國立臺灣大學管理學院財務金融學系

碩士論文

Department of Finance

College of Management

National Taiwan University

Master Thesis

努力影響產出的高階風險時，

經理人誘因如何受契約參數影響?

Managerial Motivation When Effort Improves Higher Order Risk

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中華民國 103 年 6 月

June, 2014

國立臺灣大學（碩）博士學位論文
口試委員會審定書

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本論文係**黃彥霖**君（**R01723036**）在國立臺灣大學**財務金融**研究所完成之碩士學位論文，於民國 **103** 年 **06** 月 **30** 日承下列考試委員審查通過及口試及格，特此證明

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致謝

此篇論文的完成，首先要感謝的是我的碩士班指導老師----曾郁仁教授。曾老師在保險專題及風險理論課堂上的教導使我獲益良多，也因為有了這些學習的機會，才讓我對風險理論有進一步的認識，進而促成了此篇論文。

在課堂指導之外，曾老師也開啟了我對學術研究的興趣，並在碩士班的第二年給了我許多參與研究專案的機會，不僅使我確立了以學術研究為目標的職志，更使我在繼續攻讀博士班之前能夠先累積寶貴的研究經驗，我非常感恩且珍惜這樣的學習機會。

此外，也感謝台科大黃瑞卿老師在本文寫作過程中的協助與指導，也讓我有機會在完成本文之後，有進一步發展及研究的機會

最後，也感謝所有台大財金所的同學及夥伴們，因為有與你們的互相支持及鼓勵，才讓我在碩士班的這兩年有如此的成長。

黃彥霖　於台大

2014.06

Abstract

Managerial motivation has long been an important issue in economy. There are many papers discussing the "optimal contract design" that would optimize the effort spent by managers. However, the "optimal" contract may be affected by several factors and therefore can change over time. The problem "how the managerial motivation would be affected under certain type of contracts" has long been ignored. This article aim to figure out how managerial effort would be affected by the change of the parameters of the contract provided the effort improves higher order risk of production.

Key words : higher order, managerial motivation, optimal contract, risk, risk averse

摘要

如何設計契約使經理人努力誘因最大化一直是經濟學上一個很重要的議題，目前也已有許多文獻在探討如何設計所謂的「最適契約」。然而，在現實生活中，最適的契約往往會隨時間所變動，而契約中的參數對經理人所代表的意義也可能隨外在環境不同而有所變化。如此一來，給定一個既有的契約架構，當契約參數有所變動時，經理人的努力誘因會如何受到影響就是一個值得探討的議題，而在這方面的文獻卻較少。本論文旨在探討當努力影響產出的高階風險時，經理人誘因將如何受契約參數的變化所影響。

關鍵字 ： 高階風險、最適契約、經理人誘因、風險、風險趨避

目錄

I. Introduction

Managerial motivation has long been an important issue in economy. There are many papers discussing the "optimal contract design" that would optimize the effort spent by managers. However, the "optimal" contract may be affected by several factors and therefore can change over time. The problem "how the managerial motivation would be affected under certain type of contracts" has long been ignored.

Kocabiyikoglu & Popescu(2007) aim to deal with such a question. They discussed the question "how managerial motivation would be affected when the parameters of a contract change" under the contract form $F + \lambda S$, where F is a fixed salary, S is the production of the manager, and λ is a constant. In other words, the payoff a manager would get depends on her production level and a fixed amount of payment. Kocabiyikoglu & Popescu(2007) assume that the production level of a manager is strictly increasing in her effort, or, if the managerial effort is considered an element that change the distribution of production, we can say they assume that the managerial effort would improve the first order risk of production.

As we can see, higher order risk becomes a popular issue recently. Eeckhoudt & Schlesinger(2006) provided some structure-free intuition about higher order risk and proved the equality of such intuition with the definition of higher order risk attitude under expected utility structure. Moreover, Eeckhoudt & Schlesinger(2008) and Chiu, Eeckhoudt, & Rey(2012) further discussed the intuition about relative risk aversion in the similar way. Due to the importance of higher order risk, if we consider managerial effort a factor that changes the distribution of the production, the impact should not be limited to first order risk.

This paper aim to figure out how managerial effort would be affected by the change of the parameters of the contract provided the effort improves higher order risk of production. I discuss the question under the same contract form as in Kocabiyikoglu & Popescu, and apply the concept of risk attitude raised in Eeckhoudt et al (2009) and in Chiu, Eeckhoudt, & Rey(2012.)

I show that when the fixed wage is changed, how the manager will change her effort level depends on her higher order risk attitudes and that for a risk averse manager, the degree of her risk aversion can decide how she's going to react to the change of the sensitivity of her payoff to her production level. Moreover, her decision of the effort level when an exogenous variable change the risk of her production would also be different, depending on her higher order risk attitudes.

The remaining of this thesis is organized as follows. Section 2 briefly describes the model, section 3 is the discussion of how managerial effort would be affected by the change of the contract parameters, and section 4 is the conclusion.

II. The Model

Let $\widetilde{S_\alpha}$ be the production of the manager. $\widetilde{S_\alpha}$ is a positive random variable and can be affected by the manager's effort α, where $\alpha \in [0,1]$. I assume that the more effort a manager spends, the less N_{th}-order risk her production has. The effort requires a cost $C(\alpha)$ for the manager. In order to rule out corner solutions, I assume $C'(\alpha) \geq 0$, $C''(\alpha) \geq 0$, $C'(0) = 0$, and $C'(1) = \infty$

As in Kocabiyikoglu & Popescu(2007), I set the contract form as $F + \lambda\widetilde{S_\alpha}$, where F and λ are constants. That is, at the end of a period, the manager would get a fixed payoff F and a bonus $\lambda\widetilde{S_\alpha}$ that depends on her production level. So, the manager's maximization problem would be $Max_\alpha\left[EU\left(F + \lambda\widetilde{S_\alpha}\right) - C(\alpha)\right]$

III. The Discussions

A. How would managerial effort be affected by F ?

In order to answer the question, we must first take a look at the theorem proposed by Eeckhoudt et. al(2009).

Theorem 1

Part1 :

If $\tilde{x}$ has less N_{th}-order risk than $\tilde{y}$, $\tilde{p}$ has less K_{th}-order risk than $\tilde{q}$, lottery $A_1 = [\tilde{x} + \tilde{p} \; ; \; \tilde{y} + \tilde{q}]$, $B_1 = [\tilde{x} + \tilde{q} \; ; \; \tilde{y} + \tilde{p}]$, then an agent prefers B_1 to A_1 if and only if she is $(N+K)_{th}$-degree risk averse.

Part2 :

If $\tilde{x} \succcurlyeq_{\mathrm{NSD}} \tilde{y},\ \tilde{p} \succcurlyeq_{\mathrm{KSD}} \tilde{q}$, *lottery* $A_1 = [\tilde{x} + \tilde{p}\ ;\ \tilde{y} + \tilde{q}]$, $B_1 = [\tilde{x} + \tilde{q}\ ;\ \tilde{y} + \tilde{p}]$, *then an agent prefers* B_1 *to* A_1 *if and only if she is* n_{th}*-degree risk averse, for all* $n = 1, 2, 3, \ldots\ldots, N+K$.

The intuition of part 1 is straight forward, if we define a random variable with lower risk as a variable with better distribution, or "good distribution," and a random variable with higher risk has "bad distribution," then lottery A_1 represents a lottery that combines good distribution with the good one, bad distribution with the bad one, while lottery B_1 represents a lottery that combines good distribution with bad distribution. Part 1 of the theorem is saying that an $(N+K)_{th}$-degree risk averse person would prefer a lottery that "combines good with bad." If part 1 holds, then part 2 follows.

Now we can return to the discussion. Assume that F changes from F_1 to F_2, $F_2 < F_1$. In this case, the original maximization problem would be :

$$Max_{\alpha}[EU(\mathrm{F}_1 + \lambda\widetilde{S_\alpha}) - C(\alpha)]$$

And the maximization problem after F changes would become :

$$Max_{\alpha}[EU(\mathrm{F}_2 + \lambda\widetilde{S_\alpha}) - C(\alpha)]$$

Let α_1^* be the solution of the original maximization problem and α_2^* be the solution of the maximization problem after F changes. Because α_i^* must maximize

$EU\left(\mathrm{F_i}+\lambda\widetilde{S_\alpha}\right)-C(\alpha)$, we have :

$$EU\left(\mathrm{F}_1+\lambda\widetilde{S_{\alpha_1^*}}\right)-C(\alpha_1^*)>EU\left(\mathrm{F}_1+\lambda\widetilde{S_{\alpha_2^*}}\right)-C(\alpha_2^*)\ \ \ldots\ldots\ (1)$$

and

$$EU\left(\mathrm{F}_2+\lambda\widetilde{S_{\alpha_2^*}}\right)-C(\alpha_2^*)>EU\left(\mathrm{F}_2+\lambda\widetilde{S_{\alpha_1^*}}\right)-C(\alpha_1^*)\ \ \ldots\ldots\ (2)$$

Combining the two inequality, we get :

$$EU\left(\mathrm{F}_1+\lambda\widetilde{S_{\alpha_1^*}}\right)+EU\left(\mathrm{F}_2+\lambda\widetilde{S_{\alpha_2^*}}\right)>EU\left(\mathrm{F}_1+\lambda\widetilde{S_{\alpha_2^*}}\right)+EU\left(\mathrm{F}_2+\lambda\widetilde{S_{\alpha_1^*}}\right)\ldots(3)$$

If we multiply both sides of inequality (3) by 0.5, then each side would looks like a lottery defined in Theorem 1. Because F_1 is less first-order riskier than F_2, when $\alpha_2^*>\alpha_1^*$, the left hand side is a lottery that combine good with bad, and the right hand side is a lottery that combine good with good, bad with bad. When $\alpha_2^*<\alpha_1^*$, the meaning of the lottery on each side would be the opposite. According to Theorem 1, we can conclude that :

Proposition 1

When F decreases, if a manager is Nth degree risk-averse, she would increase her effort if and only if she is (N+1)$_{th}$-degree risk averse.

Corollary 1

If the effort improves $\widetilde{S_\alpha}$ in terms of NSD, then when F decreases, if a manager is n$_{th}$-degree risk averse, for n=1, 2, 3,……,N, she would increase her effort if and only if she is (N+1)$_{th}$-degree risk averse.

B. How would managerial effort be affected by λ?

Another theorem proposed by Chiu, Eeckhoudt, & Rey(2012) would be helpful to answer the question.

Theorem2

Part 1

If $k_2 > k_1 > 0$, random variables $\tilde{x}$ and $\tilde{y} > 0$, and $\tilde{x}$ has less N_{th}-order risk than $\tilde{y}$, w is a constant, lottery $A_2 = [k_2\tilde{y} + w \;;\; k_1\tilde{x} + w]$, $B_2 = [k_1\tilde{y} + w \;;\; k_2\tilde{x} + w]$, then for an agent whose utility function satisfies $(-1)^n u^n(x) \leq 0$, for n = N, N+1, and for all x, she would prefer B_2 to A_2 if and only if $\frac{-xu^{N+1}(x+w)}{u^N(x)} \leq N$, for all x.

Part 2

If $k_2 > k_1 > 0$, random variables $\tilde{x}$ and $\tilde{y} > 0$, and $\tilde{x} \succcurlyeq_{\mathrm{NSD}} \tilde{y}$, w is a constant, lottery $A_2 = [k_2\tilde{y} + w \;;\; k_1\tilde{x} + w]$, $B_2 = [k_1\tilde{y} + w \;;\; k_2\tilde{x} + w]$, then for any agent whose utility function satisfies $(-1)^n u^n(x) \leq 0$, for all x and all n = 1, 2, 3,……, (N+1,) she would prefer B_2 to A_2 if and only if $\frac{-xu^{n+1}(x+w)}{u^n(x)} \leq n$, for all x and all n = 1, 2, 3,…… N.

The intuition of part 1 is as follows. In lottery A_2, a higher weight is imposed on the more-riskier distribution, therefore it is obvious that lottery A_2 has more N_{th}-order risk than lottery B_2. However, there's a risk apportionment effect that is in favor of A_2. Recall that $\tilde{x}$ and $\tilde{y}$ are positive random variables, so, when they're multiplied by a positive number, the expected value would increase. In lottery A_2, the riskier distribution is multiplied by a greater constant, which means that whenever an agent get a riskier distribution, she would get higher expected value. In the consequence, whether an agent would prefer lottery B_2 depends on her degree of risk aversion. If her degree of risk aversion is low, then the apportionment effect would not be significant enough for her to choose lottery A_2.

If part 1 of the theory holds, then part 2 follows.

Now, we apply the same method as in discussion A. Assume that λ changes from λ_1 to λ_2, $\lambda_1 > \lambda_2$. The original maximization problem would be :

$$Max_{\alpha}[EU\left(\mathrm{F}+\lambda_1\widetilde{S_{\alpha}}\right)-C(\alpha)]$$

And the maximization problem after λ changes would become :

$$Max_{\alpha}[EU\left(\mathrm{F}+\lambda_2\widetilde{S_{\alpha}}\right)-C(\alpha)]$$

Let α_1^* be the solution of the original maximization problem and α_2^* be the solution of the maximization problem after λ changes. Because α_i^* must maximize $EU\left(\mathrm{F}+\lambda_i\widetilde{S_{\alpha}}\right)-C(\alpha)$, applying the previous method we have :

$$EU\left(\mathrm{F}+\lambda_1\widetilde{S_{\alpha_1^*}}\right)+EU\left(\mathrm{F}+\lambda_2\widetilde{S_{\alpha_2^*}}\right)>EU\left(\mathrm{F}+\lambda_1\widetilde{S_{\alpha_2^*}}\right)+EU\left(\mathrm{F}+\lambda_2\widetilde{S_{\alpha_1^*}}\right)\ldots\ldots(4)$$

If we multiply both sides of inequality (3) by 0.5, then each side would looks like a lottery defined in Theorem 2. When $\alpha_2^* > \alpha_1^*$, the left hand side is as lottery B_2, and the right hand side is as lottery A_2. When $\alpha_2^* < \alpha_1^*$, it's the opposite. According to

Theorem 2, we can conclude that :

Proposition 2

If the manager's utility function satisfies $(-1)^n u^n(x) \leq 0$, *for all x and for n = N, N+1, then when λ rises, she would increase her effort if and only if* $\frac{-xu^{N+1}(x+F)}{u^N(x)} \leq N$, *for all x.*

Corollary 2

If the effort improves $\widetilde{S_\alpha}$ *in terms of NSD, and the manager's utility function satisfies* $(-1)^n u^n(x) \leq 0$, *for all x and for all n =1,2,……,N+1, then when λ rises, she would increase her effort if and only if* $\frac{-xu^{n+1}(x+F)}{u^n(x)} \leq n$, *for all x, and for n = 1,2,……,N.*

C. How would managerial effort be affected by an exogenous variable?

Assume that there's an exogenous variable, say, θ, which transform the distribution of the production from $\widetilde{S_\alpha}$ to $\widetilde{T_\alpha}$, where $\widetilde{T_\alpha}$ has more K_{th}-order risk than $\widetilde{S_\alpha}$, for all α. The increase of α would still decrease the N_{th}-order risk of the production, no matter the distribution is $\widetilde{S_\alpha}$ or $\widetilde{T_\alpha}$. The original maximization problem is :

$$Max_\alpha[EU(\mathrm{F} + \lambda\widetilde{S_\alpha}) - C(\alpha)]$$

and the maximization problem after the change of distribution is :

$$Max_\alpha[EU(\mathrm{F} + \lambda\widetilde{T_\alpha}) - C(\alpha)]$$

Applying the same method as in the previous discussions, we have :

$$EU\left(\mathrm{F}+\lambda\widetilde{S_{\alpha_1^*}}\right)+EU\left(\mathrm{F}+\lambda\widetilde{\mathrm{T}_{\alpha_2^*}}\right)>EU\left(\mathrm{F}+\lambda\widetilde{S_{\alpha_2^*}}\right)+EU\left(\mathrm{F}+\lambda\widetilde{\mathrm{T}_{\alpha_1^*}}\right)\ldots(5)$$

Denote the first and second term on the left hand side as term A and term B, and the first and second term on the right hand side as term C and term D. When $\boldsymbol{\alpha_1^*}<\boldsymbol{\alpha_2^*}$, we set term C as the benchmark. We compare the other three terms with term C. Term A is just like imposing an N_{th}-order risk on term C, term B is like imposing a K_{th}-order risk on term C, and term D is like simultaneously imposing an N_{th}-order risk and a K_{th}-order risk on term C.

Concluding the above, the left hand side of inequality (5) is similar to a lottery [Additional K_{th}-order risk ; Additional N_{th}-order risk] and the right hand side is similar to a lottery [0 ; Additional K_{th}-order risk + N_{th}-order risk]. We can see that the left hand side is a lottery that "combines good with bad," while the right hand side is a lottery that "combines good with good, bad with bad."

When $\boldsymbol{\alpha_1^*}>\boldsymbol{\alpha_2^*}$, we set term A as the benchmark, then we can see that the meaning of each side are the opposite. Applying Theorem 1, we have :

Proposition 3-a

When the change of θ increase K_{th}-order risk of production, if a manager is N_{th}-degree risk averse, she would increase her effort if and only if she is $(N+K)_{th}$-degree risk averse.

Proposition 3-b

When the change of θ increase the risk of the distribution of production in terms of KSD, if a manager is n_{th}-degree risk averse, for n = 1,…K and for n = N, she would increase her effort if and only if she is n_{th}-degree risk averse, for all n = (N+1), (N+2,) …(N+K).

Corollary 3-a

If the effort improves $\widetilde{S_\alpha}$ (or $\widetilde{T_\alpha}$) in terms of NSD, then when the change of θ increase K_{th}-order risk of production, if a manager is n_{th}-degree risk averse, for n = 1,2,…,N and for n = K, she would increase her effort if and only if she is r_{th}-degree risk averse, for r = (K+1),……(K+N).

Corollary 3-b

If the effort improves $\widetilde{S_\alpha}$ (or $\widetilde{T_\alpha}$) in terms of NSD, then when the change of θ increase the risk of the distribution of production in terms of KSD, if a manager is n_{th}-degree risk averse, for n = 1,2,…, max{N,K}, she would increase her effort if and only if she is r_{th}-degree risk averse, for all r =1,2,…(N+K).

IV. Conclusion

In conclusion, how the parameter would affect managerial effort depends on the risk attitude of the manager. Provided that the managerial effort improve the distribution of the production in the sense of N_{th}-order risk, when the fixed payment is lowered, the manager would increase her effort if and only if she is $(N+1)_{th}$-degree risk averse ; when the sensitivity of the payoff to the production rises, the manager would increase her effort if and only if her n_{th}-order partial risk aversion is less than n, for n = N and N+1. If an exogenous variable increase the K_{th}-order risk of the production, the manager would increase her effort if and only if she is $(N+K)_{th}$-degree risk averse.

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