國立臺灣大學管理學院資訊管理研究所

碩士論文

Department of Information Management

College of Management

National Taiwan University

Master Thesis

不完整資訊下動態頻寬交易之最佳定價方法

Optimal Pricing for Dynamic Bandwidth Trading with Incomplete Information

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中華民國 一百 年 八 月

August, 2011

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本論文係提交國立台灣大學
資訊管理學研究所作為完成碩士
學位所需條件之一部份

研究生：呂明龍　撰

中華民國一百年八月

國立臺灣大學碩士學位論文

口試委員會審定書

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本論文係呂明龍君（學號 R98725002）在國立臺灣大學資訊管理學系、所完成之碩士學位論文，於民國一百年七月二十七日承下列考試委員審查通過及口試及格，特此證明

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系主任、所長　李瑞庭

謝　詞

本論文得以順利完成，首先要感謝恩師孫雅麗教授。在論文的撰寫中，老師不斷的強調要寫出讓讀者都能看得懂的論文，訓練我將腦袋中想法轉化為文字的能力，在這學習過程中，我學到了該如何以讀者的角度來看待論文的不足之處，這讓我的論文更加具有可讀性。同時老師在其他領域的淵博知識，更指出了在宏觀上一個學術研究者所應具備的眼光，令我受益良多。也要感謝陳孟彰老師的指點，讓我在研究的過程中，學到如何先專注在問題本身，然後才專注在要用哪種方式去解決，這清晰的架構簡化了如何表達清楚自己意見的難度。此外，特別感謝蔡志宏教授，林宗男教授以及潘育群學長在口試時給予的寶貴意見，讓本論文更加完備，在此謹致最誠摯的謝意。

另外要感謝的是ANTS Lab的學長姐、同學、學弟妹們，感謝優秀的學長姐在我陷入瓶頸時給予指點，感謝一起奮鬥的同學們平時討論時給予的珍貴想法，感謝可靠的學弟妹在許多地方給予的協助，支持我完成本論文，讓我這兩年是過得十分充實。

最後要感謝我的家人們，父母給予的學習經驗，如何處理生活、學業上的眾多瑣事，兄弟姊妹時不時與我聊天，放鬆我的心情，我能夠安心寫作本論文都多虧了家人們的關懷，失去了您們的關懷，我想必沒辦法專心在學業上，謝謝您們！

呂明龍　謹識

于台大資訊管理學研究所

中華民國一百年八月

論 文 摘 要

論文題目：不完整資訊下動態頻寬交易之最佳定價方法

作者：呂明龍　　　　　　　　　　　　一百年八月

指導教授：孫雅麗　博士

頻寬是很稀少且珍貴的資源。為了增進頻寬使用效率，解決原先使用方法的低效率，感知無線電(cognitive radio)以及動態頻譜分配(dynamic spectrum allocation)的概念被提了出來。在此篇論文，我們考慮一個由單一 mobile network operator (MNO) 以及眾多有著不同類別(type)的 mobile virtual network operators (MVNOs) 組成的無線網路。我們以下提供一個由兩個階段組成的開放式動態頻寬交易模型來讓 MNO 將頻寬販賣給MVNOs。

這個開放式動態頻寬交易模型的第一個階段的目的是在一連串MNO與MVNOs的互動中，去找到參與的MVNOs的購買意願或者他們的類別，並且計算出要被販賣的頻寬的最佳價目表。計算最佳價目表的同時也會考慮到MVNOs的需求價格函數以及效用函數。最重要的是，最佳價目表必須滿足誘因相符性(incentive compatible, IC) 以及個體理性(individually rational, IR)的限制。前者確保了為某個類別的MVNO設計的數量-價格組能給該MVNO帶來最大的效用；後者確保了為其設計的數量-價格組可以給其非零的效用。我們同時也提供了一個將連續的最佳價目表轉成離散形式，以提供一個比較容易閱讀的格式；此時每個MVNO都會去選擇最靠近其在連續最佳價目表中類別的數量-價格組。在反覆進行的互動收斂且停止之後，如果全部的需求超出了可以提供的頻寬，那麼此模型就會使用背包問題的解法來將頻寬分配給一部分的MVNOs，已使得分配出去的頻寬不會超出可提通頻寬的限制。最後，我們用一個例子來說明這個開放式動態頻寬交易模型是如何運作的。

關鍵詞：動態頻譜分配、誘因相符性、頻寬交易、最佳價目、定價方法、有限頻寬分配

THESIS ABSTRACT

Optimal Pricing for Dynamic Bandwidth Trading with Incomplete Information

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AUGUST 2011
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The wireless spectrum is a limited resource. The concepts of cognitive radio and dynamic spectrum allocation (DSA) have been considered as a possible mechanism to improve the efficiency of bandwidth usage and solve the bandwidth deficiency problem. In this work, we consider a wireless network access environment comprised of a mobile network operator (MNO) and a distribution of different types of mobile virtual network operators (MVNOs). We propose an open dynamic bandwidth trading model that comprises of two phases. The goal of the phase one is to find out the distribution of the buying preferences or types of the participating MVNOs through a sequence of interactive rounds and compute the optimal price schedule for the unused bandwidth for sale. The derivation of the optimal price schedule also considers the demand and utility functions of the MVNOs. Most importantly, the optimal price schedule satisfies the incentive compatible (IC) and the individually rational (IR) constraints. The former ensures that the quantity-price pair designed for MVNO of a specific type will choose the pair that maximizes its utility; while the latter assures that the pairs cause non-negative utility. We also give an algorithm to convert the continuous optimal price schedule to a discrete one so as to provide a simple easy-to-read format for MVNOs' selection while ensuring that individual type of MVNOs will choose the pair whose corresponding utility value is closest to the value in the original function. After the iterative process converges and terminates, if the total number of bandwidth requests exceeds the total capacity constraint, the process proceeds to address the finite capacity constraint by solving a bounded knapsack problem for final bandwidth allocation. Lastly, an example is provided to explain how the proposed open dynamic bandwidth trading process with optimal incentive-compatible price schedule is derived.

keywords： dynamic spectrum sharing; incentive-compatible pricing; bandwidth trading; optimal price schedule; finite bandwidth sharing

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Chapter 1 Introduction

Since spectrum is a limited and therefore precious resource, how to achieve efficient and fair spectrum sharing and allocation among different demand groups is an important issue. Traditional spectrum allocation schemes adopt the long-term lease business model, which assigns different wireless technologies with a static amount of bandwidth to different frequency band in order to prevent interference between them. A study sponsored by the US Federal Communications Commission observed that the traditional fixed spectrum assignment model, which results in over-allocation of spectrum to some operators and applications as well as inefficient bandwidth usage, is unable to meet the growth in demand arising from today's wireless technologies [1].

One possible way to overcome the above limitations is to allow periodic trading of dynamic unused bandwidth by licensed spectrum owners, who sublet their surplus resources to service providers that need bandwidth for a short period. In this work, we consider a wireless network access environment comprised of a mobile network operator (MNO) and various mobile virtual network operators (MVNOs) of different buying preferences or types. It is assumed that an MNO is a telecommunications company that owns a frequency license and mobile infrastructure, and also provides services for mobile phone subscribers. An MVNO is a company that provides mobile phone/network services, but it does not have a frequency license or a mobile infrastructure. However, it does have access to a niche market, which an MNO finds hard to enter. The spectrum owner (MNO) periodically offers its unused bandwidth on the open market for short-term lease to MVNOs that need extra bandwidth to meet their service needs and business goals.

In dynamic bandwidth trading, how to allocate and price bandwidth among different demand groups of potential buyers are two fundamental issues. To address the issues, we propose an open dynamic bandwidth trading model that comprises of two phases. The goal of the phase one is to find out the type distribution of the participating MVNOs through a sequence of interactive rounds between an MNO and MVNOs. In each round, the MNO first revises its estimate of the distribution of the types of the participating MVNOs based on their selections submitted in the previous round. It then re-computes the optimal price schedule and announces it to the MVNOs for a new round of selection. The derivation of the optimal price schedule considers the distribution of the types of MVNOs as well as their demand and utility functions. The optimal price schedule comprises a number of pairs, each representing the optimal bandwidth quantity and the associated price designed especially for a type θ MVNO. Since the schedule is computed based on the assumption that all MVNOs are rational, each MVNO will select the quantity-price pair designed for its type in order to maximize its utility function, and the profit of the resource owner (MNO).

Through each round of interaction, the MNO learns more about the MVNOs that are interested in purchasing extra bandwidth. As a result, the MNO revises its estimate of the cumulative distribution function of the types of MVNOs to derive the optimal price schedule for the sale of the unused bandwidth. We also present an algorithm to convert the continuous optimal price schedule to a discrete form so as to provide a simple easy-to-read format for MVNOs' selection. In the second phase, we consider the finite capacity constraint in bandwidth trading. After the process converges and terminates, if the total demand for bandwidth exceeds the total capacity, we resolve the bandwidth contention problem by mapping it to a bounded knapsack problem.

The proposed open dynamic bandwidth trading model creates a win-win situation for the profit-maximizing MNO and individual MVNOs that need extra bandwidth. We believe the model achieves better spectrum utilization and meets the business goals of MNOs and MVNOs.

The remainder of the paper is organized as follows. Chapter 2 contains a review of related works. In Chapter 3, the system model of the buyers (MVNOs) and the seller (MNO) engaged in open dynamic bandwidth trading is given. In Chapter 4, we present the derivation of the continuous optimal price schedule, and the algorithm to convert the continuous optimal schedule to a discrete form to provide a simple easy-to-read format for MVNOs' selection. In Chapter 5, the proposed open dynamic interactive bandwidth trading process of the MNO and MVNOs is described in detail. We also provide an example to illustrate the key designs of the proposed open dynamic bandwidth trading process. Chapter 6 contains some concluding remarks.

Chapter 2 Related Work

A number of system models have been proposed for bandwidth allocation and pricing in wireless environments. They address the problem by considering different combinations of service providers, customers, and government regulations. Moreover, they use various concepts, such as auction [2][3][4], game theory [5][6][7][8], or economic analysis [9][10], to tackle the problem. In these works, spectrum/bandwidth buyers are typically modeled by using some simple parameters, e.g., the maximum budget or the maximum amount of money that buyers are willing to pay for a certain amount of bandwidth. In this work, we model the behavior of an MVNO by its type, demand curve and utility function to capture the essential characteristics of a potential bandwidth buyer.

Among all the methods used for spectrum management, auction is the most popular approach. For example, under the method proposed in [2], a spectrum manager periodically auctions short-term spectrum licenses to multiple CDMA network operators with the goal of maximizing revenue. Each operator determines its own price based on the amount that users are willing to bid. In [3], two auction mechanisms are proposed for spectrum sharing by a number of users based on the signal to interference-plus-noise ratio (SINR) and received power. The authors of [4] introduce a spectrum auction framework that formulates conflict-free spectrum allocation between users as an optimization problem. They also discuss the tradeoffs between the auctioneer's revenue and fairness to buyers under different pricing strategies. The main drawback of the auction approach is that it is time-consuming and it may incur high operating costs. It is also known for its unfavorable spectrum utilization [5].

Another popular approach uses game theory to solve the spectrum allocation and pricing problem. In [5], the authors consider the problem of spectrum sharing between a primary user who is eligible to access a licensed radio spectrum and a number of secondary users who have no access rights to the licensed spectrum. The problem is formulated as a Cournot game in which a pricing function is used to constrain secondary users from requesting excess spectrum. It is assumed that the total spare spectrum available for allocation is not finite, which is rather unrealistic. In [6], the authors consider the same problem with multiple primary and secondary users. Two different games are used to model the behavior of each type of users. The interaction between the games is that the spectrum and price offered by the primary users will affect the equilibrium of the secondary users' game. It continues until both of them reach equilibrium. In [7], a framework is developed to model the competition between multiple network operators for customers and the available spectrum as a non-cooperative game under the regulation of a spectrum policy server. The main disadvantage of this approach is that there is no real mediator in practice, so the players need to propose individual strategies iteratively to reach equilibrium; hence, the convergence period is often long.

There are also approaches that consider the price when allocating spectrum to users with different demands. In [9], the authors consider how a service provider sets the spectrum price to maximize profits and how users decide the amount of spectrum to purchase. They apply economic analysis techniques in a monopoly market to determine the optimum price for the service provider. The approach in [10] uses game theory to study demand-responsive pricing for radio resource management where multiple access points compete for users.

It is widely recognized that user satisfaction is an important factor that must be considered in the provision of services. Indeed, some works regard user satisfaction as the primary consideration rather than revenue-maximization. In [11], the authors propose a model of user satisfaction in which the requested QoS and the price paid are considered in radio resource management. Based on their model, the authors of [7] study how customers choose a network operator for service when multiple operators compete with each other. In contrast to the above works, we use a non-linear optimal price schedule to discriminate between different types of MVNOs. We propose an open dynamic trading model in which, through each round of interaction, the MNO learns more about the distribution of the types of the potential buyer MVNOs so to derive the optimal price schedule that will maximize its expected return.

Chapter 3 System Model

There are two kinds of players in the proposed dynamic bandwidth trading model: buyers (multiple MVNOs) and the seller (the MNO). The MNO only owns one product, i.e., surplus bandwidth, with a constant marginal cost *c*. Based on that cost, *c*, and its knowledge of the distribution of the types of buyer MVNOs, the MNO will publish an optimal price schedule for the MVNOs' selection which is specially designed so that an MVNO based on its type will choose the quantity-price pair that maximizes its utility.

3.1 MVNO: the Buyer

We assume there are different buying preferences or types of MVNOs and use the parameter θ, which is bounded by $[\theta_L, \theta_U]$, to describe them. A type θ MVNO's preference is represented by the utility function, which follows the standard consumer surplus approach:

$$U(b,T;\theta) = \int_0^b p(x;\theta)dx - T \ , \tag{1}$$

where b is the amount of bandwidth purchased, T is the total price paid, and $p(x;\theta)$ is the demand price function of a type θ MVNO. The integration of the demand price function gives the total price the buyer is willing to pay for the total number of units of bandwidth b. We also assume that there exists an efficient consumption level for which the demand price exceeds the marginal cost for type θ MVNOs, denoted by $b^e(\theta)$. We make two other assumptions:

Assumption 1. For all feasible θ, (1) the demand price function $p(b; \theta)$ is non-increasing in b and non-negative[1]; (2) $b^e(\theta) \geq 0$ and $p(b; \theta)$ is decreasing in b for $b \leq b^e(\theta)$, and $p(b; \theta) \geq c$ if and only if $b \leq b^e(\theta)$; and (3) $p(b; \theta)$ is twice continuously differentiable.

Assumption 2. Higher levels of θ are associated with higher demand. That is, $p(b; \theta)$ is strictly increasing in θ whenever $p(b; \theta)$ is positive.

It is assumed that MVNOs use the utility functions to measure and evaluate the price schedule announced by the MNO, and that each MVNO will choose the quantity-price pair that maximizes its utility. If more than one pair yields the same maximal utility, we assume that the MVNO will choose the one with the largest amount of bandwidth.

Let the types of MVNOs follow a continuous distribution represented by the cumulative distribution function (CDF) $F(\theta)$. It is assumed that the population of MVNOs are drawn independently according to $F(\theta)$. We also make the following assumption:

Assumption 3. The cumulative distribution function for θ, $F(\theta)$, is a strictly increasing, continuously differentiable function on the interval $[\theta_L, \theta_U]$ with $F(\theta_L) = 1 - F(\theta_U) = 0$.

3.2 MNO: the Seller

We assume that the demand price functions, $p(b; \theta)$, and the utility functions, $U(b, T; \theta)$, for all θ are known to the MNO because they can be obtained by analyzing historical information. However, the MNO does not know the exact types of the MVNOs participating in the trading.

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[1] The non-increasing property might not be suitable for scarce resources. Although the spectrum is scarce, we still consider this property in our work.

The MNO's objective is to construct and publish a price *schedule* (*S*) of pairs $<b_s, T_s>$ that maximizes its profit. If an MVNO chooses a pair *s* of S, it will receive b_s and pay for a total of T_s. In the next section, we explain how to derive the optimal price schedule such that each type of MVNO will choose the pair designed for it, and that pair will give the MVNO the maximal utility. First, we define the MNO's profit or "return" as follows:

$$\hat{R} = \hat{T} - c\hat{b} \,. \tag{2}$$

Let $N(b;\theta)$ denote the *social surplus* generated by the sale of a type θ MVNO, i.e.,

$$N(b;\theta) \equiv \int_0^b p(b;\theta)dx - cb \,. \tag{3}$$

The utility of a type θ MVNO that chooses pair $<b, T>$ is defined as the social surplus $N(b;\theta)$ less the MNO's profit *R*, i.e.,

$$U(b,R;\theta) = \int_0^b p(x;\theta)dx - cb - R \equiv N(b;\theta) - R \,. \tag{4}$$

Two properties are important to the MNO in the construction of the optimal price schedule. First, the MNO must ensure that the schedule contains a specific quantity-price pair for each type of MVNO. Second, the MVNOs are rational, meaning every type θ MVNO will only choose the pair $<b(\theta), T(\theta)>$ designed for it, and that pair will give the MVNO the maximal utility. That is, the price schedule will satisfy the following two constraints [12].

C1: [Incentive Compatibility (IC)]: for each θ,

$$U(b(\theta),T(\theta);\theta) \geq U(b(v),T(v);\theta), \quad \forall v \in [\theta_L, \theta_U]\,. \tag{5}$$

C2: [Individually Rational (IR)]: for each θ,

$$U(b(\theta), T(\theta); \theta) \geq 0 \,. \quad (6)$$

The IC constraint ensures that the pair <*b(θ), T(θ)*> designed for the type θ MVNO is the pair that would maximize its utility; while the IR constraint ensures that the pair <*b(θ), T(θ)*> causes non-negative utility. Given a price schedule *S* that satisfies the two constraints, every type θ MVNO will choose the pair <*b(θ), T(θ)*>. Such a quantity-based price schedule is described as *incentive compatible*. An incentive-compatible quantity-based price schedule is said to be optimal for a subinterval $[\theta_L, \theta_U]$ if it yields profits for the MNO that are at least as high as any other incentive-compatible quantity-based price schedule designed exclusively for customers in the range $[\theta_L, \theta_U]$[12].

Chapter 4 Optimal Price Schedule

In this section, we explain how to construct the optimal price schedule, S^*, of pairs $<(b_s^*, T_s^*)>$ that will maximize the MNO's profit and satisfy the IC and IR constraints. We adopt the method in [13], which satisfies the self-selection constraints. Both the self-selection and the IC constraints require that each type of MVNO will be more satisfied with the quantity-price pair designed for it than with a pair designed for any other type of MVNOs. For example, consider n different types of MVNO, $\theta_1 < \theta_2 < \ldots < \theta_n$. The optimal price schedule $\{<b(\theta), T(\theta)>\}$ that satisfies the self-selection constraints has two necessary properties. The first property is that *$b(\theta)$ is a non-decreasing function.* Let us consider any two neighboring types of MVNOs, θ_i and θ_{i+1}, and assume that the pair $<b(\theta_i), T(\theta_i)>$ designed for θ_i is known. Then, the pair $<b(\theta_{i+1}), T(\theta_{i+1})>$ designed for θ_{i+1} should be the same as the pair $<b(\theta_i), T(\theta_i)>$; otherwise, it should be located on or below the indifference curve of type θ_{i+1} through $<b(\theta_i), T(\theta_i)>$ to prevent the higher type θ_{i+1} from switching to the pair designed for the lower type θ_i. Moreover, the pair $<b(\theta_{i+1}), T(\theta_{i+1})>$ should be located on or above the indifference curve of the type θ_i through $<b(\theta_i), T(\theta_i)>$ to prevent the lower type θ_i from switching to the pair designed for the higher type θ_{i+1}. Hence, we have $b(\theta_{i+1}) \geq b(\theta_i)$ for any two neighboring types, so $b(\theta)$ is a non-decreasing function.

The second property is called the "local downward" constraint:

$$U(b(\theta_i),T(\theta_i);\theta_i) = U(b(\theta_{i-1}),T(\theta_{i-1});\theta_i), \quad i = 2,\ldots,n\,, \tag{7}$$

which implies that the lowest type would yield zero utility, i.e.,

$$U(b(\theta_1), T(\theta_1); \theta_1) = U(0, 0; \theta_1) = 0 \,. \tag{8}$$

The local downward constraint is the only self-selection constraint that is binding.

The two properties are sufficient to ensure that all self-selection constraints will be satisfied. That is, if the price schedule satisfies the two properties, then it will also satisfy the self-selection constraints. Consider two neighboring types of MVNOs, θ_i and θ_{i+1} with the known pair $<b(\theta_i), T(\theta_i)>$ designed for θ_i. The local downward constraint makes the pair $<b(\theta_{i+1}), T(\theta_{i+1})>$ locate on the indifference curve of θ_{i+1} through $<b(\theta_i), T(\theta_i)>$. Since the indifference curve of θ_{i+1} is steeper than that of θ_i and the non-decreasing property implies that $b(\theta_{i+1})$ should be equal to or greater than $b(\theta_i)$, the pair $<b(\theta_{i+1}), T(\theta_{i+1})>$ is above the indifference curve of θ_i and $U(b(\theta_{i+1}), T(\theta_{i+1}); \theta_i) < U(b(\theta_i), T(\theta_i); \theta_i)$. Therefore, the lower type θ_i would not switch to the pair designed for the higher type θ_{i+1}.

With $<b(\theta), p(\theta)>$ optimal for each type θ MVNO, we can write the maximized utility as

$$U^*(\theta) \equiv N(b(\theta); \theta) - R(\theta) \,. \tag{9}$$

According to Assumptions 1 and 2 and the "local downward" constraint, a type θ MVNO would derive no extra advantage by choosing the offers for the MVNOs with lower types than θ. Thus, we have

$$R(\theta) = N(b(\theta); \theta) - \int_{\theta_L}^{\theta} \frac{\partial N}{\partial \theta}(b(x); x)dx \,. \tag{10}$$

Therefore, the expectation of $R(\theta)$ is calculated as follows:

$$E[R]=\int_{\theta_L}^{\theta_U}\left[N(b(\theta);\theta)-\int_{\theta_L}^{\theta}\frac{\partial N}{\partial\theta}(b(x);x)dx\right]dF(\theta)\,. \tag{11}$$

After integration by parts, we have [13]:

$$E[R]=\int_{\theta_L}^{\theta_U}\left[N(b(\theta);\theta)-\frac{\partial N}{\partial\theta}(b(\theta);\theta)\frac{1-F(\theta)}{F'(\theta)}\right]dF(\theta)\,. \tag{12}$$

Let $I(b(\theta);\theta)$ denote the terms in the brackets. Then, we can rewrite (12) as follows:

$$E[R]\equiv\int_{\theta_L}^{\theta_U}I(b(\theta);\theta)dF(\theta)\,. \tag{13}$$

The goal of the MNO is to find the optimal price schedule that maximizes its expected return. The derivation of the optimal price schedule involves two steps. First is to find the optimal bandwidth $\{b^*(\theta)\}$ that would maximize the expected return. Note that the optimal price schedule must satisfy the self-selection constraints. It has been proved that, for any non-decreasing function $b(\theta)$ on $[\theta_L, \theta_U]$, there exists a unique return function, $R(\theta)$, as given in (10) such that $<b(\theta), R(\theta)>$ satisfies all the self-selection constraints and $U^*(\theta_L) = 0$, which is the maximal utility designed for the lowest potential buyer θ_L[13]. The following two assumptions are made to ensure that the non-decreasing price elasticity property of $b(\theta)$ does not offer random pairs of a schedule, and for convenience in the choice of parameterization of θ.

<u>*Assumption 4.*</u> Demand elasticity is non-decreasing in the demand price, i.e.,

$$\frac{\partial}{\partial\theta}\left(\frac{-b}{p}\frac{\partial p}{\partial b}\right)\le 0\,. \tag{14}$$

<u>*Assumption 5.*</u> The second-derivative of the demand price function with respect to θ is non-positive,

$$\frac{\partial^2}{\partial\theta^2} p(b;\theta) \le 0 \ . \tag{15}$$

After obtaining $b^*(\theta)$, in the second step we substitute $b^*(\theta)$ for $b(\theta)$ in (10) to obtain $R^*(\theta)$. We also obtain the optimal price $T^*(\theta)$ as follows:

$$T^*(\theta) = R^*(\theta) + cb^*(\theta) \ . \tag{16}$$

In the following, we explain how to derive the optimal price schedule in details.

4.1 Optimal Bandwidth Quantity

Finding the maximum expected return in (13) involves maximizing $I(b(\theta);\ \theta)$ for all feasible θ. That is, if $b^*(\theta)$ maximizes $I(b(\theta);\ \theta)$ for all feasible θ, $b^*(\theta)$ would also maximize the expected return. We prove the above statement by contradiction. Suppose that $b^*(\theta)$ maximizes $I(b(\theta);\ \theta)$ and there exists a $b^{**}(\theta)$ such that $b^{**}(\theta)$ yields a higher expected return than $b^*(\theta)$. Since $b^{**}(\theta)$ will yield a higher expected return and the expected return is the integration of $I(b^{**}(\theta);\ \theta)$, then $I(b^{**}(\theta);\ \theta)$ should be greater than $I(b^*(\theta);\ \theta)$ at some point. This means that there would exist at least one point, say θ_a, such that $I(b^{**}(\theta_a);\ \theta_a) > I(b^*(\theta_a);\ \theta_a)$. We can prove this easily because if $I(b^{**}(\theta);\ \theta) \le I(b^*(\theta);\ \theta)$ for all feasible θ, the integration of $I(b^{**}(\theta);\ \theta)$ would be smaller than that of $I(b^*(\theta);\ \theta)$. As we have $I(b^{**}(\theta_a);\ \theta_a) \le I(b^*(\theta_a);\ \theta_a)$ for all feasible θ, it would not be possible for the expected return from $b^{**}(\theta)$ to be greater than that from $b^*(\theta)$. Therefore, $b^{**}(\theta_a)$ cannot exist.

Given Assumptions 1, 2, 3, 4, and 5, we know the $b^*(\theta)$ that solves $\max_b I(b;\theta)$ must be non-decreasing. Suppose $b^*(\theta)$ maximizes $I(b;\ \theta)$ for all feasible θ. Then, $b^*(\theta)$ should satisfy the following equation:

$$\frac{\partial I}{\partial b}(b^*(\theta);\theta) = 0, \quad \forall \theta \in [\theta_L, \theta_U]. \tag{17}$$

Note that we assume the b(θ)s in the published price schedule are non-negative. To convert $b^*(\theta)$ to a non-negative function, let $b^0(\theta)$ be the optimal bandwidth function for all feasible θ. We eliminate the negative part of $b^0(\theta)$ to obtain the optimal bandwidth quantity function $b^*(\theta)$. Specifically, $b^0(\theta)$ is computed by the following equation:

$$\frac{\partial I}{\partial b}(b^0(\theta);\theta) = 0, \quad \forall \theta \in [\theta_L, \theta_U]. \tag{18}$$

We also have

$$b^*(\theta) = \max\{0, b^0(\theta)\}, \quad \forall \theta \in [\theta_L, \theta_U]. \tag{19}$$

4.2 Optimal Price

The optimal price function is computed by substituting $b^*(\theta)$ for $b(\theta)$ in (10), i.e.,

$$R^*(\theta) = N(b^*(\theta);\theta) - \int_{\theta_L}^{\theta} \frac{\partial N}{\partial \theta}(b^*(x);x)dx, \quad \forall \theta \in [\theta_L, \theta_U]. \tag{20}$$

The function gives the optimal return (profit) $R^*(\theta)$ derived by selling $b^*(\theta)$ units of bandwidth to a type θ MVNO. We also have the following optimal price function $T^*(\theta)$:

$$T^*(\theta) = R^*(\theta) + cb^*(\theta), \quad \forall \theta \in [\theta_L, \theta_U]. \tag{21}$$

Finally, we have the optimal price schedule S^*, which comprises a number of pairs $\{<b^*(\theta), T^*(\theta)>\}$, each representing the optimal bandwidth quantity and the associated price designed especially for a type θ MVNO.

4.3 Discrete Price Schedule

Thus far, we have assumed that the demand price functions and the type distribution functions of MVNOs are all continuous, and the resulting optimal price schedule is a pair of continuous functions of the MVNO type. Although the continuous forms are convenient for deriving the model and the optimal price schedule, in practice, the units of bandwidth are usually sold in a discrete format. Here, we will convert the continuous optimal price schedule to a discrete form. Assume K different quantity-price pairs are selected from the continuous optimal price schedule such that the resulting expected return is as high as possible. K is assumed to be defined by the MNO's policy. Let the resulting discrete price schedule be denoted by $S^*_{dis} = \{<b^*(\tilde{\theta}_k), T^*(\tilde{\theta}_k)>, k = 1 \sim K\}$. Given S^*_{dis}, it is obvious that a type $\tilde{\theta}_k$ MVNO will choose the pair $<b^*(\tilde{\theta}_k), T^*(\tilde{\theta}_k)>$. Next, we examine the types within the range of $\tilde{\theta}_k$ and $\tilde{\theta}_{k+1}$. Consider a θ in $(\tilde{\theta}_k, \tilde{\theta}_{k+1})$. Let us compute the difference in the utility of choosing either pair, i.e.,

$$U(b^*(\tilde{\theta}_k), T^*(\tilde{\theta}_k); \theta) - U(b^*(\tilde{\theta}_{k+1}), T^*(\tilde{\theta}_{k+1}); \theta) . \tag{22}$$

Differentiating (22) with respect to θ, we have

$$-\int_{b^*(\tilde{\theta}_k)}^{b^*(\tilde{\theta}_{k+1})} \frac{\partial}{\partial \theta} p(x; \theta) dx . \tag{23}$$

Since $p(b;\ \theta)$ is increasing in θ and $b^*(\theta)$ is non-decreasing, the derivative of the utility difference in (22) is negative, and the utility difference is decreasing in θ. That is, when θ moves outside the range of $\tilde{\theta}_k$ and $\tilde{\theta}_{k+1}$, the utility obtained by choosing $<b^*(\tilde{\theta}_k), T^*(\tilde{\theta}_k)>$ would be higher until at type θ_{k_b} or type $\theta_{(k+1)_a}$ the utility derived by choosing either $<b^*(\tilde{\theta}_k), T^*(\tilde{\theta}_k)>$ or $<b^*(\tilde{\theta}_{k+1}), T^*(\tilde{\theta}_{k+1})>$ would become the same. Thereafter, the types between $[\theta_{(k+1)_a}, \tilde{\theta}_{k+1}]$ will choose the pair θ_{k_b} that yields a higher utility than selecting $<b^*(\tilde{\theta}_k), T^*(\tilde{\theta}_k)>$. Here, we assume that if two possible price pairs render the same utility, an

MVNO would choose the one with *greater* amount of bandwidth. Hence, type θ_{k_b} will choose $< b^*(\tilde{\theta}_{k+1}), T^*(\tilde{\theta}_{k+1}) >$ instead of $< b^*(\tilde{\theta}_k), T^*(\tilde{\theta}_k) >$. For the types in the subintervals $[\theta_L, \tilde{\theta}_1)$ and $(\tilde{\theta}_K, \theta_K]$, the pairs $< b^*(\tilde{\theta}_1), T^*(\tilde{\theta}_1) >$ and $< b^*(\tilde{\theta}_K), T^*(\tilde{\theta}_K) >$ will be chosen, respectively. In summary, we assume that the discrete price schedule contains *K* pairs of bandwidth quantity and the associated price, and we want to determine which *K* pairs in the continuous optimal price schedule will maximize the expected return. The *K* pairs correspond to the *K* types (points) of MVNOs, which in turn divide the type distribution range $[\theta_L, \theta_U]$ into *K* mutually exclusive subintervals, denoted by $[\theta_{1_a}, \theta_{1_b})$, $[\theta_{2_a}, \theta_{2_b})$, ..., $[\theta_{K-1_a}, \theta_{K-1_b})$, $[\theta_{K_a}, \theta_{K_b}]$, $\theta_{1_a} = \theta_L$ and $\theta_{K_b} = \theta_U$. The selection of the set $\{\tilde{\theta}_k,\ k = 1,..,K\}$ has the property $U(b^*(\tilde{\theta}_k),\ T^*(\tilde{\theta}_k); \theta_{k_b}) = U(b^*(\tilde{\theta}_{k+1}),\ T^*(\tilde{\theta}_{k+1}); \theta_{(k+1)_a})$, $k = 1 \sim K-1$. Thus, the problem can be formulated by solving the following non-linear optimization problem.

$$\max \quad \sum_{k=1}^{K} \left(T^*(\tilde{\theta}_k) - cb^*(\tilde{\theta}_k)\right)\left(F(\theta_{k_b}) - F(\theta_{k_a})\right) \tag{24}$$

s.t.

$$\theta_{1_a} = \theta_L \tag{25}$$

$$\theta_{K_b} = \theta_U \tag{26}$$

$$\theta_{k+1_a} = \theta_{k_b}, \quad k = 1, \ldots, K-1 \tag{27}$$

$$U(b^*(\tilde{\theta}_k),\ T^*(\tilde{\theta}_k); \theta_{k_b}) = U(b^*(\tilde{\theta}_{k+1}), T^*(\tilde{\theta}_{k+1}); \theta_{(k+1)_a}), \quad k = 1, \ldots, K-1 \tag{28}$$

$$\tilde{\theta}_k < \tilde{\theta}_{k+1}, \quad k = 1 \sim K-1 \tag{29}$$

$$\theta_{k_a} \le \tilde{\theta}_k < \theta_{k_b}, \quad k = 1 \sim K \tag{30}$$

$$\tilde{\theta}_1 = \min\{\theta \mid b^*(\theta) \ge 0\} \tag{31}$$

$$b^*(\tilde{\theta}_k) \in \aleph, \quad \forall k = 1 \sim K \tag{32}$$

In the formulation, we know that when the $\tilde{\theta}_k$'s are determined, the ranges of the K intervals are also determined, i.e., $\{[\theta_{k_a}, \theta_{k_b}), k = 1,...,K\}$ based on the constraint $U(b^*(\tilde{\theta}_k), T^*(\tilde{\theta}_k); \theta_{k_b}) = U(b^*(\tilde{\theta}_{k+1}), T^*(\tilde{\theta}_{k+1}); \theta_{(k+1)_a})$. Here, the boundaries are presented as implicit functions that are parts of the constraints.

Moreover, we would like the $b^*(\tilde{\theta}_k)$ s to be integral values. Note that, in non-linear (mixed) integer programming problems, the integer property should only be applied to decision variables. However, in the formulation proposed above, the integer property is applied to $b^*(\tilde{\theta}_k)$, not $\tilde{\theta}_k$. We therefore make a conversion as follows. We convert the price function $T^*(\theta)$ to $T^*(b)$ by $T^*(b) = T^*(b^{*-1}(b))$ where $b^{*-1}(b)$ is the inverse function of $b^*(\theta)$. The problem then becomes the problem of selecting K different quantities, $\tilde{b}_k$, $k = 1 \sim K$, from a finite set $\{b \mid 0 \le b \le \lfloor b^*(\theta_U) \rfloor \,\&\, b \in N\}$ such that the expected return $\sum_{k=1}^{K} \left(T^*(\tilde{b}_k) - c\tilde{b}_k\right)\left(F(\theta_{k_b}) - F(\theta_{k_a})\right)$ is maximized. We thus formulate the problem as a non-linear mixed integer problem. Since the problem is a combinatorial optimization problem, once $\tilde{b}_k$ s have been decided, the expected return can be obtained in constant time. In addition, the problem can be solved in polynomial time $O(\lfloor b^*(\theta_U)^K \rfloor)$ by examining all possible combinations, $\binom{\lfloor b^*(\theta_U) \rfloor}{K} = \frac{1}{K!}(\lfloor b^*(\theta_U) \rfloor)(\lfloor b^*(\theta_U) \rfloor - 1)...(\lfloor b^*(\theta_U) \rfloor - K + 1)$ given a constant K.

Fig. 1 shows the K subintervals of the type distribution range. An example of utility values is also depicted to show that for the MVNOs of the types in the range of

$[\theta_{(k+1)_a}, \tilde{\theta}_{k+1})$ selecting the pair $< b^*(\tilde{\theta}_{k+1}), T^*(\tilde{\theta}_{k+1}) >$ will result in a higher utility than selecting the other pair $< b^*(\tilde{\theta}_k), T^*(\tilde{\theta}_k) >$.

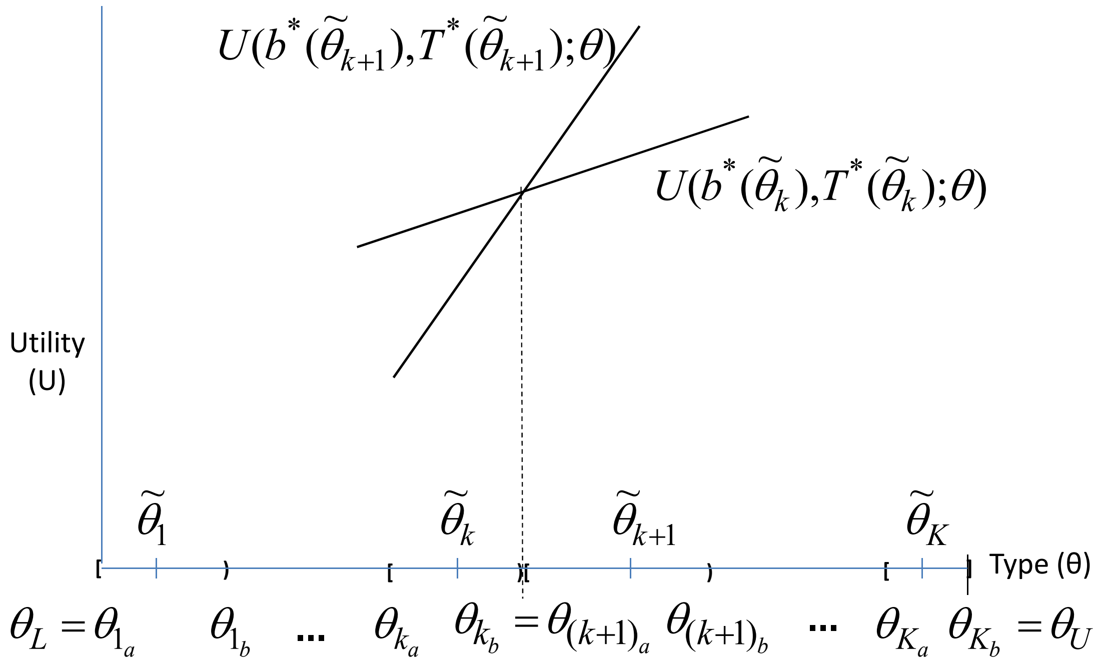

Figure 1 The K subintervals in the determination of the discrete price schedule and the illustration of the utility values in price pair selection.

Chapter 5 Open Dynamic Bandwidth Trading Model

In the previous sections, we focused on how the MNO computes the optimal price schedule based on the demand price function of MVNOs, $p(b;\theta)$; the marginal cost of bandwidth, c; and the type distribution function of the MVNO, $F(\theta)$. It is assumed that the information is based on the MNO's knowledge or estimation of the market. If any of the estimates are not accurate, the published price schedule may not give the maximum expected return. Moreover, the MNO does not know the exact population of buyers, but it does know the estimate of the type distribution function. In this section, we present an *open dynamic bandwidth trading* model, which comprises of two phases. The goal of phase one is to find out the distribution of the type of the potential buyer MVNOs who remain in the process. It is implemented as a sequence of interaction rounds between the MNO and the MVNOs. And, it is a learning-and-revising process. Through each round of interaction, the MNO learns more about the number and the type of the MVNOs based on their selection submissions. The MNO then revises its estimate of the type distribution of the MVNOs who remain in the process and computes a new optimal price schedule. At the beginning of a round, the MNO publishes an optimal price schedule based on its current knowledge of the participating MVNOs. Each MVNO then chooses the pair that maximizes its utility and submits the selection to the MNO. Note that, an MVNO may choose to leave the dynamic trading process by not submitting its selection of the price pair; however, once it leaves, it cannot re-join the process. The rule guarantees the convergence of the process. We also assume that the MNO can estimate the costs accurately. For the MVNOs' demand price functions, there always exist non-neglected estimation errors. Here, we assume they are accurate.

The phase one of the open interactive dynamic bandwidth trading process emphasizes continuous learning by the MNO so as to discover the true distribution of the types of the buyers. Initially, the MNO has the estimate $F^{(1)}(\theta)$ derived from the historical information. Based on that estimate, it computes the initial optimal price schedule $S^{*(1)}$, converts it to the discrete form $S_{dis}^{*(1)}$ and announces it to the MVNOs. Let the selections of the MVNOs submitted to the MNO be denoted by $R^{(1)}$.

5.1 Re-estimation of MVNO Type Distribution

After the MNO receives all the selections $R^{(i)}$ at the end of the ith round, let the total number of MVNOs who chose the pair $< b^*(\tilde{\theta}_k), T^*(\tilde{\theta}_k) >^{(i)}$ be denoted by $n_k^{(i)}$ and $\sum_{k=1}^{K} n_k^{(i)} = N^{(i)}$, $N^{(i)} \le N$, where N is the total population of potential buyer MVNOs estimated by the MNO initially. We use Pearson's chi-square test [14] to determine if the distribution of the types of MVNOs based on $R^{(i)}$ differs from the MNO's current estimate. Let the value of the test-statistic is

$$\chi^2 = \sum_{k=1}^{K} \frac{(n_k^{(i)} - E_k^{(i)})^2}{E_k^{(i)}}, \tag{33}$$

where $E_k^{(i)}$ is the number of MVNOs expected to choose pair k, which is computed as follows:

$$E_k^{(i)} = N^{(i)} \cdot (F^{(i)}(\theta_{k_b}) - F^{(i)}(\theta_{k_a})) . \tag{34}$$

The value of χ^2 is checked against the value of $\chi^2_{critical}$. If $\chi^2 < \chi^2_{critical}$, the observed frequency fits the estimated distribution and the MNO can proceed to solve the capacity constraint problem. Otherwise, the MNO will use the observed data to re-estimate the distribution of the types of the currently participating MVNOs.[2]

5.2 Estimation of $F^{(i)}(\theta)$ from $R^{(i)}$

To derive an estimate of $F^{(i)}(\theta)$ from $R^{(i)}$, the maximum likelihood estimation (MLE) method [15] is used. We first make an estimate of the underlying statistical model $\hat{F}^{(i)}(\theta)$. Given the price schedule $S^{*(i)}_{dis} = \{ <b^*(\tilde{\theta}_k), T^*(\tilde{\theta}_k)>^{(i)}, k = 1 \sim K\}$, the likelihood function is as follows:

$$\prod_{k=1}^{K} (\hat{F}(\theta_{k_b}) - \hat{F}(\theta_{k_a}))^{n_k^{(i)}} . \tag{35}$$

By differentiating the likelihood function with respect to each parameter and equating it to 0, we can obtain the estimates of the parameters that govern the statistical model.

Since only the MVNOs that participated in the previous round are allowed to remain in

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[2] Note that to use the chi-square goodness of fit test properly, the expected frequency in each category should be at least five. If any frequency is less than 5, it should be combined with an adjacent category. However, combining categories may have unintended consequences, e.g., there may only be one category left.

the interactive trading process, we know that the number of MVNOs participating in each round will not increase as the process continues. This ensures that the population of participating MVNOs will converge after a finite number of iterations.

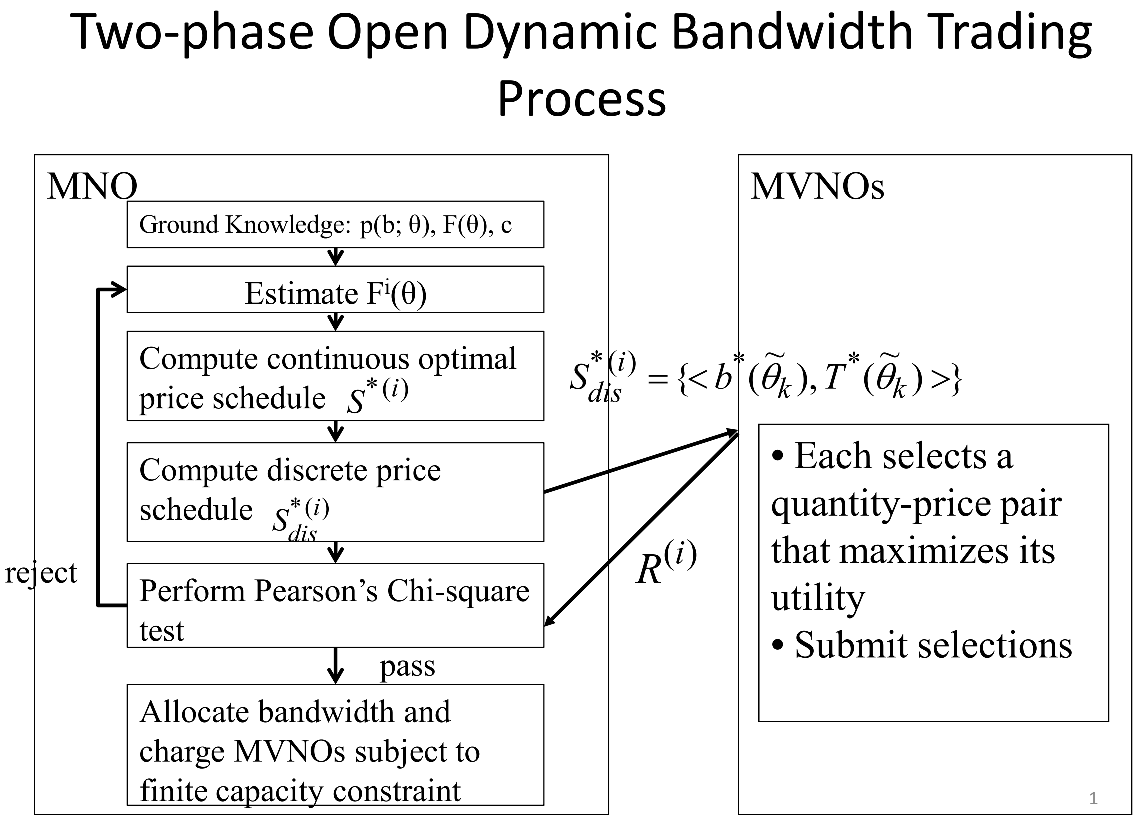

Figure 2. Flowchart of the open dynamic bandwidth trading process

5.3 Capacity Constraint

According to the Pearson's chi-square test when the observed frequency of the types of the participating MVNOs fits the estimated distribution, the first phase terminates. The bandwidth trading process proceeds to the second phase to resolve the finite capacity constraint. Note that the previous derivation of the price schedule was based on the assumption that the amount of unused bandwidth for sale was infinite. However, in practice, it is typically finite. Therefore, if the total bandwidth requested is greater than the total capacity *B,* we need to decide how to allocate the bandwidth to the buyer MVNOs given the finite capacity constraint. Otherwise, each MVNO who participated in the final round of the phase one will be satisfied with the amount as stated in the submission; and the MNO will charge them based on the discrete price schedule published in the final round.

If it is not possible to satisfy all the requests because of the finite capacity constraint, we map the bandwidth allocation problem to the bounded knapsack problem, where the size of the knapsack is the spectrum capacity *B* and the items are the MVNOs that remain in the final round. The weights and values of the items are the quantities and returns in the final discrete price schedule $S_{dis}^{*(final)} = \{ < b^*(\widetilde{\theta}_k), T^*(\widetilde{\theta}_k) >,\ k = 1 \sim K\}$; and the copies of each bandwidth-price category of items are $\{ n_k^{(final)} \}$, $\sum_{k=1}^{K} n_k^{(final)} = N^{(final)}$. The problem is formulated as:

$$\max \quad \sum_{k=1}^{K} R^*(\widetilde{\theta}_k) x_k \tag{36}$$

s.t.

$$\sum_{k=1}^{K} b^*(\widetilde{\theta}_k) x_k \le B, \quad x_k \in \{0, 1, \ldots, n_k\} \tag{37}$$

Fig. 2 shows the flowchart of the trading model and the interaction between the MNO and the MVNOs.

5.4 Example

In this section, we provide an example to illustrate the key components of the proposed open dynamic bandwidth trading process. First, we assume that the distribution of the types of MVNOs follows a uniform distribution in the range [0, 1], i.e., $F(\theta) = \theta$, and the demand price function is as follows:

$$p(b;\theta) = \begin{cases} 10 + 20\theta - b, & b \le 10 + 20\theta \\ 0, & b > 10 + 20\theta \end{cases}. \tag{38}$$

Let the distribution of MVNOs that will join the open trading process follow a triangular distribution $F(\theta \mid 0, 1, \omega = 0.9)$, i.e.,

$$F(\theta) = \begin{cases} \dfrac{\theta^2}{\omega}, & 0 \le \theta \le \omega \\ 1 - \dfrac{(1-\theta)^2}{1-\omega}, & \omega \le \theta \le 1 \end{cases}. \tag{39}$$

In addition, assume that ten MVNOs will join the process initially. The types of MVNOs are generated by using the inversion method [16]. First, we randomly generate ten values from the uniform distribution $U(0, 1)$ denoted by U_i, i = 1, …10, and substitute them in the following equation to obtain the types of MVNOs that follow the triangular distribution:

$$\theta_i = F^{-1}(U_i) = \begin{cases} \sqrt{eU_i}, & 0 \le U_i < F(e) \\ 1 - \sqrt{(1-e)(1-U_i)}, & F(e) \le U_i < 1 \end{cases}. \tag{40}$$

The distribution of the ten participating MVNOs is shown in Fig. 3.

First, let us consider the case where the MNO has perfect information about the MVNOs and its initial estimate of the type distribution function is exactly the true distribution, i.e., a uniform distribution in the range [0, 1], which means $F(\theta) = \theta$. Note that in the proposed process the MNO re-estimates the distribution of the types of the participating MVNOs in each round based on the selection submissions regardless of the initial assumption about the type of distribution. Here, it is assumed that the marginal cost c is fixed ($c = 10$).

A. Optimal Price Schedule

Based on (3), (12), and (13), we first compute the $I(b;\theta)$ of the expected return as follows:

$$I(b;\theta)=\begin{cases}(20b\theta-\frac{1}{2}b^2)-(20b)(1-\theta), & b\le 10+20\theta \\ (50+200\theta+200\theta^2-10b)-(200+400\theta)(1-\theta), & b>10+20\theta\end{cases}. \tag{41}$$

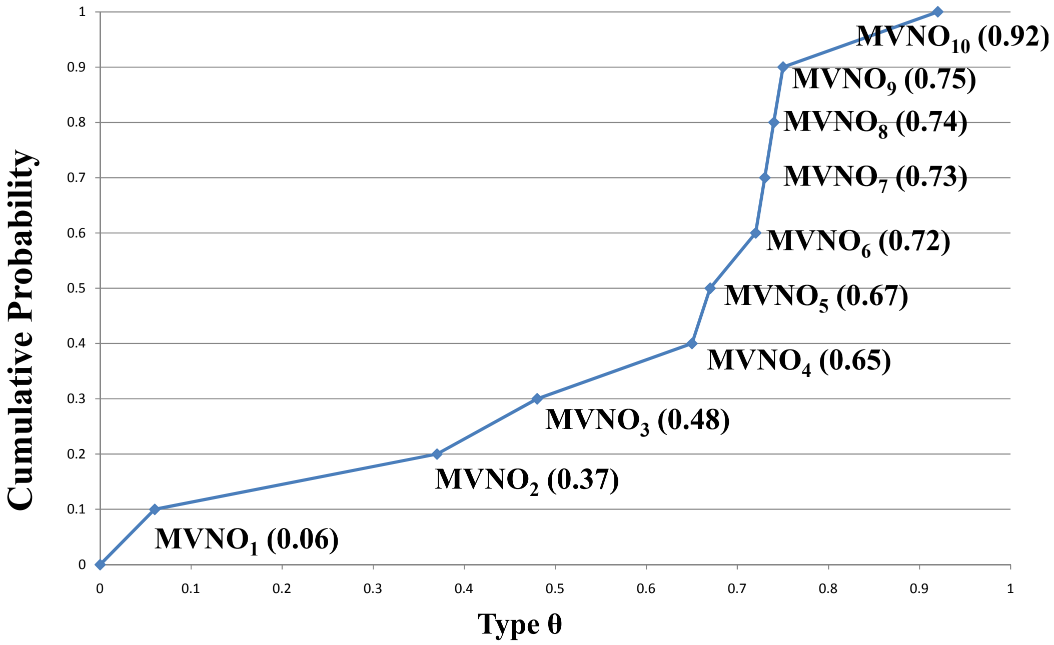

Figure 3. Distribution of the ten MVNOs that join the open dynamic bandwidth trading process initially

Then, we compute the optimal bandwidth allocation function $b^*(\theta)$

$$b^*(\theta) = \max\{0, b^0(\theta)\} = \begin{cases} 0, & \theta \le 0.5 \\ 40\theta - 20, & \theta > 0.5 \end{cases} . \tag{42}$$

Based on (20) and (21), the optimal price function $T^*(\theta)$ is derived as follows

$$T^*(\theta) = R^*(\theta) + cb^*(\theta) = -400\theta^2 + 1200\theta - 500 \quad . \tag{43}$$

In addition, the continuous optimal price schedule is computed as follows:

$$< b^*(\theta), T^*(\theta) >= \begin{cases} < 0, 0 >, & b \le 0.5 \\ < 40\theta - 20, -400\theta^2 + 1200\theta - 500 >, & b > 0.5 \end{cases} . \tag{44}$$

B. *Discrete Price Schedule*

Assume the size K of the discrete tabular price schedule is six. Using the method described in Section 4.3, we derive the discrete price schedule from the continuous optimal price schedule. The resulting discrete tabular price schedule is shown in Table I.

TABLE I. THE DISCRETE PRICE SCHEDULE AND THE TYPE SUBINTERVALS IN THE FIRST ROUND

Bandwidth	Price	$\widetilde{\theta}_k$	θ_{k_a}	θ_{k_b}
0.0	0.00	0.5000	0.0000	0.5500
4.0	76.00	0.6000	0.5500	0.6375
7.0	127.75	0.6750	0.6375	0.7125
10.0	175.00	0.7500	0.7125	0.8000
14.0	231.00	0.8500	0.8000	0.9000
18.0	279.00	0.9500	0.9000	1.0000

C. Quantity-Price Selection

Based on the IR and IC constraints, each MVNO chooses the quantity-price pair that maximizes its utility, as given in (1). For instance, for $MVNO_6$ (θ = 0.72), the utilities corresponding to the six pairs in the published price schedule table are 0, 13.6, 18.55, 19, 12.6 and -1.8, respectively. $MVNO_6$ will choose the fourth pair <10, 175>, which is exactly the one that the MNO designs for the types of MVNOs that fall in the range $0.7125 \leq \theta < 0.8$. For $MVNO_1$ (θ = 0.06), the utilities are 0, -39.2, -73.85, -113, -168.28, and -216.28 respectively. In this case, the pair <0, 0> will be chosen by $MVNO_1$, which is again the one designed for it. So do the remaining MVNOs.

D. Hypothesis Testing

We assume that all ten MVNOs respond in the first round. Table II shows the expected and observed frequencies of the six price pairs. The MNO performs hypothesis testing to determine if the current estimate of the type distribution function needs to be revised. Here, the chi-square value computed is 16.26, the degree of freedom is 5, and the critical chi-square value is 11.07 with p-value = 0.05. Since 16.26 > 11.07, the null hypothesis "The observed data and the estimated data are from the same distribution," is rejected, and the type distribution function is re-computed.

TABLE II. The FREQUNCY TABLE AFTER THE FIRST ROUND

Quantity	0	4	7	10	14	18
Price	0	76	127.75	175	231	279
Expected frequency	5.5	0.875	0.75	0.875	1	1
Observed frequency	3	0	2	4	0	1

E. Estimation of MVNO Type Distribution

After the computation, the likelihood function is formulated as $[F(0.55)-F(0)]^3$ $[F(0.7125)-F(0.6375)]^2$ $[F(0.8)-F(0.7125)]^4$ $[F(1)-F(0.9)]$, where $F(\theta)$ is the triangular distribution $F(\theta \mid 0, 1, \omega)$. Differentiating the function with respect to c and equating it to 0, we have ω= 0.9. The new estimate of the distribution is $F(\theta \mid 0, 1, \omega)$, ω= 0.9. The MNO then computes the new optimal price schedule as follows:

$$b^*(\theta)=\begin{cases}0, & 0\le\theta\le\sqrt{0.3}\\ \dfrac{30\theta^2-9}{\theta}, & \sqrt{0.3}\le\theta\le 0.9\\ 30\theta-10, & 0.9\le\theta\le 1\end{cases}, \tag{45}$$

$$T^*(\theta)=\begin{cases}0, & 0\le\theta\le\sqrt{0.3}\\ (300\theta^2-90)(1+\dfrac{1}{\theta})+180\ln\theta \\ -90\ln 0.3-\dfrac{(30\theta^2-9)^2}{2\theta^2}, & \sqrt{0.3}\le\theta\le 0.9\\ -150\theta^2+600\theta-240+90\ln 2.7, & 0.9\le\theta\le 1\end{cases}. \tag{46}$$

The corresponding discrete price schedule as shown in Table III is announced and the process enters the second round.

In the second round, $MVNO_1$ (θ = 0.06), $MVNO_2$ (θ = 0.37) and $MVNO_3$ (θ = 0.48) still choose the pair <0, 0>, which means they do not want to purchase any bandwidth. Assume they therefore decide to leave the process. We also consider the situation where, for no obvious reason, $MVNO_7$ (θ = 0.73) decides to leave the process as well in this round. Here, we wish to show that the proposed process and the associated schemes are able to quickly adapt the estimate of the type distribution function according to the MVNOs who remain in the process in the computation of the optimal price schedule that maximizes the MNO's expected return.

TABLE III. THE DISCRETE PRICE SCHEDULE AND THE TYPE SUBINTERVALS IN THE SECOND ROUND

Bandwidth	Price	$\tilde{\theta}_k$	θ_{k_a}	θ_{k_b}
0.0	0.00	0.5477	0.0000	0.5824
4.0	78.59	0.6184	0.5824	0.6571
8.0	147.16	0.6971	0.6571	0.7287

11.0	192.38	0.7609	0.7287	0.7945
14.0	232.55	0.8287	0.7945	0.8641
17.0	267.89	0.9000	0.8641	1.0000

After the second round, six MVNOs remain in the process of the type values 0.65, 0.67, 0.72, 0.74, 0.75, and 0.92. According to the model, the 2nd round submissions are as follows: <4, 78.59>, <8, 147.16>, <8, 147.16>, <11, 192.38>, <11, 192.38>, and <17, 267.89>. Table IV shows the expected and observed frequencies of the 6 sub-intervals.

TABLE IV. FREQUENCY TABLE AFTER THE SECOND ROUND

Quantity	0	4	8	11	14	17
Price	0	78.59	147.16	192.38	232.55	267.89
Expected frequency	2.26	0.62	0.66	0.67	0.77	1.02
Observed frequency	0	1	2	2	0	1

After hypothesis testing, the chi-square value is 8.63, which is less than the critical value 11.07. Thus, the observed data and the estimated data are deemed to be from the same distribution. The iterative part of the process terminates and the process proceeds to the second phased of bandwidth allocation subject to the finite capacity constraint.

F. Bandwidth Allocation

At the end of the first phase, the amount of bandwidths requested by the final remaining MVNOs are as follows: <MVNO$_1$, MVNO$_2$, MVNO$_3$, MVNO$_4$, MVNO$_5$, MVNO$_6$, MVNO$_7$,, MVNO$_8$, MVNO$_9$, MVNO$_{10}$> = <-, -, -, 4, 8, 8, -, 11, 11, 17>, where "-" means no selection because the corresponding MVNO has left the dynamic trading process. Here, the total amount requested is 59. We assume that the bandwidth capacity *B* is 30, which is less than the total demand. Thus, the knapsack problem is formulated to resolve the bandwidth allocation problem. The final allocation is as follows: <-, -, -, 4 (reject), 8 (accept), 8 (reject), -, 11 (accept), 11 (accept), 17 (reject)>. That is, only the requests from MVNOs of θ = 0.67, 0.74, 0.75 are accepted The total return is 231.9, and the bandwidth used is 30.

Chapter 6 Conclusion

In this paper, we propose an open interactive dynamic bandwidth trading model to resolve the problem of how an MNO prices and sells bandwidth to MVNOs of different buying preferences.. The model comprised of two phases. In phase one, through a sequence of interaction rounds the MNO accurately estimates the distribution of the types of the participating MVNOs and accordingly computes the optimal price schedule that satisfies the incentive compatible, individually rational and self-selection constraints. That is, an MVNO with a specific type of distribution will always select the quantity-price pair that the MNO designed for it. To achieve effective bandwidth sharing and utilization, we consider each MVNO's bandwidth request and willingness to pay as well as the expected return for the resource owner, i.e., the MNO. Although the continuous forms are convenient for deriving the model and the optimal price schedule, in practice, the units of bandwidth are usually sold in a discrete format. We present an algorithm to convert the continuous optimal price schedule to a discrete form. It is also designed to ensure that a specific type of MVNO will choose the pair whose corresponding utility value is closest to the value in the original function. After the iterative process converges and terminates, if the total number of bandwidth requests exceeds the total capacity constraint, the process proceeds to address the finite capacity constraint by solving a bounded knapsack problem for final bandwidth allocation.

Finally, we provide an example of show the derivation of the optimal price schedule and the final bandwidth allocation. It also shows that the proposed model and the associated mechanisms can quickly adapt its estimate of the type distribution function of the participating MVNOs and converges to produce the final optimal price schedule.

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