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大規模開放式線上課程之定價與多元化策略

Pricing and Diversification of Massive Open Online Course Platforms

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口試委員會審定書

Pricing and Diversification of Massive Open Online Course Platforms

大規模開放式線上課程之定價與多元化策略

本論文係李維哲（學號 R04725023）在國立臺灣大學資訊管理學系、所完成之碩士學位論文，於民國 106 年 6 月 20 日承下列考試委員審查通過及口試及格，特此證明

口試委員：

所　　長：

謝辭

很榮幸能夠在極富熱誠和抱負的指導教授孔令傑老師的引領了我享受充實的碩士班生活。這段日子裡，我學到的不僅是老師課堂上的資訊經濟和統計資料分析以及老師推薦的最佳化和賽局理論，更是在每次大大小小的開會中老師對研究的認真執著和努力不懈的態度。老師對我們的期許也成為我一路成長最大的支持和能量，讓我突破自我挑戰自己。碩一時的資訊經濟讓我們投稿決策分析研討會並且上台發表，累積我的研究能力，也開啟了碩士班研究的第一步。碩二開始研究論文，很感謝老師在研究架構上的指引、推導過程的指點，以及耐心地逐字審閱論文語句，帶領我一路從論文口試走到 PACIS 亞太資訊系統年會，開拓我的視野。另外也很感謝老師讓我有機會參與臺大開放式線上課程和 Coursera 的資料分析專案，讓我對開放式線上課程這個熱門的議題，不僅有論文上的研究，也有對資料的分析和實作呈現的學習。此外也特別感謝我的口試委員，工管所郭佳瑋教授和國企所陳聿宏教授，在口試時給予我許多讓論文更臻完善的建議。

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于臺大資訊管理研究所

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中文摘要

近年來大規模開放式線上課程（MOOCs）在高等教育領域受到極大的關注。此類型的平台在這幾年如雨後春筍般的蓬勃發展，提供全球各所知名大學傳授高品質學習內容至世界各地。然而，平台的生存必須確保其財務的持續性，故其定價策略值得我們探討。我們好奇定價策略是否會損害課程的多樣性，例如：證書購買率較低的課程是否會被排擠而消失。因此，我們採用賽局理論來研究平台之定價策略，探討學習者和大學之間的互動和策略選擇。大學會考慮競爭強度，並根據證書價格和平台決定之證書收入分潤比例，決定課程品質以吸引學習者。

在本篇研究中，我們發現，無論是高證書購買率或是低證書購買率之課程，在平台發展的整個生命週期中，無論平台發展成熟度和大學之間競爭強度如何，都將會存在，而其中一個原因是因為平台為了獲取收益會給予大學足夠誘因開設所有類型的課程。我們還發現，當大學之間發生競爭時，高品質課程的數量會隨著平台發展成熟度的上升先增後降，這是因為當平台發展進入成熟期時，其中一所大學會因為競爭過度激烈而沒有誘因提供高品質的課程。我們還發現，大學的製課成本和聲譽的差異將導致不同類型的課程的高品質課程數量出現落差。我們還探討學習者時間有限的狀況之大學的最佳策略選擇，並且發現如果低證書購買率課程的支付意願與高證書購買率課程的支付意願夠接近，能減輕不同課程類型之間的競爭造成的損失，增加大學在平台上開設高品質的課程的誘因。

關鍵字：大規模開放式線上課程、定價策略、多元化、多邊平台、賽局理論

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Thesis Abstract

Massive Open Online Courses (MOOCs) have recently received a great deal of attention in higher education. MOOCs demonstrate universities' efforts in offering high-quality digital learning materials to everyone in the world, which should be encouraged. Nevertheless, as a MOOC platform must ensure its financial sustainability, it is questionable whether a platform's profit-seeking pricing strategy will hurt the diversity of courses, such as eliminating courses with low certificate purchasing rates. To address this question, we adopt a game-theoretic framework to model the interaction and strategic choices of a MOOC platform, learners, and universities. Based on the certificate prices and revenue sharing ratios chosen by the platform for courses with various certificate purchasing rates, universities consider the competition intensity and decide their course quality levels to attract learners.

We conclude that all types of course in terms of certificate purchasing rates will exist in equilibrium throughout the lifecycle of a MOOC platform, regardless of the technology maturity and competition intensity. We also find that the number of excellent courses first increases then decreases in technology maturity when there is competition among universities. This is because the intense competition in the mature period makes one of the universities find herself suboptimal to offer an excellent course. We also find that the difference in effort level and reputation between different course types on the platform will lead to the gaps of equilibrium quality level among different types of courses. We also investigate the presence of busy learners and observe that a large willingness-to-pay of low-conversion-rate courses can somehow alleviates the disadvantages of competition between different course types brought by the presence of busy learners.

Keywords: Massive Open Online Courses (MOOCs), Pricing, Diversification, Multi-sided Platforms, Game Theory

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Chapter 1

Introduction

1.1 Background and motivation

Massive Open Online Courses (MOOCs) have recently received a great deal of attention in higher education. The scale and openness provide a new approach for expanding access to higher education and allow higher education institutions to enhance their reputation internationally. It has grown into a thriving battleground for prestigious universities competing with each other regarding reputation and course quality by putting elite courses on MOOC platforms. The rapid expansion of MOOCs has sparked considerable interest in the higher education market, leading to springing emergence of MOOC platform providers such as Coursera, edX, and Udacity.[1]

Coursera is one of the most popular MOOC platforms in the world. As a for-profit company founded in 2012 by two Stanford Computer Science professors Daphne Koller

[1] https://www.coursera.org/; https://www.edx.org/; https://www.udacity.com

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and Andrew Ng, it currently has over 1600 courses in 10 subjects from over 140 institutions, including computer science, mathematics, business, humanities, social science, medicine, engineering. edX is a non-profit and open source MOOC platform founded by Massachusetts Institute of Technology and Harvard University in 2012. It offers online courses from worldwide universities and institutions. Currently, there are a total of 30 subjects and over 950 courses including computer science, biology, engineering, architecture, data science, literature, social science, and more from about 106 institutions. Udacity is another for-profit initiative founded by Sebastian Thrun, David Stavens, and Mike Sokolsky with investment from venture capital offering computer science, programming, and related courses by industry giants Google, AT&T, Facebook, Salesforce, Cloudera, etc. Nevertheless, these three platforms all provide free access or audit alternatives.[2]

The most common revenue stream for a MOOC platform is to charge fees for certificates. Some other sources include selling learner information to potential employers or advertisers, fee-based assignment grading, access to the social networks, etc. As for cooperating universities, they may receive a proportion of revenue from the certificate fee and other value-added services for learners. For example, Young (2012) reports that Coursera shares 20% of gross revenue from certificates to partners. Partners may receive 6% to 15% of revenue for each career introduction by Coursera Career Services. edX also shares a proportion of revenue to their partners when total revenue goes beyond a threshold (Kolowich, 2013).

Currently, by November 2016, Coursera earned over 600 thousand course certificates,

[2] `https://www.coursera.org/`; `https://www.edx.org/`; `https://www.udacity.com`. Retrieved on June 13, 2017.

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and edX reached over 840 thousand certificates (Coursera, 2016a; edX, 2016). However, the profit models of these platforms are yet to be confirmed. Most of them are still following the common approach of Silicon Valley start-ups by receiving investment from venture capital. The sustainability issue and profit model are still big concerns for most MOOC platforms. Moreover, it is also questionable whether a platform's profit-seeking pricing strategy will hurt the diversity of courses, such as eliminating the courses with low certificate purchasing rates throughout the lifecycle of MOOCs.

1.2 Research objectives

As far as we know, there are quite a few studies discussing the business model of MOOCs, but rare of them adopt a theoretical framework to investigate the platform strategy. In this study, we present a game-theoretic model of the market for MOOCs. We assume that there are multiple types of course on the platform, some types are more attractive for learners to buy certificates while some types are not. In other words, we assume that the conversion rates of some types are naturally higher than the conversion rate of low type. The conversion rate somehow implies the spirit of free access of MOOCs. The learners do not need to pay for auditing the MOOCs, but only need to pay for the certificates. There may be multiple universities competing with each other, and there may be learners who are different in the amount of time to be spent on taking MOOCs. The platform decides the revenue sharing ratio and certificate price for each type of course. Universities then choose the quality of each type of course to maximize its utility. Under this setting, we investigate the platforms strategic pricing choice, platforms profit, and the induced

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course offering strategies of universities and course quality levels in equilibrium.

1.3 Research plan

In the next chapter, we review some related works with respect to MOOCs, network externality, and market of higher education. In Chapter 3, we develop a game-theoretic model to describe the competitive relationship among universities with different compositions of the course in terms of quality and effort level. The platform's strategic choice of certificate prices and revenue sharing ratios for coordinating supply and demand is also formulated. Analysis is discussed in Chapter 4. In Chapters 5 and 6, we extend our model to discuss the competition between two heterogeneous universities and the presence of busy learners. Conclusions are in Chapter 7. All proofs are in Appendix.

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Chapter 2

Literature review

2.1 Massive Open Online Courses

Massive Open Online Courses (MOOCs) are online courses aiming at unlimited participation and open access via the web (Kaplan and Haenlein, 2016). Introduced in 2008 and emerged as a popular mode of learning in 2012, MOOCs have become a popular approach to learning nowadays. In addition to traditional course materials, many MOOCs provide interactive in-video quizzes and forums to support community interactions among students, professors, and teaching team. Yuan and Powell (2013) point out that the development of MOOCs is rooted within the ideals of openness in education, knowledge should be shared freely, and the desire to learn should be met without demographic, economic, and geographical constraints. Yuan and Powell (2013) show that there are many factors which influence learners' motivation to participate in MOOCs. These include the future economic benefit, development of personal and professional identity, challenge and achievement, enjoyment, and fun. Surveys conducted by researchers at Duke University

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show that fun and enjoyment were selected as important reasons for enrolling by a large majority of learners, followed by relevance to study subject and benefits to job career, etc. (Belanger and Thornton, 2013). By October 2013, Coursera enrollment surpasses 5 million, while edX had independently reached 1.3 million (Fowler, 2013). Dellarocas and Van Alstyne (2013) indicate that education is the latest industry to face digital disruption. Industries like music, movies, and news have already built platforms that offer free service and information to attract users and their activity. These digital platforms monetize eyeballs, comments, referrals, and relationships based on two key ideas: charge for complements and charge a different group with interdependent demand. The former stressed value-added services, technical support, and consultancy to teach people how to fish so that people are willing to pay for the services; the latter explains that digital platforms would charge the group with interdependent demand. For example, TripAdvisor offers free advice to travelers and charges airlines and hotels. LinkedIn offers many free services to job seekers and charges recruiters. They expect that the digital revolution in the education industry will produce new business models and enormous social value in our increasingly connected world.

There are several studies discussing the business models and value propositions of MOOCs. Most of them hold the skeptical attitude towards the monetization of their business model. Baker and Passmore (2016) propose four pricing strategies: cross-subsidy, third-party, freemium, and nonmonetary. Under the cross-subsidy strategy, the costs of the platform are paid by using revenue earned from some other products or services. Under the third party strategy, the third party, i.e., commercial radio or advertiser, covers some or all costs of the platform. Under the freemium strategy, MOOC enrollment

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is free. However, to receive a premium, the MOOC participant must pay. Under the nonmonetary strategy, MOOCs can be viewed as gifts, freely given. However, such an act of altruism is difficult to imagine in a climate of cost-consciousness. Belleflamme and Jacqmin (2016) propose five potential monetization strategies: certification model, freemium model, advertising model, job matching model, and subcontractor model sustained based on the theory of multisided platforms. The most sustainable approach seems to be the subcontractor model which allows MOOC platforms to deliver innovative education to universities, and sell made-to-measure training programs to the private company. Burd et al. (2015) state that MOOCs potentially challenge the traditional dominance of higher education providers. The benefits for students include reduced education costs and global access to exclusive institution courses and instructors.

However, the benefits for institutions are less clear as there is a financial overhead required to develop and deliver content that is suitable for mass student consumption. The opportunities could be linking students to employers, offering certificates, blending face-to-face courses, and targeting future students. In addition, this paper holds that prestigious universities will retain the traditional degree and offer certificates of completion on a course-by-course basis, while other universities will trade these certificates of completion for course credits in long-term survivability. Nevertheless, the feasibility of monetization of MOOC business is still in the air where opportunities and challenges coexist.

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2.2 Network externality

In general, network externality, also called network effect, can be defined as a effect that there are many products for which the utility that a user derives from consumption of the good increases with the number of other agents consuming the good (Katz and Shapiro, 1985). In Armstrong (2006) and Rochet and Tirole (2006), we can see two forms of network externality: same-side and cross-side. Same-side network externality indicates that an increase in usage or increase of the group size on the platform benefits the users on the same side. This usually happens in a one-sided market where the volume of transactions realized on the platform depends only on the aggregate price level. As for cross-side network externality, the net utility on the one side increases with the number of users on the other side. This usually happens in a two-sided market as one in which the volume of transactions between groups depends not only on the overall price level but on the size of another group. Therefore, cross-side network externality is considered to be an important property of a two-sided market.

When it comes to monopoly platform cases, Armstrong (2006) develops an optimal pricing function similar to the Lerner index to depict how the price elasticity of demand and the network externality affect the platform's pricing strategy. When the price elasticity of demand is high, or the effect of network externality is strong, the platform will lower its price at any cost to attract agents as more as possible to join the platform. Hagiu (2009) introduces the consumer preferences for variety and finds that higher consumer preferences for variety lead to less substitutable among producers and greater market power of producers. The platform can then obtain more surplus from the bilateral in-

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teraction. The optimal pricing strategy is able to extract a larger share a profit from the producer than the consumer. Jing (2007) discusses how network externality affects the pricing of monopoly platform regarding vertical differentiation in quality. This paper shows that when there is network externality, the best vertical differentiation strategy is to provide the highest and the lowest quality products. The lowest-quality products are used to amplify the market base, while the highest-quality products are the main-stream of profit. When the network externality is stronger, the platform should reduce the price of the lowest-quality product even lower than the cost, and improve the price of the highest-quality product for profit. Rochet and Tirole (2006) develop a mixed model combined with these two types of charging method. In the beginning of this paper, they define the two-sided market in which they consider a platform charging per-interaction charges to the buyer and seller sides, and making the aggregate price level as a constant value. If the volume of transactions realized on the platform varies with the price for the buyer, then the market is two-sided. Similar to Armstrong (2006), the pricing function is also analogous to Lerner index. Finally, given that the market is two-sided, this pricing function could be applied to the pure membership charges, the pure usage charges or mixed of them. That is, the platform could maximize its profit by manipulating the prices for buyer and seller.

When it comes to duopoly platform cases, Armstrong (2006) depicts a duopoly platform with a two-sided single-homing environment. It concludes that neither of the two platforms would like to price too high in case of agents join the rival platform. Furthermore, they even find that the platform can increase its profit by using two-part tariffs charging method (charge fixed and per-transaction fees at the same time) so that there

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is no incentive for the platform to undercut its rival on either side of the market. Hagiu (2009) points out that there exists an additional motivation for lowering prices to consumers. Undercutting the rival platform and thereby stealing some of its consumers drives some producers away from it, resulting in even more consumers stolen, and so on. However, if consumers possess a higher preference for product variety, which means that producers possess higher market power, or producers have higher economies of scale in multi-homing, which means that the indirect competitive efficiency operating through price for consumers, it is possible that that platform will have smaller consumer price cut in equilibrium.

Despite the fact that there are different conclusion regarding different network externality settings, there is no doubt that network externality plays a crucial part to study the rapid proliferation of platform economy. In order to better clarify the competition between the types of course to produce by two universities, we leverage network externality to explain universities decisions in our study.

2.3 Market of higher education

Since that the sustainability of the business model of MOOCs remains unknown, we look to the profit model and tuition settings for traditional higher education.

Arcidiacono (2005) addresses how changing the admission and financial aid rules at colleges affects future earnings. The author constructs a structural model of the following decisions by individuals: where to submit applications, which school to attend, and what field to study. The model allows the monetary returns to different majors to vary with

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college quality and observed and unobserved ability. In the model, college quality serves as a consumption good so that high ability individuals may have preferences for particular majors independent of effort costs. The model also includes decisions by schools as to which students to accept and how much financial aid to offer. This paper provides the first step to understand how both admissions and financial aid rules affect colleges' expected future earnings. Epple et al. (2006) present an equilibrium model of the market for higher education. Their model simultaneously predicts student selection into institutions of higher education, financial aid, educational expenditures, and educational outcomes. Their model gives rise to a strict hierarchy of colleges that differ by the educational quality provided to the students. Their model defines the quality of college as a function of student ability level, expenditure per student, and mean income of student, and then defines college cost function as a function of the size of the college and expenditure per student. The decision problem of a college is to maximize their quality subject to their profit constraints and budget Constraints. Colleges seek to maximize the quality of course in consideration of its reputation. In equilibrium, the reservation price functions of each college and their beliefs about student matriculation must be consistent with utility maximization and the actions of the other colleges.

These studies have disclosed the decision procedure for the higher education market. The spirit of pursuit of quality is consistent throughout these papers. They provide comprehensive study about the higher education market competition. However, to our best knowledge, there is no research adopts a theoretic model to study MOOC business. We plan to deliver new managerial insights to complement the study in the management of modern higher education.

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Chapter 3

Model

3.1 Players and decision sequence

University and courses. Consider an MOOC platform (it) and a university (she), offering MOOCs. We assume that there are two types of courses on the MOOC platform, the high type and low type, where the high-type one has a higher conversion rate and the low-type one has a lower conversion rate. The high and low type will also be denoted by types H and L, respectively. She may offer both types of courses. The two type-j courses, $j \in \{\mathrm{H}, \mathrm{L}\}$, differ in their *conversion rate*, i.e., the proportion of auditing learners that will purchase the certificate. We assume that the conversion rate of the type-j course is $a_j - b_j p_j$, where $a_j > 0$ and $b_j > 0$ are all exogenous parameters for $j \in \{\mathrm{H}, \mathrm{L}\}$. We assume that under the same price p, the conversion rate of the high-type course is higher than that of the low one, i.e., $a_\mathrm{H} - b_\mathrm{H} p > a_\mathrm{L} - b_\mathrm{L} p$ for all $p \geq 0$.

University's decisions. University needs to determine the quality of its type-j

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course to find a balance between the benefit and cost. The benefit consists of two parts, the reputation earned from learners who audit the course and revenue shared by the platform from learners purchasing the certificate. We represent the reputation as $n_j q_j$, the number of auditing learners n_j times the course quality q_j. This captures the fact that more reputation can be earned if more learners audit the course, but the reputation is really high only if the course quality is high. The revenue earned by the platform is $(a_j - b_j p_j) p_j n_j$, where p_j is the certificate price of the type-j course and $a_j - b_j p_j$ is the corresponding conversion rate. Given the revenue sharing ratio w_j set by the platform, the university's revenue from certificate sales is $(a_j - b_j p_j) p_j w_j n_j$. Finally, as quality is costly, the university pays a cost $\frac{\alpha_j q_j^2}{2}$, where $\alpha > 0$ is an exogenous parameter scaling the cost, and the quadratic form is chosen for tractability.[1] Collectively, the utility function of the university is

$$u_j^{\mathrm{U}} = n_j q_j + (a_j - b_j p_j) p_j w_j \beta n_j - \frac{\alpha_j q_j^2}{2}, \tag{3.1}$$

where the parameter β adjusts how the university weighs the reputation and revenue. Upon observing w_js and p_js, the university then chooses its course quality levels $q_j \in [0, 1]$ to maximize its utility, where $q_j = 0$ means not offering the course and $q_j = 1$ means offering the best possible course.

Learners' decisions. We model the preference attitudes with a Hotelling line (Hotelling, 1929). Consider the type-j course. Let the university locates at 0, the end-point of a line segment $[0, 1]$, and x_j be a learner's location in respect to course j, his

[1] It can be shown that our major findings will be qualitatively unchanged as long as the cost is an increasing and convex function of q_{ij}.

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utility of taking type-j course is

$$u_j^{\mathrm{S}} = \theta_j q_j - t x_j \tag{3.2}$$

where $t > 0$ is the "transportation cost" in the Hotelling line model, measuring learners' preference over the course, and θ_j is the learners' willingness-to-pay for a unit of quality of the type-j course. As higher θ_j makes type-j course attract more learners, θ_j is also considered as the university's authority in the field of the course of type-j. The type-x_j learner will choose to audit the type-j course, or not to audit the type-j course to maximize his utility, where the utility of the last option is normalized to 0. For high-type courses, we adopt the same setting.

Platform's decision. To optimize its decision about the certificate prices p_js and revenue sharing ratios w_js, the platform must first conduct an equilibrium analysis to predict the consequence of its decision. After the prediction about the course qualities q_j and learner size n_j is done, the platform's problem is to maximize its profit.

$$\pi_j^{\mathrm{P}} = (1 - w_j)(a_j - b_j p_j) p_j n_j, \tag{3.3}$$

subject to the constraints $w_j \in [0, 1]$ and $p_j \geq 0$, $j \in \{\mathrm{H}, \mathrm{L}\}$. Note that n_j depends on the university's choices of q_j, which depends on the authority of the university θ_j, the course development cost α_j, and competition intensity (the smaller the t, the stronger the competition), etc. The platform would take these factors into consideration to set the two pricing variables w_j and p_j to induce desirable equilibrium behaviours chosen by the universities.

Decision sequence. The sequence of events is depicted in Figure 3.1. First, the platform determines the revenue sharing ratio w and the certificate price p for the type-j

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course, $j \in \{\mathrm{H}, \mathrm{L}\}$. Second, the university observes p and w and chooses its q_j. At the end, each learner makes his course auditing choice, the sizes of learners n_j are realized, and the platform earns its profit.

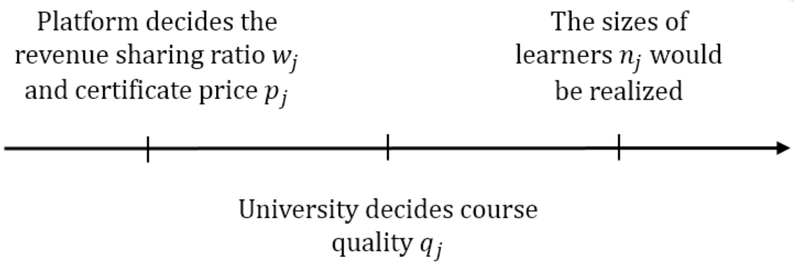

Figure 3.1: Decision sequence.

3.2 Market segmentation and assumptions

Market segmentation. After the courses are offered by the university at various quality levels, each learner independently decides which course(s) to audit. In this subsection, we will derive the learner size of type-j course, n_j, as a function of q_j, θ_j, and t.

Consider the type-j course. As a type-x_j learner sees the two type-j courses, he will be willing to take the course if $\theta_j q_j - t x_j \geq 0$, i.e., $x_j \leq \frac{\theta_{1j} q_j}{t}$. Let $\bar{x}_j = \frac{\theta_j q_j}{t}$ be the cutoff value. We assume that one university cannot cover all the market, which is $\bar{x}_j < 1$. In other words, the market is partially covered, some learners do not take any type-j courses, and $n_j = \frac{\theta_j q_j}{t}$. See Figure 3.2 for a depiction.

Assumptions. We consider the partial coverage scenarios under some mild assumptions. We assume that the universities cannot take the whole market even with the best possible course $q_j = 1$. As $n_j = \frac{\theta_j q_j}{t}$ under partial coverage, this means to assume

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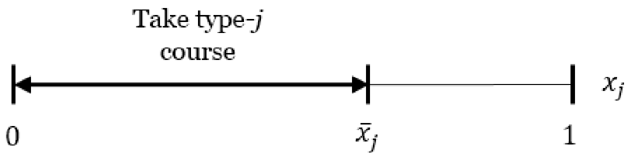

Figure 3.2: Market segmentation under partial coverage with one university.

$t > \max_{(j)}\{\theta_j\}$.

To facilitate a better understanding, we explain the model by an example. First of all, we can find that the certificate of some types of courses (like machine learning or artificial intelligence) are more popular than some other types of courses (like classic literature or history) on the platform naturally. It has nothing to do with course quality but intrinsic popularity. Therefore, we say that the type with higher certificate purchasing conversion rate is type-H and the type with lower conversion rate is type-L. The learners will choose to audit type-H course like machine learning or type-L course like classic literature independently in the basic model.

3.3 Lifecycle of MOOCs

As we mentioned above, the relationship between t and θ_j has an impact on the equilibrium market segmentation. Moreover, the value of t also determines whether a university's utility function with respect to a course is convex or concave. In the basic model, we consider a period called "expansion period" where the utility function of the university is convex, and a period called "start-up period" where the utility function of the university is concave. the university cannot take the whole market even with the best possible

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course $q_j = 1$ (cf. Figure 3.3).

1. In the *start-up period*, the transportation cost is so high ($t > \frac{2\theta_j}{\alpha_j}$) that the learner base does not contribute too much for the university. The market is partial covered. The utility function of each university is concave.

2. In the *expansion period*, we have $\max_{(j)}\{\theta_j\} < t \leq \left\{\frac{2\theta_j}{\alpha_j}\right\}$: The cost is small enough so that MOOCs are accessible to most of the learners, and the university find its utility function convex. However, the market is still partially covered.

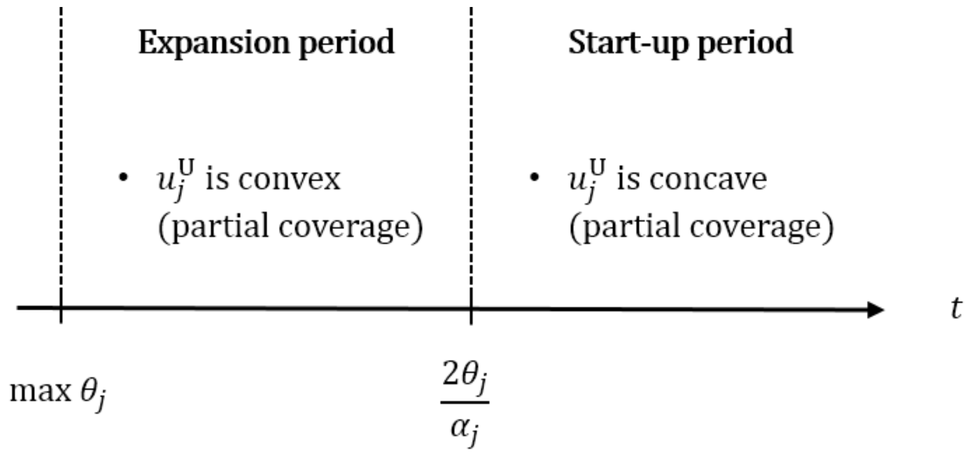

Figure 3.3: Lifecycle of MOOCs for the type-j course.

Because of the evolvement of technology and the popularity of MOOC platform, the transportation cost decreases over time, and the lifecycle of MOOCs transits from the start-up period to the expansion period. Continuing from the previous example, as the transportation cost decreases to be less than $\frac{2\theta_{\mathrm{H}}}{\alpha_{\mathrm{H}}}$, the university's utility function of offering type-H courses like machine learning changes from concave to convex. Similarly, as the transportation cost decreases to be less than $\frac{2\theta_{\mathrm{L}}}{\alpha_{\mathrm{L}}}$, the university's utility function of offering type-L course like classic literature changes from concave to convex.

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In the next section, we will first analyze the platform's pricing decisions in the period and then characterize the equilibrium certificate prices, revenue sharing ratios and course qualities. In Chapter 5, we extend the model to consider two heterogeneous universities competing in offering MOOCs. In Chapter 6, we consider one university in the market where some learners are free and some are busy, and focus on the extended expansion period to avoid tedious analysis. We assume that a busy learner can audit at most one of the type-j course at the same time, while a free learner may audit different type-j course simultaneously. We then combine the analysis of extensions to deliver our main messages.

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	Decision variables
p_j	The certificate price of type-j course
w_j	The revenue sharing ratio of j type of course
q_j	The quality of type-j course
	Parameters
H	High type course (the certificate of this type of course is more attractive)
L	Low type course (the certificate of this type of course is less attractive)
θ_j	The preference of learners of type-j course
θ_{ij}	The preference of learners of type-j course offered by university i
r	The proportion of busy learners who can audit at most one of the type-j course
t	The transportation cost, measuring learner's preference over the course
n_j	The number of learners taking type-j course
n_{ij}	The number of learners taking type-j course offered by university i
a_j	The intercept (base) of the conversion rate of certificate purchase
b_j	The slope (price sensitivity) of the conversion rate of certificate purchase
α_j	The effort level making type-j course
α_{ij}	The effort level of university i making type-j course
β_i	The importance level for university i value its revenue

Table 3.1: List of decision variables and parameters.

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Chapter 4

Analysis

We characterize the quality q_j, revenue sharing ratio w_j, certificate price p_j, and profit of the platform π_j^{P} in equilibrium where $j \in \{\mathrm{H}, \mathrm{L}\}$. The implications about market equilibrium and the platform's strategic choice will then be drawn.

As we mentioned in the model, the learner's utility taking type-j can be formulated as $u_j^{\mathrm{S}} = \theta_j q_j - t x_j$. Under partial market coverage, the size of learner taking type-j course can be calculated as $n_j = \frac{\theta_j q_j}{t}$. Therefore, the utility of university can be formulated as

$$u_j^{\mathrm{U}} = q_j^2\left(\frac{\theta_j}{t} - \frac{\alpha_j}{2}\right) + q_j\left(\frac{(a_j - b_j p_j) p_j w_j \beta \theta_j}{t}\right) \tag{4.1}$$

If $2\theta_j - \alpha_j t > 0$, the university's utility function is convex; if $2\theta_j - \alpha_j t < 0$, the utility function is concave.

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4.1 Equilibrium analysis

4.1.1 Start-up period

In the start-up period, the transportation cost is so high ($t > \frac{2\theta_j}{\alpha_j}$) that the learner base does not contribute too much for the university, and the university find its utility function concave. The first-order condition leads to the optimal course quality

$$q_j^*(w_j) = \max\left\{\frac{(a_j - b_j p_j)p_j w_j \beta\theta_j}{\alpha_j t - 2\theta_j}, 1\right\} \tag{4.2}$$

as a function of the revenue sharing ratio. Then, the size of learners n_j can be calculated. The platform's problem is to maximize its profit by determining w_j. Since that $q_j \in [0, 1]$, we can find out the constraints of $w_j \in [0, 1]$ accordingly in equilibrium.

Lemma 1. *Consider the type-j course. In the start-up period, let $B = \frac{\alpha_j t - 2\theta_j}{(a_j - b_j p_j)p_j\beta\theta_j}$. We have*

$$w_j^* = \begin{cases} \frac{1}{2}, & \text{if } \frac{1}{2} < B \\ B, & \text{if } B \leq \frac{1}{2} \end{cases}$$

as the platform's optimal revenue sharing ratio. The equilibrium qualities are

$$q_j^* = \begin{cases} q_j^*(\frac{1}{2}), & \text{if } \frac{1}{2} < B \\ 1, & \text{if } B \leq \frac{1}{2} \end{cases}$$

where $q_j^*(\frac{1}{2}) < 1$.

As not many learners have adopted MOOCs (t is large), the platform should always choose a positive revenue sharing ratio w_j to encourage the universities to participate in

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the market in the start-up period. In fact, in this period, the demand is so small so that the platform's optimal revenue sharing ratio may goes up to $\frac{1}{2}$. Half of the revenues may be given to the university. We may also observe that it is impossible for both universities to offer the courses to the highest possible quality. Fortunately, the university will quit and offer nothing. The concavity of her utility function drives them to offer a course, even if the optimal quality is low.

4.1.2 Expansion period

In the expansion period, the university's utility function is convex, and the market is partially covered because university cannot take the whole market even with the best possible course $q_j = 1$. Therefore, the university will only consider $q_j \in \{0, 1\}$ in course offering. It can be proved that $q_j = 1$ will always be the case in equilibrium: As long as the university finds it profitable to offer the course, she will offer the best possible course. This is summarized in Lemma 2.

Lemma 2. *Consider the type-j course. In the expansion period, we have $w_j^* = 0$ and $q_j^* = 1$ if $w_j \geq 0$ for all $j \in \{\mathrm{L}, \mathrm{H}\}$.*

In Lemma 2, even though the platform set the optimal revenue sharing ratio w_j^* to zero, the university will still offer the qualities to one because the transportation cost t is low so that it is easy for a university to offer a course to attract many learners. The university will drive itself to offer the best course regardless of the revenue sharing ratio because the high reputation earned through course offering is good enough, and the platform takes away all the certificate revenues.

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4.2 Discussions and implications

Having the equilibrium quality characterized in the previous section, we snow examine the relationships between the transportation cost and the optimal qualities in the lifecycle.

Proposition 1. *Regarding the relationship between the transportation cost t and the revenue sharing ratio w_j^* in equilibrium:*

(a) *In equilibrium, w_j^* decreases when t decreases. As the transportation cost decreases, learners adopt MOOCs increases, the platform can decrease the revenue sharing ratio w_j^*, and enjoy more revenue itself.*

(b) *Eventually, as the progress of technology, the transportation cost t decreases, the university can easily attract enough learners. She is comfortable with having no certificate income because the high reputation earned through course offering, and the platform takes away all the certificate revenues $w_j^* = 0$.*

Our first finding is regarding how the revenue sharing ratio changes in transportation cost.When t is large, the immaturity of technology development and the unpopularity of MOOCs enforce the platform to adjust the revenue sharing ratio to induce course offering because the platform earns revenue only when universities offer courses. When t is small, the benefit of reputation offering a course is large enough to offer the best possible course, and the maturity of technology development and the popularity of MOOCs allows the platform to decrease the revenue sharing ratio to zero. It is worth mentioned that the university should be aware of the situation when t is small. She cannot count on the revenue sharing from the platform anymore because the platform would take away all the

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certificate revenues when the maturity of technology development and the popularity of MOOCs reach a certain level.

Proposition 2. *In equilibrium, we have $q_j^* > 0$ for all j, t. The optimal qualities of both high and low type throughout the lifecycle are positive in equilibrium.*

Proposition 3. *In equilibrium, q_j^* increases when t decreases. As the transportation cost decreases, it is more profitable for the university to offer an excellent course.*

Our second and third findings are regarding the whether the diversity of the courses exists and how the course quality changes throughout the lifecycle in equilibrium. As the high transportation cost in the initial period lowers the university's intention to offer the course, it seems that under some periods some types of course will not be offered. Somewhat surprisingly, we find out that both types of course exist throughout the lifecycle. There is always enough incentives for the university to offer a course given that the platform earns revenue only when the university offer courses. However, as the transportation cost t increases, the number of learners decreases, the university may find it suboptimal to offer an excellent course anymore, and decide to decreases the quality accordingly as the period of lifecycle changes.

Proposition 4. *The certificate price p_j^* in equilibrium remains the same no matter it is decided by the platform or the university.*

As aforementioned, the utility function of the university can be formulated as (4.1). The utility function is concave in the certificate price p_j. After the first derivatives, we can find that $p_j^* = \frac{a_j}{2b_j}$ is always the case. On the other hand, the platform's profit function can be formulated as Equation 3.3. We can tell that both the university's and

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the platform's incentives of pricing certificate are aligned with each other in terms of the certificate price term of the university's utility function and the platform's profit function, so the certificate price remains the same regardless who the decision maker is.

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Chapter 5

Competition between two heterogeneous universities

Consider two heterogeneous universities competing in offering MOOCs. We will remodel the interaction and strategic choices of a MOOC platform, learners, and universities, and elaborate players and decision sequence, market segmentation and assumptions, and lifecycle of MOOCs and the four periods under competition.

5.1 Players and decision sequence

Universities and courses. Consider two heterogeneous universities (for each of them, she), university 1 and university 2, competing in offering MOOCs. We assume that there are two types of courses on the MOOC platform, the high type and low type, where the high-type one has higher conversion rate and the low-type one. The high and low type will also be denoted by types H and L, respectively. Both universities may offer both

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types of courses. To facilitate discussion, we will sometimes call the type-j course offered by university i the course (i, j), $i \in 1, 2$, $j \in \{\mathrm{H}, \mathrm{L}\}$. The two courses differ in their *conversion rate*, i.e., the proportion of auditing learners that will purchase the certificate. We assume that the conversion rate of the type-j course is $a_j - b_j p_j$, where $a_j > 0$ and $b_j > 0$ are all exogenous parameters for $j \in \{\mathrm{H}, \mathrm{L}\}$. We assume that under the same price p, the conversion rate of the high-type course is higher than that of the low one, i.e., $a_{\mathrm{H}} - b_{\mathrm{H}} p > a_{\mathrm{L}} - b_{\mathrm{L}} p$ for all $p \geq 0$.

Universities' decisions. The universities' decisions are almost the same as in the basic model except that each of the university i needs to determine the quality of its type-j course.

Learners' decisions. The learners' decisions are almost the same as in the basic model except that there are universities 1 and 2 locates at 0 and 1, the two endpoints of a line segment $[0, 1]$, and x_{L} is a learner's location in respect to course j. We assume that a learner will audit at most one course of each type and may audit two courses of different types simultaneously.

Platform's decision. The platform's decision is similar to the one in basic model.

Decision sequence. The sequence of events is the same as in the basic model.

5.2 Market segmentation and assumptions

Market segmentation. After the courses are offered by different universities at various quality levels, each learner independently decides which course(s) to audit. In this section, we will derive the learner size of course (i, j), n_{ij}, as a function of q_{ij}, θ_{ij}, and t.

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Consider the type-j course. As a type-x_j learner sees the two type-j courses, he will be willing to take course $(1, j)$ if $\theta_{1j}q_{1j} - tx_j \geq 0$, i.e., $x_j \leq \frac{\theta_{1j}q_{1j}}{t}$. Similarly, if $\theta_{2j}q_{2j} - t(1 - x_j) \geq 0$, i.e., $x_j \geq 1 - \frac{\theta_{2j}q_{2j}}{t}$, he will be willing to take course $(2, j)$. Let $\bar{x}_{1j} = \frac{\theta_{1j}q_{1j}}{t}$ and $\bar{x}_{2j} = 1 - \frac{\theta_{2j}q_{2j}}{t}$ be the two cutoff values, their relationship determines the equilibrium market segmentation. If $\bar{x}_{1j} < \bar{x}_{2j}$, the market is partially covered, some learners do not take any type-j course, and $n_{ij} = \frac{\theta_{ij}q_{ij}}{t}$. See Figure 5.1 for a depiction. On the contrary, if $\bar{x}_{1j} \geq \bar{x}_{2j}$, the market is fully covered, all learners take a type-j course from one university, and $n_{1j} = \bar{x}_{0j} = 1 - n_{2j}$, where the type-$\bar{x}_{0j}$ learner is indifferent in taking the course from either university. It then follows that $\bar{x}_{oj}$ is the unique value satisfying $\theta_{1j}q_{1j} - t\bar{x}_{0j} = \theta_{2j}q_{2j} - t(1 - \bar{x}_{0j}), i.e., \bar{x}_{0j} = \frac{\theta_{1j}q_{1j} - \theta_{2j}q_{2j} + t}{2t}$. Figure 5.2 illustrates this scenario.

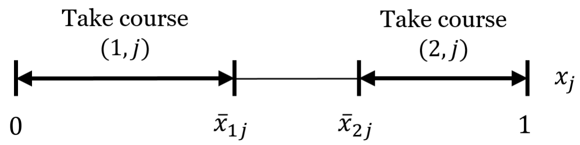

Figure 5.1: Market segmentation under partial coverage.

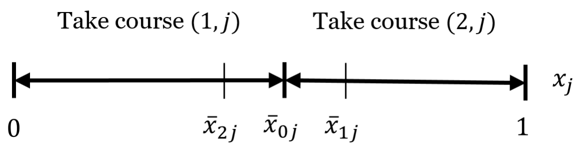

Figure 5.2: Market segmentation under full coverage.

According to the derivations above, it can be observed that when the market will be partially or fully covered depends on the value of t. When t is large, which means the cost of taking a MOOC is high, it is more likely that the market will be partially covered. As

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technology improves and an MOOC platform is more accessible to learners, t will become smaller, and it is more likely for the market to be fully covered. More precisely, the market is fully covered if and only if $\bar{x}_{1j} \geq \bar{x}_{2j}$, which is equivalent to $\theta_{1j}q_{1j} + \theta_{2j}q_{2j} \geq t$. Because $q_{ij} \leq 1$, if $\theta_{1j} + \theta_{2j} < t$, the market must be partially covered regardless of the course qualities; if $\theta_{1j} + \theta_{2j} \geq t$, it is then possible for the two universities to fully cover the market of the type-j course.

Assumptions. We consider both the full coverage and partial coverage scenarios under some mild assumptions. First, under partial coverage, we assume that none of the universities can take the whole market even with the best possible course $q_{ij} = 1$. As $n_{ij} = \frac{\theta_{ij}q_{ij}}{t}$ under partial coverage, this means to assume $t > \max_{(i,j)}\{\theta_{ij}\}$. Second, as the providers of MOOCs are usually prestigious universities and institutions, the cost of offering a course is typically an insignificant part in their annual budgets. Moreover, modern technology has diminished the difficulties to digitalize a course, which also implies that the course development cost is low. As α_{ij} is believed to be small, we assume $\theta_{1j}+\theta_{2j} < \min\left\{\frac{\theta_{ij}}{\alpha_{ij}}\right\}$ to avoid tedious comparisons that do not generate useful managerial insights.

Continue from the previous example in the basic model, now we have two universities competing in offering a type of course. Consider a type-H course as an example first. Suppose both university 1 and university 2 offer a type-H course like machine learning. We say that the two universities stand at point 0 and 1 on a Hotelling line. The learners will choose to take university 1's machine learning course or university 2's machine learning course depending on the learners' preference over the two machine learning courses offered by the two universities. Similarly, suppose both university 1 and university 2 offer a

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type-L course like classic literature. The learners will choose to take university 1's classic literature course or university 2's classic literature course depending on the learners' preference over the two classic literature courses offered by the two universities. Notice that we consider the competition between the universities offering the same type but not the competition between the two types of courses in this chapter.

5.3 Lifecycle of MOOCs and the four periods

As we mentioned above, the relationship between t and $\theta_{1j} + \theta_{2j}$ has an impact on the equilibrium market segmentation. Moreover, the value of t also determines whether a university's utility function with respect to a course is convex or concave (to be detailed below). These two factors drive us to divide the lifecycle of MOOCs into four periods depending on the value of t (cf. Figure 5.3):

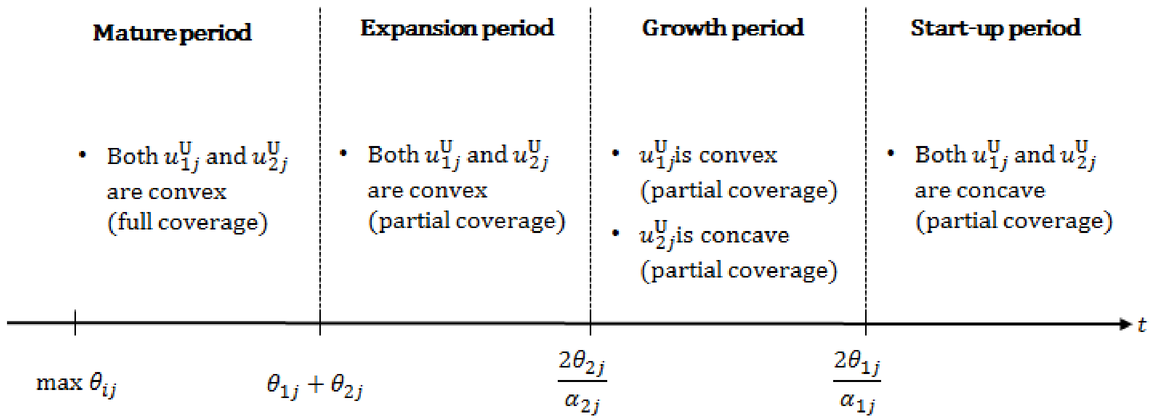

Figure 5.3: Lifecycle of MOOCs for the type-j course assuming $\theta_{1j}\alpha_{2j} > \theta_{2j}\alpha_{1j}$.

1. In the *start-up* period, we have $\max\left\{\frac{2\theta_{ij}}{\alpha_{ij}}\right\} < t$: The cost of taking a MOOC is quite large, both universities find their utility functions concave (and thus are less

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willing to offer the course to the highest possible quality level by setting $q_{ij} = 1$), and the market is partially covered.

2. In the *growth* period, we have $\min\left\{\frac{2\theta_{ij}}{\alpha_{ij}}\right\} < t \leq \max\left\{\frac{2\theta_{ij}}{\alpha_{ij}}\right\}$: The cost is still high, but one of the university's utility function becomes convex. This university will either offer the best possible course ($q_{ij} = 1$) or offer nothing. The market is still partially covered.

3. In the *expansion* period, we have $\theta_{1j} + \theta_{2j} < t \leq \min\left\{\frac{2\theta_{ij}}{\alpha_{ij}}\right\}$: The cost becomes lower, MOOCs are accessible to more learners, and both universities find their utility functions convex. However, the market is still partially covered.

4. In the *mature* period, we have $t \leq \theta_{1j} + \theta_{2j}$: The technology is well developed, platform is robust enough, and universities may attract learners easily. Both universities have convex utility functions, and it is possible for the market to be fully covered.

Because of the evolvement of technology and the popularity of MOOC platform, the transportation cost decreases over time, and the lifecycle of MOOCs transits from the start-up period to the mature period. Continuing from the previous example in the basic model, as the transportation cost decreases to be less than $\frac{2\theta_{1\mathrm{H}}}{\alpha_{1\mathrm{H}}}$, university 1's utility function of offering type-H course like machine learning course changes from concave to convex; as the transportation cost decreases to be less than $\frac{2\theta_{1\mathrm{L}}}{\alpha_{1\mathrm{L}}}$, university 1's utility function of offering type-L course like classic literature course changes from concave to convex. Similarly, as the transportation cost decreases to be less than $\frac{2\theta_{2\mathrm{H}}}{\alpha_{2\mathrm{H}}}$, university 2's utility function of offering type-H course like machine learning course changes from

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concave to convex; as the transportation cost decreases to be less than $\frac{2\theta_{2\mathrm{L}}}{\alpha_{2\mathrm{L}}}$, university 2's utility function of offering type-L course like classic literature course changes from concave to convex. Finally, as the transportation cost decreases to be less than $\theta_{1\mathrm{H}}+\theta_{2\mathrm{H}}$, the lifecycle of type-H course like machine learning course transits into the mature period, and the market of type-H course changes from partial coverage to full coverage. Similarly, as the transportation cost decreases to be less than $\theta_{1\mathrm{L}}+\theta_{2\mathrm{L}}$, the lifecycle of type-L course like classic literature course transits into the mature period, and the market of type-L course changes from partial coverage to full coverage.

Obviously, the platform's optimal pricing decisions may be different from period to period. Therefore, the platform needs to conduct a separate equilibrium analysis for each of the four periods. In the next section, we will first analyze the platform's pricing decisions in the four periods and then characterize the equilibrium certificate prices, revenue sharing ratios, and course qualities. We then combine the analyses for the four periods to deliver our main messages in this extension.

5.4 Equilibrium analysis

We characterize the quality pair (q_{1j}, q_{2j}), revenue sharing ratio w_j, certificate price p_j, and profit of the platform π_j^{P} in equilibrium under the four periods where $j \in \{\mathrm{H}, \mathrm{L}\}$. We investigate the transportation cost cut-offs between the high type and the low type and their respective quality levels. The implications about market equilibrium and the platform's strategic choice will then be drawn.

As we mentioned in the model, the utility of learner taking university 1 and university

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2 can be formulated as $u^{\mathrm{S}}_{1j} = \theta_{1j}q_{1j} - tx_j$ and $u^{\mathrm{S}}_{2j} = \theta_{2j}q_{2j} - tx_j$. Under partial market coverage scenario, the size of the learner taking university 1 and taking university 2 can be calculated as $n_{1j} = \frac{\theta_{1j}q_{1j}}{t}$ and $n_{2j} = \frac{\theta_{2j}q_{2j}}{t}$. Therefore, the utility of university 1 and university 2 can be formulated as

$$u^{\mathrm{U}}_{1j} = q^2_{1j}(\frac{\theta_{1j}}{t} - \frac{\alpha_{1j}}{2}) + q_{1j}(\frac{(a_j - b_jp_j)p_jw_j\beta_1\theta_{1j}}{t}) \tag{5.1}$$

and

$$u^{\mathrm{U}}_{2j} = q^2_{2j}(\frac{\theta_{2j}}{t} - \frac{\alpha_{2j}}{2}) + q_{2j}(\frac{(a_j - b_jp_j)p_jw_j\beta_2\theta_{2j}}{t}). \tag{5.2}$$

If $2\theta_{ij} - \alpha_{ij}t > 0$, the utility function is convex; if $2\theta_{ij} - \alpha_{ij}t < 0$, the utility function is concave. Under full market coverage scenario, the size of the learner taking university 1 and university 2 can be calculated as $n_{1j} = \frac{\theta_{1j}q_{1j} - \theta_{2j}q_{2j} + t}{t}$ and $n_{2j} = \frac{\theta_{2j}q_{2j} - \theta_{1j}q_{1j} + t}{t}$. Therefore, the utility of university 1 and university 2 can be formulated as

$$u^{\mathrm{U}}_{1j} = q^2_{1j}(\frac{\theta_{1j}}{2t} - \frac{\alpha_{1j}}{2}) + q_{1j}(\frac{(t - \theta_{2j}q_{2j}) + (a_j - b_jp_j)p_jw_j\beta_1\theta_{1j} + (t - \theta_{2j}q_{2j})(a_j - b_jp_j)p_jw_j\beta_1}{2t}) \tag{5.3}$$

and

$$u^{\mathrm{U}}_{2j} = q^2_{2j}(\frac{\theta_{2j}}{2t} - \frac{\alpha_{2j}}{2}) + q_{2j}(\frac{(t - \theta_{1j}q_{1j}) + (a_j - b_jp_j)p_jw_j\beta_2\theta_{2j} + (t - \theta_{1j}q_{1j})(a_j - b_jp_j)p_jw_j\beta_2}{2t}). \tag{5.4}$$

If $\theta_{ij} - \alpha_{ij}t > 0$, the utility function is convex; if $\theta_{ij} - \alpha_{ij}t < 0$, the utility function is concave. Since that the profit function of the platform is $\pi^{\mathrm{P}}_j = (1 - w_j)(a_j - b_jp_j)p_jn_{ij}$, the optimal p_j can be derived as $\frac{a_j}{2b_j}$, and $(a_j - b_jp_j)p_j$ is always $\frac{a_j^2}{4b_j}$. For the platform, the more challenging decision to consider is the revenue sharing ratio w_j, which will be explicitly characterized for each of the four periods below.

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To reduce tedious calculations and derivations that do not generate useful insights, we will assume that $\beta_1 = \beta_2$ throughout this paper.

5.4.1 Start-up period

In the start-up period, the transportation cost is so high ($t > \max\left\{\frac{2\theta_{ij}}{\alpha_{ij}}\right\}$) that the learner base does not contribute too much for the university. The market is partial covered. The utility function of each university is concave, and the first-order condition leads to the optimal course quality

$$q_{ij}^*(w_j) = \max\left\{\frac{(a_j - b_j p_j) p_j w_j \beta_i \theta_{ij}}{\alpha_{ij} t - 2\theta_{ij}}, 1\right\} \tag{5.5}$$

as a function of the revenue sharing ratio. Then, the size of learners n_{ij} can be calculated. The platform's problem is to maximize its profit by determining w_j. Since that $q_{ij} \in [0, 1]$, we can find out the constraints of $w_j \in [0, 1]$ accordingly in equilibrium.

Lemma 3. *Consider the type-j course. In the start-up period, let* $\frac{\theta_{1j}}{\alpha_{1j}} > \frac{\theta_{2j}}{\alpha_{2j}}$ *without loss of generality, and let* $B_1 = \frac{\alpha_{1j} t - 2\theta_{1j}}{(a_j - b_j p_j) p_j \beta_1 \theta_{1j}}$ *and* $B_2 = \frac{\alpha_{2j} t - 2\theta_{2j}}{(a_j - b_j p_j) p_j \beta_2 \theta_{2j}}$*. We have*

$$w_j^* = \begin{cases} \frac{1}{2}, & \text{if } \frac{1}{2} < B_1 \\ B_1, & \text{if } \frac{1}{2} - \frac{\theta_{1j} B_2}{2\theta_{2j}} < B_1 \le \frac{1}{2} \\ \frac{1}{2} - \frac{\theta_{1j} B_2}{2\theta_{2j}}, & \text{if } B_1 \le \frac{1}{2} - \frac{\theta_{1j} B_2}{2\theta_{2j}} \end{cases}$$

as the platform's optimal revenue sharing ratio. The equilibrium qualities are

$$(q_{1j}^*, q_{2j}^*) = \begin{cases} (q_{1j}^*(\frac{1}{2}), q_{2j}^*(\frac{1}{2})), & \text{if } \frac{1}{2} < B_1 \\ (1, q_{2j}^*(B_1)), & \text{if } \frac{1}{2} - \frac{\theta_{1j} B_2}{2\theta_{2j}} < B_1 \le \frac{1}{2} \\ (1, q_{2j}^*(\frac{1}{2} - \frac{\theta_{1j} B_2}{2\theta_{2j}})), & \text{if } B_1 \le \frac{1}{2} - \frac{\theta_{1j} B_2}{2\theta_{2j}} \end{cases}$$

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where $q^*_{2j} < 1$ *in all three cases and* $q^*_{1j} < 1$ *if* $B_1 > \frac{1}{2}$.

As not many learners have adopted MOOCs (t is large), the platform should always choose a positive revenue sharing ratio w_j to encourage the universities to participate in the market in the start-up period. In fact, just like the period in the basic model, the demand is so small so that the platform's optimal revenue sharing ratio may goes up to $\frac{1}{2}$. Half of the revenues may be given to the university. We may also observe that it is impossible for both universities to offer the courses to the highest possible quality. Fortunately, none of them will quit and offer nothing. The concavity of their utility function drives them to offer a course, even if the optimal quality is low.

5.4.2 Growth period

In the growth period, t locates between $\min\left\{\frac{2\theta_{ij}}{\alpha_{ij}}\right\}$ and $\max\left\{\frac{2\theta_{ij}}{\alpha_{ij}}\right\}$. The market is partial covered. Suppose $\frac{\theta_{1j}}{\alpha_{1j}} > \frac{\theta_{2j}}{\alpha_{2j}}$, the utility function of university 1 will be convex and that of university 2 is concave. We can identify optimal q_{1j} and q_{2j} and the constraints of w_j.

Lemma 4. *Consider type-j course. In the growth period, let* $\frac{\theta_{1j}}{\alpha_{1j}} > \frac{\theta_{2j}}{\alpha_{2j}}$ *without loss of generality, and let* $C_1 = \frac{\alpha_{1j}t-2\theta_{1j}}{(a_j-b_jp_j)p_j\beta_1\theta_{1j}}$ *and* $C_2 = \frac{\alpha_{2j}t-2\theta_{2j}}{(a_j-b_jp_j)p_j\beta_2\theta_{2j}}$. *We have*

$$w^*_j = \begin{cases} \frac{1}{2} - \frac{\theta_{2j}C_2}{2\theta_{1j}}, & \text{if } \frac{1}{2} - \frac{\theta_{2j}C_2}{2\theta_{1j}} < C_2 \\ C_2, & \text{if } C_2 < \frac{1}{2} - \frac{\theta_{2j}C_2}{2\theta_{1j}} \end{cases}$$

as the platform's optimal revenue sharing ratio. The equilibrium qualities are

$$(q^*_{1j}, q^*_{2j}) = \begin{cases} (1, q^*_{2j}(\frac{1}{2} - \frac{\theta_{2j}C_2}{2\theta_{1j}})), & \text{if } \frac{1}{2} - \frac{\theta_{2j}C_2}{2\theta_{1j}} < C_2 \\ (1, 1), & \text{if } C_2 < \frac{1}{2} - \frac{\theta_{2j}C_2}{2\theta_{1j}} \end{cases}$$

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where $q^*_{2j} < 1$ *if* $\frac{1}{2} - \frac{\theta_{2j}C_2}{2\theta_{1j}} < C_2$. *Because* $t \in (min\left\{\frac{2\theta_{ij}}{\alpha_{ij}}\right\}, max\left\{\frac{2\theta_{ij}}{\alpha_{ij}}\right\})$ *implies* $0 < C_2 < \frac{8b_j(\theta_{1j}\alpha_{2j}-\theta_{2j}\alpha_{1j})}{a_j^2\beta_2\theta_{2j}\alpha_{1j}}$, *the optimal quality pair* $(1,1)$ *exists if and only if* $\frac{8b_j(\theta_{1j}\alpha_{2j}-\theta_{2j}\alpha_{1j})}{a_j^2\beta_2\theta_{2j}\alpha_{1j}} < \frac{1}{2} - \frac{\theta_{2j}C_2}{2\theta_{1j}}$.

According to Lemma 4, the platform will still set a positive revenue sharing ratio w^*_j to stimulate the participation of the university. Compared to the optimal ratio in the start-up period, we may find that in the growth period, the optimal ratio is always lower than $\frac{1}{2}$, which is impossible in the start-up period. It is still possible that it is too expensive for the platform to make all the universities set their qualities to 1, if t is large enough. However, university 1, the university with convex utility, finds it optimal to maximize the course quality.

5.4.3 Expansion period

In the expansion period, the transportation cost becomes smaller, though the market is still partial covered. Both universities' utility functions are convex in this period. Therefore, each university will only consider the corner solutions $q_{ij} \in 0, 1$ in course offering (cf. Figure 5.4). It can be proved that in equilibrium (q_{1j}, q_{2j}) can be neither (1,0) nor (0,1): As long as one university finds it profitable to offer the course, the other would also benefit from offering a course. (1,1) is the unique equilibrium. This is summarized in Lemma 3.

Lemma 5. *Consider the type-j course. In the expansion period, we have* $w^*_j = 0$ *and* $(q^*_{1j}, q^*_{2j}) = (1, 1)$ *if* $w_j \geq 0$ *for all* $j \in \{\mathrm{L}, \mathrm{H}\}$.

In Lemma 5, even though the platform set the optimal revenue sharing ratio w^*_j to

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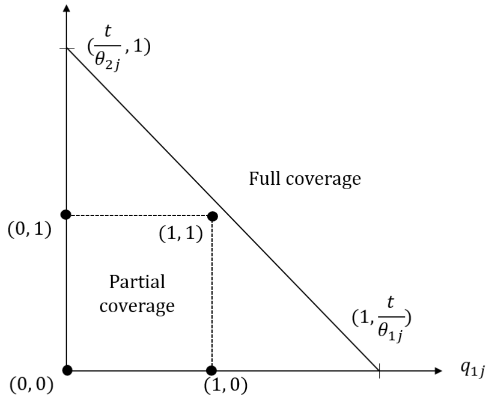

Figure 5.4: Corner solutions in the expansion period.

zero, both universities will offer the qualities to one because the transportation cost t is at the best region: it is low so that it is easy for a university to offer a course to attract many learners, and it is high enough so that the two universities' courses are not really in a competition. This may be the case, e.g., that all people in the world have high-speed free Internet access, and the concept of MOOCs has been widely adopted, but the technology of automatic translation is still imperfect. Therefore, a university may easily attract a lot of learners in its own language, and the threat from a course using a foreign language is weak. Each university will drive itself to offer the best course regardless of the revenue sharing ratio, and the platform takes away all the certificate revenues. The universities are comfortable with having no certificate income because the high reputation earned through course offering is good enough.

5.4.4 Mature period

In the mature period, the utility function of the university is convex. Moreover, now t is so small that if both universities offer their courses to the highest possible quality level,

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the market will be fully covered. In other words, in such a $(q_{1j}, q_{2j}) = (1, 1)$ situation, the two universities really compete in qualities to win learners. We can identify six corner solutions (cf. Figure 5.5), i.e., three full coverage solution $(1, 1)$, $(1, \frac{t-\theta_{1j}}{\theta_{2j}})$, and $(\frac{t-\theta_{2j}}{\theta_{1j}}, 1)$ and three partial coverage solutions (0,0), (0,1), and (1,0). We investigate the equilibrium by examining that there is no player can be better off by a unilateral change, and figure out the constraints of w_j.

Lemma 6. *Consider the type-j course. In the mature period, we have* $w_j^* = 0$ *and* $(q_{1j}^*, q_{2j}^*) \in \{(1, \frac{t-\theta_{1j}}{\theta_{2j}}), (\frac{t-\theta_{2j}}{\theta_{1j}}, 1)\}$ *if* $w_j \geq 0$ *for all* $j \in \{\mathrm{L}, \mathrm{H}\}$. *It can be verified that all the solutions along the line between* $(1, \frac{t-\theta_{1j}}{\theta_{2j}})$ *and* $(\frac{t-\theta_{2j}}{\theta_{1j}}, 1)$ *for all* $j \in \{\mathrm{L}, \mathrm{H}\}$ *are not equilibria.*

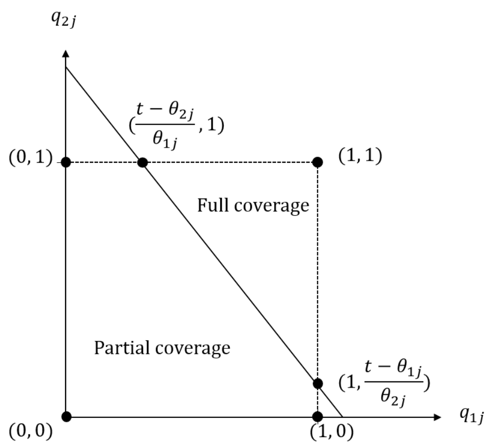

Figure 5.5: Corner solutions in the mature period.

Lemma 6 shows that the universities would set their quality so that the market is exactly full covered. The fact is that both universities find it beneficial to offer high-quality courses. However, as long as one university has set its quality to 1, the other university will find it not worthwhile to also set the quality to 1 due to the competition.

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The convexity of the utility function would then suggest the university to set the quality to exactly the level that attracts all the remaining learners. Interestingly, we have no idea which university will get the chance to be the one offering the course of higher quality, as there exist two equilibria in this case. Despite of this, both equilibria yield the same profit to the platform.

5.5 Discussions and implications

Having the equilibrium qualities characterized in the previous section, we now examine the relationships between the transportation cost and the optimal qualities in each period.

Proposition 5. *In equilibrium, we have $q^*_{ij} > 0$ for all i, j, t. The optimal qualities of both high type and low type throughout the four periods are all positive in equilibrium.*

Our first finding is regarding whether the diversity of the courses exists throughout the four periods in equilibrium. As the low-type course results in a low purchase conversion rate, and the high transportation cost in the initial period lowers the university's intention to offer the course, it seems that under some periods some types of course will not be offered. Somewhat surprisingly, we find out that both types of course exist throughout the four periods in equilibrium. This fact may be explained as follows. When t is small, even the low-type course may benefit a university by earning it reputation. When t is large, such a benefit does decrease, but the universities will at the same time find the competition between them become less intense. Given that the platform earns revenue only when universities offer courses, it will always adjust the revenue sharing proportion to induce course offering. It then follows that there is always enough incentives for both

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universities to offer both types of courses. Below we discuss the equilibrium quality levels of courses. We are particularly interested in the number of "excellent courses", which are defined as courses whose quality levels are 1.

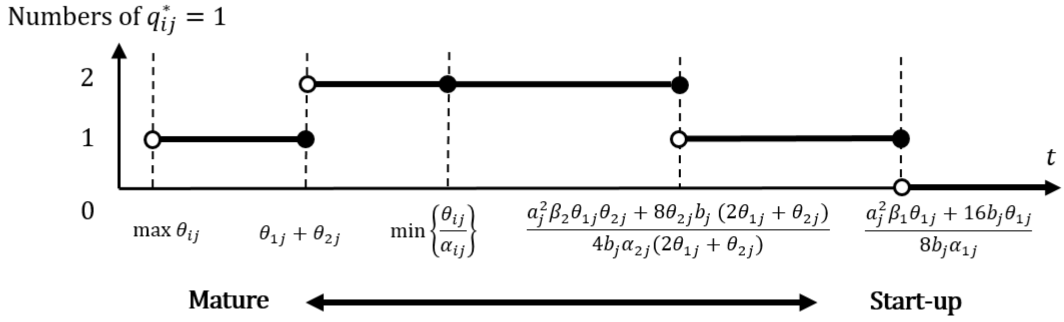

Figure 5.6: The change of the number of excellent course in t (case 1).

Proposition 6. *There is one excellent course in the mature period and two in the expansion period. In the growth period, if* $\frac{8b_j(\theta_{1j}\alpha_{2j} - \theta_{2j}\alpha_{1j})}{(a_j^2\beta_2\theta_{2j}\alpha_{1j})} < \frac{1}{2} - \frac{\theta_{2j}B_2}{2\theta_{1j}}$*, there exists a sub-period such that there are two excellent courses; otherwise, there is only one throughout the growth period. In the start-up period, there is no excellent course when* $t > \frac{a_j^2\beta_1\theta_{1j} + 16b_j\theta_{1j}}{8b_j\alpha_{1j}}$*; otherwise, there is an excellent course.*

Somewhat surprisingly, it shows that the maximum number of excellent courses locates in expansion period rather than mature period. As aforementioned, due to the intense competition and limited number of learners, in the mature period one university will find it suboptimal to offer an excellent course. Note that, in growth period, the optimal quality pair (1,1) exists if its w_j^* locates in growth period (cf. Figure 5.6). Otherwise, only $(1, q_{2j}^*)$ exists in the growth period (cf. Figure 5.7).

We now move forward to compare the quality levels of the two types of courses. Is it always the case that the high-type courses will be offered at a higher quality level? Or

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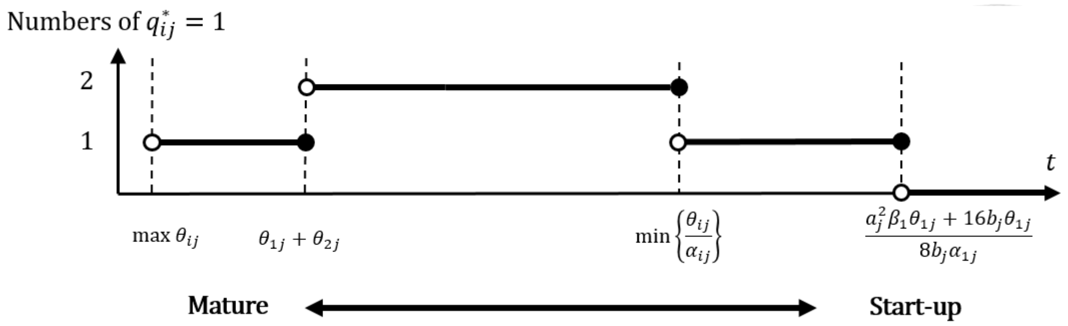

Figure 5.7: The change of the number of excellent course in t (case 2).

is it possible for a low-type course to possess better quality? The two propositions below address these questions.

Proposition 7. *If $\theta_{ij} = \theta$ for all $i \in \{1, 2\}$, $j \in \{\mathrm{L}, \mathrm{H}\}$ and $\alpha_{i\mathrm{L}} > \alpha_{i\mathrm{H}}$ for $i \in \{1, 2\}$, there exists $t > \min_{i \in \{1,2\}} \frac{2\theta}{\alpha_{i\mathrm{L}}}$ such that $q_{i\mathrm{H}}^* \geq q_{i\mathrm{L}}^*$ for all $i \in \{1, 2\}$. Moreover, if $a_\mathrm{H} > a_\mathrm{L}$ and $b_\mathrm{H} \leq b_\mathrm{L}$, there exists $t > \frac{a_\mathrm{L}^2 \beta_1 \theta_{1\mathrm{L}} + 16 b_\mathrm{L} \theta_{1\mathrm{L}}}{8 b_\mathrm{L} \alpha_{1\mathrm{L}}}$ such that $q_{i\mathrm{H}}^* \geq q_{i\mathrm{L}}^*$ for all $i \in \{1, 2\}$.*

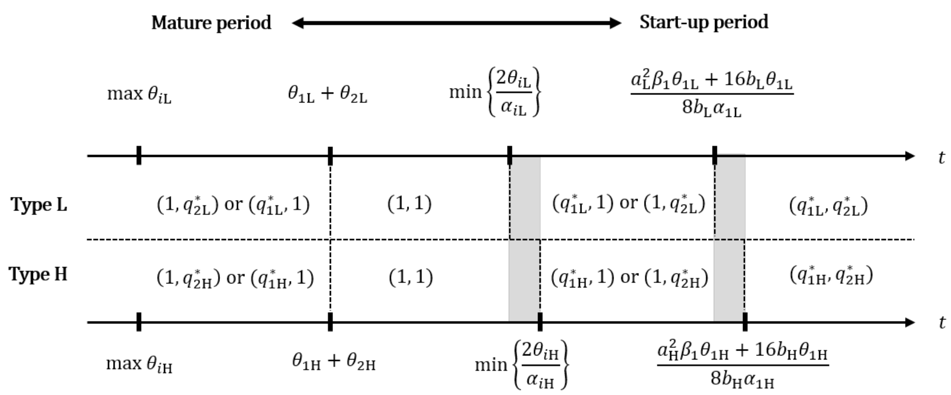

Figure 5.8: The gaps of equilibrium quality level between types H and L when $\alpha_{i\mathrm{L}} > \alpha_{i\mathrm{H}}$.

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The quality-gap periods of t mentioned in Proposition 7 are marked with grey color in Figure 5.8. If the effort cost of type-L is larger than type-H, and t locates in these quality-gap periods, then the optimal quality of type-L is smaller than or equal to type-H. Notice that the phenomenon might happen in start-up period or growth period. We suggest that the government and organization concerned should pay more attention to aid the university with higher effort cost to raise the quality in these early periods.

Proposition 8. *If $\alpha_{ij} = \alpha$ for all $i \in \{1, 2\}$, $j \in \{\mathrm{L}, \mathrm{H}\}$ and $\theta_{i\mathrm{H}} > \theta_{i\mathrm{L}}$ for $i \in \{1, 2\}$, then for all $i \in \{1, 2\}$, there exists $t > \theta_{1\mathrm{L}} + \theta_{2\mathrm{L}}$ such that $q^*_{i\mathrm{L}} \geq q^*_{i\mathrm{H}}$, and there exists $t > \min\limits_{i\in\{1,2\}} \dfrac{2\theta}{\alpha_{i\mathrm{L}}}$ such that $q^*_{i\mathrm{H}} \geq q^*_{i\mathrm{L}}$. Moreover, if $a_\mathrm{H} > a_\mathrm{L}$ and $b_\mathrm{H} \leq b_\mathrm{L}$, there exists $t > \frac{a^2_\mathrm{L}\beta_1\theta_{1\mathrm{L}} + 16b_\mathrm{L}\theta_{1\mathrm{L}}}{8b_\mathrm{L}\alpha_{1\mathrm{L}}}$ such that $q^*_{i\mathrm{H}} \geq q^*_{i\mathrm{L}}$ for all $i \in \{1, 2\}$.*

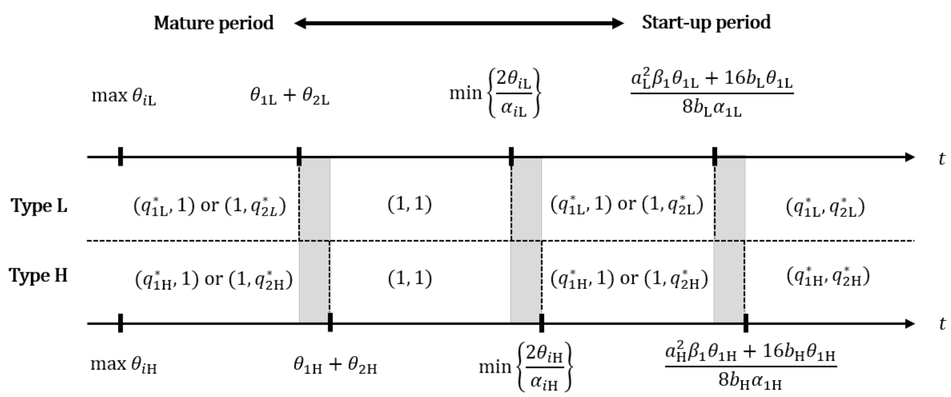

Figure 5.9: The gaps of equilibrium quality level between types H and L when $\theta_{i\mathrm{H}} > \theta_{i\mathrm{L}}$.

The quality-gap periods of t mentioned in Proposition 8 are marked with grey color in Figure 5.9. Surprisingly, if the reputation of type-H is larger than type-L, and t locates

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in the quality-gap period near $\theta_{1j} + \theta_{2j}$, then the optimal quality of type-L is larger than or equal to type-H because type-H falls in mature period earlier than type-L near this gap. The competition of type-L is more intense in the quality-gap period near $\theta_{1j} + \theta_{2j}$. If t locates in the other two quality-gap periods, the optimal quality of type-L is smaller than or equal to type-H because the reputation of type-H is larger than type-L.

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Chapter 6

Presence of busy learners

Consider one university in the market where some learners are free $(1-r)$ and some are busy (r). We remodel the interaction and strategic choices of a MOOC platform, learners, and the university, and elaborate players and decision sequence, market segmentation and assumptions, extended expansion period of MOOCs, and university's best responses in this extension.

6.1 Players and decision sequence

University and courses. Consider one university (she) in the market where some learners are free $(1-r)$ and some are busy (r). Assume that a busy learner can audit at most one of the type-j course, which is to audit at most one type-H or type-L course at the same time, while a free learner may audit one type-H and type-L course simultaneously. The university may offer both types of courses.

University's decision. The university needs to determine the quality of its type-H

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and type-L course at the same time to find a balance between the benefit and cost. The benefit consists of two parts, the reputation earned from learners who audit the course and revenue shared by the platform from learners purchasing the certificate. As quality is costly, the university pays a cost $\frac{\alpha_{\mathrm{H}} q_{\mathrm{H}}^2}{2}$ for type-H course, and $\frac{\alpha_{\mathrm{L}} q_{\mathrm{L}}^2}{2}$ for type-L course, where $\alpha > 0$ is an exogenous parameter scaling the cost, and the quadratic form is chosen for tractability. Collectively, the university's utility function is

$$u^{\mathrm{U}} = \begin{cases} u_{\mathrm{A}}^{\mathrm{U}} = n_{\mathrm{H}} q_{\mathrm{H}} + (a_{\mathrm{H}} - b_{\mathrm{H}} p_{\mathrm{H}}) p_{\mathrm{H}} w_{\mathrm{H}} \beta n_{\mathrm{H}} - \frac{\alpha_{\mathrm{H}} q_{\mathrm{H}}^2}{2} + \\ \qquad (1-r) n_{\mathrm{L}} q_{\mathrm{L}} + (1-r)(a_{\mathrm{L}} - b_{\mathrm{L}} p_{\mathrm{L}}) p_{\mathrm{L}} w_{\mathrm{L}} \beta n_{\mathrm{L}} - \frac{\alpha_{\mathrm{L}} q_{\mathrm{L}}^2}{2}, & \text{if } \theta_{\mathrm{H}} q_{\mathrm{H}} \geq \theta_{\mathrm{L}} q_{\mathrm{L}} \\ u_{\mathrm{B}}^{\mathrm{U}} = n_{\mathrm{L}} q_{\mathrm{L}} + (a_{\mathrm{L}} - b_{\mathrm{L}} p_{\mathrm{L}}) p_{\mathrm{L}} w_{\mathrm{L}} \beta n_{\mathrm{L}} - \frac{\alpha_{\mathrm{L}} q_{\mathrm{L}}^2}{2} + \\ \qquad (1-r) n_{\mathrm{H}} q_{\mathrm{H}} + (1-r)(a_{\mathrm{H}} - b_{\mathrm{H}} p_{\mathrm{H}}) p_{\mathrm{H}} w_{\mathrm{H}} \beta n_{\mathrm{H}} - \frac{\alpha_{\mathrm{H}} q_{\mathrm{H}}^2}{2}, & \text{if } \theta_{\mathrm{H}} q_{\mathrm{H}} < \theta_{\mathrm{L}} q_{\mathrm{L}} \end{cases} \tag{6.1}$$

where the parameter β adjusts how the university weighs the reputation and revenue. Upon observing w_js and p_js, the university then chooses its course quality levels $q_j \in [0, 1]$ to maximize its utility, where $q_j = 0$ means not offering the course and $q_j = 1$ means offering the best possible course. Note that when learners' preference over type-H course is larger than type-L course ($\theta_{\mathrm{H}} q_{\mathrm{H}} \geq \theta_{\mathrm{L}} q_{\mathrm{L}}$), n_{H} enters the benefit part of the utility function, and captures the positive cross-side network effect because both free $(1-r)$ and busy (r) learners prefer to take type-H course. However, only free learners $(1-r)$ are able to take more than one type of course, it follows that type-L course can only enjoy its positive cross-side network effect from $(1-r)n_{\mathrm{L}}$. Similar situation when $\theta_{\mathrm{H}} q_{\mathrm{H}} < \theta_{\mathrm{L}} q_{\mathrm{L}}$.

Learners' decisions. The learners' decisions are the same in the basic model.

Platform's decision. The platform's decision is similar to the one in basic model

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except that the platform's profit function is formulated as

$$\pi_j^{\mathrm{P}} = \begin{cases} (1-w_{\mathrm{H}})(a_{\mathrm{H}}-b_{\mathrm{H}}p_{\mathrm{H}})p_{\mathrm{H}}n_{\mathrm{H}} + (1-w_{\mathrm{L}})(a_{\mathrm{L}}-b_{\mathrm{L}}p_{\mathrm{L}})p_{\mathrm{L}}(1-r)n_{\mathrm{L}}, & \text{if } \theta_{\mathrm{H}}q_{\mathrm{H}} \geq \theta_{\mathrm{L}}q_{\mathrm{L}} \\ (1-w_{\mathrm{H}})(a_{\mathrm{H}}-b_{\mathrm{H}}p_{\mathrm{H}})p_{\mathrm{H}}(1-r)n_{\mathrm{H}} + (1-w_{\mathrm{L}})(a_{\mathrm{L}}-b_{\mathrm{L}}p_{\mathrm{L}})p_{\mathrm{L}}n_{\mathrm{L}}, & \text{if } \theta_{\mathrm{H}}q_{\mathrm{H}} < \theta_{\mathrm{L}}q_{\mathrm{L}} \end{cases}. \tag{6.2}$$

Decision sequence. The sequence of event is the same as in the basic model.

6.2 Market segmentation and assumptions

Market segmentation. After the courses are offered by the university at various quality levels, each learner independently decides which course(s) to audit. In this section, we will derive the learner size n_j, as a function of q_j, θ_j, and t.

Consider the type-j course. As a type-x_j learner sees the two type-j courses, he will be willing to take the course if $\theta_j q_j - t x_j \geq 0$, i.e., $x_j \leq \frac{\theta_{1j} q_j}{t}$. Let $\bar{x}_j = \frac{\theta_j q_j}{t}$ be the cutoff value. We assume that one university cannot cover all the market, which is $\bar{x}_j < 1$. In other words, the market is partially covered, some learners do not take any type-j courses, and $n_j = \frac{\theta_j q_j}{t}$. Recall Figure 3.2 for a depiction. Note that the market share of free learner who have free time to audit more than one type of course is $(1-r)$: the total size of learner taking type-H and type-L course is $n_{\mathrm{H}} + n_{\mathrm{L}}$ where $n_{\mathrm{H}} = \frac{\theta_{\mathrm{H}} q_{\mathrm{H}}}{t}$ and $n_{\mathrm{L}} = \frac{\theta_{\mathrm{L}} q_{\mathrm{L}}}{t}$. The market share of busy learner who can only take at most one course is r: the size of learner taking course is either n_{H} or n_{L} depends on their preference over the two types of course:

$$\begin{cases} n_{\mathrm{H}} = \frac{\theta_{\mathrm{H}} q_{\mathrm{H}}}{t}, & \text{if } \theta_{\mathrm{H}} q_{\mathrm{H}} \geq \theta_{\mathrm{L}} q_{\mathrm{L}} \\ n_{\mathrm{L}} = \frac{\theta_{\mathrm{L}} q_{\mathrm{L}}}{t}, & \text{if } \theta_{\mathrm{H}} q_{\mathrm{H}} < \theta_{\mathrm{L}} q_{\mathrm{L}} \end{cases}. \tag{6.3}$$

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Assumptions. We consider the market with type-H learners under some mild assumptions. First, as the way of learning on a MOOC plaform provides flexibility and accessibility, learners can make good use of time to structure self-imposed learning with ease in leisure time. Thus, we assume that the proportion of busy learners r is relatively small compared to the proportion of free learners. Note that the learners are all free ($r = 0$) in the market of basic model. Second, we assume that the universities cannot take the whole market even with the best possible course $q_j = 1$. As $n_j = \frac{\theta_j q_j}{t}$ under partial coverage, this means to assume $t > \max_{(j)}\{\theta_j\}$.

Continue from the previous example in the basic model, now we consider one university offering a type-H course and a type-L course. Suppose the university offers a type-H course like machine learning and a type-L course like classic literature. Her decision is to determine the optimal quality of the machine learning course and that of classic literature course to optimize her overall utility. A free learner can choose both the machine learning course and the classic literature course. A busy learner can take either the machine learning course or the classic literature course. A busy learner will choose the machine learning course if and only if the learner's utility of taking the machine learning course is higher than that of taking the classic literature course, and vice versa. Notice that we consider the competition between between type-H and type-L by the presence of busy learners in this chapter.

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6.3 Extended expansion period of MOOCs

As we mentioned above, the relationship between t and θ_j has an impact on the market segmentation. Moreover, the value of t also determines whether a university's utility function is convex or concave. Aforementioned, we assume that the proportion of busy learners r is small enough so that the cutoff of t is as shown in Figure 6.1. In the basic model, we defined the "expansion period" where $t < \frac{2\theta_L}{\alpha_L}$. In this chapter, to avoid tedious analysis, we focus on the "extended expansion period" where $\frac{2(1-r)\theta_L}{\alpha_L} < t < \frac{2\theta_L}{\alpha_L}$ because the analysis where $t < \frac{2(1-r)\theta_L}{\alpha_L}$ is the same as the analysis of "expansion period" in the basic model. Note that the university's utility is always convex in q_H, and is willing to offer the best possible type-H course in this chapter. In addition, let $\frac{\theta_H}{\alpha_H} > \frac{\theta_L}{\alpha_L}$ without loss of generality.

1. If $\theta_H q_H \geq \theta_L q_L$, none of the busy learners will take type-L course because their preference of type-H is larger than that of type-L, and only the free learners $(1-r)$ will take type-L course. It follows that the two cutoffs $\frac{2\theta_H}{\alpha_H}$ and $\frac{2(1-r)\theta_L}{\alpha_L}$ determines whether a university's utility function is convex or concave in qualities. When $\frac{2(1-r)\theta_L}{\alpha_L} < t < \frac{2\theta_H}{\alpha_H}$, the utility function is convex in q_H, and is concave in $q_{\mathring{L}}$. When $t < \frac{2(1-r)\theta_L}{\alpha_L}$, the utility function is convex in both q_H and $q_{\mathring{L}}$. Note that the two cutoffs forms strategy (A) in Figure 6.1.

2. If $\theta_H q_H < \theta_L q_L$, only the free learners $(1-r)$ will take type-H course. It follows that the two cutoffs $\frac{2(1-r)\theta_H}{\alpha_H}$ and $\frac{2\theta_L}{\alpha_L}$ determines whether a university's utility function is convex or concave in qualities. When $\frac{2\theta_L}{\alpha_L} < t < \frac{2(1-r)\theta_H}{\alpha_H}$, the utility function is

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convex in q_{H}, and is concave in $q_{\mathring{L}}$. When $t < \frac{2\theta_{\mathrm{L}}}{\alpha_{\mathrm{L}}}$, the utility function is convex in both q_{H} and $q_{\mathring{L}}$. Note that the two cutoffs forms strategy (B) in Figure 6.1.

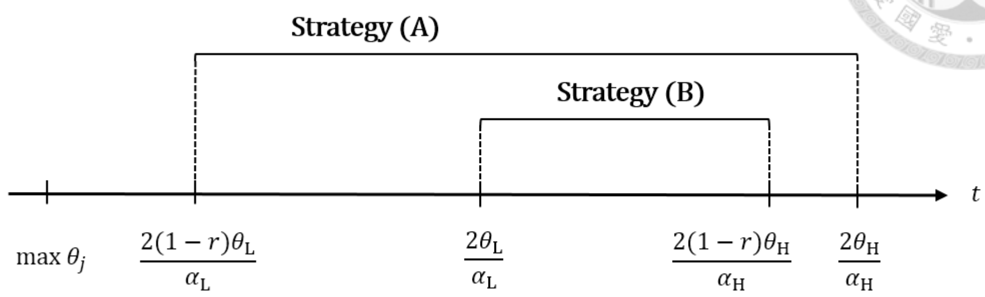

Figure 6.1: Extended expansion period of MOOCs with busy learners.

6.4 University's best responses

Consider Equation 6.3, the constraints $\theta_{\mathrm{H}} q_{\mathrm{H}} \geq \theta_{\mathrm{L}} q_{\mathrm{L}}$ and $\theta_{\mathrm{H}} q_{\mathrm{H}} < \theta_{\mathrm{L}} q_{\mathrm{L}}$ not only determine whether busy learners taking type-H or type-L course, but binds q_{H} and q_{L} on the relationship between θ_{H} and θ_{L}. We summarize the university's best responses as follows:

Lemma 7. *Let* $E_{\mathrm{L}} = \frac{(a_{\mathrm{L}} - b_{\mathrm{L}} p_{\mathrm{L}}) p_{\mathrm{L}} w_{\mathrm{L}} \beta \theta_{\mathrm{L}}}{\alpha_{\mathrm{L}} t - 2(1-r)\theta_{\mathrm{L}}}$. *Consider the period where* $\frac{2(1-r)\theta_{\mathrm{L}}}{\alpha_{\mathrm{L}}} < t < \frac{2\theta_{\mathrm{L}}}{\alpha_{\mathrm{L}}}$*:*

(a) *If* $\theta_{\mathrm{H}} \geq \theta_{\mathrm{L}}$*: Under strategy (A),* $\theta_{\mathrm{H}} q_{\mathrm{H}} \geq \theta_{\mathrm{L}} q_{\mathrm{L}}$*, the optimal quality pair* $(q_{\mathrm{H}}^*, q_{\mathrm{L}}^*) = (1, \min\{1, E_{\mathrm{L}}\})$*, and* $u^{\mathrm{U}}(q_{\mathrm{H}}^*, q_{\mathrm{L}}^*) = u_{\mathrm{A}}^{\mathrm{U}}(1, \min\{1, E_{\mathrm{L}}\})$ *is the resulting utility of the university. Under strategy (B),* $\theta_{\mathrm{H}} q_{\mathrm{H}} < \theta_{\mathrm{L}} q_{\mathrm{L}}$*, the optimal quality pair* $(q_{\mathrm{H}}^*, q_{\mathrm{L}}^*) = (\frac{\theta_{\mathrm{L}}}{\alpha_{\mathrm{H}}}, 1)$*, and* $u^{\mathrm{U}}(q_{\mathrm{H}}^*, q_{\mathrm{L}}^*) = u_{\mathrm{B}}^{\mathrm{U}}\left(\frac{\theta_{\mathrm{L}}}{\alpha_{\mathrm{H}}}, 1\right)$ *is the resulting utility of the university.*

(b) *If* $\theta_{\mathrm{H}} < \theta_{\mathrm{L}}$*: Under strategy (A),* $\theta_{\mathrm{H}} q_{\mathrm{H}} \geq \theta_{\mathrm{L}} q_{\mathrm{L}}$*, the optimal quality pair* $(q_{\mathrm{H}}^*, q_{\mathrm{L}}^*) = (1, \min\{\frac{\theta_{\mathrm{H}}}{\alpha_{\mathrm{L}}}, E_{\mathrm{L}}\})$*, and* $u^{\mathrm{U}}(q_{\mathrm{H}}^*, q_{\mathrm{L}}^*) = u_{\mathrm{A}}^{\mathrm{U}}(1, \min\{\frac{\theta_{\mathrm{H}}}{\alpha_{\mathrm{L}}}, E_{\mathrm{L}}\})$ *is the resulting utility of the*

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university. Under strategy (B), $\theta_{\mathrm{H}} q_{\mathrm{H}} < \theta_{\mathrm{L}} q_{\mathrm{L}}$*, the optimal quality pair* $(q_{\mathrm{H}}^*, q_{\mathrm{L}}^*) = (1,1)$*, and* $u^{\mathrm{U}}(q_{\mathrm{H}}^*, q_{\mathrm{L}}^*) = u_{\mathrm{B}}^{\mathrm{U}}(1,1)$ *is the resulting utility of the university.*

Overall,

$$u^{\mathrm{U}}(q_{\mathrm{H}}^*, q_{\mathrm{L}}^*) = \begin{cases} \max\left\{u_{\mathrm{A}}^{\mathrm{U}}(1, \min\{1, E_{\mathrm{L}}\}), u_{\mathrm{B}}^{\mathrm{U}}\left(\frac{\theta_{\mathrm{L}}}{\alpha_{\mathrm{H}}}, 1\right)\right\} & \text{if } \theta_{\mathrm{H}} \geq \theta_{\mathrm{L}} \\ \max\left\{u_{\mathrm{A}}^{\mathrm{U}}\left(1, \min\left\{\frac{\theta_{\mathrm{H}}}{\alpha_{\mathrm{L}}}, E_{\mathrm{L}}\right\}\right), u_{\mathrm{B}}^{\mathrm{U}}(1,1)\right\} & \text{if } \theta_{\mathrm{H}} < \theta_{\mathrm{L}} \end{cases}, \tag{6.4}$$

and

$$(q_{\mathrm{H}}^*, q_{\mathrm{L}}^*) = \begin{cases} (1, \min\{1, E_{\mathrm{L}}\}) & \text{if } \theta_{\mathrm{H}} \geq \theta_{\mathrm{L}} \text{ and } u_{\mathrm{A}}^{\mathrm{U}}(1, \min\{1, E_{\mathrm{L}}\}) \geq u_{\mathrm{B}}^{\mathrm{U}}\left(\frac{\theta_{\mathrm{L}}}{\alpha_{\mathrm{H}}}, 1\right) \\ \left(\frac{\theta_{\mathrm{L}}}{\alpha_{\mathrm{H}}}, 1\right) & \text{if } \theta_{\mathrm{H}} \geq \theta_{\mathrm{L}} \text{ and } u_{\mathrm{A}}^{\mathrm{U}}(1, \min\{1, E_{\mathrm{L}}\}) < u_{\mathrm{B}}^{\mathrm{U}}\left(\frac{\theta_{\mathrm{L}}}{\alpha_{\mathrm{H}}}, 1\right) \\ \left(1, \min\left\{\frac{\theta_{\mathrm{H}}}{\alpha_{\mathrm{L}}}, E_{\mathrm{L}}\right\}\right) & \text{if } \theta_{\mathrm{H}} < \theta_{\mathrm{L}} \text{ and } u_{\mathrm{A}}^{\mathrm{U}}\left(1, \min\left\{\frac{\theta_{\mathrm{H}}}{\alpha_{\mathrm{L}}}, E_{\mathrm{L}}\right\}\right) \geq u_{\mathrm{B}}^{\mathrm{U}}(1,1) \\ (1,1) & \text{if } \theta_{\mathrm{H}} < \theta_{\mathrm{L}} \text{ and } u_{\mathrm{A}}^{\mathrm{U}}\left(1, \min\left\{\frac{\theta_{\mathrm{H}}}{\alpha_{\mathrm{L}}}, E_{\mathrm{L}}\right\}\right) < u_{\mathrm{B}}^{\mathrm{U}}(1,1) \end{cases}. \tag{6.5}$$

6.5 Discussions and implications

Having the the university's best responses in qualities characterized in the previous section, we now observe what is the impact of r on the decision variables and what is the impact ofw_{H} and w_{L} on the university's optimal quality choice and the platform's profit.

6.5.1 Impact of revenue sharing ratios

We discuss the impact of revenue sharing ratios w_{H} and w_{L} under three circumstances. First is $\theta_{\mathrm{H}} > \theta_{\mathrm{L}}$ *with relatively large* θ_{L}. Second is $\theta_{\mathrm{H}} > \theta_{\mathrm{L}}$ *with relatively small* θ_{L}. Third

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is $\theta_{\mathrm{H}} < \theta_{\mathrm{L}}$. Note that the utility is convex in q_{H} throughout the period in this chapter, and , $r = 0$ simply represents the circumstances in the basic model.

1. $\theta_{\mathrm{H}} > \theta_{\mathrm{L}}$ with relatively large θ_{L}: As w_{H} and w_{L} are both small, strategy (B) is more likely to take place. As w_{L} increases, the possibility of strategy (A) to appear increases. When the university adopts strategy (B), the platform can enjoy all its revenue, and set w_{H}^* and w_{L}^* to zero. When the university adopts strategy (A), the platform's optimal profit appears if w_{L} is large enough and w_{H} is zero because the university's utility is concave in q_{L}, and the university will offer the best possible q_{H} without any revenue sharing from the certificate while some revenue sharing are required to offer type-L course. The observations are shown in Figure 6.2.

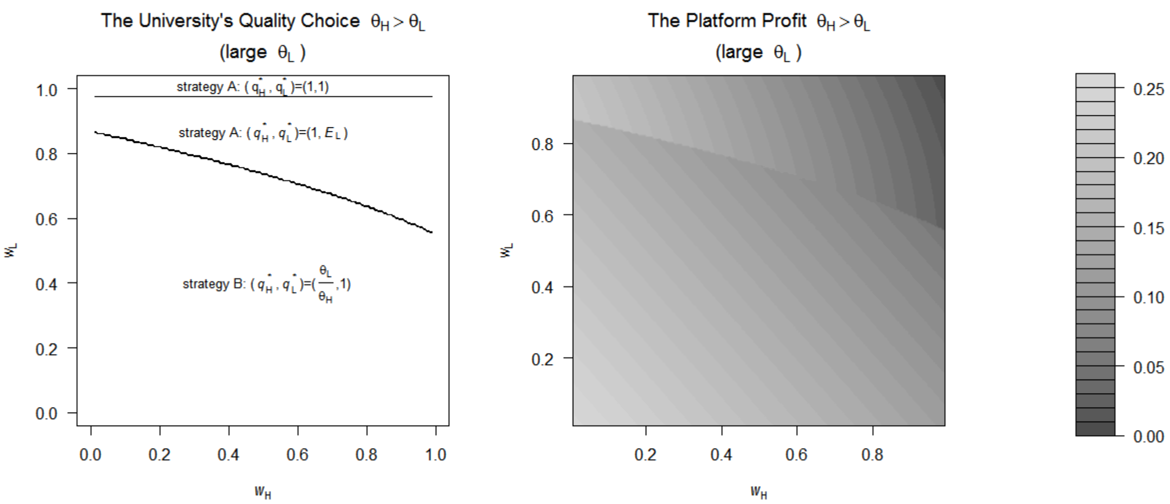

Figure 6.2: Impact of w_{H} and w_{L}: $\theta_{\mathrm{H}} > \theta_{\mathrm{L}}$ with relatively large θ_{L}.

2. $\theta_{\mathrm{H}} > \theta_{\mathrm{L}}$ with relatively small θ_{L}: When w_{L} is large, the university offers the best possible type-L course $q_{\mathrm{L}}^* = 1$. When w_{L} is small, the university offers $q_{\mathrm{L}}^* = E_{\mathrm{L}}$ accordingly. Since that the university's utility is concave in q_{L}, the platform has to

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find the balance to earn more between small w_{L} and large q_{L}. It follows that the optimal profit of the platform appears when w_{H} is zero and w_{L} is in the middle to encourage the university to offer type-L course, and at the same time earn some certificate revenue from type-L. The observations are shown in Figure 6.3.

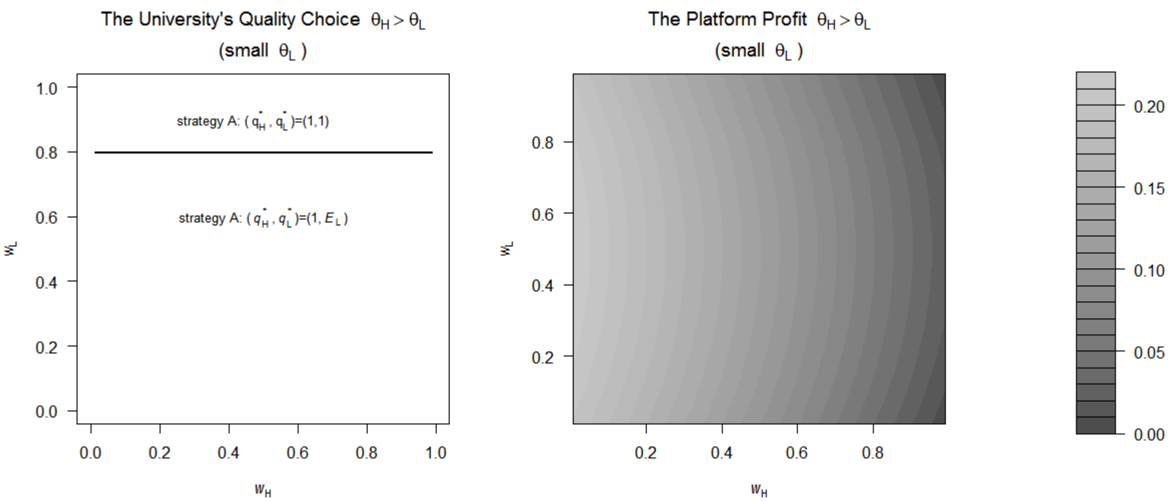

Figure 6.3: Impact of w_{H} and w_{L}: $\theta_{\mathrm{H}} > \theta_{\mathrm{L}}$ with relatively small θ_{L}.

3. $\theta_{\mathrm{H}} < \theta_{\mathrm{L}}$: We observe that strategy (B) always dominates under any w_{H}s and w_{L}s. The university is willing to offer the best possible q_{H}^* and q_{L}^* no matter how w_{H} and w_{L} change. The platform can enjoy all its revenue, and set w_{H}^* and w_{L}^* to zero. The observations are shown in Figure 6.4.

Overall, these observations regarding the impact of w_{H} and w_{L} on the university's quality choices and the platforms profit are summarized in Observation 1.

Observation 1. *Consider the extended expansion period where* $\frac{2(1-r)\theta_{\mathrm{L}}}{\alpha_{\mathrm{L}}} < t < \frac{2\theta_{\mathrm{L}}}{\alpha_{\mathrm{L}}}$*:*

(a) *When* $\theta_{\mathrm{H}} > \theta_{\mathrm{L}}$*, the university chooses strategy (A) under small* θ_{L}*, and the platform cannot take all the type-L revenue because* $w_{\mathrm{L}}^* > 0$ *for type-L course offering.*

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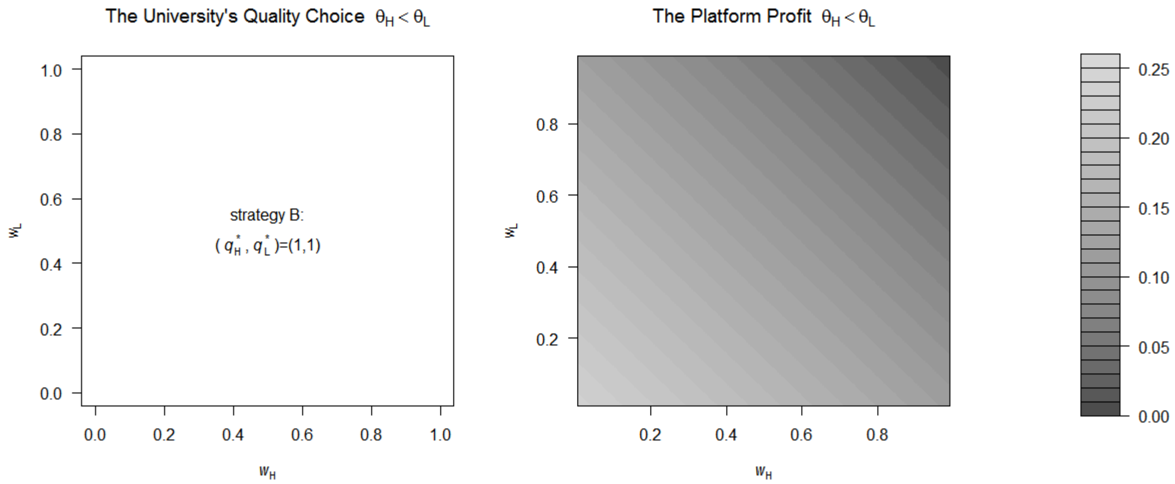

Figure 6.4: Impact of w_{H} and w_{L}: $\theta_{\mathrm{H}} < \theta_{\mathrm{L}}$.

(b) *When $\theta_{\mathrm{H}} < \theta_{\mathrm{L}}$, strategy (B) always takes place, and the platform can enjoy all the revenue and set w_{H}^* and w_{L}^* to zero.*

6.5.2 Impact of the proportion of busy learners

After discussing the impact of revenue sharing ratios w_{H} and w_{L}, we thereby discuss the impact of the proportion of busy learners r under the circumstances corresponding to the previous subsection. By observing the the proportion of busy learners r, the platform decides the revenue ratios, and the university determines the course qualities accordingly. Note that the utility is convex in q_{H} throughout the period defined in this chapter. Since that the willingness-to-pay of type-L course is lower than that of type-H course, we are thereby curious about the university's decision over the quality of type-L course when the proportion of busy learners increases. Will the university put more effort on type-L course as the proportion of busy learners increase?

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1. $\theta_{\mathrm{H}} > \theta_{\mathrm{L}}$ *with relatively large* θ_{L}: As r increases, we observe that the university first prefers strategy (A), then prefers strategy (B). Since that the profit from type-L shrinks as r increases in $u_{\mathrm{A}}^{\mathrm{U}}$ but remains the same in $u_{\mathrm{B}}^{\mathrm{U}}$, and the large θ_{L} makes type-L course profitable enough for the university to offer q_{L}^{*} without having revenue sharing from the platform, the university find herself optimal changing from strategy (A) to strategy (B) in a larger r. However, the change in strategy sacrifices the level of q_{H}^{*} from $q_{\mathrm{H}}^{*} = 1$ to $q_{\mathrm{H}}^{*} = \frac{\theta_{\mathrm{L}}}{\theta_{\mathrm{H}}}$. Nevertheless, the university is comfortable with having no revenue sharing in strategy (B). In a smaller r, the university finds herself optimal adopting strategy (A), and the platform has to make $w_{\mathrm{L}}^{*} > 0$ to induce type-L course. In a larger r, the platform can enjoy all its revenue, and set w_{H}^{*} and w_{L}^{*} to zero because the university find herself optimal changing her strategy to strategy (B). The observations are summarized in Figure 6.5.

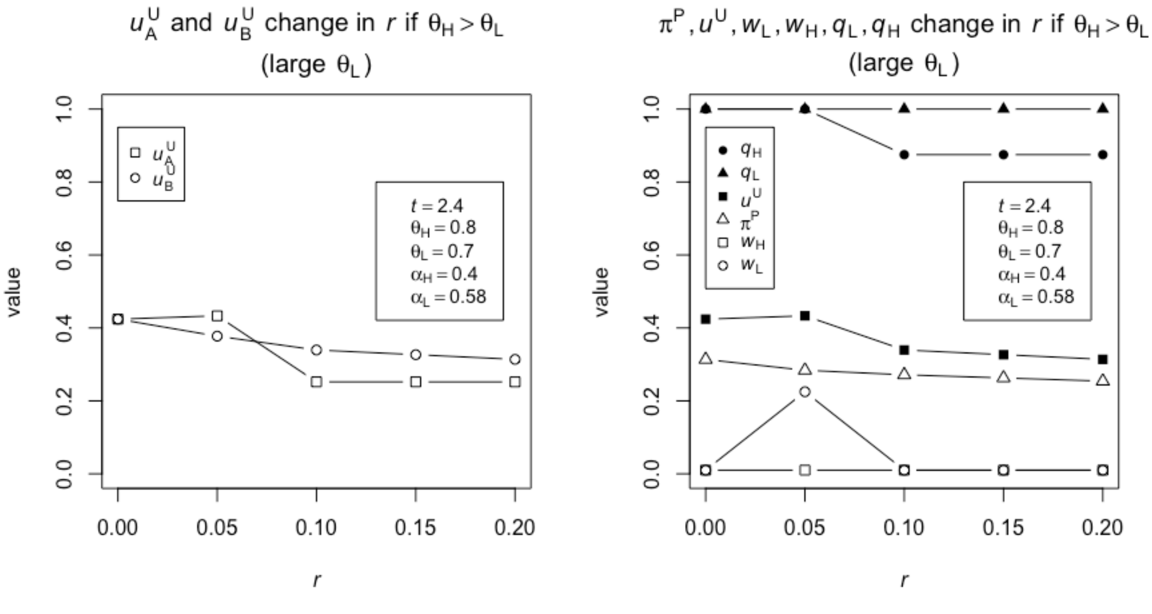

Figure 6.5: Impact of r: $\theta_{\mathrm{H}} > \theta_{\mathrm{L}}$ with relatively large θ_{L}.

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2. $\theta_{\mathrm{H}} > \theta_{\mathrm{L}}$ *with relatively small* θ_{L}: As r increases, we observe that $u_{\mathrm{A}}^{\mathrm{U}}$ always dominates $u_{\mathrm{B}}^{\mathrm{U}}$. Since that type-L course is less attractive (small θ_{L}), and $u_{\mathrm{A}}^{\mathrm{U}}$ is concave in q_{L} while $u_{\mathrm{B}}^{\mathrm{U}}$ is convex in q_{L}, the university finds herself optimal to always adopt strategy (A), and determines q_{L}^* accordingly. Obviously, the more the busy learners, the less the university is willing to offer a type-L course. It follows that q_{L}^* decreases in r. However, the platform earns only when the university offer the course. It follows that w_{L}^* increases in r to compensate the university offering type-L course. The observation are shown in Figure 6.6.

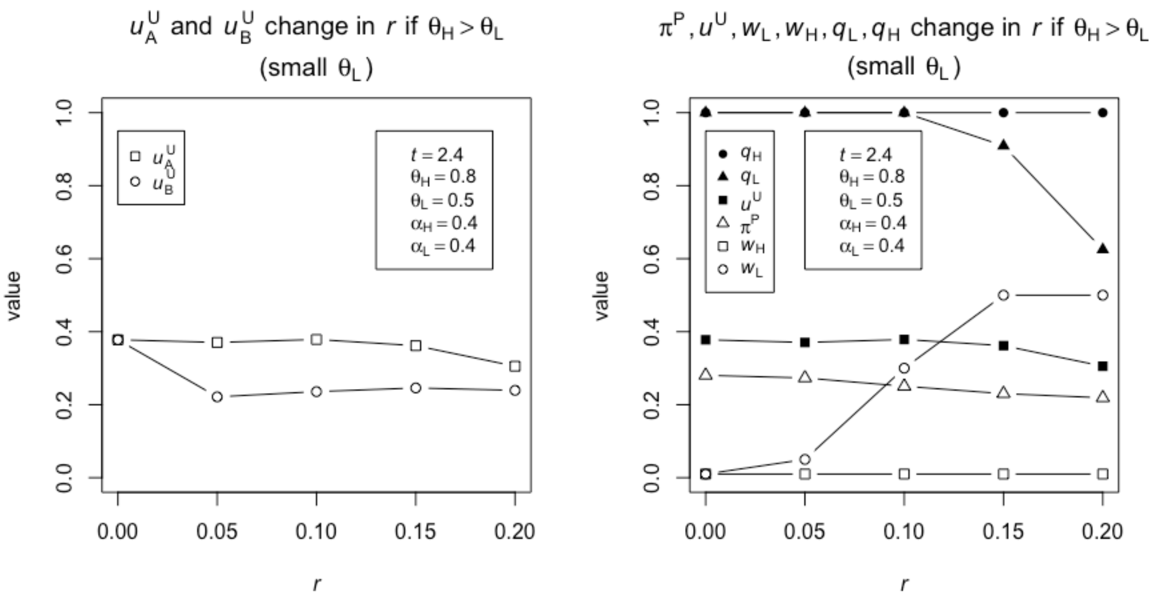

Figure 6.6: Impact of r: $\theta_{\mathrm{H}} > \theta_{\mathrm{L}}$ with relatively small θ_{L}.

3. $\theta_{\mathrm{H}} < \theta_{\mathrm{L}}$: We observe that $u_{\mathrm{B}}^{\mathrm{U}}$ always dominates $u_{\mathrm{A}}^{\mathrm{U}}$. Type-L course is so attractive that the university is always willing to adopt strategy (B), and offer the best possible type-L course even without revenue sharing. It follows that the platform can enjoy all its revenue, and set w_{H}^* and w_{L}^* to zero. Note that there is no significant difference

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in the qualitative results between different θ_{L} as long as $\theta_{\mathrm{H}} < \theta_{\mathrm{L}}$. The observations are shown in Figure 6.7.

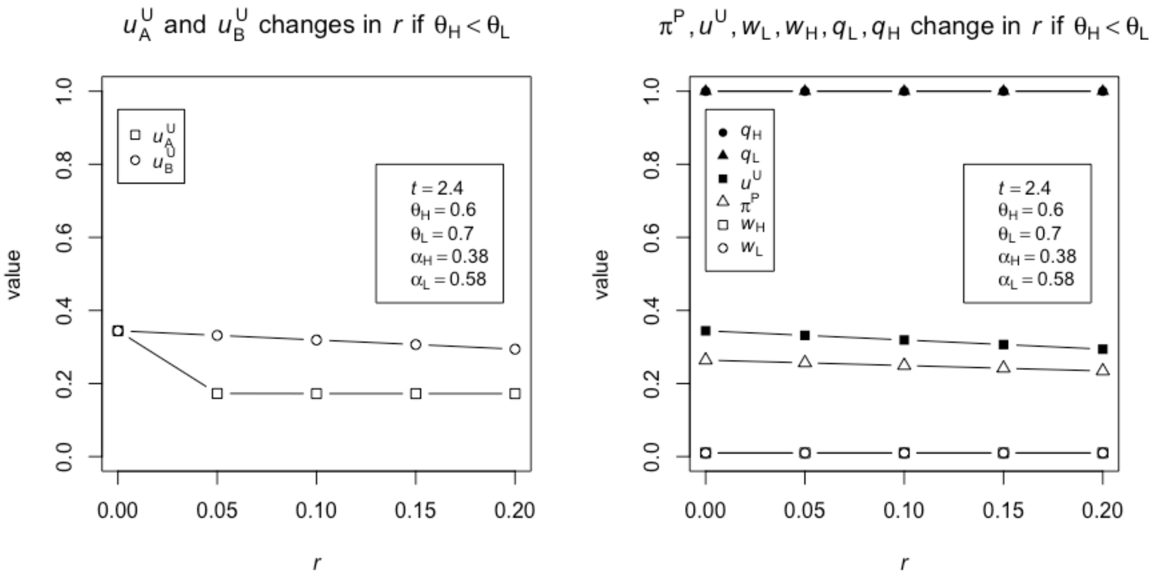

Figure 6.7: Impact of r: $\theta_{\mathrm{H}} < \theta_{\mathrm{L}}$.

Overvall, these observations regarding the impact of r on the university's and the platform's decision variables are shown in Observation 2.

Observation 2. *Consider the extended expansion period where* $\frac{2(1-r)\theta_{\mathrm{L}}}{\alpha_{\mathrm{L}}} < t < \frac{2\theta_{\mathrm{L}}}{\alpha_{\mathrm{L}}}$*:*

(a) *When* $\theta_{\mathrm{H}} > \theta_{\mathrm{L}}$*, the university finds herself optimal to first adopt strategy (A) then strategy (B) under large* θ_{L} *as* r *increases, and always adopts strategy (A) under small* θ_{L} *regardless the change in* r*.*

(b) *When* $\theta_{\mathrm{H}} < \theta_{\mathrm{L}}$*, the university finds herself optimal to always adopt strategy (B) regardless the change in* r*, and the platform enjoy all the certificate revenue.*

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Somewhat surprising, we can observe from the circumstances of $\theta_{\mathrm{H}} < \theta_{\mathrm{L}}$ and $\theta_{\mathrm{H}} > \theta_{\mathrm{L}}$ *with relatively large* θ_{L} that a large θ_{L} can somehow alleviate the disadvantages of limited learners and competition between type-H and type-L course brought by a large proportion of busy learner (large r), and the university is willing to offer the best possible type-L course almost like what basic model refers in the "expansion period". The best possible course is less likely to appear if the willingness-to-pay of type-L course is too small. In short, the university will not put more effort to offer the best possible type-L course when the willingness-to-pay of type-L course is small.

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Chapter 7

Conclusions and future works

7.1 Conclusions

In this paper, we adopt a theoretical framework to investigate an MOOC platform's strategic choice of the certificate prices and the revenue sharing ratios for coordinating supply and demand. By modeling the maximization problems of the platform's profit and the university's utility, the equilibrium quality levels and profits are then derived. Thus, the platform's optimal strategic pricing choices are determined.

In our opinion, we believe that diversity of course type make the world of MOOCs more colorful. Fortunately, we conclude that all types of courses will exist in equilibrium throughout the lifecycle on the MOOC platform in terms of certificate purchasing rates. As the improvement of technologies and the popularity of MOOCs increases, the course quality will increase while the revenue sharing ratio will decreases. We also consider some extensions which take account of the following elements: First, we discuss the competi-

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tion between two heterogeneous universities, we show that the number of courses of the highest possible quality may not always increase as technologies become more mature and MOOCs become more popular due to the potential competition among universities. Moreover, the difference in effort level and reputation between different course types on the platform will lead to the gaps of equilibrium quality level between different types of course. Second, we consider some busy learners who may not have time to take multiple types of courses at the same time to see what will happen if there is competition between different types of courses, and observe that a large willingness-to-pay of low conversion rate course can somehow alleviate the disadvantages of competition between different types of courses brought by the presence of busy learner. When the willingness-to-pay of low conversion rate course is small, the university will decide not to offer the best possible low conversion rate course rather than putting more effort on it, thus, the best possible low conversion rate course is less likely to take place.

7.2 Future works

We may further extend our research into the following directions. First, we only consider one platform in this research. The competition between multiple platforms deserves further investigation. Second, Coursera introduces its subscription business model recently that allows learners to purchase access to all content in a specialization on a periodic basis, pay for the time you actually spend learning, and earn the certificate (Coursera, 2016b). It would be thorough to compare some different pricing strategies, i.e., subscription, membership, or value-added services throughout the lifecycle of MOOCs.

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Theoretical investigations on the impact of these issues may contribute to the literature in future research.

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Appendix A

Proofs of Lemmas and Propositions

Proof of Lemma 1. In start-up period, the utility functions of the university is concave in q_j where

$$u_j^U = n_j q_{ij} + (a_j - b_j p_j) p_j w_j \beta n_j - \frac{\alpha_j q_j^2}{2}. \tag{A.1}$$

Since that $q_j \in [0, 1]$, if the q_j^* is less than or equal to zero, then $q_j^* = 0$; if q_j^* is larger than one, then $q_j^* = 1$; otherwise, after first derivatives,

$$q_j^* = \frac{(a_j - b_j p_j) p_j w_j \beta \theta_j}{\alpha_j t - 2\theta_j}, \tag{A.2}$$

and the second order condition can be verified. Let

$$B = \frac{\alpha_j t - 2\theta_j}{(a_j - b_j p_j) p_j \beta \theta_j} \tag{A.3}$$

We investigate the equilibrium by examining that there is no player can be better off by a unilateral change, and figure out the constraints of w_j respectively. The optimal quality pairs are

$$q_j^* = \begin{cases} q_j^* & \text{if and only if } B > w_j > 0 \\ 1 & \text{if and only if } w_j \geq B \end{cases}. \tag{A.4}$$

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The platform's problem is to decide w_j to maximize its profit function

$$\pi_j^{\mathrm{P}} = \frac{(1-w_j)(a_j - b_j p_j)p_j\theta_j q_j^* + \theta_j q_j^*}{t}. \tag{A.5}$$

Then, we solve the optimal w_j^*, and figure out q_j^* as a function of w_j^*. Let $\frac{\partial \pi_j^{\mathrm{P}}(q_j^*)}{\partial w_j} = 0$, we have $w_j^* = \frac{1}{2}$. We have

$$w_j^* = \begin{cases} \frac{1}{2}, & \text{if } \frac{1}{2} < B \\ B, & \text{if } B \leq \frac{1}{2} \end{cases}$$

as the platform's optimal revenue sharing ratio. The equilibrium qualities are

$$q_j^* = \begin{cases} q_j^*(\frac{1}{2}), & \text{if } \frac{1}{2} < B \\ 1, & \text{if } B \leq \frac{1}{2} \end{cases}$$

where $q_j^*(\frac{1}{2}) < 1$ because q_j^* can be formulated as $\frac{w_j^*}{B}$, we have $\frac{1}{2B} < 1$. □

Proof of Lemma 2. To prove $q_{\mathrm{L}} = 1$ is an equilibrium, we must ensure no one will be better off by a unilateral change of quality from one to zero because the university's utility function is convex in expansion period. Thus, we have $u_j^{\mathrm{U}}(q_j = 1) > u_j^{\mathrm{U}}(q_j = 0)$ which results in $w_j > \frac{\alpha_j t - 2\theta_j}{2(a_j - b_j p_j)p_j\beta\theta_j}$. Then, we examine the feasible region of w_j where $w_j \in [0,1]$ to see if the equilibrium holds. Because the university's utility function is convex in expansion period, the coefficient of the quadratic term of the university's utility is positive; thus, $(\alpha_j t - 2\theta_j) < 0$ holds. Therefore, we have $w_j > 0$ in equilibrium. □

Proof of Proposition 1. Summarize Lemmas 2 and 1 of the change of w_j^* in t. □

Proof of Proposition 2. Summarize Lemmas 2 and 1 of the change of q_j^* in t. □

Proof of Proposition 3. From Lemma 1, we have $q_j^* = q_j(\frac{1}{2})$ if $\frac{1}{2} < B$ in equilibrium in the start-up period. From Lemma 2, we have $q_j^* = 1$ in equilibrium in the expansion

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period. What we need to do is to prove if $q_j(\frac{1}{2}) < 1$ holds. Since that q_j^* can be formulated as $\frac{w_j^*}{B}$, we have $\frac{1}{2B} < 1$ because $\frac{1}{2} < B$. □

Proof of Proposition 4. The utility function of the university can be formulated as Equation 4.1. The utility function is concave in the certificate price p_j. After the first derivatives, we can find that $p_j^* = \frac{a_j}{2b_j}$ is always the case. The second order condition can be verified. □

Proof of Lemma 3. In start-up period, t is big enough ($t > \max\left\{\frac{2_{ij}}{\alpha_{ij}}\right\}$) so that the market is partial covered, and the utility functions of university 1 and university 2 are both concave. The utility function of the university is

$$u_{ij}^U = n_{ij}q_{ij} + (a_j - b_j p_j)p_j w_j \beta_i n_{ij} - \frac{\alpha_{ij} q_{ij}^2}{2} \tag{A.6}$$

Since that $q_{ij} \in [0, 1]$, if the q_{ij}^* is less than or equal to zero, then $q_{ij}^* = 0$; if q_{ij}^* is larger than one, then $q_{ij}^* = 1$; otherwise, after first derivatives,

$$q_{ij}^* = \frac{(a_j - b_j p_j)p_j w_j \beta_i \theta_{ij}}{\alpha_{ij} t - 2\theta_{ij}}, \tag{A.7}$$

and the second order condition can be verified. Let

$$B_1 = \frac{\alpha_{1j} t - 2\theta_{1j}}{(a_j - b_j p_j)p_j \beta_1 \theta_{1j}} \tag{A.8}$$

and

$$B_2 = \frac{\alpha_{2j} t - 2\theta_{2j}}{(a_j - b_j p_j)p_j \beta_2 \theta_{2j}}. \tag{A.9}$$

Since that we assume $\beta_1 = \beta_2$ in this chapter to avoid tedious calculation, and $\frac{\theta_{1j}}{\alpha_{1j}} > \frac{\theta_{2j}}{\alpha_{2j}}$ implies $\frac{\alpha_{2j} t - 2\theta_{2j}}{(a_j - b_j p_j)p_j \beta_2 \theta_{2j}} > \frac{\alpha_{1j} t - 2\theta_{1j}}{(a_j - b_j p_j)p_j \beta_1 \theta_{1j}}$, which is $B_2 > B_1$, without loss of generality. We investigate the equilibrium by examining that there is no player can be better off by a

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unilateral change, and figure out the constraints of w_j respectively. The optimal quality pairs are

$$(q_{1j}^*, q_{2j}^*) = \begin{cases} (q_{1j}^*, q_{2j}^*) & \text{if and only if } B_1 > w_j > 0 \\ (1, q_{2j}^*) & \text{if and only if } B_2 > w_j \geq B_1 \end{cases}. \tag{A.10}$$

The platform's problem is to decide w_j to maximize its profit function

$$\pi_j^{\mathrm{P}} = \frac{(1-w_j)(a_j - b_j p_j) p_j \theta_{1j} q_{1j}^* + \theta_{2j} q_{2j}^*}{t}. \tag{A.11}$$

Then, we investigate the optimal w_j^*, and figure out q_{ij}^* as a function of w_j^*. First, let $\frac{\partial \pi_j^{\mathrm{P}}(q_{1j}^*, q_{2j}^*)}{\partial w_j} = 0$, we have $w_j^* = \frac{1}{2}$. If w_j^* follows $B_1 \geq w_j > 0$, which is $B_1 > \frac{1}{2}$,

$$\begin{cases} w_j^* = \frac{1}{2} \\ q_{1j}^* = q_{1j}\left(\frac{1}{2}\right) < 1 \\ q_{2j}^* = q_{2j}\left(\frac{1}{2}\right) < 1 \end{cases}. \tag{A.12}$$

Second, let $\frac{\partial \pi_j^{\mathrm{P}}(1, q_{2j}^*)}{\partial w_j} = 0$, we have $w_j^* = \frac{1}{2} - \frac{\theta_{1j} B_2}{2\theta_{2j}}$. If w_j^* does not follow $B_2 > w_j^* \geq B_1$, which is $\frac{1}{2} \geq B_1 > \frac{1}{2} - \frac{\theta_{1j} B_2}{2\theta_{2j}}$

$$\begin{cases} w_j^* = B_1 \\ q_{1j}^* = 1 \\ q_{2j}^* = q_{2j}^*(B_1) < 1 \end{cases}. \tag{A.13}$$

Third, if w_j^* follows $B_2 > w_j^* \geq B_1$, which is $\frac{1}{2} - \frac{\theta_{1j} B_2}{2\theta_{2j}} > B_1$, we have $w_j^* = \frac{1}{2} - \frac{\theta_{1j} B_2}{2\theta_{2j}}$.

$$\begin{cases} w_j^* = \frac{1}{2} - \frac{\theta_{1j} B_2}{2\theta_{2j}} \\ q_{1j}^* = 1 \\ q_{2j}^* = q_{2j}^*\left(\frac{1}{2} - \frac{\theta_{1j} B_2}{2\theta_{2j}}\right) < 1 \end{cases}. \tag{A.14}$$

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It can be proved that $q^*_{2j} < 1$ in all three cases and $q^*_{1j} < 1$ if $B_1 > \frac{1}{2}$. From Formula (A.7), we know that q^*_{2j} can be denoted as $\frac{w^*_j}{B_2}$, and we have $B_1 < B_2$ in this chapter. Thus, in the first case, we have $\frac{w^*_j}{B_2} = \frac{1}{2B_2} < 1$ because $\frac{1}{2} < B_1 < B_2$; in the second case, we have $\frac{w^*_j}{B_2} = \frac{B_1}{B_2} < 1$; in the third case, we have $\frac{w^*_j}{B_2} = \frac{1}{B_2}\left(\frac{1}{2} - \frac{\theta_{1j}B_2}{2\theta_{2j}}\right) < 1$ because $\frac{1}{2} - \frac{\theta_{1j}B_2}{2\theta_{2j}} < B_1 < B_2$. Similarly, q^*_{1j} can be denoted as $\frac{w^*_j}{B_1}$, we have $\frac{w^*_j}{B_1} = \frac{1}{2B_2} < 1$ because $\frac{1}{2} < B_1 < B_2$ in the first case. □

Proof of Lemma 4. In growth period, we have $t \in \left(\min\left\{\frac{2\theta_{ij}}{\alpha_j}\right\}, \max\left\{\frac{2\theta_{ij}}{\alpha_j}\right\}\right)$. Let $\frac{\theta_{1j}}{\alpha_{1j}} > \frac{\theta_{2j}}{\alpha_{2j}}$ without loss of generality, the utility function of university 1 is convex and that of university 2 is concave. After first derivatives, we have $q^*_{2j} = \frac{(a_j - b_j p_j)p_j w_j \beta_2 \theta_{2j}}{\alpha_{2j}t - 2\theta_{2j}}$. Since that $q_{ij} \in [0, 1]$, if the q^*_{2j} is less than or equal to zero, then the $q^*_{2j} = 0$; if the q^*_{2j} is larger than one, then the $q^*_{2j} = 1$; otherwise, after first derivatives, $q^*_{2j} = \frac{(a_j - b_j p_j)p_j w_j \beta_2 \theta_{2j}}{\alpha_{2j}t - 2\theta_{2j}}$. Since that we assume $\beta_1 = \beta_2$ in this chapter to avoid tedious calculation, and $\frac{\theta_{1j}}{\alpha_{1j}} > \frac{\theta_{2j}}{\alpha_{2j}}$ implies

$$\frac{\alpha_{1j}t - 2\theta_{1j}}{(a_j - b_j p_j)p_j \beta_1 \theta_{1j}} < \frac{\alpha_{2j}t - 2\theta_{2j}}{(a_j - b_j p_j)p_j \beta_2 \theta_{2j}} \tag{A.15}$$

without loss of generality. Let

$$C_1 = \frac{\alpha_{1j}t - 2\theta_{1j}}{(a_j - b_j p_j)p_j \beta_1 \theta_{1j}} \tag{A.16}$$

and

$$C_2 = \frac{\alpha_{2j}t - 2\theta_{2j}}{(a_j - b_j p_j)p_j \beta_2 \theta_{2j}}. \tag{A.17}$$

We investigate the equilibrium by examining that there is no player can be better off by a unilateral change, and figure out the constraints of w_j respectively. For q^*_{1j}, since that the utility function of university 1 is convex, the candidates of q^*_{1j} are zero or one. And because $u^{\mathrm{U}}_{1j}(q_{1j} = 1) > u^{\mathrm{U}}_{1j}(q_{1j} = 0)$ holds, for any $w_j \in [0, 1]$, the only equilibrium is

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$q_{1j}^* = 1$. For q_{2j}^*, since that the utility function of university 2 is concave, we can have the optimal quality after first derivatives. Therefore, the optimal quality pairs are

$$(q_{1j}^*, q_{2j}^*) = \begin{cases} (1, q_{2j}^*) = (1, \frac{(a_j - b_j p_j) p_j w_j \beta_2 \theta_{2j}}{\alpha_{2j} t - 2\theta_{2j}}) & \text{if and only if } w_j < C_2 \\ (1, q_{2j}^*) = (1, 1) & \text{if and only if } w_j \geq C_2 \end{cases}. \quad \text{(A.18)}$$

The platform's problem is to decide w_j to maximize its profit function

$$\pi_j^{\mathrm{P}} = (1 - w_j)(a_j - b_j p_j) p_j \frac{\theta_{1j} q_{1j}^* + \theta_{2j} q_{2j}^*}{t}. \quad \text{(A.19)}$$

Then, we investigate the optimal w_j^*, and figure out q_{ij}^* as a function of w_j^*. First, let $\frac{\partial \pi_j^{\mathrm{P}}(1, q_{2j}^*)}{\partial w_j} = 0$, we have $w_j^* = \frac{1}{2} - \frac{\theta_{2j} C_2}{2\theta_{1j}}$. If w_j^* follows $w_j < C_2$, which is $\frac{1}{2} - \frac{\theta_{2j} C_2}{2\theta_{1j})} < C_2$,

$$\begin{cases} w_j^* = \frac{1}{2} - \frac{\theta_{2j} C_2}{2\theta_{1j}} \\ q_{1j}^* = 1 \\ q_{2j}^* = q_{2j}\left(\frac{1}{2} - \frac{\theta_{2j} C_2}{2\theta_{1j}}\right) < 1 \end{cases}. \quad \text{(A.20)}$$

Second, if $C_2 \leq \frac{1}{2} - \frac{\theta_{2j} C_2}{2\theta_{1j}}$, we have $w_j^* = C_2$.

$$\begin{cases} w_j^* = C_2 \\ q_{1j}^* = 1 \\ q_{2j}^* = 1 \end{cases}. \quad \text{(A.21)}$$

Notice that the optimal quality pair $(1, 1)$ exits if and only if $\frac{8b_j(\theta_{1j}\alpha_{2j} - \theta_{2j}\alpha_{1j})}{a_j^2 \beta_2 \theta_{2j} \alpha_{1j}} < \frac{1}{2} - \frac{\theta_{2j} C_2}{2\theta_{1j}}$ because $t \in \left(\min\left\{\frac{2\theta_{ij}}{\alpha_{ij}}\right\}, \max\left\{\frac{2\theta_{ij}}{\alpha_{ij}}\right\}\right)$ implies $0 < C_2 < \frac{8b_j(\theta_{1j}\alpha_{2j} - \theta_{2j}\alpha_{1j})}{a_j^2 \beta_2 \theta_{2j} \alpha_{1j}}$, and the feasible region must lie in growth period boundary. □

Proof of Lemma 5. To prove $(1, 1)$ is an equilibrium, we must ensure no one will be better off by a unilateral change of quality from one to zero. Thus, we have $u_{1j}^{\mathrm{U}}(q_{1j} =$

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$1) > u^{\mathrm{U}}_{1j}(q_{1j}=0)$ which results in $w_j > \frac{\alpha_{1j}t-2\theta_{1j}}{2(a_j-b_jp_j)p_j\beta_1\theta_{1j}}$, and $u^{U}_{1j}(q_{1j}=1) > u^{\mathrm{U}}_{1j}(q_{1j}=0)$ which results in $w_j > \frac{\alpha_{2j}t-2\theta_{2j}}{2(a_j-b_jp_j)p_j\beta_2\theta_{2j}}$. Then, we examine the feasible region of w_j where $w_j \in [0,1]$ to see if the equilibrium holds. Because the university's utility function is convex in expansion period, the coefficient of the quadratic term of the university's utility is positive; thus, $(\alpha_{ij}t - 2\theta_{ij}) < 0$ holds. Therefore, the right-hand side of the two w_j constraints mentioned above are all negative, so we have $w_j > 0$ in equilibrium. Similarly, we can prove the other corner solutions are not equilibria. Since that the university's utility function is convex, and the market is partial covered, only $(0,0)$, $(0,1)$ and $(1,0)$ are possible corner solutions. Similarly, we can prove that the three are not equilibria. □

Proof of Lemma 6. To prove $(1, \frac{t-\theta_{1j}}{\theta_{2j}})$ is an equilibrium in mature period, we must ensure no one will be better off by a unilateral change. Thus, we require $u^{\mathrm{U}}_{2j}(1, \frac{t-\theta_{1j}}{\theta_{2j}}) > u^{\mathrm{U}}_{2j}(1,0)$, which is $w_j > \frac{(\alpha_{2j}t-2\theta_{2j})(t-\theta_{1j})}{2\theta^2_{2j}(a_j-b_jp_j)p_j\beta_2}$; and $u^{\mathrm{U}}_{2j}(1, \frac{t-\theta_{1j}}{\theta_{2j}}) > u^{\mathrm{U}}_{2j}(1,1)$ which is $w_j > \frac{(\theta^2_{2j}-(t-\theta_{2j})^2)(\theta_{1j}-\alpha_{1j}t)-(t-\theta_{2j})(\theta_{1j}-(t-\theta_{1j}-\theta_{2j}))}{\theta^2_{2j}(t-\theta_{1j}-\theta_{2j})(a_j-b_jp_j)p_j\beta_2}$. Then, we examine the feasible region of w_j where $w_j \in [0,1]$ to see if the equilibrium holds. Because the university's utility function is convex and $\theta_{ij} < t \le \theta_{1j}+\theta_{2j}$ in mature period, the coefficient of the quadratic term of the university's utility is positive; thus, $(\theta_{1j} - \alpha_{1j}t) > 0$ holds, the right-hand side of the two w_j constraints mentioned above are all negative, we have $w_j > 0$ in equilibrium. Similarly, we can prove that $(\frac{t-\theta_{2j}}{\theta_{1j}}, 1)$ is the other equilibrium and the rest of the corner solutions are not equilibria.

As for the solutions along the line between $(1, \frac{t-\theta_{1j}}{\theta_{2j}})$ and $(\frac{t-\theta_{2j}}{\theta_{1j}}, 1)$, given the two known equilibrium $(\frac{t-\theta_{2j}}{\theta_{1j}}, 1)$ and $(1, \frac{t-\theta_{1j}}{\theta_{2j}})$, we assume a point P $(\lambda + (1-\lambda)\frac{t-\theta_{2j}}{\theta_{1j}}, \lambda\frac{t-\theta_{1j}}{\theta_{2j}} + (1-\lambda))$ for all $\lambda \in (0,1)$ which is a linear combination of the two equilibria. Suppose P is an equi-

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librium. We have $u^{\mathrm{U}}_{2j}(q_{1\mathrm{P}}, q_{2\mathrm{P}}) > u^{\mathrm{U}}_{2j}(q_{1\mathrm{P}}, 1)$ and $u^{\mathrm{U}}_{1j}(q_{1\mathrm{P}}, q_{2\mathrm{P}}) > u^{\mathrm{U}}_{1j}(1, q_{2\mathrm{P}})$ to ensure that no one can be better off by a unilateral change. After some algebra, $u^{\mathrm{U}}_{2j}(q_{1\mathrm{P}}, q_{2\mathrm{P}}) > u^{\mathrm{U}}_{2j}(q_{1\mathrm{P}}, 1)$ leads to $\lambda > \frac{\theta_{2j}}{(1-\frac{(t-\theta_{1j})}{\theta_{2j}})(\theta_{2j}-\alpha_{2j}t)t+\theta_{1j}(1-\frac{(t-\theta_{2j})}{\theta_{1j}})}$, and similarly $u^{\mathrm{U}}_{1j}(q_{1\mathrm{P}}, q_{2\mathrm{P}}) > u^{\mathrm{U}}_{1j}(q_{1\mathrm{P}}, 1)$ leads to $\lambda > \frac{\theta_{1j}}{(1-\frac{(t-\theta_{2j})}{\theta_{1j}})(\theta_{1j}-\alpha_{1j}t)t+\theta_{2j}(1-\frac{(t-\theta_{1j})}{\theta_{2j}})}$. Let $(1 - \frac{(t-\theta_{1j})}{\theta_{2j}})(\theta_{2j} - \alpha_{2j}t)t + \theta_{1j}(1-\frac{(t-\theta_{2j})}{\theta_{1j}})$ to be A and $(1-\frac{(t-\theta_{2j})}{\theta_{1j}})(\theta_{1j}-\alpha_{1j}t)t+\theta_{2j}(1-\frac{(t-\theta_{1j})}{\theta_{2j}})$ to be B. To disprove P is an equilibrium for all $\lambda \in (0, 1)$, we require $\lambda+(1-\lambda) > 1$, which is $\theta_{1j}A+\theta_{2j}B > BA$. By the arithmetic-geometric mean inequality, we have $\theta_{1j}A + \theta_{2j}B \geq 2\sqrt{(\theta_{1j}\theta_{2j}AB)}$, which is $4 > \frac{AB}{\theta_{1j}\theta_{2j}}$. Let $\frac{A}{\theta_{1j}} = \frac{\theta_{1j}+\theta_{2j}-t}{\theta_{1j}}\left(1 + \frac{(\theta_{2j}-\alpha_{2j}t)t}{\theta_{2j}}\right)$ and $\frac{B}{\theta_{2j}} = \frac{(\theta_{1j}+\theta_{2j}-t)}{\theta_{2j}}\left(1 + \frac{(\theta_{1j}-\alpha_{1j}t)t}{\theta_{1j}}\right)$. Because $\theta_{ij} < t \leq \theta_{1j} + \theta_{2j}$ in the mature period, we have $\frac{(\theta_{1j}+\theta_{2j}-t)}{\theta_{ij}} \in (0, 1)$ and $\left(1 + \frac{(\theta_{1j}+\theta_{2j}-t)}{\theta_{ij}}\right) \in (0, 2)$. Thus, $4 > \frac{AB}{(\theta_{1j}\theta_{2j})}$ is derived, and $\theta_{1j}A + \theta_{2j}B > BA$ holds. We prove that all the linear combinations of the two known equilibria are not equilibria. The optimal quality pair is $(1, \frac{t-\theta_{1j}}{\theta_{2j}})$ or $(\frac{t-\theta_{2j}}{\theta_{1j}}, 1)$ in equilibrium, and the market is exactly fully covered. □

Proof of Proposition 5. Summarize Lemmas 3 to 6 of the change of q^*_j in t. □

Proof of Proposition 6. In start-up period, the transition of feasible boundary of w^*_j between $(1, q^*_{2j})$ and (q^*_{1j}, q^*_{2j}) is $\frac{1}{2} = B_1$, which leads to $t = \frac{a_j^2\beta_1\theta_{1j}+16b_j\theta_{1j}}{8b_j\alpha_{1j}}$. So the optimal quality pair (q^*_{1j}, q^*_{2j}) exists when $t > \frac{a_j^2\beta_1\theta_{1j}+16b_j\theta_{1j}}{8b_j\alpha_{1j}}$. In growth period, let $C_2 = \frac{\alpha_{2j}t-2\theta_{2j}}{(a_j-b_jp_j)p_j\beta_2\theta_{2j}}$.. The optimal quality pair $(1, 1)$ exists if $w_j \geq C_2$, which leads to $\frac{8b_j(\theta_{1j}\alpha_{2j}-\theta_{2j}\alpha_{1j})}{(a_j^2\beta_2\theta_{2j}\alpha_{1j})} < \frac{1}{2} - \frac{\theta_{2j}C_2}{(2\theta_{1j})}$ Thus, there are two optimal qualities equal to one if $\frac{8b_j(\theta_{1j}\alpha_{2j}-\theta_{2j}\alpha_{1j})}{(a_j^2\beta_2\theta_{2j}\alpha_{1j})} < \frac{1}{2} - \frac{\theta_{2j}C_2}{(2\theta_{1j})}$; otherwise, only an optimal quality equal to one. □

Proof of Proposition 7. If $\theta_{ij} = \theta$ for all $i \in 1, 2, j \in \mathrm{L}, \mathrm{H}$ and $\alpha_{i\mathrm{L}} > \alpha_{i\mathrm{H}}$ for $i \in 1, 2$, the cut-offs follow $\frac{2\theta_{i\mathrm{L}}}{\alpha_{i\mathrm{L}}} < \frac{2\theta_{i\mathrm{H}}}{\alpha_{i\mathrm{H}}}$, so there exists $t > \min_{i\in 1,2} \frac{2\theta}{\alpha_{i\mathrm{L}}}$ such that the optimal quality of type-H is $(1, 1)$, while type-L is $(1, q^*_{2j})$ or $(q^*_{1j}, 1)$. Moreover, if $a_{\mathrm{H}} > a_{\mathrm{L}}$ and

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$b_{\mathrm{H}} \leq b_{\mathrm{L}}$, there exists $t > \frac{a_{\mathrm{L}}^2 \beta_1 \theta_{1\mathrm{L}} + 16 b_{\mathrm{L}} \theta_{1\mathrm{L}}}{8 b_{\mathrm{L}} \alpha_{1\mathrm{L}}}$ such that the optimal quality of type-H is $(1, q_{2j}^*)$ or $(q_{1j}^*, 1)$, while type-L is (q_{1j}^*, q_{2j}^*). □

Proof of Proposition 8. If $\alpha_{ij} = \alpha$ for all $i \in 1,2, j\mathrm{L}, \mathrm{H}$ and $\theta_{i\mathrm{H}} > \theta_{i\mathrm{L}}$ for $i \in 1,2$, the cut-off follow $_{1\mathrm{L}} + \theta_{2\mathrm{L}} < \theta_{1\mathrm{H}} + \theta_{2\mathrm{H}}$, there exists $t > \theta_{1\mathrm{L}} + \theta_{2\mathrm{L}}$ such that the optimal quality of type-H is $(1, q_{2j}^*)$ or $(q_{1j}^*, 1)$, while type-L is $(1,1)$; the cut-off also follow $\frac{2\theta_{i\mathrm{L}}}{\alpha_{i\mathrm{L}}} < \frac{2\theta_{i\mathrm{H}}}{\alpha_{i\mathrm{H}}}$, so there exists $t > \min_{i \in 1,2} \frac{2\theta}{\alpha_{i\mathrm{L}}}$ such that the optimal quality of type-H is $(1,1)$, while type-L is $(1, q_{2j}^*)$ or $(q_{1j}^*, 1)$. Moreover, if $a_{\mathrm{H}} > a_{\mathrm{L}}$ and $b_{\mathrm{H}} \leq b_{\mathrm{L}}$, there exists $t > \frac{a_{\mathrm{L}}^2 \beta_1 \theta_{1\mathrm{L}} + 16 b_{\mathrm{L}} \theta_{1\mathrm{L}}}{8 b_{\mathrm{L}} \alpha_{1\mathrm{L}}}$ such that the optimal quality of type-H is $(1, q_{2j}^*)$ or $(q_{1j}^*, 1)$, while type-L is (q_{1j}^*, q_{2j}^*). □

Proof of Lemma 7. Consider Equation 6.1, the university's utility function is concave in $q\ _{mathrmL}$ under strategy (A), which is $\theta_{\mathrm{H}} q_{\mathrm{H}} \geq \theta_{\mathrm{L}} q_{\mathrm{L}}$. Since that $q_j \in [0,1]$, if the q_j^* is less than or equal to zero, then $q_j^* = 0$; if q_j^* is larger than one, then $q_j^* = 1$; otherwise, after first derivatives, $q_j^* = \frac{(a_{\mathrm{L}} - b_{\mathrm{L}} p_{\mathrm{L}}) p_{\mathrm{L}} w_{\mathrm{L}} \beta \theta_{\mathrm{L}}}{\alpha_{\mathrm{L}} t - 2(1-r)\theta_{\mathrm{L}}}$. Let $E_{\mathrm{L}} = \frac{(a_{\mathrm{L}} - b_{\mathrm{L}} p_{\mathrm{L}}) p_{\mathrm{L}} w_{\mathrm{L}} \beta \theta_{\mathrm{L}}}{\alpha_{\mathrm{L}} t - 2(1-r)\theta_{\mathrm{L}}}$. Consider the period where $\frac{2(1-r)\theta_{\mathrm{L}}}{\alpha_{\mathrm{L}}} < t < \frac{2\theta_{\mathrm{L}}}{\alpha_{\mathrm{L}}}$. We investigate the equilibrium by examining that there is no player can be better off by a unilateral change, and figure out the constraints of strategy (A) and strategy (B) respectively. If $\theta_{\mathrm{H}} \geq \theta_{\mathrm{L}}$: Under strategy (A), $\theta_{\mathrm{H}} q_{\mathrm{H}} \geq \theta_{\mathrm{L}} q_{\mathrm{L}}$, the optimal quality pair $(q_{\mathrm{H}}^*, q_{\mathrm{L}}^*) = (1, \min\{1, E_{\mathrm{L}}\})$. Under strategy (B), $\theta_{\mathrm{H}} q_{\mathrm{H}} < \theta_{\mathrm{L}} q_{\mathrm{L}}$, the optimal quality pair $(q_{\mathrm{H}}^*, q_{\mathrm{L}}^*) = (\frac{\theta_{\mathrm{L}}}{\alpha_{\mathrm{H}}}, 1)$ because $\theta_{\mathrm{H}} q_{\mathrm{H}} < \theta_{\mathrm{L}} q_{\mathrm{L}}$ is a binding constraints for q_{L}. The university adopts strategy (A) if and only if $u_{\mathrm{A}}^{\mathrm{U}}(1, \min\{1, E_{\mathrm{L}}\}) > u_{\mathrm{B}}^{\mathrm{U}}\left(\frac{\theta_{\mathrm{L}}}{\alpha_{\mathrm{H}}}, 1\right)$, and vice versa. Similarly, if $\theta_{\mathrm{H}} < \theta_{\mathrm{L}}$: Under strategy (A), $\theta_{\mathrm{H}} q_{\mathrm{H}} \geq \theta_{\mathrm{L}} q_{\mathrm{L}}$, the optimal quality pair $(q_{\mathrm{H}}^*, q_{\mathrm{L}}^*) = (1, \min\{\frac{\theta_{\mathrm{H}}}{\alpha_{\mathrm{L}}}, E_{\mathrm{L}}\})$ because $\theta_{\mathrm{H}} q_{\mathrm{H}} \geq \theta_{\mathrm{L}} q_{\mathrm{L}}$ is a binding constraints for q_{L}. Under strategy (B), $\theta_{\mathrm{H}} q_{\mathrm{H}} < \theta_{\mathrm{L}} q_{\mathrm{L}}$, the optimal quality pair $(q_{\mathrm{H}}^*, q_{\mathrm{L}}^*) = (1,1)$. The university adopts strategy (A) if and only if $u_{\mathrm{A}}^{\mathrm{U}}(1, \min\{1, E_{\mathrm{L}}\}) < u_{\mathrm{B}}^{\mathrm{U}}\left(\frac{\theta_{\mathrm{L}}}{\alpha_{\mathrm{H}}}, 1\right)$, and vice versa. The overall best responses of university are summarized in Lemma 7. □

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