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# 高階系統性風險之研究

# Essays on High Order Systematic Risk

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# 國立臺灣大學博士學位論文
# 口試委員會審定書

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## Essays on High Order Systematic Risk

本論文係陳德峰君（D94723008）在國立臺灣大學財務金融學系暨研究所完成之博士學位論文，於民國一百零三年六月十三日承下列考試委員審查通過及口試及格，特此證明

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# 中文摘要

本論文主題在於了解高階系統性風險對於橫斷面資產定價的影響。本論文包含兩篇文章。

第一章探討股票市場波動性的波動性是否在資產定價上扮演重要的狀態變數。投資者對於市場波動度具有風險趨避的行為。除此之外，投資者也擔心市場波動度本身的波動度可能使得市場波動度變化地更劇烈。本文發展一個以股票市場為基礎的三因子模型；其中，橫斷面股票報酬率決定於市場風險、波動性風險、以及波動性的波動性風險。利用高頻率的標準普爾 500 指數選擇權資料，本文估計出股票市場波動性的波動性。實證結果支持理論，本文發現高波動性的波動性風險的公司，將有 10.5%年股票報酬率風險溢酬。本文指出，股票市場波動性的波動性是資產定價上是重要的風險因子。

第二章探討研究非線性的風險報酬抵換理論。如果市場報酬中存在高階風險溢酬，則高階風險溢酬則應該被定價於橫斷面的資產報酬補償其承擔的高階系統性風險。考慮了市場報酬的非常態性，本文提出一個無模型假設的資本資產定價模型，其模型中使用高階系統性風險來定價橫斷面的資產報酬。研究結果指出，第二階系統性風險是顯著地並

且負向地被定價；因此，結果隱含證券市場線為倒U曲線。有較高第二階系統性風險的股票，將有較高的波動性與較高的機會獲取市場波動性風險溢酬所隱含的上方波動性收益，因此將有較高的市場價格以及較低的風險溢酬。本文建構交易策略於捕抓第二階風險溢酬，實證結果指出於第一階系統性風險建構的投資組合估計出第二階風險溢酬為−12.00%、於第二階系統性風險建構的投資組合估計出第二階風險溢酬為−15.60%、以及於風險中立波動性敏感度建構的投資組合估計出第二階風險溢酬為−16.08%。本文發現實證結果與模型一致，第二階風險溢酬與市場波動性風險溢酬相關、第二階風險溢酬可以解釋橫斷面資產波動性與報酬的難題、以及解釋反向操作系統性風險的難題。本文提供對於高階系統性風險的認知，也說明了非線性的風險報酬抵換的重要性。

# Abstract

My dissertation aims at understanding the high order systematic risks in the cross-section of equity returns. It contains two chapters.

Chapter One extends Bollerslev, Tauchen, and Zhou (2009) to derive an market-based equilibrium asset pricing model in which, along with market return volatility, the volatility of market-return volatility (volatility-of-volatility) is a state variable and important for pricing individual stocks. While investors are averse to high market volatility, there is possibility that high market volatility could fluctuate even further, which could drive investors to hedge the increasing uncertainty by buying defensive stocks and dumping crash-prone stocks. To test the model, we use the high-frequency S&P 500 index option data to estimate a time series of the variance of market variance. Consistent with the model, we find that defensive stocks (i.e., returns co-move more positively with volatility-of-volatility) have lower expected returns. A hedge portfolio long in defensive stocks and short in crash-prone stocks yields a significant 10.5 percent average annual return. Furthermore, the volatility-of-volatility risk largely subsumes the valuation effect of volatility risk documented in previous studies. In sum, our model and test results provide a unified framework to better understand the importance of volatility-of-volatility risk in asset pricing.

Chapter Two studies the feature of nonlinear risk-return trade-off. If market returns have high order risk premiums, expected stock returns should comprise compensation for bearing the corresponding high order systematic risks. Allowing for non-normality in market moments, this paper presents an approximate capital asset pricing model in which high order risks are important for pricing individual stocks. Our results show that the second-order risk is significantly and negatively priced and contributes to an inverse-U shaped relation between cross-sectional expected returns and systematic risks. Stocks with high exposure to the second-order risk are volatile and are capable of earning the upside variance potential implied by the negative market variance risk premium. We develop trading strategies to mimic the second-order risk premium and we show that the resulting mimicking factor, on average, per year is −12.00% estimated from the first-order co-moment risks, −15.60% from the second-order co-moment risks, and −16.08% from the risk-neutral variance beta. Based on the mimicking factors, we find evidence consistent with our model that the second-order risk premium (1) is related to market variance risk premium, (2) accounts for the total volatility puzzle, the idiosyncratic volatility puzzle, and the MAX puzzle, and (3) helps explain the betting-against-beta premium. Our study provides a unified framework for better understanding of high order risk-return tradeoff and sheds light on the role of the second-order risk premium.

**Keywords**: Volatility-of-volatility, Expected stock returns, Variance risk premium, Model-free CAPM, Cumulants, High order risks, Nonlinear risk-return trade-off.

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# Chapter 1

# Volatility-of-Volatility Risk and Asset Prices

## 1.1 Introduction

It is well established that volatility is time-varying and tends to be high during stock market decline. The role of uncertainty during the recent financial crisis is also noted in financial press. For example,

> CRISES feed uncertainty. And uncertainty affects behaviour, which feeds the crisis. …all the indicators of uncertainty are at or near all-time highs. What is at work is not only objective, but also subjective uncertainty (e.g. the unknown unknowns).
>
> —Olivier Blanchard, *The Economist*, January 29, 2009.

The dual volatility concept has important implications for asset prices. Time-varying volatility-of-volatility affects portfolio decisions by inducing changes in investment opportunity set; it changes the expectation of future market returns and future market volatility. If volatility-of-volatility is a state variable, the Intertemporal CAPM (ICAPM; Merton, 1973) posits that volatility-of-volatility should be a priced factor in the cross-section of stocks. Intuitively, assets that co-vary positively with volatility-of-volatility are attractive to investors since these assets provide hedge for volatility risk during the market downturns. Moreover, it has been well established that market volatility is a priced factor (e.g. Coval and Shumway, 2001; Ang, Hodrick, Xing, and Zhang, 2006; and Adrian and Rosenberg, 2008) and therefore an increase in volatility-of-volatility induces volatility shock, leading to an increased required return and immediate stock price decline. Thus, investors require a return premium for a security that is suffer when the market volatility is high and when the whole market is uncertain about uncertainty.

This paper develops a market-based three-factor model that helps explain how asset prices are affected by volatility risk and volatility-of-volatility risk. The model provides a unified framework that can explain the empirical findings that aggregate volatility risk is priced in cross-sectional stock returns (e.g. Ang, Hodrick, Xing, and Zhang, 2006), that variance beta is priced in cross-sectional variance risk premiums (e.g. Carr and Wu, 2009), and that individual variance risk premiums can predict the cross-sectional stock returns (e.g. Bali and Hovakimian, 2009; Han and Zhou, 2011).

Our model begins with a macroeconomic model that incorporates the seminal long-run risks (LRR) model of Bansal and Yaron (2004) and the variance-of-variance model of Bollerslev, Tauchen, and Zhou (2009). We solve the macro-finance model explicitly and derive the equilibrium aggregate prices. Then, we use the properties of the aggregate asset prices to characterize the macroeconomic risks, transforming the underlying macro-based model to a market-based model. The market-based model developed in this paper has several advantages. First, financial data provide useful information because asset prices tell us how market participants value risks. Moreover, financial data convey information to public in a timely fashion. Hence, the empirical design of our model is compatible with a large literature of multi-factor model explaining cross-sectional monthly stock returns (see, for instance, Fama and French, 1993; Ang, Hodrick, Xing, and Zhang, 2006; Maio and Santa-Clara, 2012; among others).

In our model, the expected stock return of a security $i$ is determined by three sources of risks. These risks are associated with: (i) the return sensitivity to market return, $\mathbb{C}\text{ov}_t[r_{i,t+1}, r_{m,t+1}]$ ; (ii) the return sensitivity to market variance, $\mathbb{C}\text{ov}_t[r_{i,t+1}, V_{m,t+1}]$, where $V_{m,t} = \mathbb{V}\text{ar}_t[r_{m,t+1}]$; and, (iii) the return sensitivity to variance of market variance, $\mathbb{C}\text{ov}_t[r_{i,t+1}, Q_{m,t+1}]$, where $Q_{m,t} = \mathbb{V}\text{ar}_t[V_{m,t+1}]$. The first term measures the market risk of classical capital asset pricing model (CAPM; Sharpe, 1964; Lintner, 1965). The second term corresponds to the aggregate volatility risk of Ang, Hodrick, Xing, and Zhang (2006). The last term, which is the main focus of this paper, measures the aggregate variance of variance risk. Hereafter the paper, we refer to the variance of variance as the volatility-of-volatility.

The first goal of this paper is to investigate how market volatility-of-volatility risk affects cross-sectional stock returns. We test the predictions of the model using NYSE, AMEX, and NASDSQ listed stocks over the period 1996 to 2010. To implement our model, we develop a measure of market volatility-of-volatility using high frequency S&P 500 index option data.[1] We convert the tick-by-tick option data to equally spaced five-minute observations and then use the model-free methodology[2] to estimate the market variance implied by index option prices for each five-minute interval. Thus, for each day, we estimate the market volatility-of-volatility by calculating the realized bipower variance from a series of five-minute model-free implied market variance within the day. The bipower variation, introduced by Bardorff-Nielsen and Shephard (2004), delivers a

[1] We use the volatility index, VIX index, from the Chicago Board of Options Exchange (CBOE) as the proxy for the aggregate volatility risk, which has been shown to be a significant priced factor in the cross-sectional stock returns (e.g. Ang, Hodrick, Xing, and Zhang, 2006).

[2] It has been shown that the expectation of market variance can be inferred in a 'model-free' fashion from a collection of option prices without the use of a specific pricing model (see, for example, Carr and Madan 1998; Britten-Jones and Neuberger 2000; Bakshi, Kapadia, and Madan, 2003; Jiang and Tian 2005). The option implied information is forward-looking and the estimate can be obtained using daily or intraday option data.

consistent estimator solely for the continuous component of the volatility-of-volatility whereas the jump component is isolated.[3] In other words, our empirical results are robust to the potential jump risk embedded in volatility (see, for example, Pan, 2002; Eraker, 2008; Drechsler and Yaron 2011; among others).

Consistent with the model, by sorting stocks into quintile portfolios based on return sensitivities to market volatility-of-volatility, we find that stocks in the highest quintile have lower stock returns than stocks in the lowest quintile by 0.88 percent per month. Moreover, we also find evidence consistent with Ang, Hodrick, Xing, and Zhang (2006)'s findings that there is a significant difference of -0.87 percent per month between the stock returns with high volatility risk and the stocks with low volatility risk. Controlling for volatility risk, we still find that the market volatility-of-volatility carries a statistically significant return differentials of -0.97 percent per month. On the other hand, controlling for market volatility-of-volatility risk, we find the return difference between high volatility risk stocks and low volatility risk stocks is still large in magnitude, at -0.68 percent per month. Running the cross-sectional regressions, we find that market volatility-of-volatility carry a statistically significant negative price of risk and largely subsumes the valuation effect of volatility risk. Thus, our findings suggest that market volatility-of-volatility is indeed an independently priced risk factor in the cross-sectional stock returns.

To further explore the mechanism that volatility-of-volatility risk affects asset prices, we investigate whether the volatility-of-volatility risk contributes to the asymmetric correlations between returns and market volatility-of-volatility. We refer to the volatility-

[3] Measures of realized jump based on the difference between realized variation and bipower variation have been proposed by Barndorff-Nielsen and Shephard (2004), Huang and Tauchen (2005), and Andersen, Bollerslev, and Diebold (2007).

of-volatility feedback effect as the mechanism that if volatility-of-volatility is priced, an anticipated increase in volatility-of-volatility raises the required rate of return, implying an immediate stock price decline and higher future returns.[4] Consistent with the channel of volatility-of-volatility feedback effect, we find that stocks that co-move more negatively with market volatility-of-volatility have lower returns before the portfolio formation and earn higher post-formation returns than stocks that co-move more positively. More importantly, we find that the return differentials (e.g. the returns of negative exposure stocks minus the returns of positive exposure stocks) before the portfolio formation are negatively correlated with market volatility-of-volatility measured at the portfolio formation date while the correlations between market volatility-of-volatility and the post-formation return differentials are positive. Hence, market volatility-of-volatility seems to be the state variable that drives the feedback effect, supporting the time-varying risk premium hypothesis.

The second goal of this paper is to investigate how volatility-of-volatility risk affects cross-sectional variance risk premiums. The variance risk premium is defined as the difference between risk-neutral variance and realized variance. Define $V_{i,t}$ as the conditional variance of stock $i$ at time $t$, $V_{i,t} = \mathbb{V}\text{ar}_t\left[r_{i,t+1}\right]$. In our model, the variance risk premium of stock $i$, $VRP_{i,t} \equiv \mathbb{E}_t^{\mathbb{Q}}\left[V_{i,t+1}\right] - \mathbb{E}_t\left[V_{i,t+1}\right]$, is determined by two sources of risks: (i) the variance sensitivity to market variance, $\mathbb{C}\text{ov}_t\left[V_{i,t+1}, V_{m,t+1}\right]$; and, (ii) the variance sensitivity to variance of market variance, $\mathbb{C}\text{ov}_t\left[V_{i,t+1}, Q_{m,t+1}\right]$. The first term corresponds to the variance beta of Carr and Wu (2009). The second term measures the risk that individual stock volatility co-moves with the market volatility of volatility. As

[4] Our definition of volatility-of-volatility feedback effect follows the definition of volatility feedback effect in the literature (see, e.g. French, Schwert, and Stambaugh 1987; Campbell and Hentschel 1992; Bekaert and Wu 2000; Wu 2001; Bollerslev, Sizova, and Tauchen, 2012; among others).

shown by Carr and Wu (2009), variance risk premium corresponds to a trading strategy that shorts a swap on the realized variance; in particular, $\mathbb{E}_t^{\mathbb{Q}}[V_{i,t+1}]$ is the price for the contract and $\mathbb{E}_t[V_{i,t+1}]$ is the expected payoff. Selling a volatility asset with high the volatility sensitivity to market volatility-of-volatility requires high insurance payment since the asset can hedge away the upward market volatility-of-volatility during the market downturns.

Consistent with our model, by sorting stocks into quintile portfolios based on variance sensitivities to market volatility-of-volatility, we find that stock with high sensitivities have higher one-month variance risk premium than stocks with low sensitivities by 67.7 (in percentages squared) per month. The magnitude of the cross-sectional difference in variance risk premium is large compared to the market variance risk premium, which is 17.3 (in percentages squared) per month during our sample period. We study how volatility-of-volatility affects the variance risk premium by running the cross-sectional regressions on the 25 testing portfolios formed on the variance sensitivities to market volatility-of-volatility. We find that the risk price of variance beta with respect to variance of market variance is significantly positive. These findings suggest that market volatility-of-volatility is a priced factor in the cross-sectional variance risk premium.

Our study could also be motivated by the recent finding in Bollerslev, Tauchen, and Zhou (2009) that the variance risk premium of aggregate stock market returns has outstanding predictive power for future aggregate stock market return. The underlying mechanism in their work is that the state variable, the variance of economic variance, which affects expected market returns and solely determines the variance risk premium, delivers the predictability. Their work motivates several papers to focus on various economic mechanisms behind the return predictability afforded by variance risk premium. For example, Drechsler and Yaron (2011) show that jump shocks, in a more elaborate LRR model, capture the size and predictive power of the variance premium. Moreover, Drechsler (2013) show that model uncertainty has a large impact on variance risk premium, helping explain its power to predict stock returns. Nevertheless, none of prior studies provides evidence that volatility-of-volatility is a priced risk factor important for cross-sectional asset pricing.

Our paper is related to the pricing model with higher moments of the market return as risk factors studied by Chang, Christoffersen, and Jacobs (2013). They find that market skewness is a priced risk factor in the cross section of stock returns. Both our paper and their work extend the investigation of Ang, Hodrick, Xing, and Zhang (2006) and extract implied moments from index option prices. However, our results are robust to the inclusion of market skewness factor while the market skewness risk premium is much weaker in our sample period when we control for our market volatility-of-volatility risk.

Our paper is also related to but different from Han and Zhou (2012). They examine how firm-level variance risk premiums affect the stock returns in the cross-section, but they do not develop any theory to explain the dependencies. In contrast, our study investigates specifically the pricing of variance of market variance in the joint of cross-sectional stock returns and variance risk premium.

Finally, independent to our study, Baltussen, Van Bekkum, and Van Der Grient (2013) develop a measure of ambiguity, based on firm-level historical volatility of individual option-implied volatility (vol-of-vol). They find that vol-of-vol affects expected stock returns but their results cannot confirm that vol-of-vol is a priced risk factor. Our investigation differs with theirs in two aspects. First, our measure is based on intraday variation of market variance, resulting in a market volatility-of-volatility factor of daily frequency, while their vol-of-vol is based on historical daily information of implied volatility, resulting in a firm-level uncertainty measure of monthly frequency. Second, we find evidence for the rational pricing of market volatility-of-volatility risk, which sharply contrasts their ambiguity interpretation.

The remainder of the paper is organized as follows. The next section describes the economic dynamics and develops our market-based three-factor model for the empirical implementation. Section 3 constructs the measure of market volatility-of-volatility. Section 4 describes the data and presents the summary statistics. In section 5, we show empirical evidence on the pricing of variance of market variance risk in cross-sectional stock returns. Section 6 provides evidence in cross-sectional variance risk premium. The return predictability for the aggregate market portfolio is examined in section 7. Finally, section 8 contains our concluding remarks.

## 1.2 A three-factor model

This section describes the economic model. Our model begins with a macroeconomic model that incorporates the seminal long-run risks (LRR) model of Bansal and Yaron (2004) and the variance-of-variance model of Bollerslev, Tauchen, and Zhou (2009). We solve the macro-finance model explicitly and derive the equilibrium aggregate asset prices. Then, we use the properties of aggregate asset prices to characterize the macroeconomic risks and develop a market-based three-factor model for the cross-sectional asset prices.

### 1.2.1 Economic dynamics and equilibrium aggregate asset prices

The underlying economy is a discrete time endowment economy. The dynamics of consumption growth rate, $g_{t+1}$, and dividend growth rate, $g_{d,t+1}$, are governed by the following process:

$$\begin{aligned}
g_{t+1} &= \mu_g + x_t + \sigma_t z_{g,t+1} \\
x_{t+1} &= \rho_x x_t + \varphi_x \sigma_t z_{x,t+1} \\
\sigma_{t+1}^2 &= \mu_\sigma + \rho_\sigma \sigma_t^2 + q_t z_{\sigma,t+1} \\
q_{t+1}^2 &= \mu_q + \rho_q q_t^2 + \varphi_q z_{q,t+1} \\
g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma_t z_{d,t+1} \\
z_{g,t+1}, z_{x,t+1}, z_{\sigma,t+1}, z_{q,t+1}, z_{d,t+1} &\overset{\text{iid}}{\sim} N(0,1)
\end{aligned} \tag{1.1}$$

where $x_{t+1}$ represents the long-run consumption growth, $\sigma_{t+1}^2$ is the time-varying economic uncertainty, and $q_{t+1}^2$ is the economic volatility-of-volatility, which is the conditional variance of the economic uncertainty. The features of the long-run risk and the time-varying economic uncertainty is proposed by Bansal and Yaron (2004), while the additional feature of economic volatility-of-volatility is introduced by Bollerslev, Tauchen, and Zhou (2009). The representative agent is equipped with recursive preferences of Epstein and Zin (1989). Thus, the logarithm of the Intertemporal Marginal Rate of Substitution (IMRS), $m_{t+1}$, is

$$m_{t+1} = \theta \log(\delta) - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1}, \tag{1.2}$$

where $r_{a,t+1}$ is the return on consumption claim, and $\theta \equiv (1-\gamma)(1-1/\psi)^{-1}$. We assume that $\gamma > 1,$ and $\psi > 1$, and therefore $\theta < 0$. Based on Campbell and Shiller (1988) approximation, $r_{a,t+1} \approx \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}$, where $z_t$ is the logarithm of price–consumption ratio, which in equilibrium is an affine function of the state variables, $z_t = A_0 + A_x x_t + A_\sigma \sigma_t^2 + A_q q_t^2$.[5]

Substituting the equilibrium consumption return, $r_{a,t+1}$, into the IMRS, the innovation in the pricing kernel $m_{t+1}$ is

$$m_{t+1} - \mathbb{E}_t[m_{t+1}] = -\lambda_g \sigma_t z_{g,t+1} - \lambda_x \sigma_t z_{x,t+1} - \lambda_\sigma q_t z_{\sigma,t+1} - \lambda_q \varphi_q z_{q,t+1} \quad (1.3)$$

where $\lambda_g = \gamma > 0$, $\lambda_x = (1-\theta)A_x \kappa_1 \varphi_x > 0$, $\lambda_\sigma = (1-\theta)A_\sigma \kappa_1 < 0$, $\lambda_q = (1-\theta)A_q \kappa_1 < 0$. The parameters determine the prices for short-run risk ($\lambda_g$), long-run risk ($\lambda_x$), volatility risk ($\lambda_\sigma$), and volatility of volatility risk ($\lambda_q$).

An analogous expression holds for the stock market return, $r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1}$, where $z_{m,t}$ is the log price–dividend ratio, which in equilibrium is an affine function of the state variables, $z_{m,t} = A_{0,m} + A_{x,m} x_t + A_{\sigma,m} \sigma_t^2 + A_{q,m} q_t^2$.[6] Since we require that $\theta < 0$, we have $A_{x,m} > 0$, $A_{\sigma,m} < 0$, and $A_{q,m} < 0$. The innovation in market return can be express as

$$r_{m,t+1} - \mathbb{E}_t\left[r_{m,t+1}\right] = \varphi_d \sigma_t z_{d,t+1} + \beta_{m,x} \sigma_t z_{x,t+1} + \beta_{m,\sigma} q_t z_{\sigma,t+1} + \beta_{m,q} \varphi_q z_{q,t+1}, \quad (1.4)$$

where $\beta_{m,x} = A_{x,m} \kappa_{1,m} \varphi_x > 0$, $\beta_{m,\sigma} = A_{\sigma,m} \kappa_{1,m} < 0$, $\beta_{m,q} = A_{q,m} \kappa_{1,m} < 0$. It is straightforward now to derive the equity premium on the market portfolio,

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[5] The equilibrium solutions for the coefficients are:
$A_x = \frac{1-1/\psi}{1-\kappa_1 \rho_x} > 0, A_\sigma = \frac{\theta((1-1/\psi)^2 + A_x^2 \kappa_1^2 \varphi_x^2)}{2(1-\kappa_1 \rho_\sigma)} < 0$, and $A_q = \frac{\theta A_\sigma^2 \kappa_1^2}{2(1-\kappa_1 \rho_q)} < 0$.

[6] The equilibrium solutions for the coefficients are:
$A_{x,m} = \frac{\phi - 1/\psi}{1-\kappa_{1,m} \rho_x}$, $A_{\sigma,m} = \frac{(1-\theta)A_\sigma(1-\kappa_1 \rho_\sigma) + 0.5 H_{m,\sigma}}{1-\kappa_{1,m} \rho_\sigma}$, and $A_{q,m} = \frac{(1-\theta)A_q(1-\kappa_1 \rho_q) + 0.5 H_{m,q}}{1-\kappa_{1,m} \rho_q}$, where $H_{m,\sigma} = \gamma^2 + \varphi_d^2 + \varphi_x^2 (\lambda_x - \beta_{m,x})^2$ and $H_{m,q} = (\lambda_\sigma - \beta_{m,\sigma})^2$.

$$\mathbb{E}_t[r_{m,t+1}] - r_{f,t} + 0.5\mathbb{V}\mathrm{ar}_t[r_{m,t+1}] = \mathbb{C}\mathrm{ov}_t[r_{m,t+1}, -m_{t+1}]$$
$$= \lambda_x \beta_{m,x} \sigma_t^2 + \lambda_\sigma \beta_{m,\sigma} q_t^2 + \lambda_q \beta_{m,q} \varphi_q^2. \quad (1.5)$$

The expected market return consists of three terms. The first two terms are long-run risk premium and volatility risk premium, which are the same as in Bansal and Yaron (2004), while the last term represents the volatility-of-volatility risk premium, which corresponds to the work of Bollerslev, Tauchen, and Zhou (2009).[7]

The conditional variance of market return is readily calculated as $V_{m,t} \equiv \mathbb{V}\mathrm{ar}_t[r_{m,t+1}] = (\varphi_d^2 + \beta_{m,x}^2)\sigma_t^2 + \beta_{m,\sigma}^2 q_t^2 + \beta_{m,q}^2 \varphi_q^2$, and the process for innovations in market variance is

$$V_{m,t+1} - \mathbb{E}_t[V_{m,t+1}] = \beta_{V,\sigma} q_t z_{\sigma,t+1} + \beta_{V,q} \varphi_q z_{q,t+1}, \quad (1.6)$$

where $\beta_{V,\sigma} = \varphi_d^2 + \beta_{m,x}^2$, $\beta_{V,q} = \beta_{m,\sigma}^2$. Thus, innovations in market variance are related to both the economic volatility shock and the economic volatility-of-volatility shock. It follows that the market volatility-of-volatility (e.g. the conditional variance of market variance) is $Q_{m,t} \equiv \mathbb{V}\mathrm{ar}_t[V_{m,t+1}] = \beta_{V,\sigma}^2 q_t^2 + \beta_{V,q}^2 \varphi_q^2$, and the process for its innovations is

$$Q_{m,t+1} - \mathbb{E}_t[Q_{m,t+1}] = \beta_{Q,q} \varphi_q z_{q,t+1}, \quad (1.7)$$

where $\beta_{Q,q} = \beta_{V,\sigma}^2$. Note that innovations in market volatility-of-volatility is solely determined by economic variance of variance shock with a scaling factor, $\beta_{Q,q}$. The market volatility-of-volatility-of-volatility (e.g. the conditional variance of variance of market variance), $W_{m,t} \equiv \mathbb{V}\mathrm{ar}_t[Q_{m,t+1}] = \beta_{Q,q}^2 \varphi_q^2$, is constant in our model.

Next, we consider the market variance risk premium, which is defined as the

[7] Since we do not assume the square root process for the volatility-of-volatility as Bollerslev, Tauchen, and Zhou (2009) do, the volatility risk in the resulting equity premium does not confound with the volatility-of-volatility risk.

difference between the conditional variance under risk-neutral measure and the conditional variance under physical measure. Under the risk-neutral measure, which is characterized by the Radon-Nikodym derivative $\frac{d\mathbb{Q}}{d\mathbb{P}} = \frac{\exp(m_{t+1})}{\mathbb{E}_t[\exp(m_{t+1})]}$, the economy dynamics preserve the same structure but with a shift the mean.[8] Therefore, our model implies that both the conditional variance of market return and the conditional variance of market variance are invariant under the risk neutral measure; that is, $V_{m,t}^{\mathbb{Q}} \equiv \mathbb{V}\text{ar}_t^{\mathbb{Q}}[r_{m,t+1}] = V_{m,t}$, and $Q_{m,t}^{\mathbb{Q}} \equiv \mathbb{V}\text{ar}_t^{\mathbb{Q}}[V_{m,t+1}] = Q_{m,t}$. The market variance risk premium can be expressed as

$$\begin{aligned} VRP_{m,t} &\equiv \mathbb{E}_t^{\mathbb{Q}}[V_{m,t+1}] - \mathbb{E}_t[V_{m,t+1}] \\ &= \beta_{V,\sigma} q_t \left(\mathbb{E}_t^{\mathbb{Q}}[z_{\sigma,t+1}] - \mathbb{E}_t[z_{\sigma,t+1}]\right) \\ &\qquad + \beta_{V,q} \varphi_q \left(\mathbb{E}_t^{\mathbb{Q}}[z_{q,t+1}] - \mathbb{E}_t[z_{q,t+1}]\right) \\ &= -\lambda_\sigma \beta_{V,\sigma} q_t^2 - \lambda_q \beta_{V,q} \varphi_q^2. \end{aligned} \tag{1.8}$$

The negative volatility risk price ($\lambda_\sigma$) and the negative volatility-of-volatility risk price ($\lambda_q$) contributes to the positive market variance risk premium. Moreover, the model predicts that the market variance risk premium is time-varying and is entirely driven by the dynamics of the economic volatility-of-volatility ($q_t^2$), which corresponds to the work of Bollerslev, Tauchen, and Zhou (2009).

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[8] That is,

$$\begin{aligned} g_{t+1} &= (\mu_g - \gamma\sigma_t^2) + x_t + \sigma_t z_{g,t+1}^{\mathbb{Q}}, \\ x_{t+1} &= -\lambda_x \sigma_t^2 + \rho_x x_t + \varphi_x \sigma_t z_{x,t+1}^{\mathbb{Q}}, \\ \sigma_{t+1}^2 &= (\mu_\sigma - \lambda_\sigma q_t^2) + \rho_\sigma \sigma_t^2 + q_t z_{\sigma,t+1}^{\mathbb{Q}}, \\ q_{t+1}^2 &= (\mu_q - \lambda_q \varphi_q^2) + \rho_q q_t^2 + \varphi_q z_{q,t+1}^{\mathbb{Q}}, \\ g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma_t z_{d,t+1}^{\mathbb{Q}}, \end{aligned}$$

where $z_{g,t+1}^{\mathbb{Q}} = \gamma\sigma_t + z_{g,t+1}$, $z_{x,t+1}^{\mathbb{Q}} = \lambda_x\sigma_t + z_{x,t+1}$, $z_{\sigma,t+1}^{\mathbb{Q}} = \lambda_\sigma q_t + z_{\sigma,t+1}$, $z_{q,t+1}^{\mathbb{Q}} = \lambda_q \varphi_q + z_{q,t+1}$, and $z_{d,t+1}^{\mathbb{Q}} = z_{d,t+1}$.

### 1.2.2 Leverage effects, feedback effects, and return predictability

The model endogenously generates an asymmetric return-volatility dependency. In the literature, leverage effect (e.g. Black, 1976; Christie, 1982; among others) refers to the negative contemporaneous return-volatility correlation, while the mechanism of volatility feedback effect (see, e.g. Campbell and Hentschel 1992; Bekaert and Wu 2000; Wu 2001; Bollerslev, Sizova, and Tauchen, 2012; among others) is often used to explain the positive correlations between future returns and volatility. Our model is in line with the leverage effect and the feedback effect; that is, a straightforward calculation shows that

$$\begin{aligned} \mathbb{C}\mathrm{ov}_t\left[r_{m,t+1}, V_{m,t+1}\right] &= \beta_{m,\sigma}\beta_{V,\sigma}q_t^2 + \beta_{m,q}\beta_{V,q}\varphi_q^2 < 0, \\ \mathbb{C}ov_t\left[r_{m,t+1+j}, V_{m,t+1}\right] &= \mathbb{C}ov_t\left[\mathbb{E}_{t+1}\left[\dots\ \mathbb{E}_{t+j}\left[r_{m,t+1+j}\right]\right], V_{m,t+1}\right] \\ &= -\kappa_\sigma\ \rho_\sigma^j\beta_{m,\sigma}\beta_{V,\sigma}q_t^2 - \kappa_q\rho_q^j\ \beta_{m,q}\beta_{V,q}\varphi_q^2 > 0 \end{aligned} \tag{1.9}$$

where $\kappa_\sigma = (1-\kappa_{1,m}\rho_\sigma)/\kappa_{1,m}$ and $\kappa_q = (1-\kappa_{1,m}\rho_q)/\kappa_{1,m}$. In the absence of the time-varying economic volatility-of-volatility (e.g. when $q_t^2$ is constant and $\varphi_q^2$=0), the second term of $\mathbb{C}\mathrm{ov}_t\left[r_{m,t+1}, V_{m,t+1}\right]$ and the second term of $\mathbb{C}ov_t\left[r_{m,t+1+j}, V_{m,t+1}\right]$ are reduced to zero, leading both of the two covariances to smaller values. Thus, the dynamics of economic volatility-of-volatility amplifies the leverage effect and the volatility feedback effect.

Moreover, our model implies the existence of leverage effect and feedback effect related to market volatility-of-volatility. The contemporaneous and forward correlations between market return and market volatility-of-volatility can be expressed as

$$\mathbb{C}\text{ov}_t\left[r_{m,t+1}, Q_{m,t+1}\right] = \beta_{m,q}\beta_{Q,q}\varphi_q^2 < 0,$$
$$\mathbb{C}ov_t\left[r_{m,t+1+j}, Q_{m,t+1}\right] = -\kappa_q\rho_q^j\,\beta_{m,q}\beta_{Q,q}\varphi_q^2 > 0. \tag{1.10}$$

If volatility-of-volatility is priced, an anticipated increase in volatility-of-volatility raises the required rate of return, implying an immediate stock price decline and higher future returns. Thus, the above expressions provide important and directly testable implications for the volatility-of-volatility risk premium.

It is instructive to consider the return predictability afforded by the market variance risk premium, which is the main proposition in the pioneer work of Bollerslev, Tauchen, and Zhou (2009). In our model, the process for innovations in the market variance risk premium is

$$VRP_{m,t+1} - \mathbb{E}_t\left[VRP_{m,t+1}\right] = -\lambda_\sigma\beta_{V,\sigma}\varphi_q z_{q,t+1}, \tag{1.11}$$

which is entirely determined by economic volatility-of-volatility shock like the market volatility-of-volatility is. Thus, similar to Bollerslev, Tauchen, and Zhou (2009), market variance risk premium can predict the future market return as follows,

$$\mathbb{C}ov_t\left[r_{m,t+1+j}, VRP_{m,t+1}\right] = \kappa_q\rho_q^j\,\beta_{m,q}\lambda_\sigma\beta_{V,\sigma}\varphi_q^2 > 0. \tag{1.12}$$

In the presence of jumps, however, as indicated in Drechsler and Yaron (2011), the market variance risk premium is affected by risk of jumps that is also likely to deliver the return predictability. In which case, the market variance risk premium is no longer solely driven by the economic volatility-of-volatility, lowering the testing power of Bollerslev, Tauchen, and Zhou (2009) for the return predictability afforded by market variance risk premium against an alternative source of risk. Nevertheless, the continuous component of the market volatility-of-volatility is still corresponding to the economic volatility-of-volatility. Thus, for the testing power consideration, the empirical strategy of our model focuses on the identification of the continuous component of the market volatility-of-volatility.



### 1.2.3 A market-based three-factor model for individual stocks

We assume that the innovations in stock return *i* is

$$r_{i,t+1} - \mathbb{E}_t\left[r_{i,t+1}\right] = \beta_{i,x}\sigma_t z_{x,t+1} + \beta_{i,\sigma} q_t z_{\sigma,t+1} + \beta_{i,q}\varphi_q z_{q,t+1}. \quad (1.13)$$

Given the expression for the pricing kernel in equation(1.3), the expected stock return can be written as

$$\begin{aligned} &\mathbb{E}_t\left[r_{i,t+1}\right] - r_{f,t} + 0.5\mathbb{V}\mathrm{ar}_t\left[r_{i,t+1}\right] \\ &= \beta_{i,x}\lambda_x\sigma_t^2 + \beta_{i,\sigma}\lambda_\sigma q_t^2 + \beta_{i,q}\lambda_q\varphi_q^2. \end{aligned} \quad (1.14)$$

Thus, the expected stock return is determined by three sources of economic risks: economic long-run risk ($\beta_{i,x}$), economic volatility risk ($\beta_{i,\sigma}$), and economic volatility-of-volatility risk ($\beta_{i,q}$).

We now use the properties of the aggregate asset prices to characterize the macroeconomic risks. First of all, in equilibrium, the market volatility-of-volatility risk, which is the return covariance with respect to variance of market variance, is solely determined by the economic volatility-of-volatility risk ($\beta_{i,q}$), i.e.

$$\mathbb{C}\mathrm{ov}_t\left[r_{i,t+1}, Q_{m,t+1}\right] = \beta_{i,q}\beta_{Q,q}\varphi_q^2. \quad (1.15)$$

Furthermore, the return sensitivities with respect to market variance and with respect to market return provide additional information for the economic volatility risk and the long-run risk; that is,

$$\mathbb{C}\mathrm{ov}_t\left[r_{i,t+1}, V_{m,t+1}\right] = \beta_{i,\sigma}\beta_{V,\sigma}q_t^2 + \beta_{i,q}\beta_{V,q}\varphi_q^2, \quad (1.16)$$

$$\mathbb{C}\mathrm{ov}_t\left[r_{i,t+1}, r_{m,t+1}\right] = \beta_{i,x}\beta_{m,x}\sigma_t^2 + \beta_{i,\sigma}\beta_{m,\sigma}q_t^2 + \beta_{i,q}\beta_{m,q}\varphi_q^2. \quad (1.17)$$

Substituting out the economic risks in (1.14) with (1.15), (1.16) and (1.17) gives us the market-based three-factor model:

$$
\begin{aligned}
&\mathbb{E}_t\left[r_{i,t+1}\right] - r_{f,t} + 0.5\mathbb{V}\mathrm{ar}_t\left[r_{i,t+1}\right] \\
&\quad = \lambda_m \mathbb{C}\mathrm{ov}_t\left[r_{i,t+1}, r_{m,t+1}\right] + \lambda_V \mathbb{C}\mathrm{ov}_t\left[r_{i,t+1}, V_{m,t+1}\right] \\
&\quad\quad + \lambda_Q \mathbb{C}\mathrm{ov}_t\left[r_{i,t+1}, Q_{m,t+1}\right],
\end{aligned} \tag{1.18}
$$

where $$\lambda_m = \frac{\lambda_x}{\beta_{m,x}}, \lambda_V = \frac{\lambda_\sigma - \lambda_m \beta_{m,\sigma}}{\beta_{V,\sigma}}, \lambda_Q = \frac{\lambda_q - \lambda_V \beta_{V,q} - \lambda_m \beta_{m,q}}{\beta_{Q,q}}. \tag{1.19}$$

Thus, the expected stock return is now determined by three sources of risks related to aggregate asset prices. The first term measures the market risk of classical capital asset pricing model (CAPM; Sharpe, 1964; Lintner, 1965). The second term corresponds to the aggregate volatility risk of Ang, Hodrick, Xing, and Zhang (2006). The last term, which is the main focus of this paper, measures the aggregate volatility of volatility risk. The resulting three risk prices in our market-based model, $\lambda_m$, $\lambda_V$, and $\lambda_Q$, are related to the three economic risk prices with a linear transformation.

The market-based model developed in this paper has several advantages. First, financial data provide useful information because asset prices tell us how market participants value risks. Moreover, financial data convey information to public in a timely fashion. Hence, the empirical design of our model is compatible with a large literature of multi-factor model explaining cross-sectional monthly stock returns (see, for instance, Fama and French, 1993; Ang, Hodrick, Xing, and Zhang, 2006; Maio and Santa-Clara, 2012; among others).

It is constructive to establish the individual variance risk premiums under the proposed model. The conditional variance of the time $t$ to $t+1$ return of stock *i* ($r_{i,t+1}$) and the innovations in conditional variance *i* can be expressed as

$$V_{i,t} \equiv \mathbb{V}\mathrm{ar}_t\left[r_{i,t+1}\right] = \beta_{i,x}^2\sigma_t^2 + \beta_{i,\sigma}^2 q_t^2 + \beta_{i,q}^2\varphi_q^2$$
$$V_{i,t+1} - \mathbb{E}_t\left[V_{i,t+1}\right] = \beta_{i,\sigma}^V q_t z_{\sigma,t+1} + \beta_{i,q}^V \varphi_q z_{q,t+1} \tag{1.20}$$

where $\beta_{i,\sigma}^V = \beta_{i,x}^2$ and $\beta_{i,q}^V = \beta_{i,\sigma}^2$. It follows that

$$VRP_{i,t} \equiv \mathbb{E}_t^{\mathbb{Q}}\left[V_{i,t+1}\right] - \mathbb{E}_t\left[V_{i,t+1}\right] = -\lambda_\sigma\beta_{i,\sigma}^V q_t^2 - \lambda_q\beta_{i,q}^V\varphi_q^2, \tag{1.21}$$

which suggests that the individual variance risk premiums are determined by the conditional variance's betas with respect to the economic volatility risk ($\beta_{i,\sigma}^V$) and with respect to the economic volatility-of-volatility risk ($\beta_{i,q}^V$).

To derive a market-based variance risk premium model, we consider the variance sensitivities with respect to market variance and with respect to market volatility-of-volatility, which in the equilibrium are given by

$$\mathbb{C}\mathrm{ov}_t\left[V_{i,t+1}, Q_{m,t+1}\right] = \beta_{i,q}^V\beta_{Q,q}\varphi_q^2, \tag{1.22}$$

$$\mathbb{C}\mathrm{ov}_t\left[V_{i,t+1}, V_{m,t+1}\right] = \beta_{i,\sigma}^V\beta_{V,\sigma}q_t^2 + \beta_{i,q}^V\beta_{V,q}\varphi_q^2. \tag{1.23}$$

Similarly, substituting out the economic risks in (1.21) with (1.22) and (1.23) gives us the market-based two-factor model for the individual variance risk premium:

$$VRP_{i,t} = -\lambda_V^V\mathbb{C}\mathrm{ov}_t\left[V_{i,t+1}, V_{m,t+1}\right] - \lambda_Q^V\mathbb{C}\mathrm{ov}_t\left[V_{i,t+1}, Q_{m,t+1}\right], \tag{1.24}$$

where
$$\lambda_V^V = \frac{\lambda_\sigma}{\beta_{V,\sigma}} \text{ and } \lambda_Q^V = \frac{\lambda_q - \lambda_V^V\beta_{V,q}}{\beta_{Q,q}}. \tag{1.25}$$

Therefore, the individual variance risk premium is now determined by two sources of risks related to aggregate asset prices. The first term corresponds to the variance beta of Carr and Wu (2009). The second term measures the risk that individual stock volatility co-moves with the aggregate volatility of volatility. The resulting two risk prices associated with the individual variance risk premium, $\lambda_V^V$ and $\lambda_Q^V$, are also related to the corresponding economic risk prices, $\lambda_\sigma$ and $\lambda_q$, with a linear transformation.

Our model implies that the three aggregate asset prices are inter-dependent and so are the market-based risks in the individual expected return and variance risk premium. Moreover, the risk prices for the high moments are offset by the risk prices for the low moments. While this property is interesting, it also complicates the task of distinguishing the relative impacts from the underlying sources of risks. Nevertheless, our market-based model can be alternatively implemented using orthogonalized aggregate asset prices as risk factors. Define $\tilde{Q}_{m,t+1} = Q_{m,t+1}$, $\tilde{V}_{m,t+1} = V_{m,t+1} - \mathbb{E}[V_{m,t+1}|Q_{m,t+1}]$, and $\tilde{r}_{m,t+1} = r_{m,t+1} - \mathbb{E}[r_{m,t+1}|V_{m,t+1}, Q_{m,t+1}]$. Thus, each of the market-based risks is directly linked to the counterpart of the underlying economic risks. In which case, the expected stock return is represented by $\tilde{\lambda}_m \mathbb{C}\text{ov}_t[r_{i,t+1}, \tilde{r}_{m,t+1}] + \tilde{\lambda}_V \mathbb{C}\text{ov}_t[r_{i,t+1}, \tilde{V}_{m,t+1}] + \tilde{\lambda}_Q \mathbb{C}\text{ov}_t[r_{i,t+1}, \tilde{Q}_{m,t+1}]$ and the individual variance risk premium can also be expressed by $\tilde{\lambda}_V \mathbb{C}\text{ov}_t[V_{i,t+1}, \tilde{V}_{m,t+1}] + \tilde{\lambda}_Q \mathbb{C}\text{ov}_t[V_{i,t+1}, \tilde{Q}_{m,t+1}]$. Thus, the resulting risk prices preserve the sign of the original economic risk prices; that is, $\tilde{\lambda}_m = \lambda_x/\beta_{m,x}$, $\tilde{\lambda}_V = \lambda_\sigma/\beta_{V,\sigma}$, $\tilde{\lambda}_Q = \lambda_q/\beta_{Q,q}$.

## 1.3 Estimation of variance of market variance

In previous section, we propose a market-based three-factor model, which requires the information of market return, market variance, and variance of market variance. To proxy for the first two factors, we use CRSP value-weighted market index and CBOE VIX index, which have been widely used in the literature ( see, for example, Ang, Hodrick, Xing, and Zhang, 2006; Chang, Christoffersen, and Jacobs, 2013; Bollerslev, Tauchen, and Zhou, 2009; among others). In this study, we estimate the variance of market variance by calculating the realized bipower variation from a series of five-minute model-free implied variances, using the high-frequency S&P 500 index option data. The details of our empirical settings are described as follows.

First of all, we extract the model-free implied variance, using the spanning methodology proposed by Carr and Madan (2001), Bakshi and Madan (2000), Bakshi, Kapadia, and Madan (2003), and Jiang and Tian (2005). Bakshi, Kapadia, and Madan (2003) show that the price of a $\tau$-maturity return variance contract, which is the discounted conditional expectation of the square of market return under the risk-neutral measure, can be spanned by a collection of out-of-money call options and out-of-money put options,

$$\bar{V}_t(\tau) \equiv \mathbb{E}_t^{\mathbb{Q}}\left[\mathrm{e}^{-r_{f,t}}\mathrm{Log}\left[\frac{S_{t+\tau}}{S_t}\right]^2\right]$$
$$= \int_{S_t}^{\infty} \frac{2(1-\log[K/S_t])}{K^2} C_t(K;\tau)\, dK \qquad (1.26)$$
$$+ \int_0^{S_t} \frac{2(1+\log[K/S_t])}{K^2} P_t(K;\tau)\, dK,$$

where $C_t(K;\tau)$ and $P_t(K;\tau)$ are the prices of European calls and puts at time $t$ written on the underlying stock with strike price K and expiration date at $t+\tau$. The conditional variance of market return can be calculated by

$$IV_t(\tau) = \mathrm{e}^{r_{f,t}}\bar{V}_t(\tau) - \mu_t(\tau)^2, \qquad (1.27)$$

where $\mu_t(\tau)$ satisfies the risk-neutral valuation relationship, which is related to the first four risk-neutral moments of market returns as described in equation (39) of Bakshi, Kapadia, and Madan (2003).

Second, we use the model-free realized bipower variance, introduced by Bardorff-Nielsen and Shephard (2004), to estimate the variance of market variance. Define the intraday stock return as $r_{t+1,j} \equiv \log[S_{t+j/M}] - \log[S_{t+(j-1)/M}]$, $j = 1,\ldots,M$, where M is the sampling frequency per trading day. Bardorff-Nielsen and Shephard (2004) study two measures of realized variations; the first one is the realized variation, $RV_{t+1}$, and the second one is the bipower variation, $BV_{t+1}$:

$$RV_{t+1} = \sum_{j=1}^{M} r_{t+1,j}^2, \tag{1.28}$$

$$BV_{t+1} = \frac{\pi}{2}\left(\frac{M}{M-1}\right)\sum_{j=2}^{M} |r_{t+1,j}||r_{t+1,j-1}|. \tag{1.29}$$

Andersen, Bollerslev, and Diebold (2002) show that the realized variance converges to the integrated variance plus the jump contributions, i.e.

$$RV_{t+1} \xrightarrow{M\to\infty} \int_t^{t+1} \sigma^2(s)\, ds + \sum_{j=1}^{N_{t+1}} J_{t+1,j}^2, \tag{1.30}$$

where $N_{t+1}$ is the number of return jumps within day $t$+1 and $J_{t+1,j}^2$ is the jump size. Moreover, Bardorff-Nielsen and Shephard (2004) show that

$$BV_{t+1} \xrightarrow{M\to\infty} \int_t^{t+1} \sigma^2(s)\, ds. \tag{1.31}$$

In other words, bipower variation provides a consistent estimator of the integrated variance solely for the diffusion part.

Our measure for variance of market variance is estimated from a series of five-minute based model-free implied variances. The intraday model-free implied variances are calculated using equation (1.27), which is denoted as $IV_{t+j/M}(\tau), j = 1, \ldots, M$. Since the process of market variance is a (semi-)martingale, we apply the bipower variation formula on the changes in annualized model-free implied variances and obtain a measure for variance of market variance:

$$VoV_{t+1}(\tau) = \frac{\pi}{2}\left(\frac{M}{M-1}\right)\sum_{j=2}^{M}\left|\Delta v_{t+1,j}(\tau)\right|\left|\Delta v_{t+1,j-1}(\tau)\right| \quad (1.32)$$

where $\Delta v_{t+1,j}(\tau) \equiv \frac{365}{\tau}\left[IV_{t+j/M}(\tau) - IV_{t+(j-1)/M}(\tau)\right]$. In this way, our empirical results will not be affected by the volatility jumps (or the return jumps embedded in the volatility).

## 1.4 Data and descriptive statistics

### 1.4.1 Data description

We use the tick-by-tick quoted data on S&P 500 index (SPX) options from CBOE's Market Data Report (MDR) tapes over the time period from January 1996 to December 2010. The underlying SPX prices are also provided in the tapes. We obtain daily data from OptionMetrics for equity options and S&P 500 index options. We use the Zero Curve file, which contains the current zero-coupon interest rate curve, and the Index Dividend file, which contains the current dividend yield, from OptionMetrics to calculate the implied volatility for each tick-by-tick data from CBOE's MDR tapes. Daily and monthly stock return data are from CRSP while intraday transactions data are from TAQ data sets. Financial statement data are from COMPUSTAT. Fama and French (1993) factors and their momentum *UMD* factor are obtained from the online data library of Ken French.[9] *VIX* index is obtained from the website of CBOE.[10] While we use the 'new' *VIX* index to calculate the market variance risk premium as proposed by Bollerslev, Tauchen, and Zhou (2009), we also use the 'old' *VIX*, which is based on the S&P 100 options and Black–Scholes implied volatilities, as our volatility factor, following Ang, Hodrick, Xing, and

[9] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/
[10] http://www.cboe.com/micro/vix/historical.aspx

Zhang (2006). We use the index option prices from the Option Price file to replicate the market skewness factor and the market kurtosis factor of Chang, Christoffersen, and Jacobs (2013).

We follow the literature (see, for example, Jiang and Tian 2005; Chang, Christoffersen, and Jacobs, 2013; among others) to filter out index option prices that violate the arbitrage bounds.[11] We also eliminate in-the-money options (e.g. put options with K/S>1.03 and call options with K/S<1.03) because prior study suggests that they are less liquid. We use the daily SPX low and high prices, downloaded from Yahoo Finance,[12] to filter out the MDR data that are outside the [low, high] interval.

For the computation of the market volatility-of-volatility, we first partition the tick-by-tick S&P500 index options data into five-minute intervals. For each maturity within each interval, we linearly interpolate implied volatilities for a fine grid of one thousand moneyness levels (K/S) between 0.01% and 300%[13] and use equations (1.26) and (1.27) to estimate the model-free implied variance. We then use linearly interpolate maturities to obtain the estimate at a fixed 30-day horizon. For each day, our measure for market volatility-of-volatility ($VoV$) is calculated by using the bipower variation formula of equation (1.32) with the 81 within-day five-minute annualized 30-day model-free implied variance estimates covering the normal CBOE trading hours from 8:30 a.m. to 3:15 p.m. Central Time.

The market variance risk premium ($VRP_{m,t}$), following Bollerslev, Tauchen, and Zhou (2009), is defined as the difference between the ex-ante implied variance ($IV_{m,t}$)

[11] Moreover, we eliminate all observations for which the ask price is lower than the bid price, the bid price is equal to zero, or the average of the bid and ask price is less than 3/8.

[12] http://finance.yahoo.com/q/hp?s=^GSPC+Historical+Prices

[13] For moneyness levels below or above the available moneyness level in the market, we use the implied volatility of the lowest or highest available strike price.

and the ex-post realized variance ($RV_{m,t}$), i.e. $VRP_{m,t} \equiv IV_{m,t} - RV_{m,t}$. We focus on a fixed maturity of 30 days. Market implied variance ($IV_{m,t}$) is measured by the squared 'new' VIX index divided by 12. Summation of SPX five-minute squared logarithmic returns are used to calculate the market realized variance ($RV_{m,t}$). With eighty five-minute intervals per trading day and the overnight return, we construct the daily market realized variance, using a rolling window of 22 trading days starting from the current day.

We construct the individual model-free implied variance ($IV_{i,t}$) using equity options data from the Volatility Surface file that provides Black-Scholes implied volatilities for options with standard maturities and delta levels. The individual implied variance is estimated by applying the same methodology that we use for the index options on the equity options data with 30-day maturity.

To compute the individual realized variance ($RV_{i,t}$), we extract from TAQ database the intraday transaction and quote data within the normal trading hours from 9:30 a.m. to 4:00 p.m. Eastern Time. We first adopt the step-by-step cleaning procedures proposed by Bardorff-Nielsen, Hansen, Lunde, and Shephard (2009) to screen the TAQ high frequency data,[14] and then we follow Sadka (2006) to remove quotes in which the quoted spread is more than 25% and remove trades in which the absolute value of one-tick return is more than 25%. The resulting 78 five-minute trades and quotes per trading day in a rolling window of 22 trading days are separately used to calculate the trade-based daily individual realized variance ($RV_{i,t}^{T}$) and the quote-based daily individual realized variance ($RV_{i,t}^{Q}$). To avoid the effect from stale prices in trades or in quotes, we further require that the both the number of five-minute trades and that of quotes in the 22-day rolling window

[14] We apply the rules of P1, P2, P3, Q1, Q2, T1, T2, and T3 as described in the section 3.1 of Bardorff-Nielsen, Hansen, Lunde, and Shephard (2009) to carry out the cleaning procedures.

should be more than 78x11=858. To conserve space, we will focus on the trade-based realized variance, i.e. $RV_{i,t} = RV_{i,t}^{T}$, while the results for the quote-based measure are available upon request.

We estimate the monthly expected individual variance risk premium ($EVRP_{i,t}$) through a forecast model. We adopt a linear forecast model, following Drechsler and Yaron (2011) and Han and Zhou (2012), to estimate the expected realized variance ($ERV_{i,t}$) with the lagged realized variance and the lagged model-free implied variance measured at the end of the month.[15] Thus, the expected individual variance risk premium is defined as $EVRP_{i,t} = IV_{i,t} - ERV_{i,t}$.

To implement our empirical model, we construct innovations in market moments. First, following Ang, Hodrick, Xing, and Zhang (2006), innovations in market volatility ($\Delta VIX$) is measured by its first order difference, i.e. $\Delta VIX_{t+1} = VIX_{t+1} - VIX_{t}$. Chang, Christoffersen, and Jacobs (2013) indicate that taking the first difference is appropriate for *VIX*, whereas an ARMA(1,1) model is need to remove the autocorrelation in the their skewness and kurtosis factors. Following their approach, the innovations in market volatility-of-volatility ($\Delta VoV$) is computed as the ARMA(1,1) model residuals of the market volatility-of-volatility.

### 1.4.2 Descriptive statistics

The daily measure of *VoV* is plotted in Figure 1. 1. There are clear spikes on the graph—the Asian financial crisis in1997, the LTCM crisis in1998, September11, 2001,

[15] Specifically, for stock $i$, we run the regression: $RV_{i,t+1} = \alpha + \beta_0 IV_{i,t} + \beta_1 RV_{i,t}$ and defined the fitted value as $ERV_{i,t}$, i.e. $ERV_{i,t} \equiv \widehat{RV}_{i,t+1} = \hat{\alpha} + \hat{\beta}_0 IV_{i,t} + \hat{\beta}_1 RV_{i,t}$.

the WorldCom and Enron bankruptcies in 2001 and 2002, subprime loan crisis in 2007, the recent financial crisis in 2008, and the flash crash in 2010.

Table 1.1 reports descriptive statistics for the daily factors used in this paper. In our sample, the mean of 30-day market variance risk premium (*VRP*) is 17.260 (in percentages squared), which is slightly smaller than 18.3 in Bollerslev, Tauchen, and Zhou's (2010) sample. The mean of *VoV* is 0.054%, which is much smaller than its standard deviation, 0.563%. The mean of *SKEW* is -1.663 and the mean of *KURT* is 9.313. Thus, the risk-neutral distribution of the market return is asymmetric and has fat tails.

Panel B reports the Spearman correlations between factors, including the excess market return (*MKT*), the Fama and French (1993) *SMB* and *HML* factors, the momentum factor (*UMD*), the changes in *VIX* (*ΔVIX*; Ang, Hodrick, Xing, and Zhang, 2006), innovations in *VoV* (*ΔVoV*), and Chang, Christoffersen, and Jacobs (2013) innovations in market skewness factor (*ΔSKEW*) and market kurtosis factor (*ΔKURT)*. As expected, *MKT* is negatively correlated with both *ΔVIX* (-0.779) and *ΔVoV* (-0.044), supporting the leverage effect predicted by our model. Moreover, *VRP* is positively correlated with *ΔVoV* (0.145), consistent with our theory that the variance risk premium and the market volatility-of-volatility are both driven by the economic volatility-of-volatility. *ΔKURT* and *ΔSKEW* are highly correlated with a correlation value of -0.863, which is comparable to -0.83 reported by Chang, Christoffersen, and Jacobs (2013). *ΔVoV* shows little correlation with *ΔVIX* (0.049), *ΔSKEW* (-0.017), and *ΔKURT* (-0.010), which suggests that *ΔVoV* should be an independent state variable that cannot be explained by these market moments studied in the literature.

## 1.5 Pricing volatility-of-volatility risk in the cross-sectional stock returns

This section examines how market volatility-of-volatility risk affects cross-sectional average returns. Based on our market-based three-factor model with their empirical proxies, at the end of each month, we estimate the regression for each stock $i$ using daily returns:

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,VoV} \Delta VoV_{t+1} + \varepsilon_{i,t+1}. \tag{1.33}$$

We construct a set of testing assets that are sufficiently disperse in exposure to aggregate volatility-of-volatility innovations by sorting firms on $\beta_{i,VoV}$ loadings over the past month using the regression (1.33) with daily data. Our empirical model is an extension of Ang, Hodrick, Xing, and Zhang (2006). Following their work, we run the regression for all common stocks on NYSE, AMEX, and NASDAQ with more than 17 daily observations. After the portfolio formation, we calculate the value-weighted daily and monthly stock returns for each portfolio. If market volatility-of-volatility is a priced risk factor, we should expect to see a monotonic decreasing pattern in the portfolio returns.

### 1.5.1 Portfolios sorted on market volatility-of-volatility risk

Table 1.2 provides the performance of portfolios sorted on $\beta_{i,VoV}$. Stocks are sorted into quintile portfolios based on $\beta_{i,VoV}$, from the lowest (quintile 1) to the highest (quintile 5). Consistent with the model, we find that stocks with positive return sensitivities to market volatility-of-volatility (quintile 5) have lower stock returns than stocks with negative return sensitivities (quintile 1) by 0.88 percent per month with t-statistic of -2.32. Controlling for Fama and French four factor model, the '5-1' long-short portfolio still gives a significant alpha of -0.96 percent per month with a *t*-statistic of -2.59.

To check whether our results are robust to firm characteristics, Table 1.3 shows performance of portfolios sorted on $\beta_{i,VoV}$, controlling for market capitalization (*Size*), book-to-market ratio (*B/M*), past 11-month returns (*RET_2_12*), past 1-month returns (*RET_1*), and Amihud's illiquidity (*ILLIQ*), respectively. We first sort stocks into five quintiles based *Size*. Then, within each quintile, we sort stocks based on their $\beta_{i,VoV}$ loadings into five portfolios. All portfolios are rebalanced monthly and are value weighted. The five portfolios sorted on $\beta_{i,VoV}$ are then averaged over each of the five *Size* sorted portfolios, resulting $\beta_{i,VoV}$ quintile portfolios controlling for *Size*. *B/M*, *RET_2_12*, *RET_1*, and *ILLIQ* are analyzed with the same procedure as described above. The Fama and French four factor alpha of the '5-1' long–short portfolio remains significant controlling for these four variables, i.e. at -0.45 percent with a *t*-statistic of -2.14 controlling for *Size*, at -0.88 percent with a *t*-statistic of -3.15 controlling for *B/M*, at -0.52 percent with a *t*-statistic of -2.12 controlling for *RET_2_12*, at -0.61 percent with a *t*-statistic of -2.06 controlling for *RET_1*, and at -0.53 percent with a *t*-statistic of -2.30 controlling for *ILLIQ*. Hence, the low returns to high $\beta_{i,VoV}$ stocks are not completely driven by the existing well-known firm characteristics.

### 1.5.2 Portfolios sorted on market volatility risk

Table 1.4 provides the performance of portfolios sorted on $\beta_{i,VIX}$, using the same approach as on $\beta_{i,VoV}$. We find evidence consistent with Ang, Hodrick, Xing, and Zhang's (2006) findings that there is a significant difference of -0.87 percent per month with a $t$-statistic of -2.17 between the stock returns with high volatility risk and the stocks with low volatility risk. Controlling for Fama and French four factor model, the '5-1' long-short portfolio gives a significant alpha of -1.18 percent per month with a $t$-statistic of -3.24.

Table 1.5 considers two-way sorted portfolios on $\beta_{i,VIX}$ and $\beta_{i,VoV}$. We sort stocks into quintile portfolios based on $\beta_{i,VIX}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on $\beta_{i,VoV}$. The five portfolios sorted on $\beta_{i,VIX}$ are then averaged over each of the five $\beta_{i,VoV}$ portfolios, resulting $\beta_{i,VIX}$ quintile portfolios controlling for $\beta_{i,VoV}$. Similar approach gives $\beta_{i,VoV}$ quintile portfolios controlling for $\beta_{i,VIX}$. Controlling for volatility risk loadings ($\beta_{i,VIX}$), we still find market volatility-of-volatility risk carries a statistically significant return differential of -0.97 percent per month with a $t$-statistic of -2.84. On the other hand, controlling for market volatility-of-volatility risk loadings ($\beta_{i,VoV}$), we find that the return difference between stocks with high volatility risk and stocks with low volatility risk is still large in magnitude, at -0.68 percent per month with $t$-value of -1.94. Thus, the valuation effect of market volatility-of-volatility risk is not affected after controlling for $\beta_{i,VIX}$, suggesting that the market volatility-of-volatility risk is a pricing factor independent with the aggregate volatility factor.

### 1.5.3 Portfolios sorted on market skewness risk

At the end of each month, we estimate the model of Chang, Christoffersen, and Jacobs (2013) with ex ante higher moments of market returns for each stock $i$:

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,SKEW} \Delta SKEW_{t+1} + \beta_{i,KURT} \Delta KURT_{t+1} + \varepsilon_{i,t+1}. \quad (1.34)$$

We sort stocks into quintile portfolios based on $\beta_{i,SKEW}$, from the lowest (quintile 1) to the highest (quintile 5), and we also independently sort stocks into quintile portfolios based on $\beta_{i,VoV}$.

Panel A of Table 1.6 provides the performance of portfolios sorted on $\beta_{i,SKEW}$. We find that there is a significant difference of -0.65 percent per month with a *t*-statistic of -1.99 between the stock returns with high skewness risk and the stocks with low skewness risk. Controlling for Fama and French four factor model, the '5-1' long-short portfolio gives a significant alpha of -0.75 percent per month with a *t*-statistic of -2.28.

Panel B shows the results for the $\beta_{i,SKEW}$ quintile portfolios controlling for $\beta_{i,VoV}$ quintile portfolios. Controlling for market volatility-of-volatility risk loadings ($\beta_{i,VoV}$), we find the market skewness risk premium is much weaker, carrying a statistically insignificant return differential of -0.33 percent per month with a *t*-statistic of -1.17. On the other hand, as reported in Panel C, controlling for market skewness risk loadings ($\beta_{i,VoV}$), we find that the '5-1' long-short portfolio still gives a significant return differential of -0.82 percent per month with a *t*-statistic of -2.38. In summary, the market skewness risk is less likely to explain the market volatility-of-volatility risk, whereas part of the skewness return differential can be explained by the market volatility-of-volatility risk.

### 1.5.4 Market price of volatility-of-volatility risk

We apply the two-pass regressions of Fama-MacBeth (1973) to estimate the price of market volatility-of-volatility risk. Our set of test assets are the 25 portfolios formed on intersection of $\beta_{i,VIX}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios. For each portfolio, we estimate the time-series regression of equation (1.33) using the post-formation daily value-weighted portfolio returns to obtain the post-formation factor loadings. We then conduct the cross-sectional regression:

$$\mathbb{E}[r_p] - r_f = \lambda_{MKT}\beta_{p,MKT} + \lambda_{VIX}\beta_{p,VIX} + \lambda_{VoV}\beta_{p,VoV}. \qquad (1.35)$$

The dependent variable is the monthly value-weighted portfolio return and the independent variables are the post-ranking return betas estimated from equation (1.33) using full-sample daily portfolio returns. Robust Newey and West (1987) *t*-statistics with six lags that account for autocorrelations are used. The cross-sectional regression gives the estimates of risk prices, i.e. $\lambda_{MKT}$, $\lambda_{VIX}$, and $\lambda_{VoV}$.

Panel A of Table 1.7 reports the estimate of risk prices from the 25 portfolios sorted on $\beta_{i,VIX}$ and $\beta_{i,VoV}$. In column [2], we find that $\lambda_{VoV}$ is negative (-4.11) with a significant *t*-statistic of -3.27. Controlling for the market volatility risk, as reported in column [3], $\lambda_{VoV}$ is still significantly negative (-3.66) with a *t*-statistic of -2.35, which accounts for -3.66×0.21= -0.77 percent per month of the '5-1' return in Table 1.2. Controlling for all of the other factors, as shown in column [6], $\lambda_{VoV}$ remains significantly negative (-3.62) with a *t*-statistic of -2.68, which accounts for -3.62×0.21= -0.76 percent. In contrast, $\lambda_{VIX}$ is only significant in column [1], with *t*-value of -3.29. Thus, our empirical findings suggest that market volatility-of-volatility indeed is an independently priced risk factor relative to aggregate volatility factor.

In Panel B, we the estimate of risk prices from the 25 portfolios sorted on intersection of $\beta_{i,SKEW}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios. Consequently, the testing assets are sufficiently disperse in the exposure to aggregate volatility-of-volatility as well as in the exposure to aggregate skewness. In column [2], we find that $\lambda_{VoV}$ is negative

(-2.07) with a significant $t$-statistic of -1.77. As shown in column [6], $\lambda_{VoV}$ is significantly negative (-1.86) with a $t$-statistic of -1.75, whereas $\lambda_{SKEW}$ is positive (2.76) with insignificant $t$-statistic of 1.36. Therefore, relative to the market skewness factor, the variance of market variance remains a priced risk factor.

### 1.5.5 Leverage effect, feedback effect and volatility-of-volatility risk premium

To further explore the mechanism that volatility-of-volatility risk affects asset prices, we investigate whether the volatility-of-volatility risk contributes to the feedback effect. To identify the timely volatility-of-volatility shocks, at the end of each day, we estimate the regression of equation (1.33) using daily stock returns over the past 22 days. We then sort stocks into quintile portfolios on the estimated $\beta_{i,VoV}$ for each day and calculate the event-time daily value-weighted portfolio returns ranging from -11 to 11 in days.

If market volatility-of-volatility is priced, an anticipated increase in market volatility-of-volatility raises the required rate of return, implying an immediate stock price decline and higher future returns. As shown in Figure 1. 2, consistent with the channel of feedback effect, stocks with negative return sensitivities to market volatility-of-volatility have lower returns before the portfolio formation and earn higher post-formation returns than stocks with positive return sensitivities.

We construct a portfolio that is long the lowest quintile and short the highest quintile and we denote the portfolio as low-minus-high. The low-minus-high portfolio has, by construction, large negative exposure to innovations in market volatility-of-volatility. The theory of the leverage effect and the feedback effect predict an asymmetric cross-correlation between the aggregate volatility and the pre-formation and the post-formation low-minus-high returns.

As can be seen in the top panel of Figure 1. 3, the pre-formation low-minus-high returns are negatively correlated with *VIX* measured at the portfolio formation date while the correlations between *VIX* and the post-formation low-minus-high returns are positive, supporting the leverage effect and the feedback effect associated with the aggregate volatility.

Moreover, our theory for the leverage effect and the feedback effect similarly predicts an asymmetric cross-correlation between market volatility-of-volatility and the pre-formation and the post-formation low-minus-high returns. As can be seen in the bottom panel of Figure 1. 3, the low-minus-high return is negatively correlated with *VoV* at the portfolio formation date while the correlation between *VoV* and one-day post-formation low-minus-high return is positive. The market volatility-of-volatility carries a negative contemporaneous correlation of -0.232, which is much larger in magnitude than -0.057 for the contemporaneous leverage effect associated with market volatility. The correlation between one-day post-formation low-minus-high return and market volatility-of-volatility is 0.100, which is larger than 0.060 for the correlation between the return and the market volatility. The stronger asymmetric cross-correlation, despite less persistent, supports the leverage effect and the feedback effect associated with market volatility-of-volatility. Therefore, market volatility-of-volatility seems to be the state variable that determines the time-varying risk premium.

### 1.5.6 Robustness to volatility spreads

In this section, we check whether our results are robust to existing well-known volatility spreads that affect cross-sectional stock returns. We construct the implied-realized volatility spread (*IVOL-TVOL*), which is, as described in Bali and Hovakimian (2009), defined as the average of implied volatilities by at-the-money call and put minus the total volatility calculated using daily returns in the previous month; the call-put implied volatility spread (*CIVOL-PIVOL*), which is, as described in Bali and Hovakimian (2009) and Yan (2011), defined as the at-the-money call implied volatility minus the at-the-money put implied volatility; the expected individual variance risk premium (*EVRP*), which is, as described in the data section and in Han and Zhou (2012), defined as the difference between the model-free implied variance and the five-minute realized variance. Since we extract the volatility data from OptionMetrics Volatility Surface file as Yan (2011) do, we choose the 30-day maturity put and call options with deltas equal to -0.5 and 0.5, respectively.

Panel A of Table 1.8 shows the performance of portfolios sorted on each of the volatility spreads as well as on $\beta_{i,VoV}$ using stocks with available equity options. The Fama and French four factor alpha of the '5-1' long–short portfolio is 0.63 percent with a *t*-statistic of 1.76 for *IVOL-TVOL* quintile portfolios, 1.66 percent with a *t*-statistic of 6.70 for *CIVOL-PIVOL* quintile portfolios, 0.96 percent with a *t*-statistic of -2.21 for *EVRP* quintile portfolios, and -0.85 percent with a *t*-statistic of -2.38 for $\beta_{i,VoV}$ quintile portfolios. Hence, our results for market volatility-of-volatility risk remain significant in the options market and consistent with the literature, all of the three volatility variables carry significant premium in the cross-section.

We construct two-way sorted portfolios formed on intersection of each of the volatility spread quintile portfolios and $\beta_{i,VoV}$ quintile portfolios. Panel B shows the results for the $\beta_{i,VoV}$ quintile portfolios controlling for volatility spread quintile portfolios. The Fama and French four factor alpha of the '5-1' long–short portfolio remains significant controlling for these three variables, i.e. at -0.76 percent with a *t*-statistic of -2.38 controlling for *IVOL-TVOL*, at -0.74 percent with a *t*-statistic of -2.29 controlling for *CIVOL-PIVOL*, and at -0.66 percent with a *t*-statistic of -2.20 controlling for *EVRP*. Hence, the low returns to high $\beta_{i,VoV}$ stocks are not driven by the existing well-known volatility spreads.

As shown by Yan (2011), *CIVOL-PIVOL* is proxy for a disaster type jump risk that affects the cross-sectional stock returns. Our empirical finding that the pricing of $\beta_{i,VoV}$ is robust to *CIVOL-PIVOL* provides indirect evidence that the market volatility-of-volatility risk cannot be completely explained by a peso-problem like jump risk.

### 1.5.7 Robustness to firm-level Fama-MacBeth regressions

In this section, we examine whether the pricing of market volatility-of-volatility risk is robust to the firm-level analysis. We employ individual stocks as the set of test assets to avoid potentially spurious results that could arise when the test portfolios are constructed toward a specific model (Lewellen, Nagel, and Shanken, 2010). Furthermore, a stock-level analysis could increase the power of the test by controlling for several individual characteristics at the same time.

We test our market-based three factor model at firm-level with the following cross-sectional regression:

$$\begin{aligned} r_{i,t+1} - r_{f,t+1} = c_0 + \lambda_{MKT}\beta_{i,MKT,t} + \lambda_{VIX}\beta_{i,VIX,t} + \lambda_{VoV}\beta_{i,VoV,t} \\ + c_{FIRM}\, FirmCharac_{i,t} + c_{VOL}\, VolatilityCharac_{i,t} + \varepsilon_{i,t+1}, \end{aligned} \quad (1.36)$$

where the dependent variable is the monthly individual stock returns; $\beta_{i,MKT,t}$, $\beta_{i,VIX,t}$, and $\beta_{i,VoV,t}$ are post-ranking betas estimated from the same 25 portfolios in section 5.4 formed on intersection of $\beta_{i,VIX}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios;

$FirmCharac_{i,t}$ consists of *Size*, *B/M*, *RET_2_12*, *RET_1*, and *ILLIQ*; and $VolatilityCharac_{i,t}$ includes *IVOL-TVOL*, *CIVOL-PIVOL*, and *EVRP*. Robust Newey and West (1987) $t$-statistics with six lags that account for autocorrelations are used. Following the methodology of Fama and French (1992), we assign each of the 25 portfolio-level post-ranking beta estimates to the individual stocks within the portfolio at that time. Thus, individual stock betas vary over time because the portfolio compositions change each month.

Table 1.9 reports the results from the firm-level Fama-MacBeth regressions. In column [2], we find that $\lambda_{VoV}$ is negative (-3.07) with a significant *t*-statistic of -4.07. Controlling for the market volatility risk, as reported in column [3], $\lambda_{VoV}$ is still significantly negative (-3.07) with a *t*-statistic of -4.17, which accounts for -3.07×0.21= -0.65 percent per month of the '5-1' return in Table 1.2. Controlling for all of the other variables, as shown in column [6], $\lambda_{VoV}$ remains significantly negative (-3.12) with a *t*-statistic of -2.57, which accounts for -3.62×0.21= -0.66 percent. Thus, the firm-level evidence confirms our results that the market volatility-of-volatility is a priced risk factor in the cross-sectional stock returns.

## 1.6 Pricing market volatility-of-volatility in the cross-sectional variance risk premium

The second test in this paper is to examine whether market volatility-of-volatility is priced in the cross-sectional variance risk premium. For each stock with available equity options in each day, we calculate the 30-day model-free implied variance ($IV_{i,t+1}$). Then, at the end of each month, we estimate the variance beta with respect to market volatility-of-volatility ($\beta_{i,VoV}^{V}$) for each stock by regressing the stock's $IV_{i,t+1}$ on $VoV_{t+1}$ over the past month. We use $\beta_{i,VoV}^{V}$ to construct a set of test portfolios. Our theory suggests that the cross-sectional expected variance risk premium is determined by:

$$EVRP_p \equiv IV_p - ERV_p = -\lambda_{VIX}^{V}\beta_{p,VIX}^{V} - \lambda_{VoV}^{V}\beta_{p,VoV}^{V}. \quad (1.37)$$

We estimate $\beta_{p,VIX}^{V}$ and $\beta_{p,VoV}^{V}$ by the following time-series regression:

$$IV_{p,t+1} = \alpha_p^V + \beta_{p,VIX}^{V}\Delta\widetilde{VIX}_{t+1}^2 + \beta_{p,VoV}^{V}\Delta VoV_{t+1} + \varepsilon_{p,t+1}^{V}, \quad (1.38)$$

where $IV_{p,t+1}$ is the post-formation portfolio implied variance; $\Delta\widetilde{VIX}_{t+1}^2$ as the innovations in market variance, which is measured as the ARMA(1,1) model residuals of squared $VIX$ divided by 12, orthogonalized by $\Delta VoV_{t+1}$. While we define $\Delta VIX_{t+1} = VIX_{t+1} - VIX_t$ for the stock return beta as in Ang, Hodrick, Xing, and Zhang (2006) for the compatibility, our variance beta is estimated by $\Delta\widetilde{VIX}_{t+1}^2$ for the model consistency.

Table 1.10 provides the performance of portfolios sorted on $\beta_{i,VoV}^{V}$. Stocks are sorted into quintile portfolios based on $\beta_{i,VoV}^{V}$, from the lowest (quintile 1) to the highest (quintile 5). After the portfolio formation, we calculate monthly value-weighted expected variance risk premium and daily value-weighted model-free implied variance for each portfolio. Consistent with the model, we find that stocks with high variance sensitivities to market volatility-of-volatility (quintile 5) have higher expected variance risk premium than stocks with low variance sensitivities (quintile 1) by 67.7 (in percentages squared) per month with t- statistic of -5.15. The magnitude of the cross-sectional difference in variance risk premium is large compared to the market variance risk premium, which is 17.3 (in percentages squared) per month during our sample period. Panel B reports the performance of portfolios sorted on $\beta_{i,VIX}^{V}$. The results are very similar to the portfolios sorted on $\beta_{i,VoV}^{V}$. In fact, we find that the cross-sectional Spearman correlation between

$\beta^V_{i,VoV}$ and $\beta^V_{i,VIX}$ is 0.99, which is also part of the reason why we use the orthogonalized innovations in market variance as risk factor for individual variance.

We estimate the price of risks in cross-sectional *EVRP* using the 25 portfolios sorted on $\beta^V_{i,VoV}$. We apply the two-pass regressions of Fama-MacBeth (1973) to estimate the price of market volatility-of-volatility risk in *EVRP*. After the portfolio formation, we calculate monthly value-weighted expected variance risk premium, daily value-weighted model-free implied variance, and daily value-weighted stock returns for each portfolio. In the first stage, for each portfolio, we estimate the post-ranking variance betas by equation (1.38) using daily portfolio implied variance. For the second stage, we regress the cross-sectional monthly portfolio *EVRP* on variance betas obtained from the first stage, using Fama–MacBeth (1973) cross-sectional regression by equation (1.37).

Table 1.11 reports the estimate of risk prices in *EVRP* from the 25 portfolios sorted on $\beta^V_{i,VoV}$. In column [2], we find that $-\lambda^V_{VoV}$ is positive (5.99) with a significant *t*-statistic of 5.32. Controlling for the market volatility risk ($\beta^V_{p,VIX}$), as reported in column [3], $-\lambda^V_{VoV}$ is still significantly positive (5.20) with a *t*-statistic of 4.04, which accounts for 5.20×8.32= 43.3 (in percentages squared) per month of the '5-1' *EVRP* in Table 1.10. Controlling for all of the other factors, as shown in column [6], $-\lambda^V_{VoV}$ remains significantly positive (1.53) with a *t*-statistic of 2.27, which accounts for 1.53×8.32= 12.7 (in percentages squared). Thus, our empirical findings suggest that market volatility-of-volatility is priced risk factor in the cross-sectional variance risk premium.

## 1.7 Return predictability

In this section, we check the return predictability afforded by market volatility-of-volatility. The theoretical model suggests that market volatility-of-volatility is positively related to economic volatility-of-volatility. Hence, we should expect that our *VoV* measure can predict future stock returns as market variance risk premium does.

Panel A of Table 1.12 reports the estimates of the one-period return predictability regression using daily S&P 500 logarithmic returns multiplied by 22 on the lagged variance risk premium (*VRP*), market volatility-of-volatility (*VoV*), and innovations in market skewness (*ΔSKEW*). Robust Newey and West (1987) *t*-statistics with sixteen lags that account for autocorrelations are used. Consistent with the theory, we find that *VoV* positively predicts one-period ahead market return in all of the specifications. In panel B, we use the monthly S&P 500 logarithmic returns as the dependent variable, and the independent variables are sampled at the end of the previous month. Robust Newey and West (1987) *t*-statistics with six lags are used. The predictability afforded by *VoV* remains significant.

Overall, the return predictability supports the volatility-of-volatility feedback effect implied by our model. The evidence for the predictability afforded by the market volatility-of-volatility suggests that economic volatility-of-volatility is an important state variable that affects the aggregate asset prices.

## 1.8 Conclusions

Market volatility-of-volatility appears to be a state variable that is important for asset pricing. We develop a market-based three-factor model, in which market risk, market volatility risk, and market volatility-of-volatility risk determine the cross-sectional asset prices. We find that market volatility-of-volatility risk is priced in the cross-sectional stock returns. Stocks with negative larger return exposure to market volatility-of-volatility have substantially higher future stock returns, even after we account for exposures to the

Fama and French four factors, market skewness factor, firm characteristics and volatility characteristics. We also find that market volatility-of-volatility risk is priced in the cross-sectional variance risk premium.

Our measure of market volatility-of-volatility generates leverage effect and feedback effect. Stocks with negative larger return exposure to market volatility-of-volatility have substantially lower contemporaneous stock returns, which suggests that market volatility-of-volatility is priced such that an anticipated increase in market volatility-of-volatility risk raises the required rate of return, leading to an immediate stock price decline and higher future returns. Our evidence on return predictability for the aggregate market portfolio supports feedback effect implied by our model. The predictability evidence afforded by the market volatility-of-volatility also suggests that economic volatility-of-volatility is an important state variable.

Our study shows that market volatility-of-volatility risk affects the cross-sectional expected variance risk premium. One direction for future research is to explore whether market volatility-of-volatility risk plays a role in tradable volatility-related assets such as equity option returns or index option returns. Future research could also investigate whether our measure of market volatility-of-volatility affects the VIX option returns.

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# Table 1. 1: Properties of the daily factors

We report summary statistics and Spearman correlations for the daily factors, including Fama and French (1993) four factors (*MKT*, *SMB*, *HML*, and *UMD*), the market variance risk premium (*VRP*), the *VIX* index, our measure of variance of market variance (*VoV*), and Chang, Christoffersen, and Jacobs (2013) market skewness factor (*SKEW*) and market kurtosis factor (*KURT*). *ΔVIX* is the first difference of *VIX*. *ΔVoV*, *ΔSKEW*, and *ΔKURT* are the residuals from fitting an ARMA(1,1) regression using *VoV*, *SKEW*, and *KURT*, respectively. The sample period is from January 1996 to December 2010.

| | *Panel A: Summary statistics* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | *MKT(%)* | *SMB(%)* | *HML(%)* | *UMD(%)* | *VRP(%)* | *VIX(%)* | *VoV(%)* | *SKEW* | *KURT* |
| *Mean* | 0.023 | 0.010 | 0.016 | 0.024 | 17.260 | 23.098 | 0.054 | -1.663 | 9.313 |
| *Median* | 0.070 | 0.030 | 0.020 | 0.070 | 15.082 | 22.150 | 0.003 | -1.637 | 8.672 |
| *Std.Dev.* | 1.300 | 0.629 | 0.682 | 1.035 | 21.182 | 9.509 | 0.563 | 0.485 | 3.466 |

| | *Panel B: Spearman correlation* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | *MKT* | *SMB* | *HML* | *UMD* | *VRP* | *ΔVIX* | *ΔVoV* | *ΔSKEW* | *ΔKURT* |
| *MKT* | 1.000 | | | | | | | | |
| *SMB* | 0.038 | 1.000 | | | | | | | |
| *HML* | -0.279 | -0.082 | 1.000 | | | | | | |
| *UMD* | -0.047 | 0.053 | -0.078 | 1.000 | | | | | |
| *VRP* | -0.222 | -0.050 | 0.006 | 0.062 | 1.000 | | | | |
| *ΔVIX* | -0.779 | 0.031 | 0.210 | 0.020 | 0.180 | 1.000 | | | |
| *ΔVoV* | -0.044 | -0.003 | -0.038 | 0.036 | 0.145 | 0.049 | 1.000 | | |
| *ΔSKEW* | -0.237 | -0.022 | 0.026 | 0.034 | 0.076 | 0.248 | -0.017 | 1.000 | |
| *ΔKURT* | 0.311 | 0.014 | -0.057 | -0.017 | -0.106 | -0.307 | -0.010 | -0.863 | 1.000 |

# Table 1. 2: Portfolios sorted on $\boldsymbol{\beta_{i,VoV}}$

At the end of each month, we run the following regression for each stock using daily returns:

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,VoV} \Delta VoV_{t+1} + \varepsilon_{i,t+1}.$$

We sort stocks into quintile portfolios based on $\beta_{i,VoV}$, from the lowest (quintile 1) to the highest (quintile 5). After the portfolio formation, we calculate the value-weighted daily and monthly stock returns for each portfolio. The column "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. For each portfolio, we estimate the same time-series regression as above using the post-formation daily portfolio returns to obtain the post-formation factor loadings. We compute the risk-adjusted return of each portfolio with respect to Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*) from the intercept estimate of a time-series regression of the monthly portfolio returns on the four factors. Numbers in parentheses are *t*-statistics. *Size* reports the average market capitalization (in billion) for firms within the portfolio; *B/M* reports the average book-to-market ratios; *RET_2_12* reports the average of past 11-month returns prior to last month; *ILLIQ* reports the average of Amihud's (2002) illiquidity measure. The sample period is from January 1996 to December 2010.

| | *Portfolios ranking* | | | | | |
|---|---|---|---|---|---|---|
| | *1* | *2* | *3* | *4* | *5* | *5-1* |
| *Risk-adjusted performance of* $\beta_{i,VoV}$ *sorted portfolios (monthly return)* | | | | | | |
| *Excess return* | 0.90 | 0.64 | 0.40 | 0.34 | 0.02 | -0.88 |
| | ( 1.61) | ( 1.70) | ( 1.20) | ( 0.92) | ( 0.04) | (-2.32) |
| *α-CAPM* | 0.34 | 0.23 | 0.03 | -0.07 | -0.54 | -0.89 |
| | ( 1.21) | ( 2.01) | ( 0.33) | (-0.57) | (-2.54) | (-2.32) |
| *α-FF3* | 0.28 | 0.21 | 0.02 | -0.07 | -0.54 | -0.82 |
| | ( 1.13) | ( 1.89) | ( 0.25) | (-0.67) | (-2.59) | (-2.18) |
| *α-FF4* | 0.44 | 0.22 | 0.01 | -0.08 | -0.53 | -0.96 |
| | ( 1.88) | ( 1.99) | ( 0.05) | (-0.74) | (-2.50) | (-2.59) |
| *Post-formation factor loadings (daily return)* | | | | | | |
| $\beta_{p,MKT}$ | 1.35 | 1.00 | 0.90 | 0.96 | 1.32 | -0.02 |
| | ( 78.37) | ( 125.34) | ( 145.90) | ( 127.57) | ( 79.40) | (-0.91) |
| $\beta_{p,VIX}$ | 0.06 | -0.01 | -0.02 | -0.01 | 0.06 | -0.01 |
| | ( 5.34) | (-1.42) | (-4.70) | (-2.48) | ( 4.86) | (-0.41) |
| $\beta_{p,VoV}$ | -0.04 | -0.03 | 0.01 | 0.02 | 0.16 | 0.21 |
| | (-1.93) | (-3.00) | ( 1.47) | ( 2.27) | ( 7.35) | ( 5.95) |
| *Pre-formation characteristics* | | | | | | |
| *Size*($b) | 1.03 | 2.87 | 3.49 | 3.24 | 1.33 | 0.30 |
| *B/M* | 1.19 | 0.88 | 0.84 | 0.83 | 1.10 | -0.09 |
| *RET_2_12* | 12.31 | 14.96 | 15.11 | 14.37 | 11.76 | -0.56 |
| *ILLIQ*($10^6$) | 9.04 | 3.06 | 2.47 | 3.05 | 8.87 | -0.17 |
| *Pre-formation factor loadings* | | | | | | |
| $\beta_{i,MKT}$ | 1.23 | 0.97 | 0.91 | 0.99 | 1.28 | 0.05 |
| $\beta_{i,VIX}$ | 0.06 | 0.00 | -0.01 | -0.01 | 0.01 | -0.05 |
| $\beta_{i,VoV}(10^2)$ | -6.23 | -2.18 | -0.15 | 1.86 | 5.69 | 11.92 |

## Table 1. 3: Portfolios sorted on $\boldsymbol{\beta_{i,VoV}}$, controlling for Size, B/M, momentum, reversal, and illiquidity

This table shows performance of portfolios sorted on $\beta_{i,VoV}$, controlling for market capitalization (*Size*), book-to-market ratio (*B/M*), past 11-month returns (*RET_2_12*), past 1-month return (*RET_1*), and Amihud's illiquidity (*ILLIQ*), respectively. We first sort stocks into five quintiles based on their market capitalization (*Size*). Then, within each quintile, we sort stocks based on their $\beta_{i,VoV}$ loadings into five portfolios. All portfolios are rebalanced monthly and are value weighted. The five portfolios sorted on $\beta_{i,VoV}$ are then averaged over each of the five *Size* portfolios, resulting $\beta_{i,VoV}$ quintile portfolios controlling for *Size*. We compute the risk-adjusted return of each portfolio with respect to Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*). *B/M*, *RET_2_12*, *RET_1*, and *ILLIQ* are analyzed with the same procedure as described above. Numbers in parentheses are *t*-statistics.

| | *Portfolios ranking on* $\beta_{i,VoV}$ | | | | | | *α-FF4* |
|---|---|---|---|---|---|---|---|
| | *1* | *2* | *3* | *4* | *5* | *5-1* | *5-1* |
| *Controlling for Size* | 0.81 | 0.94 | 0.76 | 0.75 | 0.39 | -0.42 | -0.45 |
| | ( 1.30) | ( 2.12) | ( 2.00) | ( 1.80) | ( 0.70) | (-1.93) | (-2.14) |
| *Controlling for B/M* | 1.10 | 0.71 | 0.49 | 0.47 | 0.29 | -0.81 | -0.88 |
| | ( 2.19) | ( 1.95) | ( 1.48) | ( 1.31) | ( 0.59) | (-2.81) | (-3.15) |
| *Controlling for RET_2_12* | 0.53 | 0.52 | 0.41 | 0.29 | 0.04 | -0.49 | -0.52 |
| | ( 1.05) | ( 1.21) | ( 0.99) | ( 0.69) | ( 0.07) | (-2.01) | (-2.12) |
| *Controlling for RET_1* | 0.69 | 0.64 | 0.46 | 0.43 | 0.11 | -0.59 | -0.61 |
| | ( 1.29) | ( 1.54) | ( 1.20) | ( 1.08) | ( 0.20) | (-1.95) | (-2.06) |
| *Controlling for ILLIQ* | 0.79 | 0.81 | 0.75 | 0.67 | 0.28 | -0.51 | -0.53 |
| | ( 1.38) | ( 2.09) | ( 2.24) | ( 1.84) | ( 0.56) | (-2.18) | (-2.30) |

# Table 1. 4: Portfolios sorted on $\boldsymbol{\beta_{i,VIX}}$

At the end of each month, we run the following regression for each stock using daily returns:

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,VoV} \Delta VoV_{t+1} + \varepsilon_{i,t+1}.$$

We sort stocks into quintile portfolios based on $\beta_{i,VIX}$, from the lowest (quintile 1) to the highest (quintile 5). After the portfolio formation, we calculate the value-weighted daily and monthly stock returns for each portfolio. The column "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. For each portfolio, we estimate the same time-series regression as above using the post-formation daily portfolio returns to obtain the post-formation factor loadings. We compute the risk-adjusted return of each portfolio with respect to Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*) from the intercept estimate of a time-series regression of the monthly portfolio returns on the four factors. Numbers in parentheses are *t*-statistics. *Size* reports the average market capitalization (in billion) for firms within the portfolio; B/M reports the average book-to-market ratios; *RET_2_12* reports the average of past 11 month returns prior to last month; *ILLIQ* reports the average of Amihud's (2002) illiquidity measure. The sample period is from January 1996 to December 2010.

| | *Portfolios ranking* | | | | | |
|---|---|---|---|---|---|---|
| | *1* | *2* | *3* | *4* | *5* | *5-1* |
| *Risk-adjusted performance of* $\beta_{i,VIX}$ *sorted portfolios (monthly return)* | | | | | | |
| *Excess return* | 0.76 | 0.58 | 0.37 | 0.38 | -0.11 | -0.87 |
| | ( 1.55) | ( 1.69) | ( 1.12) | ( 0.95) | (-0.18) | (-2.17) |
| *α-CAPM* | 0.25 | 0.20 | 0.00 | -0.07 | -0.72 | -0.96 |
| | ( 1.17) | ( 1.99) | ( 0.01) | (-0.70) | (-2.49) | (-2.46) |
| *α-FF3* | 0.30 | 0.23 | -0.03 | -0.11 | -0.79 | -1.09 |
| | ( 1.41) | ( 2.52) | (-0.40) | (-1.20) | (-3.41) | (-2.99) |
| *α-FF4* | 0.41 | 0.25 | -0.05 | -0.14 | -0.77 | -1.18 |
| | ( 2.06) | ( 2.77) | (-0.60) | (-1.52) | (-3.27) | (-3.24) |
| *Post-formation factor loadings (daily return)* | | | | | | |
| $\beta_{p,MKT}$ | 1.20 | 0.91 | 0.89 | 1.05 | 1.47 | 0.27 |
| | ( 82.06) | ( 132.82) | ( 145.66) | ( 141.97) | ( 79.37) | ( 11.04) |
| $\beta_{p,VIX}$ | 0.02 | -0.03 | -0.03 | 0.01 | 0.12 | 0.10 |
| | ( 1.62) | (-7.08) | (-7.37) | ( 1.44) | ( 9.25) | ( 6.00) |
| $\beta_{p,VoV}$ | 0.08 | 0.00 | -0.03 | -0.02 | 0.07 | -0.02 |
| | ( 4.28) | ( 0.29) | (-4.19) | (-2.35) | ( 2.73) | (-0.48) |
| *Pre-formation characteristics* | | | | | | |
| *Size*($b) | 1.28 | 3.39 | 3.63 | 2.67 | 0.99 | -0.29 |
| *B/M* | 1.13 | 0.88 | 0.82 | 0.83 | 1.19 | 0.07 |
| *RET_2_12* | 10.88 | 14.82 | 15.39 | 15.02 | 12.33 | 1.46 |
| *ILLIQ*($10^6$) | 9.42 | 2.76 | 2.28 | 2.72 | 9.31 | -0.12 |
| *Pre-formation factor loadings* | | | | | | |
| $\beta_{i,MKT}$ | -0.08 | 0.53 | 0.95 | 1.52 | 2.75 | 2.83 |
| $\beta_{i,VIX}$ | -1.23 | -0.40 | 0.03 | 0.48 | 1.40 | 2.63 |
| $\beta_{i,VoV}$($10^2$) | -0.32 | -0.17 | 0.00 | 0.18 | -0.10 | 0.22 |

## Table 1. 5: Two-way sorted portfolios on $\boldsymbol{\beta_{i,VIX}}$ and $\boldsymbol{\beta_{i,VoV}}$

At the end of each month, we run the following regression for each stock using daily returns:

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,VIX} \Delta VIX_{t+1} + \beta_{i,VoV} \Delta VoV_{t+1} + \varepsilon_{i,t+1}.$$

We sort stocks into quintile portfolios based on $\beta_{i,VIX}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on $\beta_{i,VoV}$. All portfolios are rebalanced monthly and are value weighted. The five portfolios sorted on $\beta_{i,VIX}$ are then averaged over each of the five $\beta_{i,VoV}$ portfolios, resulting $\beta_{i,VIX}$ quintile portfolios controlling for $\beta_{i,VoV}$. Similar approach gives $\beta_{i,VoV}$ quintile portfolios controlling for $\beta_{i,VIX}$. The column "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. For each portfolio, we estimate the same time-series regression as above using the post-formation daily portfolio returns to obtain the post-formation factor loadings. We compute the risk-adjusted return of each portfolio with respect to Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*). Numbers in parentheses are *t*-statistics. Panel A and Panel B present the results for $\beta_{i,VIX}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios, respectively. The sample period is from January 1996 to December 2010.

| | *Portfolios ranking* | | | | | |
|---|---|---|---|---|---|---|
| | *1* | *2* | *3* | *4* | *5* | *5-1* |
| | | | *Panel A: Ranking on* $\beta_{i,VIX}$*, controlling for* $\beta_{i,VoV}$ | | | |
| *Excess return* | 0.66 | 0.58 | 0.48 | 0.40 | -0.02 | -0.68 |
| | ( 1.34) | ( 1.50) | ( 1.30) | ( 0.90) | (-0.03) | (-1.94) |
| *α-FF4* | 0.25 | 0.20 | 0.03 | -0.17 | -0.70 | -0.95 |
| | ( 1.38) | ( 2.14) | ( 0.38) | (-1.49) | (-3.37) | (-2.97) |
| | | | Panel B: *Ranking on* $\beta_{i,VoV}$*, controlling for* $\beta_{i,VIX}$ | | | |
| *Excess return* | 0.90 | 0.65 | 0.35 | 0.28 | -0.08 | -0.97 |
| | ( 1.57) | ( 1.56) | ( 0.94) | ( 0.70) | (-0.14) | (-2.84) |
| *α-FF4* | 0.39 | 0.18 | -0.09 | -0.20 | -0.66 | -1.05 |
| | ( 1.82) | ( 1.50) | (-0.91) | (-1.83) | (-3.43) | (-3.16) |

## Table 1. 6: Two-way sorted portfolios on $\beta_{i,SKEW}$ and $\beta_{i,VoV}$

At the end of each month, we separately run the following regressions for each stock using daily returns:

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT}MKT_{t+1} + \beta_{i,VIX}\Delta VIX_{t+1} + \beta_{i,SKEW}\Delta SKEW_{t+1} + \beta_{i,KURT}\Delta KURT_{t+1} + \varepsilon_{i,t+1}$$

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_{i,MKT}MKT_{t+1} + \beta_{i,VIX}\Delta VIX_{t+1} + \beta_{i,VoV}\Delta VoV_{t+1} + \varepsilon_{i,t+1}.$$

We sort stocks into quintile portfolios based on $\beta_{i,SKEW}$, from the lowest (quintile 1) to the highest (quintile 5), and independently sort stocks into quintile portfolios based on $\beta_{i,VoV}$. All portfolios are rebalanced monthly and are value weighted. The five portfolios sorted on $\beta_{i,SKEW}$ are then averaged over each of the five $\beta_{i,VoV}$ portfolios, resulting $\beta_{i,SKEW}$ quintile portfolios controlling for $\beta_{i,VoV}$. Similar approach gives $\beta_{i,VoV}$ quintile portfolios controlling for $\beta_{i,SKEW}$. The column "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. For each portfolio, we estimate the same time-series regression as above using the post-formation daily portfolio returns to obtain the post-formation factor loadings. We compute the risk-adjusted return of each portfolio with respect to Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*). Numbers in parentheses are *t*-statistics. Panel A presents the results for the $\beta_{i,SKEW}$ quintile portfolios. Panel B shows the results for the $\beta_{i,SKEW}$ quintile portfolios controlling for $\beta_{i,VoV}$ quintile portfolios while Panel C shows the results for the $\beta_{i,VoV}$ quintile portfolios controlling for $\beta_{i,SKEW}$ quintile portfolios. The sample period is from January 1996 to December 2010.

| | *Portfolios ranking* | | | | | |
|---|---|---|---|---|---|---|
| | *1* | *2* | *3* | *4* | *5* | *5-1* |
| | | | *Panel A: Ranking on* $\beta_{i,SKEW}$ | | | |
| *Excess return* | 0.88 | 0.43 | 0.40 | 0.37 | 0.23 | -0.65 |
| | ( 1.67) | ( 1.14) | ( 1.21) | ( 1.00) | ( 0.43) | (-1.99) |
| *α-FF4* | 0.41 | 0.04 | 0.02 | -0.08 | -0.34 | -0.75 |
| | ( 1.98) | ( 0.35) | ( 0.23) | (-0.77) | (-1.70) | (-2.28) |
| | | | *Panel B: Ranking on* $\beta_{i,SKEW}$*, controlling for* $\beta_{i,VoV}$ | | | |
| *Excess return* | 0.66 | 0.52 | 0.35 | 0.55 | 0.33 | -0.33 |
| | ( 1.23) | ( 1.22) | ( 0.94) | ( 1.32) | ( 0.62) | (-1.17) |
| *α-FF4* | 0.18 | 0.05 | -0.11 | 0.08 | -0.27 | -0.45 |
| | ( 0.93) | ( 0.50) | (-1.30) | ( 0.74) | (-1.51) | (-1.59) |
| | | | *Panel C: Ranking on* $\beta_{i,VoV}$*, controlling for* $\beta_{i,SKEW}$ | | | |
| *Excess return* | 0.88 | 0.67 | 0.38 | 0.43 | 0.05 | -0.82 |
| | ( 1.51) | ( 1.63) | ( 1.03) | ( 1.08) | ( 0.09) | (-2.38) |
| *α-FF4* | 0.31 | 0.23 | -0.06 | -0.05 | -0.51 | -0.82 |
| | ( 1.46) | ( 2.12) | (-0.58) | (-0.47) | (-2.48) | (-2.44) |

# Table 1. 7: The price of volatility-of-volatility risk

Panel A reports the Fama–MacBeth (1973) factor premiums on 25 portfolios sorted on intersection of $\beta_{i,VIX}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios, using our market-based three factors (*MKT*, *ΔVIX,* and *ΔVoV*), Chang, Christoffersen, and Jacobs (2013) market skewness factor (*ΔSKEW*), and Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*). We estimate the first stage return betas using the daily full-sample post-formation value-weighted returns. Then, we regress the cross-sectional monthly portfolio returns on daily return betas from the first stage, using Fama–MacBeth (1973) cross-sectional regression. Panel B reports the Fama–MacBeth (1973) factor premiums on 25 portfolios sorted on intersection of $\beta_{i,SKEW}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios. Robust Newey and West (1987) *t*-statistics with six lags that account for autocorrelations are reported in parentheses. The sample period is from January 1996 to December 2010.

| | *Fama-MacBeth cross-sectional regressions* | | | | | |
|---|---|---|---|---|---|---|
| | [1] | [2] | [3] | [4] | [5] | [6] |
| | *Panel A: 25 portfolios sorted on* $\beta_{i,VIX}\times\beta_{i,VoV}$ (5×5) | | | | | |
| *MKT* | 0.54 | 0.53 | 0.57 | 0.55 | 0.55 | 0.55 |
| | ( 1.34) | ( 1.28) | ( 1.41) | ( 1.37) | ( 1.36) | ( 1.36) |
| *ΔVIX* | -5.31 | | -3.14 | -3.87 | -3.97 | -5.00 |
| | (-3.29) | | (-1.61) | (-0.94) | (-0.95) | (-1.14) |
| *ΔVoV* | | -4.11 | -3.66 | -3.84 | -3.88 | -3.62 |
| | | (-3.25) | (-2.35) | (-2.68) | (-2.66) | (-2.68) |
| *SMB* | | | | -0.93 | -0.94 | -0.94 |
| | | | | (-1.25) | (-1.29) | (-1.28) |
| *HML* | | | | -0.27 | -0.17 | -0.45 |
| | | | | (-0.40) | (-0.27) | (-0.66) |
| *UMD* | | | | -1.73 | -1.65 | -1.02 |
| | | | | (-1.58) | (-1.47) | (-0.96) |
| *ΔSKEW* | | | | | 0.52 | 0.49 |
| | | | | | ( 0.50) | ( 0.48) |
| *ΔKURT* | | | | | | -15.54 |
| | | | | | | (-1.13) |
| *Adj.* $R^2$ | 0.13 | 0.10 | 0.18 | 0.24 | 0.24 | 0.25 |

## Table 1.7 (continued.)

| | *Fama-MacBeth cross-sectional regressions* | | | | | |
|---|---|---|---|---|---|---|
| | [1] | [2] | [3] | [4] | [5] | [6] |
| | *Panel B: 25 portfolios sorted on* $\beta_{i,SKEW} \times \beta_{i,VoV}$ (5×5) | | | | | |
| *MKT* | 0.48 | 0.45 | 0.61 | 0.43 | 0.64 | 0.64 |
| | ( 1.16) | ( 1.12) | ( 1.57) | ( 1.06) | ( 1.64) | ( 1.63) |
| *ΔVIX* | | -0.31 | 4.51 | -0.79 | 8.47 | 8.46 |
| | | (-0.17) | ( 1.27) | (-0.45) | ( 1.73) | ( 1.62) |
| *ΔVoV* | -2.00 | -2.07 | -1.76 | | -1.87 | -1.86 |
| | (-1.84) | (-1.77) | (-1.77) | | (-1.86) | (-1.75) |
| *SMB* | | | -0.31 | | -0.51 | -0.50 |
| | | | (-0.37) | | (-0.60) | (-0.58) |
| *HML* | | | -1.40 | | -0.87 | -0.87 |
| | | | (-2.30) | | (-1.32) | (-1.24) |
| *UMD* | | | 1.70 | | 2.33 | 2.33 |
| | | | ( 1.09) | | ( 1.44) | ( 1.43) |
| *ΔSKEW* | | | | 2.73 | 2.77 | 2.76 |
| | | | | ( 2.53) | ( 1.45) | ( 1.36) |
| *ΔKURT* | | | | 5.69 | | 0.58 |
| | | | | ( 0.59) | | ( 0.05) |
| *Adj.* $R^2$ | 0.10 | 0.15 | 0.26 | 0.13 | 0.26 | 0.26 |

## Table 1. 8: Two-way portfolios sorted on volatility spreads and $\beta_{i,VoV}$

This table shows performance of portfolios sorted on implied-realized volatility spread (*IVOL-TVOL*), the call-put implied volatility spread (*CIVOL-PIVOL*), the expected individual variance risk premium (*EVRP*), and $\beta_{i,VoV}$ using stocks with available equity options. We independently sort stocks into quintile portfolios based on each of the four variables, from the lowest (quintile 1) to the highest (quintile 5). All portfolios are rebalanced monthly and are value weighted. We compute the risk-adjusted return of each portfolio with respect to Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*). Panel A reports the results for the one-way sorted portfolios. We construct two-way sorted portfolios formed on intersection of each of the volatility spread quintile portfolios and $\beta_{i,VoV}$ quintile portfolios. Panel B shows the results for the $\beta_{i,VoV}$ quintile portfolios controlling for volatility spread quintile portfolios. Numbers in parentheses are *t*-statistics. The sample period is from January 1996 to December 2010.

| | *Portfolios ranking* | | | | | | *α-FF4* |
|---|---|---|---|---|---|---|---|
| | *1* | *2* | *3* | *4* | *5* | *5-1* | *5-1* |
| *Panel A: One-way sorted portfolios* | | | | | | | |
| *Ranking on* $\beta_{i,VoV}$ | 0.88 | 0.64 | 0.45 | 0.24 | 0.15 | -0.73 | -0.85 |
| | ( 1.68) | ( 1.80) | ( 1.31) | ( 0.63) | ( 0.28) | (-2.02) | (-2.38) |
| *Ranking on IVOL-TVOL* | -0.04 | 0.45 | 0.51 | 0.89 | 0.73 | 0.77 | 0.63 |
| | (-0.08) | ( 1.17) | ( 1.49) | ( 2.38) | ( 1.47) | ( 2.10) | ( 1.76) |
| *Ranking on CIVOL-PIVOL* | -0.24 | 0.17 | 0.44 | 0.69 | 1.15 | 1.39 | 1.66 |
| | (-0.51) | ( 0.47) | ( 1.20) | ( 1.83) | ( 2.33) | ( 5.21) | ( 6.70) |
| *Ranking on EVRP* | -0.24 | 0.29 | 0.55 | 0.73 | 0.93 | 1.16 | 0.96 |
| | (-0.39) | ( 0.76) | ( 1.50) | ( 1.65) | ( 1.41) | ( 2.50) | ( 2.21) |
| *Panel B: Two-way sorted portfolios, ranking on* $\beta_{i,VoV}$ | | | | | | | |
| *Controlling for IVOL-TVOL* | 0.97 | 0.77 | 0.45 | 0.32 | 0.28 | -0.69 | -0.76 |
| | ( 1.74) | ( 2.00) | ( 1.21) | ( 0.82) | ( 0.53) | (-2.11) | (-2.38) |
| *Controlling for CIVOL-PIVOL* | 0.82 | 0.60 | 0.50 | 0.31 | 0.16 | -0.66 | -0.74 |
| | ( 1.54) | ( 1.54) | ( 1.40) | ( 0.78) | ( 0.29) | (-2.03) | (-2.29) |
| *Controlling for EVRP* | 0.72 | 0.66 | 0.55 | 0.38 | 0.13 | -0.59 | -0.66 |
| | ( 1.33) | ( 1.52) | ( 1.33) | ( 0.87) | ( 0.25) | (-1.96) | (-2.20) |

# Table 1. 9: Firm-level Fama-MacBeth regressions

This table reports the results for the firm-level Fama-MacBeth regressions. We run the following cross-sectional regression:

$$r_{i,t+1} - r_{f,t+1} = c_0 + \lambda_{MKT}\beta_{i,MKT,t} + \lambda_{VIX}\beta_{i,VIX,t} + \lambda_{VoV}\beta_{i,VoV,t} + c_{FIRM}\, FirmCharac_{i,t} + c_{VOL}\, VolatilityCharac_{i,t} + \varepsilon_{i,t+1},$$

where the dependent variable is the monthly individual stock returns; $\beta_{i,MKT,t}$, $\beta_{i,VIX,t}$, and $\beta_{i,VoV,t}$ are post-ranking betas estimated from the 25 portfolios formed on intersection of $\beta_{i,VIX}$ quintile portfolios and $\beta_{i,VoV}$ quintile portfolios; $FirmCharac_{i,t}$ consists of *Size*, *B/M*, *RET_2_12*, *RET_1*, and *ILLIQ*; $VolatilityCharac_{i,t}$ includes *IVOL-TVOL*, *CIVOL-PIVOL*, and *EVRP*. Following the methodology of Fama and French (1992), we assign each of the 25 portfolio-level post-ranking beta estimates to the individual stocks within the portfolio at that time. Robust Newey and West (1987) *t*-statistics with six lags that account for autocorrelations are reported in parentheses. The sample period is from January 1996 to December 2010.

| | *Fama-MacBeth regressions: individual stocks* | | | | | |
|---|---|---|---|---|---|---|
| | [1] | [2] | [3] | [4] | [5] | [6] |
| *Intercept* | -1.831 | -2.715 | -2.283 | 1.447 | 1.487 | 1.394 |
| | (-1.34) | (-3.57) | (-1.74) | ( 1.11) | ( 1.12) | ( 0.99) |
| *Log(Size($b))* | -0.003 | -0.002 | -0.005 | -0.031 | -0.033 | -0.085 |
| | (-0.05) | (-0.03) | (-0.08) | (-0.40) | (-0.43) | (-0.97) |
| *Log(B/M)* | 0.302 | 0.304 | 0.300 | 0.164 | 0.156 | 0.066 |
| | ( 1.96) | ( 1.97) | ( 1.96) | ( 1.10) | ( 1.05) | ( 0.43) |
| *RET_2_12* | 0.157 | 0.166 | 0.162 | 0.269 | 0.267 | 0.208 |
| | ( 0.40) | ( 0.42) | ( 0.42) | ( 0.60) | ( 0.59) | ( 0.43) |
| *RET_1* | -3.822 | -3.785 | -3.790 | -1.881 | -1.656 | -1.496 |
| | (-5.64) | (-5.62) | (-5.61) | (-2.39) | (-2.13) | (-1.81) |
| *ILLIQ*$(10^6)$ | 0.018 | 0.018 | 0.018 | -0.031 | -0.022 | 2.315 |
| | ( 3.78) | ( 3.79) | ( 3.79) | (-0.23) | (-0.16) | ( 0.24) |
| $\beta_{i,MKT}$ | 2.231 | 3.141 | 2.707 | -0.765 | -0.773 | -0.708 |
| | ( 1.57) | ( 3.89) | ( 1.94) | (-0.55) | (-0.55) | (-0.49) |
| $\beta_{i,VIX}$ | 1.836 | | 1.774 | 8.175 | 8.252 | 5.812 |
| | ( 0.46) | | ( 0.47) | ( 1.88) | ( 1.88) | ( 1.24) |
| $\beta_{i,VoV}$ | | -3.065 | -3.059 | -2.914 | -2.853 | -3.003 |
| | | (-4.09) | (-4.12) | (-2.61) | (-2.55) | (-2.46) |
| *IVOL-TVOL* | | | | 0.543 | 0.579 | 0.729 |
| | | | | ( 1.96) | ( 2.08) | ( 2.52) |
| *CIVOL-PIVOL* | | | | | 5.083 | 6.188 |
| | | | | | ( 8.22) | ( 6.54) |
| *EVRP* | | | | | | 0.080 |
| | | | | | | ( 2.03) |
| *Adj.* $R^2$ | 0.05 | 0.05 | 0.05 | 0.08 | 0.08 | 0.10 |
| *No. obs.* | 824,426 | 824,426 | 824,426 | 310,221 | 310,221 | 241,096 |

# Table 1. 10: Portfolios sorted on $\boldsymbol{\beta^{V}_{i,VoV}}$

We sort stocks into quintile portfolios based on $\beta^{V}_{i,VoV}$, from the lowest (quintile 1) to the highest (quintile 5). After the portfolio formation, we calculate the value-weighted daily 30-day model-free implied variance and monthly 30-day variance risk premium for each portfolio. The column "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. For each portfolio, we estimate the post-ranking variance betas by running the following regression using daily portfolio implied variance:

$$\Delta IV_{p,t+1} = \alpha^{V}_{p} + \beta^{V}_{p,VIX}\Delta \widetilde{VIX}^{2}_{t+1} + \beta^{V}_{p,VoV}\Delta VoV_{t+1} + \varepsilon^{V}_{p,t+1}.$$

Numbers in parentheses are *t*-statistics. The sample period is from January 1996 to December 2010.

| | *Portfolios ranking* | | | | | |
|---|---|---|---|---|---|---|
| | *1* | *2* | *3* | *4* | *5* | *5-1* |
| | | | *Panel A: Ranking on* $\beta^{V}_{i,VoV}$ | | | |
| *EVRP(%²)* | 6.6 | 23.6 | 36.5 | 46.3 | 74.2 | 67.7 |
| | ( 4.24) | ( 9.62) | ( 9.83) | ( 7.29) | ( 5.35) | ( 5.15) |
| *Post-formation daily variance beta* | | | | | | |
| $\beta^{V}_{p,VIX}$ | 0.48 | 0.75 | 1.12 | 1.47 | 2.45 | 1.97 |
| | ( 94.24) | ( 73.65) | ( 62.04) | ( 39.01) | ( 43.77) | ( 35.80) |
| $\beta^{V}_{p,VoV}$ | 2.73 | 4.12 | 3.35 | 5.71 | 11.06 | 8.32 |
| | ( 21.96) | ( 16.52) | ( 7.62) | ( 6.18) | ( 8.09) | ( 6.19) |
| *Volatility characteristics* | | | | | | |
| *IV* | 65.0 | 115.6 | 180.5 | 284.0 | 534.4 | 469.4 |
| *ERV* | 58.4 | 92.0 | 143.9 | 237.7 | 460.1 | 401.7 |
| | | | *Panel B: Ranking on* $\beta^{V}_{i,VIX}$ | | | |
| *EVRP(%²)* | 6.6 | 23.5 | 36.9 | 47.4 | 75.5 | 68.9 |
| | ( 4.23) | ( 9.66) | ( 9.76) | ( 7.42) | ( 5.37) | ( 5.19) |
| *Post-formation daily variance beta* | | | | | | |
| $\beta^{V}_{p,VIX}$ | 0.49 | 0.78 | 1.09 | 1.56 | 2.39 | 1.90 |
| | ( 97.63) | ( 82.77) | ( 61.10) | ( 39.04) | ( 45.45) | ( 36.95) |
| $\beta^{V}_{p,VoV}$ | 2.86 | 3.79 | 4.17 | 5.00 | 10.44 | 7.58 |
| | ( 23.53) | ( 16.46) | ( 9.60) | ( 5.13) | ( 8.13) | ( 6.02) |
| *Volatility characteristics* | | | | | | |
| *IV* | 64.9 | 115.8 | 180.9 | 284.8 | 542.6 | 477.6 |
| *ERV* | 58.3 | 92.2 | 144.0 | 237.4 | 467.0 | 1.7 |

## Table 1. 11: The price of volatility-of-volatility risk in cross-sectional *EVRP*

This table reports the Fama–MacBeth (1973) factor premiums on 25 portfolios sorted on $\beta^{V}_{i,VoV}$, using our market-based three factors (*MKT*, *ΔVIX,* and *ΔVoV*), Chang, Christoffersen, and Jacobs (2013) market skewness factor (*ΔSKEW*), and Fama-French four factors (*MKT*, *SMB*, *HML*, and *UMD*). For each portfolio, we estimate the post-ranking variance betas by running the following regression using daily portfolio implied variance:

$$\Delta IV_{p,t+1} = \alpha^{V}_{p} + \beta^{V}_{p,VIX}\Delta \widetilde{VIX}^{2}_{t+1} + \beta^{V}_{p,VoV}\Delta VoV_{t+1} + \varepsilon^{V}_{p,t+1}.$$

Then, we regress the cross-sectional monthly portfolio expected variance risk premium on the post-ranking variance betas using Fama–MacBeth (1973) cross-sectional regression:

$$EVRP_{p} = -\lambda^{V}_{VIX}\beta^{V}_{p,VIX} - \lambda^{V}_{VoV}\beta^{V}_{p,VoV}.$$

In column from 4 to 6, we include the post-ranking return betas obtained from running regression using daily portfolio returns on the risk factors:

$$EVRP_{p} = -\lambda^{V}_{VIX}\beta^{V}_{p,VIX} - \lambda^{V}_{VoV}\beta^{V}_{p,VoV} + \lambda_{MKT}\beta_{p,MKT} + \lambda_{SMB}\beta_{p,SMB} + \lambda_{HML}\beta_{p,HML} + \lambda_{UMD}\beta_{p,UMD} + \lambda_{SKEW}\beta_{p,SKEW} + \lambda_{KURT}\beta_{p,KURT}.$$

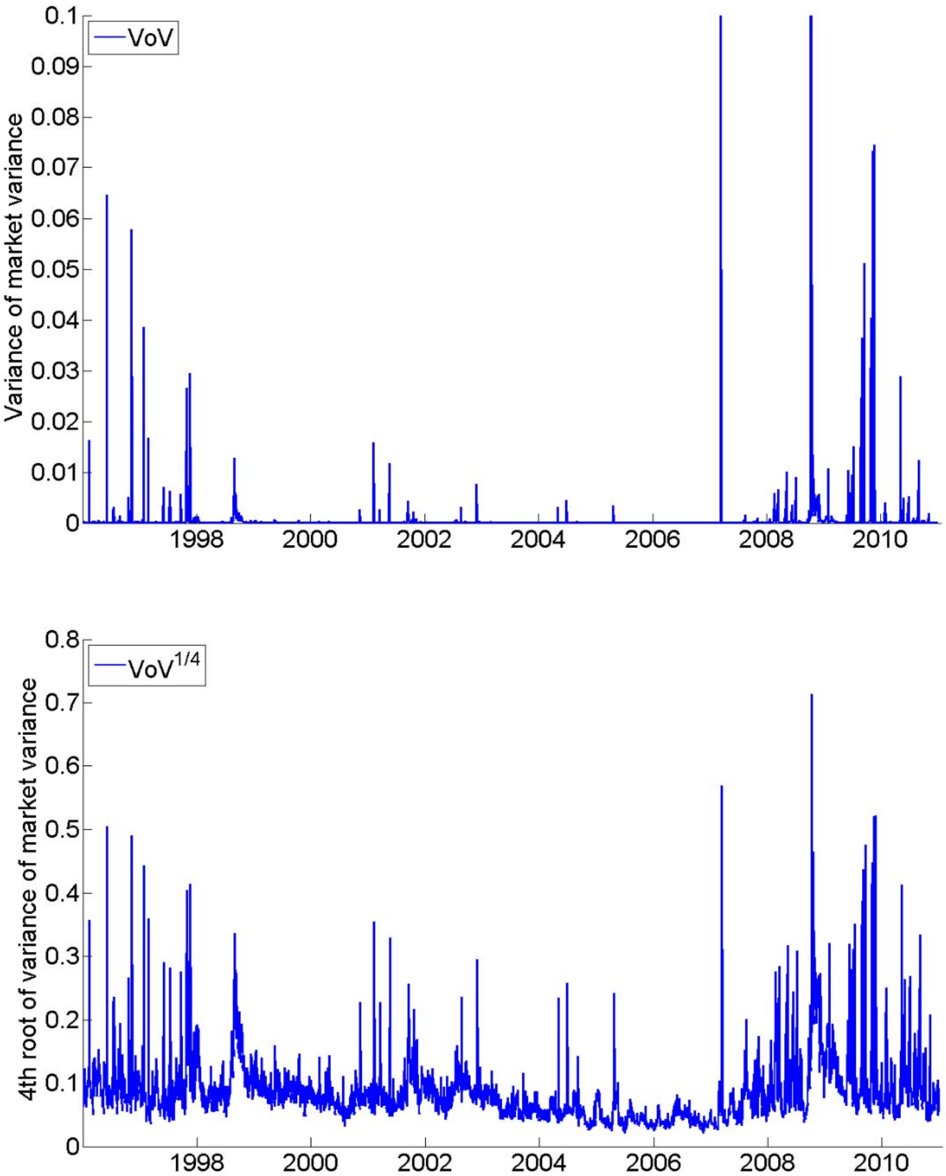
Robust Newey and West (1987) *t*-statistics with six lags that account for autocorrelations are reported in parentheses. The sample period is from January 1996 to December 2010.

| | *Fama-MacBeth cross-sectional regressions* | | | | | |
|---|---|---|---|---|---|---|
| | [1] | [2] | [3] | [4] | [5] | [6] |
| | *25 portfolios sorted on* $\beta^{V}_{i,VoV}$ | | | | | |
| $\beta^{V}_{p,VIX}$ | 34.28 | | 5.29 | -34.11 | -21.22 | -20.66 |
| | ( 5.55) | | ( 1.27) | (-5.43) | (-3.77) | (-3.67) |
| $\beta^{V}_{p,VoV}$ | | 5.99 | 5.20 | 2.74 | 1.62 | 1.53 |
| | | ( 5.32) | ( 4.04) | ( 4.54) | ( 2.36) | ( 2.27) |
| $\beta_{p,MKT}$ | | | | | | 31.20 |
| | | | | | | ( 4.58) |
| $\beta_{p,SMB}$ | | | | 9.51 | 80.73 | 71.21 |
| | | | | ( 0.72) | ( 4.53) | ( 4.29) |
| $\beta_{p,HML}$ | | | | 45.15 | 82.22 | 63.51 |
| | | | | ( 2.03) | ( 3.46) | ( 3.16) |
| $\beta_{p,UMD}$ | | | | -140.27 | -47.56 | -64.82 |
| | | | | (-6.34) | (-1.86) | (-2.27) |
| $\beta_{p,SKEW}$ | | | | | -100.91 | -151.05 |
| | | | | | (-6.76) | (-7.23) |
| $\beta_{p,KURT}$ | | | | | -528.89 | -211.78 |
| | | | | | (-3.19) | (-1.06) |
| *Adj.* $R^2$ | 0.34 | 0.39 | 0.55 | 0.73 | 0.77 | 1.1 |

# Table 1. 12: Return predictability regressions

Panel A reports the estimates of the one-period return predictability regression using daily market return on the lagged variance risk premium (*VRP*), variance of market variance (*VoV*), market skewness (*SKEW*), and market kurtosis (*KURT*). In panel B, we use the monthly market return as the dependent variable, and the independent variables are sampled at the end of the previous month. We multiply Daily market return in Panel A is multiplied by 22. Robust Newey and West (1987) *t*-statistics with sixteen lags in Panel A and with six lags in Panel B that account for autocorrelations are reported in parentheses. The sample period is from January 1996 to December 2010.

| | *Dependent variable= MKT* (t) | | | | | |
|---|---|---|---|---|---|---|
| | [1] | [2] | [3] | [4] | [5] | [6] |
| | | | *Panel A: Daily return regressions* | | | |
| *Intercept* | -2.214 | -0.970 | 0.123 | -2.302 | 0.577 | 0.674 |
| | (-2.82) | (-1.50) | ( 0.26) | (-2.36) | ( 0.30) | ( 0.35) |
| *VRP* (t-1) | 0.153 | | | 0.140 | 0.142 | 0.128 |
| | ( 3.81) | | | ( 3.86) | ( 3.89) | ( 3.45) |
| *VIX* (t-1) | | 0.027 | | 0.000 | -0.001 | -0.003 |
| | | ( 1.95) | | ( 0.00) | (-0.06) | (-0.15) |
| *VoV* (t-1) | | | 5.406 | 4.939 | 4.980 | 5.054 |
| | | | ( 2.47) | ( 2.12) | ( 2.16) | ( 2.19) |
| *SKEW* (t-1) | | | | | 2.378 | 2.304 |
| | | | | | ( 1.50) | ( 1.46) |
| *KURT*(t-1) | | | | | 0.121 | 0.136 |
| | | | | | ( 0.53) | ( 0.60) |
| *MKT* (t-1) | | | | | | -0.041 |
| | | | | | | (-2.23) |
| *Adj.* $R^2$ | 0.012 | 0.002 | 0.011 | 0.021 | 0.021 | 0.023 |
| | | | *Panel B: Monthly return regressions* | | | |
| *Intercept* | -0.369 | 0.604 | 0.280 | -0.183 | 0.796 | 0.630 |
| | (-1.11) | ( 1.13) | ( 0.68) | (-0.41) | ( 0.59) | ( 0.49) |
| *VRP* (t-1) | 0.045 | | | 0.041 | 0.041 | 0.039 |
| | ( 5.47) | | | ( 4.68) | ( 4.24) | ( 3.82) |
| *VIX* (t-1) | | -0.004 | | -0.004 | -0.004 | 0.001 |
| | | (-0.34) | | (-0.59) | (-0.62) | ( 0.18) |
| *VoV* (t-1) | | | 1.682 | 1.246 | 1.352 | 1.460 |
| | | | ( 3.06) | ( 2.44) | ( 2.61) | ( 2.53) |
| *SKEW* (t-1) | | | | | 0.008 | 0.004 |
| | | | | | ( 0.52) | ( 0.28) |
| *KURT*(t-1) | | | | | 0.000 | 0.000 |
| | | | | | ( 0.21) | (-0.23) |
| *MKT* (t-1) | | | | | | 0.124 |
| | | | | | | ( 1.52) |
| *Adj.* $R^2$ | 0.047 | -0.004 | 0.009 | 0.044 | 0.036 | 0.041 |




## Figure 1. 1: Daily market volatility-of-volatility (*VoV*)

We plot daily market volatility-of-volatility over the time period January 1996 through December 2010. We partition the tick-by-tick S&P500 index options data into five-minute intervals, and then we estimate the model-free implied variance for each interval. For each day, we use the bipower variation formula on the five-minute based annualized 30-day model-free implied variance estimates within the day, resulting in our daily measure of market volatility-of-volatility (*VoV*).

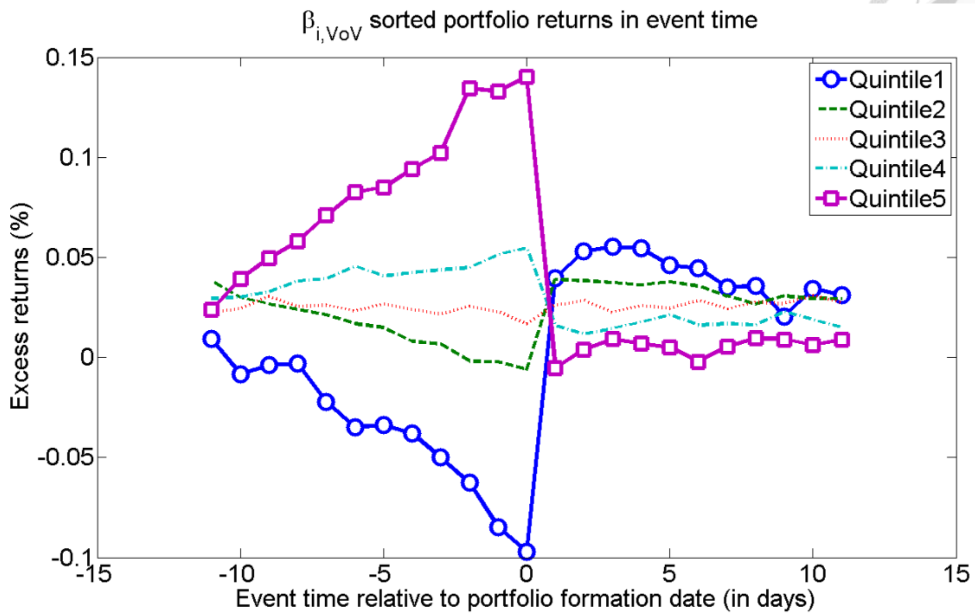
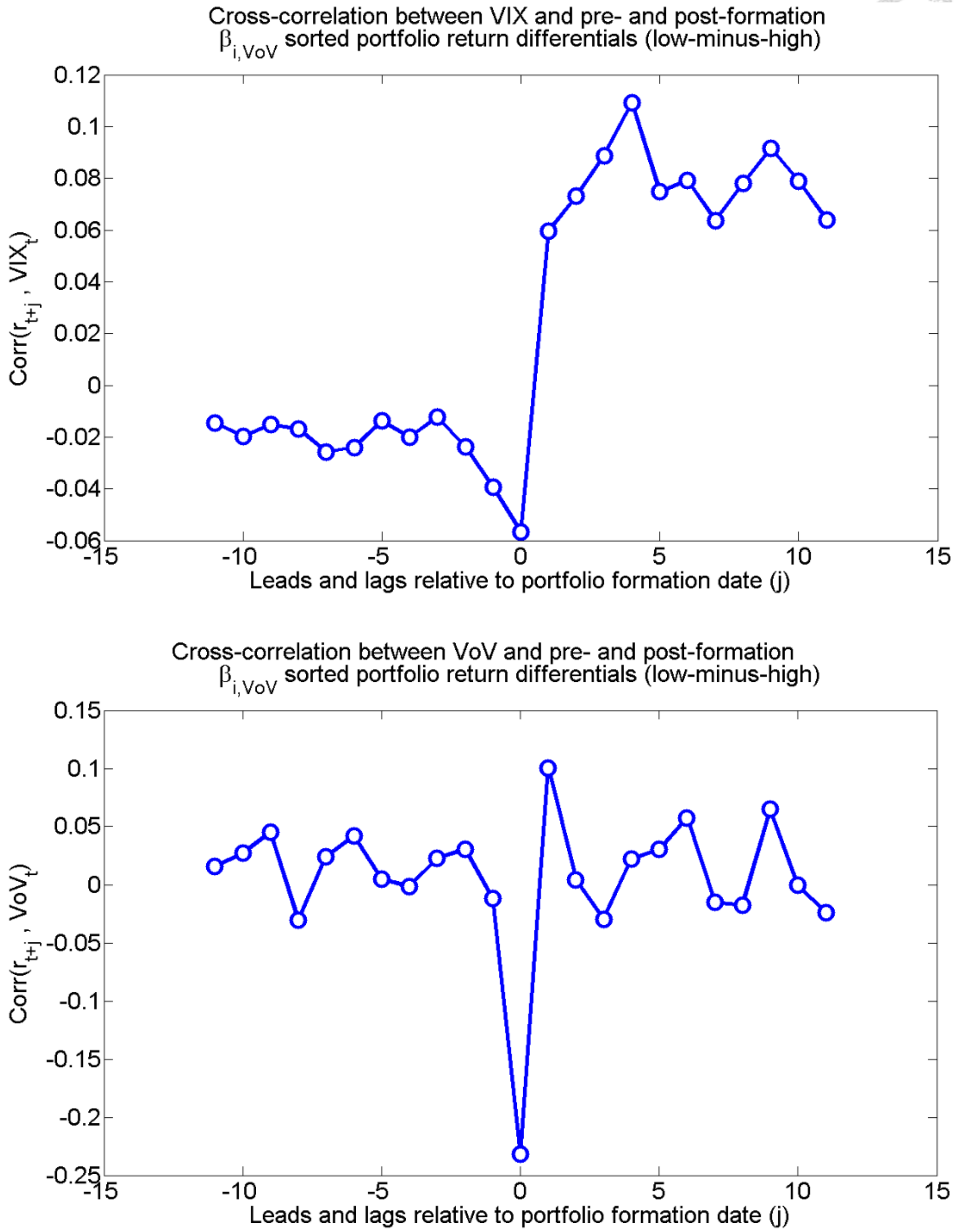



**Figure 1. 2: Performance of portfolios sorted on $\beta_{i,VoV}$ in event time**

At the end of each day, we estimate the regression of equation (1.33) using daily stock returns over the past 22 days. We then sort stocks into quintile portfolios on the estimated $\beta_{i,VoV}$ for each day and calculate the event-time daily value-weighted portfolio returns ranging from -11 to 11 in days.




## Figure 1. 3: Cross-correlations

The plots are based on the pre-formation and post-formation of quintile portfolio return differentials (low-minus-high; long the lowest quintile and short the highest quintile) formed on $\beta_{i,VoV}$. The top panel shows the sample cross-correlation between the *VIX* and portfolio formation time leads and lags of the low-minus-high ranging from -11 to 11 days. The bottom panel shows the sample cross-autocorrelations between the market volatility-of-volatility (*VoV*) and the returns.

# Chapter 2

# A Model-Free CAPM with High Order Risks

## 2.1 Introduction

The concept of linear risk-return trade-off has been the keystone in finance theory. For example, in addition to the market risk of the classical capital asset-pricing model (e.g. Sharpe (1964) and Lintner (1965)), prior literature has illustrated the important role of the stock return exposures to multiple factors (e.g. Fama and French (1992; 1993; 1995; 1996) and Carhart (1997)) and to the high order of market moments (see, for example, Kraus and Litzenberger (1976), Sears and Wei (1985), Harvey and Siddique (2000), Dittmar (2002), Chung, Johnson, and Schill (2006), and Chang, Christoffersen, and Jacobs (2013), among others) for pricing individual stocks. However, recent studies show that the traditional linear risk-return trade-off has difficulty in explaining the pricing effect embedded in higher orders of asset returns. Examples include the (idiosyncratic) volatility puzzle documented by Ang, Hodrick, Xing, and Zhang (2006; 2009) and the MAX puzzle presented by Bali, Cakici, and Whitelaw (2011) in which stocks with high volatilities have been documented to earn low future abnormal returns. In other words, existing literature restricts the investigation of systematic risk to the first-order risk, ignoring the potentially important role of the systematic components in high orders of asset returns.

Moreover, there is conflicting evidence against the linear risk-return trade-off in which stocks with high beta have been documented to earn low future returns (e.g. Baker, Bradley, and Wurgler (2011)). A recent paper by Frazzini and Pedersen (2014) further shows that a betting against beta (BAB) factor, which is long leveraged low-beta assets and short high-beta assets, produces significant positive risk-adjusted returns. Hence, to resurrect the risk-return trade-off relationship, prior literature calls for better understanding of systematic risks. One possible solution is an asset pricing model that incorporates the nonlinear pricing of the systematic risk.

This paper presents an approximate capital asset pricing model in which higher-order risks and high-order risk premiums are important for pricing individual stocks. We characterize the dynamic of market return through the cumulant generating function,[16] which provides analytical solution for our model allowing for high moments in market returns.[17] We only assume that individual stock returns in logarithm follow a simple linear model with the market returns and the linear structure is preserved in the risk-neutral measure. Then, we develop an approximate identity that provides decomposition of total risk premium for the cross-sectional stock returns, linking the market risk premium of each moment to the risk price for the systematic risk in the corresponding order of stock returns.

[16] The cumulant generating function of a random variable is defined as the logarithm of the moment generating function. The j-th cumulant, which is defined as the j-th derivative of the cumulant generating function evaluated at zero, is related to the j-th moment.

[17] Since the pioneer work of Jarrow and Rudd (1982) on approximation method for option valuation, a growing literature shows that the cumulant generating function can be used to quantify the impact of higher moments on the pricing structure of implied volatility (see, for example, Backus, Foresi, Li, and Wu (1997) and Bakshi, Kapadia, and Madan (2003), among others), identify the risk-neutral measure for heteroskedasticity volatility models (see, for example, Christoffersen, Elkamhi, Feunou, and Jacobs (2010) and Corsi, Fusari, and La Vecchia (2013), among others), and study equity premium in representative agent models with non-normal distribution (see, for example, Backus, Chernov, and Martin (2011), Martin (2012), Duan and Zhang (2013), and Backus, Chernov, and Zin (2014), among others). Our paper is complementary to this strand of literature by studying individual stock returns.

The intuition for the feature of nonlinear risk-return trade-off in our model is simple. If market returns have high order risk premiums, expected stock returns should comprise compensation for bearing the corresponding high order systematic risks. In our model, the expected stock return of a security *i* is determined by the market risk premium in each moment times the corresponding order of systematic risk:

$$\begin{aligned}
&\mathbb{E}_t\left[R_{i,t+1}\right] - R_{f,t} \\
&\quad \approx \left(\kappa_{1,t} - \kappa_{1,t}^{\mathbb{Q}}\right)\beta_{i,1} + \frac{1}{2}\left(\kappa_{2,t} - \kappa_{2,t}^{\mathbb{Q}}\right)\beta_{i,1}^{2} + \frac{1}{6}\left(\kappa_{3,t} - \kappa_{3,t}^{\mathbb{Q}}\right)\beta_{i,1}^{3} \\
&\qquad + \frac{1}{24}\left(\kappa_{4,t} - \kappa_{4,t}^{\mathbb{Q}}\right)\beta_{i,1}^{4} \\
&\quad \approx \lambda_{1,t}\beta_{i,1} + \lambda_{2,t}\beta_{i,1}^{2} + \lambda_{3,t}\beta_{i,1}^{3} + \lambda_{4,t}\beta_{i,1}^{4}
\end{aligned} \tag{2.1}$$

where $\beta_{i,1}$ is the systematic risk from the simple linear market model, and $\kappa_{j,t}$ and $\kappa_{j,t}^{\mathbb{Q}}$ are the $j$-th cumulant of the market return in the physical measure and in the risk-neutral measure, respectively. The first term, $\lambda_{1,t}\beta_{i,1}$, measures the first-order risk premium of the classical CAPM, whereas the remaining terms capture the pricing effect for market high moment risk premiums. In particular, the market variance risk premium $(\kappa_{2,t} - \kappa_{2,t}^{\mathbb{Q}})$ determines the risk price $(\lambda_{2,t})$ for the second-order systematic risk $(\beta_{i,1}^{2})$.[18]

While our model is an approximate identity for any linear market model under arbitrary economic preference, we give three specific examples to illustrate how the high-order risks are related to high-order co-moment risks. One particular example of interest is the pricing kernel with stochastic volatility that not only generates negative skewness and excess kurtosis to meet the empirical irregularity but also implies the negative market variance risk premium. In which case, the conventional covariance risk premium, the

[18] Similarly, the scaled market skewness risk premium $(\lambda_{3,t})$ and the scaled market kurtosis risk premium $(\lambda_{4,t})$ are relevant for the third-order systematic risk $(\beta_{i,1}^{3})$ and the fourth-order systematic risk $(\beta_{i,1}^{4})$, respectively.

coskewness premium, and the cokurtosis risk premium, are nested in the first-order risk premium ($\lambda_{1,t}\beta_{i,1}$). More importantly, the second-order risk premium ($\lambda_{2,t}\beta_{i,1}^2$) compensates for the second-order coskewness risk and the second-order cokurtosis risk; the third-order risk premium ($\lambda_{3,t}\beta_{i,1}^3$) corresponds to the third-order cokurtosis risk.[19] Thus, these high order co-moments, which are new to literature, are important sources of priced risks.

The first goal of this paper is to examine the market pricing of high order risks. We perform cross-sectional regressions using portfolio returns and we find evidence that the second-order risk is significantly and negatively priced, consistent with the well-documented evidence for the negative market variance risk premium (see, for example, Carr and Wu (2009), Bollerslev, Tauchen, and Zhou (2009), and Bollerslev, Marrone, Xu, and Zhou (2013), among others).[20] Thus, stocks with high exposure to the second-order risk are volatile and are capable of earning the upside variance potential implied by the negative market variance risk premium. Moreover, while the first-order risk is significantly and positively priced, and the third-order risk price and the fourth-order risk price are insignificant. Therefore, the first two order risk prices imply that the cross-sectional relation between expected return and market beta is inverse-U shaped.

The second goal of this paper is to examine the economic value of the second-order risk premium. Our model implies that the second-order risk premium corresponds to the curvature of the security market line which is the second derivative with respect to the

[19] That is, $\lambda_{1,t}\beta_{i,1} = \lambda_{11,t}\mathbb{C}ov_t[\tilde{r}_{i,t+1}, \tilde{r}_{m,t+1}] + \lambda_{12,t}\mathbb{C}ov_t[\tilde{r}_{i,t+1}, \tilde{r}_{m,t+1}^2] + \lambda_{13,t}\mathbb{C}ov_t[\tilde{r}_{i,t+1}, \tilde{r}_{m,t+1}^3]$, $\lambda_{2,t}\beta_{i,1}^2 = \lambda_{21,t}\mathbb{C}ov_t[\tilde{r}_{i,t+1}^2, \tilde{r}_{m,t+1}] + \lambda_{22,t}\mathbb{C}ov_t[\tilde{r}_{i,t+1}^2, \tilde{r}_{m,t+1}^2]$, and $\lambda_{3,t}\beta_{i,1}^3 = \lambda_{31,t}\mathbb{C}ov_t[\tilde{r}_{i,t+1}^3, \tilde{r}_{m,t+1}]$, where $\tilde{r}_{i,t+1} = r_{i,t+1} - \mathbb{E}_t[r_{i,t+1}]$ and $\tilde{r}_{m,t+1} = r_{m,t+1} - \mathbb{E}_t[r_{m,t+1}]$.

[20] While Driessen, Maenhout, and Vilkov (2009) suggest that individual options covered by S&P 100 do not embed a negative variance risk premium, Han and Zhou (2011) find that, with realized variance measured from high frequency stock prices, individual variance risk premium is significantly negative in their larger sample of equity options covered by OptionMetrics. Bali and Hovakimian (2009) and Han and Zhou (2011) also show that individual variance risk premiums can predict the cross-sectional stock returns.

first-order risk. For each the first-order co-moment sorted portfolios, we construct a curvature portfolio (*CUR1*) by the sum of the twice difference for these portfolios.[21] We find that the average annual return on the curvature portfolio is significantly negative at −12.00% for the market beta sorted portfolios, at −11.76% for the first-order coskewness sorted portfolios, and at −11.52% for the first-order cokurtosis sorted portfolios.

The second-order risk premium can also be tested directly through the second-order risks. For each the second-order co-moment sorted portfolios, we construct another curvature portfolio (*CUR2*) that longs the top portfolio and shorts the bottom portfolio. We find that the average annual performance of this curvature portfolio is significantly negative at −15.24% for the second-order coskewness-sorted portfolios, and at –12.72% for the second-order cokurtosis sorted portfolios. Moreover, to identify the second-order risk in a model-free fashion, we follow the seminal work of Bakshi, Kapadia, and Madan (2003) to estimate the model-free implied variance using individual options and index options.[22] We estimate the risk-neutral variance beta through the linear market model relation between the individual variance and the market variance and we find that the average annual return on the curvature portfolio ($CUR2_{RN}$) is significantly negative, at –16.08% for the risk-neutral variance beta sorted portfolios.

The third goal of this paper is to investigate whether the second-order risk premium helps explain cross-sectional stock returns. We construct mimicking factors for the second-order risk premiums using the curvature portfolios and we investigate whether the mimicking factors account for the anomalies documented in the literature that are

[21] That is, for *N* risk-sorted portfolios, the curvature portfolio would be $\sum_{p=3}^{N} \Delta^2 r_p$, where $\Delta r_p = r_p - r_{p-1}$ and $\Delta^2 r_p = \Delta r_p - \Delta r_{p-1}$. For example, for the 25 risk-sorted portfolios, the curvature portfolio would be the long-short portfolio between the nearby difference in the top two portfolios and the nearby difference in the bottom two portfolios, i.e., (25−24)−(2−1).

[22] It has been shown that the risk-neutral moments can be inferred in a 'model-free' fashion from a collection of option prices without use of a specific pricing model (see, for example, Carr and Madan 1998; Britten-Jones and Neuberger 2000; Bakshi, Kapadia, and Madan, 2003; Jiang and Tian 2005).

essentially exposed to the second-order systematic risk. We show that our mimicking factors accounts for the total volatility puzzle and the idiosyncratic volatility puzzle documented by Ang, Hodrick, Xing, and Zhang (2006), as well as the MAX puzzle presented by Bali, Cakici, and Whitelaw (2011). Specifically, we find that these volatility measures are closely related to the second-order systematic risks and the abnormal returns associated with these variables become insignificant once we control for the mimicking curvature factors.

Moreover, we find evidence that our mimicking factors help explain the betting-against-beta (BAB) premium of Frazzini and Pedersen (2014). While the BAB factor is by construction isolated from the first-order risk, our model suggests that the betting-against-beta strategy is exposed to high-order systematic risks. Thus, the second-order risk premium provides an alternative explanation for the betting-against-beta premium. On the other hand, since their model implies that the second-order risk price is zero, their model cannot explain the negative second-order risk premium as we find empirically.

Our paper is closely related to, but different from, Hong and Sraer (2012), in which they demonstrate that the disagreement about the market return leads to speculative overpricing for high beta stocks. Their model implies that the shape of the security market line is kinked and the slope of that decreases with the macro-disagreement. Our model, in contrast, suggests that the curvature of the security market line is determined by the market variance risk premium (e.g., the second-order risk price). More importantly, we find evidence that the market variance premium explains our curvature factors better than the macro-disagreement does. Thus, the second-order risk premium in our paper cannot be fully explained by their disagreement driven overpricing mechanism.

Our paper is complementary to Conrad, Dittmar, and Ghysels (2013), in which they focus on relative pricing of idiosyncratic risk-neutral moments and the first-order risk-neutral co-moments. In contrast, our study shed light on the importance of the second-order co-moments and the risk-neutral variance beta. Furthermore, we find evidence that our mimicking factors are priced in the firm-level cross-sectional regression and the results are robust to the inclusion of individual risk-neutral moments.

The remainder of the paper is organized as follows. The next section presents our approximate capital asset pricing model. Section 3 discusses the empirical implications of our model. Section 4 describes the data and presents the estimation of co-moment risks. In section 5, we show empirical evidence on the high order risk premiums in cross-sectional stock returns. Section 6 focuses on the construction and verification of the mimicking curvature factors. In section 7, we test the performance of the mimicking curvature factors in explaining the cross-sectional stock returns. Finally, section 8 contains our concluding remarks.

## 2.2 The Model

### 2.2.1 The market premium and the cumulant-generating function

We derive asset prices based on the pricing kernel, $M_{t+1}$. Denoted any asset return as $r_{t+1} = log\left[\frac{S_{t+1}+D_t}{S_t}\right]$, where $S_t$ ($S_{t+1}$) is the stock price at time *t* (*t*+1) and $D_{t+1}$ is dividend paid between *t* and *t*+1. Then the standard asset pricing suggests that

$$\mathbb{E}_t[M_{t+1}\, exp[r_{t+1}]] = 1, \quad (2.2)$$

where $\mathbb{E}_t$ is the expectation operator at time *t*. Exploiting the pricing condition for the risk-free return $r_{f,t}$, it follows that expected value of the pricing kernel is a discount factor with the risk-free rate, i.e. $\mathbb{E}_t[M_{t+1}] = exp\left[-\, r_{f,t}\right]$. The risk-neutral measure $\mathbb{Q}$, which is equivalent to the physical measure $\mathbb{P}$, is defined by the Radon-Nikodym derivative,

$\frac{d\mathbb{Q}}{d\mathbb{P}} = \frac{M_{t+1}}{\mathbb{E}_t[M_{t+1}]}$. The standard asset pricing condition implies that

$$\mathbb{E}_t^{\mathbb{Q}}[exp[\, r_{t+1}]] = \frac{\mathbb{E}_t[M_{t+1}\, exp[\, r_{t+1}]]}{\mathbb{E}_t[M_{t+1}]} = exp[\, r_{f,t}]. \tag{2.3}$$

Thus, the expected value of any gross return in the risk-neutral measure is the gross risk-free return.

The physical dynamics of the market return, $r_{m,t+1} = log\left[\frac{W_{t+1}}{W_t}\right]$, where $W$ is the value of the market portfolio, is defined by its cumulant generating function, (e.g., the logarithmic value of the moment generating function), $\mathbb{C}um_{m,t}[\theta] \equiv \log\left[\mathbb{E}_t\left[exp\left[\theta\, r_{m,t+1}\right]\right]\right]$. The power-series expansion yields

$$\mathbb{C}um_{m,t}[\theta] = \sum_{n=1}^{\infty} \kappa_{n,t} \frac{\theta^n}{n!}, \tag{2.4}$$

where $\kappa_{n,t}$ is the $n$-th derivative of $\mathbb{C}um_{m,t}[\theta]$ at $\theta = 0$, corresponding to $n$-th moment of $r_{m,t+1}$. In particular, $\kappa_{1,t}$ is the mean ($\mathbb{E}_t\left[r_{m,t+1}\right]$), $\kappa_{2,t}$ is the variance ($\mathbb{V}ar_t\left[r_{m,t+1}\right]$), $\kappa_{3,t}$ is the unstandardized skewness, and $\kappa_{4,t}$ is the unstandardized excess kurtosis.[23] The risk-neutral dynamics of $r_{m,t+1}$ is similarly defined by its cumulant generating function, $\mathbb{C}um_{m,t}^{\mathbb{Q}}[\theta] = \sum_{n=1}^{\infty} \kappa_{n,t}^{\mathbb{Q}} \frac{\theta^n}{n!}$, where $\kappa_{n,t}^{\mathbb{Q}}$ corresponds to $n$-th risk-neutral moment of $r_{m,t+1}$.

We define the market premium as $exret_{m,t} \equiv log[\mathbb{E}_t\left[\frac{W_{t+1}}{W_t}\right]] - r_{f,t}$. The risk-neutral pricing equation (2.3) implies that $\mathbb{C}um_{m,t}^{\mathbb{Q}}[1] = r_{f,t}$. Thus, the market premium can be conveniently expressed in terms of the cumulant generating functions:

[23] That is, $\kappa_{3,t} = \mathbb{E}_t\left[\tilde{r}_{m,t+1}^3\right]$, and $\kappa_{4,t} = \mathbb{E}_t\left[\tilde{r}_{m,t+1}^4\right] - 3\kappa_{2,t}^2$, where $\tilde{r}_{m,t+1} = r_{m,t+1} - \mathbb{E}_t\left[r_{m,t+1}\right]$.

$$\begin{aligned} exret_{m,t} &= \mathbb{C}um_{m,t}[1] - \mathbb{C}um_{m,t}^{\mathbb{Q}}[1] \\ &\approx \left(\kappa_{1,t} - \kappa_{1,t}^{\mathbb{Q}}\right) + \frac{1}{2}\left(\kappa_{2,t} - \kappa_{2,t}^{\mathbb{Q}}\right) + \frac{1}{6}\left(\kappa_{3,t} - \kappa_{3,t}^{\mathbb{Q}}\right) \\ &\quad + \frac{1}{24}\left(\kappa_{4,t} - \kappa_{4,t}^{\mathbb{Q}}\right) \\ &\approx \lambda_{1,t} + \lambda_{2,t} + \lambda_{3,t} + \lambda_{4,t} \end{aligned} \tag{2.5}$$

The result implies that the market premium, in general, comprises all of the risk premiums stemmed from the differences between the physical moments and the risk-neutral moments. Therefore, the market premium can be approximately decomposed into the market mean risk premium ($\lambda_{1,t}$), the market variance risk premium ($\lambda_{2,t}$), the market skewness risk premium ($\lambda_{3,t}$), and the market kurtosis risk premium ($\lambda_{4,t}$).

### 2.2.2 An approximate capital asset pricing model

For individual asset return $r_{i,t+1} \equiv log\left[\frac{S_{i,t+1}+D_{i,t+1}}{S_{i,t}}\right]$, we assume that

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,1}\, r_{m,t+1} + \epsilon_{i,t+1}, \tag{2.6}$$

where $\epsilon_{i,t+1}$ is the idiosyncratic component, which is independent of the market return, $\epsilon_{i,t+1} \perp r_{m,t+1}$. It follows that the cumulant generating function of $r_{i,t+1}$ is $\mathbb{C}um_{i,t}[\theta] = \theta\beta_{i,0} + \mathbb{C}um_{m,t}\left[\theta\beta_{i,1}\right] + \mathbb{C}um_t\left[\epsilon_{i,t+1}, \theta\right]$, where $\mathbb{C}um_t\left[\epsilon_{i,t+1}, \theta\right]$ is the cumulant generating function of $\epsilon_{i,t+1}$. We further assume that the idiosyncratic component is independent of the pricing kernel, $\epsilon_{i,t+1} \perp M_{t+1}$ and the linear market model structure is preserved in the risk-neutral measure. Thus, the risk-neutral dynamics of $r_{i,t+1}$ is represented by

$$\mathbb{C}um_{i,t}^{\mathbb{Q}}[\theta] = \theta\beta_{i,0} + \mathbb{C}um_{m,t}^{\mathbb{Q}}\left[\theta\beta_{i,1}\right] + \mathbb{C}um_t\left[\epsilon_{i,t+1}, \theta\right].$$

Now, we are ready to calculate the expected excess stock return, $exret_{i,t} \equiv$

$log[\mathbb{E}_t\left[\frac{S_{i,t+1}+D_{i,t+1}}{S_{i,t}}\right]] - r_{f,t} = \mathbb{C}um_{i,t}[1] - \mathbb{C}um^{\mathbb{Q}}_{i,t}[1]$. It follows that

$$exret_{i,t} = \mathbb{C}um_{m,t}[\beta_{i,1}] - \mathbb{C}um^{\mathbb{Q}}_{m,t}[\beta_{i,1}]. \tag{2.7}$$

The power-series expansion yields the following proposition for $exret_{i,t}$.

**Proposition 1.** (Nonlinear beta representation)

$$\begin{aligned} exret_{i,t} &\approx (\kappa_{1,t} - \kappa^{\mathbb{Q}}_{1,t})\beta_{i,1} + \frac{1}{2}(\kappa_{2,t} - \kappa^{\mathbb{Q}}_{2,t})\beta^2_{i,1} + \frac{1}{6}(\kappa_{3,t} - \kappa^{\mathbb{Q}}_{3,t})\beta^3_{i,1} \\ &\quad + \frac{1}{24}(\kappa_{4,t} - \kappa^{\mathbb{Q}}_{4,t})\beta^4_{i,1} \\ &\approx \lambda_{1,t}\beta_{i,1} + \lambda_{2,t}\beta^2_{i,1} + \lambda_{3,t}\beta^3_{i,1} + \lambda_{4,t}\beta^4_{i,1}. \end{aligned} \tag{2.8}$$

The result implies the feature of nonlinear risk-return tradeoff. The first term, $\lambda_{1,t}\beta_{i,1}$, measures the first-order risk premium of the classical CAPM, whereas the remaining terms capture the pricing effect for the market high moment risk premiums. In particular, the market variance risk premium $(\kappa_{2,t} - \kappa^{\mathbb{Q}}_{2,t})$ determines the risk price $(\lambda_{2,t})$ for the second-order systematic risk $(\beta^2_{i,1})$. Similarly, the scaled market skewness risk premium $(\lambda_{3,t})$ and the scaled market kurtosis risk premium $(\lambda_{4,t})$ are relevant for the third-order systematic risk $(\beta^3_{i,1})$ and the fourth-order systematic risk $(\beta^4_{i,1})$, respectively.

It is worth noting that our model does not rely on specific assumptions on the economic preference. Instead, our model is an approximate identity for any linear market model under arbitrary identification of the economic preference. In the following subsection, we examine some well-known economic preferences studied in prior literature as examples to illustrate the role of high order systematic risks.

### 2.2.3 Examples: the role of high order co-moment risks

***The power utility preference with non-normal market returns***

We assume that the market return is distributed with high moments, $\mathbb{C}um_{m,t}[\theta] \approx$

$\theta\kappa_{1,t} + \frac{\theta^2}{2}\kappa_{2,t} + \frac{\theta^3}{6}\kappa_{3,t} + \frac{\theta^4}{24}\kappa_{4,t}$. An explicit example is the skew normal distribution, $SKN(\mu_m, \sigma_m^2, \delta_m)$, which is a normal-like distribution but with higher order (excess) moments, where $\mu_m$ is the mean parameter, $\sigma_m^2$ is the variance parameter, and $\delta_m$ is the shape parameter.[24]

The pricing kernel for the power utility preference is exponentially linear in the market return, $M_{t+1} = exp[-\gamma\, r_{m,t+1}]$, with the relative risk aversion $\gamma$. The pricing kernel implies that $\mathbb{C}um_{m,t}^{\mathbb{Q}}[\theta] = \mathbb{C}um_{m,t}[\theta - \gamma] - \mathbb{C}um_{m,t}[-\gamma]$. It is straightforward to show that the premium for the stock *i* is

$$
\begin{aligned}
exret_{i,t} &= \mathbb{C}um_{m,t}[\beta_{i,1}] - \mathbb{C}um_{m,t}[\beta_{i,1} - \gamma] - \mathbb{C}um_{m,t}[-\gamma] \\
&\approx \left(\gamma\kappa_{2,t} - \frac{\gamma^2}{2}\kappa_{3,t} + \frac{\gamma^3}{6}\kappa_{4,t}\right)\beta_{i,1} + \frac{1}{2}\left(\gamma\kappa_{3,t} - \frac{\gamma^2}{2}\kappa_{4,t}\right)\beta_{i,1}^2 \\
&\quad + \frac{1}{6}(\gamma\kappa_{4,t})\beta_{i,1}^3 \\
&\approx \lambda_{1,t}\beta_{i,1} + \lambda_{2,t}\beta_{i,1}^2 + \lambda_{3,t}\beta_{i,1}^3.
\end{aligned}
\tag{2.9}
$$

Moreover, the risk premium of each order is related to the same order of co-moment risks, i.e.,

$$
\begin{aligned}
\lambda_{1,t}\beta_{i,1} &= \lambda_{11,t}\mathbb{C}ov_t[\tilde{r}_{i,t+1}, \tilde{r}_{m,t+1}] + \lambda_{12,t}\mathbb{C}ov_t[\tilde{r}_{i,t+1}, \tilde{r}_{m,t+1}^2] \\
&\quad + \lambda_{13,t}\mathbb{C}ov_t[\tilde{r}_{i,t+1}, \tilde{r}_{m,t+1}^3], \\
\lambda_{2,t}\beta_{i,1}^2 &= \lambda_{21,t}\mathbb{C}ov_t[\tilde{r}_{i,t+1}^2, \tilde{r}_{m,t+1}] + \lambda_{22,t}\mathbb{C}ov_t[\tilde{r}_{i,t+1}^2, \tilde{r}_{m,t+1}^2], \\
\lambda_{3,t}\beta_{i,1}^3 &= \lambda_{31,t}\mathbb{C}ov_t[\tilde{r}_{i,t+1}^3, \tilde{r}_{m,t+1}],
\end{aligned}
\tag{2.10}
$$

where $\tilde{r}_{i,t+1} = r_{i,t+1} - \mathbb{E}_t[r_{i,t+1}]$ and $\tilde{r}_{m,t+1} = r_{m,t+1} - \mathbb{E}_t[r_{m,t+1}]$. The results imply

---

[24] See, for example, Harvey, Liechty, Liechty, and Müller (2010) for the discussion on the skew normal distribution. The corresponding cumulant generating function is expressed by

$$
\begin{aligned}
\mathbb{C}um_{m,t}[\theta] &= log[2\Phi(\theta\delta_m\sigma_m)] + \theta\mu_m + \frac{\theta^2}{2}\sigma_m^2 \\
&\approx \theta\left(\mu_m + \frac{\sqrt{2}}{\sqrt{\pi}}\delta_m\sigma_m\right) + \frac{\theta^2}{2}\left(\sigma_m^2 - \frac{2}{\pi}\delta_m^2\sigma_m^2\right) + \frac{\theta^3}{6}\left(\frac{\sqrt{2}(4-\pi)}{\pi^{3/2}}\delta_m^3\sigma_m^3\right) \\
&\quad + \frac{\theta^4}{24}\left(\frac{8(\pi-3)}{\pi^2}\delta_m^4\sigma_m^4\right).
\end{aligned}
$$

that the first-order risk premium ($\lambda_{1,t}\beta_{i,1}$) consists of the conventional first-order co-moment risk premiums, including the covariance risk premium, the coskewness risk premium, and the cokurtosis risk premium, More importantly, the second-order risk premium ( $\lambda_{2,t}\beta_{i,1}^{2}$ ) compensates for the second-order coskewness risk ($\mathbb{C}ov_t\left[\tilde{r}_{i,t+1}^{2}, \tilde{r}_{m,t+1}\right]$) and the second-order cokurtosis risk ($\mathbb{C}ov_t\left[\tilde{r}_{i,t+1}^{2}, \tilde{r}_{m,t+1}^{2}\right]$); the third-order risk premium ( $\lambda_{3,t}\beta_{i,1}^{3}$ ) corresponds to the third-order cokurtosis risk ($\mathbb{C}ov_t\left[\tilde{r}_{i,t+1}^{3}, \tilde{r}_{m,t+1}\right]$). Thus, these high order co-moments, which are new to literature, are important sources of priced risks.

***The quadratic utility preference with normal market returns***

We examine the quadratic pricing kernel, $M_{t+1} = exp\left[-\gamma_1 r_{m,t+1} - \gamma_2\, r_{m,t+1}^{2}\right]$, in the spirit of Harvey and Siddique (2000). We assume that the market return is normally distributed, $r_{m,t+1} \sim N(\mu_m, \sigma_m^2)$ , i.e., $\mathbb{C}um_{m,t}[\theta] = \theta\mu_m + \frac{\theta^2}{2}\sigma_m^2 = \theta\kappa_{1,t} + \frac{\theta^2}{2}\kappa_{2,t}$ . Define $\mathbb{C}um_{J,t}[\theta_1, \theta_2]$ as the joint cumulant generating function of $r_{m,t+1}$ and $r_{m,t+1}^{2}$, which yields that

$$\begin{aligned} \mathbb{C}um_{J,t}[\theta_1, \theta_2] &\equiv log[\mathbb{E}[exp[\theta_1 r_{m,t+1} + \theta_2 r_{m,t+1}^{2}]]] \\ &= -\frac{1}{2} log\left[1 - 2\theta_2\kappa_{2,t}\right] + \frac{2\kappa_{1,t}\left(\theta_1 + \theta_2\kappa_{1,t}\right) + \theta_1^2\kappa_{2,t}}{2 - 4\theta_2\kappa_{2,t}}. \end{aligned} \tag{2.11}$$

The risk-neutral dynamics implied by the quadratic pricing kernel is $\mathbb{C}um_{m,t}^{\mathbb{Q}}[\theta] = \mathbb{C}um_{J,t}[\theta, -\gamma_1, -\gamma_2] - \mathbb{C}um_{J,t}[-\gamma_1, -\gamma_2]$. Thus, it is straightforward to show that the premium for the stock *i* is[25]

[25] The premium can be computed through $exret_{i,t} = \mathbb{C}um_{J,t}\left[\beta_{i,1}, 0\right] - \mathbb{C}um_{J,t}\left[\beta_{i,1}, -\gamma_1, -\gamma_2\right] + \mathbb{C}um_{J,t}[-\gamma_1, -\gamma_2]$.

$$exret_{i,t} = \left(\frac{\gamma_1\kappa_{2,t} + 2\gamma_2\kappa_{1,t}\kappa_{2,t}}{1+\gamma_2\kappa_{2,t}}\right)\beta_{i,1} + \frac{1}{2}\left(\frac{\gamma_2\left(\kappa_{2,t}\right)^2}{1+2\gamma_2\kappa_{2,t}}\right)\beta_{i,1}^2$$

$$= \lambda_{1,t}\beta_{i,1} + \lambda_{2,t}\beta_{i,1}^2, \qquad (2.12)$$

$$\lambda_{1,t}\beta_{i,1} = \lambda_{11,t}\mathbb{C}ov_t\left[\tilde{r}_{i,t+1}, \tilde{r}_{m,t+1}\right] + \lambda_{12,t}\mathbb{C}ov_t\left[\tilde{r}_{i,t+1}, r_{m,t+1}^2\right]$$

$$\lambda_{2,t}\beta_{i,1}^2 = \lambda_{22,t}\mathbb{C}ov_t\left[\tilde{r}_{i,t+1}^2, \tilde{r}_{m,t+1}^2\right].$$

The result suggests that, along with the covariance risk and the coskewness risk considered in Harvey and Siddique (2000), the second-order cokurtosis risk ($\mathbb{C}ov_t\left[\tilde{r}_{i,t+1}^2, \tilde{r}_{m,t+1}^2\right]$) is an important source of risk under the specification of the quadratic pricing kernel.

***The pricing kernel with stochastic volatility***

It has been well documented that the volatility of market return is stochastic. We examine the pricing kernel with stochastic volatility, $M_{t+1} = exp\left[-\gamma_m\, r_{m,t+1} - \gamma_m\, \sigma_{m,t+1}^2\right]$, in the spirit of Ang, Hodrick, Xing, and Zhang (2006). We assume that the market return follows a normal distribution $r_{m,t+1}|\sigma_{m,t+1}^2 \sim N(\mu_m + \rho\sigma_{m,t+1}^2, \sigma_{m,t+1}^2)$ conditionally while the market volatility itself, for simplicity, follows a normal distribution $\sigma_{m,t+1}^2 \sim N(\bar{\sigma}_m^2, q_m^2)$, where $\rho < 0$ captures the negative correlation between market return and market volatility (e.g., the leverage effect), $\bar{\sigma}_m^2$ is the long-term volatility, and $q_m^2$ is the volatility of market volatility. Define $\mathbb{C}um_{J,t}[\theta_m, \theta_v]$ as the joint cumulant generating function of $r_{m,t+1}$ and $\sigma_{m,t+1}^2$, which yields that

$$\mathbb{C}um_{J,t}[\theta_m, \theta_v] \equiv log[\mathbb{E}[exp[\theta_m r_{m,t+1} + \theta_v \sigma_{m,t+1}^2]]]$$

$$= \theta_m\left(\kappa_{1,t} + \theta_v\rho q_m^2\right) + \frac{\theta_m^2}{2}\left(\kappa_{2,t} + \theta_v q_m^2\right) \qquad (2.13)$$

$$+ \frac{\theta_m^3}{6}\kappa_{3,t} + \frac{\theta_m^4}{24}\kappa_{4,t} + \theta_v\bar{\sigma}_m^2 + \frac{\theta_v^2}{2}q_m^2,$$

where $\kappa_{1,t} = \mu_m + \rho\bar{\sigma}_m^2$, $\kappa_{2,t} = \bar{\sigma}_m^2 + q_m^2\rho^2$, $\kappa_{3,t} = 3q_m^2\rho$, and $\kappa_{4,t} = 3q_m^2$. The risk-neutral distribution of $r_{m,t+1}$ implied by the pricing kernel with stochastic volatility is $\mathbb{C}um_{m,t}^{\mathbb{Q}}[\theta_m] = \mathbb{C}um_{J,t}[\theta_m, -\gamma_m, -\gamma_v] - \mathbb{C}um_{m,v,t}[-\gamma_m, -\gamma_v]$.

Thus, it is straightforward to show that the premium for the stock $i$ is[26]

$$
\begin{aligned}
exret_{i,t} &= \left(\gamma_m \kappa_{2,t} - \left(\frac{\gamma_m^2}{2} - \frac{\gamma_v}{3}\right)\kappa_{3,t} + \left(\frac{\gamma_m^3}{6} + \frac{2\gamma_1\gamma_v}{3}\right)\kappa_{4,t}\right)\beta_{i,1} \\
&\quad + \frac{1}{2}\left(\gamma_m \kappa_{3,t} - \left(\frac{\gamma_m^2}{2} - \frac{2\gamma_v}{3}\right)\kappa_{4,t}\right)\beta_{i,1}^2 + \frac{1}{6}\left(\gamma_m \kappa_{4,t}\right)\beta_{i,1}^3 \\
&= \lambda_{1,t}\beta_{i,1} + \lambda_{2,t}\beta_{i,1}^2 + \lambda_{3,t}\beta_{i,1}^3.
\end{aligned}
\tag{2.14}
$$

The result implies the pricing of the first three order co-moment risks since the market premium comprises the market mean risk premium ($\lambda_{1,t}$), the market variance risk premium ($\lambda_{2,t}$), and the market skewness risk premium ($\lambda_{3,t}$). The relevant co-moment risks here are the same with those from the power utility preference with non-normal market returns in section 2.3.1.

It is instructive to examine the return-volatility beta of Ang, Hodrick, Xing, and Zhang (2006), which is defined as $\beta_{i,v} = \frac{\mathbb{C}ov_t[r_{i,t+1}, \sigma_{m,t+1}^2]}{\mathbb{V}ar_t[\sigma_{m,t+1}^2]} = \beta_{i,1}\rho$. Recollecting the first-order risk premium with respect to $\beta_{i,v}$ yields that:

$$
\begin{aligned}
\lambda_1\beta_{i,1} &= \lambda_{1m}\beta_{i,1} + \lambda_{1v}\beta_{i,v} \\
&= \beta_{i,1}\left\{\gamma_m \kappa_{2,t} + \left(\frac{\gamma_m^3}{2} + 2\gamma_m\gamma_v\right) q_m^2\right\} \\
&\quad + \beta_{i,v}\left\{\left(-\frac{3\gamma_m^2}{2} + \gamma_v\right) q_m^2\right\}.
\end{aligned}
\tag{2.15}
$$

More importantly, in this economy, the second-order risk price ($\lambda_{2,t}$) and the third-order risk price ($\lambda_{3,t}$) are closely related to the risk price of the return-volatility beta ($\lambda_{1v}$), i.e.,

[26] The premium can be computed through $exret_{i,t} = \mathbb{C}um_{J,t}[\beta_{i,1}, 0] - \mathbb{C}um_{J,t}[\beta_{i,1}, -\gamma_m, -\gamma_v] + \mathbb{C}um_{J,t}[-\gamma_m, -\gamma_v]$.

$$\lambda_{2,t} = \left(\frac{6\rho\gamma_m - 3\gamma_m^2 + 4\gamma_v}{-6\gamma_m^2 + 4\gamma_v}\right)\lambda_{1v},$$
$$\lambda_{3,t} = \left(\frac{\gamma_m}{-3\gamma_m^2 + 2\gamma_v}\right)\lambda_{1v}. \tag{2.16}$$

Hence, the significant negative evidence for the risk price of volatility risk in Ang, Hodrick, Xing, and Zhang (2006) is indicative of the second-order risk premium.[27]

### 2.2.4 A calibration: the risk premium implied by SPX and SPX index options

To gauge the impact of the high order risk prices, we estimate the market risk premium in each moment implied by SPX and SPX index options, following the model-free methodology proposed by the seminal work of Bakshi, Kapadia, and Madan (2003). Using the index option prices from the Option Price file, we follow the procedure of Chang, Christoffersen, and Jacobs (2013) to estimate the 30-day risk-neutral market moments for each day during the period from 1996 to 2012. We then average daily estimates from index option prices to obtain the full sample risk-neutral market moments. The physical market moments are computed using the full sample logarithmic monthly SPX returns.

Table 2. 1 presents the estimates of physical market moments, risk-neutral market moments, and their differences. The estimation result shows a positive market mean risk premium ($\kappa_1 - \kappa_1^{\mathbb{Q}}$ = 0.359%) and a positive market skewness risk premium ($\kappa_3 - \kappa_3^{\mathbb{Q}}$ = 0.049%), while the market variance risk premium ($\kappa_2 - \kappa_2^{\mathbb{Q}}$ = −0.260%) and the market kurtosis risk premium ($\kappa_4 - \kappa_4^{\mathbb{Q}}$ = −0.021%) are negative.

---

[27] In the special case of $\gamma_m = 0$, the second-order risk price is identical to the volatility risk price, i.e., $\lambda_{2,t} = \lambda_{1v} = \gamma_v q_m^2$.

We use the estimates of market moments to generate random samples and then perform the kernel smoothing density to estimate the physical density and the risk-neutral density. We estimate the pricing kernel by the ratio of the risk-neutral density to the physical density. As can be seen in the Panel A of Figure 2. 1, the risk-neutral density of the market return ($\beta$=1) is more volatile, more negatively skewed, and more fat-tailed than the physical density.

Furthermore, we find that, as shown in Panel B of Figure 2. 1, the implied pricing kernel for the market return ($\beta$=1) is U-shaped, consistent with the recent findings in the literature.[28] In particular, Christoffersen, Heston, and Jacobs (2013) show that their variance dependent pricing kernel which implies the quadratic preference of the market return can generate a U-shaped pattern. Thus, among the three pricing kernels of our examples, the quadratic preference and the stochastic volatility preference are more likely to reconcile the shape of the implied pricing kernel.

To investigate how the systematic risks interacting with the pricing kernel affects the expected stock returns, we generate random samples for individual stocks based on $r_i = \beta_{i,0} + \beta_{i,1} r_m$, where $\beta_{i,1}$ ranges from one to four and $\beta_{i,0}$ is restricted by the risk-neutral pricing relationship (e.g. $\mathbb{E}^{\mathbb{Q}}[exp[r_i]] = exp[r_f]$).[29]

In Panel A of Figure 2. 1, as $\beta_{i,1}$ increases, both the physical density and the risk-neutral density become more dispersed. Moreover, as can be seen in Panel B of Figure 2. 1, the pricing kernel puts more weight on the positive region as $\beta_{i,1}$ increases. In other words, high beta assets have volatile future payoffs and therefore are capable of earning the upside variance premium provided by the increasing region of the pricing kernel.

[28] See, for example, Bakshi, Madan, and Panayotov (2010), Christoffersen, Heston, and Jacobs (2013), Chabi-Yo, Garcia, and Renault (2008), Brown and Jackwerth (2012), and Bates (2008), among others.
[29] We do not consider the idiosyncratic randomness here since it does not affect the expected returns in our model.

Hence, high beta assets are more likely to have high prices and low expected returns, supporting the recent work of Bakshi, Madan, and Panayotov (2010) on the property of contingent claims on the upside.

We calibrate our approximate capital asset pricing model with the risk prices implied by SPX and SPX index options and plot the cross-sectional expected stock returns in Panel A of Figure 2. 2. As can be seen in the graph, the security market line is inverse U-shaped, suggesting that the first two risk prices are economically important.

The effect of the interaction between the market variance risk premium and the market beta can be confirmed in Panel B of Figure 2. 2. The figure suggests that the security market line is positive sloping in the absence of the market variance risk premium whereas the negative market variance risk premium would lead to a concave security market line. Thus, the link between the market variance premium and the curvature of the security market line is a unique feature of our model.

## 2.3 Empirical implications of the second order risk

In this section, we consider a simplified model, in which the first two order systematic risks are priced:

$$\mathbb{E}_t\left[R_{i,t+1}\right] - R_{f,t} = \lambda_{1,t}\beta_{i,1} + \lambda_{2,t}\beta_{i,1}^2, \tag{2.17}$$

where $\beta_{i,1} > 0, \lambda_{1,t} > 0$, and $\lambda_{2,t} < 0$. We then discuss the empirical implications of the second-order risk from this simplified model.

### 2.3.1 The shape of the security market line

First of all, the slope of the security market line is

$$\frac{\partial(\mathbb{E}_t[R_{i,t+1}] - R_{f,t})}{\partial \beta_{i,1}} = \lambda_{1,t} + 2\lambda_{2,t}\beta_{i,1}. \tag{2.18}$$

Thus, there exists $\beta_{*,1} \equiv \frac{\lambda_{1,t}}{-2\lambda_{2,t}}$ such that $\frac{\partial(\mathbb{E}_t[R_{i,t+1}]-R_{f,t})}{\partial \beta_{i,1}} > 0$ when $\beta_{i,1} < \beta_{*,1}$ and $\frac{\partial(\mathbb{E}_t[R_{i,t+1}]-R_{f,t})}{\partial \beta_{i,1}} \leq 0$ when $\beta_{i,1} \geq \beta_{*,1}$. The result implies that the expected excess stock return in the cross section is first increasing with $\beta_{i,1}$ and then turns decreasing with $\beta_{i,1}$. Furthermore, the curvature of the security market line is

$$\frac{\partial^2(\mathbb{E}_t[R_{i,t+1}] - R_{f,t})}{\partial \beta_{i,1}^2} = \lambda_{2,t}. \tag{2.19}$$

Thus, the curvature of the security market line corresponds to the second-order risk price ($\lambda_{2,t} < 0$), implying that the security market line is inverse U-shaped. The following corollary summarized our results.

**Corollary 1.** (Market variance risk premium and security market line)

When the market variance risk premium is negative (e.g., $\lambda_{2,t} < 0$), the slope of the security market line is decreasing in $\beta_{i,1}$. Furthermore, the security market line is inverse U-shaped and the curvature of the security market line is related to the market variance risk premium.

### 2.3.2 The cross-sectional volatility-return relationship

We now illustrate how the second-order risk price affects the cross-sectional volatility-return relation. First, define $\sigma_{i,t}^2$ as the stock return variance

$$\begin{aligned} \sigma_{i,t}^2 \equiv \mathbb{V}ar_t[r_{i,t+1}] &= \beta_{i,1}^2 \mathbb{V}ar_t[r_{m,t+1}] + \mathbb{V}ar_t[\epsilon_{i,t+1}] \\ &= \beta_{i,1}^2 \sigma_{m,t}^2 + \sigma_{i,\epsilon,t}^2. \end{aligned} \tag{2.20}$$

In our case, $\beta_{i,1} > 0$ and $\beta_{i,1} = \left(\frac{\sigma_{i,t}^2 - \sigma_{i,\epsilon,t}^2}{\sigma_{m,t}^2}\right)^{1/2}$. Thus, the first derivative of cross-sectional stock return with respect to $\sigma_{i,t}^2$ is

$$\frac{\partial\left(\mathbb{E}_t\left[R_{i,t+1}\right] - R_{f,t}\right)}{\partial \sigma_{i,t}^2} = \frac{\lambda_{2,t}}{\sigma_{m,t}^2} + \frac{\lambda_{1,t}}{2\left(\sigma_{m,t}^2\left(\sigma_{i,t}^2 - \sigma_{i,\epsilon,t}^2\right)\right)^{1/2}}. \tag{2.21}$$

Thus, there exists $\sigma_{*,t}^2 \equiv \frac{\lambda_{1,t}^2 \sigma_{m,t}^2}{4\lambda_{2,t}^2} + 1$ such that $\frac{\partial\left(\mathbb{E}_t\left[R_{i,t+1}\right] - R_{f,t}\right)}{\partial \sigma_{i,t}^2} > 0$ when $\sigma_{i,t}^2 < \sigma_{*,t}^2$ and $\frac{\partial\left(\mathbb{E}_t\left[R_{i,t+1}\right] - R_{f,t}\right)}{\partial \sigma_{i,t}^2} \leq 0$ when $\sigma_{i,t}^2 \geq \sigma_{*,t}^2$. The result yields the following corollary.

**Corollary 2.** (The second-order risk price and cross-sectional volatility-return relation) The expected excess stock return in the cross section is first increasing with $\sigma_{i,t}^2$ and then turns decreasing with $\sigma_{i,t}^2$. In particular, $\lambda_{2,t}$ contributes to the negative cross-sectional volatility-return relation.

We now show that the second-order risk price in our model can help explain the cross-sectional return differentials with respect to idiosyncratic volatility documented in Ang, Hodrick, Xing, and Zhang (2006). Define the idiosyncratic volatility as the residual variance of the stock return adjusted for the first-order risk premium,

$$IVOL_{i,t} \equiv \mathbb{V}\text{ar}\left[\mathbb{E}_t\left[R_{i,t+1}\right] - R_{f,t} - \lambda_{1,t}\beta_1\right] = \beta_{i,1}^4 \mathbb{V}\text{ar}\left[\lambda_{2,t}\right]. \tag{2.22}$$

In our case, $\beta_{i,1} > 0$ and $\beta_{i,1} = \left(\frac{IVOL_{i,t}}{\mathbb{V}\text{ar}\left[\lambda_{2,t}\right]}\right)^{1/4}$. Thus, the first derivative of cross-sectional stock return with respect to $IVOL_{i,t}$ is

$$\frac{\partial\left(E_t\left[R_{i,t+1}\right]-R_{f,t}\right)}{\partial IVOL_{i,t}} = \frac{\lambda_{2,t}}{2IVOL_{i,t}^{1/2}\mathbb{V}\mathrm{ar}\left[\lambda_{2,t}\right]^{1/2}}+\frac{\lambda_{1,t}}{4\,IVOL_{i,t}^{3/4}\mathbb{V}\mathrm{ar}\left[\lambda_{2,t}\right]^{1/4}}. \quad (2.23)$$

Thus, there exists $IVOL_{*,t} \equiv \frac{\lambda_{1,t}^4 \mathbb{V}\mathrm{ar}[\lambda_{2,t}]}{16\lambda_{2,t}^4}$ such that $\frac{\partial\left(E_t\left[R_{i,t+1}\right]-R_{f,t}\right)}{\partial IVOL_{i,t}} > 0$ when $IVOL_{i,t} < IVOL_{*,t}$ and $\frac{\partial\left(E_t\left[R_{i,t+1}\right]-R_{f,t}\right)}{\partial IVOL_{i,t}} \leq 0$ when $IVOL_{i,t} \geq IVOL_{*,t}$. The result yields the following corollary.

**Corollary 3.** (The second-order risk price and idiosyncratic volatility)

The expected excess stock return in the cross section is first increasing with $IVOL_{i,t}$ and then turns decreasing with $IVOL_{i,t}$. In particular, $\lambda_{2,t}$ contributes to the negative cross-sectional idiosyncratic volatility-return relation.

### 2.3.3 The betting-against-beta premium

Recently, Frazzini and Pedersen (2014) suggest that because constrained investors bid up high-beta assets, high-beta assets are associated with low alphas. They show that a betting-against-beta (BAB) factor, which is long leveraged low-beta assets and short high-beta assets, produces significant positive risk-adjusted returns. Our model, in contrast, suggests that the betting-against-beta strategy is exposed to high order systematic risks. To illustrate our claim, define the return on the BAB strategy as

$$\mathbb{E}_t\left[R_{BAB,t+1}\right] \equiv \frac{\mathbb{E}_t\left[R_{L,t+1}\right]-r_{f,t}}{\beta_L}-\frac{\mathbb{E}_t\left[R_{H,t+1}\right]-r_{f,t}}{\beta_H}, \quad (2.24)$$

where $\beta_L < \beta_H$, $R_{L,t+1}$ is the return for the low beta asset, and $R_{H,t+1}$ is the return for the high beta asset. In our model, it follows directly that

$$\mathbb{E}_t[R_{BAB,t+1}] = \frac{\lambda_{1,t}\beta_L + \lambda_{2,t}\beta_L^2}{\beta_L} - \frac{\lambda_{1,t}\beta_H + \lambda_{2,t}\beta_H^2}{\beta_H} \tag{2.25}$$
$$= -(\beta_H - \beta_L)\lambda_{2,t}.$$

Thus, our model implies that the second-order risk price contributes to the premium of the BAB strategy. The result yields the following corollary.

**Corollary 4.** (The second-order risk price and the betting-against-beta premium)

The BAB factor is negatively related to $\lambda_{2,t}$.

## 2.4 Data and summary statistics

### 2.4.1 Data

The sample comprises all NYSE/AMEX/NASDAQ ordinary common stocks over the period from January 1963 to December 2012. Daily and monthly stock return data (with share code =10 and 11) are from CRSP. Stocks with share prices less than $1 at the end of the previous month are excluded. Financial statement data are from COMPUSTAT. Fama and French (1993) factors, their momentum UMD factor, and their Size-B/M portfolios are obtained from the online data library of Ken French.[30] We obtain daily data from OptionMetrics for equity options and S&P 500 index options over the period from January 1996 to December 2012.

Expected market variance risk premium (*ERV−IV*) is obtained from Hao Zhou's personal website.[31] The risk-neutral expectation of variance (*IV*) is measured as the end-of-month VIX-squared de-annualized (VIX$^2$/12), whereas the realized variance (*RV*) is the sum of squared 5-minute log returns of the S&P 500 index over the month. As

30 http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/
31 https://sites.google.com/site/haozhouspersonalhomepage/

described in Drechsler and Yaron (2011) and Zhou (2012), the expected realized variance (*ERV*) is a statistical forecast of realized variance with one lag of implied variance and one lag of realized variance. Thus, the expected market variance risk premium is defined as *EVRP*= *ERV* –*IV*.

Data on government bond yields, corporate bond yields, and the TED spread are from the FRED database of the Federal Reserve Bank of St. Louis. Following the literature (see, for example, Petkova and Zhang (2005) and Petkova (2006), among the others), we construct a set of variables for macro-economy. Specifically, we use the CRSP value-weighted portfolio to measure the dividend yield (*DIV*) by the sum of dividends over the last 12 months, divided by the level of the index. The term spread (*TERM*) is measured by the difference between the yields of a 10-year and a 1-year government bond. The default spread (*DEF*) is computed by the difference between the yields of a long-term corporate Baa bond and a long-term government bond.

We use stock analyst forecasts of the long-term growth rate (*LTG*) for the earnings-per share obtained from the unadjusted I/B/E/S summary database. Following Yu (2011) and Hong and Sraer (2012), the standard deviation of the forecast for LTG is used to proxy for the firm-level disagreement. The aggregate disagreement (DIS) is measured by the cross-sectional value-weighted average of the individual stock disagreements.

### 2.4.2 Estimation of co-moment risks

For each month we estimate co-moment risks using the daily stock returns over the past month. Following Ang, Hodrick, Xing, and Zhang (2006), only stocks with more than 17 daily observations are included. First of all, the historical CAPM beta ($\hat{\beta}_{i,11}$) is estimated by the following linear regression:

$$R_{i,t+1} - R_{f,t+1} = \alpha_i + \beta_{i,11} MKT_{t+1} + \varepsilon_{i,t+1}. \tag{2.26}$$

Define $\tilde{R}_{i,t+1}$ as the residual stock return $i$, i.e., $\tilde{R}_{i,t+1} = R_{i,t+1} - R_{f,t+1} - (\hat{\alpha}_i + \hat{\beta}_{i,11} MKT_{t+1})$ and define $\tilde{R}_{m,t+1}$ as the demeaned market return, i.e., $\tilde{R}_{m,t+1} = MKT_{t+1} - \mathbb{E}[MKT_{t+1}]$. Then the first-order coskewness ($\hat{\beta}_{i,12}$) and the first-order cokurtosis ($\hat{\beta}_{i,13}$) are computed respectively by

$$\begin{aligned} \hat{\beta}_{i,12} &\equiv \mathbb{E}[\tilde{R}_i \tilde{R}_m^2]/\mathbb{E}[\tilde{R}_m^3] = \left(\mathbb{E}[\tilde{R}_i \tilde{R}_m^2]/\mathbb{E}[\tilde{R}_m^2]^{3/2}\right)/SKEW_m, \\ \hat{\beta}_{i,13} &\equiv \mathbb{E}[\tilde{R}_i \tilde{R}_m^3]/\mathbb{E}[\tilde{R}_m^4] = \left(\mathbb{E}[\tilde{R}_i \tilde{R}_m^3]/\mathbb{E}[\tilde{R}_m^2]^2\right)/KURT_m, \end{aligned} \tag{2.27}$$

where $SKEW_m = \mathbb{E}[\tilde{R}_m^3]/\mathbb{E}[\tilde{R}_m^2]^{3/2}$ and $KURT_m = \mathbb{E}[\tilde{R}_m^4]/\mathbb{E}[\tilde{R}_m^2]^2$. Our approach is similar to but different from that of Harvey and Siddique (2000).[32] In particular, our estimation for these first-order systematic risks is performed separately. We estimate the co-moments divided by market variance using the daily data from the regression whereas the market skewness ($SKEW_m$) and the market kurtosis ($KURT_m$) are computed using the full sample monthly market returns. While preserving the cross-sectional ranks of the security betas, this procedure ensures that the denominators estimated from the short regression window would be well-behaved. The second-order coskewness ($\hat{\beta}_{i,21}$) and the second-order cokurtosis ($\hat{\beta}_{i,22}$) are similarly estimated by

$$\begin{aligned} \hat{\beta}_{i,21} &\equiv \mathbb{E}[\tilde{R}_i^2 \tilde{R}_m]/\mathbb{E}[\tilde{R}_m^3] = \left|\mathbb{E}[\tilde{R}_i^2 \tilde{R}_m]/\mathbb{E}[\tilde{R}_m^2]^{3/2}\right|/|SKEW_m|, \\ \hat{\beta}_{i,22} &\equiv \mathbb{E}[\tilde{R}_i^2 \tilde{R}_m^2]/\mathbb{E}[\tilde{R}_m^4] = \left|\mathbb{E}[\tilde{R}_i^2 \tilde{R}_m^2]/\mathbb{E}[\tilde{R}_m^2]^2\right|/|KURT_m|, \end{aligned} \tag{2.28}$$

where the absolute value is required in both estimates since the second-order systematic risk should be nonnegative.

We apply the model-free approach of Bakshi, Kapadia, and Madan (2003) to

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[32] Harvey and Siddique (2000) construct a measure of coskewness, $\hat{\beta}_{i,SKD} \equiv \mathbb{E}[\tilde{R}_i \tilde{R}_m^2]/\left(\mathbb{E}[\tilde{R}_i^2]^{1/2}\mathbb{E}[\tilde{R}_m^2]\right)$.While the numerator of $\beta_{i,12}$ and that of $\beta_{i,SKD}$ in are the same, these two co-moment risks are different in which the residual volatility of the stock return is used as the denominator of their $\beta_{i,SKD}$.

estimate the 30-day risk-neutral moments for each day, following the procedure outlined in Chang, Christoffersen, and Jacobs (2013).[33] For each month we estimate the risk-neutral variance beta ($\hat{\beta}_{i,2,RN}$) with the daily estimates of risk-neutral individual variance and risk-neutral market variance, exploiting the linear market model relation for the second moment:

$$VAR_{i,RN,t+1} = \alpha_{i,RN,2} + \beta_{i,RN,2} VAR_{m,RN,t+1} + \varepsilon_{i,RN,t+1}, \tag{2.29}$$

where $VAR_{i,RN,t+1}$ is the risk-neutral variance of the stock $i$, $VAR_{m,RN,t+1}$ is the risk-neutral variance of the market. The parameters are estimated through nonnegative least squares method since both $\alpha_{i,RN,2}$ and $\beta_{i,RN,2}$ should be nonnegative.

### 2.4.3 Summary statistics

Table 2.2 table reports summary statistics for variables used in this study. We first compute in each month the cross-sectional statistics for each security and then report the time-series average. The historical CAPM beta ($\hat{\beta}_{i,11}$) ranges from −1.011 (at the 5th percentile) to 2,829 (at 95th percentile); the first-order coskewness ($\hat{\beta}_{i,12}$) ranges from −3.971 to 4.280; the first-order kurtosis ($\hat{\beta}_{i,13}$) ranges from −0.613 to 0.641. The mean values and the median values of these first-order co-moment risks tend to decrease as the orders of the market returns increase. The second-order coskewness ($\hat{\beta}_{i,21}$), ranging from 0.212 to 63.338, is more dispersed than the second-order kurtosis ($\hat{\beta}_{i,22}$) which ranges from 0.226 to 18.820. The mean and the median value of $\hat{\beta}_{i,21}$ is larger than those of $\hat{\beta}_{i,22}$. The risk-neutral variance beta ($\hat{\beta}_{i,RN,2}$), estimated from stocks with available equity

[33] Except for one thing. We use linearly interpolate implied volatilities since the cubic spline interpolation requires more available observations across moneyness and sometime produces inconsistent negative estimates for implied volatilities.

options, ranges from 0.005 to 8.675, which is less dispersed than those of the second-order co-moment risks estimated from the entire common stocks.

We also report the statistics for the average of daily estimates for risk-neutral variance ($VAR_{i,RN}$), risk-neutral skewness ($SKEW_{i,RN}$), and risk-neutral kurtosis ($KURT_{i,RN}$) as well as statistics for firm characteristics, including the book-to-market ratio (*B/M*), market capitalization (*Size*, in billion), the average of past 11-month returns prior to last month (*RET_2_12*), and Amihud's illiquidity measure (*ILLIQ*, in million).

## 2.5 Pricing high-order systematic risks

### 2.5.1 Cross-sectional regressions

We examine our approximate capital asset pricing model in the cross-section. In each month, we sort stocks into 25 portfolios based on the historical CAPM beta ($\hat{\beta}_{i,11}$) and compute the equal-weighted portfolio returns. To achieve higher testing power, we also adopt the Fama-French 25 value-weighted portfolio returns formed on size and B/M. We first estimate the following time-series regression for each portfolio on the Fama-French (1993) and Carhart (1997) four factors:

$$\begin{aligned} R_{p,t+1} - R_{f,t+1} = \alpha_p + \beta_{p,MKT} MKT_{t+1} \\ + \beta_{p,SMB} SMB_{t+1} + \beta_{p,HML} HML_{t+1} + \beta_{p,UMD} UMD_{t+1}. \end{aligned} \quad (2.30)$$

In the second stage, we use the Fama-MacBeth (1973) cross-sectional regression to estimate the prices of high order risks while controlling for common factor loadings:

$$\begin{aligned} \mathbb{E}[R_p] - R_f = \lambda_{MKT,1}\beta_{p,MKT} + \lambda_{MKT,2}\tilde{\beta}^2_{p,MKT} + \lambda_{MKT,3}\tilde{\beta}^3_{p,MKT} \\ + \lambda_{MKT,4}\tilde{\beta}^4_{MKT} + \lambda_{SMB}\beta_{SMB} + \lambda_{HML}\beta_{SMB} + \lambda_{UMD}\beta_{UMD}. \end{aligned} \quad (2.31)$$

where $\tilde{\beta}^2_{p,MKT}$, $\tilde{\beta}^3_{p,MKT}$, and $\tilde{\beta}^4_{p,MKT}$ the orthogonalized high order market risks with respect to their lower order risks. Robust Newey and West (1987) *t*-statistics with eight lags that account for autocorrelations are used.

Panel A of Table 2.3 reports the estimates for the risk prices of high order risks using equal-weighted portfolio returns formed on the historical CAPM beta. As reported in column [1], we find that $\lambda_{MKT,1}$ is positive (0.711) with a significant *t*-statistic of 3.49. Furthermore, as reported in column [2], $\lambda_{MKT,2}$ is negative (−2.372) with a significant *t*-statistic of −5.69. In column [4], we find that $\lambda_{MKT,1}$ has a significant positive value of 1.567 (with a *t*-statistic of 5.40) and $\lambda_{MKT,2}$ yields a significant negative value of −2.116 (with a *t*-statistic of −4.69) whereas $\lambda_{MKT,3}$ and $\lambda_{MKT,4}$ are insignificant.

Panel B reports the estimates using Fama-French 25 value-weighted portfolio returns formed on size and B/M. The result in Panel B is similar with that of Panel A. For example, as reported in column [4], $\lambda_{MKT,1}$ is positive (0.925) with a significant *t*-statistic of 4.87 and $\lambda_{MKT,2}$ is negative (−0.826) with a significant *t*-statistic of −2.18 whereas $\lambda_{MKT,3}$ and $\lambda_{MKT,4}$ are insignificant.

In summary, we find supporting evidence for the pricing of the first two orders of market risks. The first-order risk is significantly and positively priced while the second-order risk is significantly and negatively priced. More importantly, our findings imply that cross-sectional relation between expected return and market beta should be inverse-U shaped, which constitutes the main idea of our empirical tests in the following sections.

### 2.5.2 Evidence on the second-order risk premium

***Performance of portfolios formed based on the first-order systematic risk***

We examine the performance of portfolios formed on the first-order co-moment risks, including the market beta ($\hat{\beta}_{i,11}$), the first-order coskewness ($\hat{\beta}_{i,12}$) and the first-order cokurtosis ($\hat{\beta}_{i,13}$). In each month, stocks are sorted into 25 portfolios from the lowest (1) to the highest (25). After the portfolio formation, we calculate the equally-weighted monthly stock returns for each portfolio. For each portfolio, we compute the risk-adjusted return with respect to Fama-French (1993) and Carhart (1997) four factors (*MKT*, *SMB*, *HML*, and *UMD*) from the intercept estimate of a time-series regression. Table 2.4 presents the results for $\hat{\beta}_{i,11}$, $\hat{\beta}_{i,12}$ and $\hat{\beta}_{i,13}$ in Panel A, Panel B, and Panel C, respectively.

Consistent with an inverse-U shaped pattern implied by the model, as can be seen in Table 2.4, all of the first-order co-moment risk sorted portfolios exhibit that stocks in the bottom portfolio and in the top portfolio have lower stock returns than stocks in the middle portfolios. To exploit the economic value of the inverse-U shaped pattern, we construct a curvature portfolio (*CUR1*) by the sum of the twice difference for the portfolios formed on each of the first order co-moment risks. That is, for *N* risk-sorted portfolios, the curvature portfolio would be $\sum_{p=3}^{N} \Delta^2 r_p$, where $\Delta r_p = r_p - r_{p-1}$ and $\Delta^2 r_p = \Delta r_p - \Delta r_{p-1}$. Thus, for the 25 portfolios, *CUR1* is the curvature portfolio that longs the difference in the top portfolios (25-24) and shorts difference in the bottom portfolios (2-1).

We find that *CUR1* is significantly negative at −1.00% (with a *t*-statistic of −5.79) for $\hat{\beta}_{i,11}$ sorted portfolios, at −0.98% (with a *t*-statistic of −5.62) for $\hat{\beta}_{i,12}$ sorted portfolios, and at −0.96% (with a *t*-statistic of −6.18) for $\hat{\beta}_{i,13}$ sorted portfolios. Controlling for the Fama-French (1993) and Carhart (1997) four factor model, *CUR1* still gives a significant alpha of −1.11% with a *t*-statistic of −7.15 for $\hat{\beta}_{i,11}$ sorted portfolios, −1.01% with a *t*-statistic of −6.83 for $\hat{\beta}_{i,12}$ sorted portfolios, and −1.03% with a *t*-statistic of −7.78 for $\hat{\beta}_{i,13}$ sorted portfolios.

In summary, consistent with *Corollary* 1, we find an inverse-U shaped pattern for portfolios formed on the first-order risks. We also find the curvature portfolio based on the trading strategy exploiting the inverse-U shaped pattern generates significant abnormal returns where the the Fama-French (1993) and Carhart (1997) four factor model cannot explain. Thus, the findings suggest that the second-order risk premium is statistically and economically significant.

***Performance of portfolios formed on the second-order co-moment risks***

We examine the performance of portfolios formed on the second-order co-moment risks, including the second-order coskewness ($\hat{\beta}_{i,21}$) and the second-order cokurtosis ($\hat{\beta}_{i,22}$). In each month, stocks are sorted into 25 portfolios from the lowest (1) to the highest (25) and the portfolio returns are equal-weighted. Table 2.5 presents the results for $\hat{\beta}_{i,21}$ and $\hat{\beta}_{i,22}$ in Panel A and Panel B, respectively.

Consistent with the negative risk prices for the second-order risk, as can be seen in Table 2.4, portfolio returns exhibit decreasing pattern, albeit slightly increasing initially. To exploit the economic value of the second-order risk premium, we construct another curvature portfolio (*CUR2*) that longs the top portfolio and shorts the bottom portfolio for the portfolios formed on each of the second order co-moments risks.

We find that *CUR2* is significantly negative at −1.27% (with a *t*-statistic of −4.32) for $\hat{\beta}_{i,21}$ sorted portfolios, and at −1.06% (with a *t*-statistic of −3.05) for $\hat{\beta}_{i,22}$ sorted portfolios. Controlling for the Fama-French (1993) and Carhart (1997) four factor model, *CUR2* still gives a significant alpha of −1.47% with a *t*-statistic of −8.37 for $\hat{\beta}_{i,21}$ sorted portfolios, and −1.37% with a *t*-statistic of −6.74 for $\hat{\beta}_{i,22}$ sorted portfolios. In Panel C, we find similar results for the 15 portfolios formed on the risk-neutral variance beta ($\hat{\beta}_{i,2,RN}$). We find that the curvature portfolio (*CUR2=15-1*) is significantly negative at –

1.34% with a $t$-statistic of −2.21 and has a significant alpha of −1.38% with a $t$-statistic of −3.71 adjusted by the Fama-French (1993) and Carhart (1997) four factor model.

In summary, consistent with *Corollary* 1, we find negative relation between cross-sectional stock returns and the second-order risks. We also find that the curvature portfolios generate significant abnormal returns where the Fama-French (1993) and Carhart (1997) four factor model cannot explain. Thus, the findings confirm that the second-order risk premium is statistically and economically significant.

## 2.6 Mimicking curvature factors

### 2.6.1 Properties of the mimicking curvature factors

We construct three mimicking factors for the second-order risk premium using the curvature portfolios studied in the previous section. We construct our first mimicking factor, *FCUR1*, based on the average of the three *CUR1*s formed based on $\hat{\beta}_{i,11}$, $\hat{\beta}_{i,12}$, and $\hat{\beta}_{i,13}$, respectively. Similarly, we construct our second mimicking curvature factor, *FCUR2*, based on the average of the two *CUR2*s formed on $\hat{\beta}_{i,21}$ and $\hat{\beta}_{i,22}$, respectively. Our third mimicking curvature factor, $FCUR2_{RN}$, is measured by the curvature portfolio $CUR2_{RN}$ formed based on $\hat{\beta}_{i,2,RN}$.

Table 2.6 reports the performance of our mimicking curvature factors. In Panel B, *FCUR1* is significantly negative at −0.98% ($t$-statistic = −6.90) and *FCUR2* is also significantly negative at −1.17% ($t$-statistic = −3.66) during the sample period from January 1963 to December 2012. Moreover, *FCUR1* and *FCUR2* remain significantly negative during the sub-sample period from January 1996 to December 2012. For each curvature factor, we compute the risk-adjusted return with respect to Fama-French (1993) and Carhart (1997) four factors (*MKT*, *SMB*, *HML*, and *UMD*). Our three mimicking curvature factors, *FCUR1*, *FCUR2*, and $FCUR2_{RN}$, have significantly negative abnormal returns of −1.05% (with a *t*-statistic of = −6.90), −1.42% (with a *t*-statistic of −7.65), and −1.38% (with a *t*-statistic of −3.71), respectively.

Panel C presents the Spearman correlations. Our curvature factors are related to each other. For example, the correlation between *FCUR1* and *FCUR2* is high at 0.686 and the correlation between *FCUR2* and *FCUR2RN* is also high at 0.625. The market factor (*MKT*) has positive correlations of 0.152 with *FCUR1*, 0.531 with *FCUR2* and 0.575 with $FCUR2_{RN}$. Furthermore, the size factor (*SMB*) also shows positive correlations of 0.328 with *FCUR1*, 0.595 with *FCUR2* and 0.440 with *FCUR2N* while the value factor (*HML*) and the momentum factor (*UMD*) show smaller negative correlation with our mimicking factors. More importantly, consistent with our model, we find that the expected market variance risk premium (*ERV−IV*) yields positive correlations of 0.165 with *FCUR1*, 0.317 with *FCUR2* and 0.352 with $FCUR2_{RN}$.

### 2.6.2 The market variance risk premium and the mimicking curvature factors

To examine how our mimicking curvature factors are related to the macro-economy, we regress these factors on a set of state variables. Following the literature, we use the aggregate dividend yield (*DIV*), the default spread (*DEF*), the term spread (*TERM*), and one-month Treasury bill yield (*TB*) as explanatory variables. Moreover, we include the expected market variance risk premium (*ERV−IV*), aggregate disagreement (*DIS*), and the TED spread (*TED*). Our model implies that the second-order risk price is related to the market variance premium and therefore *ERV−IV* should explain our mimicking curvature factors.

Table 2.7 presents the results for the regressions of the mimicking factors. Consistent with our model, as reported in column [1] of Panel A, we find that *ERV−IV* has an impressive *t*-statistic of 2.19 in explaining the variation of *FCUR1* whereas all of other variables are insignificant. Controlling for *DIS* and *TED*, as reported in column [2] of Panel A, *ERV−IV* remains significant with a *t*-statistic of 1.71. Moreover, as shown in column [2] of Panel B and in column [2] of Panel C, *ERV−IV* has significant *t*-statistics of 2.47 and 2.69 in the regressions of *FCUR2* and $FCUR2_{RN}$, respectively.

Hong and Sraer (2012) demonstrate that the disagreement about the market return leads to speculative overpricing for high beta stocks. Their model implies that the shape of security market line is kinked and the slope of that decreases with the macro-disagreement. Thus, their theory implies that the aggregate disagreement should explain our mimicking factors. However, as can be seen in Table 2.7, the market variance premium explain the curvature factors better than the macro-disagreement does. Thus, the second-order risk premium in our paper cannot be fully explained by their disagreement-driven overpricing mechanism.

### 2.6.3 A curvature factor model

We construct curvature factor model based on our mimicking tradable factors. The expected excess return of asset *i* from the factor model is

$$\mathbb{E}[R_i - R_f] = \beta_{i,MKT}\mathbb{E}[MKT] + \beta_{i,FCUR}\mathbb{E}[FCUR], \tag{2.32}$$

where $\mathbb{E}[MKT]$ is the expected return on the market portfolio, $\mathbb{E}[FCUR]$ is expected return on the mimicking curvature factor, $FCUR \in \{FCUR1, FCUR2, FCUR2_{RN}\}$ and $\beta_{i,MKT}$ and $\beta_{i,FCUR}$ are the factor loadings from the time-series regression:

$$R_{i,t+1} - R_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,FCUR} FCUR_{t+1}. \quad (2.33)$$

Since our curvature factor is constructed to mimic the second-order risk premium, i.e., $\mathbb{E}_t[FCUR_{t+1}] \propto \lambda_{2,t}$, our theory implies that the curvature factor loading is related to the second-order risk, i.e., $\beta_{i,FCUR} \propto \beta_{i,1}^2$. Stocks with high curvature factor loadings, by construction, are less risky because they are more sensitive to the market variance risk premium, thereby providing hedging against market volatility risk.

We test our curvature factor model at the firm level using the cross-sectional regression:

$$\begin{aligned} R_{i,t+1} - R_{f,t+1} = c_0 + \lambda_{MKT} \beta_{i,MKT,t} + \lambda_{FCUR} \beta_{i,FCUR,t} \\ + c_{FIRM}\, FirmCharac_{i,t} + \varepsilon_{i,t+1}, \end{aligned} \quad (2.34)$$

where the dependent variable is the monthly individual stock returns; $\beta_{i,MKT,t}$ and $\beta_{i,FCUR,t}$ are post-ranking betas estimated from the 25 portfolios formed on historical CAPM beta ($\hat{\beta}_{i,11}$). $FirmCharac_{i,t}$ is a set of control variables, including book-to-market ratio *(B/M)*, market capitalization (*Size*), past 11-month return (*RET_2_12*), dividend yield (*YLD*), and illiquidity (*ILLIQ*). Following the methodology of Fama and French (1992), we assign each of the 25 portfolio-level post-ranking beta estimates to the individual stocks within the portfolio at that time.

Table 2.8 reports the results for the firm-level Fama-MacBeth regressions. In Panel A, we find that $\lambda_{FCUR1}$ is negative (−0.501) with a significant *t*-statistic of −3.30, reported in column [2]. Moreover, in column [3], $\lambda_{FCUR2}$ is also negative (−0.682) with a significant *t*-statistic of −2.59. In Panel B, we test our model with control variables of risk-neutral moments, including $VAR_{RN}$, $SKEW_{RN}$, and $KURT_{RN}$. We find similar results in which $\lambda_{FCUR1}$, $\lambda_{FCUR2}$, and $\lambda_{FCUR2RN}$ have significant negative values of −1.108 (with a *t*-statistic of −2.77), −1.307 (with a *t*-statistic of −2.75), and −1.008 (with a *t*-

statistic of −2.61), respectively.

In summary, we find significant evidence that mimicking curvature factors are priced risk factors. That is, investor required lower expected returns for holding stocks with greater exposure to the curvature factor because these assets are more sensitive to market variance risk premium, thereby providing hedge for market volatility risk. In other words, ignoring the curvature factor might omit an important source of priced risk.

## 2.7 Performance of the curvature factor model

### 2.7.1 The curvature factor model and cross-sectional volatility-return relation

We examine whether our mimicking curvature factors help explain the well-known (idiosyncratic) volatility puzzle of Ang, Hodrick, Xing, and Zhang (2006, 2009) and the MAX puzzle of Bali, Cakici, and Whitelaw (2011). Following the literature, *TVOL* is defined as the annualized past one month variance of daily stock returns; *IVOL* is defined as the annualized residual variance of the daily stock regressed on the Fama and French (1993) three factors over the past month; *MAX* is defined as the maximum daily stock return over the past one month. In our model, *Corollary* 2 and *Corollary* 3 imply that the second-order risk premium should help explain the cross-sectional return differentials with respect to *TVOL* and *IVOL*. If *MAX*, a volatility measure itself, is highly correlated to these two volatility measures, the second-order risk premium should also explain the pricing effect of *MAX*.

In Table 2.9, we report the Spearman correlations for second-order risks, and volatilities which includes *TVOL*, *IVOL*, and *MAX*. The table shows that these volatility measures are highly correlated each other with correlations ranging from 0.86 to 0.97.

Moreover, we find that these volatility measures are highly correlated with the second-order risks as well as the curvature factor loadings. Thus, results suggest that high volatility stocks could have high exposure to market variance risk premium and therefore have low expected returns.

Table 2.10 presents the performance of portfolios formed based on *TVOL*, *IVOL*, and *MAX*. Stocks are sorted into quintile portfolios, from the lowest (quintile 1) to the highest (quintile 5), and the portfolio returns are value-weighted. For each portfolio, we compute the risk-adjusted return with respect to our curvature factors as well as Fama-French (1993) and Carhart (1997) four factors (*MKT*, *SMB*, *HML*, and *UMD*) from the intercept estimate of a time-series regression. Controlling for Fama-French and Carhart four factor model, we find evidence consistent with Ang, Hodrick, Xing, and Zhang's (2006) that the '5-1' portfolios for *TVOL* and *IVOL* have significantly negative alphas of −0.93% (with a *t*-statistic of −4.72) and −0.96% (with a *t*-statistic of −5.46), respectively. Similarly, we find that the '5-1' portfolio for MAX also has a significantly negative alphas of −0.60% with a *t*-statistic of −3.63.

More importantly, after controlling for market factor and our mimicking curvature factors, we find that none of the '5-1' portfolios has significant abnormal returns. For example, the '5-1' portfolio for *TVOL* has insignificant alphas of −0.10% (with a *t*-statistic of –0.45), −0.06% (with a *t*-statistic of –0.45), and 0.27% (with a *t*-statistic of 0.67) controlling *FCUR1*, *FCUR2*, and $FCUR2_{RN}$, respectively. Furthermore, all of the '5-1' portfolios have significant exposures to our mimicking curvature factors. For example, the '5-1' portfolio for *TVOL* has significant exposures of 1.09 (with a *t*-statistic of 8.95), 0.77 (with a *t*-statistic of 22.02), and 0.92 (with a *t*-statistic of 11.84) to *FCUR1*, *FCUR2*, and $FCUR2_{RN}$, respectively.

In summary, consistent with our model, we find that these volatility measures are closely related to the second-order systematic risks and the abnormal returns associated with these variables become insignificant once we control for the mimicking curvature factors. Thus, it is the systematic second-order risk premium that accounts for these volatility puzzles.

### 2.7.2 The curvature factor model and betting-against-beta premium

We examine whether our mimicking factors help explain the betting-against-beta premium of Frazzini and Pedersen (2014). They show that a betting against beta (BAB) factor, which is long leveraged low-beta assets and short high-beta assets, produces significant positive risk-adjusted returns, supporting their theory of margin constraint. Our model, in contrast, suggests that the betting-against-beta strategy is exposed to high order systematic risks.

Table 2.11 presents the performance of portfolios formed on betting-against-beta (BAB). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of $\hat{\beta}_{i,FZ}$ at the end of the previous month, where $\hat{\beta}_{i,FZ}$ is the beta of Frazzini and Pedersen (2014). To construct the BAB factor, all stocks are assigned to one of two portfolios: low beta and high beta. Stocks are weighted by the ranked betas (lower beta security have larger weight in the low-beta portfolio and higher beta securities have larger weights in the high-beta portfolio), and the portfolios are rebalanced every calendar month. Both portfolios are rescaled to have a beta of one at portfolio formation. The betting against beta factor (BAB) is a self-financing portfolio that is long the low-beta portfolio and short the high-beta portfolio.

The BAB factor yields a significantly positive average excess return of 0.89% with a *t*-statistic of 3.82. Controlling for Fama-French and Carhart four factor model, BAB still has a significant positive alpha of 0.58% with a *t*-statistic of 2.78. Thus, our results are comparable with theirs. Controlling for the market factor and our mimicking curvature factors, we find BAB still has a significant alphas of 0.71 (with a *t*-statistic of 2.77) using *FCUR1*, a significant alphas of 0.64 (with a *t*-statistic of 2.77) using *FCUR2*, and an insignificant alphas of 0.11 (with a *t*-statistic of 0.23) using $FCUR2_{RN}$. Thus, despite the successful performance of $FCUR2_{RN}$, the market factor and our other mimicking curvature factors are insufficient to fully explain the BAB factor.

We extend the analysis using a generalized five-factor model which augments our curvature factors with Fama-French and Carhart four factors. Controlling for the five factors, the BAB premium disappears in which the BAB factor has an insignificant alphas of 0.29 (with a *t*-statistic of 1.30) using *FCUR1*, an insignificant alphas of 0.20 (with a *t*-statistic of 0.95) using *FCUR2*, and an insignificant alphas of 0.17 (with a *t*-statistic of 0.40) using $FCUR2_{RN}$. Thus, although Fama-French and Carhart four factors are similarly insufficient to explain the *BAB* factor, the extended five-factor model does capture the *BAB* premium, indicating the importance of the second-order risk premium.

In summary, we find that the second-order risk premium helps explain the *BAB* premium. Thus, the second-order risk premium provides an alternative explanation for the betting-against-beta premium other than the market friction in their paper.

### 2.7.3 Performance of portfolios formed on historical curvature factor loadings

Table 2.12 presents the performance of portfolios formed on historical curvature factor loadings ($\hat{\beta}_{FCUR1}$, $\hat{\beta}_{FCUR2}$, and $\hat{\beta}_{FCUR2RN}$). In each month, we estimate our curvature factor model using the daily stock returns over the past one month. $\hat{\beta}_{FCUR1}$,

$\hat{\beta}_{FCUR2}$, and $\hat{\beta}_{FCUR2RN}$ are the historical curvature factor loadings of *FCUR1*, *FCUR2*, and $FCUR2_{RN}$, respectively. In each month, stocks are sorted into 25 portfolios from the lowest (1) to the highest (25) and the portfolio returns are equal-weighted.

The results indicate that the portfolios formed on the historical curvature factor loadings have significant risk premiums that the current Fama-French and Carhart four factor model cannot explain. Specifically, controlling for the four factor model, the abnormal returns are −1.47 (with a *t*-statistic of −7.69) for the '25-1' portfolio formed on $\hat{\beta}_{FCUR1}$, −1.43 (with a *t*-statistic of −7.01) for the '25-1' portfolio formed on $\hat{\beta}_{FCUR2}$, and −1.12 (with a *t*-statistic of −2.61) for the '25-1' portfolio formed on $\hat{\beta}_{FCUR2RN}$. These results support the validity of our mimicking curvature factors for the second-order risk premium.

## 2.8 Conclusions

The negative market variance risk premium and the second-order risk appear to affect cross-sectional asset pricing. This paper presents an approximate capital asset pricing model, in which, along with the first-order co-moment risks in existing literature, higher order co-moment risks and high order risk premiums are important for pricing individual stocks. Stocks with high exposure to the second-order risk are more volatile and are capable of earning the upside variance premium provided by the increasing region of the pricing kernel implied by the negative market variance risk premium.

Our results show that the second-order risk is significantly and negatively priced and contributes to an inverse-U shaped relation between cross-sectional expected returns and systematic risks. We show that our mimicking curvature factors for the second-order risk premium well explain several volatility-related puzzles as well as the BAB premium. Our study provides a unified framework for better understanding of the high order risk-return tradeoff and sheds light on the role of the second-order risk premium.

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## Table 2. 1: Market moments implied by SPX and SPX index options

This table presents the estimates of physical market moments, risk-neutral market moments, and their differences. The physical market moments are computed using the full sample logarithmic monthly SPX returns. The 30-day risk-neutral market moments are estimated by the model-free approach of Bakshi, Kapadia, and Madan (2003) for each day following the procedure of Chang, Christoffersen, and Jacobs (2013). We then average daily estimates from index option prices to obtain the full sample risk-neutral market moments. The first four cumulants (*K1*, *K2*, *K3*, and *K4*) are reported. The sample period is from 1996 to 2012.

| | ***Cumulants*** | | | |
|---|---|---|---|---|
| | $\kappa_1$(%) | $\kappa_2$(%) | $\kappa_3$(%) | $\kappa_4$(%) |
| *Physical* | 0.398 | 0.218 | -0.008 | 0.002 |
| *Risk-neutral* | 0.039 | 0.479 | -0.057 | 0.023 |
| *Difference* | 0.359 | -0.260 | 0.049 | -0.021 |

# Table 2. 2: Summary statistics

This table reports the mean, standard deviation, and percentile (5th , 25th, median, 75th , and 95th) statistics for variables used in this study. We first compute in each month the cross-sectional statistics for each security and then report the time-series average. Historical CAPM beta ($\hat{\beta}_{i,11}$) is calculated using market model on the daily stock returns over the past month. We estimate $\hat{\beta}_{i,12} = \left(\mathbb{E}[\tilde{R}_i\tilde{R}_m^2]/\mathbb{E}[\tilde{R}_m^2]^{3/2}\right)/SKEW_m$ for the first order coskewness, $\hat{\beta}_{i,13} = \left(\mathbb{E}[\tilde{R}_i\tilde{R}_m^3]/\mathbb{E}[\tilde{R}_m^2]^2\right)/KURT_m$ for the first order kurtosis, $\hat{\beta}_{i,21} = \left|\mathbb{E}[\tilde{R}_i^2\tilde{R}_m]/\mathbb{E}[\tilde{R}_m^2]^{3/2}\right|/|SKEW_m|$ for the second order coskewness, and $\hat{\beta}_{i,22} = \left|\mathbb{E}[\tilde{R}_i^2\tilde{R}_m^2]/\mathbb{E}[\tilde{R}_m^2]^2\right|/|KURT_m|$ for the second order cokurtosis. We estimate the co-moments divided by market variance using the residual daily data whereas the market skewness ($SKEW_m$) and the market kurtosis ($KURT_m$) are computed using the full sample monthly market returns. We apply the model-free approach of Bakshi, Kapadia, and Madan (2003) to estimate the 30-day risk-neutral moments for each day, following the procedure outlined in Chang, Christoffersen, and Jacobs (2013). The average of daily estimates for risk-neutral variance ($VAR_{RN}$), risk-neutral skewness ($SKEW_{RN}$), and risk-neutral kurtosis ($KURT_{RN}$) are reported. The risk-neutral variance beta ($\hat{\beta}_{i,2,RN}$) is computed by the linear relation that $VAR_{i,RN,t+1} = \alpha_{i,RN,2} + \beta_{i,RN,2}VAR_{m,RN,t+1} + \varepsilon_{i,RN,t+1}$ through nonnegative least square method, where $VAR_{i,RN,t+1}$ is the risk-neutral variance of the stock *i*, and $VAR_{m,RN,t+1}$ is the risk-neutral variance of the market. B/M is the book-to-market ratio; Size is market capitalization measured in billions of dollar; *RET_2_12* reports the average of past 11-month returns prior to last month; *ILLIQ* reports the average of Amihud's (2002) illiquidity measure. The sample period is from January 1963 to December 2012.

| | ***Descriptive statistics*** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | *Mean* | *Std.Dev.* | *5%* | *25%* | *50%* | *75%* | *95%* | *Num. Obs.* |
| $\hat{\beta}_{i,11}$ | 0.751 | 1.271 | -1.011 | 0.068 | 0.652 | 1.373 | 2.829 | 4,472 |
| $\hat{\beta}_{i,12}$ | 0.128 | 2.802 | -3.971 | -1.057 | 0.110 | 1.344 | 4.280 | 4,472 |
| $\hat{\beta}_{i,13}$ | 0.006 | 0.426 | -0.613 | -0.178 | 0.002 | 0.186 | 0.641 | 4,472 |
| $\hat{\beta}_{i,21}$ | 17.440 | 86.868 | 0.212 | 1.515 | 4.758 | 14.057 | 63.338 | 4,472 |
| $\hat{\beta}_{i,22}$ | 5.303 | 17.169 | 0.226 | 0.849 | 2.098 | 5.139 | 18.820 | 4,472 |
| $\hat{\beta}_{i,RN,2}$ | 2.310 | 4.266 | 0.005 | 0.187 | 0.983 | 2.837 | 8.675 | 883 |
| $VAR_{i,RN}$ | 0.027 | 0.036 | 0.006 | 0.012 | 0.021 | 0.035 | 0.067 | 883 |
| $SKEW_{i,RN}$ | -0.405 | 0.355 | -0.983 | -0.548 | -0.354 | -0.205 | -0.003 | 883 |
| $KURT_{i,RN}$ | 3.909 | 1.349 | 3.075 | 3.296 | 3.582 | 4.107 | 5.844 | 883 |
| *B/M* | 1.213 | 8.180 | 0.142 | 0.418 | 0.724 | 1.131 | 2.198 | 3,363 |
| *Size($b)* | 1.154 | 5.686 | 0.008 | 0.034 | 0.120 | 0.483 | 4.343 | 4,505 |
| *Log(RET2_12)* | 0.039 | 0.410 | -0.647 | -0.177 | 0.054 | 0.267 | 0.671 | 4,082 |
| *ILLIQ($10^6$)* | 4.597 | 19.745 | 0.009 | 0.080 | 0.465 | 2.499 | 20.928 | 4,047 |

# Table 2. 3: The pricing of high-order risks

This table reports the estimates for the risk prices of high order risks using portfolio returns. In Panel A, we sort stocks into 25 portfolios based on the historical CAPM beta in each month and compute the equal-weighted portfolio returns. In Panel B, we adopt the Fama-French 25 value-weighted portfolio returns formed on size and B/M. We first estimate the following time-series regression for each portfolio on the Fama-French (1993) and Carhart (1997) four factors:

$$R_{p,t+1} - R_{f,t+1} = \alpha_p + \beta_{p,MKT} MKT_{t+1} + \beta_{p,SMB} SMB_{t+1} + \beta_{p,HML} HML_{t+1} + \beta_{p,UMD} UMD_{t+1}.$$

In the second stage, we use the Fama-MacBeth (1973) cross-sectional regression to estimate the prices of high order risks while controlling for common factor loadings:

$$\mathbb{E}[R_p] - R_f = \lambda_{MKT}\beta_{p,MKT} + \lambda_{MKT,2}\tilde{\beta}^2_{p,MKT} + \lambda_{MKT,3}\tilde{\beta}^3_{p,MKT} + \lambda_{MKT,4}\tilde{\beta}^4_{MKT} + \lambda_{SMB}\beta_{SMB} + \lambda_{HML}\beta_{SMB} + \lambda_{UMD}\beta_{UMD},$$

where $\tilde{\beta}^2_{p,MKT}$, $\tilde{\beta}^3_{p,MKT}$, and $\tilde{\beta}^4_{p,MKT}$ the orthogonalized high order market risks with respect to their lower order risks . Robust Newey and West (1987) *t*-statistics with eight lags that account for autocorrelations are presented in parentheses. The sample period is from January 1963 to December 2012.

| | *Cross-sectional regressions* | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\beta_{SMB}$ | $\beta_{HML}$ | $\beta_{UMD}$ | $\beta_{MKT}$ | $\tilde{\beta}^2_{MKT}$ | $\tilde{\beta}^3_{MKT}$ | $\tilde{\beta}^4_{MKT}$ | *Adj.* $R^2$ |
| | ***Panel A: 25 portfolios formed on CAPM beta*** | | | | | | | |
| [1] | -0.161 | **1.215** | 0.927 | **0.711** | | | | 0.655 |
| | (-0.54) | **( 4.18)** | ( 1.20) | **( 3.49)** | | | | |
| [2] | **-1.407** | **-0.888** | **-2.578** | **1.737** | **-2.372** | | | 0.842 |
| | **(-4.60)** | **(-2.73)** | **(-3.32)** | **( 6.04)** | **(-5.69)** | | | |
| [3] | **-1.373** | **-1.163** | **-2.388** | **1.781** | **-2.570** | -0.698 | | 0.840 |
| | **(-4.50)** | **(-2.68)** | **(-3.10)** | **( 6.14)** | **(-5.27)** | (-0.85) | | |
| [4] | **-1.245** | -0.626 | **-2.418** | **1.567** | **-2.116** | -0.260 | -3.854 | 0.841 |
| | **(-3.90)** | (-1.34) | **(-3.13)** | **( 5.40)** | **(-4.69)** | (-0.32) | (-1.53) | |
| | ***Panel B: 25 portfolios formed on Size-B/M*** | | | | | | | |
| [1] | **0.237** | **0.459** | **3.953** | **0.940** | | | | 0.712 |
| | **( 1.69)** | **( 3.15)** | **( 5.84)** | **( 4.93)** | | | | |
| [2] | 0.215 | **0.442** | **2.468** | **0.926** | **-0.855** | | | 0.729 |
| | ( 1.55) | **( 3.04)** | **( 3.46)** | **( 4.89)** | **(-2.39)** | | | |
| [3] | 0.217 | **0.442** | **2.496** | **0.925** | **-0.844** | -0.591 | | 0.715 |
| | ( 1.57) | **( 3.03)** | **( 3.30)** | **( 4.87)** | **(-2.22)** | (-0.18) | | |
| [4] | 0.220 | **0.441** | **2.564** | **0.925** | **-0.826** | -0.765 | 23.548 | 0.700 |
| | ( 1.60) | **( 3.03)** | **( 3.42)** | **( 4.87)** | **(-2.18)** | (-0.23) | ( 0.66) | |

## Table 2. 4: Performance of portfolios formed on the first order systematic risk

This table presents the performance of portfolios formed on the first order co-moment risks ($\hat{\beta}_{i,11}$, $\hat{\beta}_{i,12}$, and $\hat{\beta}_{i,13}$). In each month, stocks are sorted into 25 portfolios from the lowest (1) to the highest (25). After the portfolio formation, we calculate the equal-weighted monthly stock returns for each portfolio. $\hat{\beta}_{i,11}$ is calculated using market model on the daily stock returns over the past month. Define $\tilde{R}_i$ as the residual stock return and $\tilde{R}_m$ as the demeaned market return. We compute that $\hat{\beta}_{i,12} = \left(\mathbb{E}[\tilde{R}_i\tilde{R}_m^2]/\mathbb{E}[\tilde{R}_m^2]^{3/2}\right)/SKEW_m$ and $\hat{\beta}_{i,13} = \left(\mathbb{E}[\tilde{R}_i\tilde{R}_m^3]/\mathbb{E}[\tilde{R}_m^2]^2\right)/KURT_m$, where $SKEW_m = \mathbb{E}[\tilde{R}_m^3]/\mathbb{E}[\tilde{R}_m^2]^{3/2}$ and $KURT_m = \mathbb{E}[\tilde{R}_m^4]/\mathbb{E}[\tilde{R}_m^2]^2$ .We estimate the co-moments divided by market variance using the residual daily data whereas the market skewness ($SKEW_m$) and the market kurtosis ($KURT_m$) are computed using the full sample monthly market returns. Results for $\hat{\beta}_{i,11}$, $\hat{\beta}_{i,12}$ and $\hat{\beta}_{i,13}$ are reported in Panel A, Panel B, and Panel C, respectively. Performance of the bottom portfolios (1 and 2), the middle portfolios (12 and 13), and the top portfolios (24 and 25) are reported. The column "***CUR1***" refers to the curvature portfolio that longs the difference in the top portfolios (25-24) and shorts difference in the bottom portfolios (2-1). For each portfolio, we compute the risk-adjusted return with respect to Fama-French (1993) and Carhart (1997) four factors (*MKT*, *SMB*, *HML*, and *UMD*) from the intercept estimate of a time-series regression. Robust Newey and West (1987) *t*-statistics with eight lags that account for autocorrelations are presented in parentheses. The sample period is from January 1963 to December 2012.

| | *Portfolio ranking* | | | | | | ***CUR1*** | |
|---|---|---|---|---|---|---|---|---|
| | *1* | *2* | *12* | *13* | *24* | *25* | ***(25-24)-(2-1)*** | |
| ***Panel A: Performance of the 25 portfolios formed on*** $\hat{\boldsymbol{\beta}}_{\mathbf{11}}$ | | | | | | | | |
| *Excess returns* | 0.23 | 0.72 | 0.79 | 0.79 | 0.48 | -0.02 | **-1.00** | **(-5.79)** |
| *α-CAPM* | -0.25 | 0.29 | 0.32 | 0.31 | -0.28 | -0.84 | **-1.10** | **(-6.38)** |
| *α-FF3* | -0.53 | 0.03 | 0.05 | 0.03 | -0.41 | -0.99 | **-1.14** | **(-7.12)** |
| *α-FFC4* | -0.40 | 0.16 | 0.13 | 0.13 | -0.19 | -0.74 | **-1.11** | **(-7.15)** |
| ***Panel B: Performance of the 25 portfolios formed on*** $\hat{\boldsymbol{\beta}}_{\mathbf{12}}$ | | | | | | | | |
| *Excess returns* | 0.04 | 0.60 | 0.74 | 0.73 | 0.56 | 0.14 | **-0.98** | **(-5.62)** |
| *α-CAPM* | -0.60 | -0.02 | 0.31 | 0.30 | -0.04 | -0.49 | **-1.02** | **(-6.01)** |
| *α-FF3* | -0.88 | -0.27 | 0.05 | 0.04 | -0.25 | -0.73 | **-1.08** | **(-7.14)** |
| *α-FFC4* | -0.67 | -0.08 | 0.14 | 0.10 | -0.10 | -0.52 | **-1.01** | **(-6.83)** |
| ***Panel C: Performance of the 25 portfolios formed on*** $\hat{\boldsymbol{\beta}}_{\mathbf{13}}$ | | | | | | | | |
| *Excess returns* | 0.07 | 0.55 | 0.67 | 0.68 | 0.60 | 0.12 | **-0.96** | **(-6.18)** |
| *α-CAPM* | -0.56 | -0.07 | 0.23 | 0.24 | -0.01 | -0.51 | **-0.99** | **(-6.53)** |
| *α-FF3* | -0.85 | -0.34 | -0.01 | 0.01 | -0.20 | -0.76 | **-1.08** | **(-8.02)** |
| *α-FFC4* | -0.66 | -0.19 | 0.07 | 0.06 | 0.02 | -0.54 | **-1.03** | **(-7.78)** |

# Table 2. 5: Performance of portfolios formed on the second order systematic risk

This table presents the performance of portfolios formed on the second order co-moment risks ($\hat{\beta}_{i,21}$ and $\hat{\beta}_{i,22}$). In each month, stocks are sorted into 25 portfolios from the lowest (1) to the highest (25). After the portfolio formation, we calculate the equal-weighted monthly stock returns for each portfolio. Define $\tilde{R}_i$ as the residual stock return and $\tilde{R}_m$ as the demeaned market return. We compute that $\hat{\beta}_{i,21} = \left|\mathbb{E}[\tilde{R}_i^2\tilde{R}_m]/\mathbb{E}[\tilde{R}_m^2]^{3/2}\right|/|SKEW_m|$ and $\hat{\beta}_{i,22} = \left|\mathbb{E}[\tilde{R}_i^2\tilde{R}_m^2]/\mathbb{E}[\tilde{R}_m^2]^2\right|/|KURT_m|$ , where $SKEW_m = \mathbb{E}[\tilde{R}_m^3]/\mathbb{E}[\tilde{R}_m^2]^{3/2}$ and $KURT_m = \mathbb{E}[\tilde{R}_m^4]/\mathbb{E}[\tilde{R}_m^2]^2$ .We estimate the co-moments divided by market variance using the residual daily data whereas the market skewness ($SKEW_m$) and the market kurtosis ($KURT_m$) are computed using the full sample monthly market returns. Results for $\hat{\beta}_{i,21}$ and $\hat{\beta}_{i,22}$ are reported in Panel A and Panel B, respectively. Performance of the bottom portfolios (1 and 2), the middle portfolios (12 and 13), and the top portfolios (24 and 25) are reported. The column "***CUR2***" refers to the curvature portfolio that longs the portfolio 25 and shorts portfolio 1. Panel C presents the performance of 15 portfolios formed on the risk-neutral variance beta ($\hat{\beta}_{i,RN,2}$), which is estimated by $VAR_{i,RN,t+1} = \alpha_{i,RN,2} + \beta_{i,RN,2}VAR_{m,RN,t+1} + \varepsilon_{i,RN,t+1}$, where $VAR_{i,RN,t+1}$ is the risk-neutral variance of the stock $i$, and $VAR_{m,RN,t+1}$ is the risk-neutral variance of the market. "***CUR2$_{RN}$***" refers to the curvature portfolio that longs the portfolio 15 and shorts portfolio 1. For each portfolio, we compute the risk-adjusted return with respect to Fama-French (1993) and Carhart (1997) four factors (*MKT*, *SMB*, *HML*, and *UMD*) from the intercept estimate of a time-series regression. Robust Newey and West (1987) *t*-statistics with eight lags that account for autocorrelations are presented in parentheses. The sample period is from January 1963 to December 2012. The sample period for the results in Panel C is from January 1996 to December 2012 whereas robust *t*-statistics with six lags are used.

| | *Portfolio ranking* | | | | | | ***CUR2*** | |
|---|---|---|---|---|---|---|---|---|
| | *1* | *2* | *12* | *13* | *24* | *25* | ***25-1*** | |
| ***Panel A: Performance of the 25 portfolios formed on $\hat{\beta}_{21}$*** | | | | | | | | |
| *Excess returns* | 0.65 | 0.76 | 0.78 | 0.87 | 0.18 | -0.62 | **-1.27** | **(-4.32)** |
| *α-CAPM* | 0.32 | 0.37 | 0.26 | 0.34 | -0.44 | -1.25 | **-1.57** | **(-6.10)** |
| *α-FF3* | 0.10 | 0.14 | 0.02 | 0.11 | -0.69 | -1.53 | **-1.63** | **(-8.83)** |
| *α-FFC4* | 0.13 | 0.18 | 0.09 | 0.24 | -0.46 | -1.34 | **-1.47** | **(-8.37)** |
| ***Panel B: Performance of the 25 portfolios formed on $\hat{\beta}_{22}$*** | | | | | | | | |
| *Excess returns* | 0.48 | 0.70 | 0.86 | 0.93 | 0.09 | -0.58 | **-1.06** | **(-3.05)** |
| *α-CAPM* | 0.26 | 0.37 | 0.33 | 0.39 | -0.54 | -1.22 | **-1.48** | **(-4.98)** |
| *α-FF3* | 0.05 | 0.13 | 0.09 | 0.14 | -0.80 | -1.54 | **-1.59** | **(-7.35)** |
| *α-FFC4* | 0.05 | 0.14 | 0.17 | 0.25 | -0.59 | -1.32 | **-1.37** | **(-6.74)** |

## Table 2.5 (continued.)

| | *Portfolio ranking* | | | | | | ***$CUR2_{RN}$*** | |
|---|---|---|---|---|---|---|---|---|
| | *1* | *2* | *7* | *8* | *14* | *15* | ***15-1*** | |
| ***Panel C: Performance of the 15 portfolios formed on $\hat{\beta}_{RN,2}$*** | | | | | | | | |
| *Excess returns* | 0.69 | 0.70 | 0.78 | 0.82 | 0.15 | -0.65 | **-1.34** | **(-2.21)** |
| *α-CAPM* | 0.16 | 0.19 | 0.27 | 0.29 | -0.69 | -1.59 | **-1.76** | **(-3.50)** |
| *α-FF3* | 0.03 | 0.09 | 0.20 | 0.20 | -0.65 | -1.59 | **-1.62** | **(-4.61)** |
| *α-FFC4* | 0.05 | 0.11 | 0.17 | 0.19 | -0.48 | -1.33 | **-1.38** | **(-3.71)** |

# Table 2. 6: Properties of the mimicking curvature factors.

The three mimicking curvature factors (*FCUR1*, *FCUR2*, and *FCUR2RN*) are constructed as follows. *FCUR1* is constructed by the average of the three *CUR1*s formed on $\hat{\beta}_{i,11}$ , $\hat{\beta}_{i,12}$, and $\hat{\beta}_{i,13}$, respectively. *FCUR2* is computed by the average of the two *CUR2*s formed on $\hat{\beta}_{i,21}$ and $\hat{\beta}_{i,22}$, respectively. $FCUR2_{RN}$, is measured by the curvature portfolio $CUR2_{RN}$ formed on $\hat{\beta}_{i,2,RN}$. This table reports the performance of the Fama-French (1993) and Carhart (1997) four factors in Panel A and the performance of our mimicking curvature factors in Panel B. For each curvature factor, we compute the risk-adjusted return with respect to Fama-French (1993) and Carhart (1997) four factors (*MKT*, *SMB*, *HML*, and *UMD*). *ERV−IV* is the expected market variance risk premium; *DIS* is the aggregate disagreement. Panel C presents the Spearman correlations. Robust Newey and West (1987) *t*-statistics with eight lags that account for autocorrelations are presented in parentheses. The sample period is from January 1963 to December 2012. The sub-sample period is from January 1996 to December 2012 whereas robust *t*-statistics with six lags are used.

| | *MKT* | | *SMB* | | *HML* | | *UMD* | |
|---|---|---|---|---|---|---|---|---|
| | ***Panel A: Performance of Fama-French and Carhart four factors*** | | | | | | | |
| ***Mean (%)*** | | | | | | | | |
| *1963/01-2012/12* | **0.47** | **( 2.41)** | **0.25** | **( 1.84)** | **0.40** | **( 2.82)** | **0.70** | **( 3.91)** |
| *1996/01-2012/12* | 0.47 | ( 1.27) | 0.25 | ( 1.17) | 0.28 | ( 0.96) | 0.43 | ( 1.02) |

| | *FCUR1* | | *FCUR2* | | $FCUR2_{RN}$ | |
|---|---|---|---|---|---|---|
| | ***Panel B: Performance of the mimicking curvature factors*** | | | | | |
| ***Mean (%)*** | | | | | | |
| *1963/01-2012/12* | **-0.98** | **(-6.90)** | **-1.17** | **(-3.66)** | | |
| *1996/01-2012/12* | **-1.05** | **(-3.42)** | **-1.33** | **(-1.91)** | **-1.34** | **(-2.21)** |
| ***Fama-French and Carhart four factors adjusted performance*** | | | | | | |
| $\alpha$ | **-1.05** | **(-9.60)** | **-1.42** | **(-7.65)** | **-1.38** | **(-3.71)** |
| *β-MKT* | 0.01 | ( 0.45) | **0.44** | **( 8.88)** | **0.56** | **( 5.34)** |
| *β-SMB* | **0.45** | **( 5.59)** | **1.21** | **( 9.44)** | **0.52** | **( 4.84)** |
| *β-HML* | -0.01 | (-0.13) | -0.24 | (-1.59) | **-0.73** | **(-4.46)** |
| *β-UMD* | -0.06 | (-1.05) | **-0.21** | **(-2.23)** | **-0.35** | **(-3.90)** |
| *Adj.* $R^2$ | 0.20 | | 0.58 | | 0.57 | |
| ***Spearman correlations*** | | | | | | |
| *FCUR1* | 1.000 | | | | | |
| *FCUR2* | 0.686 | | 1.000 | | | |
| $FCUR2_{RN}$ | 0.390 | | 0.625 | | 1.000 | |
| *MKT* | 0.152 | | 0.531 | | 0.575 | |
| *SMB* | 0.328 | | 0.595 | | 0.440 | |
| *HML* | -0.006 | | -0.182 | | -0.325 | |
| *UMD* | -0.036 | | -0.093 | | -0.204 | |
| *ERV−IV* | 0.165 | | 0.317 | | 0.352 | |
| *DIS* | -0.034 | | -0.026 | | -0.002 | |

# Table 2. 7: Market variance risk premium and the mimicking curvature factors

This table presents the regressions of the mimicking curvature factors on the macroeconomic state variables. The dependent variables, *FCUR1*, *FCUR2*, and $FCUR2_{RN}$, are used in Panel A, Panel B, and Panel C, respectively. *DIV* is the aggregate dividend yield; *DEF* is the default spread, which is measured by the difference between the yields of a long-term corporate Baa bond and a long-term Aaa bond; *TERM* is the term spread, which is measured by the difference between the yields of a 10-year and a 1-year government bond; *TB* is one-month Treasury-bill yield; *ERV−IV* is the expected market variance risk premium; *DIS* is the aggregate disagreement. Robust Newey and West (1987) *t*-statistics with six lags that account for autocorrelations are presented in parentheses. The sample period is from January 1990 to December 2012.

| | *Dependent variable = FCUR* $\in$ *{ FCUR1, FCUR2,* $FCUR2_{RN}$ *}* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | *Constant* | *DIV* | *DEF* | *TERM* | *TB* | *ERV-IV* | *DIS* | *TED* | *Adj.* $R^2$ |
| *Panel A: Dependent variable = FCUR1* | | | | | | | | | |
| [1] | 0.034 | -0.176 | -0.169 | 0.225 | -1.173 | **0.024** | | | 0.019 |
| | (0.03) | (-0.35) | (-0.22) | (0.92) | (-0.56) | **(2.19)** | | | |
| [2] | 1.606 | -0.195 | 0.221 | 0.064 | -1.567 | **0.021** | -0.345 | -0.756 | 0.017 |
| | (0.62) | (-0.44) | (0.32) | (0.21) | (-0.55) | **(1.71)** | (-0.54) | (-1.07) | |
| *Panel B: Dependent variable = FCUR2* | | | | | | | | | |
| [1] | -0.003 | -1.734 | 2.420 | **1.066** | 4.182 | **0.091** | | | 0.076 |
| | (-0.00) | (-1.41) | (1.11) | **(2.12)** | (0.87) | **(3.02)** | | | |
| [2] | 3.942 | **-1.820** | 3.289 | 0.709 | 3.055 | **0.084** | -0.863 | -1.544 | 0.075 |
| | (0.71) | **(-1.73)** | (1.52) | (1.22) | (0.47) | **(2.47)** | (-0.61) | (-1.04) | |
| *Panel C: Dependent variable =* $FCUR2_{RN}$ | | | | | | | | | |
| [1] | 1.784 | -0.238 | 0.921 | -0.282 | -6.448 | **0.078** | | | 0.060 |
| | (0.50) | (-0.16) | (0.35) | (-0.30) | (-0.94) | **(2.87)** | | | |
| [2] | 10.650 | -0.952 | 1.350 | -0.562 | -11.478 | **0.077** | **-1.720** | 0.681 | 0.062 |
| | (1.86) | (-0.69) | (0.50) | (-0.62) | (-1.53) | **(2.69)** | **(-1.67)** | (0.40) | |

# Table 2. 8: Market price of curvature factor loadings: firm-level cross-sectional regressions

This table reports the results for the firm-level Fama-MacBeth regressions. We run the following cross-sectional regression:

$$R_{i,t+1} - R_{f,t+1} = c_0 + \lambda_{MKT}\beta_{i,MKT,t} + \lambda_{FCUR}\beta_{i,FCUR,t} + c_{FIRM}\ FirmCharac_{i,t} + \varepsilon_{i,t+1},$$

where the dependent variable is the monthly individual stock returns; $\beta_{i,MKT,t}$ and $\beta_{i,FCUR,t}$ are post-ranking betas estimated from the 25 portfolios formed on historical CAPM beta ($\hat{\beta}_{i,11}$); $FirmCharac_{i,t}$ is a set of control variables. B/M is the book-to-market ratio; Size is market capitalization measured in billions of dollar; *RET_2_12* reports the average of past 11-month returns prior to last month; *YLD* is the dividend yield measured by the sum of all dividends paid over the past 12 month; *ILLIQ* reports the average of Amihud's (2002) illiquidity measure. Risk-neutral moments, $VAR_{RN}$, $SKEW_{RN}$, and $KURT_{RN}$ are included as control variables in Panel B. Following the methodology of Fama and French (1992), we assign each of the 25 portfolio-level post-ranking beta estimates to the individual stocks within the portfolio at that time. Robust Newey and West (1987) *t*-statistics with eight lags that account for autocorrelations are presented in parentheses. The sample period is from January 1963 to December 2012. The sample period for the results in Panel B is from January 1996 to December 2012 whereas robust *t*-statistics with six lags are used.

| | *Fama-MacBeth regressions: individual stocks* | | | | | |
|---|---|---|---|---|---|---|
| | ***Panel A: January 1963- December 2012*** | | | | | |
| | [1] | | [2] | | [3] | |
| *Constant* | **0.490** | **( 2.07)** | **0.537** | **( 2.41)** | **0.507** | **( 2.30)** |
| *log(Size) ($b)* | **-0.141** | **(-3.94)** | **-0.145** | **(-4.04)** | **-0.145** | **(-4.04)** |
| *log(B/M)* | **0.224** | **( 3.56)** | **0.219** | **( 3.65)** | **0.220** | **( 3.65)** |
| *RET_2_12* | **0.876** | **( 4.74)** | **0.865** | **( 4.81)** | **0.866** | **( 4.83)** |
| *log(1+YLD)* | -2.078 | (-0.92) | -1.958 | (-0.97) | -1.945 | (-0.96) |
| *ILLIQ* | 0.023 | ( 1.05) | 0.027 | ( 1.28) | 0.027 | ( 1.25) |
| $\beta_{MKT}$ | **1.445** | **( 2.24)** | 0.117 | ( 0.64) | 0.189 | ( 1.01) |
| $\beta_{MKT}^2$ | **-1.627** | **(-2.30)** | | | | |
| $\beta_{FCUR1}$ | | | **-0.501** | **(-3.30)** | | |
| $\beta_{FCUR2}$ | | | | | **-0.682** | **(-2.59)** |
| *Adj.* $R^2$ | 0.050 | | 0.056 | | 0.056 | |
| *Nobs.* | 1,762,462 | | 1,762,462 | | 1,762,462 | |

## Table 2.8 (continued.)

| | *Fama-MacBeth regressions: individual stocks* | | | | | |
|---|---|---|---|---|---|---|
| | ***Panel B: January 1996- December 2012*** | | | | | |
| | [1] | | [2] | | [3] | |
| *Constant* | 0.332 | ( 0.53) | 0.392 | ( 0.61) | 0.142 | ( 0.22) |
| *logSZ($b)* | -0.072 | (-1.01) | -0.073 | (-1.02) | -0.067 | (-0.93) |
| *logBM* | 0.112 | ( 1.04) | 0.111 | ( 1.03) | 0.115 | ( 1.06) |
| *RET_2_12* | **0.627** | **( 1.69)** | **0.634** | **( 1.71)** | **0.617** | **( 1.67)** |
| *log(1+YLD)* | -3.570 | (-0.64) | -3.876 | (-0.69) | -3.706 | (-0.66) |
| *ILLIQ* | 0.521 | ( 0.33) | 0.534 | ( 0.34) | 0.528 | ( 0.34) |
| $\beta_{MKT}$ | 0.551 | ( 1.15) | 0.544 | ( 1.14) | 0.722 | ( 1.28) |
| $\beta_{FCUR1}$ | **-1.108** | **(-2.77)** | | | | |
| $\beta_{FCUR2}$ | | | **-1.307** | **(-2.75)** | | |
| $\beta_{FCUR2RN}$ | | | | | **-1.008** | **(-2.61)** |
| $VAR_{RN}$ | **-19.592** | **(-2.41)** | **-19.846** | **(-2.44)** | **-19.834** | **(-2.44)** |
| $SKEW_{RN}$ | **0.470** | **( 1.86)** | **0.471** | **( 1.85)** | **0.456** | **( 1.80)** |
| $KURT_{RN}$ | 0.051 | ( 0.81) | 0.052 | ( 0.82) | 0.050 | ( 0.79) |
| *Adj.* $R^2$ | 0.090 | | 0.089 | | 0.090 | |
| *Nobs.* | 161,815 | | 161,815 | | 161,815 | |

## Table 2. 9: Correlations for second order risks and volatilities

This table reports the Spearman correlations for second order risks, and volatilities. In each month, we estimate the time-series regression

$$R_{i,t+1} - R_{f,t+1} = \alpha_i + \beta_{i,MKT} MKT_{t+1} + \beta_{i,FCUR} FCUR_{t+1},$$

using the daily stock returns over the past one month. $\hat{\beta}_{FCUR1}$, $\hat{\beta}_{FCUR1}$, and $\hat{\beta}_{FCUR1}$ are the historical curvature factor loadings of *FCUR1*, *FCUR2*, and $FCUR2_{RN}$, respectively. *TVOL* is the total volatility, which is defined as the annualized standard deviation of daily stock returns over the past month; *IVOL* is the idiosyncratic volatility, which is defined as the annualized residual variance of the daily stock regressed on the Fama and French (1993) three factors over the past month. The sample period is from January 1963 to December 2012, except for correlations involving $\hat{\beta}_{RN,2}$ and $\hat{\beta}_{FCUR2RN}$, which are computed over the sample period from January 1996 to December 2012.

| | *Spearman correlations* | | | | | |
|---|---|---|---|---|---|---|
| | *TVOL* | *IVOL* | *MAX* | $\hat{\beta}_{FCUR1}$ | $\hat{\beta}_{FCUR2}$ | $\hat{\beta}_{FCUR2RN}$ |
| *TVOL* | 1.000 | 0.970 | 0.890 | 0.561 | 0.574 | 0.561 |
| *IVOL* | 0.970 | 1.000 | 0.866 | 0.562 | 0.562 | 0.544 |
| *MAX* | 0.890 | 0.866 | 1.000 | 0.502 | 0.512 | 0.502 |
| $\hat{\beta}_{11}^2$ | 0.519 | 0.393 | 0.462 | 0.243 | 0.255 | 0.230 |
| $\hat{\beta}_{12}^2$ | 0.535 | 0.532 | 0.484 | 0.322 | 0.331 | 0.311 |
| $\hat{\beta}_{13}^2$ | 0.532 | 0.530 | 0.480 | 0.316 | 0.324 | 0.303 |
| $\hat{\beta}_{21}$ | 0.752 | 0.759 | 0.702 | 0.443 | 0.449 | 0.434 |
| $\hat{\beta}_{22}$ | 0.909 | 0.911 | 0.813 | 0.530 | 0.543 | 0.521 |
| $\hat{\beta}_{RN,2}$ | 0.260 | 0.206 | 0.213 | 0.123 | 0.146 | 0.158 |

# Table 2. 10: Curvature factor adjusted performance of portfolios formed on volatilities

This table presents the performance of portfolios formed on volatilities (*TVOL*, *IVOL*, and *MAX*). In each month, stocks are sorted into quintile portfolios from the lowest (1) to the highest (5). After the portfolio formation, we calculate the value-weighted monthly stock returns for each portfolio. "5-1" refers to the hedge portfolio that longs portfolio 5 and shorts portfolio 1. For each portfolio, we compute the risk-adjusted return with respect to our curvature factors as well as Fama-French (1993) and Carhart (1997) four factors (*MKT*, *SMB*, *HML*, and *UMD*) from the intercept estimate of a time-series regression. Robust Newey and West (1987) *t*-statistics with eight lags that account for autocorrelations are presented in parentheses. The sample period is from January 1963 to December 2012. The sample period for the results involving $FCUR2_{RN}$ is from January 1996 to December 2012 whereas robust *t*-statistics with six lags are used.

| | *Portfolio ranking* | | | | | | |
|---|---|---|---|---|---|---|---|
| | *1* | *2* | *3* | *4* | *5* | ***5-1*** | |
| ***Panel A: Performance of portfolios formed on TVOL*** | | | | | | | |
| *Excess returns* | 0.49 | 0.53 | 0.59 | 0.43 | -0.32 | **-0.80** | **(-2.33)** |
| *α-CAPM* | 0.15 | 0.08 | 0.02 | -0.26 | -1.09 | **-1.24** | **(-4.21)** |
| *α-FF3* | 0.08 | 0.03 | 0.00 | -0.23 | -1.10 | **-1.18** | **(-5.95)** |
| *α-FFC4* | 0.03 | 0.05 | 0.06 | -0.14 | -0.89 | **-0.93** | **(-4.72)** |
| ***Panel B: Performance of portfolios formed on IVOL*** | | | | | | | |
| *Excess returns* | 0.48 | 0.53 | 0.60 | 0.33 | -0.35 | **-0.82** | **(-2.68)** |
| *α-CAPM* | 0.10 | 0.04 | 0.01 | -0.35 | -1.08 | **-1.19** | **(-4.36)** |
| *α-FF3* | 0.06 | 0.01 | 0.00 | -0.33 | -1.17 | **-1.23** | **(-6.97)** |
| *α-FFC4* | 0.03 | 0.05 | 0.04 | -0.21 | -0.93 | **-0.96** | **(-5.46)** |
| ***Panel C: Performance of portfolios formed on MAX*** | | | | | | | |
| *Excess returns* | 0.51 | 0.50 | 0.59 | 0.42 | 0.01 | **-0.49** | **(-1.76)** |
| *α-CAPM* | 0.16 | 0.05 | 0.05 | -0.22 | -0.68 | **-0.84** | **(-3.47)** |
| *α-FF3* | 0.08 | 0.02 | 0.04 | -0.20 | -0.68 | **-0.76** | **(-4.63)** |
| *α-FFC4* | 0.05 | 0.03 | 0.09 | -0.13 | -0.54 | **-0.60** | **(-3.63)** |

# Table 2.10 (continued.)

| | *CAPM plus FCUR adjusted performance of 5-1* <br> *FCUR* ∈ { *FCUR1*, *FCUR2*, *FCUR2<sub>RN</sub>* } | | | | | |
|---|---|---|---|---|---|---|
| | ***FCUR1*** | | ***FCUR2*** | | ***FCUR2_RN*** | |
| ***Panel D: CAPM plus FCUR adjusted performance of 5-1 formed on TVOL*** | | | | | | |
| *α* | -0.10 | (-0.45) | -0.06 | ( -0.39) | 0.27 | ( 0.67) |
| *β-MKT* | **0.80** | **( 9.69)** | **0.34** | **( 5.15)** | **0.58** | **( 5.67)** |
| *β-FCUR* | **1.09** | **( 8.95)** | **0.77** | **( 22.02)** | **0.92** | **( 11.84)** |
| *Adj.* $R^2$ | 0.54 | | 0.69 | | 0.72 | |
| ***Panel E: CAPM plus FCUR adjusted performance of 5-1 formed on IVOL*** | | | | | | |
| *α* | -0.06 | (-0.26) | 0.01 | ( 0.04) | 0.25 | ( 0.61) |
| *β-MKT* | **0.65** | **( 8.27)** | **0.17** | **( 3.02)** | **0.42** | **( 3.75)** |
| *β-FCUR* | **1.09** | **( 8.70)** | **0.78** | **( 20.96)** | **0.86** | **( 9.40)** |
| *Adj.* $R^2$ | 0.51 | | 0.69 | | 0.66 | |
| ***Panel F: CAPM plus FCUR adjusted performance of 5-1 formed on MAX*** | | | | | | |
| *α* | 0.09 | ( 0.46) | 0.15 | ( 1.11) | 0.34 | ( 1.04) |
| *β-MKT* | **0.64** | **( 9.41)** | **0.25** | **( 4.67)** | **0.42** | **( 4.42)** |
| *β-FCUR* | **0.90** | **( 6.93)** | **0.65** | **( 18.22)** | **0.76** | **( 9.25)** |
| *Adj.* $R^2$ | 0.50 | | 0.66 | | 0.61 | |

# Table 2. 11: Curvature factor adjusted performance of portfolios formed on betting-against-beta

This table presents the performance of portfolios formed on betting-against-beta (BAB). At the beginning of each calendar month, stocks are ranked in ascending order on the basis of $\hat{\beta}_{i,FZ}$ at the end of the previous month, where $\hat{\beta}_{i,FZ}$ is the beta of Frazzini and Pedersen (2014). To construct the BAB factor, all stocks are assigned to one of two portfolios: low beta and high beta. Stocks are weighted by the ranked betas (lower beta security have larger weight in the low-beta portfolio and higher beta securities have larger weights in the high-beta portfolio), and the portfolios are rebalanced every calendar month. Both portfolios are rescaled to have a beta of one at portfolio formation. The betting against beta factor (BAB) is a self-financing portfolio that is long the low-beta portfolio and short the high-beta portfolio. For each portfolio, we compute the risk-adjusted return with respect to our curvature factors as well as Fama-French (1993) and Carhart (1997) four factors (*MKT*, *SMB*, *HML*, and *UMD*) from the intercept estimate of a time-series regression. Robust Newey and West (1987) *t*-statistics with eight lags that account for autocorrelations are presented in parentheses. The sample period is from January 1963 to December 2012. The sample period for the results involving $FCUR2_{RN}$ is from January 1996 to December 2012 whereas robust *t*-statistics with six lags are used.

***Panel A: Performance of portfolios formed on betting-against-beta***

| | $Ret^L$-$R_f$ | $Ret^H$-$R_f$ | $(Ret^L$-$R_f)/\beta^L$ | $(Ret^H$-$R_f)/\beta^H$ | ***BAB*** |
|---|---|---|---|---|---|
| *Excess returns* | 0.84 | 0.64 | 1.64 | 0.75 | **0.89 ( 3.82)** |
| *α-CAPM* | 0.51 | -0.07 | 1.02 | -0.04 | **1.06 ( 4.24)** |
| *α-FF3* | 0.24 | -0.34 | 0.53 | -0.28 | **0.81 ( 3.81)** |
| *α-FFC4* | 0.27 | -0.09 | 0.57 | -0.01 | **0.58 ( 2.78)** |

***Panel B: FCUR adjusted performance of BAB***

$FCUR \in \{ FCUR1, FCUR2, FCUR2_{RN} \}$

| | ***FCUR1*** | | ***FCUR2*** | | ***$FCUR2_{RN}$*** | |
|---|---|---|---|---|---|---|
| *CAPM plus FCUR adjusted* | | | | | | |
| *α* | **0.71** | **( 2.77)** | **0.64** | **( 2.73)** | 0.11 | ( 0.23) |
| *β-MKT* | **-0.33** | **(-4.34)** | **-0.15** | **(-2.28)** | **-0.26** | **(-1.92)** |
| *β-FCUR* | **-0.36** | **(-3.26)** | **-0.29** | **(-5.68)** | **-0.50** | **(-10.96)** |
| *Adj. $R^2$* | 0.20 | | 0.28 | | 0.52 | |
| *FFC4 plus FCUR adjusted* | | | | | | |
| *α* | 0.29 | ( 1.30) | 0.20 | ( 0.95) | 0.17 | ( 0.40) |
| *β-MKT* | **-0.17** | **(-2.62)** | -0.05 | (-0.82) | **-0.25** | **(-2.05)** |
| *β-SMB* | -0.06 | (-0.81) | 0.14 | ( 1.50) | -0.12 | (-1.37) |
| *β-HML* | **0.64** | **( 5.96)** | **0.58** | **( 6.24)** | **0.61** | **( 3.75)** |
| *β-UMD* | **0.24** | **( 3.50)** | **0.20** | **( 3.35)** | **0.19** | **( 2.67)** |
| *β-FCUR* | **-0.28** | **(-3.67)** | **-0.27** | **(-5.38)** | **-0.29** | **(-4.06)** |
| *Adj. $R^2$* | 0.38 | | 0.41 | | 0.60 | |

## Table 2. 12: Performance of portfolios formed on pre-ranking curvature factor loadings

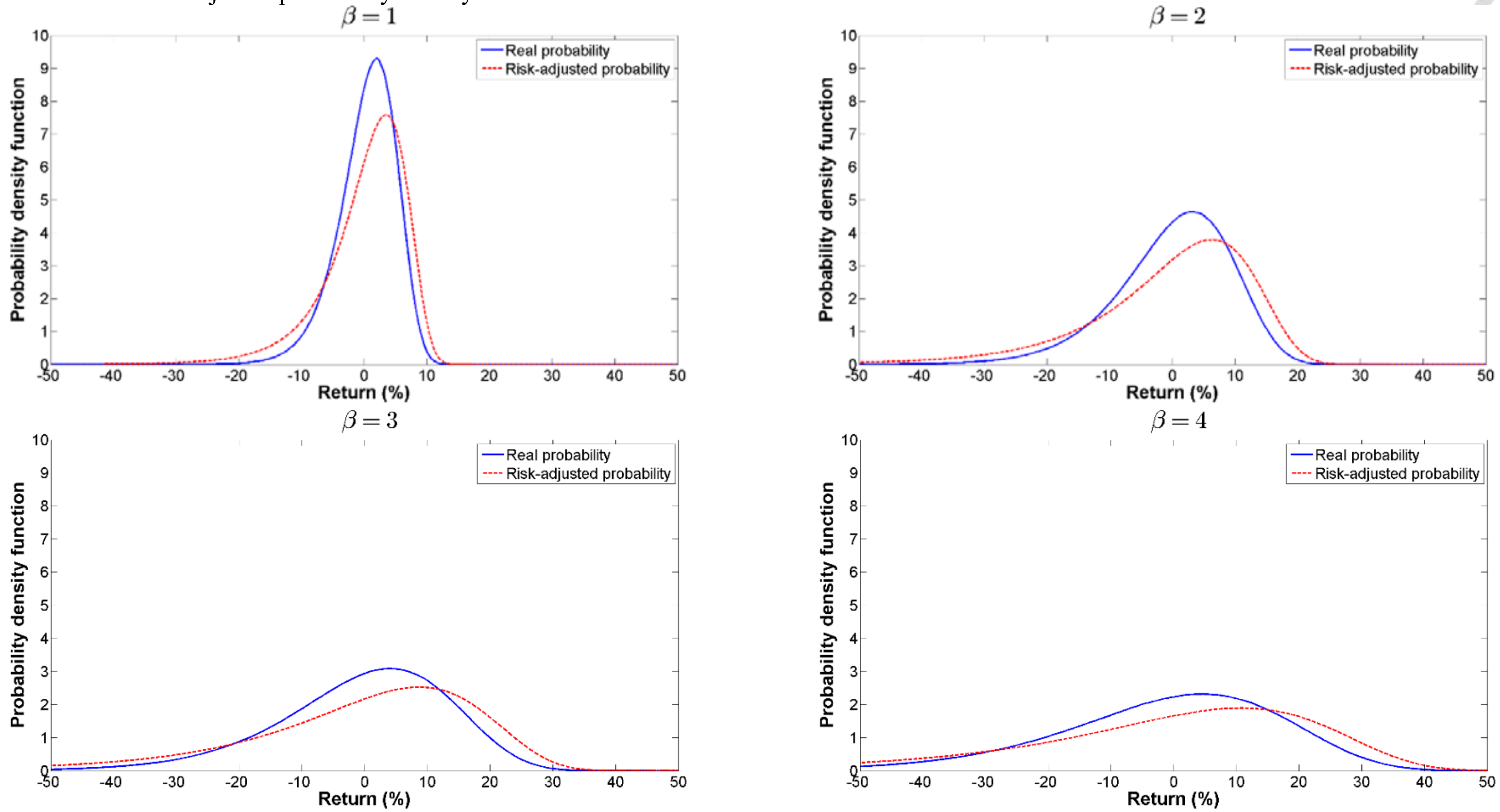
This table presents the performance of portfolios formed on historical curvature factor loadings ($\hat{\beta}_{FCUR1}$, $\hat{\beta}_{FCUR2}$, and $\hat{\beta}_{FCUR2RN}$). In each month, we estimate the time-series regression,

$$R_{i,t+1} - R_{f,t+1} = \alpha_i + \beta_{i,MKT}MKT_{t+1} + \beta_{i,FCUR}FCUR_{t+1},$$

using the daily stock returns over the past one month. $\hat{\beta}_{FCUR1}$, $\hat{\beta}_{FCUR2}$, and $\hat{\beta}_{FCUR2RN}$ are the historical curvature factor loadings of *FCUR1*, *FCUR2*, and $FCUR2_{RN}$, respectively. In each month, stocks are sorted into 25 portfolios from the lowest (1) to the highest (25). After the portfolio formation, we calculate the equally-weighted monthly stock returns for each portfolio. Performance of the bottom portfolios (1 and 2), the middle portfolios (12 and 13), and the top portfolios (24 and 25) are reported. For each portfolio, we compute the risk-adjusted return with respect to Fama-French (1993) and Carhart (1997) four factors (*MKT*, *SMB*, *HML*, and *UMD*) from the intercept estimate of a time-series regression. Robust Newey and West (1987) *t*-statistics that account for autocorrelations are reported in parentheses. The sample period, in Panel A and Panel B, is from January 1963 to December 2012, where robust *t*-statistics with eight lags are used. In Panel C, robust *t*-statistics with six lags are used in the sample period from January 1996 to December 2012.

| | *Portfolio ranking* | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | *1* | *2* | *12* | *13* | *24* | *25* | ***25-1*** | |
| ***Panel A: Performance of the 25 portfolios formed on*** $\widehat{\boldsymbol{\beta}}_{FCUR1}$ | | | | | | | | |
| *Excess returns* | 0.82 | 0.66 | 0.73 | 0.70 | 0.21 | -0.53 | **-1.34** | **(-4.54)** |
| *α-CAPM* | 0.41 | 0.24 | 0.23 | 0.19 | -0.44 | -1.19 | **-1.60** | **(-5.89)** |
| *α-FF3* | 0.17 | 0.01 | -0.02 | -0.04 | -0.71 | -1.44 | **-1.61** | **(-7.91)** |
| *α-FFC4* | 0.28 | 0.07 | 0.07 | 0.10 | -0.49 | -1.19 | **-1.47** | **(-7.69)** |
| ***Panel B: Performance of the 25 portfolios formed on*** $\widehat{\boldsymbol{\beta}}_{FCUR2}$ | | | | | | | | |
| *Excess returns* | 0.75 | 0.75 | 0.79 | 0.80 | 0.24 | -0.57 | **-1.32** | **(-4.28)** |
| *α-CAPM* | 0.35 | 0.33 | 0.28 | 0.28 | -0.43 | -1.22 | **-1.57** | **(-5.56)** |
| *α-FF3* | 0.11 | 0.08 | 0.02 | 0.07 | -0.66 | -1.43 | **-1.54** | **(-6.98)** |
| *α-FFC4* | 0.19 | 0.16 | 0.13 | 0.19 | -0.46 | -1.24 | **-1.43** | **(-7.01)** |
| ***Panel C: Performance of the 25 portfolios formed on*** $\widehat{\boldsymbol{\beta}}_{FUCR2RN}$ | | | | | | | | |
| *Excess returns* | 0.76 | 0.80 | 0.71 | 0.75 | 0.31 | -0.36 | -1.12 | (-1.64) |
| *α-CAPM* | 0.37 | 0.40 | 0.23 | 0.24 | -0.47 | -1.17 | **-1.54** | **(-2.59)** |
| *α-FF3* | 0.13 | 0.18 | 0.03 | 0.04 | -0.51 | -1.18 | **-1.31** | **(-3.34)** |
| *α-FFC4* | 0.27 | 0.22 | 0.19 | 0.19 | -0.20 | -0.85 | **-1.12** | **(-2.61)** |

Panel A: Real and risk-adjusted probability density functions



Panel B: Implied pricing kernels

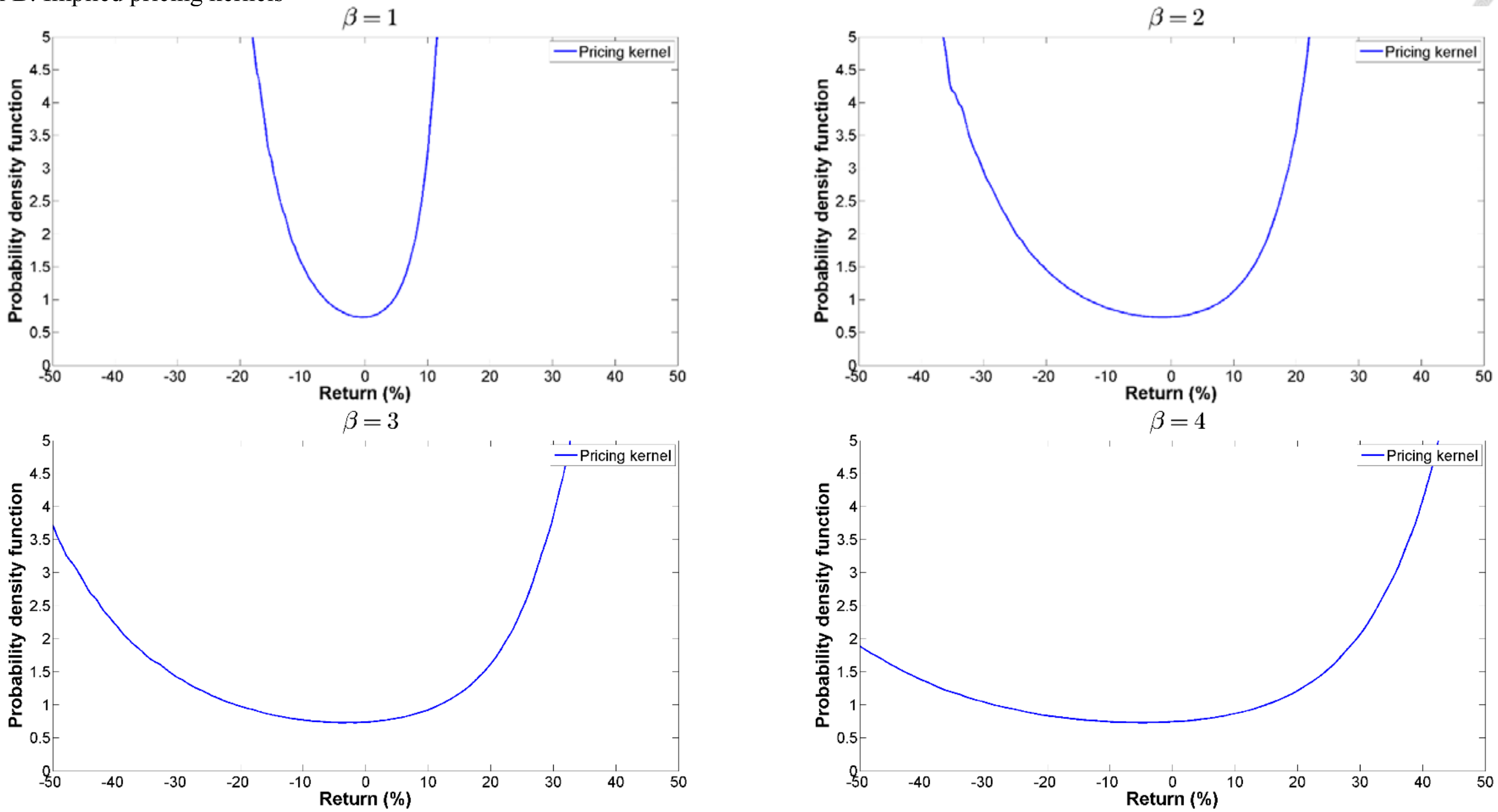
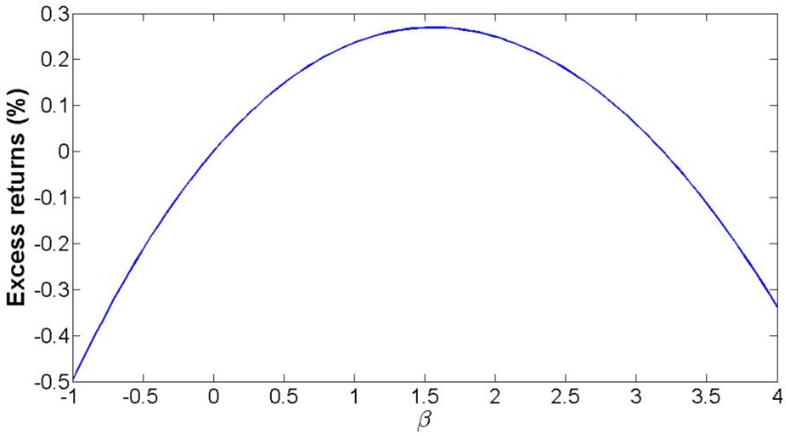



**Figure 2. 1: Real, risk-adjusted distribution, and pricing kernel implied by SPX**

Pane A: Security market line with respect to the market beta




Panel B: Surface of expected returns on the market variance risk premium and the market beta

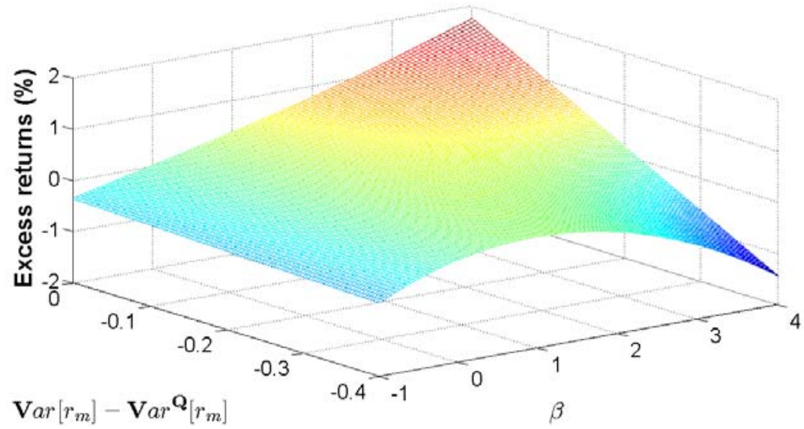



**Figure 2. 2: Security market line implied by SPX**