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Master Thesis

平台式配送服務：

基於賽局理論分析一共享經濟下的新型態配送模型

Platform Delivery: A Game-theoretic Analysis of a New Delivery Model in the Sharing Economy

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謝辭

回想起兩年的碩班生涯，每個學期初會出現新的挑戰，然後在每個學期末都能感受到自己的成長，最開心的事莫過於看著陪伴自己成長的論文逐漸成形。

這段日子裡，我很幸運能夠被我的指導教授孔令傑老師所帶領。入學前，老師為我安排了補充最佳化知識的課程；碩一時，老師給了我實習與參與國科會計畫的機會，強化了我的程式能力。並安排我們將「資訊經濟」的期末報告投稿至 2015 年決策分析研討會。在台積電半導體大數據分析競賽，以及擔任統計類課程助教的過程中，老師給予我學習資料分析的環境。我的研究能力在老師的訓練下逐漸養成。碩一那年著實是我人生中成長最快速，也最難忘的一年；碩二開始進行論文撰寫，我很感謝老師的陪伴，在我迷糊的時候指點我，並及時修正我所有問題，注重細節的老師還常常逐字審閱我的文字，在論文口試以及 PACIS 研討會前夕甚至陪我們一起練習到深夜。此外，我也要特別感謝口試委員，工工所的洪一薰教授與資管系的李瑞庭教授，給予我許多能使論文更完善的建議。

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在這個即將揮別校園生活的時刻，我回憶完這兩年的碩士班生涯，發現身旁一直有一位重要的舊識，何禾，有妳這些年來的陪伴，我才能夠順利地走到這裡。

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摘要

資訊科技的逐漸進步，使得許多新型態的商業模式如雨後春筍般出現，其中包含了以共享經濟為主要精神的配送產業。Instacart 在共享經濟的精神下遠近馳名，它沒有倉儲系統，也不雇用任何全職員工，而是建立一個媒合平台，媒合「想要利用空閒時間賺取額外收入的代購配送員」與「有代購需求的消費者」這兩群人，直接利用實體生鮮業者的倉儲和店面，將生鮮食品配送至顧客手中。我們建立了一個賽局理論模型去探討這種「平台式的配送服務」，並期望能夠找到使平台最大化自身利益的訂價策略。在本論文中討論三種常見的訂價策略，分別是「會員費策略」、「手續費策略」以及「交叉補貼策略」。

在本論文中，我們建立一個賽局模型，包含了網路外部性與共享經濟的特性，試圖回答我們的研究問題。我們討論的情境如下：市場中存在一個提供媒合服務的平台商、互相存在網路外部性的一群潛在顧客與一群潛在代購配送員。平台商將在三種定價策略下最大化自身利益，並找出何種定價策略能產生最大利潤。

我們發現在某些情況下，三種定價策略不只一樣好，而且都是最好的。然而，在考量平台希望能夠盡量早收到現金的需求後，我們發現會員費策略能最大化平台利潤。而在消費者在每次會員期間的使用量，會隨著平台制定的每次交易手續費用上升而遞減時，交叉補貼策略將能最大化平台利潤。

關鍵字：共享經濟、網路外部性、配送服務、賽局理論、定價策略

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Abstract

Thanks to the advances in technology, new types of service delivery spring up in the sharing economy. Owning no warehouse and hiring no full-time shoppers, Instacart runs its grocery delivery service by delivering grocery from independent retailers by independent contractors to its consumers. This “platform delivery” model is formulated as a game-theoretic model and investigated. We discuss the profitability of three common pricing policies: membership-based pricing, transaction-based pricing, and cross subsidization. We wonder which policy is the best for the platform.

In this study, we construct a game-theoretic model featuring network externality and sharing economy to address our research questions. There are three types of players in the market: a group of potential consumers placing orders, a group of potential shoppers providing delivery services, and a platform connecting consumers and shoppers. There exists positive cross-side network externality between consumers and shoppers. The major purpose of our work is to study the profitability of the three pricing strategies and figure out factors that affect the platform’s choice.

Our main result shows that all the three strategies are equivalent in some situations: They result in the same per-transaction subsidy for shoppers, numbers of shoppers and consumers, and profits in equilibrium. However, when the platform care about how fast it can receive money, we find that membership-based pricing is the best and transaction-based pricing is the worst. Furthermore, if a consumer’s consumption in each membership period would be negatively affected by the per-transaction fee charged from consumers, we find that the cross-subsidization strategy is better than the transaction-based pricing one for platform to implement.

Keywords: sharing economy, network externality, delivery service, game theory, pricing strategy

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Chapter 1

Introduction

1.1 Background and motivation

Owing to the advances in technology, different types of delivery services spring up in recent years. In the grocery delivery industry, AmazonFresh adopts *integrated delivery* and delivers grocery from self-owned warehouse to its consumers. On the contrary, Instacart owns no warehouse and runs its service by delivering grocery from independent retailers to its consumers. Moreover, instead of hiring full-time employees, Instacart relies on independent contractors to provide the deliveries.[1] As this type of service shares the same multi-sided platform idea with Uber and Airbnb, we call it *platform delivery.*

In general, the success of an Internet-based platform relies on its installed base, and the benefit of using a service provided by a platform increases as the number of its user raises up. This is known as positive network externality. With network externality, a

[1]For more details about Instacarts model, please refer to, e.g., http://nextjuggernaut.com/blog/how-instacart-works-makes-money-revenue-business-model.

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platform could not only make its service more valuable but also leverage this effect to enter a new market. One common approach of entering a new market is to subsidize users at the beginning. Take Uber for example, it gave new users 200 TWD when it expanded its business to Taipei in mid-2013, and the numbers of drivers and passengers had increased by an average of 30 percent a month up to the end of march 2015.[2] Similarly, Instacart offers a free first delivery to attract as many consumers as possible, in the hope that the initial users will attract more consumers in a virtuous cycle.

There might be some perceptible problems resulting from the self-scheduling characteristic of Instacart. One of them is a shortage of resources (contractors). Because the contractors are actually not employed by Instacart, every contractor could decide when to work. Thus, if most of the contractors decide not to work at the same time, or a sudden demand for grocery delivery takes place, the shortage of the resources will occur. Moreover, the part-time contractor lacked of experience may deliver the wrong groceries to the consumers or deliver them in a bad condition. On the other hand, AmazonFresh's full-time employees empower AmazonFresh to provide a stable and reliable service. AmazonFresh might have the ability to prevent it from the shortage problem to a certain degree. In short, while Instacart may save money from owning no warehouse and full-time employees, it faces the challenge of attracting enough contractors to provide good enough services to attract customers. Its pricing strategy is therefore critical for running a financially sustainable business.

In this study, we investigate such a matching platform's pricing strategy. While this

[2] Information source: http://topics.amcham.com.tw/2015/03/uber-taiwan-transportation-or-information-company/.

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study is motivated by the observation on Instacart and AmazonFresh, we would like to study the pricing strategy of all similar platforms for sharing economy. We hope our study may help explain the rationale behind the selection of pricing strategies adopted by these platforms in practice.

1.2 Research objectives

While in theory there can be all kinds of pricing mechanisms, in practice three kinds of strategies are common. If a company adopts the *membership-based pricing* strategy, the platform sustains losses in every transaction but charges every consumer a fixed membership fee at the beginning. On the opposite, the platform may charge a per transaction fee but no fixed fee. This is the *transaction-based pricing* strategy. In either case, the platform needs to pay the shopper a per transaction fee. This introduces the third strategy, the *cross-subsidization* strategy, under which the platform simply subsidizes the shopper exactly the amount collected from the customer in each transaction. It is worthwhile to investigate which pricing strategy may generate the highest profit for the platform.

In this study, we construct a game-theoretic model featuring network externality and sharing economy to address our research questions. There are three types of players in the market, a firm providing platform delivery service, a group of potential consumers, and a group of potential shoppers. The major purpose of our work is to study the profitability of the three pricing strategies and figure out factors that affect the firm's choice.

Note that the matching platform we discuss matches "suppliers" and "customers," like Instacart, Uber, etc. This kind of platform is different from a platform matching two

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parties who both need each other, such as friend making websites. As it will become clear when we describe the model, in this study a supplier's utility is based on the monetary earning minus the cost of providing the service/product, but a customer's utility is based on the utility of being served/getting the product minus the monetary payment. Matching platforms without a clear supplier-customer relationship are not discussed in this study.

1.3 Research plan

In the next chapter, we review some related works with respect to sharing economy, network externality, and delivery service competition. In Chapter 3, we develop a game-theoretic model that addresses the interaction among the platform, customers, and shoppers. The analysis and results of the basic model are then presented in Chapter 4. Chapter 5 extends the basic model and delivers further managerial insights. Chapter 6 concludes. All proofs are in the appendix.

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Chapter 2

Literature review

2.1 Sharing economy and crowdsourcing

Uber, an Internet-based platforms in the transportation industry, has swept across the whole world. Many companies want to copy their business model, or at least find out the critical success factors making Uber become a classic paradigm shift. Specifically, quite a few people attribute the success of Uber to "sharing economy," which emphasizes how to make good use of idle resources spreading in the market. For instance, Santi et al. (2014) claim that the cumulative trip length could be reduced by roughly 40 percent when using ride sharing like Uber related to traditional taxis. Furthermore, since the similarity between Uber and platform delivery in terms of sharing resources, we believe that the use of idle resources would be one of the reasons which leads to the difference between integrated delivery and platform delivery. The rest of this section would present some related works about sharing economy.

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Andersson et al. (2013) investigate ways ride sharing could improve the use of idle resources, and classify the business model of sharing economy into three kinds according to the properties of trade matching: First, in the *deferred sharing pattern*, every matching is independent, and needs long planning time, such as Zimride; second, in the *recurrent sharing pattern*, after matching one trade successfully, the rest of other trade matching do not need matching platform anymore, such as Rideshare.com; finally, in the *immediate sharing pattern*, every matching is independent, and the planning time is usually short, such as SideCar and Lyft. In our opinion, Instacart should be one of the immediate sharing patterns.[1]

In Felländer et al. (2015), their definition of sharing economy focus on the peer-to-peer exchange of tangible assets and intangible assets which involve information exchange through the Internet or mobile phones. Moreover, they mention that sharing economy has some benefits like almost zero marginal cost caused by digitalization, the high quality of trade matching using the Internet, etc. It seems like that the platform delivery has more advantage over the integrated delivery. However, integrated delivery is not replaced completely by platform delivery in current market structure. What is the limit of platform delivery? It deserves our further study.

Zervas et al. (2015) analyze the competition relationship between Airbnb and hotel chains, and obtain the following main conclusion: Since the little marginal cost, Airbnb can expand their service coverage rapidly, and pose a threat to the traditional hotel

[1] Official website of Zimride: http://zimride.com. Official website of Rideshare.com: http://www.rideshare.com. Official website of SideCar: http://www.side.cr. Official website of Lyft: http://www.lyft.com.

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chains.[2] Nevertheless, hotel chains could weaken the competition with Airbnb by product/service differentiation.

When it comes to the use of idle resources, utilization of resources is one of the most important issues. Both (Teresa and Christy, 2015) and (Rougés and Montreuil, 2004) study the paradigm change of crowdsourcing/crowdsourcing delivery, a delivery solution which outsource the delivery business to anyone who is willing to fulfill it. To customers, crowdsourcing delivery gives them lower cost and flexibility to apply the service. To retailers, crowdsourcing delivery can lower their delivery and operation costs. To click-and-mortar retailers, it can even eliminate the requirements of inventory management. To the whole society, crowdsourcing delivery reduces the total travel distance, and thus achieves the purpose of reducing the wasteful resources.

Gurvich et al. (2015) build up a newsvendor model to investigate the benefits of a firm using self-scheduling, which allow its workers/agents decide when and whether to work. Eventually, they arrive at the result below: Self-scheduling can impose excess costs on a firm, and then lower the service level. If the firm's resources (number of workers/agents) are sufficient enough, the firm could keep the service level well. In this situation, the gap between self-scheduling and without self-scheduling is smaller, but agents' benefits are invaded.

Due to the growth of Internet-based platforms, the concept of sharing economy was mentioned in many recent studies. Most of them using qualitative research and statistical analysis try to explain how the new type of business model changes the game rules in existing industry. However, none of the above mentioned studies conduct rigorous

[2] Official website: www.airbnb.com.

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economic modeling to test the profitability of sharing economy. Therefore, we try to build up a game-theoretic model with sharing economy and network externality to discuss the competition between the two types of delivery service. Furthermore, keep the insight from (Zervas et al., 2015) in mind, we wonder if the platform delivery (Airbnb) would focus on the low-end market, while the integrated delivery (hotel chains) would focus on the high-end market.

2.2 Network externality and multi-sided platform

In general, network externality (also called network effect) can be classified into two forms: direct and indirect. Regarding the direct network externality, there are one platform with one group of agents in the market, and each agent's profit is effected by the group size. From this, we can see that direct network externality usually happens in a "one-sided market." For indirect network externality, there is one platform and two groups of agents in the market. The size of one group would effect the benefits of agents in another group from joining the platform. Therefore, indirect network externality is seen as an important property of a "two-sided market."

According to Katz and Shapiro (1985), the pioneers who study network externality, the sources of network externality can be summarized as following: (1) A consumer's utility of purchasing a good would be directly effected by the number of the other consumers consuming the same good. For example, the more people using a phone, the more profit a consumer would get when she/he owns one. (2) The consumption of compatible goods from the other consumers may indirectly give rises to a consumer's utility. For example,

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the larger number of people using a computer does not directly give any benefit to a software company, but make it more profitable to develop a software. (3) The quality and availability of postpurchase service for a good depends on the market size, and would effect the benefit of consuming the good. For example, the postpurchase service for cars is better in a large market than a small market.

The main purpose of Jing (2007) is to delve into how network externality impacts on the product line design. Consider the existence of network externality, the author find that price discrimination is a beneficial strategy to increase a firm's profit. Namely, the firm has an incentive to expand its market by lowering the price of low-end products and raising the price of high-end ones. The firm might even sell the low-end product at a loss when the network effect is strong. In this case, the purposes of providing the low-end and high-end products are to expand its network size and make it as a primary source of profits respectively.

In consideration of network externality, Fudenberg and Tirole (2000) study the competition between a monopolistic incumbent and a potential entrant in a two-sided market, and develop a model representing the incumbent's pricing strategy to deter the threat of entrant. In the end, they obtain two major conclusions. First, if the incumbent dose not face with the entry threat, then the incumbent will focus on the high-value consumers. Second, the entrant will enter the market if and only if its entry cost is less enough. After that, the incumbent will take "limit pricing" strategy which lower the product price at any cost to deter entry.

When it comes to the case of a monopoly platform, Armstrong (2006) develop an optimal pricing function similar to the Lerner index to depict how the price elasticity of

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demand and the network externality affect the platform's pricing strategy. In the long run, they have the following conclusion: When the price elasticity of demand is high, or the effect of network externality is strong, the platform would lower its price at any cost to attract agents as more as possible to join the platform. When it comes to the case of a duopoly platform in a two-sided single-homing environment, they have the following conclusion: There is no platform would like to price its service too high. Furthermore, they even find that the platform can increase its profit by using "two-part tariffs" charging method (charge fixed and per-transaction fees at the same time).

By 2006, there are two trends of literature discussing the pricing strategy with network externality, which are pure membership and pure usage charges. Rochet and Tirole (2006) develop a mixed model combined with these two types of charging methods. In the beginning of this paper, they define the "two-sided market" as following: Consider a platform charging per-interaction charges a^B and a^S to the buyer and seller sides, and make $a = a^B + a^S$ as a constant value. If the volume of transactions realized on the platform varies with a^B, then the market is two-sided. Similar to Armstrong (2006), the authors then build up a pricing function which is analogous to Lerner index. Finally, given that the market is two-sided, this pricing function could be applied to the pure membership charges, the pure usage charges or mixed of them. That is, the platform could maximize its profit by manipulating a^B and a^S.

When we take network externality into account, many of past studies might have different conclusions. Furthermore, we have no doubt that this effect would also change the traditional business model, or even create a paradigm-shift. These works related to network externality would help us to penetrate those emerging business models. In order

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to better clarify the competition between the two types of delivers services, we leverage network externality and sharing economy to explain consumers' behavior in our study.

2.3 Delivery service competition

Traditionally, Bertrand and Cournot competition are two fundamental modeling methods to represent the competitive relationship between two players. Watson (2013) illustrates these two kinds of competition in detail. To put it briefly, the players decide their optimal price in Bertrand competition, and decide optimal quantity in Cournot competition. However, with the progress of the times, more and more potential factors which may change the competitive relationship appear. Particularly, response time has been a most common weapon to compete with other competitors. Below we show some papers related to this topic.

Li and Lee (1994) develop a model to depict a time-based duopolistic competition and address the optimal choice of price, quality, responsiveness and technology by a firm in this environment. There is an important assumption in their model: the customers have information about the delivery speed and current workload of the two firms. Finally, they have the conclusion as follows: The firm with faster processing speed always enjoys a price premium and acquires a larger market share.

With the assumption that demands are effected by both prices and time guarantees, So (2000) studies how firms compete with each other by setting their prices and time guarantees. The conclusion in this paper suggest that no matter in an oligopolistic or a monopolistic market, the optimal price and time guarantee decisions would be identical.

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But if the firms are heterogeneous, these firms would expand their service differentiation.

McGuire and Staelin (1983) study the key question in distribution channel selection: The number of levels of intermediary to distribute products. There are two manufacturers in the market, each of them decides to sell its product through a company store (integration) or through a franchise store (decentralization). According to the conclusion of this paper, when the products are quite similar or the competition is quite intense, both manufacturers may want do decentralization to shield themselves from an intense competition environment. However, when the competition is intense but not intense enough, it will result in a prisoners dilemma, which means that both manufacturers would do integration even if both of them doing decentralization is better.

With the ever-changing nature of technology, the gap between recent logistics and the related works we mention above becomes more and more distant. For example, the assumption of Li and Lee (1994) which assumes that the customers have logistics information may not be accepted optionally in the 1990s, but it is quite reasonable with logistics tracking status online technology in this time. Besides, put the concepts of network externality and sharing economy into these timed-based and channel selection models, we expect that we can explain more economic phenomenon.

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Chapter 3

Model

We consider a market with two groups of people, consumers (for each of them, she) and shoppers (for each of them, he), and a monopolistic platform (it) who provides platform delivery services to match consumers and shoppers. To join the platform, a consumer pays a membership fee F_{C} to the platform. She may then order on the platform and let the platform find a shopper for her. After matching a transaction successfully, a shopper is in duty bound to deliver groceries to a consumer. In every match, the platform charges a per transaction fee r_{C} from the consumer and gives the shopper a per matching subsidy r_{S}. The per matching cost incurred by the platform is $c \geq 0$. In our basic model, we assume that $c = 0$. This will be relaxed in Section 5.2.

Because shoppers are independent contractors and are not forced to work for the platform, the number of shoppers cannot be controlled by the platform. Therefore, the service quality depends on the number of shoppers on the market in equilibrium. Let Q be the service quality and n_{S} be the number of shoppers on the market, in this study we assume that $Q = \sqrt{n_{\mathrm{S}}}$. This setting captures the fact that the quality increases as

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the number of shoppers becomes larger, and the marginal improvement is decreasing.[1] Consumers are heterogeneous on their type θ, the willingness to pay for high-quality services. We assumed that θ is uniformly distributed in $[0, 1]$. Let $N > 0$ be the number of orders that a consumer will order in one membership period, a type-θ consumer's utility is thus

$$u_{\mathrm{C}} = N(\theta Q - r_{\mathrm{C}}) - F_{\mathrm{C}}. \tag{3.1}$$

In our basic model, we will assume that N is a constant that is not affected by the prices. While this may be true for services like grocery delivery, this may be inappropriate for services like transportation. We adopt a price-sensitive number of orders in Section 6.2 to examine this situation.

To complete a transaction, the shopper incurs a per transaction cost η from the platform, where η is assumed to distribute uniformly within 0 and 1. Therefore, his net earning for completing one transaction is $-\eta + r_{\mathrm{S}}$. If there are n_{C} customers being members of the platform, there will be in total Nn_{C} orders in a membership period. Given that there are n_{S} shoppers in the market, each shopper in expectation will get $\frac{Nn_{\mathrm{C}}}{n_{\mathrm{S}}}$ orders. Therefore, a type-η shopper's utility in a membership period is

$$u_{\mathrm{S}} = \frac{Nn_{\mathrm{C}}}{n_{\mathrm{S}}}(-\eta + r_{\mathrm{S}}) - F_S, \tag{3.2}$$

[1] Note that this setting does not consider the negative externality among customers (due to, e.g., competition for getting services). In Uber's case, such a component is required because as more customers using Uber to find a driver, the chances of getting the driving service becomes lower. For grocery delivery, however, this issue is not that critical, as the grocery delivery service is typically not so urgently needed. We thus assume that Q is not affected by the number of customers in this study. In fact, this setting is also adopted by Armstrong (2006) to emphasize the impact of network externality on platform's optimal strategy.

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where F_S is the fixed membership fee for a shopper to join the platform. As a fixed membership fee for a service provider is quite uncommon in practice, we will assume that $F_S = 0$ in the sequel. We examine the impact of adopting a nonzero F_S in Section 5.3.

It is assumed that a consumer or shopper will join the platform if $u_{\mathrm{C}} \geq 0$ or $u_{\mathrm{S}} \geq 0$, respectively. According to our setting, there exists a critical value θ^* which divides consumers into two groups: A consumer would join the platform if and only if $\theta > \theta^*$. Similarly, there exists a critical value η^* such that a shopper would join the platform if and only if $\eta < \eta^*$. In our notation, this means

$$n_{\mathrm{C}} = 1 - \theta^* \quad \text{and} \quad n_{\mathrm{S}} = \eta^*. \tag{3.3}$$

A visualization is provided in Figure 3.1.

Figure 3.1: When will the consumers and shoppers join the platform.

The platform's problem is to maximize its profit

$$\pi = Nn_{\mathrm{C}}a(r_{\mathrm{C}} - r_{\mathrm{S}} - c) + n_{\mathrm{C}}F_{\mathrm{C}} - n_{\mathrm{S}}F_{\mathrm{S}}, \tag{3.4}$$

where $a \in [0, 1]$ is the discounting factor. In our basic model, we will assume that $a = 1$, i.e., there is no time discounting. We investigate the impact of discounting in Section 5.1.

If a platform wants to maximize its profit, it should solve the above optimization problem with three decision variables (F_{C}, r_{C}, and r_{S}). As this may be too complicated, many companies in practice adopt more restricted pricing strategies. In this study, we

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investigate the profitability of three pricing strategies commonly adopted in practice. By adopting the *membership-based pricing* strategy, the platform only charges consumers a fixed membership fee, i.e., $r_{\mathrm{C}} = 0$. On the opposite, under the *transaction-based pricing* strategy, the platform does not charge any fixed membership fee, i.e., $F_{\mathrm{C}} = 0$, and rely on transaction fees to generate revenue. The third strategy, the *cross-subsidization* strategy, lets the platform subsidize the shopper by the entire transaction fee collected from the consumer, i.e., $r_{\mathrm{C}} = r_{\mathrm{S}}$. We are interested in understanding the profitability of these three pricing strategies.

The sequence of events is as follows. First, the platform decides the per transaction fee r_{C}, the per matching subsidy r_{S}, and the membership fees F_{C}. Second, potential consumers and shoppers observe the prices of using the service and decide whether to join the platform or not independently. In the end of this stage, the sizes of the two groups will be realized, and the platform can calculate its optimal profit in equilibrium.

A list of notations is provided in table 3.1.

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	Decision variables
r_{C}	The per-transaction fee
r_{S}	The per-transaction subsidy
F_{C}	The membership fee
F_{S}	The fixed subsidy in one membership period
	Parameters
n_{C}	The number of consumers
n_{S}	The number of shoppers
θ	Consumers' valuation for per-quality service
η	Shoppers' per-transaction cost
N	Consumption of each consumer in one membership period
Q	Quality of platform's service
c	The platform's marginal cost
a	The platform's discount factor

Table 3.1: List of decision variables and parameters

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Chapter 4

Analysis

In this section, we analyze the maximization problems of the platform delivery company. We present the platform company's optimal profits respectively under the three pricing strategies. We then reveal some characteristics of these strategies by comparing them. Finally, we compare their profitability.

We first derive the profit function of platform. Given r_{C}, r_{S}, and F_{C}, (3.1), (3.2), (3.3), and $Q = \sqrt{n_{\mathrm{S}}}$ together imply that

$$F_{\mathrm{C}} = N(\theta^* \sqrt{\eta^*} - r_{\mathrm{C}}) \quad \text{and} \quad N\frac{1-\theta^*}{\eta^*}(-\eta^* + r_{\mathrm{S}}) = 0, \tag{4.1}$$

where the former and latter are for the type-θ^* customer's and type-η^* shopper's utilities to be 0, respectively. By solving the system, we get a unique solution of θ^* and η^*

$$\theta^* = \frac{\sqrt{r_{\mathrm{S}}}(r_{\mathrm{C}}N + F_{\mathrm{C}})}{r_{\mathrm{S}}N} \quad \text{and} \quad \eta^* = r_{\mathrm{S}}. \tag{4.2}$$

Substituting θ^* and η^* into (3.4), we have the platform's profit function as

$$\pi = \left(1 - \frac{\sqrt{r_{\mathrm{S}}}(r_{\mathrm{C}}N + F_{\mathrm{C}})}{r_{\mathrm{S}}N}\right)(F_{\mathrm{C}} + N(r_{\mathrm{C}} - r_{\mathrm{S}})) \tag{4.3}$$

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4.1 Optimal profits

We now examine each of the three pricing strategies one by one. When the platform adopts membership-based pricing, it earns profits only from membership fees, i.e., $r_{\mathrm{C}} = 0$. Now, the platform's objective function reduces to

$$\pi^{\mathrm{M}} = (1 - \frac{\sqrt{r_{\mathrm{S}}}F_{\mathrm{C}}}{r_{\mathrm{S}}N})(F_{\mathrm{C}} + N(-r_{\mathrm{S}})). \tag{4.4}$$

The optimal solution can be found as follows.

Lemma 1. *The optimal solution of membership-based pricing strategy is*

$$r_{\mathrm{S}}^{\mathrm{M}} = \frac{1}{9} \quad \textit{and} \quad F_{\mathrm{C}}^{\mathrm{M}} = \frac{2}{9}N. \tag{4.5}$$

Both $r_{\mathrm{S}}^{\mathrm{M}}$ and $F_{\mathrm{C}}^{\mathrm{M}}$ are non-negative.

When the platform employs transaction-based pricing, it earns profits only from per-transaction fee, i.e., $F_{\mathrm{C}} = 0$. Now, the platform's objective function reduces to

$$\pi^{\mathrm{T}} = (1 - \frac{\sqrt{r_{\mathrm{S}}}r_{\mathrm{C}}}{r_{\mathrm{S}}})N(r_{\mathrm{C}} - r_{\mathrm{S}}) \tag{4.6}$$

The optimal solution can be found as follows.

Lemma 2. *The optimal solution of transaction-based pricing strategy is*

$$r_{\mathrm{S}}^{\mathrm{T}} = \frac{1}{9} \quad \textit{and} \quad r_{\mathrm{C}}^{\mathrm{T}} = \frac{2}{9}. \tag{4.7}$$

Both $r_{\mathrm{S}}^{\mathrm{T}}$ and $r_{\mathrm{C}}^{\mathrm{T}}$ are non-negative.

When the platform adopts cross-subsidization strategy, it subsidizes a shopper by a transaction fee charged from a consumer in every matching, i.e., $r_{\mathrm{C}} = r_{\mathrm{S}}$. Now, formula

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(4.3) can be rewritten as

$$\pi^{\mathrm{X}} = (1 - \frac{\sqrt{r_{\mathrm{S}}}(r_{\mathrm{S}}N + F_{\mathrm{C}})}{r_{\mathrm{S}}N})F_{\mathrm{C}} \tag{4.8}$$

The optimal solution can be found as follows.

Lemma 3. *The optimal solution of cross-subsidization strategy is*

$$r_{\mathrm{S}}^{\mathrm{X}} = \frac{1}{9}, \quad r_{\mathrm{C}}^{\mathrm{X}} = \frac{1}{9} \quad \textit{and} \quad F_{\mathrm{C}}^{\mathrm{X}} = \frac{1}{9}N. \tag{4.9}$$

$r_{\mathrm{S}}^{\mathrm{X}}$, $r_{\mathrm{C}}^{\mathrm{X}}$ *and* $F_{\mathrm{C}}^{\mathrm{X}}$ *are all non-negative.*

4.2 Comparisons

Up until this point, we have the profit-maximizing prices of three possible pricing strategies. We would like to do some comparison on these strategies to see which one is the platform's best pricing strategy. Furthermore, we hope our findings could explain the revenue model of platforms in sharing economy, to some extent.

First of all, we compare the fees under the three pricing strategies. The results are shown in Proposition 1.

Proposition 1. *By comparing the optimal ways of implementing membership-based pricing, transaction-based pricing, and cross subsidization, we have*

$$\begin{cases} r_{\mathrm{S}}^{\mathrm{X}} = r_{\mathrm{S}}^{\mathrm{M}} = r_{\mathrm{S}}^{\mathrm{T}} \\ r_{\mathrm{C}}^{\mathrm{T}} > r_{\mathrm{C}}^{\mathrm{X}} > 0 \\ F_{\mathrm{C}}^{\mathrm{M}} > F_{\mathrm{C}}^{\mathrm{X}} > 0 \end{cases} . \tag{4.10}$$

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Proposition 1 demonstrates some interesting findings. No matter which pricing strategy the platform employs, the amount (r_{S}) that the platform subsidizes a shopper in each transaction are all the same. This finding also means that the platform would induce the same number of shoppers joining it.

If we only investigate the relative magnitude of each kind of fees, it is still insufficient to see which strategy is the best for platform. Thus, we further discuss the platform's profit-maximizing strategy (among the three strategies considered in this study) in Proposition 2.

Proposition 2. *The platform's profits under three strategies are all the same, i.e.,*

$$\pi^{\mathrm{M}} = \pi^{\mathrm{T}} = \pi^{\mathrm{X}}. \tag{4.11}$$

Proposition 2 is an interesting and even surprising discovery. It shows that the three pricing strategies are equally good for the platform to maximize profit. In fact, it can be analytically verified that $1-\theta^*$ and η^*, i.e., the numbers of participating consumers and shoppers, are all the same. This means that the three pricing strategies are equivalent.

The only question that is still unsolved is that whether the three strategies are indeed optimal among all possible strategies. To address this question, in Proposition 3 we derive a necessary and sufficient condition for a solution $(r_{\mathrm{C}}, r_{\mathrm{S}}, F_{\mathrm{C}})$ to be optimal. It turns out that a family of pricing strategies are all optimal and all the three pricing strategies satisfy the condition in Proposition 3.

Proposition 3. *A solution $(r_{\mathrm{C}}, r_{\mathrm{S}}, F_{\mathrm{C}})$ is optimal to the platform's problem in (4.3) if and only if*

$$r_{\mathrm{S}} = \frac{1}{9} \quad \text{and} \quad r_{\mathrm{C}}N + F_{\mathrm{C}} = \frac{2}{9}N. \tag{4.12}$$

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According to Proposition 3, for the platform to optimize its profit, it should set r_{S} to a single level regardless of the values of r_{C} and F_{C}. This implies that there exists a most profitable equilibrium shopper volume, and the ability to choose r_{S} allows the platform to induce exactly that number of shoppers to join. Moreover, the membership fee F_{C} and transaction fee r_{C} satisfy a linear equation, which means that the platform may freely adjust these two fees to achieve the most profitable equilibrium customer volume. For example, if it wants to increase r_{C}, all it needs to do is to reduce F_{C} accordingly. As long as a customer's annual payments sum up to $\frac{2}{9}N$, an optimal solution is reached. Even if we take away one pricing variable (by following any of the three strategies), the platform may still perfectly match supply and demand to maximize its profit.

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Chapter 5

Extensions

5.1 Discount factor

When it comes to the asset turnover, it is necessary to take consideration of how sensitive the platform is to cash flow in each transaction. Thus, we add a parameter $a \in [0, 1]$ in our model to demonstrate it, and the platform's profit function would be

$$\pi = Nan_{\mathrm{C}}(r_{\mathrm{C}} - r_{\mathrm{S}} - c) + n_{\mathrm{C}}F_{\mathrm{C}}. \tag{5.1}$$

When a is small, the platform would like to collect money as soon as possible. In other words, the platform is more impatient.

Substituting n_C and n_S into (5.1), we have the general platform's profit function:

$$\pi_{\text{discount}} = \left(1 - \frac{\sqrt{r_{\mathrm{S}}}(r_{\mathrm{C}}N + F_{\mathrm{C}})}{r_{\mathrm{S}}N}\right)(F_{\mathrm{C}} + Na(r_{\mathrm{C}} - r_{\mathrm{S}})). \tag{5.2}$$

Similar to how we derive lemma 1, 2, and 3, we have following lemmas which demonstrate the platform's optimal solutions under three pricing strategies.

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Lemma 4. *The optimal solution of membership-based pricing strategy:*

$$r_{\mathrm{S}}^{\mathrm{M}} = \frac{a^2+1}{18} \text{ and} \tag{5.3}$$

$$F_{\mathrm{C}}^{\mathrm{M}} = \frac{(\sqrt{1+a^2}(a+a^2)+3\sqrt{2}(1+a^2))N}{36\sqrt{1+a^2}}. \tag{5.4}$$

Both $r_{\mathrm{S}}^{\mathrm{M}}$ *and* $F_{\mathrm{C}}^{\mathrm{M}}$ *are non-negative.*

Lemma 5. *The optimal solution of transaction-based pricing strategy:*

$$r_{\mathrm{S}}^{\mathrm{T}} = \frac{1}{9} \text{ and} \tag{5.5}$$

$$r_{\mathrm{C}}^{\mathrm{T}} = \frac{2}{9}. \tag{5.6}$$

Both $r_{\mathrm{S}}^{\mathrm{T}}$ *and* $r_{\mathrm{C}}^{\mathrm{T}}$ *are non-negative.*

Lemma 6. *The optimal solution of cross-subsidization strategy:*

$$r_{\mathrm{S}}^{\mathrm{X}} = \frac{1}{9}, \tag{5.7}$$

$$r_{\mathrm{C}}^{\mathrm{X}} = \frac{1}{9} \text{ and} \tag{5.8}$$

$$F_{\mathrm{C}}^{\mathrm{X}} = \frac{1}{9}N. \tag{5.9}$$

All $r_{\mathrm{S}}^{\mathrm{X}}$, $r_{\mathrm{C}}^{\mathrm{X}}$ *and* $F_{\mathrm{C}}^{\mathrm{X}}$ *are non-negative.*

As with how we get Proposition 1 and 2, we have the following two propositions.

Proposition 4.

$$\begin{cases} r_{\mathrm{S}}^{\mathrm{T}} = r_{\mathrm{S}}^{\mathrm{X}} > r_{\mathrm{S}}^{\mathrm{M}} \\ r_{\mathrm{C}}^{\mathrm{T}} > r_{\mathrm{C}}^{\mathrm{X}} > 0 \\ F_{\mathrm{C}}^{\mathrm{M}} > F_{\mathrm{C}}^{\mathrm{X}} > 0 \end{cases}. \tag{5.10}$$

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In the general case ($a < 1$), the platform would subsidize shoppers the least under the membership-based pricing strategy and the same under the others. Regarding the membership fee F_{C}, unsurprisingly the platform adopting membership-based pricing strategy would charge membership fee on consumers more than the other strategies. Interestingly, the platform implementing the transaction-based pricing strategy would always charge the per-transaction fee from consumers more than the other strategies.

Proposition 5. *In the general case* ($a < 1$)*, the platform's optimal profits under three strategies*

$$\pi^{\mathrm{M}}_{\mathrm{discount}} > \pi^{\mathrm{X}}_{\mathrm{discount}} > \pi^{\mathrm{T}}_{\mathrm{discount}}. \tag{5.11}$$

Figure 5.1 provides a visualization for Proposition 2 and 5. Proposition 5 is an inter-

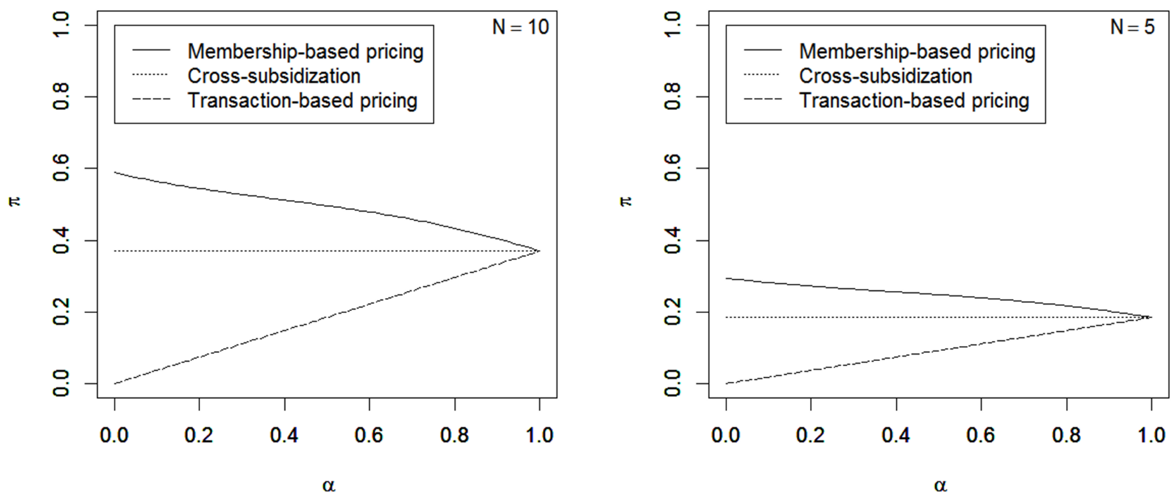

Figure 5.1: Platform optimal profits under three strategies.

esting discovery. It shows that the three pricing strategies we discuss in this study are all equivalent when the platform is perfectly patient ($a = 1$). However, as long as the

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platform is somewhat impatient ($a < 1$), membership-based pricing outperforms cross-subsidization, which outperforms transaction-based pricing. Given that the three policies are equally good when there is no time discounting, it is intuitive that collecting money as early as possible is profitable if there is time discounting. This explains why membership-based pricing is the best and transaction-based pricing is the worst. Cross-subsidization then lies in between. Our result may partly explain why Instacart tries hard to promote Instacart Express, its membership program.[1]

5.2 Marginal transaction cost

In this section, we furthermore investigate the optimal profit for platform when we take platform's cost into account and set $a = 1$ to challenge the solidification of Proposition 1 and 2. Here, we have the general platform's profit function:

$$\pi_{\text{cost}} = \left(1 - \frac{\sqrt{r_{\text{S}}}(r_{\text{C}}N + F_{\text{C}})}{r_{\text{S}}N}\right)(F_{\text{C}} + N(r_{\text{C}} - r_{\text{S}} - c)). \tag{5.12}$$

Similar to how we derive lemma 1, 2, and 3, we have the following lemmas which demonstrate the platform's optimal solutions under three pricing strategies.

Lemma 7. *The optimal solution of membership-based pricing strategy:*

$$r_{\text{S}}^{\text{M}} = \frac{6c + 1 + \sqrt{12c + 1}}{18} \text{ and} \tag{5.13}$$

[1]By giving members unlimited free 2-hour and scheduled deliveries over \$35, Instacart can collect \$149 per year as the membership fee at the beginning of a membership cycle. Information source: https://news.instacart.com/2015/12/29/weve-updated-our-delivery-prices/.

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$$F_{\mathrm{C}}^{\mathrm{M}} = \frac{\left(\sqrt{\sqrt{12c+1}+6c+1}\left(\sqrt{12c+1}+24c+1\right)+3\sqrt{2}(\sqrt{12c+1}+6c+1)\right)N}{36\sqrt{\sqrt{12c+1}+6c+1}}. \tag{5.14}$$

Both $r_{\mathrm{S}}^{\mathrm{M}}$ and $F_{\mathrm{C}}^{\mathrm{M}}$ are non-negative.

Lemma 8. *The optimal solution of transaction-based pricing strategy:*

$$r_{\mathrm{S}}^{\mathrm{T}} = \frac{6c+1+\sqrt{12c+1}}{18} \text{ and} \tag{5.15}$$

$$r_{\mathrm{C}}^{\mathrm{T}} = \frac{6c+1+\sqrt{12c+1}}{9}. \tag{5.16}$$

Both $r_{\mathrm{S}}^{\mathrm{T}}$ and $r_{\mathrm{C}}^{\mathrm{T}}$ are non-negative.

Lemma 9. *The optimal solution of cross-subsidization strategy:*

$$r_{\mathrm{S}}^{\mathrm{X}} = \frac{6c+1+\sqrt{12c+1}}{18}, \tag{5.17}$$

$$r_{\mathrm{C}}^{\mathrm{X}} = \frac{6c+1+\sqrt{12c+1}}{18} \text{ and} \tag{5.18}$$

$$F_{\mathrm{C}}^{\mathrm{X}} = \frac{6c+1+\sqrt{12c+1}}{18}N. \tag{5.19}$$

All $r_{\mathrm{S}}^{\mathrm{X}}$, $r_{\mathrm{C}}^{\mathrm{X}}$ and $F_{\mathrm{C}}^{\mathrm{X}}$ are non-negative.

Similar to Proposition 1, we then compare every kind of fees respectively in these three pricing strategies. The results are shown in Proposition 6.

Proposition 6. *When we take platform's marginal cost into account and set $a = 1$, we have*

$$\begin{cases} r_{\mathrm{S}}^{\mathrm{X}} = r_{\mathrm{S}}^{\mathrm{M}} = r_{\mathrm{S}}^{\mathrm{T}} \\ r_{\mathrm{C}}^{\mathrm{T}} > r_{\mathrm{C}}^{\mathrm{X}} > 0 \\ F_{\mathrm{C}}^{\mathrm{M}} > F_{\mathrm{C}}^{\mathrm{X}} > 0 \end{cases}. \tag{5.20}$$

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Furthermore, we put optimal solutions of three pricing strategies into (5.12) and then compare them. The result is presented in Proposition 7.

Proposition 7. *The platform's optimal profits under three strategies are all the same:*

$$\pi_{\text{cost}}^{\text{M}} = \pi_{\text{cost}}^{\text{T}} = \pi_{\text{cost}}^{\text{X}}. \tag{5.21}$$

The results in Proposition 6 and 7 are the same as the results in Proposition 1 and 2. Therefore, we can judge that the results in Proposition 1 and 2 are solid even though we do not take consideration of the platform's marginal cost.

The services Instacart provides are a kind of information services. Moreover, quite a few researchers claim that the marginal costs of information goods/services should be zero. For example, iTunes is an online music platform which matches music manufactures and its consumers. In every matching, it basically cost nothing for Apple Inc. Thus, the platform's marginal cost is not only mathematically negligible, but also quite reasonable to be ignored in economic aspect.

5.3 Fixed shopper subsidization

In the previous discussion, we investigate the platform's profitability based on the assumption that shoppers make profit only from per transaction fee subsidized by the platform, i.e., $r_{\text{S}} \neq 0$ and $F_{\text{S}} = 0$. In this section, we wonder the result when the platform employs every shopper with a fixed payment rather than a per-transaction one, i.e., $F_{\text{S}} \neq 0$ and $r_{\text{S}} = 0$. Then we have consumers' and shoppers' utility functions

$$u_{\text{C}} = N(\theta\sqrt{\eta} - r_{\text{C}}) - F_{\text{C}} \text{ and} \tag{5.22}$$

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$$u_{\mathrm{S}} = \frac{Nn_{\mathrm{C}}}{n_{\mathrm{S}}}(-\eta) + F_{\mathrm{S}}. \tag{5.23}$$

The platform's profit function here is

$$\pi = Nan_{\mathrm{C}}(r_{\mathrm{C}}) + n_{\mathrm{C}}F_{\mathrm{C}} - n_{\mathrm{S}}F_{\mathrm{S}}. \tag{5.24}$$

With (5.22) and (5.23), we get the indifference points of consumers and shoppers as $\theta^* = \frac{-F_{\mathrm{S}}+N}{N}$ and $\eta^* = \left(\frac{F_{\mathrm{C}}+Nr_{\mathrm{C}}}{-F_{\mathrm{S}}+N}\right)^2$. The platform's profit function would then be calculated as

$$\pi_{fixed} = \frac{F_{\mathrm{S}}}{N}(Nar_{\mathrm{C}} + F_{\mathrm{C}}) - \left(\frac{Nr_{\mathrm{C}} + F_{\mathrm{C}}}{-F_{\mathrm{S}} + N}\right)^2 F_{\mathrm{S}}. \tag{5.25}$$

Similar to previous discussions, we would still like to investigate some commonly-used pricing strategies. The only difference is that the cross-subsidization strategy is excluded here since somehow we have already set r_{S} be zero. The main results are presented in Proposition 8 and 9.

Proposition 8. *When we set $a = 1$ and $c = 0$, a plan $(r_{\mathrm{C}}, F_{\mathrm{C}}, F_{\mathrm{S}})$ is the platform's optimal solution if and only if*

$$F_{\mathrm{S}} = \frac{N}{3} \text{ and } r_{\mathrm{C}}N + F_{\mathrm{C}} = \frac{2}{9}N. \tag{5.26}$$

Furthermore, no matter the platform subsidizes shoppers with fixed or per-transaction subsidization, the platform's profits are the same (cf. proposition 3).

Even though in Proposition 6 we have mathematically proven that it is indifferent between fixed and per-transaction subsidies for the platform to employ when $a = 1$, it may still be better for the platform to implement per-transaction subsidy in practical situation. If the platform adopts fixed subsidy, it may need to deal with the following

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screening problem: The shoppers may loaf on their jobs after they take the fixed subsidy. On contrary, adopting per-transaction subsidy is relatively easy to handle; after all, the platform can substantially get what it pays for in each transaction by this way.

Proposition 9. *When we set $a < 1$ and $c = 0$, the platform's optimal solution under transaction-based and membership-based pricing strategies would be*

$$F_{\mathrm{S}}^{\mathrm{T}} = \frac{N}{3},\ r_{\mathrm{C}}^{\mathrm{T}} = \frac{2}{9}a \text{ and} \tag{5.27}$$

$$F_{\mathrm{S}}^{\mathrm{M}} = \frac{N}{3},\ F_{\mathrm{C}}^{\mathrm{M}} = \frac{2}{9}N. \tag{5.28}$$

And the platform's optimal profits under these two strategies would have the following relation

$$\pi_{\text{fixed}}^{\mathrm{M}} > \pi_{\text{fixed}}^{\mathrm{T}}. \tag{5.29}$$

Furthermore, no matter the platform employs which pricing strategy, subsidizing shoppers with per-transaction subsidization is better for it, i.e,

$$\pi_{\text{discount}}^{\mathrm{M}} > \pi_{\text{fixed}}^{\mathrm{M}} \text{ and } \pi_{\text{discount}}^{\mathrm{T}} > \pi_{\text{fixed}}^{\mathrm{T}}. \tag{5.30}$$

Obviously, solutions (5.27) and (5.28) are all optimal, since they all satisfy the condition in proposition 8. We also show that no matter the platform charges consumers per-transaction or fixed membership fees when $a < 1$, it can earn more by subsidizing shoppers with per-transaction subsidy. The parameter a in this study can also be presented as the degree of requirement for the platform to holding cash, a smaller a means that the more urgent the platform need cash. In the financial aspect, holding cash is conducive to enhancing a company's ability to not only face financial risk but also future invest, especially R&D as Bates et al. (2009) mention. In conclusion, a smaller a makes

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the platform more eager to hold cash, i.e., the platform would like to receive money from its consumers as soon as possible and subsidize shoppers as late as possible. The results of Proposition 2, 5, and 9 perfectly demonstrate above phenomenon: Charging membership fees for consumers is strictly better than charging per-transaction ones when $a < 1$; Subsidizing per-transaction subsidies to shoppers is strictly better than Subsidizing fixed ones when $a < 1$.

5.4 Price-sensitive number of orders

The previous discussions focus on a fixed number of orders (N) that a consumer will order in one membership period. It is sufficient for the case of Instacart, since the amount of daily necessities a family consumes would unlikely vary significantly across every membership period. However, in the case of Uber, a cheaper per-transaction fee (r_{C}) may attract consumers to use the service more often, i.e., the number of orders would decrease in the per-transaction fee in a certain fashion. Hence, we let each consumer's total consumption in one membership period be $\frac{N}{r_{\mathrm{C}}}$ here to demonstrate the above phenomenon. Theoretically, we can expect that if it is almost free in each transaction, then the consumers would have incentive to use the service unlimitedly ($\lim_{r_{\mathrm{C}} \to 0} \frac{N}{r_{\mathrm{C}}} = \infty$). We also set a capacity constraint for shoppers:

$$\frac{N n_{\mathrm{C}}}{r_{\mathrm{C}} n_{\mathrm{S}}} \leq K, \tag{5.31}$$

which means every shopper can only serve up to K times in one membership period.

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The platform's maximization problem now would be

$$\begin{aligned}
\max \quad & \frac{N}{r_{\mathrm{C}}} n_{\mathrm{C}}(r_{\mathrm{C}} - r_{\mathrm{S}}) + n_{\mathrm{C}} F_{\mathrm{C}} \\
\text{s.t.} \quad & u_{\mathrm{C}} = \frac{N}{r_{\mathrm{C}}}(\theta\sqrt{\eta} - r_{\mathrm{C}}) - F_{\mathrm{C}} \geq 0 \ \forall\ \theta > \theta^* \\
& u_{\mathrm{S}} = \frac{N}{r_{\mathrm{C}}} \frac{1-\theta}{\eta}(-\eta + r_{\mathrm{S}}) \geq 0 \ \forall\ \eta < \eta^* \\
& \frac{N n_{\mathrm{C}}}{r_{\mathrm{C}} n_{\mathrm{S}}} \leq K,
\end{aligned}$$

where θ^* and η^* are the indifference points. According to previous settings, we have $n_{\mathrm{S}} = \eta^*$ and $n_{\mathrm{C}} = 1 - \theta^*$. We also know the platform's optimal solution would let the first two constraints be binding, then we have $\theta^* = \frac{\frac{F_{\mathrm{C}} r_{\mathrm{C}}}{N} + r_{\mathrm{C}}}{\sqrt{r_{\mathrm{S}}}}$ and $\eta^* = r_{\mathrm{S}}$. Plug θ^* and η^* into the platform's profit function and (5.31), the maximization problem would be

$$\begin{aligned}
\max \quad & \left(1 - \frac{\frac{F_{\mathrm{C}} r_{\mathrm{C}}}{N} + r_{\mathrm{C}}}{\sqrt{r_{\mathrm{S}}}}\right)\left(\frac{N}{r_{\mathrm{C}}}(r_{\mathrm{C}} - r_{\mathrm{S}}) + F_{\mathrm{C}}\right) \\
\text{s.t.} \quad & r_{\mathrm{C}} \geq \frac{\sqrt{r_{\mathrm{S}}} N}{r_{\mathrm{S}}^{\frac{3}{2}} K + N + F_{\mathrm{C}}}
\end{aligned}$$

Here, we only conduct analysis on pure-transaction pricing and cross-subsidization strategies, since the membership pricing one must let the platform go in the red. Under pure-transaction pricing strategy, the maximization problem is

$$\max \quad \left(1 - \frac{r_{\mathrm{C}}}{\sqrt{r_{\mathrm{S}}}}\right)\left(\frac{N}{r_{\mathrm{C}}}(r_{\mathrm{C}} - r_{\mathrm{S}})\right) \tag{5.32}$$

$$\text{s.t.} \quad r_{\mathrm{C}} \geq \frac{\sqrt{r_{\mathrm{S}}} N}{r_{\mathrm{S}}^{\frac{3}{2}} K + N}. \tag{5.33}$$

Take a closer look at the objective function (5.32), we find that the platform could get a positive profit if and only if $1 - \frac{r_{\mathrm{C}}}{\sqrt{r_{\mathrm{S}}}} > 0$ and $r_{\mathrm{C}} - r_{\mathrm{S}} > 0$. With these conditions, we have $r_{\mathrm{S}} < r_{\mathrm{C}} < \sqrt{r_{\mathrm{S}}}$, which means r_{S} must be between 0 and 1. Hence, we search r_{S} from 0 to 1, and each r_{S} would decide an upper and a lower bounds of r_{C} according to $r_{\mathrm{S}} < r_{\mathrm{C}} < \sqrt{r_{\mathrm{S}}}$ and equation (5.31). We further search r_{C} between its bounds and calculate the profits.

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Figure 5.2 provides a part of our numerical study to find the optimal solution under pure-transaction pricing strategy. Eventually, we find that given an arbitrary set of N and K, the optimal solution under pure-transaction pricing strategy would always bind to (5.31). In fact, we can get a similar result under cross-subsidization strategy. Under cross-subsidization strategy, the maximization problem is

$$\max \quad \left(1 - \frac{\frac{F_{\mathrm{C}} r_{\mathrm{S}}}{N} + r_{\mathrm{S}}}{\sqrt{r_{\mathrm{S}}}}\right) F_{\mathrm{C}} \tag{5.34}$$

$$\text{s.t.} \quad F_{\mathrm{C}} \geq \frac{N}{\sqrt{r_{\mathrm{S}}}} - r_{\mathrm{S}}^{\frac{3}{2}} K - N. \tag{5.35}$$

Similar to previous analysis, we have $0 < F_{\mathrm{C}} < \frac{\sqrt{r_{\mathrm{S}}} - r_{\mathrm{S}}}{r_{\mathrm{S}}} N$, which means $0 < r_{\mathrm{S}} < 1$. Here, we also find that given an arbitrary set of N and K, the optimal solution under cross-subsidization strategy would always bind to (5.31). We summarize the main result in Observation 1. This observation illustrates that the shoppers' capacity is critical to the platform's profitability.

Observation 1. *Given an arbitrary set of N and K, the optimal solution would bind to (5.31) under pure-transaction pricing and cross-subsidization strategies.*

Compare each fee under the strategies we focus here, we have Observation 2. A visualization is presented in Figure 5.3. Since the number of shopper n_{S} is equal to r_{S}, we further know the number of shoppers is more under the cross-subsidization strategy than the pure-transaction pricing one ($n_{\mathrm{S}}^{\mathrm{X}} > n_{\mathrm{S}}^{\mathrm{T}}$).

Observation 2. $r_{\mathrm{C}}^{\mathrm{T}} > r_{\mathrm{C}}^{\mathrm{X}} = r_{\mathrm{S}}^{\mathrm{X}} > r_{\mathrm{S}}^{\mathrm{T}}$.

Examine the platform's profit functions (5.32) and (5.34) under two strategies, we can find that the functions consist of two parts: the number of consumers, and earnings from

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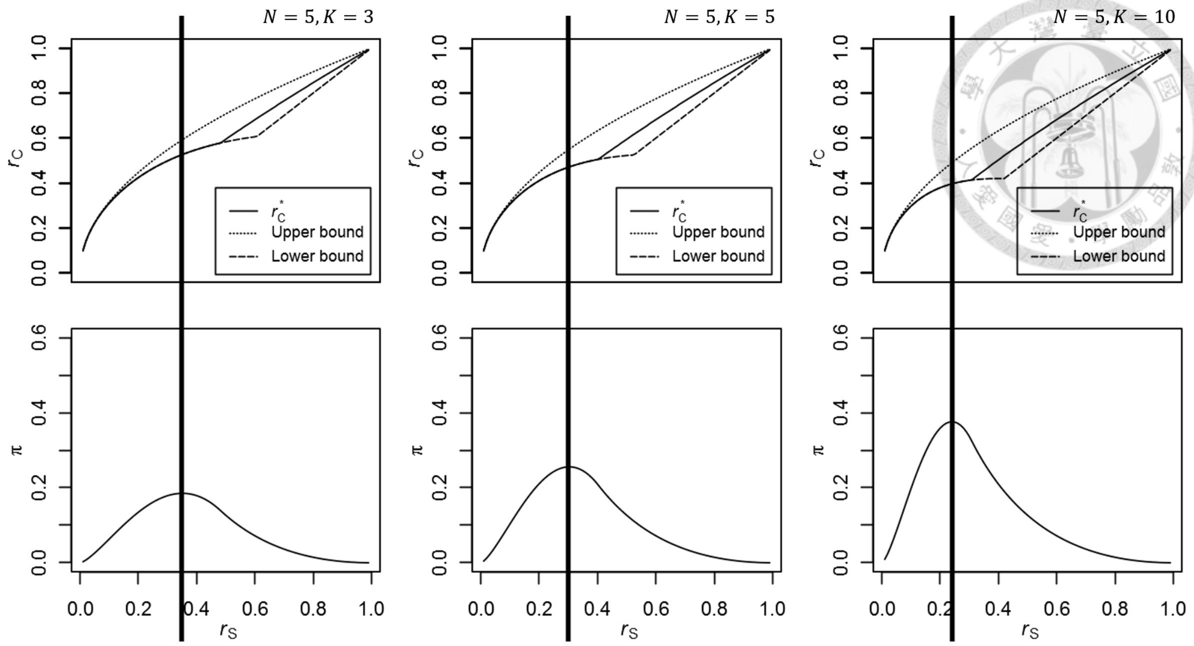

Figure 5.2: The optimal solution under pure-transaction pricing strategy.

each consumer. We further define m as the earnings part. Hence, (5.32) can be rewritten as $n_C^T m^T$, where $n_C^T = 1 - \frac{r_C}{\sqrt{r_S}}$ and $m^T = \frac{N}{r_C}(r_C - r_S)$. Similarly, (5.34) can be rewritten as $n_C^X m^X$, where $n_C^X = 1 - \frac{\frac{F_C r_S}{N} + r_S}{\sqrt{r_S}}$ and $m^X = F_C$. Moreover, we define TC as the total consumption of consumers, i.e., $TC = \frac{N}{r_C} n_C$. Compare the above three elements, the results are presented in the Observation 3, and Figure 3 provides a visualization.

Observation 3. *The number of consumers is more under the pure-transaction pricing strategy than the cross-subsidization one, i.e.,* $n_C^T > n_C^X$. *However, the earnings from each consumer and the total consumption are both more under the cross-subsidization strategy, i.e.,* $m^X > m^T$ *and* $TC^X > TC^T$.

Eventually, we compare the platform's optimal profits under these two strategies and get the Observation 4. A visualization is presented in Figure 5.5.

Observation 4. *The platform's profit would increase in* K*, and it can earn more by*

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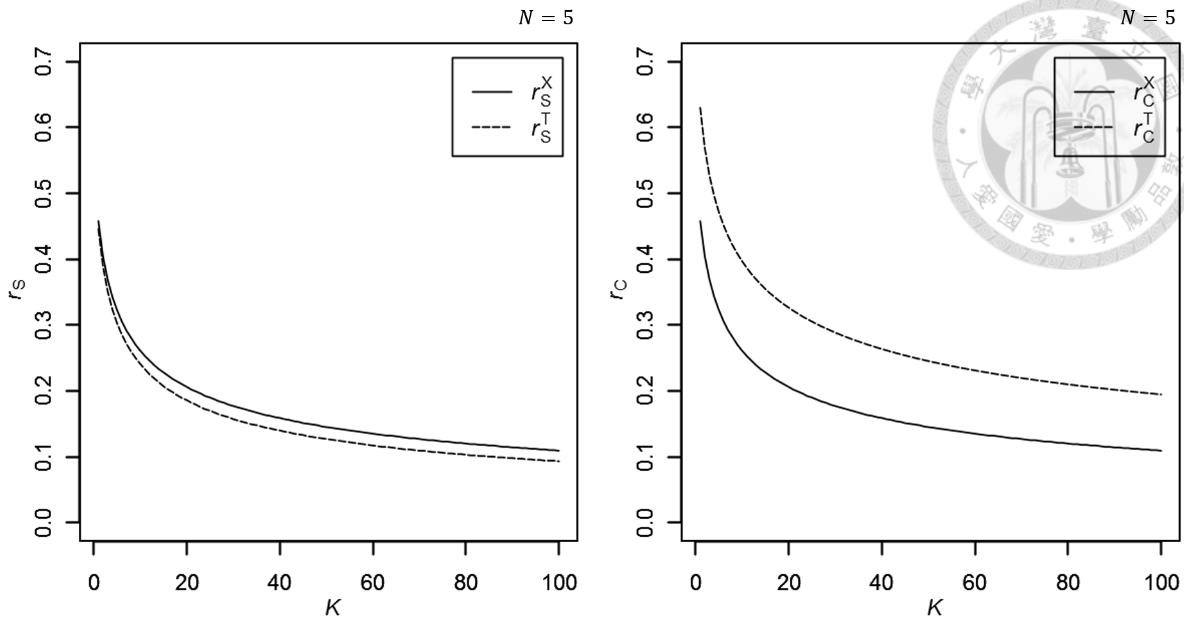

Figure 5.3: Compare each fee under these two strategies.

	r_S	r_C	n_S	n_C	m	TC	π
Cross-subsidization	higher		more		more	more	higher
Per-transaction based pricing		higher		more			

Table 5.1: Summary of Observation 2, 3 and 4

adopting the cross-subsidization strategy, i.e., $\pi^{X} > \pi^{T}$.

Table 5.1 is a summary of the above comparisons: We first find that even though the number of consumers is less under cross-subsidization strategy, the total consumption is more under this strategy. That is why the number of shoppers is more under cross-subsidization strategy to cope with the more requirement of delivery. Furthermore, we find that the platform can even earn more from each consumer under cross-subsidization strategy. Finally, implementing cross-subsidization strategy is better for the platform.

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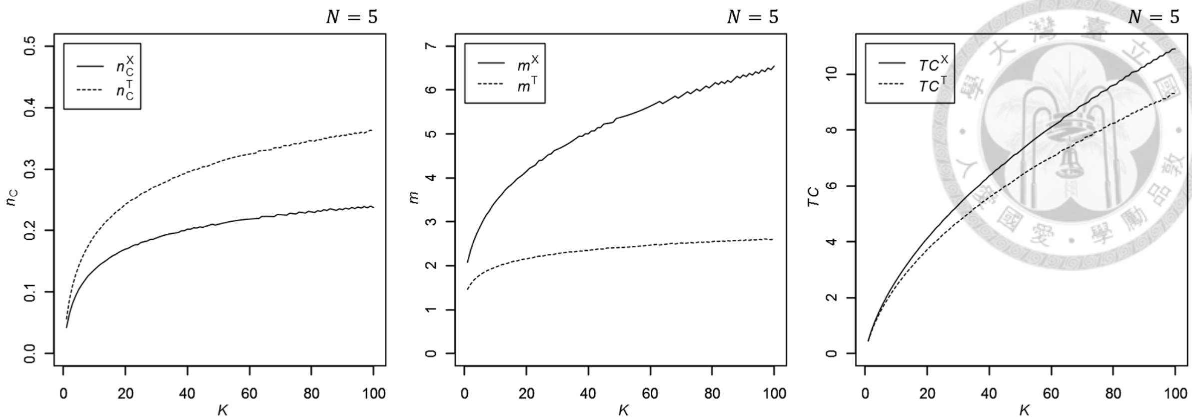

Figure 5.4: Compare the number of consumers, earnings from each consumer, and total consumption in one membership period.

5.5 Distribution of shoppers' costs

In the previous discussion, we assume that the shoppers and consumers are both uniformly distributed in 0 and 1. However, the shoppers may be distributed more sparsely or densely than consumers. Hence, we define a parameter b which represents the diversity of shoppers' distribution. Under this setting, we know that θ is uniformly distributed in 0 and 1, and η is uniformly distributed in 0 and b. The shoppers are distributed more densely than the consumers when $b < 1$; On the other hand, the shoppers are distributed more sparsely than the consumers when $b > 1$. We can find a consumer whose valuation is θ^* and a shopper whose cost is η^*, they are indifferent to join the platform. Then we have $n_C = 1 - \theta^*$ and $n_S = \frac{\eta^*}{b}$.

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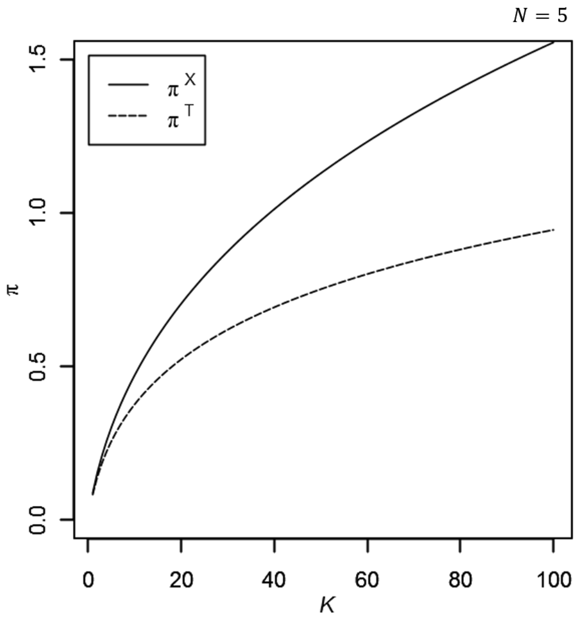

Figure 5.5: Compare the platform's optimal profits.

The platform's maximization problem now would be

$$\begin{aligned} \max \quad & Nn_{\mathrm{C}}(r_{\mathrm{C}} - r_{\mathrm{S}}) + n_{\mathrm{C}}F_{\mathrm{C}} \\ \text{s.t.} \quad & u_{\mathrm{C}} = N(\theta\sqrt{n_{\mathrm{S}}} - r_{\mathrm{C}}) - F_{\mathrm{C}} \geq 0 \ \forall\ \theta > \theta^* \\ & u_{\mathrm{S}} = \frac{Nn_{\mathrm{C}}}{n_{\mathrm{S}}}(-\eta + r_{\mathrm{S}}) \geq 0 \ \forall\ \eta < \eta^*. \end{aligned}$$

We know that the platform's optimal solution would let all constraints be binding, then we have $\theta^* = \frac{(r_{\mathrm{C}}N + F_{\mathrm{C}})\sqrt{b}}{N\sqrt{r_{\mathrm{S}}}}$ and $\eta^* = r_{\mathrm{S}}$. Plug θ^* and η^* into the platform's profit function, the general maximization problem would be

$$\max \quad \left(1 - \frac{(r_{\mathrm{C}}N + F_{\mathrm{C}})\sqrt{b}}{N\sqrt{r_{\mathrm{S}}}}\right)\left(F_{\mathrm{C}} + N(r_{\mathrm{C}} - r_{\mathrm{S}})\right).$$

Examine this profit function, we can find that the function can be divided into two parts: the number of consumers, and earnings from each consumer. We define m as the earnings part. Hence, this profit function can be rewritten as $n_{\mathrm{C}}m$, where $n_{\mathrm{C}} = 1 - \frac{(r_{\mathrm{C}}N + F_{\mathrm{C}})\sqrt{b}}{N\sqrt{r_{\mathrm{S}}}}$ and $m = F_{\mathrm{C}} + N(r_{\mathrm{C}} - r_{\mathrm{S}})$.

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After solving this problem, we have Proposition 10. The main discovery of Proposition 10 is the following: when b is decreasing, which means the shoppers are distributed more and more densely, we can first find that the number of consumers would not be change. However, the shoppers would be more, which directly lead to a better quality of service. At the same time, even though the platform should subsidize its shoppers more to attract more shoppers to join the platform, the consumers are also willing to pay the platform much more for a better service, which means the platform can earn more from each consumer. Eventually, we find that the platform's profit is decreasing in b, which means that when the shoppers are distributed more and more densely, i.e., the consumers are distributed more and more sparsely, the platform would be more profitable.

Proposition 10. *A plan $(r_{\mathrm{C}}, F_{\mathrm{C}}, r_{\mathrm{S}})$ is the platform's optimal solution if and only if*

$$r_{\mathrm{S}} = \frac{1}{9b} \text{ and } r_{\mathrm{C}}N + F_{\mathrm{C}} = \frac{2}{9b}N. \tag{5.36}$$

Furthermore, the number of consumers and shoppers would be $n_{\mathrm{C}} = \frac{1}{3}$ and $n_{\mathrm{S}} = \frac{1}{9b^2}$. Then, the earnings from each consumer would be $m = \frac{N}{9b}$ .Finally, the optimal profit would be $\frac{N}{27b}$.

In the end, it can be conducted that all the three common pricing policies, i.e., membership-based pricing, transaction-based pricing and cross subsidization, are all the best and all the same for the platform to implement, since the optimal condition in Proposition 10 can be satisfied under these policies. For example, let r_{C} be 0, we have the optimal solution under membership-based pricing strategy $F_{\mathrm{C}}^{\mathrm{M}} = \frac{2}{9b}N$ and $r_{\mathrm{C}}^{\mathrm{M}} = \frac{1}{9b}$.

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5.6 General service quality function

In this section, we generalize the platform's service quality as an increasing concave function of n_{S}, i.e., $Q = f(n_{\mathrm{S}})$. The platform's maximization problem now would be

$$\begin{aligned} \max \quad & N n_{\mathrm{C}}(r_{\mathrm{C}} - r_{\mathrm{S}}) + n_{\mathrm{C}} F_{\mathrm{C}} \\ \text{s.t.} \quad & u_{\mathrm{C}} = N(\theta f(n_{\mathrm{S}}) - r_{\mathrm{C}}) - F_{\mathrm{C}} \geq 0 \ \forall \ \theta > \theta^* \\ & u_{\mathrm{S}} = \frac{N n_{\mathrm{C}}}{n_{\mathrm{S}}}(-\eta + r_{\mathrm{S}}) \geq 0 \ \forall \ \eta < \eta^*. \end{aligned}$$

We know that the platform's optimal solution would let all constraints be binding, then we have $\theta^* = \frac{r_{\mathrm{C}} N + F_{\mathrm{C}}}{N f(r_{\mathrm{S}})}$ and $\eta^* = r_{\mathrm{S}}$. Plug θ^* and η^* into the platform's profit function, the maximization problem would be

$$\max \quad \left(1 - \frac{r_{\mathrm{C}} N + F_{\mathrm{C}}}{N f(r_{\mathrm{S}})}\right)\left(F_{\mathrm{C}} + N(r_{\mathrm{C}} - r_{\mathrm{S}})\right).$$

Define a variable $y = r_{\mathrm{C}} N + F_{\mathrm{C}}$, we can further rewrite the above problem as

$$\max \quad \left(1 - \frac{y}{N f(r_{\mathrm{S}})}\right)\left(y - N r_{\mathrm{S}}\right).$$

Make sure that the quality function $f(r_{\mathrm{S}})$ would let the objective function be concave, we can find the optimal solution when the conditions $\frac{\partial \pi}{\partial y} = 0$ and $\frac{\partial \pi}{\partial r_{\mathrm{S}}} = 0$ are satisfied. Any pricing policy which is satisfied the above condition would be optimal for the platform to adopt. After putting the constraints of three pricing policies into the problem, the optimal conditions can also be fulfilled. Hence, it can be claimed that the three common pricing policies are all the best and all the same under a rational service quality function.

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Chapter 6

Conclusions

6.1 Conclusions

In this study, we present a game-theoretic model featuring network externality and sharing economy to investigate three pricing strategies in platform delivery, i.e., membership-based pricing, transaction-based pricing, and cross-subsidization. We analytically calculate the optimal prices of these three strategies and then investigate the relative magnitude among fees charged with every strategy. Our main result shows that all the three strategies are equivalent: They result in the same per-transaction subsidy for shoppers, numbers of shoppers and consumers, and profits in equilibrium.

We also consider some extensions which take account of the following elements: First, when we take a discount factor into consideration, which means that the platform care about how fast it can receive money back, we find that membership-based pricing is the best and transaction-based pricing is the worst; Furthermore, we take the platform's

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marginal cost into account. The results in this extension shows that it is solid even though we do not take account of platform's cost; Moreover, in the extension of subsidizing shoppers with fixed fees rather than per-transaction fees, we find that subsidizing shoppers with per-transaction fees is better for platform to adopt; In the extension of regarding N as a decreasing function of r_C, which means that a consumer's consumption in each membership period would be negatively affected by the per-transaction fee charged from consumers, we find that cross-subsidization strategy is better than pure-transaction pricing one for platform to implement; In the extension of taking the distribution of shoppers' costs, the main result shows that when the shoppers are distributed more and more densely, the platform would be more profitable; Finally, we generalize the service quality function in the last extension, and find that the three common pricing strategies are all the best and all the same under a rational service quality function.

6.2 Future works

Our study certainly has its limitations. First, it would be interesting to compare this delivery model with the traditional approach, i.e., shipping from one's own warehouse by one's own full-time shoppers like AmazonFresh. Conditions under which platform delivery is a better model call for further investigation. Moreover, we have not consider the case of Uber's *surge pricing* yet, which means the platform may dynamically adjust the service price according to the idle resources.

Recently, the CEO of Walmart, Doug McMillon, announced their experimental program which outsources their groceries delivery business to Uber, Lyft and Deliv rather

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than self-owned trucks. [1] It means that AmazonFresh can also make the best of sharing economy to reach cost advantage similar to Instacart by using this way. How serious the impact is to Instacart? It deserves our future research.

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[1] Information source: http://blog.walmart.com/business/20160603/piloting-delivery-with-uber-lyft-and-deliv.

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Appendix A

Proofs of Lemmas and Propositions

Proof of Lemma 1. First of all, we let r_{P} be an opposite number of r_{S}, i.e., $r_{\mathrm{S}} = -r_{\mathrm{P}}$. Formula (4.4) is equal to

$$\pi^{\mathrm{M}} = \left(1 + \frac{\sqrt{-r_{\mathrm{P}}}F_{\mathrm{C}}}{r_{\mathrm{P}}N}\right)\Big(F_{\mathrm{C}} + Nr_{\mathrm{P}}\Big) \tag{A.1}$$

The derivatives of formula (A.1) with respect to F_{C} and r_{P} can be deduced as $\frac{\partial \pi^{\mathrm{M}}}{\partial F_{\mathrm{C}}} = \frac{\sqrt{-r_{\mathrm{P}}}(Nr_{\mathrm{P}}+2F_{\mathrm{C}})}{r_{\mathrm{P}}N} + 1$ and $\frac{\partial \pi^{\mathrm{M}}}{\partial r_{\mathrm{P}}} = \frac{2\sqrt{-r_{\mathrm{P}}}r_{\mathrm{P}}N^2+(-r_{\mathrm{P}})F_{\mathrm{C}}N+F_{\mathrm{C}}^2}{2\sqrt{-r_{\mathrm{P}}}r_{\mathrm{P}}N}$. According to $\frac{\partial^2 \pi^{\mathrm{M}}}{\partial F_{\mathrm{C}}^2} = \frac{2\sqrt{-r_{\mathrm{P}}}}{Nr_{\mathrm{P}}} < 0$ and $\frac{\partial^2 \pi^{\mathrm{M}}}{\partial r_{\mathrm{P}}^2} = -\frac{F_{\mathrm{C}}(Nr_{\mathrm{P}}-3F_{\mathrm{C}})}{4N(-r_{\mathrm{P}})^{\frac{3}{2}}r_{\mathrm{P}}} < 0$, we know that formula (A.1) is a concave function. The optimal solution $F_{\mathrm{C}}^{\mathrm{M}}$ and $r_{\mathrm{P}}^{\mathrm{M}}$ can be found when $\frac{\partial \pi^{\mathrm{M}}}{\partial F_{\mathrm{C}}} = 0$ and $\frac{\partial \pi^{\mathrm{M}}}{\partial r_{\mathrm{P}}} = 0$ are satisfied. $\frac{\partial \pi^{\mathrm{M}}}{\partial F_{\mathrm{C}}} = 0$ implies

$$F_{\mathrm{C}} = -\frac{\Big((\sqrt{-r_{\mathrm{P}}}+1)r_{\mathrm{P}}\Big)N}{2\sqrt{-r_{\mathrm{P}}}}. \tag{A.2}$$

$\frac{\partial \pi^{\mathrm{M}}}{\partial r_{\mathrm{P}}} = 0$ implies

$$F_{\mathrm{C}} = \frac{r_{\mathrm{P}}N \pm \sqrt{r_{\mathrm{P}}^2 - 8\sqrt{-r_{\mathrm{P}}}r_{\mathrm{P}}N}}{2}. \tag{A.3}$$

Let $x = \sqrt{-r_{\mathrm{P}}}$, we can rewrite (A2) and (A3) as

$$F_{\mathrm{C}} = -\frac{\Big((x+1)(-x^2)\Big)N}{2x} \tag{A.4}$$

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and

$$F_{\mathrm{C}} = \frac{(-x^2)N \pm \sqrt{x^4 + 8x^3}N}{2} \tag{A.5}$$

Solve the system (A.4) and (A.5), we can get four possible solutions. $x = \frac{1 \pm 1}{6}$ or $\frac{1 \pm 1}{2}$. With $r_{\mathrm{P}} = -x^2$, we can further get $r_{\mathrm{P}} = \frac{-1 \pm 1}{18}$. It can easily be verified that $r_{\mathrm{P}} = -\frac{1}{9}$ would always be optimal. Finally, we have optimal solution

$$r_{\mathrm{S}}^{\mathrm{M}} = \frac{1}{9} \tag{A.6}$$

and

$$F_{\mathrm{C}}^{\mathrm{M}} = \frac{2}{9}N. \tag{A.7}$$

□

Proof of Lemma 2. First of all, we let r_{P} be an opposite number of r_{S}, i.e., $r_{\mathrm{S}} = -r_{\mathrm{P}}$. Furthermore, we let $r_{\mathrm{P}} = r'_{\mathrm{P}} - r'_{\mathrm{C}}$ and $r_{\mathrm{C}} = r'_{\mathrm{C}}$. Formula (11) is equal to

$$\pi^{\mathrm{T}} = \left(1 - \frac{\sqrt{r'_{\mathrm{C}} - r'_{\mathrm{P}}}r'_{\mathrm{C}}}{r'_{\mathrm{C}} - r'_{\mathrm{P}}}\right)Nar'_{\mathrm{P}} \tag{A.8}$$

The derivatives of formula (A8) with respect to r'_{C} and r'_{P} can be deduced as $\frac{\partial \pi^{\mathrm{T}}}{\partial r'_{\mathrm{C}}} = r'_{\mathrm{P}}\left(\frac{r'_{\mathrm{C}}}{2(r'_{\mathrm{C}}-r'_{\mathrm{P}})^{\frac{3}{2}}} - \frac{1}{\sqrt{r'_{\mathrm{C}}-r'_{\mathrm{P}}}}\right)N$ and $\frac{\partial \pi^{\mathrm{T}}}{\partial r'_{\mathrm{P}}} = \frac{\left(2(r'_{\mathrm{C}}-r'_{\mathrm{P}})^{\frac{3}{2}} - 2r'^{2}_{\mathrm{C}} + r'_{\mathrm{P}}r'_{\mathrm{C}}\right)N}{2(r'_{\mathrm{C}}-r'_{\mathrm{P}})^{\frac{3}{2}}}$. The optimal solution r'_{C} and r'_{P} can be found when $\frac{\partial \pi^{\mathrm{T}}}{\partial r'_{\mathrm{C}}} = 0$ and $\frac{\partial \pi^{\mathrm{T}}}{\partial r'_{\mathrm{P}}} = 0$ are satisfied. $\frac{\partial \pi^{\mathrm{T}}}{\partial r'_{\mathrm{C}}} = 0$ implies

$$r'_{\mathrm{C}} = 2r'_{\mathrm{P}}. \tag{A.9}$$

Put A.9 into $\frac{\partial \pi^{\mathrm{T}}}{\partial r'_{\mathrm{P}}} = 0$, we have

$$r'_{\mathrm{P}} = \frac{1}{9}. \tag{A.10}$$

According to $\frac{\partial^2 \pi^{\mathrm{T}}}{\partial r'^{2}_{\mathrm{C}}} = \frac{Nr'_{\mathrm{P}}(r'_{\mathrm{C}}-4r'_{\mathrm{P}})}{4(r'_{\mathrm{C}}-r'_{\mathrm{P}})^{\frac{5}{2}}} < 0$ and $\frac{\partial^2 \pi^{\mathrm{T}}}{\partial r'^{2}_{\mathrm{P}}} = \frac{r'_{\mathrm{C}}N(r'_{\mathrm{P}}-4r'_{\mathrm{C}})}{4(r'_{\mathrm{C}}-r'_{\mathrm{P}})^{\frac{5}{2}}} < 0$, we know that formula (A.8) is a concave function. With $r_{\mathrm{S}} = r'_{\mathrm{C}} - r'_{\mathrm{P}}$ and $r_{\mathrm{C}} = r'_{\mathrm{C}}$, we have optimal solution

$$r_{\mathrm{S}}^{\mathrm{T}} = \frac{1}{9} \text{ and} \tag{A.11}$$

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$$r_{\mathrm{C}}^{\mathrm{T}} = \frac{2}{9}. \tag{A.12}$$

□

Proof of Lemma 3 and 6. The derivatives of formula (14) with respect to F_{C} and r_{S} can be deduced as $\frac{\partial \pi^{\mathrm{X}}}{\partial F_{\mathrm{C}}} = \frac{-Nr_{\mathrm{S}}-2F_{\mathrm{C}}}{\sqrt{r_{\mathrm{S}}N}} + 1$ and $\frac{\partial \pi^{\mathrm{X}}}{\partial r_{\mathrm{S}}} = F_{\mathrm{C}}\left(\frac{r_{\mathrm{S}}N+F_{\mathrm{C}}}{2r_{\mathrm{S}}^{\frac{3}{2}}N} - \frac{1}{\sqrt{r_{\mathrm{S}}}}\right)$. The optimal solution $F_{\mathrm{C}}^{\mathrm{X}}$ and $r_{\mathrm{S}}^{\mathrm{X}}$ can be found when $\frac{\partial \pi^{\mathrm{X}}}{\partial F_{\mathrm{C}}} = 0$ and $\frac{\partial \pi^{\mathrm{X}}}{\partial r_{\mathrm{S}}} = 0$ are satisfied. $\frac{\partial \pi^{\mathrm{X}}}{\partial F_{\mathrm{C}}} = 0$ implies

$$r_{\mathrm{C}} = \frac{-r_{\mathrm{S}} - \sqrt{r_{\mathrm{S}}}}{2}. \tag{A.13}$$

$\frac{\partial \pi^{\mathrm{X}}}{\partial r_{\mathrm{S}}} = 0$ implies

$$F_{\mathrm{C}} = 0 \text{ or} \tag{A.14}$$

$$r_{\mathrm{S}} = \frac{F_{\mathrm{C}}}{N}. \tag{A.15}$$

Solve system (A.13) and (A.14), we can get following solution

$$\begin{cases} r_{\mathrm{S}} = \frac{1\pm 1}{2} \\ F_{\mathrm{C}} = 0 \end{cases}. \tag{A.16}$$

Solve system (A13) and (A15), we can get following solution

$$\begin{cases} r_{\mathrm{S}} = \frac{1\pm 1}{18} \\ F_{\mathrm{C}} = \frac{1\pm 1}{18}N \end{cases}. \tag{A.17}$$

It can easily be verified that $r_{\mathrm{S}} = \frac{1}{9}$ would always be optimal. Finally, we have optimal solution

$$r_{\mathrm{S}}^{\mathrm{X}} = \frac{1}{9} \text{ and} \tag{A.18}$$

$$F_{\mathrm{C}}^{\mathrm{X}} = \frac{1}{9}N. \tag{A.19}$$

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According to $\frac{\partial^2 \pi^{\mathrm{X}}}{\partial F_{\mathrm{C}}^2} = -\frac{2}{N\sqrt{r_{\mathrm{S}}}} < 0$ and $\frac{\partial^2 \pi^{\mathrm{X}}}{\partial r_{\mathrm{S}}^2} = \frac{F_{\mathrm{C}}(Nr_{\mathrm{S}} - 3F_{\mathrm{C}})}{4Nr_{\mathrm{S}}^{\frac{5}{2}}} < 0$, we know that formula (14) is a concave function. □

Proof of Proposition 1. To begin with, we can prove $r_{\mathrm{S}}^{\mathrm{M}} = r_{\mathrm{S}}^{\mathrm{X}} = r_{\mathrm{S}}^{\mathrm{T}}$ in that formulas (4.5), (4.7) and (4.9) are all equivalent. Furthermore, we can find the relation $r_{\mathrm{C}}^{\mathrm{T}} = 2r_{\mathrm{C}}^{\mathrm{X}}$, which yields $0 < r_{\mathrm{C}}^{\mathrm{X}} < r_{\mathrm{C}}^{\mathrm{T}}$. Finally, we have $F_{\mathrm{C}}^{\mathrm{M}} = \frac{2}{9}N$ and $F_{\mathrm{C}}^{\mathrm{M}} = 2F_{\mathrm{C}}^{\mathrm{X}}$, which means $F_{\mathrm{C}}^{\mathrm{M}} > F_{\mathrm{C}}^{\mathrm{X}} > 0$ is true. □

Proof of Proposition 2. When we put optimal solutions of three pricing strategies into formula (4.3), it can be calculated that platform's profits under these three strategies would all be $\frac{N}{27}$. □

Proof of Proposition 3. Fist of all, we set $r_{\mathrm{C}}N + F_{\mathrm{C}} = y$ then equation (4.3) could be rewritten as $\pi = (1 - \frac{y}{\sqrt{r_{\mathrm{S}}}N})(y - r_{\mathrm{S}}N)$. Thus, the maximization problem now is

$$\begin{aligned} \max_{r_{\mathrm{S}}, y} \quad & (1 - \frac{y}{\sqrt{r_{\mathrm{S}}}N})(y - r_{\mathrm{S}}N) \\ \text{s.t.} \quad & 0 \leq \frac{y}{\sqrt{r_{\mathrm{S}}}N} \leq 1 \\ & 0 \leq y \leq \sqrt{r_{\mathrm{S}}}N. \end{aligned}$$

The derivatives of π with respect to y and r_{S} can be deduced as $\frac{\partial \pi}{\partial y} = -\frac{2y - r_{\mathrm{S}}N}{\sqrt{r_{\mathrm{S}}}N} + 1$ and $\frac{\partial \pi}{\partial r_{\mathrm{S}}} = \frac{y(y - r_{\mathrm{S}}N)}{2r_{\mathrm{S}}^{\frac{3}{2}}N} - (1 - \frac{y}{\sqrt{r_{\mathrm{S}}}N})N$. $\frac{\partial \pi}{\partial y} = 0$ implies

$$y = \frac{r_{\mathrm{S}} + \sqrt{r_{\mathrm{S}}}}{2}N. \tag{A.20}$$

$\frac{\partial \pi}{\partial r_{\mathrm{S}}} = 0$ implies

$$y = \frac{-r_{\mathrm{S}}N \pm \sqrt{r_{\mathrm{S}}^2 + 8r_{\mathrm{S}}^{\frac{3}{2}}}}{2}N. \tag{A.21}$$

Solve equations (A.20) and (A.21), we have following necessary condition $r_{\mathrm{S}}(3r_{\mathrm{S}} - 4\sqrt{r_{\mathrm{S}}} + 1) = 0$. Solve this necessary condition, we have three candidate stationary points $r_{\mathrm{S}} =$

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$0, 1$ or $\frac{1}{9}$. Finally, we know that the platform's optimal solution must satisfies one of the following points:

$$r_{\mathrm{S}} = 1 \text{ and } r_{\mathrm{C}}N + F_{\mathrm{C}} = N. \tag{A.22}$$

$$r_{\mathrm{S}} = 0 \text{ and } r_{\mathrm{C}}N + F_{\mathrm{C}} = 0. \tag{A.23}$$

$$r_{\mathrm{S}} = \frac{1}{9} \text{ and } r_{\mathrm{C}}N + F_{\mathrm{C}} = \frac{2}{9}N. \tag{A.24}$$

Put points (A.22) and (A.23) into the firm's general profit function, the results are all zero. Thus, we know the only reasonable stationary point of platform's optimization problem is point (A.24).

To get sufficient condition, we first figure out the hessian matrix

$$\nabla^2\pi = \begin{bmatrix} \frac{\partial^2\pi}{\partial y^2} & \frac{\partial^2\pi}{\partial y \partial r_{\mathrm{S}}} \\ \frac{\partial^2\pi}{\partial r_{\mathrm{S}} \partial y} & \frac{\partial^2\pi}{\partial r_{\mathrm{S}}^2} \end{bmatrix} = \begin{bmatrix} -\frac{2}{\sqrt{r_{\mathrm{S}}}N} & \frac{1}{\sqrt{r_{\mathrm{S}}}} - \frac{r_{\mathrm{S}}N-2y}{2r_{\mathrm{S}}^{\frac{3}{2}}N} \\ \frac{2y-r_{\mathrm{S}}N}{2r_{\mathrm{S}}^{\frac{3}{2}}N} + \frac{1}{\sqrt{r_{\mathrm{S}}}} & -\frac{3y(y-r_{\mathrm{S}}N)}{4r_{\mathrm{S}}^{\frac{5}{2}}N} - \frac{y}{r_{\mathrm{S}}^{\frac{3}{2}}} \end{bmatrix}.$$

Hence the stationary points are at $\frac{\partial\pi}{\partial y} = 0$ and $\frac{\partial\pi}{\partial r_{\mathrm{S}}} = 0$. This gives us the stationary point $(y, r_{\mathrm{S}}) = (\frac{2}{9}N, \frac{1}{9})$, and hence its value at the only stationary point is

$$\begin{bmatrix} \frac{-6}{N} & \frac{15}{2} \\ \frac{15}{2} & -\frac{27}{2}N \end{bmatrix},$$

which is negative semidefinite ($\mathrm{D}_1 = \frac{-6}{N}, \mathrm{D}_2 = \frac{99}{4}$). In conclusion, $(y, r_{\mathrm{S}}) = (\frac{2}{9}N, \frac{1}{9})$ is the only maximum solution. Furthermore, it can be easily verified that both η^* and θ^* are all in 0 and 1. □

Proof of Lemma 4. First of all, we let r_{P} be an opposite number of r_{S}, i.e., $r_{\mathrm{S}} = -r_{\mathrm{P}}$. Formula (5.2) is equal to

$$\pi^{\mathrm{M}} = \left(1 + \frac{\sqrt{-r_{\mathrm{P}}}F_{\mathrm{C}}}{r_{\mathrm{P}}N}\right)\Big(F_{\mathrm{C}} + N(ar_{\mathrm{P}})\Big) \tag{A.25}$$

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The derivatives of formula (A.25) with respect to F_{C} and r_{P} can be deduced as $\frac{\partial \pi^{\mathrm{M}}}{\partial F_{\mathrm{C}}} = \frac{\sqrt{-r_{\mathrm{P}}}(N(ar_{\mathrm{P}})+2F_{\mathrm{C}})}{r_{\mathrm{P}}N} + 1$ and $\frac{\partial \pi^{\mathrm{M}}}{\partial r_{\mathrm{P}}} = \frac{2a\sqrt{-r_{\mathrm{P}}}r_{\mathrm{P}}N^2+(-ar_{\mathrm{P}})F_{\mathrm{C}}N+F_{\mathrm{C}}^2}{2\sqrt{-r_{\mathrm{P}}}r_{\mathrm{P}}N}$. The optimal solution $F_{\mathrm{C}}^{\mathrm{M}}$ and $r_{\mathrm{P}}^{\mathrm{M}}$ can be found when $\frac{\partial \pi^{\mathrm{M}}}{\partial F_{\mathrm{C}}} = 0$ and $\frac{\partial \pi^{\mathrm{M}}}{\partial r_{\mathrm{P}}} = 0$ are satisfied. $\frac{\partial \pi^{\mathrm{M}}}{\partial F_{\mathrm{C}}} = 0$ implies

$$F_{\mathrm{C}} = -\frac{\Big((a\sqrt{-r_{\mathrm{P}}}+1)r_{\mathrm{P}}\Big)N}{2\sqrt{-r_{\mathrm{P}}}}. \tag{A.26}$$

$\frac{\partial \pi^{\mathrm{M}}}{\partial r_{\mathrm{P}}} = 0$ implies

$$F_{\mathrm{C}} = \frac{(ar_{\mathrm{P}})N \pm \sqrt{a^2 r_{\mathrm{P}}^2 - 8a\sqrt{-r_{\mathrm{P}}}r_{\mathrm{P}}N}}{2}. \tag{A.27}$$

Let $x = \sqrt{-r_{\mathrm{P}}}$, we can rewrite (A.26) and (A.27) as

$$F_{\mathrm{C}} = -\frac{\Big((ax+1)(-x^2)\Big)N}{2x} \tag{A.28}$$

and

$$F_{\mathrm{C}} = \frac{(-ax^2)N \pm \sqrt{a^2x^4 + 8ax^3}N}{2} \tag{A.29}$$

Solve the system (A.28) and (A.29), we can get four possible solutions. $x = \frac{1\pm 1}{6a}$ or $\frac{1\pm 1}{2a}$. With $r_{\mathrm{P}} = -x^2$, we can further get $r_{\mathrm{P}} = \frac{-a^2\pm 1}{18}$. It can easily be verified that $r_{\mathrm{P}} = \frac{-a^2-1}{18}$ would always be optimal. Finally, we have optimal solution

$$r_{\mathrm{S}}^{\mathrm{M}} = \frac{a^2+1}{18} \tag{A.30}$$

and

$$F_{\mathrm{C}}^{\mathrm{M}} = \frac{\Big(\sqrt{1+a^2}(a+a^3) + 3\sqrt{2}(1+a^2)\Big)N}{36\sqrt{1+a^2}} \tag{A.31}$$

□

Proof of Lemma 5. First of all, we let r_{P} be an opposite number of r_{S}, i.e., $r_{\mathrm{S}} = -r_{\mathrm{P}}$. Furthermore, we let $r_{\mathrm{P}} = r'_{\mathrm{P}} - r'_{\mathrm{C}}$ and $r_{\mathrm{C}} = r'_{\mathrm{C}}$. Formula (5.2) is equal to

$$\pi^{\mathrm{T}} = \left(1 - \frac{\sqrt{r'_{\mathrm{C}} - r'_{\mathrm{P}}}r'_{\mathrm{C}}}{r'_{\mathrm{C}} - r'_{\mathrm{P}}}\right)Nar'_{\mathrm{P}} \tag{A.32}$$

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The derivatives of formula (A.32) with respect to r'_{C} and r'_{P} can be deduced as $\frac{\partial \pi^{\mathrm{T}}}{\partial r'_{\mathrm{C}}} = a r'_{\mathrm{P}} \left(\frac{r'_{\mathrm{C}}}{2(r'_{\mathrm{C}} - r'_{\mathrm{P}})^{\frac{3}{2}}} - \frac{1}{\sqrt{r'_{\mathrm{C}} - r'_{\mathrm{P}}}} \right) N$ and $\frac{\partial \pi^{\mathrm{T}}}{\partial r'_{\mathrm{P}}} = \frac{\left(2a(r'_{\mathrm{C}} - r'_{\mathrm{P}})^{\frac{3}{2}} - 2a r'^2_{\mathrm{C}} + a r'_{\mathrm{P}} r'_{\mathrm{C}} \right) N}{2(r'_{\mathrm{C}} - r'_{\mathrm{P}})^{\frac{3}{2}}}$. The optimal solution r'_{C} and r'_{P} can be found when $\frac{\partial \pi^{\mathrm{T}}}{\partial r'_{\mathrm{C}}} = 0$ and $\frac{\partial \pi^{\mathrm{T}}}{\partial r'_{\mathrm{P}}} = 0$ are satisfied. $\frac{\partial \pi^{\mathrm{T}}}{\partial r'_{\mathrm{C}}} = 0$ implies

$$r'_{\mathrm{C}} = 2r'_{\mathrm{P}}. \tag{A.33}$$

$\frac{\partial \pi^{\mathrm{T}}}{\partial r'_{\mathrm{P}}} = 0$ implies

$$r'_{\mathrm{P}} = \frac{1}{9}. \tag{A.34}$$

With $r_{\mathrm{S}} = r'_{\mathrm{C}} - r'_{\mathrm{P}}$ and $r_{\mathrm{C}} = r'_{\mathrm{C}}$, we have optimal solution

$$r^{\mathrm{T}}_{\mathrm{S}} = \frac{1}{9} \text{ and} \tag{A.35}$$

$$r^{\mathrm{T}}_{\mathrm{C}} = \frac{2}{9}. \tag{A.36}$$

□

Proof of Proposition 4. When $a < 1$. It can be verified that $r^{\mathrm{T}}_{\mathrm{S}} = r^{\mathrm{X}}_{\mathrm{S}} > r^{\mathrm{M}}_{\mathrm{S}}$ from formulas (5.3), (5.5) and (5.7). It can also be verified that $r^{\mathrm{T}}_{\mathrm{C}} > r^{\mathrm{X}}_{\mathrm{C}} > 0$ from formulas (5.6) and (6.8). Then, we have $F^{\mathrm{M}}_{\mathrm{C}} = \frac{\left(\sqrt{1+a^2}(a+a^3)+3\sqrt{2}(1+a^2)\right)N}{36\sqrt{1+a^2}} = \frac{(1+a^2)a+3\sqrt{2(1+a^2)}}{36}N$ and $F^{\mathrm{X}}_{\mathrm{C}} = \frac{1}{9}N = \frac{4}{36}N$. Since $(1+a^2)a + 3\sqrt{2(1+a^2)}$ is strictly increasing in a and $a \geq 0$, which means $a = 0$ would minimize $F^{\mathrm{M}}_{\mathrm{C}}$. If we put $a = 0$ into $F^{\mathrm{M}}_{\mathrm{C}}$, we can get $F^{\mathrm{M}}_{\mathrm{C}} = \frac{3\sqrt{2}}{36}N$. Here, we notice that the minimum $F^{\mathrm{M}}_{\mathrm{C}}$ is larger than $F^{\mathrm{X}}_{\mathrm{C}}$, which means $F^{\mathrm{M}}_{\mathrm{C}} > F^{\mathrm{X}}_{\mathrm{C}} > 0$ is true. Furthermore, it can be easily verified that both η^* and θ^* are all in 0 and 1. □

Proof of Proposition 5. In general ($a < 1$) it can be analytically proved that $\pi^{\mathrm{M}} > \pi^{\mathrm{X}} > \pi^{\mathrm{T}}$ as following. We first calculate the platform's optimal profits in respect of three pricing strategies: With lemma 4, we have $\pi^{\mathrm{M}} = \left(1 - \frac{\sqrt{18}(\sqrt{1+a^2}a+3\sqrt{2})}{36}\right)\left(\frac{-a^3-a+3\sqrt{2(1+a^2)}}{36}\right)N = \frac{\sqrt{2(a^2+1)}(a^4+a^2+18)-12a^3-12a}{432}N$; With lemma 5, we have $\pi^{\mathrm{T}} = \frac{1}{27}Na$; With lemma 6, we have

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$\pi^{\mathrm{X}} = \frac{1}{27}N$. After that, it can be shown that $\pi^{\mathrm{X}} > \pi^{\mathrm{T}}$ when $0 < a < 1$. Finally we show that $\pi^{\mathrm{M}} > \pi^{\mathrm{X}}$ when $0 < a < 1$: First of all, we have already known that $\pi^{\mathrm{M}} = \pi^{\mathrm{X}}$ when $a = 1$ in the beginning of this proposition; Next, if π^{M} is strictly decreasing in a when $0 < a < 1$, then this proof is done. In next paragraph, we show that π^{M} is strictly decreasing in a when $0 < a < 1$.

Let $g(a)$ represent the numerator of π^{M}, i.e., $g(a) = \sqrt{2(a^2+1)}(a^4 + a^2 + 18) - 12a^3 - 12a$. Then we can get $\frac{\partial g(a)}{\partial a} = \frac{\sqrt{2}a(a^4+a^2+18)}{\sqrt{a^2+1}} + \sqrt{2(a^2+1)}(4a^3 + 2a) - 36a^2 - 12 = \frac{\sqrt{2}a(5a^4+7a^2+20)}{\sqrt{a^2+1}} - 12(3a^2 + 1)$. $\frac{\partial g(a)}{\partial a} = 0$ implies $2a^2(5a^4 + 7a^2 + 20)^2 = (a^2 + 1)144(3a^2 + 1)^2$. Solve the above equation, we can get two real roots $a = \pm\sqrt{\frac{\sqrt{73}-1}{2}}$ and the other eight complex roots. Put the real roots into $g'(a)$, we have $g'(\sqrt{\frac{\sqrt{73}-1}{2}}) = 0$ and $g'(-\sqrt{\frac{\sqrt{73}-1}{2}}) \approx -295.58$. Up to this point, we know π^{M} is decreasing in a when $a < \sqrt{\frac{\sqrt{73}-1}{2}}$. Furthermore, with $\sqrt{\frac{\sqrt{73}-1}{2}} \approx 1.94$, we can claim that π^{M} is decreasing in a when $0 < a < 1$. $\square$

Proof of Lemma 7. First of all, we let r_{P} be an opposite number of r_{S}, i.e., $r_{\mathrm{S}} = -r_{\mathrm{P}}$. Formula (5.12) is equal to

$$\pi^{\mathrm{M}}_{\mathrm{cost}} = \left(1 + \frac{\sqrt{-r_{\mathrm{P}}}F_{\mathrm{C}}}{r_{\mathrm{P}}N}\right)\Big(F_{\mathrm{C}} + N(r_{\mathrm{P}} - c)\Big) \tag{A.37}$$

The derivatives of formula (A.37) with respect to F_{C} and r_{P} can be deduced as $\frac{\partial \pi^{\mathrm{M}}_{\mathrm{cost}}}{\partial F_{\mathrm{C}}} = \frac{\sqrt{-r_{\mathrm{P}}}(N(r_{\mathrm{P}}-c)+2F_{\mathrm{C}})}{r_{\mathrm{P}}N} + 1$ and $\frac{\partial \pi^{\mathrm{M}}_{\mathrm{cost}}}{\partial r_{\mathrm{P}}} = \frac{2\sqrt{-r_{\mathrm{P}}}r_{\mathrm{P}}N^2+(-r_{\mathrm{P}}-c)F_{\mathrm{C}}N+F_{\mathrm{C}}^2}{2\sqrt{-r_{\mathrm{P}}}r_{\mathrm{P}}N}$. The optimal solution $F^{\mathrm{M}}_{\mathrm{C}}$ and $r^{\mathrm{M}}_{\mathrm{P}}$ can be found when $\frac{\partial \pi^{\mathrm{M}}_{\mathrm{cost}}}{\partial F_{\mathrm{C}}} = 0$ and $\frac{\partial \pi^{\mathrm{M}}_{\mathrm{cost}}}{\partial r_{\mathrm{P}}} = 0$ are satisfied. $\frac{\partial \pi^{\mathrm{M}}_{\mathrm{cost}}}{\partial F_{\mathrm{C}}} = 0$ implies

$$F_{\mathrm{C}} = -\frac{\Big((\sqrt{-r_{\mathrm{P}}} + 1)r_{\mathrm{P}} - c\sqrt{-r_{\mathrm{P}}}\Big)N}{2\sqrt{-r_{\mathrm{P}}}}. \tag{A.38}$$

$\frac{\partial \pi^{\mathrm{M}}_{\mathrm{cost}}}{\partial r_{\mathrm{P}}} = 0$ implies

$$F_{\mathrm{C}} = \frac{(r_{\mathrm{P}} + c)N \pm \sqrt{r_{\mathrm{P}}^2 - 8\sqrt{-r_{\mathrm{P}}}r_{\mathrm{P}} + 2cr_{\mathrm{P}} + c^2}N}{2}. \tag{A.39}$$

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Let $x = \sqrt{-r_{\rm P}}$, we can rewrite (A.38) and (A.39) as

$$F_{\rm C} = -\frac{\Big((x+1)(-x^2) - cx\Big)N}{2x} \tag{A.40}$$

and

$$F_{\rm C} = \frac{(-x^2+c)N \pm \sqrt{x^4+8x^3-2cx^2+c^2}N}{2} \tag{A.41}$$

Solve the system (A.40) and (A.41), we can get four possible solutions. $x = \frac{1\pm\sqrt{12c+1}}{6}$ or $\frac{1\pm\sqrt{1-4c}}{2}$. With $r_{\rm P} = -x^2$, we can further get $r_{\rm P} = \frac{-6c-1\pm\sqrt{12c+1}}{18}$ or $\frac{2c-1\pm\sqrt{1-4c}}{18}$. It can easily be verified that $r_{\rm P} = \frac{-6c-1-\sqrt{12c+1}}{18}$ would always be optimal. Finally, we have optimal solution

$$r_{\rm S}^{\rm M} = \frac{6c+1+\sqrt{12c+1}}{18} \tag{A.42}$$

and

$$F_{\rm C}^{\rm M} = \frac{\left(\sqrt{\sqrt{12c+1}+6c+1}\Big(\sqrt{12c+1}+24c+1\Big) + 3\sqrt{2}(\sqrt{12c+1}+6c+1)\right)N}{36\sqrt{\sqrt{12c+1}+6c+1}} \tag{A.43}$$

$\square$

Proof of Lemma 8. First of all, we let $r_{\rm P}$ be an opposite number of $r_{\rm S}$, i.e., $r_{\rm S} = -r_{\rm P}$. Furthermore, we let $r_{\rm P} = r'_{\rm P} - r'_{\rm C}$ and $r_{\rm C} = r'_{\rm C}$. Formula (5.12) is equal to

$$\pi_{\rm cost}^{\rm T} = \left(1 - \frac{\sqrt{r'_{\rm C} - r'_{\rm P}}\, r'_{\rm C}}{r'_{\rm C} - r'_{\rm P}}\right) N(r'_{\rm P} - c) \tag{A.44}$$

The derivatives of formula (A.44) with respect to $r'_{\rm C}$ and $r'_{\rm P}$ can be deduced as $\frac{\partial \pi_{\rm cost}^{\rm T}}{\partial r'_{\rm C}} = (r'_{\rm P} - c)\left(\frac{r'_{\rm C}}{2(r'_{\rm C}-r'_{\rm P})^{\frac{3}{2}}} - \frac{1}{\sqrt{r'_{\rm C}-r'_{\rm P}}}\right)N$ and $\frac{\partial \pi_{\rm cost}^{\rm T}}{\partial r'_{\rm P}} = \frac{\left(2(r'_{\rm C}-r'_{\rm P})^{\frac{3}{2}} - 2r'^2_{\rm C} + (r'_{\rm P}+c)r'_{\rm C}\right)N}{2(r'_{\rm C}-r'_{\rm P})^{\frac{3}{2}}}$. The optimal solution $r'_{\rm C}$ and $r'_{\rm P}$ can be found when $\frac{\partial \pi_{\rm cost}^{\rm T}}{\partial r'_{\rm C}} = 0$ and $\frac{\partial \pi_{\rm cost}^{\rm T}}{\partial r'_{\rm P}} = 0$ are satisfied. $\frac{\partial \pi_{\rm cost}^{\rm T}}{\partial r'_{\rm C}} = 0$ implies

$$r'_{\rm C} = 2r'_{\rm P}. \tag{A.45}$$

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$\frac{\partial \pi^{\mathrm{T}}_{\mathrm{cost}}}{\partial r'_{\mathrm{P}}} = 0$ implies

$$r'_{\mathrm{P}} = \frac{6c + 1 \pm \sqrt{12c+1}}{18}. \tag{A.46}$$

It can easily be verified that $r'_{\mathrm{P}} = \frac{6c+1+\sqrt{12c+1}}{18}$ would always be optimal. With $r_{\mathrm{S}} = r'_{\mathrm{C}} - r'_{\mathrm{P}}$ and $r_{\mathrm{C}} = r'_{\mathrm{C}}$, we have optimal solution

$$r^{\mathrm{T}}_{\mathrm{S}} = \frac{6c + 1 + \sqrt{12c+1}}{18} \text{ and} \tag{A.47}$$

$$r^{\mathrm{T}}_{\mathrm{C}} = \frac{6c + 1 + \sqrt{12c+1}}{9}. \tag{A.48}$$

□

Proof of Lemma 9. The derivatives of formula (5.12) with respect to F_{C} and r_{S} can be deduced as $\frac{\partial \pi^{\mathrm{X}}_{\mathrm{cost}}}{\partial F_{\mathrm{C}}} = \frac{N(c-r_{\mathrm{S}})-2F_{\mathrm{C}}}{\sqrt{r_{\mathrm{S}}}N} + 1$ and $\frac{\partial \pi^{\mathrm{X}}_{\mathrm{cost}}}{\partial r_{\mathrm{S}}} = F_{\mathrm{C}}\left(\frac{r_{\mathrm{S}}N+F_{\mathrm{C}}}{2r_{\mathrm{S}}^{\frac{3}{2}}N} - \frac{1}{\sqrt{r_{\mathrm{S}}}}\right)$. The optimal solution $F^{\mathrm{X}}_{\mathrm{C}}$ and $r^{\mathrm{X}}_{\mathrm{S}}$ can be found when $\frac{\partial \pi^{\mathrm{X}}_{\mathrm{cost}}}{\partial F_{\mathrm{C}}} = 0$ and $\frac{\partial \pi^{\mathrm{X}}_{\mathrm{cost}}}{\partial r_{\mathrm{S}}} = 0$ are satisfied. $\frac{\partial \pi^{\mathrm{X}}_{\mathrm{cost}}}{\partial F_{\mathrm{C}}} = 0$ implies

$$F_{\mathrm{C}} = \frac{-r_{\mathrm{S}} - \sqrt{r_{\mathrm{S}}} - c}{2}N. \tag{A.49}$$

$\frac{\partial \pi^{\mathrm{X}}_{\mathrm{cost}}}{\partial r_{\mathrm{S}}} = 0$ implies

$$F_{\mathrm{C}} = cN \text{ or} \tag{A.50}$$

$$r_{\mathrm{S}} = \frac{F_{\mathrm{C}}}{N}. \tag{A.51}$$

Solve system (A.49) and (A.50), we can get following solution

$$\begin{cases} r_{\mathrm{S}} = \frac{1-2c\pm\sqrt{1-4c}}{2} \\ F_{\mathrm{C}} = cN \end{cases}. \tag{A.52}$$

Solve system (A.49) and (A.51), we can get following solution

$$\begin{cases} r_{\mathrm{S}} = \frac{1+6c\pm\sqrt{1+12c}}{18} \\ F_{\mathrm{C}} = \frac{1+6c\pm\sqrt{1+12c}}{18}N \end{cases}. \tag{A.53}$$

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It can easily be verified that $r_{\mathrm{S}} = \frac{1+6c+\sqrt{1+12c}}{18}$ would always be optimal. Finally, we have optimal solution

$$r_{\mathrm{S}}^{\mathrm{X}} = r_{\mathrm{C}}^{\mathrm{X}} = \frac{1+6c+\sqrt{1+12c}}{18} \text{ and} \tag{A.54}$$

$$F_{\mathrm{C}}^{\mathrm{X}} = \frac{1+6c+\sqrt{1+12c}}{18} N. \tag{A.55}$$

$\square$

Proof of Proposition 6. To begin with, we can prove $r_{\mathrm{S}}^{\mathrm{M}} = r_{\mathrm{S}}^{\mathrm{X}} = r_{\mathrm{S}}^{\mathrm{T}}$ in that formulas (5.13), (5.15) and (5.17) are all equivalent. Furthermore, we can find the relation $r_{\mathrm{C}}^{\mathrm{T}} = 2r_{\mathrm{C}}^{\mathrm{X}}$ from formulas (5.16) and (5.18), which yields $r_{\mathrm{C}}^{\mathrm{M}} < r_{\mathrm{C}}^{\mathrm{X}} < r_{\mathrm{C}}^{\mathrm{T}}$. Finally, we show that $F_{\mathrm{C}}^{\mathrm{M}} = 2F_{\mathrm{C}}^{\mathrm{X}}$, which means $F_{\mathrm{C}}^{\mathrm{M}} > F_{\mathrm{C}}^{\mathrm{X}} > F_{\mathrm{C}}^{\mathrm{T}}$ is true. Below we show $F_{\mathrm{C}}^{\mathrm{M}} = 2F_{\mathrm{C}}^{\mathrm{X}}$. We have $F_{\mathrm{C}}^{\mathrm{M}} = \frac{\left(\sqrt{\sqrt{12c+1}+6c+1}\left(\sqrt{12c+1}+24c+1\right)+3\sqrt{2}(\sqrt{12c+1}+6c+1)\right)N}{36\sqrt{\sqrt{12c+1}+6c+1}}$ and $2F_{\mathrm{C}}^{\mathrm{X}} = \frac{1+6c+\sqrt{1+12c}}{9}N$. $F_{\mathrm{C}}^{\mathrm{M}} = 2F_{\mathrm{C}}^{\mathrm{X}}$ yields $\left(\sqrt{\sqrt{12c+1}+6c+1}\left(\sqrt{12c+1}+24c+1\right)+3\sqrt{2}(\sqrt{12c+1}+6c+1)\right)N = 4(1+6c+\sqrt{1+12c})^{\frac{3}{2}}$. Divide both sides of the equation by $\sqrt{1+6c+\sqrt{1+12c}}$ we get $\sqrt{12c+1}+24c+1+3\sqrt{2}\sqrt{\sqrt{12c+1}+6c+1} = 4(\sqrt{12c+1}+6c+1)$. Minus $\sqrt{12c+1}+6c+1$ to both sides, the equation would be $\sqrt{2}\sqrt{\sqrt{12c+1}+6c+1} = \sqrt{12c+1}+6c+1$. After squaring both sides of the equation, we have $2(\sqrt{12c+1}+6c+1) = 12c+2+2\sqrt{12c+1}$. Therefore, $F_{\mathrm{C}}^{\mathrm{M}} = 2F_{\mathrm{C}}^{\mathrm{X}}$ is established. Furthermore, it can be easily verified that both η^* and θ^* are all in 0 and 1. $\square$

Proof of Proposition 7. Formula (5.12) can be rewritten as

$$\pi_{\mathrm{cost}}^{i} = \frac{1}{\sqrt{r_{\mathrm{S}}^{i}N}}\left(\sqrt{r_{\mathrm{S}}^{i}}N - (r_{\mathrm{C}}^{i}N + F_{\mathrm{C}}^{i})\right)(r_{\mathrm{C}}^{i}N + F_{\mathrm{C}}^{i} - r_{\mathrm{S}}^{i}N - cN), \tag{A.56}$$

which represents the platform's general profit function adopting strategies $i \in \{\mathrm{M}, \mathrm{T}, \mathrm{X}\}$. Moreover, we have already known that $r_{\mathrm{S}}^{\mathrm{M}} = r_{\mathrm{S}}^{\mathrm{X}} = r_{\mathrm{S}}^{\mathrm{T}}$ in proposition 6. It means that the relative magnitude among $\pi_{\mathrm{cost}}^{\mathrm{M}}$, $\pi_{\mathrm{cost}}^{\mathrm{T}}$, and $\pi_{\mathrm{cost}}^{\mathrm{X}}$ depend on $r_{\mathrm{C}}^{i}N + F_{\mathrm{C}}^{i}$.

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In the proof of proposition 6, we have already shown that $F_{\mathrm{C}}^{\mathrm{M}} = 2F_{\mathrm{C}}^{\mathrm{X}} = \frac{1+6c+\sqrt{1+12c}}{9}N$. According to the above equation, we have $r_{\mathrm{C}}^{M}N + F_{\mathrm{C}}^{M} = \frac{1+6c+\sqrt{1+12c}}{9}N$. With formula (5.16), we have $r_{\mathrm{C}}^{T}N + F_{\mathrm{C}}^{T} = \frac{6c+1+\sqrt{12c+1}}{9}N$. With formulas (5.18) and (5.19), we have $r_{\mathrm{C}}^{X}N + F_{\mathrm{C}}^{X} = \frac{6c+1+\sqrt{12c+1}}{9}N$. In conclusion, $\pi_{\mathrm{cost}}^{\mathrm{M}} = \pi_{\mathrm{cost}}^{\mathrm{T}} = \pi_{\mathrm{cost}}^{\mathrm{X}}$. □

Proof of Proposition 8. In the case of $a = 1$. Define $x = Nr_{\mathrm{C}} + F_{\mathrm{C}}$, we can rewrite equation (5.25) as $\pi = \frac{F_{\mathrm{S}}}{N}x - \left(\frac{x}{-F_{\mathrm{S}}+N}\right)^2 F_{\mathrm{S}}$. The derivatives of π with respect to x and F_{S} can then be deduced as $\frac{\partial\pi}{\partial x} = \frac{-2F_{\mathrm{S}}x}{(-F_{\mathrm{S}}+N)^2} + \frac{F_{\mathrm{S}}}{N}$ and $\frac{\partial\pi}{\partial F_{\mathrm{S}}} = \frac{x^2}{(-F_{\mathrm{S}}+N)^2} + \frac{2x^2F_{\mathrm{S}}}{(-F_{\mathrm{S}}+N)^3} - \frac{x}{N}$. Solve the system of $\frac{\partial\pi}{\partial x} = 0$ and $\frac{\partial\pi}{\partial F_{\mathrm{S}}} = 0$, we have four candidate optimal solutions: (1.) $F_{\mathrm{S}} = 0$ and $x = 0$; (2.) $F_{\mathrm{S}} = N$ and $x = 0$; (3.) $F_{\mathrm{S}} = 0$ and $x = N$; (4.) $F_{\mathrm{S}} = \frac{N}{3}$ and $x = \frac{2}{9}N$. The first three solutions would lead to zero profit, hence the unique optimal solution is $F_{\mathrm{S}} = \frac{N}{3}$ and $Nr_{\mathrm{C}} + F_{\mathrm{C}} = \frac{2}{9}N$. Put the optimal solution here into equation (5.25), we can get $\pi_{\mathrm{cost}} = \frac{N}{27}$ which is exact equivalent to the optimal profit in proposition 2 when $a = 1$. Furthermore, it can be easily verified that both η^* and θ^* are all in 0 and 1. □

Proof of Proposition 9. In the case of $a < 1$. The platform's profit function is $\pi = \frac{F_{\mathrm{S}}}{N}Nar_{\mathrm{C}} - (\frac{Nr_{\mathrm{C}}}{-F_{\mathrm{S}}+N})^2 F_{\mathrm{S}}$ when it implements transaction-base pricing strategy. Then we have $\frac{\partial\pi}{\partial F_{\mathrm{S}}} = (\frac{Nr_{\mathrm{C}}}{-F_{\mathrm{S}}+N})^2 + \frac{2(Nr_{\mathrm{C}})^2F_{\mathrm{S}}}{(-F_{\mathrm{S}}+N)^3} - \frac{Nar_{\mathrm{C}}}{N}$ and $\frac{\partial\pi}{\partial r_{\mathrm{C}}} = \frac{-2NF_{\mathrm{S}}(Nr_{\mathrm{C}})}{(-F_{\mathrm{S}}+N)^2} + aF_{\mathrm{S}}$. Solve the system of the above equations to be zero, we have the optimal solution $F_{\mathrm{S}}^{\mathrm{T}} = \frac{N}{3}$ and $r_{\mathrm{C}}^{\mathrm{T}} = \frac{2a}{9}$. On the other hand, the platform's profit function is $\pi = \frac{F_{\mathrm{S}}}{N}F_{\mathrm{C}} - (\frac{F_{\mathrm{C}}}{-F_{\mathrm{S}}+N})^2 F_{\mathrm{S}}$ when it implements membership-base pricing strategy. Then we have $\frac{\partial\pi}{\partial F_{\mathrm{S}}} = (\frac{F_{\mathrm{C}}}{-F_{\mathrm{S}}+N})^2 + \frac{2F_{\mathrm{C}}^2F_{\mathrm{S}}}{(-F_{\mathrm{S}}+N)^3} - \frac{F_{\mathrm{C}}}{N}$ and $\frac{\partial\pi}{\partial F_{\mathrm{C}}} = -\frac{2F_{\mathrm{S}}F_{\mathrm{C}}}{(-F_{\mathrm{S}}+N)^2} + \frac{F_{\mathrm{S}}}{N}$. Solve the system of the above equations to be zero, we have the optimal solution $F_{\mathrm{S}}^{\mathrm{M}} = \frac{N}{3}$ and $F_{\mathrm{C}}^{\mathrm{M}} = \frac{2}{9}N$. Put the optimal solutions here into equation (5.25), we can get $\pi^{\mathrm{T}} = \frac{a^2}{27}N$ and $\pi^{\mathrm{M}} = \frac{1}{27}N$. In proposition 5, we have already known that the profits employing transaction-based and membership-based strategies are $\frac{a}{27}N$

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and $\frac{\sqrt{2(a^2+1)}(a^4+a^2+18)-12a^3-12a}{432}N$ respectively. Eventually we find that no matter the platform employs which pricing strategy here, subsidizing shoppers with per-transaction subsidization is better for platform. Furthermore, it can be easily verified that both η^* and θ^* are all in 0 and 1. $\square$

Proof of Proposition 9. The platform's profit function is $\pi = (1 - \frac{(r_C N + F_C)\sqrt{b}}{N\sqrt{r_S}})(F_C + N(r_C - r_S))$. Define a new variable $y = r_C N + F_C$, we have $\pi = (1 - \frac{y\sqrt{b}}{N\sqrt{r_S}})(y - N r_S)$. Then we have $\frac{\partial \pi}{\partial r_S} = \frac{-2N^2 r_S^{3/2} + \sqrt{b} N y r_S + \sqrt{b} y^2}{2N r_S^{3/2}}$ and $\frac{\partial \pi}{\partial y} = -\frac{2\sqrt{b}y - \sqrt{b}N r_S - N\sqrt{r_S}}{N\sqrt{r_S}}$. Let $\frac{\partial \pi}{\partial r_S} = 0$ and $\frac{\partial \pi}{\partial y} = 0$ be satisfied, we can get the optimal solution $y = r_C N + F_C = \frac{2N}{9b}$ and $r_S = \frac{1}{9b}$. $\square$

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Appendix B

Summary of lemmas, propositions and observations

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Lemma 1 *The optimal solution of membership-based pricing strategy is* $r_{\mathrm{S}}^{\mathrm{M}} = \frac{1}{9}$ *and* $F_{\mathrm{C}}^{\mathrm{M}} = \frac{2}{9}N$. *Both* $r_{\mathrm{S}}^{\mathrm{M}}$ *and* $F_{\mathrm{C}}^{\mathrm{M}}$ *are non-negative.*

Lemma 2 *The optimal solution of transaction-based pricing strategy is* $r_{\mathrm{S}}^{\mathrm{T}} = \frac{1}{9}$ *and* $r_{\mathrm{C}}^{\mathrm{T}} = \frac{2}{9}$. *Both* $r_{\mathrm{S}}^{\mathrm{T}}$ *and* $r_{\mathrm{C}}^{\mathrm{T}}$ *are non-negative.*

Lemma 3 *The optimal solution of cross-subsidization strategy is* $r_{\mathrm{S}}^{\mathrm{X}} = \frac{1}{9}$, $r_{\mathrm{C}}^{\mathrm{X}} = \frac{1}{9}$ *and* $F_{\mathrm{C}}^{\mathrm{X}} = \frac{1}{9}N$. $r_{\mathrm{S}}^{\mathrm{X}}$, $r_{\mathrm{C}}^{\mathrm{X}}$ *and* $F_{\mathrm{C}}^{\mathrm{X}}$ *are all non-negative.*

Proposition 1 *By comparing the optimal ways of implementing membership-based pricing, transaction-based pricing, and cross subsidization, we have*

$$\begin{cases} r_{\mathrm{S}}^{\mathrm{X}} = r_{\mathrm{S}}^{\mathrm{M}} = r_{\mathrm{S}}^{\mathrm{T}} \\ r_{\mathrm{C}}^{\mathrm{T}} > r_{\mathrm{C}}^{\mathrm{X}} > 0 \\ F_{\mathrm{C}}^{\mathrm{M}} > F_{\mathrm{C}}^{\mathrm{X}} > 0 \end{cases}.$$

Proposition 2 *The platform's profits under three strategies are all the same, i.e.,*

$$\pi^{\mathrm{M}} = \pi^{\mathrm{T}} = \pi^{\mathrm{X}}.$$

Proposition 3 *A solution* $(r_{\mathrm{C}}, r_{\mathrm{S}}, F_{\mathrm{C}})$ *is optimal to the platform's problem in (4.3) if and only if*

$$r_{\mathrm{S}} = \frac{1}{9} \quad \textit{and} \quad r_{\mathrm{C}}N + F_{\mathrm{C}} = \frac{2}{9}N.$$

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Section 5.1: Discount factor

Lemma 4 *The optimal solution of membership-based pricing strategy:* $r_{\mathrm{S}}^{\mathrm{M}} = \frac{a^2+1}{18}$ *and* $F_{\mathrm{C}}^{\mathrm{M}} = \frac{(\sqrt{1+a^2}(a+a^2)+3\sqrt{2}(1+a^2))N}{36\sqrt{1+a^2}}$. *Both* $r_{\mathrm{S}}^{\mathrm{M}}$ *and* $F_{\mathrm{C}}^{\mathrm{M}}$ *are non-negative.*

Lemma 5 *The optimal solution of transaction-based pricing strategy:* $r_{\mathrm{S}}^{\mathrm{T}} = \frac{1}{9}$ *and* $r_{\mathrm{C}}^{\mathrm{T}} = \frac{2}{9}$. *Both* $r_{\mathrm{S}}^{\mathrm{T}}$ *and* $r_{\mathrm{C}}^{\mathrm{T}}$ *are non-negative.*

Lemma 6 *The optimal solution of cross-subsidization strategy:* $r_{\mathrm{S}}^{\mathrm{X}} = \frac{1}{9}$, $r_{\mathrm{C}}^{\mathrm{X}} = \frac{1}{9}$ *and* $F_{\mathrm{C}}^{\mathrm{X}} = \frac{1}{9}N$. *All* $r_{\mathrm{S}}^{\mathrm{X}}$, $r_{\mathrm{C}}^{\mathrm{X}}$ *and* $F_{\mathrm{C}}^{\mathrm{X}}$ *are non-negative.*

Proposition 4

$$\begin{cases} r_{\mathrm{S}}^{\mathrm{T}} = r_{\mathrm{S}}^{\mathrm{X}} > r_{\mathrm{S}}^{\mathrm{M}} \\ r_{\mathrm{C}}^{\mathrm{T}} > r_{\mathrm{C}}^{\mathrm{X}} > 0 \\ F_{\mathrm{C}}^{\mathrm{M}} > F_{\mathrm{C}}^{\mathrm{X}} > 0 \end{cases}.$$

Proposition 5 *In the general case* $(a < 1)$*, the platform's optimal profits under three strategies*

$$\pi_{\text{discount}}^{\mathrm{M}} > \pi_{\text{discount}}^{\mathrm{X}} > \pi_{\text{discount}}^{\mathrm{T}}.$$

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Lemma 7 *The optimal solution of membership-based pricing strategy:* $r_{\mathrm{S}}^{\mathrm{M}} = \frac{6c+1+\sqrt{12c+1}}{18}$ *and* $F_{\mathrm{C}}^{\mathrm{M}} = \frac{\left(\sqrt{\sqrt{12c+1}+6c+1}\left(\sqrt{12c+1}+24c+1\right)+3\sqrt{2}(\sqrt{12c+1}+6c+1)\right)N}{36\sqrt{\sqrt{12c+1}+6c+1}}$. *Both* $r_{\mathrm{S}}^{\mathrm{M}}$ *and* $F_{\mathrm{C}}^{\mathrm{M}}$ *are non-negative.*

Lemma 8 *The optimal solution of transaction-based pricing strategy:* $r_{\mathrm{S}}^{\mathrm{T}} = \frac{6c+1+\sqrt{12c+1}}{18}$ *and* $r_{\mathrm{C}}^{\mathrm{T}} = \frac{6c+1+\sqrt{12c+1}}{9}$. *Both* $r_{\mathrm{S}}^{\mathrm{T}}$ *and* $r_{\mathrm{C}}^{\mathrm{T}}$ *are non-negative.*

Lemma 9 *The optimal solution of cross-subsidization strategy:* $r_{\mathrm{S}}^{\mathrm{X}} = \frac{6c+1+\sqrt{12c+1}}{18}$, $r_{\mathrm{C}}^{\mathrm{X}} = \frac{6c+1+\sqrt{12c+1}}{18}$ *and* $F_{\mathrm{C}}^{\mathrm{X}} = \frac{6c+1+\sqrt{12c+1}}{18}N$. *All* $r_{\mathrm{S}}^{\mathrm{X}}$, $r_{\mathrm{C}}^{\mathrm{X}}$ *and* $F_{\mathrm{C}}^{\mathrm{X}}$ *are non-negative.*

Proposition 6 *When we take platform's marginal cost into account and set* $a = 1$, *we have*

$$\begin{cases} r_{\mathrm{S}}^{\mathrm{X}} = r_{\mathrm{S}}^{\mathrm{M}} = r_{\mathrm{S}}^{\mathrm{T}} \\ r_{\mathrm{C}}^{\mathrm{T}} > r_{\mathrm{C}}^{\mathrm{X}} > 0 \\ F_{\mathrm{C}}^{\mathrm{M}} > F_{\mathrm{C}}^{\mathrm{X}} > 0 \end{cases}.$$

Proposition 7 *The platform's optimal profits under three strategies are all the same:*

$$\pi_{\mathrm{cost}}^{\mathrm{M}} = \pi_{\mathrm{cost}}^{\mathrm{T}} = \pi_{\mathrm{cost}}^{\mathrm{X}}.$$

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Proposition 8 *When we set $a = 1$ and $c = 0$, a plan $(r_{\mathrm{C}}, F_{\mathrm{C}}, F_{\mathrm{S}})$ is the platform's optimal solution if and only if*

$$F_{\mathrm{S}} = -\frac{N}{3} \text{ and } r_{\mathrm{C}}N + F_{\mathrm{C}} = \frac{2}{9}N.$$

Furthermore, no matter the platform subsidizes shoppers with fixed or per-transaction subsidization, the platform's profits are the same (cf. proposition 3).

Proposition 9 *When we set $a < 1$ and $c = 0$, the platform's optimal solution under transaction-based and membership-based pricing strategies would be*

$$F_{\mathrm{S}}^{\mathrm{T}} = -\frac{N}{3},\ r_{\mathrm{C}}^{\mathrm{T}} = \frac{2}{9}a \text{ and}$$

$$F_{\mathrm{S}}^{\mathrm{M}} = -\frac{N}{3},\ F_{\mathrm{C}}^{\mathrm{M}} = \frac{2}{9}N.$$

And the platform's optimal profits under these two strategies would have the following relation

$$\pi_{\text{fixed}}^{\mathrm{M}} > \pi_{\text{fixed}}^{\mathrm{T}}.$$

Furthermore, no matter the platform employs which pricing strategy, subsidizing shoppers with per-transaction subsidization is better for it, i.e,

$$\pi_{\text{discount}}^{\mathrm{M}} > \pi_{\text{fixed}}^{\mathrm{M}} \text{ and } \pi_{\text{discount}}^{\mathrm{T}} > \pi_{\text{fixed}}^{\mathrm{T}}.$$

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Section 5.4: Price-sensitive number of orders

Observation 1 *Given an arbitrary set of N and K, the optimal solution would bind to (5.31) under pure-transaction pricing and cross-subsidization strategies.*

Observation 2 $r_{\mathrm{C}}^{\mathrm{T}} > r_{\mathrm{C}}^{\mathrm{X}} = r_{\mathrm{S}}^{\mathrm{X}} > r_{\mathrm{S}}^{\mathrm{T}}$.

Observation 3 *The number of consumers is more under the pure-transaction pricing strategy than the cross-subsidization one, i.e., $n_{\mathrm{C}}^{\mathrm{T}} > n_{\mathrm{C}}^{\mathrm{X}}$. However, the earnings from each consumer and the total consumption are both more under the cross-subsidization strategy, i.e., $m^{\mathrm{X}} > m^{\mathrm{T}}$ and $TC^{\mathrm{X}} > TC^{\mathrm{T}}$.*

Observation 4 *The platform's profit would increase in K, and it can earn more by adopting the cross-subsidization strategy, i.e., $\pi^{\mathrm{X}} > \pi^{\mathrm{T}}$.*

Section 5.5: Diversity of shoppers' distribution

Proposition 10 *A plan $(r_{\mathrm{C}}, F_{\mathrm{C}}, r_{\mathrm{S}})$ is the platform's optimal solution if and only if*

$$r_{\mathrm{S}} = \frac{1}{9b} \text{ and } r_{\mathrm{C}}N + F_{\mathrm{C}} = \frac{2}{9b}N.$$

Furthermore, the number of consumers and shoppers would be $n_{\mathrm{C}} = \frac{1}{3}$ and $n_{\mathrm{S}} = \frac{1}{9b^2}$. Then, the earnings from each consumer would be $m = \frac{N}{9b}$ .Finally, the optimal profit would be $\frac{N}{27b}$.

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