theta_k \exp(-\phi N_0 m_k^{-1}) \exp(-p_k \rho m_k^{-\delta}) \tag{4.1}$$

Since each user in the network chooses its access strategy such that its own utility is maximized, the SNE $\mathbf{p}^* = [p_1^*, \ldots, p_K^*]$ could be characterized by

$$U(\mathbf{p}^*, \mathbf{p}^*) = \max_{\substack{\mathbf{0} \leq \mathbf{p}' \leq \mathbf{1} \\ \sum_{k \in \mathcal{K}} p_k' \leq 1}} U(\mathbf{p}', \mathbf{p}^*) \tag{4.2}$$

Lemma 2. *The symmetric mixed strategy Nash equilibrium $\mathbf{p}^*$ of the multi-channel random access game satisfies*

$$\sum_{k \in \mathcal{S}^*} p_k^* = 1 \tag{4.3}$$

where $\mathcal{S}^*$ is the support of the equilibrium.

Proof. This property can be proved by contradiction. Assume that $\sum_{k \in \mathcal{S}^*} p_k^* < 1$, and all the users in the network other than the a typical user plays $\mathbf{p}^*$, and the typical player adopts a strategy $\mathbf{p}'$ similar to $\mathbf{p}^*$ except for channel $i \in \mathcal{S}^*$ such that

$$p_k' = \begin{cases} p_k^* + (1 - \sum_{i \in \mathcal{S}^*} p_i^*), & k = i \\ p_k^*, & k \neq i \end{cases}, \quad k \in \mathcal{K} \tag{4.4}$$

we would have

$$\begin{aligned} U(\mathbf{p}', \mathbf{p}^*) &= \sum_{k \in \mathcal{S}^*} p_k' \theta_k \exp(-\phi N_0 m_k^{-1}) \exp(-p_k^* \rho m_k^{-\delta}) \\ &< \sum_{k \in \mathcal{S}^*} p_k^* \theta_k \exp(-\phi N_0 m_k^{-1}) \exp(-p_k^* \rho m_k^{-\delta}) \\ &= U(\mathbf{p}^*, \mathbf{p}^*) \end{aligned}$$

which leads to a contradiction to (4.2). Thus, we must have $\sum_{k \in \mathcal{S}^*} p_k^* = 1$. □

With the property in Lemma 1, we can characterize the SNE of the game and its corresponding performance with the following theorem.

Theorem 2. *The symmetric mixed strategy Nash equilibrium* $\mathbf{p}^*$ *of the multi-channel random access game satisfies*

$$p_k^* = \begin{cases} \frac{m_k^\delta}{\rho}\left(\ln\theta_k - \phi N_0 m_k^{-1} - \ln E(\mathcal{S}^*)\right), & k \in \mathcal{S}^* \\ 0, & k \in \mathcal{K} \setminus \mathcal{S}^* \end{cases} \tag{4.5}$$

where

$$E(\mathcal{S}) \triangleq \exp\left(-\left(\sum_{k\in\mathcal{S}} m_k^\delta\right)^{-1}\left(\rho - \sum_{k\in\mathcal{S}} m_k^\delta\left(\ln\theta_k - \phi N_0 m_k^{-1}\right)\right)\right) \tag{4.6}$$

and $\mathcal{S}^*$ *is the support of the equilibrium. The throughput of a user at the equilibrium is*

$$\mathcal{T} = U(p^*, p^*) = E(\mathcal{S}^*) \tag{4.7}$$

Proof. By Lemma 1, the user would be indifferent between the actions in the support of the mixed strategy Nash equilibrium, which means that the utility of selecting the channels in the support $\mathcal{S}^*$ would be equalized to a same value say E. We thus have

$$\mathcal{T}_k = \theta_k \exp(-\phi N_0 m_k^{-1})\exp(-p_k^*\rho m_k^{-\delta}) = E, \quad \forall k \in \mathcal{S}^* \tag{4.8}$$

By taking natural logarithm on both sides and rearranging terms, we have for $k \in \mathcal{S}^*$,

$$p_k^* = \frac{m_k^\delta}{\rho}\left(\ln\theta_k - \phi N_0 m_k^{-1} - \ln E\right) \tag{4.9}$$

With Lemma 2, we have $\sum_{k\in\mathcal{S}^*} p_k^* = 1$. Thus, we can obtain the constant E as

$$E = \exp\left(-\left(\sum_{k\in\mathcal{S}^*} m_k^\delta\right)^{-1}\left(\rho - \sum_{k\in\mathcal{S}^*} m_k^\delta\left(\ln\theta_k - \phi N_0 m_k^{-1}\right)\right)\right) \tag{4.10}$$

The throughput of a user at the equilibrium $\mathbf{p}^*$ is then obtained as

$$\mathcal{T} = U(p^*, p^*) = \sum_{k \in \mathcal{S}^*} p_k^* E = E, \tag{4.11}$$

$\square$

Given Theorem 2, the Nash equilibrium strategy of a user in the network and the corresponding performance could be evaluated. By the fact that the utility of selecting each channel k in the support $\mathcal{S}^*$ (i.e. the set of channels that will be chosen with positive probability) is equalized to $E(\mathcal{S}^*)$ at the equilibrium, we have the following relationship

$$\theta_k \exp(-\phi N_0 m_k^{-1}) \exp(-p_k^* \rho m_k^{-\delta}) = E(\mathcal{S}^*), \quad k \in \mathcal{S}^*$$

which can be rearranged to

$$-\ln \theta_k + \phi N_0 m_k^{-1} + p_k^* \rho m_k^{-\delta} = -\ln E(\mathcal{S}^*), \quad k \in \mathcal{S}^*$$

and can be explained by a water-filling concept as shown in Fig. 4.1. Each white block corresponds to a channel, and the length and height of them are related to the channel qualities m_k and θ_k. The water level corresponds to the fact that the utility for each channel in the support is equalized at the equilibrium, and the area of the blue region above each white block is the allocation of access probability p_k^* for channel k at the equilibrium. We can thus see how the access probability of each channel at the equilibrium is related to its channel qualities. The channels not in the support $\mathcal{S}^*$ are those whose corresponding white blocks with heights above the water level. Note that the width of each block is also related to the density of user λ by the definition of ρ in (3.9). As λ grows, the width of each white block shrinks, and since the area of the blue regions sums to one (i.e. $\sum_{k \in \mathcal{S}^*} p_k^* = 1$), the water level also rises and more channels might then be included in the support.

The support $\mathcal{S}^*$ could be obtained as described in Algorithm 1 with $E(\mathcal{S})$ defined

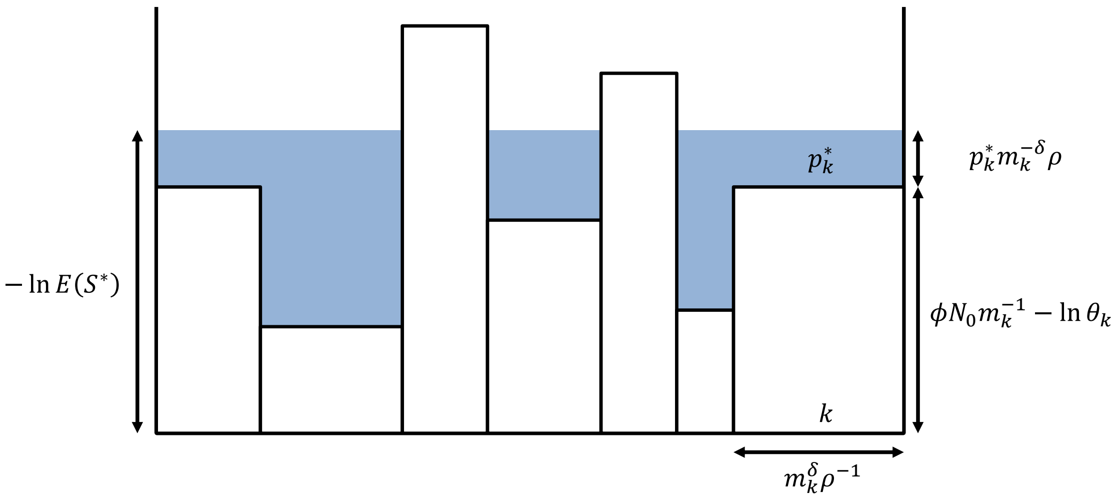


Figure 4.1: The water-filling concept of the game-theoretic solution at the equilibrium.

as in (4.6), and is explained in the following. The algorithm starts with the candidate set $\mathcal{M}$ for the support which is in the beginning the set $\mathcal{K}$. Finding the support $\mathcal{S}^*$ is equivalent to finding which channels would be flooded along with the water-filling concept. In the first iteration of the while loop at Step 3, the algorithm equivalently chooses the channel that has the lowest height of white block in the water-filling diagram. At Step 4, the chosen channel k^* is added to the support $\mathcal{S}^*$ and removed from the candidate set $\mathcal{M}$. With the current set for support $\mathcal{S}^*$, Step 5 excludes the channels that will never be chosen to be in the support for that the heights of their corresponding white blocks are already greater than or equal to the current water level. The while loop continues until the candidate set $\mathcal{M}$ is empty (i.e. when all the K channels have been checked for eligibility to be in the support), in each iteration adding a channel to the support and excluding channels that will never be added.

Algorithm 1 Computing the support of the symmetric mixed strategy Nash equilibrium

Input: Set of all channels $\mathcal{K} = [1, \ldots, K]$.

Output: Set of channels $\mathcal{S}^*$ in the support of the Nash equilibrium.

1: $\mathcal{S}^* \leftarrow \varnothing, \mathcal{M} \leftarrow \mathcal{K}$
2: **while** $\mathcal{M} \neq \varnothing$ **do**
3: Pick one channel $k^* \in \arg\max_{i \in \mathcal{M}} \theta_i \exp\left(-\phi N_0 m_i^{-1}\right)$
4: $\mathcal{S}^* \leftarrow \mathcal{S}^* \cup \{k^*\}$, $\mathcal{M} \leftarrow \mathcal{M} \setminus \{k^*\}$
5: $\mathcal{M} \leftarrow \mathcal{M} \setminus \left\{k \in \mathcal{M} : \theta_k \exp(-\phi N_0 m_k^{-1}) \leq E(\mathcal{S}^*)\right\}$
6: **end while**
7: **return** $\mathcal{S}^*$

4.2 Centralized optimal channel access strategy

When there's a central entity or when all the users in the network are cooperative, the channel selection probabilities of each user can be controlled such that network-wide performance is optimized. In particular, we assume that the network operator is aimed at maximizing the system throughput $\lambda\mathcal{T}$ as defined in Section 3.2. Since the density of users λ in the network is not affected by the channel selection probabilities, the equivalent optimization problem of the central entity becomes

$$\max_{\mathbf{p}} \quad \sum_{k\in\mathcal{K}} p_k \mathcal{T}_k = \sum_{k\in\mathcal{K}} p_k \theta_k \exp(-\phi N_0 m_k^{-1}) \exp(-p_k \rho m_k^{-\delta}) \tag{4.12}$$

$$\text{s.t.} \quad p_k \geq 0,\ k \in \mathcal{K} \tag{4.13}$$

$$\sum_{k\in\mathcal{K}} p_k \leq 1 \tag{4.14}$$

Note that the inequality in (4.14) includes the case when a user might not access any channel with some nonzero probability, which is not observed in the SNE of the game in Chapter 4.1. The optimal solution of the centralized multi-channel random access problem is obtained in the following theorem.

Theorem 3. *The optimal channel access strategy of the centralized multi-channel random access problem is*

$$p_k^* = \begin{cases} \frac{m_k^\delta}{\rho}, & \sum_{i\in\mathcal{K}} m_i^\delta < \rho \\ \frac{m_k^\delta}{\rho}\left[1 - W\left(\gamma\theta_k^{-1} e^{1+\phi N_0 m_k^{-1}}\right)\right]^+, & \sum_{i\in\mathcal{K}} m_i^\delta \geq \rho \end{cases} \tag{4.15}$$

where γ is the constant that satisfies $\sum_{i\in\mathcal{K}} p_i^* = 1$, $W(z)$ is the Lambert W function with defining equation $z = W(z)e^{W(z)}$, and $[x]^+ \triangleq \max\{0, x\}$. The corresponding *throughput of a user with the optimal access strategy is*

$$\mathcal{T} = \begin{cases} \frac{1}{\rho e} \sum_{k\in\mathcal{K}} m_k^\delta \theta_k e^{-\phi N_0 m_k^{-1}}, & \sum_{k\in\mathcal{K}} m_k^\delta < \rho \\ \frac{1}{\rho} \sum_{k\in\mathcal{K}} m_k^\delta \theta_k e^{-\phi N_0 m_k^{-1}} & \\ \quad \cdot \left[1 - W\left(\gamma \theta_k^{-1} e^{1+\phi N_0 m_k^{-1}}\right)\right]^+, & \sum_{i\in\mathcal{K}} m_i^\delta \geq \rho \\ \quad \cdot e^{-\left[1-W\left(\gamma\theta_k^{-1} e^{1+\phi N_0 m_k^{-1}}\right)\right]^+} & \end{cases} \tag{4.16}$$

Proof. The Karush-–Kuhn–-Tucker (KKT) conditions for optimality gives

$$\begin{aligned} (p_k^* \rho m_k^{-\delta} - 1)\theta_k \exp(-\phi N_0 m_k^{-1}) \exp(-p_k^* \rho m_k^{-\delta}) - \xi_k + \gamma = 0,\ k \in \mathcal{K} \\ \xi_k p_k^* = 0,\ k \in \mathcal{K} \\ \left(\sum_{k\in\mathcal{K}} p_k^* - 1\right) \gamma = 0, \\ \sum_{k\in\mathcal{K}} p_k^* \leq 1, \\ p_k^* \geq 0,\ k \in \mathcal{K} \\ \xi_k \geq 0,\ k \in \mathcal{K} \\ \gamma \geq 0 \end{aligned} \tag{4.17}$$

where ξ_k and γ are the KKT multipliers.

Consider first the case when $\sum_{k\in\mathcal{K}} p_k < 1$. By complementary slackness, we have $\gamma = 0$. After solving $(p_k^* \rho m_k^{-\delta} - 1)\theta_k \exp(-\phi N_0 m_k^{-1}) \exp(-p_k^* \rho m_k^{-\delta}) - \xi_k = 0$ along with $\xi_k p_k^* = 0$, we obtain $\xi_k = 0$ and $p_k = \frac{m_k^\delta}{\rho}$ for all $k \in \mathcal{K}$. The case now corresponds to $\sum_{k\in\mathcal{K}} p_k = \sum_{k\in\mathcal{K}} \frac{m_k^\delta}{\rho} < 1$.

Now consider the other case when $\sum_{k\in\mathcal{K}} p_k = 1$. Noting that $\xi_k = 0$ acts as an slack variable, the conditions can be eliminated to

$$
\begin{aligned}
\gamma \geq (1 - p_k^* \rho m_k^{-\delta}) \theta_k \exp(-\phi N_0 m_k^{-1}) \exp(-p_k^* \rho m_k^{-\delta}),\ k \in \mathcal{K} \\
p_k^* \left[\gamma + (p_k^* \rho m_k^{-\delta} - 1) \theta_k \exp(-\phi N_0 m_k^{-1}) \exp(-p_k^* \rho m_k^{-\delta}) \right] = 0,\ k \in \mathcal{K} \\
\sum_{k \in \mathcal{K}} p_k^* = 1, \\
p_k^* \geq 0,\ k \in \mathcal{K} \\
\gamma \geq 0
\end{aligned}
\tag{4.18}
$$

If $\gamma \geq \theta_k \exp(-\phi N_0 m_k^{-1})$, then $p_k^* > 0$ is impossible since it would imply that

$$
\begin{aligned}
\gamma &\geq \theta_k \exp(-\phi N_0 m_k^{-1}) \\
&> (1 - p_k^* \rho m_k^{-\delta}) \theta_k \exp(-\phi N_0 m_k^{-1}) \exp(-p_k^* \rho m_k^{-\delta})
\end{aligned}
$$

which violates the second condition in (4.18). Thus, we have $p_k^* = 0$ if $\gamma \geq \theta_k \exp(-\phi N_0 m_k^{-1})$. If $\gamma < \theta_k \exp(-\phi N_0 m_k^{-1})$, we must have $p_k^* > 0$ to satisfy the first condition in (4.18), which by the second condition implies that $\gamma + (p_k^* \rho m_k^{-\delta} - 1) \theta_k \exp(-\phi N_0 m_k^{-1}) \exp(-p_k^* \rho m_k^{-\delta}) = 0$. To solve for p_k^*, we observe that

$$
\begin{aligned}
& (p_k^* \rho m_k^{-\delta} - 1) \theta_k \exp(-\phi N_0 m_k^{-1}) \exp(-p_k^* \rho m_k^{-\delta}) + \gamma = 0 \\
\Rightarrow & (1 - p_k^* \rho m_k^{-\delta}) \exp(1 - p_k^* \rho m_k^{-\delta}) = \gamma \theta_k^{-1} \exp(1 + \phi N_0 m_k^{-1}) \\
\Rightarrow & 1 - p_k^* \rho m_k^{-\delta} = W\left(\gamma \theta_k^{-1} \exp(1 + \phi N_0 m_k^{-1})\right) \\
\Rightarrow & p_k^* = \frac{m_k^{\delta}}{\rho} \left[1 - W\left(\gamma \theta_k^{-1} \exp(1 + \phi N_0 m_k^{-1})\right)\right]
\end{aligned}
\tag{4.19}
$$

where in the third line we have used the fact that the equation $we^w = z$ has the solution $w = W(z)$ where $W(z)$ is the Lambert W function. Therefore, we have

$$
p_k^* = \begin{cases} \frac{m_k^{\delta}}{\rho} \left[1 - W\left(\gamma \theta_k^{-1} \exp(1 + \phi N_0 m_k^{-1})\right)\right], & \gamma < \theta_k \exp(-\phi N_0 m_k^{-1}) \\ 0, & \gamma \geq \theta_k \exp(-\phi N_0 m_k^{-1}) \end{cases}
\tag{4.20}
$$

for the case, where γ is the constant such that $\sum_{k\in\mathcal{K}} p_k^* = 1$. By noting that

$$1 - W\left(\gamma\theta_k^{-1}\exp(1+\phi N_0 m_k^{-1})\right) > 0 \Longleftrightarrow \gamma < \theta_k \exp(-\phi N_0 m_k^{-1})$$

we can write

$$p_k^* = \frac{m_k^\delta}{\rho}\left[1 - W\left(\gamma\theta_k^{-1}\exp(1+\phi N_0 m_k^{-1})\right)\right]^+ \tag{4.21}$$

where $[x]^+ \triangleq \max\{0, x\}$. Note that this case corresponds to

$$\sum_{k\in\mathcal{K}} p_k^* = 1 = \sum_{k\in\mathcal{K}} \frac{m_k^\delta}{\rho}\left[1 - W\left(\gamma\theta_k^{-1}\exp(1+\phi N_0 m_k^{-1})\right)\right]^+ \le \sum_{k\in\mathcal{K}} \frac{m_k^\delta}{\rho}$$

After combining the results for both cases, we have the theorem. Note that the unique solution satisfying the KKT conditions is globally optimal for that the optimization problem satisfies linear constraint qualification. $\square$

Theorem 3 gives the channel access strategy for a user with a centralized optimization perspective that maximizes the system performance. Thus, by noting that $\rho \triangleq \lambda\pi r^2(2^R-1)^\delta\Gamma(1+\delta)\Gamma(1-\delta)$ as defined in (3.9), we can see that the centralized access strategy not only takes the channel qualities into consideration but also the density of network. When the network is congested with a high density of users (i.e. when $\sum_{i\in\mathcal{K}} m_i^\delta < \rho$), the sum access probability of a user on all channels are reduced to less than one (as can be seen in the proof) to keep the interference in the network at the best condition. On the other hand, when the network is not so congested (i.e. when $\sum_{i\in\mathcal{K}} m_i^\delta \ge \rho$), the sum access probability of a user is one and the allocation of access probabilities on channels are optimized according to various channel characteristics.

4.3 Special cases

4.3.1 With non-i.i.d. channel fading and i.i.d. channel availability

Consider the case when the availability statistics of channels are i.i.d. and without loss of generality we assume that $\theta_k = 1$ for all $k \in \mathcal{K}$. Furthermore, assume that the network operates under the interference-limited regime ($N_0 = 0$). Fig. 4.2 gives the throughput versus λ of the game-theoretic and centralized solution. We can see that as the network density grows, the throughput degrades. At high user density, the throughput of the game-theoretic solution is lower than the centralized one because of the selfish behavior of users leading to a social suboptimal result. However, an interesting finding is that the throughput of both schemes are identical for user density below a threshold $\widetilde{\lambda}$. This is not a coincidence and we found that in fact, the access strategy at the equilibrium of the game is actually the same as the centralized optimal solution. This can be shown by considering the following two corollaries.

Corollary 2. *When $\theta_k = 1, \forall k \in \mathcal{K}$, the access strategy of the multi-channel random access game in the interference-limited regime ($N_0 = 0$) at the equilibrium is*

$$p_k^* = \frac{m_k^\delta}{\sum_{i \in \mathcal{K}} m_i^\delta}, \quad \forall k \in \mathcal{K} \tag{4.22}$$

with support $\mathcal{K}$. The corresponding user throughput at the equilibrium is

$$\mathcal{T} = \exp\left(-\frac{\rho}{\sum_{i \in \mathcal{K}} m_i^\delta}\right) \tag{4.23}$$

Proof. We first show that the support of the equilibrium is the set of all channels $\mathcal{K}$ by contradiction. Assume that the Nash equilibrium $\mathbf{p}^*$, which all the users in the network other than a typical user play, is with support $\mathcal{S}^* \subset \mathcal{K}$. Consider the

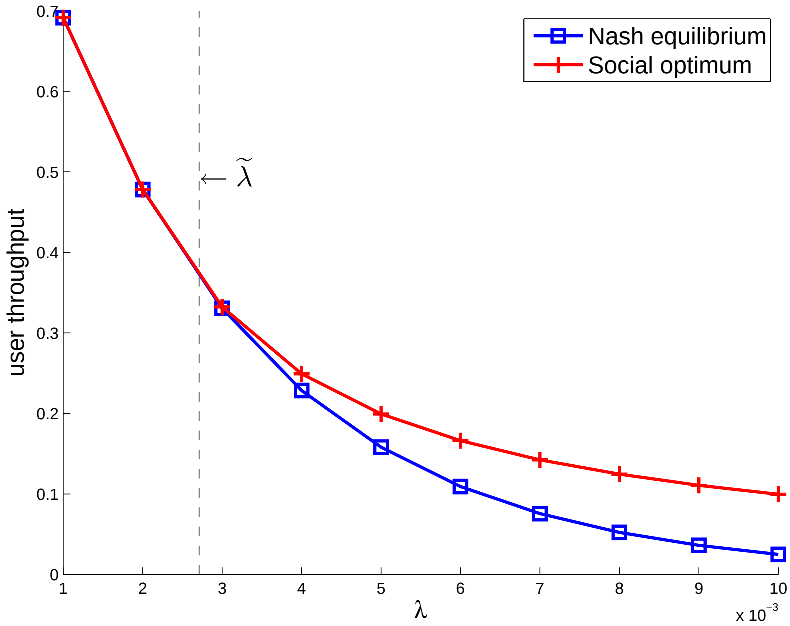


Figure 4.2: Throughput versus user density when channel availability statistics are i.i.d. ($\theta_k = 1$) of the game-theoretic solution and the centralized solution in the interference-limited regime ($N_0 = 0$). Before the user density reaches $\widetilde{\lambda}$, the performance of the game-theoretic solution and the centralized solution are identical. The other network parameters are set as $\alpha = 4$, $R = 2$ bits/s/Hz, $r = 13$m, and $P = 1$. There are $K = 5$ channels with $\mathbf{m} = [0.1, 0.3, 0.7, 0.9, 1.6]$.

strategy $\mathbf{p}'$, which except for an $i \in \mathcal{S}^*$ and $j \in \mathcal{K} \setminus \mathcal{S}^*$ such that $\forall k \in \mathcal{K}$,

$$p'_k = \begin{cases} p_i^*, & k = j \\ 0, & k = i \\ p_k^*, & \text{otherwise} \end{cases}$$

The difference of the utility for the typical user between playing $\mathbf{p}'$ and playing the equilibrium strategy $\mathbf{p}^*$ is

$$\begin{aligned} U(\mathbf{p}', \mathbf{p}^*) - U(\mathbf{p}^*, \mathbf{p}^*) = & \left[p'_j \exp\left(-p_j^* \rho m_j^{-\delta}\right) + p'_i \exp\left(-p_i^* \rho m_i^{-\delta}\right)\right] \\ & - \left[p_j^* \exp\left(-p_j^* \rho m_j^{-\delta}\right) + p_i^* \exp\left(-p_i^* \rho m_i^{-\delta}\right)\right] \\ = & \left[p_i^* \cdot 1 + 0\right] - \left[0 + p_i^* \exp\left(-p_i^* \rho m_i^{-\delta}\right)\right] \\ = & p_i^* \left(1 - \exp\left(-p_i^* \rho m_i^{-\delta}\right)\right) \geq 0 \end{aligned}$$

which contradicts (4.2). Thus, we must have the support $\mathcal{S}^* = \mathcal{K}$. As a result of Theorem 2 with $\theta_k = 1$ and $N_0 = 0$, we have the theorem. □

Corollary 3. *When $\theta_k = 1, \forall k \in \mathcal{K}$, the centralized optimal access strategy of the multi-channel random access problem in the interference-limited regime ($N_0 = 0$) is*

$$p_k^* = \begin{cases} \frac{m_k^\delta}{\rho}, & \sum_{i\in\mathcal{K}} m_i^\delta < \rho \\ \frac{m_k^\delta}{\sum_{i\in\mathcal{K}} m_i^\delta}, & \sum_{i\in\mathcal{K}} m_i^\delta \geq \rho \end{cases} \tag{4.24}$$

The corresponding throughput of a user with the optimal access strategy is

$$\mathcal{T} = \begin{cases} \frac{\sum_{k\in\mathcal{K}} m_k^\delta}{\rho e}, & \sum_{k\in\mathcal{K}} m_k^\delta < \rho \\ \exp\left(-\frac{\rho}{\sum_{k\in\mathcal{K}} m_k^\delta}\right), & \sum_{k\in\mathcal{K}} m_k^\delta \geq \rho \end{cases} \tag{4.25}$$

Proof. According to Theorem 3, we have when $N_0 = 0$, $\theta_k = 1$ and $\sum_{k\in\mathcal{K}} m_k^\delta \geq \rho$ that

$$p_k^* = \frac{m_k^\delta}{\rho}[1 - W(\gamma e)] \tag{4.26}$$

since $\sum_{k\in\mathcal{K}} p_k^* = 1$ in this case and thus we have $[1 - W(\gamma e)]^+ = \frac{\rho}{\sum_{k\in\mathcal{K}} m_k^\delta} > 0$, and the operator $[\cdot]^+$ can be removed. Now $W(\gamma e) = 1 - \frac{\rho}{\sum_{k\in\mathcal{K}} m_k^\delta}$ and the constant γ times e can be obtained as

$$\gamma e = W(\gamma e)e^{W(\gamma e)} = \left(1 - \frac{\rho}{\sum_{k\in\mathcal{K}} m_k^\delta}\right) e^{1-\frac{\rho}{\sum_{k\in\mathcal{K}} m_k^\delta}}$$

by the definition of the Lambert W function, and the optimal solution is obtained by plugging in $W(\gamma e) = 1 - \frac{\rho}{\sum_{k\in\mathcal{K}} m_k^\delta}$. □

We can see from Corollary 2 that with i.i.d. channel availability statistics in the interference limited regime, the access probability for a channel given by (4.22) only depends on the fading statistics of a channel (m_k), and is proportional to its δth power. Each of the K channels are in the support of the equilibrium, and thus every channel will be accessed with a nonzero probability. Corollary 3 tells us that under the aforementioned case, the optimal solution of the centralized scheme has the same form as that for the game-theoretic scheme when $\rho \leq \sum_{k\in\mathcal{K}} m_k^\delta$. Recall that $\rho \triangleq \lambda\pi r^2(2^R-1)^\delta\Gamma(1+\delta)\Gamma(1-\delta)$ as defined in (3.9), we can see that this happens when the user density in the network is less than a threshold, say $\widetilde{\lambda}$. This result is summarized in Theorem 4.

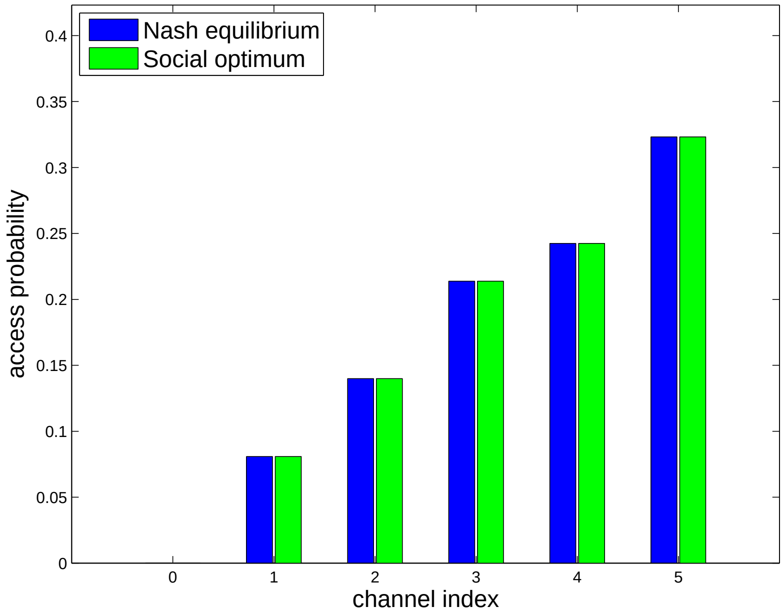
Theorem 4. *The game-theoretic solution of the multi-channel random access problem with $\theta_k = 1$ is socially optimal in the interference-limited regime ($N_0 = 0$) when the density of users λ in the network satisfies*

$$\lambda \leq \widetilde{\lambda} \triangleq \frac{\sum_{k\in\mathcal{K}} m_k^\delta}{\pi r^2(2^R-1)^\delta\Gamma(1+\delta)\Gamma(1-\delta)} \tag{4.27}$$

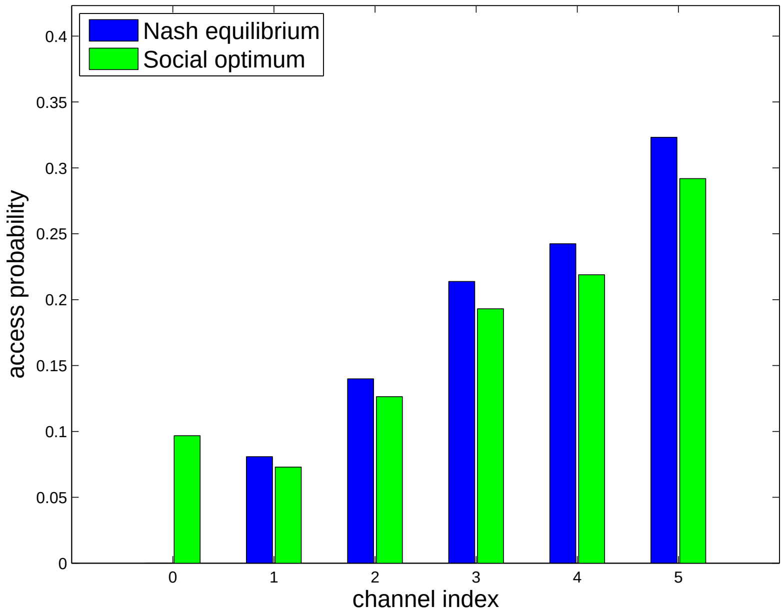
Proof. This can be observed from Corollary 2 and Corollary 3 with the constraint $\sum_{k\in\mathcal{K}} m_k^\delta \geq \rho$ and the definition of ρ in (3.9). □

As the user density grows higher than $\widetilde{\lambda}$, it would be socially optimal for users not to access any channel with some nonzero probability, for it will otherwise deteriorate the performance of channels by enhancing the interference in the network. However, with the selfish behavior of users, the probability of not accessing any channel would be zero as implied by Lemma 2, since otherwise it would have incentive to put more access probability on some channels and then gain more throughput by doing so unilaterally.

Fig. 4.3a and 4.3b depict the channel access probabilities of both the game-theoretic and centralized scheme with the same network parameters as for Fig. 4.2 at user density $\lambda = 1.2 \times 10^{-3}/\text{m}^2$ and $\lambda = 3 \times 10^{-3}/\text{m}^2$, respectively, where the threshold density is $\widetilde{\lambda} = 2.7 \times 10^{-3}/\text{m}^2$ as given by (4.27). It can be seen in Fig. 4.3a that below the threshold density $\widetilde{\lambda}$, the game-theoretic access strategy is the same as that with the centralized scheme, and is thus socially optimal. The probabilities of not accessing any channel are zero for both schemes. When the user density is above $\widetilde{\lambda}$, the centralized strategy reduced the access probabilities on channels and put some probability on not accessing any channel while the game-theoretic access strategy stays the same. The reason why the channel access probabilities are reduced in the centralized scheme is that the network is now so crowded that each user should somehow lower its access attempt. On the other hand, when the users behave selfishly in high user densities, each of them spend all their efforts for channel access which makes the network remains crowded and ends in a socially suboptimal result.



(a) $\lambda = 1.2 \times 10^{-3}/\mathrm{m}^2 < \widetilde{\lambda}$



(b) $\lambda = 3 \times 10^{-3}/\mathrm{m}^2 > \widetilde{\lambda}$

Figure 4.3: Channel access probabilities under different user densities, where channel index 0 is for the probability of not accessing any channel. The threshold density is $\widetilde{\lambda} = 2.7 \times 10^{-3}/\mathrm{m}^2$. The other network parameters are set to the same as in Fig. 4.2, where $\mathbf{m} = [0.1, 0.3, 0.7, 0.9, 1.6]$.

4.3.2 With i.i.d. channel fading and non-i.i.d. channel availability

Now, consider another case when the channel fading coefficients are i.i.d. (without loss of generality with $m_{S,k} = m_{I,k} = 1$) but the channel availability probabilities are non-i.i.d.. In this case, it is very similar to the study of Opportunistic Spectrum Access (OSA) in Cognitive Radio Networks (CRN), where licensed primary users have the priority to utilize the channels and the secondary users can only access the vacant spectrum opportunities. Analogously, the users in our model are the secondary users, and can only utilize the channels when they are sensed to be available for secondary usage. The game-theoretical and centralized access strategy in this case follows from Theorem 2 and Theorem 3, respectively and are given in the following corollaries.

Corollary 4. *When $m_k = 1, \forall k \in \mathcal{K}$, the access strategy of the multi-channel random access game in the interference-limited regime ($N_0 = 0$) at the equilibrium is*

$$p_k^* = \begin{cases} \frac{1}{|\mathcal{S}^*|} + \frac{1}{\rho} \ln \frac{\theta_k}{\left(\prod_{i \in \mathcal{S}^*} \theta_i\right)^{\frac{1}{|\mathcal{S}^*|}}}, & k \in \mathcal{S}^* \\ 0, & k \in \mathcal{K} \setminus \mathcal{S}^* \end{cases} \tag{4.28}$$

with support $\mathcal{S}^*$ computed by Algorithm 1. The corresponding throughput of a user at the equilibrium is

$$\mathcal{T} = \exp\left(-\frac{\rho - \sum_{k \in \mathcal{S}^*} \ln \theta_k}{|\mathcal{S}^*|}\right) \tag{4.29}$$

Proof. The results directly follows from Theorem 2 with some algebraic manipulations. □

Corollary 5. *When $m_k = 1, \forall k \in \mathcal{K}$, the centralized optimal access strategy of the* *multi-channel random access problem in the interference-limited regime ($N_0 = 0$) is*

$$p_k^* = \begin{cases} \frac{1}{\rho}, & \rho > K \\ \frac{1}{\rho}\left[1 - W\left(\gamma\theta_k^{-1}e\right)\right]^+, & \rho \leq K \end{cases} \tag{4.30}$$

where γ is the constant that satisfies $\sum_{i\in\mathcal{K}} p_i^* = 1$, $W(z)$ is the Lambert W function with defining equation $z = W(z)e^{W(z)}$, and $[x]^+ \triangleq \max\{0, x\}$. The corresponding throughput of a user with the optimal access strategy is

$$\mathcal{T} = \begin{cases} \frac{\sum_{k\in\mathcal{K}}\theta_k}{\rho e}, & \rho > K \\ \\ \frac{1}{\rho}\sum_{k\in\mathcal{K}}\theta_k\left[1 - W\left(\gamma\theta_k^{-1}e\right)\right]^+ e^{-\left[1-W\left(\gamma\theta_k^{-1}e\right)\right]^+}, & \rho \leq K \end{cases} \tag{4.31}$$

Proof. The results directly follows from Theorem 3 with some algebraic manipulations. □

Fig. 4.4 gives the throughput versus user density of the game-theoretic and centralized solution. The phenomenon of identical performance with density less than a threshold is no longer present. The loss of throughput due to selfish behavior is small when the density is low but large when the density is high.

As can be seen in Corollary 5, there is still a threshold behavior of the access strategy for the centralized scheme by noting that the access probability on a channel is $\frac{1}{\rho}$ for $\rho > K$ and $\frac{1}{\rho}\left[1 - W\left(\gamma\theta_k^{-1}e\right)\right]^+$ for $\rho \leq K$. With the definition of ρ as given in (3.9), this in fact gives the same density threshold $\widetilde{\lambda}$ as given in (4.27) but with $m_k = 1$ for all $k \in K$. When the user density is lower than $\widetilde{\lambda}$, the centralized optimal access strategy is dependent on the channel available probabilities θ_k, but when the user density is higher than $\widetilde{\lambda}$, it is actually oblivious of the channel available probabilities of the channels. This can be observed in Fig. 4.5, and is explained in the following. With low user density, it would be beneficial for users to put more probabilities on accessing channels since the interference problem in the network is still not so obvious. Thus, a user should take careful considerations to on which channels should it put access probabilities, depending on the channel available probabilities θ_k. However, when the user density are higher than $\widetilde{\lambda}$, it would be better that each channel be accessed with a optimal probability that maximizes expected throughput in that channel and users not access any channels with a nonzero probability. And since each user in the network can only access a channel when it is available, which will be effective throughout the whole network, the access strategy can be oblivious of the channel available probabilities in this case.

As for the game-theoretic access strategy, we can see from Fig. 4.5 that it takes more channels into the support of the equilibrium as the network density grows and more channels are utilized. However, it does not put any probability on not accessing any channel because of the selfish behavior of users, leading to a suboptimal result compared to the centralized scheme.

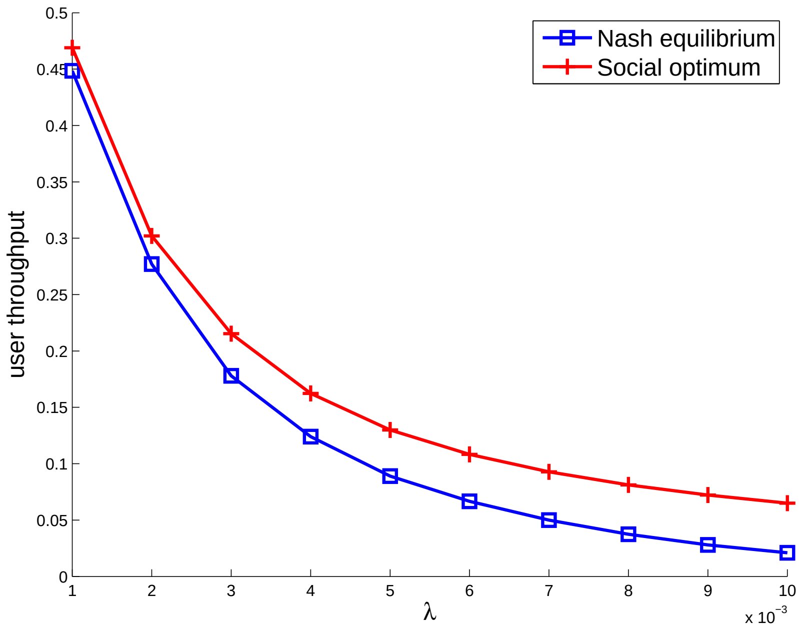
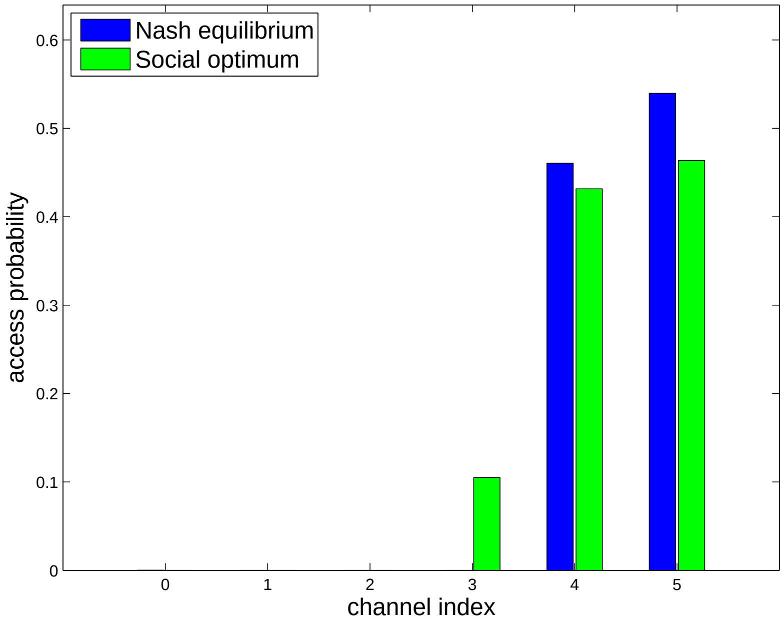
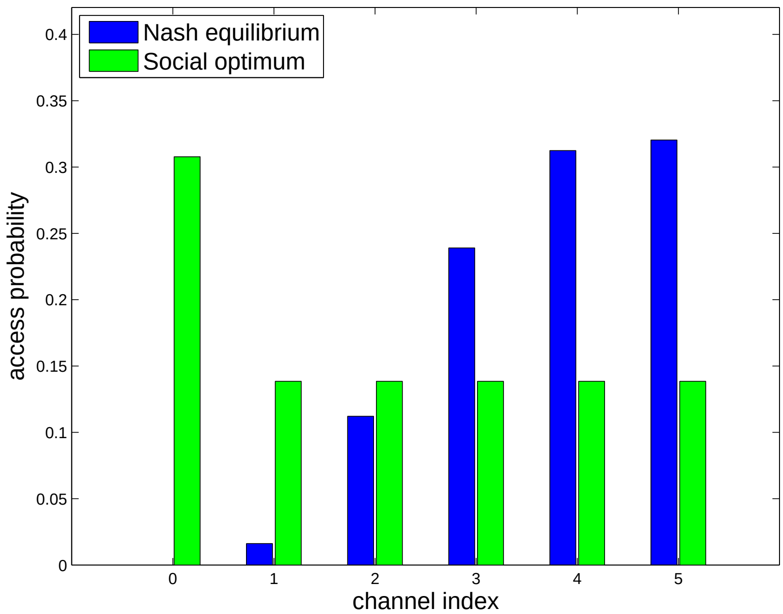


Figure 4.4: Throughput versus user density when channel fading statistics are i.i.d. ($m_k = 1$) of the game-theoretic solution and the centralized solution in the interference-limited regime ($N_0 = 0$). The other network parameters are set as $\alpha = 4$, $R = 2$ bits/s/Hz, $r = 13$m, and $P = 1$. There are $K = 5$ channels with $\Theta = [0.1, 0.2, 0.5, 0.85, 0.9]$.



(a) $\lambda = 5.0 \times 10^{-4}/\text{m}^2 < \widetilde{\lambda}$



(b) $\lambda = 5.0 \times 10^{-3}/\text{m}^2 > \widetilde{\lambda}$

Figure 4.5: Channel access probabilities under different user densities, where channel index 0 is for the probability of not accessing any channel. The threshold density is $\widetilde{\lambda} = 3.5 \times 10^{-3}/\text{m}^2$. The other network parameters are set to the same as in Fig. 4.4, where $\Theta = [0.1, 0.2, 0.5, 0.85, 0.9]$.

4.3.3 With i.i.d. channel fading and i.i.d. channel availability

Now, consider the case with i.i.d. channel fading and i.i.d. channel availability statistics. We have the following corollaries.

Corollary 6. *When $\theta_k = 1, m_k = 1, \forall k \in \mathcal{K}$, the access strategy of the multi-channel random access game in the interference-limited regime ($N_0 = 0$) at the equilibrium is*

$$p_k^* = \frac{1}{K}, \quad \forall k \in \mathcal{K} \tag{4.32}$$

with support $\mathcal{K}$. The corresponding throughput at the equilibrium is

$$\mathcal{T} = \exp\left(-\frac{\rho}{K}\right) \tag{4.33}$$

Proof. The results directly follows from Corollary 2. □

Corollary 7. *When $\theta_k = 1, m_k = 1, \forall k \in \mathcal{K}$, the centralized optimal access strategy of the multi-channel random access problem in the interference-limited regime ($N_0 = 0$) is*

$$p_k^* = \begin{cases} \frac{1}{\rho}, & \rho > K \\ \frac{1}{K}, & \rho \leq K \end{cases} \tag{4.34}$$

The corresponding throughput of a user with the optimal access strategy is

$$\mathcal{T} = \begin{cases} \frac{K}{\rho e}, & \rho > K \\ \exp\left(-\frac{\rho}{K}\right), & \rho \leq K \end{cases} \tag{4.35}$$

Proof. The results directly follows from Corollary 3. □

As we can see from Corollary 6 and 7, the access strategies reduce to uniform random selection among the K channels for the game-theoretic scheme, and also for the centralized scheme when $\rho \leq K$.

4.4 Remarks

With stochastic geometry, we are now able to capture the nature of spatial distribution of users in wireless ad hoc networks with our system model in Chapter 3. Furthermore, we have proposed channel selection strategies for multi-channel random access for users with a game-theoretic view for the interactions between users, and a centralized optimization perspective for the system performance. In the following, we try to show some connection of our work with previous ones.

4.4.1 Connection with the non-spatial model

As mentioned in Section 3.4.1, a connection can be found between our model and the *non-spatial multi-channel random access with collision channels* defined previously. In this section, we aim to show this relationship.

First, we derive similarly the the game-theoretic channel access strategy for the non-spatial model. The players are the N users, the action set is the set of channels $\mathcal{K}$, and the utility of a action is the expected user throughput associated with the channel selection as defined in (3.11). The results are given in the following.

Theorem 5. *The symmetric Nash equilibrium for non-spatial multi-channel random access game with collision channels is*

$$p_k^* = 1 - (|\mathcal{S}^*| - 1) \frac{\theta_k^{\frac{-1}{N-1}}}{\sum_{i \in \mathcal{S}^*} \theta_i^{\frac{-1}{N-1}}} \tag{4.36}$$

where $\mathcal{S}^*$ is the support of the equilibrium. The expected utility of a user at the equilibrium is

$$\widetilde{E} = \left(\frac{|\mathcal{S}^*| - 1}{\sum_{i \in \mathcal{S}^*} \theta_i^{\frac{-1}{N-1}}} \right)^{N-1} \tag{4.37}$$

Proof. The proof is similar to that for Theorem 2. By Lemma 1, we have

$$\widetilde{\mathcal{T}}_k = \theta_k (1 - p_k^*)^{N-1} = \widetilde{E}, \quad \forall k \in \mathcal{S}^* \tag{4.38}$$

After some manipulations, we have for $k \in \mathcal{S}^*$,

$$p_k^* = 1 - \widetilde{E}^{\frac{1}{N-1}} \theta_k^{\frac{-1}{N-1}} \tag{4.39}$$

By Lemma 2, we have $\sum_{i \in \mathcal{S}^*} p_k^* = 1$ and thus

$$\widetilde{E} = \left(\frac{|\mathcal{S}^*| - 1}{\sum_{i \in \mathcal{S}^*} \theta_i^{\frac{-1}{N-1}}} \right)^{N-1} \tag{4.40}$$

The Nash equilibrium can readily be solved by plugging back $\widetilde{E}$. $\square$

The game-theoretic solution given above also follows a concept similar to water-filling, with the relation

$$-\ln \theta_k - (N-1) \ln(1 - p_k^*) = -\ln \widetilde{E}, \quad \forall k \in \mathcal{S}^* \tag{4.41}$$

where the water level is now $\ln \frac{1}{\widetilde{E}}$, the height of each white block being $\ln \frac{1}{\theta_k}$, and the height of water above each block being $(N-1) \cdot \ln \frac{1}{(1-p_k^*)}$.

The connection of the non-spatial model to ours can be found in some limiting case of both models. Specifically, when we ignore the fading statistics of the channels by setting $m_k = 1$ for all $k \in \mathcal{K}$ and consider for $N_0 = 0$, the game-theoretic solution of our model would be as given in Corollary 4. When $\lambda \to \infty$, the channel access strategy at the Nash equilibrium becomes $p_k^* \to \frac{1}{K}$ for that we have the support $\mathcal{S}^*$ being $\mathcal{K}$ in the limiting case. As for the non-spatial model, we consider the limiting case when $N \to \infty$. The Nash equilibrium also becomes $p_k^* \to \frac{1}{K}$ since

$$\frac{\theta_k^{\frac{-1}{N-1}}}{\sum_{i \in \mathcal{S}^*} \theta_i^{\frac{-1}{N-1}}} \to \frac{1}{|\mathcal{S}^*|}$$

as $N \to \infty$ and that the support $\mathcal{S}^*$ also becomes $\mathcal{K}$, which is obvious from the water-filling concept. This shows that as the number of users becomes large, the game-theoretic strategies for the spatial and non-spatial models will coincide, both being oblivious of the exact value of θ_i.

4.4.2 Relationship to previous works with spatial analysis

It should be noted that when there is only one channel for access (i.e. $K = 1$), the channel availability issue is not considered (i.e. $\theta_1 = 1$), and the average power of fading is set to $m_1 = 1$, the results in our work reduce to those in [42], where both game-theoretic and centralized optimization perspectives are considered for medium access with a single channel and with spatially distributed users. The results are given below.

Corollary 8. *With $K = 1$, $\theta_1 = 1$, $m_1 = 1$, and $N_0 = 0$, the access strategy of the multi-channel random access game at the equilibrium is*

$$p_1^* = 1 \tag{4.42}$$

The corresponding throughput at the equilibrium is

$$\mathcal{T} = \exp\left(-\rho\right) \tag{4.43}$$

where ρ is given by (3.9).

Corollary 9. *With $K = 1$, $\theta_1 = 1$, $m_1 = 1$, and $N_0 = 0$, the centralized optimal access strategy of the multi-channel random access problem is*

$$p_1^* = \begin{cases} \frac{1}{\rho}, & \rho > 1 \\ 1, & \rho \leq 1 \end{cases} \tag{4.44}$$

where ρ is given by (3.9). *The corresponding throughput is*

$$\mathcal{T} = \begin{cases} \frac{1}{e\rho}, & \rho > 1 \\ e^{-\rho}, & \rho \leq 1 \end{cases} \tag{4.45}$$

It can be easily seen that our game-theoretic solution in Corollary 8 is exactly the same as the one provided in [42] without pricing, and the centralized solution in Corollary 9 is also exactly the same as that in [42]. Our work is a non-trivial extension for that the nonhomogeneous characteristics of multiple channels are further considered, and the solution follows a water-filling concept.

Another thing to be noted is that the performance metrics used in our work are closely related to those defined in Section 2.2 for single channel slotted ALOHA. Despite the fact that we further considered the context with multiple channels, there is still another difference of our study to those on transmission capacity. The study of transmission capacity aims at characterizing the maximum spatial density of transmission subject to an outage probability constraint ν^*. With the density of potential transmitters being λ, if we have $\lambda \cdot (1-\nu^*)$ exceeding the transmission capacity $\mathrm{TC}(\nu^*)$, then some throttling of transmission attempt is needed such that each transmitter transmits with probability

$$p_{\mathrm{tx}} = \frac{\mathrm{TC}(\nu^*)}{\lambda \cdot (1-\nu^*)} \tag{4.46}$$

so that the outage constraint would be satisfied [22]; otherwise, if we have $\lambda \cdot (1 - \nu^*) \leq \mathrm{TC}(\nu^*)$, no throttling of transmission attempt is needed. On the contrary, although the system throughput considered in our work has the same unit of successful transmission per unit area as the transmission capacity, it has a different meaning. Without specifying any outage probability constraint, we dealt with a given density λ of users that may decide their access strategies (i.e. channel access probabilities), and study the performance by measuring the system throughput as a result of the access strategies of users. Since the goal here is to devise distributed access strategies from a user perspective, the transmission capacity context is not directly used in this thesis.

4.4.3 The local delay

In this section, we give a brief discussion on another basic performance metric, the *local delay*, that is related to the quality-of-service provided by a network. The local delay is generally defined as the mean time until a packet is successfully transmitted over a communication link [20]. In [51], local delay with nearest-neighbor communications is studied in Poisson networks in both the static and highly mobile cases. It was found that the local delay is always finite in the high mobility case while it exhibits phase transition in the static network case where the local delay grows to infinity depending on the network parameters.

When we assume that the node locations of the whole network is independently re-sampled in every slot, which is a reasonable assumption for highly mobile networks [20], the local delay in our scenario is just the reciprocal of the user throughput defined as in Section 3.2, which gives $\frac{1}{\mathcal{T}}$. In all other cases, it serves as a lower bound [20, 42].

Chapter 5

Multi-channel Random Access With Channel Side Information

In this chapter, we consider the case when some channel side information could be obtained. By channel side information, we specifically mean the channel state information (CSI) and channel availability. Given the channel side information, the access strategy would change. In the following, we devise the optimal access strategy and derive the resulting expected user throughput. Finally, we compare the results with those in the cases without such channel side information.

5.1 With channel state information

We assume that a user can obtain its own CSI $\mathbf{h_S} = [h_{S,1}, \ldots, h_{S,K}]$, which is the realization of the channel fading gain of the desired signal, and consider the interference-limited regime ($N_0 = 0$) for simplicity. We further assume that the channel availabilities are i.i.d. and without loss of generality set $\theta_k = 1$ for all channels to focus on the effect of CSI. Now, the throughput of a typical user in a channel would become

$$\mathcal{T}_k = \mathsf{Pr}\left(\log_2\left(1 + \frac{h_{S,k}Pr^{-\alpha}}{I_k}\right) \geq R\right)$$
$$= \mathsf{Pr}\left(I_k \leq \phi^{-1}h_{S,k}\right)$$
$$= F_{I_k}(\phi^{-1}h_{S,k}) \tag{5.1}$$

where $I_k = \sum_{X_i \in \Psi_k} H_{i0,k}P|X_i|^{-\alpha}$ is the interference power that would be perceived by the typical user in channel k, and $F_{I_k}(h)$ is the cdf of I_k. Note that since the user knows its own CSI only, the fading coefficient of the interfering links are unknown and random. Due to the homogeneity of PPP, and since that the uncoordinated users know only the statistics of the network except for its own CSI, we shall assume that the users adopt the same channel access strategy. Although we say that the users in the network adopts the same access strategy (which we refer to as symmetric strategy), different users in the network can however choose different channels for access in a time slot for that the CSI seen by different users may vary.

Let q_k denote the probability that a channel is chosen for access given the symmetric access strategy of the users. Not that this probability q_k is different from the concept of a mixed strategy in Section 4.1, where a channel is chosen for access according to a probabilistic distribution over the set $\mathcal{K}$. Here, q_k results from channels selected according to the symmetric access strategy where the randomness is due to different CSI realizations, and it means that an arbitrary user would be transmitting in a channel k with probability q_k. According to the thinning property of PPP, the interference I_k in a channel k can now be written as $I_k = \sum_{X_i \in \Pi(q_k\lambda)} H_{i0,k}P|X_i|^{-\alpha}$.

With their own CSI at hand, the uncoordinated users in the network now aims at selecting a proper channel for access that maximizes its own throughput given the CSI. That is, a channel k is chosen by a user if

$$F_{I_k}(\phi^{-1}h_{S,k}) = \max_{i \in \mathcal{K}} F_{I_i}(\phi^{-1}h_{S,i}) \tag{5.2}$$

where $\mathbf{h_S} = [h_{S,1}, \ldots, h_{S,K}]$ is the CSI.

Lemma 3. *In the multi-channel random access game with CSI, where* $N_0 = 0$ *and* $\theta_k = 1$*, the strategy that channel* k *is chosen if*

$$F_{I_k}(\phi^{-1}h_{S,k}) = \max_{i\in\mathcal{K}} F_{I_i}(\phi^{-1}h_{S,i}) \tag{5.3}$$

is a pure strategy Nash equilibrium.

Proof. Consider a typical user who deviates from the strategy while all other users in the network follow the strategy. The utility of choosing a channel k for access of that typical user is $F_{I_k}(\phi^{-1}h_{S,k})$. Deviating from the strategy would only leads to a strict degradation of performance to the user. Thus, the strategy is a pure strategy Nash equilibrium. □

In the following, we first consider the case when the channel statistics are i.i.d. across different channels, and then discuss the more general case when the channels are non-i.i.d..

5.1.1 The special case under i.i.d. channels

When the fading statistics of the K channels are i.i.d. (without loss of generality we set $m_k = 1$), and due to the fact that the users in the network adopt the symmetric strategy, the statistics of the interference power over different channels shall be the same, and thus we can drop the subscript k and represent the interference power by I. The throughput of a user in channel k now becomes

$$\mathcal{T}_k = F_I\left(\phi^{-1}h_{S,k}\right) \tag{5.4}$$

which depends only on the cdf of I and the CSI of each channel k. By the nondecreasing property of the cdf, the following corollary is obvious.

Corollary 10. *When the channel fading statistics are i.i.d. over the* K *channels in* *addition to the assumptions in Lemma 3, the strategy that channel k is chosen if*

$$h_{S,k} = \max_{i \in \mathcal{K}} h_{S,i} \tag{5.5}$$

is a pure strategy Nash equilibrium.

Corollary 10 suggests that a user should choose the channel with the best instantaneous channel quality $h_{S,k}$ for access when CSI is available. Under such a case, the following Theorem holds.

Theorem 6. *The throughput of a user in multi-channel random access game with CSI in i.i.d. channels is*

$$\mathcal{T} = \sum_{i=1}^{K} \binom{K}{i} (-1)^{i+1} \exp(-\frac{\rho}{K} i^{\delta}) \tag{5.6}$$

at the equilibrium, where ϕ is given by (3.6) *and ρ is given by* (3.9)*. The probability that a channel k is chosen by a user is*

$$q_k = \frac{1}{K} \tag{5.7}$$

Proof. The probability that a channel k is chosen by a user can be obtained as

$$\begin{aligned} q_k &= \mathsf{Pr}\left(H_{S,k} = \max_{i \in \mathcal{K}} H_{S,i} \right) \\ &= \int_0^{\infty} f_{H_{S,k}}(h) \prod_{\substack{i \neq k \\ i \in \mathcal{K}}} \mathsf{Pr}\left(h \geq H_{S,i}\right) dh \\ &= \int_0^{\infty} e^{-h} \left(1 - e^{-h}\right)^{K-1} dh \\ &= \frac{1}{K} \end{aligned} \tag{5.8}$$

By the thinning property of PPP, we have the interference

$$I = \sum_{X_i \in \Pi(q_k \lambda)} H_{i0,k} P |X_i|^{-\alpha}$$

The Laplace transform of the interference I is

$$\mathsf{E}\left[e^{-sI}\right] = \exp\left(-\frac{\lambda}{K}\pi P^{\delta}\Gamma(1+\delta)\Gamma(1-\delta)s^{\delta}\right) \tag{5.9}$$

Let H_S^* denote the equivalent channel fading gain experienced by a user adopting this strategy, which is defined as $H_S^* \triangleq \max_{k\in\mathcal{K}} H_{S,k}$. The throughput of a user can be characterized as

$$\begin{aligned}
\mathcal{T} &= \mathsf{Pr}\left(\log_2\left(1+\frac{H_S^* P r^{-\alpha}}{I}\right) \geq R\right) \\
&= \mathsf{Pr}\left(H_S^* \geq \phi I\right) \\
&\overset{(a)}{=} \mathsf{E}\left[\int_{-\infty}^{\infty} \exp(-\phi I t)\widetilde{h}_S^*(t)dt\right] \\
&= \int_{-\infty}^{\infty} \mathsf{E}\left[\exp(-\phi I t)\right]\widetilde{h}_S^*(t)dt \\
&\overset{(b)}{=} \int_{-\infty}^{\infty} \exp\left(-\frac{\lambda}{K}\pi r^2(2^R-1)^{\delta}\Gamma(1+\delta)\Gamma(1-\delta)t^{\delta}\right)\widetilde{h}_S^*(t)dt
\end{aligned} \tag{5.10}$$

where (a) follows by using $\mathsf{Pr}\left(H_S^* \geq s\right) = \int_{-\infty}^{\infty} e^{-st}\widetilde{h}_S^*(t)dt$ in which $\widetilde{h}_S^*(t)$ is the inverse Laplace transform of the ccdf of H_S^*, ϕ is given by (3.6), and (b) follows by using (5.9). The cdf of H_S^* is

$$\begin{aligned}
F_{H_S^*}(s) &= \mathsf{Pr}\left(H_S^* \leq s\right) \\
&= \mathsf{Pr}\left(\max_{k\in\mathcal{K}} H_{S,k} \leq s\right) \\
&= \mathsf{Pr}\left(H_{S,1} \leq s, H_{S,2} \leq s, \ldots, H_{S,K} \leq s\right) \\
&= \prod_{k\in\mathcal{K}} \mathsf{Pr}\left(H_{S,i} \leq s\right) \\
&= \left(1-e^{-s}\right)^K
\end{aligned} \tag{5.11}$$

and the inverse Laplace transform of the ccdf of H_S^* is

$$\begin{aligned}\widetilde{h}_S^*(t) &= \mathcal{L}^{-1}\left\{1-\left(1-e^{-s}\right)^K\right\} \\ &= \mathcal{L}^{-1}\left\{1-\sum_{i=0}^{K}\binom{K}{i}(-1)^i e^{-si}\right\} \\ &= \mathcal{L}^{-1}\left\{\sum_{i=1}^{K}\binom{K}{i}(-1)^{i+1} e^{-si}\right\} \\ &= \sum_{i=1}^{K}\binom{K}{i}(-1)^{i+1}\delta(t-i) \end{aligned} \tag{5.12}$$

where $\delta(t)$ is the Dirac delta function. The throughput of a user can thus be obtained as

$$\mathcal{T} = \sum_{i=1}^{K}\binom{K}{i}(-1)^{i+1}\exp(-\frac{\lambda}{K}\pi r^2(2^R-1)^\delta\Gamma(1+\delta)\Gamma(1-\delta)i^\delta) \tag{5.13}$$

With ρ as defined in (3.9), we thus have the theorem. $\square$

Theorem 6 shows that when the channels are i.i.d., although CSI is provided to each user, the probability q_k of a channel being selected is the same as the case when no CSI is provided as given by p_k^* in Corollary 6, both being $\frac{1}{K}$. Since the statistics of interference in each channel is only determined by the density of active transmitters in that channel which is $q_k\lambda = p_k^*\lambda = \frac{\lambda}{K}$ for both cases, the statistics of interference in each channel is not affected whether CSI is provided or not in this case. However, the statistics of the desired signal when CSI is provided will be with a better characteristics than that without CSI since it is with the maximum of the fading coefficients for each channel. Thus, when each user is provided with its own CSI, the throughput performance can be better.

Fig. 5.1 gives the throughput versus user density of case with and without CSI provided to the transmitter. We can see that with CSI provided, there's a significant improvement over that without CSI. Fig. 5.2 depicts the probabilities of channels being selected, which are the same for both cases.

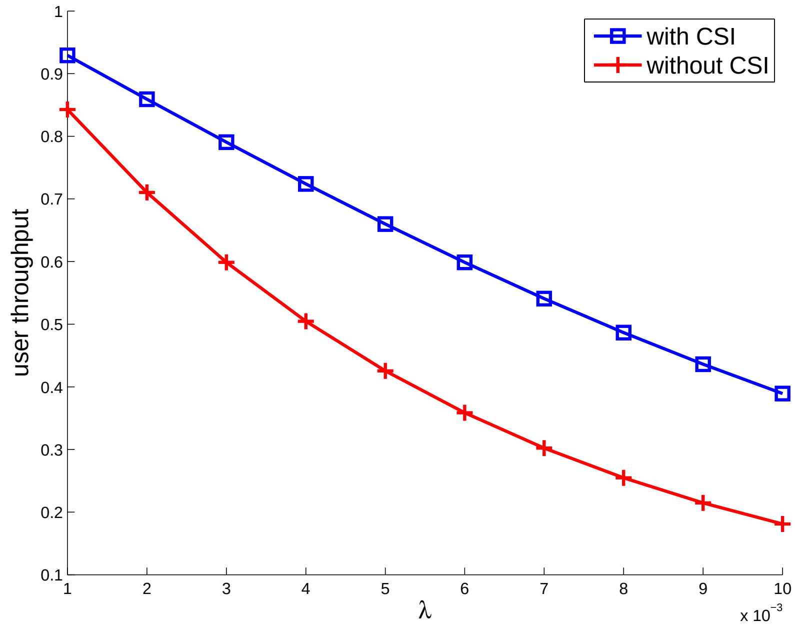


Figure 5.1: Throughput versus user density for the case with and without CSI when channel fading statistics are i.i.d. ($m_k = 1$) in the interference-limited regime ($N_0 = 0$). The other network parameters are set as $\alpha = 4$, $R = 2$ bits/s/Hz, $r = 13$m, $P = 1$, and there are $K = 5$ channels.

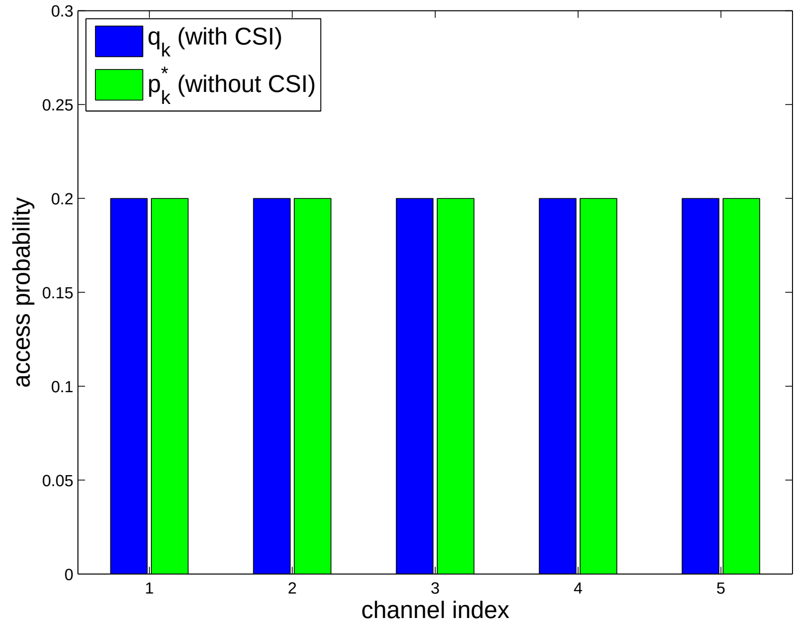


Figure 5.2: Probability that a channel is accessed by a user for the case with and without CSI when channel fading statistics are i.i.d. ($m_k = 1$) in the interference-limited regime ($N_0 = 0$). The other network parameters are set to the same as in Fig. 5.1.

5.1.2 The general case with non-i.i.d. channels

Now, we proceed to discuss the case when the channel fading statistics are non-i.i.d., where $\mathsf{E}[H_{S,k}] = m_k$. In such a case, each user selects channels according to the strategy described in Lemma 3, where the cdf $F_{I_k}(h)$ of interference I_k in channel k is used. In the following, we assume the path loss exponent is $\alpha = 4$ (i.e. $\delta = \frac{1}{2}$) where the cdf of I_k can be expressed as

$$F_{I_k}(s) = \begin{cases} \mathrm{erfc}(\frac{q_k \lambda \pi^2 P^{\frac{1}{2}}}{4\sqrt{s}}), & s \geq 0 \\ 0, & s < 0 \end{cases} \tag{5.14}$$

where $\mathrm{erfc}(s) = \frac{2}{\sqrt{\pi}} \int_s^\infty \exp(-t^2)dt$ is the standard complementary error function and $q_k\lambda$ is the density of users transmitting in channel k.

Theorem 7. *The throughput of a user in multi-channel random access game with CSI in non-i.i.d. channels with $\alpha = 4$ is*

$$\mathcal{T} = \sum_{i=1}^{K} (-1)^{i+1} \sum_{\omega_j \in \Omega_i^{\mathcal{K}}} e^{-2\kappa \left(\sum_{l \in \omega_j} \frac{q_l^2}{m_l} \right)^{\frac{1}{2}}} \tag{5.15}$$

at the equilibrium, where

$$\kappa \triangleq \frac{1}{4} \lambda \pi^2 r^2 (2^R - 1)^{\frac{1}{2}} \tag{5.16}$$

and $\Omega_n^{\mathcal{M}} \triangleq \left\{ \omega_1, \ldots, \omega_{\binom{|\mathcal{M}|}{n}} \right\}$ is defined as the set of all subsets with cardinality n of the set $\mathcal{M}$. The probability that a channel k is chosen by a user is the solution of the following simultaneous equations:

$$q_k = 1 + \sum_{i=1}^{K-1} (-1)^i \sum_{\omega_j \in \Omega_i^{\mathcal{K} \setminus \{k\}}} \left(1 + \frac{m_k}{q_k^2} \sum_{l \in \omega_j} \frac{q_l^2}{m_l} \right)^{-1}, \quad \forall k \in \mathcal{K} \tag{5.17}$$

along with the fact that $\sum_{k \in \mathcal{K}} q_k = 1$.

Proof. First, we derive for the probability that a channel k is chosen by a user, which with the strategy given by Lemma 3 can be obtained as

$$
\begin{aligned}
q_k &= \mathsf{Pr}\left(F_{I_k}(\phi^{-1}H_{S,k}) = \max_{i\in\mathcal{K}} F_{I_i}(\phi^{-1}H_{S,i}) \right) \\
&= \int_0^\infty \mathsf{Pr}\left(F_{I_k}(\phi^{-1}h \geq F_{I_i}(\phi^{-1}H_{S,i}), \forall i \in \mathcal{K}\setminus\{k\} | H_{S,k} = h \right) f_{H_{S,k}}(h)dh \\
&= \int_0^\infty \left(\prod_{i\in\mathcal{K}\setminus\{k\}} \mathsf{Pr}\left(F_{I_k}(\phi^{-1}h) \geq F_{I_i}(\phi^{-1}H_{S,i}) | H_{S,k} = h \right) \right) f_{H_{S,k}}(h)dh \qquad (5.18)
\end{aligned}
$$

When $\alpha = 4$, we apply (5.14) and thus

$$
\begin{aligned}
q_k &= \int_0^\infty \left(\prod_{i\in\mathcal{K}\setminus\{k\}} \mathsf{Pr}\left(\mathrm{erfc}\left(\frac{q_k\lambda\pi^2 P^{\frac{1}{2}}}{4\sqrt{\phi^{-1}h}} \right) \geq \mathrm{erfc}\left(\frac{q_i\lambda\pi^2 P^{\frac{1}{2}}}{4\sqrt{\phi^{-1}H_{S,i}}} \right) | H_{S,k} = h \right) \right) f_{H_{S,k}}(h)dh \\
&\overset{(a)}{=} \int_0^\infty \left(\prod_{i\in\mathcal{K}\setminus\{k\}} \mathsf{Pr}\left(\frac{p_k^2}{h} \leq \frac{p_i^2}{H_{S,i}} | H_{S,k} = h \right) \right) f_{H_{S,k}}(h)dh \\
&= \int_0^\infty \left(\prod_{i\in\mathcal{K}\setminus\{k\}} \mathsf{Pr}\left(H_{S,i} \leq \frac{p_i^2}{p_k^2} h | H_{S,k} = h \right) \right) f_{H_{S,k}}(h)dh \\
&= \int_0^\infty \left(\prod_{i\in\mathcal{K}\setminus\{k\}} \left(1 - e^{-\frac{1}{m_i}\frac{p_i^2}{p_k^2}h} \right) \right) \frac{1}{m_k} e^{-\frac{1}{m_k}} dh \qquad (5.19)
\end{aligned}
$$

where (a) follows by the property that $\mathrm{erfc}(s)$ is strictly decreasing. Define $\Omega_n^{\mathcal{M}} \triangleq \left\{ \omega_1, \ldots, \omega_{\binom{|\mathcal{M}|}{n}} \right\}$ as the set of all subsets with cardinality n of the set $\mathcal{M}$. For example, when the set $\mathcal{M} = \{1, 2, 3\}$, we would have $\Omega_2^{\mathcal{M}} = \{\{1, 2\}, \{2, 3\}, \{1, 3\}\}$.

With this definition, we can continue as

$$
\begin{aligned}
q_k &= \frac{1}{m_k}\int_0^\infty \left[1+\sum_{i=1}^{K-1}(-1)^i \sum_{\omega_j\in\Omega_i^{\mathcal{K}\setminus\{k\}}} e^{-\frac{h}{q_k^2}\sum_{l\in\omega_j}\frac{q_l^2}{m_l}}\right] e^{-\frac{1}{m_k}h}dh \\
&= 1+\frac{1}{m_k}\sum_{i=1}^{K-1}(-1)^i \sum_{\omega_j\in\Omega_i^{\mathcal{K}\setminus\{k\}}} \int_0^\infty e^{-h\left(\frac{1}{m_k}+\frac{1}{q_k^2}\sum_{l\in\omega_j}\frac{q_l^2}{m_l}\right)}dh \\
&= 1+\frac{1}{m_k}\sum_{i=1}^{K-1}(-1)^i \sum_{\omega_j\in\Omega_i^{\mathcal{K}\setminus\{k\}}} \left(\frac{1}{m_k}+\frac{1}{q_k^2}\sum_{l\in\omega_j}\frac{q_l^2}{m_l}\right)^{-1} \\
&= 1+\sum_{i=1}^{K-1}(-1)^i \sum_{\omega_j\in\Omega_i^{\mathcal{K}\setminus\{k\}}} \left(1+\frac{m_k}{q_k^2}\sum_{l\in\omega_j}\frac{q_l^2}{m_l}\right)^{-1}
\end{aligned}
\tag{5.20}
$$

As for the throughput $\mathcal{T}$ of a user, we have

$$
\begin{aligned}
\mathcal{T} &= \mathsf{E}\left[\max_{k\in\mathcal{K}} F_{I_k}\left(\phi^{-1}H_{S,k}\right)\right] \\
&= \mathsf{E}\left[\max_{k\in\mathcal{K}}\left\{\operatorname{erfc}\left(\frac{q_k\lambda\pi^2P^{\frac{1}{2}}}{4\sqrt{\phi^{-1}H_{S,k}}}\right)\right\}\right] \\
&\overset{(a)}{=} \mathsf{E}\left[\operatorname{erfc}\left(\frac{\lambda\pi^2P^{\frac{1}{2}}}{4\sqrt{\phi^{-1}}}\min_{k\in\mathcal{K}}\left\{\frac{q_k}{\sqrt{H_{S,k}}}\right\}\right)\right] \\
&\overset{(b)}{=} \int_0^\infty \operatorname{erfc}\left(\kappa g\right) f_G(g)dg \\
&\overset{(c)}{=} \int_0^\infty \left(\frac{2}{\sqrt{\pi}}\int_{\kappa g}^\infty e^{-t^2}dt\right) f_G(g)dg \\
&\overset{(d)}{=} \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-t^2}\underbrace{\int_0^{\frac{t}{\kappa}} f_G(g)dg}_{F_G\left(\frac{t}{\kappa}\right)}\,dt
\end{aligned}
\tag{5.21}
$$

where (a) follows by the fact that $\operatorname{erfc}(s)$ is strictly decreasing. (b) follows by defining the random variable $G \triangleq \min_{k\in\mathcal{K}}\left\{\frac{q_k}{\sqrt{H_{S,k}}}\right\}$ with pdf $f_G(g)$ and constant $\kappa \triangleq \frac{\lambda\pi^2P^{\frac{1}{2}}}{4\sqrt{\phi^{-1}}} = \frac{1}{4}\lambda\pi^2r^2(2^R-1)^{\frac{1}{2}}$ with ϕ given by (3.6). (c) follows by using the definition $\operatorname{erfc}(s) = \frac{2}{\sqrt{\pi}}\int_s^\infty \exp(-t^2)dt$ and (d) follows by interchanging the order of integration. The second integral in the last line can be identified as the cdf of G evaluated at $\frac{t}{\kappa}$, where the cdf of G can be obtained as

$$
\begin{aligned}
F_G(g) &= 1 - \mathsf{Pr}\left(\min_{k\in\mathcal{K}}\left\{\frac{q_k}{\sqrt{H_{S,k}}}\right\} \geq g\right) \\
&= 1 - \prod_{k\in\mathcal{K}} \mathsf{Pr}\left(\frac{q_k}{\sqrt{H_{S,k}}} \geq g\right) \\
&= 1 - \prod_{k\in\mathcal{K}} \mathsf{Pr}\left(H_{S,k} \leq \frac{q_k^2}{g^2}\right) \\
&= 1 - \prod_{k\in\mathcal{K}} \left(1 - e^{-\frac{1}{m_k}\frac{q_k^2}{g^2}}\right) \\
&= \sum_{i=1}^{K} (-1)^{i+1} \sum_{\omega_j \in \Omega_i^{\mathcal{K}}} e^{-\frac{1}{g^2}\sum_{l\in\omega_j}\frac{q_l^2}{m_l}}
\end{aligned}
\tag{5.22}
$$

Thus, we have

$$
\begin{aligned}
\mathcal{T} &= \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-t^2} \left(\sum_{i=1}^{K}(-1)^{i+1} \sum_{\omega_j\in\Omega_i^{\mathcal{K}}} e^{-\frac{\kappa^2}{t^2}\sum_{l\in\omega_j}\frac{q_l^2}{m_l}}\right) dt \\
&= \frac{2}{\sqrt{\pi}} \sum_{i=1}^{K}(-1)^{i+1} \sum_{\omega_j\in\Omega_i^{\mathcal{K}}} \left(\int_0^\infty e^{-t^2-\frac{1}{t^2}\kappa^2\sum_{l\in\omega_j}\frac{q_l^2}{m_l}} dt\right) \\
&\overset{(a)}{=} \sum_{i=1}^{K}(-1)^{i+1} \sum_{\omega_j\in\Omega_i^{\mathcal{K}}} e^{-2\kappa\left(\sum_{l\in\omega_j}\frac{q_l^2}{m_l}\right)^{\frac{1}{2}}}
\end{aligned}
\tag{5.23}
$$

where (a) follows by using the integral

$$
\int_0^\infty \exp\left(-ax^2 - \frac{b}{x^2}\right) dx = \frac{1}{2}\sqrt{\frac{\pi}{a}} \exp\left(-2\sqrt{ab}\right)
$$

in which $a > 0$, $b > 0$ [52, eq. (3.325)].

□

As an example, when we have $K = 2$,

$$
\begin{aligned}
q_1 &= 1 - \left(1 + \frac{m_1}{q_1^2}\frac{q_2^2}{m_2}\right) = \frac{m_1 q_2^2}{m_1 q_2^2 + m_2 q_1^2} \\
q_2 &= \frac{m_2 q_1^2}{m_1 q_2^2 + m_2 q_1^2}
\end{aligned}
\tag{5.24}
$$

Substituting $q_2 = 1 - q_1$ into (5.24), we have

$$(m_1 + m_2)q_1^3 - 3m_1q_1^2 + 3m_1q_1 - m_1 = 0 \tag{5.25}$$

and q_1, q_2 can be obtained by solving the cubic equation above. As for the expected user throughput when $K = 2$, we have

$$\mathcal{T} = \exp\left(-2\kappa\frac{q_1}{\sqrt{m_1}}\right) + \exp\left(-2\kappa\frac{q_2}{\sqrt{m_2}}\right) - \exp\left(-2\kappa\left(\frac{q_1^2}{m_1} + \frac{q_2^2}{m_2}\right)^{\frac{1}{2}}\right) \tag{5.26}$$

which can be evaluated with closed form given q_k.

When the channels are i.i.d. and without loss of generality assume $m_k = 1$, we have from Theorem 7 that $q_k = \frac{1}{K}$ by symmetry and

$$\begin{aligned}\mathcal{T} &= \sum_{i=1}^{K}(-1)^{i+1}\sum_{\omega_j\in\Omega_i^K} e^{-2\kappa\left(\sum_{l\in\omega_j}\frac{1}{K^2}\right)^{\frac{1}{2}}} \\ &= \sum_{i=1}^{K}(-1)^{i+1}\sum_{\omega_j\in\Omega_i^K} e^{-2\kappa\left(\frac{i}{K^2}\right)^{\frac{1}{2}}} \\ &= \sum_{i=1}^{K}(-1)^{i+1}\binom{K}{i} e^{-\frac{2\kappa}{K}i^{\frac{1}{2}}}\end{aligned} \tag{5.27}$$

which reduces to the result given by Theorem 6 by noting that when $\delta = \frac{1}{2}$, we have $\Gamma(1+\delta) = \frac{\sqrt{\pi}}{2}$, $\Gamma(1-\delta) = \sqrt{\pi}$, and $\rho = 2\kappa$.

Again, Fig. 5.3 gives the throughput versus user density of case with and without CSI provided to the transmitter. We can see that in the case when the channels are non-i.i.d. and with CSI provided, there is also a significant improvement over that without CSI. Fig. 5.2 depicts the probabilities of channels being selected.

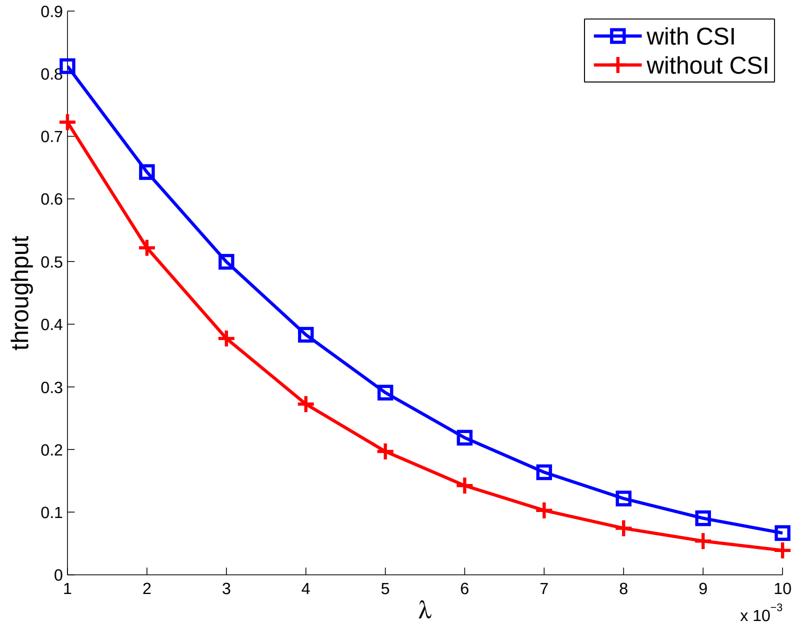


Figure 5.3: Throughput versus user density for the case with and without CSI when there are $K = 2$ channels and the channel fading statistics are non-i.i.d. with $m_1 = 1.0$ and $m_2 = 2.5$ in the interference-limited regime ($N_0 = 0$). The other network parameters are set as $\alpha = 4$, $R = 2$ bits/s/Hz, $r = 10$m, and $P = 1$.

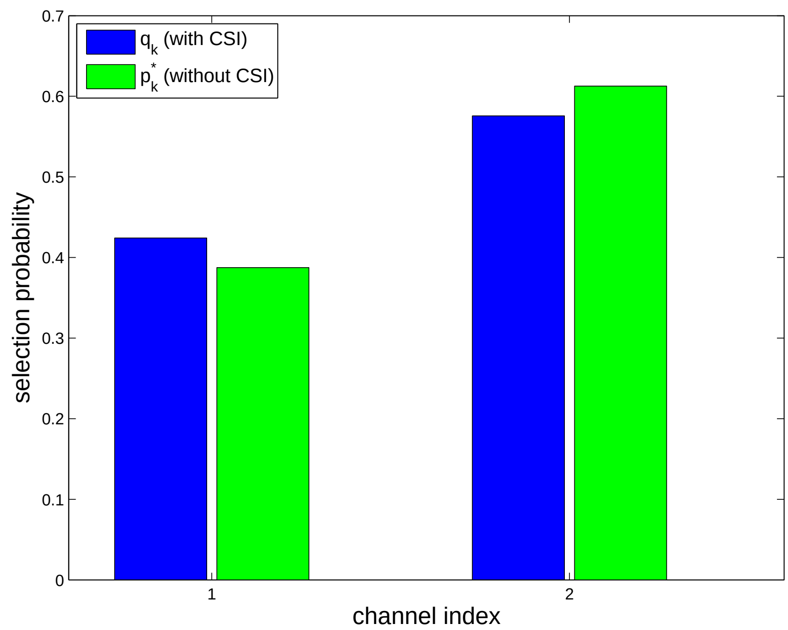


Figure 5.4: Probability that a channel is accessed by a user for the case with and without CSI when there are $K = 2$ channels and the channel fading statistics are non-i.i.d. with $m_1 = 1.0$ and $m_2 = 2.5$ in the interference-limited regime ($N_0 = 0$). The other network parameters are set to the same as in Fig. 5.3.

5.1.3 Remarks on multi-channel random access with CSI

We have seen in the previous sections that by providing CSI to the transmitter, the throughput performance of a user can be greatly improved. This is made by utilizing the channel diversity with the help of CSI. The signal distribution seen by a receiver is improved with CSI for the i.i.d. fading channels, for its statistics will become $H_S^* \triangleq \max_{k \in \mathcal{K}} H_{S,k}$ which has mean $\sum_{i=1}^{K} \frac{1}{i}$ compared to the unit mean for the case without CSI, while the statistics of the interference are the same for both cases.

The findings of our work here are different from those in [41], where random access game of users sharing a common communication channel and willing to access a single base station was studied. The authors of [41] found that when local CSI is provided to the selfish users in the interference limited regime, the performance (throughput) of homogeneous users will be worse compared to that when no CSI is provided. This phenomenon is called a Braess-like paradox, where the performance degrades when more information is provided to a system of noncooperative users. The reason is that the users in the scenario of [41] aim to access the same base station selfishly, and although the received signal power at the based station from a user is improved when CSI is provided, the strategy of the users at the Nash equilibrium also increases the average interference power nonetheless. In contrast, in our scenario with ad hoc networks and i.i.d. fading channels, the average interference power is remained the same while the average received signal power increases when CSI is provided, and the performance is therefore improved.

The gain in performance, however, needs the CSI at the transmitter side, and in turn it means that some feedback information to the transmitter is needed. Therefore, there might be a tradeoff between the gain that can be achieved by the CSI and the cost for feedback information.

5.2 With channel availability information

Now, we consider the case when the availability of each channel can be known before a user makes the channel selection decision for channel access. This corresponds to the case in CRN when user are capable of performing multi-channel spectrum sensing (e.g. as the wideband sensing technique in [53]). In this case, a user shall access one among the channels that are available in a slot for that accessing a channel that is not available is not allowed or will result in an access failure. Recall that A_k is the indicator variable that equals 1 if channel k is available and equals 0 otherwise, therefore we can represent the set of channels that are available in a slot as $\{k \in \mathcal{K} : A_k = 1\}$. Since that with the available probability θ_k, A_k is a Bernoulli random variable with parameter θ_k, we can define the probability of user seeing a realization $\mathcal{C} \subseteq \mathcal{K}$ of the channel availabilities as

$$\Upsilon_{\mathcal{C}} \triangleq \mathsf{Pr}(\{k \in \mathcal{K} : A_k = 1\} = \mathcal{C}) = \prod_{i \in \mathcal{C}} \theta_i \prod_{j \in \mathcal{K} \setminus \mathcal{C}} \theta_j \tag{5.28}$$

When the channel availability statistics are i.i.d. over different channels, or more specifically when $A_k \sim Bernoulli(\theta)$ for all $k \in \mathcal{K}$, we have $\Upsilon_{\mathcal{C}} = \binom{K}{|\mathcal{C}|}\theta^{|\mathcal{C}|}(1-\theta)^{K-|\mathcal{C}|}$.

Given the set of available channels $\mathcal{C} \subseteq \mathcal{K}$ in a time slot, a user could naively choose the channel with the best channel quality (m_k) among the set $\mathcal{C}$. However, as discussed in Section 4.1, this might not be a good option when the effect of the other users in the network needs to be taken into consideration. Therefore, we can again formulate the problem of *multi-channel random access with channel availability information* $\mathcal{C}$ in each time slot as a game. The players are the users in the network. Since all the players observe the same set of available channels $\mathcal{C}$, the action of a player is the channel selection for access which is constrained to be in the set $\mathcal{C}$. The utility associated with each action is the throughput achieved by accessing that channel. With similar arguments of homogeneity as in Section 4.1, we shall look for the symmetric mixed strategy Nash equilibrium $\mathbf{p}_{\mathcal{C}}{}^* = [p_{\mathcal{C},1}, \ldots, p_{\mathcal{C},K}]$ of this game, where $p_{\mathcal{C},k}$ is the probability of choosing channel k for access given $\mathcal{C}$. The results are given in the following theorem.

Theorem 8. *For multi-channel random access with channel availability information, the channel selection probability at the equilibrium given $\mathcal{C}$ is*

$$p^*_{\mathcal{C},k} = \begin{cases} \frac{m_k^\delta}{\sum_{i\in\mathcal{S}^*_{\mathcal{C}}} m_i^\delta}, & k \in \mathcal{S}^*_{\mathcal{C}} \\ 0, & k \in \mathcal{K} \setminus \mathcal{S}^*_{\mathcal{C}} \end{cases} \tag{5.29}$$

where where $\mathcal{S}^*_{\mathcal{C}}$ is the support of the equilibrium given $\mathcal{C}$. The throughput of multi-channel random access with channel availability information is

$$\mathcal{T} = \sum_{\mathcal{C}\subseteq\mathcal{K}} \Upsilon_{\mathcal{C}} \widetilde{E}(\mathcal{S}^*_{\mathcal{C}}) \tag{5.30}$$

where

$$\widetilde{E}(\mathcal{S}) \triangleq \exp\left(-\frac{\rho}{\sum_{k\in\mathcal{S}} m_k^\delta}\right) \tag{5.31}$$

Proof. The proof is similar as that for Theorem 2, but now the utility to be equalized in the support is the throughput in channel k given the availability $\mathcal{C}$

$$\mathcal{T}_{\mathcal{C},k} = \exp(-\phi N_0 m_k^{-1}) \exp(-p^*_{\mathcal{C},k}\rho m_k^{-\delta}) = E_{\mathcal{C}}, \quad \forall k \in \mathcal{S}^*_{\mathcal{C}}$$

By taking natural logarithm on both sides and rearranging terms, we have for $k \in \mathcal{S}^*_{C}$,

$$p^*_{\mathcal{C},k} = -\frac{m_k^\delta}{\rho} \ln E_{\mathcal{C}} \tag{5.32}$$

With arguments similar as in Lemma 2, we have $\sum_{k\in\mathcal{S}^*_{C}} p^*_{C,k} = 1$. Thus, we can obtain the equalizing constant as

$$E_{\mathcal{C}} = \exp\left(-\frac{\rho}{\sum_{k\in\mathcal{S}^*_{\mathcal{C}}} m_k^\delta}\right)$$

Noting that a channel that is not available should not be accessed, we have the access strategy

$$p^*_{\mathcal{C},k} = \begin{cases} \frac{m_k^\delta}{\sum_{i \in \mathcal{S}^*_{\mathcal{C}}} m_i^\delta}, & k \in \mathcal{S}^*_{\mathcal{C}} \\ 0, & k \in \mathcal{K} \setminus \mathcal{S}^*_{\mathcal{C}} \end{cases}$$

The throughput of a user given channel availability information $\mathcal{C}$ at the equilibrium $\mathbf{p}_{\mathcal{C}}{}^*$ is then obtained as

$$\mathcal{T}_{\mathcal{C}} = \sum_{k \in \mathcal{S}^*_{\mathcal{C}}} p^*_{\mathcal{C},k} E_{\mathcal{C}} = E_{\mathcal{C}},$$

Note that the throughput of a user should be averaged over all possible realizations of channel availability $\mathcal{C}$, thus we have

$$\mathcal{T} = \sum_{\mathcal{C} \subseteq \mathcal{K}} \Upsilon_{\mathcal{C}} E_{\mathcal{C}}$$

$\square$

Theorem 8 shows how a channel should be accessed given the channel availability $\mathcal{C}$ according to its channel quality (m_k). The support $\mathcal{S}^*_{\mathcal{C}}$ of the equilibrium can be obtained by an algorithm similar to Algorithm 1, and is given in Algorithm 2 with $\widetilde{E}(\mathcal{S})$ defined as in (5.31).

Algorithm 2 Computing the support of the symmetric mixed strategy Nash equilibrium given $\mathcal{C}$

Input: Set of available channels $\mathcal{C}$.
Output: Set of channels $\mathcal{S}^*_C$ in the support of the Nash equilibrium given $\mathcal{C}$.

$\mathcal{S}^*_C \leftarrow \varnothing, \mathcal{M} \leftarrow \mathcal{C}$
while $\mathcal{M} \neq \varnothing$ **do**
    Pick one channel $k^* \in \arg\max_{i \in \mathcal{M}} \exp\left(-\phi N_0 m_i^{-1}\right)$
    $\mathcal{S}^*_C \leftarrow \mathcal{S}^*_C \cup \{k^*\}$, $\mathcal{M} \leftarrow \mathcal{M} \setminus \{k^*\}$
    $\mathcal{M} \leftarrow \mathcal{M} \setminus \left\{k \in \mathcal{M} : \exp(-\phi N_0 m_k^{-1}) \leq \widetilde{E}(\mathcal{S}^*)\right\}$
end while
return $\mathcal{S}^*_C$

Fig. 5.3 gives the throughput versus user density of case with and without channel availability information. At low user density, the scheme when the availability of each channels can be obtained gives a better performance than that without such information, for this information avoids the occurrence of a channel that is not available being accessed. However, such information leads to a worse performance when the user density is high. This is because that when the user density is high and the users are not provided with the channel availability information, users that have chosen for access a channel that is not available are somehow "muted" and thus the active users in the channels that are available will be less dense. But when channel availability information are provided to each user, they will only choose the channels that are available for access, and thus the channels that are available will be more crowded. This gives a Braess-like paradox [41] that when more information is provided to a noncooperative network, the performance however degrades.

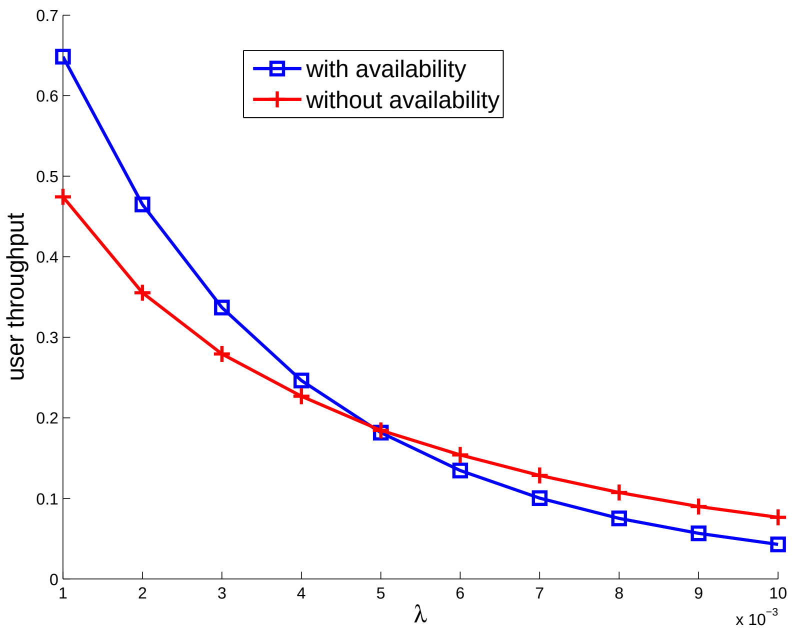


Figure 5.5: Throughput versus user density for the case with and without channel availability information. The network parameters are set as $\alpha = 4$, $R = 2$ bits/ s/Hz, $r = 13$m, $P = 1$, and $N_0 = 10^{-6}$. There are $K = 9$ channels with $\Theta = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]$.

Chapter 6

Advanced Mechanisms

6.1 Access barring for performance stabilization

As was shown in Chapter 4, it is socially optimal in a system perspective that users reduce their access attempts (i.e. not accessing any channel with a nonzero probability) when the network user density λ is above a threshold $\widetilde{\lambda}$. However, the selfish nature of users makes them always choosing a channel when they attempt to access. This causes a substantial degradation of system throughput when the user density grows due to excessive access attempts which leads to a high level of interference in the network.

The performance gap between the game-theoretic solution (i.e. Nash equilibrium) and the centralized optimal solution can be eliminated by schemes of pricing as in [42, 54] and is treated in the context of multi-channel random access in [1]. Here, on the other hand, we introduce a simple user access baring scheme that eliminates the performance gap and keeps the system throughput at an optimal level even at high user densities.

Consider the scenario when no channel side information is provided to the transmitters, the channel availability statistics are i.i.d. with $\theta_k = 1$ for all $k \in \mathcal{K}$, and the network is interference-limited ($N_0 = 0$) for simplicity. The performance of the system can be further improved by performing access barring in the time domain where in each slot a user are allowed to perform channel access with probability $\frac{1}{T}$, which reduces the access attempts of users by a factor of $\frac{1}{T}$ mandatorily.

Equivalently, with access barring with parameter T, each user can only perform channel access once every T time slots[1]. Although this results in a loss of transmission opportunity by $\frac{1}{T}$ for a user, the density of interferers seen by a user for each access is also reduced by the same factor which leads to a lower level of interference encountered by each user, indicating a nontrivial tradeoff.

By the thinning property of PPP, the set of transmitting users in a slot becomes a PPP with density $\frac{\lambda}{T}$. Since the users still have the freedom in making channel selections and we assume that the users are only mandated to follow the rule for reducing access attempts, the users accessing a specific time slot still face a the same game-theoretic channel selection problem as described in Section 4.1 but now with the set of active users $\Psi = \Pi(\frac{\lambda}{T})$ in a time slot. Recall that without access barring (i.e. the original case, when $T = 1$), the throughput of a user is $\mathcal{T} = \exp\left(-\frac{\rho}{\sum_{k\in\mathcal{K}} m_k^\delta}\right)$ as given by (4.23) in Corollary 2. Now, we the throughput of a user with access barring with parameter T becomes

$$\widehat{\mathcal{T}}(T) \triangleq \frac{1}{T}\exp\left(-\frac{1}{T}\frac{\rho}{\sum_{k\in\mathcal{K}} m_k^\delta}\right) \tag{6.1}$$

where the $\frac{1}{T}$ outside the exponential is due to the loss of transmission opportunity and by noting that $\rho \triangleq \lambda\pi r^2(2^R - 1)^\delta\Gamma(1+\delta)\Gamma(1-\delta)$ defined in (3.9) captures the effect of user density with λ, the $\frac{1}{T}$ inside the exponential is due to the alleviated density of simultaneous transmissions.

As mentioned earlier about the tradeoff, we aim to find the optimal number of time slots T^* for access barring that will maximize the system throughput, which solves the following optimization problem.

$$\max_{T\geq 1} \quad \lambda\widehat{\mathcal{T}}(T) \tag{6.2}$$

The results are given in the following theorem.

[1]Note that the number of slots T here can be any real number with $T \geq 1$. A fraction T say 1.5 can be thought of that a user transmits in every 3 slots.

Theorem 9. *The optimal number of time slots T^* that maximizes the throughput of a user with access barring is*

$$T^* = \begin{cases} 1, & \rho < \sum_{i\in\mathcal{K}} m_i^\delta \\ \frac{\rho}{\sum_{i\in\mathcal{K}} m_i^\delta}, & \rho \geq \sum_{i\in\mathcal{K}} m_i^\delta \end{cases} \tag{6.3}$$

with ρ given by (3.9)*. The throughput of a user is be given by*

$$\widehat{\mathcal{T}}(T^*) = \begin{cases} \exp\left(-\frac{\rho}{\sum_{i\in\mathcal{K}} m_i^\delta}\right), & \rho < \sum_{i\in\mathcal{K}} m_i^\delta \\ \frac{\sum_{i\in\mathcal{K}} m_i^\delta}{e\rho}, & \rho \geq \sum_{i\in\mathcal{K}} m_i^\delta \end{cases} \tag{6.4}$$

and the system throughput is given by

$$\lambda\widehat{\mathcal{T}}(T^*) = \begin{cases} \lambda\exp\left(-\frac{\rho}{\sum_{i\in\mathcal{K}} m_i^\delta}\right), & \rho < \sum_{i\in\mathcal{K}} m_i^\delta \\ \frac{\sum_{i\in\mathcal{K}} m_i^\delta}{e\pi r^2(2^R-1)^\delta\Gamma(1+\delta)\Gamma(1-\delta)}, & \rho \geq \sum_{i\in\mathcal{K}} m_i^\delta \end{cases} \tag{6.5}$$

Proof. It can be easily seen that the objective function $\lambda\widehat{\mathcal{T}}(T)$ is quasiconcave for $T > 0$ with only one critical point $\widetilde{T}^*$, which is the global maximizer [55]. Since the optimization problem is with variable T, the optimizer can be found by taking the derivative of $\widehat{\mathcal{T}}(T)$ with respect to T and equating to zero, where we temporarily ignored the constraint on T and have

$$\begin{aligned} &\left.\frac{\partial\widehat{\mathcal{T}}(T)}{\partial T}\right|_{T=\widetilde{T}^*} \\ &= \left.\left\{\left[-\frac{1}{T^2} + \frac{1}{T}\left(-\frac{1}{T^2}\right)\left(-\frac{\rho}{\sum_{i\in\mathcal{K}} m_i^\delta}\right)\right]\exp(-\frac{1}{T}\frac{\rho}{\sum_{i\in\mathcal{K}} m_i^\delta})\right\}\right|_{T=\widetilde{T}^*} \\ &= \quad 0 \end{aligned} \tag{6.6}$$

Thus, the global maximizer is

$$\widetilde{T}^* = \frac{\rho}{\sum_{i\in\mathcal{K}} m_i^\delta} \tag{6.7}$$

By noting that the number of time slots T should be a greater than one, we would have the optimal number of time slots as

$$T^* = \max\left\{1, \frac{\rho}{\sum_{i\in\mathcal{K}} m_i^\delta}\right\} \tag{6.8}$$

The user throughput follows directly by plugging T^* in to $\widehat{\mathcal{T}}(T)$ and the system throughput is then obtained as $\lambda\widehat{\mathcal{T}}(T^*)$. □

Theorem 9 gives the optimal number of time slots T^* for access barring, and also the corresponding user throughput and system throughput. By noting the definition of ρ as given by (3.9), we can divide the solution into two parts as before with the same threshold $\widetilde{\lambda}$ as given by (4.27) in Section 4.3.1. For $\lambda < \widetilde{\lambda}$, the users can employ full access attempt, and the game-theoretic solution is already socially optimal in this case so the barring parameter T^* is equal to 1. But for $\lambda \geq \widetilde{\lambda}$, access barring brings down the access attempts of users so that the interference in the network will not keep growing with the user density λ. In fact, the density of transmitting users with access barring is kept at $\frac{\lambda}{T^*} = \widetilde{\lambda}$ for all $\lambda \geq \widetilde{\lambda}$ where the channels are best utilized, and the system throughput is kept at a constant with respect to user density λ.

Fig. 6.1 gives the user throughput versus user density λ for cases with and without access barring. At density $\lambda < \widetilde{\lambda}$, the user throughput with and without access barring are the same. When the user density λ reaches above $\widetilde{\lambda}$, it is optimal to lower the access attempts of users so that the interference in the network can be lowered, and thus with access barring the network can provide a better throughput performance for each user. Fig. 6.2 gives the system throughput versus user density for cases with and without access barring. We can see that without access barring, the system throughput degrades with λ after $\widetilde{\lambda}$; with access barring, the system throughput is actually kept at an optimum even when the density of users is high, which shows a great benefit of access barring. However, it should be noted that the throughput of a user will still degrade due to high user density.

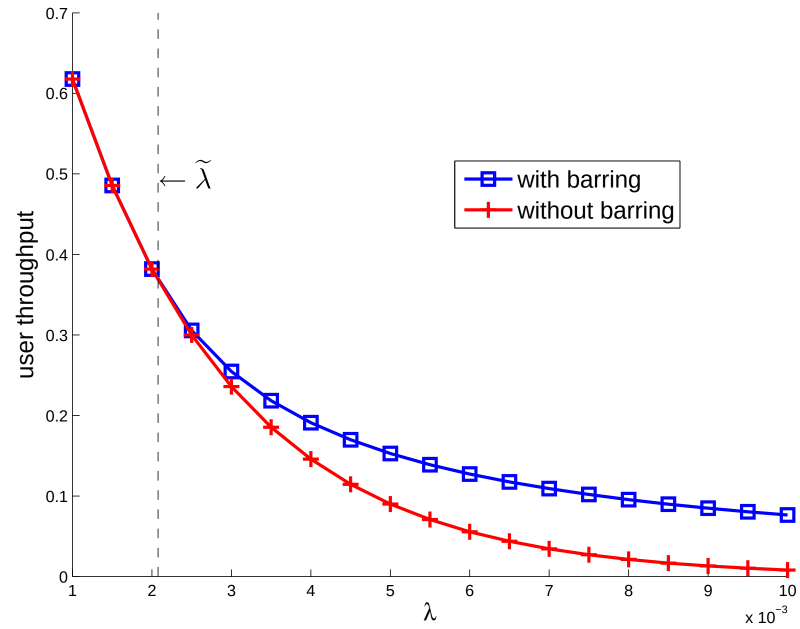


Figure 6.1: User throughput versus user density with and without access barring. The network parameters are set as $\alpha = 4$, $R = 2$ bits/s/Hz, $r = 13$m, and $P = 1$. There are $K = 3$ channels with $\mathbf{m} = [1, 1, 1]$.

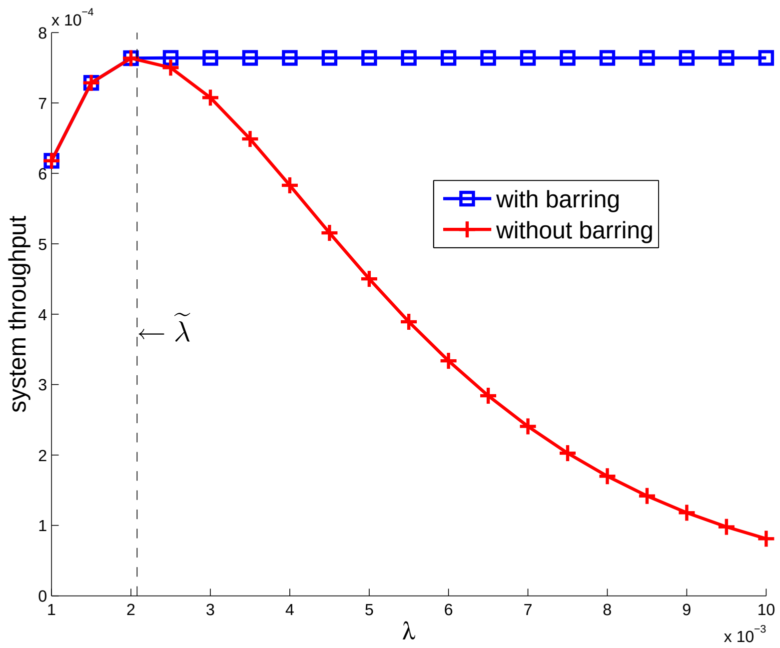


Figure 6.2: System throughput versus user density with and without access barring. The network parameters are set to the same as in Fig. 6.1.

It should be noted that the optimal barring parameter T^* is proportional to $\lambda \pi r^2$ for $\lambda \geq \widetilde{\lambda}$, which is the average number of users in the disc formed by a transmitter-receiver pair. A connection to the case with traditional collision model analysis can be found by considering the limiting case when $\alpha \to \infty$. Under such a scenario, transmission by a typical user in channel k is successful only when there are no other user transmitting in the same channel within the disc of radius r centered at the desired receiver of the typical user. The access probability of channel k becomes $p_k^* \to \frac{1}{K}$ as given by (4.22) and the access attempt of a user becomes $\frac{1}{T^*} \to \frac{K}{\lambda \pi r^2}$. Let $N = \frac{\lambda \pi r^2}{K}$ denote the expected number of users in the disc that select a specific channel. Now, we can obtain the mean number of attempted transmission per slot in a specific channel within any disc of radius r as $N \cdot \frac{1}{T^*} = 1$. This is in fact the the desired operating point of traditional slotted ALOHA with star topology and users attempts to access a common receiver [7], and it shows that the access barring scheme has the same effect as stabilization, which also controls the access attempts of users. However, as mentioned earlier that most previous studies considers collision models but ours consider the spatial context of wireless networks.

6.2 Devices with multi-channel transmission capability

When the availability of each channels could be known and a user device has the capability of taking transmission simultaneously over multiple channels, it is possible to take the advantage of the frequency diversity over multiple available channels. We assume that the users know the set of available channels $\mathcal{C}$ at the beginning of each slot, and the transmitter transmit the same information over the channels in $\mathcal{C}$. The receiver adds up the signals from the channels in $\mathcal{C}$ and then performs decoding. This is similar to performing Maximal Ratio Combining (MRC) with a diversity in the frequency domain. Assume the fading statistics over different channels are i.i.d. Rayleigh faded with unit average power ($m_k = 1$) for simplicity. Successful decoding at the receiver with combining happens when

$$\log_2\left(1 + \frac{\left(\sum_{k\in\mathcal{C}} H_{S,k}\right) Pr^{-\alpha}}{N_0 + I}\right) \geq R \tag{6.9}$$

where the equivalent fading gain $H_{\mathcal{C}} \triangleq \sum_{k\in\mathcal{C}} H_{S,k}$ of the signal is the sum of $|\mathcal{C}|$ exponential random variables and the equivalent fading gain for the interference remains the same [23]. This is because that the fading coefficients of the desired signal are combined coherently over different channels, while those for interference signal are not . Thus, we have $I = \sum_{X_i\in\Pi(\lambda)} H_{i0} P|X_i|^{-\alpha}$ where H_{i0} is an exponential random variable with unit mean. The throughput of a user after coherently combining signals of the channels in $|\mathcal{C}|$ can be computed as

$$\mathcal{T}_{\mathcal{C}} = \mathsf{Pr}\left(\log_2(1 + \frac{H_{\mathcal{C}} Pr^{-\alpha}}{N_0 + I}) \geq R\right) \tag{6.10}$$

The results are given in the following theorem.

Theorem 10. *When the users in the network with i.i.d. fading ($m_k = 1$) transmit in the set of available channels $\mathcal{C}$ and the receivers perform Maximal Ratio Combining* *of the signals in $\mathcal{C}$, the throughput of a user is*

$$\mathcal{T} = \sum_{\mathcal{C} \subseteq \mathcal{K}} \Upsilon_{\mathcal{C}} \mathcal{T}_{\mathcal{C}} \tag{6.11}$$

where $\Upsilon_{\mathcal{C}}$ is given in (5.28)*, and*

$$\mathcal{T}_{C} = \sum_{k=0}^{|\mathcal{C}|-1} \frac{(-1)^k \varphi^{(k)}(1)}{k!} \tag{6.12}$$

in which $\varphi^{(k)}(c)$ is the kth derivative of $\varphi(t)$ evaluated at c where

$$\varphi(t) \triangleq \exp(-\phi N_0 t) \exp(-\rho t^{\delta}) \tag{6.13}$$

with ρ given by (3.9)*. When the network is in the interference-limited regime ($N_0 =$ 0), the throughput with available channel set $\mathcal{C}$ can be further obtained as*

$$\mathcal{T}_{C} = e^{-\rho} \left(1 + \sum_{k=1}^{|\mathcal{C}|-1} \frac{(-1)^k}{k!} \sum_{n=1}^{k} \frac{\beta_{n,k}}{n!} \rho^n \right) \tag{6.14}$$

where $\beta_{n,k} \triangleq \sum_{m=1}^{n} (-1)^m \binom{n}{m} (\delta m)_k$ and $(\delta m)_k \triangleq (\delta m) \cdots (\delta m - k + 1)$ is the falling factorial.

Proof. We have

$$\begin{aligned}
\mathcal{T}_{\mathcal{C}} &= \mathsf{Pr}\left(H_{\mathcal{C}} \geq \phi(N_0 + I)\right) \\
&\stackrel{(a)}{=} \mathsf{E}\left[\int_{-\infty}^{\infty} \exp(-\phi(N_0 + I)t) \widetilde{h}_{\mathcal{C}}(t) dt\right] \\
&= \int_{-\infty}^{\infty} \exp(-\phi N_0 t) \mathsf{E}[\exp(-\phi I t)] \widetilde{h}_{\mathcal{C}}(t) dt \\
&\stackrel{(b)}{=} \int_{-\infty}^{\infty} \exp(-\phi N_0 t) \exp(-\lambda \pi r^2 (2^R - 1)^{\delta} \mathsf{E}[H_{i0}^{\delta}] \Gamma(1-\delta) t^{\delta}) \widetilde{h}_{C}(t) dt
\end{aligned} \tag{6.15}$$

where (a) follows by using $\mathsf{Pr}\left(H_{\mathcal{C}} \geq s\right) = \int_{-\infty}^{\infty} e^{-st} \widetilde{h}_{\mathcal{C}}(t) dt$ and $\widetilde{h}_{\mathcal{C}}(t) = \mathcal{L}^{-1}\{\overline{F}_{H_{\mathcal{C}}}(s)\}$ is the inverse Laplace transform of the ccdf $\overline{F}_{H_{\mathcal{C}}}(s)$ $H_{\mathcal{C}}$. (b) follows by using the

Laplace transform of I that

$$\mathsf{E}[e^{-sI}] = \exp(\lambda\pi P^{\delta}\mathsf{E}[H_{i0}]\Gamma(1-\delta)s^{\delta}) \tag{6.16}$$

Since each link in each channel is Rayleigh faded with unit average power, H_{i0} is exponentially distributed with unit mean. We have $\mathsf{E}[H_{i0}] = \Gamma(1+\delta)$. The random variable $H_{\mathcal{C}}$ is gamma-distributed with shape $|\mathcal{C}|$ and rate 1, and its ccdf is

$$\begin{aligned}\overline{F}_{H_{\mathcal{C}}}(s) &= 1 - \frac{1}{\Gamma(|\mathcal{C}|)}\gamma(|\mathcal{C}|, s) \\ &= e^{-s}\sum_{k=0}^{|\mathcal{C}|-1}\frac{s^k}{k!} \end{aligned} \tag{6.17}$$

where $\gamma(k,s) = \int_0^s t^{k-1}e^{-t}dt$ is the lower incomplete gamma function, and note that when k is a positive integer, we have $\Gamma(k) = (k-1)!$ and $\gamma(k,s) = (k-1)!e^{-s}\sum_{n=0}^{k-1}\frac{s^n}{n!}$. The inverse Laplace transform of $\overline{F}_{H_{\mathcal{C}}}(s)$ can then be obtained as

$$\widetilde{h}_{\mathcal{C}}(t) = \sum_{k=0}^{|\mathcal{C}|-1}\frac{\delta^{(k)}(t-1)}{k!} \tag{6.18}$$

where $\delta^{(k)}(t)$ is the kth derivative of the Dirac delta function. Thus, we have

$$\begin{aligned}\mathcal{T}_{\mathcal{C}} &= \int_{-\infty}^{\infty}\exp(-\phi N_0 t)\exp(-\lambda\pi r^2(2^R-1)^{\delta}\Gamma(1+\delta)\Gamma(1-\delta)t^{\delta})\sum_{k=0}^{|\mathcal{C}|-1}\frac{\delta^{(k)}(t-1)}{k!}dt \\ &= \sum_{k=0}^{|\mathcal{C}|-1}\frac{(-1)^k\varphi^{(k)}(1)}{k!} \end{aligned} \tag{6.19}$$

where in the second equation we have used the identity $\int_{-\infty}^{\infty}\delta^{(k)}(t-c)\varphi(t)dt = (-1)^k\varphi^{(k)}(c)$ with the test function

$$\varphi(t) \triangleq \exp(-\phi N_0 t)\exp(-\rho t^{\delta}) \tag{6.20}$$

where ρ is given by (3.9) and $\varphi^{(k)}(c)$ is the kth derivative of $\varphi(t)$ evaluated at c.

The throughput averaged over all possible realizations of channel availability $\mathcal{C}$ is

$$\mathcal{T} = \sum_{\mathcal{C} \subseteq \mathcal{K}} \Upsilon_{\mathcal{C}} \mathcal{T}_{\mathcal{C}} \tag{6.21}$$

with $\Upsilon_{\mathcal{C}}$ given by (5.28).

In the interference-limited regime ($N_0 = 0$), the kth derivative of $\varphi(t)$ for $k \geq 1$ can be obtained using similar techniques as identified in [56] as

$$\varphi^{(k)}(t) = t^{-k} e^{-\rho t^{\delta}} \sum_{n=1}^{k} \frac{\beta_{n,k}}{n!} (\rho t^{\delta})^{n} \tag{6.22}$$

where $\beta_{n,k} \triangleq \sum_{m=1}^{n} (-1)^m \binom{n}{m} (\delta m)_k$ and $(\delta m)_k \triangleq (\delta m) \cdots (\delta m - k + 1)$ is the falling factorial and $\delta = \frac{2}{\alpha}$. By plugging back to the expression for $\mathcal{T}_{\mathcal{C}}$, we have (6.14). $\square$

Fig. 6.3 gives the throughput versus user density of the case with and without the capability of transmitting over multiple channels, denoted by "with multi-ch tx" and "without multi-ch tx", respectively. At low density, the throughput performance with multi-channel transmission is better for that frequency diversity of multiple channels are utilized. However, when the user density is high, the gain provided by frequency diversity would be outplayed by the high interference in every available channels. On the other hand, when a user only transmit in one of its available channels, the interference on each channel is dispersed, thus giving a better performance at high user density.

We have shown that network of devices with simultaneous multi-channel transmission capability can improve throughput performance by MRC with frequency diversity. However, it should be noted that the gain is only seen in an environment with small density of users. Otherwise, when the density is high, this capability will actually degrade the performance for it cause a higher interference in the network.

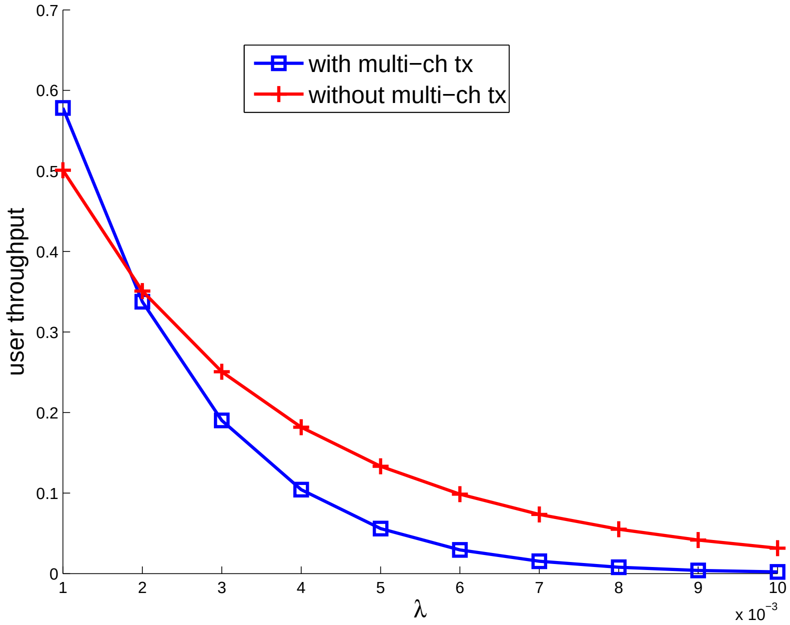


Figure 6.3: Throughput versus user density with and without the capability of transmitting over multiple channels, denoted by "with multi-ch tx" and "without multi-ch tx", respectively. The network parameters are set as $\alpha = 4$, $R = 2$ bits/s/Hz, $r = 10$m, $P = 1$, and $N_0 = 10^{-5}$. There are $K = 5$ channels with $\Theta = [0.1, 0.3, 0.5, 0.7, 0.9]$.

Chapter 7

Conclusion

In this thesis, we studied multi-channel random access in wireless ad hoc networks with the help of tools from stochastic geometry and game theory. With stochastic geometry, the spatial aspects of wireless ad hoc network considering node distribution, channel fading, and path loss are taken into account more realistically than traditional graph models of networks, and the interaction between and the behavior of users that distributively and selfishly make channel access decisions are captured with a game-theoretic model, where suitable access strategy at the equilibrium and the corresponding performance in terms of user throughput are obtained. The socially optimal solution when the users are cooperative in making channel access decisions is also studied, and the results are compared with that when the game-theoretic scheme is used. We found that selfishness is actually socially optimal in some network scenarios, and the conditions for optimality is derived.

We also studied the impact of channel side information to the access strategies and corresponding performance when they are provided to the users in the network. Specifically, when CSI is available at the transmitter, channel diversity can be exploited and the throughput performance of users can be greatly improved. On the other hand, when channel availability information is provided to users as in the case of CRN with multi-channel sensing capability, a Braess-like paradox is found that the performance degrades when more information is provided. Mechanisms that are able to further improve the performance of the network are also studied.

By regulating in the time domain the access of users with access barring, the interference problem between users with high user density can be alleviated. When the transmitters are capable of performing multi-channel transmission, MRC with frequency diversity can improve throughput performance when the user density is not high. The proposed framework gives some insights for multi-channel random access in wireless ad hoc networks, and facilitates the understanding of design and analysis of such networks.

Appendix A

Table of important notations

Table A.1: Important notations used in this thesis

notation	description
λ	Density of users in the network
α	Path loss exponent
P	Transmission power
N_0	Background noise power
r	The distance between a transmitter and the desired receiver
K	Number of orthogonal frequency channels
$\mathcal{K}$	Set of K orthogonal frequency channels
$\mathcal{S}^*$	The support of a Nash equilibrium
R	Information rate of a transmitter and receiver pair
$\Pi(\lambda)$	Poisson point process with density λ
$\Psi = \{X_i\}$	Set of locations of transmitters in the network
Ψ_k	Set of locations of transmitters transmitting in channel k
A_k	Indicator for availability of channel k
θ_k	Available probability of channel k
$\mathcal{C}$	Set of channels that are available in a time slot
$\Upsilon_{\mathcal{C}}$	Probability of occurrence with set of available channels $\mathcal{C}$
$m_{S,k}$, m_k	Average power of fading over desired signal in channel k
$m_{I,k}$	Average power of fading over interference link in channel k
p_k	Channel selection probability for channel k of a strategy
q_k	The probability that a user will select channel k
$\mathcal{T}_k$	Throughput of a user in channel k
$\mathcal{T}$	Throughput of a user averaged over its channel selection strategy
$\delta(\cdot)$	Dirac delta function
$\Gamma(\cdot)$	Gamma function
$\gamma(\cdot,\cdot)$	Lower incomplete gamma function
$W(\cdot)$	Lambert W function
$\mathrm{erfc}(\cdot)$	Complementary error function
δ	$\frac{2}{\alpha}$
ϕ	$(2^R - 1)P^{-1}r^{\alpha}$
ρ	$\lambda\pi r^2(2^R-1)^{\delta}\Gamma(1+\delta)\Gamma(1-\delta)$
κ	$\frac{1}{4}\lambda\pi^2 r^2(2^R-1)^{\frac{1}{2}}$

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