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勞動供給三維與政策分析

Three Margins of Labor Supply and Policy Analysis

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摘 要

歐洲的勞動供給在過去的 30 年間—70 年代初期到 2000 年代初期—相對於美國下降了 30%，其中部分來自於每位工人勞動工時的下降，部分來自於就業人數的減少。目前已有許多文獻探討其成因，Prescott (2002, 2004)認為此差異全為歐洲的高勞動所得稅所造成，Ljungqvist and Sargent(2007, 2008)則認為歐洲優厚的失業救濟金導致居高不下的失業率。然而絕大多數文獻的討論都專注在政策對工時的影響或政策對就業的影響，卻鮮少有文獻探討政策對兩者的相對影響。又就業的變化同時受失業率和勞動參與率變動的影響，因此本博士論文試圖建構一套完整的理論模型涵蓋勞動供給的三個維度：每工人工時、失業率和勞動參與率，以供政策分析之用。

既有文獻中，目前已知涵蓋勞動供給的工時和就業兩個面向的只有 Fang and Rogerson (2009)所提出的理論模型。Fang and Rogerson (2009)將工時植入標準的 Pissarides 配對模型中，得到勞動所得稅會同時降低每工人工時和就業的結果，但他們並未討論救濟金的影響，也未作定量分析。本論文第一篇延伸 Fang and Rogerson (2009)的架構，將勞動搜尋的特性加入新古典成長模型的設定中，分析勞動稅和失業救濟金對勞動供給兩個面向的相對影響，並作定量分析。我們發現勞動稅提高的確會同時降低工時和就業；而較高的失業救濟金則會降低就業，但可能提高工時；兩者同時增加約能解釋 75%歐美勞動供給的差異，解釋力則受勞動供給彈性和勞動尋職強度左右。

第一篇採用 Fang and Rogerson (2009)的設定，每工人工時和工資皆由勞資雙方談判決定，然而文獻上工時有不同的決定機制，即使是在談判工時時，相對談判能力也會有所不同。第二篇延續 Fang and Rogerson (2009)的架構，變動勞資雙方的相對工時談判能力。我們發現工人的工時談判能力越大，提高勞動稅對工時的負向影響越大，而對就業的負向影響越輕微。當工人的工時完全由家計單位決定，也就是談判能力為 100%時，勞動稅對工時的影響達到最大，搭配效用函數中若是線性於工時，則對就業完全無影響。而工時若是由官方管制，勞資無法隨意更動時，勞動稅提高，想當然耳只會降低就業。

談完工時，第三篇分析就業。工人要就業必須先想工作而後找工作，前者決定於工人主觀的勞動參與意願，後者受限於勞動市場客觀的結構限制，主觀意願與客觀限制的不同連帶影響著政策的效力。因此第三篇在第一章的架構下內生化勞動參與，將就業分離為勞動參與率和失業率兩個維度，同樣以提高勞動稅和失業救濟金為例，分析在勞動參與內生與外生下，政策對每工人工時和就業的影響。本章發現勞動稅提高雖會升高保留工資而降低配對，但當勞動參與內生時，勞動參與的意願也降低了，勞動市場上尋職者減少，配對成功機率相對提高，就業反而較勞動參與外生時降得少。失業救濟金在勞動參與外生時為尋職者的考量，但當勞動參與內生時，工人在決定是否想工作時就已列入考慮，尋職時反而已不是保留工資的一部分。因為失業救濟金的提高鼓勵了勞動參與，就業也跟著增加，得到與傳統勞動搜尋模型截然相反的結果。而兩者同時提高的定量分析在勞動參與內生時對工時和就業的解釋度較佳。

綜合三篇的研究結果顯示，勞動供給可分為三個維度，各個維度的決定機制各有不同，連帶影響政策對各維度的運作效果，又各維度之間互有牽引，如若未將勞動供給的三個維度放在一個完整的架構裡討論，則政策分析的結果可能失真。

關鍵詞：搜尋與配對、工時、失業、勞動參與、勞動稅和失業救濟金

Abstract

This dissertation decomposes labor supply into three margins step by step and studies the relative effects of two adverse labor market institutes on labor supply. Labor supply in Europe declined about 30% relative to the US over the past 3 decades. The decline in labor supply comes from both hours worked per worker and employment. Some studies attributed the declining hours worked to higher labor taxes, while other studies accredited high unemployment rates in Europe to generous non-employment benefits. Fang and Rogerson (2009) is the only exception which incorporates two margins of labor supply.

Fang and Rogerson (2009) embedded working hours into Pissarides matching model and found that higher labor taxes decrease both hours per worker and employment. The first essay of this dissertation starts from Fang and Rogerson (2009) to compares the relative effects of increases in labor taxes and non-employment benefits on hours per worker and employment and quantifies them. We find that increases in labor taxes decrease hours per worker and employment, with an overstated adverse effect on hours per worker if extensive margins are not taken into account. Moreover, increases in non-employment benefits decrease employment and increase hours per worker, with an understated adverse effect on employment if intensive margins are not considered. In the baseline parameterization, we find that increases in labor taxes and non-employment benefits together explain about 75% of declining labor supply in Europe, with the fraction accounted for being increasing in the labor supply elasticity and decreasing in the labor's contribution in matching.

The second essay adopts the same setup of Fang and Rogerson (2009) but varies

the relative bargaining power of workers on working hours. We find that the mechanisms shaping the supply of hours per worker play an important role. In the mechanism when the working hour is bargained by matched job-worker pairs, a higher labor income tax reduces both employment and hours per worker. When the laborer's hour bargaining power is larger, the negative effect on employment is smaller while the negative effect on hours is larger. In the mechanism when labor hours are decided exclusively by the household, i.e., the laborer's hour bargaining power is 100%, the negative effects on hours per worker approach to the maximum. In extremis, when the utility of leisure is linear in hours, there is no any effect on employment. In the mechanism when the working hour is effectively regulated by an authority, a higher labor tax only reduces employment without any effect on hours.

The third essay further splits employment into unemployment rates and labor force participation which is endogenous, and compares with the model with exogenous LFP. Because of discouraging LFP, labor taxes decrease employment in our model less than the model with exogenous LFP, have ambiguous effects on hours, and decrease less labor supply in our model. Due to boosting LFP, unemployment compensation increases employment in our model and decreases in the model with exogenous LFP, but with opposite effects on hours, labor supply is ambiguous in both models. With endogenous LFP, the quantitative result explains the difference in labor supply better than the model with exogenous LFP.

Keywords: search and matching, hours worked, unemployment, labor force participation, labor taxes and unemployment benefits.

Content

1 Introduction

In the early 1970s, average labor supply in Europe was roughly the same as that in the US, but then declined by about 30% over the past 3 decades from the early 1970s to the early 2000s. Data indicates that differences in average labor supply in the EU relative to the US are due to differences along three margins: *hours worked per worker*, *unemployment rates* and *labor force participation*. A growing body of literature has tried to figure out the relative importance of the various policies and institutional factors that have been proposed as competing explanations. In particular, two important labor market policies are blamed for causing declining labor supply in the EU. First, Prescott (2002, 2004) and his followers attributed the large difference in hours worked to higher labor income taxes in Europe. Conversely, Ljungqvist and Sargent (2007, 2008) and their followers accredited Europe's high unemployment rates to generous non-employment benefits. While the former only differentiates working from leisure hours, the latter distinguishes only employment from non-employment. The only exception is Fang and Rogerson (2009) who took both margins into account. While there are models that incorporate endogenous labor forces, no paper incorporates all three margins when explaining declining labor supply in the EU relative to the US. The purpose of this dissertation is to makes the first attempt to envisage the effects of the two adverse labor market institutions on labor supply along all three margins in one unified general equilibrium framework.

Fang and Rogerson (2009) embedded working hours into the standard Pissarides matching model and found that higher labor income taxes decrease both hours per worker and employment, but they did not discussed the effects of non-employment benefits nor conducted the quantitative analysis. The first essay of this dissertation starts from Fang and Rogerson (2009) to compares the relative effects of increases in labor taxes and non-employment benefits on hours worked per worker and employment and quantifies them. We find that increases in labor taxes decrease hours per worker and employment, with an overstated adverse effect on hours per worker if extensive margins are not taken into account. Moreover, increases in non-employment benefits decrease employment and increase hours per worker, with an understated adverse effect on employment if intensive margins are not considered. In the baseline parameterization, we find that increases in labor taxes and non-employment benefits together explain about 75% of declining labor supply in Europe relative to the US over the past 3 decades, with the fraction accounted for being increasing in the labor supply elasticity and decreasing in the labor's contribution in matching.

The second essay adopts the same setup of Fang and Rogerson (2009) but varies the relative bargaining power of workers on working hours. We find that the mechanisms shaping the supply of hours per worker play an important role. In the mechanism when the worker's supply of hours is bargained by matched job-worker pairs, a higher labor income tax reduces both employment and hours per worker. When the laborer's share in the hour bargaining is larger, the negative effect on employment is smaller while the negative effect on hours is larger. In the mechanism when labor hours are decided exclusively by the household, i.e., the laborer has a one-hundred percent share in the hour bargaining, the negative effects on hours per worker approach to the maximum. In extremis, when the utility of leisure is linear in hours, there is no any effect on employment. In the mechanism when the worker's supply of hours is effectively regulated by an authority, a higher labor tax only reduces employment with a zero effect on hours.

The third essay further splits employment into unemployment rates and labor force participation rates which is endogenous. We compare the long-run effects of increases in labor taxes and unemployment benefits on labor supply in models with and without endogenous labor force. With increases in labor taxes, due to discouraging labor-force participation, employment in our model is reduced less than the model with exogenous labor forces and, with ambiguous effects on hours worked in both models, labor supply is decreased by less in our model. With higher unemployment compensation, thanks to boosting labor-force participation, employment increases in our model but decreases in the model with exogenous labor forces and, with effects on hours worked being opposite to those on employment, labor supply is ambiguous in both models, depending on whether the effect on employment or that on hours per worker dominates.

The quantitative results show that the model with exogenous labor forces explains too much of the decreases in employment and labor supply between the EU and the US. In particular, this model predicts an increase in hours per worker, but the data indicates a decrease. By contrast, with endogenous labor forces, our model explains a more reasonable decrease in labor supply, with a sensible decrease in employment and a modest decrease rather than an increase in hours per worker in the EU relative to the US. Thus, with the participation margin, the model explains the difference in labor supply better than the model with exogenous labor forces.

2 Relative Effects of Labor Taxes and Non-employment Benefits on Hours Worked per Worker and Employment

2.1 Introduction

In the early 1970s, labor supply in Europe was roughly the same as that in the US. While labor supply remained to be unchanged in the US, it declined by about 30% in Europe relative to the US over the past 3 decades from the early 1970s to the early 2000s. Data indicates that declines in labor supply come from both hours worked per worker and employment rates. A growing body of literature has sought to understand reasons behind declining labor supply in Europe relative to the US. A number of papers pointed to adverse labor market institutions in Europe.[1] In particular, Europe has witnessed steadily higher labor taxes and more generous non-employment benefits than the US. There are two contrasting viewpoints concerning the effects of the two types of adverse labor market institutions on labor supply in Europe. First, Prescott (2002, 2004) and his followers attributed the large difference in hours worked per worker to higher labor income taxes in Europe.[2] Conversely, Ljungqvist and Sargent (2007, 2008a) and their followers accredited Europe's high unemployment rates to generous

[1] See Nickell (1997), Blanchard and Wolfers (2000), and Blanchard and Giavazzi (2003) that underlined the role of adverse labor market institutions in Europe. There are also other kinds of explanation, like leisure references in Europe (Blanchard, 2004; Azariadis et al, 2013) and home production in Europe (Ngai and Pissarides, 2008).

[2] Other papers that have stressed the role of labor taxes in probing hours of work differences between Europe and the US include Ohanian *et al*. (2008), Rogerson (2008), Jacobs (2009) and Rogerson and Wallenius (2009).

non-employment benefits: "*an important aspect of the European landscape that Prescott ignored: Government supplied non-employment benefits in the form of a replacement ratio times foregone labor income*" (Ljungqvist and Sargent, 2007, pp. 181-182).[3]

The former strand of research only differentiates working from leisure hours with neither employment nor unemployment. In contrast, the latter school of research distinguishes only employment from non-employment with neither working nor leisure hours. They do not analyze the effects on labor supply along both intensive and extensive margins. The purpose of this paper is to study a matching model so as to envisage the effects on labor supply along both intensive and extensive margins in one unified general equilibrium framework. We use the model to investigate and compare the relative effects of increases in labor income taxes and more generous non-employment benefits on hours worked per worker and employment rates and thus, labor supply.

Specifically, this paper studies a model that considers labor search within the neoclassical growth framework. There are a representative large household and a representative large firm. The large household decides consumption and savings and pools all resources for its members. These members include the employed who engage in work or leisure and the non-employed who undertake job search or leisure. The large firm creates and maintains vacancies. The firm rents capital and hires labor to produce output by using a neoclassical technology that is concave in capital, employment and hours worked per worker. In the model, the non-employed choose search effort so as to equate the marginal cost of search and the marginal gain of employment from a

[3] Other studies that have underscored the role of non-employment benefits in understanding higher unemployment in Europe include Mortensen (1977), Layard et al. (1991), Mortensen and Pissarides (1999) and Garibaldi and Wasmer (2005).

successful match. The firm creates vacancies so as to equate the marginal cost of vacancies and the marginal benefit of employment from a successful match. Job seekers and vacancies are brought together by a matching technology. Upon a successful match, the wage and hours worked per worker are determined by the two sides of a match. We analyze the steady-state search equilibrium in terms of the optimal work-hour condition and the firm's vacancy-employment condition which link hours worked per worker to employment. We use these equilibrium conditions to investigate the relative effects of increases in labor income taxes and more generous non-employment benefits on hours worked per worker and employment/unemployment.

Our main results are summarized as follow. First, an increase in the labor tax decreases both hours worked per worker and employment rates in the long run because it increases the household's net marginal cost of working hours and decreases the firm's net marginal benefit of employment. If only an intensive margin is taken into account as is in Prescott (2002, 2004), the adverse effect on hours worked per worker is overstated as only the household's net marginal cost of working hours increases and the adverse effect on employment is neglected. Next, an increase in non-employment benefits decreases employment and increases hours worked per worker since it decreases the firm's net marginal benefit of employment but it also decreases search effort which in turn lowers the household's marginal cost of working hours. If only an extensive margin is taken into consideration as is in Ljungqvist and Sargent (2007, 2008a), the adverse effect on employment rates is understated as only the firm's net marginal benefit of employment falls and the positive effect on hours worked per worker is overlooked. Finally, by feeding into the model the data of increases in labor income taxes and non-employment benefits in Europe relative to the US, we find that an increase in labor income taxes has a more detrimental effect on hours worked per worker but has a less harmful effect on employment rates than an increase in non-employment benefits. In the baseline parameterization, these increases in labor taxes and non-employment benefits can account for about 75% of declining labor supply in Europe relative to the US over the past 30 years, with the fraction accounted for being increasing in the labor supply elasticity and decreasing in the labor's contribution in matching.

The closest paper to ours is Fang and Rogerson (2009) which has embedded working hours into the standard Pissarides matching model. In their model, the production of a worker-job pair is concave in working hours, with aggregate output simply summing over the number of jobs and thus linear in employment. Our paper may be thought of as an extension of the Fang and Rogerson (2009) model with three different perspectives. First, we consider labor search within the neoclassical growth framework with capital accumulation and leisure of the non-employed. By doing so, the non-employed are not necessarily better-off than the employed, as opposed to the standard Pissarides matching model. Secondly, we employ a representative large firm instead of a worker-job pair as in the standard search model. Thus, as opposed to linear aggregate production in employment in Fang and Rogerson (2009), in our model aggregate production is concave in employment which is consistent with a diminishing marginal product. Thirdly, we include non-employment benefits which are not analyzed by Fang and Rogerson (2009). In particular, we compare the relative effects of two types of adverse labor market institutions and find that labor taxes are more detrimental to hours worked per worker while non-employment benefits are more harmful to unemployment. The former results are consistent with Prescott (2002, 2004) who attributed Europe's lower working hours to higher labor taxes and the latter results lend support to Ljungqvist and Sargent (2007, 2008a) who accredited Europe's higher unemployment to generous non-employment benefits.

The remainder of this paper is organized as follow. In the next section, we document relevant data concerning differences in labor supply between Europe and the US. In Section 3, we set up a labor-search and neoclassical-growth model. In Section 4, we characterize the steady state equilibrium. Section 5 studies the effects of higher labor income taxes and more generous non-employment benefits. Finally, we offer some concluding remarks in Section 6.

2.2 Relevant Data

Before proceeding to the model, we briefly summarize the evidence concerning differences in labor supply (hours worked per person), employment rates and hours worked per worker in Europe relative to the US. Table 1 presents the data for eleven European countries (EU-11), along with Belgium, France, Germany, with the US data normalized at 100 in 1970-73 and 2000-03.[4] According to Table 1, in the early 1970s, hours worked per person in Germany were 30% and those in France were 9% higher than those in the US. Although hours worked per person in Belgium were lower than those in the US in the early 1970s, hours worked per person in the EU-11 on average were 9% higher than those in the US. In the early 2000s, however, hours worked per person in Belgium, France and Germany were 20%-30% and in the EU-11 were 19% lower than those in the US. These numbers indicate that, relative to the US, hours worked per person were dropped by 55% in Germany, 35% in France, 20% in Belgium and 28% on average in the EU-11 over the period from the early 1970s to the early 2000s.

The fall in hours worked per person comes from decreasing employment rates and hours worked per worker. First, Germany, France and the EU-11 had higher

[4] To calculate the statistics, we employ the same method as those used in Prescott (2004) and Rogerson (2006).

employment rates than the US in the early 1970s, while Belgium had a slightly smaller employment rate than the US in the early 1970s. In the early 2000s, all these European countries had lower employment rates than the US. Over the past 30 years from the early 1970s to the early 2000s, relative to the US, the employment rate was dropped by 12% in Belgium, 14% in France, 18% in Germany and 13% in the EU-11. Next, for hours worked per worker, in the early 1970s, Germany, France and the EU-11 had higher hours worked per worker than the US, while Belgium had about the same hours worked per worker as the US. In the early 2000s, these European countries all had lower hours worked per worker. Over the period from the early 1970s to the early 2000s, relative to the US, hours worked per worker were dropped by 11% in Belgium, 22% in France, 37% in Germany and 16% in the EU.

To summarize the data, over the past 30 years from the early 1970s to the early 2000s, hours worked per person in the EU-11 on average were declined by 28% relative to the US. The decline in labor supply is from both decreasing hours worked per worker and falling employment rates.

2.3 The Model

The economy is populated by the representative large household and the representative large firm. As in Andolfatto (1996) and Fang and Rogerson (2009), we adopt the assumption of the large household setup. Family members in a larger household pool all resources regardless of their labor market status which assures perfect consumption insurance. The large household comprises a continuum of members (of measure one), who are either employed or non-employed. Like Fang and Rogerson (2009), the employed engage in work or leisure and obtain a wage when working. Yet, unlike Fang and Rogerson (2009), the non-employed take on job search or leisure and the cost of job search is foregone leisure. Also, unlike these authors, there is a large firm. The large firm creates and maintains multiple vacancies and rents capital and hires labor to produce goods using a technology that is concave in employment. The job finding and recruitment rates are endogenous, depending on the masses of both matching parties. Unfilled vacancies and job seekers are met bilaterally through the matching technology. Filled vacancies and employed workers are separated at an exogenous rate. Finally, there is a fiscal authority that levies taxes and offers non-employment benefits.

2.3.1 Households

The representative household has a unified preference and pools all resources for its members. In a period t, a fraction e_t of the members is employed and the remaining fraction $(1-e_t)$ is non-employed. Given a fixed time endowment normalized at unity, each employed member allocates a fraction l_t of the total time to work and the remaining fraction $(1-l_t)$ to leisure. Non-employed members devote a fraction s_t of their time to job search and the remaining fraction $(1-s_t)$ to leisure. From the household's perspective, the employment changes according to

$$e_{t+1} - e_t = \mu_t \left(s_t (1 - e_t) \right) - \psi e_t, \tag{1}$$

where $s_t(1\text{-}e_t)$ is an aggregate search made by the non-employed, μ_t is the effective job finding rate and ψ is the (exogenous) job separation rate. Thus, the change in employment $(e_{t+1}-e_t)$ is equal to the inflow of non-employed workers into the employment pool $(\mu_t s_t(1-e_t))$ net of the outflow as a result of separation (ψe_t).

Denote c_t as consumption and k_t as capital with δ the depreciation rate. Further, denote by w_t and r_t the wage rate and the interest rate, respectively. Let the profit be π_t, non-employment benefits be b, the labor income tax rate be τ and the lump-sum tax per household be T_t. The household's budget constraint is

$$c_t + \left[k_{t+1} - (1-\delta)k_t\right] + T_t = r_t k_t + (1-\tau) w_t e_t l_t + b(1-e_t) + \pi_t. \tag{2}$$

The large household has four sources of income: capital rental, after-tax wage earned by employed members, the compensation received by non-employed members, and profits remitted from firms. It allocates income to consumption, investment and lump-sum taxes. The household obtains utility from consumption and leisure. Following Andolfatto (1996), the utility of an employed member is $u(c_t) + \chi_1 V(1-l_t)$ and the utility of a non-employed member is $u(c_t) + \chi_2 V(1-s_t)$, where χ_1 and χ_2 are the degree of leisure utilities for an employed and a non-employed member, respectively. We assume that u and V exhibit the standard concavity property of positive and decreasing marginal utilities.[5] If $\chi_2 V(1-s_t) \le \chi_1 V(1-l_t)$, a non-employed member is not better-off than an employed member, as opposed to the standard Pissarides matching model adopted in Fang and Rogerson (2009). The utility of the large household is the sum across all household members and thus $u(c_t) + e_t \chi_1 V(1-l_t) + (1-e_t)\chi_2 V(1-s_t)$.

The household's optimal control problem is written as the following Bellman equation,

$$U(k_t, e_t) = \max_{k_{t+1}, s_t}\left[u(c_t) + e_t \chi_1 V(1-l_t) + (1-e_t)\chi_2 V(1-s_t) + \frac{1}{1+\rho} U(k_{t+1}, e_{t+1})\right], \tag{3}$$

subject to the constraints (1) and (2), where ρ>0 is the time preference rate.

The first-order conditions with respect to k_{t+1} and s_t and the Benveniste-Scheinkman conditions for k_t and e_t are, respectively,

$$u'(c_t) = \frac{1}{1+\rho} U_k\left(k_{t+1}, e_{t+1}\right), \tag{4a}$$

$$\chi_2 V'(1-s_t) = \frac{\mu_t}{1+\rho} U_e\left(k_{t+1}, e_{t+1}\right). \tag{4b}$$

[5] For simplicity, we use the same form of the leisure utility for employed and non-employed members in the household. Results are the same if different forms are used.

$$U_k\left(k_t, e_t\right) = u'(c_t)\left(1-\delta + r_t\right), \tag{4c}$$

$$U_e(k_t, e_t) = u'(c_t)\left[\left(1-\tau\right)w_t l_t - b\right] + \chi_1 V(1-l_t) - \chi_2 V(1-s_t) + \frac{1}{1+\rho}U_e(k_{t+1}, e_{t+1})\left(1-\psi - s_t \mu_t\right). \tag{4d}$$

While (4a) is standard, (4b) equates the marginal cost of search effort in terms of foregone leisure to the expected marginal gain of employment from a successful match in the next period. The last two conditions are the representative household's marginal gain of capital and employment, respectively, in the beginning of the period. Forwarding (4c) by one period and substituting it into (4a) gives the following standard Euler equation

$$u'(c_t) = \frac{1}{1+\rho}u'(c_{t+1})\left(1-\delta + r_{t+1}\right). \tag{5}$$

2.3.2 Firms

The representative large firm rents capital and hires labor in order to produce a single final good y_t. The production technology is neoclassical, given by the following function.

$$y_t = A k_t^{\alpha}(e_t l_t)^{1-\alpha}, \tag{6}$$

where $A>0$ is a productivity parameter and $\alpha \in (0, 1)$ is the share of capital. The production function is concave in employment, as opposed to that in the standard Pissarides matching model adopted by Fang and Rogerson (2009) wherein aggregate production is linear in employment.

From the firm's perspective, employment is increased by the inflow of employees and decreased by the outflow due to separation.

$$e_{t+1} - e_t = \eta_t v_t - \psi e_t, \tag{7}$$

where η_t is the (endogenous) recruitment rate and v_t is (endogenously) created vacancies.

To create and maintain vacancies, firms need to pay a cost to adjust the vacancy numbers. We assume the following quadratic cost function: $\Lambda(v_t)=\lambda_0 v_t+\lambda_1 v_t^2$, where $\lambda_0>0$, $\lambda_1>0$. Hence, firm's flow profits in t equal the output net of the costs of labor, capital, and vacancy creation; i.e.

$$\pi_t = Ak_t^{\alpha}(e_t l_t)^{1-\alpha} - w_t e_t l_t - r_t k_t - \Lambda(v_t). \tag{8a}$$

The representative large firm chooses capital and vacancies in order to maximize the discounted sum of flow profits. The Bellman equation associated with the firm is

$$\Pi(e_t)=\max_{k_t,v_t}\left[\pi_t+\frac{1}{1+r_t}\Pi(e_{t+1})\right], \tag{8b}$$

subject to constraint (7).

The first-order conditions with respect to k_t and v_t and the Benveniste-Scheinkman condition for e_t are, respectively,

$$\alpha A(\frac{k_t}{e_t l_t})^{\alpha-1}=r_t, \tag{9a}$$

$$\lambda_0+2\lambda_1 v_t=\frac{\eta_t}{1+r_t}\Pi_e(e_{t+1}), \tag{9b}$$

$$\Pi_e(e_t)=[(1-\alpha)A(\frac{k_t}{e_t l_t})^{\alpha}-w_t]l_t+\frac{1-\psi}{1+r_t}\Pi_e(e_{t+1}). \tag{9c}$$

Capital is determined by the marginal product of capital equal the rental rate in (9a). (9b) is the vacancy-employment condition which equates a firm's marginal cost of vacancies in this period to the expected marginal benefit of employment/recruitment from a successful match in the next period. A firm's marginal benefit of employment in (9c) is the sum of the marginal product of labor net of the wage rate multiplied by hours worked per worker and the discounted future marginal benefit.

It is straightforward to rewrite (9a) as

$$q_t \equiv \frac{k_t}{e_t l_t} = \left(\frac{\alpha A}{r_t}\right)^{\frac{1}{1-\alpha}}. \tag{9d}$$

Thus, the market effective capital-labor ratio, denoted by q, is decreasing in the rental rate.

2.3.3 Labor Matching and Bargaining

The labor market exhibits search frictions with aggregate flow matches depending on the masses of job seekers and vacancies. Following Diamond (1982), we assume pair-wise random matching. The matching technology takes the constant-returns form:[6] $M_t = m\left(s_t(1-e_t)\right)^{\gamma}\left(v_t\right)^{1-\gamma}$, where m>0 measures the degree of matching efficacy and $\gamma \in (0, 1)$ the contribution of job seekers in matching. Aggregate search and recruitment behave like two inputs in the matching function and the output is the aggregate matched pair M_t. The matching function facilitates the endogenous determination of job finding rates and recruitment rates. As in Andolfatto (1996), since s_t is search effort per job seeker, aggregate search effort by job seekers is $s_t(1- e_t)$.

A job seeker's surplus acquired from a successful match is evaluated by its augmenting value from employment U_e in (4d), whereas a vacant job's surplus of a successful match is gauged by its incremental value from recruit Π_e in (9c). In a frictionless Walrasian world, taking the wage as given, the household maximizes U_e and the firm maximizes Π_e in order to decide their supply of and demand for labor. There is implicitly an auctioneer in the labor market which sets an equilibrium wage so as to equate labor supply to labor demand. In a frictional labor market, however, there is no

[6] In a survey of micro foundations underlying the matching function and its empirical success, Petrongolo and Pissarides (2001) referred to the matching function as a useful modeling device for building labor market frictions into equilibrium macroeconomic models of wages, employment, and unemployment that occupies the same place in the macroeconomist's tool kit as other aggregate functions such as the production function.

auctioneer and a job seeker would meet at most one unfilled job one time and similarly, an unfilled job would meet at most one job seeker one time. This creates a bilateral monopoly.

Following conventional wisdom, the wage rate and working hours are determined simultaneously by a matched worker-job pair through a cooperative bargaining game. Like Fang and Rogerson (2009), an employed worker does not devote all the time endowment to work and thus the pair of a successful match also bargains over working hours. In the game, the following joint surplus is maximized: $[U_e(k_t, e_t)]^{\beta}[\Pi_e(e_t)]^{1-\beta}$, where $\beta \in (0, 1)$ measures a labor's bargaining power. In solving the bargaining problem, the worker-job pair treats matching rates (μ_t and η_t), the beginning-of-period level of employment (e_t), and the market interest rate (r_t) as given. The worker also takes as given the wage and working hours of all others. The first-order conditions are

$$\frac{\beta}{U_e(k_t, e_t)} \frac{dU_e(k_t, e_t)}{dw_t} = -\frac{1-\beta}{\Pi_e(e_t)} \frac{d\Pi_e(e_t)}{dw_t}, \tag{10a}$$

$$\frac{\beta}{U_e(k_t, e_t)} \frac{dU_e(k_t, e_t)}{dl_t} = -\frac{1-\beta}{\Pi_e(e_t)} \frac{d\Pi_e(e_t)}{dl_t}. \tag{10b}$$

2.3.4 The Government

The government's behavior is passive; it levies labor income and lump-sum taxes and offers non-employment benefits. The government budget constraint is

$$T_t + \tau w_t e_t l_t = b\left(1 - e_t\right). \tag{11}$$

In order to isolate the effects of policy changes carried out later, we include lump-sum taxes T_t. When the labor tax rate τ is changed, with non-employment benefits b being held unchanged, lump-sum taxes/subsidies T will change accordingly in order to balance the budget. Similarly, when non-employment benefits are increased, with the labor tax rate being held constant, lump-sum taxes will adjust to balance the budget.

2.4 Equilibrium

A search equilibrium is a tuple of individual quantity variables, $\{e_t, l_t, s_t, v_t, c_t, k_t, y_t\}$, a pair of aggregate quantities, $\{M_t, T_t\}$, a pair of matching rates, $\{\mu_t, \eta_t\}$, and a pair of prices, $\{w_t, r_t\}$, such that: (i) all households and firms optimize; (ii) all employment evolutions hold, (iii) labor-market matching and wage and hours bargaining conditions are met; (iv) the government budget is balanced; and (v) the goods market clears.

2.4.1 Steady State

A steady state is a search equilibrium when all variables do not change over time. First, in a steady state the Euler equation in (5) gives the following interest rate: $r = \rho + \delta$. Substituting the rate into (9d) yields the effective capital-labor ratio: $q = (\frac{\alpha A}{\rho+\delta})^{\frac{1}{1-\alpha}}$, which is constant in a steady state.

Next, if we use the household's budget (2) and the firm's flow profit (8a), along with the government's budget (11), the goods market clearing condition in a steady state is

$$y = c + \delta k + \Lambda(v). \tag{12}$$

Moreover, in a steady state the labor market must satisfy the following matching relationships (Beveridge curve) given by

$$m\left(s(1-e)\right)^{\gamma}(v)^{1-\gamma} = \mu\left(s(1-e)\right) = \eta v = \psi e. \tag{13}$$

Thus, the number of successful job matches equals the employment inflow from the household side, $\mu(s(1-e))$, the employment inflow from the firm side, ηv, and is equal to the employment outflow in a steady state. These relationships enable us to solve matching rates and equilibrium vacancies as functions of *e* and *s*.

$$\mu = \frac{\psi e}{s(1-e)} \equiv \mu(\underset{+}{e}, \underset{-}{s}), \tag{14a}$$

$$v = \left[\frac{\psi e}{m\left(s(1-e)\right)^{\gamma}}\right]^{\frac{1}{1-\gamma}} \equiv v(\underset{+}{e}, \underset{-}{s}), \tag{14b}$$

$$\eta = \frac{\psi e}{v(e,s)} = \left[m\left(\frac{s(1-e)}{\psi e}\right)^{\gamma}\right]^{\frac{1}{1-\gamma}} \equiv \eta(\underset{-}{e}, \underset{+}{s}). \tag{14c}$$

Thus, the effective job finding rate and the equilibrium vacancy are positively related to employment and negatively related to search effort, while the recruitment rate is negatively related to employment and positively related to search effort.

In steady state, the household's surplus accrued from a successful match in (4d) is

$$U_e = \frac{1+\rho}{\rho+\psi+s\mu}\left\{u'(c)\left((1-\tau)wl-b\right)+\left[\chi_1 V(1-l)-\chi_2 V(1-s)\right]\right\}. \tag{15}$$

Moreover, using $r=\rho+\delta$, the firm's surplus accrued from a successful match in (9c) is

$$\Pi_e = \frac{1+\rho+\delta}{\rho+\delta+\psi}\left(MPL-w\right)l, \tag{16}$$

where $MPL \equiv (1-\alpha)Aq^{\alpha}$ denotes the marginal product of labor which is constant.

Following Andolfatto (1996), the parametric forms are used for utility.

$$u(c)=\ln c \text{ and } V(1-x)=\frac{(1-x)^{1-\sigma}}{1-\sigma}, \text{ where } x=l,\ s,$$

in which $\sigma>0$ is the reciprocal of the elasticity of leisure.[7] These forms are consistent with the balanced growth path.

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[7] For simplicity, we use the same elasticity of leisure for the employed and the non-employed. Allowing elasticity of leisure of the non-employed to be different from that of the employed will not change the results.

We are ready to derive equilibrium conditions in a steady state. First, by using the effective capital-labor ratio and (14b), (12) gives the following consumption

$$c=(Aq^{\alpha}-\delta q)el-\Lambda(v(e,s))\equiv c(\underset{+}{e},\underset{+}{l},\underset{+}{s}), \tag{17}$$

which is increasing in employment, hours worked per worker and search effort.[8] Intuitively, employment and hours worked per worker both increase output and thus, consumption. Moreover, larger search effort reduces the vacancy creation cost which increases disposable income and thus, consumption.

Next, by using (15) and (16), we rewrite (10a) as

$$\beta(1-\tau)l\left[(1-\tau)wl-b+\left(\frac{\chi_1(1-l)^{1-\sigma}-\chi_2(1-s)^{1-\sigma}}{1-\sigma}\right)c\right]^{-1}=1-\beta\left[MPL-w\right]^{-1},$$

where the left-hand side of the equation is the household's marginal benefit of wage and is decreasing in the wage, while the right-hand side of the equation is the firm's marginal cost of wage and is increasing in the wage. With the use of (17), the condition above gives the following bargained wage

$$w=\beta MPL+(1-\beta)\left[\frac{b+MRS^{e}(e,l,s)}{(1-\tau)l}\right]\equiv W(\underset{+}{e},\underset{?}{l},\underset{+}{s};\underset{+}{\tau},\underset{+}{b}), \tag{18}$$

where $MRS^{e}(\underset{+}{e},\underset{+}{l},\underset{+}{s})\equiv\left[\frac{\chi_2(1-s)^{1-\sigma}-\chi_1(1-l)^{1-\sigma}}{1-\sigma}\right]c(\underset{+}{e},\underset{+}{l},\underset{+}{s})$ is the difference of the marginal rate of substitution (MRS) for leisure and consumption between non-employment and employment; thus, the loss in leisure utilities in the consumption term from non-employment to employment. The bargained wage is a weighted average of the marginal product of labor and the reservation wage; the reservation wage is the sum of non-employment benefits and losses in leisure utilities from non-employment to employment. As the marginal product of labor is constant in a steady state, policy changes affect the steady-state bargained wage via their effects on the reservation wage.

[8] Notice that c_e>0 when e>0.

We characterize the bargained wage in (18). First, with all other things being equal, a higher employment e raises the bargained wage since it increases consumption which increases the reservation wage.[9] Secondly, a larger working hour l has an ambiguous effect on the bargained wage. In the special case of b=0, a larger working hour raises the bargained wage since it increases losses in leisure utilities in the consumption term per hour (MRS^e/l). However, when non-employment benefits are large, the offsetting effect from non-employment benefits is substantial and a larger working hour may reduce the wage. Thirdly, higher search effort s increases consumption which decreases the marginal utility of consumption and thus increases losses in leisure utilities in the consumption term from non-employment to employment and thus the wage, but it may also decrease the marginal utility of leisure and hence losses in leisure utilities from non-employment to employment and thus the wage. As the positive effect dominates, higher search effort increases the bargained wage. Finally, higher labor income taxes (τ) and higher non-employment benefits (b) both increase the reservation wage. Thus, the bargained wage is increasing in labor income taxes and non-employment benefits.

Moreover, by using the equilibrium interest rate, (14a)-(14c) and (15)-(18), we rewrite a non-employed member's optimal search effort condition in (4b) as

$$\underbrace{\frac{\mu(e,s)}{\rho+\psi+s\mu(e,s)}\beta\Big[\big(1-\tau\big)MPL\cdot l-b-MRS^{e}(e,l,s)\Big]}_{\substack{\parallel \\ MB^{s}(\underset{-}{e},\underset{-}{l},\underset{-}{s};\underset{-}{\tau},\underset{-}{b})}} = \underbrace{\chi_2\frac{c(e,l,s)}{\big(1-s\big)^{\sigma}}}_{\substack{\parallel \\ MRS^{s}(\underset{+}{e},\underset{+}{l},\underset{+}{s})}}, \qquad (19)$$

which equates a non-employed member's discounted (after-tax) marginal benefit from a successful match, denoted as MB^s, to the marginal cost of search which is a non-employed member's MRS between leisure and consumption, denoted as MRS^s. It is clear that the condition gives unique search effort as higher search effort decreases the marginal benefit of search effort and increases the marginal cost. Moreover, higher

[9] To save the space, all algebra below is delegated in the Appendix.

employment increases the marginal cost of search effort but has an ambiguous effects on the marginal benefit and thus has a negative or an ambiguous effect on search effort. Further, a higher working hour increases the marginal cost of search effort and decreases the marginal benefit and thus decreases search effort. Finally, labor taxes and non-employment benefits both decrease the marginal benefit of search effort and thus decrease search effort. Therefore, the condition gives the following optimal search effort.

$$s = S(\underset{-or?}{e}, \underset{-}{l}; \underset{-}{\tau}, \underset{-}{b}). \tag{20}$$

2.4.2 Simplified Steady-State Equilibrium Conditions

Now, we simplify equilibrium conditions in a steady state in terms of employment and hours worked per worker. First, we rewrite the optimal working hour condition in (10b) as

$$\beta\left[(1-\tau)w - \chi_1(1-l)^{-\sigma}c\right]\left[(1-\tau)wl - b + \left(\frac{\chi_1(1-l)^{1-\sigma} - \chi_2(1-s)^{1-\sigma}}{1-\sigma}\right)c\right]^{-1} = -\frac{1-\beta}{l}$$

Substituting (18) into the condition above and rearranging terms yields

$$\frac{MRS^l(\overset{+}{e}, \overset{+}{l}, \overset{+}{s})}{(1-\tau)} = MPL, \tag{21}$$

where $MRS^l(e,l,s) \equiv \chi_1(1-l)^{-\sigma}c(e,l,s)$ is an employed member's MRS between leisure hours and consumption. The left-hand side is the marginal cost of working hours. The right-hand side is the marginal product of labor which is the marginal benefit of working hours. As the marginal cost of working hours is increasing in working hours and the marginal product of labor is constant, this condition determines a unique hour worked per worker. To characterize hours worked per worker, it is clear to see that employment and search effort both increase the marginal cost of working hours due to higher consumption. Moreover, a higher labor tax also increases the marginal cost of working hours due to a lower post-tax wage rate. Thus, employment, search effort and labor taxes all decrease hours worked per worker. With the search effort in (20), the condition above gives the following optimal working hour.

$$l = L(\underset{-}{e}, \underset{-}{S}(\underset{-or?}{e}, \underset{-}{l}; \underset{-}{\tau}, \underset{-}{b}); \underset{-}{\tau}).$$

In the relationship above, although l and e also exert indirect effects via search effort S in (20) that may offset the direct effect on the net marginal cost of working hours, we find that these indirect effects are dominated by the direct effects. Thus, the working hour function above is written as

$$l = \tilde{L}(\underset{-}{e}; \underset{-}{\tau}, \underset{+}{b}), \tag{22}$$

which is negatively sloping in the (e, l) plane. See Figure 1 wherein the hour locus is referred to as Locus H (Hours).

Next, by using the equilibrium interest rate, (14a)-(14c), and (15)-(18), we can rewrite the firm's vacancy-employment condition in (9b) as

$$\underbrace{\frac{\eta(e,s)}{\rho+\delta+\psi}(1-\beta)\left(MPL \cdot l - \frac{b + MRS^{e}(e,l,s)}{1-\tau}\right)}_{\underset{MB^{v}(\underset{-}{e},\underset{-}{l},\underset{?}{s})}{\parallel}} = \underbrace{\lambda_0 + 2\lambda_1 v(e,s)}_{\underset{MC^{v}(\underset{+}{e},\underset{-}{s})}{\parallel}}. \tag{23}$$

The condition equates a firm's discounted marginal benefit of employment from a successful match, denoted as MB^{v}, to the marginal cost of vacancy, denoted as MC^{v}. It is clear that the firm's marginal benefit of employment is decreasing in employment and the marginal cost is increasing in employment. Thus, this condition determines unique employment. To characterize the employment function, it is clear that a higher working hour decreases the firm's marginal benefit of employment and thus decreases employment. Moreover, higher search effort decreases the marginal cost but has an ambiguous effect on the marginal benefit. Thus, higher search effort may increase or have an ambiguous effect on employment. Further, higher taxes, more generous non-employment benefits and larger vacancy creation costs all decrease the firm’s net marginal benefit of employment and thus decrease employment. Therefore, the condition above gives the following employment function.

$$e = E(\underset{-}{l}, \underset{+or?}{S}(\underset{-or?}{e}, \underset{-}{l}; \underset{-}{\tau}, \underset{-}{b}); \underset{-}{\tau}, \underset{-}{b}, \underset{-}{\lambda_0}).$$

In the employment function above, l and e exert indirect ambiguous effects via search effort S on the firm’s net marginal benefit of employment. Yet, it is easy to show the direct effects of l and e always dominate these indirect effects. Accordingly, we obtain the following employment function

$$e = \tilde{E}(\underset{-}{l}; \underset{-}{\tau}, \underset{-}{b}, \underset{-}{\lambda_0}), \tag{24}$$

which is negatively sloping in the (e, l) plane. See Figure 1 wherein the employment locus is referred to as Locus E (Employment).

Thus, the steady state is determined by the interaction of Loci H and E. By exploring the effects of a higher cost of vacancy creation, it is clear that Locus H needs to be always flatter than Locus E.[10] As Locus H is flatter than Locus E, this implies that the two curves have at most one intersection. See Q_0 in Figure 1. The two loci determine steady-state employment (e_0) and hours worked per worker (l_0), and thus labor supply ($e_0 l_0$).

2.5 Policy Analysis

Although the simplicity of our model confines the breadth of the policies that can be envisaged, two policies of pervasive interest can be studied within our model: a tax on the employed which is proportional to labor income and is used to make a lump-sum

[10] A higher cost of vacancy creation λ_0 shifts the employment locus down without shifting the hour locus; should the employment locus be less steep than the hour locus, employment would be increased, not decreased, which is inconsistent.

transfer; and a benefit to the unemployed which is proportional to labor income as financed by a lump-sum tax. While the former policy has been stressed by Prescott (2002, 2004) in explaining lower hours worked per worker in Europe than the US, the latter policy has been emphasized by Ljungqvist and Sargent (2007, 2008a) in accounting for higher unemployment rates in Europe than the US. We start with the analysis of increases in labor income taxes, followed by increases in non-employment benefits. The comparative-static analysis is delegated in the Appendix. Here, we offer graphical illustrations.

2.5.1 Effects of Labor Taxes

First, we analyze the effects of increases in the labor tax rate (higher τ). Suppose that the initial steady state is at Q_0 in Figure 2. Thus, the initial hour worked per worker is l_0, initial employment is e_0, initial unemployment is $(1-e_0)$ and initial labor supply is $(e_0 l_0)$.

When the labor tax rate (τ) is increased, the household's net marginal cost of working hours increases and thus working hours are decreased; the firm's net marginal benefit of employment is decreased and thus employment is decreased. Then, Loci H is shifted to Locus H_1 and Locus E is shifted to Locus E_1 in Figure 2. Moreover, with given employment levels, Locus E_1 is shifted downward more than that of Locus H_1. The reasons are that a higher labor tax rate yields direct effects to decrease working hours in both Loci H and E. However, in Locus H, a higher labor tax rate also generates an offsetting effect via decreasing search effort which reduces the net marginal cost of working hours and thus increases working hours. Hence, Locus H_1 is shifted downward less than Locus E_1. The new steady state is at Q_1 in Figure 2. As a result, hours worked per worker l_1 and employment e_1 are lower than their initial levels l_0 and e_0, respectively.

Accordingly, hours worked per person ($e_1 l_1$) are lower than the initial level ($e_0 l_0$).

Note that in Prescott (2002, 2004), there is only an intensive margin (i.e., work hours and leisure hours) and not an extensive margin (i.e., employment and non-employment). The equilibrium condition in Prescott (2002, 2004) may be thought of as involving only Locus H without Locus E, with the initial steady state Q_0 being determined by Locus H and the initial employment level e_0 in Figure 2. In this case, a higher labor tax rate (τ) shifts Locus H downward to Locus H_1. The new steady state is at Q_3. Thus, compared to the case with both intensive and extensive margins, hours worked per worker here are reduced by more to the level $l_2 < l_1$. Therefore, without an extensive margin in Prescott (2002, 2004), as the adverse effect on employment is not taken into account, the adverse effects on hours worked per worker are overstated. To summarize the results,

Proposition 1 *An increase in labor taxes decreases both hours worked per worker and employment. With fixed employment, the adverse effect on hours worked per person is overstated.*

2.5.2 Effects of Non-employment Benefits

Next, we analyze the effects of increases in non-employment benefits (higher b). Suppose that the initial steady state is at Q_0 in Figure 3.

When non-employment benefits are increased, the firm’s net marginal benefit of employment is decreased. With given work hours, employment decreases and thus the Locus E is shifted leftward to Locus E_2 in Figure 3. Moreover, more generous non-employment benefits also decrease search effort which reduces the household’s marginal cost of working hours. With given employment, hours worked per worker increase and thus Locus H is shifted upward to Locus H_2. The new steady state is at Q_2 in Figure 3. As a result, employment is lower but hours worked per worker are higher.

In Ljungqvist and Sargent (2007, 2008a), there is only an extensive margin and not an intensive margin (i.e., fixed working hours). The equilibrium condition in Ljungqvist and Sargent (2007, 2008a) may be interpreted as involving only Locus E without Locus H, with the initial steady state Q_0 being determined by Locus E and the initial work-hour level l_0 in Figure 3. In this case, more generous non-employment benefits only shift Loci E downward to Loci E_2, and thus the new steady state is at Q_3. Compared to the case with both intensive and extensive margins, employment here is reduced by less to the level $e_1 > e_2$. Therefore, without an intensive margin in Ljungqvist and Sargent (2007, 2008a), as the positive effect on hours worked per worker is not taken into account, the adverse effect on employment is understated. To summarize the results,

Proposition 2 *An increase in non-employment benefits decreases employment and increases hours worked per worker. With fixed hours worked per worker, the adverse effect on employment is understated.*

2.5.3 Quantitative Analysis

We now quantify the effects of increases in labor taxes and non-employment benefits on labor supply. We are particularly interested in understanding the effects on hours worked per worker and employment and thus labor supply in Europe relative to the US over the past 3 decades from the early 1970s to the early 2000s. To this end, we calibrate our model in a steady state to the US economy. We assume that all parameters values in Europe are the same as those in the US except for labor income taxes and non-employment benefits. Then, we feed in the data of increases in labor income taxes and non-employment benefits in Europe relative to the US in the early 2000s and quantify the effects.

We calibrate parameters and variables at a quarterly frequency. With the annual depreciation rate of capital in the range of 6%-8% and the annual time preference rate of 4%, we follow Ljungqvist and Sargent (2008b) to set the quarterly capital depreciation rate to δ=0.02 and the quarterly time preference rate to ρ=0.01. The data gives the steady-state interest rate at r=0.03. The coefficient of technology is normalized to A=1. The capital share is about one-third and we follow Prescott (2004) to use the value α=0.3224. With the values of A and α, we compute the effective capital-labor ratio as $q=(\frac{\alpha A}{\rho+\delta})^{\frac{1}{1-\alpha}}=33.2622$, which in turn gives MPL=2.0973 and, via (6), the quarterly capital-output ratio k/y=10.7467 which is consistent with a capital-output ratio of 2.5-3 in annual data.

The fraction of employment in the working-age population is about 75% (cf. Kydland and Prescott 1991) and thus we set e=0.75. The fraction of time allocated to the market (el) is 25% as pointed out by Prescott (2006). This implies l=0.3333. For the average fraction of time spent to search, we follow Andolfatto (1996) to set $s=0.5\times l=0.1667$. According to Shimer (2005), the monthly job finding rate is 0.45. We go along this rate and translate it into a quarterly value of $s\mu=1-(1-0.45)^3=0.8336$, implying μ=5.0016. We employ (13) to compute the quarterly separation rate as a fraction of employment at $\psi=(s\mu(1-e))/e=0.2779$. Moreover, we follow Shimer (2005) by normalizing the steady-state ratio of vacancies to searching workers to one ($v/(1-e)=1$) which implies the vacancy at v=0.25 in a steady state. Then, we utilize (13) to calibrate $\eta=(s\mu(1-e))/v=0.8336$.

By setting the consumption-output ratio at c/y=0.67 and normalizing λ_1=1, we use

(17) to calibrate the coefficient of vacancy costs λ_0=0.1061. We compute the wage at w=1.4257 from (18). In accordance with Prescott (2004), non-employment benefits are 0.319 times forgone labor income, and hence we calibrate b=0.319×w×l which gives b=0.1516. Based on the data in McDaniel (2007), Rogerson (2008) used the labor taxes in Belgium, France, Germany, Italy, and the Netherlands to represent the tax in Europe.[11] We follow this method and calculate the population-weighted average effective tax rate on labor income for these five countries. We find that the average effective tax rate in years 1970-73 is 0.3982 which leads us to set the benchmark labor tax rate to τ=0.4, a rate similar to that of the US as noted in Prescott (2004).

Finally, for the utility function adopted here, the labor supply elasticity is LSE=(1-l)/(σl). The LSE estimated in MaCurdy (1981) ranged from 0.1 to 0.5 for men and is likely higher for women, while Andolfatto (1996) set LSE=1. For present purposes, we choose an intermediate value: LSE=0.65, which implies σ=3.0769. Given this value, (21) is solved for χ_1=0.6971 and (19) is solved for χ_2=1.6813.[12] We obtain the bargaining share β=0.7183 from (23), which is close to the value of 0.72 used by Shimer (2005). Assuming that Hosios' rule holds (Hosios, 1990), a search worker's contribution in matching is pinned down by the labor's share in the wage bargaining, γ=β. Then, from matching relationships we calibrate m=3.0193. The parameter values, observables and calibrated values are listed in Table 2. Under the benchmark parameter values, we obtain a unique steady state.

Now, we quantify the effects of increases in tax rates and non-employment benefits. We start by measuring the increase in labor taxes and non-employment benefits in Europe relative to the US in the early 2000s. For labor taxes, based on McDaniel (2007),

[11] McDaniel (2007) calculated a series of average tax rates on consumption, investment, labor and capital using national account statistics in 15 OECD countries. The data has been used by Rogerson (2008) and Ohanian *et al*. (2008).

[12] These parameter values indicate that the employed are better off than the non-employed.

we calculate the population-weighted average effective tax rate on labor income in the five European countries under concern in 2000-03 and obtain the tax rate 0.5168. With the data that the effective labor tax rate increased a little bit in the US in the past 30 years,[13] this indicates an increase of labor tax rates by about 30% in Europe relative to the US from that in 1970-73. Next, based on the data in OECD (1999, Table 2.2), the population-weighted average unemployment payment rate is 69.72% in the five European countries under concern and 50% in the US in the late 1990s. These data suggest that non-employment benefits in Europe are roughly 40% higher than the US. Given the data, we quantify the effects of increases in the value of τ by 30% and the value of b by 40% from their baselines. In each exercise, the government budget is balanced by adjusting lump-sum taxes or transfers. Quantitative results are illustrated in Table 3.

First, the quantitative effects of increases in the labor income tax are in the first row of Table 3. The results indicate that when the labor income tax rate is increased by 30%, hours worked per worker are decreased from 0.333 to 0.310 which means a drop by 6.85%. The employment rate is decreased from 0.75 to 0.708 which indicates a decrease by 5.55%; thus, the unemployment rate is increased by 5.55%. As a result, labor supply is decreased by 12.02%. Next, the quantitative effects of increases in non-employment benefits are reported in the second row of Table 3. The results suggest that when non-employment benefits are increased by 40%, the employment rate is decreased from 0.75 to 0.703, which is a decrease by 6.26%; thus, the unemployment rate is increased by 6.26%. Hours worked per worker grow slightly from 0.333 to 0.337, which is an increase by 1.13%. As a result, labor supply is decreased by 5.2%.

Our foregoing results indicate that a 30% increase in labor income taxes in Europe

[13] Based on the data in McDaniel (2007), the effective labor tax rate (on household income and payroll) in the US increased from 0.1775 in 1970-73 to 0.22475 in 2000-03.

relative to the US has a large adverse effect on hours worked per worker, which is consistent with the claim made by Prescott (2002, 2004). Yet, there is also a substantial adverse effect on employment rates. Moreover, our results suggest that a 40% increase in non-employment benefits has a large adverse effect on employment which is consistent with the argument made by Ljungqvist and Sargent (2007, 2008a). These quantitative effects imply that a 30% increase in labor income taxes has a more detrimental effect on hours worked per worker but has a less harmful effect on employment than a 40% increase in non-employment benefits.

To see the combined effects of these two adverse labor market institutions, we increase the labor income tax and non-employment benefits at the same time, with the effects shown in the last row of Table 3. The results reveal that the employment rate is decreased from 0.75 to 0.609, which indicates a large drop by 18.73%. Hours worked per worker are decreased from 0.333 to 0.323, which implies a decrease by 3.08%. As a result, these two adverse labor market institutions decrease labor supply by 21.23%. Compared to the data of a decrease by 28.23% in the EU-11 relative to the US over the past 30 years in Table 1, our quantitative results suggest that higher labor income taxes and more generous non-employment benefits in the EU than the US both can account for about 75% of the declining labor supply in the EU relative to the US over the past 30 years from the early 1970s to the early 2000s.

Finally, we investigate the robustness of the foregoing quantitative results by carrying out two types of sensitivity analysis. First, we vary the *LSE* by increasing its value to 1 and decreasing its value to 0.5.[14] Next, we envisage whether or not the results are robust when the Hosios' rule does not hold. In this exercise, we fix the labor's bargaining share at β=0.7183 and vary the labor's contribution in matching γ to

[14] The value of LSE cannot be smaller than 0.5 as then the calibrated value of χ_2 is negative.

take alternative values {0.235, 0.54, 0.72} used by Hall (2005), Hall and Milgrom (2008) and Shimer (2005), respectively. In the first sensitivity analysis, we recalibrate the model and find that all parameter values are the same as those in Table 2 except for the values of σ, χ_1, χ_2, m and β. In the second sensitivity analysis, we recalibrate the model and find that all parameter values are the same as those in Table 2 except for the value of m. Overall, we find that our foregoing results are robust in that an increase in the labor tax reduces both hours worked per worker and employment rates, and an increase in non-employment benefits lowers employment rates with a small increase in hours worked per worker. The quantitative results indicate that the two adverse labor market institutions explain declining labor supply by more when the labor supply elasticity is larger and the labor's contribution in search γ is smaller.[15]

2.6 Concluding Remarks

Over the past 30 years from the early 1970s to the early 2000s, labor supply in Europe was declined by about 30% relative to the US. The decline in labor supply comes from hours worked per worker and employment rates. Europe has witnessed steadily higher labor taxes and more generous government-supplied non-employment benefits than the US. Some studies attributed declining hours worked per worker in Europe relative to the US to higher labor taxes, while other studies accredited high unemployment rates in Europe to more generous non-employment benefits. This paper studies a model that consider labor search within the neoclassical growth framework so as to investigate the effects on labor supply along both intensive and extensive margins in one unified general equilibrium framework. We use the model to envisage and compare the relative effects of increases in labor taxes and more generous

[15] Labor supply is decreased by 30.14% when LSE=1 and by 12.74% when LSE=0.5. Moreover, labor supply is decreased by 36.1% when γ=0.235, by 24.12% when γ=0.54 and by 21.21% when γ=0.72.

non-employment benefits on hours worked per worker and employment rates.

We find that an increase in the labor tax decreases hours worked per worker and employment rates with an overstated adverse effect on hours worked per worker if employment is fixed as is in Prescott (2002, 2004). Moreover, more generous non-employment benefits decrease employment rates and increase hours worked per worker, with an understated adverse effect on employment rates if hours worked per worker are fixed as are in Ljungqvist and Sargent (2007, 2008a). In the baseline parameterization, we find that increases in labor taxes and non-employment benefits together explain about 75% of declining labor supply in Europe relative to the US over the past 3 decades, with the fraction accounted for being increasing in the labor supply elasticity and decreasing in the labor's contribution in matching.

Finally, our model has a limitation. The labor force is fixed in our model wherein people who are not employed are treated as the non-employed who are entitled to non-employment benefits. In reality, the labor force is variable and people may be out of the labor force. An extension of our research is to compare the effects of labor taxes and unemployment benefits on employment rates and hours worked per worker in a context with an endogenous labor force. In particular, male labor force participation had declined and female labor force participation had risen over the period under study. The aggregate effects may be different between Europe and the US which suggest an alternative mechanism.

Mathematical Appendix

1. The Wage Equation

The relationship $w = W(\underset{+}{e}, \underset{?}{l}, \underset{+}{s}; \underset{+}{\tau}, \underset{+}{b})$ in (18) can be derived as follows.

$$W_e = \frac{(1-\beta)}{(1-\tau)l} MRS_e^e > 0\,, \tag{A1a}$$

$$W_l = \frac{(1-\beta)}{(1-\tau)l}\left[MRS_l^e - \frac{MRS^e}{l} - \frac{b}{l}\right] \gtrless 0\,,$$ [16] if b is $\substack{\text{small} \\ \text{large}}$, (A1b)

$$W_s = \frac{(1-\beta)}{(1-\tau)l} MRS_s^e > 0\,, \tag{A1c}$$

$$W_\tau = (1-\beta)\left[\frac{b + MRS^e}{(1-\tau)^2 l}\right] > 0\,, \tag{A1d}$$

$$W_b = \frac{(1-\beta)}{(1-\tau)l} > 0\,, \tag{A1e}$$

where $MRS_e^e = Xc_e > 0$, $MRS_l^e = MRS^l + Xc_l > 0$, $MRS_s^e = -MRS^s + Xc_s > 0$, [17] and $X \equiv \frac{\chi_2(1-s)^{1-\sigma} - \chi_1(1-l)^{1-\sigma}}{1-\sigma} > 0.$

2. The Search Effort Equation

The relationship $s = S(\underset{-or?}{e}, \underset{-}{l}; \underset{-}{\tau}, \underset{-}{b})$ in (20) is derived as follows.

$$\left(MRS_s^s - MB_s^s\right)ds = \left(-MRS_e^s + MB_e^s\right)de + \left(-MRS_l^s + MB_l^s\right)dl + MB_\tau^s d\tau + MB_b^s db, \tag{A2a}$$

where $$MRS_e^s = \chi_2 (1-s)^{-\sigma} c_e > 0\,, \tag{A2b}$$

$$MRS_l^s = \chi_2 (1-s)^{-\sigma} c_l > 0\,, \tag{A2c}$$

$$MRS_s^s = \chi_2 (1-s)^{-\sigma} c_s + \sigma\chi_2 (1-s)^{-\sigma-1} c > 0\,, \tag{A2d}$$

---

[16] Due to $MRS_{ll}^e > 0$, $MRS_l^e - \frac{MRS^e}{l} > 0.$

[17] If the worker devoted more effort in searching, it would increase his outside option or reservation wage. Hence, we suppose $MRS_s^e > 0.$

$$MB_e^s = \frac{(\rho+\psi)\mu_e}{(\rho+\psi+s\mu)^2}\beta\left[(1-\tau)MPL\cdot l - b - MRS^e\right] - \frac{\mu}{\rho+\psi+s\mu}\beta MRS_e^e < 0 \quad ,[18] \tag{A2e}$$

$$MB_l^s = -\frac{\mu}{\rho+\psi+s\mu}\beta X c_l < 0\,, \tag{A2f}$$

$$MB_s^s = \frac{\mu_s}{(\rho+\psi+s\mu)}\beta\left[(1-\tau)MPL\cdot l - b - MRS^e\right] - \frac{\mu}{\rho+\psi+s\mu}\beta MRS_s^e < 0\,, \tag{A2g}$$

$$MB_\tau^s = -\frac{\mu}{\rho+\psi+s\mu}\beta MPLl < 0\,, \tag{A2h}$$

$$MB_b^s = -\frac{\mu}{\rho+\psi+s\mu}\beta < 0. \tag{A2i}$$

3. The Hour Equation

The relationship $l = L(\underset{-}{e},\underset{-}{s};\underset{-}{\tau})$ in (22) is derived as follows.

$$MRS_e^l de + MRS_l^l dl + MRS_s^l ds + MPLd\tau = 0, \tag{A3a}$$

where $$MRS_e^l = \chi_1(1-l)^{-\sigma}c_e > 0\,, \tag{A3b}$$

$$MRS_l^l = \chi_1(1-l)^{-\sigma}c_l + \sigma\chi_1(1-l)^{-\sigma-1}c > 0\,, \tag{A3c}$$

$$MRS_s^l = \chi_1(1-l)^{-\sigma}c_s > 0\,. \tag{A3d}$$

4. The Employment Equation

The relationship $e = E(\underset{-}{l},\underset{+or?}{s};\underset{-}{\tau},\underset{-}{b},\underset{-}{\lambda_0}) = 0$ is derived as follows.

$$\left(MB_e^v - 2\lambda_1 v_e\right)de + MB_l^v dl + \left(MB_s^v - 2\lambda_1 v_s\right)ds + MB_\tau^v d\tau + MB_b^v db + MB_\beta^v d\beta - d\lambda_0 = 0, \tag{A4a}$$

where $$MB_e^v = \frac{(1-\beta)}{\rho+\delta+\psi}\left[\eta_e\left(MPL\cdot l - \frac{b+MRS^e}{1-\tau}\right) - \eta\frac{MRS_e^e}{1-\tau}\right] < 0\,, \tag{A4b}$$

[18] We assume that the direct effect dominates in order to ensure the diminishing marginal benefit.

$$MB_l^{\nu}=\frac{\eta(1-\beta)}{\rho+\delta+\psi}\left[MPL-\frac{MRS_l^e}{1-\tau}\right]=-\frac{\eta(1-\beta)}{\rho+\delta+\psi}\frac{Xc_l}{1-\tau}<0\,, \tag{A4c}$$

$$MB_s^{\nu}=\frac{(1-\beta)}{\rho+\delta+\psi}\left[\eta_s\left(MPL\cdot l-\frac{b+MRS^e}{1-\tau}\right)-\eta\frac{MRS_s^e}{1-\tau}\right]>(<)0, \tag{A4d}$$

$$MB_\tau^{\nu}=-\frac{\eta(1-\beta)}{\rho+\delta+\psi}\frac{b+MRS^e}{(1-\tau)^2}<0\,, \tag{A4e}$$

$$MB_b^{\nu}=-\frac{\eta}{\rho+\delta+\psi}\frac{(1-\beta)}{1-\tau}<0. \tag{A4f}$$

5. The Slope of Loci E and H

The signs of $l=\tilde{L}(\underset{-}{e};\underset{-}{\tau},\underset{-}{b})$ in (22) and $e=\tilde{E}(\underset{-}{l};\underset{-}{\tau},\underset{-}{b},\underset{-}{\lambda_0})$ in (24) in the (e, l) plan is derived as follows. By substituting (A2a), we rewrite (A3a) and (A4a) as follows.

$$\tilde{L}_e de+\tilde{L}_l dl=\tilde{L}_\tau d\tau+\tilde{L}_b db, \tag{A5a}$$

$$\tilde{E}_e de+\tilde{E}_l dl=\tilde{E}_\tau d\tau+\tilde{E}_b db+d\lambda_0, \tag{A5b}$$

where $\tilde{L}_e\equiv MRS_e^l-MRS_s^l\dfrac{MRS_e^s-MB_e^s}{MRS_s^s-MB_s^s}>0\,,$

$$\tilde{L}_l\equiv MRS_l^l-MRS_s^l\frac{MRS_l^s-MB_l^s}{MRS_s^s-MB_s^s}>0\,,$$

$$\tilde{L}_\tau\equiv -MPL-MRS_s^l\frac{MB_\tau^s}{MRS_s^s-MB_s^s}<0\,,$$

$$\tilde{L}_b\equiv -MRS_s^l\frac{MB_b^s}{MRS_s^s-MB_s^s}>0\,,$$

$$\tilde{L}_\beta\equiv -MRS_s^l\frac{MB_\beta^s}{MRS_s^s-MB_s^s}<0\,,$$

$$\tilde{E}_e\equiv\left(MB_e^{\nu}-2\lambda_1 v_e\right)-\left(MB_s^{\nu}-2\lambda_1 v_s\right)\frac{MRS_e^s-MB_e^s}{MRS_s^s-MB_s^s}<0\,,$$

$$\tilde{E}_l\equiv MB_l^{\nu}-\left(MB_s^{\nu}-2\lambda_1 v_s\right)\frac{MRS_l^s-MB_l^s}{MRS_s^s-MB_s^s}<0\,,$$

$$\tilde{E}_\tau \equiv -MB_\tau^v - \left(MB_s^v - 2\lambda_1 v_s\right)\frac{MB_\tau^s}{MRS_s^s - MB_s^s} > 0\,,$$

$$\tilde{E}_b \equiv -MB_b^v - \left(MB_s^v - 2\lambda_1 v_s\right)\frac{MB_b^s}{MRS_s^s - MB_s^s} > 0.$$ [19]

Thus, Loci E and H are both negatively sloping in the (*e*, *l*) plane.

Moreover, a standard result is that a higher unit cost of vacancy creation λ_0 leads to less vacancies and thus less employment, i.e. $\frac{de}{d\lambda_0} < 0$. Let $D \equiv \tilde{L}_e \tilde{E}_l - \tilde{L}_l \tilde{E}_e$ denote the determinant of the Jacobean matrix in (A5a)-(A5b). Straightforward calculation gives $\frac{de}{d\lambda_0} = -\frac{\tilde{L}_l}{D} < 0$, which requires $-\frac{\tilde{L}_e}{\tilde{L}_l} > -\frac{\tilde{E}_e}{\tilde{E}_l}$ and *D*>0. Therefore, the two curves have at most one intersection.

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[19] We assumed that the direct effects of all these derivatives dominated the indirect effects resulted from the changes of searching effort.

Table Appendix

Table 1: Hours and Employment in the EU Relative to the US, 1970-73 and 2000-2003.

	Hours worked per person			Employment rate			Hours worked per worker		
	70-73	00-03	diff.	70-73	00-03	diff.	70-73	00-03	diff.
Belgium	92.86	72.5	-20.36	95.44	83.65	-11.79	97.29	86.7	-10.59
France	109.63	74.87	-34.76	103.36	89.52	-13.84	106.07	83.65	-22.42
Germany	132.79	77.42	-55.37	107.91	90.34	-17.57	123.04	85.7	-37.34
EU-11	109.63	81.4	-28.23	101.51	88.91	-12.60	107.99	91.57	-16.42
						(43.42%)			(56.58%)
United States	100	100	0	100	100	0	100	100	0

Note: 1. The hours worked per person are the total hours worked divided by the number of the population aged 15-64; the employment rate is the number of the employed divided by the number of the population aged 15-64; the hours worked per worker are the total hours worked divided by the number of the employed.

2. All US values are normalized to 100 in 1970-73 and 2000-03. All EU data in 1970-73 and 2000-03 are normalized to the U.S. values in the respective period. EU-11 includes Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, Spain, Sweden and the UK. We use the population of a country as the weight of the country in calculating the data for the EU-11.

3. Numbers in parenthesis are the composition of differences in hours worked per person in EU-11 into employment and hours worked per worker.

Sources: Data on total numbers of hours worked and total numbers of the employed are taken from OECD (2010a), whereas data on total numbers of the population aged 15-64 are taken from OECD (2010b).

Table 2 Benchmark parameter values and calibration

Benchmark Parameters and Observables		quarterly
physical capital's depreciation rate	δ	0.0200
time preference rate	ρ	0.0100
aggregate consumption-aggregate output ratio	c/y	0.6700
capital's share	α	0.3224
job finding rate per job seeker	$s\mu$	0.8336
fraction of employment	e	0.7500
vacancy-searching worker ratio	$v/(1\text{-}e)$	1.0000
coefficient of goods technology	A	1.0000
coefficient of the cost of vacancy creation and management	λ_1	1.0000
fraction of time devote to work of the employed	el	0.2500
effective tax rate on labor income	τ	0.4000
labor supply elasticity	LSE	0.6500
<u>**Calibration**</u>		
rate of return of capital	r	0.0300
effective capital-labor ratio	q	33.2622
marginal product of labor	MPL	2.0973
capital-output ratio	k/y	10.7467
hours worked per worker	l	0.3333
fraction of time spend on search of the non-employed	s	0.1667
effective job finding rate	μ	5.0016
job separation rate	ψ	0.2779
vacancy creation	v	0.2500
employee recruitment rate	η	0.8336
coefficient of the cost of vacancy creation and management	λ_0	0.1061
equilibrium wage	w	1.4257
unemployment compensation	b	0.1516
inverse of intertemporal elasticity of substitution of leisure	σ	3.0769
utility weight of leisure for the employed	χ_1	0.6971
utility weight of leisure for the non-employed	χ_2	1.6813
labor searcher's bargaining power	β	0.7183
labor searcher's share in matching technology	γ	0.7183
coefficient of matching efficacy	m	3.0193

Table 3: Quantitative Results

	el		e		l	
Benchmark	**0.25000**	**100%**	**0.75000**	**100%**	**0.33333**	**100%**
$\tau\uparrow$ 30%	0.21996	-12.02%	0.70841	-5.55%	0.31050	-6.85%
$b\uparrow$ 40%	0.23699	-5.20%	0.70302	-6.26%	0.33710	1.13%
$\tau\uparrow$ 30% and $b\uparrow$ 40%	0.19691	-21.23%	0.60953	-18.73%	0.32306	-3.08%

Figure Appendix

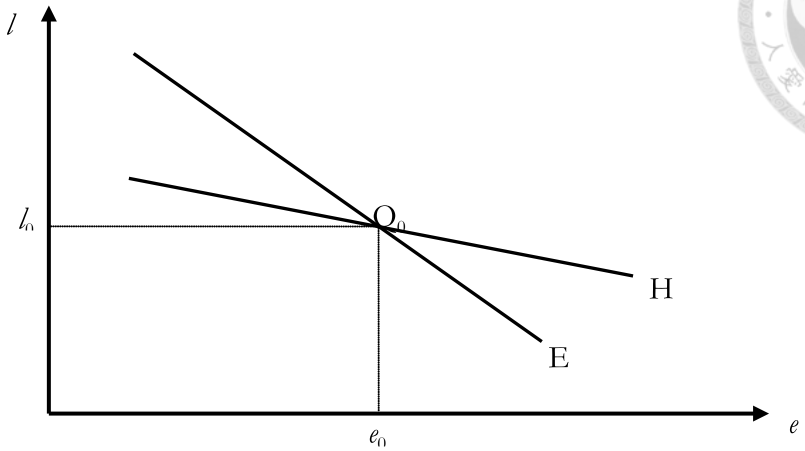

Figure 1: Steady state

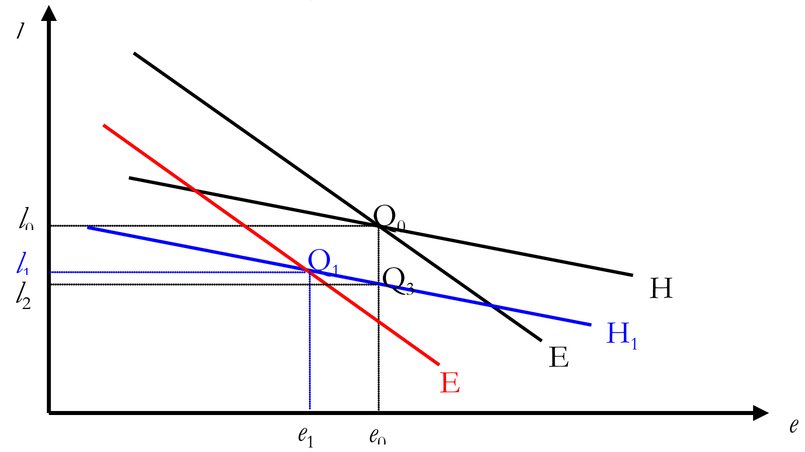

Figure 2: Long-run effects of higher wage taxes (τ)

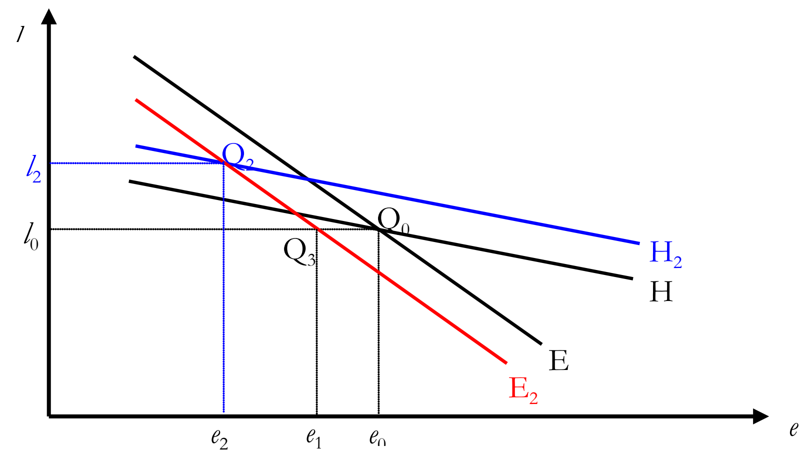

Figure 3: Long-run effects of higher non-employment benefits (b)

3 Relative Effects of Labor Taxes on Working Hours and Employment: Role of Mechanisms Shaping Working Hours

3.1 Introduction

One striking observation in the labor market is that, relative to the US, the labor supply in Europe declined by about 30% from the early 1970s to the early 2000s (cf., Prescott, 2002, 2004). A declining labor supply may come from either decreasing hours worked per worker or falling employment rates. A growing body of literature has found higher labor income taxes in Europe than in the US to be an important reason resulting in the declining labor supply in Europe.[20] Prescott (2002, 2004, and 2006) attributed Europe's large declining hours per employed worker relative to the US entirely to Europe's higher labor income taxes and, as a result, provoked a heated debate. Most studies in this line of research are based on the neoclassical tradition in which the labor supply is determined by households who trade off between leisure and consumption.[21] There is also a large body of literature on European employment, wherein the

[20] According to McDaniel (2007) and Rogerson (2008), in the early 1970s the average tax rate on labor income in Belgium, France, Germany, Italy, and the Netherlands was about 39% which was higher than the 18% average rate in the US. In the early 2000s, the average tax rate on labor in these European countries was about 51% which was still much higher than the 22% average rate in the US.

[21] Prescott (2004, 2006), Ohanian et al. (2008), Rogerson (2008), Jacobs (2009) and Rogerson and Wallenius (2009) argued that the differences in taxes explain much of the variations in hours of work. Alesina et al. (2006) found that European labor market regulation explained the bulk of the difference between the U.S. and Europe. Other kinds of explanation include entry cost (Fonseca *et al*., 2001), preferences (Blanchard, 2004), changes in technology and government (Rogerson, 2006), working-time regulation and employment protection (Causa, 2008), home production (Ngai and Pisssarides, 2008; Olovsson, 2009) and different attitudes toward leisure and leisure externalities (Azariadis *et al*., 2013).

framework in this strand of research implicitly assumes that the labor supply is represented by jobs with fixed hours and then inquires about the reasons underlying the rise of unemployment in Europe. In particular, Ljungqvist and Sargent (2007a, 2008b) argued that the disposition of taxes like non-employment benefits accounted for high unemployment in Europe. Moreover, Daveri and Tabellini (2000) and Krusell et al. (2010) found that higher labor taxes led to higher unemployment in Europe.[22]

Data indicate that in Europe the declining labor supply comes mostly from falling employment in some countries but mainly from decreasing hours per worker in some other countries.[23] Although some existing studies used higher labor taxes to explain declining working hours in Europe and other studies drew upon higher labor taxes to account for falling employment in Europe, they have not answered why higher labor income taxes have stronger negative effects on hours in some countries but stronger negative effects on employment in some other countries. This is an important question. This paper explores the relative effects of labor taxes on hours per worker and employment. A novel feature is to show that different mechanisms shaping the supply of hours will trigger very different effects of labor income taxes on hours vis-à-vis on employment.

To this end, we study a representative agent, labor search model embedded with

[22] To explain the rise of unemployment in Europe, Ljungqvist and Sargent (1998) highlighted the role of skill accumulation on the job and skill loses during unemployment, and Ljungqvist and Sargent (2008a) drew attention to higher dismissal costs and more generous unemployment compensation in Europe. Moreover, Ljungqvist and Sargent (2007b) stressed heterogeneous skills and adverse congestion externalities due to search and match and suggested that unemployment was caused by increased turbulence. Mortensen and Pissarides (1999) emphasized skill-biased shocks, Blanchard and Wolfers (2000) underscored interactions between prices, technology shocks and an adverse labor market, and Blanchard and Giavazzi (2003) pointed out interactions between monopolistic product markets and labor unions.

[23] Available data in OECD countries indicate that in some of the OECD countries, declining hours per person relative to the US from the early 1970s to the early 2000s come from both declining hours per worker and falling employment rates relative to the US. However, in other countries, declining hours per person in the same period come essentially from decreasing hours per worker which accounts for more than 90%. On the other hand, in some other countries, declining hours per person in the same period basically are from falling employment rates which also accounts for more than 90%. See Appendix Table 1 for details.

the supply of workers and hours per worker. Specifically, our model has a representative large household that comprises a continuum of members, who are either employed or unemployed. An employed member chooses to work with pay or to take leisure without pay. An unemployed member enjoys leisure and also searches for jobs. There is a representative firm which pays costs to create job vacancies. Unfilled jobs and job seekers are met bilaterally through a matching technology, with endogenous job finding and recruitment rates that depend on the masses of both matching parties. In the model, a worker's supply of hours is governed by one of the following three mechanisms employed in existing literature. The supply of hours may be bargained by a matched job-worker pair, decided exclusively by the household, or regulated effectively by the authority. The mechanisms have been taken up by, among others, Fang and Rogerson (2009), Prescott (2004), and Marimon and Zilibotti (2000), respectively.[24] Using each of the three mechanisms, we investigate the relative effects of labor taxes on employment and hours in steady state. The steady state is analyzed in terms of two loci linking hours to employment. One of the loci is the firm's demand for hours per worker derived from the free-entry condition that is negatively related to employment which is positively related to vacancy. The other locus is the worker's supply of hours that is also negatively related to employment. With other things being equal, a higher labor tax reduces hours demanded by the firm, but the effects on the hour supply depend on the different mechanisms determining hours. Our main findings are as follows.

First, when the supply of hours is governed by a matched job-worker pair, a cooperative bargaining game agrees on hours. In this mechanism, given employment, a

[24] Marimon and Zilibotti (2000) envisaged employment and distributional effects of regulating (reducing) working time in a general equilibrium model with search-matching frictions. Fang and Rogerson (2009) set up a matching model of labor supply and examined the effects of tax and transfer policies on the two margins of labor supply. While Fang and Rogerson (2009) and Marimon and Zilibotti (2000) used labor search models, Prescott (2004) used a neoclassical growth model. Moreover, Prescott (2004) and Marimon and Zilibotti (2000) considered capital adjustment, whereas there is no capital in Fang and Rogerson (2009).

higher labor tax reduces a worker's supply of hours but the decrease is less than the reduction in the firm's demand for hours per worker, thereby decreasing both employment and hours. When the laborer's share in the hour bargaining is larger, the negative effect on hours is stronger and the negative effect on employment is weaker. Second, when labor hours are decided exclusively by the household who trades off between leisure and consumption, this is the special case that arises when the laborer has a one-hundred percent share in the hour bargaining. In this mechanism, the effect on employment is smallest and on hours is largest. In extremis, when the utility of leisure is linear in hours, the negative effect is completely on hours without reducing employment. Finally, at the other extreme, when a worker's supply of hours is effectively regulated, labor taxes only lower employment without reducing hours.

The reasons of these results can be easily understood. When a worker's supply of hours is governed by a matched job-worker pair, given employment, a higher labor tax decreases the supply of hours per worker less than the reduction of the demand for hours per worker, thereby reducing both employment and hours. With an increasing laborer's share in the hour bargaining, in response to a higher labor tax, the representative household can reduce more of the supply of hours per worker for given employment, thereby increasing the negative effect on hours per employed member while decreasing the negative effect on employment of its members. In the mechanism when the supply of hours is determined exclusively by the household, the household will reduce more hours per worker in response to higher taxes. When the utility of leisure is flatter in hours, the Frisch hour elasticity is larger and thus the effect on hours is larger. With a linear utility of leisure in hours, the Frisch hour elasticity is infinite and the household decreases the supply of hours per worker exactly to the reduced level demanded by the firm. Hence, the negative effect of labor taxes is entirely on hours with a zero effect on employment. Conversely, when hours per worker are effectively regulated, labor taxes only reduce employment.

Finally, we also study a calibrated version of our model with the average effective labor income tax rate being calibrated to European countries in the early 1970s. Data indicates that the average effective labor income tax rate in EU was increased by about 30% from the early 1970s to the early 2000s. We quantify the effects of such an increase in the average effective labor income tax rate. In the benchmark parameterization, we find that the effects on employment and hours are both quantitatively large which are consistent with existing literature. However, when the labor's hour-bargaining power increases, the quantitative effect on employment is diminishing while the effect on hours per worker is increasing. In particular, when the labor's hour-bargaining power approaches to 100%, the negative effect on hours comes up to a maximum and the negative effect on employment is reduced to a minimum which would move toward zero if the utility of leisure in hours is parameterized to be linear. These different hour-shaping mechanisms thus help understand why, in facing higher labor tax rates in Europe over the past thirty years, some countries experienced a more severe increase in unemployment rates while others underwent a sharper decrease in hours per worker.

The remainder of this paper is as follows. In Section 2, we set up a labor search model without capital adjustment. We study the effects of higher labor taxes under different mechanisms governing the supply of hours in Section 3. In Section 4 we calibrate the model to quantify the effects of labor income taxes under different mechanisms governing the supply of hours. Finally, we offer some concluding remarks in Section 5.

3.2 A Simple Labor Search Model

Our simple model embeds the labor supply into a standard Pissarides labor search framework. The labor supply model distinguishes working hours from leisure, and the Pissarides labor search framework separates employment from unemployment.

Following Andolfatto (1996) and Fang and Rogerson (2009), the economy is populated by a continuum of identical infinitely-lived large households. The adoption of the large household setup assures full insurance which eases unnecessary complexity involved in tracking the distribution of the employed and the unemployed. The large household comprises a continuum of members (of measure one), who are either (i) employed, by engaging in work or leisure, or (ii) unemployed, by enjoying leisure only. By assuming that job search is costless, the cost of a household's (endogenously determined) job search is a foregone earning cost. A firm can create a job vacancy for each unfilled job by paying an up front vacancy creation cost ϕ, measured in units of output. Unfilled vacant jobs and active job seekers are met bilaterally through a matching technology, with each vacancy being filled by exactly one job seeker. The matching flow rates (job finding and recruitment rates) are endogenous, depending on the masses of both matching parties. In every period, filled jobs and employed workers are separated at an exogenous rate. Finally, the fiscal authority levies labor income taxes and transfers the tax revenue to households in a lump sum.

3.2.1 Households

The representative large household has a unified preference and pools all resources and enjoyment for its members. In period t, a fraction e_t of the members is employed and the remaining fraction $(1-e_t)$ is unemployed. Given a fixed time endowment normalized at unity, each employed member allocates a fraction h_t of the total time to work and the remaining fraction $(1-h_t)$ to leisure. Unemployed members devote their entire time to leisure. From the household's perspective, the employment changes according to the following birth-death process

$$e_{t+1} - e_t = p_t\left(1 - e_t\right) - \lambda e_t, \tag{1}$$

where p denotes the (endogenous) job finding rate and λ is the (exogenous) job separation rate. Thus, the change in employment ($e_{t+1}-e_t$) is equal to the inflow of workers into the employment pool ($p_t(1-e_t)$) net of the outflow as a result of separation (λe_t).

Denote w_t and τ as the wage rate and the income tax rate, respectively. The household receives after-tax wage income, profits Π_t, and lump-sum transfers T_t and spends on consumption c_t. Thus, the representative household's budget constraint is

$$c_t = \left(1 - \tau\right) w_t e_t h_t + \Pi_t + T_t. \tag{2}$$

The household seeks to maximize the total utility of its members. Let $U(e_t)$ denote the lifetime value of the household in period t when the state is e_t. Denote as $\rho>0$ the time preference rate. The Bellman equation of the household's optimal control problem is

$$U\left(e_t\right) = \max\left[u\left(c_t, l_t\right) + \frac{1}{1+\rho} U\left(e_{t+1}\right)\right], \tag{3}$$

subject to constraints (1) and (2). In (3), $u(.)$ is the large household's periodic utility and l_t denotes leisure. The literature often assumes that an agent's preference is separable in consumption and leisure in order to be consistent with a balanced growth path (e.g., King and Rebelo, 1999). Moreover, the real business cycle literature often assumes indivisible labor (Hansen, 1985).[25] Following this strand of wisdom, we may think of the utility $u(c_t, l_t) = \tilde{u}(c_t) + \mu(l_t) = \bar{\mu} + \tilde{u}(c_t) - \bar{\mu} e_t \tilde{g}(h_t),$ in which the utility of leisure is

[25] Although the fraction of a household's time devoted to work in the Hansen (1985) model looks like employment in the search model here, the former is determined by lottery contracts traded in a complete market while the latter is due to labor market frictions.

$\mu(l_t) = \bar{\mu}[1 - e_t \tilde{g}(h_t)],\ \bar{\mu} > 0,$ where $\tilde{g} : [0,1] \to R$ is continuously differentiable, increasing from zero, and weakly convex; i.e., $\tilde{g}(0) = 0,\ \tilde{g}'(h) > 0$ and $\tilde{g}''(h) \geq 0.$ [26] Based on these considerations, we assume that $u_c > 0 > u_{cc},\ u_l > 0 = u_{ll}$ and $u_{cl} = u_{lc} = 0.$

The Benveniste-Scheinkman condition for e_t is[27]

$$U_e(e_t) = \left[u_c(c_t, l_t)(1-\tau)w_t h_t - u_l(c_t, l_t)\tilde{g}(h_t)\right] + \frac{1}{1+\rho} U_e(e_{t+1})(1 - \lambda - p_t). \tag{4}$$

3.2.2 Firms

Firms are owned by households. Following Fang and Rogerson (2009), the unit of production is a matched job-worker pair. Output from a matched job-worker pair is given by

$$y_{it} = A k_i^{\alpha} (h_t)^{1-\alpha} = f(h_t),$$

where A>0 is a parameter. As in Marimon and Zilibotti (2000), k_i is a firm-specific productive factor that firm i is endowed with, and its supply is fixed. Moreover, following Marimon and Zilibotti (2000), we assume that all firms in the economy have an identical endowment of the fixed factor, i.e., k_i=k.[28]

As in Fang and Rogerson (2009), the value of individual jobs to the firm is independent of the number of jobs a firm already has. Denote π_{vt} and π_{et} as the lifetime value of an unfilled job and a filled job, respectively, in period t. We assume that the

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[26] An example of the utility is the separable utility used in Fang and Rogerson (2009): $\tilde{u}(c_t) + 1 - \tilde{g}(h_t)$. Taking an average over all employed and unemployed members in the large household gives $e_t\left[\tilde{u}(c_t) + 1 - \tilde{g}(h_t)\right] + (1 - e_t)\left[\tilde{u}(c_t) + 1 - \tilde{g}(0)\right] = \tilde{u}(c_t) + 1 - e_t\tilde{g}(h_t) = \tilde{u}(c_t) + \mu(l_t).$ The properties we impose are consistent with this form.

[27] Conditions determining working hours are studied later as different mechanisms give different conditions.

[28] To ease analysis, we present a model without capital adjustment. In the Appendix, we modify the interpretation of k to general capital whose quantity can be adjusted by firms.

vacancy creation cost is a one-time up front cost ϕ.[29] When a firm with an unfilled job matches with an unemployed worker, it has two options: (i) to accept the worker and fill the job, or (ii) to reject the worker and retain the unfilled job into the next period.

If a job is filled in period t, its lifetime value is

$$\pi_{et} = [\, f(h_t) - w_t h_t\,] + \frac{1}{1+r_t}(1-\lambda)\pi_{e(t+1)}, \tag{5a}$$

where r_t is the interest rate. Thus, the lifetime value of a filled job in t equals the flow value in t, ($f(h_t)$-$w_t h_t$), plus the discounted future value when the match is not separated.

Conversely, if the job is not filled in period t, its lifetime value in period t is

$$\pi_{vt} = \frac{1}{1+r_t}\Big[q_t \pi_{e(t+1)} + \big(1-q_t\big)\pi_{v(t+1)}\Big], \tag{5b}$$

where q_t is the recruitment rate. Thus, the lifetime value of an unfilled job is the discounted weighted average of the value of becoming a filled job and the value of retaining an unfilled job. The recruitment rate is taken as given by a firm with an unfilled job, but it is endogenously determined in equilibrium.

Denote v_t as (endogenously) created job vacancies. Then, from the firms' perspective, the employment is increased by the inflow ($q_t v_t$) and decreased by the outflow (λe_t),

$$e_{t+1} - e_t = q_t v_t - \lambda e_t. \tag{6}$$

3.2.3 Labor Matching and Bargaining

In each period t, unemployed workers (1-e_t) and unfilled jobs v_t meet each other through the Diamond (1982) type pair-wise random matching function M_t=M(1-e_t,v_t) which operates like a production function with given inputs (1-e_t) and v_t. Like Fang and

[29] The sufficient condition for any equilibrium with positive employment is that the vacancy creation cost be not too large. The surplus from a match is always positive under our assumptions on the function u and f.

Rogerson (2009), we assume that this function is of constant returns and increasing and strictly concave in each of the two inputs. In order to express the number of vacant jobs *v* as a function of employment *e* so as to ease analysis in equilibrium, we simply adopt the standard Cobb-Douglas form in the literature

$$M_t = M(1-e_t, v_t) = m(1-e_t)^{\gamma}(v_t)^{1-\gamma}, \tag{7}$$

where $m>0$ is the degree of matching efficacy and $\gamma \in (0, 1)$ the contribution of job seekers in matching.

After a successful match, both sides of the match determine the effective wage rate in a cooperative bargaining game that maximizes the following joint surplus

$$\max_{w_t} \; \left[U_e(e_t)\right]^{\beta}\left[\pi_{et}-\pi_{vt}\right]^{1-\beta},$$

where $\beta \in (0, 1)$ measures a labor's bargaining power.[30] In the joint surplus, the household's surplus is evaluated by the value of a marginal increase in employment (U_e) whereas the firm's surplus is gauged by recruiting a worker to fill an unfilled job (π_e-π_v).

In solving the bargaining problem, the job-worker pair treats matching rates (p_t and q_t), the beginning-of-period employment (e_t), the bargains of all other household members in the current period, and all future bargains as given. The first-order condition of the bargaining problem is

$$\frac{\beta}{U_e(e_t)}\frac{\partial U_e(e_t)}{\partial w_t} = -\frac{1-\beta}{\pi_{et}-\pi_{vt}}\left(\frac{\partial \pi_{et}}{\partial w_t}-\frac{\partial \pi_{vt}}{\partial w_t}\right), \tag{8}$$

where $\frac{\partial U_e(e_t)}{\partial w_t} = u_c(c_t, l_t)(1-\tau)h_t$,[31] $\frac{\partial \pi_{et}}{\partial w_t} = -h_t$ and $\frac{\partial \pi_{vt}}{\partial w_t} = 0$.

[30] Under the Hosios (1990) rule, β=γ and the bargaining is efficient. The results in our paper hold no matter whether the bargaining is efficient or not.

[31] The household takes profits and future values as given when bargaining over current values. An individual worker also takes all other members' bargains in the current period as given. See Fang and Rogerson (2009).

3.2.4 The Government

The government's behavior is passive: it levies flat labor income taxes to finance transfers.

$$\tau w_t e_t h_t = T_t. \tag{9}$$

As noted in Rogerson (2006) and Ljungqvist and Sargent (2007a, 2008b), the impact of the labor income tax on the labor supply may depend on the way of using the tax revenue. In order to isolate the effects of taxes from different ways of government spending, in this paper we simply rebate the tax revenue to households in a lump-sum fashion.

3.2.5 Equilibrium

A search equilibrium is a tuple of individual quantity variables, $\{e_t, h_t, v_t, c_t, y_t\}$, a pair of aggregate quantities, $\{M_t, T_t\}$, a pair of matching rates, $\{p_t, q_t\}$, and wage rates, $\{w_t\}$, such that: (i) households optimize; (ii) firms freely enter; (iii) employment evolutions hold; (iv) labor-market matching and wage bargaining conditions are met; (v) the government budget is balanced; and (vi) the goods market clears.

A steady state is search equilibrium when all variables do not change over time. In a steady state, $e_{t+1}=e_t=e$ and the interest rate r is equal to the time preference rate ρ. Moreover, the labor market satisfies the following matching relationships (Beveridge curve)

$$p(1-e) = qv = M = \lambda e. \tag{10}$$

Thus, in equilibrium, the employment inflow from the household side, $p(1-e)$, equals the employment inflow from the firm side, qv, and equals the number of successful matches and, in a steady state, equals the employment outflow due to job separation. The above relationships enable us to solve the two matching rates and the vacant jobs as functions of e.

$$p(e)=\frac{\lambda e}{1-e},\quad \text{where} \quad p'(e)>0, \tag{11a}$$

$$q(e)=m^{\frac{1}{1-\gamma}}(\frac{\lambda e}{1-e})^{\frac{-\gamma}{1-\gamma}}=m^{\frac{1}{1-\gamma}}[p(e)]^{\frac{-\gamma}{1-\gamma}},\quad \text{where} \quad q'(e)<0, \tag{11b}$$

$$v(e)=\left[\frac{\lambda e}{m(1-e)^{\gamma}}\right]^{\frac{1}{1-\gamma}}=m^{\frac{-1}{1-\gamma}}[p(e)]^{\frac{\gamma}{1-\gamma}}\lambda e,\quad \text{where} \quad v'(e)>0. \tag{11c}$$

In the foregoing relationships, the job finding rate p and the vacant jobs v are positively related to employment and the employee recruitment rate q is negatively related to employment.

Aggregate output is ey and total profits are $\Pi=e(y\text{-}wh\text{-}\phi\lambda)$. The profits are remitted to households as households are owners of firms. If we use the household's budget constraint (2) and the government's budget constraint (9), the goods market clearing condition is rewritten as

$$c=e[f(h)-\phi\lambda]\equiv c(e,h), \tag{12}$$

where positive consumption requires $f(h)-\phi\lambda>0.$ Straightforward differentiation gives $c_e=f(h)-\phi\lambda>0$ and $c_h=ef_h(h)>0.$

Then, from (4), the household's surplus accrued from a successful match in a steady state is

$$U_e(e)=\frac{1+\rho}{\rho+\lambda+p}\left[u_c(c,l)\left(1-\tau\right)wh-u_l(c,l)\tilde{g}(h)\right]. \tag{13a}$$

From (5a), the value of a filled job in a steady state is

$$\pi_e=\frac{1+\rho}{\rho+\lambda}\left(f(h)-wh\right). \tag{13b}$$

Obviously, $f(h)>wh$ if employment is positive.

Moreover, in a steady state, the free-entry condition implies that a firm will create vacant jobs until $\pi_v=\phi$. Using (5b) and (13b), the free-entry condition implies

$$\pi_v = \frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda}\left(f(h)-wh\right)=\phi. \tag{13c}$$

Hence, by using (13a)-(13c), the first-order condition of the wage bargaining in (8) becomes

$$\underbrace{\beta\left[\tfrac{1+\rho}{\rho+\lambda+p}\left(wh-\tfrac{u_l(c,l)\tilde{g}(h)}{u_c(c,l)}\tfrac{1}{(1-\tau)}\right)\right]^{-1}}_{MB_w}=\underbrace{\left(1-\beta\right)\left[\tfrac{1+\rho}{\rho+\lambda}\left(f(h)-wh\right)-\phi\right]^{-1}}_{MC_w}. \tag{14}$$

In (14), the left-hand side is the marginal benefit of the household from asking higher wages, referred to as MB_w, which is decreasing in the wage rate. The right-hand side of (14) is the marginal cost of the firm from accepting higher wages, referred to as MC_w, which is increasing in the wage. Thus, given e and h, there is a unique bargained wage in equilibrium. See E_0 in Figure 1.

Rearranging terms in (14) and using (12) give the bargained wage as a function of e and h,

$$w=w(\underset{(+)}{e},\underset{(+)}{h};\underset{(+)}{\tau})\equiv\beta\frac{\left(\rho+\lambda+p\right)}{\rho+\lambda+\beta p}[AP(h)]+\left(1-\beta\right)\frac{\left(\rho+\lambda\right)}{\rho+\lambda+\beta p}[\frac{MRS(c(e,h),1-e\tilde{g}(h))}{1-\tau}\frac{\tilde{g}(h)}{h}], \tag{15}$$

where $MRS(c(e,h),1-e\tilde{g}(h))\equiv\frac{u_l(c,l)}{u_c(c,l)}$ and $AP(h)=\frac{1}{h}[f(h)-\frac{\rho+\lambda}{1+\rho}\phi]$ is a worker's average output net of the vacancy creation cost.

The bargained wage in (15) is a weighted average of the average product of hours net of the vacancy creation cost (the value of working hours) and the marginal utility of leisure in terms of the marginal utility of consumption (the value of leisure hours). To characterize the bargained wage, it is easy to see from Figure 1 that a higher employment level e shifts the MB_w upward and thus the firm needs to pay a higher wage. Next, a longer hour h shifts both the MB_w and MC_w upward and has an ambiguous effect on the wage. Since the utility function implies that the value of leisure is greater for unemployed members than for employed members,[32] near the steady state it is harder to ask employed workers to work more hours. Thus, the MB_w is shifted more, so the bargained wage is increasing in h. Finally, a higher labor tax rate reduces a worker's after-tax wage and shifts the MB_w upward and the firm needs to pay a higher wage rate.

By using (15), the firm's free-entry condition in (13c) is written as

$$\Omega(\underset{(-)}{e}, \underset{(-)}{h}; \underset{(-)}{\tau}) \equiv \frac{q(e)}{\rho + q(e)} \frac{1+\rho}{\rho+\lambda}\left[f(h) - w(e,h;\tau)h \right] - \phi = 0. \tag{16}$$

In (16), the marginal cost of a vacant job is constant at ϕ while the marginal benefit is decreasing in employment e. Thus, with given hours, there is a unique employment level in (16). Moreover, a longer work hour h increases the output and thus the marginal benefit, but a longer work hour also increases the labor cost and reduces the marginal benefit. As the effect on the labor cost dominates, the marginal benefit is decreasing in h (See Appendix A). Thus, (16) is downward-sloping in the h - e space, referred to as the free-entry (FE) curve in Figure 2, indicating a trade-off between e and h from the firm perspective. The net marginal benefit of vacancies is decreasing in τ as a higher labor tax increases the bargained wage and pushes up the labor cost.

In addition to the free-entry condition, we need the worker's supply of hours in order to solve employment and hours in a steady state. Three kinds of mechanisms are studied in the next section.

3.3 Effects of Labor Taxes on Employment and Working Hours

In this section, we explore the relative effects of a higher labor tax on employment and hours. We start with the general mechanism when the worker's supply of hours is negotiated by a matched job-worker pair. We then envisage the mechanism when the

[32] This is a property in search and match models; see, for example, Cheron and Langot (2004).

household chooses the supply of hours per worker which is the special case when the worker has a one-hundred percent share in the labor hour bargaining game. Finally, we study the mechanism in the other extreme when the supply of hours per worker is regulated effectively.

As the curvature of the leisure utility affects relative effects, to ease analysis we use the following form

$$\tilde{g}(h) = gh^{1+\varepsilon}, \quad g>0,\ \varepsilon\geq 0 \tag{17}$$

which satisfies $\tilde{g}(0)=0$, $\tilde{g}'(h)>0$ and $\tilde{g}''(h)\geq 0$. Then, $\mu(l_t)=\bar{\mu}\left[1-e_t\tilde{g}(h_t)\right]=\bar{\mu}\left(1-ge_th_t^{1+\varepsilon}\right)$ and the utility of leisure is weakly convex in hours. In the limit when ε=0, $\frac{\tilde{g}(h)}{h}=\tilde{g}'(h)=g$, $\mu(l_t)=\bar{\mu}\left(1-ge_th_t\right)$ and the leisure utility is linear in both employment and working hours.

3.3.1 Hours Bargained by Job-Worker Pairs

When a worker's supply of hours is governed by a matched job-worker pair, it is negotiated in a cooperative bargaining game. Fang and Rogerson (2009) and others used bargaining to determine hours.[33] In this mechanism, a matched job-worker pair negotiates not only the wage but also hours. Although they bargain wage and hours simultaneously, we assume they could have different bargaining power across wage and hours.[34] While the wage bargaining gives the wage rate in (15), the hour bargaining leads to

$$\frac{\beta_h}{U_e(e_t)}\frac{\partial U_e(e_t)}{\partial h_t}=-\frac{1-\beta_h}{\pi_{et}-\pi_{vt}}\left(\frac{\partial \pi_{et}}{\partial h_t}-\frac{\partial \pi_{vt}}{\partial h_t}\right), \tag{18a}$$

[33] See also Rocheteau (2002) and Shimer (2008), among others.

[34] Since the government has different regulations on price and quantity, workers may have different bargaining power across wage and hours.

where $\frac{\partial U_e(e_t)}{\partial h_t} = u_c(c_t,l_t)(1-\tau)w_t - u_l(c_t,l_t)\tilde{g}'(h_t),$ [35] $\frac{\partial \pi_{et}}{\partial h_t} = f'(h_t) - w_t$ and $\frac{\partial \pi_{vt}}{\partial h_t} = 0.$ We denote the laborer's power in the hour bargaining in (18a) as β_h, which may or may not be the laborer's power β in the wage bargaining in (15).

In (18a), the left-hand side is the household's utility from supplying a marginal unit of hours per worker, and the right-hand side is the firm's gain from recruiting the marginal unit of hours. Condition (18a) will be replaced by other conditions later when a worker's supply of hours is determined by other mechanisms. In steady state, by using (13a)-(13c), equation (18a) becomes

$$-\beta_h \frac{u_c(c,l)(1-\tau)w - u_l(c,l)\tilde{g}'(h)}{\frac{1+\rho}{\rho+\lambda+p}\left[u_c(c,l)(1-\tau)wh - u_l(c,l)\tilde{g}(h)\right]} = (1-\beta_h)\frac{f'(h)-w}{\frac{1+\rho}{\rho+\lambda}\left(f(h)-wh\right)-\phi}. \tag{18b}$$

Substituting the bargained wage rate in (15) into (18b) and rearranging terms yields

$$\Gamma(e,h;\tau) \equiv MRS(c(e,h),1-e\tilde{g}(h))\tilde{g}'(h) - \tfrac{\beta}{\beta_h}\tfrac{1-\beta_h}{1-\beta}(1-\tau)MP(h) - \tfrac{\beta_h-\beta}{\beta_h(1-\beta)}(1-\tau)w(e,h) = 0, \tag{19a}$$

where $MP(h) \equiv f'(h)$ is the marginal product of hours per worker. To start, consider the case when $\beta_h=\beta$ and thus the laborer's share in the hour bargaining is the same as that in the wage bargaining. Then, (19a) is

$$\Gamma(\underset{(+)}{e},\underset{(+)}{h};\underset{(+)}{\tau}) = MRS(c(e,h),1-e\tilde{g}(h))\cdot\tilde{g}'(h) - (1-\tau)MP(h) = 0. \tag{19b}$$

Then, the supply of hours is determined by the marginal cost of hours $MRS\cdot\tilde{g}'$ equal the after-tax marginal gain of hours. Eq. (19b) is referred to as the bargained hour (BH) curve. In (19b), given employment, the net marginal cost of hours is increasing in hours ($\partial\Gamma/\partial h>0$) as longer hours increase the marginal cost but decrease the marginal

[35] To obtain the expression, we follow Fang and Rogerson (2009, p. 1158) and consider the case with finite family members. Let E_t denote the number of members that are employed in period t. In the bargaining over hours, we take the derivatives of $U(E_t)$-$U(E_t$-1) with respect to the current hours of the E^{th} worker, taking as given the hours of all other (E_t-1) workers in the family. Thus, working hours of the E^{th} worker only enter into the current period utility in $U(E_t)$ and do not enter into $U(E_t$-1). Therefore, if the E^{th} worker works one more hour, consumption is increased by the unit of (1-τ)w_t while leisure is decreased by $\tilde{g}'(h_t)$, which would change the value of $U(E_t)$ by $u_c(1-\tau)w_t - u_l\tilde{g}'(h_t)$.

gain. Moreover, the net marginal cost of hours is increasing in employment ($\partial\Gamma/\partial e>0$) because higher employment augments the marginal cost, as resulted from higher pooled consumption and lower pooled leisure in the large household which increases the marginal utility of leisure relative to the marginal utility of consumption (See Appendix A). Therefore, the BH curve is downward-sloping in the h - e space, indicating an underlying trade-off between e and h from the household perspective.

The BH curve (19b) and the FE curve (16) together characterize the allocation of e and h in steady state. Although both the FE and the BH curves are downward-slopping in the h - e space, we have shown that the BH curve is always flatter than the FE curve at any point of intersection, implying that there is at most one intersection. See E_0 in Figure 2. Once we determine the unique pair of hours (h_0) and employment (e_0) in a steady state, we can use other conditions to solve for other variables. In particular, the product of employment and hours per worker gives working hours per person (e_0h_0).

We now analyze the effect of a higher labor tax on hours in both curves, given employment.

$$\left.\frac{dh}{d\tau}\right|_{FE}=-\frac{\Omega_\tau}{\Omega_h}=-\frac{MRS\cdot\tilde{g}/(1-\tau)}{MRS_h\cdot\tilde{g}+MRS\cdot\tilde{g}'-(1-\tau)f'}\overset{\tilde{g}(h)=gh}{=}-\frac{f'}{MRS_h\cdot g}<0, \tag{20a}$$

$$\left.\frac{dh}{d\tau}\right|_{BH}=-\frac{\Gamma_\tau}{\Gamma_h}\overset{\beta_h=\beta}{=}-\frac{f'}{MRS_h\cdot\tilde{g}'+MRS\cdot\tilde{g}''-(1-\tau)f''}\overset{\tilde{g}(h)=gh}{=}-\frac{f'}{MRS_h\cdot g-(1-\tau)f''}<0, \tag{20b}$$

were $\Gamma_\tau=\frac{\beta}{\beta_h}\frac{1-\beta_h}{1-\beta}f'+\frac{\beta_h-\beta}{\beta_h(1-\beta)}\frac{\beta(\rho+\lambda+p)}{\rho+\lambda+\beta p}AP(h)>0,$

$$\Gamma_h=MRS_h\cdot\tilde{g}'+MRS\cdot\tilde{g}''-\frac{\beta}{\beta_h}\frac{1-\beta_h}{1-\beta}(1-\tau)f''-\frac{\beta_h-\beta}{\beta_h(1-\beta)}(1-\tau)w_h>0.$$

A higher labor tax shifts the FE curve downward, because it increases the bargained wage and decreases a firm's marginal benefit of vacant jobs. In optimum, given employment, hours per worker need to lower in order to increase the marginal benefit of vacant jobs thereby shifting the FE curve downward. Moreover, a higher labor tax also shifts the BH curve downward, since it decreases a household's after-tax marginal gain of hours. In optimum, given employment, the household needs to decrease hours per worker in order to decrease the net marginal cost of hours per worker. First, note that a linear utility in leisure helps to pin down the relative shift. With $\varepsilon=0$ and thus $\tilde{g}(h)=gh,$ the FE curve is shifted downward more than the BH curve, as $MRS_h \cdot g - (1-\tau)f'' > MRS_h \cdot g > 0$. The relative shifts are similar when $\tilde{g}(h)$ is concave. See E_2 in Figure 2. It follows that a higher labor income tax reduces both hours and employment.

Intuitively, given employment, a higher labor tax increases the hourly bargained wage and lowers the firm's demand for hours per worker. Similarly, given employment, a higher labor tax reduces the net hourly bargained wage and lowers the supply of hours per worker. Now, the household cannot flexibly change but needs to negotiate hours per worker with the matched firm. Then, the household is not able to reduce the supply of hours sufficiently even if the leisure utility is linear in hours. As a result, a higher labor tax reduces both hours per worker and employment, like those in Fang and Rogerson (2009).

When $\beta_h \neq \beta$, different laborer's shares in the hour bargaining β_h affect the relative effects a higher labor tax has on employment and hours. To see this, we carry out the exercise of increasing β_h while holding the laborer's share in the wage bargaining fixed at β. While such a change does not influence the FE curve, the effect on the BH curve is affected as follows.

$$\frac{d\Gamma_\tau}{d\beta_h}=\frac{\beta}{1-\beta}\frac{1}{\beta_h^{\,2}}\left(\frac{\beta\left(\rho+\lambda+p\right)}{\rho+\lambda+\beta p}AP(h)-f'\right)>0,^{36}\quad \frac{d\Gamma_h}{d\beta_h}=\frac{\beta}{1-\beta}\frac{\left(1-\tau\right)}{\beta_h^{\,2}}\left(f''-w_h\right)<0.$$

[36] The sign holds when β is not too small or the production $f(h)$ is not too flat.

Obviously, as the value of β_h is increased from β, the value in the denominator in (20b) is reduced and the value in the numerator in (20b) is increased. Thus, with a larger laborer's share in the hour bargaining, a higher labor tax unambiguously shifts the BH curve downward more. See E_4 in Figure 2. Thus, with a larger laborer's share in the hour bargaining, the negative effect of a higher labor tax on employment is diminished while the negative effect on hours is enhanced. Intuitively, with a larger laborer's share in the hour bargaining, the household is able to reduce more of the supply of hours per worker. To summarize our finding,

Proposition 1. *Let the worker's supply of hours be determined by a cooperative bargaining game. Then,*

(i) a higher labor tax reduces both hours per worker and employment;

(ii) as the laborer's share in the hour bargaining increases, the negative effect on employment is smaller and the negative effect on hours per worker is larger.

3.3.2 Hours Determined by Households

When the supply of hours is determined exclusively by the household, the household takes wage as given and trades off between consumption and leisure. Prescott (2004) and many other studies in the neoclassical growth model use the same mechanism to determine the supply of hours.[37] By substituting (1) and (2) into (3) and taking derivatives with respect to h_t, we obtain

$$MRS(c_t, l_t)\tilde{g}' \equiv \frac{u_l(c_t, l_t)\tilde{g}'}{u_c(c_t, l_t)} = (1-\tau)w_t. \tag{21a}$$

In the left-hand side of (21a) is the marginal rate of substitution between leisure

[37] See also Rogerson (2008) and Azariadis et al. (2012), among others.

and consumption (hereafter, *MRS*) which is the marginal cost of hours. The right-hand side is the after-tax wage rate which is the marginal gain of hours. It is clear that (21a) is a special case of (18b) in subsection 3.1 which emerges when β_h=1. As β_h is the largest, the negative effect of labor taxes on employment is minimum and the negative effect on hours per worker is maximum. By using the bargained wage (15) and consumption (12), (21a) is rewritten as a form of a zero net marginal cost of hours as in (19b).[38]

$$\Gamma(\underset{(+)}{e}, \underset{(+)}{h}; \underset{(+)}{\tau}) \equiv MRS(c(e,h), 1-geh)g - (1-\tau)AP(h) = 0. \tag{21b}$$

which is referred to as the flexible hours (FH) curve.

Like the BH curve in (19b), here the FH curve is also downward-sloping in the h - e space.[39] The FE curve in (16) and the FH curve in (21b) together determine a unique pair of e and h in a steady state. See E_0 in Figure 3. As in subsection 3.1, a higher labor tax shifts both the FE and the FH curves downward (See Appendix B). In particular, when ε=0, given employment, both curves decrease hours at the same level and as a result, a higher labor tax reduces only hours without affecting employment.

$$\left.\frac{dh}{d\tau}\right|_{FE} = -\frac{\Omega_\tau}{\Omega_h} = -\frac{\frac{MRS\cdot g}{1-\tau}}{\left[MRS_h \cdot g - (1-\tau)\frac{1}{h}\left(f' - \frac{MRS\cdot g}{1-\tau}\right)\right]} = -\frac{AP(h)}{MRS_h \cdot g - (1-\tau)AP_h(h)} < 0,$$

$$\left.\frac{dh}{d\tau}\right|_{FH} = -\frac{\Gamma_\tau}{\Gamma_h} = -\frac{AP(h)}{MRS_h \cdot g - (1-\tau)AP_h(h)} < 0,$$

where $MRS_h = \frac{u_{lc}c_h - u_{ll}ge}{u_c} - \frac{u_l(u_{cc}c_h - u_{cl}ge)}{(u_c)^2} = -\frac{u_l u_{cc} c_h}{(u_c)^2} > 0$ and $AP_h(h) = \frac{1}{h}\left(f'(h) - AP(h)\right) < 0.$

The result can be understood. The Frisch hour elasticity is $1/\varepsilon>0$. In the case when ε=0, the Frisch hour elasticity is the infinite. Now, the household can flexibly choose hours per worker. When the Frisch hour elasticity is infinite, given employment, the

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38 If e=1, there is no friction in the labor market and the wage rate is determined solely by the marginal product of labor as it is in Prescott (2004).

39 Note the difference between the condition $MRS \cdot \tilde{g}' = (1-\tau)MP(h)$ in (19b) and the condition $MRS \cdot \tilde{g}' = (1-\tau)AP(h)$ in (21b). Under a utility of leisure linear in hours, $\tilde{g}'(h) = g$.

household would reduce the supply of hours per worker exactly to the level the firm demands. As a result, employment is not changed and all the effects are on hours. Conversely, when ε>0, the Frisch hour elasticity is less than infinite. Then, given employment, the household will not reduce the supply of hours per worker to exactly the level the firm demands. Thus, a higher labor tax also lowers hours in a steady state. Nevertheless, when ε is smaller, the Frisch hour elasticity is larger. The household reduces more of the supply of hours and thus the negative effect on hours is larger and the negative effect on employment is smaller. To summarize results, we obtain

Proposition 2. *Let the supply of hours be determined by the household via the leisure-consumption tradeoff. Then,*

(i) under a linear utility of leisure in hours, a higher labor tax reduces only hours with no impact on employment;

(ii) with a strictly convex utility of leisure in hours, the flatter the utility in hours, the smaller the negative effect of a higher labor tax on employment.

3.3.3 Hours Regulated by Authorities

When working hours are regulated by the union and the government, a worker's supply of hours is fixed. Marimon and Zilibotti (2000) and others used regulation to determine hours.[40] Following Marimon and Zilibotti (2000), we assume that there is a regulation of maximum work time which is reduced from the level of bargained hours in subsection 3.1 and the regulation is effectively enforced. Suppose that hours are initially at the level of the market equilibrium. In this case, $h_t = \bar{h}$ is referred to as the regulated hour (RH) curve. In steady state, while the horizontal RH curve $h = \bar{h}$ determines

[40] See Calmfors (1985), Hoel and Vale (1986) and Marimon and Zilibotti (2000), among others.

hours, the downward-sloping FE curve (16) determines employment *e*. In Appendix C, we have shown that there exists a unique steady state. With $h = \bar{h}$ in Figure 4, then E_0 is the steady state and $e=e_0$.

To analyze the effect of a higher labor income tax, it is obvious that the RH curve is not affected while the FE curve is shifted downward in the h - e space. As a result, hours are unchanged but the employment is reduced. See E_1 in Figure 4. Intuitively, a higher labor tax drives up the bargained wage and thus depresses the value of an unfilled job. As hours are fixed, firms will respond to a higher labor income tax by creating less vacant jobs so employment is reduced in steady state. Indeed, this model matches exactly the standard Mortensen-Pissarides search-and-match model. Thus, when hours are regulated, the labor tax only reduces employment without affecting hours.[41]

In Ljungqvist and Sargent (2007a, 2008b), the labor supply is represented by jobs with some fixed hours and thus the labor supply is adjusted only by employment. Thus, when the labor income tax is increased, firms respond only by adjusting job creation and households only by changing job search. As a result, the effect of a higher labor tax is entirely on employment.

To recapitulate our results in this section, we find that the relative effects of a higher labor income tax on employment vis-à-vis depends on the mechanism determining the supply of hours per worker. The effect on employment changes from a small negative effect when the worker's supply of hours is determined exclusively by the household, to a partial negative effect when the worker's supply of hours is governed by a bargaining game, and finally to a full negative effect when the worker's

[41] It is worth noting that when regulated hours are reduced, say from $\bar{h}$ to h_5 in Figure 4, with other things being equal, the steady state changes from E_0 to E_5. Thus, a working time reducing policy can increase employment that achieves the goal "work less, work all."

supply of hours is effectively regulated. Our analysis above abstracts from the capital adjustment. We have shown that these results are robust if capital is adjustable (See Appendix E).

3.4 Quantitative Analysis

Our results in Section 3 indicate that the relative effects of a higher labor tax on employment and hours are different under different hour's determination mechanism. This section studies a calibrated version of our model at a quarterly frequency and quantifies these effects. European countries increased their labor tax rates over past three decades. In calibration, we use as a baseline parameterization the average effective tax rate on the labor income in Europe in the early 1970s. The average effective tax rate was increased by about 30% in the early 2000s. We quantify the effect of a 30% increase in the average effective labor tax rate on employment and hours and envisage how the quantitative effect changes as the labor's hour-bargaining power increases.

3.4.1 Calibration

The fraction of employment in the working-age population is about 75 percent (cf. Kydland and Prescott, 1991) and thus we set e=0.75. As pointed out by Prescott (2006), the fraction of productive time allocated to the market ($L=eh$) is 25% and this implies h=0.3333. According to Shimer (2005), the quarterly separation rate is λ=0.1. We employ (10) to compute the quarterly job finding rate at $p=\lambda e/(1-e)$ =0.3. Moreover, we follow Shimer (2005) by normalizing the steady-state ratio of vacancies to searching workers to unity ($v/(1-e)=1$) which implies the vacancy creation in a steady state at v=0.25. Then, we utilize (10) to calibrate the recruitment rate $q=\lambda e/v$=0.3. Because the depreciation rate of capital is assumed to be zero in the model, we set a higher quarterly time preference rate at ρ=0.015 to target the annual real interest rate of 6% which is used by Bils *et al.* (2011). The coefficient of technology is normalized to A=1. The capital share is about one-third and we use the value α=0.333. (cf. Ljungqvist and Sargent, 2007b, 2008b). Since the interest rate is equal to the time preference rate in the steady state, we use the production function to compute the level of capital at k=34.7856.

By setting the aggregate consumption-output ratio at $c/(ey)$=0.6, we use (12) to calibrate ϕ=6.2677. We can then calibrate the wage rate at w=2.4638 from (16). McDaniel (2007) calculated a series of tax rates in OECD countries. On the basis of average tax rates calculated by McDaniel (2007), Rogerson (2008) used the labor taxes in Belgium, France, Germany, Italy, and the Netherlands to represent the tax in Europe.[42] We follow this strand and calculate the population-weighted average effective tax rate on the labor income for these five European countries. We find that the average effective tax rate in 1970-73 is 0.3982 which gives the labor tax rate at τ=0.3982 in the baseline.

Finally, for the utility function adopted in our paper, the parameter ε is the reciprocal of the labor supply elasticity (henceforth, *LSE*). The *LSE* for men estimated by MaCurdy (1981) was about 0.3, while the estimated *LSE* in Heckman and Macurdy (1980, 1982) was about 2.2. Within the range 0.3-2.2, 1 is taken as a reasonable value which implies ε=1.[43] Given this value and under an equal bargaining power of the wage and the hour in the baseline $\beta_h=\beta$, (20b) is used to solve for g=4.014. We then use (15) to obtain β=0.6998, which is close to the value of 0.72 used by Shimer (2005). Assuming that Hosios' rule holds (Hosios, 1990), $\gamma=\beta$ and hence a search worker's

[42] McDaniel (2007) calculated a series of average tax rates on consumption, investment, labor and capital using national account statistics in 15 OECD countries. The data has been used by Rogerson (2008) and Ohanian *et al*. (2008).

[43] Andolfatto (1996) also set *LSE*=1.

contribution to matching is pinned down by the labor's share in the wage bargaining. Then, from (10) we calibrate m=0.3. The parameter values, observables and calibrated values are listed in Table 1. Under the benchmark parameter values, we obtain a unique steady state.

3.4.2 Quantitative Effects of Higher Labor Taxes on Employment and Hours

We now quantify the effect of a higher labor income tax (τ) on employment e and hour per worker h and thus hours per person eh. Using the data in McDaniel (2007), the population-weighted average effective tax rate on the labor income in Belgium, France, Germany, Italy, and the Netherlands is 0.5168 in 2000-03, which is about an increase by 30% from the baseline tax rate in 1970-73.

First, to see the effect of a higher labor tax rate, we hold all parameter values unchanged except for the effective labor income tax rate τ which is increased by 30% from the baseline. See Table 2. In Row 1 wherein β_h=β, we find that as results of a higher labor tax rate, the employment e is decreased by 1.58% while hours per worker h is decreased by 7.99%. The results indicate that in a standard calibration model, labor income taxes have quantitatively large negative effects on employment and hours per worker. The results are consistent with existing studies on employment as well as those on hours.

Next, to see how a different labor's hour-bargaining power β_h affects the relative effect of a higher labor tax rate on employment and hours per worker, we quantify the effect of a deviation of the value of β_h from a labor's wage bargaining power β. See Table 2. It is clear that given β, when the labor's hour bargaining power β_h is increased from 0.6998, the negative effect on employment is quantitatively diminishing while the negative effect on hours per worker are quantitatively increasing (Rows 2-7). In particular, in the limit case when β_h is increased to 100% and thus the supply of hours are completely determined by households, the negative effect on hours per worker is as high as 22.7% and the negative effect on employment is 0.29% which is close to zero.

Finally, we may wonder how sensitive our quantitative relative effects on employment and hours are with respect to the labor supply elasticity. To see the effect, we carry out quantitative exercises both by lowering the value of *LSE* from 1 in the baseline to 0.8 (cf. Table 3) and by raising the value of *LSE* to 1.2 (cf. Table 4). While a smaller *LSE* reduces the negative effects on employment and hours and a larger *LSE* increases the negative effects on employment and hours, overall the results are similar to those in Table 2. These results are thus supportive of the Propositions 1 and 2.

3.5 Concluding Remarks

The past thirty years have witnessed large declines in the labor supply in Europe relative to the US. High labor taxes are considered an important reason resulting in either falling employment or decreasing hours per worker. This paper studies the relative detrimental effects of higher labor taxes on hours worked per worker and employment. We have shown that the relative effects depend on the mechanisms shaping the supply of hours per worker.

We find that in the mechanism when the worker’s supply of hours is bargained by matched job-worker pairs, a higher labor income tax reduces both employment and hours per worker. When the laborer’s share in the hour bargaining is larger, the negative effect on employment is smaller while the negative effect on hours is larger. In the mechanism when the supply of hours is decided exclusively by the household who trades off between leisure and consumption, this is the special case that arises when the laborer has a one-hundred percent share in the hour bargaining. In extremis, when the utility of leisure is linear in hours, the negative effect on employment is zero and all negative effects are on hours per worker. At the other extreme, in the mechanism when the worker's supply of hours is effectively regulated by the authority, a higher labor tax only reduces employment with a zero effect on hours. The quantitative results in a calibrated version of model support the findings. Thus, these different hour-shaping mechanisms help understand the underlying mechanisms why, in facing higher labor tax rates in Europe over the past thirty years, some countries experienced more severe increases in unemployment rates while some other countries underwent sharper decreases in hours per worker.

3.6 Mathematical Appendix

A. Derivation of the model without capital when hours are determined by households

When the disutility of hours is linear, the steady-state equilibrium conditions are (16) and (17c). Differentiating (16) and (17c) yields

$$\Gamma_e = MRS_e \cdot g > 0,$$

$$\Gamma_h = MRS_h \cdot g - (1-\tau) AP_h > 0,$$

$$\Gamma_\tau = AP = \frac{MRS \cdot g}{1-\tau} > 0,$$

$$\Omega_e = \underbrace{-\frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}MRS_e \cdot gh}_{\Omega_e^1<0} + \underbrace{\frac{1+\rho}{\rho+q}\left[\frac{\rho q'}{\rho+q}\frac{f-wh}{\rho+\lambda} - \frac{q(1-\beta)\beta p'}{(\rho+\lambda+\beta p)^2}\overbrace{\left(AP - \frac{MRS \cdot g}{1-\tau}\right)}^{=0}h\right]}_{\Omega_e^2<0} < 0,$$

$$\Omega_h = \frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}\left[(1-\tau) f' - MRS \cdot g - MRS_h \cdot gh\right] < 0,$$

$$\Omega_\tau = -\frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}\frac{MRS \cdot g}{1-\tau}h < 0,$$

where $MRS_e = \frac{u_{lc}c_e - u_{ll}\tilde{g}}{u_c} - \frac{u_l\left(u_{cc}c_e - u_{cl}\tilde{g}\right)}{u_c^2} = -\frac{u_l u_{cc}c_e}{u_c^2} > 0$,

$$MRS_h = \frac{u_{lc}c_h - u_{ll}e\tilde{g}'}{u_c} - \frac{u_l\left(u_{cc}c_h - u_{cl}e\tilde{g}'\right)}{u_c^2} = -\frac{u_l u_{cc}c_h}{u_c^2} > 0,$$

$$AP_h = \tfrac{1}{h}\left(f'(h) - AP\right) < 0 .$$[44]

Since both $-\frac{\Gamma_e}{\Gamma_h} < 0$, and $-\frac{\Omega_e}{\Omega_h} < 0$, the flexible hours curve and the free-entry curve are both downward sloping in the $h-e$ space. By noting that $\Gamma_e\Omega_h = \Gamma_h\Omega_e^1$, we obtain $\Gamma_e\Omega_h - \Gamma_h\Omega_e = -\Gamma_h\Omega_e^2 > 0$, which implies $-\frac{\Gamma_e}{\Gamma_h} > -\frac{\Omega_e}{\Omega_h}$ and thus, the flexible hours curve is always flatter than the free-entry curve at any point of intersection. Hence, the two curves have at most one intersection.

For a given e, when τ is increased, the FH and the FE curves are shifted downward at the same level as follows.

$$\left.\frac{dh}{d\tau}\right|_{FE} = -\frac{\Omega_\tau}{\Omega_h} = -\frac{\frac{MRS\cdot g}{1-\tau}}{\left[MRS_h\cdot g - (1-\tau)\frac{1}{h}\left(f' - \frac{MRS\cdot g}{1-\tau}\right)\right]} = -\frac{AP}{MRS_h\cdot g - (1-\tau)AP_h} < 0,$$

$$\left.\frac{dh}{d\tau}\right|_{FH} = -\frac{\Gamma_\tau}{\Gamma_h} = -\frac{AP}{MRS_h\cdot g - (1-\tau)AP_h} < 0.$$

Further, by noting that $\Gamma_e\Omega_\tau = \Gamma_\tau\Omega_e^1$, we have $\Gamma_\tau\Omega_h - \Gamma_h\Omega_\tau = 0$ and $\Gamma_e\Omega_\tau - \Gamma_\tau\Omega_e =$ $-\Gamma_\tau\Omega_e^2 > 0$. It follows that

$$\frac{de}{d\tau} = -\frac{\Gamma_\tau\Omega_h - \Gamma_h\Omega_\tau}{\Gamma_e\Omega_h - \Gamma_h\Omega_e} = 0,$$

$$\frac{dh}{d\tau} = -\frac{\Gamma_e\Omega_\tau - \Gamma_\tau\Omega_e}{\Gamma_e\Omega_h - \Gamma_h\Omega_e} < 0.$$

B. Derivation of the model without capital when hours are bargained by job-worker pairs

The stead-state conditions are (16) and (20a). Differentiating (16) and (20a) gives

$$\Gamma_e = MRS_e\cdot\tilde{g}' - \frac{\beta_h - \beta}{\beta_h(1-\beta)}(1-\tau)w_e \overset{\beta_h=\beta}{=} MRS_e\cdot\tilde{g}' > 0,$$

[44] We assume that the vacancy creation cost is not too large.

$$\Gamma_h = MRS_h \cdot \tilde{g}' + MRS \cdot \tilde{g}'' - \frac{\beta}{\beta_h}\frac{1-\beta_h}{1-\beta}(1-\tau) f'' - \frac{\beta_h - \beta}{\beta_h(1-\beta)}(1-\tau) w_h$$

$$\overset{\beta_h=\beta}{=} \underbrace{MRS_h \cdot \tilde{g}'}_{\Gamma_h^1>0} + \underbrace{MRS \cdot \tilde{g}'' - (1-\tau) f''}_{\Gamma_h^2>0} > 0,$$

$$\Gamma_\tau = \frac{\beta}{\beta_h}\frac{1-\beta_h}{1-\beta} f' + \frac{\beta_h - \beta}{\beta_h(1-\beta)} \frac{\beta(\rho+\lambda+p)}{\rho+\lambda+\beta p} AP \overset{\beta_h=\beta}{=} f' > 0,$$

$$\Omega_e = \underbrace{-\frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau} MRS_e \tilde{g}}_{\Omega_e^1<0} + \underbrace{\frac{1+\rho}{\rho+q}\left[\frac{\rho q'}{\rho+q}\frac{f-wh}{\rho+\lambda} - \frac{q(1-\beta)\beta p'}{(\rho+\lambda+\beta p)^2}\left(AP - \frac{MRS}{1-\tau}\frac{\tilde{g}}{h}\right)h\right]}_{\Omega_e^2<0} < 0,$$

$$\Omega_h = \frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}\left[\underbrace{(1-\tau) f' - MRS \cdot \tilde{g}'}_{=0 \text{ if } \beta_h=\beta} - MRS_h \cdot \tilde{g}\right] < 0,$$

$$\Omega_\tau = -\frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}\frac{MRS \cdot \tilde{g}}{1-\tau} < 0.$$

We note that $\Omega_e^2 < 0$, because $\left(AP - \frac{MRS}{1-\tau}\frac{\tilde{g}}{h}\right)$ is the surplus of a filled job net of the cost of the worker, which must be positive in order for a firm with a vacancy to fill a worker. Under a utility of leisure linear in hours, $\left(AP - \frac{MRS}{1-\tau}\frac{\tilde{g}}{h}\right) = AP - MP$.

Since both $-\frac{\Gamma_e}{\Gamma_h} < 0$ and $-\frac{\Omega_e}{\Omega_h} < 0$, the BH curve and the FE curve are both downward sloping in the $h-e$ space.

Moreover, by noting that $\Gamma_e \Omega_h = \Gamma_h^1 \Omega_e^1$, we have $\Gamma_e \Omega_h - \Gamma_h \Omega_e = -\Gamma_h^1 \Omega_e^2 - \Gamma_h^2 \Omega_e > 0$, which implies $-\frac{\Gamma_e}{\Gamma_h} > -\frac{\Omega_e}{\Omega_h}$. Since the BH curve is always flatter than the FE curve at any point of intersection, there is at most one intersection.

For a given e, when τ is increased, the BH and the FE curves shift downward, respectively, as follows.

$$\left.\frac{dh}{d\tau}\right|_{FE} = -\frac{\Omega_\tau}{\Omega_h} = -\frac{MRS \cdot \tilde{g}/(1-\tau)}{MRS_h \cdot \tilde{g} + MRS \cdot \tilde{g}' - (1-\tau) f'} \overset{\tilde{g}(h)=gh}{=} -\frac{f'}{MRS_h \cdot g} < 0,$$

$$\left.\frac{dh}{d\tau}\right|_{BH} = -\frac{\Gamma_\tau}{\Gamma_h} = -\frac{\frac{\beta}{\beta_h}\frac{1-\beta_h}{1-\beta} f' + \frac{\beta_h-\beta}{\beta_h(1-\beta)}\frac{\beta(\rho+\lambda+p)}{\rho+\lambda+\beta p} AP}{MRS_h \cdot \tilde{g}' + MRS \cdot \tilde{g}'' - \frac{\beta}{\beta_h}\frac{1-\beta_h}{1-\beta}(1-\tau) f'' - \frac{\beta_h-\beta}{\beta_h(1-\beta)}(1-\tau) w_h}$$

$$\overset{\beta_h=\beta}{=} -\frac{f'}{MRS_h \cdot \tilde{g}' + MRS \cdot \tilde{g}'' - (1-\tau) f''} \overset{\tilde{g}(h)=gh}{=} -\frac{f'}{MRS_h \cdot g - (1-\tau) f''} < 0.$$

Although it is difficult to compare the relative downward shift of these two curves, a utility of leisure linear in hours helps to pin down the relative magnitude. Under $\tilde{g}(h) = gh$, since $(1\text{-}\tau)f''(h)<0$, the BH curve is unambiguously shifted downward less than the FE curve. The following comparative state confirms this conjecture.

By noting that $\Gamma_\tau \Omega_h = \Gamma_h^1 \Omega_\tau$ and $\Gamma_e \Omega_\tau = \Gamma_\tau \Omega_e^1$, we have $\Gamma_\tau \Omega_h - \Gamma_h \Omega_\tau =$ $-\Gamma_h^2 \Omega_\tau > 0$ and $\Gamma_e \Omega_\tau - \Gamma_\tau \Omega_e = -\Gamma_\tau \Omega_e^2 > 0$. It follows that

$$\frac{de}{d\tau} = -\frac{\Gamma_\tau \Omega_h - \Gamma_h \Omega_\tau}{\Gamma_e \Omega_h - \Gamma_h \Omega_e} < 0,$$

$$\frac{dh}{d\tau} = -\frac{\Gamma_e \Omega_\tau - \Gamma_\tau \Omega_e}{\Gamma_e \Omega_h - \Gamma_h \Omega_e} < 0.$$

C. Derivation of the model without capital when hours are regulated by authorities

The equilibrium conditions are (16) and

$$\Gamma(e, h; \bar{h}, \tau) = h - \bar{h} = 0.$$

Differentiating the above two conditions gives

$$\Gamma_e = 0, \quad \Gamma_h = 1, \quad \Gamma_{\bar{h}} = -1, \quad \Gamma_\tau = 0, \quad \Omega_{\bar{h}} = 0,$$

$$\Omega_e = \underbrace{-\frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau} MRS_e \cdot \tilde{g}}_{\Omega_e^1<0} + \underbrace{\frac{1+\rho}{\rho+q}\left[\frac{\rho q'}{\rho+q}\frac{f-wh}{\rho+\lambda} - \frac{q(1-\beta)\beta p'}{(\rho+\lambda+\beta p)^2}\left(AP - \frac{MRS}{1-\tau}\frac{\tilde{g}}{h}\right)h\right]}_{\Omega_e^2<0} < 0,$$

$$\Omega_h = \frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}\left[\underbrace{(1-\tau) f' - MRS \cdot \tilde{g}'}_{=0} - MRS_h \cdot \tilde{g}\right] < 0,$$

$$\Omega_\tau = -\frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}\frac{MRS \cdot \tilde{g}}{1-\tau} < 0.$$

Since $-\frac{\Gamma_e}{\Gamma_h} = 0$ and $-\frac{\Omega_e}{\Omega_b} < 0$, the RH curve is horizontal and the FE curve is downward sloping in the $h - e$ space. There is obviously a unique steady state.

Moreover, the comparative-static exercises give

$$\frac{de}{d\bar{h}} = -\frac{\Gamma_{\bar{h}} \Omega_h - \Gamma_h \Omega_{\bar{h}}}{\Gamma_e \Omega_h - \Gamma_h \Omega_e} = -\frac{-\Omega_h}{-\Omega_e} < 0,$$

$$\frac{dh}{d\bar{h}} = -\frac{\Gamma_e \Omega_{\bar{h}} - \Gamma_{\bar{h}} \Omega_e}{\Gamma_e \Omega_h - \Gamma_h \Omega_e} = -\frac{\Omega_e}{-\Omega_e} = 1,$$

$$\frac{de}{d\tau} = -\frac{\Gamma_\tau \Omega_h - \Gamma_h \Omega_\tau}{\Gamma_e \Omega_h - \Gamma_h \Omega_e} = -\frac{-\Omega_\tau}{-\Omega_e} < 0,$$

$$\frac{dh}{d\tau} = -\frac{\Gamma_e \Omega_\tau - \Gamma_\tau \Omega_e}{\Gamma_e \Omega_h - \Gamma_h \Omega_e} = 0.$$

D Model with Capital Adjustments

Our analysis in Section 3 abstracts from capital. In this appendix, we show that the results in Section 3 are robust if capital is adjustable in the same way as was in Marimon and Zilibotti (2000).[45]

We assume that the production function is now $y_t = Ak_t^{\alpha}(h_t)^{1-\alpha} = f(h_t, k_t)$ and capital k_t is accumulated by firms. As in Marimon and Zilibotti (2000), we think of final-goods producing firms that take the output from worker firms and combine it with capital. Hence, capital is separate from the wage bargaining process. The result will be the same if the firm rents capital from the household since the capital market is perfect. By assuming that capital k does not depreciate in order to simplify our analysis, then the interest rate equals the marginal product of capital: $r_t = f_k(h_{t+1}, k_{t+1})$.

The representative household's problem and the optimization conditions all remain the same as the model above. The government's behavior remains the same as (9). While the lifetime value of an unfilled job is also the same as (5b), the lifetime value of a filled job in (5a) is modified as

$$\pi_{et} = [f(h_t, k_t) - w_t h_t + k_t] + \frac{1}{1+r_t}(1-\lambda)\pi_{e(t+1)}. \tag{D1}$$

Note that different from (5a), here the flow value in t includes the value of capital.

In a steady state, the interest rate satisfies $r=\rho$ and hence $f_k(h,k) = \rho$. This implies that

[45] While the case with capital adjustment in Marimon and Zilibotti (2000) was carried out in a small open economy when the interest rate is taken as given, we will maintain the closed-economy setup and thus the interest rate is endogenously determined.

the capital-hour ratio in a steady state is constant and thus k is in proportion to h,

$$k = \kappa h \equiv k(h), \text{ where } \kappa = \left(\frac{\alpha A}{\rho}\right)^{\frac{1}{1-\alpha}}.$$

The goods market clearing condition is now

$$c = e\big(f(h,k(h)) - \phi\lambda\big) \equiv \widehat{c}(e,h), \tag{D2}$$

where $\widehat{c}_e = f(h,k(h)) - \phi\lambda > 0$ and $\widehat{c}_h = e(f_h + f_k k_h) > 0$.

From (D1), the value of a filled job in a steady state is

$$\pi_e = \frac{1+\rho}{\rho+\lambda}\big(f(h,k(h)) - wh + k(h)\big),$$

and then the free-entry condition in a steady state is

$$\pi_v = \frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda}\big(f(h,k(h)) - wh + k(h)\big) = \phi\,. \tag{D3}$$

From the first order condition of the wage bargaining problem, the bargained wage rate is

$$w = \widehat{w}(e,h;\tau) \equiv \frac{\beta(\rho+\lambda+p)}{\rho+\lambda+\beta p}[\widehat{AP}(h)] + \frac{(1-\beta)(\rho+\lambda)}{\rho+\lambda+\beta p}[\frac{MRS(\widehat{c},l)}{1-\tau}\frac{\tilde{g}(h)}{h}], \tag{D4}$$

where $\widehat{AP}(h) \equiv \frac{1}{h}\Big(f(h,k(h)) + k(h) - \frac{\rho+\lambda}{1+\rho}\phi\Big)$. Notice that the steady-state matching relationships given by (10) still hold. Thus, p and q both are functions of e as stated in (11a) and (11b).

Substituting the bargained wage rate in (D4) into (D3) yields the free-entry condition

$$\Omega(\underset{(-)}{e},\underset{(-)}{h};\underset{(-)}{\tau}) \equiv \frac{q(e)}{\rho+q(e)}\frac{1+\rho}{\rho+\lambda}\big(f(h,k(h)) - \widehat{w}(e,h)h + k(h)\big) - \phi = 0, \tag{D5}$$

which relates employment negatively to hours. As in Section 3, (D5) is referred to as the FE curve.

D.1 Hours Determined by Households

First, consider the mechanism wherein, given employment, the supply of hours is exclusively decided the household, like that in Prescott (2004). The leisure-consumption tradeoff condition is (17a). By using the bargained wage in (D4), consumption in (D2) and the utility of leisure linear in hours $\tilde{g}(h_t) = gh_t$, (17a) is rewritten to yield the following FH curve

$$\Gamma(\underset{(+)}{e}, \underset{(+)}{h}; \underset{(+)}{\tau}) \equiv MRS(\hat{c}(e,h), l)g - (1-\tau)\widehat{AP}(h) = 0. \tag{D6}$$

The steady-state conditions of model are (D5) and (D6) wherein p and q are functions of e, defined by (11a) and (11b), and c is a function of e and h, given by (D2). Below, we show that when the labor tax rate is increased, both the FE and the FH curves are shifted downward to the same level as it is in Figure 2. Thus, even though capital is adjusted, when the supply of hours is determined exclusively by the household, under the leisure utility linear in hours, a higher labor income tax only reduces hours per worker without affecting employment, a result the same as proposition 1.

Differentiating (D5) and (D6) gives

$$\Gamma_e = MRS_e \cdot g > 0,$$

$$\Gamma_h = MRS_h \cdot g - (1-\tau)\widehat{AP}_h > 0,$$

$$\Gamma_\tau = \widehat{AP} > 0,$$

$$\Omega_e = \underbrace{-\frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}MRS_e \cdot gh}_{\Omega_e^1<0} + \underbrace{\frac{1+\rho}{\rho+q}\left[\frac{\rho q'}{\rho+q}\frac{f-wh+k}{\rho+\lambda} - \frac{q(1-\beta)\beta p'}{(\rho+\lambda+\beta p)^2}\overbrace{\left(\widehat{AP}-\frac{MRS\cdot g}{1-\tau}\right)}^{=0}h\right]}_{\Omega_e^2<0} < 0,$$

$$\Omega_h = \frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}\left[(1-\tau)\left(f_h + f_k k_h + k_h\right) - MRS\cdot g - MRS_h \cdot gh\right] < 0,$$

$$\Omega_\tau = -\frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}\frac{MRS\cdot g}{1-\tau}h < 0,$$

where $\widehat{AP}_h = \frac{1}{h}\left(f_h + f_k k_h + k_h - \widehat{AP}\right)$.

By noting that $\Gamma_e\Omega_h = \Gamma_h\Omega_e^1$, we have $\Gamma_e\Omega_h - \Gamma_h\Omega_e = -\Gamma_h\Omega_e^2 > 0$, which implies $-\frac{\Gamma_b}{\Gamma_e} > -\frac{\Omega_h}{\Omega_e}$. Since the FH curve is always flatter than the FE curve at any point of intersection, there is at most one intersection. Further, by noting that $\Gamma_e\Omega_\tau = \Gamma_\tau\Omega_e^1$, we obtain $\Gamma_\tau\Omega_h - \Gamma_h\Omega_\tau = 0$ and $\Gamma_e\Omega_\tau - \Gamma_\tau\Omega_e = -\Gamma_\tau\Omega_e^2 > 0$. It follows that

$$\frac{de}{d\tau} = -\frac{\Gamma_\tau \Omega_h - \Gamma_h \Omega_\tau}{\Gamma_e \Omega_h - \Gamma_h \Omega_e} = 0,$$

$$\frac{dh}{d\tau} = -\frac{\Gamma_e \Omega_\tau - \Gamma_\tau \Omega_e}{\Gamma_e \Omega_h - \Gamma_h \Omega_e} < 0.$$

D.2 Hours Bargained by Job-Worker Pairs

Next, consider the mechanism when a worker's supply of hours is determined by a matched job-worker pair in a cooperative bargaining game. When the laborer's share in the hour bargaining is β_h, the hour is determined by

$$-\beta_h \frac{u_c(c,l)(1-\tau)w - u_l(c,l)\tilde{g}'(h)}{\frac{1+\rho}{\rho+\lambda+p}\left[u_c(c,l)(1-\tau)wh - u_l(c,l)\tilde{g}(h)\right]} = (1-\beta_b)\frac{f_h(h,k(h)) - w}{\frac{1+\rho}{\rho+\lambda}\left(f(h,k(h)) - wh + k(h)\right) - \phi}.$$

Substituting (D4) and (D2) into the above expression yields

$$\Gamma(e,h;\tau) \equiv MRS(\hat{c},l)\tilde{g}'(h) - \frac{\beta}{\beta_h}\frac{1-\beta_h}{1-\beta}(1-\tau)\, f_h(h,k(h)) - \frac{\beta_h - \beta}{\beta_h(1-\beta)}(1-\tau)\,\hat{w}(e,h) = 0, \qquad \text{(D7)}$$

which is the same as (20a) in subsection 3.2 except for taking into account the effect of capital on hours, $k(h)$.

When $\beta_h = \beta$, (D7) yields the following BH curve.

$$\Gamma(e,h;\tau) \equiv MRS(\hat{c}(e,h), 1 - e\tilde{g}(h))\tilde{g}'(h) - (1-\tau)\widehat{MP}(h) = 0, \qquad \text{(D8)}$$

where $\widehat{MP}(h) \equiv f_h(h,k(h))$. The expression (D8) is the same as (20b) in subsection 3.2.

Hence, the steady-state condition includes the FE curve (D5) and the BH curve (D8) and determines e and h. Below, we differentiate (D5) and (D8) with respect to τ and find that the two curves are shifted in the same way as they are in Figure 3. Thus, a higher labor tax reduces both hours per worker and employment.

Differentiating (D5) and (D8) gives

$$\Gamma_e = MRS_e \cdot \tilde{g}' - \frac{\beta_h - \beta}{\beta_h(1-\beta)}(1-\tau)\hat{w}_e \overset{\beta_h=\beta}{=} MRS_e \cdot \tilde{g}' > 0,$$

$$\Gamma_h = MRS_h \cdot \tilde{g}' + MRS \cdot \tilde{g}'' - \frac{\beta}{\beta_h}\frac{1-\beta_h}{1-\beta}(1-\tau)\underbrace{\left(f_{hh} + f_{hk}k_h\right)}_{=0} - \frac{\beta_h - \beta}{\beta_h(1-\beta)}(1-\tau)\widehat{w}_h$$

$$\overset{\beta_h=\beta}{=} \underbrace{MRS_h \cdot \tilde{g}'}_{\Gamma_h^1>0} + \underbrace{MRS \cdot \tilde{g}''}_{\Gamma_h^2>0} > 0,$$

$$\Gamma_\tau = \frac{\beta}{\beta_h}\frac{1-\beta_h}{1-\beta} f_h + \frac{\beta_h - \beta}{\beta_h(1-\beta)}\frac{\beta(\rho+\lambda+p)}{\rho+\lambda+\beta p}\widehat{AP} \overset{\beta_h=\beta}{=} f_h > 0,$$

$$\Omega_e = \underbrace{-\frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}MRS_e \cdot \tilde{g}}_{\Omega_e^1<0} + \underbrace{\frac{1+\rho}{\rho+q}\left[\frac{\rho q'}{\rho+q}\frac{f-wh+k}{\rho+\lambda} - \frac{q(1-\beta)\beta p'}{(\rho+\lambda+\beta p)^2}\left(\widehat{AP} - \frac{MRS}{1-\tau}\frac{\tilde{g}}{h}\right)h\right]}_{\Omega_e^2<0} < 0,$$

$$\Omega_h = \underbrace{\frac{q}{\rho+q}\frac{(1+\rho)^2(1-\beta)}{\rho+\lambda+\beta p}k_h}_{\Omega_h^1>0} + \underbrace{\frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}\left[\underbrace{(1-\tau)f_h - MRS \cdot \tilde{g}'}_{=0} - MRS_h \cdot \tilde{g}\right]}_{\Omega_h^2<0},$$

$$\Omega_\tau = -\frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}\frac{MRS \cdot \tilde{g}}{1-\tau} < 0.$$

By noting that $\Gamma_e\Omega_h^2 = \Gamma_h^1\Omega_e^1$, we have $\Gamma_e\Omega_h - \Gamma_h\Omega_e = \Gamma_e\Omega_h^1 - \Gamma_h^1\Omega_e^2 - \Gamma_h^2\Omega_e > 0$, which implies $-\frac{\Gamma_e}{\Gamma_h} > -\frac{\Omega_e}{\Omega_h}$. Since the BH curve is always flatter than the FE curve at any point of intersection, there is a unique steady state.

For a given e, when τ is increased, the BH and the FE curves shift downward, respectively, as follows.

$$\left.\frac{dh}{d\tau}\right|_{FE} = -\frac{\Omega_\tau}{\Omega_h} = -\frac{MRS \cdot \tilde{g}/(1-\tau)}{MRS_h \cdot \tilde{g} + MRS \cdot \tilde{g}' - (1-\tau)f_h - (1-\tau)(1+\rho)k_h}$$

$$\overset{\tilde{g}(h)=gh}{=} -\frac{f_h}{MRS_h \cdot g - (1-\tau)(1+\rho)k_h / h} < 0,$$

$$\left.\frac{dh}{d\tau}\right|_{BH} = -\frac{\Gamma_\tau}{\Gamma_h} = -\frac{\frac{\beta}{\beta_h}\frac{1-\beta_h}{1-\beta} f_h + \frac{\beta_h-\beta}{\beta_h(1-\beta)}\frac{\beta(\rho+\lambda+p)}{\rho+\lambda+\beta p}\widehat{AP}}{MRS_h \cdot \tilde{g}' + MRS \cdot \tilde{g}'' - \frac{\beta}{\beta_h}\frac{1-\beta_h}{1-\beta}(1-\tau)\widehat{w}_h}$$

$$\overset{\beta_h=\beta}{=} -\frac{f_h}{MRS_h \cdot \tilde{g}' + MRS \cdot \tilde{g}''} \overset{\tilde{g}(h)=gh}{=} -\frac{f_h}{MRS_h \cdot g} < 0.$$

Thus, even under a linear utility in hours, the BH curve is shifted downward less than the FE curve and thus hours and employment both are reduced.

Finally, by noting that $\Gamma_\tau\Omega_h^2 = \Gamma_h^1\Omega_\tau$ and $\Gamma_e\Omega_\tau = \Gamma_\tau\Omega_e^1$, we have $\Gamma_\tau\Omega_h - \Gamma_h\Omega_\tau = \Gamma_\tau\Omega_h^1 - \Gamma_h^2\Omega_\tau > 0$ and $\Gamma_e\Omega_\tau - \Gamma_\tau\Omega_e = -\Gamma_\tau\Omega_e^2 > 0$. It follows that

$$\frac{de}{d\tau} = -\frac{\Gamma_\tau \Omega_h - \Gamma_h \Omega_\tau}{\Gamma_e \Omega_h - \Gamma_h \Omega_e} < 0,$$

$$\frac{dh}{d\tau} = -\frac{\Gamma_e \Omega_\tau - \Gamma_\tau \Omega_e}{\Gamma_e \Omega_h - \Gamma_h \Omega_e} < 0.$$

When $\beta_h > \beta$, we find

$\frac{d\Gamma_\tau}{d\beta_h} = \frac{\beta}{1-\beta} \frac{1}{\beta_h^2} \left(\frac{\beta(\rho + \lambda + p)}{\rho + \lambda + \beta p} \widehat{AP} - f_h \right) > 0$ if β is not too small or productivity diminishes more in h, and

$$\frac{d\Gamma_h}{d\beta_h} = -\frac{\beta}{1-\beta} \frac{(1-\tau)}{\beta_h^2} \widehat{w}_h < 0.$$

Hence, when β_h increases, the BH curve is shifted downward more.

When $\beta_h \neq \beta$, we increase the laborer's share in the hour bargaining β_h while holding the laborer's share in the wage bargaining fixed at β. We find that as in subsection 3.2, the FE curve is not affected but the BH curve in (D8) is shifted downward more in response to a higher labor tax. Thus, as β_h is increased from β, the negative effect of a higher labor tax on employment is lessened and the negative effect on hours is strengthened. In the limit as β_h goes to 1, when the leisure utility is linear in hours, a higher labor tax has no effect on employment.

Hence, even though capital is adjustable, when the worker's supply of hours is determined by a bargaining game, the relative effect of a higher labor tax on the intensive and extensive margins of labor supply in proposition 2 continues to hold.

D.3 Hours Regulated by Authorities

Finally, when the worker's supply of hours is regulated effectively, the hour curve is replaced by $h = \bar{h}$. Then, the steady state is characterized by $h = \bar{h}$ and (D5). Below, we show that a higher labor tax rate only reduces employment. Thus, the results in proposition 3 continue to hold.

The steady-state conditions are (D5) and

$$\Gamma(e,h;\overline{h},\tau) \equiv h-\overline{h}=0. \tag{D9}$$

Differentiating (D5) and (D9) gives

$$\Gamma_e=0,\quad \Gamma_h=1,\quad \Gamma_{\overline{h}}=-1,\quad \Gamma_\tau=0,\quad \Omega_{\overline{h}}=0,$$

$$\Omega_e=\underbrace{-\frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}MRS_e\cdot\tilde{g}}_{\Omega_e^1<0}+\underbrace{\frac{1+\rho}{\rho+q}\left[\frac{\rho q'}{\rho+q}\frac{f-wh+k}{\rho+\lambda}-\frac{q(1-\beta)\beta p'}{(\rho+\lambda+\beta p)^2}\left(\widehat{AP}-\frac{MRS}{1-\tau}\frac{\tilde{g}}{h}\right)h\right]}_{\Omega_e^2<0}<0,$$

$$\Omega_h=\underbrace{\frac{q}{\rho+q}\frac{(1+\rho)^2(1-\beta)}{\rho+\lambda+\beta p}k_h}_{\Omega_h^1>0}+\underbrace{\frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}\left[\underbrace{(1-\tau)f_h-MRS\cdot\tilde{g}'}_{=0}-MRS_h\cdot\tilde{g}\right]}_{\Omega_h^2<0},$$

$$\Omega_\tau=-\frac{q}{\rho+q}\frac{1+\rho}{\rho+\lambda+\beta p}\frac{(1-\beta)}{1-\tau}\frac{MRS\cdot\tilde{g}}{1-\tau}<0.$$

The comparative-static results are as follow.

$$\frac{de}{d\overline{h}}=-\frac{\Gamma_{\overline{h}}\Omega_h-\Gamma_h\Omega_{\overline{h}}}{\Gamma_e\Omega_h-\Gamma_h\Omega_e}=-\frac{-\Omega_h}{-\Omega_e}\gtrless 0,$$

$$\frac{dh}{d\overline{h}}=-\frac{\Gamma_e\Omega_{\overline{h}}-\Gamma_{\overline{h}}\Omega_e}{\Gamma_e\Omega_h-\Gamma_h\Omega_e}=-\frac{\Omega_e}{-\Omega_e}=1,$$

$$\frac{de}{d\tau}=-\frac{\Gamma_\tau\Omega_h-\Gamma_h\Omega_\tau}{\Gamma_e\Omega_h-\Gamma_h\Omega_e}=-\frac{-\Omega_\tau}{-\Omega_e}<0,$$

$$\frac{dh}{d\tau}=-\frac{\Gamma_e\Omega_\tau-\Gamma_\tau\Omega_e}{\Gamma_e\Omega_h-\Gamma_h\Omega_e}=0.$$

Table Appendix

Table 1: Benchmark Parameter Values and Calibration

Benchmark parameters and	Variable	Quarterly	Sources
fraction of employment	e	0.7500	Kydland and Prescott (1991)
hours of work	eh	0.2500	Prescott (2006)
vacancy-searching worker ratio	$v/(1-e)$	1.0000	Shimer (2005)
job separation rate	λ	0.1000	Shimer (2005)
time preference rate	ρ	0.0150	Bils *et al.* (2011)
capital's share	α	0.3330	Ljungqvist and Sargent
coefficient of goods technology	A	1.0000	Normalization
aggregate consumption-output ratio	$c/(ey)$	0.6000	Data
labor tax rate	τ	0.3982	McDaniel (2007), Rogerson (2008)
labor supply elasticity (*LSE*)	$1/\varepsilon$	1.0000	Andolfatto (1996)
Calibration			
hours worked per worker	h	0.3333	
vacancy creation	v	0.2500	
job finding rate	p	0.3000	
employee recruitment rate	q	0.3000	
capital	k	34.7856	
unit cost of vacancy creation	□	6.2677	
equilibrium wage	w	2.4638	
disutility of hours for the employed	g	4.0140	
labor's bargaining power for wage	β	0.6998	
labor's bargaining power for hour	β_h	0.6998	
labor's share in matching	γ	0.6998	
coefficient of matching technology	m	0.3000	

Table 2: Numerical results when τ is increased by 30% (*LSE*=1)

	e		h		eh	
Benchmark	**0.75000**	**100%**	**0.33333**	**100%**	**0.25000**	**100%**
$\beta_h=\beta$	0.73815	-1.58%	0.30670	-7.99%	0.22639	-9.44%
$\beta_h=0.75$	0.74084	-1.22%	0.29890	-10.33%	0.22144	-11.42%
$\beta_h=0.80$	0.74304	-0.93%	0.29117	-12.65%	0.21635	-13.46%
$\beta_h=0.85$	0.74482	-0.69%	0.28339	-14.98%	0.21107	-15.57%
$\beta_h=0.90$	0.74622	-0.50%	0.27543	-17.37%	0.20553	-17.79%
$\beta_h=0.95$	0.74724	-0.37%	0.26713	-19.86%	0.19961	-20.16%
$\beta_h=1.00$	0.74786	-0.29%	0.25827	-22.52%	0.19315	-22.74%

Note: Parameter values are in Table 1.

Table 3 : Numerical results when τ is increased by 30% (LSE=0.8)

	e		h		eh	
Benchmark	**0.75000**	**100%**	**0.33333**	**100%**	**0.25000**	**100%**
β_h=β	0.74160	-1.12%	0.30852	-7.44%	0.22880	-8.48%
β_h=0.75	0.74211	-1.05%	0.30639	-8.08%	0.22738	-9.05%
β_h=0.80	0.74369	-0.84%	0.29893	-10.32%	0.22231	-11.08%
β_h=0.85	0.74496	-0.67%	0.29138	-12.59%	0.21707	-13.17%
β_h=0.90	0.74593	-0.54%	0.28365	-14.91%	0.21158	-15.37%
β_h=0.95	0.74661	-0.45%	0.27557	-17.33%	0.20574	-17.70%
β_h=1.00	0.74698	-0.40%	0.26694	-19.92%	0.19940	-20.24%

Note: Parameter values are in Table 1 except for ε=1.25, g=4.6958 and β=γ=0.7358.

Table 4: Numerical results when τ is increased by 30% (LSE=1.2)

	e		h		eh	
Benchmark	**0.75000**	**100%**	**0.33333**	**100%**	**0.25000**	**100%**
β_h=β	0.73389	-2.15%	0.30564	-8.31%	0.22430	-10.28%
β_h=0.75	0.74045	-1.27%	0.29160	-12.52%	0.21591	-13.63%
β_h=0.8	0.74330	-0.89%	0.28376	-14.87%	0.21091	-15.63%
β_h=0.85	0.74561	-0.59%	0.27589	-17.23%	0.20571	-17.72%
β_h=0.9	0.74744	-0.34%	0.26788	-19.64%	0.20022	-19.91%
β_h=0.95	0.74881	-0.16%	0.25957	-22.13%	0.19437	-22.25%
β_h=1	0.74970	-0.04%	0.25072	-24.78%	0.18796	-24.81%

Note: Parameter values are in Table 1 except for ε=0.8333, g=3.6462 and β=γ=0.6622.

Figure Appendix

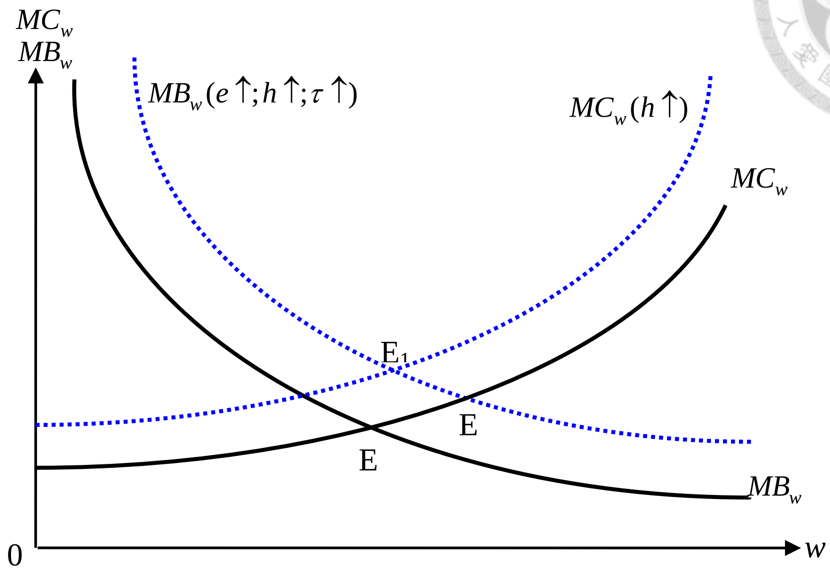

Figure 1: Equilibrium wage

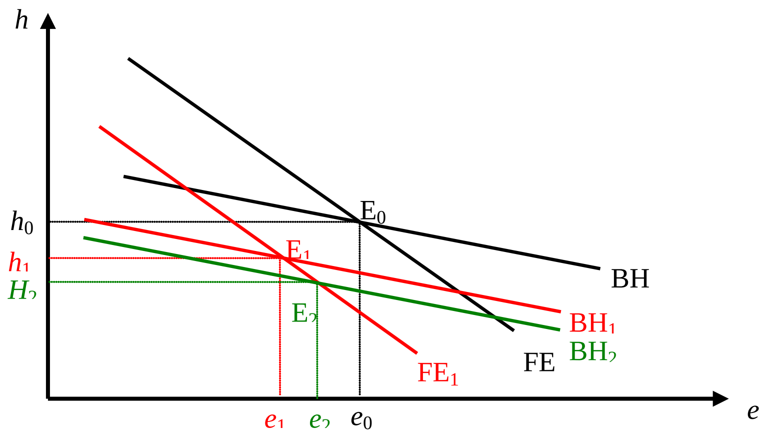

Figure 2: Steady state and effects of higher wage taxes when the supply of hours is bargained by job-worker pairs

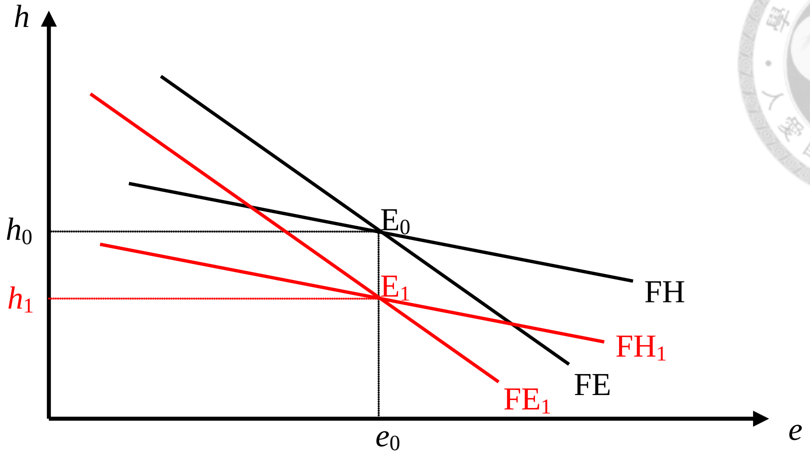

Figure 3: Steady state and effects of higher wage taxes when the supply of hours is determined by households

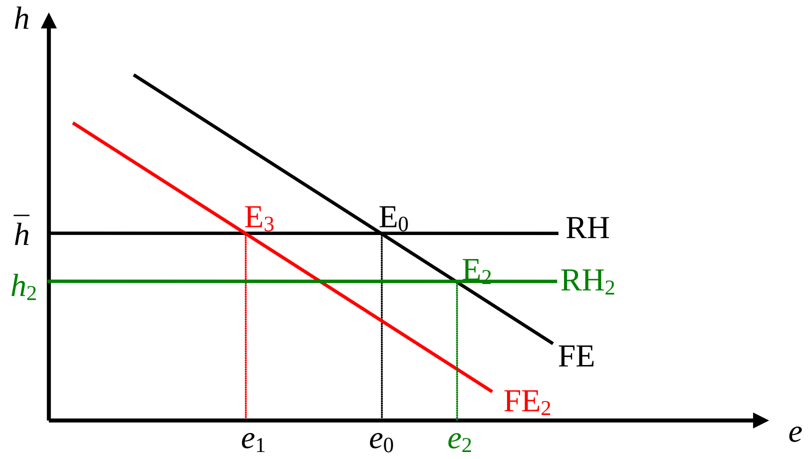

Figure 4： Steady state and effects of higher wage taxes when the supply of hours is regulated by authorities

Appendix Table 1 Working hours and Employment in OECD Relative to the US, 1970-73 and 2000-03.

	Hours worked per person			Employment rate			Hours worked per worker		
	70-73	00-03	diff.	70-73	00-03	diff.	70-73	00-03	diff.
Mostly intensive margin									
Ireland	110.14	85.69	**-24.45**	89.97	86.78	-3.19	122.41	98.75	**-23.65**
Netherlands	96.80	80.80	**-16.00**	98.27	98.20	-0.07	98.49	82.29	**-16.20**
Norway	99.99	84.66	**-15.33**	100.14	102.37	2.23	99.85	82.69	**-17.16**
Mostly extensive margin									
Australia	103.03	92.58	**-10.45**	101.16	91.26	**-9.90**	101.85	101.45	-0.40
New Zealand	120.23	100.02	**-20.21**	114.32	94.71	**-19.62**	105.16	105.63	0.46
Turkey	119.57	75.83	**-43.74**	103.91	67.28	**-36.63**	115.07	112.74	-2.33
Both margins									
Belgium	92.86	72.50	**-20.36**	90.42	79.56	**-10.86**	102.70	91.13	**-11.57**
Denmark	115.02	91.14	**-23.88**	114.10	100.78	**-13.32**	100.83	90.45	**-10.38**
Finland	117.89	87.39	**-30.50**	109.28	86.88	**-22.39**	107.88	100.60	**-7.28**
France	109.64	74.87	**-34.77**	98.01	86.04	**-11.97**	111.86	87.02	**-24.84**
Germany	132.79	77.41	**-55.39**	126.69	91.69	**-35.00**	104.81	84.43	**-20.38**
Italy	92.42	84.21	**-8.21**	83.59	78.82	**-4.77**	110.56	106.87	**-3.69**
Spain	103.26	78.92	**-24.34**	90.68	78.98	**-11.70**	113.87	99.94	**-13.93**
Switzerland	125.05	105.06	**-19.99**	115.69	110.50	**-5.19**	108.10	95.09	**-13.01**
United Kingdom	110.00	91.42	**-18.58**	104.01	93.12	**-10.89**	105.75	98.19	**-7.57**
United States	100	100	**0**	100	100	**0**	100	100	**0**

Note: All US values were normalized to 100 in 1970-73 and 2000-03 and all other data in 1970-73 and 2000-03 normalized to the U.S. values in the respective period. We divide total hours worked by the number of the employed to obtain hours per worker. The employment rate is the number of the employed divided by the number of the population aged 15-64. The product of these two values provides a measure of working hours per person of working age which can also be calculated by dividing total hours worked by the number of the population aged 15-64.[46]

Sources: Data on total hours of work and total employment numbers are taken from OECD (2010a), whereas data on annual total numbers of the population aged 15-64 are taken from OECD (2010b).

[46] The calculation method we employ is the same as those used in Prescott (2004) and Rogerson (2006). We thank Rogerson for sharing the calculation method.

4 Effects of Labor Taxes and Unemployment Benefits on Labor Supply in a Search Model with Endogenous Labor Force Participation

4.1 Introduction

Average labor supply in the EU declined about one fourth relative to those in the US from the early 1970s to the early 2000s. A growing body of literature has sought to understand the relative importance of the various policies and institutional factors that have been proposed as competing explanations. In particular, two important labor market policies are blamed for causing declining labor supply in the EU relative to the US over the past 30 years. One of these is higher labor taxes that were advocated by Prescott (2002, 2004) and his followers (e.g., Ohanian *et al.*, 2008; Jacobs, 2009; and Rogerson and Wallenius, 2009) and the other is generous unemployment benefits that were stressed by Alesina et al. (2006) and Ljungqvist and Sargent (2007a, 2008). The former studies involve only an intensive margin (working hours per worker), whereas the latter papers include only an employment margin. The only exception is Fang and Rogerson (2009) who took both margins into account.[47] Thus, these existing models considered either the intensive margin, the employment margin, or both margins of labor supply.

[47] Fang and Rogerson (2009) is the Andolfatto (1996) model that abstracted from capital but allowed for an employee to choose between working time and leisure time. Their paper analyzed the implications of increases in the labor tax and increases in the cost of job creation on labor supply of the intensive and extensive margin in a steady state.

A notable feature in the data is that differences in average labor supply in the EU relative to the US are due to differences along three margins: the intensive margin, the employment margin and the participation margin (the labor force). According to OECD (2010a, 2010b), the US added more to labor forces than the EU over the past 30 years. See Table 1. While there are models that incorporate endogenous labor forces, no paper incorporates all three margins when explaining declining labor supply in the EU relative to the US.[48] In this paper we attempt to fill the gap by studying a model with all these three margins. Our paper compares the long-run effects on labor supply of increases in labor taxes and unemployment compensation in models with and without participation margins. These two policies may not fully explain the difference in labor supply between the EU and the US in the past 3 decades, because there are differences in other labor market policies and institutions.[49] Yet, by considering the participation, our model serves as a first step in understanding the effects of the two major labor market policies on labor supply.

Specifically, our model is the large household model of Fang and Rogerson (2009) extended to consider the participation margin. The large household pools all resources for its members and decides between consumption and savings. Employment is a predetermined state and the employed members choose between working and leisure time. The large firm creates and maintains multiple vacancies and produces goods. Job vacancies and job seekers are brought together by the matching technology and, upon a

[48] There are existing papers that studied different topics with endogenous labor forces. Early theoretic analyses of labor force participation include Burdett et al. (1984) and Andolfatto and Gomme (1996). Pissarides (2000, Ch. 7) developed a general equilibrium matching model with labor force participation wherein there were no flows in and out of the labor market. Garibaldi and Wasmer (2005), Pries and Rogerson (2009) and Krusell et al. (2011) extended this model to generate flows into and out of the labor market. These models did not analyze changes in an average labor supply. Moreover, in these papers the participation margin is a state with exogenous random arrival rates such that the participation decision is a discrete, binary choice.

[49] Other policies and institutions that were argued to cause declining labor supply in the EU include working-time regulation and employment protection (Causa, 2008), home production (Ngai and Pisssarides, 2008; Olovsson, 2009) and preferences (Blanchard, 2004; Azariadis *et al*., 2013).

successful match, bargain over wage and working hours. Unlike Fang and Rogerson (2009), here the nonemployed are free to choose between searching for jobs and engaging in nonmarket activities. A novel feature of our work is that the participation margin is modeled as a control variable, not a state variable, and thus can be introduced into the framework within a representative large household. Our model renders as a special case the model studied by Fang and Rogerson (2009) wherein the labor-force participation is exogenous.

In analyzing the long-run effects regarding the policies of increases in labor tax rates and unemployment compensation on labor supply, main results are as follows. First, with increases in labor taxes, due to discouraging labor-force participation, the employment in our model is decreased less than that in the model with exogenous labor forces and, with ambiguous effects on hours worked per worker in both models, labor supply is decreased by less in our model. Next, with increases in unemployment compensation, due to inducing labor forces, employment increases in our mode but decreases in the model with exogenous labor forces and, with effects on hours worked per worker opposite to those on employment, the effects on labor supply are ambiguous in both models, depending on whether the effect on employment or that on hours worked per worker dominates.

To quantify the net effect on labor supply, we calibrate our model to the US economy. By feeding in the data of increases in the labor tax and unemployment compensation in the EU relative to the US from the early 1970s to the early 2000s, we find that the model with exogenous labor forces explains too much of the decreases in employment and labor supply between the EU and the US. In particular, this model predicts an increase in hours worked per worker, but the data indicates a decrease. By contrast, with endogenous labor forces, our model explains a more reasonable decrease in labor supply, with a sensible decrease in employment and a modest decrease rather than an increase in hours worked per worker in the EU relative to the US. Thus, with the participation margin, our model explains the difference in labor supply better than the model with exogenous labor forces.

We must point out that Tripier (2003) and Shimer (2011) have considered non-participation as a control wherein the nonemployed decide to be unemployed or inactive.[50] Tripier (2003) used his model to quantitatively account for the allocation of time among employment, unemployment and non-participation in the US. Shimer (2011) applied his model to study counter-cyclical unemployment rates and persistent fluctuations in the vacancy-unemployment ratio in the US. Unlike these two papers, our paper explores the effects of increases in labor taxes and unemployment compensation on labor supply as a result from changes in hours per worker, employment and labor forces. Kim (2008) is also close to our paper in that he analyzed the effect of unemployment benefits in a search model with endogenous labor forces. However, non-participation is a state rather than a control in Kim (2008), so it is difficult to offer analytical analysis.

This paper is outlined as follow. In Section 2, we document differences in the aggregate labor supply between the US and EU along intensive, employment and participation margins. In Section 3, we set up a matching model with the three margins and then characterize the steady state equilibrium in models with and without an endogenous participation margin. In Section 4, we analyze the effects of increases in labor taxes and unemployment compensation on labor supply and then offer quantitative results. Finally, we provide concluding remarks in Section 5.

[50] Tripier (2003) and Shimer (2011) are large household models a la Merz (1995) with standard preferences and technologies.

4.2 The Model

Our model is based on Fang and Rogerson (2009) and extended to include an active participation margin. The economy is composed by a continuum of households and firms, and a passive fiscal authority. The details of the environment follow.

4.2.1 Households

The economy is populated by a continuum of "large" households of unit mass. The setup of large households is convenient in that all family members pool all resources regardless of their labor market status. This useful method of modeling perfect consumption insurance in general-equilibrium search models has been common since Merz (1995) and Andolfatto (1996). A representative large household consists of a continuum of family members (of measure one). Family members are either employed, by engaging in productive activities, or nonemployed, by engaging either in job search activities or in other nonmarket activities. Employment is a predetermined state in each period. If we denote e as the fraction of employed members (referred to as employed workers) in the representative large household, then the fraction of nonemployed members is $(1–e)$. Employed members choose between working time h and leisure time $(1–h)$. Nonemployed members decide to search for jobs (referred to as job seekers or unemployed workers) or to engage in other nonmarket activities (referred to as non-participants). If n is the fraction of members engaging in other non-market activities, the fraction of members in the labor force is $(1–n)$ and thus $(1–e–n)$ is the fraction in the labor force not working but searching for jobs. See the labor allocation in Figure 1.[51] Given the basics of the environment, the unemployment rate is the fraction

[51] Our model does not allow for a direct transit from out of labor force to employment, because the direct flows from out of labor force to employment in the data are due to misclassification problems in a time aggregation bias, as argued by Garibaldi and Wasmer (2005) and others.

of unemployment in the labor force and is thus u≡(1–*n*–*e*)/(1–*n*).

Let μ_t denote the (endogenous) job finding rate and λ the (exogenous) job separation rate. Then, changes of employment from the household's perspective are

$$e_{t+1}-e_t=\mu_t\left(1-e_t-n_t\right)-\lambda e_t. \tag{1}$$

Let w denote the wage per employed worker,[52] r the capital rental, τ the labor income tax rate, b unemployment benefits received by unemployed members, π profits remitted from firms and T lump-sum taxes. The large household's budget constraint is

$$c_t+\left[k_{t+1}-(1-\delta)k_t\right]=r_tk_t+(1-\tau)w_te_t+b\left(1-e_t-n_t\right)+\pi_t-T_t, \tag{2}$$

where c is consumption, k physical capital and δ the depreciation rate of capital.

An agent obtains utility from consumption and leisure depending on the labor-market status. The utility of an employed member is $u(c_t)+\chi_1V(1-h_t)$, the utility of an unemployed member is $u(c_t)+\chi_2$, and the utility of a member outside the labor force is $u(c_t)+\chi_3$, where χ_1, χ_2 and χ_3 are parameters. The representative household's utility simply sums up utilities over its members and is thus

$$u(c_t)+e_t\chi_1V(1-h_t)+\left(1-e_t-n_t\right)\chi_2+n_t\chi_3.$$

Some remarks about the utility of the representative household follow. Following Garibaldi and Wasmer (2005), Pries and Rogerson (2009) and Krusell et al. (2011), we use a linear utility of leisure for members outside the labor force as well as members in the labor force not working but actively searching for a job. Moreover, as these studies, we restrict $\chi_3>\chi_2$ in order to allow for a non-degenerated fraction of members outside the labor force.[53] Different from the linear utility of consumption adopted by Garibaldi and

52 The wage per worker w equals the wage per hour ω multiplied by working hours per worker h: $w_t=\omega_th_t$. The pair of a successful match bargains over the wage and working hours. No matter whether the wage is paid in terms of per worker or per hour, our results are the same.

53 See Pissarides (2000, Ch7, p167) who also assumed that the leisure utility of non-participants is demonstrably greater than that of unemployed workers. Note that implicit in the assumption $\chi_3>\chi_2$ is the

Wasmer (2005) and Pries and Rogerson (2008), which implicitly impose assumptions on income and substitution effects that govern labor supply that are not consistent with standard specifications, we follow Krusell et al. (2011) and employ a concave utility of consumption. Unlike Krusell et al. (2011) wherein an employed worker devotes all the time endowment to work so h=1, we follow Fang and Rogerson (2009) to assume a concave utility of leisure for an employed worker so as to give interior work and leisure time. To ease the analysis, we follow Andolfatto (1996) and use the parametric forms of utility given by

$$u(c) = \ln c \quad \text{and} \quad V(1-h) = \frac{(1-h)^{1-\sigma}}{1-\sigma}, \tag{3}$$

in which σ>0 is the reciprocal of the elasticity of leisure. These forms of utility are consistent with a balanced growth path.

The household chooses a path for consumption c_t and a path for employment n_t to maximize its lifetime utility subject to the flow budget constraint (2), taking as given the law of motion for employment (1) as well as the job-finding rate, the capital rental rate, the wage rate, the income tax rate and the unemployment benefit. Let $U(k_t, e_t)$ be the lifetime value of the representative household. The household's optimization problem is written as

$$U(k_t, e_t) = \max_{\{k_{t+1}, n_t\}} \left\{ \left[u(c_t) + e_t \chi_1 V(1-h_t) + (1-e_t-n_t)\chi_2 + n_t \chi_3 \right] + \frac{1}{1+\rho} U(k_{t+1}, e_{t+1}) \right\},$$

subject to the constraints (1) and (2), where ρ>0 is the time preference rate. The first-order conditions with respect to c_t and n_t and the Benveniste-Scheinkman conditions for k_t and e_t are

$$u'(c_t) = \frac{1}{1+\rho} U_k(k_{t+1}, e_{t+1}), \tag{4a}$$

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notion that because of a job search, an unemployed worker has a lower utility of leisure than one who does not search for a job.

$$u'(c_t)b+\chi_2+\frac{1}{1+\rho}\mu_t U_e\left(k_{t+1},e_{t+1}\right)=\chi_3, \tag{4b}$$

$$U_k\left(k_t,e_t\right)=u'(c_t)\left(1-\delta+r_t\right), \tag{4c}$$

$$U_e(k_t,e_t)=\left[u'(c_t)\left(1-\tau\right)w_t+\chi_1 V(1-h_t)+\frac{1}{1+\rho}(1-\lambda)U_e(k_{t+1},e_{t+1})\right]-\left[u'(c_t)b+\chi_2+\frac{1}{1+\rho}\mu_t U_e(k_{t+1},e_{t+1})\right]. \tag{4d}$$

In these conditions, (4c) is the marginal value of capital and, with the use of (4a), we obtain the standard consumption Euler equation

$$u'(c_t)=\frac{1-\delta+r_{t+1}}{1+\rho}u'(c_{t+1}). \tag{5a}$$

Condition (4d) gives the marginal value of employment which is the difference in the marginal utility between employment and unemployment. If labor forces are exogenous, (4d) indicates that a higher unemployment benefit b increases the marginal utility of unemployment which gives a smaller marginal value of employment. Conversely, if labor forces are endogenous, (4b) is the condition which states that, in optimum, the marginal utility of unemployment is equal to the marginal utility of non-participation. In this case, if we replace the last brackets in (4d) by terms in (4b), we obtain

$$U_e(k_t,e_t)=[u'(c_t)\left(1-\tau\right)w_t+\chi_1 V(1-h_t)+\frac{1}{1+\rho}(1-\lambda)U_e(k_{t+1},e_{t+1})]-[\chi_3]. \tag{5b}$$

Thus, with endogenous labor forces, a higher unemployment benefit b does not affect the marginal value of employment. Intuitively, because the marginal utility of unemployment is equal to the fixed marginal utility of non-participation, a higher unemployment benefit cannot affect the marginal utility of unemployment and thus the marginal value of employment is not changed.

4.2.2 Firms

The production side of the economy includes a continuum of representative firms that creates job vacancies, rent capital and hire labor in order to produce final goods. The representative firm is "large" in the sense that it operates many jobs and consequently has many individual workers attached to it through those jobs. The production technology is neoclassical, represented by

$$y_t = Ak_t^{\alpha}(e_t h_t)^{1-\alpha}, \tag{6}$$

where A>0 is a productivity parameter and $\alpha \in (0, 1)$ is capital's income share.

From the firm's perspective, employment is increased by the inflow of employment and decreased by the outflow due to separation.

$$e_{t+1} - e_t = \eta_t v_t - \lambda e_t, \tag{7}$$

where η_t is the (endogenous) recruitment rate and v_t is the number of job vacancies.

As in Fang and Rogerson (2009), we assume that creating and maintaining one vacant job has a constant up-front cost of ϕ>0. Hence, firm's flow profits in t equal the output net of the costs of capital, labor, and vacancy creation; i.e.,

$$\pi_t = Ak_t^{\alpha}(e_t h_t)^{1-\alpha} - r_t k_t - w_t e_t - \phi v_t. \tag{8}$$

The firm maximizes its value taking as given the law of motion for employment as well as the recruitment rate, the capital rental rate and the wage rate. Let $\Pi(e_t)$ denote the firm's lifetime value and $\frac{1}{1+\xi_t} \equiv \frac{1}{1+\rho}\frac{u'(c_{t+1})}{u'(c_t)}$ denote its discount factor.[54]. The Bellman equation associated with the firm is

$$\Pi(e_t) = \max_{\{k_t, v_t\}}\left[\pi_t + \frac{1}{1+\xi_t}\Pi(e_{t+1})\right],$$

subject to the constraint (7). The first-order conditions are

$$\alpha A(\frac{k_t}{e_t h_t})^{\alpha-1} = r_t, \tag{9a}$$

[54] This is the discount factor of firms because households are the ultimate owners of firms. Using (5a), the discount factor is $\frac{1}{1+\xi_t} = \frac{1}{1-\delta+r_{t+1}}$.

$$\phi = \frac{\eta_t}{1+\xi_t}\Pi_e(e_{t+1}), \tag{9b}$$

$$\Pi_e(e_t) = \left[(1-\alpha)A(\frac{k_t}{e_t h_t})^{\alpha} h_t - w_t\right] + \frac{1-\lambda}{1+\xi_t}\Pi_e(e_{t+1}). \tag{9c}$$

Condition (9a) determines the demand for capital which gives the effective capital-labor ratio as $q_t \equiv \frac{k_t}{e_t h_t} = (\frac{\alpha A}{r_t})^{\frac{1}{1-\alpha}}$. Condition (9b) shows that the firm creates the number of vacancies up to the margin when the marginal cost of vacancies equals the expected discounted marginal value of recruitment in the next period. The marginal value of recruitment in this period given by (9c) is the sum of the marginal product of labor net of the wage and the discounted marginal value of recruitment in the next period.

4.2.3 Labor Matching and Bargaining

Following Diamond (1982), we assume pair-wise random matching. The matching technology takes the following constant-returns form: $M_t = m(1-e_t-n_t)^{\gamma}(v_t)^{1-\gamma}$, where $m>0$ measures the degree of matching efficacy and $\gamma \in (0, 1)$ the contribution of job seekers in random matching. Aggregate job seekers ($1-e_t-n_t$) and unfilled vacancies v_t behave like two inputs in the matching function and the output is aggregate matched pairs M_t. The matching function facilitates the endogenous determination of job finding rates and recruitment rates.

A job seeker's surplus from a successful match is evaluated by the marginal value of employment U_e in (5b), whereas a vacant job's gain from a successful match is gauged by the marginal value of recruitment Π_e in (9c). In a frictionless Walrasian world, taking the wage as given, the household maximizes U_e and the firm maximizes Π_e in order to decide their supply of and demand for labor. There is implicitly an auctioneer in the labor market which sets an equilibrium wage so as to equate labor supply to labor demand. Yet, there is no auctioneer in a frictional labor market and a job seeker would meet at most one unfilled job one time and similarly, an unfilled job would meet at most one job seeker one time. This situation creates a bilateral monopoly.

Following conventional wisdom, the wage is determined by a matched pair through a Nash bargaining game. Like Fang and Rogerson (2009), a worker does not devote all the time endowment to work in our model and thus the pair of a successful match also bargains over working hours. In a cooperative bargaining game, the following joint surplus is maximized: $[U_e(k_t, e_t)]^{\beta}[\Pi_e(e_t)]^{1-\beta}$, where $\beta \in (0, 1)$ measures a labor's bargaining power. In solving the bargaining problem, the worker-job pair treats as given matching rates (μ_t and η_t), the beginning-of-period level of employment e_t, and the market interest rate r_t. The worker also takes as given the wage and working hours of all others. The first-order conditions with respect to the wage and working hours are such that the resulting changes in the marginal value of employment and the marginal value of recruitment are summed to zero.[55]

4.2.4 The Government

The government's behavior is passive. The government levies labor income taxes in order to pay unemployment benefits that satisfy the following budget constraint.

$$T_t + \tau w_t e_t = b\left(1 - e_t - n_t\right). \tag{10}$$

In order to isolate the effects of policy changes carried out later, we include lump-sum taxes T_t. When the labor tax rate is increased, with unemployment benefits being held constant, lump-sum taxes will change accordingly in order to balance the government budget. Similarly, when unemployment compensation is increased, with the

[55] The conditions are $\frac{\beta}{U_e(k_t,e_t)}\frac{dU_e(k_t,e_t)}{dx_t} + \frac{1-\beta}{\Pi_e(e_t)}\frac{d\Pi_e(e_t)}{dx_t} = 0$, x_t=w_t, h_t.

labor tax rate being held constant, lump-sum taxes will adjust accordingly to balance the budget.

4.2.5 Equilibrium

A search equilibrium is a tuple of individual quantity variables, $\{e_t, h_t, n_t, v_t, c_t, k_t, y_t\}$, a pair of aggregate quantities, $\{M_t, T_t\}$, a pair of matching rates, $\{\mu_t, \eta_t\}$, and a pair of prices, $\{w_t, r_t\}$, such that: (i) all households and firms optimize; (ii) all employment evolutions hold, (iii) labor-market matching and wage and hours bargaining conditions are met; (iv) the government budget is balanced; and (v) the goods market clears. We focus on a steady state which is search equilibrium when all variables do not change over time. In a steady state, the consumption Euler equation (5a) gives $r = \rho + \delta$, and thus $\xi = r - \delta = \rho$. With this result, the effective capital-labor ratio in a steady state is $q \equiv \frac{k}{eh} = (\frac{\alpha A}{\rho+\delta})^{\frac{1}{1-\alpha}}$ which is constant. If we use the household's budget (2) and the firm's flow profit (8), along with the government's budget (10), the goods market clearing condition in a steady state is

$$y = c + \delta k + \phi v. \tag{11}$$

Moreover, in a steady state the labor market satisfies the following matching relationships (Beveridge curve) given by $\mu(1-e-n) = \eta v = m(1-e-n)^{\gamma}(v)^{1-\gamma} = \lambda e.$ These relationships enable us to solve matching rates and equilibrium vacancies as functions of e and n.

$$\mu = \frac{\lambda e}{(1-n)-e} \equiv \mu(\underset{+}{e}, \underset{-}{1-n}), \tag{12a}$$

$$v = \left[\frac{\lambda e}{m(1-n-e)^{\gamma}}\right]^{\frac{1}{1-\gamma}} \equiv v(\underset{+}{e}, \underset{-}{1-n}), \tag{12b}$$

$$\eta = \frac{\lambda e}{v(e,1-n)} = \left[m \left(\frac{1-n-e}{\lambda e} \right)^{\gamma} \right]^{\frac{1}{1-\gamma}} \equiv \eta(\underset{-}{e}, 1-\underset{+}{n}). \tag{12c}$$

Intuitively, more employment *e* decreases and higher labor-force participation (1-*n*) increases job seekers. Thus, in these relationships, the job finding rate and the equilibrium vacancy are increasing in the number of employment and decreasing in the number of participants, while the firm's recruitment rate is decreasing in the number of employment and increasing in the number of participants. These relationships give $\frac{\mu}{\eta} = \frac{v}{1-n-e}$ which indicates that the ratio of job finding rates to recruitment rates is equal to the ratio of job vacancies to job seekers, a measure of the labor market tightness.

With the parametric forms of utility in (3), the steady-state conditions are as follows. First, (11) and (12b) give consumption as a function of employment, labor-force participation and work hours.[56]

$$c = (Aq^{\alpha} - \delta q)eh - \phi v(e,1-n) \equiv c(e,1-n,h), \;\; c_e > 0, c_{1-n} > 0, c_h > 0, \tag{13}$$

where higher employment and working hours increase output available for consumption. Moreover, higher labor-force participation reduces vacancies and hence more output is available for consumption.

The firm's gain from a successful match is (9c) and in a steady state, with ξ=ρ, is

$$\Pi_e = \frac{1+\rho}{\rho+\lambda}\left(MPL \cdot h - w\right), \tag{14}$$

where $MPL \equiv (1-\alpha)Aq^{\alpha}$ is the marginal product of labor which is fixed in a steady state. The firm's gain is the capitalized value of working hours-augmented marginal product of labor net of the wage.

4.3 Two Models

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56 c_e>0 if ϕ is not too large.

If the labor-force participation is exogenous, n_t is exogenously given by $\bar{n}$. Then, it is the model studied by Fang and Rogerson (2009). We will first study the steady state of the Fang and Rogerson model and then our model with endogenous labor forces.

4.3.1 Model with Exogenous Labor-force Participation

First, as the labor force is exogenously given at $1-\bar{n}$, the consumption function c in (13) can be expressed as $c(e,h;1-\bar{n})$. In a steady state, the household's surplus from a match in (4d) is

$$U_e = \frac{1+\rho}{\rho+\lambda+\mu}\left\{[u'(c)(1-\tau)w+\chi_1 V(1-h)]-[u'(c)b+\chi_2]\right\}. \tag{15}$$

Thus, the household's surplus from a match is the capitalized value of the difference in the marginal value between employment and unemployment.

By using the firm's gain from a match in (14) and the household's gain from a match in (15), we maximize the joint surplus of a match to determine the bargained wage as follows.[57]

$$w = \beta[MPL\cdot h]+(1-\beta)\left[\frac{1}{1-\tau}[(\chi_2-\chi_1\tfrac{(1-h)^{1-\sigma}}{1-\sigma})c(e,h;1-\bar{n})+b]\right] \equiv w(e,h;\tau,b,1-\bar{n}), \tag{16}$$

where $w_e > 0, w_h > 0, w_\tau > 0$ and $w_b > 0$. The wage is the weighted average of the marginal product of labor and the tax-adjusted opportunity cost of employment. In addition to unemployment compensation, the opportunity cost is $MRS^{se}(e,h;1-\bar{n}) \equiv (\chi_2-\chi_1\tfrac{(1-h)^{1-\sigma}}{1-\sigma})c(e,h;1-\bar{n}) > 0$, the difference in the marginal rate of substitution (*MRS*) between leisure and consumption from searching a job to being employed.[58] In view of the c function in (13), the bargained wage is increasing in

[57] The derivations concerning the signs of the expressions discussed here and below are relegated to the Appendix.

[58] To ensure a loss of leisure utilities from unemployment to employment, we assume

employment e, working hours per worker h and the labor force $1-\overline{n}$. Moreover, a higher labor tax rates τ and a higher unemployment benefit b increase the opportunity cost and thus raises the wage.

Moreover, we maximize the joint surplus to attain the condition for hours worked per worker. Dividing this condition by the first-order condition for the bargained wage gives

$$\chi_1(1-h)^{-\sigma}c(e,h;1-\overline{n})=(1-\tau)MPL, \tag{17a}$$

where the marginal cost of working hours is an employee's *MRS* between leisure and consumption and the marginal benefit of working hours is the after-tax *MPL*. The condition relates hours worked per worker h to employment e and exogenous factors in the way as follows.

$$h=h(e;1-\overline{n},\tau), \tag{17b}$$

where $h_e<0$, $h_{1-n}<0$ and $h_\tau<0$. These signs emerge because working hours increase the marginal cost of working hours, while employment increases the marginal cost of working hours and the labor force and the labor tax rate both decrease the marginal benefit of working hours.

Finally, we use $r=\rho+\delta$ and the Π_e in (14) to rewrite the vacancy creation condition in (9b) as $\frac{\eta(e,1-\overline{n})}{\rho+\lambda}(MPL\cdot h-w)=\phi.$ The condition equates the marginal cost ϕ to the firm's capitalized value of the marginal product of recruitment net of the wage. Notice that a higher recruitment rate η increases the capitalized value. By using the w function in (16), this condition is rewritten as

$$\frac{\eta(e;1-\overline{n})}{\rho+\lambda}(1-\beta)\left[MPL\cdot h-\frac{1}{1-\tau}\left(MRS^{se}(e,h;1-\overline{n})+b\right)\right]=\phi, \tag{18}$$

$\chi_2>\chi_1(1-h)^{1-\sigma}\frac{1}{1-\sigma}$ near the steady state, so the leisure utility of unemployed workers is larger than that of employed workers. See also Cheron and Langot (2004) for the same assumption.

where the terms in the brackets are the flow gain from a match of which the firm's share is 1-β. The left-hand side is the firm's capitalized value of the gain from a match and thus, the firm's marginal benefit of employment. With a given labor force $1-\overline{n}$, the condition relates employment e to hours worked per worker h.

Equations (17b) and (18) are the most simplified conditions in the model with exogenous labor forces. They determine a unique pair of employment e and hours worked per worker h in the steady state. With employment, if we use (12a) and (12b), the ratio of vacancy to unemployment is determined. As we will focus on a simultaneous determination of employment and labor forces in the next section, here we substitute hours worked per worker in (17b) into (18) in order to obtain an expression that relates employment as a function of labor forces as follows.

$$\frac{\eta(e;1-\overline{n})}{\rho+\lambda}(1-\beta)\left[MPL\cdot h(\underset{-}{e};\underset{-}{\tau},1-\underset{-}{\overline{n}})-\frac{1}{1-\tau}\left(MRS^{se}(e,h(\underset{-}{e};\underset{-}{\tau},1-\underset{-}{\overline{n}});1-\overline{n})+b\right)\right]-\phi=0. \quad (19)$$

Thus, given n, (19) determines employment in the steady state. As indicated in Figure 2, with $\overline{n}=n_0$ (19) determines the steady state Q_0 with unique employment given by the level e_0. A different value of n would give a different level of employment. In particular, an increase in n (and thus a decrease in labor forces) raises the firm's capitalized value of the marginal product of recruitment net of the wage. Then, employment will decrease so as to decrease the firm's capitalized value of the marginal product of recruitment net of the wage. Thus, n and e are inversely related in (19). As a result, we may perceive (19) as a negatively-sloping locus in the (e, n) plane which, for convenience, is referred to as Locus E (employment).

4.3.2 Our Model with Endogenous Labor-force Participation

Next, we study our model. Now, the household's surplus from a successful match is not (4d) but is (5b). In a steady state, (5b) gives

$$U_e = \frac{1+\rho}{\rho+\lambda}\left\{[u'(c)(1-\tau)w + \chi_1 V(1-h)] - [\chi_3]\right\}, \tag{20}$$

which is the capitalized value of the difference in the marginal value between employment and non-labor force.

Note that the job finding rate μ reduces the household's surplus from a match in (15) but not here. The reason is that without choices of labor-force participation in (15), the outside option of employment is unemployment. The value of unemployment includes the prospect of employment which is increasing in job finding rates. However, with choices of labor-force participation here, the outside option of employment is non-employment and unemployment benefits are not a value of non-employment.

Next, by using the firm's gain from a match in (14) and the household's surplus from a match in (20), we maximize the joint surplus of a match and obtain the following bargained wage.

$$w = \beta[MPL \cdot h] + (1-\beta)\left[\frac{1}{1-\tau}(\chi_3 - \chi_1 \tfrac{(1-h)^{1-\sigma}}{1-\sigma})c(e,1-n,h)\right] \equiv w(e,1-n,h;\tau), \tag{21}$$

where $w_e > 0, w_n < 0, w_h > 0$ and $w_\tau > 0$. The bargained wage is otherwise the same as (16) except for the opportunity cost of employment, $MRS^{ne}(e,1-n,h) \equiv (\chi_3 - \chi_1 \frac{(1-h)^{1-\sigma}}{1-\sigma})c(e,1-n,h) > 0$. Because of choices of labor-force participation, the opportunity cost here is the difference in the *MRS* from non-employment to employment and thus, unemployment benefits do not directly affect the bargaining game here. In view of the c function in (13), like (16), the bargained wage is increasing in employment e, labor forces 1-n and hours worked per worker h. Clearly, a higher labor tax rates τ leads to an increase in the wage.

For the condition determining hours worked per worker, even though the household's surplus from a match is (20) instead of (15), except for n being endogenous, the condition is still the same as (17b) and thus, $h = h(e, 1-n; \tau)$.

As the bargained wage is (21), the vacancy creation condition is not (19) but is

$$\frac{\eta(e,1-n)}{\rho+\lambda}(1-\beta)\left[MPL \cdot h(\underset{-}{e},1\underset{-}{-}n;\underset{-}{\tau}) - \frac{1}{1-\tau}MRS^{ne}(e,1-n,h(\underset{-}{e},1\underset{-}{-}n;\underset{-}{\tau}))\right] - \phi = 0. \tag{22}$$

Like (19), the condition equates the marginal benefit of employment to the marginal cost. Yet, the marginal benefit is the capitalized value of the gain of a match from non-labor force to employment, rather than from unemployment to employment. The condition determines employment *e* as a function of labor forces (1-*n*), also referred to as Locus E in the (*e*, *n*) plane. Like (19), it is downward sloping in the (*e*, *n*) plane as seen in Figure 2. The reasons are that more employment (a higher *e*) decreases recruitment rates and increases employees' outside options MRS^{ne}, so the net marginal benefit of employment is increased. Moreover, a smaller labor force (a higher *n*) decreases both recruitment rates and outside options so the effect on the net marginal benefit is ambiguous, but it also increases work hours and decreases the net marginal benefit of employment which dominates the other ambiguous effects (see the Appendix).

Besides, there is a labor-force participation condition. By using $r = \rho + \delta$, the μ function in (12a), the *h* function in (17b), the U_e function in (20) and the *w* function in (21), the participation condition (4) is rewritten as

$$\begin{aligned}&\left\{\frac{\mu(e,1-n)}{\rho+\lambda}\beta\left[MPL \cdot h(e,1-n;\tau) - \frac{1}{1-\tau}MRS^{ne}(e,1-n,h(e,1-n;\tau))\right] + b\right\} \\ &\qquad - \left[\frac{1}{1-\tau}MRS^{ns}(e,1-n,h(e,1-n;\tau))\right] = 0.\end{aligned} \tag{23}$$

In the condition above, the terms in the large brackets are the marginal benefit of labor-force participation which includes the capitalized value of the gain of a match from non-labor force to employment and unemployment benefits. The marginal cost of participation is the tax-adjusted loss in leisure utilities from non-labor force to search, $MRS^{ns}(e,1-n,h) \equiv (\chi_3 - \chi_2)c(e,1-n,h) > 0.$ The condition determines the labor force 1-n as a function of employment e. In the (e, n) plane, the condition is referred to as Locus P.

To study the slope of Locus P, with given work hours, a smaller labor force (a higher n) directly increases the net marginal gain of participation as the resulting higher job finding rate μ and lower employees' outside option MRS^{ne} increase the gain of a match from non-labor force to employment and the resultant smaller loss in leisure utilities from non-participation to participation MRS^{ns} decreases the marginal cost of participation. By increasing work hours, a smaller labor force indirectly exerts a negative effect, but the effect is dominated by the positive direct effect, so the net marginal gain of participation is increased. Moreover, with given work hours, more employment (a higher e) has an ambiguous direct effect on the net marginal gain of participation as it increases both the marginal gain from a match (via a higher job finding rate) and the marginal cost of participation. By reducing work hours per worker, more employment indirectly brings in a positive effect that dominates the ambiguous direct effect, so the net marginal gain of participation is increased. Thus, Locus P is downward sloping in the (e, n) plane.

Although Loci E and P are both downward sloping, the two loci intersect only once. In the Appendix, we have shown that as n goes to 0, h goes to the lowest value h_L and Locus E and Locus P approach to the e axis at e_E and e_P in Figure 2, respectively. The value of e cannot go to 0, as then h does not exist. There is thus a minimum value e_L to which e can approach. As e decreases to e_L, h goes to the highest value h_H and Locus E and Locus P approach to n_E and n_P, respectively. We have shown that a larger value of A decreases h_L and increases h_H. We also show that under a minor condition, $e_E<e_P$ and

$n_E > n_P$ and thus, there exists an intersection of Loci E and P. We then show that if A is larger, the value h_L is smaller and the value h_H is larger, so that it is easier to meet the condition that assures $e_E < e_P$ and $n_E > n_P$. Furthermore, it is required that Locus P be flatter than Locus E in each intersection.[59] Therefore, there exists a unique steady state.

In Figure 2, the unique steady state is Q_0, with the employment at e_0 and the labor force at $1-n_0$. With unemployment at $(1-e_0-n_0)$, the unemployment rate is $u_0=(1-e_0-n_0)/(1-n_0)=1-e_0/(1-n_0)$. Substituting e_0 and n_0 into (17b) gives working hours per worker h_0. Thus, labor supply in the economy, or equivalently hours worked per person, is $L^s_0=e_0h_0$.

4.4 Policy Analysis

This section analyzes two policies of pervasive interest to compare the long-run effect on labor supply in our model and the model with exogenous labor forces. These two policies are a tax on the employed which is proportional to labor income and is used to make a lump-sum transfer; and a benefit to the unemployed which is proportional to labor income as financed by a lump-sum tax. We start with the analysis of increases in labor income taxes, followed by increases in unemployment compensation. Here, we offer graphical illustrations with the comparative-static analysis being delegated in the Appendix.

4.4.1 Effects of Labor Taxes

4.4.1.1 Model with exogenous labor-force participation

[59] Suppose that the unit cost of vacancies increases. A higher unit cost of vacancies reduces the net marginal benefit of vacancy and shifts Locus E in Figure 2 leftward without shifting Locus P. It is reasonable to expect that job vacancies decrease and thus employment decreases. However, should Locus P of Figure 2 be steeper than Locus E, employment would have had increased.

Now, we analyze the effects of increases in the labor tax rate (higher τ). When the labor-force participation is exogenously given, Locus E is the only relevant equilibrium condition. In Figure 3, the intersection of the Locus E with the horizontal line $n=n_0$ determines the initial steady state Q_0. When the labor tax is increased, with given working hours, a higher labor tax decreases the net marginal benefit of employment, thereby shifting Locus E leftward. Yet, by way of reducing working hours per worker, a higher labor tax generates an indirect effect that increases the net marginal benefit of employment. Because the indirect effect is dominated by the direct effect, Locus E in Figure 3 is shifted leftward to Locus E_1. Thus, that employment is reduced from e_0 to e_1. Then, unemployment is increased from $(1-n_0-e_0)$ to $(1-n_0-e_1)$ and, with a given labor force, the unemployment rate is increased from $\mathbf{u}_0=(1-e_0-n_0)/(1-n_0)$ to $\mathbf{u}_1=(1-n_0-e_1)/(1-n_0)$.

4.4.1.2 Our model with endogenous labor-force participation

With endogenous labor-force participation, the initial steady state Q_0 is the intersection of Locus E with Locus P in Figure 3. Like the model with exogenous labor forces above, a higher labor tax decreases the net marginal benefit of employment here. In order to increase the net marginal benefit of employment, given a labor force level, employment will decrease. Thus, the Locus E shifts leftward toward Locus E_2. When labor-force participation is endogenous, the Locus E is also affected by labor forces. With a sufficiently large value of b, the Locus E here is shifted leftward toward Locus E_2 that is less than Locus E_1.

Moreover, as the household chooses labor-force participation, a higher labor tax affects Locus P. With given working hours, a higher labor tax directly shifts Locus P upward, because the marginal benefit of participation is decreased which shrinks labor forces. Yet, by reducing hours worked per worker, a higher labor tax generates an indirect effect that increases the marginal benefit of participation. As the direct effect dominates the indirect effect, Locus P is shifted upward to Locus P_2.

The new steady state is at Q_2 in Figure 3. In this steady state, the labor force is decreased from $(1-n_0)$ to $(1-n_2)$, so employment is decreased from e_0 to e_2, a level less than that in the model with exogenous labor forces. As unemployment is $(1-n_2-e_2)$, the unemployment rate is $\mathbf{u}_2=(1-e_2-n_2)/(1-n_2)$ which may decrease or increase that depends on whether labor forces $(1-n_2)$ are decreased more or less than employment (e_2).

With labor forces and employment, hours worked per worker are in turn determined. When the labor force is exogenous, the effect of a higher labor tax rate on hours worked per worker is ambiguous, because, with unemployment compensation, the indirect positive effect from much lower employment may offset the direct adverse effect.[60] When the labor force is endogenous, the effects on hours worked per worker are still ambiguous because the indirect positive effects from smaller labor forces and lower employment both may offset the direct adverse effects. Nevertheless, as the adverse effect on employment is strong, no matter whether labor forces are endogenous or not, the labor supply ($L^s=eh$) is likely to decrease. Yet, in the model with exogenous labor forces, as employment is reduced by more, labor supply is reduced by more.

To summarize the effects of a higher labor tax rate, we obtain

Proposition 1. *When the labor tax is increased, because of a decrease in labor forces, the employment in the model with endogenous labor forces is reduced less than the model with exogenous labor forces and, with ambiguous effects on hours per worker in both models, labor supply in the model with endogenous labor forces is reduced less*

[60] When unemployment compensation is zero, the effect of a higher labor tax rate on hours worked per worker is negative. Thus is the situation studied in Fang and Rogerson (2009),

than that in the model with exogenous labor forces.

4.4.2 Effects of Unemployment Compensation

4.4.2.1 Model with exogenous labor-force participation

Next, we envisage the effects of increases in unemployment compensation (a higher b). Suppose that the initial steady state is Q_0 in Figure 4. With exogenous labor forces, the outside option of employment is unemployment. With given working hours, a higher unemployment benefit directly raises the opportunity cost from unemployment to employment which decreases the net marginal benefit of employment. Thus, Locus E is shifted leftward which reduces employment. Besides, by increasing working hours per worker, the net marginal benefit of employment is decreased further, thereby generating an indirect effect to shift Locus E leftward even more (cf. Locus E_1). With a given labor force $(1-n_0)$, the new steady state is Q_1 and employment is decreased from e_0 to e_1. Unemployment is increased from $(1-n_0-e_0)$ to $(1-n_0-e_1)$ which causes the unemployment rate to increase from $\mathbf{u}_0=(1-e_0-n_0)/(1-n_0)$ to $\mathbf{u}_1=(1-e_1-n_0)/(1-n_0)$.

4.4.2.2 Our model with endogenous labor-force participation

When the labor-force participation is endogenous, a higher unemployment benefit does not affect the gain of a match and thus does not shift Locus E. Yet, a higher unemployment benefit increases the gain from non-participation to participation. With given working hours, a higher unemployment benefit raises the marginal benefit of participation which increases labor forces and thus shifts Locus P downward. In addition, by decreasing working hours per worker, a larger labor force generates two further effects. First, the marginal benefit of participation is increased which in turn induces more labor forces and thus shifts Locus P downward more (cf. Locus P_2).

Second, a lower working hour per worker makes Locus E flatter which is rotated counter-clockwise (cf. Locus E_2).

As a result, the labor force is increased from $(1\text{-}n_0)$ to $(1\text{-}n_2)$ and the employment is increased from e_0 to e_1. Moreover, it is clear that unemployment $(1\text{-}e_2\text{-}n_2)$ and the unemployment rate $\mathbf{u}_1=(1\text{-}e_2\text{-}n_2)/(1\text{-}n_2)$ are both ambiguous; both may decrease or increase depending on whether labor forces $(1\text{-}n_2)$ are increased more or less than employment (e_2).

Unemployment compensation has no direct effect on hours worked per worker. When the labor force is exogenous, higher unemployment compensation decreases employment which indirectly increase hours worked per worker. Thus, the effect on labor supply is ambiguous. When the labor force is endogenous, higher unemployment compensation increases both labor forces and employment which indirectly reduce hours worked per worker. As a result, the effect on labor supply is also unambiguous, depending on whether the effect on employment or the effect on hours worked per worker dominates.

To summarize the effects of higher unemployment benefits, we obtain

Proposition 2. *A higher unemployment benefit decreases employment in the model with exogenous labor forces but increases both labor forces and employment in the model with endogenous labor forces. As the effects on hours worked per worker are opposite to those of the effects on employment and labor forces, the effects on labor supply are ambiguous under both models with and without endogenous labor forces.*

4.4.3 Quantitative Analysis

We have analyzed the effects of changes in labor taxes and unemployment compensation on labor supply in our model and the model without endogenous labor forces. Yet, as the theoretical effects on hours worked per worker are ambiguous or opposite to those of the effects on employment and labor forces, the net effects on labor supply are ambiguous. In this subsection, we carry out quantitative exercises so as to pin down the net effects. In the quantitative exercise, we assume that the economic structure in the EU and the US is the same except for labor tax rates and unemployment benefits.

4.4.3.1 Calibration

We start by calibrating parameters and variables in a quarterly frequency. First, the productivity coefficient is normalized to unity (A=1) and the capital share is set at α=0.36. With the annual time preference rate of 4% in the US data, we set the quarterly time preference rate to ρ=0.01. Then, we calibrate the capital depreciation rate to target a quarterly capital-output ratio of k/y=12. We obtain δ=0.02 which is in the range of a 3%-8% annual depreciation rate of capital. These values give the quarterly interest rate equal to r=0.03 and the effective capital-labor ratio equal to q=48.5535.

Next, we use the employment rate and the labor force participation rate in the US to compute the fraction of employment in the working-age population and the average unemployment rate. We obtain e=72.03% and u=5.1%, respectively, the former value close to the value 71.9% calculated by Alesina et al. (2006) and the latter the same as the value obtained by Krusell et al. (2011). These values give n=0.2410 and thus a labor force participation rate of 1-n=0.7590, a value close to the data in the OECD. Then, we calibrate h=0.3471 in order to target a 25% productive time allocated to the market, L=eh (Prescott, 2006). These parameter values yield an output level of y=1.0115.

According to Shimer (2005), the monthly job finding rate is 0.45. We go along with this rate and translate it into a quarterly value of μ=1-(1-0.45)^3=0.8336. Then, we employ the matching relationships to compute the quarterly separation rate λ=μ(1-n-e)/e=0.0448. Moreover, we calibrate the steady-state vacancies v=0.0387 in order to target a unit degree of the labor market tightness in a steady state as proposed by Shimer (2005). This value in turn gives a quarterly recruitment rate at η=μ(1-n-e)/v=0.8336.

By using (13), we calibrate a unit vacancy creation cost of ϕ=1.5679 in order to target a 70% consumption-output ratio (c/y=0.7). Then, we use (14), together with (9b), to compute and obtain the wage per worker of w=0.7957. Shimer (2005) estimated a 40% unemployment replacement rate. We go along with Shimer (2005) and calibrate b to target the 40% unemployment replacement rate. We get b=0.3183 which is set as the baseline unemployment compensation. Utilizing the data compiled by McDaniel (2007), Rogerson (2008) used the labor tax in Belgium, France, Germany, Italy, and the Netherlands to represent the tax in the EU. We follow this invention and compute the population-weighted average effective tax rate on labor income for these five European countries in 1970-73. The average effective tax rate is τ=0.3982 which is set as the baseline labor tax rate.

In our utility function, the parameter σ governs the intertemporal elasticity of leisure for the employed and is negatively related to the Frisch labor supply elasticity (*LSE*): (1-h)/(σh). The *LSE* for men estimated by MaCurdy (1981) ranged from 0.1 to 0.5 and that for women was higher. Conversely, Greenwood et al. (1988) suggested that *LSE*=1.7 was reasonable. For the present purpose, we go along with Andolfatto (1996) and set *LSE*=1 which is within the estimates above. This gives σ=1.8812.[61] We then calibrate χ_1=0.987 in order to be consistent with the hour bargaining condition.

[61] Our results remain unchanged for a large rage of the value of *LSE*.

Finally, following Blanchard and Diamond (1989), we set the search worker's contribution in matching at γ=0.4. By assuming that Hosios' rule holds (Hosios, 1990), we pin down labor's bargaining share at $\beta=\gamma$. Then, from the matching relationships we can compute m=0.8336. With the values above, we compute other parameters in the utility of leisure. When labor forces are endogenous, we compute and obtain χ_3=-1.0129 and χ_2=-2.3507 so as to be consistent with the participation condition in (23) and the bargained wage condition in (21). By contrast, when labor forces are exogenous, we compute and obtain χ_2=-1.4624 so as to be consistent with the bargained wage condition in (16).

The benchmark parameter values, observables and calibrated values are listed in Table 2. Under the benchmark parameter values, we obtain a unique steady state.

4.4.3.2 Quantifying the effects of increases in tax rates and unemployment compensation

To quantify the effects of increases in tax rates and unemployment compensation, we start by measuring changes in labor taxes and unemployment compensation in the EU relative to the US from the early 1970s to the early 2000s.

For labor taxes, based on the data in McDaniel (2007), we follow Rogerson (2008) and calculate the population-weighted average effective tax rate on labor income in Belgium, France, Germany, Italy, and the Netherlands in 2000-03. We obtain the tax rate 0.5168. Together with the data that the effective labor tax rate increased a little bit in the US in the past 30 years, this indicates about a 30% increase in the labor tax rate in the EU relative to the US from the early 1970s to the early 2000s.[62]

For unemployment compensation, based upon the dataset compiled by van Vliet

[62] Based on the data in McDaniel (2007), the effective labor tax rate (on household income and payroll) in the US increased from 0.1775 in 1970-73 to 0.22475 in 2000-03.

and Caminada (2012), we calculate the net unemployment replacement rate for one earner couple in Belgium, France, Germany, Italy, and the Netherlands in the EU and the US in 1971 and 2001. We find that the ratio of the net unemployment replacement rate between the EU and the US was increased from 0.85 in 1971 to 1.24 in 2001 which indicates about a 40% increase from 1971 to 2001.[63]

Given these data, we quantify the effects of an increase in the value of τ by 30% and an increase in the value of b by 40% from their baselines. In each exercise, the government balances the budget in each period by adjusting lump-sum taxes or transfers. First, the effects of an increase in the labor tax by 30% are reported in the upper panel of Table 3. In the model with an exogenous labor force, the labor force is fixed at the baseline level of $1\text{-}n$=0.7590. In this model, employment is reduced largely by 18.9 percentage points; thus, unemployment is increased largely. Hours worked per worker change little, though it is increased due to the dominance effect of lower employment. Because of a large decrease in employment, labor supply is decreased by 6.38 percentage points which, if we normalize the baseline value to 100%, amounts to a reduction by 25.51% as seen in the parenthesis.

By contrast, in our model with endogenous labor forces, as an increase in the labor tax rate by 30% also reduces labor forces by 13.71 percentage points, employment is reduced less that is by 13.22 percentage points. Because the labor force and employment are reduced by about the same size, unemployment changes little. As the effect from small labor forces offsets the effects from lower employment, hours worked per worker also changes little. Because of a decrease in labor forces, labor supply in our model is decreased less than the model with exogenous labor forces.

[63] Based on the data in van Vliet and Caminada (2012), the net unemployment replacement rate for one earner couple was 0.5001 for these five countries in the EU and 0.59 in the US in 1971. In 2001, the corresponding rate was 0.6813 for these five countries in the EU and 0.55 in the US

Next, the effects of an increase in unemployment compensation by 40% are demonstrated in the lower panel of Table 3. When labor forces are exogenously fixed at 1-n=0.7590, employment is decreased largely by 29.67 percentage points; thus, unemployment is increased largely. Because of a large decrease in employment, as a substitute, hours worked per worker are increased substantially by 8 percentage points. As the employment effect dominates, labor supply is decreased by 6.86 percentage points which means a reduction by 27.45% from the baseline.

By contrast, in our model with endogenous labor forces, as an increase in unemployment compensation by 40% enhances labor forces, employment is increased slightly. Because the effect from larger labor forces offsets the effects from higher employment, unemployment changes little and so do hours worked per worker and labor supply.

Moreover, we quantify the total effect by simultaneously increasing the tax rate by 30% and unemployment compensation by 40%. See the results in Table 4. In the model with exogenous labor forces, employment is reduced very largely by 54.18 percentage points and as a result, the unemployment rate is increased by 71.384 percentage points. Hours worked per worker are also increased largely. As the employment effect dominates, labor supply is reduced by 15.38 percentage points which amounts to a reduction by 61.51% from the baseline.

By contrast, in our model with endogenous labor forces, because the favorable effect of increases in unemployment compensation lessens the adverse effect of a higher tax on labor forces, the employment is decreased by less and the labor force and employment are both decreased by about 10 percentage points. The unemployment rate is slightly increased and hours worked per worker are slightly decreased. As a result, labor supply is reduced by 4.46 percentage points which amounts to a decrease of

17.84% from the baseline.

We find that these results above are robust for different values of *LSE*. Moreover, these results hold even when the Hosios' rule does not hold. Specifically, we have fixed the labor's contribution in search at γ=0.4 and varied the labor's bargaining share β to take alternative values {0.235, 0.54, 0.72} used by Hall (2005), Hall and Milgrom (2008) and Shimer (2005), respectively. To save the space, we do not report these robustness analyses.

Overall, we find that the model with exogenous labor forces explains too much of the decrease in employment and the decrease in labor supply in the data in the EU relative to the US. In particular, in response to these two important sources of labor market regulation, the model with exogenous labor forces predicts an increase in hours worked per worker as opposed to a decrease in the data. By contrast, our model takes account of endogenous changes in labor forces, so it explains a reasonable 17% decrease in labor supply in the UE relative to the US, which is close to a 26% decrease in the data. Our model explains 10 percentage-point decreases in both employment and labor force which is also close to the data, along with a decrease rather than an increase in hours worked per worker. Because of other differences in labor market characteristics and regulations between the EU and the US, our model cannot explain all the difference in labor supply in the EU relative to the US. Yet, our model explains the difference in labor supply better than the model with exogenous labor forces.

4.5 Concluding Remarks

Labor supply in the EU declined over one fourth relative to that in the US from the early 1970s to the early 2000s. The existing papers have used increases in labor taxes and unemployment benefits to explain declining labor supply in the EU relative to the

US. These existing models include the intensive margin, the employment margin, or both margins of labor supply, but they did not take into account the participation margin. Our article extends the existing model to the one with the participation margin. We compare the long-run effects on labor supply of increases in labor taxes and unemployment benefits in the models with and without the participation margin.

We find that with increases in labor taxes, thanks to discouraging labor forces, employment in our model is reduced less than that in the model without endogenous labor forces and, with ambiguous effects on hours per worker, labor supply is decreased by less in our model. In the case of increases in unemployment benefits, due to inducing labor forces, employment increases in our model but decreases in the model with exogenous labor forces and, with the effect on hours per worker being opposite to that on employment, the effect on labor supply is ambiguous in both models, depending on whether the effect on employment or that on hours worked per worker dominates.

To quantify the net effect on labor supply, we calibrate our model to the US economy. By feeding in the data of increases in labor taxes and unemployment benefits, we find that the model without endogenous labor forces explains too much of the decreases in employment and labor supply in the EU relative to the US from the early 1970s to the early 2000s. In particular, the model without endogenous labor forces predicts an increase in hours per worker which is at odd with the data. By contrast, due to endogenous labor forces, our model explains a reasonable decrease in labor supply, along with a reasonable decrease in employment and a moderate decrease rather than an increase in hours per worker. Overall, because of considering endogenous labor forces, our model explains the difference in labor supply better than the model with exogenous labor forces.

Table Appendix

Table 1 Labor Supply in EU Relative to US, 1970-73 and 2000-03

	Labor supply			Hours per worker			Employment rate (%)			Participation rate (%)		
	70-73	00-03	diff.(%)	70-73	00-03	diff.(%)	70-73	00-03	diff.	70-73	00-03	diff.
EU	1227 (109.4)	1034 (83.39)	-15.71	1940 (107.7)	1613 (93.67)	-16.84	97.4 (102.9)	92.2 (97.17)	-5.32	65 (98.78)	69.5 (91.63)	7.06
US	1122 (100)	1240 (100)	10.57	1802 (100)	1722 (100)	-4.43	94.7 (100)	94.9 (100)	0.23	65.8 (100)	75.9 (100)	15.41

Sources: OECD (2010a; 2010b).

Note: The hours per worker are annual hours of market work per worker. The employment rate is the number of the employed divided by the number in the labor force. The participation rate is the number of the labor force divided by the number of the population aged 15-64. Finally, the labor supply is annual hours of market work per capita and is calculated by the annual work hours per worker times the number of the employed divided by the working-age population. In a cell with two values, the tops are the original values and the bottoms in parenthesis are relative to the US with the value in the US normalized to 100% in both 1970-73 and 2000-03. The difference is a percentage difference of a value in 2000-2003 to a value in 1970-1973. The EU includes Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, Spain, Sweden and the UK wherein the data are available in both periods.

Table 2 Benchmark parameter values and calibration **Quarters**

Benchmark Parameters and Observables	Variables	Quarterly	Source
coefficient of production technology	A	1.0000	normalization
capital's share	α	0.3600	Kydland and Prescott (1982, 1991)
time preference rate	ρ	0.0100	Kydland and Prescott(1991)
fraction of employment	e	0.7203	OECD (2010b)
unemployment rate	u	0.0510	OECD (2010b)
fraction of non-participants	n	0.2410	OECD (2010b)
job finding rate	μ	0.8336	Shimer (2005)
labor tax rate	τ	0.3982	McDaniel (2007)
labor's share in matching function	γ	0.4000	Blanchard and Diamond (1989)
Calibration			Target
depreciation rate of capital	δ	0.0200	Capital-output ratio = 12
hours worked per worker	h	0.3471	Hours of work per person = 25%
job separation rate	λ	0.0448	Matching relationship
vacancy creation	v	0.0387	Vacancy-search worker ratio = 1
unit cost of vacancy creation	ϕ	1.5679	Consumption-output ratio = 0.7
unemployment compensation	b	0.3183	Unemploy. replacement rate = 40%
the intertemporal elasticity of leisure	σ	1.8812	Frisch labor supply elasticity = 1
coefficient of worker's leisure	χ_1	0.9870	Bargaining hour condition
leisure utility of unemployed (endo. n)	χ_2	-2.3507	Bargained wage condition
leisure utility of non-participants	χ_3	-1.0129	Participation condition
leisure utility of unemployed (exog. n)	χ_2	-1.4624	Bargained wage condition
labor's bargaining power	β	0.4000	Hosios' rule
coefficient of matching function	m	0.8336	Matching technology

Table 3 Effects of Increases in Labor Tax Rate and Unemployment Compensation (%)

	e		$1\text{-}n$		$u=(1\text{-}n\text{-}e)/(1\text{-}n)$		h		$L^s=eh$	
Benchmark	**72.03**	**0.00♦**	**75.90**	**0.00♦**	**5.10**	**0.00♦**	**34.71**	**0.00♦**	**25.00**	**0.00♦**
$\tau\uparrow$ 30%										
exog LF	53.09	–18.94	**75.90**	0.00	30.06	+24.96	35.08	+0.37	18.62	– 6.38
		(-0.26%)		(0.00%)		(4.89%)		(0.01%)		(–25.51*)
endo LF	58.81	–13.22	62.20	–13.71	5.45	+ 0.35	34.34	– 0.37	20.19	– 4.81
		(-0.18%)		(-0.18%)		(0.06%)		(-0.01%)		(-19.23*)
$b\uparrow$ 40%										
exog LF	42.36	–29.67	**75.90**	0.00	44.19	+39.09	42.81	+8.11	18.14	–6.86
		(0.41%)		(0.00%)		(7.66%)		(0.23%)		(–27.45)
endo LF	74.26	+2.23	78.74	+2.84	5.69	+0.59	34.05	-0.66	25.32	+0.32
		(0.03%)		(0.04%)		(0.11%)		(-0.01%)		(+1.28)

Note: All changes under columns with ♦ are changes in percentage points from the baseline except for those in the parenthesis are percent changes from their baseline values that are normalized to 100%.

Table 4 Effects of Increases in Labor Tax Rate and Unemployment Compensation (%)

	e		$1\text{-}n$		$u=(1\text{-}n\text{-}e)/(1\text{-}n)$		h		$L^s=eh$	
Benchmark	**72.03**	**0.00♦**	**75.90**	**0.00♦**	**5.10**	**0.00♦**	**34.71**	**0.00♦**	**25.00**	**0.00♦**
$\tau\uparrow$ and $b\uparrow$										
exog LF	17.85	–54.18	75.90	0	76.48	+71.38	53.90	+19.19	9.62	–15.38
		(-0.75%)		(0.00%)		(13.99%)		(0.55%)		(-61.51*)
endo LF	61.31	–10.72	65.49	-10.42	6.37	+ 1.27	33.50	– 2.21	20.54	– 4.46
		(-0.15%)		(-0.14%)		(0.25%)		(-0.06%)		(–17.84*)

Note: See Table 3.

Figure Appendix

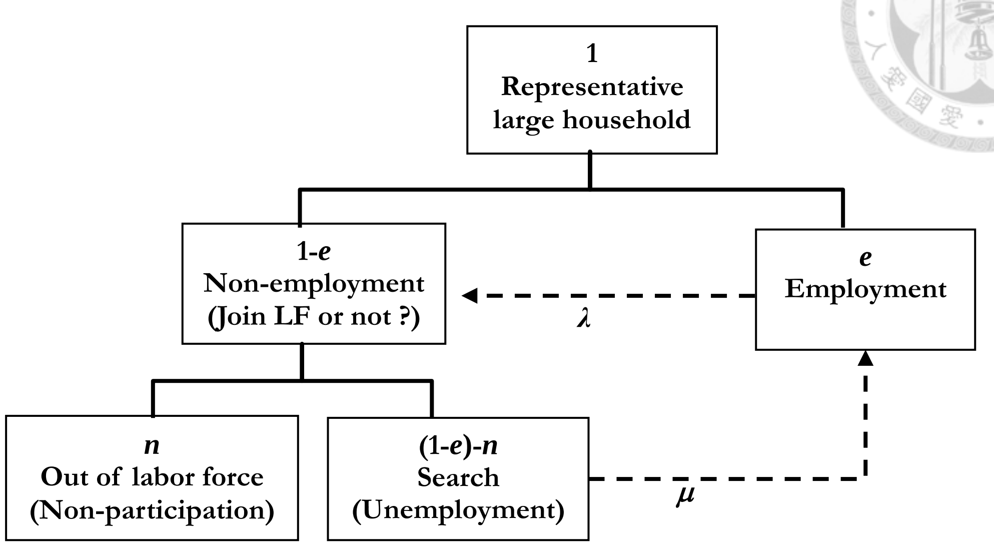


Figure 1: Labor allocation for the large household

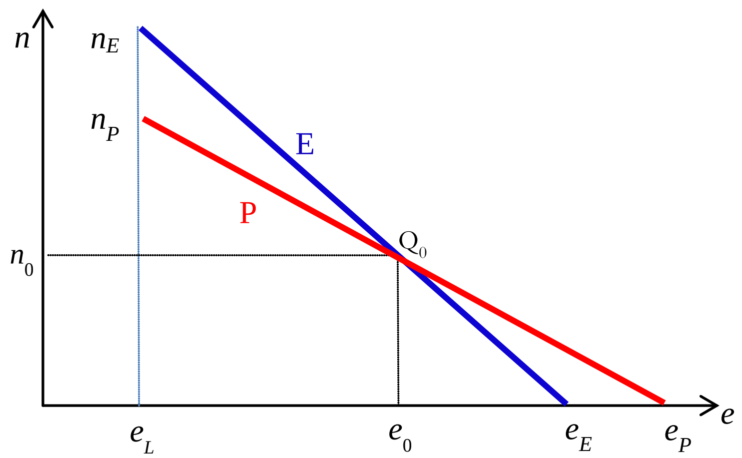


Figure 2：Existence of steady state

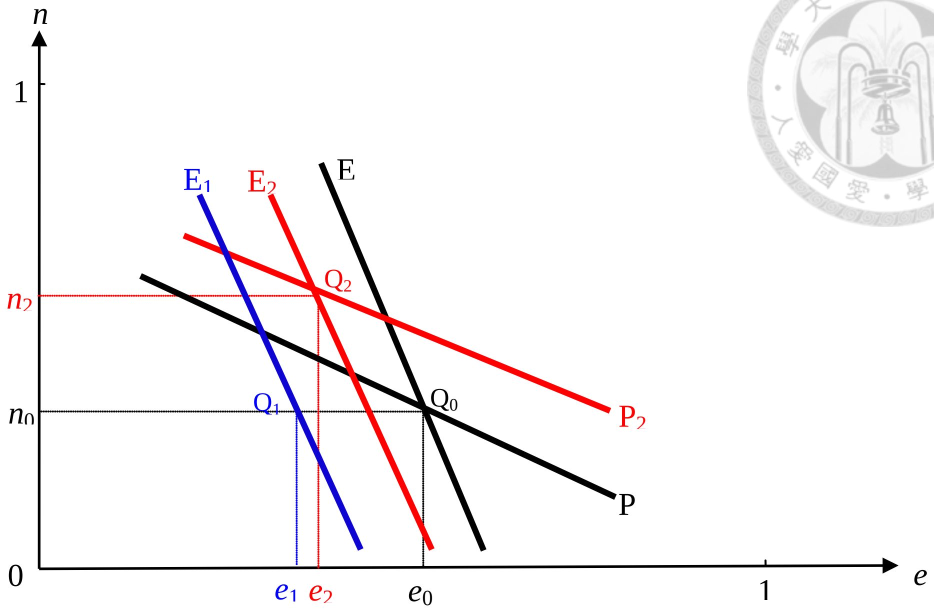


Figure 3：Steady-state effects of increases in wage taxes (τ↑)

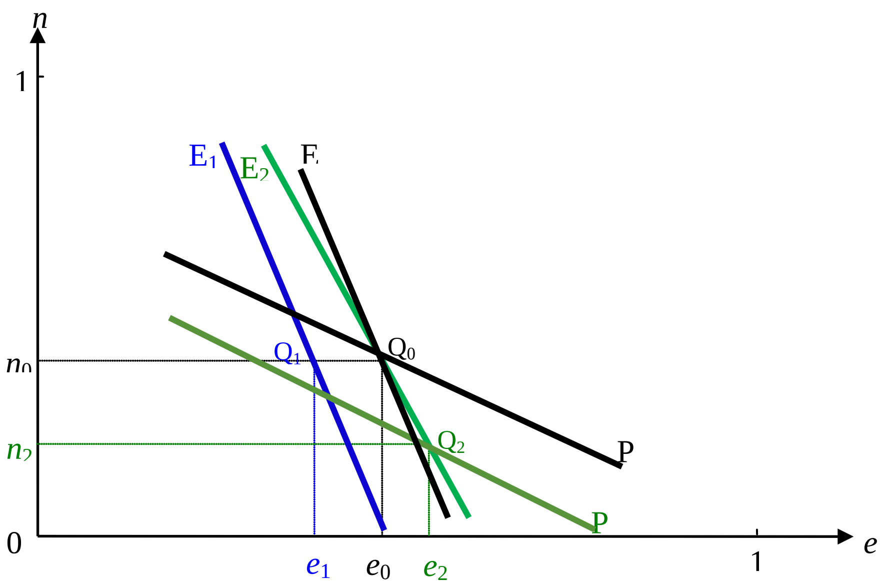


Figure 4：Steady-state effects of increases in unemployment compensation (b↑)

Mathematical Appendix

A1 The wage equation

In the wage equation $w = \beta[MPL \cdot h] + (1-\beta)\left[\dfrac{b + MRS^{se}(e,\bar{n},h)}{1-\tau}\right] \equiv w(\underset{+}{e},\underset{-}{\bar{n}},\underset{+}{h};\underset{+}{\tau},\underset{+}{b})$ in (16), the signs in the arguments are derived as follows.

(A1a) $$w_e = (1-\beta)\frac{MRS_e^{se}}{1-\tau} > 0\,,$$

(A1b) $$w_h = \beta \cdot MPL + (1-\beta)\frac{b + MRS_h^{se}}{1-\tau} > 0$$

(A1c) $$w_\tau = (1-\beta)\frac{b + MRS^{se}}{(1-\tau)^2} > 0\,,$$

(A1d) $$w_b = (1-\beta)\frac{1}{1-\tau} > 0\,,$$

where $MRS_e^{se} = \left(\chi_2 - \chi_1 \frac{(1-h)^{1-\sigma}}{1-\sigma}\right)c_e > 0$ and $MRS_h^{se} = MRS^h + \left(\chi_2 - \chi_1 \frac{(1-h)^{1-\sigma}}{1-\sigma}\right)c_h > 0$.

In the wage equation $w = \beta[MPL \cdot h] + (1-\beta)\left[\dfrac{MRS^{ne}(e,n,h)}{1-\tau}\right] \equiv w(\underset{+}{e},\underset{-}{n},\underset{+}{h};\underset{+}{\tau})$ in (21), the signs in the arguments are derived as follows.

(A1e) $$w_e = (1-\beta)\frac{\left(\chi_3 - \chi_1 \frac{(1-h)^{1-\sigma}}{1-\sigma}\right)c_e}{1-\tau} > 0\,.$$

(A1f) $$w_n = (1-\beta)\frac{\left(\chi_3 - \chi_1 \frac{(1-h)^{1-\sigma}}{1-\sigma}\right)c_n}{1-\tau} < 0\,,$$

(A1g) $$w_h = \beta \cdot MPL + (1-\beta)\frac{MRS^h + \left(\chi_3 - \chi_1 \frac{(1-h)^{1-\sigma}}{1-\sigma}\right)c_h}{1-\tau} > 0\,,$$

(A1h) $$w_\tau = (1-\beta)\frac{\left(\chi_3 - \chi_1 \frac{(1-h)^{1-\sigma}}{1-\sigma}\right)c}{(1-\tau)^2} > 0\,.$$

A2 The hour equation

The signs in the arguments of the hour equation $h = h(\underset{-}{e},\underset{+}{n};\underset{-}{\tau})$ in (17b) are derived as follows. Rewriting (17a) as $H \equiv \chi_1(1-h)^{-\sigma}c(e,h;1-\bar{n}) - (1-\tau)MPL$ and totally differentiation yields

(A2a) $$H_h dh = -H_e de - H_n dn - H_\tau d\tau,$$

where

(A2b) $$H_h = \underbrace{\chi_1(1-h)^{-\sigma}c_h}_{H_h^1>0} \underbrace{+\sigma\chi_1(1-h)^{-\sigma-1}c}_{H_h^2>0} > 0\,,$$

(A2c) $$H_e = \chi_1(1-h)^{-\sigma}c_e > 0\,,$$

(A2d) $$H_n = \chi_1(1-h)^{-\sigma}c_n < 0\,,$$

(A2e) $$H_\tau = MPL > 0\,.$$

A3 The employment equation

When the labor-force participation is exogenous, the signs of the employment equation $E^{\bar n}(\underset{-}{e},\underset{-}{h};\underset{-}{\tau},\underset{-}{b},\underset{-}{\phi}) \equiv MB^v(e,h) - \phi = 0$ in (18) are derived as follows. Totally differentiating (18) yields

(A3a) $$E_e^{\bar n} de + E_h^{\bar n} dh + E_\tau^{\bar n} d\tau + E_b^{\bar n} db - d\phi = 0,$$

where

(A3b) $$E_e^{\bar n} = MB_e^v = \underbrace{\frac{\eta_e(1-\beta)}{\rho+\lambda}GFM_{exog}}_{E_e^{\bar n 1}<0} \underbrace{-\frac{\eta(1-\beta)}{\rho+\lambda}\frac{MRS_e^{se}}{1-\tau}}_{E_e^{\bar n 2}<0} < 0\,,$$

(A3c) $$E_h^{\bar n} = \frac{\eta(1-\beta)}{\rho+\lambda}[\underbrace{MPL - \frac{\chi_1(1-h)^{-\sigma}c}{1-\tau}}_{=0} - \frac{(\chi_2 - \chi_1\frac{(1-h)^{1-\sigma}}{1-\sigma})c_h}{1-\tau}] < 0$$

(A3d) $$E_\tau^{\bar n} = MB_\tau^v = -\frac{\eta(1-\beta)}{\rho+\lambda}\frac{b+MRS^{se}}{(1-\tau)^2} < 0\,,$$

(A3e) $$E_b^{\bar n} = MB_b^v = -\frac{\eta(1-\beta)}{\rho+\lambda}\frac{1}{1-\tau} < 0\,,$$

and $GFM_{exog} \equiv MPL \cdot h - \frac{1}{1-\tau}(MRS^{se}+b)$ is the *Gain From Match* when the participation is exogenous.

When the labor force participation is endogenous, the signs of the employment equation

$E(\underset{-}{e}, \underset{-or\,?}{n}, \underset{-}{h}; \underset{-}{\tau}, \underset{-}{\phi}) \equiv MB^{v}(e,n,h) - \phi = 0$ in (22) are derived as follows. Totally differentiating (22) yields

(A3f) $$E_e de + E_n dn + E_h dh + E_\tau d\tau + E_b db - d\phi = 0,$$

where

(A3g) $$E_e = \underbrace{[\frac{\eta_e(1-\beta)}{\rho+\lambda} GFM_{endo}]}_{E_e^1<0} + \underbrace{[-\frac{\eta(1-\beta)}{\rho+\lambda}\frac{MRS_e^{ne}}{1-\tau}]}_{E_e^2<0} < 0,$$

(A3h) $$E_n = \underbrace{[\frac{\eta_n(1-\beta)}{\rho+\lambda} GFM_{endo}]}_{E_n^1<0)} + \underbrace{[-\frac{\eta(1-\beta)}{\rho+\lambda}\frac{MRS_n^{ne}}{1-\tau}]}_{E_n^2>0} < 0(\gtreqless 0),$$

(A3i) $$E_h = \frac{\eta(1-\beta)}{\rho+\lambda}[\underbrace{MPL - \frac{\chi_1(1-h)^{-\sigma}c}{1-\tau}}_{=0} - \frac{(\chi_3 - \chi_1\frac{(1-h)^{-\sigma}}{1-\sigma})c_h}{1-\tau}] < 0,$$

(A3j) $$E_\tau = -\frac{\eta(1-\beta)}{\rho+\lambda}\frac{MRS^{ne}}{(1-\tau)^2} < 0,$$

and $GFM_{endo} \equiv MPL \cdot h - \frac{1}{1-\tau}MRS^{ne}$ is the gain from match when the participation is endogenous,

and $MRS_e^{ne} = (\chi_3 - \chi_1\frac{(1-h)^{-\sigma}}{1-\sigma})c_e > 0$ and $MRS_n^{ne} = (\chi_3 - \chi_1\frac{(1-h)^{-\sigma}}{1-\sigma})c_n < 0.$

A4 The participation equation

The signs of the participation equation $P(\underset{+or\,?}{e}, \underset{+}{n}, \underset{-}{h}; \underset{-}{\tau}, \underset{+}{b}) \equiv MB^{p}(e,n,h) - MC^{p}(e,n,h) = 0$ in (23) are derived as follows. Totally differentiating (23) yields

(A4a) $$P_e de + P_n dn + P_h dh + P_\tau d\tau + db = 0,$$

where

(A4b) $$P_e = \underbrace{\frac{\mu_e\beta}{\rho+\lambda}(1-\tau)GFM_{endo}}_{P_e^1>0} \underbrace{-\frac{\mu\beta}{\rho+\lambda}MRS_e^{ne}}_{P_e^2<0} \underbrace{-(\chi_3-\chi_2)c_e}_{P_e^3<0} > 0(\gtreqless 0),$$ [64]

(A4c) $$P_n = \underbrace{\frac{\mu_n\beta}{\rho+\lambda}(1-\tau)GFM_{endo}}_{P_n^1>0} + \underbrace{[-\frac{\mu\beta}{\rho+\lambda}MRS_n^{ne}]}_{P_n^2>0} + \underbrace{[-(\chi_3-\chi_2)c_n]}_{P_n^3>0} > 0,$$

[64] We assume that the labor market externality effect through job finding rate dominates based on the simulation results.

(A4d) $$P_h = \frac{\mu\beta}{\rho+\lambda}\left[\underbrace{(1-\tau)MPL-\chi_1(1-h)^{-\sigma}}_{=0} - \left(\chi_3-\chi_1\frac{(1-h)^{1-\sigma}}{1-\sigma}\right)c_h\right] - (\chi_3-\chi_2)c_h < 0,$$

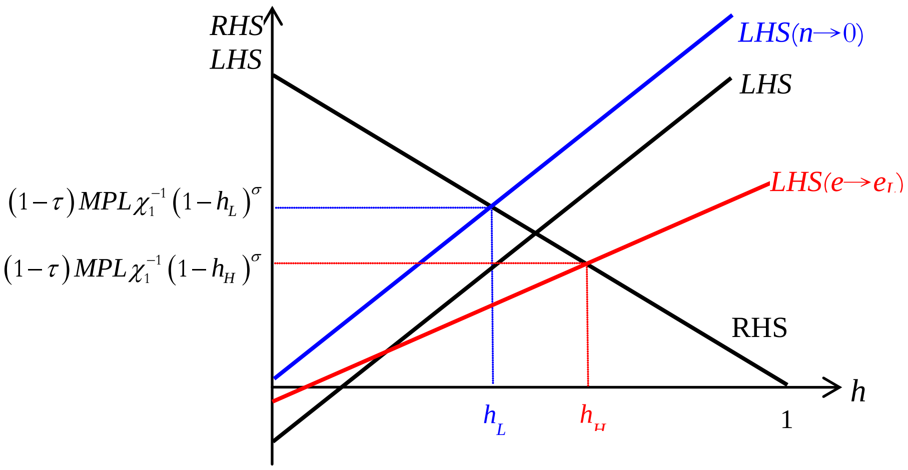
(A4e) $$P_\tau = -\frac{\mu\beta}{\rho+\lambda}MPL\cdot h < 0.$$

A5 Existence of steady state

Using (13) and rearranging (17a) gives

(A5a) $$LHS \equiv [(Aq^{\alpha}-\delta q)eh-\phi(\frac{\lambda e}{m(1-e-n)^{\gamma}})^{\frac{1}{1-\gamma}}] = (1-\tau)MPL\chi_1^{-1}(1-h)^{\sigma} \equiv RHS.$$

The value of *RHS* decreases in *h*, while, for given *e* and *n*, the value of *LHS* increases in *h* and thus the *RHS* and *LHS* loci determine *h*. When *n* decreases, locus *LHS* shifts upward. In the limit when *n* goes to 0, locus *LHS* shifts to the highest level and as a result, *h* goes to the lowest value h_L>0 such that $LHS = RHS = (1-\tau)MPL\chi_1^{-1}(1-h_L)^{\sigma}$. See the figure below. Note that if the value of *A* is larger, locus *LHS* shifts upward more and thus h_L is smaller.



Conversely, when *e* decreases, locus *LHS* rotates clockwise with a flatter slope. However, *e* cannot go to 0, as then the value of *LHS* would be negative and *h* does not exist. There is a lowest value of *e*, denoted by e_L. As *e* goes to e_L, locus *LHS* rotates and reaches the smallest slope. As a result, *h* goes to the highest value h_H>0 such that $LHS = RHS = (1-\tau)MPL\chi_1^{-1}(1-h_H)^{\sigma}$. Note that if the value of *A* is larger, *LHS* rotates more and thus h_H is larger.

Rewriting (23) and (23) gives, respectively,

(A5b) $$\frac{m^{\frac{1}{1-\gamma}}(\frac{1-e-n}{\lambda e})^{\frac{\gamma}{1-\gamma}}}{\rho+\lambda}(1-\beta)\left[MPL\cdot h-\frac{(\chi_3-\chi_1\frac{(1-h)^{1-\sigma}}{1-\sigma})\cdot LHS}{1-\tau}\right]-\phi=0,$$

(A5c) $$\frac{\frac{\lambda e}{1-e-n}}{\rho+\lambda}\beta\left[(1-\tau)MPL\cdot h-(\chi_3-\chi_1\frac{(1-h)^{1-\sigma}}{1-\sigma})\cdot LHS\right]+b-(\chi_3-\chi_2)\cdot LHS=0.$$

Substituting (A5a) into (A5b) and (A5c), respectively, gives two expressions of (A5b) and (A5c). First, when $n\to 0$, these two new expressions of (A5b) and (A5c) lead to, respectively,

(A5b) $$\frac{\lambda e_E}{1-e_E}=\left\{\frac{m^{\frac{1}{1-\gamma}}}{\rho+\lambda}(1-\beta)\frac{MPL}{\phi}\left[h_L-(\frac{\chi_3}{\chi_1}-\frac{(1-h_L)^{1-\sigma}}{1-\sigma})(1-h_L)^{\sigma}\right]\right\}^{\frac{1-\gamma}{\gamma}},$$

(A5c)

$$\frac{\lambda e_P}{1-e_P}=\frac{\rho+\lambda}{\beta}\left[(\frac{\chi_3-\chi_2}{\chi_1})(1-h_L)^{\sigma}-\frac{b}{(1-\tau)MPL}\right]\left[h_L-(\frac{\chi_3}{\chi_1}-\frac{(1-h_L)^{1-\sigma}}{1-\sigma})(1-h_L)^{\sigma}\right]^{-1},$$

which yield e_E and e_P, respectively.

Second, when e decreases to e_L, the two new expressions of (A5b) and (A5c) give, respectively,

(A5d) $$n_E=1-e_L-\frac{\lambda e_L}{\{\frac{m^{\frac{1}{1-\gamma}}}{\rho+\lambda}(1-\beta)\frac{MPL}{\phi}[h_H-(\frac{\chi_3}{\chi_1}-\frac{(1-h_H)^{1-\sigma}}{1-\sigma})(1-h_H)^{\sigma}]\}^{\frac{1-\gamma}{\gamma}}},$$

(A5e) $$n_P=1-e_L-\frac{\lambda e_L[h_H-(\frac{\chi_3}{\chi_1}-\frac{(1-h_H)^{1-\sigma}}{1-\sigma})(1-h_H)^{\sigma}]}{\frac{\rho+\lambda}{\beta}\left[(\frac{\chi_3-\chi_2}{\chi_1})(1-h_H)^{\sigma}-\frac{b}{(1-\tau)MPL}\right]},$$

which yield n_E and n_P, respectively.

Denote $\Psi(h)\equiv\frac{[(\frac{\chi_3-\chi_2}{\chi_1})(1-h)^{\sigma}-\frac{b}{(1-\tau)MPL}]^{\gamma}}{h-(\frac{\chi_3}{\chi_1}-\frac{(1-h)^{1-\sigma}}{1-\sigma})(1-h)^{\sigma}}$. Then, $e_E<e_P$ and $n_E>n_P$ if the following conditions are met.

Condition E (Existence) $\Psi(h_H)<\frac{m}{\rho+\lambda}\beta^{\gamma}(1-\beta)^{1-\gamma}\left(\frac{MPL}{\phi}\right)^{1-\gamma}<\Psi(h_L).$

Under Condition E, there exists an intersection of (22) and (23) that determines e and n as illustrated in Figure 2. As a larger value of A decreases h_L which increases $\Psi(h_L)$ and increases h_H which decreases $\Psi(h_H)$, Condition E is easier to meet if A is larger.

A6 Comparative-static Effects

A6.1 Exogenous participation with given working hours

When $n = \bar{n}$, (4b) and (10b) do not exist and (19) alone determines e. The differentiation of (19) is in (A3a) which, under $h = \bar{h}$, is rewritten as $E_e^{\bar{n}} de + E_\tau^{\bar{n}} d\tau + E_b^{\bar{n}} db = 0$. Straightforward calculation gives the following comparative-static results:

$$\frac{de}{d\tau} = -\frac{E_\tau^{\bar{n}}}{E_e^{\bar{n}}} < 0 \quad \text{and} \quad \frac{de}{db} = -\frac{E_b^{\bar{n}}}{E_e^{\bar{n}}} < 0 \,.$$

A6.2 Exogenous participation with variable working hours

When $n = \bar{n}$ and h is endogenous, (19) alone determines e too. Now, h is endogenous, so (A2a) and (A3a) determine the steady state levels of e and h. Denoting $\tilde{D}^{\bar{n}} \equiv E_e^{\bar{n}} H_h - E_h^{\bar{n}} H_e$ as the determinant of the Jacobian matrix in the system (A2a) and (A3a), using (A2b) and (A3b) and noting $E_e^{\bar{n}2} H_h^1 = E_h^{\bar{n}} H_e$, we have $\tilde{D}^{\bar{n}} = E_e^{\bar{n}1} H_h + E_e^{\bar{n}2} H_h^2 < 0$. Straightforward calculation gives the following comparative-static results:

$$\frac{de}{d\tau} = -\frac{1}{\tilde{D}^{\bar{n}}}\left(E_\tau^{\bar{n}} H_h - E_h^{\bar{n}} H_\tau\right) < 0 \,,$$

$$\frac{dh}{d\tau} = -\frac{1}{\tilde{D}^{\bar{n}}}\left(E_e^{\bar{n}} H_\tau - E_\tau^{\bar{n}} H_e\right) \gtrless 0 \,,$$

$$\frac{de}{db} = -\frac{E_b^{\bar{n}} H_h}{\tilde{D}^{\bar{n}}} < 0 \,,$$

$$\frac{dh}{db} = -\frac{-E_b^{\bar{n}} H_e}{\tilde{D}^{\bar{n}}} > 0 \,.$$

A6.3 Endogenous participation with given working hours

When n is endogenous, (22) and (23) are the equilibrium conditions. The results of total differentiation of (22) and (23) are (A3f) and (A4a) which, with $h = \overline{h}$, are rewritten as follows.

(A6a) $\quad E_e de + E_n dn + E_\tau d\tau = 0,$

(A6b) $\quad P_e de + P_n dn + P_\tau d\tau + db = 0.$

Noting that $E_e^1 P_n^1 = E_n^1 P_e^1$ and $E_e^2\left(P_n^2 + P_n^3\right) = E_n^2\left(P_e^2 + P_e^3\right)$, we have

$$D \equiv E_e P_n - E_n P_e = E_e^1\left(P_n^2 + P_n^3\right) + E_e^2 P_n^1 - E_n^1\left(P_e^2 + P_e^3\right) - E_n^2 P_e^1 < 0 .$$

Then, $-\frac{E_e}{E_n} < -\frac{P_e}{P_n} < 0$ follows from the results that the Locus E and Locus P are both downward sloping and Lucas P is always flatter than Locus E in the intersection.

Note $E_\tau P_n^2 - E_n^2 P_\tau = -GFM_{endo} \cdot \Phi \cdot MRS_n^{ne} < 0$ and $E_e^2 P_\tau - E_\tau P_e^2 = GFM_{endo} \cdot \Phi \cdot MRS_e^{ne} > 0$, where $\Phi \equiv \frac{\eta(1-\beta)}{(\rho+\lambda)^2}\frac{\mu\beta}{1-\tau} > 0$. We thus obtain

$$E_\tau P_n - E_n P_\tau < 0 \quad \text{and} \quad E_e P_\tau - E_\tau P_e = \underbrace{E_e^1 P_\tau - E_\tau P_e^1}_{+} \underbrace{+ E_e^2 P_\tau - E_\tau P_e^2}_{>0} \underbrace{- E_\tau P_e^3}_{-} > 0 (\gtrless 0).$$

Standard analysis implies that comparative-static results are given by

$$\frac{de}{d\tau} = -\frac{1}{D}\left(E_\tau P_n - E_n P_\tau\right) < 0 ,$$

$$\frac{dn}{d\tau} = -\frac{1}{D}\left(E_e P_\tau - E_\tau P_e\right) < 0 ,$$

$$\frac{de}{db} = -\frac{-E_n}{D} > 0 ,$$

$$\frac{dn}{db} = -\frac{E_e}{D} < 0 .$$

A6.4 Endogenous participation with variable working hours

When n and h are endogenous, by substituting (A2a), we rewrite (A3f) and (A4a) as follows

(A6c) $\quad \tilde{E}_e de + \tilde{E}_n dn = -\tilde{E}_\tau d\tau + d\phi ,$

(A6d) $\quad \tilde{P}_e de + \tilde{P}_n dn = -\tilde{P}_\tau d\tau - db$,

where $\quad \tilde{E}_e \equiv E_e - E_h \dfrac{MRS_e^h}{MRS_h^h} < 0$, $\tilde{E}_n \equiv E_n - E_h \dfrac{MRS_n^h}{MRS_h^h} < 0$, $\tilde{E}_\tau \equiv E_\tau - E_h \dfrac{MPL}{MRS_h^h} < 0$,

$\tilde{P}_e \equiv P_e - P_h \dfrac{MRS_e^h}{MRS_h^h} > 0$, $\tilde{P}_n \equiv P_n - P_h \dfrac{MRS_n^h}{MRS_h^h} > 0$, $\tilde{P}_\tau \equiv P_\tau - P_h \dfrac{MPL}{MRS_h^h} < 0$.[65]

Let $\tilde{D} \equiv \tilde{E}_e \tilde{P}_n - \tilde{E}_n \tilde{P}_e$ denote the determinant of the Jacobian matrix in (A6c)-(A6d). Then, $-\frac{\tilde{E}_e}{\tilde{E}_n} < -\frac{\tilde{P}_e}{\tilde{P}_n} < 0$ follows from the results that the Locus *E* and Locus *P* are both downward sloping and Lucas P is always flatter than Locus E in the intersection.

Standard analysis implies that comparative-static results are given by

$$\frac{de}{d\tau} = -\frac{1}{\tilde{D}}\left(\tilde{E}_\tau \tilde{P}_n - \tilde{E}_n \tilde{P}_\tau\right) < 0 ,$$

$$\frac{dn}{d\tau} = -\frac{1}{\tilde{D}}\left(\tilde{E}_e \tilde{P}_\tau - \tilde{E}_\tau \tilde{P}_e\right) > 0 ,$$

$$\frac{de}{db} = -\frac{1}{\tilde{D}}\left(\tilde{E}_b \tilde{P}_n - \tilde{E}_n \tilde{P}_b\right) > 0 ,$$

$$\frac{dn}{db} = -\frac{1}{\tilde{D}}\left(\tilde{E}_e \tilde{P}_b - \tilde{E}_b \tilde{P}_e\right) < 0 .$$

[65] The sign assumes that the direct effects dominate those indirect effects via changes of work hours per worker which is met in quantitative analysis.

5 Conclusion

This dissertation decomposes labor supply into three margins step by step and studies the relative effects of two adverse labor market institutes on labor supply. Average labor supply in the EU declined over 25% relative to that in the US over the past 30 years from the early 1970s to the early 2000s. Europe has also witnessed steadily higher labor income taxes and more generous government-supplied unemployment benefits, among others, than the US. Some studies attributed declining hours worked in Europe relative to the US to higher labor taxes, while other studies accredited high unemployment rates in Europe to more generous non-employment benefits. However, data indicates the difference of labor supply between the EU and the US comes from both the intensive (hours) and the extensive (employment) margins.

The first essay begins from studying a model that consider labor search within the neoclassical growth framework so as to investigate the effects on labor supply along both intensive and extensive margins in one unified general equilibrium framework. The theoretical results find that an increase in the labor tax decreases hours per worker and employment with an overstated adverse effect on hours worked if employment is fixed as is in Prescott (2002, 2004). Moreover, more generous non-employment benefits decrease employment and increase hours per worker, with an understated adverse effect on employment if hours per worker are fixed as are in Ljungqvist and Sargent (2007, 2008a). The numerical results show that increases in labor taxes and non-employment benefits together explain about 75% of declining labor supply in Europe relative to the US over the past 3 decades, with the fraction accounted for being increasing in the labor supply elasticity and decreasing in the labor’s contribution in matching.

Individually, labor supply declined more from employment in some countries and more from hours per worker in other countries. The second essay studies the relative detrimental effects of higher labor taxes on hours per worker and employment and finds that the relative effects depend on the mechanisms shaping the supply of hours per worker. When hours per worker are bargained by matched job-worker pairs, a higher labor income tax reduces both employment and hours per worker. As the laborer's the hour bargaining power is larger, the negative effect is smaller on employment and larger on hours. When the supply of hours is decided exclusively by the household, together with the utility of leisure is linear in hours, the negative effect on employment is zero and all negative effects are on hours per worker. At the other extreme, when the worker's supply of hours is effectively regulated by the authority, a higher labor tax only reduces employment with a zero effect on hours. Thus, these different hour-shaping mechanisms help understand the underlying mechanisms why, in facing higher labor tax rates in Europe over the past thirty years, some countries experienced more severe increases in unemployment rates while some other countries underwent sharper decreases in hours per worker.

The third essay takes the further step to consider the participation margin and compare the long-run effects of increases in labor taxes and unemployment benefits on labor supply in the models with and without the participation margin. With increases in labor taxes, thanks to discouraging labor forces, the employment is reduced less than that in the model without endogenous labor forces and, with ambiguous effects on hours per worker, labor supply is decreased by less. In the case of increases in unemployment benefits, due to inducing labor forces, employment increases instead of decreases in the model with exogenous labor forces and, with the effect on hours per worker being opposite to that on employment, the effect on labor supply is ambiguous in both models.

The quantitative results reveal that the model without endogenous labor forces explains too much of the decreases in employment and labor supply in the EU relative to the US over the past 3 decades. In particular, it predicts an increase in hours per worker which is at odd with the data. By contrast, due to endogenous labor forces, our model explains a reasonable decrease in labor supply, along with a reasonable decrease in employment and a moderate decrease rather than an increase in hours per worker. Relatively, the model with endogenous labor forces explains the difference in labor supply better than the model with exogenous labor forces.

Finally, we should mention that differences in labor-force participation may come from older and younger workers and female labor-force participation. Moreover, differences in labor supply may also reflect differences in workweeks, full and part-time jobs, holidays and vacation days. Our model and the models studied by Ljungqvist and Sargent (2007a, 2007b), Fang and Rogerson (2009) and Shimer (2011) consider neither life-cycle elements nor female and male labor-force participation, because these models are aimed at understanding differences in the labor supply or employment for a representative agent with full-time employment instead of the choice of part-time versus full-time and female versus male employment. Although there are some variations in the EU relative to the US, the key pattern these existing papers wish to emphasize is that the very large differences in average labor supply per person in the past decades are due to large differences in hours per worker and employment. Our model adds value to these existing studies in that by taking account of endogenous labor forces, it explains the difference in labor supply in the EU relative to the US better than the model with exogenous labor forces.

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