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不定均衡在具生產外部性之兩部門模型應用 - 以 Benhabib and Farmer (1994) 爲基礎

Indeterminacy in a Two-sector Model with Factors of Production Externalities - Based on Benhabib and Farmer (1994)

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摘要

本篇論文主要係探討在不定均衡於具有規模報酬遞增生產特性之兩部門實質景氣循環模型中之動態性質。全文以Benhabib and Farmer (1994)之模型架構為基礎, 並將傳統休閒財定義為需經勞力及資本投入生產過程而取得, 試圖構建合理模型參數範圍以出現不定均衡之現象。本文主要發現為於傳統參數範圍設定下, 本模型設定並不會出現不定均衡之結果。更由甚者, 由上揭結果暗示傳統文獻中不定均衡易於兩部門模型架構下呈現之結論, 容有討論及證實之空間。

關鍵字*:*不定均衡*;* 生產外部性*;* 預期自我實現*;* 太陽黑子均衡*;* 模型校正

Abstract

In this paper I present a two-sector real business cycle (RBC) model where one sector has a production function with increasing return-to-scale occurring as a consequence of production externalities. I also redefine leisure as production output from both labor and capital input. My findings show that indeterminacy would not exist in this specific way of setting the model. Moreover, my results suggest that the common notion where a two-sector model would be more prone to generating indeterminacy than a one-sector model would not hold for some special settings.

Keywords: *Indeterminacy; Production externalities; Self-fulfilling expectations;Sunspot equilibrium; Calibration*

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1 Introduction

Since the real business cycle (RBC) model was first introduced in Kydland and Prescott (1982), many extended RBC models have been developed[1]. Today, the research scope has already been extended from the simple representative agent and perfect competitive equilibrium to imperfect competitive market and multiple equilibrium models. This study would mainly focus on models with multiple equilibrium paths (local indeterminacy[2]).

These models are mostly relevant to increasing returns to scale, imperfect competition or some specific habits of consumption through which sunspot equilibrium would be generated. The major concern toward indeterminacy is that models with sunspot equilibrium can fit the data better than the standard RBC models; furthermore, these models may provide a criterion for determining the policy which brings the maximum welfare. The reason why these models can provide a better match for the data is that they can catch some fluctuations supported not only by exogenous technical shocks but also by endogenous impact resulting from self-fulfilment of expectations. For instance, the mechanism of how indeterminacy works in an increasing return to scale model starts from a common belief[3] that there should be higher return in investment. The agents would accordingly divert their consumption to investment. Next, in the presence of increasing return to scale, over-accumulating investment does increase its rate of return. However, to reach equilibrium, another belief that the economy eventually declines to its steady state value which is determined by underlying fundamentals would gradually grow. This would force the agnets to choose investment and consumption at the original equilibrium level[4]. Secondly, if the equilibrium of the model with indeterminacy can be Pareto-ranked, it then could help us decide which equilibrium is more beneficial , thus these models might guide the agents to approach better equilibrium. On the contrary, there are also constraints in such models as the plausible assumption that all agents response identically to changes in random exogenous variables.

As the literature has developed, Benhabib and Farmer (1994) may be considered as one of the pioneering research in this field. They used a one-sector model with production externalities and monopolistic competition to calibrate the magnitude of

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[1]See Williamson (1996) and more formal details are in Cooley (1995).

[2]This term is for comparing with global indeterminacy whose model has more than one steady state, such as Galor (1996).

[3]Note that this belief needs not be incorrect to create fluctuation as traditional model.

[4]See more in Benhabib and Farmer (1999).

increasing returns. There were two main findings in their analysis, one of which was that indeterminacy would appear only if the production externalities were large enough (over than 1.43). They also derived a necessary and sufficient condition for indeterminacy under which the labor demand curve must be upward-sloping and it should be steeper than the labor supply curve. Nonetheless, their research suffered from two major limitations. The first one was that such a high degree of production externalities was rarely observable in real economic environments. Some earlier estimates of production externalities, i.e. Hall (1988), Hall (1991), Caballero and Lyons (1992), and Baxter and King (1991), offered a relatively larger range of magnitudes of production externalities. However, subsequent research by Basu and Fernald (1995), Basu and Fernald (1997), and Burnside (1996) criticized the estimation methodology adopted by previous studies and turned to use disaggregated U.S. data to estimate the level of increasing returns, and then found that the magnitude was pretty small[5]. The second limitation happened with the necessary condition under which an aggregate labor demand curve was upward-sloping. This condition implied that the higher wage rate led firms to hire more labor; apparently, this relation was inconsistent with normal economic intuition.

Based on the above facts, much recent research on indeterminate equilibrium has focused on how to reduce the magnitudes of increasing returns (production externalities) in such models. Wen (1998) retained the one-sector model structure, but his capital depreciation rate depended on the rate of capacity utilization. Wen (1998) took advantage of this mechanism to amplify the technology shock so as to imitate increasing returns, and he found that the threshold level of production externalities would be 1.144. Moreover, some studies adopted a two-sector model structure and all their results indicated that indeterminacy is much easier to obtain in two-sector models than in one-sector models. Benhabib and Farmer (1996) used a model with sector-specific externalities that produced indeterminacy with even lower magnitudes of production externalities (around 1.07). Later, Harrison (2001) relaxed the restriction set in Benhabib and Farmer (1996) on the identical sector-specific externalities. He further replaced the utility function with logarithm form in consumption with the constant risk-averse utility function. Harrison found that indeterminacy could appear with a minimum externality existing uniquely in the investment sector and with the zero externality in the consumption sector. Furthermore, when the parameter for the relative

[5]The best estimate among these studies was 1.03; namely, the parameters corresponding to our model θ equals to 0.03

risk aversion increased, the necessary level of production externalities also increased. Some research such as Benhabib and Nishimura (1998) then used a three-sector model to evaluate the threshold level of production externalities for indeterminacy; the minimum level was, as expected, further reduced.

A major purpose of this study is to find a model that can obtain indeterminacy under empirically plausible parametrizations. Here I still follow the basic structure in Benhabib and Farmer (1994); however, to amend the flaw of Benhabib and Farmer (1994), I set the aggregate labor supply to be one unit constantly; in other words, the representative agent is available only in deciding the ratio of labor input between two sectors. This setting ensures that the slope of the labor demand curve would not exceed the slope of the labor supply curve in every sector. In addition, how this study introduces another sector is by redefinition of the leisure term. This reconstruction came mainly from the idea of home production in Perli (1998) which I found in accordance with intuitions. One reason is that the time when the agents did not work would definitely not be used for leisure; in fact, the agents might embark on some productions beneficial to raise the utility when being at leisure. Also, according to Rupert et al. (1994) and Eisner (1988), the estimated size of the home production sector may range from 20% to 50% of U.S. GNP. This statistics imply that the home sector did play an important role to some extent in aggregate production, though most studies usually ignore this sector. These two arguments may justify our redefinition of the leisure in the work of Benhabib and Farmer (1994).

There are two findings in this paper: One is that we can not generate indeterminacy in our model with a two-sector structure under the reasonable parametrizations. The other is that the claim of indeterminacy by Benhabib and Farmer (1994) would not exist when we introduced the home production sector to that model; in other words, the conclusion drawn by Benhabib and Farmer (1994) might not be robust in the presence of another sector. The main results surprisingly contradict previous literature. The cause of these results may relate to the adjustment mechanism of labor supply in my model.

The paper is structured as follows. Section 2 presents the theoretical model structure. Calibration results of production externalities are presented in Section 3. Section 4 contains a discussion and then section 5 concludes. The appendix includes the derivations of each condition.

2 Model

The economy that I construct in this paper basically follows Benhabib and Farmer (1994) , i.e. both production functions follow a Cobb-Douglas setting and the utility function is separable and in continuous-time form. However, Benhabib and Farmer (1994) only has one production sector, but my model contains two production sectors which can simply be imagined as a capital-intensive and a labor-intensive sector. And then, for each production factor, capital and labor inputs are allocated among the two sectors by some ratio decided by first order conditions. The aggregate labor supply equals one unit for every period. The major difference between the original model and my model is that mine has production externality only in the sector referred to as the capital-intensive sector, the other sector has not. The model can be described as follows.

2.1 Firms

Let k be the aggregate stock of capital, s be the ratio of capital input allocation between two sectors, and n be the labor input in the capital-intensive sector. The production functions can be expressed as

$$\begin{aligned} y_1 &= A(k_1)^{\alpha} n^{1-\alpha} (\overline{k_1})^{\alpha\theta_1} (\overline{n})^{(1-\alpha)\theta_2} \\ &= A(sk)^{\alpha} n^{1-\alpha} (\overline{sk})^{\alpha\theta_1} (\overline{n})^{(1-\alpha)\theta_2} \qquad (2.1) \\ y_2 &= (k_2)^{\beta} (1-n)^{1-\beta} \\ &= [(1-s)k]^{\beta} (1-n)^{1-\beta} \qquad (2.2) \end{aligned}$$

where α and β are the capital share in each sector and θ_1 and θ_2 represent the magnitude of the production externalities for capital and labor input. When θ_1 and θ_2 both equal zero, there are no production externalities.

Notice that the ratio of capital input allocation is different from that of labor input allocation, unlike Benhabib and Farmer (1996). Since I assume that the production factors markets are perfectly competitive and there is no adjustment cost for production inputs between the two factors, the first order conditions can be described as follows:

$$\begin{aligned} r &= \frac{\alpha y_1}{k_1} = P\frac{\beta y_2}{k_2} \qquad (2.3) \\ w &= \frac{(1-\alpha)y_1}{n_1} = P\frac{(1-\beta)y_2}{n_2} \qquad (2.4) \end{aligned}$$

where r represents the rental rate of capital, w is the real wage rate and P is the relative price of y_1 and y_2.

2.2 Households

There is an assumption that infinity-lived representative agents maximize their discounted present value in the utility function separated in two goods. The maximizing problem is:

$$\max \int_0^\infty \left[\log c_1 + \varphi \frac{c_2^{1-\chi} - 1}{1-\chi} \right] e^{-\rho t} dt. \tag{2.5}$$

Here I replace the leisure term by consumption of the second sector(c_2) as one source of utility gain; furthermore, I assume that y_2 is a non-storable goods. This assumption implies that all production goods of y_2 will be consumed in the current period. That means c_2 must equal to y_2. The maximizing problem is subject to the law of motion for capital accumulation,

$$\dot{k} \;=\; y_1 - c_1 - \delta k \tag{2.6}$$

where δ is the depreciation rate of capital. Thus, the first order condition is:

$$-\frac{\dot{\lambda}}{\lambda} \;=\; \left[\alpha(\frac{y_1}{k_1}) - (\rho+\delta)\right] = \left[\alpha(\frac{y_1}{sk}) - (\rho+\delta)\right] \tag{2.7}$$

where λ is the multiplier of the Hamiltonian equation of the optimization problem. Furthermore, since $c_1 = \frac{1}{\lambda}$ is obtained from another first order condition, one could infer that

$$\dot{c_1} \;=\; c_1 \left[\alpha(\frac{y_1}{sk}) - (\rho+\delta)\right]. \tag{2.8}$$

Besides, the transversality condition satisfies, $\lim_{t\to\infty} e^{-\rho t} \frac{k}{c_1} = 0$. After rearrangement of the equation(2.3), the relative price of the two goods is:

$$P \;=\; \varphi c_1 c_2^{-\chi} \;=\; \varphi \frac{U_{c_2}}{U_{c_1}} \;=\; \frac{MPK_{y_1}}{MPK_{y_2}} \;=\; \frac{MPL_{y_1}}{MPL_{y_2}} \tag{2.9}$$

which implies in equilibrium that the relation of $P = MRS$ would still hold even if the economy has production externalities. From Equation (2.3) and (2.4), the relation between the ratio of capital input allocation and the labor input for the capital-intensive sector under steady state can be obtained as follows:

$$\left(\frac{1-s}{s}\right) = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\beta}{1-\beta}\right)\left(\frac{1-n}{n}\right). \tag{2.10}$$

This condition would decide the equilibrium level of the labor input in the capital-intensive sector and the ratio of capital input allocation. Combining equation (2.3), (2.4) and (2.7), one can get another relation between the ratio of capital input allocation and the labor input for the capital-intensive sector as follows

$$\alpha = \left[\frac{\beta - \alpha\beta + \alpha n - \beta n}{n(1-\beta)}\right](s). \tag{2.11}$$

In the next section, Equation (2.10) and (2.11) will be used to calculate the steady state value for $\{s, n\}$.

2.3 Equilibrium and Local Dynamics

In this section I will discuss the properties of local dynamics of Equation (2.6) and (2.8) around the steady state. When the economy reaches the steady state, Equation (2.6) and (2.8) must equal to zero; hence, one can compute the following steady state ratio.

$$\frac{y_1}{k_1} = \left(\frac{\rho+\delta}{\alpha}\right) \tag{2.12}$$

$$\frac{c_1}{k_1} = \left[\frac{\delta(s-\alpha)+s\rho}{\alpha}\right] \tag{2.13}$$

Note that any equilibrium path must follow these two equations above and satisfy the transversality condition. Now I can compute the Jacobian Matrix of Equation (2.6) and (2.8) at the steady state, it would follow this following system:

$$\begin{bmatrix} \partial \dot{c}_1 \\ \partial \dot{k} \end{bmatrix} = J \begin{bmatrix} \partial c_1 \\ \partial k \end{bmatrix} \tag{2.14}$$

where J is the Jacobian Matrix of partial derivatives of the system above. (The computations of the elements of the Jacobian Matrix are in Appendix.) Once each element of the Jacobian Matrix are obtained, the trace and determinant at the steady state would be:

$$Tr = -\delta + s\left[\frac{y_1}{k_1}\right]\left\{[\alpha(1+\theta_1)] + \left[\frac{1+\phi_1}{\phi_1-\phi_2}\right]\left[\beta(1-\chi) - \alpha(1+\theta_1) + (\frac{\alpha}{s})\right]\right\} \quad (2.15)$$

$$Det = (\delta+\rho)\left[\frac{c_1}{k}\right]\left\{[\alpha(1+\theta_1)]\left[\frac{-1-\phi_2}{\phi_1-\phi_2}\right] + [1+\beta(1-\chi)]\left[\frac{1+\phi_1}{\phi_1-\phi_2}\right] - 1\right\} \quad (2.16)$$

where

$$\phi_1 = [(1-\alpha)(1+\theta_2) - 1]$$
$$\phi_2 = (\frac{-n}{1-n})[(1-\beta)(1-\chi) - 1]$$

Because there is only one state variable (k) in my model and the dimension of the Jacobian Matrix is two by two, the number of negative roots would decide how this model's dynamic properties exhibit. This model would demonstrate saddle-path stability if there exists one negative root and one positive root; on the contrary, the model would have no convergence path if there are two positive roots, which is also known as the unstable equilibrium (source). Please note that this situation still belongs to determinacy because, although this model does not have a convergence path, it comes to the steady state as long as the initial allocation of capital was exactly the same as in the equilibrium. Finally, indeterminate equilibrium appears only when two roots are both negative; that is, it will have a negative trace (the sum of two roots) and a positive determinant (the product of two roots).

2.4 Comparison with Benhabib and Farmer (1994)

Before I advance toward the calibration process, this subsection will display how my model can be reduced to the model of Benhabib and Farmer (1994). First of all, I re-transform our non-storable goods back to the normal "leisure" term; thus, the economy would become the original one with a unique production sector. Now there is no need for agents to choose their optimal ratio of capital allocation between two sectors; meaning that the capital now only be consumed for the first sector. Therefore, the ratio of capital allocation (s) would equal one, and the capital share of producing non-storable goods (β) must equal zero.

With this specific parametrization, the following demonstrates that the identical condition corresponded to Benhabib and Farmer (1994). First, the maximizing problem would become

$$\max \int_0^\infty \left[\log c + \varphi\frac{(1-n)^{1-\chi} - 1}{1-\chi}\right] e^{-\rho t} dt. \quad (2.17)$$

Similarly, the law of motion for capital accumulation and the other dynamic equation would be exactly consistent with the original dynamic equation system (see Equation (2.6) and (2.7)). Following the same procedure, one can obtain the elements of the Jacobian Matrix and its trace and determinant are as following[6]

$$Tr^D = \rho + \alpha[\frac{y}{k}]\theta_1 \left[\frac{-1-\phi_2^D}{\phi_1^D - \phi_2^D}\right] \tag{2.18}$$

$$Det^D = \alpha[\frac{y}{k}][\frac{c}{k}] \left\{[\alpha(1+\theta_1)-1]\left[\frac{-1-\phi_2^D}{\phi_1^D - \phi_2^D}\right]\right\} \tag{2.19}$$

Where

$$\phi_1^D = [(1-\alpha)(1+\theta_2)-1]$$
$$\phi_2^D = (\frac{-n}{1-n})[-\chi] = \frac{n\chi}{1-n}$$

Indeterminacy would appear under the circumstance that the trace is negative and the determinant is positive. After proper derivations(see details in Appendix), for producing indeterminacy, this following condition must be satisfied; that is,

$$\phi_1^D - \phi_2^D > 0. \tag{2.20}$$

Recall that the necessary and sufficient condition for indeterminacy in Benhabib and Farmer (1994) just represented as the difference of the slope between labor demand and supply curve[7]. In fact, here ϕ_1^D and ϕ_2^D also stand for the slope of labor demand and supply curve respectively. Besides, this condition exhibits a positive sign as in Benhabib and Farmer (1994).

[6]Here, $(\frac{y}{k})$ and $(\frac{c}{k})$ came from Equation (2.12) and (2.13) under the parametrization that $\beta = 0$ and $s = 1$. Also, since now the economy has been already reduced to a one-sector model structure, we remove the subscript of these ratio at steady state.

[7]The original necessary and sufficient condition was $(\beta - 1) - (-\chi) > 0$ whose notation for the slope of labor demand and supply curve were $(\beta - 1)$ and $-\chi$ respectively.

3 Results

3.1 Calibration of $\{\theta_1, \theta_2\}$

In order to see the conditions of an indeterminate equilibrium, this section first calibrates the baseline economy without production externalities under a specific set of parameters. Next, I turn to calibrate the threshold externalities level of each production input under the parametrization of the steady state. Last, it is followed by the comparison between previous papers' findings and my results.

As mentioned in the previous section, there are two different production sectors in my model. One of which is set as capital-intensive, and the other one is the opposite, labor-intensive. I abide by the setting in Ákos Valentinyi and Herrendorf (2008) and let the capital share in the capital-intensive sector, α in my model, be set at 0.32. On the other hand, the word, labor-intensive, here has a much broader meaning, it contains not only the manufacturing industry that produces daily necessities or some recreational facilities but also service industry such as restaurants, movie theaters, record shops, and some other entertainment services. Lee and Wolpin (2006) used the U.S. data collected by the Bureau of Economic Analysis and then estimated the capital and labor share in the service sector from 1968 to 2000. The average of their estimated value of the labor share during 1985 to 2000 was around 72%, which also fitted the real data extremely well.

Therefore, here I set the value of β equal to 0.28. The rest of parameters follows Benhabib and Farmer (1994) setting, i.e. the discount rate, ρ, was set at 0.065 per year, the elasticity of labor substitution was set at 2.0; hence, the corresponding parameter, χ, must equal 0.25. The depreciation rate, δ, equals 0.1 per year. Also, for simplicity, here is an assumption that φ and total factor productivity (A) both equal one. Table 1 are as follows:

Table 1: Parameters settings

Parameter	α	β	χ	ρ	δ	φ
Calibrated value	0.32	0.28	.25	0.065	0.1	1

Table 2: Steady State Value

Variable	n	s	$\frac{c_1}{k}$	$\frac{k_1}{y_1}$	$\frac{c_1}{y_1}$	$\frac{k}{y_1}$
Steady State	0.682	0.722	0.272	1.939	0.731	2.686

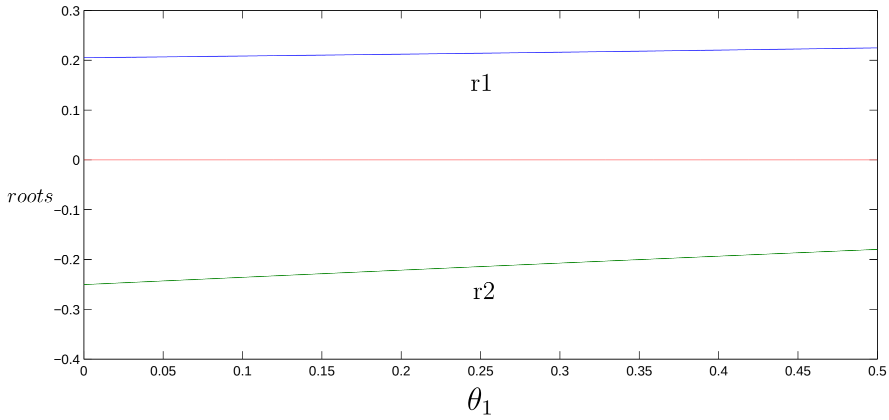

Figure 1: θ_1 v.s. roots under $\theta_2 = 0$

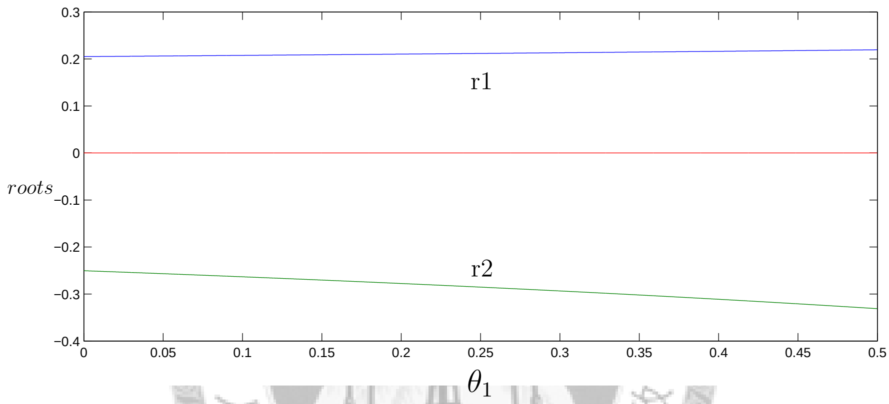

Figure 2: θ_2 v.s. roots under $\theta_1 = 0$

Plugging these parameters into first order conditions, from equation (2.10) and (2.11), first I compute the steady state value of the ratio of capital input allocation and the labor input for the capital-intensive sector, $s = 0.722$ and $n = 0.682$ respectively.

At the steady state, equation (2.7) and (2.6) must equal zero; consequently, this implies that the ratio of $\frac{y_1}{k_1}$ and $\frac{c_1}{k}$ should be some constant depending on the parameters, the ratio of capital input allocation and labor input in the capital-intensive sector. Table 2 indicates the computed steady state value of each variable. [8]

Finally, I plug these steady state values and parameters into equation (2.15) and (2.16) and use these two equations to do calibrations, the following are the calibrated results.

Figure 1 shows that the two roots' value of the Jacobian Matrix along with the production externalities of capital input (θ_1). The red curve is a base line where its

[8]Be cautious that $\delta\frac{k_1}{y_1} + \frac{c_1}{y_1} \neq 1$ because the capital in the law of motion is related to the aggregate capital input, instead of the capital input in the capital-intensive sector.

value equals zero and the blue and green curves indicate that each roots' associated with θ_1 respectively. As shown in Figure 1, the values of the two roots are of different sign, and this indicates that this model belongs to the saddle-path case in such setting. In addition, since the two roots both present a positive trend with an increasing value of θ_1, this pattern may suggest that this model eventually leads to the source case if the magnitude of production externalities of capital input continues to rise.

On the other hand, Figure 2 indicates the relation between the value of the two roots and the production externalities of labor input (θ_2). It is obvious that this model still belongs to the saddle-path case; however, the pattern of each root exhibits different direction with the increasing of the production externalities in labor input for the capital-intensive sector. Following this pattern, the difference between the value of the roots would increase and the two roots also remain in different sign as θ_2 rises.

From the calibrated result under the previous setting, it might be suggested that this model would not produce indeterminacy even if the economy exhibits such an unrealistic level of production externalities. As mentioned before, this model mainly follows the basic setting of Benhabib and Farmer (1994). Precisely to say, under specific values of parameters, this model would degenerate to the original model above. However, once I extend the original one-sector model to a two-sector model, the previous conclusions of Benhabib and Farmer (1994) may not hold as before; in other words, their statement might not be robust under different sets of parametrizations.

3.2 Duplication Result of Benhabib and Farmer (1994)

To examine the robustness of Benhabib and Farmer (1994)'s conclusion, I first use my model structure to reproduce their original result, and then testify whether their conclusion still holds or not when I make an extension of their model.

The setting of model parameters were consistent with Benhabib and Farmer (1994) which was displayed in the Table 3. Figure 3 is the duplicated results of Figure 3 in Benhabib and Farmer (1994). Before I increase the production externalities to the level at 0.4286, the sign of the two roots are different (i.e. The dynamic system remains in saddle until $\theta_1 = \theta_2 \simeq 0.4285$). Once the production externalities exceed this critical value, the two roots' real part both exhibit negative signs with the increasing of production externalities. If the production externalities continue to increase, two roots would contain imaginary parts and their real parts are still negative as described above and shown in Two stable complex.

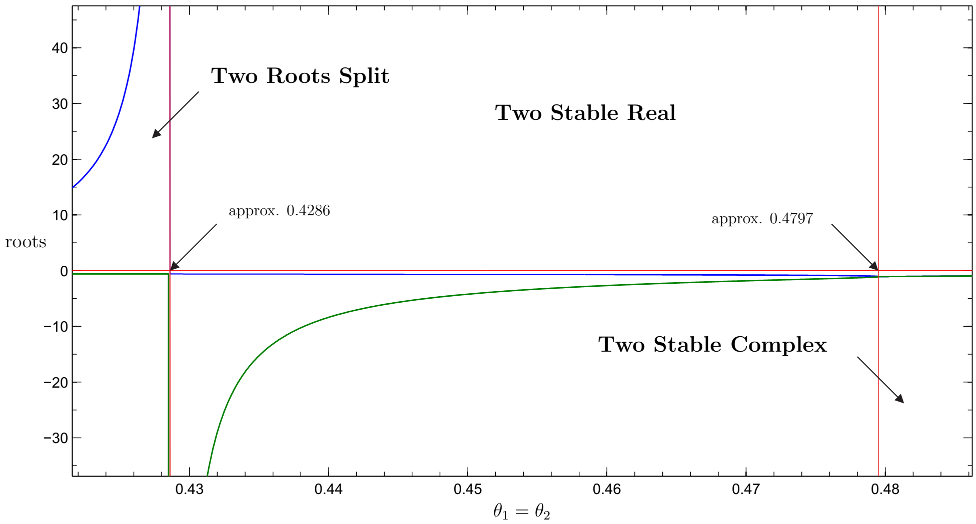

Figure 3: Duplication of Benhabib and Farmer (1994)

Table 3: Parameters settings for duplication

Parameter	α	β	χ	ρ	δ	φ
Calibrated value	0.30	0	0	0.065	0.1	1

3.3 Calibration of $\{\theta_1, \theta_2\}$ while $\beta \to 0$

As shown in the previous section, my model did duplicate the result of Benhabib and Farmer (1994). Next, this section is going to testify whether the model in Benhabib and Farmer (1994) was robust or not. If I let $\beta = 0$ and $s = 1$, then my model would transform back to the original model in Benhabib and Farmer (1994)(see Appendix D). Now let β be very close to zero and the rest of parameters remain unchanged, and then the calibration result would be shown in Figure 4. Table 4 indicates the steady state values under a new combination of parameters. Generally speaking, the patterns of the two roots shown in Figure 4 are very similar to the previous case. This outcome may suggest that there is still no indeterminate equilibrium in this model even if the model has $\beta \to 0$.

Also, note that the steady state value of labor input in the capital-intensive sector declines from 0.682 to 0.478; that is, the labor input in the labor-intensive sector rises to 0.522. Since the labor input in the labor-intensive sector $(1 - n)$, in fact, represents the leisure of consumers in normal RBC models if $\beta = 0$, this value is still quite far away from the common estimation from the data (approximately equal to 0.7).

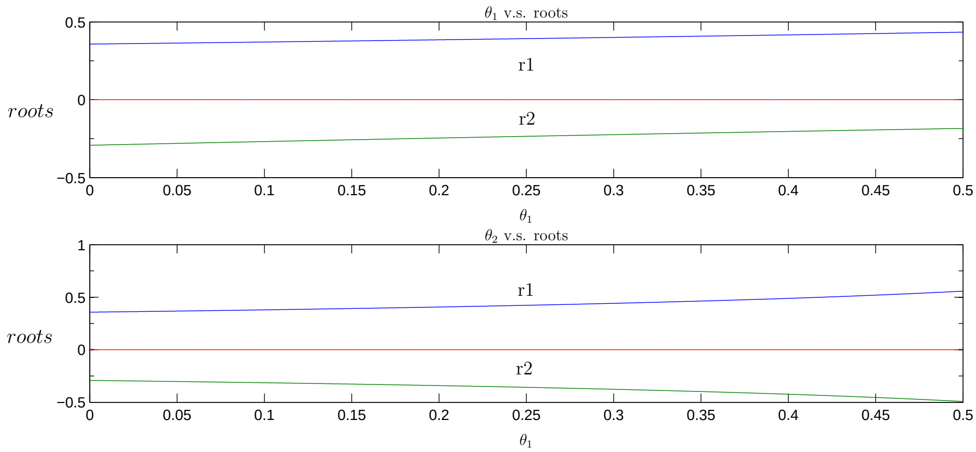

Figure 4: $\theta_1(\theta_2)$ v.s. roots under $\theta_2(\theta_1) = 0$ and $\beta \to 0$

Table 4: Steady State Value when $\beta \to 0*$

Variable	n	s	$\frac{c_1}{k}$	$\frac{k_1}{y_1}$	$\frac{c_1}{y_1}$	$\frac{k}{y_1}$
Steady State	0.478	0.999	0.416	1.939	0.806	1.939

* Here, we let β equal to 0.00001 as the approximated value for convergence to zero.

3.4 Calibration of χ and $\theta_1(\theta_2)$

According to the results above, one might conclude that this model could not generate indeterminacy under a traditionally plausible range of each parameter. Considering this situation, I depict the region of indeterminacy which is associated with the inverse of elasticity of labor supply in the capital-intensive sector (χ) and the production externalities (θ_1, θ_2)[9].

Figure 5 displays the region of indeterminacy in the $\chi - \theta$ space. The shaded area represents the numerical range of χ and θ within which indeterminacy would appear. One can observe that in a small amount production externality, indeterminacy can be produced as long as χ is high enough, say, around 7. Though with an increasing of the degree of production externalities the necessary scale of χ also declines, it still remains such an implausible high level as above. This unpleasant result might infer that indeterminate equilibrium, indeed, would not happen based on plausible parameters in this model.

[9] For simplicity, here I assume the production externalities in each production input are the same, i.e. $\theta_1 = \theta_2$.

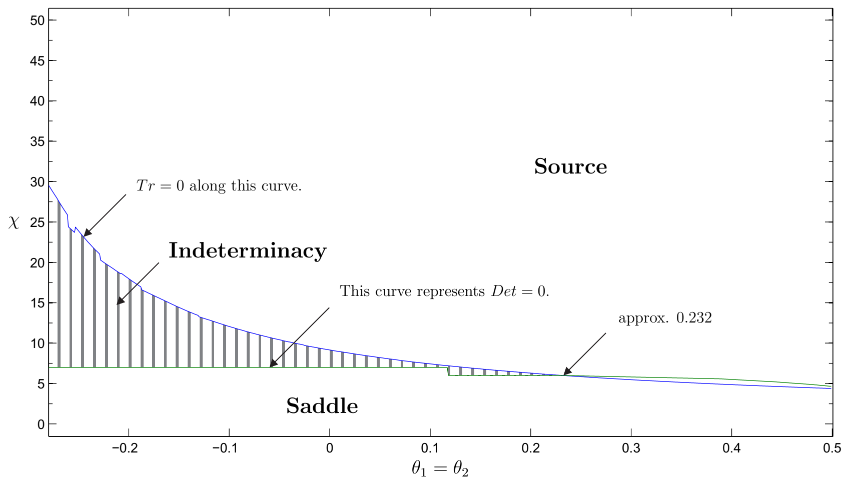

Figure 5: Range of χ and θ for indeterminacy

4 Discussion

In this section, we want to address two main issues of this paper: the first one is the intuition of this model structure and in what way mine differs from Benhabib and Farmer (1994) and Benhabib and Farmer (1996). The second issue is about our "unexpected" calibration results and some probable causes for absence of indeterminacy in our model.

4.1 The Intuition of the Model Structure

As addressed in the previous section, my labor-intensive sector can be viewed as the combination of home production and leisure. In most economic models, the agents can gain their utility by increasing the time of leisure. This means that the agents , in fact, only can do nothing but taking "more" rest; however, people do not spend the whole day just sitting there in their spare time. For example, drinking espresso and tasting some dessert functions as relaxing activities for most people. Making espresso at least needs a stove for sure, and the stove represents the capital input for the production of "leisure". This explains why this paper did not directly follow the definition of home production in Perli (1998); instead, I add another sector into Benhabib and Farmer (1994)'s model by transforming the traditional leisure structure.

Moreover, this transformation also accounts for why we did not follow the same way in which Benhabib and Farmer (1996) introduced the second sector by dividing one identical goods into two functions (consumption and investment). Instead, this model could have two different goods whose production processes follow each specific pattern. That is, this methodology of forming a two-sector economy not only preserves the characteristics of different usage of production output but also draws a parallel between the real world and a theoretical model. For example, the production procedures of computers and DVD players is impossiblely perfectly identical. Furthermore, such a setting still easily duplicate the original model in Benhabib and Farmer (1994).

4.2 Discussion of Calibration Results

In the previous section, the calibration shows that there is no indeterminacy in my model under traditional parametrization. However, according to previous studies, indeterminacy should be more likely to be generated in a two-sector model than in a one-sector model. The reason is that a two-sector model allows the agents in the economy to much more freely choose the allocation of their resources; hence, the impact from self-fulfilment expectation would be enhanced in a two-sector model.

For instance, if the agents believe that it would be higher return in the first sector, then the agents would increase the capital and labor inputs in the first sector. As for labor allocation, since the aggregate labor supply of the agents remains constant, increasing the labor supply in one sector must crowd out the labor supply in the other sector.

This crowding-out phenomenon results in the decline of the marginal product of capital in the second sector; thus, the agents now would put more capital into the higher-return sector. This belief would lead to a new adjustment of the optimal allocation of capital across the two sectors. However, since the steady state is determined by the fundamental economy variables, the agents would finally realize that such an increase is only temporary and expect a depreciation in the "higher-return" sector. By then the agents start to re-allocate their resources to the original steady state level. The whole process would create a new convergence path toward the equilibrium, thus indeterminacy appears.

In my model, I transform the traditional leisure term into a second sector production. This modification should have made our model much prone to obtaining an indeterminate equilibrium; nonetheless, the calibration results indicate that the dynamic characteristics of the equilibrium would be a saddle under the traditional combination of parameters. Consequently, in section 3.4, Figure 5 illustrates the numerical range of χ and θ for indeterminacy. However, the numerical range for indeterminacy is unobservable in reality and section 3.3 also shows the unrobustness of Benhabib and Farmer

(1994)'s model. Apparently, once I increase the value of β by a very small amount, the original conclusion of Benhabib and Farmer (1994) can not continue to hold. A possible reason why the result of Benhabib and Farmer (1994) was not robust might be due to the adjustment mechanism of labor supply. It is because the agents are allowed to reallocate the quantity of labor supply in each sector that an endogenous shock, say a belief, could lead to a indeterminate equilibrium in a two-sector model.

Compared with one-sector models, leisure can not directly increase the agents' utility in my model since there is no "pure" leisure term in the utility function; in other words, an increase in GDP resulting from increasing returns would also lead to an increase in the two production goods (y_1, y_2). Thus, no stabilizer such as leisure in one-sector model drives the agents to forming the reverse belief forcing the economy to return to the steady state level determined by fundamental factors. On the other hand, in comparison with two-sector models such as Benhabib and Farmer (1996) or Harrison (2001), the second sector production, in fact, simultaneously has the role of consumption and investment. Once some expect that there is higher return in one sector, this will drive the labor supply out from the lower-return sector and marginal production of capital in that sector would decrease, too. This crowding out effect may induce a large fall in this sector production; thus, the welfare gain from a higher-return sector may not be able to compensate for the welfare loss caused by the decrease in a lower-return sector. For this reason, an optimistic belief might not have equally strong impact like in other two-sector models.

5 Summary and Conclusion

Most previous literature claims that a two-sector model would generate indeterminacy much more easily than a one-sector model. However, this study has examined a model with different production functions in each sector and it turns out that indeterminacy is absent under a reasonable parametrization. Furthermore, I found that the result of Benhabib and Farmer (1994) was not robust as long as the second sector was introduced having a role as a home production sector. These results may suggest that the plausible minimum level obtained from previous studies do not hold in general cases.

Nonetheless, due to lack of a suitable stabilizer and enough utility compensation in our model structure, a self-fulfilment effect might not be able to drive the business cycles, thus producing indeterminacy.

Appendices

A Maximizing Problem

Since this paper follows Benhabib and Farmer (1994)'s model structure, here I use Hamiltonian Equation to solve the maximizing problem as follows:

$$
\begin{aligned}
H &= e^{-\rho t}\left\{[\log c_1 + \varphi\frac{c_2^{1-\chi}-1}{1-\chi}] + \lambda[y_1 - c_1 - \delta k]\right\} \\
&= e^{-\rho t}\left[\log c_1 + \varphi\frac{\{[(1-s)k]^{\beta}(1-n)^{1-\beta}\}^{1-\chi}-1}{1-\chi}\right] \\
&\qquad + e^{-\rho t}\lambda\left[A(sk)^{\alpha}l^{1-\alpha}(\overline{sk})^{\alpha\theta_1}(\overline{n})^{(1-\alpha)\theta_2} - c_1 - \delta k\right].
\end{aligned} \tag{A.1}
$$

In this model the endogenous variables are $\{c_1, n, s, k\}$, thus four first order conditions are

$$c_1 = \frac{1}{\lambda} \tag{A.2}$$

$$\varphi\left\{[(1-s)k]^{\beta}(1-n)^{1-\beta}\right\}^{1-\chi}\frac{(1-\beta)}{(1-n)} = \lambda\left[\frac{(1-\alpha)}{n}A(sk)^{\alpha}n^{1-\alpha}(\overline{sk})^{\alpha\theta_1}(\overline{n})^{(1-\alpha)\theta_2}\right] \tag{A.3}$$

$$\varphi\left\{[(1-s)k]^{\beta}(1-n)^{1-\beta}\right\}^{1-\chi}[\frac{\beta}{(1-s)k}] = \lambda\left[(\frac{\alpha}{sk})A(sk)^{\alpha}n^{1-\alpha}(\overline{sk})^{\alpha\theta_1}(\overline{n})^{(1-\alpha)\theta_2}\right] \tag{A.4}$$

$$-\frac{\dot{\lambda}}{\lambda} = \alpha\left[A(sk)^{\alpha-1}n^{1-\alpha}(\overline{sk})^{\alpha\theta_1}(\overline{n})^{(1-\alpha)\theta_2}\right] - (\rho+\delta). \tag{A.5}$$

Note that equation (A.4) is exactly the same as equation (2.3); similarly, equation (A.3) is the same as equation (2.4). Besides, by rearranging the equations above one can derive the relative price (P) of y_1 and y_2, here take equation (A.4) as example :

$$
\begin{aligned}
\varphi c_2^{-\chi}(\frac{1}{\lambda})\beta\left[\frac{c_2}{(1-s)k}\right] &= \alpha\frac{y_1}{sk} \\
[\varphi c_1 c_2^{-\chi}]\beta\frac{y_2}{k_2} &= \alpha\frac{y_1}{k_1} = P(\beta)\frac{y_2}{k_2} \\
P &= [\varphi c_1 c_2^{-\chi}]
\end{aligned} \tag{A.6}
$$

B Steady State

Now that the first order conditions were obtained, one can decide the steady state conditions. Since all of the dynamics equations should equal zero under steady state; hence, the first steady state condition is the law of motion

$$
\begin{aligned}
c_1^* + \delta k^* &= [A(s^*k^*)^{\alpha(1+\theta_1)}(n^*)^{(1-\alpha)(1+\theta_2)}] \\
c_1^* + \delta k^* &= y_1^*.
\end{aligned} \tag{B.1}
$$

Next, because equation (A.2) implies $\frac{\dot{c}_1}{c_1} = \frac{\dot{\lambda}}{\lambda}$, the second steady state condition is

$$
\begin{aligned}
(\rho+\delta) &= \alpha[A(s^*k^*)^{\alpha(1+\theta_1)-1}(n^*)^{(1-\alpha)(1+\theta_2)}] \\
(\rho+\delta) &= \alpha[\frac{y_1^*}{s^*k^*}] \\
(\frac{\rho+\delta}{\alpha}) &= \frac{y_1^*}{s^*k^*}. \qquad \text{(B.2)}
\end{aligned}
$$

I combine equation (B.1) and (B.2) to obtain the ratio of consumption of the capital-intensive sector over the aggregate capital input under steady state that

$$
\frac{c_1^*}{k^*} = \left[\frac{\delta(s^*-\alpha)+s\rho}{\alpha}\right] \qquad \text{(B.3)}
$$

C Dynamic Process

For analyzing the dynamic properties of this model, it needs to compute the elements of the Jacobian Matrix. First, I combine equation (A.2) and (A.3) to derive the partial derivative of labor with respect to consumption of the capital-intensive sector and the aggregate capital input, the answers are as follows:

$$
\begin{aligned}
\frac{\partial n}{\partial c_1} &= \frac{(\frac{1}{c_1})}{\left\{(\frac{1}{1-n})[(1-\chi)(1-\beta)-1]+(\frac{1}{n})[(1-\alpha)(1+\theta_2)-1]\right\}} \\
&= (\frac{1}{c_1})\left[\frac{n(1-n)}{\Omega}\right] \qquad \text{(C.1)} \\
\frac{\partial n}{\partial k} &= \frac{(\frac{1}{k})\{[\beta(1-\chi)]-[\alpha(1+\theta_1)]\}}{\left\{(\frac{1}{1-n})[(1-\chi)(1-\beta)-1]+(\frac{1}{n})[(1-\alpha)(1+\theta_2)-1]\right\}} \\
&= (\frac{1}{k})\left[\frac{n(1-n)}{\Omega}\right][\beta(1-\chi)-\alpha(1+\theta_1)] \qquad \text{(C.2)}
\end{aligned}
$$

where

$$
\begin{aligned}
\Omega &= (1-n)[(1-\alpha)(1+\theta_2)-1]+n[(1-\beta)(1-\chi)-1] \\
&= (1-n)(\phi_1-\phi_2) \\
\phi_1 &= [(1-\alpha)(1+\theta_2)-1] \\
\phi_2 &= (\frac{-n}{1-n})[(1-\beta)(1-\chi)-1]
\end{aligned}
$$

Note that Ω is, in fact, identical to the necessary condition for indeterminacy in Benhabib and Farmer (1994); that is, Ω represents the difference between the slope of labor demand curve and the slope of labor supply curve, the details will be introduced in the following section.

Secondly, I differentiate equation (2.6) and (2.8), thus the elements of the Jacobian Matrix are:

$$
\begin{aligned}
J_{11} &= \alpha[(1-\alpha)(1+\theta_2)]\left[A(sk)^{\alpha(1+\theta_1)-1}n^{(1-\alpha)(1+\theta_2)}\right]\left[\frac{(1-n)}{(1-n)(\phi_1-\phi_2)}\right] \\
&= \alpha[\frac{y_1}{sk}][\frac{1+\phi_1}{\phi_1-\phi_2}] \qquad \text{(C.3)}
\end{aligned}
$$

$$
\begin{aligned}
J_{12} &= (\frac{c_1}{k})[(\alpha)A(sk)^{\alpha(1+\theta_1)-1}n^{(1-\alpha)(1+\theta_2)}]\times \\
&\qquad \left\{[\alpha(1+\theta_1)-1]+\frac{[(1-\alpha)(1+\theta_2)][\beta(1-\chi)-\alpha(1+\theta_1)](1-n)}{(1-n)(\phi_1-\phi_2)}\right\} \\
&= (\frac{c_1}{k})[\alpha(\frac{y_1}{sk})]\left\{[\alpha(1+\theta_1)-1]+\frac{(1+\phi_1)}{(\phi_1-\phi_2)}[\beta(1-\chi)-\alpha(1+\theta_1)]\right\} \qquad \text{(C.4)}
\end{aligned}
$$

$$
\begin{aligned}
J_{21} &= [(1-\alpha)(1+\theta_2)][A(sk)^{\alpha(1+\theta_1)-1}n^{(1-\alpha)(1+\theta_2)}](s)(\frac{k}{c_1})\left[\frac{(1-n)}{(1-n)(\phi_1-\phi_2)}\right]-1 \\
&= [\frac{y_1}{sk}](s)(\frac{k}{c_1})\left[\frac{1+\phi_1}{\phi_1-\phi_2}\right]-1 \qquad \text{(C.5)}
\end{aligned}
$$

$$
\begin{aligned}
J_{22} &= (-\delta)+s[A(sk)^{\alpha(1+\theta_1)-1}n^{(1-\alpha)(1+\theta_2)}]\times \\
&\qquad \left\{[\alpha(1+\theta_1)]+[(1-\alpha)(1+\theta_2)]\frac{[\beta(1-\chi)-\alpha(1+\theta_1)](1-n)}{(1-n)(\phi_1-\phi_2)}]\right\} \\
&= (-\delta)+s[\frac{y_1}{sk}]\left\{[\alpha(1+\theta_1)]+\frac{(1+\phi_1)}{\phi_1-\phi_2}][\beta(1-\chi)-\alpha(1+\theta_1)]\right\}. \qquad \text{(C.6)}
\end{aligned}
$$

Finally, I use these elements to calculate the trace and determinant of the Jacobian Matrix. The results are the following:

$$
TR = -\delta+s[\frac{y_1}{k_1}]\left\{[\alpha(1+\theta_1)]+\left[\frac{1+\phi_1}{\phi_1-\phi_2}\right]\left[\beta(1-\chi)-\alpha(1+\theta_1)+(\frac{\alpha}{s})\right]\right\} \qquad \text{(C.7)}
$$

$$
Det = (\delta+\rho)[\frac{c_1}{k}]\left\{[\alpha(1+\theta_1)]\left[\frac{-1-\phi_2}{\phi_1-\phi_2}\right]+[1+\beta(1-\chi)]\left[\frac{1+\phi_1}{\phi_1-\phi_2}\right]-1\right\}. \qquad \text{(C.8)}
$$

In the section 3, these two equation are what we use to compute the calibration results.

D Duplication of Benhabib and Farmer (1994)

Since this model follows the basic settings of Benhabib and Farmer (1994), we can simply duplicate their model by proper setting of the parameters. Keeping in mind that there is only one production sector in Benhabib and Farmer (1994), this implies that the representative agent has no need to decide how to allocate the capital allocation.

That is, $s = 1$. In addition, the production function of y_2 also abides by the Cobb-Douglas setting. As long as the capital share equals to zero, y_2 can be also considered as the term of leisure in Benhabib and Farmer (1994). That is, I let β be equal to zero. The representative agent maximizing problem becomes

$$\max \int_0^\infty \left[\log c + \varphi \frac{(1-n)^{1-\chi} - 1}{1-\chi}\right] e^{-\rho t} dt.. \tag{D.1}$$

And the one-sector production function would be as follows

$$y = Ak^\alpha n^{1-\alpha} (\overline{k})^{\alpha\theta_1} (\overline{n})^{(1-\alpha)\theta_2}. \tag{D.2}$$

In addition, choose variables in this model are now consumption (c), labor (n), capital (k) and the shadow price. From the first order conditions for labor market, one could derive that the slope of the aggregate labor demand and supply curve are

$$\frac{\partial \log w}{\partial \log n_d} = [(1-\alpha)(1+\theta_2) - 1] \tag{D.3}$$

$$\frac{\partial \log w}{\partial \log n_s} = (-\chi)(\frac{1}{(1-n_s)})(-1)(n_s) = \frac{n_s \chi}{(1-n_s)} \tag{D.4}$$

Next, for the dynamic analysis I directly plug $\beta = 0$ and $s = 1$ into Equation (C.7) and (C.8), thus obtaining the duplicated trace and determinant as follows

$$\begin{aligned} TR^D &= -\delta + \alpha(\frac{y}{k}) + [\frac{y}{k}] \left\{ [\alpha(1+\theta_1)] + \left[\frac{1+\phi_1}{\phi_1 - \phi_2} - 1\right] \right\} \\ &= \rho + \alpha[\frac{y}{k}]\theta_1 \left[\frac{-1-\phi_2^D}{\phi_1^D - \phi_2^D}\right] \end{aligned} \tag{D.5}$$

$$\begin{aligned} Det^D &= (\delta+\rho)[\frac{c_1}{k}] \left\{ [\alpha(1+\theta_1)] \left[\frac{-1-\phi_2}{\phi_1-\phi_2}\right] + \left[\frac{1+\phi_1}{\phi_1-\phi_2} - 1\right] \right\} \\ &= \alpha[\frac{y}{k}][\frac{c}{k}] \left\{ [\alpha(1+\theta_1) - 1] \left[\frac{-1-\phi_2^D}{\phi_1^D - \phi_2^D}\right] \right\}. \end{aligned} \tag{D.6}$$

Where

$$\begin{aligned} \phi_1^D &= [(1-\alpha)(1+\theta_2) - 1] \\ \phi_2^D &= (\frac{-n}{1-n})[-\chi] = \frac{n\chi}{1-n} \end{aligned}$$

Note that ϕ_1^D and ϕ_2^D are just coincident with the slope of labor demand and supply curve severally. Hence, in this section I will present how to get the identical condition for indeterminacy as Benhabib and Farmer (1994). Since indeterminacy would be generated only if $Tr^D < 0$ and $Det^D > 0$ are simultaneously satisfied; thus, this requirement could be described as the following inequality system:

$$\left[\frac{-1-\phi_2^D}{\phi_1^D-\phi_2^D}\right] < 0$$

$$\left\{[\alpha(1+\theta_1)-1]\left[\frac{-1-\phi_2^D}{\phi_1^D-\phi_2^D}\right]\right\} > 0.$$

At first, I could simplify these two inequalities as one single condition; that is,

$$\left[\frac{1+\phi_2^D}{\phi_1^D-\phi_2^D}\right] > 0. \tag{D.7}$$

To decide how this inequality would be satisfactory, there are two cases that needs to be discussed: (1) $1+\phi_2^D < 0$ and $\phi_1^D - \phi_2^D < 0$ or (2) $1+\phi_2^D > 0$ and $\phi_1^D - \phi_2^D > 0$. As mentioned before, ϕ_2^D is also the slope of the labor supply curve. Besides, following the model setting, $\chi > 0$ and $n \in (0,1)$ need to be satisfied, too. This indicates that $1+\phi_2^D$ should always be greater than zero; hence, the former case just contradicts and the latter case would lead to indeterminacy. That is, $\phi_1^D - \phi_2^D > 0$ would be the necessary and sufficient condition for indeterminacy. One can notice that this term is just consistent with the condition which Benhabib and Farmer (1994) proposed.

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