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使用賽局理論及學習演算法在裝置對裝置通訊系統中的中央節點選擇

Device-to-Device Central Entity Election using Game Theory and Learning Algorithm

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本論文係廖冠傑君（學號 R01921036）在國立臺灣大學電機工程學系完成之碩士學位論文，於民國 103 年 7 月 8 日承下列考試委員審查通過及口試及格，特此證明。

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摘要

Device-to-Device (D2D) 是一種近距離的通訊服務技術，消耗較少的能量且能有較高的頻譜使用效率，是最近幾年興起的熱門研究主題。在我們的研究中，我們考慮有許多的裝置 (device) 需要從同一個基地台 (Base Station) 下載同樣的資料，較好的方式是將裝置們分成許多的集團 (cluster)，每個集團選擇一個裝置做為中央節點 (central entity) 來接收基地台的資料，再由中央節點廣播資料給其他的裝置。但是中央節點會因此消耗較多的能量所以會有較多的花費 (cost)，也會降低裝置成為中央節點的意願。當所有裝置都是自私且理性的時候，賽局理論是個相當有用的工具來分析這些裝置的行為模式。我們把這樣的問題設計成了一個賽局的問題。為了解決這個問題，我們設計了一個拍賣機制來協助中央節點的選擇最適合的裝置來成為中央節點。在我們的機制設計及賽局理論的分析之下，整個系統可以有許多良好的特性。能達到惟一主宰的納許均衡 (dominating Nash Equilibrium)，所有的裝置會誠實回報他們的花費 (strategy-proofness)，整個系統可以達到最大的系統效能 (maximize social welfare)。另外也分析了個人理性 (individual rationality) 和預算平衡 (budget balance) 達到的條件。個人理性的意思當裝置選擇參加集團的時候參加者的效用 (utility) 必須是非負的。預算平衡的意思則是基地台的收支相減也是非負的。另一方面，

我們證明了多個集團的時候，集中式 (centralized) 系統會是一個非多項式複雜度 (non-polynomial hard) 的問題，為了避開這樣的複雜度，我們考慮的分散式 (distributed) 系統並設計了一個分散式中央節點選擇學習演算法 (distributed central entity election (DCEE) learning algorithm)。這個演算法在我們的改進之後，可以確保整個演算法的結果一定會收斂到純策略 (pure strategy)，且此演算法能協助尋找納許均衡。我們也研究了關於此演算法的其他性質包含預算平衡和個人理性。更進一步的，這種分散式學習演算法可以不只應用在我們的系統之中，更可以應用到各種不同的分散式系統之中。最後我們在模擬結果中印證了我們推導出的理論結果，也說明了 D2D 通訊服務技術在無線網路的系統中仍有極大的潛力。

Abstract

Device-to-Device (D2D) communications provides a proximity service, consuming less energy and having higher spectrum reuse. It has become more and more popular in recent years. In our work, we consider that the devices in a cell request the same data from a base station (BS). The devices will form some clusters to receive data. Every cluster will have one device be central entity. The central entity in a cluster receives the data from the BS, and then broadcasts the data to all other devices in the same cluster. The central entity suffers from the cost of transmit power consumption, which discourages the devices from being the central entity. As the devices are selfish in maximizing their own utility, game theory serve as a powerful technique for analyzing the behavior of the devices. We formulate the selfish and non-cooperative interaction of the devices under the system as a game problem. To solve this problem, we propose a central-entity-election mechanism that motivates the devices to report the true transmission costs, and elects the most appropriate devices as the central entities to reach the maximum system utility (social welfare). On the other way, we prove that the multiple-cluster central entity election is a NP hard problem. To avoid the NP hard problem, we propose the distributed central entity election learning (DCEE) algorithm to form clusters.

We prove the DCEE algorithm can always converge and have many desirable properties as budget balance and individual rationality. In the simulation part, we verify the theoretical analysis in a real LTE system setting. With the proposed mechanism and the simulation results, D2D communications is shown to have the potential to improve the performance of wireless networks.

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Chapter 1

Introduction

Device-to-device (D2D) wireless communication and proximity service is more and more important and has been well discussed in recent years. Like the machine-to-machine (M2M) technology, D2D communications also has the direct connections among devices without a main server. This has the advantages of higher spectrum utilization and power saving. Different from M2M communications, devices in D2D communications may be smaller and have a higher mobility. Central entity[1] for D2D communications could be used to coordinate resource allocation and management. A central entity in a cluster could be one of the devices or the base station (BS).

Sometimes, the devices may have the same objective. They can form a cluster to request the common target. Under this circumstance, a central entity plays a leader role in a cluster. The central entity represents the cluster, communicates with a base station, and relays the data from the BS to other devices. On the other hand, the devices are usually selfish and non-cooperative. Each device is selfish in optimizing its own performance and makes its own decision like joining the cluster and becoming the central entity. The selfish and non-cooperative behavior of the devices may lead severe competition and result in poor system performance. Therefore, game theory is a good mathematical tool for analyzing the non-cooperative system and helping design an incentive mechanism to achieve optimal system performance.

In this paper, we investigate some D2D devices where each device requires the same data from the BS. The devices form clusters and elect central entities. The central entities receive data from the BS by unicasting and then broadcasting the data to the devices in the same cluster. The central entities suffer from the cost of transmit power consumption, which discourages the devices from being the central entity. Although the devices in the cluster have the same objective, they are selfish in becoming or not becoming the central entity in order to maximize their own utility. Regarding the above issues, we use the concept of a mechanism design to design a mechanism for the central entities election for a centralized control system. In a general central election system, the centralized control is a NP hard problem. To reduce the complexity, 2e propose a distributed central entity election (DCEE) learning algorithm for the multiple central entities election scenario. We also use game theory to analyze the selfish behavior of the devices. Our contributions are stated as follows:

1. We first investigate in the simple one cluster system. Considering the private information of the devices on the transmission costs, we propose a truth-revealing central-entity-election mechanism. The proposed mechanism incites the devices to truthfully report their private information on the transmission costs in the unique truth-revealing dominant-strategy Nash equilibrium (strategy-proofness).

2. The election of the central entity maximizes the overall cluster utility (social welfare). In addition, we investigate and propose the condition that each device obtains non-negative utility when joining the cluster (individual rationality) and the condi-

tion that the BS receives non-negative transfer from the devices (budget balance).

3. In the multiple-cluster central entities election system, we prove the optimal solution of a centralized system is a NP hard problem. To reduce the complexity, we propose a distributed central entity election learning (DCEE) algorithm. Though the DCEE algorithm updates the strategy of every device with probability, we still prove the convergence of the DCEE algorithm to ensure that it will always converge to a pure strategy combination with probability 1.

4. We derive many desirable properties of DCEE algorithm. There are some previous work investigate on similar learning algorithm when step size $b \to 0$ as [2] and [3], but we derive the different theoretical results with the previous work.

5. In the simulation, we verify the theoretical analysis and show the above desirable properties in a real LTE system setting.

Chapter 2

Related Work

Seppälä et al. introduce a reliable multicast concept for D2D communication in [4]. They investigate the reliable multicast with HARQ in a D2D cluster. Wang et al. propose a novel joint radio resource and power allocation scheme in a uplink D2D cluster in [5]. Hakola et al. study how a direct-communication device group can improve the performance in a conventional cellular in [6]. They also investigate how devices can form clusters with each other.

Many literatures also study D2D clustering and relay mechanisms. Du et al. investigate the topic that a D2D cluster needs to relay and broadcast data to devices in [7]. They mention that HARQ feedback may have better performance by using a 2-bit ACK/NACK feedback. Zhou, et al. consider a cooperative cluster without a central entity which needs many devices relay data to all other devices in [8]. They give an algorithm to find the optimal solution.

There are also many work study in clustering in wireless ad hoc networks. Chatterjee et al. propose a weighted clustering algorithm for multi-hop packet radio networks, which can dynamically adapt itself with the ever changing topology of ad hoc networks in [9].

Huang et al. design a protocol using split and merge maintain a logical connectivity graph in ad hoc mobile systems in [10]. Badia et al. survey and review the routing algorithms in heterogeneous wireless networks in [11].

Game theory [12] is a useful mathematical method to analyze D2D communications where devices are selfish. The auction and mechanism design is also a useful approach that can be applied in D2D communications. Xu et al. investigate interference-aware resource allocation in D2D communications by a sequential second price auction in [13]. They also propose a reverse iterative combinatorial auction for resource allocation in D2D communications in [14] [15].

Learning algorithm is also a useful tool in a distributed system. Wu et al. investigate a coalition formation game for the energy-efficient uplink resource sharing scheme in [16]. They propose a distributed coalition formation algorithm with the merge-and-split rule and the Pareto order to solve the problem. Mastronarde et al. consider a cooperative cellular D2D network, where devices can use token to exchange some relay data for other users. They propose a simple learning algorithm to learn its optimal cooperation strategy for each device in [17]. Zhou et al. propose a reinforcement learning algorithm for power control in cognitive radio networks in [18] . They model an incomplete-information repeated game and the reinforcement algorithm will converge to the Nash equilibrium.

Some work investigate on the cooperative system for cluster head election in wireless sensor networks. Kang and Thinh propose a distributed cluster head selection algorithm that optimally balances the energy consumption among the sensors in [19]. Kim et al. propose another algorithm to maximize energy efficiency by the frequent information exchange in wireless sensor network in [20]. In order to prevent the attack or jamming from others, Buttyan and Holczer propose a private cluster head election method in wireless sensor networks in [21].

No previous work analyzes how to elect a central entity for a cluster from a game-theoretical point of view. They only consider cooperative scenarios. On the other hand, our work models the non-cooperative scenario of a cluster and takes into account the devices' private information on the transmission costs. In Section 3, we model the framework of the cluster election system.

From Section 4 to Section 6, we consider the simple one cluster problem. We design a mechanism for the central entity election problem in Section 4, propose an auction game for solving the problem in Section 5, and analyze the Nash equilibrium and some properties in Section 6.

Furthermore, we investigate in multiple-cluster central entities in Section 7 to Section 10. In Section 7, we prove the optimal solution of the multiple-cluster central entity election should be a NP hard problem for a centralized system, and we propose a distributed central entity election (DCEE) learning algorithm in Section 8. The proposed DCEE algorithm has more properties such as convergence, budget balance, individual rationality, which will be discussed in Section 9. When step size $b \to 0$, we also compare our different theory results with [2] and [3] in Section 10.

In the simulation Section 11, we verify our theoretical part. Finally, we compare some different central entity election approaches with our proposed approaches and show it outperforms the other approaches with a better performance.

Chapter 3

D2D System Framework

3.1 System Model

We assume there are n selfish and non-cooperative devices in a cellular network. Each device is denoted by device D_i where $i = 1, 2, ..., n$. The devices request the same data from the base station (BS). They prefer to form some clusters to receive the data rather than a direct link connect to the BS for every device. Every cluster elects one of the devices as the central entity. The BS first unicasts the data to the central entity. Then the central entity broadcasts the data to the other devices in the same cluster. We can choose $\mathcal{H} \subseteq \mathcal{D}$ as the set of central entities, every central entity can form a cluster. Every other device may join the cluster with a highest SINR with central entity in order to receive data, and the device does not join any cluster that will not receive the data. In any cluster, like the Equation (3.1), the cluster incurs more cost

$$C_i = f(d_{BS,i}) + g(d_{i,j|j \in N_i}) \tag{3.1}$$

The function $f(d_{BS,i})$ represents the cost of the transmit power consumption from the BS to the central entity, and the BS will transfer this cost to the central entity. The function $f(d_{BS,i})$ is related to the distance between the BS and device D_i. It should be an increasing and convex function. On the other hand, the function $g(d_{i,j|j\in N_i})$ represents the cost of broadcast power consumption from the central entity to the other devices. The function $g(d_{i,j|j\in N_i})$ is related to the distance between devices D_i and D_j, $j = 1, 2, ..., N_i$, where N_i is the cluster members of central entity i's cluster. It should be an increasing and convex function as well. In a special case, if $N_i = \phi$, $g(d_{i,j|j\in N_i}) = 0$, each device can calculate its cost of the transmit power consumption. On the other hand, the BS and each device cannot calculate cost of the transmit power consumption of any other device. Therefore, the cost $g(d_{i,j|j\in N_i})$ is the private information for device i and j, and the cost $f(d_{BS,i})$ is also the private information for the BS and device i.

3.2 User's Utility

When receiving the data, each device is assumed to obtain utility μ which can be considered as the throughput. Note that the cluster needs to elect a central entity to first receive the data from the BS and then relay the data to the other devices. If device D_i becomes the central entity D_{CE}, the cluster incurs the cost C_i. We can formulate the utility function of each device D_i as follows:

$$\bar{u}_i = \begin{cases} \mu - C_{CE} = \mu - C_i & \text{if } i = CE \\ \mu & \text{otherwise} \end{cases} \tag{3.2}$$

We also define the overall cluster utility, i.e., the social welfare [22], as follows.

Definition 1. [social welfare] *The overall cluster utility (social welfare) is the sum of the* *utility of all devices minus the cost of the central entity. Mathematically, social welfare is* $\sum_{i=1}^{n} \bar{u}_i(k)$.

Chapter 4

Central-Entity-Election Mechanism For One Cluster System

In this section, assume that devices are close enough to listen to each other, so they are able to form a cluster and only need to elect one central entity, denoted by device D_{CE}, as shown in Fig. 4.1. With the assumption for one cluster system, every device can receive the data. We can derive the social welfare as $\sum_{i=1}^{n} \bar{u}_i(k) = n\mu - C_{CE}$. The cost function can be rewritten as follows:

$$C_i = f(d_{BS,i}) + g(d_{i,j|\forall j \neq i}). \tag{4.1}$$

The cost $g(d_{i,j|\forall j \neq i})$ and $f(d_{BS,i})$ are also the private information for themselves.

We design a mechanism to elect the central entity in the cluster. In the mechanism, the BS first announce a charge parameter λ that will be used to define the transfers later. The BS requires each device D_i to report its private transmission cost C_i. The reported cost is denoted by S_i. Note that $S_i = C_i$ only in truth telling. The BS rearranges the order of the

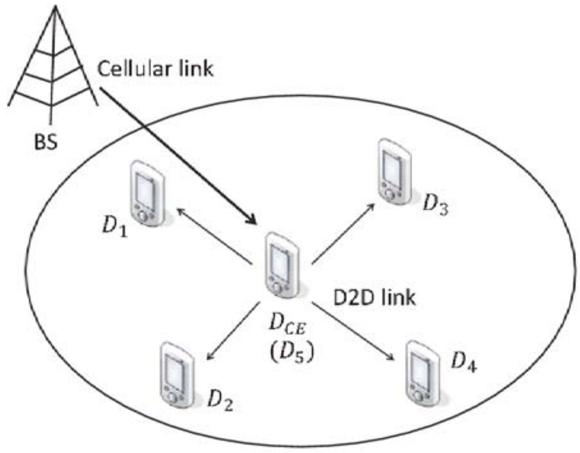

Figure 4.1: A downlink D2D cluster in a cellular network.

received cost reports such that $S_{j_1} \leq S_{j_2} \leq ... \leq S_{j_n}$. The BS elects device D_{j_1} with the lowest reported cost S_{j_1} as the central entity, i.e., $D_{CE} = D_{j_1}$. To motivate the devices to become the central entity, the BS gives the central entity the transfer T_{CE} in (4.2), and charges any other device the transfer T_{UE} in (4.3),

$$T_{CE} = \frac{\lambda S_{j_2}}{1+\lambda} \tag{4.2}$$

$$T_{UE} = \frac{S_{j_1}}{1+\lambda} \tag{4.3}$$

where λ is the charge parameter that the BS announces in the beginning of the mechanism.

Our proposed central-entity-election mechanism is summarized in Table 4.1.

Table 4.1: Central-Entity-Election Mechanism

Step 1.	The BS announces a charge parameter λ.
Step 2.	Each device D_i reports its cost S_i to the BS. Note that $S_i = C_i$ only in truth telling. The BS rearranges the order of the received cost reports such that $S_{j_1} \leq S_{j_2} \leq ... \leq S_{j_n}$.
Step 3.	The BS elects device D_{j_1} with the lowest reported cost S_{j_1} as the central entity, i.e., $D_{CE} = D_{j_1}$.
Step 4.	The BS gives transfer T_{CE} to the central entity and charges the transfer T_{UE} from any other devices, where T_{CE} and T_{UE} are given respectively in (4.2) and (4.3).

We can rewrite the utility function of each device D_i under the proposed mechanism as follows:

$$U_i = \begin{cases} \mu + T_{CE} - C_{CE} & \text{if } i = CE \\ \mu - T_{UE} & \text{otherwise} \end{cases} \tag{4.4}$$

All of the notations are summarized in Table 4.2.

Table 4.2: Notations

D_i	Devices in the cluster, $i = 1, 2, ..., n$.
D_{CE}	The central entity of the cluster, where $CE \in \{1, 2, ..., n\}$.
C_i	The incurred cost of transmit power consumption when device D_i becomes the central entity.
μ	The utility of receiving the data.
T_{CE}	The transfer from the BS to the central entity D_{CE}. T_{CE} is given in (4.2).
T_{UE}	The transfer from any other devices to the BS. T_{UE} is given in (4.3).
λ	The charge parameter set by the BS.

Chapter 5

Auction Game in Mechanism

Since all devices are selfish and non-cooperative, they may not report the true transmission cost to the BS in general. We apply game theory to construct a game model for the proposed mechanism. We will further analyze the game model in the next section. The constructed game, denoted by $\mathbf{G} = (\mathbf{N}, \mathbf{C}, \mathbf{S}, \mathbf{U})$, has four main components, each of which is specified as follows.

1. Player set $\mathbf{N} = \{1, 2, ..., n\}$: Each device D_i, $i \in \{1, 2, ..., n\}$ in the cluster is a player in the game.

2. Cost function set $\mathbf{C} = \{C_1, C_2, ..., C_n\}$: When device D_i becomes the central entity $D_{CE} = D_i$, device D_i has an additional transmission cost C_i which is the private information.

3. Strategy space $\mathbf{S} = \{S_1, S_2, ..., S_n\}$: Each device D_i reports its private transmission cost S_i to the BS. Therefore, the strategy of each device is S_i. Note that $S_i = C_i$ only in truth telling.

4. Utility function set $\mathbf{U} = \{U_1, U_2, ..., U_n\}$: The utility of each device D_i under the

proposed mechanism is given in (4.4).

With the components above, we can treat the proposed game **G** as a *Auction Game*. Devices D_i, where $i \in$ **N**, are the bidders, the costs **C** are private information, and the reported costs **S** are the bid prices, and becoming the central entity is the bid object for devices. The central entity receives the transfer $T_{CE} = (\lambda S_{j_2})/(1+\lambda)$, which is related to the second lowest bid S_{j_2}. The other device pays the transfer $T_{UE} = S_{j_1}/(1+\lambda)$ related to the lowest bid S_{j_1}. The proposed game model is summarized in Table 5.1.

Table 5.1: Auction Game in Mechanism

	Game model $\mathbf{G} = (\mathbf{N}, \mathbf{C}, \mathbf{S}, \mathbf{U})$
Player set	$\mathbf{N} = \{1, 2, ..., n\}$. The devices are players in the game.
Cost function set	$\mathbf{C} = \{C_1, C_2, ...C_n\}$. The incurred transmission cost when the device becomes the central entity. The cost is private information.
Strategy space	$\mathbf{S} = \{S_1, S_2, ..., S_n\}$. Each device reports its cost to the BS.
Utility function set	$\mathbf{U} = \{U_1, U_2, ..., U_n\}$. The utility of each device is given in (4.4).

Chapter 6

Analysis – the Equilibrium and the Desirable Properties

We prove that the proposed mechanism incites each device to truthfully report its cost since truthfully reporting the cost always brings the highest utility.

Theorem 1. *For each device D_i, truthfully reporting the cost $S_i = C_i$ is the unique dominant strategy that maximizes the utility.*

Proof. For any device D_i, we know that the utility U_i is independent of its strategy S_i. When device D_i plays the truthful strategy $S_i = C_i$, we consider the following two situations:

1. Device D_i plays $S_i = C_i$ and is elected as the central entity.

2. Device D_i plays $S_i = C_i$ and is not elected as the central entity.

In situation 1, we know that $C_{CE} = C_i = S_{j_1}$. To show that device D_i cannot obtain higher utility by deviating its strategy to not become the central entity, we need to show that

$$U_i = \mu + T_{CE} - C_i \geq \mu - T'_{UE} \iff \mu + \frac{\lambda S_{j_2}}{1+\lambda} - S_{j_1} \geq \mu - \frac{S_{j_2}}{1+\lambda} \iff S_{j_2} \geq S_{j_1} \tag{6.1}$$

which is always true.

In situation 2, we know that $C_i = S_{j_k}$ for some $k \geq 2$. To show that device D_i cannot obtain higher utility by deviating its strategy to become the central entity, we need to show that

$$\begin{aligned} U_i = \mu - T_{UE} \geq \mu + T'_{CE} - C_i &\iff \mu - \frac{S_{j_1}}{1+\lambda} \geq \mu + \frac{\lambda S_{j_1}}{1+\lambda} - S_{j_k} \\ &\iff S_{j_k} \geq S_{j_1}\ (k \geq 2) \end{aligned} \tag{6.2}$$

which is always true.

So we know that it is a dominant strategy for each device D_i to play the truthful strategy $S_i = C_i$. Furthermore, when a device plays $S_i > C_i$ or $S_i < C_i$, it is possible to meet the situation 1 or situation 2 respectively, as the cost function of other devices are private information, we say the truthful strategy $S_i = C_i$ is the unique dominant strategy. □

In Theorem 1, each device D_i plays the unique truthful dominant strategy $S_i = C_i$. A direct result of the dominant-strategy Nash equilibrium [23] is given as follows.

Proposition 1. *The cluster reaches the unique truth-telling dominant-strategy Nash equilibrium,* $\mathbf{S} = \mathbf{C}$.

Proposition 1 states that the unique dominant-strategy Nash equilibrium that our proposed mechanism reaches has a desirable property of truth telling. In addition to this truth-telling property, the proposed mechanism achieves more desirable properties at the unique dominant-strategy Nash equilibrium, such as the social welfare maximization, individual rationality [22], and (weak) budget balance [22]. The definition of the social welfare is given in Definition 1 in Section 3.2. The definitions of individual rational and (weak) budget balance are given as follows:

Definition 2. [individual rationality] *A mechanism achieves individual rationality if and only if all devices join the mechanism and obtain the non-negative utility. Mathematically,* $U_i \geq 0 \, \forall i \in \mathbf{N}$.

Definition 3. [budget balance] *A mechanism achieves (weak) budget balance if and only if the total transfer of the BS, i.e., the transfers the BS receiveing from the non-central-entity devices minus the transfers the BS paying the central entity, is non-negative. Mathematically,* $(n-1)T_{UE} - T_{CE} \geq 0$.

We prove the above properties of the social welfare maximization, individual rationality, and (weak) budget balance as follows. We first show in a lemma that the utility of the central entity is higher than those of the other devices. This lemma helps prove the property of individual rationality.

Lemma 1. *The utility of the central entity is greater than or equal to that of the other devices in the unique dominant-strategy Nash equilibrium.*

Proof.

$$\begin{aligned} U_{CE} \geq U_i, \, i \neq CE \iff & \mu + \frac{\lambda C_{j_2}}{1+\lambda} - C_{j_1} \geq \mu - \frac{C_{j_1}}{1+\lambda} \iff \frac{\lambda C_{j_2}}{1+\lambda} \geq \frac{\lambda C_{j_1}}{1+\lambda} \\ \iff & C_{j_2} \geq C_{j_1} \end{aligned} \tag{6.3}$$

which is true. □

Theorem 2. *The social welfare is maximized in the unique dominant-strategy Nash equilibrium.*

Proof. By Definition 1, the social welfare is maximized when the cost is the lowest. Since the proposed mechanism elects the device with the lowest cost as the central entity, the social welfare is maximized. □

Theorem 3. *The proposed mechanism achieves individual rationality when the charge parameter satisfies* $\lambda \geq (C_{j_1} - \mu)/\mu$

Proof. By Lemma 1, we have $U_{CE} \geq U_i \ \forall i \neq CE$. We only have to show that $U_i \geq 0$ $\forall i \neq CE$ as follows:

$$U_i \geq 0 \iff \mu - \frac{C_{j_1}}{1+\lambda} \geq 0 \iff \frac{C_{j_1}}{1+\lambda} \leq \mu \iff 1+\lambda \geq \frac{C_{j_1}}{\mu} \iff \lambda \geq \frac{C_{j_1} - \mu}{\mu} \tag{6.4}$$

which is the condition given. □

Theorem 4. *The proposed mechanism achieves (weak) budget balance when the charge parameter satisfies* $\lambda \leq (n-1)C_{j_1}/C_{j_2}$.

Proof. The total transfer of the BS is

$$(n-1)T_{UE} - T_{CE} = (n-1)\frac{C_{j_1}}{1+\lambda} - \frac{\lambda C_{j_2}}{1+\lambda} \tag{6.5}$$

The total transfer of the BS is non-negative if and only if

$$(n-1)T_{UE} - T_{CE} \geq 0 \iff (n-1)\frac{C_{j_1}}{1+\lambda} - \frac{\lambda C_{j_2}}{1+\lambda} \geq 0 \iff \lambda \leq \frac{(n-1)C_{j_1}}{C_{j_2}} \tag{6.6}$$

which is the condition given. □

Combining Theorems 3 and 4, we may also state the condition that the proposed mechanism achieves both individual rationality and (weak) budget balance.

Proposition 2. *The proposed mechanism achieves both individual rationality and (weak) budget balance when the charge parameter satisfies* $(C_{j_1} - \mu)/\mu \leq \lambda \leq (n-1)C_{j_1}/C_{j_2}$.

Although the BS is not easy to decide a proper charge parameter λ before the auction, if we assume the data utility μ is important enough as $\mu \geq C_{j_1}$, then the BS can choose $\lambda = 0$ to satisfy both individual rationality and budget balance. In fact, the Vickrey Clarke Groves (VCG) auction mechanism [24] can also apply in our system. But in the result of VCG mechanism, the BS will pay transfer for the central entity. Compare the results of our mechanism and VCG mechanism, we can know that VCG mechanism is a special case in our mechanism of $\lambda \to \infty$, and the VCG mechanism always cannot satisfy the budget balance.

Chapter 7

Extension From One-Cluster to Multiple-Cluster System

7.0.1 System Model

In the previous sections, we proposed a centralized mechanism to achieve the optimal solution in the one cluster central entity election system. We will extension the scenario to a general multiple-cluster scenario. In the following research, we do not assume the devices are close enough to listen to each other, so they may form more than one clusters to receive the data.

7.1 Centralized System Analysis

In a centralized system, the BS plays an important role to choose the central entities. We prove an important centralized system theorem as follows:

Theorem 5. *In a general centralized system, the optimal solution to elect the central entities is a NP hard problem.*

Proof. To show that optimal solution is a NP hard problem, we should compare this problem with a well-known NP hard problem. Consider a special scenario with data utility $\mu >> f(d_{BS,i}) >> g(d_{i,j|j\in N_i})\forall i$ and $f(d_{BS,i}) = f(d_{BS,i_2}), g(d_{i,j|j\in N_i}) = g(d_{i_2,j|j\in N_i})\forall i, i_2$. Data utility μ is a dominating term in the social welfare, so the optimal solution should let every device receive data. The cost function terms $f(d_{BS,i}) >> g(d_{i,j|j\in N_i})$ means that the cost from BS to central entity is much more than that of D2D broadcast, so the optimal solution also needs to minimize the central entity numbers. In a graph theory, the problem which finds minimum number of nodes such that their neighbors coverage whole the nodes is called ”minimum node cover” problem, which is a well-known NP hard problem. As a special scenario is a NP hard problem, the general problem that finding optimal solution of electing the central entities is also a NP hard problem. □

We extend our proposed mechanism design in Section 4 to the multiple-cluster system. The transfers can be designed as follows:

$$T_{CE,j} = \frac{\lambda}{1+\lambda}\sum_{i\in\mathcal{H}'} C_i' - \sum_{i\in\mathcal{H}, i\neq j} C_i \tag{7.1}$$

$$T_{UE,j} = \frac{1}{1+\lambda}\sum_{i\in\mathcal{H}_i} \tag{7.2}$$

where $T_{CE,j}$ represents that the device j is a central entity, and $T_{UE,j}$ represents the device j is not a central entity. The term $\sum_{i\in\mathcal{H}'} C_i'$ means the system cost if the CE_j becomes UE_j. $\sum_{i\in\mathcal{H},i\neq j} C_i$ is the system cost of the other CEs, and $\sum_{i\in\mathcal{H}_i}$ is the system cost.

In such transfer mechanism design, the system also possesses the properties of the optimal social welfare, truth-telling, individual rationality and budget balance, but we don’t prove them here. Consider the Theorem. 5, the system complexity will be NP hard and may cause an excessive loading to the BS; therefore, it is better to find a heuristic solution with regard to the application. For example, if the devices locate in a limited region, they are assumed to form at most three clusters. By utilizing the brute-force algorithm, the system complexity is $T(P) = (\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3}) = \Theta(n^3)$, can be effectively reduce. Though the mechanism design still works, we consider another approach to reduce the system complexity. We will investigate in a distributed system to solve the election problem of the multiple-cluster central entities.

7.2 Distributed System Analysis

In a distributed D2D communication system, any device will make decisions in a distributed manner. As the devices are selfish and non-cooperative, their decisions may have system performance unsatisfactorily fail to achieve the Nash equilibrium. We design the multiple-cluster central entities election distributed system as follows:

7.2.1 User's Utility

When receiving the data, each device is assumed to obtain utility μ which can be considered as the throughput. Note that the central entity has a cost C_i. We assume that the BS set a price $T < \mu$ as the transfer. The BS charges T from every device and pays $|N_i| \times T$ to the central entities, where $|N_i|$ is the devices number which connect to the central entity D_i. So we can rewrite the utility function as follows:

$$\bar{u}_i(k) = \begin{cases} \mu + |N_i| \times T - C_i & \text{if i is central entity} \\ \mu - T & \text{if i is a cluster member} \\ 0 & \text{otherwise} \end{cases} \tag{7.3}$$

In this distributed system, we assume that every device has two strategies "yes" and "no" to choose. The devices choosing "yes" will become central entities, whereas the other devices choosing "no" will connect to the central entity will highest SINR for receiving data and the central entity cannot reject the connection request. However, if a device chooses "no" and it cannot connect to any central entity, it will not receive the data and thus has utility 0. In the transmission process of the clusters, we can use time division duplex(TDD) to avoid the interference. We write the action combination as $\underline{\mathbf{a}} = (a_1, a_2, ..., a_n) \in \mathcal{A}$, which $\mathcal{A} = \{yes, no\}^n$, so that a action combination will correspond to an unique communication scenario. We use $u_i(\underline{\mathbf{a}})$ represents the utility of device i in such scenario, and we summarize the game model of the distributed system in table. 7.1.

Table 7.1: Multiple Central-Entity Election Game Model in Distributed System

	Game model $\mathbf{G} = (\mathbf{N}, \mathbf{C}, \underline{\mathbf{a}}, \underline{\mathbf{u}})$
Player set	$\mathbf{N} = \{1, 2, ..., n\}$. The devices are players in the game.
Cost function set	$\mathbf{C} = \{C_1, C_2, ...C_n\}$. The incurred transmission cost when the device becomes the central entity. The cost is private information.
Strategy space	$\underline{\mathbf{a}} = (a_1, a_2, ..., a_n) \in \mathcal{A}$, which $a_i \in \{yes, no\}$.
Utility function set	$\underline{\mathbf{u}} = \{u_1(\underline{\mathbf{a}}), u_2(\underline{\mathbf{a}}), ..., u_n(\underline{\mathbf{a}})\}$. The utility of each device is given in (7.3).

To analyze this problem, we will use game theory to analyze the behavior of the devices, and the Nash equilibrium [23] is defined as follows:

Definition 4. [Nash equilibrium] *An action combination* $\underline{\mathbf{a}} = (a_1, a_2, ..., a_n) \in \mathcal{A}$ *is Nash equilibrium (NE) if and only if* $u_i(\underline{\mathbf{a}}) \geq u_i(a_i', a_{-i}) \forall i$*, which* $a_{-i} = (a_1, a_2, ..., a_{i-1}, a_{i+1}, ..., a_n)$.

In other words, no one can change its strategy unilaterally and has better utility in the Nash equilibrium.

Then we prove an important theorem for the distributed system as follows:

Theorem 6. *A distributed D2D communication system may not admitting any pure Nash equilibrium.*

Proof. To show that a distributed D2D communication system may not have an Equilibrium, we only need to give an example that does not admit any pure NE. We propose a rotationally symmetric scenario as Fig. 7.1, where the circles represent the D2D broadcast area of D_1, D_2, and D_3. Considering the three devices D_1, D_2, and D_3, if D_1 and D_3 are both central entities but D_2 is not, then D_2 will prefer to connect with D_1 due to the closer distance and higher SINR, and so do D_3, D_4, and D_5 as well as D_5, D_6, and D_1. We assume the detailed parameters are data utility $\mu = 5$, price $T = 4$, cost function $C_1 = C_3 = C_5 = 10$, and $C_2 = C_4 = C_6 = 100$ are constants. Recall the utility function of Equation (7.3), we have

$$\bar{u}_i(k) = \begin{cases} 5 + |N_i| \times 4 - C_i & \text{if } a_i = yes \\ 5 - 4 & \text{if } a_i = no \text{ and receives data} \\ 0 & \text{otherwise} \end{cases} \tag{7.4}$$

We know that $\forall i \; a_i = no$ always has a non-negative utility. For D_2, D_4, and D_6, they have too much cost C_i and will have negative utility if choosing $a_i = yes$. As a result, we know that D_2, D_4, and D_6 have dominating strategy $a_2 = a_4 = a_6 = no$. For D_1, D_3, and D_5, if none of them or three of them are central entities, all of them would like to deviate this strategy. If only one of them is the central entity, without loss of generality, assume

D_1 is the only central entity, then D_5 would like to become a central entity to have better utility. If two of them are both central entities, without loss of generality, assume D_1 and D_5 are central entities, then D_1 will have a negative utility and prefer not to be a central entity to have better utility. After the discussion above, we know that the topology Fig. 7.1 has no pure strategy Nash equilibrium in any condition.

□

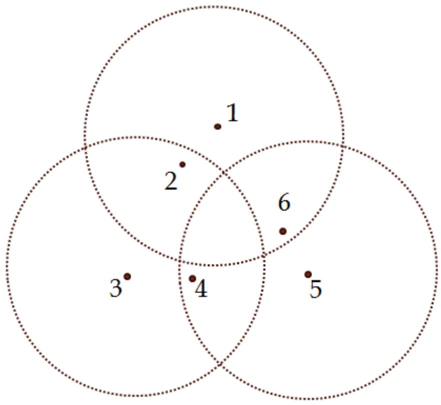

Figure 7.1: An example which does not admit any Nash equilibrium

Although a distributed system may not submit any Nash equilibrium, we still investigate finding the Nash equilibriums. We propose a Distributed Central Entity Election Algorithm to help to find Nash equilibriums in Section 8.

Chapter 8

Distributed Central Entity Election(DCEE) Algorithm

8.1 Distributed Central Entity Election(DCEE) Algorithm

In this section, we will propose a distributed learning algorithm to give an approach to form the clusters. Device i will have a probability $p_i(k)$ and utility $u_i(k)$ in time instant k, the probability represent whether device i will become central entity, and $p_i(k+1)$ will update from $p_i(k)$ and utility function $u_i(k)$ at time instant k. When the probability $p_i(k)$ of all the devices converge to 0 or 1, the algorithm will stop.

We propose the distributed central entity election (DCEE) Algorithm as the table. 8.1

In fact, the stopping criterion **Step 5** cannot be met in finite times, we can use $(p_i(k+1) \in [0, \epsilon_c] \cap [1-\epsilon_c, 1])\forall i$ as the stopping criterion, where the ϵ_c should be a small positive constant.

$0 < b < 1$ is the step size of the updating rule and u_i is normalized utility function. We normalize the utility $\{u_i\}$ as

Table 8.1: Central-Entity-Election Learning Algorithm

Step 1.	Set the initial probability $0 < p_i(0) < 1, \forall i$.
Step 2.	At every time instant k, each device chooses whether to be central entity according to its action probability $p_i(k)$. Thus, the device i chooses action $a_i = yes$ at instant k with probability $p_i(k)$, and chooses action $a_i = no$ with probability $1 - p_i(k)$.
Step 3.	Each player can obtain a normalized utility $u_i(k)$ based on the set of all actions at time instant k.
Step 4.	Each player updates its action probability according to the rule $$\begin{aligned} p_i(k+1) &= p_i(k) + bu_i(k)(1 - p_i(k)) \text{ if } a_i(k) = yes \\ p_i(k+1) &= p_i(k) - bu_i(k)p_i(k) \text{ if } a_i(k) = no, \\ & i = 1, 2, ..., n, \end{aligned} \quad (8.1)$$
Step 5.	Stopping criterion met $(p_i(k+1) = p_i(k))\forall i$; else, go to **Step 2**.

$$u_i(k) = \begin{cases} \hat{u}_i(k) & \text{if } \hat{u}_i(k) \geq 0 \\ 0 & \text{if } \hat{u}_i(k) < 0 \end{cases} \quad (8.2)$$

$$\text{where } \hat{u}_i(k) = \frac{\bar{u}_i(k) + \epsilon_1 \times max_{1 \leq t \leq k}\{|\bar{u}_i(t)|, \epsilon_2\}}{(1 + \epsilon_1) \times max_{1 \leq t \leq k}\{|\bar{u}_i(t), \epsilon_2|\}} \quad (8.3)$$

where $\bar{u}_i(k)$ is the original utility of user i in time instant k, $\hat{u}_i$ normalize the utility function to $[-1, 1]$, and u_i truncate the minus part let utility region is $[0, 1]$. The purpose of adding a small positive real number ϵ_1 is to ensure that when a device choose strategy $a_i(k) = no$, $u_i(k)$ always takes a positive value, which will help the convergence of this algorithm. The ϵ_2 ensures the denominator will not become 0, and we can ignore the ϵ_2 most times. We show the two main properties of the normalized utility function as follows:

Property 1: $\forall i, k \in \mathbb{N}$, we have $u_i(k) \in [0, 1]$, and $u_i(k)$ is an increasing function of $\bar{u}_i(k)$ when $max_{1 \leq t \leq k}\{|\bar{u}_i(t)|\}$ is constant.

Proof. The $u_i(k)$ only takes non-negative values, so $u_i(k) \geq 0$. On the other way, recall the equation of $\hat{u}_i(k)$, $\bar{u}_i(k) \leq max_{1 \leq t \leq k}\{|\bar{u}_i(t)|\}$, so $\hat{u}_i(k) \leq 1$, which implies $u_i(k) \leq 1$.

The increasing property is also easy to know in the equation of $\hat{u}_i(k)$. □

Property 2: $\forall i, k$, we will have $(u_i(k) \in [\epsilon_0, 1] | a_i(k) = no)$, where $\epsilon_0 = \epsilon_1/(1+\epsilon_1)$.

Proof. First, we know when $a_i = no$, device i will have only two case of receiving or not receiving data. $\bar{u}_i(k)$ will be 0 or $(\mu - T)/max_{1 \le t \le k}\{|\bar{u}_i(t)|\}$. Recall the equation of $\hat{u}_i(k)$, $\bar{u}_i(k) = 0$ is corresponding to the $\hat{u}_i(k) = \epsilon_0$, with the increasing property of $\hat{u}_i(k)$, we can know that $\epsilon_0 \le \hat{u}_i(k) \le 1$. □

8.2 The Convergence of the DCEE Algorithm

In this section, we prove the DCEE algorithm will converge to a pure strategy combination with probability 1. Before beginning to prove the convergence, we need some useful lemmas.

Lemma 2.

$$\forall 0 < a, \epsilon < 1, \exists \rho > 0 \; s.t. \prod_{k=0}^{\infty} (1 - a(1-\epsilon)^k) > \rho \tag{8.4}$$

Proof. We choose a positive integer $k_0 > ln(\epsilon/a)/ln(1-\epsilon)$, which implies $1 > (a(1-\epsilon)^{k_0})/\epsilon$, so

$$\begin{aligned}
&\prod_{k=0}^{\infty} (1 - a(1-\epsilon)^k) = \prod_{k=0}^{k_0-1} (1 - a(1-\epsilon)^k) * \prod_{k=k_0}^{\infty} (1 - a(1-\epsilon)^k) \\
>&\rho_0 * (1 - \sum_{k=k_0}^{\infty} (a(1-\epsilon)^k)) = \rho_0 * (1 - \frac{a(1-\epsilon)^{k_0}}{\epsilon}) > 0
\end{aligned}$$

where the proof is derived by using an easy inequality $(1-x)(1-y) > (1-x-y) \forall 0 < x, y < 1$, and $\rho_0 = \prod_{k=0}^{k_0-1} (1 - a(1-\epsilon)^k)$ is a constant, we can choose constant $\rho =$

$\rho_0 * (1 - a(1-\epsilon)^{k_0}/\epsilon)$ to complete the proof. $\square$

Lemma 3. *In a markov chain, when a state X_0 has a probability $\rho > 0$ to jump in an absorbing state, this state X_0 is a transient state(i.e. the markov chain comes back to state X_0 infinite times with probability 0).*

Proof. Define that markov chain as $X(t)$, and $X(0) = X_0$, we will prove by contradiction. Assume that markov chain has a probability $\rho_0 > 0$ to visit the state X_0 infinite times, let T be the number of visits to state X_0 given $X(0) = X_0$. Calculate the expectation of T, we have

$$E[T|X(0) = X_0] \geq \rho_0 * \infty = \infty \tag{8.5}$$

On the other way, we know state X_0 has a probability ρ to jump in an absorbing state, we define

$$\rho_1 = Prob\{\exists i \in \mathbb{N}, X(i) = X_0 | X(0) = X_0\} \tag{8.6}$$

which ρ_1 means the probability to visit X_0 again, so we know $\rho_0 + \rho_1 \leq 1$, which is $\rho_1 \leq 1 - \rho_0$. Then we can have the distribution of T with $P(T = k) = \rho_1^k(1-\rho_1)$ $\forall k \geq 0, k \in \mathbb{Z}$, which is similar to a geometrically distribution, so the expectation of T is

$$E[T|X(0) = X_0] = \sum_{i=0}^{\infty} i\rho_1^i(1-\rho_1) = \sum_{i=0}^{\infty}(i+1)\rho_1^i(1-\rho_1) - \sum_{i=0}^{\infty}\rho_1^i(1-\rho_1)$$
$$= \sum_{j=1}^{\infty} j\rho_1^{(j-1)}(1-\rho_1) - 1 = \frac{1}{1-\rho_1} - 1 \tag{8.7}$$

Compare $E[T|X(0) = X_0]$ in Equation (8.5) and (8.7), we have a contradiction. $\square$

To prove the convergence of the DCEE algorithm, we need a corollary first.

Corollary 1: Consider any one of person $a_i(k)$ in the learning algorithm, the strategy of $a_i(k)$ will converge to yes or no with probability 1, i.e. $p_i(k)$ will also converge to 0 or converge to 1 with probability 1.

Proof. We will prove this corollary using utility properties 2, Lemma 2, and Lemma 3. First, we will construct a markov chain of the p_i. In fact, p_i can take uncountable infinite values, and transition probability will also affect by $\mathbf{a}(k) = (a_1(k), a_2(k), ..., a_n(k))$ so there are too many states. We will merge all the states with similar properties to some compound states. Whatever condition $\mathbf{a}(k)$ is in, if $p_i(k) \in [0, 1 - (b\epsilon_0/2)]$, they are compound state 1. On the other way, if $p_i(k) \in (1 - (b\epsilon_0/2), 1]$, they are compound state 2. And we define a special absorbing compound state 0 when $p_i(k) \in [0, 1 - (b\epsilon_0/2)]$ and satisfies $\forall k_1 \geq k, a_i(k_1) = no$. We say that is compound state 0.

Observe the compound state 0, we know $p_i(k)$ will converge to 0 because $\forall k_1 \geq k, a_i(k_1) = no$ and $\forall k_1, u_i(k_1) \geq \epsilon_0$ by utility property 2. The compound state 0 will not have any probability to transit its state to compound state 1 or compound state 2, so we call the compound state 0 as an absorbing state.

Then we observe the compound state 2, which is a very small region when $p_i(k) \in (1 - (b\epsilon_0/2), 1]$. Consider a case of $p_i(k) \in (1 - (b\epsilon_0/2), 1]$ and $a_i(k) = no$, we know that $u_i(k) \geq \epsilon_0$ by utility property 2. Then we can derive $p_i(k+1) = p_i(k) * (1 - bu_i(k)) \leq 1 - bu_i(k) \leq 1 - b\epsilon_0$, so $p_i(k+1)$ will transit into compound state 1(or absorbing compound state 3).

Then we observe the compound state 1, $p_i(k) \leq 1 - (b\epsilon_0/2)$ implies the probability $a_i(k) = no$ is at least $1 - p_i(k) \geq 1 - (1 - (b\epsilon_0/2))$. Therefore, we can derive the probability for device i satisfying $\forall k_1 \geq k, a_i(k_1) = no$ as

$$P(\forall k_1 \geq k, a_i(k_1) = no) \geq (1 - (1 - \frac{b\epsilon_0}{2}))(1 - (1 - \frac{b\epsilon_0}{2})(1 - \epsilon_0))...$$
$$= \prod_{k=0}^{\infty} (1 - (1 - \frac{b\epsilon_0}{2})(1 - \epsilon_0)^k) > \rho \tag{8.8}$$

where we can choose a positive constant ρ to satisfy (11) by Lemma 2. This fact implies that in any state in the compound state 1, there is always a transition probability to transit it state to absorbing state 0 with probability at least ρ. We draw a state graph sketch map as Fig. 8.1.

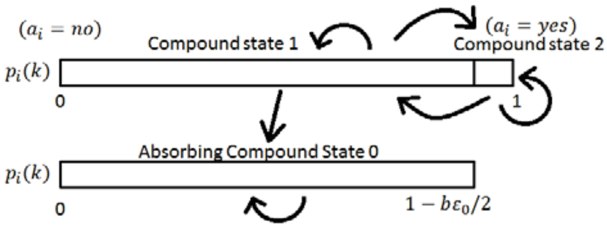

Figure 8.1: Compound States Sketch Map

Now we will prove corollary 1 by contradiction. Assume the $a_i(k)$ will not converge to yes or no, which implies $p_i(k)$ also cannot converge to 0 or 1. So it is clear that the transition state can not visit absorbing state 0. We consider the three different case as follows:

1. Assume the transition process visits the absorbing compound state 0. Then we know that transition process will converge to $p_i = 0$ and $a_i = no$.

2. Assume the transition process never visits compound state 0, and it meets compound state 1 infinite times. Recall the Lemma 3, we know that compound state 1 has a probability to transit into absorbing compound state 0, so compound state 1 should

be a transient state. We also know that transition process will visit a transient state infinite times with probability 0. In other words, this case is impossible.

3. Assume the transition process never visits compound state 0, and it meets compound state 1 finite times. Then $k_1 \in \mathbb{N}$ such that $\forall k \geq k_1$ exists, the transition process meets compound state 2. So $\forall k \geq k_1, a_i(k) = yes$. Now we prove this implies $p_i(k)$ will also converge to 1 with probability 1. First, we know $\forall k \geq k_1, a_i(k) = yes$, which implies $p_i(k)$ will increase when $k \geq k_1$. Prove by contradiction, if $p_i(k)$ is increasing $\forall k \geq k_1$ and $p_i(k)$ does not converge to 1, then $\epsilon' > 0$ such that $\forall k \geq k_1, p_i(k) \leq 1 - \epsilon'$ exists. So we can derive the probability $P(\forall k \geq k_1, a_i(k) = yes) \leq (1 - \epsilon')^\infty = 0$, which is a contradiction. So we proved that $p_i(k)$ will also converge to 1 with probability 1.

In previous discussion, we prove that $a_i(k)$ and $p_i(k)$ will always converge in the DCEE algorithm with probability 1 in all cases. Furthermore, corollary 1 implies the theorem 7.

□

Theorem 7. $\forall i, p_i$ *in the DCEE algorithm will converge to 0 or 1 with probability 1, and* a_i *also converges to a pure strategy* yes *or* no *with probability 1.*

Proof. By corollary 1, we know that every device's a_i and p_i will converge will probability 1, so it is obviously that the whole DCEE algorithm will converge with probability 1. □

In previous discussion, we note that normalization of utility u_i doesn't affect the DCEE algorithm result much, but ϵ_0 plays an important role to assist the convergence. Without the normalized utility function u_i and ϵ_0, if there are devices always take utility 0, that devices will always unable to converge to a pure strategy.

Chapter 9

Properties and Theorems of the DCEE Algorithm

In the previous section, we know the DCEE algorithm will always converge to pure strategies, but we do not know exactly which strategy will be converged. In this section, we will derive not only the strategies that is capable of being converged, but some other properties of this algorithm.

Corollary 2: If a strategy combination $(a_1^*, a_2^*, ..., a_n^*)$ satisfies $\forall i, u_i(a_1^*, a_2^*, ..., a_n^*) > 0$, then the DCEE algorithm may converge to this strategy combination. Furthermore, the DCEE algorithm may converge to different strategy combinations with the same initial state.

Proof. By Lemma 2, we know that $\forall i, \exists \rho_i$ s.t. $P(\forall j \geq 0, a_i(j) = a_i^*) \geq \rho_i$,
so $P(\forall j \geq 0, (a_1(j), a_2(j), ..., a_n(j)) = (a_1^*, a_2^*, ..., a_n^*) \geq \prod_{h=1}^{n} \rho_h$, which means the DCEE algorithm at least has probability $\prod_{h=1}^{n} \rho_h$ to converge to the strategy combination $(a_1^*, a_2^*, ..., a_n^*)$. □

Corollary 3: If the DCEE algorithm converges to a strategy combination $(a_1^*, a_2^*, ..., a_n^*)$, then $\forall i, u_i(a_1^*, a_2^*, ..., a_n^*) > 0$

Proof. Proof by contradiction, we assume there $\exists i$ s.t. $u_i(a_1^*, a_2^*, ..., a_n^*) = 0$. On the other way, the DCEE algorithm converges to $(a_1^*, a_2^*, ..., a_n^*)$ means $\exists k_0$ s.t. $\forall k \geq k_0, j \in \{1, 2, ...n\}, a_j(k) = a_j^*$. So $\exists p_i^* < 1$ s.t. $\forall k \geq k_0, p_i(k) = p_i^*$, implies $\prod_{k \geq k_0} P[a_i(k) = a_i^*] = (p_i^*)^\infty = 0$, which indicates that the learning algorithm converges to $(a_1^*, a_2^*, ..., a_n^*)$ with probability 0, which is a contradiction. □

Definition 5. [Positive Strategy Set] *We say a strategy combination $a \in \mathcal{A}$ is a positive strategy if a satisfies $\forall i, u_i(a) > 0$, and we define set $\mathcal{S}$ as $a \in \mathcal{S}$ if and only if $u_i(a) > 0 \forall i$.*

Theorem 8. *The strategy set the DCEE algorithm can converge to is $\mathcal{S}$*

Proof. We can know that Corollary 2 and Corollary 3 are contrary propositions to each other, and we can combine them to have Theorem 4. □

In the previous discussion, we know though that the DCEE algorithm always converges, but it may not always converge to a specific strategy combination with the same initial condition. The converge pattern may form a probability distribution in $\mathcal{S}$, and the probability distribution mainly depends on the topology and parameters such as u_i,$p_i(0)$,ϵ_1 and step size b.

Theorem 9. *The DCEE algorithm always satisfies the budget balance property.*

Proof. Consider any cluster, every cluster member pays the transfer T, and the central entity receives the transfer $|N_i| \times T$. So it is obvious that the total transfer of all the devices is 0, which means the budget balance. □

According to the budget balance, we can derive the social welfare $\sum_{i=1}^{n} \bar{u}_i(k) = n_0\mu - \sum_{i \in \mathcal{H}} C_i$, where the $n_0 \leq n$ represents the total device number that can receive data.

Definition 6. **[$\epsilon-$ individual rationality:]** *Individual rationality means the player joining a cluster or becoming a central entity will have a non-negative utility, and we define that $\epsilon-$individual rationality means the player joining a cluster or becoming a central entity will have utility $\geq -\epsilon$.*

Theorem 10. *For any $\epsilon > 0$, we can choose the parameter ϵ_1 small enough to ensure the DCEE algorithm have $\epsilon-$individual rationality for every device in any learning result.*

Proof. By, theorem 8, we know that converge strategy provides $u_i > 0$ for every device i in any learning result. We recall the utility normalized Equation (8.2) and (8.3) to calculate the truth device utility $\bar{u}_i$. We will derive the relationship u_i and $\bar{u}_i$ as follows:

$$u_i > 0 \iff \hat{u}_i > 0 \iff \frac{\bar{u}_i + \epsilon_1 * max_t|\bar{u}_i(t)|}{(1+\epsilon_1) * max_t|\bar{u}_i(t)|} > 0 \iff \bar{u}_i > -\epsilon_1 * max_t|\bar{u}_i(t)| \tag{9.1}$$

□

we can choose $\epsilon_1 = \epsilon / max_{i,t}\{max|\bar{u}_i(t)|\}$, then $\bar{u}_i > -\epsilon_1 * max_t|\bar{u}_i(t)| \geq -\epsilon \forall i$. So we know that when ϵ_1 is a very small positive number, it almost can not affect the individual rationality.

Chapter 10

Further Investigation of the DCEE Algorithm and Discussion

10.1 Theoretical Analysis in Small Step Size b

In this section, we will discuss the algorithm behavior when step size b used in equation (8.1) is small enough. By using the ordinary differential equation (ODE) whose solution approximates the asymptotic behavior of $p_i(k)$ as in [3], we can rewrite the learning algorithm updating process in equation (8.1) as

$$P(k+1) = P(k) + bG(P(k), a(k), u(k)) \tag{10.1}$$

where $a(k) = (a_1(k), ..., a_n(k))$, and $u(k) = (u_1(k), ..., u_n(k))$. $G(.)$ represents the updating process by equation (8.1). When $b \to 0$, we define a function f by the conditional expectation:

$$f(P) = E[G(P(k), a(k), u(k))|P(k)] \tag{10.2}$$

Lemma 4. *In the DCEE algorithm, considering one player D_1 scenario (or assuming the strategies of other n-1 players are static), two strategies, $a_{11} = yes$ or $a_{12} = no$, are to be chosen. Then $P[a_1(k) \to a_{11}] \to 1$ when $b \to 0$ and $u_1(a_{11}) > u_1(a_{12})$, or $P[a_1(k) \to a_{12}] \to 1$ when $b \to 0$ and $u_1(a_{11}) < u_1(a_{12})$. It means the player will have a higher probability to converge to a better strategy when b is small.*

Proof. Without loss of generality, we assume $u_1(a_{11}) > u_1(a_{12})$, and then

$$f_{11} = \frac{dp_{11}}{dt} = p_1 u_1(a_{11})(1-p_1) - (1-p_1)u_1(a_{12})p_1 = p_1(1-p_1)(u_1(a_{11}) - u_1(a_{12})) > 0 \tag{10.3}$$

$p_{11}(t)$ is a monotonically increasing function of t, which indicates $a_1(k) \to a_{11}$, so we know that $P[a_1(k) \to a_{11}] \to 1$. On the other way of $u_1(a_{11}) < u_1(a_{12})$, we can still obtain similar results.

□

The notation a_{ij} represents the jth strategy for the player i, and a property could be derived as follows:

Theorem 11. *In the DCEE algorithm, if a player D_j has a dominant strategy a_{j1}, then $P[a_j(k) \to a_{j1}] \to 1$ when $b \to 0$.*

The proof of theorem. 11 is similar to lemma 4 and we will not prove it here.

10.2 Discussion and Comparison to Related Work

There are some similar learning algorithms proposed in [2] and [3]. They also adopt ODE to have the asymptotic solution for the learning algorithm, but we think some theories in the previous work are wrong. They admit the weak convergence of the learning algorithm satisfies $\frac{dP}{dt} = 0$, which is $\frac{dp_{ij}}{dt} = 0 \forall i, j$. The weak convergence result admits the learning algorithm to converge to mixed strategies, which are also Nash equilibriums. However, in the theorem 7 of our DCEE algorithm, we prove the learning algorithm cannot converge to any mixed strategy even if step size b is very small. In a macro view, the DCEE algorithm may be close to the mixed Nash equilibriums, but the mixed Nash equilibrium cannot be a stable point. The learning process may oscillate around the mixed Nash equilibrium for a long time, but finally converge to a pure strategy even if step size b is very small.

Chapter 11

Simulation Results

11.1 Simulation Scenario

Our simulation scenario follows 3GPP TR 36.843 V1.0.0, Urban macro(500m ISD) $+1$ RRH/Indoor Hotzone per cell in [25]. The synchronization reference is derived from the timing of a cell. The BS that transmits data to the devices is located at the origin. The devices are uniformly distributed in a bounded square area 200*200m^2 to form a cluster. The cluster is 200m away from the BS. Other 18 BSs that cause interference are located in the inter-BS distance of 500m. Each device can measure the signal strength of the BS and the central entity, and calculate the SINR.

In the following, we define the cost function of each device. Recall that the cost function is derived in (4.1). We assume that a data block can be received correctly if the block error probability is less than a small number $\epsilon = 0.001$. The BS and the central entity will adjust their transmit power, denoted by P_{BS} and P_{CE}, to guarantee the block error probability to be less than ϵ. Assuming that the normal transmit power of the BS is 46dBm(40W) and the normal D2D transmit power is 23dBm(200mW), we define the cost function as the normalized transmit power consumption $P_{BS}/40W + P_{CE}/200mW$. In addition, we assume that each device obtains utility $\mu = 1$ when receiving data. Then we can rewrite the utility function as follows:

$$U_i = \begin{cases} 1 + T_{CE} - \frac{P_{BS}}{40W} - \frac{P_{CE}}{200mW} & \text{if } i = CE \\ 1 - T_{UE} & \text{otherwise} \end{cases} \tag{11.1}$$

All of the simulation parameters are showed in Table 11.1.

Table 11.1: Simulation Parameters

System bandwidth	10MHz
Carrier frequency	2GHz
Number of BSs	19
Inter-BS distance	500m
BS transmit power(P_{BS})	46dBm(40W)
D2D transmit power(T_{CE})	23dBm(200mW)
Cluster region	200*200m^2
Minimum distance from the cluster to the BS	200m
Number of D2D devices	uniform 10 drops
Mobility	static scenario
Charge parameter λ	7.2
Utility μ	1
Block error probability ϵ	0.001

11.2 Verification of the Theoretical Analysis in the Auction Mechanism Design

We verify the theoretical analysis of the auction mechanism design in Section 4 in the following three performance metrics:

1. Truth telling: Each device maximizes its utility by truthfully reporting its cost.

2. Maximum cluster utility: The proposed mechanism maximizes the overall cluster utility by choosing the device with lowest cost to be the central entity.

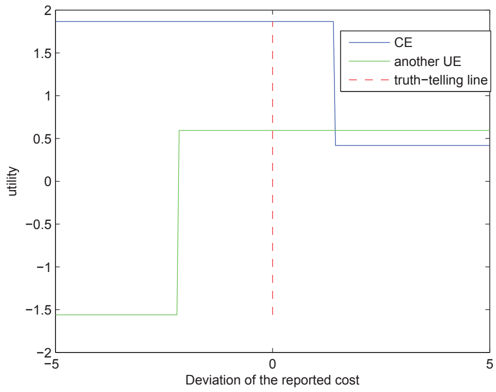

Figure 11.1: The utility of the devices under different values of the reported costs.

3. Effect of the charge parameter λ: We analyze the effect of the charge parameter λ on the properties of the individual rationality and the budget balance.

11.2.1 Truth Telling

In Fig.11.1, we show the utility of the central entity and another device denoted by the UE. We find that the central entity and the UE have the maximum utility 1.8 and 0.8 when reporting the true costs. If the central entity reports a higher cost to not become the central entity, its utility decreases to 0.6. On the other hand, if the other device reports a lower cost to become the central entity, its utility decreases to -0.3. In other words, both the central entity and the UE maximize their utility by truthfully reporting their costs.

11.2.2 Maximum Cluster Utility

Fig. 11.2 shows the overall cluster utility and the cost when each device D_i becomes the central entity. Since our proposed mechanism elects the device with the lowest cost, i.e., D_1, as the central entity, the overall cluster utility is maximized as depicted in the figure.

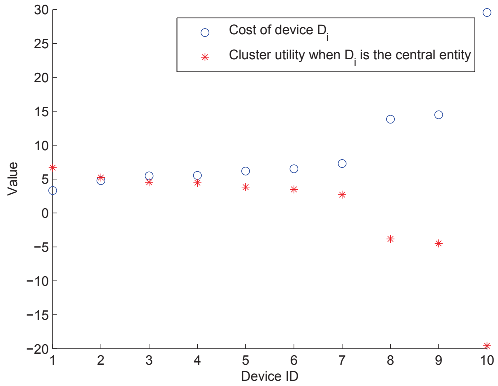

Figure 11.2: The cost and the cluster utility when device D_i is the central entity.

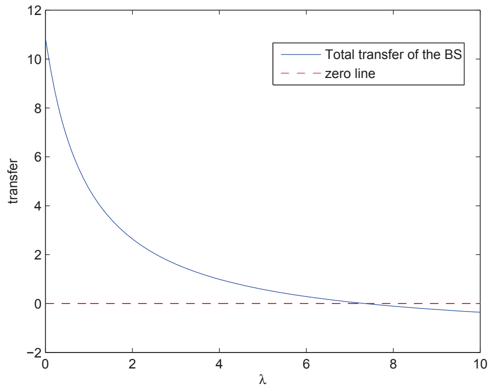

Figure 11.3: The total transfer of the BS under different values of the charge parameter λ

11.2.3 Effect of the charge parameter λ

In Fig. 11.3, the proposed mechanism achieves (weak) budget balance when $\lambda \leq 8$. When λ increases, the total transfer of the BS defined in Equation (6.5) decreases. In Fig. 11.4, the proposed mechanism achieves individual rationality when $\lambda \geq 1$. The utility of the central entity and that of the UE increases with λ. Combining the results in Fig. 11.3 and Fig. 11.4, we know that when $1 \leq \lambda \leq 8$, the proposed mechanism achieves both the individual rationality and (weak) budget balance.

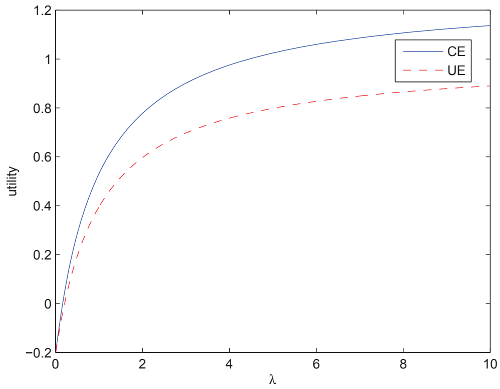

Figure 11.4: The utility of the central entity and the UE under different values of the charge parameter λ

11.3 Verification of the Theoretical Analysis in DCEE algorithm

In this section, we drop devices uniformly distributed in a bounded rectangular area 100*300m^2 to form a cluster. The cluster is 300m away from the BS. In the following, we define the cost function of each device. Recall that the cost function is derived in (7.3). Then we can rewrite the utility function as follows:

$$\bar{u}_i(k) = \begin{cases} 1 + T * |N_i| - \frac{P_{BS}}{46dBm} - \frac{P_{CE}}{23dBm} & \text{if } i = CE \\ 1 - T & \text{if } i \text{ is a cluster member} \\ 0 & \text{otherwise} \end{cases} \tag{11.2}$$

We show the different simulation parameters in Table 11.2.

11.4 Observation in Different Parameters

In this section, we will conduct simulation with fixed scenario, and change the setting of step size b, transfer price T and initial condition $p_i(0)$. We do 3000 times simulation

Table 11.2: Simulation Parameters2

Device Region	100*300m^2
Number of D2D devices	uniform 20 drops
Mobility	static scenario
Data utility	1
Step size b	0.1
Transfer price T	0.1
Initial condition $p_i(0), \forall i$	0.5
Stopping criteria	0.01
Utility normalized parameters (ϵ_1, ϵ_2)	(0.1,0.01)

in each setting and plot the curves of convergence time, converge to NE probability, the DCEE algorithm efficiency U_L/U_O, and the average cluster numbers, where U_L is the average social welfare of the DCEE algorithm, and U_O is the optimal social welfare.

11.4.1 Change Step Size b

In this simulation part, we change the step size b from 0.02 to 0.5 as in Fig. 11.5 and Fig. 11.6. We can find that when b becomes small, the NE rate and the algorithm efficiency will increase simultaneously. It represents a better performance but with the trade-off in rapid increase of the convergence time. We can also find that the appropriate cluster numbers is about 4 clusters in this scenario. When step size is too large, DCEE algorithm may have a higher probability to converge to bad results and form more clusters.

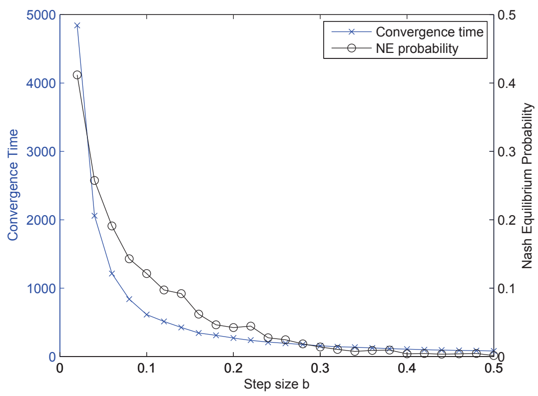

Figure 11.5: Change Step Size b

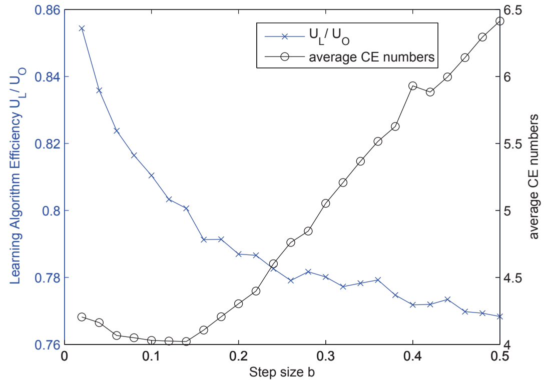

Figure 11.6: Change Step Size b

11.4.2 Change Transfer Price T

In this simulation part, we change the transfer price T from 0 to 1 as in Fig. 11.7 and Fig. 11.8. In Fig. 11.7, we can find that algorithm will be easier to converge to NE when T is extremely high or extremely low in that the device can be easier to dominant strategy and easy to decide whether to be central entity. On the other way, when T is middle value, many device may hesitate and wait for others' decisions. The convergence time does not have trend with transfer price T, but when T is in some specific values, there has a peak in convergence time. The reason of the peak is that few devices almost have same utility whether it becomes a central entity or not. That is a coincidence from the specific scenario characteristic and transfer price value T. We can also find that when a peak for convergence time exists, there is also a hollow of NE probability. In Fig. 11.8, when transfer price T is increasing, every device will prefer to receive data from itself rather than join other's cluster, so cluster numbers is also increasing. The DCEE algorithm efficiency becomes bad when T is too low, because low transfer discourage devices from becoming a central entity, and that may cause some devices can't receive data. In addition, high transfer price is also not good for too many central entities will increase more transmission cost among the BS and central entities.

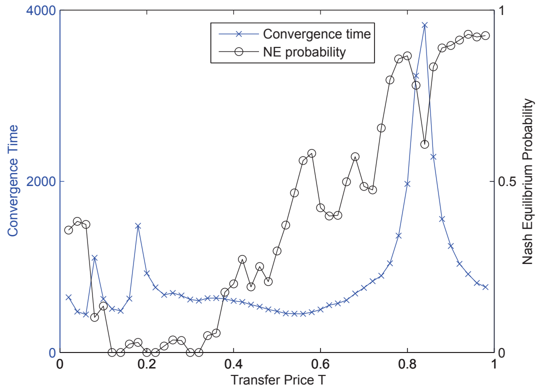

Figure 11.7: Change Transfer Price T

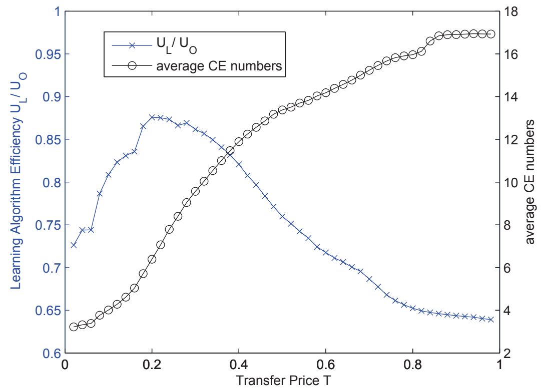

Figure 11.8: Change Transfer Price T

11.4.3 Change Initial condition $p_i(0)$

In this simulation part, we change the initial condition $p_i(0)$ for all devices from 0.02 to 0.98 as Fig. 11.9 and Fig. 11.10. Four curves in these two figures have the same property of going down when $p_i(0)$ is extremely small because it is too close to the stop criteria. Exclude this reason, four curves have its own trend when $p_i(0)$ changes. When initial condition $p_i(0)$ becomes larger, the cluster number is also increasing, whereas other three curves are decreasing. For the system doesn't need too many central entities, we can know that lower initial condition $p_i(0)$ is better in the most cases.

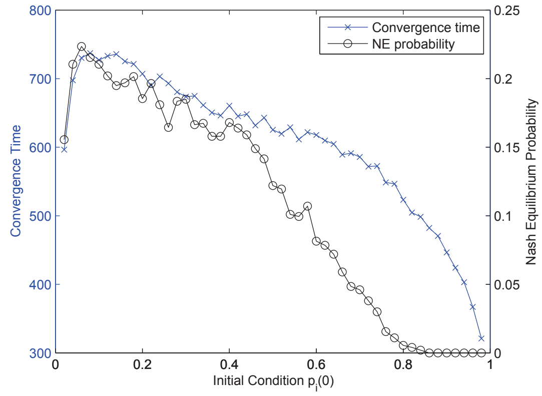

Figure 11.9: Change Initial condition $p_i(0)$

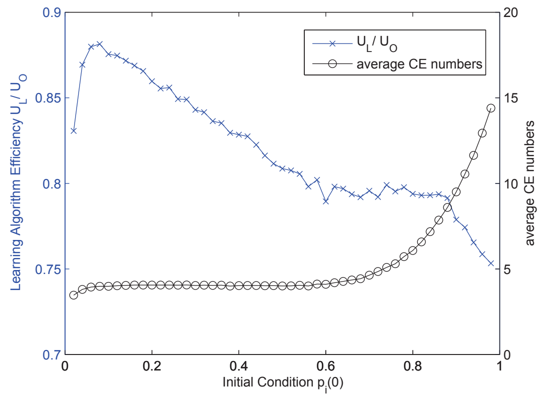

Figure 11.10: Change Initial condition $p_i(0)$

11.5 Oscillation Phenomenon

In this section, we will verify the oscillation property addressed in Section 10.2. We plot a DCEE algorithm process as Fig. 11.11, where the twenty curves represent the respective probability of the corresponding devices. We can find that the probability of the most device converge to 0 or 1 rapidly, only that of few devices oscillate in the middle. The reason for them to hesitate whether to become a central entity may due to the existance of a mixed strategy Nash equilibrium, so as we discussed.

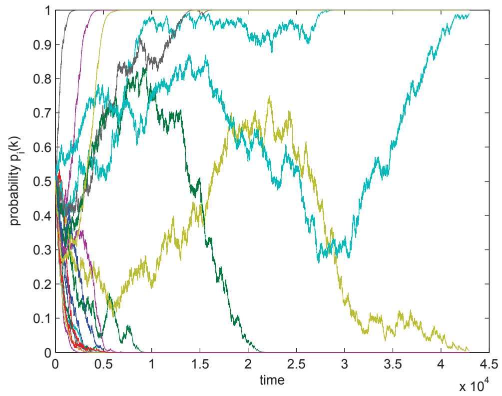

Figure 11.11: Oscillation of DCEE Algorithm

11.6 Compare Social Welfare

In this section, we compare the social welfare in some different central entity election approaches as shown in Fig. 11.12. We use our centralized mechanism design to solve the optimal solution. Although the result of centralized mechanism design can achieve the optimal social welfare, it spends too much time, and the optimal solution may not be a Nash equilibrium making every device satisfied. The random approach represents every device decides whether to be a central entity with a probability $P_r = 0.3$. The greedy approach means the devices decide whether to become central entity one by one. And the all central entity means every device will just connect to the BS by itself. In the simulation results, the random approach is certainly worst. Our DCEE algorithm is better than the greedy algorithm. Despite of disadvantage in social welfare, our DCEE algorithm possess great advantages in convergence and finding Nash equilibriums. On the other way, greedy algorithm needs to decide a decision order among the devices, but they cannot easy to have a decision order in a distributed system. By comparing the learning algorithm, greedy, and all central entity results, we know that system performance will have obvious improvement by using D2D transmission rather than just unicast.

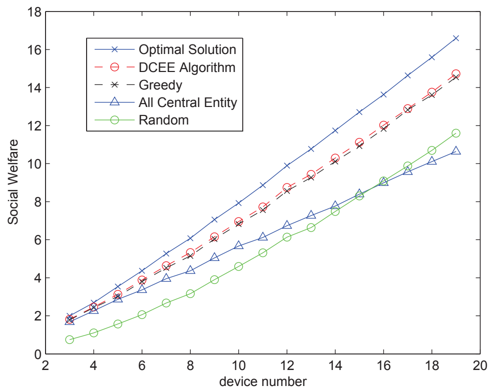

Figure 11.12: Compare Social Welfare from different approaches

Chapter 12

Conclusion

When many devices request the same data, it is more efficient for the devices to form clusters and to elect central entities to receive the data from the BS and then broadcasts the data to all the other devices. We first consider a one-cluster simple system. As the devices are selfish and non-cooperation in nature, we propose a mechanism to elect a central entity. The interaction among the devices in the proposed mechanism is formulated into an auction game. In the game-theoretical analysis, the proposed mechanism induces the true information on the transmission costs from the devices. The cluster reaches the unique truth-telling dominant-strategy Nash equilibrium, in which the device with the lowest cost is elected as the central entity. In addition to the property of truth telling, the proposed mechanism maximizes the social welfare and achieves the properties of the individual rationality and budget balance.

In a multiple-cluster central entity election system, the centralized mechanism design is a NP hard problem. We propose the distributed DCEE algorithm to avoid the NP hard problem. We proved that DCEE algorithm can always converge with many desirable properties, and we generate different results when step size $b \to 0$ with previous work. We observe an oscillation phenomenon of the DCEE algorithm and show the phenomenon in simulations. The centralized mechanism design and the distributed algorithm have different advantages, and the system performance and the complexity is a trade-off.

Our simulation results verify the theoretical analysis in a real LTE system setting. With the proposed mechanism and the simulation results, D2D communications is shown to have the potential in improving the performance of wireless networks.

Actually, the DCEE algorithm can extend to many distributed system, it is useful when a distributed system satisfies the following conditions:

1. Every member in the system is selfish and non-cooperative.

2. Every member has finite strategies.

3. After every member decides a strategy, everyone can calculate his own utility.

In such a distributed system, for example, a distributed D2D power control game, or a D2D channel selection system. The distributed systems can also apply the DCEE algorithm. Though the DCEE algorithm only has two strategies yes and no, it can be further extend to more than two strategies. After the extension, the theorems we proved for the DCEE algorithm will still remain true. The DCEE algorithm can also have the convergence property and can help many other distributed systems to find the Nash equilibriums.

Bibliography

[1] 3GPP R2-134096, ”Centralized D2D transmission for out-of-coverage”, Nov 2013 .

[2] Y. Xing and R. Chandramouli. Stochastic learning solution for distributed discrete power control game in wireless data networks. *IEEE/ACM Transactions on Networking*, 16(4):932–944, Aug 2008.

[3] P. S. Sastry, V. V. Phansalkar, and M. Thathachar. Decentralized learning of nash equilibria in multi-person stochastic games with incomplete information. *IEEE Transactions on Systems, Man and Cybernetics*, 24(5):769–777, May 1994.

[4] J. Seppala, T. Koskela, T. Chen, and S. Hakola. Network controlled device-to-device (d2d) and cluster multicast concept for lte and lte-a networks. In *IEEE Wireless Communications and Networking Conference (WCNC)*, pages 986–991, 2011.

[5] F. Wang, C. Xu, L. Song, Han Z, and B. Zhang. Energy-efficient radio resource and power allocation for device-to-device communication underlaying cellular networks. In *International Conference on Wireless Communications Signal Processing (WCSP)*, 2012.

[6] T. Koskela, S. Hakola, T. Chen, and J. Lehtomaki. Clustering concept using device-to-device communication in cellular system. In *IEEE Wireless Communications and Networking Conference (WCNC)*, 2010.

[7] J. Du, W. Zhu, J. Xu, Z. Li, and H. Wang. A compressed harq feedback for device-to-device multicast communications. In *IEEE Vehicular Technology Conference (VTC Fall)*, 2012.

[8] B. Zhou, H. Hu, S.Q. Huang, and H.H. Chen. Intracluster device-to-device relay algorithm with optimal resource utilization. *IEEE Transactions on Vehicular Technology*, 62(5):2315–2326, 2013.

[9] M. Chatterjee, S.K. Das, and D. Turgut. Wca: A weighted clustering algorithm for mobile ad hoc networks. *Cluster Computing*, 5(2):193–204, 2002.

[10] Q. Huang, C. Julien, and G.C. Roman. Relying on safe distance to achieve strong partitionable group membership in ad hoc networks. *IEEE Transactions on Mobile Computing*, 3(2):192–205, 2004.

[11] L. Badia, M. Miozzo, M. Rossi, and M. Zorzi. Routing schemes in heterogeneous wireless networks based on access advertisement and backward utilities for qos support. *IEEE Communications Magazine*, 45(2):67–73, 2007.

[12] Y. Narahari, D. Garg, R. Narayanam, and H. Prakash. *Game Theoretic Problems in Network Economics and Mechanism Design Solutions,*. Spinger, 2009.

[13] C. Xu, L. Song, Z. Han, Q. Zhao, X. Wang, and B. Jiao. Interference-aware resource allocation for device-to-device communications as an underlay using sequential second price auction. In *IEEE International Conference on Communications (ICC)*, pages 445–449, 2012.

[14] C. Xu, L. Song, Z. Han, Q. Zhao, X. Wang, X. Cheng, and B. Jiao. Efficiency resource allocation for device-to-device underlay communication systems: A reverse

iterative combinatorial auction based approach. *IEEE Journal on Selected Areas in Communications*, 31(9):348–358, 2013.

[15] C. Xu, L. Song, Z. Han, D. Li, and B. Jiao. Resource allocation using a reverse iterative combinatorial auction for device-to-device underlay cellular networks. In *IEEE Global Communications Conference (GLOBECOM)*, pages 4542–4547, 2012.

[16] D. Wu, J. Wang, R.Q. Hu, Y. Cai, and L. Zhou. Energy-efficient resource sharing for mobile device-to-device multimedia communications. *IEEE Transactions on Vehicular Technology*, 63(5):2093–2103, Jun 2014.

[17] N. Mastronarde, V. Patel, J. Xu, and M. Schaar. Learning relaying strategies in cellular d2d networks with token-based incentives. In *IEEE Globecom Workshops (GC Wkshps).*, pages 163–169, Dec 2013.

[18] P. Zhou, Y. Chang, and J.A. Copeland. Reinforcement learning for repeated power control game in cognitive radio networks. *IEEE Journal on Selected Areas in Communications*, 30(1):54–69, Jan 2012.

[19] S.H. Kang and N. Thinh. Distance based thresholds for cluster head selection in wireless sensor networks. *IEEE Communications Letters*, 16(9):1396–1399, Sep 2012.

[20] J.S. Kim, S.Y. Choi, S.J. Han, J.H. Choi, J.H. Lee., and K.W. Rim. Alternative cluster head selection protocol for energy efficiency in wireless sensor networks. In *Software Technologies for Future Dependable Distributed Systems.*, pages 159–163, Mar 2009.

[21] L. Buttyan and T. Holczer. Private cluster head election in wireless sensor networks.

In *IEEE 6th International Conference on Mobile Adhoc and Sensor Systems.*, pages 1048–1053, Oct 2009.

[22] D. C. Parkes and L. H. Ungar. Iterative combinatorial auctions: Theory and practice. In *AAAI/IAAI*, pages 74–81, 2000.

[23] M.J. Osborne. *An introduction to game theory*, volume 3.

[24] W. Vickrey. Counterspeculation, auctions, and competitive sealed tenders. *The Journal of finance*, 16(1):8–37, 1961.

[25] 3GPP TR 36.843 1.0.0, ”Study on LTE Device to Device Proximity Services-Radio Aspects,” Oct 2013.